Alma Mater Studiorum · Università di Bologna

**Scuola di Scienze**
**Corso di Laurea Magistrale in Fisica**

# MORPHOLOGICAL INSTABILITY ANALYSIS OF A MISFIT STRAINED CORE-SHELL NANOWIRE FOR THE GROWTH OF QUANTUM DOTS

**Relatore:**
**Prof. Fabio Ortolani**

**Correlatore:**
**Prof. Jean Noel Aqua**

**Presentata da:**
**Danilo Messinese**

**Sessione I**
**Anno Accademico 2013/2014**

*To my strong and sweet mum,*

*To my funny and supportive dad,*

*To my crazy and smart brother.*

*Failure is the opportunity to begin again more intelligently*

*[Henry Ford]*

# Contents

# Abstract (Italian version)

I meandri di un fiume o le dune di sabbia hanno una struttura osservata correntemente a scale di spazio geologiche. Con le moderne tecnologie, scale nanometriche di superfici cristalline possono essere osservate direttamente, mostrando organizzazioni stranamente simili.

La loro comprensione, controllo e manipolazione formano un campo di studi in piena espansione, dove la ricerca teorica e l'applicazione tecnologica si intersecano.

In questo lavoro di tesi il mio interesse è rivolto alla modellizzazione della crescita cristallina su scale nanometriche, e più precisamente della crescita per (etero)epitassia di isole quantiche sulla superficie di un core-shell (Si-Ge) nanofilo. Queste isole sono 'scatole' di dimensioni sufficientemente piccole tali che le loro proprietà elettroniche e magnetiche mostrino effetti quantistici.

*Epitassia* (deriva dalle radici greche *epi* che significa 'sopra' e *taxis* che significa 'una maniera ordinata') consiste nella crescita , per deposizione atomica, di uno strato cristallino, chiamato film, tale che il materiale depositato adotti un'orientazione ben definita rispetto alla struttura cristallina del substrato. Secondo i materiali e condizioni di crescita, le superfici possono auto-organizzarsi in strutture differenti. Le isole quantiche ottenute per eteroepitassia sono conseguenza

della deformazione elastica dovuta al misfit tra le maglie del substrato e del film in accordo con il metodo di crescita di Stransky-Krastanov.

Infatti, se esiste un misfit tra le maglie del film depositato e del substrato di supporto, l'energia elastica si accumulerà nel film. A una certa altezza critica questa energia libera del film può essere ridotta se il film si organizza in isole, dove la tensione può essere rilassata lateralmente. L'altezza critica dipende dai moduli di Young (E, $\upsilon$), dal misfit tra le maglie (m) e dalla tensione superficiali ($\gamma$).

Il trasporto di materiale nel film è portato avanti per diffusione superficiale.

Come detto in precedenza, ho analizzato l'instabilità morfologica di un nanofilo cilindrico core-shell deformato a causa del misfit: naturalmente può essere preso in considerazione il caso piano[18], ma in questo siamo interessati al caso cilindrico a causa di interessanti applicazioni tecnologiche.

Lo studio è stato svolto mediante un'analisi lineare di stabilità: vedremo come la teoria elastica, potente e delicata, può fornire risultati molto interessanti e inaspettati ad un primo sguardo.

I nanofili core-shell non sono in realtà perfettamente cilindrici: possono mostrare deviazioni locali dalla geometria cilindrica. Ragionando in termini di spazio di Fourier, queste deviazioni locali corrispondono ad una distribuzione di modulazioni sinusoidali della superficie che possono essere descritte da il numero di vettore d'onda *q* nella direzione assiale (direzione **z**) e dal numero di modo normale *n* nella direzione della circonferenza.

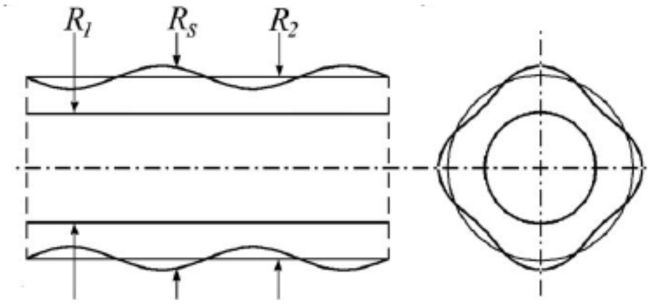


FIG 1 Schematizzazione di un nanofilo core-shell con una perturbazione sinusoidale della superficie; $R_1$ e $R_2$ sono rispettivamente il raggio del core e il raggio dello shell.

Perciò se il raggio esterno del nanofilo imperturbato è $R_2$ e $\delta$ è l'ampiezza della perturbazione, allora la superficie effettiva di raggio $R_S$ può essere espressa come

$$R_s = R_2 + \delta e^{i(n\theta + qz)}$$

Naturalmente queste modulazioni dello spessore dello shell influenzano anche le distribuzioni delle deformazioni e degli sforzi nel nanofilo e causano variazioni dell'energia elastica.
Queste variazioni possono guidare la diffusione di superficie, e la domanda cui trovare risposta è quali condizioni (raggio del core, spessore dello shell e modo normale) portano al modo di crescita di quantum dots sulla superficie del nanofilo più rapido nel tempo.
L'analisi sarà condotta risolvendo le equazioni all'equilibrio in modo da esplicitare le espressioni degli spostamenti, del tensore delle deformazioni e degli sforzi fino al primo ordine in $\delta$.
Una caratteristica fondamentale di quest'analisi è la scelta dello stato di riferimento degli spostamenti nello shell visti dal core: dimostreremo come questa scelta influenza notevolmente la ricerca del modo di crescita più veloce.

A tal fine metteremo a confronto i risultati dell'analisi condotta con diverse scelte dello stato di riferimento.
L'obiettivo di questo lavoro è dunque la ricerca di caratteristiche e regole (raggio del core, spessore dello shell e modo normale) del modo di crescita più veloce dei quantum dots sulla superficie del nanofilo con determinate ipotesi e condizioni al contorno. Questi risultati teorici saranno usati per la fabbricazione in laboratorio del modello proposto di nanofilo core-shell, presso l'Istituto di NanoScienze di Parigi (INSP).

# Abstract

The meanders of a river or the sand dunes have a structure currently observed at geological spatial scales. With the modern technologies, nanometric scales of crystalline surfaces can be observed directly, showing strangely resembling organizations.

Their understanding, control and manipulation form a domain in full expansion where theoretical research and technology application are mixed.

In this thesis I am interested in modelling the crystalline growth at nanometric scales, and more precisely in the growth by (hetero) *epitaxy* of quantum islands on the surface of a core-shell (Si-Ge) nanowire. These islands are boxes of dimensions sufficiently small in the way that their electronic and electromagnetic properties show quantum effects.

*Epitaxy* (comes from the Greek roots *epi* meaning 'above' and *taxis* meaning 'an ordered manner') consists in the growth, by atomic deposition, of a crystalline overlayer, called film, such that the deposited material adopts a well-defined orientation with respect to the substrate crystal structure. Depending on materials and growth conditions, surfaces can self-organize in different structures. Quantum islands obtained by heteroepitaxy are consequence of the

elastic deformation due to the misfit between the substrate and the film lattices according with the Stransky-Krastanov growth mode.
In fact, if there is a mismatch between the lattices of the growing film and the supporting substrate, elastic energy will be accumulated in the growing film. At some critical thickness, the free energy of the film can be decreased if the film breaks into isolated islands, where the tension can be relaxed laterally. The critical height depends on Young's moduli (E, $\upsilon$), lattice misfit (m) and surface tension ($\gamma$).
The material transport in the film is carried out by surface diffusion.
As said before, I have investigated the morphological instability of a misfit-strained cylindrical core-shell nanowire: of course a flat case substrate can be taken into account (this had already been done at INSP labs by J.N. Aqua group), but in this work we are interested in the cylindrical shaped substrate due to technological applications.
I did my investigation by performing a linear stability analysis: we will see how the theory of elasticity can lead to physical results not always expected at a first look.
Core-shell nanowires are in reality not perfectly cylindrical. Instead they show local deviation of cylindrical geometry. Thinking in Fourier space, these local deviations correspond to a broad distribution of sinusoidal surface modulations, which can be characterized by their wave number $q$ in axial direction (**z** direction) and the mode number $n$ in circumferential direction.

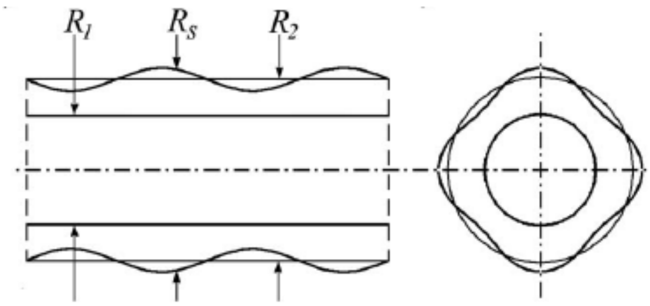


FIG 1 Schematic of core-shell nanowire with a sinusoidal perturbation of the surface; $R_1$ and $R_2$ are the initial core and shell radii, respectively and $R_s$ is the surface radius

Thus, if the outer radius of the unperturbed core-shell nanowire is $R_2$ and $\delta$ is the amplitude of the perturbation, then the actual surface radius $R_S$ can be expressed as

$$R_s = R_2 + \delta e^{i(n\theta + qz)}$$

Of course such modulations of the shell thickness also affects the stress and strain distributions within the nanowire and cause variations of the elastic energy.

These variations can drive surface diffusion, and the question to be answered is which conditions (core radius, thickness, n mode) lead to the fastest growing mode for the quantum dots on nanowire surface.

I will perform my analysis by solving equilibrium equations in order to have explicit expressions for strain and stress tensors and displacements up to first order in $\delta$.

A crucial feature of this analysis is the choice of the reference state of displacements in the shell with respect to the core due to the misfit: we will see how this choice affects equations and the research of the fastest growing mode.

We shall perform a comparison between the results computed with different reference state choices.

The goal of this work is the detection of general physical features (core radius, shell thickness, normal mode number) of the fastest growing mode of quantum islands on the nanowire surface. The theoretical results will be used by experimental researchers of the Institute of NanoScience of Paris (INSP).

Chapter 1

# Quantum nanostructures and crystal growth


## Abstract

In this chapter we will introduce the concept of nanostructure, the nature of quantum dot, also know as artificial atoms, and nanowires. After it we will describe briefly (a deeper description is beyond our need) the most important technique of crystalline growth called epitaxy, with a focus on the Stranski-Krastanov method: in this process instabilities (Asaro-Tiller-Grinfeld) are generated in the crystal. Next chapters will be dedicated to the analysis of this instabilities.


## 1.1 What are nanostructures?

The word "nano" means a billionth ($10^{-9}$) part of a unit in general. In our case, it refers specifically to the length scale: *nanometer* or nm,

$$1\text{nm} = 10^{-9}\text{ m} = 10^{-3}\ \mu\text{m} = 10\ \text{Å}$$

*Nanostructures* refer to materials systems with length scale in the range of ~ 1-100 nm in at least one dimension. In a nanostructure, electrons are confined in the nanoscale dimension(s), but are free to move in other dimension(s). One way to classify nanostructures is based on the dimensions in which electrons move freely:

*Quantum well*: electrons are confined in one dimension (1D), free in other 2D. It can be realized by sandwiching a narrow-bandgap semiconductor layer between the wide-gap ones. A quantum well is often called a 2D electronic system.

*Quantum nanowires*: confined in two dimensions, free in 1D (so it is called a 1D electronic system). Real quantum wires include polymer chains, nanowires and nanotubes.

*Quantum dots*: electrons are confined in all dimensions, as in clusters and nano-crystallites

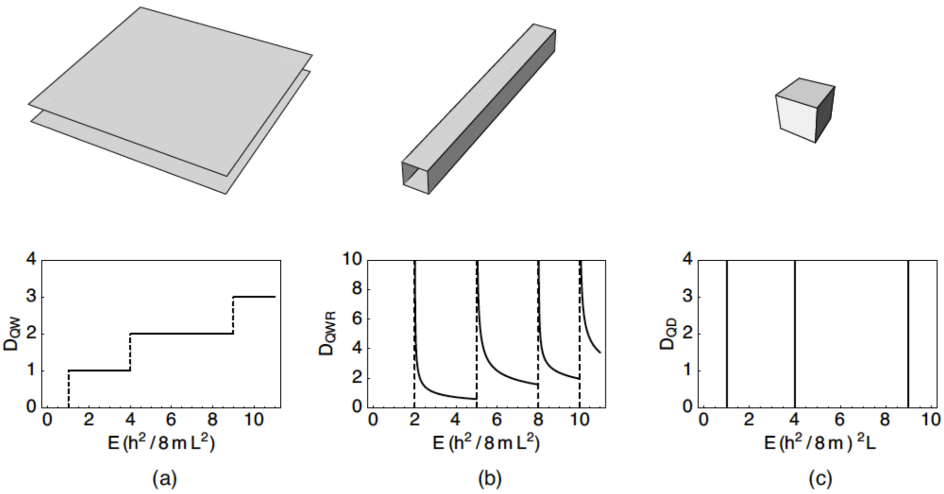


FIG 1.1 Schematic illustration of (a) a quantum well, (b) a quantum wire, and (c) a quantum dot. The planar surfaces represent infinite potential barriers separated by a distance L. Electrons are thereby confined in one direction in (a), in two directions in (b), and in three directions in (c). The corresponding densities of states of free electrons within these structures are displayed in the bottom panels. The broken lines in (a) and (b) indicate the discrete levels of the confined states. Associated with each such level is a subband of continuous states corresponding to the unconfined motion.

Nanostructures are unique as compared with both individual atoms/molecules at a smaller scale and the macroscopic bulk materials. They are also called *mesoscopic* structures.
Nanoscience research focuses on the unique properties of nanoscale structures and materials that do not exist (or only very weakly exist) in structures of same material composition but at other scale ranges.

### 1.1.1. What makes nanostructures unique and interesting?

For bulk materials (e.g., a Cu wire, a cup of water), their intrinsic physical properties, such as density, conductivity and chemical reactivity, are independent of their sizes. For example, if a one-meter Cu wire is cut into a few pieces, those intrinsic properties of the shorter wires remain the same as in the original wire. If the dividing process is repeated again and again, this invariance cannot be kept indefinitely. Certainly, we know that the properties are changed greatly when the wire is divided into individual Cu atoms (even more at the level of electrons, protons and neutrons). Significant property changes often start when we get down to the nanoscales. The following phenomena critically affect the properties of nanostructural materials:

*Quantum confinement*: the confinement of electrons in the nanoscale dimensions result in quantization of energy and momentum, and reduced dimensionality of electronic states

*Quantum coherence*: certain phase relation of wave function is preserved for electrons moving in a nanostructure, so wave interference effect must be considered. But in nanostructures, generally the quantum coherence is not maintained perfectly as in atoms and molecules. The coherence is often disrupted to some extent by defects in the nanostructures. Therefore, both quantum coherent and de-coherent effects have to be considered, which often makes the description of electronic motion in a nanostructure more complicated than in the extreme cases.

*Surface/interface effects*: a significant fraction (even the majority) of atoms in nanostructure is located at and near the surfaces or interfaces. The mechanic, thermodynamic, electronic, magnetic, optical and chemical states of these atoms can be quite different than those interior atoms.

These factors play roles to various degrees (but not 100%) of importance. For example, the confinement and the coherent effects are not as complete as that in an atom. Both the crystalline (bulk) states and the surface/interface states cannot be ignored in nanoscale structures. The different mixture of atomic/molecular, mesoscopic and macroscopic characters make the properties of nanostructures vary dramatically. Nanostructural materials are often in a metastable state.

Their detailed atomic configuration depends sensitively on the kinetic processes in which they are fabricated. Therefore, the properties of nanostructures can be widely adjustable by changing their size, shape and processing conditions. The situation is similar to molecular behavior in chemistry (e.g., N vs. $N_2$) in certain aspect. Because of the rich and often surprising outcomes, it will be extremely interesting and challenging to play with nanostructural systems. Nanoscience and nano-engineering have been an area where many breakthroughs have been and will continue to be produced.

## 1.2 Quantum dots: artificial atoms

Quantum dots are man-made "droplets" of charge that can contain anything from a single electron to a collection of several thousand.

The physics of quantum dots shows many parallels with the behaviour of naturally occurring quantum systems in atomic and nuclear physics. Indeed, quantum dots exemplify an important trend in condensed-matter physics in which researchers study man-made objects rather than real atoms or nuclei.

As in an atom, the energy levels in a quantum dot become quantized due to the confinement of electrons. With quantum dots, however, an experimentalist can scan through the entire periodic table by simply changing a voltage.

Many of these phenomena can be studied by allowing single electrons to tunnel into and out of the dot, since this reveals the quantized energy levels of the device.

What is most exciting is that many of the quantum phenomena observed in real atoms and nuclei - from shell structure in atoms to quantum chaos in nuclei - can be observed in quantum dots. And rather than having to study different elements or isotopes, these effects can be investigated in a quantum dot by simply changing its size or shape.

## 1.3 Core-shell nanowires

One-dimensional semiconductor nanostructures, such as Si and Ge nanowires, have attracted extensive research efforts over the past decade. They are expected to play important roles as both interconnects and functional components in future nanoscale electronic and optical devices, such as light-emitting diodes (LEDs), ballistic field-effect transistors (FETs) and nanoscale sensors. Experimental and theoretical investigations showed that in these nanoscale wires the charge carriers are confined in the lateral direction of the wires, thus quantum confinement effects are expected to play an important role on the electronic properties.

Researchers also found that the band gap of Si and Ge nanowires depends on several factors, such as size, crystalline orientation, surface chemistry and doping.

Recently, a particular attention has been given to Si/Ge core-shell nanowires, in which factors, such as heterostructure composition and interface geometry, can be further manipulated to tune the electronic properties of the nanowires. Compared to the single composition for Si or Ge nanowires, the core-shell structure has some superior properties.

For instance, a better conductance and higher mobility of charge carriers can be obtained, due to the band offsets in the core-shell nanowires. Strain is another factor that has been demonstrated to critically affect the electronic properties of various nanostructures.

Beside electronic properties, strain is a crucial factor that allows the growth of quantum dots on nanowire surface, due to the lattice misfit: in this way devices using both quantum dots and nanowires physical features can be studied.

In the next section we will study the most important method to grow quantum dots on nanowires called *heteroepitaxy.*

## 1.4 Crystal growth

A crystal is a solid material whose constituent atoms, molecules, or ions are arranged in an orderly repeating pattern extending in all three spatial dimensions. Crystal growth is a major stage of a crystallization process, and consists in the addition of new atoms, ions, or polymer strings into the characteristic arrangement of a crystalline Bravais lattice. The growth typically follows an initial stage of either homogeneous or heterogeneous (surface catalysed) nucleation, unless a "seed" crystal, purposely added to start the growth, was already present. We are going to study the case where the crystal growth is due to a self-organization of the system.

Self-organization is a process where some form of global order or coordination arises out of the local interactions between the components of an initially disordered system. This process is spontaneous: it is not directed or controlled by any agent or subsystem inside or outside of the system; however, the laws followed by the process and its initial conditions may have been chosen or caused by an agent. In the next section we will study briefly a method called "(hetero)epitaxy" where a material is deposited atom by atom on a crystalline substrate: this system will grow towards an equilibrium by self-organization.

### 1.4.1 Stranski-Krastanov epitaxial growth

Epitaxial growth is a process during which a crystal is formed on an underlying crystalline surface as the result of deposition of new material onto that surface. The term 'epitaxy', which is a combination of a Greek words *epi,* meaning 'upon', and *taxis,* meaning 'order', was coined by Royer in the 1920s to convey the notion of growing a new crystal whose orientation is determined by a crystalline substrate and to distinguish epitaxial growth from polycristalline and amorphous growth. Numerous experiments have revealed that, for small amounts of one material deposited onto the surface of a possibly different material, the epitaxial morphology falls into one of three distinct categories. By convention, these are referred to as: *Frank-van der Merwe* morphology, with flat single crystal films consisting of successive largely complete layers, *Volmer-Weber* morphology, with 3D islands that leave part of the substrate exposed, and *Stranski-Krastanov* morphology, with 3D islands atop a thin flat 'wetting' film that completely covers the substrate.

These morphologies are illustrated in Fig 1.2.

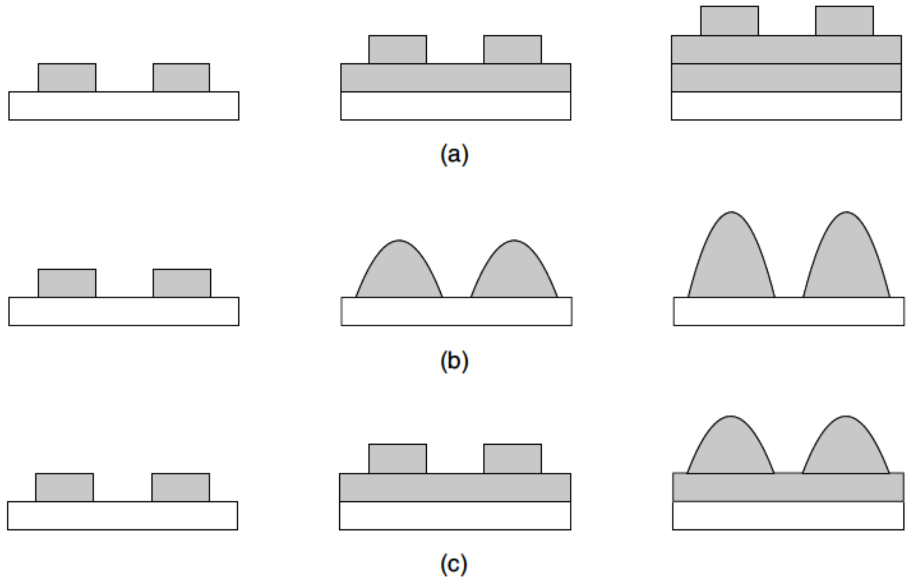

FIG1.2 Schematic evolution of the (a) Frank-van der Merwe, (b) Volmer-Weber, and (c) Stranski-Krastanov heteroepitaxial growth morphologies. The unshaded regions represent the substrate and the shaded region the deposited material. In each panel, time increases from left to right as more material is deposited. In (a) growth proceeds by the formation of successive layers, while in (b), the 3D islands that appear at the onset of growth eventually merge into a complete layer. In (c), a wetting film, typically consisting of a few monolayers is formed initially, after which 3D islands appear.

The Stranski-Krastanov morphology is observed in systems where there is appreciable lattice mismatched between the deposited material and the substrate.

The classical rationale for this growth mode is based on the accomodation of misfit strain, which changes the balance between the surface and interfacial free energies as the strain energy increases with the film thickness. Although the growth of wetting layers is favored initially, the build-up of strain energy eventually makes subsequent layer growth energically unfavorable. The deposition of material beyond this point leads

to the formation of 3D islands within which strain is relaxed by the formation of misfit dislocations. However, there is another scenario within the Stranski-Krastanov morphology: the formation of islands *without* dislocations - called *coherent* islands because their atomic structures are coherent with the substrate and the wetting layers - atop one or more wetting layers. This will be discussed in the next section.

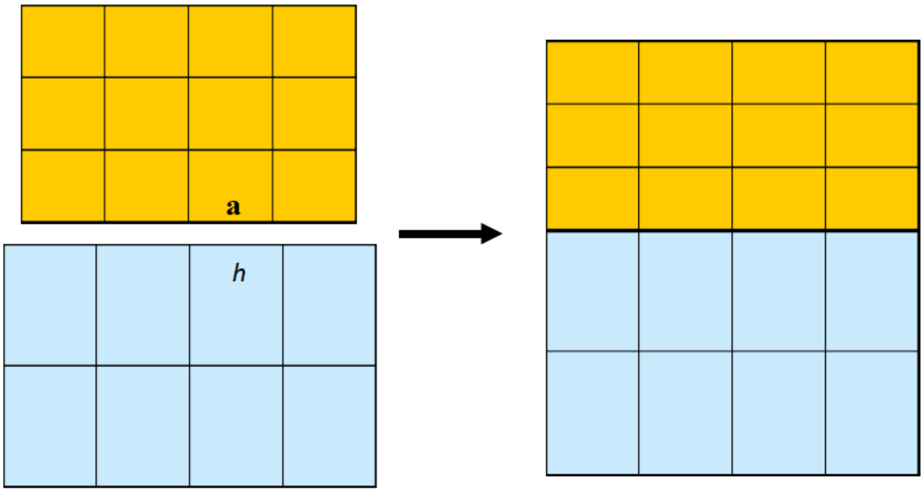


FIG 1.3 Strain energy accumulates in an epitaxial film due to the lattice misfit $m = \frac{a-h}{h}$ : the film is under stress

The prototypical cases for the Stranski-Krastanov growth of coherent 3D islands are InAs on GaAs (7% lattice mismatch), and Ge on Si, which has a 4% lattice mismatch.

