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STRING INFLATIONARY MODELS WITH NON-MONOTONIC SLOW-ROLL AND DETECTABLE TENSOR MODES

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Contents

Abstract

The first chapter of this work has the aim to provide a brief overview of the history of our Universe, in the context of string theory and considering inflation as its possible application to cosmological problems. We then discuss type IIB string compactifications, introducing the study of the inflaton, a scalar field candidated to describe the inflation theory. The Large Volume Scenario (LVS) is studied in the second chapter paying particular attention to the stabilisation of the Kähler moduli which are four-dimensional gravitationally coupled scalar fields which parameterise the size of the extra dimensions. Moduli stabilisation is the process through which these particles acquire a mass and can become promising inflaton candidates. The third chapter is devoted to the study of Fibre Inflation which is an interesting inflationary model derived within the context of LVS compactifications. The fourth chapter tries to extend the zone of slow-roll of the scalar potential by taking larger values of the field φ. Everything is done with the purpose of studying in detail deviations of the cosmological observables, which can better reproduce current experimental data. Finally, we present a slight modification of Fibre Inflation based on a different compactification manifold. This new model produces larger tensor modes with a spectral index in good agreement with the date released in February 2015 by the Planck satellite.

Introduction

From the Big Bang Nucleosynthesis era to today the history of the universe is based on well understood and experimentally tested laws of particle physics and gravity. It is therefore justified to have some confidence about the events shaping the universe during that time.For earlier times, we can only conjecture the evolution of our universe based on prosiming theoretical ideas. Under some general assumptions, the key events of the history of the universe and their corresponding time and energy scales can be summarised as follows[1]:

- Planck Epoch: $< 10^{-43}$ s $\longrightarrow$ 2,4$\times 10^{18}$ GeV
- String Scale: $\geq 10^{-43}$ s $\longrightarrow$ $\leq 10^{18}$ GeV
- Grand Unification: $\sim 10^{-36}$ s $\longrightarrow$ $\sim 10^{16}$ GeV
- Inflation: $\geq 10^{-34}$ s $\longrightarrow$ $\sim 10^{16}$ GeV
- SUSY Breaking: $< 10^{-10}$ s $\longrightarrow$ > 1 TeV
- Baryogenesis: $< 10^{-10}$ s $\longrightarrow$ > 1 TeV
- Electroweak symmetry breaking: 10^{-10} s $\longrightarrow$ 100 GeV
- Quark-Hadron Transition: 10^{-4} s $\longrightarrow$ 100 GeV
- Nucleon Freeze-Out: 0.01 s $\longrightarrow$ 10 MeV
- Neutrino Decoupling: 1 s $\longrightarrow$ 1 MeV
- Big Bang Nucleosynthesis: 3 min $\longrightarrow$ 0.1 MeV
- Matter-Radiation Equality: 10^4 yrs $\longrightarrow$ 1 eV
- Recombination: 10^5 yrs $\longrightarrow$ 0.1 eV

- Galaxy Formation: $\sim 6 \times 10^8$ yrs
- Dark Energy: $\sim 10^9$ yrs
- Solar System: 8×10^9 yrs

As we can see from this list, around 0.1 eV (380,000 yrs) protons and electrons combine to form neutral hydrogen atoms, while photons decouple and freestream forming the cosmic microwave background. 13.7 billion years later these photons give us the earliest snapshot of the universe. Anisotropies in the CMB temperature provide evidence for fluctuations in the primordial matter density. At this point a question arises: what is the fundamental microphysical origin of the CMB fluctuations? It is clear that the answer to this question would give us explanations about the origin of all large scale structures in the universe.
It is here that a period of cosmic *inflation* plays a fundamental role. This period is characterized by an exponential expansion in the very early universe that is believed to have taken place about 10^{-34} seconds after the Big Bang singularity. Inflation is thought to be responsible both for the flatness and the large scale homogeneity of the universe and for the small fluctuations that were the seeds for the formation of structures like our own galaxy. Indeed, these small fluctuations grow via gravitational instability to form the large-scale structures observed in the late universe. A competition between the background pressure and the universal attraction of gravity determines the details of the growth of structure. Thus, inflation turns microscopic quantum fluctuations into macroscopic fluctuations in the energy density of the universe: a stunning connection between the 'physics of the large and the small'. This is the reason why the study of inflation is important and can guide us in the search of new physics beyond our present knowledge.

Chapter 1

Cosmology and inflation

The current view of our Universe is based on two theories with robust theoretical basis and solid experimental evidence:

- Standard Model of particle physics,
- Standard Model of Cosmology.

1.1 Standard Model of Particle Physics and Beyond

The Standard Model of Particle Physics is a four dimensional relativistic quantum field theory based on the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$.
This Model produces theoretical predictions in excellent agreement with experimental results that make it one of the great triumphs of the XX century and it is divided into two kinds of particles, according to different statistical laws and spin:

- **Fermions**

  The fermions of the Standard Model are classified according to how they interact. There are six quarks (up, down, charm, strange, top, bottom), and six leptons (electron, electron neutrino, muon, muon neutrino, tau, tau neutrino), the building blocks of matter. Pairs from each classification are grouped together to form a generation, with corresponding particles exhibiting similar physical behavior. The lightest and most stable particles make up the first generation, whereas the heaviest and less stable particles belong to the second and third generations. All stable matter in the universe is made from particles that belong to the first generation; any heavier particle quickly decays to the next most stable level.

- **Bosons**

The bosons of the Standard Model mediate the strong, weak, and electromagnetic fundamental interactions. The gauge bosons of the Standard Model all have spin 1. As a result, they do not follow the Pauli exclusion principle that constrains fermions: thus bosons (e.g. photons) do not have a theoretical limit on their spatial density (number per volume). The different types of gauge bosons are described below.

1. Photons mediate the electromagnetic force between electrically charged particles. The photon is massless and is well-described by the theory of quantum electrodynamics
2. The W^+, W^- and Z gauge bosons mediate the weak interactions between particles of different flavors (all left-handed quarks and leptons). They are massive, with the Z being more massive than the W's. The weak interactions involving the W's exclusively act on left-handed particles. Furthermore, the W's carry an electric charge of +1 and -1 and couple to the electromagnetic interaction. The electrically neutral Z boson also interacts only with left-handed particles. These three gauge bosons along with the photons are grouped together, as collectively mediating the electroweak interaction
3. The eight gluons mediate the strong interactions between color charged particles (the quarks). Gluons are massless. The eightfold multiplicity of gluons is labeled by a combination of color and anticolor charge (e.g. red-antigreen). Because the gluons have an effective color charge, they can also interact among themselves. The gluons and their interactions are described by the theory of quantum chromodynamics

However, the mechanism that breaks electroweak symmetry in the SM has not been completely tested experimentally [33], [34], [35], [31], [8], [5]. This mechanism, which gives mass to all elementary particles, implies the existence of a scalar particle, the SM Higgs boson.

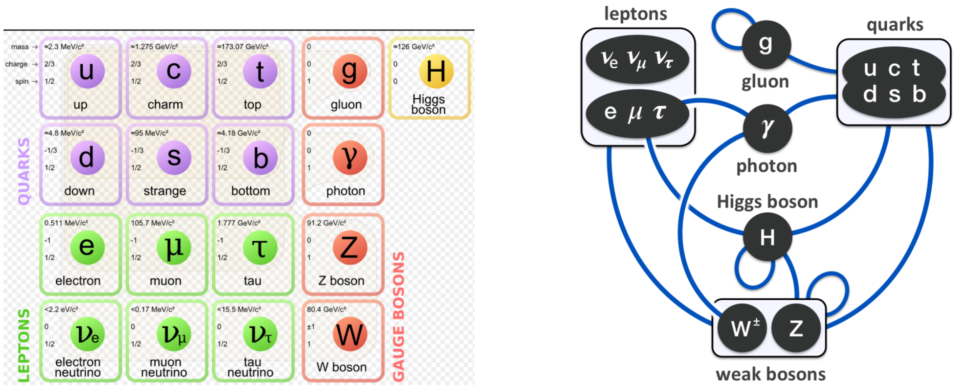

Figure 1.1: The interactions between all the particles described by the Standard Model.[26],[27].

According to the Goldstone theorem, theories with spontaneously broken global symmetries exhibit massless particle which however have not been seen in Nature [10],[4]. Higgs has shown how these massless Goldstone particles become additional lngitudinal degrees of freedom of massive gauge bosons in the presence of broken local symmetries [5].
The Higgs boson has spin 0 and plays a unique role in the Standard Model, by explaining why the other elementary particles, except the photon and gluon, are massive. In particular, the Higgs boson explains why the photon has no mass, while the W and Z bosons are very heavy.
On 4 July 2012, the ATLAS and CMS experiments at CERN's Large Hadron Collider (LHC) announced the observation of a new particle in the mass region around 126 GeV. This particle is consistent with the Higgs boson On 14 March 2013 the Higgs Boson was tentatively confirmed to exist[28]. The fact that our universe is almost flat, homogeneous and isotropic is often considered as a strong indication that the Standard Model (SM) of elementary particles is not complete. We argue however that the Higgs boson of the Standard Model can lead to inflation and produce cosmological perturbations in accordance with observations [29].
Although the Standard Model accurately describes the phenomena within its domain, it is still incomplete. Perhaps it is only a part of a bigger picture that includes new physics hidden deep in the subatomic world or in the dark regions of the universe. New information from experiments at the LHC will help us to find more of these missing pieces[30].
The Standard Model describes gravity only at the classical level. As a matter of fact, gravitational interactions are studied in the framework of the classical

theory of General Relativity that leads to the famous Einstein-Hilbert action of the form:

$$S_{\mathrm{EH}} = \frac{M_{\mathrm{P}}}{2} \int \sqrt{-g}\, R\, d^4x \tag{1.1}$$

but it cannot be quantised in the usual fashion.
The modern framework is that Einstein theory should be regarded as an effective field theory, which is a good approximation at energies below the Planck mass $\mathrm{M_P}$.
The Planck mass can be derived approximately by setting it as the mass whose Compton wavelength and Schwarzschild radius are equal[32]. The Compton wavelength is, loosely speaking, the length-scale where quantum effects start to become important for a particle. M_p defines the Planck energy, according to the famous formula $E = Mc^2$, that is about 10^{18} GeV. Setting $\mathrm{c} = 1$ we can use M_p as an energy scale.
There should be an underlying, quantum mechanically well defined, theory which exists for all ranges of energy, and reduces to General Relativity at low energies, below the cutoff scale.
Some proposals for extending the physics of the Standard Model are:

- **Supersymmetry**

  1. Supersymmetry [3] is a spacetime symmetry, despite the fact that it is seen only as a transformation that exchanges bosons and fermions. Supersymmetry solves the naturalness issue of the hierarchy problem due to cancellations between the contributions of bosons and fermions to the Higgs mass. Combined with the Grand Unified Theory (GUT) idea, it yields the unification of the three gauge couplings at one single point at larger energies of order 10^{16} GeV. Furthermore, the lightest superymmetric particle, if stable, could behave as dark matter. For all these reasons, theories with low energy supersymmetry have emerged as the strongest candidate for new physics beyond the Standard Model at the TeV scale.

  2. However, unbroken supersymmetry implies a mass degeneracy between superpartners, a possibility which is clearly forbidden by experiment. Hence exact supersymmetry is not viable, but we can keep all its desirable good properties, if it is broken softly. As we have seen, supersymmetry has several appealing features but, even though supersymmetry is responsible for gauge couplings unification, the Minimal Supersymmetric Standard Model(MSSM) does

not treat the three non-gravitational interactions in a unified manner, and, on top of that, it keeps not containing gravity.

- **Grand Unified Theory**

  1. This theory includes the Standard Model group as the low-energy remnant of a larger gauge group (G_{GUT}) that is broken spontaneously by a Higgs mechanism at a high scale $M_{GUT} \sim 10^{16}$ Gev. Besides this nice feature, grand unified theories do not address the fundamental problem of gravity at the quantum level, or the relation between gravity and the other interactions.

- **Supergravity**

  1. A very important feature of supergravity theories is the presence of a new supermultiplet, the gravity multiplet, including a spin-2 graviton $G_{\mu\nu}$ and its spin 3/2 superpartner ${\psi_\mu}^\alpha$, the gravitino, which is the gauge field of local supersymmetry. This is the first attempt to construct a theory that incorporates both gravity and all the non-gravitational interactions within the same description. However, it does not provide an ultra-violet completion of Einstein gravity, since it is neither finite nor renormalisable.

- **Kaluza-Klein theories**

  1. The number of space-time dimensions is intuitively 3+1 since this is what our common sense tells us. However, Theodor Kaluza, in 1919, formulated his original hypothesis and sent his results to Einstein. Independently, Klein worked on the same idea. The result was the beautiful proposal of unifying gravity and gauge interactions thanks to the presence of tiny compactified extra dimensions. We know that the surface of the Earth is a two-dimensional surface inside a three-dimensional space. Kaluza and Klein wondered if physical space dimensions are not extended in space but could be curved like a circle. However, although the idea involves gravity, it still suffers from quantum inconsistencies.

- **Brane-world scenario**

  1. This is another fascinating idea concerning the existence of extra dimensions. It is based on the assumption that only gravity can propagate in the extra dimensions whereas all other non-gravitational interactions are trapped on a four-dimensional brane embedded in a higher dimensional space-time. Unfortunately, this setup does not provide an ultra-violet completion of General Relativity, since gravity is treated classically

1.2 Standard Model of Cosmology

We are in the framework of the classical theory of Einstein gravity in which the cosmological evolution of our Universe is determined by considering a perfect fluid with pressure p and energy density ρ in a homogeneous and isotropic space-time. This assumption forces the metric to take the famous Friedmann-Robertson-Walker (FRW) form:

$$ds^2 = -dt^2 + a^2(t)\ (\ \frac{dr^2}{1-kr^2} + r^2\ (d\theta^2 + \sin^2\theta d\phi^2)\ ) \tag{1.2}$$

a(t) is the scale factor which measures the evolution of the Universe in cosmological time

k is a discrete parameter, k = -1, 0, 1, that determines the curvature of the spatial sections at fixed t, corresponding to an open, flat or closed Universe, respectively

The most important implication of the FRW ansatz, is that the space-time has an initial singularity at t = 0, nothing but the big-bang from which the Universe started its expansion. This is a clear sign that quantum effects start playing a crucial role and we are using General Relativity in a region beyond its regime of validity.
Putting the metric (1.2) into the Einstein equations, one obtains the so-called Friedmann equations:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_p^2} - \frac{kc^2}{a^2} \tag{1.3}$$

$$\frac{\ddot{a}}{a} = \frac{\rho + 3p}{6M_p^2} \tag{1.4}$$

where H is the Hubble scale and the energy conservation equation:

$$\dot{\rho} = -3H(\rho + p) \tag{1.5}$$

Let us make some comments here:

- $(\rho + 3p) >$ 0 the Universe expands decelerating
- $k = -1$; $\rho >$ 0 the Universe keeps expanding forever, whereas for $k = 1$ there can be a value of a, for which the curvature term compensates the energy density term, and $\dot{a} = 0$.
  After this time, a decreases and the Universe re-collapses.
- It is not possible to have a closed Universe ($k = 1$) that re-collapses, if it is always accelerating.

At this point we can introduce the parameter Ω, which is the ratio between the energy density of our universe and the critical energy density ρ_c:

$$\Omega = \frac{\rho}{\rho_{\rm c}}; \qquad \rho_{\rm c} = 3H^2 M_P^2 \tag{1.6}$$

For $\Omega = 1$, $\Omega <$ 1, $\Omega >$ 1 one can have a flat, open or closed universe respectively.
In this Standard Cosmological Model, the early Universe is supposed to be undergone a cosmological evolution through the following main steps:

1. Approximately 13 billion years ago the universe began expanding from an almost inconceivably hot, dense state and it continued its long process of expansion and cooling, reaching the cold, sparse state we see today.

2. The early universe was a soup of matter and energy, in which particle/antiparticle pairs were constantly being born and annihilating. As the Universe became cold enough to prevent the production of certain kinds of particles, these species dropped out of thermal equilibrium and are said to have *frozen out*. After this point, the only reason we have any matter in the universe at all is because of a poorly-understood process called *baryogenesis* that should explain the observed asymmetry between matter and antimatter.

3. Big Bang Nucleosynthesis (BBN)took place during the first 3 minutes or so. At this stage, free protons and neutrons, created in baryogenesis, combined together to give rise to various light elements. During this period the Universe was opaque to light due to the very efficient absorption of photons by the large number of free electrons.

4. Recombination: 10^5 years after the big-bang, after the formation of atoms, the Universe became transparent and the light released at this time is perceived today as the cosmic microwave background (CMB). At the same time, the Universe also changed from being radiation dominated to matter dominated and galaxies and stars began to form when the baryonic gas and dust collapsed to the centre of the pre-existing dark matter halos.

