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MODULI STABILISATION AND SOFT SUPERSYMMETRY BREAKING IN STRING COMPACTIFICATIONS

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Abstract

String theory is one of the most active branches of theoretical physics and constitutes the most promising candidate for a consistent fundamental theory describing our Universe. Its structure provides a unified description of all known particles and forces of Nature, including gravity, in a single quantum mechanical framework.
Its low-energy limit, determined by compactifications of extra dimensions on Calabi-Yau orientifolds, yields an effective theory which represents the key tool in order to make contact with experiments and observable physics.
This theory is Supergravity, with the natural presence of a moduli hidden sector, which consists in a collection of gravitationally coupled scalar fields inherited from Kaluza-Klein dimensional reduction.
In order to avoid non-observed long range fifth-forces mediated by massless moduli, these fields have to develop a large mass via the process of moduli stabilisation. The stabilisation of the moduli provides also a dynamical supersymmetry-breaking mechanism which can be mediated from the hidden to the visible sector by gravitational interactions.
Moduli physics inherited from strings turns out to explain and motivate TeV-scale supersymmetry-breaking, giving rise to important predictions for the masses of superpartners.

In this thesis, we shall work in the framework of type IIB Calabi-Yau flux compactifications and present a detailed review of moduli stabilisation studying in particular the phenomenological implications of the LARGE-volume scenario (LVS). All the physical relevant quantities such as moduli masses and soft-terms, are computed and compared to the phenomenological constraints that today guide the research.

The structure of this thesis is the following.

The first chapter introduces the reader to the fundamental concepts that are essentially supersymmetry-breaking, supergravity and string moduli, which represent the basic framework of our discussion.
In the second chapter we focus our attention on the subject of moduli stabilisation. Starting from the structure of the supergravity scalar potential, we point out the main features of moduli dynamics, we analyse the KKLT and LARGE-volume scenario and we compute moduli masses and couplings to photons which play an important role in the early-universe evolution since they are strictly related to the decay rate of moduli particles.
The third chapter is then dedicated to the calculation of soft-terms, which arise dynamically from gravitational interactions when moduli acquire a non-zero vacuum expectation value (VeV).
In the last chapter, finally, we summarize and discuss our results, underling their phenomenological aspects. Moreover, in the last section we analyse the implications of the outcomes for standard cosmology, with particular interest in the cosmological moduli problem.

Contents

Chapter 1

Introduction

Before going into details of string moduli dynamics and its physical consequences, it is necessary to focus on the main concepts that allow and guide such a profound construction.
We start from the review of the Standard Model and the many questions it cannot answer, then we present a brief review of the key aspects of string compactifications (expecially through the simple case of Kaluza-Klein theory) and introduce the reader to origin of the moduli sector and the moduli stabilisation problem.
Finally, in the last section we examine the main issues about supersymmetry and its local extension, supergravity, in order to build a solid framework to refer to later.

1.1 Physics beyond the Standard Model

The Standard Model (SM) is an impressive success of twentieth century physics and constitutes one of the cornerstones of all science.

The SM is a particular solution of Quantum Field Theory based on the gauge group

$$G_{SM} \;=\; SU(3)_c \;\times\; SU(2)_L \;\times\; U(1)_Y \tag{1.1}$$

where $SU(3)_c$ describes strong interactions through Quantum Chromodynamics (QCD), and $SU(2)_L \times U(1)_Y$ describes electroweak interactions.

Matter fields are organized into three generations (or families) of quarks and leptons, represented by left-handed Weyl fermions which transform under the SM group as showed in Fig.1.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	1/6
U	$\bar{3}$	1	1/3
D	$\bar{3}$	1	−2/3
L	1	2	−1/2
E	1	1	1

Fig.1 Gauge quantum numbers of SM quarks and leptons. [1]

A crucial and deep feature of fermioni spectrum is *chirality* (parity-violation), whose implication is that explicit Dirac mass terms $m\bar{f}_R f_L + h.c.$ are forbidden by gauge invariance.
In order to give masses to fermions and weak gauge bosons, the electroweak symmetry must be spontaneously broken down to $U(1)_{EM}$. This is achieved through the interaction with a complex scalar Higgs field ϕ which gets a non-zero vacuum expectation value (VeV) thanks to a potential of the form:

$$V(\phi) = -m^2\phi^*\phi + \lambda(\phi^*\phi)^2 \tag{1.2}$$

whose plot (in Fig.2) shows the degeneracy of the ground state under phase transformation.

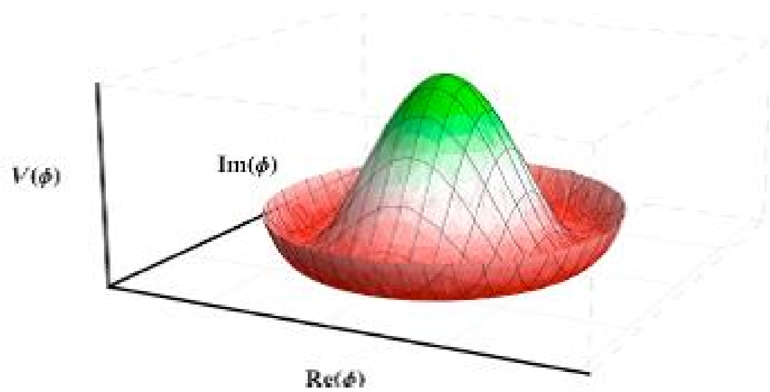


Fig.2 Typical mexican-hat form of the potential [2].

This mechanism generates $W^{\pm}$ and Z^0 massive bosons, which mediate the weak force, and gives mass to quarks and leptons through Yukawa couplings:

$$L_{Yuk} = Y_U^{ij}\bar{Q}_L^i U_R^j H^* \ + \ Y_D^{ij}\bar{Q}_L^i D_R^j H \ + Y_L^{ij}\bar{L}^i E_R^j H \ + \ h.c. \tag{1.3}$$

Hence the scale of fermion masses is related to the electro-weak symmetry breaking scale, which is of the order

$$M_{EW} \simeq \langle\phi\rangle \simeq \frac{m}{\lambda^{1/2}} \simeq 10^2 GeV \tag{1.4}$$

Within this framework, the Standard Model describes elementary particles and their electormagnetic, weak and strong interactions in a remarkable wide range of energies, and with unprecedented precision.

However, there are a number of theoretical and phenomenological issues that the Standard Model still fails to answer properly:

- ***Quantum Gravity***.

Gravity is not included in the Standard Model picture, which embraces only three of the four fundamental forces in Nature.
Gravitational interactions are described by the classical theory of General Relativity and are encoded in the Einstein-Hilbert action, given by (in obvious notation)

$$S_{EH} = \frac{M_P}{2}\int d^4x\sqrt{-g}R \tag{1.5}$$

which is invariant under general coordinates reparametrisations.
This theory cannot be quantised in the usual fashion and is not well defined in the ultraviolet, being perturbatively non-renormalizable.
Quantum gravity should be regarded as an effective field theory, which constitutes a good approximation at energies below M_P (or some other cutoff scale at which four dimensional classical Einstein theory ceases to be valid).

- **Hierarchy problem**.

This is divided in two parts. The first is related to the prediction about the Higgs mass. From the theoretical point of view, it is quadratically divergent because of loop quantum corrections, and it turns out to be of the order M_P, such as the 'cutoff' scale of the model.
The second is associated to the difficulty of explaining the huge difference between the elctroweak and the Planck scale:

$$M_{EW} \sim 10^2 GeV \ , \quad M_P = \sqrt{\frac{\hbar c}{8\pi G}} \sim 10^{18} GeV \tag{1.6}$$

- ***Fundamental questions***

What is the origin of SM gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$? Why are there four interactions and exactly three families of fermions? Why $3+1$ spacetime dimensions Why are there 20 free parameters, such as masses and gauge couplings between particles, that are introduced by hand without theoretical understanding of their experimental values?

- ***Electro-weak symmetry breaking***.

We need an explanation for the dynamics which causes electro-weak symmetry breaking. The Higgs field potential is just introduced by hand.

- ***Cosmological challenges***

First of all, taking into account quantum corrections to the vacuum energy, it is not possible to reproduce the observed value of the cosmological constant.

Moreover, there is no explanation for the baryon asymmetry of the Universe and there is no candidate particle, within the SM, for cold dark matter and for inflaton (ignoring non-minimal couplings of the Higgs field to gravity).

We understand the puzzle is far to be completed.
The Standard Model is not the fundamental theory of the Universe but only an effective theory describing the fundamental one at low energy.
Along the avenue beyond the SM, researches have been made about symmetry enhancement. More general spacetime symmetries produce interesting extension of the Standard Model and provide possible solutions to the above listed problems:

- *Supersymmetry* is a symmetry under the exchange of bosons and fermions. It gives an elegant explanation of the Higgs mass divergence due to cancellations between boson and fermion radiative corrections. Also it provides the best examples of dark matter candidates (the so called superpartners) and suggests the possibility to have a Grand Unified Theory (GUT) of all the SM forces at high energy.

- *Extra dimensions* introduce new spacetime 'directions' with consequent Lorentz symmetries enhancement. They are a fascinating possibility which could give a different perspective of the hierarchy problem and force unification.

Presently the best hope for a fundamental theory of quantum gravity which gives also an explanation to the open questions above, is String Theory.
Its basic assumption is that the fundamental objects which compose the profound architecture of Nature are not pointlike, like particles, but have at least one dimension.
Surprisingly, this simple feature, gives rise to a deep quantum mechanical structure (see [3] and [4]) able to describe all particles and forces requiring nothing else that one single 'tiny' paramater: string lenght l_s.
But the road to the final theory is still complicated since, from experimental point of view, the string energy scale $M_s \sim 1/l_s$ seems to be out of reach.

However the hope is to be able to make some important and testable predictions from string theory about low-energy particle physics.

1.2 String Compactifications

String theory, for internal consistency, requires the existence of six extra spatial dimensions. Studying compactification techniques is then extremely important in order to make contact with our observable world.
The physical space in which the fundamental theory lives, is the product $\mathcal{M}^4 \times Y_6$ of the usual four dimensional space-time with some compact six dimensional manifold.
In order to connect this structure with the experiments, we need an N = 1 supersymmetry theory in four space-time dimensions. This requirement forces the internal manifold Y_6 to be a very complicated 'Calabi-Yau' space (see [1] for details) .
The landscape of all the choices for compactification manifold, is very extended. Moreover, the topology of the extra dimensions affect the physical features of the effective model. Each of the Calabi-Yau choices leads to a different four dimensional physics.
The great challenge, which is called *model building*, is to find a string compactification able to reproduce the Standard Model and its phenomenology [5].
In this paper we refer to type IIB string compactifications. This kind of models, from recent years studies, seems to be the most promising framework to find the correct low-energy effective theory. Their phenomenological potentiality relies on the concept of D-branes (discovered by Polchinski in 1995) which extends the notion of string, being D-dimensional surfaces on which open strings end points are constrained to move [6]. These non-perturbative objects provide non-Abelian gauge symmetries and chirality constructions, allow background fluxes, which are an essential ingredient for moduli stabilisation, and yields the possibility to have interesting large extra dimensions models in the context of the so called 'brane world scenario'.
The latter gives a fascinating picture of the geometry of the Universe, based on the idea that the Standard Model, and the real world with it, is placed and constrained to live on a particular D-brane, while only gravity (associated with closed strings) is able to travel across the other dimensions.

Basics of Kaluza-Klein Theory

The idea that extra dimensions could provide an elegant framework describing fundamental physics comes from T.Kaluza and O.Klein, who independently proposed the brilliant idea to unify gravity and electromagnetism through the addition of a tiny rolled fifth dimension.
From the mathematical point of view, they imagined to build a physical theory on a $\mathcal{M}^4 \times S^1$ space-time, endowed with a 5-dimensional metric g_{MN}.
The geometrical picture is that the extra dimension is compactified on a circle (S^1) of radius R at each point of Minkowski space-time.

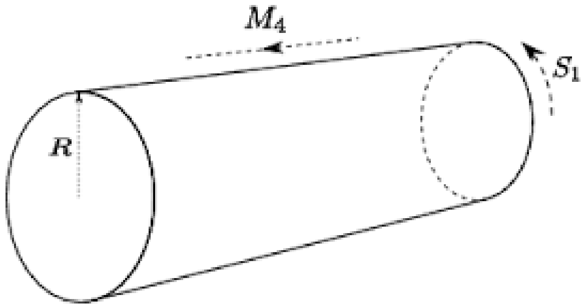


Fig.3 Geometrical interpretation of Kaluza-Klein (KK) compactification on the circle S^1. [7]

Let us discover the physical implications of this fascinating model.
We consider the gravitational field in 5D described by the Kaluza-Klein ansatz metric

$$g_{MN} = \phi^{-1/3} \begin{pmatrix} (g_{\mu\nu} - k^2\phi A_\mu A_\nu) & -k\phi A_\mu \\ -k\phi A_\nu & \phi \end{pmatrix} \tag{1.7}$$

where $M, N = 0, 1, 2, 3, 4$.
Thanks to the periodicity in the extra dimension, the effective fields can be written through a discrete Fourier expansion

$$g_{MN} = \sum_{n=-\infty}^{\infty} g^n_{MN}(x^\mu) exp\left(\frac{iny}{R}\right) \tag{1.8}$$

where we indicate with y the fifth compactified dimension.

Taking into account (1.7), the Fourier expansion then gives [2]

$$g_{MN} = \phi^{(0)-1/3} \begin{pmatrix} (g^{(0)}_{\mu\nu} - k^2\phi^{(0)} A^{(0)}_\mu A^{(0)}_\nu) & -k\phi^{(0)} A^{(0)}_\mu \\ -k\phi A^{(0)}_\nu & \phi^{(0)} \end{pmatrix} + \infty \text{ tower of massive modes} \tag{1.9}$$

Now, at energies much lower than the compactification scale $M_{KK} = 1/R$, it is possible to integrate out all the massive terms remaining only with the zero-modes.
This goes under the name of *dimensional reduction*.
By doing this we are left with the following decomposition for the gravitational field in 5D:

$$g_{MN} = g_{\mu\nu} \oplus g_{\mu 4} \oplus g_{44} \tag{1.10}$$

that is, the zero-modes contains a 4D gravitational field (graviton), a massless vector and a real scalar.
The unified theory of gravity, electromagnetism and scalar fields takes an explicit form when substituting (1.9) into the Einstein-Hilbert 5D action

$$S = \int d^5x \, \sqrt{|g_{MN}|}^{(5)} R \tag{1.11}$$

where ${}^{(5)}R_{MN} = 0$.
The remarkable result is:

$$S_{4D} = \int d^4x \, \sqrt{|g|} \left[M_P^2 \, R - \frac{1}{4} \phi^{(0)} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \frac{1}{6(\phi^{(0)})^2} \partial^\mu \phi^{(0)} \partial_\mu \phi^{(0)} \right] \tag{1.12}$$

Unfortunately this theory had no phenomenological success, nevertheless, with the advent of string theory, the technique to treat extra dimensions devoleped in that context, has turned out to play a crucial role.
String compactifications, in fact, are a generalisation of the Kaluza-Klein dimensional reduction.

Moduli Stabilisation

In String Theory, the compactified manifold is not simply a sphere S^N, but as we pointed out before, is a complicated Calabi-Yau manifold.
Performing a Kaluza-Klein reduction of the low-energy limit of the ten-dimensional type IIB superstring theory, one obtains a huge number of massless scalars (gravitationally coupled to ordinary fields) that compose the so called hidden sector of the effective theory.
This scalar fields are called moduli and parametrise the shape and the size of the compactified extra dimensions. In the specific, we deal with:

1. Kahler moduli, which parametrise the deformations in size of the Calabi-Yau
2. Axio-dilaton, whose VeV sets the string coupling
3. Complex structure moduli, which parametrise the shape of the extra dimensions.

Since the massless moduli would mediate unobserved long-range fifth forces, it is of primary importance to develop a potential for these particles, and give them a mass. This problem is called 'moduli stabilisation'.
Moreover, from a physical point of view, the issue becomes more interesting since it gives the possibility to do realistic phenomenology. The low-energy parameters, such as coupling constants and mass scales, are fixed by moduli vacuum expectation values.
The key point that has to be underlined is that the presence of the moduli sector is a model-independent feature of string compactifications and then must represents an essential link to the observable physics.

1.3 Supersymmetry

Despite its incompleteness, the Standard Model and QFT in general, has pointed out that the guide tool in studying and learning about elementary processes is the concept of *symmetry*.
In 1967 Coleman and Mandula proved a theorem which affirms that in a generic quantum field theory, under some reasonable assumptions (such as locality, causality, positivity of

energy, finiteness of number of particles, etc...), the only possible continous symmetries of the S-matrix are those generated by Poincaré group generators, P_μ and $M_{\mu\nu}$, plus some internal symmetry group G commuting with them.

$$[P_\mu, G] \;=\; [M_{\mu\nu}, G] \;=\; 0 \tag{1.13}$$

Nevertheless, the Coleman-Mandula theorem can be eluded by weakening one or more of its assumptions. For example, one is that the symmetry algebra only involves commutators, which implies that all generators are bosonic generators.
This assumption does not have any particular physical reason not to be relaxed.
If then we consider fermionic generators, which satisfy anti-commutation relations, we find that the set of allowed symmetries can be enlarged. Indeed, in 1975 Haag, Lopuszanski and Sohnius, following this idea, proved that *supersymmetry* is the only possible option.

Supersymmetry Algebra

Supersymmetry (SUSY) is a space-time symmetry mapping fields of integer spin (bosons) into fields of half integer spin (fermions), and viceversa. From the theoretical view point, the generators Q act as

$$Q\,|fermion\rangle \;=\; |boson\rangle \quad \text{and} \quad Q\,|boson\rangle \;=\; |ferion\rangle \tag{1.14}$$

The operator Q is a fermionic object which transforms as a Weyl spinor under Lorentz transformations and satisfies the following commutation relations:

$$[Q, P_\mu] \;=\; [Q, G] \;=\; 0 \;\;,\;\; [Q, M_{\mu\nu}] \;\neq\; 0 \tag{1.15}$$

Let us see a simple toy model in details.
The simplest 4D system invariant under supersymmetry transformation is a free theory with a fermion ψ_α and a complex scalar ϕ, whose action is given by

$$S = \int d^4x(-\partial^\mu \phi^* \partial_\mu \phi - i\bar{\psi}_\alpha \bar{\sigma}^\mu \partial_\mu \psi_\alpha) \tag{1.16}$$

In this model, making use of the equation of motion, it is not difficult to find the so called conserved super-current:

$$J^\mu_\alpha = (\partial_{nu}\phi^* \sigma^\nu \bar{\sigma}^\mu \psi)_\alpha \quad , \quad \partial_\mu J^\mu_\alpha = 0 \tag{1.17}$$

which yields the conservation of super-charges

$$Q_\alpha = \int d^3x J^0_\alpha \quad , \quad \bar{Q}_{\dot{\alpha}} = \int d^3x \bar{J}^0_{\dot{\alpha}} \tag{1.18}$$

The algebra of the generators is given by anticommutation relations; since both Q and $\bar{Q}$ are fermionic, their anticommutator is expected to be a bosonic conserved quantity. The only possible choice is the spacetime 4-momentum P_μ contracted with σ^μ in order to have the right spinorial structure.
Thus we have:

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}\beta}\} \; = \; 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \tag{1.19}$$

The other (anti)commutator between these three operator vanish.