In both cases, the substrate exerts a compressive strain on the deposited material. When the 3D islands are embedded within epitaxial layers of a material with a wider band gap, the carriers within the islands are confined by the potential barriers that surround each island, forming an array of quantum dots because these quantum dots are obtained directly by growth, with no additional processing, they are referred to as *self-organized* or *self-assembled* structures.

### 1.4.2 Asaro-Tiller-Grinfeld instability

As I said before, the substrate exerts on the film a biaxial constraint in the interfacial plane, that a morphological evolution in the normal direction to the interface permits to relax.

When the constraint is "enough" small, the evolution starts from instability called of *Asaro-Tiller-Grinfeld* (ATG) developing on the whole film surface with his characteristic wavelength.

I have considered a continuous approach to study the ATG instability. This is linked to the *surface diffusion* that is the only one to be activated at work temperature. Then, diffusion flows can be linked to the chemical potential gradients, where the flow of matter on the surface $\mathbf{j_S}$ is proportional to the gradient of surface of the surface chemical potential. The matter conservation implies a diffusion equation like

$$\partial h/\partial t = D\Delta_S \mu_S$$

where $D$ is a coefficient of diffusion linking the evolution of the film surface to the surface laplacian of the chemical potential of surface $\mu_S$ and $h$ is the height of the film surface.

The chemical potential brings a first term equal to the elastic energy density on the surface, which favors the transfer of matter towards the relaxed zones where the concavity of the surface is positive, and that contributes as $+|\mathbf{q}|^3$ to the growing rates of a perturbation of wave vector **q**. This effect is balanced by a second term associated to the cost of surface energy and equal to the product of energy of surface by the surface local curvature, contributing as $-\mathbf{q}^4$ to the growing rates. These two terms are classically present in the ATG instability description.

In the case of thin film, we need to add to this model the effects of interaction between the film surface and the substrate on which it is deposited in a coherent mode.

The combination of these three effects allows me to give a basic model of ATG instability when the system is isotropic.

In the last chapter we will analyse a linear model of surface diffusion and ATG instability in the cylindrical case.

Chapter 2

# Basic elements of theory of elasticity

**Abstract**

In this chapter we will introduce the main features of the theory of elasticity in order to have the necessary tools to investigate the morphological instability of our misfit-strained cylindrical core-shell nanowire by performing a linear stability analysis.

## 2.1 The strain tensor

The mechanics of solid bodies, regarded as continuous media, forms the content of the theory of elasticity. Under the action of applied forces, solid bodies exhibit deformation to some extent, i.e. they change in shape and volume. The deformation of a body is described mathematically in the following way. The position of any point in the body is defined by its radius vector $\boldsymbol{r}$ (with components $x_1 = x$, $x_2 = y,\ \ x_3 = z$) in some coordinate system. When the body is deformed, every point in it is in general displaced. Let us consider some particular point; let its radius vector before the deformation be r, and after the deformation have a different value $\boldsymbol{r}'$ (with components $x_i{}'$). The displacement of this point due to the deformation is then given by the vector $\boldsymbol{r}' - \boldsymbol{r}$, which we shall denote by u:

$$u_i = x_i{}' - x_i \tag{2.1}$$

The vector u is called the displacement vector. The coordinates $x_i{}'$ of the displaced point are, of course, functions of the coordinates $x_i$ of the point before displacement. The displacement vector $u_i$ is therefore also a function of the coordinates $x_i$. If the vector u is given as a function of $x_i$, the deformation of the body is entirely determined. When a body is deformed, the distances between its points change. Let us consider two points very close together. If the radius vector joining them before the deformation is $dx_i{}' = dx_i + du_i$. The distance between the points is $dl = \sqrt{(dx_1{}^2 + dx_2{}^2 + dx_3{}^2)}$ before the deformation, and $dl' = \sqrt{(dx'_1{}^2 + dx'_2{}^2 + dx'_3{}^2)}$ after it. Using the general summation rule, we can write $dl^2 = dx_i{}^2$, $dl'^2 = dx'_i{}^2 = (dx_i + du_i)^2$. Substituting $du_i = (\partial u_i/\partial x_k)dx_k$ we can write

$$dl'^2 = dl^2 + 2\frac{\partial u_i}{\partial x_k}dx_i dx_k + \frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_l}dx_l dx_k$$

Since the summation is taken over both suffixes $i$ and $k$ in the second term on the right, we can put $(\partial u_i/\partial x_k)dx_l dx_k$ $(\partial u_k/\partial x_i)dx_i dx_k$. In the third term, we interchange the suffixes $i$ and $l$.

Then $dl'^2$ takes the final form

$$dl'^2 = dl^2 + 2\varepsilon_{ij} dx_i dx_k \tag{2.2}$$

where the tensor $\varepsilon_{ij}$ is defined as

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right). \tag{2.3}$$

$$\varepsilon_{ij} = \varepsilon_{ji} \ . \tag{2.4}$$

This result has been obtained by writing the term $2\frac{\partial u_i}{\partial x_k} dx_i dx_k$ in $\mathrm{dl}'^2$ in the explicitly symmetrical form

$$\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) dx_i dx_k$$

Like any symmetrical tensor, $\varepsilon_{\mathrm{ij}}$ can be diagonalised at any given point. This means that, at any given point, we can choose coordinate axes (the principal axes of the tensor) in such a way that only the diagonal components $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}$ of the tensor $\varepsilon_{ij}$ are different from zero. If the strain tensor is diagonalised at a given point, the element of length (2.2) near it becomes

$$dl'^2 = (\delta_{ik} + 2u_{ik}) dx_i dx_k$$

$$= (1 + 2\varepsilon_{11}) {dx_1}^2 + (1 + 2\varepsilon_{22}) {dx_2}^2 + (1 + 2\varepsilon_{33}) {dx_3}^2.$$

We see that the expression is the sum of three independent terms. This means that the strain in any volume element may be regarded as composed of independent strains in three mutually perpendicular directions, namely those of the principal axes of the strain tensor. Each of these strains is a simple extension (or compression) in the corresponding direction: the length $\mathrm{dx}_1$ along the first principal axis becomes $dx'_1 = \sqrt{(1+2\varepsilon_{11})}dx_1$, and similarly for the other two axes. The quantity $\sqrt{(1+2\varepsilon_{11})} - 1$ is consequently equal to the relative extension $(dx'_i - dx_i)/dx_i$ along the ith principal axis. In almost all cases occurring in practice, the strains are small. This means that the change in any distance in the body is small compared with the distance itself. In other words, the relative extensions are small compared with unity. In what follows we shall suppose that all strains are small. If a body is subjected to a small deformation, al the components of the strain tensor are small, since they give, as we have seen, the relative changes in lengths in the body. Except in such special cases, the displacement vector for a small deformation is itself small, and we can therefore neglect the last term in the general expression (2.3), as being of the second order of smallness. Thus, for small deformations, the strain tensor is given by

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right). \tag{2.5}$$

while in direct vector/matrix notation reads as:

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T] \tag{2.6}$$

where $\varepsilon$ is the strain matrix and $\nabla u$ is the displacement gradient matrix and $(\nabla\mathbf{u})^{\mathrm{T}}$ is its transpose.

The strain is therefore a symmetric second-order tensor and is commonly written in matrix format:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}.$$

### 2.1.1 Cylindrical coordinates

In the next chapter we will analyse a cylindrical nanowire, so it is convenient to give the corresponding formulae in cylindrical coordinates $r, \vartheta, z$,

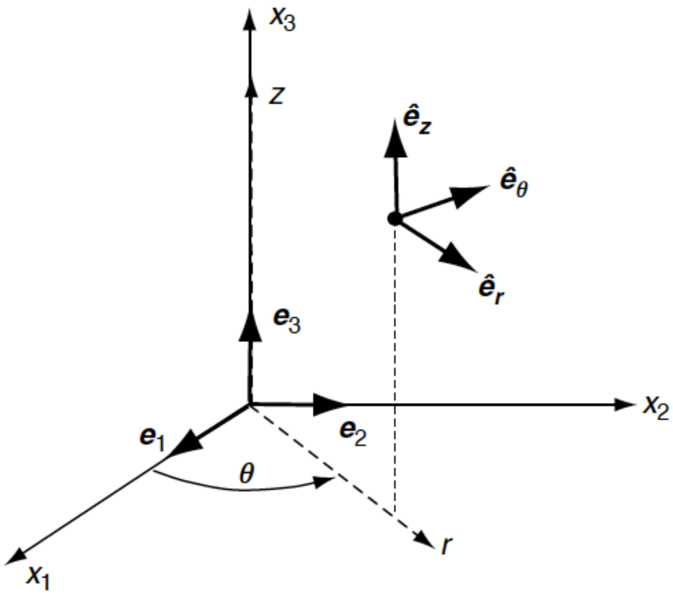


FIG 2.1 Cylindrical coordinate system

$$\boldsymbol{u} = u_r \boldsymbol{e_r} + u_\vartheta \boldsymbol{e_\vartheta} + u_z \boldsymbol{e_z}$$

$$\begin{pmatrix} \varepsilon_{rr} = \frac{\partial u_r}{\partial r} & \varepsilon_{r\vartheta} = \frac{1}{2}\left(\frac{\partial u_\vartheta}{\partial r} - \frac{u_\vartheta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \vartheta}\right) & \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \\ \varepsilon_{\vartheta r} = \frac{1}{2}\left(\frac{\partial u_\vartheta}{\partial r} - \frac{u_\vartheta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \vartheta}\right) & \varepsilon_{\vartheta\vartheta} = \frac{1}{2}\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\vartheta}{\partial \vartheta}\right) & \varepsilon_{\vartheta z} = \frac{1}{2}\left(\frac{\partial u_\vartheta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \vartheta}\right) \\ \varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) & \varepsilon_{z\vartheta} = \frac{1}{2}\left(\frac{\partial u_\vartheta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \vartheta}\right) & \varepsilon_{zz} = \frac{\partial u_z}{\partial z} \end{pmatrix}$$

(2.7)

where we have used equation (2.6) with the gradient equal to

$$\nabla \mathbf{u} = \frac{\partial u_r}{\partial r}\boldsymbol{e_r}\boldsymbol{e_r} + \frac{\partial u_\vartheta}{\partial r}\boldsymbol{e_r}\boldsymbol{e_\vartheta} + \frac{\partial u_z}{\partial r}\boldsymbol{e_r}\boldsymbol{e_z} + \frac{1}{r}\left(\frac{\partial u_r}{\partial \vartheta} - u_\vartheta\right)\boldsymbol{e_\vartheta}\boldsymbol{e_r}$$

$$+\frac{1}{r}\left(\frac{\partial u_\vartheta}{\partial \vartheta} + u_r\right)\boldsymbol{e_\vartheta}\boldsymbol{e_\vartheta} + \frac{1}{r}\frac{\partial u_z}{\partial \vartheta}\boldsymbol{e_\vartheta}\boldsymbol{e_z} + \frac{\partial u_r}{\partial z}\boldsymbol{e_z}\boldsymbol{e_r} + \frac{\partial u_\vartheta}{\partial z}\boldsymbol{e_z}\boldsymbol{e_\vartheta} + \frac{\partial u_z}{\partial z}\boldsymbol{e_z}\boldsymbol{e_z}$$

## 2.2 The stress tensor

In a body that is not deformed, the arrangement of the molecules corresponds to a state of thermal equilibrium. All parts of the body are in mechanical equilibrium. This means that, if some portion of the body is considered, the resultant of the forces on that portion is zero. When a deformation occurs, the arrangement of the molecules is changed, and the body ceases to be in its original state of equilibrium. Forces therefore arise which tend to return the body to equilibrium. These internal forces, which occur when a body is deformed, are called internal stresses. If no deformation occurs, there are no occurs, there are no internal stresses.

The internal stresses are due to molecular forces, i.e. the forces of interaction between the molecules. An important fact in the theory of elasticity is that the molecular forces have a very short range of action. Their effect extends only to the neighbourhood of the molecule exerting them, over a distance of the same order as that between the molecules, whereas in the theory of elasticity, which is a macroscopic theory, the only distances considered are those large compared with the distances between molecules.

The range of action of the molecular forces should therefore be taken as zero in the theory of elasticity. we can say that the forces which cause the internal stresses are, as regards the theory of elasticity, "near-action" forces, which act from any point only to neighbouring points. Hence it follows that the forces exerted on any part of the body by surrounding parts act only on the surface of that part. Let us consider the total force on some portion of the body. Firstly, this total force is equal to the sum of all the forces on all the volume elements in that portion of the body, i.e. it can be written as the volume integral $\int F dV$, where $F$ is the force per unit volume and $FdV$ the force on the volume element $dV$. Secondly, the forces with which various parts of the portion considered act on one another cannot give anything but zero in the total resultant force, since they cancel by Newton's third law. The required total force can therefore be regarded as the sum of the forces exerted on the given portion of the body by portions surrounding it. From above, however these forces act on the surface of that portion, and so the resultant force can be represented as the sum of forces acting on all the surface elements, i.e. as an integral over the surface. Thus, for any portion of the body, each of the three components $\int F_i dV$ of the resultant of all the internal stresses can be transformed into an integral over the surface. As we know from vector analysis, the integral of a scalar over an arbitrary volume can be transformed into an integral over the surface if the scalar is the divergence of a vector. In the present case we have the integral of a vector, and not of a scalar. Hence the vector $F_i$ must be the divergence of a tensor of rank two, i.e. be of the form

$$F_i = \frac{\partial \sigma_{ik}}{\partial x_k}$$

Then the force on any volume can be written as an integral over the closed surface bounding that volume:

$$\int F_i dV = \int \frac{\partial \sigma_{ik}}{\partial x_k} dV = \oint \sigma_{ik} df_k$$

where $df_k$ are the components of the surface element vector $d\boldsymbol{f}$, directed(as usual) along the outward normal. The tensor $\sigma_{ik}$ is called the *stress tensor*. As we see $\sigma_{ik} df_k$ is the $i$th component of the force on the surface element $d\boldsymbol{f}$. By taking elements of area in the planes of $xy, yz, zx$, we find that the component $\sigma_{ik}$ of the stress tensor is in the $i$th component of the force on unit area perpendicular to the $x_k$-axis. For instance, the force on unit area perpendicular to the $x$-axis, normal to the area is $\sigma_{xx}$, and the tangential forces are $\sigma_{yx}$ and $\sigma_{zx}$. As the strain tensor, it is possible to demonstrate that also the stress tensor is symmetric: $\sigma_{ij} = \sigma_{ji}$.

## 2.3 The linear stress-strain relationship

Stress and strain are linked in elastic media by a stress-strain relationship. The most general linear relationship between the stress and the strain tensors can be written

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \tag{2.8}$$

where $c_{ijkl}$ is termed the elastic tensor. Here we begin using the summation convention in our index notation. Any repeated index in a product indicates that the sum is to be taken as the index varies from 1 to 3. Equation (2.8) assumes perfect elasticity; there is no energy loss or attenuation as the material deforms in response to the applied stress. The elastic tensor $c_{ijkl}$ is a fourth-order tensor with 81 ($3^4$) components. However, because of the symmetry of the stress and

strain tensors and thermodynamic considerations, only 21 of these components are independent. These 21 components are necessary to specify the stress-strain relationship for the most general form of elastic solid. The properties of such a solid may vary with direction; if they do, the material is termed anisotropic. In contrast, the properties of an isotropic solid are the same in all directions.
In our case of study, we will consider the isotropic case.
If we assume isotropy ($c_{ijkl}$ is invariant with respect to rotation), it can be shown that the number of independent parameters is reduced to two:

$$c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}\right)$$

where $\lambda$ and $\mu$ are called the Lamé parameters of the material and $\delta_{ij}$ is the Kronecker delta. The stress-strain equation (2.8) for an isotropic solid is

$$\sigma_{ij} = \left[\lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}\right)\right]\varepsilon_{kl} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij}$$

where we have used the symmetry of the strain tensor to combine the $\mu$ terms. Note that $\varepsilon_{kk} = tr[\varepsilon]$, the sum of the diagonal elements of $\varepsilon$. Using this equation, we can directly write the components of the stress tensor in terms of the strains:

$$\boldsymbol{\sigma} = \begin{pmatrix} \lambda tr[\varepsilon] + 2\mu\varepsilon_{11} & 2\mu\varepsilon_{12} & 2\mu\varepsilon_{13} \\ 2\mu\varepsilon_{21} & \lambda tr[\varepsilon] + 2\mu\varepsilon_{22} & 2\mu\varepsilon_{23} \\ 2\mu\varepsilon_{31} & 2\mu\varepsilon_{32} & \lambda tr[\varepsilon] + 2\mu\varepsilon_{33} \end{pmatrix}$$

The two Lamé parameters completely describe the linear stress-strain relation within an isotropic solid. $\mu$ is termed the shear modulus and is a measure of the resistance of the material to shearing. Its value is given by half of the ratio between the applied shear stress and the resulting shear strain, that is, $\mu = \sigma_{xy}/2\varepsilon_{xy}$.

The other Lamé parameter, $\lambda$, does not have a simple physical explanation. Other commonly used elastic constants for isotropic solids include:

*Young's modulus E*: The ratio of extensional stress to the resulting extensional strain for a cylinder being pulled on both ends.

$$E = \frac{(3\lambda + 2\mu)\mu}{(\lambda + \mu)}$$

*Poisson's ratio* $\upsilon$: The ratio of the lateral contraction of a cylinder (being pulled on its ends) to its longitudinal extension. It can be expressed as

$$\upsilon = \frac{\lambda}{2(\lambda + \mu)}$$

Using these two parameters, we can write the stress tensor as:

$$\sigma_{ij} = \frac{E}{(1-2\upsilon)(1+\upsilon)}((1 - 2\upsilon)\varepsilon_{ij} + \upsilon\varepsilon_{kk}\delta_{ij}) \tag{2.9}$$

## 2.4 Equilibrium equations

The stress field in an elastic solid is continuously distributed within the body and uniquely determined from the applied loadings. Because we are dealing primarily with bodies in equilibrium, the applied loadings satisfy the equations of static equilibrium; that is, the summation of forces and moments is zero. If the entire body is in equilibrium, then all parts must also be in equilibrium. Thus, we can partition any solid into an appropriate subdomain and apply the equilibrium principle to that region. Following this approach, equilibrium equations can be developed that express the vanishing of the resultant force and moment at a continuum point in the material. These equations can be developed by using an arbitrary finite subdomain. Consider a closed subdomain with volume V and surface S within a body in equilibrium. In equilibrium the internal stresses in every volume element must balance, i.e. we must have

$$\int \frac{\partial \sigma_{ik}}{\partial x_k} dV = \oint \sigma_{ik} df_k = 0$$

Because the region V is arbitrary and the integrand is continuous, then by the zero-value theorem, the integrand must vanish:

$$\frac{\partial \sigma_{ik}}{\partial x_k} = 0 \tag{2.10}$$

This result represents three scalar relations called the equilibrium equations or Lamé equations. Written in scalar notation they are

$$\begin{gathered}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} = 0 \\
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} = 0 \\
\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0
\end{gathered} \tag{2.11}$$

Thus, all elasticity stress fields must satisfy these relations in order to be in static equilibrium.

### 2.4.1 Cylindrical coordinates

As before, it is now convenient to transform these equations in cylindrical coordinates. By using a direct vector/matrix notation, the equilibrium equations can be expressed as

$$\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} = \mathbf{0} \tag{2.12}$$

where $\boldsymbol{\sigma} = \sigma_{ij}\boldsymbol{e}_i\boldsymbol{e}_j$ is the stress matrix, $\boldsymbol{e}_i$ are the unit basis vectors in the cylindrical system. The desired expressions can be obtained from (2.12) by using the appropriate form for $\boldsymbol{\nabla} \cdot \boldsymbol{\sigma}$.

In a vector/matrix notation the stress tensor reads as:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{rr} & \sigma_{r\vartheta} & \sigma_{rz} \\ \sigma_{\vartheta r} & \sigma_{\vartheta\vartheta} & \sigma_{\vartheta z} \\ \sigma_{zr} & \sigma_{z\vartheta} & \sigma_{zz} \end{pmatrix}$$

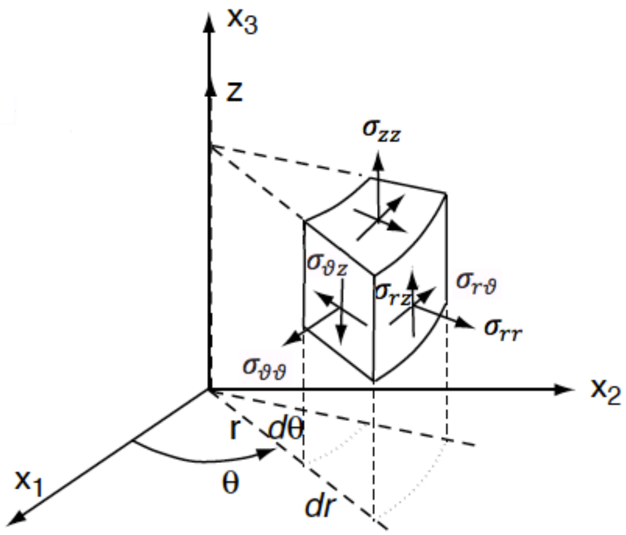


FIG 2.2 Stress components in cylindrical coordinates

Now the stress can be expressed in terms of the traction components as

$$\boldsymbol{\sigma} = \boldsymbol{e}_r\boldsymbol{T}_r + \boldsymbol{e}_\vartheta\boldsymbol{T}_\vartheta + \boldsymbol{e}_z\boldsymbol{T}_z$$

where

$$\boldsymbol{T}_r = \boldsymbol{e}_r\boldsymbol{\sigma}_{rr} + \boldsymbol{e}_\vartheta\boldsymbol{\sigma}_{r\vartheta} + \boldsymbol{e}_z\boldsymbol{\sigma}_{rz}$$

$$\boldsymbol{T}_\vartheta = \boldsymbol{e}_r\boldsymbol{\sigma}_{r\vartheta} + \boldsymbol{e}_\vartheta\boldsymbol{\sigma}_{\vartheta\vartheta} + \boldsymbol{e}_z\boldsymbol{\sigma}_{\vartheta z}$$

$$\boldsymbol{T_z} = \boldsymbol{e_r}\boldsymbol{\sigma_{zr}} + \boldsymbol{e_\vartheta}\boldsymbol{\sigma_{\vartheta z}} + \boldsymbol{e_z}\boldsymbol{\sigma_{zz}}$$

Using the divergence operation of a vector, we get:

$$\boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = \frac{\partial \boldsymbol{T_r}}{\partial r} + \frac{1}{r}\boldsymbol{T_r} + \frac{1}{r}\frac{\partial \boldsymbol{T_\vartheta}}{\partial \vartheta} + \frac{\partial \boldsymbol{T_z}}{\partial z}$$

$$= \frac{\partial \sigma_{rr}}{\partial r}\boldsymbol{e_r} + \frac{\partial \sigma_{r\vartheta}}{\partial r}\boldsymbol{e_\vartheta} + \frac{\partial \sigma_{rz}}{\partial r}\boldsymbol{e_z} + \frac{1}{r}(\boldsymbol{e_r}\sigma_{rr} + \boldsymbol{e_\vartheta}\sigma_{r\vartheta} + \boldsymbol{e_z}\sigma_{rz})$$

$$+\frac{1}{r}\left(\frac{\partial \sigma_{r\vartheta}}{\partial \vartheta}\boldsymbol{e_r} + \boldsymbol{e_\vartheta}\sigma_{r\vartheta} + \frac{\partial \sigma_{\vartheta\vartheta}}{\partial \vartheta}\boldsymbol{e_\vartheta} - \boldsymbol{e_r}\sigma_{\vartheta\vartheta} + \frac{\partial \sigma_{\vartheta z}}{\partial \vartheta}\boldsymbol{e_z}\right)$$

$$+\frac{\partial \sigma_{rz}}{\partial z}\boldsymbol{e_r} + \frac{\partial \sigma_{\vartheta z}}{\partial z}\boldsymbol{e_\vartheta} + \frac{\partial \sigma_{zz}}{\partial z}\mathbf{e_z}$$

Combining this result into (2.12) gives the vector equilibrium equation in cylindrical coordinates. Three scalar equations expressing equilibrium in each coordinate direction then become

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\vartheta}}{\partial \vartheta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r}(\sigma_{rr} - \sigma_{\vartheta\vartheta}) = 0$$

$$\frac{\partial \sigma_{r\vartheta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\vartheta\vartheta}}{\partial \vartheta} + \frac{\partial \sigma_{\vartheta z}}{\partial z} + \frac{2}{r}\sigma_{r\vartheta} = 0$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\vartheta z}}{\partial \vartheta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{r}\sigma_{rz} = 0$$

It is now useful to express the equilibrium equations in term of displacements. Putting (2.4) in (2.9) we get the stress tensor in terms of displacements:

$$\sigma_{ij} = \frac{E}{(1-2\upsilon)(1+\upsilon)}\left((1-2\upsilon)\frac{1}{2}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right)+\upsilon\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\vartheta}{\partial \vartheta}+\frac{\partial u_z}{\partial z}\right)\delta_{ij}\right)$$

(2.14)

Substituting (2.14) in (2.13) we obtain the explicit form of Lamé equations.