Despite many theoretical and experimental successes like the Hubble law, Big Bang Nucleosynthesis and the Cosmic Microwave Background, the Standard Model of Cosmology still faces several severe problems like the initial singularity, the flatness and horizon problems, horizon problem, the origin of the CMB anisotropies and the excess of matter over antimatter. Furthermore, present observations show that ordinary matter accounts just about the 5% of the total energy density. The remaining 95% is presently unknown!
There should be other contributions to the total energy density of our Universe, in particular the remaining 95% should contain *cold dark matter* (25%), a mysterious form of matter that has not yet been identified, and *dark energy* (70%), some kind of 'antigravity' effect which causes the Universe to accelerate. This acceleration could be caused by the so called 'Quintessence', a time varying scalar field in contrast with the idea of the static cosmological constant explanation of dark energy.

1.3 Inflation

As already written above, the most compelling solution to these cosmological problems is achieved by requiring that in the first 10^{-34} seconds or so, the Universe underwent a brief period of exponentially fast expansion, known as inflation. The most beautiful feature of this period is that it can render quantum effects visible in the sky and explain why some regions could be in causal contact with each other. A significant confirmation to this theory was received by the detailed observations of the cosmic microwave background made by the Wilkinson Microwave Anisotropy Probe (WMAP) spacecraft[6].

The simplest models of inflation involve a single scalar field ϕ below $\mathrm{M_p}$, whose dynamics, coupled to gravity, is governed by the action:

$$S = \int \sqrt{-g}\, [\frac{R}{2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)]\, d^4x = S_{\mathrm{EH}} + S_\phi \tag{1.7}$$

(with $M_p = 1$)This is the sum of the gravitational Einstein-Hilbert action, $\mathrm{S_{EH}}$, and the action of a scalar field with canonical kinetic term, S_ϕ. The potential $\mathrm{V}(\phi)$ describes the self-interactions of the scalar field. The energy-momentum tensor is:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu}\, [\frac{1}{2} \partial^\sigma \phi \partial_\sigma \phi + V(\phi)] \tag{1.8}$$

Assuming the FRW metric for $\mathrm{g}^{\mu\nu}$ and restricting to the case of a homogeneous field ϕ(t; x)$\equiv$ ϕ(t), the scalar energy-momentum tensor takes the form of a perfect fluid with the resulting equation of state:

$$\omega_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \tag{1.9}$$

This shows how a scalar field can lead to negative pressure($\omega_\phi < 0$) and accelerated expansion ($\omega_\phi <$ - 1/3) if the potential energy V dominates over the kinetic energy $\dot{\phi}^2/2$ The best present model of inflation, referring to a scalar field called *inflaton*, relies on the slow roll of this field down a very shallow potential. This implies a very small mass of the inflaton but we are going to study it in detail in the next chapters.
Now, given the notorious ultra-violet sensitivity of scalar masses, how is it possible to keep the inflaton mass low protecting it from getting large quantum corrections?
Furthermore, the ultra-violet sensitivity of inflation gets even worse when one looks at possible models that would generate observable tensor modes, since they require the inflaton to travel a trans-Planckian distance in field space [20].

1.4 String theory

This section is not supposed to be a comprehensive review of string theory but its purpose is only to show that all the previous proposals for physics beyond the Standard Model can be embedded into string theory, which represents also the natural framework where inflation should be derived. This

theory is, at present, the most promising candidate for a consistent theory of quantum gravity and it is also able to incorporate all the four known interactions and matter in a beautiful unified framework whose dinamic selects a particular vacuum state which, in turn, determines all the masses and couplings. String theory was born about fifty years ago and, although it still lacks an experimental evidence, it has the potential to provide an answer to most of the fundamental questions beyond the Standard Cosmological Model However, in the last years, there is a new set of experimental data coming from two crucial experiments for fundamental physics: the PLANCK satellite, which has been launched by the European Space Agency in May 2009, and the Large Hadron Collider at CERN in Geneva. Furthermore, consistent and well-defined low energy limits of string theory have been found. It's time, therefore, to try to make contact of string theory with our real world but, first, we have to put things in the right order.
String theory can be introduced as the theory describing tiny one-dimensional objects, called *strings*, that move through D-dimensional space-time and sweep out two-dimensional *world sheets* which may be viewed as thickened Feynman diagrams. The two-dimensional field theories living on such Riemann surfaces define different perturbative string theories based on free two-dimensional bosons and fermions, which correspond to the bosonic and fermionic coordinates of the string in D-dimensional space-time. Combinations of these two-dimensional fields give rise to a finite number of massless spacetime fields, plus an infinite tower of stringy excitations with arbitrary high masses and spins. The string scale M_s adjusts the size of the string and governs the level spacing of the excited states. Moreover, thanks to the presence in the massless theory spectrum of a spin-2 particle, the *graviton*, we realize that the string theory includes the existence of gravity. Indeed, in the study of string perturbation theory, the loop expansion does not contain any ultra-violet divergence, so making the theory a perfect candidate for a consistent treatment of quantum gravity. The fact that the quantum theory of strings is naturally defined in more than four spacetime dimensions, has led us to study the compactification methods for the extra spatial dimensions, trying to reach ordinary four-dimensional physics. For concreteness, we will focus on the compactification of the critical ten-dimensional type IIB string theory[1]. The detailed explanation of how the ten-dimensional sources determine the four-dimensional effective theory is a central task in string theory model-building.

[1] There are five ten-dimensional string theories which have completely different spectra, number of supersymmetries and gauge symmetries at the perturbative level but they turned out to be all connected by various dualities

We shall, therefore, assume that the space-time manifold is not simply $\mathbb{R}^{1,9}$, but $\mathbb{R}^{1,3} \times \mathrm{Y}^6$, where Y^6 is some compact six-dimensional manifold. The compactification is usually demanded to yield a supersymmetric low energy effective theory mainly because the supersymmetry breaking scale is supposed to be low in order to solve the hierarchy problem and further supersymmetry simplifies the calculations.
The requirement of obtaining $\mathrm{N} = 1$ supersymmetry in $\mathrm{D} = 4$, forces the internal manifold Y^6 to be a very complicated *Calabi-Yau* space.

For our purposes, the most important degrees of freedom of the effective theory are four-dimensional scalar fields known as *moduli*. They arise from deformations of the compactification manifold, typically numbering in the hundreds for the Calabi-Yau spaces under consideration, and from the positions, orientations, and gauge field configurations of any D-branes. These non-perturbative objects, discovered by Polchinski in 1995, have changed our view of type II compactifications that until then seemed to be much less interesting. Some D-brane features are:

- Non-Abelian gauge and matter fields are open strings whose end-points are constrained to move on the brane
- D-branes allow to circumvent an existing *no-go theorem* that until then prevented the turning on of background fluxes, interesting candidate energy sources to stabilise the moduli
- The Standard Model lives on a particular D-brane whereas the closed string sector, including gravity and the dilaton whose vacuum expectation value sets the string coupling, probes all the extra dimensions

Now, one can compute the kinetic terms and scalar potentials of the moduli whose vacuum expectation values parameterise the shape and the size of the extra dimensions.
These moduli are massless uncharged scalar particles but in the presence of generic ten-dimensional sources of stress-energy, such as D-branes and quantized fuxes, there is an energy cost for deforming the compactification, and many (though not always all) of the moduli fields become massive: it is of primary importance to develop a potential for these particles and to give them a mass, this is the so called *moduli stabilisation*. There is also a cosmological constraint over the moduli masses, $\mathrm{m}_{\mathrm{mod}} \geq 10$ TeV, so that they decay before BBN evading the cosmological moduli problem.

It is useful to divide the scalar fields into a set of light fields with masses below the Hubble scale and a set of heavy fields with masses much greater than the Hubble scale. Here one of the light fields, denoted ϕ, has been identified as the *inflaton* candidate. To understand whether successful inflation can occur, one must understand all the scalar fields, both heavy and light. It must be stressed that light scalars absorb energy during inflation and, if they persist after inflation, they can release this energy during or after Big Bang nucleosynthesis, spoiling the successful predictions of the light element abundances. To avoid this problem it suffices to ensure that their mass is larger than 30 TeV, as in this case the moduli decay before Big Bang nucleosynthesis.

Chapter 2

A simple string model

As we have explained in chapter 1, type IIB string theory seems to be, at present, the most promising way to connect string theory with particle physics and cosmology. Assuming the space-time to be a product of the form but $\mathbb{R}^{1,3} \times \mathrm{X}$, where X is a Calabi-Yau three-fold, one obtains an $\mathrm{N} = 2$ supersymmetric field in four dimensions. Then taking Calabi-Yau orientifolds, the number of supersymmetries can be reduced from $\mathrm{N} = 2$ to $\mathrm{N} = 1$. In the *braneworld scenario*, the Standard Model, or any of its possible generalisations, lives on a stack of space-time filling D-branes in the bulk. $\mathrm{N} = 1$ supersymmetry can then be spontaneously broken by additionally turning on background fluxes in the orientifold bulk, which render the compactification manifold conformally Calabi-Yau. The internal fluxes also generate a potential that freezes the scalar fields except for the Kähler moduli which parameterise the size of the extra dimensions. Hence additional perturbative and non-perturbative effects have to be considered in order to fix all the moduli and construct a stable vacuum. Behind everything there is always the question: What is the fundamental microscopic origin of inflation?
As said previously, inflation is believed to have occurred at an enormous energy scale, far out of reach of terrestrial particle accelerators. Any description of the inflationary era therefore requires a considerable extrapolation of the known laws of physics, and until recently, only a phenomenological parameterization of the inflationary dynamics was possible[1]. In this approach, a suitable inflationary potential function $\mathrm{V}(\varphi)$ is postulated and details of the primordial fluctuation spectra will depend on its precise shape. Having a model with stabilised moduli means that we are able to do realistic phenomenology and compute all the relevant scales: the Kaluza-Klein mass, the

[1] Recently, progress has been made both in a systematic effective field theory description of inflation and in top-down derivations of inflationary potentials from string theory.

gravitino mass and also the masses of different particles in the moduli sector. This issue has been successfully solved in the context of type IIB string theory. In this framework there are Kähler moduli, moduli which parameterise the size of the extra dimensions, complex structure moduli which control the shape of the Calabi-Yau and the dilaton whose vacuum expectation values sets the string couping. Most of the geometric moduli are stabilized by fluxes and for the remainig moduli it was at first proposed the KKLT scenario, then ameliorated and extended in the Large Volume Scenario. The next sections have the purpose of trying to explain a concrete example of large field inflation in the context of moduli stabilisation within the well studied IIB string compactifications. Working within such a framework allows us to use the well-understood properties of low-energy 4D supergravity.

2.1 Type IIB moduli stabilisation

The first question to ask is how to work out the number of supersymmetries of the effective four-dimensional theory. This can be done by looking at the decomposition of the spinor representation of the ten-dimensional Lorentz group SO(1,9) and then counting the number of singlets under the structure group of the compactification manifolds. The number of singlets gives the number of supersymmetries in the effective four-dimensional theory.

2.1.1 Basic features

Given that we shall be interested in compactification manifolds that preserve the minimal amount of supersymmetry in four dimensions, we focus on the case of manifolds X with SU(3) holonomy group. Moreover, it turns out that these spaces are Ricci-flat Kähler manifolds, corresponding to the famous case of Calabi-Yau manifolds. This is equivalent to prove the presence of a globally defined (1, 1)-form J and a complex holomorphic (3, 0)-form Ω, which are both closed. In order now to work out the particle spectrum in four dimensions, one has to study the splitting of the ten-dimensional equations of motion in a compactified space-time background of the form $\mathbb{R}^{1,3} \times$ X. Starting from the 10-dimensional metric, one obtains that the massless modes of the D = 4 theory are in one-to-one correspondence with the harmonic forms on the Calabi-Yau X. The number of these harmonic forms is given by the so-called Hodge numbers $h^{1,1}$ and $h^{1,2}$. We can put them in the so-called Hodge diamond, since it renders all the possible symmetries manifest and show that this Hodge diamond has three symmetries which are the complex conjugation (reflection about the central vertical axis), the Hodge- duality, also called Poincaré duality (reflection about the central horizontal axis), and the mirror symmetry (reflection about the central diagonal axis). The massless modes counted by these Hodge numbers are particular deformations of the Calabi-Yau metric which do not deform the Calabi-Yau condition. These deformations of the metric correspond to scalar fields in the low energy effective action, which are called moduli. They can be viewed as the coordinates of the geometrical moduli space of the Calabi-Yau manifold We shall now focus on deformations of the metric δg that preserve Ricci-flatness. Given that we are not interested in changes of coordinates, we need to eliminate them by fixing the diffeomorphism invariance. This implies to see the variation metric expanded in terms of harmonic (1,1) and (1,2)-forms. In the case of harmonic (1,1)-forms, they are Poincare dual to volumes of internal 4-cycles which are given by the Kähler moduli. As we shall see in the next sections, these moduli are promising inflaton candidates. Kähler moduli stabilisation is made in the framework of the previously mentioned LVS, the simplest example of a broad new family of inflationary constructions within the rich class of IIB stabilisation mechanisms.

2.1.2 The Large Volume Framework

We know that the low-energy 4D theory obtained at low energies is described by an N = 1 supergravity, characterised by the Kähler potential K, the superpotential W and the gauge kinetic function f. Within this framework complex structure moduli are fixed semiclassically through the presence of branes and fluxes, while Kähler moduli are stabilised by an interplay between non-perturbative corrections to the low-energy superpotential, W , and perturbative corrections to the Kähler potential, K, of the effective low-energy 4D supergravity. In particular, the LARGE volume that defines these scenarios naturally arises as an exponentially large function of the small parameters that control the calculation. It is important to note that large volumes imply a low string scale, M_s, and this drives down the inflationary scale M_{inf}. This is interesting because it may lead to inflation even at low energy scales but could be a problem inasmuch as it makes it more difficult to obtain large enough scalar fluctuations to account for the primordial fluctuations seen in the CMB.

Lowest-order expressions

Type IIB string theory is defined in 10 dimensions and has 32 supercharges. The ten dimensional bosonic massless fields consist of the metric (g_{MN}), the dilaton (ϕ), RR antisymmetric forms (C_0,C_2,C_4 with the self-dual field strength) and an NS-NS antisymmetric tensor (B_2). To obtain the four dimensional model we compactify this theory on a Calabi-Yau orientifold. Fluxes for the RR 3-form F_3=dC_2 and NS-NS 3-form H_3=dB_2 can be turned on and the following quantisation conditions must be imposed [11]:

$$\frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_{\rm a}} F_3 = n_{\rm a} \in \mathbb{Z} \qquad\qquad \frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_{\rm b}} H_3 = m_{\rm b} \in \mathbb{Z} \tag{2.1}$$

where $\Sigma_{\rm a,b}$ represent the 3-cycles of the Calabi-Yau manifold.
The superpotential at three level is independent of the Kähler moduli and is given by the Gukov-Vafa-Witten superpotential:

$$W = \int_{CY} G_3 \wedge \Omega = W(S,U) \tag{2.2}$$

where $\mathrm{G}_3 = \mathrm{F}_3 + \mathrm{iSH}_3$, being $\mathrm{S} = e^{-\phi} + \mathrm{iC}_0$, the axion-dilaton field and Ω the already seen holomorphic form (3,0) of Calabi-Yau.
To leading order in the string-loop and α' expansions, the resulting low-energy Kähler potential has the form:

$$K_{\rm tree} = -2ln V - ln(\, S + \overline{S}\,) - ln\Big(-i\int_X \Omega\wedge\overline{\Omega}\,\Big) \tag{2.3}$$

where V is the Calabi-Yau volume, measured with an Einstein frame metric $\mathrm{g}^{\mathrm{E}}_{\mu\nu} = \mathrm{e}^{-\phi/2}\mathrm{g}^{\mathrm{s}}_{\mu\nu}$, and expressed in units of the string length, $\mathrm{l_s} = 2\,\pi\sqrt{\alpha'}$. In general, the complex fields of the 4D theory include S, the complex structure moduli, U_α, $\alpha = 1,...,h_{2,1}(X)$, and the Kähler moduli $\mathrm{T_i}$, i = 1,...,$\mathrm{h}_{1,1}$(X). In eq. (2.3) Ω is to be read as implicitly depending on the U_α's, and V as depending implicitly on the $\mathrm{T_i}$'s. The values of S and the complex structure moduli, U_α, can become fixed once background fluxes are turned on. These fluxes may, but need not, break the remaining 4D N = 1 supersymmetry, corresponding to whether or not the resulting scalar potential is minimized where $\mathrm{D}_\alpha\mathrm{W} = \partial_\alpha\mathrm{W} + \mathrm{W}\partial_\alpha\mathrm{K}$ vanishes at the minumum.