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\alpha}\beta}\} = [Q_\alpha, P_\mu] = \{\bar{Q}_{\dot{\alpha}\beta}, P_\mu\} = 0 \tag{1.20}$$

It is important to remark that, since they intertwine with Poincarè algebra, Q and $\bar{Q}$ are not generators of internal symmetries.

The physical implication of a supersymmetric theory is that each one-particle state has a superpartner. More specifically, in a SUSY model, there is an equal number of fermionic and bosonic degrees of freedom and, instead of single particle states, one has to deal with (super)multiplets of particle states. Besides, since the generators commute with P_μ and with internal symmetries, particles in the same multiplet are expected to have different spin, but same mass and internal quantum numbers.

There are many reasons to believe that supersymmetry is a real symmetry of Nature. From a purely theoretical point of view, supersymmetric theories actually aim, for construction, at a unified description of radiation and matter. In other words, SUSY looks like the most natural framework where to formulate a theory able to describe all known interactions in a unified way. Given that, it is not a surprise that string theory requires it as a necessary component of consistent ultraviolet physics.

For these reasons, and due to the fact that it provides natural solutions to problems like dark matter, forces unification, the hierarchy problem and radiative electroweak symmetry breaking, supersymmetry is one of the most studied candidates for physics beyond the Standard Model.

Superfields formalism

In order to write down more general supersymmetric field theory actions, it is necessary to introduce the concept of superfields.
Superfields are functions of the so called *superspace*, which is Minkowski space-time augmented with additional fermionic coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$:

$$\Phi = \Phi(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}) \tag{1.21}$$

In this formalism, θ is a left-handed spinor with two anticommuting component $(\theta^{(1)}, \theta^{(2)})$ which by definition satisfy:

$$\theta^{(1)}\theta^{(2)} = -\theta^{(2)}\theta^{(1)} \quad , \quad \theta^{(1)}\theta^{(1)} = \theta^{(2)}\theta^{(2)} = 0 \tag{1.22}$$

from which we can define

$$\theta^2 = \theta^\alpha\theta^\beta\epsilon_{\alpha\beta} = 2\theta^{(1)}\theta^{(2)} \tag{1.23}$$

Thanks to the Grassmann anticommuting property, it is easy to notice that all the expressions which contain more than two powers of either θ or $\bar{\theta}$ vanish. This allows us to write a generic superfield as a finite power-expansion in the fermionic coordinates, which leads to a finite number of ordinary fields, filling out supermultiplets.

The irreducible representation of SUSY transformations are extracted from the general form of the scalar superfield [8]. Introducing the SUSY covariant derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{1.24}$$

we define chiral and antichiral superfields respectively by the condition :

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \qquad D_\alpha\bar{\Phi} = 0 \tag{1.25}$$

Concretely, looking at the chiral superfield, the constraint is solved by the following structure

$$\Phi(x,\theta,\bar{\theta}) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \tag{1.26}$$

where we performed the reparametrisation $y = x + i\theta\sigma^\mu\bar{\theta}$.
In this expression ϕ is a scalar field, ψ is a Weyl spinor and F is called the scalar auxiliary field, which eventually can be integrated out via its equation of motion.
Remarkably, under a supersymmetry transformation, the components of the supermultiplets (ϕ, ψ, F) transform into each other as expected from the physical point of view.

The other fundamental representation is the vector superfield, which contains as components the gauge boson fields and the superpartners gauginos.
The vector multiplet is constructed without using SUSY covariant derivatives. The constraint we impose is simply $V = V^\dagger$, that means V must be a real superfield.
In the so called Wess-Zumino gauge (see [8] for further details) the vector superfield takes the simple form:

$$V(x,\theta,\bar{\theta}) = -\theta\sigma^\mu\bar{\theta}A_\mu(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\lambda(x) + \frac{1}{2}\theta^2\bar{\theta}^2 D(x) \tag{1.27}$$

where A_μ is a gauge field, λ is its fermionic superpartner (the gaugino) and D is a real auxiliary scalar field.

A remarkable fact is that under a supersymmetry transformation the F and D auxiliary fields, transform as a total space-time derivative.
This important feature allows the construction of general supersymmetric Lagrangians through the following argument.

In order to have $\delta S = 0$ under SUSY transformation, $\delta\mathcal{L}$ must contain total derivatives of superfields. Thus, we can write for the chiral superfield Φ

$$\mathcal{L} = K(\Phi, \Phi^\dagger)|_D \; + \; W(\Phi)|_F + h.c. \;) \tag{1.28}$$

where K, known as the Kähler potential, is a real function of the superfield, while W is called the superpotential and is an holomorphic function of Φ (and therefore a chiral superfield itself).
Moreover, thanks to the fact that for Grassmann variables one gets

$$\int d\theta = \frac{\partial}{\partial\theta} \tag{1.29}$$

we find

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger) \; + \; \int d^2\theta (W(\Phi) + h.c.) \tag{1.30}$$

The superfield formalism is a useful tool which simplifies calculation, while keeping supersymmetry manifest.
The Kähler potential K and the superpotential W determine entirely the chiral supermultiplet free theory.
Let us see how to add interactions and realise a consistent supersymmetric extension of the Standard Model.

Minimal Supersymmetric Standard Model

The superfields formalism provides a simple and powerful procedure to build general supersymmetric Lagrangians, through the knowledge of two functions of the supermultiplets: the superpotential W and the Kähler potential K.
In order to construct the supersymmetric extension of the SM action, we have to introduce a generalisation of gauge symmetry invariance, which describes interactions.
This is realised by a simple argument involving vector and chiral superfields.

First of all, it is necessary to generalise the gauge invariant field strenght $F_{\mu\nu}$. This is provided by a chiral superfield W_α defined in the abelian case as

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V \tag{1.31}$$

where D is the auxiliary field, while D_α is the SUSY-covariant derivative.
The kinetic term for gauge bosons and gauginos then arises from:

$$\mathcal{L} = \frac{1}{4} Tr \int d^2\theta W^\alpha W_\alpha + h.c. \; = \; Tr\left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu \bar{\lambda} + \frac{1}{2} D^2\right) \tag{1.32}$$

Now, the interactions between a gauge multiplet V with a chiral superfield Φ are described by the following Kähler potential

$$\int d^2\theta d^2\bar{\theta} \; \Phi^\dagger e^V \Phi \tag{1.33}$$

The terms (1.32) and (1.33) are invariant under the following generalised gauge transformation

$$\Phi \to e^{-i\;\Lambda}\Phi \quad , \quad V \to V + i(\Lambda - \Lambda^\dagger) \tag{1.34}$$

which includes the ordinary gauge transformation.
The total Lagrangian of the so called Minimal Supersymmetric Model (MSSM) then is given by:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \; \Phi^\dagger e^V \Phi \; + \; \int d^2\theta (W(\Phi) + h.c.) + \left(Tr \int d^2 \; \theta f(\Phi) W^\alpha W_\alpha + h.c.\right) \tag{1.35}$$

where $f(\Phi)$ is an holomorphic function of the chiral superfield and is called the 'kinetic gauge function'.
This construction represents the simplest supersymmetric model, where the matter consists of three generations of quark and lepton supermultiplets plus two Higgs doublets superfields (supersymmetry requires two higgs doublets in order to avoid anomalies), and the gauge sector is given by $SU(3)_c \times SU(2)_L \times U(1)_Y$ vector superfields.
The associated superpotential, in an obvious notation, takes the form:

$$W = \sum_{generations} [Y_u Q_L H_2 \bar{u}^c_L + Y_d Q_L H_1 \bar{d}^c_L + Y_e Q_L H_1 \bar{e}^c_L] + \mu H_1 H_2 \tag{1.36}$$

where the first three terms correspond to standard Yukawa couplings giving masses to up quarks, down quarks and leptons, while the fourth term is a mass for the two Higgs fields.

Further terms are forbidden by the so called R-parity (see [2],[8]) for more details) which ensures baryon and lepton number conservation.

MSSM considers the (minimum) number of new particles and new interactions consistent with phenomenology and provides important physical consequences:

• **Naturalness and Hierarchy Problem**: the Higgs mass divergence can be solved by scalar superpartners radiative corrections (see [2] for more details on miraculous cancellation).

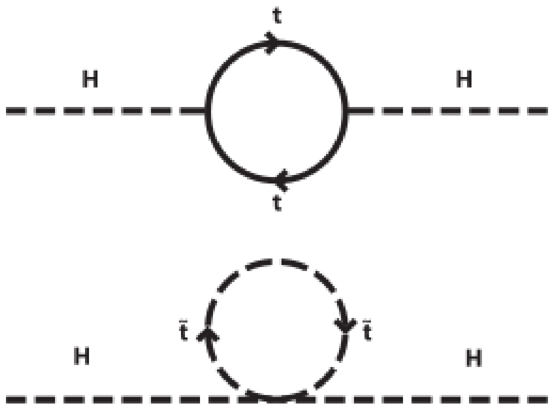


Fig.4 Cancellation of the Higgs boson quadratic mass renormalization between fermionic top quark loop and scalar top squark Feynman diagrams in the MSSM. [2]

• **Dark Matter**: one of the s-particles of the MSSM (the lightest netralino in particular) falls into the category of a Weakly Interacting Massive Particle (WIMP), which constitutes a promising candidates for dark matter.

• **Grand Unification**: if the superpartners of the Standard Model were near the TeV scale, then the gauge couplings of the three gauge groups are expected to unify at energies of order 10^{16} GeV.

Nevertheless, from experimental ground, supersymmetry is not an exact symmetry of Nature.

SUSY-Breaking and Soft-Terms

As pointed out before, supersymmetry implies that all fields in the same multiplets have the same mass.
Nevertheless, this cannot correspond to our real world, because super-partners of ordinary particles have not been detected. Thus, supersymmetry must be broken at an energy higher than the electroweak scale.
In analogy with other symmetries in particle physics, we expect SUSY to be broken spontaneously. This means that the complete Lagrangian of the theory would be invariant under supersymmetry transformations, but the vacuum state would not.
In mathematical language, this means

$$Q_\alpha \mid vacuum \rangle \neq 0 \tag{1.37}$$

In more details, let us consider the infinitesimal transformation laws under SUSY for components of a chiral superfield Φ

$$\delta\phi = \sqrt{2}\,\epsilon\psi \tag{1.38}$$

$$\delta\psi = \sqrt{2}\,\epsilon F + i\sqrt{2}\,\sigma^\mu\bar{\epsilon}\,\partial_\mu\phi \tag{1.39}$$

$$\delta F = i\sqrt{2}\,\bar{\epsilon}\,\bar{\sigma}^\mu\partial_\mu\psi \tag{1.40}$$

If, in the ground state of the theory, one of these expressions is non-zero, then SUSY is broken.
Now, to preserve Lorentz invariance, it is easy to conclude that the only possibility is [2]

$$\langle F\rangle \neq 0 \quad \rightarrow \quad \delta\psi \neq 0 \tag{1.41}$$

If we call ψ a Goldstone fermion (or *goldstino*), we find an analogous of the Goldstone theorem for non-supersymmetric model whith the VeV of the auxiliary field playing the role of the order parameter of the transition.

Moreover, from the supersymmetric Lagrangian (1.28), we can extract the scalar potential V_F of the theory given by the F-term of the superpotential $W(\Phi)$. From explicit calculation [2], it takes the form

$$V_F = K^{-1}_{i\bar{j}}\frac{\partial W}{\partial\phi_i}\frac{\partial W^*}{\partial\phi^*_{\bar{j}}} \tag{1.42}$$

where ϕ is the superfield scalar component, while $K^{-1}_{i\bar{j}}$ is the inverse matrix of the Kähler metric given by $\frac{\partial K}{\partial\phi_i \partial\phi^*_{\bar{j}}}$.
Then integrating out the auxiliary field F using the equations of motion

$$\frac{\delta S}{\delta F^i} = 0 \quad \rightarrow \quad F^{*i} = -\frac{\partial W}{\partial\phi_i} \qquad , \qquad \frac{\delta S}{\delta F^{*i}} = 0 \quad \rightarrow \quad F^i = -\frac{\partial W^*}{\partial\phi^{*i}}$$

the scalar potential becomes simply:

$$V_F = K^{-1}_{i\bar{j}} F^i F^{*j} \tag{1.43}$$

so that SUSY breaking condition implies $\langle V_F \rangle > 0$.

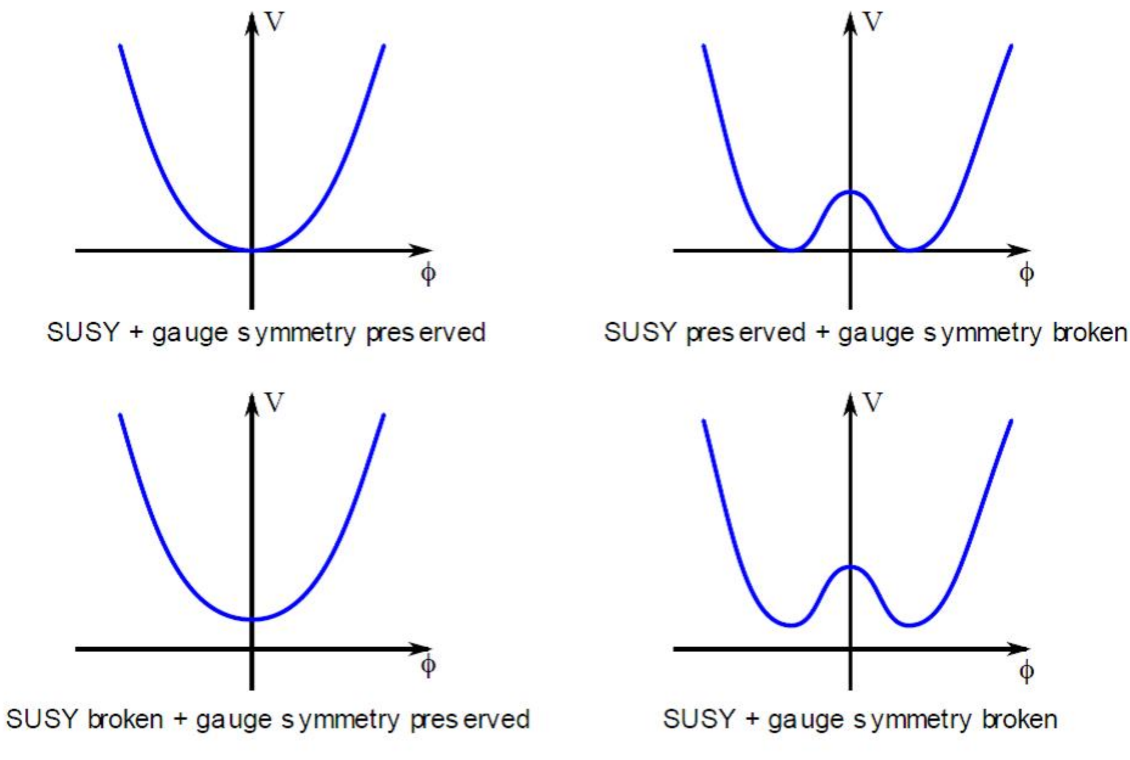


Fig.5 Various symmetry breaking scenarios. [2]

Since no consistent model can be constructed in which SUSY is dynamically broken in the observable fields sector, the most promising possibility is to assume the existence of a hidden sector inside the MSSM framework, in which the dynamics of some scalar fields

breaks SUSY. Then SUSY-breaking has to be communicated to the observable sector through the action of some messenger interaction.
In all scenarios, in order to break SUSY, the effective Lagrangian of the MSSM must receive a contribution from the hidden sector of the form

$$\begin{aligned}\mathcal{L}_{soft} = & \frac{1}{2}\left(M_a\ \hat{\lambda}^a\hat{\lambda}^a + h.c.\right)\ -\ m_\alpha^2\phi^{*\alpha}\phi^\alpha\ - \\ & -\Big(\ A_{\alpha\beta\gamma}\hat{Y}_{\alpha\beta\gamma}\phi^\alpha\phi^\beta H^\gamma\ +\ B\hat{\mu}H_1H_2\ +\ h.c.\Big)\end{aligned} \tag{1.44}$$

where ϕ^α denotes the scalar components of the chiral multiplets; M_a, m_α^2, $A_{\alpha\beta\gamma}$ and $B\hat{\mu}$ are called *soft breaking terms*.
These terms are crucial not only because they determine the supersymmetric spectrum (gaugino, higgsino, squark and sleptons masses), but also because they contribute to the Higgs potential generating the radiative break-down of the electroweak symmetry.
It has been calculated that, in order to reproduce the correct electroweak scale, the soft terms should be of the order of TeV energy scale.

Although in principle supersymmetry breaking may look arbitrary, it can arise naturally and dynamically in the context of local SUSY (i.e. supergravity).

Supergravity

The classical theory of gravity can be thought as the gauge theory of global space-time transformations. In this section we want to point out that supergravity, as well, can be viewed as the gauge theory of global supersymmetry.
From relation (1.19) it is visible the connection between SUSY transformations and space-time translation generated by P_μ. It is intuitive that promoting supersymmetry to a local symmetry, space-time translations which differ from point to point are generated. Therefore local SUSY implies general coordinate transformation, i.e. gravity.