# Chapter 3

# Linear stability analysis


## Abstract

In this chapter we will perform the morphological instability analysis of a core-shell misfit strained nanowire of infinite length with a sinusoidally perturbed surface. Using the mathematical tools of the theory of elasticity we will solve equilibrium equations before at zero order and after at first order with different hypothesis and boundary conditions. We will see how the choice of the reference state for displacements in the shell is crucial. The outcome of the stability analysis is threefold: first, the choice of a "Cylindrical" reference state, instead of a "Cartesian" one, strongly affects the analysis. Second, the fastest growing mode of perturbation is found for axially symmetric mode. Third, the smaller core radius and shell thickness are, the faster perturbation grows.

## 3.1 Introduction

Developing epitaxial and dislocation free core-shell Si-Ge nanowires is an experimentally challenging but delicate task. Delicate because, for large misfits, high stresses and strains can be achieved in the core which could cause strong changes in the band structure. A strongly strained core-shell nanowire, as described in chapter 1, tends to relax the misfit strain by developing a modulation of the surface that leads to the growth of islands on the shell surface. The creation of islands is related to the ATG instability. One can describe this mechanism by stating that for certain surface modulation, the energetic costs of increasing the surface area are smaller than the gain of reducing the elastic energy. Taking into account the work of J.N. Aqua concerning the growth of islands on misfit-strained films on flat substrates, those results cannot be directly applicable to misfit-strained core-shell nanowires. Besides the pure geometrical differences, this is mainly caused by the fact that a semi-infinite bulk substrate is much less susceptible to the deformation by a strained layer than a nanowire. In case of a misfit-strained core-shell nanowire, one expects that a considerable part of the elastic energy is stored in the core, whereas in case of a bulk substrate the strained film basically contains the whole elastic energy.

## 3.2 Theory

The system under investigation is a cylindrical core-shell (Si-Ge) nanowire with the shell grown epitaxially as described in the first chapter. The shell material is assumed to have a lattice misfit $m$ with respect to the core material: in this way both core and shell are elastically strained.

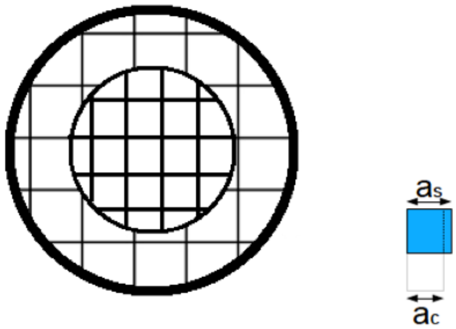


FIG 3.1 Misfit strained core-shell nanowire section

### 3.2.1 Reference state

Before solving the equilibrium equations at zero and first order with different geometrical hypothesis, we must introduce a crucial concept: the *reference state* of the displacements. But what does it mean? We must choose an appropriate reference state of displacements in the shell $\boldsymbol{u}^S$ with respect to the core. In order to better understand this concept we can take into account the flat case according to the J.N.Aqua work, where the substrate is taken semi-infinite in the y direction:

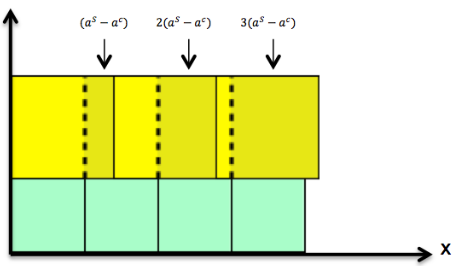


FIG 3.2 Substrate-film lattice misfit in the flat case

Finding the right reference state in flat case is easy: how we can see from figure 3.2, we can label the difference between the core lattice cell and the shell one, going towards x direction, by a quantity equal to

$$i(a^S - a^C) \qquad \text{with} \qquad \begin{cases} S, C = shell, core \\ i = 1,2,3.. \end{cases}$$

An analogous reasoning is applicable in the y direction: $j\,(a^S - a^C)$. Then we can try to do a mental exercise: we may think about labelling the $i$th film (shell) cell starting from a generic cell in the substrate (core), e.g. the first considering x axis, by counting how many steps we have to do throughout core cells. In other words we are looking for an expression to describe shell displacements as seen from the core. The correct relation is of course:

$$\boldsymbol{u}_{ref}^{C} = \boldsymbol{u}_{ref}^{S} - i(a^S - a^C)\frac{a^C}{a^C} - j(a^S - a^C)\frac{a^C}{a^C} = \boldsymbol{u}_{ref}^{S} - m(x\boldsymbol{e}_{\boldsymbol{x}} + y\boldsymbol{e}_{\boldsymbol{y}})$$

where

$$m = \frac{(a^S - a^C)}{a^C}, \qquad \begin{cases} x\boldsymbol{e}_{\boldsymbol{x}} = ia^C \\ y\boldsymbol{e}_{\boldsymbol{y}} = ja^C \end{cases}$$

So the shell in its state of lattice parameter $a^S$ is characterised by a displacement with respect to the reference state equal to $-m\,(x\boldsymbol{e}_{\boldsymbol{x}} + y\boldsymbol{e}_{\boldsymbol{y}})$.

### 3.2.2 Cylindrical reference state choice

Now we are ready to study the cylindrical case. In their work, Schmidt-McIntyre-Gosele use the same reference state of the flat case also to describe the cylindrical nanowire case. This is their 'Cartesian' choice:

$$\boldsymbol{u}^{C}_{ref} = \boldsymbol{u}^{S}_{ref} - m\boldsymbol{r}$$

In my opinion, this is not the correct choice: it doesn't take into account the fact that in the flat case the substrate is semi-infinite in the y direction, instead in the cylindrical case, the core has precise dimensions. This means that we must expect a dependence on the core radius in the choice of the reference state in the nanowire case. Now let's introduce the *'Cylindrical'* reference state.

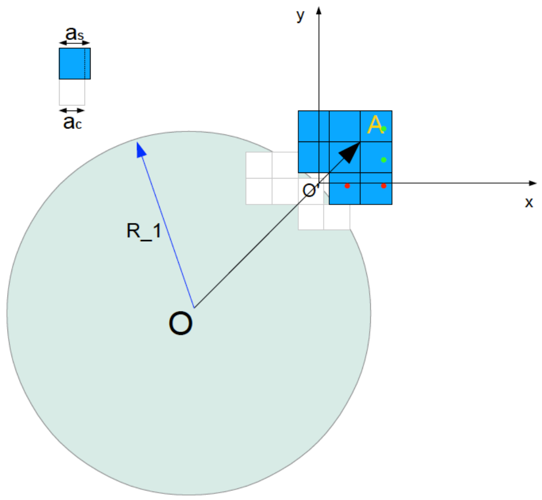


FIG 3.3 Core-shell misfit-strained nanowire: the reference state of shell displacements with respect to the core is dependent on the core radius

We can consider the reference state with his origin standing on the core surface, i.e. $r = R_1$, with $R_1$ being the core radius. So when we compute the positions of the elementary cells of the shell, we can start counting taking as variable $r' = r - R_1$ instead of $r$. Now we can see that, due to the misfit between the core and the shell, each cell in the shell 'feels' displacements propagating in the radial direction. To do this we can use a 'trick'. By considering the radial symmetry, we can start our reasoning from any point of the circumference of radius $R_1$. In particular, seeing fig (3.3), we can consider the position vector of *A* respect to *O* and then we can focus our attention on the point *O'*. Now, we may think to build a second cartesian frame with its origin standing in *O'.* Therefore, to reach *A*, we have to mode in the $\boldsymbol{e}_{x'}$ and $\boldsymbol{e}_{y'}$ directions:

$$\begin{cases} i(a^S - a^C)\dfrac{a^C}{a^C} = m\boldsymbol{x}_i' \\ j(a^S - a^C)\dfrac{a^C}{a^C} = m\boldsymbol{y}_j' \end{cases}$$

Again, we have divided and multiplied by $a^C$ and called $ia^C = \boldsymbol{x}_i'$, $ja^C = \boldsymbol{y}_j'$.

So we obtain:

$$\boldsymbol{u}_{ref}^C = \boldsymbol{u}_{ref}^S - m(x'\boldsymbol{e}_{x'} + y'\boldsymbol{e}_{y'} + z\boldsymbol{e}_z) = \boldsymbol{u}_{ref}^S - m((r - R_1)\boldsymbol{e}_r + z\boldsymbol{e}_z). \quad (3.1)$$

As seen before, to reach any cell in the shell we have just to rotate the frame built on the core surface using the radial symmetry. Even if the construction of this reference state seems to be more complex, it does include the dependence on the core radius and it has a very simple form. We shall see how this simple adjustment of reference state (the constant core radius) leads to very strong consequences both in the equilibrium equations and in the physical results fro the fastest growing mode, influencing also the linear stability analysis.

## 3.3 Linear stability analysis

Our task is studying morphological instability during islands growth by performing a linear stability analysis. The scheme of the linear stability analysis, following the works of Mullins and Spencer *et al.,* is simple. Core-shell nanowires are in reality not perfectly cylindrical. Instead, they show local deviations of cylindrical geometry. Thinking in Fourier space, these local deviations correspond to a broad distribution of sinusoidal surface modulations, which can be characterized by their wave number $q$ in axial direction (the $\boldsymbol{e_z}$ direction) and the mode number *n* in circumferential direction. Thus, if the outer radius of the unperturbed core-shell nanowire is $R_2$, and $\delta$ is the amplitude of the perturbation, then the actual surface radius $R_S$ can be expressed as

$$R_S = R_2 + \delta \cos(qz)\cos(n\vartheta) \tag{3.2}$$

this is schematically depicted in fig 3.4 for a perturbation with *n=4* and $q = 2\pi/R_2$.

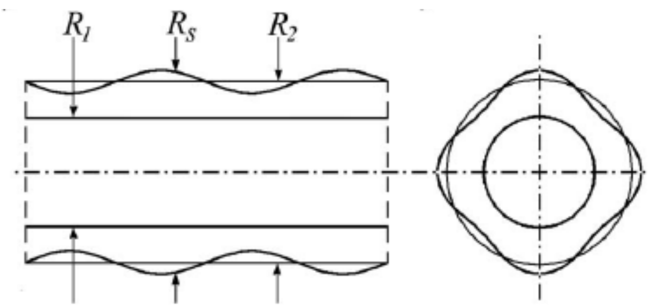


FIG 3.4 Schematic of core-shell nanowire with a sinusoidal perturbation of the surface; $R_1$ and $R_2$ are the initial core and shell radii, respectively and $R_s$ is the surface radius

Of course, such modulations of the shell thickness also affect the stress and strain distributions within the nanowire and cause variations of the elastic energy. These variations can drive surface diffusion, and the question to be answered is whether surface diffusion ultimately leads to an increase or a decrease in the initial amplitude of modulation.

In order to answer this question the stress and strain distributions are calculated in a perturbative fashion up to the first order in $\delta$, where all first order quantities will be marked by a tilde, symbolizing the waviness of the surface. Thus:

$$u_i^\alpha = \bar{u}_i^\alpha + \tilde{u}_i^\alpha + \mathcal{O}(\delta^2)$$

$$\varepsilon_{ij}^\alpha = \bar{\varepsilon}_{ij}^\alpha + \tilde{\varepsilon}_{ij}^\alpha + \mathcal{O}(\delta^2) \tag{3.3}$$

$$\sigma_{ij}^\alpha = \bar{\sigma}_{ij}^\alpha + \tilde{\sigma}_{ij}^\alpha + \mathcal{O}(\delta^2)$$

The index $\alpha$ takes values C or S, where C stands for the nanowire core and S for the shell. Terms that are of second order in $\delta$ will be neglected. To determine the zeroth and first order stress and strain distributions, we assume that the system is in equilibrium, so that in the absence of external body forces the displacement vector **u** has to fulfill the equations of equilibrium (2.12) that in terms of displacements read as:

$$(1-2\upsilon)\Delta\boldsymbol{u}^{\boldsymbol{\alpha}} + \nabla(\nabla \cdot \boldsymbol{u}^{\boldsymbol{\alpha}}) = 0$$

where

$$\boldsymbol{u}^\alpha = u_r^\alpha \boldsymbol{e_r} + u_\vartheta^\alpha \boldsymbol{e_\vartheta} + u_z^\alpha \boldsymbol{e_z}$$

Of course we will need proper boundary conditions to solve these equations. To simplify the problem, we use the radial symmetry cutting off the dependence on $\vartheta$.

### 3.3.1 Zeroth order

Let's solve Lamé equations at zeroth order. Firstly, let's modify expressions (2.5) and (2.14) inserting the reference state displacement $\boldsymbol{u}^{ref} = -m\,((r - R_1)\boldsymbol{e_r} + z\boldsymbol{e_z})$:

$$\bar{\boldsymbol{u}}^{\alpha} = \bar{\boldsymbol{u}}^{\alpha} + \bar{\boldsymbol{u}}^{ref}\delta_{\alpha s}$$

So that

$$\bar{\varepsilon}_{ij}^{\alpha} = \frac{1}{2}\left(\frac{\partial \bar{u}_i^{\alpha}}{\partial x_j} + \frac{\partial \bar{u}_j^{\alpha}}{\partial x_i}\right) - m\delta_{\alpha s}(\delta_{ir} + \delta_{i\vartheta}\left(1 - \frac{R_1}{r}\right) + \delta_{iz})\delta_{ij} \tag{3.4}$$

As we can see, if we had used the Cartesian reference state we would have the unit instead of $\left(1 - \frac{R_1}{r}\right)$ term. Thus, the 'Cylindrical' reference state influences in a strong way also the strain tensor. Now let's compute the stress tensor: from (2.14), inserting (3.4) we get:

$$\bar{\sigma}_{ij}^{\alpha} = \frac{E}{(1+\upsilon)(1-2\upsilon)}\Bigg[(1-2\upsilon)\left(\frac{1}{2}\left(\frac{\partial \bar{u}_i^{\alpha}}{\partial x_j} + \frac{\partial \bar{u}_j^{\alpha}}{\partial x_i}\right) - m\delta_{\alpha s}(\delta_{ir} + \delta_{i\vartheta}\left(1 - \frac{R_1}{r}\right) + \delta_{iz})\delta_{ij}\right) + \upsilon\left(\bar{u}_{kk}^{\alpha} - \delta_{\alpha s} m\left(3 - \frac{R_1}{r}\right)\right)\delta_{ij}\Bigg] \tag{3.5}$$

where

$$\bar{u}_{kk}^{\alpha} = \frac{\partial \bar{u}_r^{\alpha}}{\partial r} + \frac{\bar{u}_r^{\alpha}}{r} + \frac{\partial \bar{u}_z^{\alpha}}{\partial z}$$

We are now ready to solve Lamé equations:

$$\begin{cases} \dfrac{\partial \bar{\sigma}_{rr}^{\alpha}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \bar{\sigma}_{r\vartheta}^{\alpha}}{\partial \vartheta} + \dfrac{\partial \bar{\sigma}_{rz}^{\alpha}}{\partial z} + \dfrac{1}{r}(\bar{\sigma}_{rr}^{\alpha} - \bar{\sigma}_{\vartheta\vartheta}^{\alpha}) = 0 \\ \dfrac{\partial \bar{\sigma}_{r\vartheta}^{\alpha}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \bar{\sigma}_{\vartheta\vartheta}^{\alpha}}{\partial \vartheta} + \dfrac{\partial \bar{\sigma}_{\vartheta z}^{\alpha}}{\partial z} + \dfrac{2}{r}\bar{\sigma}_{r\vartheta}^{\alpha} = 0 \\ \dfrac{\partial \bar{\sigma}_{rz}^{\alpha}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \bar{\sigma}_{\vartheta z}^{\alpha}}{\partial \vartheta} + \dfrac{\partial \bar{\sigma}_{zz}^{\alpha}}{\partial z} + \dfrac{1}{r}\bar{\sigma}_{rz}^{\alpha} = 0 \end{cases}$$

Cutting the dependence on $\vartheta$ we have:

$$\begin{cases} \dfrac{\partial \bar{\sigma}_{rr}^{\alpha}}{\partial r} + \dfrac{\partial \bar{\sigma}_{rz}^{\alpha}}{\partial z} + \dfrac{1}{r}(\bar{\sigma}_{rr}^{\alpha} - \bar{\sigma}_{\vartheta\vartheta}^{\alpha}) = 0 \\ \dfrac{\partial \bar{\sigma}_{rz}^{\alpha}}{\partial r} + \dfrac{\partial \bar{\sigma}_{zz}^{\alpha}}{\partial z} + \dfrac{1}{r}\bar{\sigma}_{rz}^{\alpha} = 0 \end{cases}$$

In term of displacements:

$$\begin{cases} (2-2\upsilon)\left(\dfrac{\partial^2 \bar{u}_r^{\alpha}}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \bar{u}_r^{\alpha}}{\partial r} - \dfrac{\bar{u}_r^{\alpha}}{r^2} - m\delta_{\alpha s}\dfrac{R_1}{r^2}\right) + (1-2\upsilon)\dfrac{\partial^2 \bar{u}_r^{\alpha}}{\partial z^2} + \dfrac{\partial^2 \bar{u}_z^{\alpha}}{\partial r \partial z} = 0 \\ (2-2\upsilon)\dfrac{\partial^2 \bar{u}_z^{\alpha}}{\partial z^2} + (1-2\upsilon)\dfrac{\partial^2 \bar{u}_z^{\alpha}}{\partial r^2} + \dfrac{\partial^2 \bar{u}_r^{\alpha}}{\partial r \partial z} + \dfrac{1}{r}\dfrac{\partial \bar{u}_r^{\alpha}}{\partial z} + (1-2\upsilon)\dfrac{1}{r}\dfrac{\partial \bar{u}_z^{\alpha}}{\partial r} = 0 \end{cases}$$

### 3.3.2 'Fixed' hypothesis

Before carrying on with the solutions, we impose some strong hypothesis. Beside the independence on $\vartheta$ due to the radial symmetry (both $\bar{u}_{\vartheta}^{\alpha}$ as well as the derivatives being null), we consider the problem to be translationally invariant with respect to translation in the $\boldsymbol{e_z}$ direction:

$$\frac{\partial \bar{u}_r^{\alpha}}{\partial z} = 0$$

We can further simplify the problem with another strong, not geometrical, hypothesis: it is instructive and experimentally interesting considering the nanowire limited at the end faces by two planes perpendicular to the $\boldsymbol{e_z}$ direction. Of course this is a mental exercise because the nanowire is taken with an infinite length. But from an experimental point of view this is applicable in laboratory by setting a fixed plane (see fig 3.5). What kind of physical consequences could we expect from this hypothesis? If we limit the length of the nanowire (with no dependence on the length), we can expect that during the deposition of material on the substrate this material, instead of being distributed along the infinite z-axis, tends to accumulate in the $\boldsymbol{e}_r$ direction accelerating the process of growth. So, what we can expect from this hypothesis is a faster growth of islands.

In quantitative terms, the hypothesis is:

$$\frac{\partial \bar{u}_z^\alpha}{\partial z} = 0$$

In these hypothesis Lamé equations reduce to:

$$\begin{cases} \dfrac{\partial^2 \bar{u}_r^\alpha}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \bar{u}_r^\alpha}{\partial r} - \dfrac{\bar{u}_r^\alpha}{r^2} - m\delta_{\alpha s}\dfrac{R_1}{r^2} = 0 \\ \dfrac{\partial^2 \bar{u}_z^\alpha}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \bar{u}_z^\alpha}{\partial r} = 0 \end{cases}$$

where the solutions are:

$$\bar{u}_r^\alpha = A_\alpha r + \frac{B_\alpha}{r} - mR_1$$

$$\bar{u}_z^\alpha = C_\alpha + D_\alpha \log r$$

with $A_\alpha,\ B_\alpha,\ C_\alpha, D_\alpha$ being eight undetermined constants. We can eliminate directly $B_c$ and $D_c$ imposing the condition that $\bar{u}_r^c$ has to be finite at $r = 0$. Thus the problem is reduced to determining the six unknown $A_c,\ A_s,\ B_s,\ C_c,\ C_s, D_s$ by imposing proper boundary conditions.

The conditions to be imposed are as follows.

i. Equality of displacements at the core-shell interface. This ensures the epitaxial nature of the core-shell interface and leads to

$$\left. \begin{matrix} u_r^c = u_r^s \\ u_z^c = u_z^s \end{matrix} \right|_{r=R_1}$$

ii. Zero net normal face at the core-shell interface. Neglecting interface stress and using the outward normal $\bar{n}_i$ of the shell-core interface, this gives

$$\bar{\sigma}_{ij}^c \bar{n}_j = \bar{\sigma}_{ij}^s \bar{n}_j \Big|_{r=R_1}$$

The outward normal is computed in this way: the cylinder can be parameterized as

$$C(\vartheta, t) = (r\cos\vartheta\,, r\sin\vartheta\,, t)$$

by definition the normal is

$$\boldsymbol{n} = \left(\frac{\partial C}{\partial \vartheta}\right) \wedge \left(\frac{\partial C}{\partial t}\right)$$

then

$$\boldsymbol{n} = (-r\sin\vartheta\,, r\cos\vartheta\,, 0) \wedge (0,0,1) = (r\cos\vartheta\,, r\sin\vartheta\,, 0)$$

Normalizing we obtain

$$n = \frac{\boldsymbol{n}}{\|\boldsymbol{n}\|} = \frac{(r\cos\vartheta\,, r\sin\vartheta\,, 0)}{\sqrt{r^2\cos^2\vartheta + r^2\sin^2\vartheta + 0}} = (\cos\vartheta, \sin\vartheta, 0)$$

expanding in power series, at the 0 order the normal to the surface is:

$$n = (1,0,0)$$

The boundary condition then reads as

$$\begin{cases}\bar{\sigma}_{rr}^{c} = \bar{\sigma}_{rr}^{s} \\ \bar{\sigma}_{zr}^{c} = \bar{\sigma}_{zr}^{s}\end{cases}\Bigg|_{r=R_1}$$

iii. Zero net normal force at the surface:

$$\bar{\sigma}_{ij}^{s}\bar{n}_j = 0\Big|_{r=R_1}$$

where at zero order in $\delta$ the core and shell surfaces are taken smooth. Then:

$$\begin{matrix}\bar{\sigma}_{rr}^{s} = 0 \\ \bar{\sigma}_{zr}^{s} = 0\end{matrix}\Bigg|_{r=R_1}$$

Solving these boundary conditions we get:

$$A_s = \frac{m\left((v-2v^2)R_1^2-(2-2v^2)R_2^2\right)}{2(v-1)R_2^2} \qquad B_s = \frac{mvR_1^2}{2(v-1)}$$

$$A_c = \frac{m(2v-1)(vR_2^2-vR_1^2)}{2(v-1)R_2^2} \qquad D_S = 0 \qquad C_c = C_s = constant$$

imposing constant to be zero for simplicity.