The Kähler moduli T_i do not appear in the eq.(2.2) and so remain precisely massless at leading semiclassical order. To obtain the supergravity descibring this massless sector, we elimate the heavier fields S and U_α at the classical level. This can be done in a supersymmetric way by solving $\partial_\alpha W = 0$. At this point the supergravity description of the remaining Kähler moduli is specified by a constant superpotential, $W = W_0 = \langle W_{\text{tree}} \rangle$, and the Kähler potential $K = K_{cs} - 2\ln(2/g_s) + K_0$, with

$$K_0 = -2\ln V \qquad\qquad e^{-K_{cs}} = \left\langle -i\int_X \Omega\wedge\overline{\Omega} \right\rangle \tag{2.4}$$

To express K_0 explicitly in terms of the fields T_i, referring to what we said earlier, we write the volume in terms of the Kähler form, J, expanded in a basis of harmonic (1, 1)-forms $\hat{D}_i$ of $H^{1,1}(X, \mathbb{Z})$ and we obtain:

$$J = \textstyle\sum_{k=1}^{h_{1,1}} t^i \hat{D}_i \qquad \Longrightarrow \qquad V = \frac{1}{6}\int_X J\wedge J\wedge J = \frac{1}{6}k_{ijk}t^i t^j t^k \tag{2.5}$$

where k_{ijk} are the triple intersection numbers of X and the t^i are 2-cycle which are Poincaré dual to the Kahler moduli T_i defined as

$$T_i = \tau_i + ib_i \tag{2.6}$$

where τ_i turns out to be the Einstein-frame volume (in units of l_s) of the divisor D_i, the Poincaré dual to $\hat{D}_i$. Its axionic partner is b_i, the component of the RR 4-form C_4 along this cycle: $\int_{D_i} C_4 = b_i$ [11]. Furthermore, the 4-cycle volumes τ_i are related to the 2-cycle volumes as follows:

$$\tau_i = \frac{\partial V}{\partial t^i} = \frac{1}{2}k_{ijk}t^j t^k \tag{2.7}$$

Now, we can use the eq.(2.4) and (2.5) and solve these equations for the t^i as functions of $\tau_i = \frac{1}{2}(\ T_i + \overline{T_i}\ )$ obtaining K_0 as a function of T_i.
The supergravity scalar potential for T_i in terms of K and W is:

$$V = e^K[\ K^{i\bar{j}} D_i W D_{\bar{j}}\overline{W} - 3|W|^2\ ] \tag{2.8}$$

where $K^{i\bar{j}}$ is the inverse of Kähler metric $K_{i\bar{j}}$ = $\partial_{\bar{i}}\partial_j$K and D$_\alpha$W = ∂_αW + W∂_αK. Thanks to the no-scale identity of K$_0$, we obtain a completely flat potential, V≡0 as is required for agreement with the microscopic compactification since the fluxes did not stabilize the Kähler moduli to leading order.

Non-perturbative corrections

To stabilize the Kähler moduli, non perturbative correction to superpotential have to be included. The full non-perturbative superpotential is expected to be

$$W = W_0 + \textstyle\sum_k A_i e^{-a_i T_i} \tag{2.9}$$

where a_i and A_i are model-dependent costants.
The no-scale structure is broken and this non perturbative effects allow T moduli to be stabilized by solving D_TW = 0 [12]. To understand better the situation, let's consider only one modulus, denoted by τ and the corresponding axion set to zero. The Kähler potential, the superpotential and the scalar potential are given by:

$$K = -3ln\big(\, T + \overline{T}\,\big) \tag{2.10}$$

$$W = W_0 + Ae^{\,-aT} \tag{2.11}$$

$$V = e^K[\; K^{T\overline{T}}|D_T W|^2 - 3|W|^2\;] \tag{2.12}$$

The condition of unbroken SUSY, D_TW = 0, allows to find an expression for W_0 and the following scalar potential minimum:

$$V = -3e^K|W|^2 = -\frac{a^2A^2e^{-2a\tau}}{3\tau} \tag{2.13}$$

This is the famous KKLT minimum which is SUSY and Anti-de Sitter.
The important feature is that non-perturbative contribution, in the example above W_{np} = A$e^{\,-aT}$, breaks the no-scale. Since each term in W_{np} is exponentially suppressed in terms of the Kähler moduli, we generally expect a similar suppression also for the in scalar potential. However, this is not consistent with the neglecting of α' and g_s corrections because these go as some powers of the Kähler moduli and so dominate exponentially suppressed

terms coming from W_{np}. Their neglection can be justified if complex structure and dilaton moduli are stabilized at a very small value of W_0, so one has to fine-tune W_0 to a very small value. This means that the stabilization only works for a small parameters range. At this point we have to uplift this minimum to a de Sitter one and also in a way to give a vanishing cosmological constant.

Perturbative α' effects

As mentioned earlier, there are several problems in this scenario. These are overcome in the so-called Large Volume Scenario where perturbative α' corrections are included in the Kähler potential which takes the form .[2]

$$K = -2ln\big[\; V + \frac{\xi(\; S + \overline{S}\;)^{3/2}}{2}\;\big] - ln(\; S + \overline{S}\;) - ln\big(-i\int_{CY}\Omega\wedge\overline{\Omega}\,\big) \quad (2.14)$$

where $\xi = \chi/2(2\pi)^3$ with χ the Euler number of the Calabi-Yau three-fold. For large volume, corrections go as inverse powers in the volume:

$$ln\big[\; V + \frac{\xi(\; S + \overline{S}\;)^{3/2}}{2}\;\big] \sim lnV + \frac{\xi(\; S + \overline{S}\;)^{3/2}}{V} - \frac{\xi^2(\; S + \overline{S}\;)^2}{2V^2} + O(\frac{1}{V^3}) \quad (2.15)$$

and will dominate in the scalar potential expression whose full analytic expression is reached by splitting it into three terms:

$$V = e^K\;(\; V_{np1} + V_{np2} + V_{`alpha'}\;) \quad (2.16)$$

with

$$V_{np1} = K^{i\overline{j}}\partial_i W_{np}\partial_{\overline{j}}\overline{W}_{np} \quad (2.17)$$

$$V_{np2} = K^{i\overline{j}}[\;\partial_i W_{np} K_{\overline{j}}(\;\overline{W}_0 + \overline{W}_{np} + K_i(\;\overline{W}_0 + \overline{W}_{np}\;)\partial_{\overline{j}} W_{np}\;] \quad (2.18)$$

$$V_{`alpha'} = (\; K^{i\overline{j}} K_i K_{\overline{j}} - 3\;)|W|^2 \quad (2.19)$$

At large volume we can take only the leading terms in the scalar potential.

[2]There are also string loop corrections to V in LVS but they do not appreciably alter the minimum.

For this purpose, we can use the $\mathbb{C}P^4_{[1,1,1,6,9]}$ Calabi-Yau with two Kähler moduli:

$$T_b = \tau_b + ib_b \qquad and \qquad T_s = \tau_s + ib_s \tag{2.20}$$

in which the τ_b modulus is stabilised at large values whereas the small modulus τ_s is fixed at much smaller values. The Calabi-Yau volume can be written as follows[13]:

$$V = \frac{1}{9\sqrt{2}}\left({\tau_b}^{3/2} - {\tau_s}^{3/2} \right) \tag{2.21}$$

The Kähler potential and the superpotential become:

$$K = -2ln\left[\frac{1}{9\sqrt{2}}\left({\tau_b}^{3/2} - {\tau_s}^{3/2} \right) + \frac{\xi}{2{g_s}^3/2} \right] \tag{2.22}$$

$$W = W_0 + A_s e^{-a_s\tau_s} \tag{2.23}$$

As it is possible to see, in the Kähler potential there are terms like ξ, that takes into account perturbative corrections, and g_s, the string coupling. The supergravity scalar potential at large volume, at leading order, is as follows:

$$V = \frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{3\tau_b^{3/2}} - \frac{\mu a_s|W_0|\tau_s e^{-a_s\tau_s}}{\tau_b^3} + \frac{\nu|W_0|^2}{\tau_b^{9/2}}$$

where $\lambda = 8\ (a_sA_s)^2$ and $\mu = 4A_s$. Now, in the absence of fine tuning, the tree-level superpotential is of order $W_0 \sim O(1)$ and so the α' and non-perturbative corrections compete naturally to give an exponentially large volume (AdS) minimum that breaks SUSY, located at

$$\mathcal{V} \sim W_0\, e^{a_s\tau_s} \gg \tau_s \sim \xi^{2/3} \gg 1$$

For phenomenological applications it is usually necessary to up-lift this minimum from AdS to allow Minkowski (or slightly de Sitter) 4D geometries. This can be done by adding suitable new contributions to either K or W. The stabilized exponentially large volume can generate hierarchies because to small variations of $a_s\tau_s$ correspond large variations of the Calabi-Yau volume. The gravitino mass $m_{3/2}$ is given by:

$$m_{3/2} = e^{\mathcal{K}/2}|W_0| = \frac{|W_0|}{\mathcal{V}} M_{Pl}$$

Phenomenological reasons associated with the solution of the hierarchy problem based on low energy SUSY require $m_{3/2} \sim O(TeV)$ from which the Calabi-Yau volume is about 10^{15} in string unit and the string scale is

$$M_s = \frac{M_{Pl}}{\mathcal{V}^{1/2}}$$

From the supergravity potential expression we can compute the moduli mass[3] $m_b^2 \sim K^{bb}V_{bb}$ and $m_s^2 \sim K^{ss}V_{ss}$:

$$m_{\tau_b} \sim \frac{M_{Pl}}{\mathcal{V}^{3/2}}$$
$$m_{\tau_s} \sim \frac{M_{Pl}\ln\mathcal{V}}{\mathcal{V}}$$

[3] b_s has the same mass of τ_s while b_b is essentially massless

An immediate generalisation of the $\mathbb{C}P^4_{[1,1,1,6,9]}$ model is the so called 'Swiss-cheese' Calabi-Yaus, whose volume is:

$$\mathcal{V} = \alpha \left(\tau_b^{3/2} - \sum_{i=1}^{N_{small}} \lambda_i \tau_i^{3/2} \right), \quad \alpha > 0, \quad \lambda_i > 0 \ \ \forall i = 1, ..., N_{small}$$

where τ_i control the size of the 'holes' of the Swiss-chees and τ_b controls the overall size of the Calabi-Yau.

2.1.3 Canonical normalization

Now we expand the lagrangian around the minimum, see appendix B. Starting from

$$\begin{pmatrix} \tau_b \\ \tau_s \end{pmatrix} = \boldsymbol{\tau} = \langle \boldsymbol{\tau} \rangle + \boldsymbol{\delta\tau} = \begin{pmatrix} \langle \tau_b \rangle + \delta\tau_b \\ \langle \tau_s \rangle + \delta\tau_s \end{pmatrix}$$

where

$$\langle \boldsymbol{\tau} \rangle = \langle \tau_i \rangle \ (i = b, s) \qquad \boldsymbol{\delta\tau} = (\delta\tau)_i \ (i = b, s)$$

are the VEVs and the real fields respectively. The lagrangian is:

$$\mathcal{L}_{free} = \partial_\mu \boldsymbol{\delta\tau}^T \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu \boldsymbol{\delta\tau} - V_0 - \boldsymbol{\delta\tau}^T \cdot \boldsymbol{M}^2 \cdot \boldsymbol{\delta\tau} - O(\boldsymbol{\delta\tau})^3$$

with mass matrix

$$M_{ij}^2 = \frac{1}{2}\begin{pmatrix} \frac{\partial^2 V}{\partial \tau_b^2} & \frac{\partial^2 V}{\partial \tau_b \partial \tau_s} \\ \frac{\partial^2 V}{\partial \tau_s \partial \tau_b} & \frac{\partial^2 V}{\partial \tau_s^2} \end{pmatrix}$$

and K*ä*hler matrix

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \mathcal{K}_{b\bar{b}} & \mathcal{K}_{b\bar{s}} \\ \mathcal{K}_{s\bar{b}} & \mathcal{K}_{s\bar{s}} \end{pmatrix}$$

For the explicit computation see Appendices A and C.
Let us write the above Lagrangian in terms of the canonical normalized fields ϕ and χ which are related to τ_b and τ_s in this way

$$\boldsymbol{\delta\tau} = \boldsymbol{v}_\Phi \frac{\Phi}{\sqrt{2}} + \boldsymbol{v}_\chi \frac{\chi}{\sqrt{2}}$$

The normalisation condition for the kinetic terms and the eigenvalue equation give:

$$\boldsymbol{v}_\Phi \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{v}_\chi = \delta_{\Phi,\chi}$$
$$\boldsymbol{\mathcal{K}}^{-1} M^2 \boldsymbol{v}_{\boldsymbol{\Phi}} = m_\Phi^2 \boldsymbol{v}_{\boldsymbol{\Phi}}$$
$$\boldsymbol{\mathcal{K}}^{-1} M^2 \boldsymbol{v}_{\boldsymbol{\chi}} = m_\chi^2 \boldsymbol{v}_{\boldsymbol{\chi}}$$

with

$$\boldsymbol{v_\Phi} = \begin{pmatrix} (v_\Phi)_b \\ (v_\Phi)_s \end{pmatrix} \qquad \boldsymbol{v_\chi} = \begin{pmatrix} (v_\chi)_b \\ (v_\chi)_s \end{pmatrix}$$

The Lagrangian in terms of ϕ and χ is (as shown in appendix B):

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - V_0 - \frac{1}{2} m_\Phi^2 \Phi^2 - \frac{1}{2} m_\chi^2 \chi^2$$

Adding to the lagrangian, an interaction term between the small modulus and the electromagnetic field, one has:

$$\mathcal{L} = \partial_\mu \delta\tau^T \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu \delta\tau - V_0 - \delta\tau^T \cdot M^2 \cdot \delta\tau - O(\delta\tau)^3 - \tau_s F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - V_0 - \frac{1}{2} m_\Phi^2 \Phi^2 - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{(\Phi (v_\Phi)_s + \chi (v_\chi)_s)}{4\sqrt{2}\langle \tau_s \rangle M_{Pl}} F_{\mu\nu} F^{\mu\nu}$$

with the resulting moduli coupling to photons

$$\lambda_{\Phi\gamma\gamma} = \frac{(v_\Phi)_s}{\sqrt{2}\langle \tau_s \rangle}$$
$$\lambda_{\chi\gamma\gamma} = \frac{(v_\chi)_s}{\sqrt{2}\langle \tau_s \rangle}$$

Now, the explicit expression, eigenvalues and normalized eigenvectors of $\mathbf{K^{-1}M^2}$ are:

$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2 = \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\begin{pmatrix} -9(1-7\epsilon) & 6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2) \\ -\frac{6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(1-5\epsilon+4\epsilon^2) & \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(1-3\epsilon+6\epsilon^2)\end{pmatrix}$$

$$m_\Phi^2 \simeq \mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) \simeq \frac{8a_s^2|W_0|^2\langle\tau_s\rangle^{1/2}\nu}{3\langle\tau_b\rangle^3} \sim \left(\frac{\ln\mathcal{V}}{\mathcal{V}}\right)^2 M_{Pl}^2$$

$$m_\chi^2 \simeq \frac{\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)}{\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)} \simeq \frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}} \sim \frac{M_{Pl}^2}{\mathcal{V}^3\ln\mathcal{V}}$$

$$\delta\tau_b = \left(\sqrt{6}\langle\tau_b\rangle^{1/4}\langle\tau_s\rangle^{3/4}\right)\frac{\Phi}{M_{Pl}\sqrt{2}} + \left(\sqrt{\frac{4}{3}}\langle\tau_b\rangle\right)\frac{\chi}{M_{Pl}\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{1/6})\frac{\Phi}{M_{Pl}} + \mathcal{O}(\mathcal{V}^{2/3})\frac{\chi}{M_{Pl}}$$

$$\delta\tau_s = \left(\frac{2\sqrt{6}}{3}\langle\tau_b\rangle^{3/4}\langle\tau_s\rangle^{1/4}\right)\frac{\Phi}{M_{Pl}\sqrt{2}} + \left(\frac{\sqrt{3}}{a_s}\right)\frac{\chi}{M_{Pl}\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{1/2})\frac{\Phi}{M_{Pl}} + \mathcal{O}(1)\frac{\chi}{M_{Pl}}$$

Chapter 3

A string inflationary model

As written in the previous sections, inflation, a very early period in the history of our Universe where the scale factor a(t) increased exponentially, can provide a solution to the flatness, horizon and monopole problems of the Standard Cosmological Model, and, at the same time, it can explain the origin of the CMB anisotropies, so providing a beautiful mechanism for large scale structure formation[18]. This chapter is focused on the case in which the exponential expansion is driven by[17] a scalar field φ with a flat potential V(φ). In a flat Universe, the Friedmann equation takes the form:

$$H^2 = \frac{1}{3M_p^2}\left(V + \frac{\ddot{\varphi}^2}{2} \right) \tag{3.1}$$

When the potential energy dominates over the kinetic energythe following condition is satisfied:

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1.$$

The Friedmann equation becomes:

$$H^2 \simeq \frac{V}{3M_p^2} \tag{3.2}$$

For a very flat potential, the Friedmann equation admits an inflationary solution of the form $a \sim e^{Ht}$ and the equation of motion for the inflaton reads:

$$\ddot{\varphi} - 3H\dot{\varphi} = -\frac{dV}{d\varphi}$$

One can guarentee that the inflationary period lasts for some time if the friction term $-3H\dot{\varphi}$ dominates the LHS of φ equation, forcing the inflaton to roll slowly on the potential V(φ). This is the case if:

$$\eta \equiv M_P^2 \frac{V''}{V} \ll 1.$$

The conditions $\epsilon \ll 1$ and $\eta \ll 1$ are named 'slow roll conditions' and, if satisfied, guarantee the presence of an inflationary expansion, whose amount is measured in terms of the number of e-foldings, defined as follows:

$$N_e(t) \equiv \int_{t_{\rm in}}^{t_{\rm end}} H(t')dt' = \int_{\varphi_{\rm in}}^{\varphi_{\rm end}} \frac{H}{\dot{\varphi}} d\varphi = \frac{1}{M_P^2} \int_{\varphi_{\rm end}}^{\varphi_{\rm in}} \frac{V}{V'} d\varphi.$$

where $\varphi_e nd$ is defined as the point in field space where the slow-roll conditions cease to be valid, corresponding to $\epsilon(\varphi_{end}) \sim 1$ In most models of inflation one needs, at least, $N_e \geq 50$ to solve the horizon problem, but it depends both on the inflationary scale and the reheating temperature. The observed CMB temperature fluctuations δT/T, corresponding to density perturbations $\delta\rho/\rho$, are generated by the quantum fluctuations of the inflaton. These give rise to scalar and tensor perturbations to the metric with a primordial spectrum of the form:

$$\mathcal{P}_{\mathcal{R}}(k) \propto k^{n_s - 1} \qquad \mathcal{P}_{\rm grav}(k) \propto k^{n_{\rm grav}}$$

The constants n_s and n_{grav} are the 'scalar spectral index' and the 'gravitational spectral index', respectively. They represent two important CMB observables, like also the COBE normalisation of the scalar density perturbations. Another important CMB observable is r which is defined as the ratio of the amplitude of tensor fluctuations to the amplitude of scalar fluctuations. All these four CMB observables, are only sensitive to essentially three numbers in any slow-roll inflationary model: the inflationary Hubble scale, H_{inf} , and the two small slow-roll parameters, ϵ and η, evaluated at 'horizon exit', that is at 50 or 60 e-foldings before the end of inflation when the relevant scales left the horizon and got frozen. Since tensor fluctuations have not been detected yet, it is not possible to provide a single amongst the four CMB observables. One may instead constrain the inflationary scale H_{inf} by demanding to match the COBE normalisation for the density fluctuations. Subsequently, one can work out the value of the spectral index n_s and the tensor-to-scalar ratio r in terms of ϵ and η as:

$$n_s = 1 + 2\eta - 6\epsilon \qquad\qquad r = 16\,\epsilon$$

The hope is to use experimental data to be able to distinguish between the predictions of different broad classes of models that can be divided in:

Large-Field Models , for which the inflaton travels a trans-Planckian distance in field space during inflation

Small-Field Models , for which the inflaton travels a sub-Planckian distance in field space during inflation

Hybrid Models , for which the field evolution at the end of inflation involves at least a two-dimensional field space, and for which the slow-roll parameters depend on parameters in the potential which govern the couplings between these fields.

Finally, the fact that the inflaton is nothing but a carefully-chosen modulus, whose potential will generically be affected by any mechanism of moduli stabilisation, implies a problem, the so-called 'η problem'[24]. We have the following relation between m_φ and H:

$$\eta = \frac{m_\varphi^2}{3H^2} \tag{3.3}$$

The η problem is related to the fact that, any mechanism which lifts the flat directions of all the other moduli, will in general also lift the inflaton flat direction, implying $m_\varphi \sim$ H and so $\eta \sim 1$ ruining the corresponding slow-roll condition. The no-scale property of the Kähler potential in type IIB compactifications, helps to evade the η problem, which, however, can find a definite solution only via the discovery of the extended no-scale structure.

3.1 Fibre inflation

Over the past few years there has been substantial progress towards the goal of finding cosmological inflation within the controlled solutions of string theory[21]. The hope to find generic prediction which coudl hold for all (or many) realizations of inflation in string theory. The amplitude of primordial gravity waves has recently emerged as a possible observable of this kind. Since the observational constraints on primordial tensor fluctuations are about to improve considerably it is important to identify precisely how fatal to string theory would be the observation of primordial gravity waves at this level. This has launched a search amongst theorists either to prove a no-go theorem for observable r from string theory, or to derive explicit string-inflationary scenarios that can produce observably large values of r. It was found that for K3-fibred Calabi-Yaus, LVS moduli stabilisation only fixes the overall volume and blow-up modes if string loop corrections to K are ignored. The fibre modulus then remains with a flat potential, only lifted once string loop corrections are also included. The proposal here is to explain this flatness mechanism and describe the so called *Fibre Inflation*. Now, one can use LVS results to explicitly derive the inflaton potential in this scenario, where the range of field values is large enough to easily give rise to $N_e = 60$.
The values for $N_e = 60$, between horizon exit and inflation end, completely determine the slow-roll parameters. Elimination of N_e then implies the slow-roll parameters are related by $\epsilon \sim 3/2\eta^2$, implying a relation for r: $r \sim 6(n_s - 1)^2$. Furthermore, since the value of N_e depends somewhat on the post-inflationary reheat history, the precise values of r and n_s are slightly model dependent.

3.1.1 The Lyth bound

In 1996 David Lyth[17] derived a general correlation between the ratio r and the range of values through which the (canonically normalized) inflaton field, φ, rolls in single-field slow-roll models:

$$r = 16\,\varepsilon = \frac{8}{N_{\text{eff}}^2}\left(\frac{\Delta\varphi}{M_P}\right)^2$$

$$N_{\text{eff}} = \int_{t_{\text{he}}}^{t_{\text{end}}} \left(\frac{\xi}{r}\right)^{1/2} H\,\mathrm{d}t$$

with $\epsilon = 1/2(V'/V)^2$ and $\xi(t) = 8(\frac{\dot{\varphi}}{HM_P})^2$ is the quantity whose value at horizon exit gives the observed tensor-to-scalar ratio r. Note that $N_{eff} = N_e$ if ξ is a constant. The validity of slow roll and measurements of the scalar spectral index, constrain $N_{eff} \geq 50$, and so r ≥ 0.01 requires the inflaton to roll through a trans-Planckian range $\Delta\varphi \geq M_P$. The inflaton usually has some sort of a geometrical interpretation when inflationary models are embedded into string theory, and this allows the calculation of its maximum range of variation. This considerations shows that the distance travelled by the inflaton is too small to allow $r \geq 0.01$. However, in the absence of a no-go theorem, there is strong motivation to find stringy examples which evade these kinds of constraints.
In Kähler moduli inflation,the starting point is a Swiss cheese Calabi-Yau manifold. The interest is in that part of moduli space where $\tau_b \gg \tau \gg \tau_s$. Then one finds that fixing τ_b and τ_s to their stabilised values and considering the subdominant dependence on τ, the potential gives rise to slow-roll inflation, without the need for fine-tuning parameters in the potential.

3.1.2 The simplest K3 fibration Calabi-Yau

K3-Fibration Inflation is a rapresentative model of a larger class of constructions (Fibre Inflation), which rely on choosing the inflaton to be one of those Kähler moduli whose potential is first generated at the string-loop level. In this work, we shall focus on a simplified version of these constructions. We start from a volume with two Kähler moduli[14],[19]:

$$\mathcal{V} = \tilde{t}_1\tilde{t}_2^2 + \tfrac{2}{3}\tilde{t}_2^3$$
$$\mathcal{V} = \tfrac{1}{2}\sqrt{\tilde{\tau}_1}\left(\tilde{\tau}_2 - \tfrac{2}{3}\tilde{\tau}_1\right)\,, \qquad \tilde{\tau}_1 = \tilde{t}_2^2\,, \quad \tilde{\tau}_2 = 2(\tilde{t}_1+\tilde{t}_2)\tilde{t}_2$$
$$\mathcal{V} = t_1 t_2^2 = \frac{1}{2}\sqrt{\tau_1}\,\tau_2 \quad \Leftrightarrow \quad \mathcal{V} = t_1\tau_1\,, \qquad \tau_1 = \tilde{\tau}_1, \quad \tau_2 = \tilde{\tau}_2 - \frac{2}{3}\tilde{\tau}_1$$

In the above equations the CY volume is expressed in terms of 2-cycle [15] and 4-cycle volumes. A third Kähler modulus is required for inflationary purposes:

$$\mathcal{V} = \lambda_1 t_1 t_2^2 + \lambda_3 t_3^3 = \alpha \left(\sqrt{\tau_1}\tau_2 - \gamma \tau_3^{3/2} \right) = t_1 \tau_1 - \alpha\gamma\tau_3^{3/2}$$

where the constants α and γ are given in terms of the model-dependent numbers, λ_i :

$$\alpha = \tfrac{1}{2}\,\lambda_1^{-1/2}$$
$$\gamma = (4\lambda_1/27\lambda_3)^{1/2}$$

We shall work in the parameter regime in which

$$\mathcal{V}_0 := \alpha\sqrt{\tau_1}\,\tau_2 \gg \alpha\gamma\tau_3^{3/2} \gg 1$$

Now, including the leading α' corrections to the Kähler potential, as well as including nonperturbative corrections to the superpotential, we have:

$$K = K_0 + \delta K_{(\alpha')} = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) \qquad W = W_0 + \sum_{k=1}^{3} A_k \mathrm{e}^{-a_k T_k}$$

Because our interest is in large volume regime, it is useful to consider the following approximation:

$$W \simeq W_0 + A_3 \mathrm{e}^{-a_3 T_3}$$

In this regime, the Kähler metric and its inverse are:

$$K^0_{i\bar{j}} = \frac{1}{4\tau_2^2}\begin{pmatrix} \frac{\tau_2^2}{\tau_1^2} & \gamma\left(\frac{\tau_3}{\tau_1}\right)^{3/2} & -\frac{3\gamma}{2}\frac{\sqrt{\tau_3}}{\tau_1^{3/2}}\tau_2 \\ \gamma\left(\frac{\tau_3}{\tau_1}\right)^{3/2} & 2 & -3\gamma\frac{\sqrt{\tau_3}}{\sqrt{\tau_1}} \\ -\frac{3\gamma}{2}\frac{\sqrt{\tau_3}}{\tau_1^{3/2}}\tau_2 & -3\gamma\frac{\sqrt{\tau_3}}{\sqrt{\tau_1}} & \frac{3\alpha\gamma}{2}\frac{\tau_2^2}{\mathcal{V}\sqrt{\tau_3}} \end{pmatrix}$$

$$K_0^{\bar{i}j} = 4\begin{pmatrix} \tau_1^2 & \gamma\sqrt{\tau_1}\tau_3^{3/2} & \tau_1\tau_3 \\ \gamma\sqrt{\tau_1}\tau_3^{3/2} & \frac{1}{2}\tau_2^2 & \tau_2\tau_3 \\ \tau_1\tau_3 & \tau_2\tau_3 & \frac{2}{3\alpha\gamma}\mathcal{V}\sqrt{\tau_3} \end{pmatrix}$$

In particular, here (and below), the volume is only the part

$$\mathcal{V}_0 = \alpha\sqrt{\tau_1}\tau_2$$

The resulting scalar potential simplifies to:

$$V = \frac{8\,a_3^2A_3^2}{3\alpha\gamma}\left(\frac{\sqrt{\tau_3}}{\mathcal{V}}\right)\mathrm{e}^{-2a_3\tau_3} - 4W_0a_3A_3\left(\frac{\tau_3}{\mathcal{V}^2}\right)\mathrm{e}^{-a_3\tau_3} + \frac{3\,\hat{\xi}W_0^2}{4\mathcal{V}^3}$$

with $W_0 > 0$.
Thanks to the relation

$$K_0^{3\bar{1}}K^0_{\bar{1}} + K_0^{3\bar{2}}K^0_{\bar{2}} + \text{c.c.} = -3\tau_3$$

the scalar potential dipends only on two Kähler moduli. Setting $a_1\tau_1$ large enough to switch off its non-perturbative dependence in W, there is a modulus (a proper combination of τ_1 and τ_2) describing a direction along which V is flat. This modulus plays the role of the inflaton.

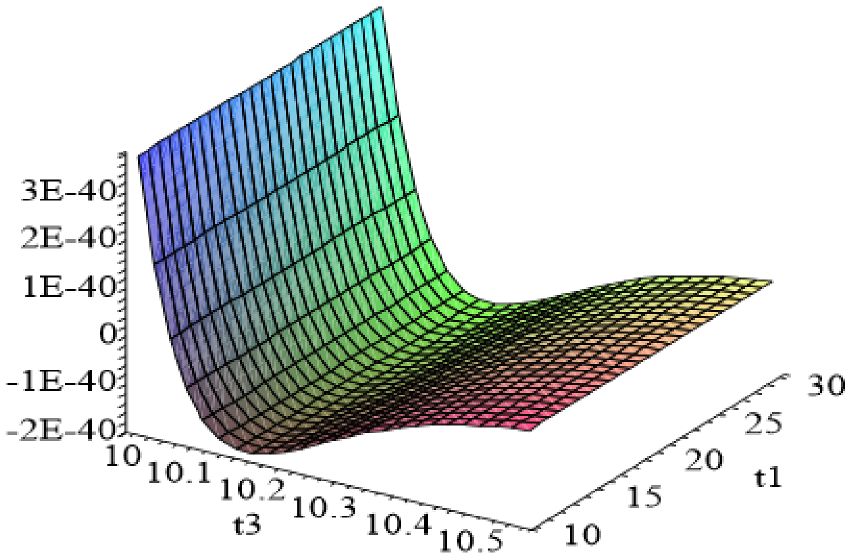

Figure 3.1: V (arbitrary units) versus τ_1 and τ_3 for one of the parameter sets discussed in the text, with volume fixed at its minimum[7].

The minimum for V is at[1]

$$\langle \tau_3 \rangle = \left(\frac{\hat{\xi}}{2\,\alpha\gamma} \right)^{2/3} \qquad \langle \mathcal{V} \rangle = \left(\frac{3\,\alpha\gamma}{4a_3 A_3} \right) W_0 \sqrt{\langle \tau_3 \rangle}\, \mathrm{e}^{a_3 \langle \tau_3 \rangle}$$

Now, the purpose is to follow some concrete numerical choices for the various underlying parameters. There are several sets of choices:[16] LVS corresponds to a volume $\sim 10^{13}$ and $M_s \sim 10^{12}$ GeV, whereas in other two models (SV), with different numerical choices, volume $\sim 10^{3}$ and $M_s \sim 10^{16}$ GeV. The choices for LVS give an inflationary potential whose energy scale is too small to provide observable primordial density fluctuations.