From a technical point of view, if we consider for simplicity the action (1.16) we find that is not invariant under local supersymmetry. In fact, when the supersimmetric infinitesimal parameter ϵ becomes a function of the coordinates of space, one gets

$$\delta\mathcal{L} = \partial_\mu \epsilon^\alpha K^\mu_\alpha + h.c. \tag{1.45}$$

where

$$K^\mu_\alpha = -\partial_\mu \phi^* \psi^\alpha - \frac{i}{2}\psi^\beta (\sigma_\mu \bar{\sigma}^\nu)^\alpha_\beta \partial \nu \phi^* \tag{1.46}$$

Thus, in analogy with ordinary gauge theory, a gauge field has to be introduced in order to keep the action invariant. In this case we have to include:

$$\mathcal{L}_{3/2} = k K^\alpha_\mu \Psi^\mu_\alpha \tag{1.47}$$

where k is a constant introduced to give the correct dimension and Ψ is a Majorana spinor field, with spin 3/2, called gravitino, which transforms as

$$\Psi^\mu_\alpha \;\rightarrow\; \Psi^\mu_\alpha + \frac{1}{k}\partial^\mu \epsilon_\alpha \tag{1.48}$$

Nonetheless, if we look now at the total Lagrangian behaviour, we get:

$$\delta(\mathcal{L} + \mathcal{L}_{3/2}) = k \Psi^\mu_\alpha \gamma_\nu \; \epsilon T^{\mu\nu} \tag{1.49}$$

where $T^{\mu\nu}$ is the energy-momentum tensor.
One may be surprised by this result, but the physical intuition suggests that, since we are working in the context of SUSY, we have not to introduce a stand alone field, but a supermultiplet. In fact, the Lagrangian is invariant only if we add a new term of the form:

$$\mathcal{L}_g = -g_{\mu\nu} T^{\mu\nu} \tag{1.50}$$

where, remarkably, the field $g_{\mu\nu}$ transforms under SUSY as:

$$\delta g_{\mu\nu} = k \Psi^\mu_\alpha \gamma_\nu \; \epsilon \tag{1.51}$$

Any local supersymmetric theory has to include gravity through the supermultiplet $(g_{\mu\nu}, \Psi^\mu_\alpha)$ with graviton and gravitino respectively.

Since here we have more symmetry than in pure quantum gravity, one could reasonably think about an improvement in the high-energy behaviour. Although this is essentially

true, supersymmetry is not sufficient to cancel all divergences of the theory, which again has to be considered as a low-energy limit of a more profound structure.
Supergravity, in fact, thanks to the compactification of the higher-dimensional theory, is the connecting link between superstring theory and low-energy phenomenology.
Moreover, due to the Kaluza-Klein dimensional reduction, supergravity turns out to have a natural hidden sector built-in which is given by the moduli fields.
Within this framework, it is possible to study spontaneous supergravity breaking in the moduli sector, which generates, thanks to gravity-mediation, the soft breaking terms in (1.44).

In this regard, let us point out some technical essential details and focus on the entire Lagrangian of the model.
The $N=1$ 4D Supergravity Lagrangian for interacting supermultiplets, can be obtained following the same *Noether* approach we have used in the case of free chiral multiplet model.
Starting from the MSSM Lagrangian given by (1.35), one can generalize to local-supersymmetry and obtain the entire supergravity action in four space-time dimensions (see [9]).
Since we are interested in supersymmetry breaking, we concentrate on the form of the scalar potential obtained by integrating out the F auxiliary fields through their equation of motion. Explicit calculations (in units of $M_P = 1$) give:

$$V_F = e^K \left[(K^{-1})^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \; - \; 3|W|^2 \right] \tag{1.52}$$

where $D_i W = \partial_i W \; + \; (\partial_i K)\; W$, is called Kähler covariant derivative.
The structure of the scalar potential determines the vacuum state of the theory and then the possibility to have spontaneous SUSY-breaking.
As in the case of global SUSY, it is possible to write the potential (1.52) in terms of the F fields:

$$V_F = K_{i\bar{j}} F^i F^{\bar{j}} - 3e^K |W|^2 \tag{1.53}$$

with

$$F^i = e^{K/2} (K^{-1})^{i\bar{j}} D_{\bar{j}} \bar{W} \quad , \quad \bar{F}^{\bar{j}} = e^{K/2} (K^{-1})^{\bar{j}i} D_i W \tag{1.54}$$

We remark that, due to the presence of the last terms in (1.53) (coming directly from the auxiliary fields of the gravity multiplet), in a local supersymmetric theory both $\langle V_F \rangle = 0$ and $\langle V_F \rangle \neq 0$ are possible after supergravity breaking. This is very important for the cosmological constant problem, since it allows to break SUSY and at the same time to keep the vacuum energy zero.
By studying the minima of this potential, it is possible to reveal which fields break supersymmetry and the magnitude of the relevant F-terms.
In this context the super-Higgs mechanism is the following: scalar fields acquire a non-zero VeV giving rise at spontaneous SUSY-breaking; then the goldstino is eaten up by the massless gravitino to obtain a massive spin 3/2 particle.
From the analysis of the supergravity Lagrangian it's easy to find the gravitino mass term

$$m_{3/2} = e^{K/2}|W| \tag{1.55}$$

which, as we will see, is strictly related to the soft terms expressions.
Soft terms are generated through the gravitational interaction between moduli and observable fields.
The structure of the vacuum of (1.52) then provides a concrete connection between the fundamental theory of our universe and the low-energy physics which might be tested experimentally.

Chapter 2

Type IIB Moduli Stabilisation

In this section we investigate the structure of the effective scalar potential resulting from type IIB flux compactifications. The task here is to give a comprehensive description of the fundamental scenarios of moduli stabilisation, focusing our interest on the ground state of the theory as well as on the emergent possibility to break SUSY dynamically in the hidden sector.

We start from a detailed analysis, in the context of Supergravity, of the tree-level structure of the scalar moduli potential obtained from turning on fluxes in the internal manifold.

Then, in order to stabilise Kahler moduli, we study the effects of leading corrections to the Kahler potential and the superpotential (perturbative for the former and non-perturbative for the latter).

Finally we get through some considerations about the physical picture that emerges from these models and the possibility to connect the elegant LARGE-volume model, using string loop corrections, to the MSSM and chiral visible matter.

2.1 Tree-Level Moduli Potential

As previously pointed out, taking the low-energy limit of String Theory and compactifying the extra dimensions through Kaluza-Klein reduction, we are left with a local supersymmetric extension of the Standard Model.

We are interested in the moduli sector dynamics and, for this reason, we analyse the scalar potential (1.52) generated by the moduli chiral superfields:

$$V_F = e^K \left[\ (K^{-1})^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \ \right] \tag{2.1}$$

From this expression it is manifest that the Kahler potential and the superpotential play a crucial role in the theory.

Let us inherit from String Theory the explicit form of $K(\phi, \phi^\star)$ and $W(\phi)$.

We assume the following standard notation: call T, U, S respectively the Kahler, complex structure and axio-dilaton moduli.
In agreement with the results in type IIB compactifications, the Kähler potential at the leading order in α' and g_s takes the block diagonal form

$$K_{tree} = \ -2ln(\mathcal{V}) - ln(S+\bar{S}) - ln\left(-i\int_M \Omega(U) \wedge \bar{\Omega}(\bar{U})\right) \tag{2.2}$$

where $\mathcal{V}$ is the volume of M, which is the entire Calabi-Yau manifold, while Ω is the Calabi-Yau holomorphic $(3,0)$-form.
From turning on background fluxes, we have instead for the superpotential:

$$W_{tree} = \lambda(S,U) \ = \int_M G_3 \wedge \Omega \tag{2.3}$$

Notice that $G_3 = F_3 - SH_3$ with F_3 and H_3 being respectively RR and NS-NS 3-forms fluxes ([10] , [11]).
It is to be underlined here that the superpotential does not depend on the Kähler moduli. Since W is holomorfic, a dependence on T would violate the shift symmetry (see for example [12]). The consequence of this fact will be clear in a moment.

Let us compute the explicit form of the scalar potential V_F and see what happens.

$$\begin{aligned} V_F &= e^K[\ (K^{-1})^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2] \\ &= e^K[\sum_T (K^{-1})^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} + \sum_{U,S} (K^{-1})^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} - 3|W|^2] \end{aligned} \tag{2.4}$$

Applying the definition of Kähler covariant derivative, we have:

$$D_T W = \partial_T W + W \partial_T K$$

but, since W does not depend on the modulus T:

$$\begin{aligned} V_F &= e^K [\sum_T (K^{-1})^{i\bar{j}} K_i K_{\bar{j}} |W|^2 + \sum_{U,S} (K^{-1})^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} - 3|W|^2]] \\ V_F &= e^K [(\sum_T (K^{-1})^{i\bar{j}} K_i K_{\bar{j}} - 3)|W|^2 + \sum_{U,S} (K^{-1})^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W}] \end{aligned} \tag{2.5}$$

Now, recalling the so called *no-scale* property for a generic Calaby-Yau manifold

$$(K^{-1})^{T\bar{T}} K_T K_{\bar{T}} = 3$$

we obtain:

$$\begin{aligned} V_F &= e^K [\ \sum_{U,S} (K^{-1})^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W}\ ] \\ &= \ e^K\ [\ (K^{-1})^{U\bar{U}} |D_U W|^2 + (K^{-1})^{S\bar{S}} |D_S W|^2\ ] \end{aligned} \tag{2.6}$$

This is the final generic form of the scalar moduli potential at tree-level.
Now it is easy to work out the minimum of the potential, because of:

$$V_F \ = \ e^K\ [\ (K^{-1})^{U\bar{U}} |D_U W|^2 + (K^{-1})^{S\bar{S}} |D_S W|^2\ ]\ \geq\ 0 \tag{2.7}$$

Hence we have a Minkowski ground state when U and S acquire a vacuum expectation value (VeV) $< U >$ and $< S >$ such that respectively:

$$D_U W = 0 \ \ , \ \ D_S W = 0. \tag{2.8}$$

Let us analyse how these results affect the supersymmetric properties of the theory and in particular of the ground state.

As seen in section 1.3, SUSY-breaking occurs when the F-term, in the vaccum state, takes a non zero value $< F > \neq \ 0$.

In this context, recalling the expression (1.54), we know the F-term takes the form:

$$F^I = e^{K/2} \ (K^{-1})^{I\bar{J}} D_{\bar{J}} W \tag{2.9}$$

Given the explicit form of the local supersymmetry transformations, indeed, this expression emerges easily due to the preservation of Lorentz invariance.

The F-terms referred to U and S moduli are zero, because of the relations (2.8), while the F-term of Kahler moduli becomes, after calculation:

$$F^T = -2\tau \ e^{K/2} |W| \tag{2.10}$$

where the relation $K^{T\bar{T}} K_{\bar{T}} = -2\tau$, that holds for a generic Calabi-Yau space, has been used with $\tau = Re(T) = \frac{T+\bar{T}}{2}$.
Notice that, since the gravitino mass term in supergravity models is given by

$$m_{3/2} = e^{K/2} |W|$$

we deduce that the F-term (2.10) which breaks supersymmetry is proportional to the gravitino mass. The latter, indeed, is strictly connected with spontaneous SUSY-breaking thanks to the so called super-Higgs mechanism, in which the gravitino "eats" a Goldstone fermion mode and becomes massive. The relation (2.10) gives us a hint: this mass, as we will see later in section 3, enters directly in the structure of soft terms and then determines approximatively the energy scale at which supersymmetry is broken.

Returning to our discussion, we have learned that, while on one hand at tree-level supersymmetry can be broken dynamically in the hidden sector, on the other we are still far from achieving a satisfying model: the T-moduli have no hope to be stabilised here. Although the complex structure moduli and the axio-dilaton are just fixed thanks to the superpotential induced by the background fluxes, for Kahler moduli the potential is

clearly flat.
If we want a physical acceptable model we must find a technique which provides a stable ground state for the T-moduli.
A simple model which addresses this issue is realised and discussed in the next paragraph.

2.2 KKLT scenario

In the previous paragraph we have learned that supersymmetry can be spontaneously broken by Kahler moduli.
Nevertheless the T-moduli have no minimum and thus we have to introduce some corrections for K and W in order to solve the problem.

First, let's focus on the superpotential, employing a dependence on T-moduli (we actually ignore the α' correction to the Kahler potential, which instead will be added next): we have to introduce non-perturbative corrections to W, due the the fact that W is not renormalised at any order in perturbation theory.
From a physical point of view, in some circumstances in the context of string compactification, which we do not investigate here, W acquires a non-perturbative dependence on some or all of the Kahler moduli through D3-brane instantons or gaugino condensation from wrapped D7-branes. W then takes the form:

$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i} \tag{2.11}$$

where A_i only depends on the complex structure moduli, $a_i = \frac{2\pi}{n}$ (with n positive integer).

To sum up we want to calculate the new scalar potential when K and W take the form:

$$K = -2 \, ln \, \mathcal{V} \tag{2.12}$$
$$W = W_0 + \sum_i A_i e^{-a_i T_i}$$

where W_0 is the value of the superpotential fixed by the minimization of S and U moduli already seen from the tree-level analysis.

From the relation $\mathcal{V} \;\cong\; \tau^{3/2}$, considering for simplicity only one T-modulus, we can easily simplify and rewrite the previous expression in terms of T, we obtain:

$$\begin{aligned} K &= -3\; ln\left(\frac{T+\bar{T}}{2}\right) \\ W &= W_0 \;+\; A\, e^{-a\;T} \end{aligned} \tag{2.13}$$

Taking into account the new W dependence on T we obtain:

$$V_F = e^K\left[K^{T\bar{T}}(W_T\bar{W}_{\bar{T}} + K_T W\bar{W}_{\bar{T}} + K_T\bar{W}W_T + K_T K_{\bar{T}}\bar{W}W) - 3|W|^2\right] \tag{2.14}$$

Now, making use of the *no-scale* structure seen previously in the tree-level model, last two terms cancel and we are left with:

$$V_F = e^K\left[K^{T\bar{T}}W_T\bar{W}_{\bar{T}} + K^{T\bar{T}}K_T(W\bar{W}_{\bar{T}} + \bar{W}W_T)\right] \tag{2.15}$$

Substituting the explicit form of the superpotential and recovering the useful relations $K^{T\bar{T}}K_T \;=\; -2\tau$ and $T \;=\; \tau + i\rho$, we have:

$$K^{T\bar{T}}K_T(W\bar{W}_{\bar{T}} + \bar{W}W_T) \;=\; 2\tau\left[2aA^2e^{-2a\tau} - AW_0e^{-a\tau}(e^{ia\rho} + e^{-ia\rho})\right] \tag{2.16}$$

in which, we notice :

$$e^{ia\rho} + e^{-ia\rho} \;=\; 2cos(a\rho)$$

At this point, since our final goal is to work out the vacuum structure, becomes convenient to minimize now with respect to $Im(T) \;=\; \rho$ obtaining:

$$<\rho> = \;\pi/a \quad\rightarrow\quad cos(a\rho) \;=\; -1 \tag{2.17}$$

Carrying on with calculation:

$$V_F = e^K \left[K^{T\bar{T}} W_T \bar{W}_{\bar{T}} + 4\tau a A^2 e^{-2a\tau} - 4\tau a A W_0 e^{-a\tau}\right] = \tag{2.18}$$

$$= \frac{1}{\tau^3}\left[\frac{4}{3}\tau^2 A^2 a^2 e^{-2a\tau} + +4\tau a A^2 e^{-2a\tau} - 4\tau a A W_0 e^{-a\tau}\right]$$

which finally becomes:

$$V_F \ = \ \frac{4}{3}A^2 a^2 \frac{e^{-2a\tau}}{\tau} \ + \ 4aA^2\frac{e^{-2a\tau}}{\tau^2} \ - \ 4aAW_0\frac{e^{-a\tau}}{\tau^2} \tag{2.19}$$

To discover the ground state value of the T-modulus, we have to study the minimum of this potential.
Let us perform the derivative to find stationary points:

$$\frac{\partial V}{\partial \tau} \ = \ 0 \tag{2.20}$$

From (2.19) we have:

$$-\frac{4}{3}A^2 a^2\left(\frac{e^{-2a\tau}}{\tau^2} + 2a\frac{e^{-2a\tau}}{\tau}\right) - 8aA^2\left(a\ \frac{e^{-2a\tau}}{\tau^2} + \frac{e^{-2a\tau}}{\tau^3}\right) +$$

$$+ \ 4aAW_0\left(a\ \frac{e^{-a\tau}}{\tau^2} + 2\frac{e^{-a\tau}}{\tau^3}\right) \ = \ 0$$

Multiplying then by τ^3, this becomes:

$$-\frac{4}{3}A^2 a^2\left(e^{-2a\tau}\tau + 2ae^{-2a\tau}\tau^2\right) - 8aA^2\left(a\ e^{-2a\tau}\tau + e^{-2a\tau}\right) +$$

$$+ \ 4aAW_0\left(a\ e^{-a\tau}\tau + 2e^{-a\tau}\right) \ = \ 0$$

Now, in general we have to consider higher instanton contributions to W, which represent a non-perturbative correction of the form:

$$W_{np} = \sum_n A_n e^{-naT} \tag{2.21}$$

Then, comparing with (2.13), in order to ignore terms as e^{-2aT}, e^{-3aT} and so on, we need $a\tau \gg 1$.
Assuming that limit, we can simplify the equation neglecting subleading terms. After some simple calculations, we are left with

$$4a^2 A W_0 e^{-a\tau} - \frac{8}{3} a^3 A^2 e^{-2a\tau}\tau \;=\; 0$$

which becomes finally:

$$e^{-a\tau} \;\sim\; \frac{W_0}{\tau} \tag{2.22}$$

We conclude that T-moduli can be stabilised through non-perturbative corrections to the superpotential, but as shown in (2.22), this occur only when W_0 **is exponentially small** (tipically $< 10^{-4}$).
This model goes under the name of KKLT scenario (from Kachru, Kallosh, Linde, Trivedi) and represent the prototype of convincing models of moduli stabilisation derived from string theory [13].

Let us discuss some important aspects about KKLT solutions.
In the context of this class of minima, all the moduli are stabilised, but supersymmetry holds in the ground state, where indeed we have:

$$D_T W = \partial_T W + W \partial_T K \;=\; 0$$

with as immediate consequence

$$F^T = e^{K/2}\, (K^{-1})^{T\bar{T}} D_{\bar{T}} W = 0 \tag{2.23}$$

In order to have supersymmetry breaking, one has to perform an uplift of the minimum. In fact by looking at the scalar potential in (2.1) and taking into account that we have $D_I W = 0$, it is easy to recognize we are in an Anti-deSitter scenario.