Then displacements are:

$$\bar{\boldsymbol{u}}^{s} = \left(\frac{m(2v-1)(vR_2^2-vR_1^2)}{2(v-1)R_2^2}r + \frac{mvR_1^2}{2(v-1)}\frac{1}{r} - mR_{1,} \quad 0, \quad 0\right)$$

$$\bar{\boldsymbol{u}}^{c} = \left(\frac{m(2v-1)(vR_2^2-vR_1^2)}{2(v-1)R_2^2}r, \quad 0, \quad 0\right)$$

### 3.3.3 First order

Next we examine how adding a sinusoidal perturbation changes the stress and/or strain distribution. To arrive at expressions for the first order contributions, $\tilde{\varepsilon}_{ij}^{\alpha}$ or $\tilde{\sigma}_{ij}^{\alpha}$ is in principle as straightforward as seen before. However, instead of solving the equations of equilibrium for the displacements, we will look for a solution in terms of the so-called Papkovich-Neuber potentials $\boldsymbol{\psi}$ and $\xi$ with $\boldsymbol{\psi} = \psi_r \boldsymbol{e}_r + \psi_\vartheta \boldsymbol{e}_\vartheta + \psi_z \boldsymbol{e}_z$ being a vector and $\xi$ being a scalar potential. From these, the displacements can then be derived by using the following relation:

$$\boldsymbol{u}^{\alpha} = 4(1-\upsilon)\boldsymbol{\psi}^{\alpha} - \nabla(\boldsymbol{R}\cdot\boldsymbol{\psi}^{\alpha} + \xi^{\alpha}) \tag{3.6}$$

with $R = r\boldsymbol{e_r} + z\boldsymbol{e_z}$.

The main advantage of the Papkovic-neuber potentials is that they simplify solving the equations of equilibrium, as these are fulfilled if both potentials separately satisfy Laplace's equation

$$\Delta \cdot \boldsymbol{\psi}^{\alpha} = 0$$

$$\Delta \xi^{\alpha} = 0$$

For a radially symmetric problem the solutions are

$$\xi^{\alpha} = (d_\alpha I_n(qr) + e_\alpha K_n(qr))e^{i(n\vartheta + qz)}$$

$$\psi_r^{\alpha} = (f_\alpha I_{n+1}(qr) + g_\alpha K_{n-1}(qr) + h_\alpha K_{n+1}(qr) + i_\alpha K_{n-1}(qr))e^{i(n\vartheta + qz)}$$

$$\psi_\vartheta^{\alpha} = (-if_\alpha I_{n+1}(qr) + ig_\alpha I_{n-1}(qr) - ih_\alpha K_{n+1}(qr) + ii_\alpha K_{n-1}(qr))e^{i(n\vartheta + qz)}$$

$$\psi_z^{\alpha} = (j_\alpha I_n(qr) + k_\alpha K_n(qr))e^{i(n\vartheta + qz)}$$

where $I_n(qr)$ and $K_n(qr)$ denote the modified Bessel functions of order $n$. Again the task is to determine the values of the constants $d_\alpha$, $e_\alpha$, $f_\alpha$, $g_\alpha$, $h_\alpha$, $i_\alpha$, $j_\alpha$, $k_\alpha$. Fortunately , one can greatly reduce the number of unknowns by taking both the symmetry and the finiteness of the solution into account.

Consider the diagonal strain component in the $\boldsymbol{e_z}$ direction, which according to (3.6) and (2.5) is given by:

$$\varepsilon_{zz} = (3-4\upsilon)\partial_z\psi_z - r\partial_z^2\psi_r - \partial_z^2\xi - \partial_z\psi_z - z\partial_z^2\psi_z \tag{3.7}$$

This strain component must exhibit the same periodicity as the perturbation. However, due to the last term in Eq. (3.7), this is only possible if $\psi_z = 0$ or equivalently if $j_\alpha = k_\alpha = 0$. So Eq. (3.5) simplifies to

$$\tilde{u}_r = 4(1-\upsilon)\psi_r^\alpha - \partial_r(r\psi_r^\alpha + \xi^\alpha) \tag{3.8}$$

$$\tilde{u}_z = -\partial_z(r\psi_r^\alpha + \xi^\alpha) \tag{3.9}$$

$$\tilde{u}_\vartheta = 4(1-\upsilon)\psi_\vartheta^\alpha - \frac{1}{r}\partial_\vartheta(r\psi_r^\alpha + \xi^\alpha) \tag{3.10}$$

One is temped now to directly insert these relations in the boundary conditions. Yet one has to be careful here, as this is only correct for modes with $n \neq 1$. The problem with the $n = 1$ mode arises when one tries to directly derive the strain using Eq. (2.5). Due to the $r^{-1}$ terms these expressions would give infinite strains unless the displacements $\tilde{u}_r$ and $\tilde{u}_\vartheta$ are zero at $r = 0$.

$$\tilde{u}_{r\,|r=0} = \left((3-4\upsilon)g_c - \frac{q}{2}d_c\right)\delta_{n1}e^{i(n\vartheta+qz)} \tag{3.11}$$

$$\tilde{u}_{\vartheta\,|r=0} = \left((3-4\upsilon)g_c - \frac{q}{2}d_c\right)\delta_{n1}e^{i(n\vartheta+qz)} \tag{3.12}$$

Note that due to $\delta_{n1}$, the displacements $\tilde{u}_r$ and $\tilde{u}_\vartheta$ at $r = 0$ are nonzero only in case of the $n = 1$ mode. In order to eliminate this potential divergence of the strain, we have to subtract expressions (3.11) (3.12) from (3.8) (3.10) respectively. Why we only obtain nonzero displacements at r=0 in case of the n=1 mode can be seen in fig 3.5, which shows a schematic of the first three modes, n=0, n=1 and n=2. Due to symmetry reason, the radially symmetric n=0 perturbation cannot effect any nonzero displacements $\tilde{u}_r$ and $\tilde{u}_\vartheta$ at r=0, i.e., at the center of symmetry center. Also in case of the n=1 mode, where you have one bump on one side and a recess on the opposite side of the wire $\tilde{u}_r$ and $\tilde{u}_\vartheta$ can become nonzero at r=0. One can interpret this nonzero displacement as a bending of the wire as a whole, resulting in case of the n=1 mode in a slight S shape. Let us come back now to determining the unknowns in general solutions. We can eliminate three constants by demanding that the solution has to be finite at r=0, wherefore $e_c$, $h_c$ and $i_c$ are bound to be zero. The remaining nine constants we have to determine with the help of an equal number of boundary conditions.

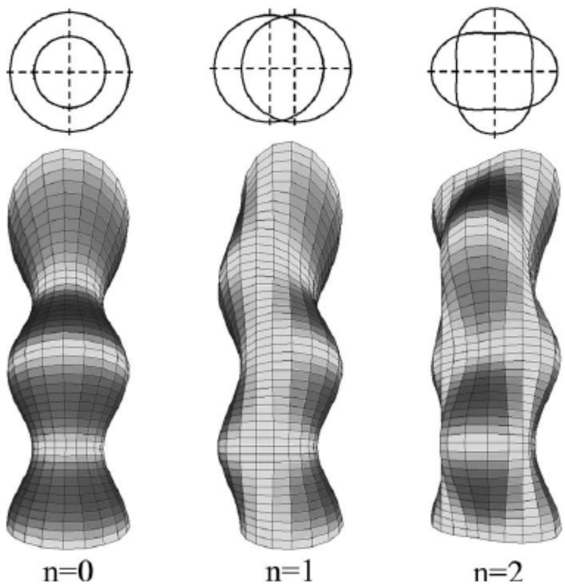


FIG 3.5 Schematic cross section and 3D view of the three first modes n=0, n=1 and n=2; the amplitude of the perturbation was chosen to be $\boldsymbol{R_2/4}$. Only in case of the n=1 mode the center of the cross section shifts with z

For this aim it is useful to introduce the normal vectors $\tilde{n}_i^c$ and $\tilde{n}_i^s$of the core-shell interface and of the surface, respectively.

Neglecting the second order terms, the surface normal is

$$\tilde{\boldsymbol{n}}^s = \begin{pmatrix} 1 \\ \eta_\vartheta \\ \eta_z \end{pmatrix} \tag{3.13}$$

The vector $\boldsymbol{R}_s = R_s \boldsymbol{e_r} + z\boldsymbol{e_z}$ with $R_s$ being the perturbed shell radius, defines the position of a point on the surface of the nanowire. Following the work of Weatherburn, let us define two vectors $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$,

$$\boldsymbol{r_1} = \frac{1}{R_s}\partial_\vartheta \boldsymbol{R}_s = (\eta_\vartheta \boldsymbol{e}_r + \boldsymbol{e}_\vartheta + 0\boldsymbol{e}_z)$$

$$\boldsymbol{r_2} = \partial_z \boldsymbol{R}_s = (\eta_z \boldsymbol{e}_r + 0\boldsymbol{e}_\vartheta + \boldsymbol{e}_z)$$

Substituting the expression of $R_s$:

$$\boldsymbol{r_1} = (\frac{\delta i n e^{i(n\vartheta+qz)}}{R_2 + \delta e^{i(n\vartheta+qz)}}\boldsymbol{e}_r + \boldsymbol{e}_\vartheta + 0\boldsymbol{e}_z)$$
$$\boldsymbol{r_2} = (\delta i q e^{i(n\vartheta+qz)}\boldsymbol{e}_r + 0\boldsymbol{e}_\vartheta + \boldsymbol{e}_z)$$

so neglecting the second order in $\delta$

$$\eta_\vartheta = \frac{\delta i n}{R_2} e^{i(n\vartheta+qz)}$$
$$\eta_z = \delta i q e^{i(n\vartheta+qz)}$$

The outward normal of the perturbed shell surface is given by

$$\tilde{\boldsymbol{n}}^s = H^{-1}\boldsymbol{r}_1 \times \boldsymbol{r}_1 = H^{-1}\begin{pmatrix} 1 \\ \eta_\vartheta \\ \eta_z \end{pmatrix}$$

where $H = \sqrt{1 - \eta_\vartheta^2 + \eta_z^2}$ is a normalization factor that, however, in a linear approximation becomes equal to 1. The curvature $\kappa$ of the nanowire surface is given by

$$\kappa = \nabla_s \cdot \tilde{\boldsymbol{n}}^s$$

where the surface gradient $\nabla_s$ can now be obtained

$$\nabla_s = H^{-2}(\boldsymbol{r}_2 \times \tilde{\boldsymbol{n}}^s)\frac{1}{R_s}\partial_\vartheta + H^{-2}(\tilde{\boldsymbol{n}}^s \times \boldsymbol{r}_1)\partial_z =$$

$$= \frac{1}{H^2}\begin{pmatrix} -\eta_\vartheta \\ 1+\eta_z^2 \\ -\eta_\vartheta\eta_z \end{pmatrix}\frac{1}{R_s}\partial_\vartheta + \frac{1}{H^2}\begin{pmatrix} -\eta_z \\ -\eta_\vartheta\eta_z \\ 1+\eta_\vartheta^2 \end{pmatrix}\partial_z$$

Ignoring terms of second and higher order in $\delta$ gives the linearized surface gradient

$$\nabla_s = \begin{pmatrix} -\eta_\vartheta R_s^{-1}\partial_\vartheta - \eta_z\partial_z \\ R_s^{-1}\partial_\vartheta \\ \partial_z \end{pmatrix}.$$

Using above results the linearized surface divergence acting on the vector $\boldsymbol{A} = A_r\boldsymbol{e}_r + A_\vartheta\boldsymbol{e}_\vartheta + A_z\boldsymbol{e}_z$ reads

$$\nabla_s \cdot \boldsymbol{A} = \left(\frac{1}{R_s} - \frac{\eta_\vartheta}{R_s}\partial_\vartheta - \eta_z\partial_z\right)A_r + \frac{1}{R_s}(\partial_\vartheta + \eta_\vartheta)A_\vartheta + \partial_z A_z.$$

Then the curvature of the surface can most easily be obtained by calculating the divergence of the normal vector:

$$\kappa = \nabla_s \cdot \tilde{\boldsymbol{n}}^s = \nabla_s \cdot \begin{pmatrix} 1 \\ \eta_\vartheta \\ \eta_z \end{pmatrix} =$$

$$= \left(\frac{1}{R_s} - \frac{\eta_\vartheta}{R_s}\partial_\vartheta - \eta_z\partial_z\right)(1) + \frac{1}{R_s}(\partial_\vartheta + \eta_\vartheta)\eta_\vartheta + \partial_z\eta_z =$$

$$= \frac{1}{R_2} + \delta\left(\frac{n^2-1}{R_2^2} + q^2\right)e^{i(n\vartheta+qz)} \qquad (3.14)$$

where we have inserted the expression for $R_s$, used the first order expansion of $(1+x)^{-1}=(1-x)$ and neglected second order terms in $\delta$.

Now we can compute the boundary conditions:

i. Equality of displacements at the core-shell interface:

$$\left.\begin{array}{l}\tilde{u}_r^c=\tilde{u}_r^s\\ \tilde{u}_\vartheta^c=\tilde{u}_\vartheta^s\\ \tilde{u}_z^c=\tilde{u}_z^s\end{array}\right|_{r=R_1} \tag{3.15}$$

ii. Zero net force at the core-shell interface

$$\tilde{\sigma}_{ij}^c\tilde{n}_j^c=\tilde{\sigma}_{ij}^s\tilde{n}_j^c\Big|_{r=R_1}$$

which by using $\boldsymbol{\sigma}_{ij}=\overline{\boldsymbol{\sigma}}_{ij}+\widetilde{\boldsymbol{\sigma}}_{ij}$ together with the fact that the interface normal $\tilde{n}_i^c=\bar{n}_i=\boldsymbol{e}_r$ gives

$$\begin{array}{l}\tilde{\sigma}_{rr}^c=\tilde{\sigma}_{rr}^s\\ \tilde{\sigma}_{r\vartheta}^c=\tilde{\sigma}_{r\vartheta}^s\\ \tilde{\sigma}_{rz}^c=\tilde{\sigma}_{rz}^s\end{array}\qquad \text{with } r=R_1 \tag{3.16}$$

iii. Zero net normal force at the surface,

$$\tilde{\sigma}_{ij}^s\tilde{n}_j^s=0 \qquad \text{with } r=R_2$$

That is, using the expression of $\tilde{n}_j^s$ and $\boldsymbol{\sigma}_{ij}=\overline{\boldsymbol{\sigma}}_{ij}+\widetilde{\boldsymbol{\sigma}}_{ij}$,

$$\begin{array}{c}\tilde{\sigma}_{rr}^s=0\\ \tilde{\sigma}_{\vartheta r}^s+\eta_\vartheta\bar{\sigma}_{\vartheta\vartheta}^s=0\\ \tilde{\sigma}_{zr}^s+\eta_z\bar{\sigma}_{zz}^s=0\end{array}\qquad \text{with } r=R_2 \tag{3.17}$$

This set of equations has now to be solved for the nine unknown constants. However, one shows that (3.15) and (3.16) are fulfilled if $e_s=h_s=i_s=0$, $d_c=d_s=d$, and $g_c=g_s=g$. Thus, it boils down to determining the values of $d_s$, $g_s$ and $h_s$ by solving Eqs (3.17).

The solutions are displayed in the Appendix. With the expressions for $d_s$, $g_s$ and $h_s$ at hand, one can derive the displacements, the strain and the stress tensors. These results can then combined with the corresponding zeroth order results to obtain the full stress and strain distributions to first order in $\delta$.

### 3.3.4 Stability parameter

Now with the solutions at zeroth and first order we are ready to perform the linear stability analysis. According to Spencer et al. the diffusion induced surface flux can be expressed as

$$J_s = \frac{-D_s \Gamma}{kT} \nabla_s M_v \tag{3.18}$$

with $D_s$, $\Gamma$, $\nabla_s$ being the surface diffusion constant, the area density of lattice sites, and the surface gradient , respectively; $kT$ has its usual meaning. $M_v$ denotes the diffusion potential

$$M_v = \Omega\,\gamma'\kappa' + \frac{1}{2}\sigma_{ij}\varepsilon_{ij}\Big|_{r=R_s}$$

with $\gamma'$ and $\kappa'$ being the surface energy density and the curvature of the (unstrained) reference state and $\Omega$ being the volume per atom. The diffusion potential has to be evaluated at the surface of the nanowire. Following the suggestion by Spencer et al. we will neglect the difference between the actual state and the reference state and take the corresponding quantities $\gamma$ and $\kappa$ of the strained state instead. The $\frac{1}{2}\sigma_{ij}\varepsilon_{ij}$ term is the energy density of the stress and/or strain field. In addition to surface diffusion, we also consider the deposition of atoms onto the nanowire surface, going on at a rate $Q$, defined as the numbers of atoms per unit area and unit time. By employing the continuity equation, one can show that a radial vector $\boldsymbol{R}_s$ to a position on the surface will change with time as

$$\dot{\boldsymbol{R}}_s = \Omega Q \tilde{\boldsymbol{n}}^s - \Omega(\nabla_s J_s)\tilde{\boldsymbol{n}}^s$$

with the surface normal $\tilde{\boldsymbol{n}}^s$ defined in Eq (3.13). Inserting Eq (3.18), the radial component of this vector equation becomes, to first order in $\delta$, equal to

$$\dot{R}_s = \Omega Q + \frac{D_s \Gamma \Omega^2}{kT} \Delta_s \left(\gamma\kappa + \frac{1}{2}\sigma_{ij}\varepsilon_{ij}\right)_{r=R_s} \qquad (3.19)$$

with $\Delta_s$ being the surface Laplace operator, whose first order expression is given by

$$\Delta_s = \nabla_s \cdot \nabla_s = \frac{\eta_\vartheta}{R_s^2}\partial_\vartheta + \frac{1}{R_s^2}\partial_\vartheta^2 - \frac{\eta_z}{R_s}\partial_z + \partial_z^2 \qquad (3.20)$$

Using $\sigma_{ij} = \bar{\sigma}_{ij} + \tilde{\sigma}_{ij}$ and $\varepsilon_{ij} = \bar{\varepsilon}_{ij} + \tilde{\varepsilon}_{ij}$ in the above equation and expanding the product would give four terms. Yet, the $\bar{\sigma}_{ij}\bar{\varepsilon}_{ij}$ term is independent of z and $\vartheta$ and can therefore be omitted. Also the $\tilde{\sigma}_{ij}\tilde{\varepsilon}_{ij}$ can be neglected, since it is of second order in $\delta$, and the other two terms can be combined by using $\bar{\sigma}_{ij}\tilde{\varepsilon}_{ij} = \tilde{\sigma}_{ij}\bar{\varepsilon}_{ij}$. That this equality holds can be shown in a straightforward calculation by inserting the explicit relations and simplifying the expressions. Since all terms in parentheses in Eq (3.20) are at least of order $\delta$, it is sufficient to use the zeroth order approximation for the Laplacian $\Delta_s$

$$\Delta_s^0 = \frac{1}{R_2^2}\partial_\vartheta^2 + \partial_z^2$$

Since we are only interested in first order contributions, we can evaluate the expression in parentheses at $R_2$ instead of $R_s$,

$$\dot{R}_s = \Omega Q + \frac{D_s \Gamma \Omega^2}{kT} \Delta_s^0 \left(\gamma\kappa + \bar{\sigma}_{ij} \tilde{\varepsilon}_{ij}\right)_{r=R_2}$$

Due to the linear approximation, $\tilde{\varepsilon}_{ij}$ must be proportional to $\delta$. In addition, one can show using (3.8)-(3.10) and (3.4) that $\tilde{\varepsilon}_{ij}$ is proportional to $e^{i(n\vartheta+qz)}$, so that we can define a quantity $\tilde{\varepsilon}_{ij}$ that is independent of z and $\vartheta$ as

$$\tilde{\epsilon}_{ij} = \tilde{\varepsilon}_{ij} e^{i(n\vartheta+qz)}$$

We can now insert the explicit forms of $R_s$ and $\kappa$ and perform $\Delta_s^0$,

$$\dot{R}_2 + \dot{\delta} e^{i(n\vartheta+qz)} = \Omega Q + \frac{D_s \Gamma \Omega^2 \gamma}{kT} \left(\frac{n^2}{R_2^2} + q^2\right) \left(\frac{n^2-1}{R_2^2} + q^2 + \frac{\bar{\sigma}_{ij} \tilde{\varepsilon}_{ij}}{\gamma}\right) \delta e^{i(n\vartheta+qz)}$$

Separating the terms that are proportional to the exponential from those that are not then gives

$$\dot{R}_2 = \Omega Q \qquad (3.21)$$

$$\dot{\delta} = \frac{D_s \Gamma \Omega^2 \gamma}{kT} S \delta \qquad (3.22)$$

with the stability parameter (or growing rate)

$$S = \left(\frac{n^2}{R_2^2} + q^2\right) \left(\frac{1-n^2}{R_2^2} - q^2 - \frac{\bar{\sigma}_{ij} \tilde{\varepsilon}_{ij}}{\gamma}\right)$$

One can see that a uniform deposition on the surface, characterized by the parameter $Q$, causes an increase in the outer radius $R_2$. This change in $R_2$ would in principle also affect the magnitude of stability parameter S. However, if we assume that the radius change caused by the deposition happens on a much longer time scale, i.e., considering low deposition rates, we can decouple Eqs (3.21) and (3.22) and consider the growth of the instability separately.

In this case, the solution becomes

$$\delta(t) = \delta_0 e^{\frac{D_S \Gamma \Omega^2 \gamma}{kT} S t} \tag{3.23}$$

with $\delta_0$ being the amplitude of the perturbation at $t = 0$. This simple exponential time dependence corresponds to an increase in $\delta$ if $S > 0$. That is the surface is unstable with respect to this particular perturbation. For $S < 0$ the initial amplitude of perturbation decays exponentially with time, i.e., the cylindrical surface is stable with respect to this particular perturbation.

## 3.4 Results and discussion

In this section we want to find the features ($R_1, R_2 - R_1, n$) that lead to the fastest growing mode. In order to find the fastest growing mode we perform the stability parameter as a function of the wave number $q$ for $m = -0.043$, $\upsilon = 0.26$, $\gamma = 1.5\ \mathrm{Jm}^{-2}$ $E = 115{,}92\ \mathrm{GPa}$ for different values of the core radius $R_1$, shell thickness $R_2 - R_1$ and mode number $n$. In the figure (3.6) we can see the behaviour of the curve: the higher is the maximum, the faster is growing mode.