Inclusion of string loops

There are three relevant kinds of loop corrections which turn out to generate a potential for the remaining flat direction:

[1]These two relations do not take into account the shift in the volume minimum due to the up-lifting term

$$\delta V_{(g_s)} = \delta V^{KK}_{(g_s),\tau_1} + \delta V^{KK}_{(g_s),\tau_2} + \delta V^{W}_{(g_s),\tau_1\tau_2},$$
$$\delta V^{KK}_{(g_s),\tau_1} = g_s^2 \frac{\left(C_1^{KK}\right)^2}{\tau_1^2}\frac{W_0^2}{\mathcal{V}^2},$$
$$\delta V^{KK}_{(g_s),\tau_2} = 2g_s^2 \frac{\left(C_2^{KK}\right)^2}{\tau_2^2}\frac{W_0^2}{\mathcal{V}^2},$$
$$\delta V^{W}_{(g_s),\tau_1\tau_2} = -\left(\frac{2\,C_{12}^{W}}{t_*}\right)\frac{W_0^2}{\mathcal{V}^3}$$

Here the 2-cycle t_* denotes the intersection locus of the two 4-cycles whose volumes are given by τ_1 and τ_2. The corresponding corrections to the scalar potential are:

$$\delta V_{(g_s)} = \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\sqrt{\tau_1}} + \frac{C\tau_1}{\mathcal{V}^2}\right)\frac{W_0^2}{\mathcal{V}^2},$$

$$A = \left(g_s C_1^{KK}\right)^2 > 0,$$
$$B = 2\,C_{12}^{W}\lambda_1^{-1/2} = 4\alpha C_{12}^{W},$$
$$C = 2\,\left(\alpha g_s C_2^{KK}\right)^2 > 0.$$

Here τ_1 will turn out to be mostly the inflaton.
Minimizing these corrections of scalar potential with respect to τ_1 at fixed volume and τ_3 we obtain:

$$\frac{1}{\tau_1^{3/2}} = \left(\frac{B}{8A\mathcal{V}}\right)\left[1 + (\text{sign}\,B)\sqrt{1+\frac{32AC}{B^2}}\right]$$

$$32AC \ll B^2$$

$$\tau_1 \simeq \left(-\frac{B\mathcal{V}}{2C}\right)^{2/3} \quad \text{if } B<0 \qquad \text{or} \qquad \tau_1 \simeq \left(\frac{4A\mathcal{V}}{B}\right)^{2/3} \quad \text{if } B>0$$

Furthermore, any meaningful minimum must lie within the Kähler cone defined by the conditions that no 2-cycle or 4-cycle shrink to zero and that the overall volume be positive. Since τ_1 and τ_2 both have been considered much larger than τ_3 we have

$$\mathcal{V} \simeq \alpha\sqrt{\tau_1}\tau_2 = \lambda_1 t_1 t_2^2$$

$$t_1 = \frac{\mathcal{V}}{\tau_1}, \quad t_2 = \left(\frac{\tau_1}{\lambda_1}\right)^{1/2}, \qquad \tau_2 = 2\mathcal{V}\left(\frac{\lambda_1}{\tau_1}\right)^{1/2}$$

and the Kähler cone is given by $0 < \tau_1 < \infty$. To discuss dynamics and masses we need to consider the kinetic terms in addition to the potential, in terms of the volume and τ_1:

$$\begin{aligned} -\mathcal{L}_{kin} &= K^0_{i\bar{j}}\left(\partial_\mu T_i\, \partial^\mu \overline{T}_j\right) = \frac{1}{4}\frac{\partial^2 K_0}{\partial\tau_i \partial\tau_j}\left(\partial_\mu \tau_i\, \partial^\mu \tau_j + \partial_\mu b_i\, \partial^\mu b_j\right) \\ &= \frac{\partial_\mu \tau_1 \partial^\mu \tau_1}{4\tau_1^2} + \frac{\partial_\mu \tau_2 \partial^\mu \tau_2}{2\tau_2^2} + \cdots \\ &= \frac{3}{8\tau_1^2}\,\partial_\mu \tau_1 \partial^\mu \tau_1 - \frac{1}{2\tau_1 \mathcal{V}}\,\partial_\mu \tau_1 \partial^\mu \mathcal{V} + \frac{1}{2\mathcal{V}^2}\,\partial_\mu \mathcal{V} \partial^\mu \mathcal{V} + \cdots . \end{aligned}$$

After having established the existence of a consistent LVS minimum of the potential for all fields, the purpose is to explore the inflationary possibilities that can arise when some of these fields are displaced from these minima,in particular τ_1, beacause of its systematically flat potential in the absence of string loop corrections (it could be a good candidate for a slow-roll inflaton).

Inflationary potential

Fixing the volume and τ_3 to their τ_1-independent minimum τ_1 is displaced far away from its minimum and it rolls towards it from initially larger values. This is the approximation of single field τ_1 [2]. Choosing τ_1 as the coordinate along the inflationary direction, the relevant dynamics reduces to

$$\mathcal{L}_{inf} = -\frac{3}{8}\left(\frac{\partial_\mu \tau_1 \partial^\mu \tau_1}{\tau_1^2}\right) - V_{inf}(\tau_1)\,,$$

$$V_{inf} = V_0 + \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\sqrt{\tau_1}} + \frac{C\tau_1}{\mathcal{V}^2}\right)\frac{W_0^2}{\mathcal{V}^2}$$

$$V_0 = \frac{8\,a_3^2 A_3^2 \sqrt{\langle\tau_3\rangle}}{3\alpha\gamma\langle\mathcal{V}\rangle}\, e^{-2a_3\langle\tau_3\rangle} - \frac{4W_0 a_3 A_3 \langle\tau_3\rangle}{\langle\mathcal{V}\rangle^2}\, e^{-a_3\langle\tau_3\rangle} + \frac{3\,\hat{\xi} W_0^2}{4\langle\mathcal{V}\rangle^3} + \delta V_{up}$$

$$\delta V_{up} \sim \delta_{up}/\mathcal{V}^{4/3}$$

where it was included also an up-lifting potential, which might be produced by the tension of an anti D3 brane in a warped region somewhere in the extra dimensions. The canonically normalised inflaton is therefore given by:

$$\varphi = \frac{\sqrt{3}}{2}\ln\tau_1\,, \qquad \tau_1 = e^{\kappa\varphi}\,, \qquad \kappa = \frac{2}{\sqrt{3}}\,.$$

and the Kähler cone is:

$$0 < \tau_1 < \infty \Longleftrightarrow -\infty < \varphi < +\infty.$$

thus the inflationary dynamics can in principle take place over an infinite range in field space.
Shifting φ from its vacuum value, the potential becomes:

$$V_{inf} = V_0 + \frac{W_0^2}{\mathcal{V}^2}\left(A\,e^{-2\kappa\varphi} - \frac{B}{\mathcal{V}}\,e^{-\kappa\varphi/2} + \frac{C}{\mathcal{V}^2}\,e^{\kappa\varphi}\right)$$
$$= \frac{1}{\langle\mathcal{V}\rangle^{10/3}}\left(\mathcal{C}_0\,e^{\kappa\hat\varphi} - \mathcal{C}_1\,e^{-\kappa\hat\varphi/2} + \mathcal{C}_2\,e^{-2\kappa\hat\varphi} + \mathcal{C}_{up}\right)$$
$$\varphi = \langle\varphi\rangle + \hat\varphi$$
$$V_0 = \mathcal{C}_{up}/\langle\mathcal{V}\rangle^{10/3} \quad\Rightarrow\quad V_{inf}(\langle\varphi\rangle) = 0$$
$$\langle\varphi\rangle = \frac{1}{\sqrt{3}}\ln(\zeta\mathcal{V})$$

$$\zeta \simeq -B/2C \quad \text{if } B<0$$
$$\zeta \simeq 4A/B \quad \text{if } B>0$$

$$\mathcal{C}_0 = CW_0^2\zeta^{2/3}, \quad \mathcal{C}_1 = BW_0^2\zeta^{-1/3}, \quad \mathcal{C}_2 = AW_0^2\zeta^{-4/3} \quad \text{and} \quad \mathcal{C}_{up} = \mathcal{C}_1 - \mathcal{C}_0 - \mathcal{C}_2$$

A tipical plot for scalar potential as a function of φ is:

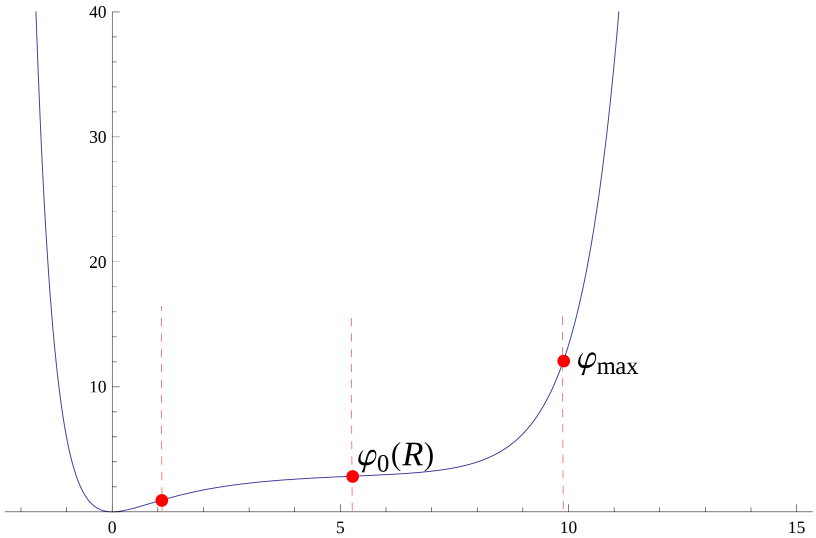

Figure 3.2: V(φ) with A,C $\ll$ B, R $= C_0/C_2 = 10^{-4}$ and volume and τ_3 fixed at their minimum

Inflationary slow-roll

Working in the most natural limit with A,C $\ll$ B and B $>$ 0 that implies $0 < C_0 \ll C_1 = 4C_2$, the potential is well approximed by:

$$V \simeq \frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}}\left[(3-R) - 4\left(1+\frac{1}{6}R\right)e^{-\kappa\hat{\varphi}/2} + \left(1+\frac{2}{3}R\right)e^{-2\kappa\hat{\varphi}} + R\,e^{\kappa\hat{\varphi}}\right]$$

$$\mathcal{C}_{up} \simeq \mathcal{C}_1 - \mathcal{C}_0 - \mathcal{C}_2 \qquad \mathcal{C}_1/\mathcal{C}_2 \simeq 4,$$

$$R := \frac{\mathcal{C}_0}{\mathcal{C}_2} = 2g_s^4\left(\frac{C_1^{KK}C_2^{KK}}{C_{12}^W}\right)^2 \ll 1 \qquad .$$

In the potential, in practice, the powers of R can be neglected in all but the last term which eventually becomes important for sufficiently large $\hat{\varphi}$. For smaller $\hat{\varphi}$ R is completely negligible. We seek inflationary slow roll focusing on the situation in which $\hat{\varphi}$ rolls down to its minimum (at $\hat{\varphi} = 0$) from positive values, with the following slow-roll parameters:

$$\varepsilon = \frac{1}{2V^2}\left(\frac{\partial V}{\partial\hat{\varphi}}\right)^2, \qquad \eta = \frac{1}{V}\left(\frac{\partial^2 V}{\partial\hat{\varphi}^2}\right)$$

with $k^2 = 4/3$ and keeping R only when it comes multiplied by $e^{k\hat{\varphi}}$, it follows that:

$$\varepsilon \simeq \frac{8}{3}\left(\frac{e^{-\kappa\hat{\varphi}/2} - e^{-2\kappa\hat{\varphi}} + \frac{1}{2}R\,e^{\kappa\hat{\varphi}}}{3 - 4\,e^{-\kappa\hat{\varphi}/2} + e^{-2\kappa\hat{\varphi}} + R\,e^{\kappa\hat{\varphi}}}\right)^2,$$

$$\eta \simeq -\frac{4}{3}\left(\frac{e^{-\kappa\hat{\varphi}/2} - 4\,e^{-2\kappa\hat{\varphi}} - R\,e^{\kappa\hat{\varphi}}}{3 - 4\,e^{-\kappa\hat{\varphi}/2} + e^{-2\kappa\hat{\varphi}} + R\,e^{\kappa\hat{\varphi}}}\right)$$

Plots of these expressions are:

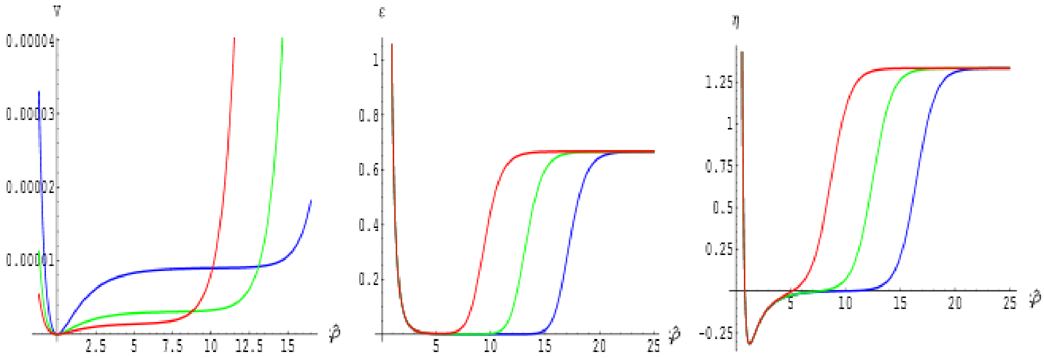

Figure 3.3: Plots of the potential and the slow-roll parameters ϵ and η with $\mathrm{R} = 10^{-8}$ (blue), $\mathrm{R} = 10^{-6}$(green), $\mathrm{R} = 10^{-4}$(red)[7].

There are three qualitatively different regions in the potential:

slow-roll regime , both ϵ and η are small and $e^{-k\hat{\varphi}/2}$ dominates in the potential. The approximation in this regime is:

$$V \simeq \frac{\mathcal{C}_2}{\langle \mathcal{V} \rangle^{10/3}} \left(3 - 4\, e^{-\kappa\hat{\varphi}/2}\right)$$

The slow-roll parameters are as follows, with an interesting relation

$$\varepsilon \simeq \frac{8}{3\left[3\, e^{\kappa\hat{\varphi}/2} - 4\right]^2}$$

$$\eta \simeq -\frac{4}{3\left[3\, e^{\kappa\hat{\varphi}/2} - 4\right]}$$

$$\varepsilon \simeq \frac{3\,\eta^2}{2}$$

small $\hat{\varphi}$ regime , the slow-roll conditions break down when $\hat{\varphi}$ is small enough that the two negative exponentials, in the potential expression, are comparative in size to produce $\eta = 0$. Accordingly, there is an inflection point in this regime.

large $\hat{\varphi}$ regime , the positive exponential dominates the potential, which becomes:

$$V \simeq \frac{m_\varphi^2}{4} R\, e^{\kappa\hat{\varphi}}$$

and the slow-roll parameters plateau at constant values $\eta \simeq 2\epsilon \simeq k^2 = 4/3$. The slow-roll condition also breaks down when k$\hat{\varphi} \simeq ln(1/R)$, providing an upper limit to the distance where slow roll occurs and, accordingly, to N_e. Going to this region η changes sign. This shows that, while ϵ is still small, $\eta \gg \epsilon > 0$. Furthermore, unlike generic single-field inflationary models, in this regime $n_s > 1$. Due to the current observational preference for $n_s < 1$, horizon exit is choosen to occur before this value of $\hat{\varphi}$.
η vanishes due to the competition between $e^{k\hat{\varphi}}$ and $e^{-k\hat{\varphi}/2}$ in the potential. This occurs when $\hat{\varphi}_0(R) = -ln(R)/\sqrt{3}$

The number of e-foldings N_e occurring during the slow-roll regime can be computed using the approximate potential for that regime:

$$N_e = \int_{\hat{\varphi}_{end}}^{\hat{\varphi}_*} \frac{V}{V'}\, \mathrm{d}\hat{\varphi} \simeq \frac{\sqrt{3}}{4} \int_{\hat{\varphi}_{end}}^{\hat{\varphi}_*} \left[3\, e^{\kappa\hat{\varphi}/2} - 4\right] \mathrm{d}\hat{\varphi} = \left[\frac{9}{4}\, e^{\kappa\hat{\varphi}/2} - \sqrt{3}\,\hat{\varphi}\right]_{\hat{\varphi}_{end}}^{\hat{\varphi}_*}$$

where $\hat{\varphi}_{end}$ represents the end of inflation and $\hat{\varphi}_*$ the value of $\hat{\varphi}$ at horizon exit.
Figure 3.4 shows how the number of e-foldings depends on $\hat{\varphi}_*$ as well as the insensitivity of this result to $\hat{\varphi}_{end}$.

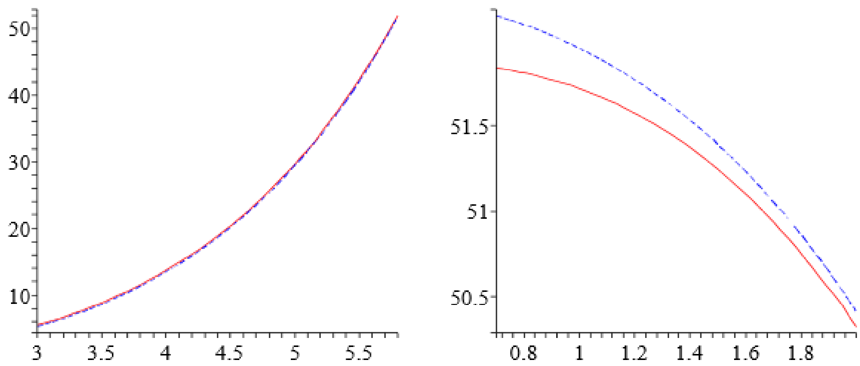

Figure 3.4: N_e vs $\hat{\varphi}_*$ (left) and N_e vs $\hat{\varphi}_{end}$ (right) for R $= 0$; The solid (red) curves are computed using the full potential while the dashed (blue) curves are computed using the approximate potential in the slow-roll regime[7].