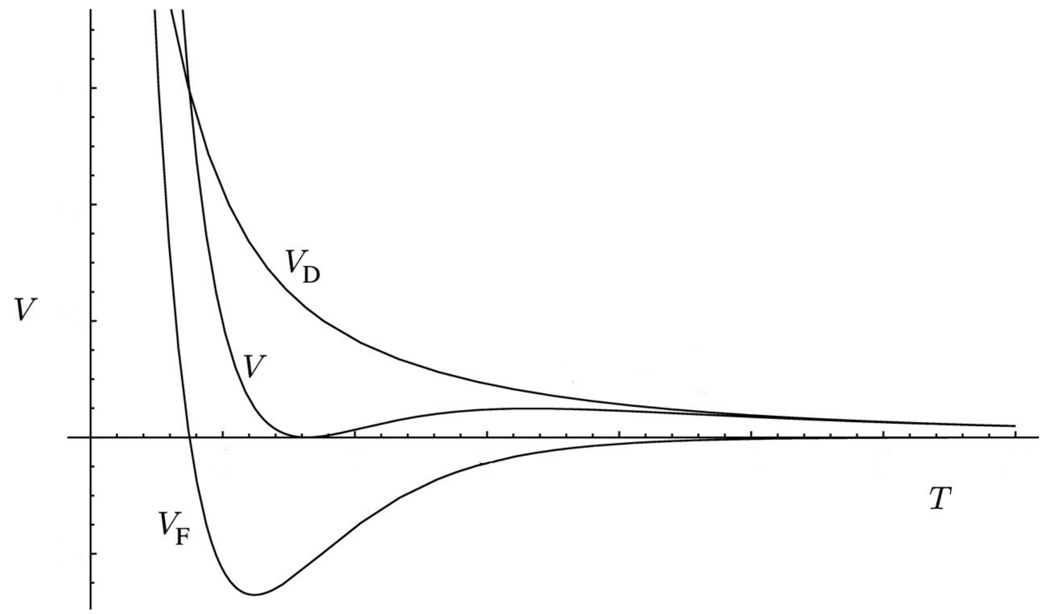


Fig. 6. Uplifting of ground state in KKLT scenario.

A well known problem of the KKLT scenario is exactly the uplift of the vacuum energy to zero or positive values. This has relevant consequences not only for SUSY-breaking, but also for cosmology.
The original proposal was to use antibranes which relies essentially on non-linearly realised supersymmetry, whereas other attempts to uplift vacuum energy are made by using D-term and lead generically to very heavy (close to the Planck mass) gravitino mass.

The second main problem of a such scenario is that, in the flux "landscape", values of W_0 of $O(10)$ are more common than those of the order $O(10^{-4})$ by a factor $\sim 10^{10}$ [10] !
It would be interesting to have example models of large volume minima of the potential for large values of W_0.
We now turn our attention to this.

2.3 LARGE Volume Scenario

In KKLT scenario, τ parametrizes extra dimensions, and in particular Calabi-Yau volume, according with the relation $\mathcal{V} = \tau^{3/2}$.
What we suppose now, is that exists another parameter, inherited from Kaluza-Klein dimensional reduction, that controls the volume of a small blow-up cycle in the bulk space.
For the calculation we specify that we refer to the expicit form of the orientifold of the Calabi-Yau space given by the degree 18 hypersurface in $\mathbf{P}^4_{[1,1,1,6,9]}$ which has been built and studied deeply [1]. In order to grasp a geometrical meaning of such construction, we can visualise it as a huge swiss-cheese with the small blow-up cycle being a "hole".
Let us introduce two T-moduli T_b for the large bulk and T_s for the small hole. As we're going to show, with this construction we gain large-volume minima with natural values of W_0.

But a question may (should) arise now, where is the observable world in this strange picture?

When the volume of the bulk is very large (as we will see $\sim 10^{15}$ in strings units), N D-7 branes wrapped on the small cycles (in general one could have a stack of them) support a non-Abelian $SU(N)$ theory and the corrisponding gauge coupling is qualitatively similar to those of the Standard Model [14]. If the branes are magnetised, SM chiral matter can arise from strings stretching between stacks of D7-branes.
We are ready to explore this fascinating scenario.

The Kahler geometry is specified by:

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_b^{3/2} - \tau_s^{3/2}\right) \tag{2.24}$$

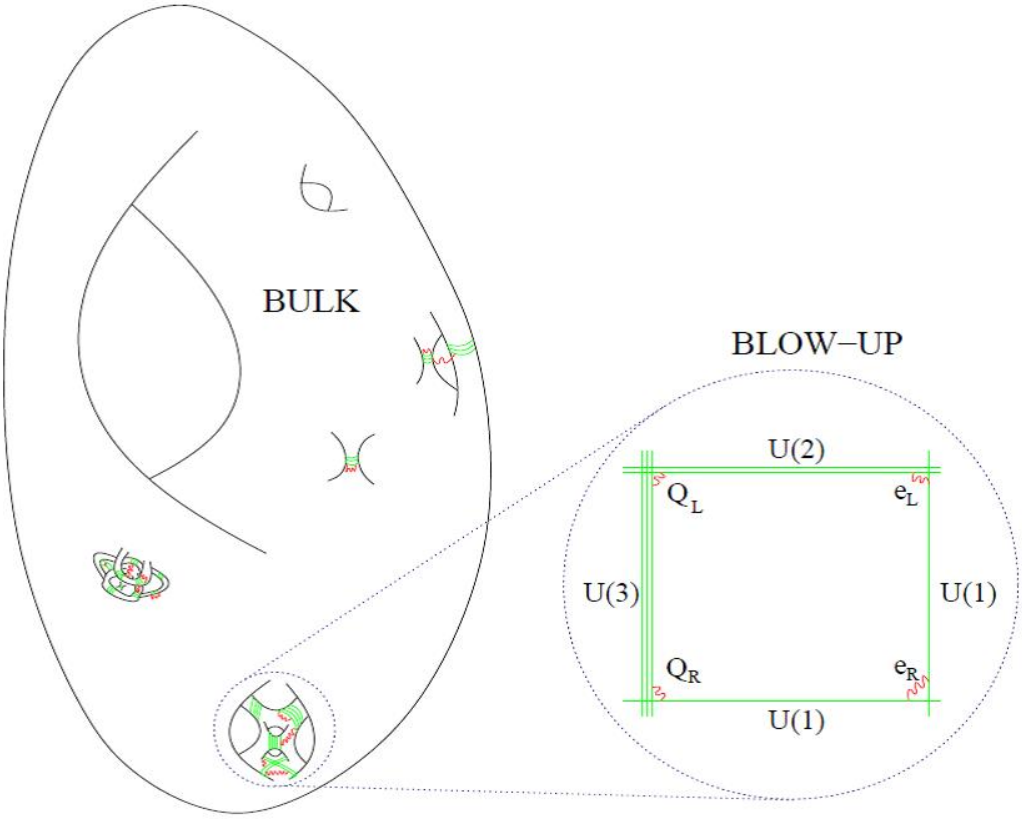


Fig.7 The physical picture: Standard Model matter is supported on a small blow-up cycle located within the bulk of a very large Calabi-Yau manifold [14].

The scalar potential V_F is, as usually, determined by the form of K and W. In this case we have:

$$\begin{aligned} K &= -2\ ln\left(\mathcal{V}+\frac{\hat{\xi}}{2}\right) \simeq -2ln(\mathcal{V})-\frac{\hat{\xi}}{\mathcal{V}}+O(\mathcal{V}^{-2}) \\ W &= W_0+A_b e^{-a_b T_b}+A_s e^{-a_s T_s} \end{aligned} \quad (2.25)$$

where this time we consider α' perturbative corrections to the Kahler potential, which instead were neglected in the previous case. These are higher derivative corrections to the 4D effective action which are proportional to $\hat{\xi}=\xi/g_s^{3/2}$ where ξ is an $\mathcal{O}(1)$ constant that depends on the topological features of the Calabi-Yau, whereas g_s is the string coupling

which is set by the VeV of $\mathrm{Re}(S)$.
As in the precedent model, we specify real and immaginary part of T-moduli; we have

$$T_b = \tau_b + i\rho_b \quad , \quad T_s = \tau_s + i\rho_s \tag{2.26}$$

The computation of the potential is shown below.

Let us start from the usual expression:

$$V_F \;=\; e^K \left[K^{T_i \bar{T}_j} D_{T_i} W D_{\bar{T}_j} \bar{W} - 3|W|^2 \right] \tag{2.27}$$

If we substitute the form of K and W we obtain:

$$V_F \;=\; V_{np} \;+\; V_{\alpha'} \tag{2.28}$$

with

$$\begin{aligned} V_{np} \;&=\; e^K \left[K^{i\bar{j}} (W_{T_i} \bar{W}_{\bar{T}_j} + K_{T_i} W \bar{W}_{T_j} + K_{\bar{T}_j} \bar{W} W_{T_i}) \right] \\ V_{\alpha'} \;&=\; e^K \left[3\hat{\xi} \frac{(\hat{\xi}^2 + 7\hat{\xi}\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} |W|^2 \right] \end{aligned}$$

No-scale structure is lost, due to the presence of the $\hat{\xi}$ correction.
To develop calculations, let us first work out the following relations:

$$\begin{aligned} K_{T_s} &= \frac{1}{2} \frac{\partial K}{\partial \tau_s} = \frac{3\sqrt{2\tau_s}}{2\mathcal{V}} \\ K_{T_b} &= \frac{1}{2} \frac{\partial K}{\partial \tau_b} = -\frac{3\sqrt{2\tau_b}}{2\mathcal{V}} \end{aligned} \tag{2.29}$$

together with the inverse 2x2 symmetric matrix elements:

$$K^{T_i \bar{T}_j} = -\frac{2}{9}(2\mathcal{V} + \hat{\xi}) k_{ijk} t^k + \frac{4\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}} \tau_i \tau_j \tag{2.30}$$

which in the large Volume limit becomes:

$$K^{T_s\bar{T}_s} \simeq \sqrt{\tau_s}\mathcal{V} \tag{2.31}$$

$$K^{T_s\bar{T}_b} = 4\tau_s\tau_b \simeq \tau_s\mathcal{V}^{2/3}$$
$$K^{T_b\bar{T}_b} = \frac{4}{3}\tau_b^2$$

Assuming $W = W_0 + O(1/\mathcal{V})$ from integration of U and S moduli, the non-perturbative term of the potential takes the form:

$$\begin{aligned} V_{np} \;=\; & e^K[K^{T_i\bar{T}_j}W_{T_i}\bar{W}_{T_j} \;+\; 2\tau_s a_s A_s W_0 e^{-a_s\tau_s}(e^{-ia_s\rho_s} + e^{ia_s\rho_s}) + \\ & + 2\tau_b a_b A_b W_0 e^{-a_b\tau_b}(e^{-ia_b\rho_b} + e^{ia_b\rho_b})] \end{aligned} \tag{2.32}$$

using the explicit form of the inverse of Kähler metric, the first term into the brackets lead to:

$$\begin{aligned} K^{T_i\bar{T}_j}W_{T_i}\bar{W}_{T_j} &= K^{T_s\bar{T}_s}W_{T_s}\bar{W}_{T_s} + K^{T_b\bar{T}_b}W_{T_b}\bar{W}_{T_b} + K^{T_s\bar{T}_b}(W_{T_s}\bar{W}_{T_b} + W_{T_b}\bar{W}_{T_s}) = \\ &= a_s^2A_s^2\sqrt{\tau_s}Ve^{-a_sT_s}e^{-a_s\bar{T}_b} \;+\; a_b^2A_b^2\frac{4}{3}\tau_b^2e^{-a_bT_b}e^{-a_b\bar{T}_b} \;+ \\ &+ a_ba_sA_bA_s\tau_sV^{2/3}(e^{-a_sT_s}e^{-a_b\bar{T}_b} + e^{-a_s\bar{T}_s}e^{-a_bT_b}) = \\ &= a_s^2A_s^2\sqrt{\tau_s}Ve^{-2a_s\tau_s} + \frac{4}{3}\tau_b^2e^{-2a_b\tau_b} + \tau_sV^{2/3}e^{-a_b\tau_b-a_s\tau_s}[2cos(a_s\rho_s - a_b\rho_b)] \end{aligned}$$

Now, we perform the LARGE volume limit $\mathcal{V} >> 1$ and we can neglect the terms which are suppressed by $\simeq e^{-a_b\tau_b}$, obtaining:

$$K^{T_i\bar{T}_j}W_{T_i}\bar{W}_{T_j} = a_s^2A_s^2\sqrt{\tau_s}\mathcal{V}e^{-2a_s\tau_s} \tag{2.33}$$

and

$$V_{np} = e^K\left[a_s^2A_s^2\sqrt{\tau_s}\mathcal{V}e^{-2a_s\tau_s} + 4\tau_sa_sA_sW_0e^{-a_s\tau_s}cos(a_s\rho_s)\right] \tag{2.34}$$

Now we note that $e^K \simeq \frac{1}{\mathcal{V}^2}$ and we approximate $V_{\alpha'}$ in the LARGE volume limit [10]. Finally we obtain the total expression for the scalar potential:

$$V_F \simeq \frac{a_s^2 A_s^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} + \frac{a_s A_s W_0 \tau_s e^{-a_s\tau_s} cos(a_s\rho_s)}{\mathcal{V}^2} + \frac{\hat{\xi}}{\mathcal{V}^3}|W_0|^2 \tag{2.35}$$

From this expression we start evaluate minimum conditions in order to verify T-moduli stabilisation.
First of all we minimize with respect to $Im(T) = \rho$.
We immediately find:

$$\rho_s = \frac{n\pi}{a_s} \tag{2.36}$$

in order to have $cos(a_s\rho_s) = -1$.
For the immaginary part of T_b holds a similar relation, but because of the approximation we performed, it does not affect the minimum investigation.
From fixing ρ_s, we are left with:

$$V_F \simeq \frac{a_s^2 A_s^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s A_s W_0 \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\hat{\xi}}{\mathcal{V}^3}|W_0|^2 \tag{2.37}$$

Since V_F depends explicitly on τ_s and $\mathcal{V}$, it is worthwhile evaluating the following equation:

$$\frac{\partial V_F}{\partial \tau_s} = 0 \ , \ \frac{\partial V_F}{\partial V} = 0 \tag{2.38}$$

Where V_F is given by:

$$V_F \simeq \frac{\lambda\sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{\mu\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\nu}{\mathcal{V}^3} \tag{2.39}$$

with $\lambda = a_s^2 A_s^2$; $\mu = a_s A_s W_0$; $\nu = \hat{\xi} W_0^2$.

$$\frac{\partial V_F}{\partial \mathcal{V}} = -\frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{\mathcal{V}^2} + 2\frac{\mu\tau_s e^{-a_s\tau_s}}{\mathcal{V}^3} - 3\frac{\nu}{\mathcal{V}^4} = 0 \tag{2.40}$$

which can be easily rearranged into a quadratic:

$$-\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}\mathcal{V}^2 + 2\mu\tau_s e^{-a_s\tau_s}\mathcal{V} - 3\nu = 0 \tag{2.41}$$

and be solved for:

$$\mathcal{V} = \frac{\mu}{\lambda}\tau_s^{1/2}e^{a_s\tau_s}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right) \tag{2.42}$$

The equation for τ_s leads to:

$$\frac{\partial V_F}{\partial \tau_s} = \frac{\lambda\mathcal{V}e^{-a_s\tau_s}}{\tau_s^{1/2}}\left(\frac{1}{2} - 2a_s\tau_s\right) - \mu(1 - a_s\tau_s) = 0 \tag{2.43}$$

We then use (2.42) to obtain an implicit equation for τ_s:

$$\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)\left(\frac{1}{2} - 2a_s\tau_s\right) = (1 - a_s\tau_s) \tag{2.44}$$

Now, by requiring $a_s\tau_s >> 1$ to be able to ignore higher order corrections, we can simplify and solve for τ_s.
We have, in conclusion:

$$\begin{aligned} \left\langle \tau_b^{3/2} \right\rangle \ \simeq \ \langle \mathcal{V} \rangle \ &= \ \frac{\mu}{2\lambda}\left\langle \tau_s \right\rangle^{1/2} e^{a_s\langle\tau_s\rangle} \\ \langle \tau_s \rangle \ &= \ \left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3} \end{aligned} \tag{2.45}$$

Recalling the explicit form of λ, μ, ν :

$$\langle \tau_s \rangle \ \simeq \ \hat{\xi}^{2/3} \ \simeq \ \frac{\xi^{2/3}}{g_s} \ \simeq \ O(1/g_s) \ \simeq \ 10$$

$$\langle \mathcal{V} \rangle \simeq W_0 e^{a_s \tau_s} \simeq e^{1/g_s} >> 1$$

As emerges from the computation, this model allows T-moduli stabilisation with natural values of $W_0 \simeq O(1)$ [10]. The vacuum of the potential can generally be at exponentially large volume, as it is expressed in the relation above.
In this context we have a non-supersymmetric AdS vacuum state, with cosmological constant of order $1/\mathcal{V}^3$. An uplift to dS vacuum state is viable through different techniques and is necessary in order to reproduce the correct value of the cosmological constant.

It is important to remark that the mechanism described results in internal spaces that are exponentially large in string units; this implies a realization of the large extra dimensions scenario in which the fundamental string scale is hierarchically smaller than the Planck scale since dimensional reduction gives $M_s \sim M_p/\sqrt{\mathcal{V}}$.
For the particular example discussed here one finds an intermediate string scale $M_s \sim 10^{12} GeV$, which, as we will see in the section on SUSY-breaking, leads to TeV-scale soft terms.

2.4 Moduli Masses and Coupling to Photons

String moduli are naively massless particles. Since this would give rise to unobserved fifth force, it is necessary that they receive mass from the flux-induced moduli scalar potential V_F. Let us see in details how to canonically normalise the moduli and also compute their masses and couplings to photons in the context of the large volume scenario.