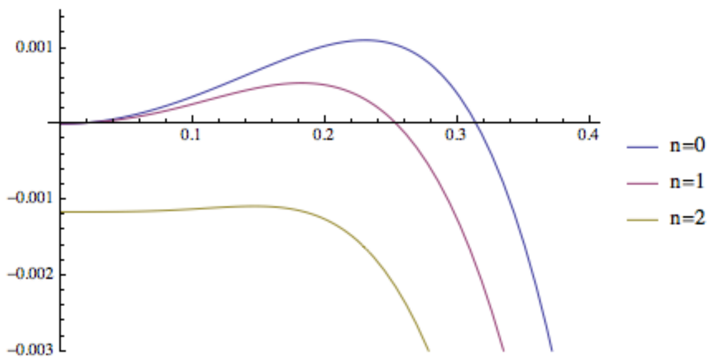


FIG 3.6 Stability parameter S in function of the wave number $q$, with $R_1 = 7\mathrm{nm}$ and $R_2 = 10\mathrm{nm}$, misfit $m = -0.043$

| | R=R1 | R1=0.1 | R1=1 | R1=5 | R1=10 | R1=20 |
|---|---|---|---|---|---|---|
| n=0 | Smax | 2674 | 0,4592 | 0,0040 | 0,0014 | 0,00115 |
| | qmax | 7,2042 | 0,8388 | 0,2830 | 0,2491 | 0,242 |
| n=1 | Smax | 20,7195 | 0,02072 | 0,00076 | 0,00066 | 0,00084 |
| | qmax | 0.0000776964 | 0,00005 | 0,18 | 0,1946 | 0,23306 |
| n=2 | Smax | -119834,00 | -11,83 | -0,01787 | -0,00092 | 0,00078 |
| | qmax | -3.99248*10^-10 | 2,1914*10^-9 | 1.00487*10^-11 | 0,15848 | 0,21041 |
| | | | | | | |
| | R=2R1 | R1=0.1 | R1=1 | R1=5 | R1=10 | R1=20 |
| n=0 | Smax | 177 | 0,0443587 | 0,000992725 | 0,00074102 | 0,00067741 |
| | qmax | 3,65863 | 0,474184 | 0,221886 | 0,217378 | 0,211207 |
| n=1 | Smax | 0,00006 | 0,000161871 | 0,000504022 | 0,000515724 | 0,000650607 |
| | qmax | 0,09911 | 0,000025591 | 0,177373 | 0,204709 | 0,20995 |
| n=2 | Smax | -7498,71 | -0.74902 | -0,00111394 | 0,000534737 | 0,000604238 |
| | qmax | 1,88753*10^-11 | 6,88408*10^-9 | 0,14396 | 0,187743 | 0,20219 |
| | | | | | | |
| | R=10R1 | R1=0.1 | R1=1 | R1=5 | R1=10 | R1=20 |
| n=0 | Smax | 0,437549 | 0,000895596 | 0,000575161 | 0,000553451 | 0,000543665 |
| | qmax | 0,82530 | 0,213785 | 0,202948 | 0,201361 | 0,200704 |
| n=1 | Smax | 2,0719*10^-6 | 0,000479318 | 0,000563346 | 0,000550374 | 0,00054284 |
| | qmax | 6,63728*10^-6 | 0,173242 | 0,201819 | 0,201003 | -0.200615 |
| n=2 | Smax | -12 | -0.00111536 | 0,000533876 | 0,000541068 | 0,000540362 |
| | qmax | 2,71567*10^-9 | 0.143096 | 0,197116 | 0,199927 | -0.200347 |

Table 3.1 Maximum values of the stability parameter S in function of normal mode $\boldsymbol{n}$ and $\boldsymbol{R_1}$ in the three cases: shell thickness being null, equal to $\boldsymbol{R_1}$ and large compared with $\boldsymbol{R_1}$ ('Cylindrical' reference state)

Let's analyse the table (3.1). As we can see we have the maximum values of S and the relative $q$ values with $R_1 = R_2$ and so with shell thickness being null, this is a limit case, with $R_2 = 2R_1$ that is with the shell thickness equal to core radius and with $R_2 = 10R_1$ that is with a very large shell thickness. For each of these three cases we have taken the core radius $R_1$ going from 0.1nm, that is really small, to 20nm, that is quite large; and of course the mode number $n$ being equal to 0,1,2.

What may we conclude from this data?

1. The larger the core radius, the smaller $S_{max}$. It means that, in order to reach the fastest growing modes, we should take small values of $R_1$
2. For each value of $R_1$ and $R_2$ for $n = 0$ we find the fastest growing mode.
3. The smaller the thickness of the shell, the larger the stability parameter

These are very strong features from an experimental point of view. It's instructive stress that in the limit case of shell thickness being null, the $S_{max}$ value are several order of magnitude larger than the cases with large thickness. This means that, looking at Eq (3.23), the perturbation, for small shell thickness, grows really quickly: the growth process is therefore delicate in the initial phases of atomic deposition.

### 3.4.1 Comparison with 'Cartesian' reference state

In order to better understand how the choice of the reference state affects the linear stability analysis it has been done all the analysis with the 'Cartesian' reference state $\boldsymbol{u}_{ref}^{C} = \boldsymbol{u}_{ref}^{S} - m\boldsymbol{r}$

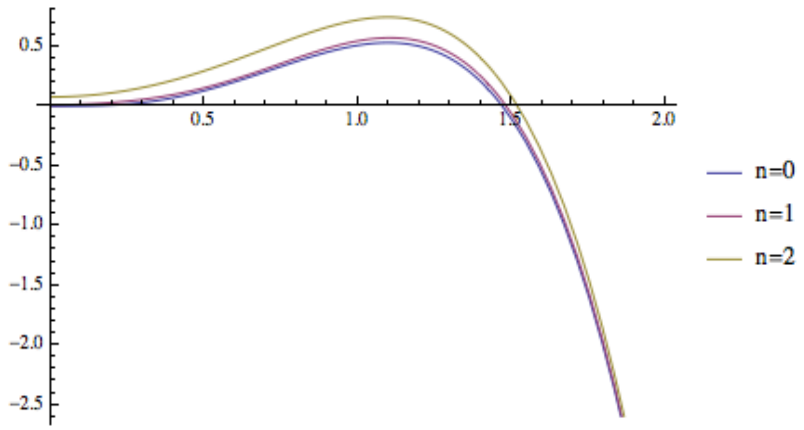


FIG 3.7 Stability parameter S in function of the wave number $q$, with $R_1 = 7\text{nm}$ and $R_2 = 10\text{nm}$ ('Cartesian' reference state)

From the table (3.2) we can see strong differences from the previous results:

1. $n = 0$ normal mode isn't yet the fastest growing mode.
2. The maximum values of stability parameters S are greater than the 'Cylindrical' reference state case

The great result, from an experimental point of view, of the choice of the 'Cylindrical' reference state is the possibility to set clear conditions to build a misfit-strained core-shell nanowire with the fastest growing mode.

Besides this we can see from fig (3.5) that for the $n = 1$ mode, that is in most cases the fastest growing mode for the 'Cartesian' reference state case, the centre of the cross section of the nanowire shift with z; instead the $n = 0$ mode is the simpler one. All these features are really useful from an experimental point of view, considering the delicateness of the growth process.

| | R=R1 | R1=0.1 | R1=1 | R1=5 | R1=10 | R1=20 |
|---|---|---|---|---|---|---|
| n=0 | Smax | 3074 | 1,6236 | 0,5918 | 0,4826 | 0.,425649 |
| | qmax | 7,5434 | 1,3300 | 1,1305 | 1,0768 | 1,052 |
| n=1 | Smax | 3527.86 | 5,57378 | 0,63588 | 0,50724 | 0,43186 |
| | qmax | 0,00 | 1,24228 | 1,14 | 1,0780 | 1,05180 |
| n=2 | Smax | -91777.2 | 16,22 | 1,02511 | 0,58495 | 0,45050 |
| | qmax | -0,0000001 | 4,25*10^-7 | 1,12 | 1,07700 | 1,05210 |
| | | | | | | |
| | R=2R1 | R1=0.1 | R1=1 | R1=5 | R1=10 | R1=20 |
| n=0 | Smax | 241 | 0.808039 | 0,549422 | 0,488978 | 0,459726 |
| | qmax | 4,04590 | 1,276 | 1,114 | 1,090 | 1,078 |
| n=1 | Smax | 391,56200 | 1,250 | 0,568955 | 0,493963 | 0,46095 |
| | qmax | 0,00112 | 1,105 | 1,114 | 1,090 | 1,078 |
| n=2 | Smax | -4367,51 | 3,3255 | 0,63069 | 0,508934 | 0,464655 |
| | qmax | 2,63063*10^-8 | 1,107 | 1,110 | 1,089 | 1,078 |
| | | | | | | |
| | R=10R1 | R1=0.1 | R1=1 | R1=5 | R1=10 | R1=20 |
| n=0 | Smax | 1,82 | 0.567902 | 0.471451 | 0.459987 | 0,441013 |
| | qmax | 1,37210 | 1,123 | 1,086 | 1,082 | 1 |
| n=1 | Smax | 4,68 | 0.585765 | 0.472176 | 0.460167 | 0,441058 |
| | qmax | 1,19890 | 1,123 | 1,086 | 1,082 | 1 |
| n=2 | Smax | 12 | 0.642323 | 0,474349 | 0,460708 | 0,441195 |
| | qmax | -0,0000002 | 1,118 | 1,086 | 1,082 | 1 |

Table 3.2 Maximum values of the stability parameter S in function of normal mode $\boldsymbol{n}$ and $\boldsymbol{R_1}$ in the three cases: shell thickness being null, equal to $\boldsymbol{R_1}$ and large compared with $\boldsymbol{R_1}$ ('Cartesian' reference state)

# Conclusion

In this work we have learned to play with the theory of elasticity also using interesting mathematical tools like Papkovich-Neuber[13] potential to solve equilibrium equations.

It has been really instructive and exciting realizing that this theory presents surprises in the results.

This work started from the work of Schmidt-McIntyre-Gosele[19]. Their morphological instability analysis of misfit-strained core-shell nanowire has been developed with the 'Cartesian' reference state (but with different hypothesis and boundary conditions) giving features of the fastest growing mode that in some cases didn't fit with experimental data.

This made me think that the problem could have been in the reference state choice for reasons that I have explained in section 3.2.2. After having analysed how the core of the nanowire is built in laboratory (in order to have a model of the disposition of lattice elementary cells), the simpler reference state has been chosen, denoted as 'Cylindrical', using the radial symmetry and introducing a dependence on the core radius.

This choice revealed to be lucky because solved the problems with experimental data of Schmidt-McIntyre-Gösele work (results in Appendix B).

After having identified the correct reference state, equilibrium equations have been solved with different hypothesis and boundary conditions. The case solved has been called 'fixed', because, by taking displacements independent on z, reproduced the case of a nanowire limited at the end faces, with no dependence on the nanowire length.

The linear stability analysis has been performed also with the 'Cartesian' reference state in order to compare the results and realize the consequences of this choice.

Using the diffusion equation, the stability parameter (growth rate) S has been written in an explicit way and displayed in function of vector wave number. By definition it has to show a maximum value indicating the fastest growing mode for a precise value of vector wave number.

After having displayed S with different choices of core radius and shell thickness with $n = 0,1,2$ we found the precious features $(R_1, R_2 - R_1, n)$ of the fastest growing mode: $R_1 \to 0$, $R_2 - R_1 \to 0$, $n = 0$.

These three general rules are different from the 'Cartesian' case that didn't show a fixed tendency for the fastest growing mode.

In conclusion, we can say that, beside technical results, in this thesis it become evident that the theory of elasticity is a very delicate and powerful theory: by changing the basic hypothesis of a peculiarity, the consequences can be broad and physically relevant.

# Appendix A

In this section first order parameters $d, f, g$ are displayed (see next page) in the 'Cylindrical' reference state case.

Y is the Young's modulus, R the shell radius and $\rho$ the core radius.

*Bessel I* is the modified Bessel function and KroneckerDelta has an obvious meaning.

The parameters have been computed with Mathematica 9.0.

**d**

$$\Bigg(-1/\big((1-2\nu)(1+\nu)\big)\, i\, q\, Y \Bigg(-m(1-2\nu)+\nu\Bigg(-2m-m\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\left(-\frac{m\nu\rho^2}{R^2(-1+\nu)}+\frac{m\left(2R^2(-1+\nu^2)+(1-2\nu)\nu\rho^2\right)}{R^2(-1+\nu)}\right)+\frac{-m\rho+\frac{m\nu\rho^2}{2R(-1+\nu)}+\frac{m\left(2R^2(-1+\nu^2)+(1-2\nu)\nu\rho^2\right)}{2R(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(\frac{n^2 Y\nu\, \text{BesselI}[-1+n, qR]}{R(1-2\nu)(1+\nu)}+\frac{q^2 R Y\nu\, \text{BesselI}[-1+n, qR]}{(1-2\nu)(1+\nu)}+\frac{Y(3-4\nu)\nu\, \text{BesselI}[-1+n, qR]}{R(1-2\nu)(1+\nu)}-\frac{4nY(1-\nu)\nu\, \text{BesselI}[-1+n, qR]}{R(1-2\nu)(1+\nu)}-$$

$$\frac{qY(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])}{2(1+\nu)}+\frac{qY(3-4\nu)(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])}{2(1+\nu)}-$$

$$\frac{qY\nu(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])}{(1-2\nu)(1+\nu)}+\big(qY(3-4\nu)\nu(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])\big)/\big(2(1-2\nu)(1+\nu)\big)-$$

$$\frac{1}{2(1+\nu)}qRY\left(\frac{1}{2}q(\text{BesselI}[-3+n, qR]+\text{BesselI}[-1+n, qR])+\frac{1}{2}q(\text{BesselI}[-1+n, qR]+\text{BesselI}[1+n, qR])\right)-$$

$$\left(qRY\nu\left(\frac{1}{2}q(\text{BesselI}[-3+n, qR]+\text{BesselI}[-1+n, qR])+\frac{1}{2}q(\text{BesselI}[-1+n, qR]+\text{BesselI}[1+n, qR])\right)\right)\Big/\big(2(1-2\nu)(1+\nu)\big)-$$

$$\frac{Y(3-4\nu)\nu\, \text{KroneckerDelta}[1, n]}{R(1-2\nu)(1+\nu)}+\frac{nY(3-4\nu)\nu\, \text{KroneckerDelta}[1, n]}{R(1-2\nu)(1+\nu)}\Bigg)$$

$$\Bigg(\Bigg(\frac{i n Y\, \text{BesselI}[1+n, qR]}{2R(1+\nu)}+\frac{i n Y(3-4\nu)\, \text{BesselI}[1+n, qR]}{2R(1+\nu)}+\frac{2iY(1-\nu)\, \text{BesselI}[1+n, qR]}{R(1+\nu)}-\frac{i n q Y(\text{BesselI}[n, qR]+\text{BesselI}[2+n, qR])}{2(1+\nu)}-$$

$$\frac{1}{1+\nu}iqY(1-\nu)(\text{BesselI}[n, qR]+\text{BesselI}[2+n, qR])\Bigg)\Bigg(-\frac{iqY\, \text{BesselI}[-1+n, qR]}{2(1+\nu)}+\frac{iqY(3-4\nu)\, \text{BesselI}[-1+n, qR]}{2(1+\nu)}-$$

$$\frac{iq^2RY(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])}{2(1+\nu)}-\frac{iqY(3-4\nu)\, \text{KroneckerDelta}[1, n]}{2(1+\nu)}\Bigg)-$$

$$\Bigg(-\frac{iqY\, \text{BesselI}[1+n, qR]}{2(1+\nu)}+\frac{iqY(3-4\nu)\, \text{BesselI}[1+n, qR]}{2(1+\nu)}-\frac{iq^2RY(\text{BesselI}[n, qR]+\text{BesselI}[2+n, qR])}{2(1+\nu)}\Bigg)$$

$$\Bigg(\frac{inY\, \text{BesselI}[-1+n, qR]}{2R(1+\nu)}+\frac{inY(3-4\nu)\, \text{BesselI}[-1+n, qR]}{2R(1+\nu)}-\frac{2iY(1-\nu)\, \text{BesselI}[-1+n, qR]}{R(1+\nu)}-\frac{inqY(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])}{2(1+\nu)}+$$

$$\frac{1}{1+\nu}iqY(1-\nu)(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])+\frac{iY(3-4\nu)\, \text{KroneckerDelta}[1, n]}{2R(1+\nu)}-\frac{inY(3-4\nu)\, \text{KroneckerDelta}[1, n]}{2R(1+\nu)}\Bigg)\Bigg)+$$

$$\Bigg(-1/\big(R(1-2\nu)(1+\nu)\big)\, i\, n\, Y\Bigg((1-2\nu)\Bigg(-m\left(1-\frac{\rho}{R}\right)+\frac{-m\rho+\frac{m\nu\rho^2}{2R(-1+\nu)}+\frac{m\left(2R^2(-1+\nu^2)+(1-2\nu)\nu\rho^2\right)}{2R(-1+\nu)}}{R}\Bigg)+$$

$$\nu\Bigg(-2m-m\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\left(-\frac{m\nu\rho^2}{R^2(-1+\nu)}+\frac{m\left(2R^2(-1+\nu^2)+(1-2\nu)\nu\rho^2\right)}{R^2(-1+\nu)}\right)+\frac{-m\rho+\frac{m\nu\rho^2}{2R(-1+\nu)}+\frac{m\left(2R^2(-1+\nu^2)+(1-2\nu)\nu\rho^2\right)}{2R(-1+\nu)}}{R}\Bigg)\Bigg)\Bigg(-\frac{iqY\, \text{BesselI}[-1+n, qR]}{2(1+\nu)}+$$

$$\frac{iqY(3-4\nu)\, \text{BesselI}[-1+n, qR]}{2(1+\nu)}-\frac{iq^2RY(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])}{2(1+\nu)}-\frac{iqY(3-4\nu)\, \text{KroneckerDelta}[1, n]}{2(1+\nu)}\Bigg)+$$

$$1/\big((1-2\nu)(1+\nu)\big)\, i\, q\, Y\Bigg(-m(1-2\nu)+\nu\Bigg(-2m-m\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\left(-\frac{m\nu\rho^2}{R^2(-1+\nu)}+\frac{m\left(2R^2(-1+\nu^2)+(1-2\nu)\nu\rho^2\right)}{R^2(-1+\nu)}\right)+\frac{-m\rho+\frac{m\nu\rho^2}{2R(-1+\nu)}+\frac{m\left(2R^2(-1+\nu^2)+(1-2\nu)\nu\rho^2\right)}{2R(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(\frac{inY\, \text{BesselI}[-1+n, qR]}{2R(1+\nu)}+\frac{inY(3-4\nu)\, \text{BesselI}[-1+n, qR]}{2R(1+\nu)}-\frac{2iY(1-\nu)\, \text{BesselI}[-1+n, qR]}{R(1+\nu)}-\frac{inqY(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])}{2(1+\nu)}+$$

$$\frac{1}{1+\nu}iqY(1-\nu)(\text{BesselI}[-2+n, qR]+\text{BesselI}[n, qR])+\frac{iY(3-4\nu)\, \text{KroneckerDelta}[1, n]}{2R(1+\nu)}-\frac{inY(3-4\nu)\, \text{KroneckerDelta}[1, n]}{2R(1+\nu)}\Bigg)\Bigg)$$

$$\Bigg(\Bigg(\frac{n^2Y\nu\, \text{BesselI}[1+n, qR]}{R(1-2\nu)(1+\nu)}+\frac{q^2RY\nu\, \text{BesselI}[1+n, qR]}{(1-2\nu)(1+\nu)}+\frac{Y(3-4\nu)\nu\, \text{BesselI}[1+n, qR]}{R(1-2\nu)(1+\nu)}+\frac{4nY(1-\nu)\nu\, \text{BesselI}[1+n, qR]}{R(1-2\nu)(1+\nu)}-$$

$$\frac{qY(\text{BesselI}[n, qR]+\text{BesselI}[2+n, qR])}{2(1+\nu)}+\frac{1}{2(1+\nu)}qY(3-4\nu)(\text{BesselI}[n, qR]+\text{BesselI}[2+n, qR])-$$

$$\frac{qY\nu(\text{BesselI}[n, qR]+\text{BesselI}[2+n, qR])}{(1-2\nu)(1+\nu)}+\big(qY(3-4\nu)\nu(\text{BesselI}[n, qR]+\text{BesselI}[2+n, qR])\big)/\big(2(1-2\nu)(1+\nu)\big)-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\Big)-$$

$$\Big(q\,R\,Y\,\nu\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\Big)\Big)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))\Big)$$

$$\Big(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Big)-$$

$$\Big(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\Big)$$

$$\Big(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+(q\,Y\,(3-4\,\nu)\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]))\,/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\Big)-$$

$$\Big(q\,R\,Y\,\nu\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\Big)\Big)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\Big)\Big)\Big)\Big/$$

$$\Big(\Big(-\Big(\frac{i\,n\,Y\,\text{BesselI}[n,\,q\,R]}{R^2\,(1+\nu)}-\frac{1}{2\,R\,(1+\nu)}\,i\,n\,q\,Y\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])-\frac{i\,q\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)}+\frac{i\,n\,q\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)}\Big)$$

$$\Big(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Big)+$$

$$\Big(-\frac{1}{2\,(1+\nu)}\,i\,q^2\,Y\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{i\,q^2\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,(1+\nu)}\Big)$$

$$\Big(\frac{i\,n\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Big)\Big)$$

$$\Big(\Big(\frac{n^2\,Y\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+(q\,Y\,(3-4\,\nu)\,\nu\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]))\,/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\Big)-$$

$$\Big(q\,R\,Y\,\nu\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\Big)\Big)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))\Big)$$

$$\Big(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Big)-$$

$$\Big(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\Big)$$

$$\Big(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+(q\,Y\,(3-4\,\nu)\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]))\,/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\Big)-$$

$$\Big(q\,R\,Y\,\nu\,\Big(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\Big)\Big)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(\Bigg(\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])\Bigg)$$

$$\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-$$

$$\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\Bigg)$$

$$\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])+\frac{i\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)$$

$$\Bigg(\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)$$

$$\Bigg(\frac{q^2\,Y\,\nu\,\mathrm{BesselI}[n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[n,\,q\,R]}{R^2\,(1-2\,\nu)\,(1+\nu)}-\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])}{2\,R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{1}{2\,(1+\nu)}\,q\,Y\,\Bigg(\frac{1}{2}\,q\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])\Bigg)-$$

$$\Bigg(q\,Y\,\nu\,\Bigg(\frac{1}{2}\,q\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])\Bigg)\Bigg)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))+\frac{q\,Y\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{n\,q\,Y\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1-2\,\nu)\,(1+\nu)}\Bigg)-\Bigg(-\frac{1}{2\,(1+\nu)}\,i\,q^2\,Y\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{i\,q^2\,Y\,\mathrm{KroneckerDelta}[1,\,n]}{4\,(1+\nu)}\Bigg)$$

$$\Bigg(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+(q\,Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]))\,/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\,\Bigg(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\Bigg)-$$

$$\Bigg(q\,R\,Y\,\nu\,\Bigg(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\Bigg)\Bigg)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\Bigg)\Bigg)\Bigg)$$