Moreover, in the slow-roll regime we can use $\hat{\varphi}_* = \hat{\varphi}_0(R)$ and this leads to an upper limit to N_e, plotted in the here below vs x $= log_{10}$(R): Ultimately,

$$N_e^{max} \simeq \frac{9}{4}\left(R^{-1/3} - 2\right) - \left[\ln\left(\frac{1}{R}\right) - \ln 8\right]$$

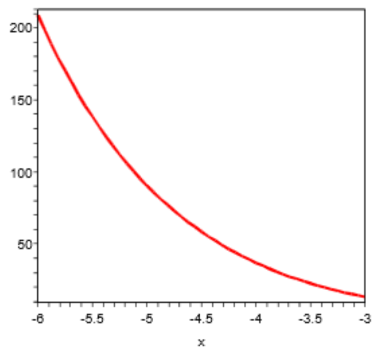

Figure 3.5: N_e, plotted in the here below vs x $= log_{10}$(R)[7].

the most robust predictions are for those observables whose values depend only on the slow roll parameters, such as the spectral index and the tensor-to-scalar ratio, which are:

$$n_s = 1 + 2\eta_* - 6\epsilon_* \qquad and \qquad r = 16\epsilon_* \tag{3.4}$$

where, in general, η_* and ϵ_* are function of $\hat{\varphi}_*$ and R; hence $n_s = n_s(\hat{\varphi}_*$,R) and $r = r(\hat{\varphi}_*$,R). For R $\ll 1$, which is required for a significant number of e-foldings, a good approximation is $n_s = n_s(\hat{\varphi}_*)$ and $r = r(\hat{\varphi}_*)$, unless $\hat{\varphi}_*$ is large enough that $Re^{k\hat{\varphi}_*}$ cannot be neglected. This is studied in depth in the next chapter. For small R we find a robust correlation between r,n_s and N_e:

$$r \simeq 6(n_s - 1)^2 \tag{3.5}$$

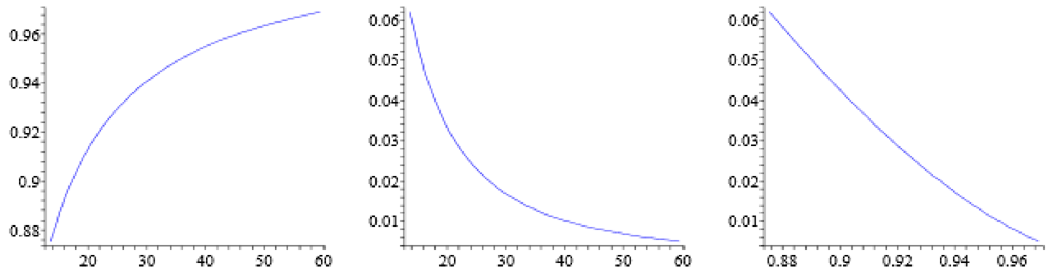

Figure 3.6: Plot of n_s vs N_e, r vs N_e and r vs vs n_s rescpectively[7].

Deviations([20]) from this correlation arise for large $\hat{\phi}_*$. This is the topic on which the next chapter is based.

Chapter 4

New slow-roll results

As explained in the previous chapter, the scalar potential is divided into three regimes

small $\hat{\varphi}$ regime

slow-roll regime

large $\hat{\varphi}$ regime

The last two listed above are of particular importance. Let us see in detail what happens in the transition between these two regimes. In the slow-roll regime the dominant term is $e^{-k\hat{\varphi}/2}$ and we have already seen the corresponding approximation for the scalar potential, that is

$$V \simeq \frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}}\left(3 - 4\,e^{-\kappa\hat{\varphi}/2}\right)$$

with $k = 2/\sqrt{3}$.
In this frame, in addition, the following relationship between the slow-roll parameters holds

$$\epsilon \simeq 3\frac{\eta^2}{2} \tag{4.1}$$

We have also seen how the number of e-foldings is calculated in this region

$$N_e = \int_{\hat{\varphi}_{end}}^{\hat{\varphi}_*} \frac{V}{V'}\,\mathrm{d}\hat{\varphi} \simeq \frac{\sqrt{3}}{4}\int_{\hat{\varphi}_{end}}^{\hat{\varphi}_*}\left[3\,e^{\kappa\hat{\varphi}/2} - 4\right]\,\mathrm{d}\hat{\varphi} = \left[\frac{9}{4}\,e^{\kappa\hat{\varphi}/2} - \sqrt{3}\,\hat{\varphi}\right]_{\hat{\varphi}_{end}}^{\hat{\varphi}_*}$$

where $\hat{\varphi}_{end}$ represents the end of inflation and $\hat{\varphi}_*$ the value of $\hat{\varphi}$ at horizon exit. N_e is, therefore, very sensitive to the value of $\hat{\varphi}_*$ and we placed as the upper limit the value $\hat{\varphi}_* = \hat{\varphi}_0(R)$. This value represents, consequently, the entry in the large $\hat{\varphi}$ regime where the terms $e^{k\hat{\varphi}}$ and $e^{-k\hat{\varphi}/2}$ become comparable. For large values of $\hat{\phi}$, the potential is completely dominated by the positive exponential term and, the slow-roll conditions are broken. Consequently, the cosmological observables, previously considered only as functions of $\hat{\varphi}_*$, now also become dipendent on R.
We tried to stretch the area of slow-roll, taking higher values for $\hat{\varphi}_*$, which still preserve the slow-roll conditions. Now, the full expression of the inflationary potential involves all the exponential terms, apart from corrections in R. We studied the following issues and their dependence on R:

- **Scalar potential**
- **Slow-roll parameters ϵ and η**
- **Cosmological observables $\mathbf{N_e}$, r and $\mathbf{n_s}$**

4.1 Scalar potential

The explicit expression for the potential is

$$V \simeq \frac{\mathcal{C}_2}{\langle\mathcal{V}\rangle^{10/3}} \left[(3-R) - 4\left(1+\frac{1}{6}\,R\right)\,e^{-\kappa\hat{\varphi}/2} + \left(1+\frac{2}{3}\,R\right)\,e^{-2\kappa\hat{\varphi}} + R\,e^{\kappa\hat{\varphi}}\right]$$

taking into account all the exponential terms.
As can be seen from Figure 4.1, three crucial values of $\hat{\phi}$ are $\hat{\varphi}_{end}$, $\hat{\varphi}_0(R)$ and $\hat{\varphi}_{max}$. The latter has been fixed as the largest possible value for $\hat{\varphi}_*$ which corresponds to the point where the slow-roll conditions are no longer valid for ϵ or η. The lower end of this range is fixed by $\hat{\varphi}_0(R)$. As already said, $\hat{\varphi}_{end}$ is the point where inflation ends and is about the same for all values of R.
Plotting the potential for different values of R, it is possible to note that, for smaller values of R, the slow-roll range increases (Figure 4.2).
This extension of the slow-roll area, allows us to study the parameters and the cosmological observables at large values of $\hat{\phi}$.

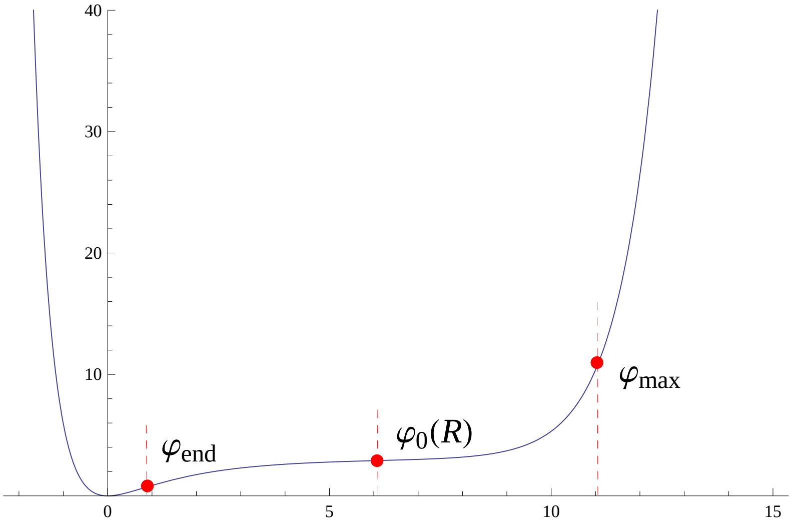

Figure 4.1: Plot of V vs $\hat{\varphi}$, with all exponential terms and R = 2,25×10^{-5}

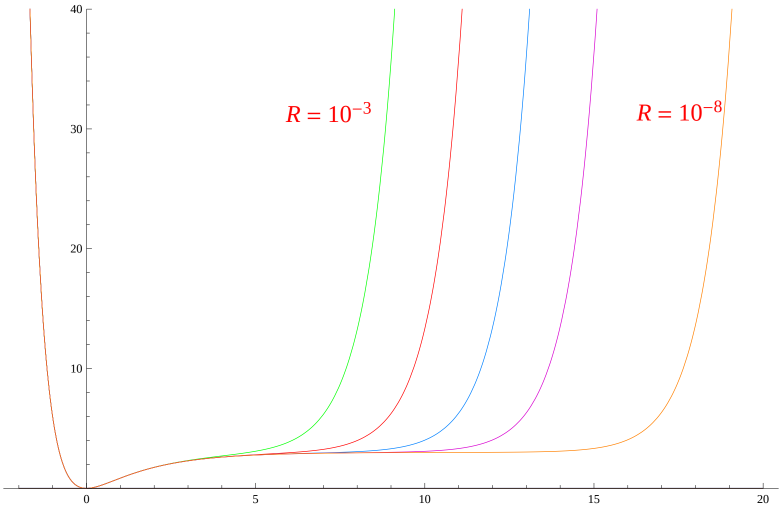

Figure 4.2: V($\hat{\varphi}$), with all exponential terms, with R = 10^{-3}, R = 10^{-4}, R = 10^{-6}, R = 10^{-8} from left to right respectively.

4.2 Slow-roll parameters

Using the standard definition of the slow-roll parameters

$$\varepsilon = \frac{1}{2V^2}\left(\frac{\partial V}{\partial \hat{\varphi}}\right)^2 , \qquad \eta = \frac{1}{V}\left(\frac{\partial^2 V}{\partial \hat{\varphi}^2}\right)$$

with $k^2 = 4/3$ and keeping R only when it comes multiplied by $e^{k\hat{\varphi}}$, we obtain

$$\varepsilon \simeq \frac{8}{3}\left(\frac{e^{-\kappa\hat{\varphi}/2} - e^{-2\kappa\hat{\varphi}} + \frac{1}{2}R\,e^{\kappa\hat{\varphi}}}{3 - 4\,e^{-\kappa\hat{\varphi}/2} + e^{-2\kappa\hat{\varphi}} + R\,e^{\kappa\hat{\varphi}}}\right)^2 ;$$

$$\eta \simeq -\frac{4}{3}\left(\frac{e^{-\kappa\hat{\varphi}/2} - 4\,e^{-2\kappa\hat{\varphi}} - R\,e^{\kappa\hat{\varphi}}}{3 - 4\,e^{-\kappa\hat{\varphi}/2} + e^{-2\kappa\hat{\varphi}} + R\,e^{\kappa\hat{\varphi}}}\right)$$

In the following plots for ϵ and η it is possible to note the same extension of the slow-roll area taking values of R between 10^{-3} and 10^{-8}:

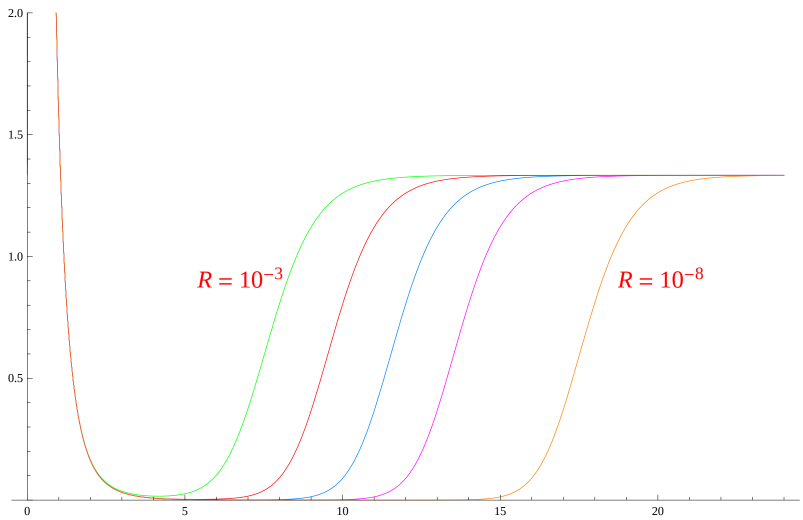

Figure 4.3: $\epsilon(\hat{\varphi})$ with $R = 10^{-3}$, $R = 10^{-4}$, $R = 10^{-6}$, $R = 10^{-8}$ from left to right respectively.

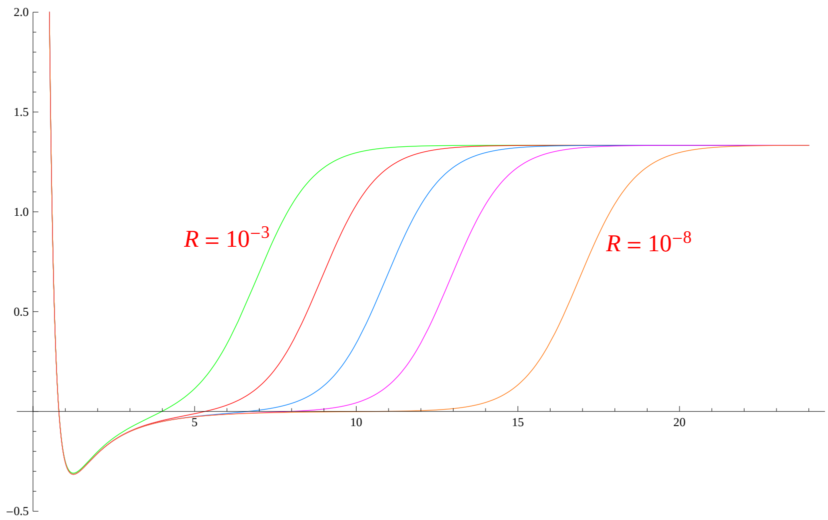

Figure 4.4: $\eta(\hat{\varphi})$ with R $= 10^{-3}$, R $= 10^{-4}$, R $= 10^{-6}$, R $= 10^{-8}$ from left to right respectively.

Finally, we plot in Figure 4.5 $\epsilon(\hat{\varphi})$ and $\eta(\hat{\varphi})$ for a given value of R (intuitively the relation 4.1 between ϵ and η is not valid for large values of $\hat{\varphi}$):

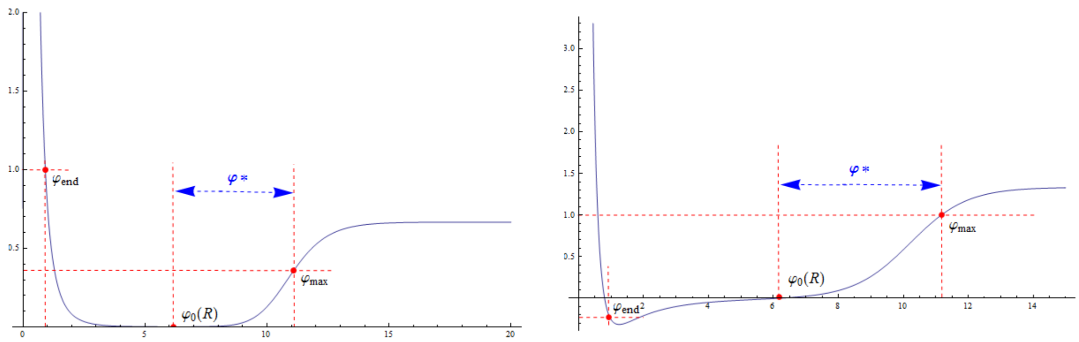

Figure 4.5: Left:$\eta(\hat{\varphi})$ with R $= 2{,}25\times10^{-5}$. Right: $\eta(\hat{\varphi})$ with R $= 2{,}25\times10^{-5}$.

From Figure 4.5 it is possibile to note the range choosen for $\hat{\varphi}_*$. The relevant points in field space for $R = 2.25 \times 10^{-5}$ are $\hat{\varphi}_{end} = 0.91$, $\hat{\varphi}_0(R) = 6.17$, $\hat{\varphi}_{max} = 11.16$ for which we have: $\eta(\hat{\varphi}_0(R)) = 0.043$, $\eta(\hat{\varphi}_{max}) = 1{,}24$, $\eta(\hat{\varphi}_{end}) =$ -0.18. $\epsilon(\hat{\varphi}_0(R)) = 5\times10^{-4}$, $\epsilon(\hat{\varphi}_{max}) = 0.37$, $\epsilon(\hat{\varphi_{end}}) = 1$.

4.3 Cosmological observables

First of all, we compute the number of e-foldings N_e as

$$N_e = \int_{\hat{\varphi}_{end}}^{\hat{\varphi}_*} \frac{V}{V'} \, d\hat{\varphi}$$

with $\hat{\varphi}_0(R) < \hat{\varphi}_* < \hat{\varphi}_{max}$. The resulting plot of $N_e(\hat{\varphi})$ for R $= 5\times10^{-5}$ is shown in Figure 4.6.