First of all, we assume that the vacuum of the potential has been located as shown in the previous paragraph. By writing $\tau_i = \langle \tau_i \rangle + \delta\tau_i$, we are able to expand the Lagrangian about the minimum of the moduli potential.
In the neighbourhood of the minimum, we can write (with obvious notation):

$$L = K_{i\bar{j}}\partial_\mu(\delta\tau_i)\partial^\mu(\delta\tau_j) - V_0 - (M^2)_{ij}(\delta\tau_i)(\delta\tau_j) - O(\delta\tau^3) - k\frac{\tau_s}{M_P}F_{\mu\nu}F^{\mu\nu}. \tag{2.46}$$

where we take $Re(f_{U(1)}) = k\tau_s$, the gauge kinetic function, from string compactification. In order to express the Lagrangian in terms of normalised fields, we have to diagonalize simultaneously $K_{i\bar{j}} = \frac{1}{4}\frac{\partial^2 K}{\partial\tau_i\partial\tau_j}|_{min}$ and $(M^2)_{i\bar{j}} = \frac{1}{4}\frac{\partial^2 V}{\partial\tau_i\partial\tau_j}|_{min}$.
For this scope, let us introduce two fields Φ and χ in terms of which we write:

$$\begin{pmatrix}\delta\tau_b\\ \delta\tau_s\end{pmatrix} = \begin{pmatrix}(v_\Phi)_b\\ (v_\Phi)_s\end{pmatrix}\frac{\Phi}{\sqrt{2}} + \begin{pmatrix}(v_\chi)_b\\ (v_\chi)_s\end{pmatrix}\frac{\chi}{\sqrt{2}} \tag{2.47}$$

where we impose $v = \begin{pmatrix}v_b\\ v_s\end{pmatrix}$ to be normalised, satisfying the relation $v_\alpha^T \cdot K \cdot v_\beta = \delta_{\alpha\beta}$.
Now, thanks to the commutation of (K^{-1}) and (M^2), we have to solve a generalized eigenvalue problem:

$$\left((K^{-1})(M^2)\right) v_i = m_i^2 v_i \tag{2.48}$$

where $i = \Phi, \chi$ and so m_i^2 are respectively the eigenvalues m_ϕ^2 and m_χ^2, with $m_\Phi > m_\chi$.
Calculations begin by inverting Kahler metric.
The explicit form of Kähler potential is

$$K = -2ln\left(\frac{1}{9\sqrt{2}}(\tau_b^{3/2} - \tau_s^{3/2}) + \frac{\xi}{2g_s^{3/2}}\right) .$$

We recall $\frac{\partial}{\partial\tau_i} = \frac{1}{2}\frac{\partial}{\partial T_i}$ and ignore α' correction and terms that are suppressed in the large volume limit, to obtain

$$K_{i\bar{j}} = \begin{pmatrix}K_{b\bar{b}} & K_{b\bar{s}}\\ K_{s\bar{b}} & K_{s\bar{s}}\end{pmatrix} = \begin{pmatrix}\frac{3}{4\tau_b^2} & -\frac{9\tau_s^{1/2}}{8\tau_b^{5/2}}\\ -\frac{9\tau_s^{1/2}}{8\tau_b^{5/2}} & \frac{3}{8\tau_s^{1/2}\tau_b^{3/2}}\end{pmatrix} \tag{2.49}$$

The inverse matrix then turns out to be:

$$K_{i\bar{j}}^{-1} = \begin{pmatrix}\frac{4\tau_b^2}{3} & 4\tau_b\tau_s\\ 4\tau_b\tau_s & \frac{8\tau_s^{1/2}\tau_b^{3/2}}{3}\end{pmatrix} \tag{2.50}$$

To compute second derivatives of scalar potential (2.39), instead, it is useful to perform an approximation at the second order in series of $\epsilon = 1/(4a_s\tau_s)$.
On this line it is not hard to rearrange the equation (2.45) for τ_s e $\mathcal{V}$ obtaining that in the zero point energy holds [15]

$$\tau_s^{3/2}\left(\frac{\mu^2}{4\lambda}\right) = \nu\left(1+\frac{1}{2a_s\tau_s}+\frac{9}{(4a_s\tau_s)^2}\right) \tag{2.51}$$
$$e^{-a_s\tau_s} = \frac{\mu}{2\lambda}\frac{W_0}{\tau_b^{3/2}}\frac{\sqrt{\tau_s}}{a_s}\left(1-\frac{3}{4a_s\tau_s}-\frac{3}{(4a_s\tau_s)^2}\right)$$

Then for second derivatives of the potential, we have

$$\frac{\partial^2 V}{\partial\tau_b^2} = \frac{15}{4}\frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{\tau_b^{7/2}} - \frac{12\mu\tau_s e^{-a_s\tau_s}}{\tau_b^5} + \frac{99\nu}{4\tau_b^{13/2}}$$
$$\frac{\partial^2 V}{\partial\tau_s^2} = \frac{4a_s^2\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{\tau_b^{3/2}} - \frac{2\lambda\tau_s^{-1/2}e^{-2a_s\tau_s}}{\tau_b^{3/2}} - \frac{\lambda\tau_s^{-3/2}e^{-2a_s\tau_s}}{4\tau_b^{3/2}} + \frac{2a_s\mu e{-}a_s\tau_s}{\tau_b^3} - \frac{a_s^2\mu\tau_s e^{-a_s\tau_s}}{\tau_b^3}$$
$$\frac{\partial^2 V}{\partial\tau_b\partial\tau_s} = \frac{3\lambda\sqrt{\tau_s}a_s e^{-2a_s\tau_s}}{\tau_b^{5/2}} - \frac{3\lambda\tau_s^{-1/2}e^{-2a_s\tau_s}}{4\tau_b^{5/2}} + \frac{3\mu e^{-a_s\tau_s}}{\tau_b^4} - \frac{3\mu a_s\tau_s e^{-a_s\tau_s}}{\tau_b^4}$$

and using (2.51)

$$\frac{\partial^2 V}{\partial\tau_b^2} = \frac{9W_0^2\nu}{2\tau_b^{13/2}}\left(1+\frac{1}{2a_s\tau_s}\right), \tag{2.52}$$

$$\frac{\partial^2 V}{\partial\tau_s^2} = \frac{2a_s^2W_0^2\nu}{\tau_b^{9/2}}\left(1-\frac{3}{4a_s\tau_s}+\frac{6}{(4a_s\tau_s)^2}\right), \tag{2.53}$$

$$\frac{\partial^2 V}{\partial\tau_b\partial\tau_s} = -\frac{3a_sW_0^2\nu}{\tau_b^{11/2}}\left(1-\frac{5}{4a_s\tau_s}+\frac{4}{(4a_s\tau_s)^2}\right) \tag{2.54}$$

Now we can easily build both the mass matrix, given by

$$(M^2)_{i\bar{j}} \ = \ \begin{pmatrix} \frac{9W_0^2\nu}{4\tau_b^{13/2}}\left(1+\frac{1}{2a_s\tau_s}\right) & \frac{3a_sW_0^2\nu}{2\tau_b^{11/2}}\left(1-\frac{5}{4a_s\tau_s}+\frac{4}{(4a_s\tau_s)^2}\right) \\ \frac{3a_sW_0^2\nu}{2\tau_b^{11/2}}\left(1-\frac{5}{4a_s\tau_s}+\frac{4}{(4a_s\tau_s)^2}\right) & \frac{a_s^2W_0^2\nu}{\tau_b^{9/2}}\left(1-\frac{3}{4a_s\tau_s}+\frac{6}{(4a_s\tau_s)^2}\right) \end{pmatrix} \tag{2.55}$$

and, by multiplying (2.50) and (2.55) :

$$K^{-1}(M^2) \ = \ \frac{2a_s\langle\tau_s\rangle W_0^2\nu}{3\langle\tau_b\rangle^{9/2}}\begin{pmatrix} -9(1-7\epsilon) & 6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2) \\ -\frac{6a_s\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(1-5\epsilon+4\epsilon^2) & \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(1-3\epsilon+6\epsilon^2) \end{pmatrix} \tag{2.56}$$

This matrix exhibit one large and one relatively small eigenvalue.
With an appropriate change of base (2.56) transforms into

$$K^{-1}(M^2) = \begin{pmatrix} m_\Phi^2 & 0 \\ 0 & m_\chi^2 \end{pmatrix}$$

The computation of m_Φ^2 and m_χ^2 is performed by taking advantage of trace and determinant invariance under similarity transformations.
Calculations lead to [15]

$$m_\Phi^2 \simeq Tr(K^{-1}(M^2)) \simeq \frac{8\nu W_0^2 a_s^2\langle\tau_s\rangle^{1/2}}{3\langle\tau_b\rangle^3} \ = \tag{2.57}$$

$$= \ 2m_{3/2}ln(M_P/m_{3/2})^2 \sim \left(\frac{ln\mathcal{V}}{\mathcal{V}}\right)^2$$

$$m_\chi^2 \simeq \frac{Det(K^{-1}M^2)}{Tr(K^{-1}M^2)} \simeq \frac{27W_0^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}} \sim \frac{1}{\mathcal{V}^3 ln\mathcal{V}} \tag{2.58}$$

where in the last steps we have made use of the relations (2.45).
The canonically normalised fields Φ and χ are the ones which give rise to real observable particles descending from T-moduli. These results show numerically the large mass hierarchy among these two kind of particles, with Φ heavier than gravitino mass and χ lighter by a factor of $\mathcal{V}^{1/2}$.

Once the eigenvectors of $(K^{-1}(M^2)$ have been found [15], we can write original fields in terms of Φ and χ. From (2.47):

$$\delta\tau_b = \left(\sqrt{6}\,\langle\tau_b\rangle^{1/4}\,\langle\tau_s\rangle^{3/4}\,(1-2\epsilon)\right)\frac{\Phi}{\sqrt{2}} + \left(\sqrt{\frac{4}{3}}\,\langle\tau_b\rangle\right)\frac{\chi}{\sqrt{2}} \tag{2.59}$$

$$\delta\tau_b \;\sim\; O(\mathcal{V}^{1/6})\Phi \;+\; O(\mathcal{V}^{2/3})\chi$$

$$\delta\tau_s = \left(\frac{2\sqrt{6}}{3}\,\langle\tau_b\rangle^{3/4}\,\langle\tau_s\rangle^{1/4}\right)\frac{\Phi}{\sqrt{2}} + \left(\frac{\sqrt{3}}{a_s}(1-2\epsilon)\right)\frac{\chi}{\sqrt{2}} \tag{2.60}$$

$$\delta\tau_s \;\sim\; O(\mathcal{V}^{1/2})\Phi \;+\; O(1)\chi$$

This result shows an important mixing in which, as one can expected, τ_b **is mostly** χ **and** τ_s **is mostly** Φ.
This fact, together with the mass hierarchy, allows us to roughly estimate moduli masses by using $m_b^2 \sim K_{bb}^{-1}\partial^2 V/\partial\tau_b^2$ and $m_s^2 \sim K_{ss}^{-1}\partial^2 V/\partial\tau_s^2$, which give

$$m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}}, \quad m_{\tau_s} \sim \frac{M_P ln(M_P/m_{3/2})}{\mathcal{V}} \tag{2.61}$$

in agreement with previous outcomes.

Another important fact that emerges from (2.59) and (2.60) is that, although τ_b exhibits no coupling to photons (see Lagrangian (2.46)), the light field χ, due to the small component in the τ_s direction, does have a measurable coupling to photons. The physical intuition, in fact, impose that both of the particles have to interact gravitationally with photons.
The couplings become manifest by writing down the full Lagrangian in terms of canonically normalised fields we find:

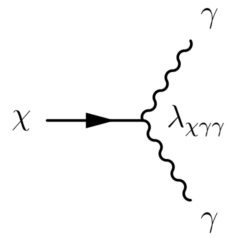
$$L = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V_0 - \frac{1}{2}m_\Phi^2\Phi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} -$$

$$- \frac{((v_\Phi)_s\Phi + (v_\chi)_s\chi)}{4\sqrt{2}\,\langle\tau_s\rangle\, M_P}F_{\mu\nu}F^{\mu\nu}$$

where Planck mass dependence is included.
We can easily read from the last terms the couplings $\chi\gamma\gamma$ and $\Phi\gamma\gamma$.

$$\lambda_{\Phi\gamma\gamma} = \frac{(v_\Phi)_s}{\sqrt{2}\,\langle\tau_s\rangle} \tag{2.62}$$

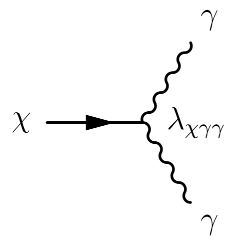
associated with the vertex:



while

$$\lambda_{\chi\gamma\gamma} = \frac{(v_\chi)_s}{\sqrt{2}\,\langle\tau_s\rangle} \tag{2.63}$$

which corresponds to



Making use of (2.59) and (2.60) we find [15]

$$\lambda_{\chi\gamma\gamma} = \frac{\sqrt{6}}{2M_P ln(M_P/m3/2)} \sim \frac{1}{M_P ln(\mathcal{V})} \tag{2.64}$$

and

$$\lambda_{\Phi\gamma\gamma} \sim \frac{2\,\langle\tau_b\rangle^{3/4}}{\sqrt{3}\,\langle\tau_s\rangle^{3/4} M_P} \sim \frac{\sqrt{\mathcal{V}}}{M_P} \sim 1/m_s \tag{2.65}$$

Notice that the coupling of χ to photons appear to be slightly weaker than standard moduli coupling to matter. In fact, it is not only suppressed by the Planck scale, as one naively might expected, but it also has a further suppression factor proportional to

$ln(\mathcal{V})$. On the other hand, expression (2.65) implies that interaction of Φ with photons are only suppressed by the string scale $m_s << M_P$ and therefore the decay rates of the heavy fields Φ (which is substantially τ_s) are much faster than is usually assumed for moduli fields.
From a physical point of view, this is in agreement with the fact that couplings between moduli and photons are obtained through the overlap integral of the particles wavefunctions. Indeed, while the wavefunction associated to τ_b spreads all over the bulk, the probability density for τ_s shares the maximum amplitude location with the ones associated to observable matter which lives on D7-branes wrapped on the τ_s-cycle.

As we will see later in Section 4, moduli masses and couplings are crucial features when studying the evolution of the early Universe.

2.5 String Loop Corrections

In the earlier section we have described a theoretically robust and very promising framework to make contact with experiments; now, the next step is to try to embed local brane constructions in this scenario.
As said earlier, the plan is to take one intersecting brane realisation of the MSSM and embed it in LARGE-volume model by wrapping these branes around some 4-cycles.
For example, we've seen that the cycle supporting the MSSM could be a small blow-up cycle, τ_s, so that the corresponding gauge coupling, $g_s^2 = 1/\tau_s$, would not be exponentially small.

Nevertheless it has been pointed out in recent researches that the original plan of stabilising the moduli without any concern about the local construction, and then embedding an intersecting brane realisation of the MSSM, is unfortunately too naive.
In fact, it has been discovered another source of problems which is the **tension between moduli stabilisation and chirality**.
More precisely, there is a problem with any stabilisation technique which relies on non-

perturbative effects to fix the 4-cycle supporting visible chiral matter.
The generic presence of chiral intersections between the instantons and observable sector divisors induces a prefactor for the non-perturbative superpotential which depends on chiral matter VEVs.

$$W \sim \prod_i \langle \phi_i \rangle \, e^{-T} \tag{2.66}$$

In order not to break any visible sector gauge symmetry, the VEVs of these fields have to vanish, killing the instanton contribution to the superpotential (see [16] for further details).

Fortunately a possible way-out exists and relies on loop corrections coming from strings. The solution consists in the concrete possibility to stabilize the modulus that controls the cycle on which we construct brane realisation of MSSM, through string loop corrections. Following this line of thought, a picture of Calabi-Yau manifold could be a sort of multiple hole swiss-cheese. One (or even an intersection of more) of the small blow-up cycles we consider, namely τ_{SM} supports MSSM and for the reason above has to be fixed using loop corrections to the Kahler potential; the others moduli fields instead can be stabilized as usual by non-perturbative corrections in order to keep untouched the structure of LARGE-volume scenario, whose results, as we will see in chapters 3 and 4, seem to be very promising from phenomenological point of view.

Let us have a brief look at the mathematical treatment for one simple model describing these ideas.

We consider one cycle, τ_s fixed by non-perturbative correction, and another, τ_{SM}, supporting visible chiral matter, which receives loop corrections.

We examine the case in which the volume is simply given by:

$$\mathcal{V} \simeq \tau_b^{3/2} - \tau_s^{3/2} - \tau_{SM}^{3/2} \tag{2.67}$$

together with Kahler potential and superpotential which take the following form

$$W = W_0 + A_s e^{-a_s T_s} \tag{2.68}$$
$$K = -\ 2\ ln(\mathcal{V} + \hat{\xi}) + K_{loop}$$

The scalar potential takes the same form as LARGE-volume scenario, except for a further term coming from the presence of K_{loop}:

$$V = V_{LVS}(\tau_b, \tau_s) + V_{loop}(\tau_b, \tau_{SM}) \tag{2.69}$$

From calculations, similar to the previous one small cycle case, we find ([17],[18])

$$V_{loop} = \left(\frac{\mu_1}{\tau_{SM}^{1/2}} - \frac{\mu_2}{\tau_{SM}^{1/2} - \mu_3} \right) \frac{W_0^2}{\mathcal{V}^3} \tag{2.70}$$

where μ_1 and μ_2 are positive constants dependent on the complex structure moduli, while μ_3 depends on the VEVs of τ_s.

The stabilisation of T_{SM} leads to a large volume vacuum state in which

$$\langle \tau_{SM} \rangle = \frac{\mu_1 \mu_3^2}{(\sqrt{\mu_1} + \sqrt{\mu_2})^2} \ \sim \ O(10) \tag{2.71}$$

T_{SM}, which cannot receive non-perturbative corrections (killed by zero VEVs of visible matter), is fixed by loop correction.

In next section we turn our attention to supersymmetry breaking aspects and phenomenological predictions of these scenarios.

Chapter 3

Soft Terms Computation

In this section we study supersymmetry breaking and its features in the context of the LARGE-volume scenario studied before.

Gravity-mediated supersymmetry breaking leads to non-zero soft-terms which arise dynamically from moduli stabilisation.
Starting from general aspects, we develop and determine here the explicit expressions for soft terms; first in the case of one small cycle supporting visible matter, and then, in the case of multiple-cycles with one of which being stabilised through loop corrections and wrapped by brane MSSM constructions as seen in section 2.5.

3.1 SUSY-Breaking from Moduli Dynamics

Our viewpoint, illustrated in the first chapter, is that first supersymmetry is spontaneously broken in the hidden sector of the MSSM, for example in a large-volume scenario, and then the observable sector feels the breaking indirectly due to the appearance of soft terms through gravitational interactions.

Let us see in details how to extract and compute soft terms.

The starting point is the N=1 4D Supergravity Lagrangian obtained from dimensional reduction as the low-energy limit of string compactifications.
Once we know the dynamics of the hidden sector, studied from the V_F moduli potential, by replacing moduli and auxiliary fields with their VeVs, we can obtain from the entire action of both observable and hidden fields the effective MSSM Lagrangian given by the sum

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{soft} \tag{3.1}$$

where $\mathcal{L}_{susy}$ is the usual supersymmetric extension of the SM Lagrangian, while $\mathcal{L}_{soft}$ is the soft supersymmetry-breaking Lagrangian which takes the form

$$\begin{aligned}\mathcal{L}_{soft} = \; & \frac{1}{2}\,(M_a\;\hat{\lambda}^a\hat{\lambda}^a + h.c.)\; - \; m_\alpha^2 C^\alpha \bar{C}^{\bar{\alpha}} \; - \\ & - \left(\frac{1}{6}\;A_{\alpha\beta\gamma}\hat{Y}_{\alpha\beta\gamma}C^\alpha C^\beta C^\gamma \; + \; B\hat{\mu}\hat{H}_1\hat{H}_2 \; + \; h.c.\right)\end{aligned} \tag{3.2}$$

where C^α denotes generic matter fields and for convenience, we have separated Higgs fields from the rest of the observable fields and specialised to the MSSM by assuming two Higgs doublets.