**f**

$$\Bigg(1\,/\,(R\,(1-2\,\nu)\,(1+\nu))\,i\,n\,Y\,\Bigg((1-2\,\nu)\,\Bigg(-m\,\left(1-\frac{\rho}{R}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)+$$

$$\nu\,\Bigg(-2\,m-m\,\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\,\Bigg(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{R^2\,(-1+\nu)}\Bigg)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-$$

$$1\,/\,((1-2\,\nu)\,(1+\nu))\,i\,q\,Y\,\Bigg(-m\,(1-2\,\nu)+\nu\,\Bigg(-2\,m-m\,\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\,\Bigg(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{R^2\,(-1+\nu)}\Bigg)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)\Big/$$

$$\Bigg(\Bigg(\Bigg(\frac{i\,n\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{1+\nu}\Bigg)$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)-$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\right)$$

$$\Bigg(\frac{i\,n\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)+$$

$$\Bigg(\Bigg(\Bigg(\frac{i\,n\,Y\,\text{BesselI}[n,\,q\,R]}{R^2\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])}{2\,R\,(1+\nu)}-\frac{i\,q\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)}+\frac{i\,n\,q\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)}\Bigg)$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)-$$

$$\left(-\frac{1}{2\,(1+\nu)}\,i\,q^2\,Y\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{i\,q^2\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,(1+\nu)}\right)$$

$$\Bigg(\frac{i\,n\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)$$

$$\left(1/((1-2\,\nu)\,(1+\nu))\,i\,q\,Y\left(-m\,(1-2\,\nu)+\nu\left(-2\,m-m\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\left(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{R^2\,(-1+\nu)}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\right)\right)\right)$$

$$\Bigg(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+(q\,Y\,(3-4\,\nu)\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]))\,/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\right)-$$

$$\left(q\,R\,Y\,\nu\left(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\right)\right)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\Bigg)$$

$$\Bigg(\Bigg(\frac{i\,n\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])\Bigg)\Bigg(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-$$

$$\frac{1}{2\,(1+\nu)}\,i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\right)\Bigg(\frac{i\,n\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+$$

$$\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,n\,q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])+$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)-$$

$$\Bigg(-1\Big/\big(R\,(1-2\,\nu)\,(1+\nu)\big)\ \mathrm{i}\,n\,Y\,\Bigg((1-2\,\nu)\,\Bigg(-m\left(1-\frac{\rho}{R}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\left(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2\right)}{2\,R\,(-1+\nu)}}{R}\Bigg)+\nu\,\Bigg(-2\,m-m\left(1-\frac{\rho}{R}\right)+$$

$$\frac{1}{2}\left(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\left(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2\right)}{R^2\,(-1+\nu)}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\left(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2\right)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)\Bigg(-\frac{\mathrm{i}\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+$$

$$\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,\mathrm{i}\,q^2\,R\,Y\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)-\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)+$$

$$1\Big/\big((1-2\,\nu)\,(1+\nu)\big)\ \mathrm{i}\,q\,Y\,\Bigg(-m\,(1-2\,\nu)+\nu\,\Bigg(-2\,m-m\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\left(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\left(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2\right)}{R^2\,(-1+\nu)}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\left(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2\right)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(\frac{\mathrm{i}\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{\mathrm{i}\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,\mathrm{i}\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,\mathrm{i}\,n\,q\,Y\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,$$

$$q\,R]\big)+\frac{1}{1+\nu}\,\mathrm{i}\,q\,Y\,(1-\nu)\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)+\frac{\mathrm{i}\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{\mathrm{i}\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)$$

$$\Bigg(\Bigg(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)-$$

$$\frac{q\,Y\,\nu\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)}{(1-2\,\nu)\,(1+\nu)}+\big(q\,Y\,(3-4\,\nu)\,\nu\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)\big)\Big/\big(2\,(1-2\,\nu)\,(1+\nu)\big)-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}\,q\,\big(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\big)+\frac{1}{2}\,q\,\big(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R]\big)\right)-$$

$$\left(q\,R\,Y\,\nu\left(\frac{1}{2}\,q\,\big(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\big)+\frac{1}{2}\,q\,\big(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R]\big)\right)\right)\Big/\big(2\,(1-2\,\nu)\,(1+\nu)\big)\Bigg)$$

$$\Bigg(-\frac{\mathrm{i}\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,\mathrm{i}\,q^2\,R\,Y\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)-$$

$$\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-\Bigg(-\frac{\mathrm{i}\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-$$

$$\frac{\mathrm{i}\,q^2\,R\,Y\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)}{2\,(1+\nu)}\Bigg)\Bigg(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{q\,Y\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)-$$

$$\frac{q\,Y\,\nu\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)}{(1-2\,\nu)\,(1+\nu)}+\big(q\,Y\,(3-4\,\nu)\,\nu\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)\big)\Big/\big(2\,(1-2\,\nu)\,(1+\nu)\big)-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}\,q\,\big(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R]\big)+\frac{1}{2}\,q\,\big(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\big)\right)-$$

$$\left(q\,R\,Y\,\nu\left(\frac{1}{2}\,q\,\big(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R]\big)+\frac{1}{2}\,q\,\big(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\big)\right)\right)\Big/\big(2\,(1-2\,\nu)\,(1+\nu)\big)-$$

$$\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\Bigg)\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\Bigg(\Bigg(\frac{\mathrm{i}\,n\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{\mathrm{i}\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,\mathrm{i}\,Y\,(1-\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{\mathrm{i}\,n\,q\,Y\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)}{2\,(1+\nu)}-$$

$$\frac{1}{1+\nu}\,\mathrm{i}\,q\,Y\,(1-\nu)\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)\Bigg)$$

$$\Bigg(-\frac{\mathrm{i}\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{\mathrm{i}\,q^2\,R\,Y\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)}{2\,(1+\nu)}-\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-$$

$$\Bigg(-\frac{\mathrm{i}\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{\mathrm{i}\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{\mathrm{i}\,q^2\,R\,Y\,\big(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\big)}{2\,(1+\nu)}\Bigg)$$

$$\Bigg(\frac{\mathrm{i}\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{\mathrm{i}\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,\mathrm{i}\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{\mathrm{i}\,n\,q\,Y\,\big(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\big)}{2\,(1+\nu)}+$$

$$\frac{1}{1+\nu} i q Y (1-\nu) (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) + \frac{i Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 R (1+\nu)} - \frac{i n Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 R (1+\nu)} ) )$$

$$( ( - ( \frac{i n Y\, \text{BesselI}[n, q R]}{R^2 (1+\nu)} - \frac{1}{2 R (1+\nu)} i n q Y (\text{BesselI}[-1+n, q R] + \text{BesselI}[1+n, q R]) - \frac{i q Y\, \text{KroneckerDelta}[1, n]}{4 R (1+\nu)} + \frac{i n q Y\, \text{KroneckerDelta}[1, n]}{4 R (1+\nu)} )$$

$$( - \frac{i q Y\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} + \frac{i q Y (3-4\nu)\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} - \frac{1}{2 (1+\nu)} i q^2 R Y (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) -$$

$$\frac{i q Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 (1+\nu)} ) + ( - \frac{1}{2 (1+\nu)} i q^2 Y (\text{BesselI}[-1+n, q R] + \text{BesselI}[1+n, q R]) + \frac{i q^2 Y\, \text{KroneckerDelta}[1, n]}{4 (1+\nu)} )$$

$$( \frac{i n Y\, \text{BesselI}[-1+n, q R]}{2 R (1+\nu)} + \frac{i n Y (3-4\nu)\, \text{BesselI}[-1+n, q R]}{2 R (1+\nu)} - \frac{2 i Y (1-\nu)\, \text{BesselI}[-1+n, q R]}{R (1+\nu)} - \frac{1}{2 (1+\nu)} i n q Y (\text{BesselI}[-2+n, q R] + \text{BesselI}[n,$$

$$q R]) + \frac{1}{1+\nu} i q Y (1-\nu) (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) + \frac{i Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 R (1+\nu)} - \frac{i n Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 R (1+\nu)} ) )$$

$$( ( ( \frac{n^2 Y \nu\, \text{BesselI}[1+n, q R]}{R (1-2\nu) (1+\nu)} + \frac{q^2 R Y \nu\, \text{BesselI}[1+n, q R]}{(1-2\nu) (1+\nu)} + \frac{Y (3-4\nu) \nu\, \text{BesselI}[1+n, q R]}{R (1-2\nu) (1+\nu)} + \frac{4 n Y (1-\nu) \nu\, \text{BesselI}[1+n, q R]}{R (1-2\nu) (1+\nu)} -$$

$$\frac{q Y (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R])}{2 (1+\nu)} + \frac{1}{2 (1+\nu)} q Y (3-4\nu) (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R]) -$$

$$\frac{q Y \nu (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R])}{(1-2\nu) (1+\nu)} + (q Y (3-4\nu) \nu (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R])) / (2 (1-2\nu) (1+\nu)) -$$

$$\frac{1}{2 (1+\nu)} q R Y ( \frac{1}{2} q (\text{BesselI}[-1+n, q R] + \text{BesselI}[1+n, q R]) + \frac{1}{2} q (\text{BesselI}[1+n, q R] + \text{BesselI}[3+n, q R]) ) -$$

$$( q R Y \nu ( \frac{1}{2} q (\text{BesselI}[-1+n, q R] + \text{BesselI}[1+n, q R]) + \frac{1}{2} q (\text{BesselI}[1+n, q R] + \text{BesselI}[3+n, q R]) ) ) / (2 (1-2\nu) (1+\nu)) )$$

$$( - \frac{i q Y\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} + \frac{i q Y (3-4\nu)\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} - \frac{1}{2 (1+\nu)} i q^2 R Y (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) -$$

$$\frac{i q Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 (1+\nu)} ) - ( - \frac{i q Y\, \text{BesselI}[1+n, q R]}{2 (1+\nu)} + \frac{i q Y (3-4\nu)\, \text{BesselI}[1+n, q R]}{2 (1+\nu)} -$$

$$\frac{i q^2 R Y (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R])}{2 (1+\nu)} ) ( \frac{n^2 Y \nu\, \text{BesselI}[-1+n, q R]}{R (1-2\nu) (1+\nu)} + \frac{q^2 R Y \nu\, \text{BesselI}[-1+n, q R]}{(1-2\nu) (1+\nu)} + \frac{Y (3-4\nu) \nu\, \text{BesselI}[-1+n, q R]}{R (1-2\nu) (1+\nu)} -$$

$$\frac{4 n Y (1-\nu) \nu\, \text{BesselI}[-1+n, q R]}{R (1-2\nu) (1+\nu)} - \frac{q Y (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R])}{2 (1+\nu)} + \frac{1}{2 (1+\nu)} q Y (3-4\nu) (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) -$$

$$\frac{q Y \nu (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R])}{(1-2\nu) (1+\nu)} + (q Y (3-4\nu) \nu (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R])) / (2 (1-2\nu) (1+\nu)) -$$

$$\frac{1}{2 (1+\nu)} q R Y ( \frac{1}{2} q (\text{BesselI}[-3+n, q R] + \text{BesselI}[-1+n, q R]) + \frac{1}{2} q (\text{BesselI}[-1+n, q R] + \text{BesselI}[1+n, q R]) ) -$$

$$( q R Y \nu ( \frac{1}{2} q (\text{BesselI}[-3+n, q R] + \text{BesselI}[-1+n, q R]) + \frac{1}{2} q (\text{BesselI}[-1+n, q R] + \text{BesselI}[1+n, q R]) ) ) / (2 (1-2\nu) (1+\nu)) -$$

$$\frac{Y (3-4\nu) \nu\, \text{KroneckerDelta}[1, n]}{R (1-2\nu) (1+\nu)} + \frac{n Y (3-4\nu) \nu\, \text{KroneckerDelta}[1, n]}{R (1-2\nu) (1+\nu)} ) ) +$$

$$( ( \frac{i n Y\, \text{BesselI}[1+n, q R]}{2 R (1+\nu)} + \frac{i n Y (3-4\nu)\, \text{BesselI}[1+n, q R]}{2 R (1+\nu)} + \frac{2 i Y (1-\nu)\, \text{BesselI}[1+n, q R]}{R (1+\nu)} - \frac{i n q Y (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R])}{2 (1+\nu)} -$$

$$\frac{1}{1+\nu} i q Y (1-\nu) (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R]) ) ( - \frac{i q Y\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} + \frac{i q Y (3-4\nu)\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} -$$

$$\frac{1}{2 (1+\nu)} i q^2 R Y (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) - \frac{i q Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 (1+\nu)} ) -$$

$$( - \frac{i q Y\, \text{BesselI}[1+n, q R]}{2 (1+\nu)} + \frac{i q Y (3-4\nu)\, \text{BesselI}[1+n, q R]}{2 (1+\nu)} - \frac{i q^2 R Y (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R])}{2 (1+\nu)} ) ( \frac{i n Y\, \text{BesselI}[-1+n, q R]}{2 R (1+\nu)} +$$

$$\frac{i n Y (3-4\nu)\, \text{BesselI}[-1+n, q R]}{2 R (1+\nu)} - \frac{2 i Y (1-\nu)\, \text{BesselI}[-1+n, q R]}{R (1+\nu)} - \frac{1}{2 (1+\nu)} i n q Y (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) +$$

$$\frac{1}{1+\nu} i q Y (1-\nu) (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) + \frac{i Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 R (1+\nu)} - \frac{i n Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 R (1+\nu)} ) )$$

$$( ( - \frac{i q Y\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} + \frac{i q Y (3-4\nu)\, \text{BesselI}[-1+n, q R]}{2 (1+\nu)} - \frac{1}{2 (1+\nu)} i q^2 R Y (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) -$$

$$\frac{i q Y (3-4\nu)\, \text{KroneckerDelta}[1, n]}{2 (1+\nu)} ) ( \frac{q^2 Y \nu\, \text{BesselI}[n, q R]}{(1-2\nu) (1+\nu)} + \frac{n^2 Y \nu\, \text{BesselI}[n, q R]}{R^2 (1-2\nu) (1+\nu)} - (q Y \nu (\text{BesselI}[-1+n, q R] + \text{BesselI}[1+n, q R])) /$$

$$(2 R (1-2\nu) (1+\nu)) - \frac{1}{2 (1+\nu)} q Y ( \frac{1}{2} q (\text{BesselI}[-2+n, q R] + \text{BesselI}[n, q R]) + \frac{1}{2} q (\text{BesselI}[n, q R] + \text{BesselI}[2+n, q R]) ) -$$

$$
\left(q\,Y\,\nu\left(\frac{1}{2}q\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)+\frac{1}{2}q\left(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]\right)\right)\right)\Big/\left(2\,(1-2\nu)\,(1+\nu)\right)+
\frac{q\,Y\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1-2\nu)\,(1+\nu)}-\frac{n\,q\,Y\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1-2\nu)\,(1+\nu)}\Bigg)\Bigg)-\left(-\frac{1}{2\,(1+\nu)}\,i\,q^2\,Y\left(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\right)+\right.
$$

$$
\left.\frac{i\,q^2\,Y\,\mathrm{KroneckerDelta}[1,\,n]}{4\,(1+\nu)}\right)\left(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\nu)\,(1+\nu)}+\frac{Y\,(3-4\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\right.
$$

$$
\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\frac{q\,Y\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\nu)\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)-
$$

$$
\frac{q\,Y\,\nu\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)}{(1-2\nu)\,(1+\nu)}+\left(q\,Y\,(3-4\nu)\,\nu\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)\right)/\left(2\,(1-2\nu)\,(1+\nu)\right)-
$$

$$
\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}q\left(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\right)\right)-
$$

$$
\left(q\,R\,Y\,\nu\left(\frac{1}{2}q\left(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\right)\right)\right)\Big/\left(2\,(1-2\nu)\,(1+\nu)\right)-
$$

$$
\left.\frac{Y\,(3-4\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\nu)\,(1+\nu)}\right)\Bigg)\Bigg)\Bigg)
$$

# g

$$
\begin{aligned}
&(2\,(2\,m\,q\,R^3\,\mathrm{BesselI}[n,\,q\,R]+m\,n\,q\,R^3\,\mathrm{BesselI}[n,\,q\,R]-2\,m\,q\,R^3\,\nu\,\mathrm{BesselI}[n,\,q\,R]-2\,m\,q\,R^3\,\nu^2\,\mathrm{BesselI}[n,\,q\,R]-m\,n\,q\,R^3\,\nu^2\,\mathrm{BesselI}[n,\,q\,R]+\\
&\quad 2\,m\,q\,R^3\,\nu^3\,\mathrm{BesselI}[n,\,q\,R]-m\,n\,q\,R\,\nu\,\rho^2\,\mathrm{BesselI}[n,\,q\,R]+2\,m\,q\,R\,\nu^2\,\rho^2\,\mathrm{BesselI}[n,\,q\,R]+m\,n\,q\,R\,\nu^2\,\rho^2\,\mathrm{BesselI}[n,\,q\,R]-2\,m\,q\,R\,\nu^3\,\rho^2\,\mathrm{BesselI}[n,\,q\,R]-\\
&\quad 4\,m\,R^2\,\mathrm{BesselI}[1+n,\,q\,R]-4\,m\,n\,R^2\,\mathrm{BesselI}[1+n,\,q\,R]+4\,m\,R^2\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]+4\,m\,n\,R^2\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]+4\,m\,R^2\,\nu^2\,\mathrm{BesselI}[1+n,\,q\,R]+\\
&\quad 4\,m\,n\,R^2\,\nu^2\,\mathrm{BesselI}[1+n,\,q\,R]-4\,m\,R^2\,\nu^3\,\mathrm{BesselI}[1+n,\,q\,R]-4\,m\,n\,R^2\,\nu^3\,\mathrm{BesselI}[1+n,\,q\,R]+2\,m\,n\,\nu\,\rho^2\,\mathrm{BesselI}[1+n,\,q\,R]-4\,m\,\nu^2\,\rho^2\,\mathrm{BesselI}[1+n,\,q\,R]-\\
&\quad 8\,m\,n\,\nu^2\,\rho^2\,\mathrm{BesselI}[1+n,\,q\,R]+4\,m\,\nu^3\,\rho^2\,\mathrm{BesselI}[1+n,\,q\,R]+4\,m\,n\,\nu^3\,\rho^2\,\mathrm{BesselI}[1+n,\,q\,R]+2\,m\,q\,R^3\,\mathrm{BesselI}[2+n,\,q\,R]+m\,n\,q\,R^3\,\mathrm{BesselI}[2+n,\,q\,R]-\\
&\quad 2\,m\,q\,R^3\,\nu\,\mathrm{BesselI}[2+n,\,q\,R]-2\,m\,q\,R^3\,\nu^2\,\mathrm{BesselI}[2+n,\,q\,R]-m\,n\,q\,R^3\,\nu^2\,\mathrm{BesselI}[2+n,\,q\,R]+2\,m\,q\,R^3\,\nu^3\,\mathrm{BesselI}[2+n,\,q\,R]-\\
&\quad m\,n\,q\,R\,\nu\,\rho^2\,\mathrm{BesselI}[2+n,\,q\,R]+2\,m\,q\,R\,\nu^2\,\rho^2\,\mathrm{BesselI}[2+n,\,q\,R]+m\,n\,q\,R\,\nu^2\,\rho^2\,\mathrm{BesselI}[2+n,\,q\,R]-2\,m\,q\,R\,\nu^3\,\rho^2\,\mathrm{BesselI}[2+n,\,q\,R]))\,/\\
&(R^2\,(-1+\nu)\,(-4\,q^2\,R^2\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]+4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]+8\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]-\\
&\quad 2\,n\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]-16\,q\,R\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]+8\,q\,R\,\nu^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]-\\
&\quad 4\,q^2\,R^2\,\mathrm{BesselI}[n,\,q\,R]^2+4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[n,\,q\,R]^2+8\,q\,R\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+\\
&\quad 2\,n\,q\,R\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-16\,q\,R\,\nu\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+8\,q\,R\,\nu^2\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-\\
&\quad 16\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+48\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-32\,\nu^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+\\
&\quad 8\,q\,R\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+2\,n\,q\,R\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-16\,q\,R\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+\\
&\quad 8\,q\,R\,\nu^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-4\,q^2\,R^2\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+\\
&\quad 8\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-2\,n\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-16\,q\,R\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+\\
&\quad 8\,q\,R\,\nu^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-4\,q^2\,R^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-\\
&\quad 9\,q\,R\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+18\,q\,R\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-8\,q\,R\,\nu^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+\\
&\quad 18\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+6\,n\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-48\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-\\
&\quad 8\,n\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+32\,\nu^2\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-9\,q\,R\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+\\
&\quad 18\,q\,R\,\nu\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-8\,q\,R\,\nu^2\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]))+
\end{aligned}
$$

$$
\begin{aligned}
&\Big(\big(-4\,q^2\,R^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]+4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]-4\,n\,q\,R\,\mathrm{BesselI}[n,\,q\,R]^2+\\
&\quad 8\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+4\,n\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-8\,q\,R\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+\\
&\quad 8\,n\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-4\,q^2\,R^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-16\,n\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+\\
&\quad 4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+8\,q\,R\,\mathrm{BesselI}[1+n,\,q\,R]^2+4\,n\,q\,R\,\mathrm{BesselI}[1+n,\,q\,R]^2-8\,q\,R\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]^2-\\
&\quad 4\,q^2\,R^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-4\,n\,q\,R\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-\\
&\quad 4\,q^2\,R^2\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+4\,q^2\,R^2\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+3\,q^2\,R^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-\\
&\quad 2\,q^2\,R^2\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-6\,q\,R\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-2\,n\,q\,R\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+\\
&\quad 8\,q\,R\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+3\,q^2\,R^2\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-2\,q^2\,R^2\,\nu\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]\big)
\end{aligned}
$$

$$
\left(1\Big/\left((1-2\nu)\,(1+\nu)\right)\,i\,q\,Y\left(-m\,(1-2\nu)+\nu\left(-2\,m-m\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\left(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\left(2\,R^2\,(-1+\nu^2)+(1-2\nu)\,\nu\,\rho^2\right)}{R^2\,(-1+\nu)}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\left(2\,R^2\,(-1+\nu^2)+(1-2\nu)\,\nu\,\rho^2\right)}{2\,R\,(-1+\nu)}}{R}\right)\right)\right)
$$

$$
\left(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\nu)\,(1+\nu)}+\frac{Y\,(3-4\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\right.
$$

$$
\frac{q\,Y\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\nu)\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)-
$$

$$
\frac{q\,Y\,\nu\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)}{(1-2\nu)\,(1+\nu)}+\left(q\,Y\,(3-4\nu)\,\nu\left(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]\right)\right)/\left(2\,(1-2\nu)\,(1+\nu)\right)-
$$

$$
\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}q\left(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\right)\right)-
$$

$$
\left(q\,R\,Y\,\nu\left(\frac{1}{2}q\left(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]\right)\right)\right)\Big/\left(2\,(1-2\nu)\,(1+\nu)\right)-
$$

$$
\left.\frac{Y\,(3-4\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\nu)\,(1+\nu)}\right)
$$

$$\Bigg(\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])\Bigg)\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-$$

$$\frac{1}{2\,(1+\nu)}\,i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-$$

$$\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\Bigg)\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+$$

$$\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])+$$

$$\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])+\frac{i\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)-$$

$$\Bigg(-1/(R\,(1-2\,\nu)\,(1+\nu))\,i\,n\,Y\,\Bigg((1-2\,\nu)\,\Bigg(-m\left(1-\frac{\rho}{R}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)+\nu\,\Bigg(-2\,m-m\left(1-\frac{\rho}{R}\right)+$$

$$\frac{1}{2}\left(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{R^2\,(-1+\nu)}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+$$

$$\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)+$$

$$1/((1-2\,\nu)\,(1+\nu))\,i\,q\,Y\,\Bigg(-m\,(1-2\,\nu)+\nu\,\Bigg(-2\,m-m\left(1-\frac{\rho}{R}\right)+\frac{1}{2}\left(-\frac{m\,\nu\,\rho^2}{R^2\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{R^2\,(-1+\nu)}\right)+\frac{-m\,\rho+\frac{m\,\nu\,\rho^2}{2\,R\,(-1+\nu)}+\frac{m\,(2\,R^2\,(-1+\nu^2)+(1-2\,\nu)\,\nu\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,$$

$$q\,R])+\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])+\frac{i\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)$$

$$\Bigg(\Bigg(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+(q\,Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R]))\,/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R])\right)-$$

$$\left(q\,R\,Y\,\nu\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R])\right)\right)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))\Bigg)$$

$$\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])-$$

$$\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-\Bigg(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-$$

$$\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\Bigg)\Bigg(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\,\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+(q\,Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R]))\,/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\right)-$$

$$\left(q\,R\,Y\,\nu\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\right)\right)\Big/\,(2\,(1-2\,\nu)\,(1+\nu))-$$

$$\left.\left.\left.\frac{Y\,(3-4\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R\,(1-2\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R\,(1-2\nu)\,(1+\nu)}\right)\right)\right)\right)\Big/$$

$$\Big(2\,R\,\big(-4\,q^2\,R^2\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[n,\,q\,R]+4\,q^2\,R^2\,\nu\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[n,\,q\,R]+8\,q\,R\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[n,\,q\,R]-$$

$$2\,n\,q\,R\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[n,\,q\,R]-16\,q\,R\,\nu\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[n,\,q\,R]+$$

$$8\,q\,R\,\nu^2\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[n,\,q\,R]-4\,q^2\,R^2\,\text{BesselI}[n,\,q\,R]^2+4\,q^2\,R^2\,\nu\,\text{BesselI}[n,\,q\,R]^2+$$

$$8\,q\,R\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]+2\,n\,q\,R\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]-$$

$$16\,q\,R\,\nu\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]+8\,q\,R\,\nu^2\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]-$$

$$16\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]+48\,\nu\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]-$$

$$32\,\nu^2\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]+8\,q\,R\,\text{BesselI}[n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]+$$

$$2\,n\,q\,R\,\text{BesselI}[n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]-16\,q\,R\,\nu\,\text{BesselI}[n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]+$$

$$8\,q\,R\,\nu^2\,\text{BesselI}[n,\,q\,R]\,\text{BesselI}[1+n,\,q\,R]-4\,q^2\,R^2\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]+$$

$$4\,q^2\,R^2\,\nu\,\text{BesselI}[-2+n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]+8\,q\,R\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]-$$

$$2\,n\,q\,R\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]-16\,q\,R\,\nu\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]+$$

$$8\,q\,R\,\nu^2\,\text{BesselI}[-1+n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]-4\,q^2\,R^2\,\text{BesselI}[n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]+$$

$$4\,q^2\,R^2\,\nu\,\text{BesselI}[n,\,q\,R]\,\text{BesselI}[2+n,\,q\,R]-9\,q\,R\,\text{BesselI}[n,\,q\,R]\,\text{KroneckerDelta}[1,n]+$$

$$18\,q\,R\,\nu\,\text{BesselI}[n,\,q\,R]\,\text{KroneckerDelta}[1,n]-8\,q\,R\,\nu^2\,\text{BesselI}[n,\,q\,R]\,\text{KroneckerDelta}[1,n]+$$

$$18\,\text{BesselI}[1+n,\,q\,R]\,\text{KroneckerDelta}[1,n]+6\,n\,\text{BesselI}[1+n,\,q\,R]\,\text{KroneckerDelta}[1,n]-$$