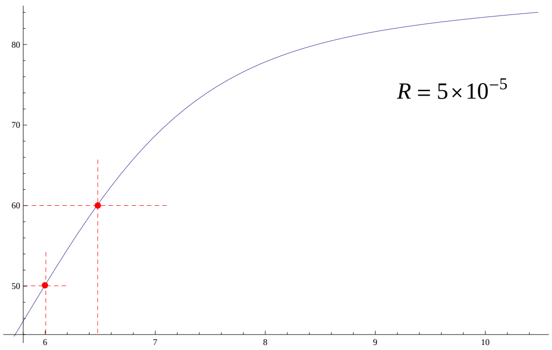

Figure 4.6: Plot of $N_e(\varphi)$

As shown in Figure 4.6, $N_e \simeq 50$ for $\hat{\varphi} = 6$, and $N_e \simeq 60$ for $\hat{\varphi} = 6.47$. We remind that, for this value of R, $\hat{\varphi}_0(R) = 5.71$ and $\hat{\varphi}_{max} = 10.47$. For this reason, R $= 5\times10^{-5}$ could be a good value for studying r and n_s. Figure 4.7 shows $N_e(\hat{\varphi})$ for R $= 10^{-3}$, R $= 10^{-4}$, R $= 10^{-6}$, R $= 10^{-8}$:

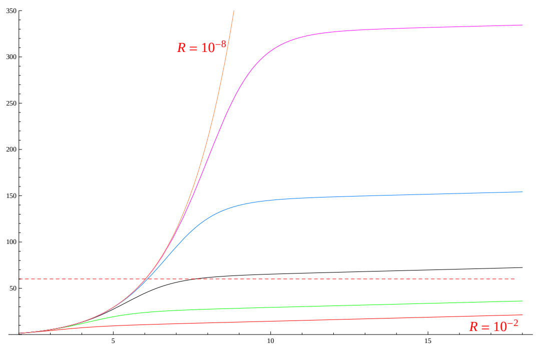

Figure 4.7: Plot of $N_e(\varphi)$ for R $= 10^{-3}$, R $= 10^{-4}$, R $= 10^{-6}$, R $= 10^{-8}$.

Now, using eq.3.1 for the spectral index n_s and the tensor-to-scalar ratio r we obtain (taking the best value for R, which in this model is $R = 2.25 \times 10^{-5}$):

- $n_s = 0.98$ and r=0.01 for $N_e \simeq 50$
- $n_s = 0.99$ and r=0.01 for $N_e \simeq 60$

These values for n_s are in slight disagreement with Planck 2015 results which predict $ns = 0.9655 \pm 0.0062$ (68% CL) for the base ΛCDM model [9]. However the value of the present Hubble scale inferred from Planck data is in 2.4σ tension with the direct measurement of the Hubble Space Telescope. This could be an intruguing indication of new physics beyond the base ΛCDM model. A very well theoretically motivated extension of the base ΛCDM model is dark radiation.
Dark radiation is usually parameterized by the effective number of neutrino-like species N_{eff}, defined so that the total relativistic energy density in neutrinos and any other dark radiation is given in terms of the photon density ρ_γ at T $\ll$ 1 MeV as

$$\rho = N_{\rm eff} \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \rho_\gamma$$

The standard cosmological prediction is $N_{eff} = 3.046$, since neutrinos are not completely de-coupled at electron-positron annihilation and are subsequently slightly heated[25]. In addition to massless sterile neutrinos, a variety of other particles could contribute to N_{eff}. We assume that the additional massless particles are produced well before recombination, and neither interact nor decay, so that their energy density scales with the expansion exactly like massless neutrinos.

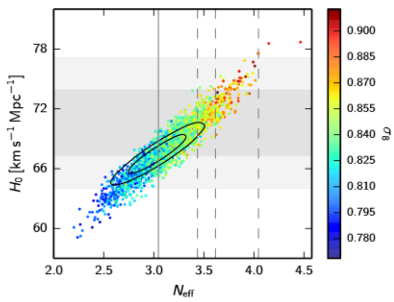

Figure 4.8: H_0 versus N_{eff} [9].

For $N_{eff} > 3$ the Planck data favour higher values of the Hubble parameter than the Planck base Λ CDM value which is in better agreement with direct measurements of H_0. As shown in Figure 4.8, larger N_{eff} corresponds to a region of parameter space with significantly higher Hubble parameter which, in turn, gives a larger values of the spectral index in perfect agreement with the prediction of our inflationary model:

$$\left.\begin{aligned} \sigma_8 &= 0.850 \pm 0.015 \\ n_s &= 0.983 \pm 0.006 \\ H_0 &= 70.6 \pm 1.0 \end{aligned}\right\} \textit{Planck}\ \text{TT+lowP};\ \Delta N_{\rm eff} = 0.39.$$

Figure 4.9 shows also $n_s(\hat{\varphi})$ for R $= 10^{-3}$, R $= 10^{-4}$, R $= 10^{-6}$, R $= 10^{-8}$

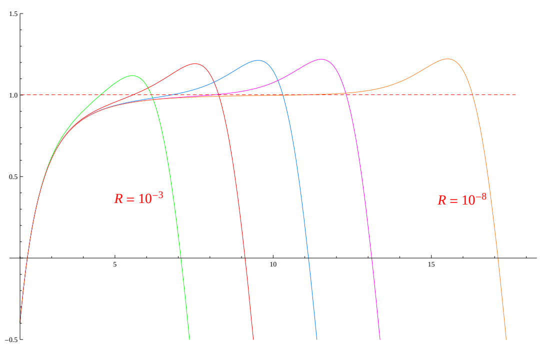

Figure 4.9: Plot of $n_s(\varphi)$ with R $= 10^{-3}$, R $= 10^{-4}$, R $= 10^{-6}$, R $= 10^{-8}$.

In Figure 4.9, the red horizontal line shows the deviations from the slow-roll regime where $n_s < 1$.
All these calculations have been performed for $k = 2/\sqrt{3}$ corresponding to the K3-fibration model, described in chapter 3. The previous expression for the volume was

$$V \simeq \alpha\sqrt{\tau_1}\tau_2 \tag{4.2}$$

Let us now investigate if a different value of k might give rise to different phenomenological results which are more compatible with recent Planck data. In order to do that, we consider a slightly different form of the volume:

$$V \simeq \sqrt{\tau_1\tau_2\tau_3} \tag{4.3}$$

We now assume that τ_3 is fixed at $\tau_3 = \alpha\tau_1$ with $\alpha > 0$ due to some stabilisation effects (like D-terms for example). By requiring then a constant volume during inflation, that is:

$$0 = \partial_\mu V = \frac{\partial V}{\partial \tau_1} + \frac{\partial V}{\partial \tau_2} + \frac{\partial V}{\partial \tau_3} = \frac{\sqrt{\tau_2 \tau_3}}{2\sqrt{\tau_1}} \partial \tau_1 + \frac{\sqrt{\tau_1 \tau_3}}{2\sqrt{\tau_2}} \partial \tau_2 + \frac{\sqrt{\tau_1 \tau_2}}{2\sqrt{\tau_3}} \partial \tau_3 \quad (4.4)$$

we obtain

$$\frac{\partial \tau_2}{\tau_2} = -2 \frac{\partial \tau_1}{\tau_1} \quad (4.5)$$

This yields:

$$\frac{1}{4} \frac{\partial_\mu \tau_1 \partial^{\mu \tau_1^2}}{\tau_1^2} + \frac{1}{4} \frac{\partial_\mu \tau_2 \partial^\mu \tau_2^2}{\tau_2^2} + \frac{1}{4} \frac{\partial_\mu \tau_3 \partial^\mu \tau_3^2}{\tau_3^2} \quad (4.6)$$

$$= \frac{1}{4} \frac{\partial_\mu \tau_1 \partial^\mu \tau_1^2}{\tau_1^2} + \frac{\partial_\mu \tau_1 \partial^\mu \tau_1^2}{\tau_1^2} \quad (4.7)$$

$$= \frac{1}{2} \frac{\partial \tau_1}{\tau_1}^2 + \frac{\partial \tau_1}{\tau_1}^2 = \frac{3}{2} \frac{\partial \tau_1}{\tau_1}^2 \quad (4.8)$$

and from

$$\frac{3}{2} \frac{\partial \tau_1}{\tau_1}^2 = \frac{1}{2} \partial \varphi^2 \quad (4.9)$$

$$\varphi = \sqrt{3} ln \tau_1 \qquad \Longrightarrow \qquad k = \frac{1}{\sqrt{3}} \quad (4.10)$$

This change in k yields the following values for n_s and r:

- $n_s = 0.967$ and r $= 0.019$ for $N_e \simeq 50$
- $n_s = 0.973$ and r $= 0.015$ for $N_e \simeq 60$

with $\epsilon_{max} = 0.16$ and $\eta_{max} = 0.33$.

In this new framework, n_s is in perfect agreement with Planck 2015 results without the need to add extra neutrinos-like species, as it was written before.

Conclusions

As we have explained in chapter 1, type IIB string theory seems to be, at present, the most promising way to connect string theory with particle physics and cosmology. An intriguing connection is represented by inflation. Inflation is believed to have occurred at an enormous energy scale, far out of reach of terrestrial particle accelerators. Any description of the inflationary era therefore requires a considerable extrapolation of the known laws of physics. In this approach, a suitable inflationary potential $V(\varphi)$ is postulated and the details of the primordial fluctuation spectra depend on its precise shape.
This thesis is based on type IIB moduli stabilisation and it is focused on the Large Volume Scenario. Within this framework the dilaton and the complex structure moduli are fixed semiclassically through the presence of branes and fluxes, while Kähler moduli are stabilised by an interplay between non-perturbative corrections to the low-energy superpotential, W , and perturbative α' and string-loop corrections to the Kähler potential, K, of the effective low-energy 4D supergravity. Furthermore, the LARGE volume that defines these scenarios naturally arises as an exponentially large function of the small parameters that control the calculation. Starting from the leading order expression of K and W, we added subdominant corrections and we showed how an AdS minimum emerges which breaks SUSY. This minimum can be uplifted to dS due to an appropriate mechanism. Moreover, we also explicitly performed a full canonical normalisation of all Kähler moduli. Subsequently, we described a promising string inflationary model called Fibre Inflation. The goal is to find cosmological inflation amongst the controlled solutions of string theory. The hope is to find observable predictions which can be fully trustable from the theoretical point of view. The amplitude of primordial gravity waves has recently emerged as a possible observable of this kind[22][23]. This has launched a search amongst theorists either to prove a no-go theorem for observable r from string theory, or to derive explicit string-inflationary scenarios that can produce observably large values of r. Ultimately, it was found that for K3-fibred Calabi-Yaus, LVS moduli

stabilisation only fixes the overall volume and blow-up modes while leaving a flat direction, if string loop corrections to K are ignored. In chapter 3 we review a string inflationary model where the remaining flat direction behaves as an inflaton field. We analysed different properties of this model taking all the exponential terms into account. In particular we focused on:

- **Scalar potential**
- **Slow-roll parameters ϵ and η**
- **Cosmological observables $\mathbf{N_e}$, n_s and r**

The results obtained in this way, were compared with those obtained in the case where the Calabi-Yau volume takes a slightly different form.
As it is possible to see from the values obtained for n_s and r (corresponding to $50 < N_e < 60$), the new Fibre Inflation model is compatible with the data recently reported by the PLANCK satellite without using a number of neutrino-like species different from the one of the Standard Model, overcoming the difficulties of the standard Fibre Inflation shown in the section 4.3, whose predictions approach the PLANCK 2015 results only if one includes new neutrino-like degrees of freedom.
The PLANCK 2015 results are in very good agreement with the 2013 analysis of the Planck nominal-mission temperature data, but with increased precision. The Planck temperature data combined with Planck lensing give a spectral index $n_s = 0.968 \pm 0.006$ [9]. Furthermore, combining Planck observations with other astrophysical data they found $N_{eff} = 3.15\pm\ 0.23$ for the effective number of relativistic degrees of freedom, consistent with the value 3.046 of the Standard Model of particle physics. The spatial curvature of our Universe is found to be very close to zero. Adding a tensor component, as a single-parameter extension to base ΛCDM, they found an upper limit on the tensor-to-scalar ratio of order $r_{0.002} < 0.09$ consistent with the B-mode polarization constraints from a joint analysis of BICEP2, Keck Array, and Planck (BKP) data.

Appendix A

Kähler metric

Starting from the expression of the Kähler potential in the framework of τ_b and τ_s (2.22) we obtain:

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \mathcal{K}_{b\bar{b}} & \mathcal{K}_{b\bar{s}} \\ \mathcal{K}_{s\bar{b}} & \mathcal{K}_{s\bar{s}} \end{pmatrix}$$

with

$$K_{i\bar{j}} \equiv \frac{\partial^2 K}{\partial T_i \partial T_{\bar{j}}} \tag{A.1}$$

in which i and j run over Kähler moduli. Let us find the elements of the Kähler matrix:

$$\begin{aligned} \mathcal{K}_b &= \frac{\partial \mathcal{K}}{\partial T_b} = \frac{\partial \mathcal{K}}{\partial \tau_b}\frac{\partial \tau_b}{\partial T_b} = \frac{1}{2}\frac{\partial \mathcal{K}}{\partial \tau_b} \\ &= \frac{1}{2}(-2)\frac{3/2\,\tau_b^{1/2}}{\tau_b^{3/2} - \tau_s^{3/2} + \xi'} \\ &= -\frac{3}{2}\frac{\tau_b^{1/2}}{\tau_b^{3/2} - \tau_s^{3/2} + \xi'} \end{aligned}$$

and

$$
\begin{aligned}
\mathcal{K}_{b\bar{b}} &= \frac{\partial \mathcal{K}_b}{\partial \bar{T}_{\bar{b}}} = \frac{\partial \mathcal{K}_b}{\partial \tau_b}\frac{\partial \tau_b}{\partial \bar{T}_{\bar{b}}} = \frac{1}{2}\frac{\partial \mathcal{K}_b}{\partial \tau_b} \\
&= -\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left[\frac{(1/2)\tau_b^{-1/2}(\tau_b^{3/2} - \tau_s^{3/2} + \xi') - \tau_b^{1/2}(3/2)\tau_b^{1/2}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^2}\right] \\
&\sim -\left(\frac{3}{4}\right)\frac{(1/2)\tau_b - (3/2)\tau_b}{\tau_b^3} \\
&= \frac{3}{4\tau_b^2}
\end{aligned}
$$

Since the Kähler metric is symmetric, one needs to calculate only one element in the diagonal:

$$
\begin{aligned}
\mathcal{K}_{s\bar{b}} &= \frac{\partial \mathcal{K}_{\bar{b}}}{\partial T_s} = \frac{\partial \mathcal{K}_{\bar{b}}}{\partial \tau_s}\frac{\partial \tau_s}{\partial T_s} = \frac{1}{2}\frac{\partial \mathcal{K}_{\bar{b}}}{\partial \tau_s} \\
&= \left(\frac{1}{2}\right)\left(-\frac{3\tau_b^{1/2}}{2}\right)(-1)(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^{-2}\left(-\frac{3}{2}\tau_s^{1/2}\right) \\
&= -\left(\frac{9}{8}\right)\frac{\tau_b^{1/2}\tau_s^{1/2}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^2} \\
&\sim -\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}
\end{aligned}
$$

Now, starting from

$$\mathcal{K}_s = \frac{3}{2}\frac{\tau_s^{1/2}}{\tau_b^{3/2} - \tau_s^{3/2} + \xi'}$$

$$\begin{aligned}\mathcal{K}_{s\bar{s}} &= \frac{\partial \mathcal{K}_s}{\partial \bar{T}_{\bar{s}}} = \frac{\partial \mathcal{K}_s}{\partial \tau_s}\frac{\partial \tau_s}{\partial \bar{T}_{\bar{s}}} = \frac{1}{2}\frac{\partial \mathcal{K}_s}{\partial \tau_s} \\ &= \left(\frac{3}{4}\right)\frac{(1/2)\tau_s^{-1/2}(\tau_b^{3/2} - \tau_s^{3/2} + \xi') - \tau_s^{1/2}(-3/2)\tau_s^{1/2}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^2} \\ &\sim \frac{3}{8}\frac{\tau_b^{3/2}\tau_s^{-1/2}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^2} \\ &\sim \frac{3}{8\tau_b^{3/2}\tau_s^{1/2}}\end{aligned}$$

The Kähler matrix and the corresponding inverse $K^{i\bar{j}}$ are as follows:

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \frac{3}{4\tau_b^2} & -\frac{9}{8}\frac{\tau_s^{1/2}}{\tau_b^{5/2}} \\ -\frac{9}{8}\frac{\tau_s^{1/2}}{\tau_b^{5/2}} & \frac{3}{8\tau_b^{3/2}\tau_s^{1/2}} \end{pmatrix}$$

$$\mathcal{K}^{i\bar{j}} \begin{pmatrix} \frac{4\tau_b^2}{3} & 4\tau_b\tau_s \\ 4\tau_b\tau_s & \frac{8\tau_b^{3/2}\tau_s^{1/2}}{3} \end{pmatrix}$$