The soft terms are the gaugino masses M_a, the soft-scalar masses m_α^2, the trilinear terms $A_{\alpha\beta\gamma}$ which are multiplied by the Yukawa couplings $\hat{Y}_{\alpha\beta\gamma}$ and finally the B-term $B\mu$ related to the Higgs fields.
These parameters are computed starting from the Kahler potential K, the superpotential W and gauge kinetic function f_a, which all three together completely determine the theory.

Gaugino masses are produced by the gauge kinetic function through the term

$$\frac{1}{4}\int d^2\theta\; f(\Phi_i)Tr(W^\alpha W_\alpha)\; + \; h.c. \tag{3.3}$$

which appears into the vector-multiplet part of the supergravity Lagrangian.

After performing the canonical normalization of the gauginos

$$\hat{\lambda}^a = (Ref_a)^{1/2}\lambda^a \tag{3.4}$$

one can read from the explicit form of their mass term [9]:

$$M_a = \frac{1}{2}\frac{F^m \partial_m f_a}{Ref_a} \tag{3.5}$$

Soft scalar masses and trilinear terms, instead, can be derived directly from the structure of the scalar potential. In particular, in the expression (3.2) the last three terms can be viewed as an effective potential

$$V_{soft} \ = \ m_\alpha^2 C^\alpha \bar{C}^{\bar{\alpha}} \ + \ \frac{1}{6}\ A_{\alpha\beta\gamma}\hat{Y}_{\alpha\beta\gamma}C^\alpha C^\beta C^\gamma \ + \ B\hat{\mu}\hat{H}_1\hat{H}_2 \ + \ h.c. \tag{3.6}$$

which arises from

$$V = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \tag{3.7}$$

by making an expansion of K and W in powers of the matter fields.

Let us analyse in details.
The series expansions we perform, are

$$W = \hat{W}(\Phi_m) + \mu(\Phi_m)H_1H_2 + \frac{1}{6}\hat{Y}_{\alpha\beta\gamma}(\Phi_m)C^{\alpha_m}C^{\beta_m}C^{\gamma_m} + ... \tag{3.8}$$

$$K = \hat{K}(\Phi_m, \bar{\Phi_m}) + \tilde{K}_{\bar{\alpha}\beta}(\Phi_m, \bar{\Phi_m})C^{\bar{\alpha}}C^\beta + [Z(\Phi_m, \bar{\Phi_m})H_1H_2 + h.c.] + ... \tag{3.9}$$

where $\hat{Y}_{\alpha\beta\gamma}(\Phi_m)$ are the Yukawa couplings,$\tilde{K}_{\bar{\alpha}\beta}(\Phi_m, \bar{\Phi}_m)$ and $Z(\Phi_m, \bar{\Phi}_m)$ are the Kähler metrics with respect to matter fields and, again, $\{\Phi_m\}$ is a set of arbitrary moduli fields.

Note that in general the Kähler metric for the observable sector may be non-diagonal causing phenomenological problems with flavour changing neutral currents. Happily strings compactifications often lead to diagonal metrics [14], such that

$$\tilde{K}_{\bar{\alpha}\beta}(\Phi_m, \bar{\Phi}_m) \;=\; \delta_{\bar{\alpha}\beta}\tilde{K}_{\alpha}(\Phi_m, \bar{\Phi}_m) \tag{3.10}$$

Thanks to this simplification we are allowed to rewrite the power expansion of K as

$$K = \hat{K}(\Phi_m, \bar{\Phi}_m) + \tilde{K}_{\alpha}(\Phi_m, \bar{\Phi}_m)C^{\bar{\alpha}}C^{\alpha} + [Z(\Phi_m, \bar{\Phi}_m)H_1H_2 + h.c.] + ... \tag{3.11}$$

Substituting (3.8) and (3.11) into (3.7), and replacing dynamical moduli fields with their VeVs, we obtain the expression (3.6) for V_{soft} with [14],[19]

$$m_{\alpha}^2 = (m_{3/2}^2 + V_0) - F^{\bar{m}}F^n\partial_{\bar{m}}\partial_n log\tilde{K}_{\alpha} \tag{3.12}$$

$$A_{\alpha\beta\gamma} = F^m\left[\hat{K}_m + \partial_m logY_{\alpha\beta\gamma} - \partial_m log(\tilde{K}_{\alpha}\tilde{K}_{\beta}\tilde{K}_{\gamma})\right] \tag{3.13}$$

$$\begin{aligned} B\hat{\mu} = (\tilde{K}_{H_1}\tilde{K}_{H_2})^{-1/2} \;\; \{ \;\; & e^{\hat{K}/2}\mu(F^m[\hat{K}_m + \partial_m log\mu - \partial_m log(\tilde{K}_{H_1}\tilde{K}_{H_2})] - m_{3/2}) + \\ & + Z(2m_{3/2}^2 + V_0) \; - m_{3/2}\bar{F}^{\bar{m}}\partial_{\bar{m}}Z \; + \; m_{3/2}F^m[\partial_m Z - Z\partial_m log(\tilde{K}_{H_1}\tilde{K}_{H_2})] - \\ & - \bar{F}^{\bar{m}}F^n[\partial_m\partial_n Z - (\partial_m Z)\partial_n log(\tilde{K}_{H_1}\tilde{K}_{H_2})]] \;\; \} \end{aligned} \tag{3.14}$$

To perform soft-terms computation then we must know explicit form of F-terms, Yukawa couplings and Kahler metrics $\tilde{K}_{\alpha}$ for observable matter.
While the F-terms are perfectly known from the study of the ground state of the moduli potential, $\tilde{K}_{\alpha}(\Phi, \bar{\Phi})$ and $\hat{Y}_{\alpha\beta\gamma}(\Phi_m)$ are not; fortunately, thanks to locality and scaling arguments valid in the context of IIB compactifications, we are able to take control over their modular dependence.
As we will see in the proceeding, this will be enough for computations.

3.2 'Single-Hole' Model

From the study of the large-volume model we inherit the results described in section 2.3 for two Kähler moduli stabilised by a combination of α' corrections and non-perturbative superpotential. Their interaction, we have seen, produces one exponentially large cycle controlling the overall volume together with one small blow-up cycle which could support MSSM realisation.

Moduli F-terms

First of all, soft terms computation requires knowledge of the F-terms, Kahler metric and Yukawa couplings.

We start from the computation of the F-terms

$$F^m = e^{K/2} K^{m\bar{n}} \left(\partial_{\bar{n}} \bar{\hat{W}} + (\partial_{\bar{n}} \hat{K}) \bar{\hat{W}} \right) \tag{3.15}$$

They can be obtained directly from the results of large-volume scenario.
For the large modulus we have

$$F^b = e^{K/2} K^{b\bar{n}} \left(\partial_{\bar{n}} \bar{\hat{W}} + (\partial_{\bar{n}} \hat{K}) \bar{\hat{W}} \right) \tag{3.16}$$

As already seen, it is a property of the Kahler potential $K = -2ln\mathcal{V}$ that

$$K^{mn} \partial_n K = -2\tau_m \tag{3.17}$$

which then implies

$$F^b = e^{K/2} \left(K^{bb} \partial_b W + K^{bs} \partial_s W - 2\tau_b W \right) \tag{3.18}$$

Accounting of the results of moduli stabilisation in the large-volume scenario, we have

$$\partial_s W \sim e^{-T_s} \sim \mathcal{V}^{-1} \tag{3.19}$$

which together with $\tilde{K}^{bs} \sim \mathcal{V}^{2/3}$, gives

$$\tilde{K}^{bs} \partial_s W \sim \tilde{K}^{bs} e^{-T_s} \sim \mathcal{V}^{-1/3} \tag{3.20}$$

so that, neglecting terms proportional to the exponential e^{-T_b}, we find

$$F^b = -2\tau_b m_{3/2}(1 + O(\mathcal{V}^{-1})) \tag{3.21}$$

We read here that any soft breaking terms solely depending on F^b would be of the order of the gravitino mass $m_{3/2}$.

As regards the F-terms of the small Kahler moduli, instead, we are going to show they are hierarchically smaller.
From relation (3.15), in the simple case with solely one modulus associated with a small cycle, we are left with

$$F^s = e^{\tilde{K}/2}\left(\tilde{K}^{ss} a_s A_s e^{-a_s\tau_s} - 2\tau_s W\right) \tag{3.22}$$

Let us recall now formula (2.51) obtained in section 2.4:

$$e^{-a_s\tau_s} = \frac{\mu}{2\lambda}\frac{W_0}{\tau_b^{3/2}}\frac{\sqrt{\tau_s}}{a_s}\left(1 - \frac{3}{4a_s\tau_s} - \frac{3}{(4a_s\tau_s)^2}\right) \tag{3.23}$$

which is valid in the large volume limit and turn out to be very useful here.
It is easy to see that, by substituting it in (3.22) and accounting $\tilde{K}^{ss} \sim \sqrt{\tau_s}\mathcal{V}$, terms of (3.22) cancel with each other at leading order in the power of $\epsilon = \frac{1}{4a_s\tau_s}$ [20].
By using (2.51) we dropped subleading terms suppressed by

$$4a_s\tau_s \ \sim \ ln\mathcal{V} \ \sim \ ln\left(\frac{M_P}{m_{3/2}}\right) \tag{3.24}$$

being

$$m_{3/2} = e^{K/2}W \ \sim \ \frac{W_0}{\mathcal{V}} \ \sim \ \frac{M_P}{\mathcal{V}} \tag{3.25}$$

and so we expect the cancellation to fail at this order, giving

$$F^s \sim 2\tau_s \frac{m_{3/2}}{M_P \ log \ m_{3/2}} \tag{3.26}$$

This is an important result; in facts, as will occur, if the dependence of the soft terms on F^b is cancelled, the soft parameters will naturally be smaller than the gravitino mass by a factor of $log\left(\frac{M_P}{m_{3/2}}\right)$.
Moreover, it emerges here that supersymmetry is essentially broken by F^b-term. Hence, during the spontaneous SUSY-breaking mechanism, we understand that the role of the Goldstino, which is eaten by the gravitino, is played by the the τ_b superpartner, called τ_b modulino.

Matter Kähler metric and Yukawa couplings

Before going into details of soft parameters calculation, we are required to analyse the matter Kähler metric and the Yukawa coupling, which appear in the soft terms.
First of all we want to understand modular dependence of $\tilde{K}_\alpha$. Since in the large-volume compactification one of the Kahler moduli controlling the 'bulk' size is much larger than the other one, we can expand Kahler matter metrics in power series of $\tau_i = Re(T_i)$ and concentrate on the leading order of the inverse volume, obtaining

$$\tilde{K}_\alpha = \tau_b^{-p_\alpha} k(\phi, \tau_s) \tag{3.27}$$

where ϕ are the complex structure moduli and p_α is an integer we will determine later on through scaling argument and locality properties of the model.

As regards Yukawa couplings, we first define the canonically normalised matter fields as

$$\hat{C}^\alpha = \tilde{K}_\alpha^{1/2}(\Phi, \bar{\Phi})\varphi^\alpha \tag{3.28}$$

thanks to which we can simplify the second term in (83), being

$$\tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi}) C^\alpha C^{\bar{\beta}} = \varphi^\alpha \bar{\varphi}^{\bar{\beta}} \tag{3.29}$$

Therefore, normalising the superpotential $\hat{W} = e^{K/2}W$, we can rewrite the third term of

W expansion as

$$\hat{W} \supset \hat{Y}_{\alpha\beta\gamma}\varphi^\alpha\varphi^\beta\varphi^\gamma \tag{3.30}$$

in which Yukawa couplings take the form:

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2}\frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)^{1/2}}. \tag{3.31}$$

As the relation shows, the $\hat{Y}_{\alpha\beta\gamma}$ couplings are strictly related to $\tilde{K}_\alpha$. The latter plays a crucial role in the computation, because it determines both the normalisation of the matter fields and their mass basis.

Gaugino Masses

Let us approach soft terms computation and start from gaugino masses given by equation (3.5).
In IIB compactification, if T_i is the Kahler modulus corresponding to a particular 4-cycle, DBI action reduction for an unmagnetised brane wrapped on that cycle, gives [14]:

$$f_i = \frac{T_i}{2\pi} \tag{3.32}$$

Since we are interested in magnetised branes wrapped on 4-cycles, we should include a further term so that (3.33) becomes:

$$f_i = h_i(F)S + \frac{T_i}{2\pi} \tag{3.33}$$

where h_i depends on the fluxes F present on that stack.
In our model we have only one cycle with wrapping branes, paratrized by T_s. Thus we obtain

$$f_s = h_s(F)S + \frac{T_s}{2\pi} \tag{3.34}$$

Since the 'bulk modulus' does not appear in this expression, we solve (3.5) and find:

$$M_i = \frac{1}{2}\frac{F^s\partial_s f_s}{Re f_s}$$

which, together with (3.34), gives

$$M_i = \frac{F^s}{2} \frac{1}{Re(T_s + 2\pi h(F)S)} \tag{3.35}$$

In the limit of the large cycle volume the flux becomes diluted and can be neglected. This implies we get

$$M_i = \frac{1}{2} \frac{F^s}{\tau_s} \sim \frac{m_{3/2}}{M_P \; log \; m_{3/2}} \tag{3.36}$$

Note that if several small cycle are involved, the expression for gaugino masses may become more complicated.

Scalar masses and A-terms

Soft scalar masses and trilinear terms, instead, are determined from equations (3.12)-(3.14).
Now that we control the modular dependence of $\tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})$ through relation (3.27), we can develop explicit calculations for soft parameters.

Using (3.21) and (3.27), the expression (3.12) for scalar masses easily becomes

$$m_\alpha^2 = (1 - p_\alpha) m_{3/2} - \bar{F}^{\bar{s}} F^s \partial_{\bar{s}} \partial_s log \; k_\alpha(\tau_s, \phi) \tag{3.37}$$

Now consider the A-terms given by (3.13). It is well known that, at perturbative level, shift symmetry and holomorphy together entail that superpotential does not depend on the Kahler moduli T_i. We neglect any non-perturbative e^{-T} dependence, since we are in LVS and they are volume-suppressed.
Taking this into account it is possible to simplify the expression by evaluate

$$F^m \partial_m log Y_{\alpha\beta\gamma} = 0 \tag{3.38}$$

due to the fact that only Kahler moduli have a non vanishing F-term.
Therefore we are left with

$$A_{\alpha\beta\gamma} = F^m \left[\hat{K}_m - \partial_m log(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma) \right] \tag{3.39}$$

in which we calculate

$$F^m K_m = e^{\hat{K}/2}\hat{K}^{m\bar{n}}D_{\bar{n}}\bar{\hat{W}}\hat{K}_m \tag{3.40}$$

$$= e^{\hat{K}/2}\left[\sum_n -2\tau_n(\partial_{\bar{n}}\bar{\hat{W}} + (\partial_{\bar{n}}\hat{K})\bar{\hat{W}})\right]$$

where the index 'n' runs over b and s moduli. Since $K = -2log\ \mathcal{V}$ we have

$$\partial_{\bar{n}}\hat{K} = -2\frac{\partial_{\bar{n}}\mathcal{V}}{\mathcal{V}} \tag{3.41}$$

Then, being $\mathcal{V}$ homogeneus of degree 3/2 in τ_n, implies

$$\sum_n \tau_n\partial_{\bar{n}}\mathcal{V} = \frac{3\mathcal{V}}{4} \tag{3.42}$$

and substituting in (3.39) gives

$$F^m\tilde{K}_m = e^{\hat{K}/2}\left[-2\tau_s\partial_{\bar{s}}\bar{W} - 2\tau_b\partial_{\bar{b}}\bar{W} + 3\right] \tag{3.43}$$

Neglecting, as usual, the second term and recalling that $\partial_{\bar{s}}\bar{W} \sim \mathcal{V}^{-1}$ in the large volume limit, we get

$$F^m\hat{K}_m = 3m_{3/2}(1 + O(\mathcal{V}^{-1})) \tag{3.44}$$

We evaluate then expression (3.38), which gives

$$A_{\alpha\beta\gamma} = 3m_{3/2} - F^m\partial_m log(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma) \tag{3.45}$$

$$= 3m_{3/2} - F^b\partial_b(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma) - F^s\partial_s\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)$$

Now,

$$log(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma) = -(p_\alpha + p_\beta + p_\gamma)log\tau_b \ + \ log(k_\alpha k_\beta k_\gamma(\tau_s, \phi)) \tag{3.46}$$

which becomes

$$\partial_b log(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma) = -\frac{p_\alpha + p_\beta + p_\gamma}{2\tau_b} \tag{3.47}$$

and finally gives

$$A_{\alpha\beta\gamma} = (3 - (p_\alpha + p_\beta + p_\gamma))\, m_{3/2} - F^s \partial_s log(k_\alpha k_\beta k_\gamma(\tau_s, \phi)) \tag{3.48}$$

B-term

Let us turn our attention on B-term connected with Higgs doublet and described by equation (3.14).
First, we set $\mu = 0$. This hail from Giudice-Masiero mechanism, which is proposed as a solution to the μ-problem and is based on the hypothesis that μ could arise dynamically through spontaneous susy-breaking without appearing explicitly in the Lagrangian .
Following this proposal, we have [21]

$$\mu = 0 \quad ; \quad \hat{\mu} \;=\; m_{3/2} Z - F^j \partial_j Z \tag{3.49}$$

This considerably simplifies the expression (3.14) since many terms vanish.
We get

$$\begin{gathered} B\hat{\mu} = (\tilde{K}_{H_1}\tilde{K}_{H_2})^{-1/2}[(2m_{3/2}^2 + V_0)Z \\ -m_{3/2} Z F^m \partial_m log(\tilde{K}_{H_1}\tilde{K}_{H_2}) + m_{3/2}(F^m \partial_m Z - F^{\bar{n}} \partial_{\bar{n}} Z) \\ -\bar{F}^{\bar{m}} F^n (\partial_{\bar{m}} \partial_n Z - \partial_{\bar{m}} Z \partial_n log(\tilde{K}_{H_1}\tilde{K}_{H_2})] \end{gathered} \tag{3.50}$$

which becomes, in our 'single-hole' LVS model,

$$\begin{gathered} B\hat{\mu} = (\tilde{K}_{H_1}\tilde{K}_{H_2})^{-1/2}[(2m_{3/2}^2 + V_0)Z \\ -m_{3/2} Z (F^b \partial_b log(\tilde{K}_{H_1}\tilde{K}_{H_2}) + F^s \partial_s log(\tilde{K}_{H_1}\tilde{K}_{H_2})) \\ -\bar{F}^{\bar{b}} F^b (\partial_{\bar{b}} \partial_b Z - \partial_{\bar{b}} Z \partial_b log(\tilde{K}_{H_1}\tilde{K}_{H_2})) \\ -\bar{F}^{\bar{b}} F^s (\partial_{\bar{b}} \partial_s Z - \partial_{\bar{b}} Z \partial_s log(\tilde{K}_{H_1}\tilde{K}_{H_2})) \end{gathered}$$

$$-\bar{F}^{\bar{s}}F^{b}(\partial_{\bar{s}}\partial_{b}Z - \partial_{\bar{s}}Z\partial_{b}log(\tilde{K}_{H_1}\tilde{K}_{H_2}))$$
$$-\bar{F}^{\bar{s}}F^{s}(\partial_{\bar{s}}\partial_{s}Z - \partial_{\bar{s}}Z\partial_{s}log(\tilde{K}_{H_1}\tilde{K}_{H_2}))\ ] \qquad (3.51)$$

and again, assume a vanishing cosmological constant and using (3.27) which is also valid for Z, we get

$$B\hat{\mu} = \frac{\tau_b^{\frac{p_1+p_2}{2}}}{(k_{H_1}k_{H_2})^{1/2}}\ [\ m_{3/2}^2 z\tau_b^{-p_z}\,[2-2(p_1+p_2)-pz(p_z+1)+2p_z(p_1+p_2)]$$
$$+2(1-p_z)\tau_b^{-p_z}m_{3/2}F^s\partial_s z + (p_z-1)\tau_b^{-p_z}m_{3/2}zF^s\partial_s log(\tilde{k}_1\tilde{k}_2)]$$
$$-\frac{\tau_b^{\frac{p_1+p_2}{2}-pz}}{(k_{H_1}k_{H_2})^{1/2}}\left[F^sF^s(\partial_s\partial_s z - \partial_s Z\partial_s log(\tilde{k}_{H_1}\tilde{k}_{H_2})\right] \qquad (3.52)$$

We notice that in the particular case in which $p_\alpha = 1$, in all the expression (3.36), (3.47), (3.51) of the soft parameters would arise very interesting cancellations which would considerably simplify the results.
Let us show that $p_\alpha = 1$ is truly the right case.