$$48\,\nu\,\text{BesselI}[1+n,\,q\,R]\,\text{KroneckerDelta}[1,n]-8\,n\,\nu\,\text{BesselI}[1+n,\,q\,R]\,\text{KroneckerDelta}[1,n]+$$

$$32\,\nu^2\,\text{BesselI}[1+n,\,q\,R]\,\text{KroneckerDelta}[1,n]-9\,q\,R\,\text{BesselI}[2+n,\,q\,R]\,\text{KroneckerDelta}[1,n]+$$

$$18\,q\,R\,\nu\,\text{BesselI}[2+n,\,q\,R]\,\text{KroneckerDelta}[1,n]-8\,q\,R\,\nu^2\,\text{BesselI}[2+n,\,q\,R]\,\text{KroneckerDelta}[1,n]\big)$$

$$\Bigg(\bigg(-\left(\frac{i\,n\,Y\,\text{BesselI}[n,\,q\,R]}{R^2\,(1+\nu)}-\frac{1}{2\,R\,(1+\nu)}\,i\,n\,q\,Y\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])-\frac{i\,q\,Y\,\text{KroneckerDelta}[1,n]}{4\,R\,(1+\nu)}+\frac{i\,n\,q\,Y\,\text{KroneckerDelta}[1,n]}{4\,R\,(1+\nu)}\right)$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])-\right.$$

$$\left.\frac{i\,q\,Y\,(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2\,(1+\nu)}\right)+\left(-\frac{1}{2\,(1+\nu)}\,i\,q^2\,Y\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{i\,q^2\,Y\,\text{KroneckerDelta}[1,n]}{4\,(1+\nu)}\right)$$

$$\left(\frac{i\,n\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,n\,q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\right.$$

$$\left.q\,R])+\frac{1}{1+\nu}\,i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])+\frac{i\,Y\,(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2\,R\,(1+\nu)}\right)\bigg)$$

$$\Bigg(\bigg(\frac{n^2\,Y\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[1+n,\,q\,R]}{(1-2\nu)\,(1+\nu)}+\frac{Y\,(3-4\nu)\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\nu)\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{(1-2\nu)\,(1+\nu)}+(q\,Y\,(3-4\nu)\,\nu\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]))\,/\,(2\,(1-2\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\right)-$$

$$\left(q\,R\,Y\,\nu\left(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\right)\right)\Big/\,(2\,(1-2\nu)\,(1+\nu))\bigg)$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{1}{2\,(1+\nu)}\,i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])-\right.$$

$$\left.\frac{i\,q\,Y\,(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2\,(1+\nu)}\right)-\left(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\right.$$

$$\left.\frac{i\,q^2\,R\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\right)\left(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{(1-2\nu)\,(1+\nu)}+\frac{Y\,(3-4\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\right.$$

$$\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\frac{q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{1}{2\,(1+\nu)}\,q\,Y\,(3-4\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{(1-2\nu)\,(1+\nu)}+(q\,Y\,(3-4\nu)\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]))\,/\,(2\,(1-2\nu)\,(1+\nu))-$$

$$\frac{1}{2\,(1+\nu)}\,q\,R\,Y\left(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\right)-$$

$$\left(q\,R\,Y\,\nu\left(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\right)\right)\Big/\,(2\,(1-2\nu)\,(1+\nu))-$$

$$\left.\frac{Y\,(3-4\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R\,(1-2\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R\,(1-2\nu)\,(1+\nu)}\right)\Bigg)+$$

$$\Bigg(\bigg(\frac{i\,n\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{1}{1+\nu}\, i\, q\, Y\, (1-\nu)\, (\mathrm{BesselI}[n,\, q R] + \mathrm{BesselI}[2+n,\, q R])\Big)\Big(-\frac{i\, q\, Y\, \mathrm{BesselI}[-1+n,\, q R]}{2\,(1+\nu)} + \frac{i\, q\, Y\,(3-4\nu)\, \mathrm{BesselI}[-1+n,\, q R]}{2\,(1+\nu)} -$$

$$\frac{1}{2\,(1+\nu)}\, i\, q^2 R\, Y\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]) - \frac{i\, q\, Y\,(3-4\nu)\, \mathrm{KroneckerDelta}[1,\, n]}{2\,(1+\nu)}\Big) -$$

$$\Big(-\frac{i\, q\, Y\, \mathrm{BesselI}[1+n,\, q R]}{2\,(1+\nu)} + \frac{i\, q\, Y\,(3-4\nu)\, \mathrm{BesselI}[1+n,\, q R]}{2\,(1+\nu)} - \frac{i\, q^2 R\, Y\,(\mathrm{BesselI}[n,\, q R] + \mathrm{BesselI}[2+n,\, q R])}{2\,(1+\nu)}\Big)\Big(\frac{i\, n\, Y\, \mathrm{BesselI}[-1+n,\, q R]}{2 R\,(1+\nu)} +$$

$$\frac{i\, n\, Y\,(3-4\nu)\, \mathrm{BesselI}[-1+n,\, q R]}{2 R\,(1+\nu)} - \frac{2\, i\, Y\,(1-\nu)\, \mathrm{BesselI}[-1+n,\, q R]}{R\,(1+\nu)} - \frac{1}{2\,(1+\nu)}\, i\, n\, q\, Y\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]) +$$

$$\frac{1}{1+\nu}\, i\, q\, Y\,(1-\nu)\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]) + \frac{i\, Y\,(3-4\nu)\, \mathrm{KroneckerDelta}[1,\, n]}{2 R\,(1+\nu)} - \frac{i\, n\, Y\,(3-4\nu)\, \mathrm{KroneckerDelta}[1,\, n]}{2 R\,(1+\nu)}\Big)\Big)$$

$$\Big(\Big(\Big(-\frac{i\, q\, Y\, \mathrm{BesselI}[-1+n,\, q R]}{2\,(1+\nu)} + \frac{i\, q\, Y\,(3-4\nu)\, \mathrm{BesselI}[-1+n,\, q R]}{2\,(1+\nu)} - \frac{1}{2\,(1+\nu)}\, i\, q^2 R\, Y\,(\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]) -$$

$$\frac{i\, q\, Y\,(3-4\nu)\, \mathrm{KroneckerDelta}[1,\, n]}{2\,(1+\nu)}\Big)\Big(\frac{q^2\, Y\, \nu\, \mathrm{BesselI}[n,\, q R]}{(1-2\nu)\,(1+\nu)} + \frac{n^2\, Y\, \nu\, \mathrm{BesselI}[n,\, q R]}{R^2\,(1-2\nu)\,(1+\nu)} - (q\, Y\, \nu\, (\mathrm{BesselI}[-1+n,\, q R] + \mathrm{BesselI}[1+n,\, q R]))\, /$$

$$(2 R\,(1-2\nu)\,(1+\nu)) - \frac{1}{2\,(1+\nu)}\, q\, Y\, \Big(\frac{1}{2} q\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]) + \frac{1}{2} q\, (\mathrm{BesselI}[n,\, q R] + \mathrm{BesselI}[2+n,\, q R])\Big) -$$

$$\Big(q\, Y\, \nu\, \Big(\frac{1}{2} q\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]) + \frac{1}{2} q\, (\mathrm{BesselI}[n,\, q R] + \mathrm{BesselI}[2+n,\, q R])\Big)\Big) \Big/ \, (2\,(1-2\nu)\,(1+\nu)) +$$

$$\frac{q\, Y\, \nu\, \mathrm{KroneckerDelta}[1,\, n]}{2 R\,(1-2\nu)\,(1+\nu)} - \frac{n\, q\, Y\, \nu\, \mathrm{KroneckerDelta}[1,\, n]}{2 R\,(1-2\nu)\,(1+\nu)}\Big) - \Big(-\frac{1}{2\,(1+\nu)}\, i\, q^2\, Y\, (\mathrm{BesselI}[-1+n,\, q R] + \mathrm{BesselI}[1+n,\, q R]) +$$

$$\frac{i\, q^2\, Y\, \mathrm{KroneckerDelta}[1,\, n]}{4\,(1+\nu)}\Big)\Big(\frac{n^2\, Y\, \nu\, \mathrm{BesselI}[-1+n,\, q R]}{R\,(1-2\nu)\,(1+\nu)} + \frac{q^2 R\, Y\, \nu\, \mathrm{BesselI}[-1+n,\, q R]}{(1-2\nu)\,(1+\nu)} + \frac{Y\,(3-4\nu)\, \nu\, \mathrm{BesselI}[-1+n,\, q R]}{R\,(1-2\nu)\,(1+\nu)} -$$

$$\frac{4\, n\, Y\,(1-\nu)\, \nu\, \mathrm{BesselI}[-1+n,\, q R]}{R\,(1-2\nu)\,(1+\nu)} - \frac{q\, Y\,(\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R])}{2\,(1+\nu)} + \frac{1}{2\,(1+\nu)}\, q\, Y\,(3-4\nu)\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]) -$$

$$\frac{q\, Y\, \nu\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R])}{(1-2\nu)\,(1+\nu)} + (q\, Y\,(3-4\nu)\, \nu\, (\mathrm{BesselI}[-2+n,\, q R] + \mathrm{BesselI}[n,\, q R]))\, /\, (2\,(1-2\nu)\,(1+\nu)) -$$

$$\frac{1}{2\,(1+\nu)}\, q R\, Y\, \Big(\frac{1}{2} q\, (\mathrm{BesselI}[-3+n,\, q R] + \mathrm{BesselI}[-1+n,\, q R]) + \frac{1}{2} q\, (\mathrm{BesselI}[-1+n,\, q R] + \mathrm{BesselI}[1+n,\, q R])\Big) -$$

$$\Big(q R\, Y\, \nu\, \Big(\frac{1}{2} q\, (\mathrm{BesselI}[-3+n,\, q R] + \mathrm{BesselI}[-1+n,\, q R]) + \frac{1}{2} q\, (\mathrm{BesselI}[-1+n,\, q R] + \mathrm{BesselI}[1+n,\, q R])\Big)\Big) \Big/ \, (2\,(1-2\nu)\,(1+\nu)) -$$

$$\frac{Y\,(3-4\nu)\, \nu\, \mathrm{KroneckerDelta}[1,\, n]}{R\,(1-2\nu)\,(1+\nu)} + \frac{n\, Y\,(3-4\nu)\, \nu\, \mathrm{KroneckerDelta}[1,\, n]}{R\,(1-2\nu)\,(1+\nu)}\Big)\Big)\Big)\Big)$$

# Appendix B

In this section strain, stress tensors at zeroth order and first order parameters $d, f, g$ are displayed in the 'Cartesian' case: all the instability analysis have been made by choosing the 'Cartesian' reference state $\boldsymbol{u}_{ref}^{C} = \boldsymbol{u}_{ref}^{S} - m\boldsymbol{r}$.

Again Y is the Young's modulus, R the shell radius and $\rho$ the core radius. *Bessel I* is the modified Bessel function and KroneckerDelta has an obvious meaning.

The parameters have been computed with Mathematica 9.0 (see next page)

```
(*ZEROTH ORDER PARAMETERS*)
```

$$\alpha = -\frac{\mathbf{m}\ (\mathbf{1}+\nu)\ \left(\mathbf{2\ R^2}\ (\mathbf{-1}+\nu) + (\mathbf{1}-\mathbf{2}\ \nu)\ \rho^2\right)}{\mathbf{2\ R^2}\ (\mathbf{-1}+\nu)};$$

$$\beta = \frac{\mathbf{m}\ (\mathbf{1}+\nu)\ \rho^2}{\mathbf{2}\ (\mathbf{-1}+\nu)};$$

$$\mathbf{a} = \frac{\mathbf{m}\ (\mathbf{1}+\nu)\ (\mathbf{-1}+\mathbf{2}\ \nu)\ \left(\mathbf{R^2}-\rho^2\right)}{\mathbf{2\ R^2}\ (\mathbf{-1}+\nu)};$$

```
(*Core - 0 order*)

uc[r_] = {a*r, 0, c*r};
ϵc[r_] =
  (1/2) * (Grad[uc[r], {r, θ, z}, "Cylindrical"] + Transpose[Grad[uc[r], {r, θ, z}, "Cylindrical"]]);
```

$$\sigma\mathbf{c[r\_]} = \frac{\mathbf{Y}}{(\mathbf{1}-\mathbf{2}\ \nu)\ (\mathbf{1}+\nu)} * ((1-2\ \nu)\ \epsilon c[r] + \text{Table}[\nu * \text{KroneckerDelta}[i,\ j] * \text{Tr}[\epsilon c[r]],\ \{i,\ 3\},\ \{j,\ 3\}]);$$

```
(*SHELL - 0 ORDER*)

us[r_] = {α*r + β/r, 0, 0};
ϵs[r_] =
  (1/2) * (Grad[us[r], {r, θ, z}, "Cylindrical"] + Transpose[Grad[us[r], {r, θ, z}, "Cylindrical"]]) -
   Table[m * (KroneckerDelta[i, 1] + KroneckerDelta[i, 2] + KroneckerDelta[i, 3]) *
     KroneckerDelta[i, j], {i, 3}, {j, 3}];
```

$$\sigma\mathbf{s[r\_]} = \frac{\mathbf{Y}}{(\mathbf{1}-\mathbf{2}\ \nu)\ (\mathbf{1}+\nu)} * ((1-2\ \nu)\ \epsilon s[r] + \text{Table}[\nu * \text{KroneckerDelta}[i,\ j] * \text{Tr}[\epsilon s[r]],\ \{i,\ 3\},\ \{j,\ 3\}]);$$

```
(*FIRST ORDER PARAMETERS*)
```

In[46]:= **d**

Out[46]=

$$\left(-1/\left(\left(1-2\ \nu\right)\ \left(1+\nu\right)\right)\ \mathbb{i}\ q\ Y\ \left(-m\ \left(1-2\ \nu\right)+\nu\ \left(-3\ m+\frac{1}{2}\ \left(-\frac{m\ \left(1+\nu\right)\ \rho^2}{R^2\ \left(-1+\nu\right)}-\frac{m\ \left(1+\nu\right)\ \left(2\ R^2\ \left(-1+\nu\right)+\left(1-2\ \nu\right)\ \rho^2\right)}{R^2\ \left(-1+\nu\right)}\right)+\frac{\frac{m\ \left(1+\nu\right)\ \rho^2}{2\ R\ \left(-1+\nu\right)}-\frac{m\ \left(1+\nu\right)\ \left(2\ R^2\ \left(-1+\nu\right)+\left(1-2\ \nu\right)\ \rho^2\right)}{2\ R\ \left(-1+\nu\right)}}{R}\right)\right)\right)$$

$$\left(\frac{n^2\ Y\ \nu\ \text{BesselI}[-1+n,\ q\ R]}{R\ \left(1-2\ \nu\right)\ \left(1+\nu\right)}+\frac{q^2\ R\ Y\ \nu\ \text{BesselI}[-1+n,\ q\ R]}{\left(1-2\ \nu\right)\ \left(1+\nu\right)}+\frac{Y\ \left(3-4\ \nu\right)\ \nu\ \text{BesselI}[-1+n,\ q\ R]}{R\ \left(1-2\ \nu\right)\ \left(1+\nu\right)}-\frac{4\ n\ Y\ \left(1-\nu\right)\ \nu\ \text{BesselI}[-1+n,\ q\ R]}{R\ \left(1-2\ \nu\right)\ \left(1+\nu\right)}-\right.$$

$$\frac{q\ Y\ \left(\text{BesselI}[-2+n,\ q\ R]+\text{BesselI}[n,\ q\ R]\right)}{2\ \left(1+\nu\right)}+\frac{q\ Y\ \left(3-4\ \nu\right)\ \left(\text{BesselI}[-2+n,\ q\ R]+\text{BesselI}[n,\ q\ R]\right)}{2\ \left(1+\nu\right)}-$$

$$\frac{q\ Y\ \nu\ \left(\text{BesselI}[-2+n,\ q\ R]+\text{BesselI}[n,\ q\ R]\right)}{\left(1-2\ \nu\right)\ \left(1+\nu\right)}+\frac{q\ Y\ \left(3-4\ \nu\right)\ \nu\ \left(\text{BesselI}[-2+n,\ q\ R]+\text{BesselI}[n,\ q\ R]\right)}{2\ \left(1-2\ \nu\right)\ \left(1+\nu\right)}-$$

$$\frac{q\ R\ Y\ \left(\frac{1}{2}\ q\ \left(\text{BesselI}[-3+n,\ q\ R]+\text{BesselI}[-1+n,\ q\ R]\right)+\frac{1}{2}\ q\ \left(\text{BesselI}[-1+n,\ q\ R]+\text{BesselI}[1+n,\ q\ R]\right)\right)}{2\ \left(1+\nu\right)}-$$

$$\frac{q\ R\ Y\ \nu\ \left(\frac{1}{2}\ q\ \left(\text{BesselI}[-3+n,\ q\ R]+\text{BesselI}[-1+n,\ q\ R]\right)+\frac{1}{2}\ q\ \left(\text{BesselI}[-1+n,\ q\ R]+\text{BesselI}[1+n,\ q\ R]\right)\right)}{2\ \left(1-2\ \nu\right)\ \left(1+\nu\right)}-$$

$$\left.\frac{Y\ \left(3-4\ \nu\right)\ \nu\ \text{KroneckerDelta}[1,\ n]}{R\ \left(1-2\ \nu\right)\ \left(1+\nu\right)}+\frac{n\ Y\ \left(3-4\ \nu\right)\ \nu\ \text{KroneckerDelta}[1,\ n]}{R\ \left(1-2\ \nu\right)\ \left(1+\nu\right)}\right)\ \left(\left(\left(\frac{\mathbb{i}\ n\ Y\ \text{BesselI}[1+n,\ q\ R]}{2\ R\ \left(1+\nu\right)}+\frac{\mathbb{i}\ n\ Y\ \left(3-4\ \nu\right)\ \text{BesselI}[1+n,\ q\ R]}{2\ R\ \left(1+\nu\right)}+\right.\right.\right.$$

$$\left.\frac{2\ \mathbb{i}\ Y\ \left(1-\nu\right)\ \text{BesselI}[1+n,\ q\ R]}{R\ \left(1+\nu\right)}-\frac{\mathbb{i}\ n\ q\ Y\ \left(\text{BesselI}[n,\ q\ R]+\text{BesselI}[2+n,\ q\ R]\right)}{2\ \left(1+\nu\right)}-\frac{\mathbb{i}\ q\ Y\ \left(1-\nu\right)\ \left(\text{BesselI}[n,\ q\ R]+\text{BesselI}[2+n,\ q\ R]\right)}{1+\nu}\right)$$

$$\left(-\frac{\mathbb{i}\ q\ Y\ \text{BesselI}[-1+n,\ q\ R]}{2\ \left(1+\nu\right)}+\frac{\mathbb{i}\ q\ Y\ \left(3-4\ \nu\right)\ \text{BesselI}[-1+n,\ q\ R]}{2\ \left(1+\nu\right)}-\frac{\mathbb{i}\ q^2\ R\ Y\ \left(\text{BesselI}[-2+n,\ q\ R]+\text{BesselI}[n,\ q\ R]\right)}{2\ \left(1+\nu\right)}-\frac{\mathbb{i}\ q\ Y\ \left(3-4\ \nu\right)\ \text{KroneckerDelta}[1,\ n]}{2\ \left(1+\nu\right)}\right)-$$

$$\left(-\frac{\mathbb{i}\ q\ Y\ \text{BesselI}[1+n,\ q\ R]}{2\ \left(1+\nu\right)}+\frac{\mathbb{i}\ q\ Y\ \left(3-4\ \nu\right)\ \text{BesselI}[1+n,\ q\ R]}{2\ \left(1+\nu\right)}-\frac{\mathbb{i}\ q^2\ R\ Y\ \left(\text{BesselI}[n,\ q\ R]+\text{BesselI}[2+n,\ q\ R]\right)}{2\ \left(1+\nu\right)}\right)$$

$$\left(\frac{\mathbb{i}\ n\ Y\ \text{BesselI}[-1+n,\ q\ R]}{2\ R\ \left(1+\nu\right)}+\frac{\mathbb{i}\ n\ Y\ \left(3-4\ \nu\right)\ \text{BesselI}[-1+n,\ q\ R]}{2\ R\ \left(1+\nu\right)}-\frac{2\ \mathbb{i}\ Y\ \left(1-\nu\right)\ \text{BesselI}[-1+n,\ q\ R]}{R\ \left(1+\nu\right)}-\frac{\mathbb{i}\ n\ q\ Y\ \left(\text{BesselI}[-2+n,\ q\ R]+\text{BesselI}[n,\ q\ R]\right)}{2\ \left(1+\nu\right)}+\right.$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)+$$

$$\Bigg(-1/(R\,(1-2\,\nu)\,(1+\nu))\,i\,n\,Y\,\Bigg((1-2\,\nu)\,\Bigg(-m+\frac{\frac{m\,(1+\nu)\,\rho^2}{2\,R\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\,R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)+$$

$$\nu\,\Bigg(-3\,m+\frac{1}{2}\,\Bigg(-\frac{m\,(1+\nu)\,\rho^2}{R^2\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\,R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{R^2\,(-1+\nu)}\Bigg)+\frac{\frac{m\,(1+\nu)\,\rho^2}{2\,R\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\,R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)+$$

$$1/((1-2\,\nu)\,(1+\nu))\,i\,q\,Y\,\Bigg(-m\,(1-2\,\nu)+\nu\,\Bigg(-3\,m+\frac{1}{2}\,\Bigg(-\frac{m\,(1+\nu)\,\rho^2}{R^2\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\,R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{R^2\,(-1+\nu)}\Bigg)+\frac{\frac{m\,(1+\nu)\,\rho^2}{2\,R\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\,R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{2\,R\,(-1+\nu)}}{R}\Bigg)\Bigg)$$

$$\Bigg(\frac{i\,n\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)$$

$$\Bigg(\Bigg(\frac{n^2\,Y\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,R\,Y\,\left(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\right)}{2\,(1+\nu)}-$$

$$\frac{q\,R\,Y\,\nu\,\left(\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R])\right)}{2\,(1-2\,\nu)\,(1+\nu)}\Bigg)$$

$$\Bigg(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)-$$

$$\Bigg(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\Bigg)$$

$$\Bigg(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$

$$q\,R\,Y\,\left(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\right)-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}$$

$$q\,R\,Y\,\nu\,\left(\frac{1}{2}\,q\,(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])\right)-$$

$$\frac{Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\Bigg(-\Bigg(\frac{i\,n\,Y\,\text{BesselI}[n,\,q\,R]}{R^2\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R])}{2\,R\,(1+\nu)}-\frac{i\,q\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)}+\frac{i\,n\,q\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)}\Bigg)$$

$$\Bigg(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\Bigg)+$$

$$\left(-\frac{i\,q^2\,Y\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])}{2\,(1+\nu)}+\frac{i\,q^2\,Y\,\mathrm{KroneckerDelta}[1,\,n]}{4\,(1+\nu)}\right)$$

$$\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)$$

$$\Bigg(\Bigg(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,R\,Y\,\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R])\right)}{2\,(1+\nu)}-$$

$$\frac{q\,R\,Y\,\nu\,\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R])\right)}{2\,(1-2\,\nu)\,(1+\nu)}\Bigg)$$

$$\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)-$$

$$\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\right)$$

$$\Bigg(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$

$$q\,R\,Y\,\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\right)-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}$$

$$q\,R\,Y\,\nu\,\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\right)-$$

$$\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\Bigg)\Bigg)+$$

$$\Bigg(\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{1+\nu}\Bigg)$$

$$\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)-$$

$$\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\right)$$

$$\Bigg(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$

$$\frac{i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\Bigg)\Bigg)$$

$$\Bigg(\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)$$

$$\Bigg(\frac{q^2\,Y\,\nu\,\mathrm{BesselI}[n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[n,\,q\,R]}{R^2\,(1-2\,\nu)\,(1+\nu)}-\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])}{2\,R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\,\left(\frac{1}{2}\,q\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])\right)}{2\,(1+\nu)}-$$

$$\frac{qY\nu\left(\frac{1}{2}q\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])+\frac{1}{2}q\,(\text{BesselI}[n,qR]+\text{BesselI}[2+n,qR])\right)}{2(1-2\nu)(1+\nu)}+\frac{qY\nu\,\text{KroneckerDelta}[1,n]}{2R(1-2\nu)(1+\nu)}-$$

$$\frac{nqY\nu\,\text{KroneckerDelta}[1,n]}{2R(1-2\nu)(1+\nu)}\Bigg)-\left(-\frac{iq^2Y\,(\text{BesselI}[-1+n,qR]+\text{BesselI}[1+n,qR])}{2(1+\nu)}+\frac{iq^2Y\,\text{KroneckerDelta}[1,n]}{4(1+\nu)}\right)$$

$$\Bigg(\frac{n^2Y\nu\,\text{BesselI}[-1+n,qR]}{R(1-2\nu)(1+\nu)}+\frac{q^2RY\nu\,\text{BesselI}[-1+n,qR]}{(1-2\nu)(1+\nu)}+\frac{Y(3-4\nu)\,\nu\,\text{BesselI}[-1+n,qR]}{R(1-2\nu)(1+\nu)}-\frac{4nY(1-\nu)\,\nu\,\text{BesselI}[-1+n,qR]}{R(1-2\nu)(1+\nu)}-$$

$$\frac{qY\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1+\nu)}+\frac{qY(3-4\nu)\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1+\nu)}-$$

$$\frac{qY\nu\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{(1-2\nu)(1+\nu)}+\frac{qY(3-4\nu)\,\nu\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1-2\nu)(1+\nu)}-\frac{1}{2(1+\nu)}$$

$$qRY\left(\frac{1}{2}q\,(\text{BesselI}[-3+n,qR]+\text{BesselI}[-1+n,qR])+\frac{1}{2}q\,(\text{BesselI}[-1+n,qR]+\text{BesselI}[1+n,qR])\right)-\frac{1}{2(1-2\nu)(1+\nu)}$$

$$qRY\nu\left(\frac{1}{2}q\,(\text{BesselI}[-3+n,qR]+\text{BesselI}[-1+n,qR])+\frac{1}{2}q\,(\text{BesselI}[-1+n,qR]+\text{BesselI}[1+n,qR])\right)-$$

$$\frac{Y(3-4\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R(1-2\nu)(1+\nu)}+\frac{nY(3-4\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R(1-2\nu)(1+\nu)}\Bigg)\Bigg)\Bigg)$$