Appendix B

Minimum of the scalar potential

Starting from the explicit expression of the scalar potential:

$$V = \frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{3\tau_b^{3/2}} - \frac{\mu a_s|W_0|\tau_s e^{-a_s\tau_s}}{\tau_b^3} + \frac{\nu|W_0|^2}{\tau_b^{9/2}}$$

the scalar potential is minimised in this way:

$$\frac{\partial V}{\partial\tau_b} = 0 = \frac{\partial V}{\partial\tau_s} \tag{B.1}$$

From the first of above equations:

$$\begin{aligned} 0 = \frac{\partial V}{\partial\tau_b} &= -\frac{3\lambda a_s^2\tau_s^{1/2}e^{-2a_s\tau_s}}{2\tau_b^{5/2}} + \frac{3\mu|W_0|a_s\tau_s e^{-a_s\tau_s}}{\tau_b^4} - \frac{9\nu|W_0|^2}{2\tau_b^{11/2}} \\ &= -\left(\frac{3\lambda a_s^2\tau_s^{1/2}e^{-2a_s\tau_s}}{2\tau_b^{11/2}}\right)\left(\tau_b^3 - \frac{2\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\tau_b^{3/2} + \frac{3\nu|W_0|^2}{\lambda a_s^2\tau_s^{1/2}e^{-2a_s\tau_s}}\right) \end{aligned}$$

Setting x = ${\tau_b}^3/2$

$$x^2 - \frac{2\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\,x + \frac{3\nu|W_0|^2}{\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}} = 0$$

We obtain the solutions:

$$\begin{aligned} x &= \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}} \pm \sqrt{\left(\frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\right)^2 - \frac{3\nu|W_0|^2}{\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}} \\ &= \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}} \pm \sqrt{\left(\frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\right)^2 \left(1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}\right)} \\ &= \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}} \left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right) \end{aligned}$$

Now, taking into consideration the right hand side of the equation (B.1)

$$\begin{aligned} 0 = \frac{\partial V}{\partial \tau_s} &= \frac{\lambda a_s^2}{\tau_b^{3/2}} e^{-2a_s\tau_s}\left[(1/2)\tau_s^{-1/2} - 2a_s\tau_s^{1/2}\right] - \frac{\mu|W_0|a_s}{\tau_b^3}\, e^{-a_s\tau_s}(1 - a_s\tau_s) \\ &= \frac{\lambda a_s^2}{\tau_b^{3/2}} e^{-2a_s\tau_s}\left[\frac{1}{2\tau_s^{1/2}}\,(1 - 4a_s\tau_s) - \frac{\mu|W_0|}{\lambda a_s \tau_b^{3/2} e^{-a_s\tau_s}}\,(1 - a_s\tau_s)\right] \end{aligned}$$

That is equivalent to this expression

$$\frac{1-4a_s\tau_s}{2\tau_s^{1/2}} = \frac{\mu|W_0|}{\lambda a_s \tau_b^{3/2} e^{-a_s\tau_s}}\left(1-a_s\tau_s\right)$$
$$e^{-a_s\tau_s} = \frac{2\mu|W_0|\tau_s^{1/2}}{\lambda a_s \tau_b^{3/2}}\frac{1-a_s\tau_s}{1-4a_s\tau_s}$$

For $\tau_s \gg 1$, setting y $= a_s\tau_s$ and performing a Taylor expansion in the limit y $\gg 1$, one has:

$$\frac{1-y}{1-4y} = \frac{1}{4} - \frac{3}{16y^2} - \frac{3}{64y^2} + \mathcal{O}\left(\frac{1}{y^3}\right)$$

which leads to

$$e^{-a_s\tau_s} = \frac{\mu|W_0|\tau_s^{1/2}}{2\lambda a_s \tau_b^{3/2}}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]$$

Now, inserting the above equation in the previous result for τ_b, one has:

$$\tau_b^{3/2} = \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)$$

$$\Downarrow$$

$$\tau_b^{3/2} = \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)\frac{2\lambda a_s\tau_b^{3/2}}{\mu|W_0|\tau_s^{1/2}}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]^{-1}$$

$$1 = 2\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]^{-1}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)$$

Multiplying both sides of the last equation for the square-bracket term

$$\frac{1}{2}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right] = 1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}$$

$$\frac{1}{2}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right] - 1 = \pm\sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}$$

$$-\frac{1}{2} - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right) = \pm\sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}$$

and squaring both sides

$$\frac{1}{4}\left[1+\frac{3}{2a_s\tau_s}+\frac{9}{(4a_s\tau_s)^2}+\frac{6}{(4a_s\tau_s)^2}+\mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]=1-\frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}$$

A little algebra yields:

$$\frac{\mu^2}{4\lambda}\,\tau_s^{3/2}=\nu\left[1-\frac{1}{2a_s\tau_s}-\frac{5}{(4a_s\tau_s)^2}+\mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]^{-1}$$

Recalling that y $= a_s\tau_s \gg 1$, we can expand the term in the square bracket and, finally, obtain:

$$\frac{\mu^2}{4\lambda}\,\tau_s^{3/2}=\nu\left(1+\frac{1}{2a_s\tau_s}+\frac{9}{(4a_s\tau_s)^2}+\dots\right)$$

Appendix C

Mass matrix

From the definition of the mass matrix we are going to calculate the second derivatives of the scalar potential, with respect to τ_b and τ_s, evaluated at the minimum:

$$
\begin{aligned}
\frac{\partial^2 V}{\partial^2 \tau_b} &= \frac{\partial}{\partial \tau_b}\left[-\frac{3\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}{2\tau_b^{5/2}} + \frac{3\mu |W_0| a_s \tau_s e^{-a_s\tau_s}}{\tau_b^4} - \frac{9\nu |W_0|^2}{2\tau_b^{11/2}} \right] \\
&= \frac{15\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}{4\tau_b^{7/2}} - \frac{12\mu |W_0| a_s \tau_s e^{-a_s\tau_s}}{\tau_b^5} + \frac{99\nu |W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15\lambda a_s^2 \tau_s^{1/2}}{4\tau_b^{7/2}} \left[\frac{\mu |W_0| \tau_s^{1/2}}{2\lambda a_s \tau_b^{3/2}} \left(1 - \frac{3}{4a_s\tau_s} - \frac{3}{16 a_s^2 \tau_s^2}\right)\right]^2 \\
&- \frac{12\mu |W_0| a_s \tau_s}{\tau_b^5} \left[\frac{\mu |W_0| \tau_s^{1/2}}{2\lambda a_s \tau_b^{3/2}} \left(1 - \frac{3}{4a_s\tau_s} - \frac{3}{16 a_s^2 \tau_s^2}\right)\right] + \frac{99\nu |W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15\mu^2 |W_0|^2 \tau_s^{3/2}}{16\lambda \tau_b^{13/2}} \left(1 - \frac{3}{2a_s\tau_s} + \frac{3}{(4a_s\tau_s)^2}\right) - \frac{12\mu^2 |W_0|^2 \tau_s^{3/2}}{2\lambda \tau_b^{13/2}} \left(1 - \frac{3}{2a_s\tau_s} - \frac{3}{(4a_s\tau_s)^2}\right) \\
&+ \frac{99\nu |W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15 |W_0|^2 \nu}{4\tau_b^{13/2}} \left(1 + \frac{1}{2a_s\tau_s} + \frac{9}{(4a_s\tau_s)^2}\right)\left(1 - \frac{3}{2a_s\tau_s} + \frac{3}{(4a_s\tau_s)^2}\right) \\
&- \frac{24 |W_0|^2 \nu}{\tau_b^{13/2}} \left(1 + \frac{1}{2a_s\tau_s} + \frac{9}{(4a_s\tau_s)^2}\right)\left(1 - \frac{3}{2a_s\tau_s} - \frac{3}{(4a_s\tau_s)^2}\right) \\
&+ \frac{99\nu |W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15 |W_0|^2 \nu}{4\tau_b^{13/2}} \left(1 - \frac{1}{2a_s\tau_s}\right) - \frac{24 |W_0|^2 \nu}{\tau_b^{13/2}} \left(1 - \frac{1}{4a_s\tau_s}\right) + \frac{99\nu |W_0|^2}{4\tau_b^{13/2}}
\end{aligned}
$$

that yields:

$$\frac{\partial^2 V}{\partial \tau_b^2} = \frac{9|W_0|^2 \nu}{2\tau_b^{13/2}} \left(1 + \frac{1}{2a_s\tau_s}\right)$$

Similarly for others matrix elements we obtain:

$$\frac{\partial^2 V}{\partial \tau_s^2} = \frac{2a_s^2|W_0|^2 \nu}{\tau_b^{9/2}} \left(1 - \frac{3}{4a_s\tau_s} + \frac{6}{(4a_s\tau_s)^2}\right)$$
$$\frac{\partial^2 V}{\partial \tau_b \tau_s} = -\frac{3a_s|W_0|^2 \nu}{\tau_b^{11/2}} \left(1 - \frac{5}{4a_s\tau_s} + \frac{4}{(4a_s\tau_s)^2}\right)$$

At this point the mass matrix is as follows

$$M_{ij}^2 = \begin{pmatrix} \frac{9|W_0|^2\nu}{4\tau_b^{13/2}}\left(1 + \frac{1}{2a_s\tau_s}\right) & -\frac{3a_s|W_0|^2\nu}{2\tau_b^{11/2}}\left(1 - \frac{5}{4a_s\tau_s} + \frac{4}{(4a_s\tau_s)^2}\right) \\ -\frac{3a_s|W_0|^2\nu}{2\tau_b^{11/2}}\left(1 - \frac{5}{4a_s\tau_s} + \frac{4}{(4a_s\tau_s)^2}\right) & \frac{a_s^2|W_0|^2\nu}{\tau_b^{9/2}}\left(1 - \frac{3}{4a_s\tau_s} + \frac{6}{(4a_s\tau_s)^2}\right) \end{pmatrix}$$

Appendix D

Lagrangian in terms of canonically normalised fields

Let us compute kinetic and mass terms using the normalization condition:

$$\begin{aligned}\partial_\mu \boldsymbol{\delta\tau}^T \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu \boldsymbol{\delta\tau} &= \partial_\mu \left[\frac{\Phi}{\sqrt{2}} (\boldsymbol{v}_\Phi)^T + \frac{\chi}{\sqrt{2}} (\boldsymbol{v}_\chi)^T \right] \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu \left[\frac{\Phi}{\sqrt{2}} \boldsymbol{v}_\Phi + \frac{\chi}{\sqrt{2}} \boldsymbol{v}_\chi \right] \\ &= \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi\end{aligned}$$

$$\begin{aligned}\boldsymbol{\delta\tau}^T \cdot \boldsymbol{M}^2 \cdot \boldsymbol{\delta\tau} &= \boldsymbol{\delta\tau}^T \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{\mathcal{K}}^{-1} \boldsymbol{M}^2 \cdot \boldsymbol{\delta\tau} \\ &= \left[\frac{\Phi}{\sqrt{2}} (\boldsymbol{v}_\Phi)^T + \frac{\chi}{\sqrt{2}} (\boldsymbol{v}_\chi)^T \right] \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{\mathcal{K}}^{-1} \boldsymbol{M}^2 \left[\frac{\Phi}{\sqrt{2}} \boldsymbol{v}_\Phi + \frac{\chi}{\sqrt{2}} \boldsymbol{v}_\chi \right] \\ &= \left[\frac{\Phi}{\sqrt{2}} (\boldsymbol{v}_\Phi)^T + \frac{\chi}{\sqrt{2}} (\boldsymbol{v}_\chi)^T \right] \cdot \boldsymbol{\mathcal{K}} \cdot \left[m_\Phi^2 \frac{\Phi}{\sqrt{2}} \boldsymbol{v}_\Phi + m_\chi^2 \frac{\chi}{\sqrt{2}} \boldsymbol{v}_\chi \right] \\ &= \frac{1}{2} m_\Phi^2 \Phi^2 + \frac{1}{2} m_\chi^2 \chi^2\end{aligned}$$

Then we have to recover the Maxwell lagrangian and the interaction term

$$\begin{aligned}\kappa\, \tau_s\, F_{\mu\nu} F^{\mu\nu} &= \kappa \left(\langle \tau_s \rangle + \delta\tau_s \right) F_{\mu\nu} F^{\mu\nu} \\ &= \kappa \langle \tau_s \rangle F_{\mu\nu} F^{\mu\nu} + \kappa\, \delta\tau_s\, F_{\mu\nu} F^{\mu\nu}\end{aligned}$$

($M_{PL} = 1$)

Furthermore, setting

$$\kappa \langle \tau_s \rangle F_{\mu\nu} F^{\mu\nu} = \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

$$F_{\mu\nu} F^{\mu\nu} = \frac{1}{4\kappa \langle \tau_s \rangle} G_{\mu\nu} G^{\mu\nu}$$

we get(renaming G with F and recalling the expression of $\delta\tau_s$ in terms of ϕ and χ):

$$\begin{aligned} \kappa \langle \tau_s \rangle F_{\mu\nu} F^{\mu\nu} + \kappa \, \delta\tau_s \, F_{\mu\nu} F^{\mu\nu} &= \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \kappa \, \delta\tau_s \frac{1}{4\kappa \langle \tau_s \rangle} G_{\mu\nu} G^{\mu\nu} \\ &= \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{\delta\tau_s}{\langle \tau_s \rangle} G_{\mu\nu} G^{\mu\nu} \end{aligned}$$

$$\kappa \, \tau_s \, F_{\mu\nu} F^{\mu\nu} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{(\Phi(v_\Phi)_s + \chi(v_\chi)_s)}{4\sqrt{2}\langle \tau_s \rangle}$$

Now, one can multiply the matrix $\mathbf{K^{-1}}$ by $\mathbf{M^2}$, knowing that $m_\phi^2 \gg m_\chi^2$, at the leading order in ϵ

$$\begin{aligned}
&\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) = (m_\Phi^2 + m_\chi^2) \simeq m_\Phi^2 \\
&\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) = m_\Phi^2 m_\chi^2 \\
&\frac{\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)}{\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)} \simeq m_\chi^2
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) &= \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\left[-9\,(1-7\epsilon) + \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}\,(1-3\epsilon+6\epsilon^2)\right] \\
&\simeq \left(\frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\right)\left(\frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}\right) \\
&= \frac{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}{3\langle\tau_b\rangle^3} \\
\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) &= \frac{4a_s^2\langle\tau_s\rangle^2|W_0|^4\nu^2}{9\langle\tau_b\rangle^9}\Bigg[-9\,(1-7\epsilon)\cdot\frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}\,(1-3\epsilon+6\epsilon^2) - \\
&\quad + 6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2)\cdot(1-5\epsilon+16\epsilon^2)\,\frac{-6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}\Bigg] \\
&= \frac{18a_s\langle\tau_s\rangle^2|W_0|^4\nu^2}{\langle\tau_b\rangle^{15/2}\langle\tau_s\rangle^{1/2}} \\
\frac{\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)}{\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)} &= \left(\frac{18a_s\langle\tau_s\rangle^2|W_0|^4\nu^2}{\langle\tau_b\rangle^{15/2}\langle\tau_s\rangle^{1/2}}\right)\left(\frac{3\langle\tau_b\rangle^3}{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}\right) \\
&= \frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}
\end{aligned}$$

At this point we have to solve

$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2 \begin{pmatrix} (v_\Phi)_b \\ (v_\Phi)_s \end{pmatrix} = m_\Phi^2 \begin{pmatrix} (v_\Phi)_b \\ (v_\Phi)_s \end{pmatrix}$$
$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2 \begin{pmatrix} (v_\chi)_b \\ (v_\chi)_s \end{pmatrix} = m_\chi^2 \begin{pmatrix} (v_\chi)_b \\ (v_\chi)_s \end{pmatrix}$$

Employing the normalization condition for $\mathbf{v}_\phi$ and $\mathbf{v}_\xi$, it is easy to obtain:

$$\boldsymbol{v_\Phi} = \begin{pmatrix} (v_\Phi)_b \\ (v_\Phi)_s \end{pmatrix} \begin{pmatrix} \sqrt{6}\langle\tau_b\rangle^{1/4}\langle\tau_s\rangle^{3/4} \\ \frac{2\sqrt{6}}{3}\langle\tau_b\rangle^{3/4}\langle\tau_s\rangle^{1/4} \end{pmatrix}$$

$$\boldsymbol{v_\chi} = \begin{pmatrix} (v_\chi)_b \\ (v_\chi)_s \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{4}{3}}\langle\tau_b\rangle \\ \frac{\sqrt{3}}{a_s} \end{pmatrix}$$

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