τ_b-dependence of matter Kähler metric

From relation (3.31) we see that informations about $\tilde{K}_\alpha$ are encoded in the modular dependence of Yukawa terms.

The physical origin of Yukawa couplings is through the interaction and overlap of the quantum wavefunctions associated with the different matter fields.
Since matter fields are localised on the branes, thus the wavefunctions for Standard Model matter all have support in the local geometry on the small 4-cycle. As the interactions are determined only locally, in the large-volume limit the physical Yukawa couplings should be independent of the overall volume, that is it should be invariant under rescalings $\tau_b \to \lambda\tau_b$.
Now, since Yukawa couplings are given by

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2}\frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)^{1/2}}$$

we obtain, being $\tilde{K}_\alpha = \tau_b^{-p_\alpha} k_\alpha(\tau_s, \phi)$,

$$\hat{Y}_{\alpha\beta\gamma} = \tau_b^{-3/2} \frac{Y_{\alpha\beta\gamma} \quad \tau_b^{\frac{p_\alpha+p_\beta+p_\gamma}{2}}}{\left(\tilde{k}_\alpha \tilde{k}_\beta \tilde{k}_\gamma\right)^{1/2}} \tag{3.53}$$

Invariance under τ_b rescaling then implies:

$$p_\alpha + p_\beta + p_\gamma \;=\; 3 \tag{3.54}$$

for all matter fields present in the Yukawa couplings.
We therefore expect p_α to be universal; giving [12]

$$p_\alpha \;=\; 1 \quad \forall\alpha \tag{3.55}$$

Simplified Expressions for the Soft-Terms

Turning back to the expressions of soft terms, we take advantage of the cancellations and we get:

$$m_\alpha^2 = -F^s F^s \partial_s^2 log\ \tilde{k}_\alpha \tag{3.56}$$

$$A_{\alpha\beta\gamma} = -F^s \partial_s log(k_\alpha k_\beta k_\gamma) \tag{3.57}$$

$$B\mu \;=\; -\frac{1}{(k_{H_1} k_{H_2})^{1/2}} \left[F^s F^s [\partial_s^2 z - \partial_s z \partial_s log(\tilde{k}_{H_1} \tilde{k}_{H_2})\right] \tag{3.58}$$

Therefore, thanks to the Giudice-Masiero mechanism and formula (3.48), we write also

$$\hat{\mu} = -\frac{F^s \partial_s z}{(k_{H_1} k_{H_2})^{1/2}} \tag{3.59}$$

where the contribution of F^b disappears due to

$$F^m \partial_m Z = F^b \partial_b (z \tau_b^{-1}) + F^s \partial_s z$$

$$= 2\tau_b^{-1} m_{3/2} z + F^s \partial_s z$$

Notice the important cancellations of F^b terms in all the calculations we made. They do not contribute to soft parameters value.

Computations are not concluded yet.
We further simplify these expressions by expanding $k_\alpha(\tau_\alpha, \phi)$ and $z(\tau_\alpha, \phi)$ as a power series in τ_s

$$k_\alpha(\tau_\alpha, \phi) = \tau_s^\lambda \; k_\alpha^0(\phi) + O(\tau_s^{\lambda-1}) k_\alpha^1(\phi) + ... \tag{3.60}$$

$$z_\alpha(\tau_\alpha, \phi) = \tau_s^\lambda \; z_\alpha^0(\phi) + O(\tau_s^{\lambda-1}) z_\alpha^1(\phi) + ... \tag{3.61}$$

The assumption here is that both k_α and z_α scale with same power λ.

Scalar masses, trilinears and B-term then become:

$$m_\alpha^2 = \lambda \left(\frac{F^s}{2\tau_s}\right)\left(\frac{F^s}{2\tau_s}\right) \tag{3.62}$$

$$A_{\alpha\beta\gamma} = -3\lambda \left(\frac{F^s}{2\tau_s}\right) \tag{3.63}$$

$$B\mu = \left(\frac{F^s}{2\tau_s}\right)\left(\frac{F^s}{2\tau_s}\right) \lambda(\lambda+1) \frac{z^0(\phi)}{(k_{H_1}^0 k_{H_2}^0)^{1/2}} \tag{3.64}$$

together with the expression of $\hat{\mu}$ and Yukawa couplings that we rewrite for convenience:

$$\hat{\mu} = -\frac{F^s}{2\tau_s} \frac{z^0(\phi)}{(k_{H_1}^0 k_{H_2}^0)^{1/2}} \tag{3.65}$$

$$\hat{Y} = \frac{Y_{\alpha\beta\gamma}}{\tau_s^{\frac{3\lambda}{2}} (k_\alpha^0 k_\beta^0 k_\gamma^0)^{\frac{1}{2}}} \tag{3.66}$$

Computations now depends entirely on F^s and λ.
We focus now on the latter.

τ_s-dependence of Yukawa couplings

Our key tool here is the viewpoint that physical Yukawa couplings arise from the triple overlap of normalised wavefunctions. Because of the constraints of shift symmetry and holomorphy, the Kahler moduli can only affect the physical Yukawa couplings through the power λ, which corresponds purely to an overall scaling of these matter field wavefunctions and not to a change in the shape.
If we knew scaling properties of the wavefunction, we could find the modular dependence of $\hat{Y}$ and in particular the value of λ from equation (3.31).

The idea is that the overlap integral has a simple dependence on the Kahler moduli and its scaling can be computed without concerns about the explicit functional form of the wavefunctions.

Let us briefly describe how calculations arise.
We start directly from the dimensional reduction of low-energy limits of magnetised brane constructions.
Accounting of DBI action reduction, we are left with super Yang-Mills Lagrangian from which we extract the fermionic term:

$$\int_{M_4 \times \Sigma} \bar{\lambda}\ \Gamma^i\ (\partial_i + A_i)\ \lambda \tag{3.67}$$

with M_4 representing the observable 4-dimensional space, while Σ is the 4-cycle wrapped by the stack of D7-branes.
The higher dimensional gauge fields (A_i) and gaugino (λ_i) can be decomposed a là Kaluza-Klein in:

$$A \ = \ \sum_i \phi_{4,i} \otimes \phi_{6,i} \qquad \lambda = \sum_i \psi_{4,i} \otimes \psi_{6,i} \tag{3.68}$$

We are interested here in the spectrum of massless chiral fermions living in four dimensions. A full calculation of the magnitude of Yukawa couplings requires the explicit scalar and fermion wavefunctions. So, for completeness, suppose to solve the Dirac and Laplace equation:

$$\Gamma^i D_i \psi = 0 \qquad \nabla^2 \phi = 0 \tag{3.69}$$

where D_i and ∇^2 are the appropriate differential operators on the fluxed 4-cycle. Decomposition of (3.66) gives the kinetic term

$$\left(\int_\Sigma \psi^\dagger_{6,\alpha} \psi_{6,\alpha} \right) \int_{M_4} \bar{\psi}_{4,\alpha} \Gamma^\mu \partial_\mu \psi_{4,\alpha} \tag{3.70}$$

whose normalisation requires

$$\int_\Sigma \psi^\dagger_6 \psi_6 \ = \ \int_\Sigma \phi^\star_6 \phi_6 \ = \ 1 \tag{3.71}$$

and the Yukawa couplings

$$\left(\int_\Sigma \bar{\psi}_\alpha \Gamma^i \phi_{i,\gamma} \psi_\beta \right) \int_{M_4} \phi_\gamma \bar{\psi}_\alpha \psi_\beta \tag{3.72}$$

from which it is clear that, defining

$$\hat{Y}_{\alpha\beta\gamma} = \int_\Sigma \bar{\psi}_\alpha \Gamma^i \phi_{i,\gamma} \psi_\beta \tag{3.73}$$

the magnitude is given by the triple overlap integral of normalised wavefunctions on Σ. Our interest is on the scaling of (3.72) with the τ_s cycle volume.
Under rescaling $\tau_s \to \beta \tau_s$, from normalization condition (3.70), we have:

$$\psi \to \frac{\psi}{\sqrt{\beta}} \tag{3.74}$$

Then we deduce that the physical Yukawas scale as

$$\hat{Y}_{\alpha\beta\gamma} \ \to \ \int_\Sigma \beta \frac{\psi_\alpha}{\sqrt{\beta}} \ \frac{\phi_\gamma}{\sqrt{\beta}} \ \frac{\psi_\beta}{\sqrt{\beta}} \ = \ \frac{\hat{Y}_{\alpha\beta\gamma}}{\sqrt{\beta}} \tag{3.75}$$

which means [12]:

$$\hat{Y}_{\alpha\beta\gamma} \sim \frac{\hat{Y}_{\alpha\beta\gamma}}{\sqrt{\tau_s}} \tag{3.76}$$

Finally, comparing to the form

$$\hat{Y}_{\alpha\beta\gamma} = \frac{Y_{\alpha\beta\gamma}}{\tau_s^{\frac{3\lambda}{2}}(k^0_\alpha k^0_\beta k^0_\gamma)^{\frac{1}{2}}}$$

we conclude that

$$\lambda = 1/3 \tag{3.77}$$

Final Expression for the Soft-Terms

Now, once we finally substitute $\lambda = \frac{1}{3}$ in the expressions of the soft terms we derive [14]

$$m_\alpha = \frac{M_s}{\sqrt{3}} \tag{3.78}$$

$$A_{\alpha\beta\gamma} = -M_s \tag{3.79}$$

$$B = -\frac{4}{3}M_s \tag{3.80}$$

where

$$M_s = \frac{F^s}{2\tau_s} \simeq \frac{m_{3/2}}{log\ (M_P/m_{3/2})}$$

All soft terms are proportional to $\frac{m_{3/2}}{log\ (M_P/m_{3/2})}$ and thus reduced with respect to the gravitino mass.

Phenomenological implication and discussions are postponed to chapter 4.

3.3 'Multiple-Hole' Model

Due to arguments seen in the previous chapter, we know that we cannot stabilise T_s by means of non-perturbative corrections if this one is the modulus which supports the MSSM realisation.
Hence, the purpose of this section is to perform soft terms computation in the geometrical picture described in section 2.5, where we consider two different small cycles stabilised through different type of corrections.
The first, namely T_s, receive non-perturbative corrections to the superpotential

$$W = W_0 + A_s e^{-a_s T_s} \tag{3.81}$$

while the second, which we call T_{SM}, is stabilised through strings loop corrections in order to admit wrapped branes intersection and gives rise to observable matter fields.

Let us resume the information we need from section 2.5.
First, we look at the volume which is given by

$$\mathcal{V} \simeq \tau_b^{3/2} - \tau_s^{3/2} - \tau_{SM}^{3/2} \tag{3.82}$$

It enters directly in the computations through the form of Kahler potential:

$$K = -\ 2\ ln(\mathcal{V} + \hat{\xi}) + K_{loop} \tag{3.83}$$

where K_{loop} is responsible of T_{SM} stabilisation.

All the ingredients we require for soft parameters calculation are the F-terms and the modular dependence of $\tilde{K}_\alpha$ and $\hat{Y}_{\alpha\beta\gamma}$.

Moduli F-terms

Let us start from analysing the F-term expression.
First of all, we note that F-terms for τ_b and τ_s remain the same we calculated for the

'single-hole' picture.
We study the expression for F^{SM}.

$$F^{SM} = e^{K/2} K^{SM\ j} D_j W \tag{3.84}$$

where we recall $D_j W = (\partial_j W + W \partial_j)$ and $K^{SM\ j}\partial_j K = -2\tau_{SM}$ to obtain:

$$\begin{aligned} F^{SM} &= e^{K/2}\left(K^{SM\ j}\partial_j W - 2\tau_{SM} W\right) \\ &= e^{K/2}\left(-K^{SM\ s} a_s A_s e^{-a_s T_s} - K^{SM\ b} a_b A_b e^{-a_b T_b} - 2\tau_{SM} W\right) \end{aligned} \tag{3.85}$$

thanks to relation (3.80).
Neglecting the second term which is subleading in the large volume limit, we are left with

$$F^{SM} = e^{K/2}\left(-K^{SM\ s} a_s A_s e^{-a_s T_s} - 2\tau_{SM} W\right) \tag{3.86}$$

All we need now is the explicit form of the 3x3 matrix K^{ij}, which is the inverse of the Kahler moduli metric K_{ij} where index i and j can take the values b , s and SM, which stands respectively for T_b , T_s and T_{SM}.

Kahler metric is given by

$$K_{ij} = \begin{pmatrix} \partial_b^2 K & \partial_b \partial_s K & \partial_b \partial_{SM} K \\ \partial_s \partial_b K & \partial_s^2 K & \partial_s \partial_{SM} K \\ \partial_{SM} \partial_b K & \partial_{SM} \partial_s K & \partial_{SM}^2 K \end{pmatrix} \tag{3.87}$$

that is, evaluating derivatives:

$$K_{ij} = \begin{pmatrix} \frac{3}{4\tau_b^2} & -\frac{9\tau_s^{\frac{1}{2}}}{8\tau_b^{\frac{5}{2}}} & -\frac{9\tau_{SM}^{\frac{1}{2}}}{8\tau_b^{\frac{5}{2}}} \\ -\frac{9\tau_s^{\frac{1}{2}}}{8\tau_b^{\frac{5}{2}}} & \frac{3}{8\tau_s^{\frac{1}{2}}\tau_b^{\frac{3}{2}}} & \frac{9\tau_s^{\frac{1}{2}}\tau_{SM}^{\frac{1}{2}}}{8\tau_b^3} \\ -\frac{9\tau_{SM}^{\frac{1}{2}}}{8\tau_b^{\frac{5}{2}}} & \frac{9\tau_s^{\frac{1}{2}}\tau_{SM}^{\frac{1}{2}}}{8\tau_b^3} & \frac{9\tau_{SM}}{8\tau_b^3} + \frac{3\tau_{SM}^{-\frac{1}{2}}}{8\tau_b^{\frac{3}{2}}} \end{pmatrix} \tag{3.88}$$

The inverse can be obtained without much difficulty, however in our circumstance we notice we are interested only in one term appearing in equation (3.85), that is F^{SMs}. Recalling a bit of matrix algebra, we find:

$$K^{SM\ s}\ \ (Det\ K_{ij})\ = \begin{vmatrix} \frac{3}{4\tau_b^2} & -\frac{9\tau_{SM}^{\frac{1}{2}}}{8\tau_b^{\frac{5}{2}}} \\ -\frac{9\tau_s^{\frac{1}{2}}}{8\tau_b^{\frac{5}{2}}} & \frac{9\tau_s^{\frac{1}{2}}\tau_{SM}^{\frac{1}{2}}}{8\tau_b^3} \end{vmatrix} = -\frac{27}{64}\frac{\tau_{SM}^{1/2}\tau_s^{1/2}}{\tau_b^5} \tag{3.89}$$

where the determinant of K_{ij} is given by the usual formula for 3x3 matrices that we recall in an obvious notation:

$$\begin{aligned} DetA_{ij} = &(a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) \\ &-(a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{21}a_{12}a_{33}) \end{aligned} \tag{3.90}$$

At leading order in the inverse power of the volume $\mathcal{V} \sim \tau_b^{3/2}$, we get

$$Det\ K_{ij}\ \simeq\ \frac{27}{64\cdot 4}\,\frac{1}{\tau_s^{1/2}\tau_{SM}^{1/2}\tau_b^5} \tag{3.91}$$

Substituting (3.90) in (3.88), we immediately obtain:

$$K^{SM\ s}\ \simeq\ -4\tau_s\tau_{SM} \tag{3.92}$$

Now we can finally turn on equation (3.85) and calculate the F-term for T_{SM}. Since $a_sA_se^{-a_s\tau_s} \sim \mathcal{V}^{-1}$, from τ_s stabilisation, we obtain:

$$F^{SM}\ =\ e^{K/2}\left(\frac{4\tau_s\tau_{SM}}{\mathcal{V}} - 2\tau_{SM}W\right)\ =\ 2\tau_{SM}m_{3/2}\ \left(1 + O(\mathcal{V}^{-1})\right) \tag{3.93}$$

The minus sign disappears due to the relation $m_{3/2} = e^{K/2}|W|$, which involves the modulus of W. The choice of positive sign is performed in order not to have negative soft-masses.

Soft-Terms

Let us proceed to soft parameters computation.

In doing so, we underline that considerations about the form of kinetic gauge function $f_a(\phi)$ or the modular dependence of $\tilde{K_\alpha}$ and $\hat{Y}_{\alpha\beta\gamma}$ we made in section 3.2, are still valid in this context with the only change of T_s with T_{SM}. This is because T_{SM} has taken here the role which was of T_s in the other model, of supporting MSSM observable fields. For this reason we assume:

$$f_i = \frac{T_{SM}}{2\pi} \tag{3.94}$$

$$\tilde{K} = \frac{\tau_{SM}^{\lambda}}{\tau_b^p} \tag{3.95}$$

with $p = 1$ and $\lambda = 1/3$.
From this point, the development of soft term expressions follows exactly the same path of section 3.2. The calculations lead to the same results because T_s has been substituted with T_{SM} and then both Kahler matter metrics and Yukawa couplings do not depend on τ_s due to locality of branes intersection.
Therefore, the computation steps are the same and in particular the F-term of T_b lead to the same previously encountered cancellations.

The only (very important) difference here resides in the fact that $F^s \neq F^{SM}$.
F^s is reduced by a factor of $log\ m_{3/2}$, while F^{SM} is not.

The consequences of this are that in the current scenario soft terms are directly proportional to the gravitino mass.
In fact, after computation, we have:

$$m_\alpha = \frac{M_{SM}}{\sqrt{3}} = \frac{m_{3/2}}{\sqrt{3}} \tag{3.96}$$

$$A_{\alpha\beta\gamma} = -M_{SM} = -m_{3/2} \tag{3.97}$$

$$B = \frac{4}{3}M_{SM} = \frac{4}{3}m_{3/2} \tag{3.98}$$

thanks to

$$M_{SM} \; = \; \frac{F^{SM}}{2\tau_{SM}} \; \simeq \; m_{3/2}$$

From a phenomenological point of view this means that SUSY-breaking scale would be about gravitino mass.

If we recall that gravitino mass is equal to

$$m_{3/2} \; \sim \; M_P/\mathcal{V} \tag{3.99}$$

we conclude that the only knowledge of the overall volume sets all the parameters of the theory and quantifies supersymmetry-breaking with important implications and predictions about low-energy physics.

Chapter 4

Discussion

In this section we discuss the results obtained as well as their phenomenological implications. As we have already pointed out, the theoretical scenarios we have studied, lead to viable constructions of dynamical susy-breaking through gravity-mediation.

The soft parameters we have computed fix the energy scale of supersymmetry breaking and also determine the masses of squarks and sleptons. If these predictions were confirmed by the experiments at the LHC, it would be a huge success towards a deeper knowledge about Nature. Moreover it would represent a great indication and support to String Theory.

In the first part of this section, making use of previous calculations, we build and analyse the low-energy mass spectrum that emerges from the two examined models. Then, in the last part, we deal with the cosmological moduli problem focusing our attention on its constraints for moduli masses.

4.1 Mass Hierarchies

From the results of section 3, we learn that the soft-terms are entirely determined by the gravitino mass.

Hence, being

$$m_{3/2} \sim \frac{M_P}{\mathcal{V}}\ , \quad m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}} \tag{4.1}$$

respectively gravitino and string scales, the exponentially large volume obtained through moduli stabilisation allows both TeV-scale soft masses, with consequent generation of hierarchies, and accessible string scale.
In fact, we can take $\mathcal{V} \sim 10^{14}$ (corresponding to a dimensionful volume of order $Vol(CY) = \mathcal{V}\ l_s^6$) and get TeV-scale soft-terms as it is required from hierarchy problem. Let us summarise our resulting mass spectrum with a table.

For the 'single-hole' model, in which we have $M_{soft} = \frac{m_{3/2}}{log\ m_{3/2}}$, by recalling also the calculations made in section 2.4 about moduli masses, we get the following hiererchies:

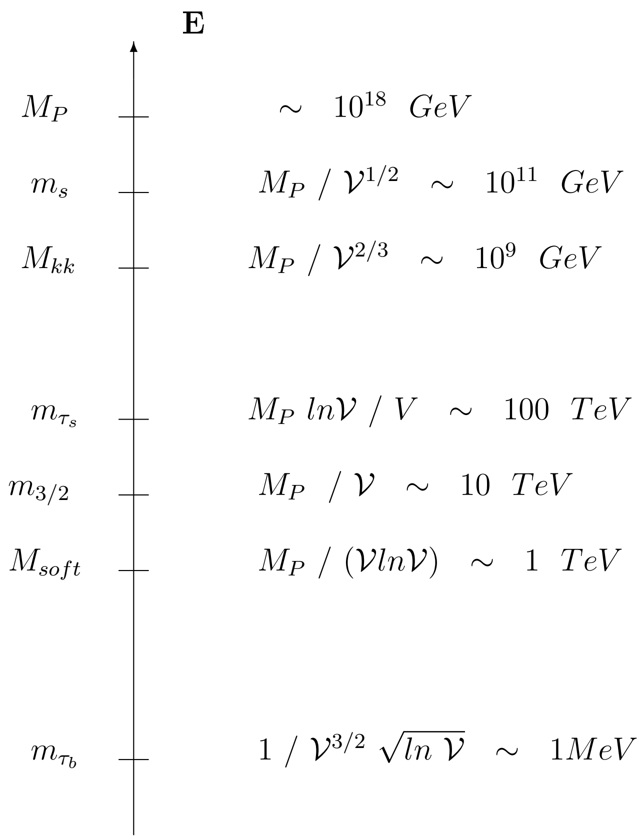


Fig.8 Mass Hierarchy for 'single-hole' model.

where, from string compactification, we have inherited the appropriate Kaluza-Klein energy scale

$$M_{kk} \;=\; \frac{m_s}{\mathcal{V}^{1/6}} \;\sim\; \frac{M_P}{\mathcal{V}^{2/3}} \tag{4.2}$$

As regards the 'multiple-hole' scenario, since $M_{soft} = m_{3/2}$ we take:

$$\mathcal{V} \sim 10^{15} \tag{4.3}$$

in order to obtain low-energy supersymmetry.

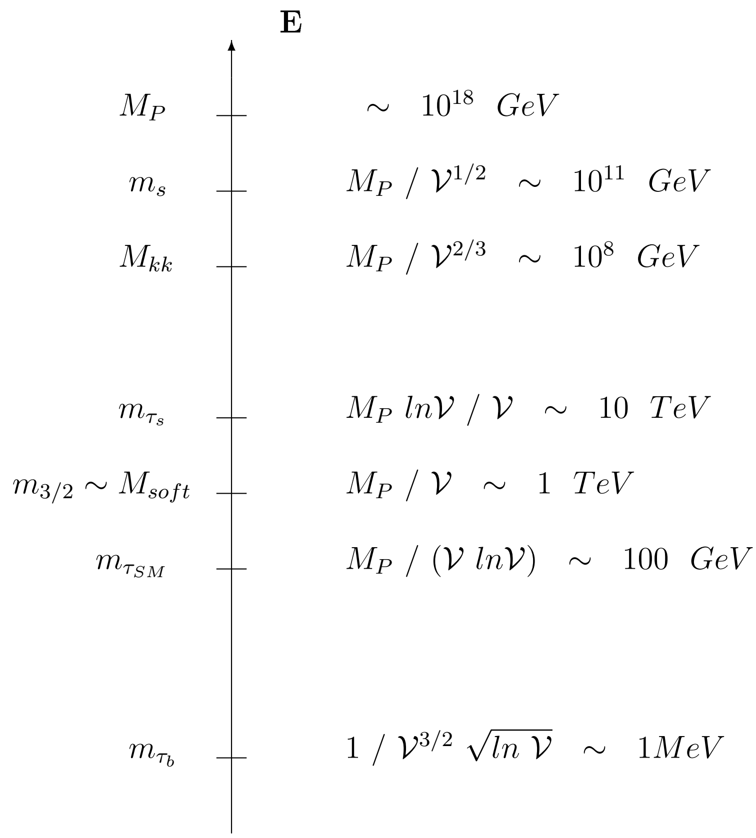


Fig.9 Mass Hierarchy for 'multiple-hole' model.

Moduli masses are computed in the approximation

$$m_\tau^2 \;\simeq\; K^{-1}_{\tau\tau}\frac{\partial^2 V}{\partial \tau^2} \tag{4.4}$$

which follows from canonical normalization.

Taking into account that

$$V_{loop} = \left(\frac{\mu_1}{\tau_{SM}^{1/2}} - \frac{\mu_2}{\tau_{SM}^{1/2} - \mu_3} \right) \frac{W_0^2}{\mathcal{V}^3}$$

for τ_{SM} we obtain:

$$m_{\tau_{SM}}^2 \;\simeq\; K_{SMSM}^{-1} \frac{\partial^2 V}{\partial \tau_{SM}^2} \;\simeq\; \tau_{SM}^{1/2} \tau_b^{3/2} \cdot \frac{W_0^2}{\tau_b^{9/2} \tau_{SM}^{5/2}} \;\simeq\; \frac{W_0^2}{\mathcal{V}^2 \tau_{SM}^2} \tag{4.5}$$

which, using (2.71)

$$\langle \tau_{SM} \rangle \;\sim\; O(10) \tag{4.6}$$

gives

$$m_{\tau_{SM}} \;\simeq\; \frac{W_0}{\mathcal{V} \tau_{SM}} \;\simeq\; \frac{1}{\mathcal{V}\; ln \mathcal{V}} \tag{4.7}$$

It is remarkable that the only parameter that has to be placed by hand is the string coupling g_s (or equivalently the VeV of the dilaton $\langle Re(S) \rangle = 1/g_s$).
This can be viewed from moduli stabilisation and in particular from section 3.3, where equations

$$\langle \tau_s \rangle \;\simeq\; \hat{\xi}^{2/3} \;\simeq\; \frac{\xi^{2/3}}{g_s} \;\simeq\; O(1/g_s) \;\simeq\; 10$$

in which string coupling determines the VeV of τ_s and hence the overall volume size through

$$\langle \mathcal{V} \rangle \;\simeq\; W_0 e^{a_s \tau_s} \;\simeq\; e^{1/g_s} >> 1$$

Notice that we must have $g_s < 1$ in order to rely on string perturbation theory and in particular on the corrections to the Kähler potential.
In light of this, if supersymmetry were found at LHC, experimental data about soft-masses combined with the theoretical construction we have developed here, could be used to set the only parameter that is required in string theory to represent our real world.

4.2 Cosmological Moduli Problem

It is well-known that generic moduli with mass $m \sim O(1)\ TeV$ cause problems for early-universe cosmology. Such moduli masses are unavoidable in the picture of gravity-mediated supersymmetry breaking, where, as seen, moduli get masses of the order of $m \sim M_{soft} \sim m_{3/2}$.
This is among the most pressing problems facing low-scale supersymmetry.

Let us briefly analyse the origin of the problem.

The hot big bang model together with the inflationary paradigm provides a highly attractive framework for cosmology. Typically, it is assumed that after inflation the visible sector degrees of freedom reheat and evolve adiabatically, following the well known relations from FRW model:

$$\rho_{rad} \sim T^4 , \quad \rho_m \sim T^3 \tag{4.8}$$

where ρ_{rad} and ρ_m represent respectively the energy density of radiation and matter.

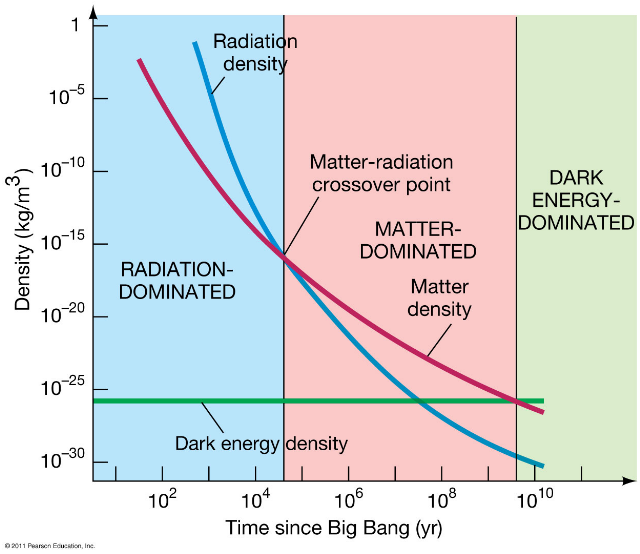


Fig.10 Energy density evolution for matter and radiation in standard cosmology. [22]

But what happens to the hidden sector fields?.

After inflation, moduli are expected to be displaced far from the low energy minimum; in particular we are left with $\phi_{in} \sim M_P$ which implies $\rho_{rad} \sim \rho_\phi$. When the Hubble constant reaches values equal to the moduli masses, they begin to behave as harmonic oscillators [23],[24],[25]. Energy stored in moduli oscillations then begins to redshift like non-relativistic matter, hence at a rate significantly slower than radiation.
This implies that moduli quickly come to dominate the energy density of the universe.
If moduli were completely stable, the associated density ρ_ϕ would give a huge contribution to the today's energy density and would overclose the universe. However, although they are suppressed by a factor of M_P, the gravitational couplings studied in section 2.4 can provide a moduli decay into photons and visible matter reheating the universe and essentially fixing the "initial conditions" for cosmological evolution.
But another problem arises because of their weak couplings and consequent long lifetime. The non thermal matter dominated universe, indeed, must end prior to Big Bang Nucleosynthesis. This is because we know with great confidence that at the time of the primordial nucleosynthesis the universe was radiation dominated.
This produces an important constraint on the moduli masses, which are strictly related to their decay rates.

Let us describe the situation in more details.

The evolution of the Universe is parametrized by the Hubble constant

$$H = \frac{\dot{a}}{a} \tag{4.9}$$

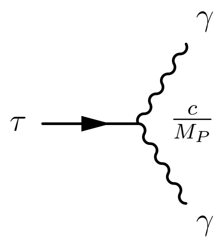
where a is the scale factor.
From this expression we easily extract the estimate of the age of the Universe, which is roughly

$$t_U \ \sim \ 1/H \tag{4.10}$$

Moduli decay when H, during the expansion, reaches the value of $\Gamma_{\tau\to\gamma\gamma}$, which represents the decay rate of moduli fields.
The latter can be easily obtained from the knowledge of the moduli couplings to photons. In fact, [15]

$$\Gamma_{\tau\to\gamma\gamma} \;\sim\; \frac{c^2}{M_P^2} m_\tau^3 \tag{4.11}$$

where $\frac{c^2}{M_P^2}$ is associated with the following Feynman vertex:



Now, recalling from cosmology the relation

$$H \;\sim\; \frac{T^2}{M_P} \tag{4.12}$$

we impose

$$\Gamma \sim H \quad \to \quad \frac{c^2}{M_P^2} m_\tau^3 = \frac{T^2}{M_P} \tag{4.13}$$

and we get

$$T_{decay} = c\; m_\tau\; \sqrt{m_\tau / M_P} \tag{4.14}$$

which is the temperature at which moduli are expected to decay into photons and then into observable matter.
Hence, the constraint associated with the cosmological moduli problem, can be written

as:

$$c\, m_\tau \sqrt{m_\tau/M_P} \;>\; T_{BBN} = O(1)\ MeV \tag{4.15}$$

in order not to spoil Big Bang Nucleosinthesis.

Let us analyse the implication of this argument for the mass spectrum we derived in the previous section.
During the calculation we use the formulae (2.64) and (2.65) from section 2.4 thanks to which we are able to estimate the coupling c appearing in (4.15).
We first consider the heavier moduli, which are expected to be less problematic, since we know their masses to be larger than $O(1)TeV$.

In the first model we have studied, the 'single-hole swiss-cheese', one obtains

$$m_{\tau_s} \sim 100\ TeV$$

The coupling between τ_s and photon is not simply $\frac{1}{M_P}$, but, from normalisation in section 2.4, is given by

$$\lambda_{\phi\gamma\gamma} \sim \frac{\sqrt{\mathcal{V}}}{M_P}$$

which, compared with (2.65), allows to extract $c = \sqrt{\mathcal{V}} \sim 10^7\ GeV$.
Now, computing the reheating (decay) temperature for the small modulus, we are left with:

$$T_{\tau_s} \sim 10^7 10^5 \sqrt{\frac{10^5}{10^{18}}} \;\sim\; 10^5 GeV \tag{4.16}$$

which abundantly exceeds the lower-bound limit of $O(1)\ MeV$.

As regards the second picture, we have to deal with two Kähler moduli τ_s and τ_{SM}.

For the latter, which supports the MSSM, same arguments are valid and the coupling to photon is much larger than $1/M_P$ being multiplied again by $c \sim \sqrt{\mathcal{V}}$. Then we have

$$T_{\tau_{SM}} \sim 10^8 10^2 \sqrt{\frac{10^2}{10^{18}}} \sim 10^2 \; GeV \tag{4.17}$$

which again saves Big Bang Nucleosinthesis.

In the computation of T_{τ_s}, instead, we have to accounting that by geometrical construction of the model, the cycle associated to τ_s is nearly isolated from Standard Model matter fields. The wavefunctions of visible matter is indeed localised around τ_{SM} blow-up cycle.
Taking this into account, the coupling between τ_s and photons results to be simply of the order $1/M_P$, giving rise to:

$$T_{\tau_s} \sim 10^4 \sqrt{\frac{10^4}{10^{18}}} \sim 10^{-3} \; GeV \sim O(1) \; MeV \tag{4.18}$$

which is a borderline outcome, quite satisfying.

Finally we deal with the light modulus τ_b, which we are going to see has no hope to decay in sufficient time.
In both scenarios, being the modulus which controls the size of the overall volume, τ_b is very weakly coupled to visible sector and therefore, from $\lambda_{\chi\gamma\gamma}$ in (2.64), we obtain $c \sim 1/ln\mathcal{V}$ in the expression (4.15). This implies:

$$T_{\tau_b} \sim 10^{-4} \sqrt{\frac{10^{-3}}{10^{18}}} \sim 10^{-14} \; GeV \sim 10^{-5} eV \tag{4.19}$$

which is an interesting result, because of

$$10^{-5} eV \;\; \sim \;\; T_{CMB} \tag{4.20}$$

where T_{CMB} is the today temperature of the background cosmic radiation!
In light of this, τ_b may still be stable today and could represent a good candidate for dark matter dominating the energy density of the Universe.
If this was the scenario, in order to avoid unobserved too large values of ρ_{DM} density, we should conclude that the neutralinos (and in general other candidates from s-particles) do not contribute to dark matter. If we want to preserve the R-parity conservation, this could happen only if we assume that neutralinos are under-abundant; this condition strictly depends on the annihilation cross-section values and need an explicit check.

By these considerations, we learn how deeply the supersymmetry breaking scale affects the evolution and the structure of our Universe.
Moduli fields dynamics really plays a key role in the understanding of the vacuum structure in which we live and may contribute to answer the fundamental questions about low-energy physics and cosmology.

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