In[47]:= **f**

Out[47]=

$$\Bigg(1/\left(R(1-2\nu)(1+\nu)\right)\,i\,n\,Y$$

$$\left((1-2\nu)\left(-m+\frac{\frac{m(1+\nu)\rho^2}{2R(-1+\nu)}-\frac{m(1+\nu)\left(2R^2(-1+\nu)+(1-2\nu)\rho^2\right)}{2R(-1+\nu)}}{R}\right)+\nu\left(-3m+\frac{1}{2}\left(-\frac{m(1+\nu)\rho^2}{R^2(-1+\nu)}-\frac{m(1+\nu)\left(2R^2(-1+\nu)+(1-2\nu)\rho^2\right)}{R^2(-1+\nu)}\right)+\frac{\frac{m(1+\nu)\rho^2}{2R(-1+\nu)}-\frac{m(1+\nu)\left(2R^2(-1+\nu)+(1-2\nu)\rho^2\right)}{2R(-1+\nu)}}{R}\right)\right)$$

$$\left(-\frac{iqY\,\text{BesselI}[-1+n,qR]}{2(1+\nu)}+\frac{iqY(3-4\nu)\,\text{BesselI}[-1+n,qR]}{2(1+\nu)}-\frac{iq^2RY\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1+\nu)}-\frac{iqY(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2(1+\nu)}\right)-$$

$$1/\left((1-2\nu)(1+\nu)\right)\,i\,q\,Y\left(-m(1-2\nu)+\nu\left(-3m+\frac{1}{2}\left(-\frac{m(1+\nu)\rho^2}{R^2(-1+\nu)}-\frac{m(1+\nu)\left(2R^2(-1+\nu)+(1-2\nu)\rho^2\right)}{R^2(-1+\nu)}\right)+\frac{\frac{m(1+\nu)\rho^2}{2R(-1+\nu)}-\frac{m(1+\nu)\left(2R^2(-1+\nu)+(1-2\nu)\rho^2\right)}{2R(-1+\nu)}}{R}\right)\right)$$

$$\left(\frac{inY\,\text{BesselI}[-1+n,qR]}{2R(1+\nu)}+\frac{inY(3-4\nu)\,\text{BesselI}[-1+n,qR]}{2R(1+\nu)}-\frac{2iY(1-\nu)\,\text{BesselI}[-1+n,qR]}{R(1+\nu)}-\frac{inqY\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1+\nu)}+\right.$$

$$\left.\frac{iqY(1-\nu)\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{1+\nu}+\frac{iY(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2R(1+\nu)}-\frac{inY(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2R(1+\nu)}\right)\Bigg)\Bigg/$$

$$\Bigg(\Bigg(\frac{inY\,\text{BesselI}[1+n,qR]}{2R(1+\nu)}+\frac{inY(3-4\nu)\,\text{BesselI}[1+n,qR]}{2R(1+\nu)}+\frac{2iY(1-\nu)\,\text{BesselI}[1+n,qR]}{R(1+\nu)}-\frac{inqY\,(\text{BesselI}[n,qR]+\text{BesselI}[2+n,qR])}{2(1+\nu)}-$$

$$\frac{iqY(1-\nu)\,(\text{BesselI}[n,qR]+\text{BesselI}[2+n,qR])}{1+\nu}\Bigg)$$

$$\left(-\frac{iqY\,\text{BesselI}[-1+n,qR]}{2(1+\nu)}+\frac{iqY(3-4\nu)\,\text{BesselI}[-1+n,qR]}{2(1+\nu)}-\frac{iq^2RY\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1+\nu)}-\frac{iqY(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2(1+\nu)}\right)-$$

$$\left(-\frac{iqY\,\text{BesselI}[1+n,qR]}{2(1+\nu)}+\frac{iqY(3-4\nu)\,\text{BesselI}[1+n,qR]}{2(1+\nu)}-\frac{iq^2RY\,(\text{BesselI}[n,qR]+\text{BesselI}[2+n,qR])}{2(1+\nu)}\right)$$

$$\left(\frac{inY\,\text{BesselI}[-1+n,qR]}{2R(1+\nu)}+\frac{inY(3-4\nu)\,\text{BesselI}[-1+n,qR]}{2R(1+\nu)}-\frac{2iY(1-\nu)\,\text{BesselI}[-1+n,qR]}{R(1+\nu)}-\frac{inqY\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1+\nu)}+\right.$$

$$\left.\frac{iqY(1-\nu)\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{1+\nu}+\frac{iY(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2R(1+\nu)}-\frac{inY(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2R(1+\nu)}\right)\Bigg)+$$

$$\Bigg(\Bigg(\Bigg(\frac{inY\,\text{BesselI}[n,qR]}{R^2(1+\nu)}-\frac{inqY\,(\text{BesselI}[-1+n,qR]+\text{BesselI}[1+n,qR])}{2R(1+\nu)}-\frac{iqY\,\text{KroneckerDelta}[1,n]}{4R(1+\nu)}+\frac{inqY\,\text{KroneckerDelta}[1,n]}{4R(1+\nu)}\Bigg)$$

$$\left(-\frac{iqY\,\text{BesselI}[-1+n,qR]}{2(1+\nu)}+\frac{iqY(3-4\nu)\,\text{BesselI}[-1+n,qR]}{2(1+\nu)}-\frac{iq^2RY\,(\text{BesselI}[-2+n,qR]+\text{BesselI}[n,qR])}{2(1+\nu)}-\frac{iqY(3-4\nu)\,\text{KroneckerDelta}[1,n]}{2(1+\nu)}\right)-$$

$$\left(-\frac{i\, q^2\, Y\, (\mathrm{BesselI}[-1+n,\, q\, R]+\mathrm{BesselI}[1+n,\, q\, R])}{2\,(1+\nu)}+\frac{i\, q^2\, Y\, \mathrm{KroneckerDelta}[1,\, n]}{4\,(1+\nu)}\right)$$

$$\Big(\frac{i\, n\, Y\, \mathrm{BesselI}[-1+n,\, q\, R]}{2\, R\,(1+\nu)}+\frac{i\, n\, Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{2\, R\,(1+\nu)}-\frac{2\, i\, Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{R\,(1+\nu)}-\frac{i\, n\, q\, Y\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1+\nu)}+$$

$$\frac{i\, q\, Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{1+\nu}+\frac{i\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\, R\,(1+\nu)}-\frac{i\, n\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\, R\,(1+\nu)}\Big)\Big)$$

$$\Big(1/((1-2\,\nu)\,(1+\nu))\, i\, q\, Y\,\Big(-m\,(1-2\,\nu)+\nu\,\Big(-3\, m+\frac{1}{2}\Big(-\frac{m\,(1+\nu)\,\rho^2}{R^2\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\, R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{R^2\,(-1+\nu)}\Big)+\frac{\frac{m\,(1+\nu)\,\rho^2}{2\, R\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\, R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{2\, R\,(-1+\nu)}}{R}\Big)\Big)\Big)$$

$$\Big(\frac{n^2\, Y\,\nu\,\mathrm{BesselI}[-1+n,\, q\, R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\, R\, Y\,\nu\,\mathrm{BesselI}[-1+n,\, q\, R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[-1+n,\, q\, R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\, n\, Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\, q\, R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\, Y\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1+\nu)}+\frac{q\, Y\,(3-4\,\nu)\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1+\nu)}-$$

$$\frac{q\, Y\,\nu\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{(1-2\,\nu)\,(1+\nu)}+\frac{q\, Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$

$$q\, R\, Y\left(\frac{1}{2}\, q\,(\mathrm{BesselI}[-3+n,\, q\, R]+\mathrm{BesselI}[-1+n,\, q\, R])+\frac{1}{2}\, q\,(\mathrm{BesselI}[-1+n,\, q\, R]+\mathrm{BesselI}[1+n,\, q\, R])\right)-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}$$

$$q\, R\, Y\,\nu\left(\frac{1}{2}\, q\,(\mathrm{BesselI}[-3+n,\, q\, R]+\mathrm{BesselI}[-1+n,\, q\, R])+\frac{1}{2}\, q\,(\mathrm{BesselI}[-1+n,\, q\, R]+\mathrm{BesselI}[1+n,\, q\, R])\right)-\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\, n]}{R\,(1-2\,\nu)\,(1+\nu)}+$$

$$\frac{n\, Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\, n]}{R\,(1-2\,\nu)\,(1+\nu)}\Big)\Big(\Big(\frac{i\, n\, Y\,\mathrm{BesselI}[1+n,\, q\, R]}{2\, R\,(1+\nu)}+\frac{i\, n\, Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\, q\, R]}{2\, R\,(1+\nu)}+\frac{2\, i\, Y\,(1-\nu)\,\mathrm{BesselI}[1+n,\, q\, R]}{R\,(1+\nu)}-$$

$$\frac{i\, n\, q\, Y\,(\mathrm{BesselI}[n,\, q\, R]+\mathrm{BesselI}[2+n,\, q\, R])}{2\,(1+\nu)}-\frac{i\, q\, Y\,(1-\nu)\,(\mathrm{BesselI}[n,\, q\, R]+\mathrm{BesselI}[2+n,\, q\, R])}{1+\nu}\Big)\Big(-\frac{i\, q\, Y\,\mathrm{BesselI}[-1+n,\, q\, R]}{2\,(1+\nu)}+$$

$$\frac{i\, q\, Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{2\,(1+\nu)}-\frac{i\, q^2\, R\, Y\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1+\nu)}-\frac{i\, q\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\,(1+\nu)}\Big)-$$

$$\left(-\frac{i\, q\, Y\,\mathrm{BesselI}[1+n,\, q\, R]}{2\,(1+\nu)}+\frac{i\, q\, Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\, q\, R]}{2\,(1+\nu)}-\frac{i\, q^2\, R\, Y\,(\mathrm{BesselI}[n,\, q\, R]+\mathrm{BesselI}[2+n,\, q\, R])}{2\,(1+\nu)}\right)$$

$$\Big(\frac{i\, n\, Y\, \mathrm{BesselI}[-1+n,\, q\, R]}{2\, R\,(1+\nu)}+\frac{i\, n\, Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{2\, R\,(1+\nu)}-\frac{2\, i\, Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{R\,(1+\nu)}-\frac{i\, n\, q\, Y\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1+\nu)}+$$

$$\frac{i\, q\, Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{1+\nu}+\frac{i\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\, R\,(1+\nu)}-\frac{i\, n\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\, R\,(1+\nu)}\Big)\Big)-$$

$$\Big(-1/(R\,(1-2\,\nu)\,(1+\nu))\, i\, n\, Y\,\Big((1-2\,\nu)\Big(-m+\frac{\frac{m\,(1+\nu)\,\rho^2}{2\, R\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\, R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{2\, R\,(-1+\nu)}}{R}\Big)+\nu\,\Big(-3\, m+\frac{1}{2}\Big(-\frac{m\,(1+\nu)\,\rho^2}{R^2\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\, R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{R^2\,(-1+\nu)}\Big)+$$

$$\frac{\frac{m\,(1+\nu)\,\rho^2}{2\, R\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\, R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{2\, R\,(-1+\nu)}}{R}\Big)\Big)\Big)\Big(-\frac{i\, q\, Y\,\mathrm{BesselI}[-1+n,\, q\, R]}{2\,(1+\nu)}+$$

$$\frac{i\, q\, Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{2\,(1+\nu)}-\frac{i\, q^2\, R\, Y\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1+\nu)}-\frac{i\, q\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\,(1+\nu)}\Big)+$$

$$1/((1-2\,\nu)\,(1+\nu))\, i\, q\, Y\,\Big(-m\,(1-2\,\nu)+\nu\,\Big(-3\, m+\frac{1}{2}\Big(-\frac{m\,(1+\nu)\,\rho^2}{R^2\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\, R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{R^2\,(-1+\nu)}\Big)+\frac{\frac{m\,(1+\nu)\,\rho^2}{2\, R\,(-1+\nu)}-\frac{m\,(1+\nu)\,(2\, R^2\,(-1+\nu)+(1-2\,\nu)\,\rho^2)}{2\, R\,(-1+\nu)}}{R}\Big)\Big)$$

$$\Big(\frac{i\, n\, Y\, \mathrm{BesselI}[-1+n,\, q\, R]}{2\, R\,(1+\nu)}+\frac{i\, n\, Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{2\, R\,(1+\nu)}-\frac{2\, i\, Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\, q\, R]}{R\,(1+\nu)}-\frac{i\, n\, q\, Y\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{2\,(1+\nu)}+$$

$$\frac{i\, q\, Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\, q\, R]+\mathrm{BesselI}[n,\, q\, R])}{1+\nu}+\frac{i\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\, R\,(1+\nu)}-\frac{i\, n\, Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\, n]}{2\, R\,(1+\nu)}\Big)\Big)$$

$$\Big(\Big(\frac{n^2\, Y\,\nu\,\mathrm{BesselI}[1+n,\, q\, R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\, R\, Y\,\nu\,\mathrm{BesselI}[1+n,\, q\, R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[1+n,\, q\, R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{4\, n\, Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[1+n,\, q\, R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\, Y\,(\mathrm{BesselI}[n,\, q\, R]+\mathrm{BesselI}[2+n,\, q\, R])}{2\,(1+\nu)}+\frac{q\, Y\,(3-4\,\nu)\,(\mathrm{BesselI}[n,\, q\, R]+\mathrm{BesselI}[2+n,\, q\, R])}{2\,(1+\nu)}-$$

$$\frac{q Y \nu\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{(1-2\nu)\ (1+\nu)} + \frac{q Y\ (3-4\nu)\ \nu\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{2\ (1-2\nu)\ (1+\nu)} - \frac{1}{2\ (1+\nu)}$$

$$q R Y \left(\frac{1}{2} q\ (\mathrm{BesselI}[-1+n, q R] + \mathrm{BesselI}[1+n, q R]) + \frac{1}{2} q\ (\mathrm{BesselI}[1+n, q R] + \mathrm{BesselI}[3+n, q R])\right) - \frac{1}{2\ (1-2\nu)\ (1+\nu)}$$

$$q R Y \nu \left(\frac{1}{2} q\ (\mathrm{BesselI}[-1+n, q R] + \mathrm{BesselI}[1+n, q R]) + \frac{1}{2} q\ (\mathrm{BesselI}[1+n, q R] + \mathrm{BesselI}[3+n, q R])\right)\Big)\Big) \Big(-\frac{i q Y \mathrm{BesselI}[-1+n, q R]}{2\ (1+\nu)} +$$

$$\frac{i q Y\ (3-4\nu)\ \mathrm{BesselI}[-1+n, q R]}{2\ (1+\nu)} - \frac{i q^2 R Y\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1+\nu)} - \frac{i q Y\ (3-4\nu)\ \mathrm{KroneckerDelta}[1, n]}{2\ (1+\nu)}\Big) -$$

$$\Big(-\frac{i q Y \mathrm{BesselI}[1+n, q R]}{2\ (1+\nu)} + \frac{i q Y\ (3-4\nu)\ \mathrm{BesselI}[1+n, q R]}{2\ (1+\nu)} - \frac{i q^2 R Y\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{2\ (1+\nu)}\Big)$$

$$\Big(\frac{n^2 Y \nu \mathrm{BesselI}[-1+n, q R]}{R\ (1-2\nu)\ (1+\nu)} + \frac{q^2 R Y \nu \mathrm{BesselI}[-1+n, q R]}{(1-2\nu)\ (1+\nu)} + \frac{Y\ (3-4\nu)\ \nu \mathrm{BesselI}[-1+n, q R]}{R\ (1-2\nu)\ (1+\nu)} - \frac{4 n Y\ (1-\nu)\ \nu \mathrm{BesselI}[-1+n, q R]}{R\ (1-2\nu)\ (1+\nu)} -$$

$$\frac{q Y\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1+\nu)} + \frac{q Y\ (3-4\nu)\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1+\nu)} -$$

$$\frac{q Y \nu\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{(1-2\nu)\ (1+\nu)} + \frac{q Y\ (3-4\nu)\ \nu\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1-2\nu)\ (1+\nu)} - \frac{1}{2\ (1+\nu)}$$

$$q R Y \left(\frac{1}{2} q\ (\mathrm{BesselI}[-3+n, q R] + \mathrm{BesselI}[-1+n, q R]) + \frac{1}{2} q\ (\mathrm{BesselI}[-1+n, q R] + \mathrm{BesselI}[1+n, q R])\right) - \frac{1}{2\ (1-2\nu)\ (1+\nu)}$$

$$q R Y \nu \left(\frac{1}{2} q\ (\mathrm{BesselI}[-3+n, q R] + \mathrm{BesselI}[-1+n, q R]) + \frac{1}{2} q\ (\mathrm{BesselI}[-1+n, q R] + \mathrm{BesselI}[1+n, q R])\right) -$$

$$\frac{Y\ (3-4\nu)\ \nu \mathrm{KroneckerDelta}[1, n]}{R\ (1-2\nu)\ (1+\nu)} + \frac{n Y\ (3-4\nu)\ \nu \mathrm{KroneckerDelta}[1, n]}{R\ (1-2\nu)\ (1+\nu)}\Big)\Big)\Big)\Big)\Big) \Big/$$

$$\Big(\Big(\Big(\frac{i n Y \mathrm{BesselI}[1+n, q R]}{2 R\ (1+\nu)} + \frac{i n Y\ (3-4\nu)\ \mathrm{BesselI}[1+n, q R]}{2 R\ (1+\nu)} + \frac{2 i Y\ (1-\nu)\ \mathrm{BesselI}[1+n, q R]}{R\ (1+\nu)} - \frac{i n q Y\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{2\ (1+\nu)} -$$

$$\frac{i q Y\ (1-\nu)\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{1+\nu}\Big)$$

$$\Big(-\frac{i q Y \mathrm{BesselI}[-1+n, q R]}{2\ (1+\nu)} + \frac{i q Y\ (3-4\nu)\ \mathrm{BesselI}[-1+n, q R]}{2\ (1+\nu)} - \frac{i q^2 R Y\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1+\nu)} - \frac{i q Y\ (3-4\nu)\ \mathrm{KroneckerDelta}[1, n]}{2\ (1+\nu)}\Big) -$$

$$\Big(-\frac{i q Y \mathrm{BesselI}[1+n, q R]}{2\ (1+\nu)} + \frac{i q Y\ (3-4\nu)\ \mathrm{BesselI}[1+n, q R]}{2\ (1+\nu)} - \frac{i q^2 R Y\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{2\ (1+\nu)}\Big)$$

$$\Big(\frac{i n Y \mathrm{BesselI}[-1+n, q R]}{2 R\ (1+\nu)} + \frac{i n Y\ (3-4\nu)\ \mathrm{BesselI}[-1+n, q R]}{2 R\ (1+\nu)} - \frac{2 i Y\ (1-\nu)\ \mathrm{BesselI}[-1+n, q R]}{R\ (1+\nu)} - \frac{i n q Y\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1+\nu)} +$$

$$\frac{i q Y\ (1-\nu)\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{1+\nu} + \frac{i Y\ (3-4\nu)\ \mathrm{KroneckerDelta}[1, n]}{2 R\ (1+\nu)} - \frac{i n Y\ (3-4\nu)\ \mathrm{KroneckerDelta}[1, n]}{2 R\ (1+\nu)}\Big)\Big)$$

$$\Big(\Big(-\Big(\frac{i n Y \mathrm{BesselI}[n, q R]}{R^2\ (1+\nu)} - \frac{i n q Y\ (\mathrm{BesselI}[-1+n, q R] + \mathrm{BesselI}[1+n, q R])}{2 R\ (1+\nu)} - \frac{i q Y \mathrm{KroneckerDelta}[1, n]}{4 R\ (1+\nu)} + \frac{i n q Y \mathrm{KroneckerDelta}[1, n]}{4 R\ (1+\nu)}\Big)$$

$$\Big(-\frac{i q Y \mathrm{BesselI}[-1+n, q R]}{2\ (1+\nu)} + \frac{i q Y\ (3-4\nu)\ \mathrm{BesselI}[-1+n, q R]}{2\ (1+\nu)} - \frac{i q^2 R Y\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1+\nu)} -$$

$$\frac{i q Y\ (3-4\nu)\ \mathrm{KroneckerDelta}[1, n]}{2\ (1+\nu)}\Big) + \Big(-\frac{i q^2 Y\ (\mathrm{BesselI}[-1+n, q R] + \mathrm{BesselI}[1+n, q R])}{2\ (1+\nu)} + \frac{i q^2 Y \mathrm{KroneckerDelta}[1, n]}{4\ (1+\nu)}\Big)$$

$$\Big(\frac{i n Y \mathrm{BesselI}[-1+n, q R]}{2 R\ (1+\nu)} + \frac{i n Y\ (3-4\nu)\ \mathrm{BesselI}[-1+n, q R]}{2 R\ (1+\nu)} - \frac{2 i Y\ (1-\nu)\ \mathrm{BesselI}[-1+n, q R]}{R\ (1+\nu)} - \frac{i n q Y\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{2\ (1+\nu)} +$$

$$\frac{i q Y\ (1-\nu)\ (\mathrm{BesselI}[-2+n, q R] + \mathrm{BesselI}[n, q R])}{1+\nu} + \frac{i Y\ (3-4\nu)\ \mathrm{KroneckerDelta}[1, n]}{2 R\ (1+\nu)} - \frac{i n Y\ (3-4\nu)\ \mathrm{KroneckerDelta}[1, n]}{2 R\ (1+\nu)}\Big)\Big)$$

$$\Big(\Big(\frac{n^2 Y \nu \mathrm{BesselI}[1+n, q R]}{R\ (1-2\nu)\ (1+\nu)} + \frac{q^2 R Y \nu \mathrm{BesselI}[1+n, q R]}{(1-2\nu)\ (1+\nu)} + \frac{Y\ (3-4\nu)\ \nu \mathrm{BesselI}[1+n, q R]}{R\ (1-2\nu)\ (1+\nu)} + \frac{4 n Y\ (1-\nu)\ \nu \mathrm{BesselI}[1+n, q R]}{R\ (1-2\nu)\ (1+\nu)} -$$

$$\frac{q Y\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{2\ (1+\nu)} + \frac{q Y\ (3-4\nu)\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{2\ (1+\nu)} -$$

$$\frac{q Y \nu\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{(1-2\nu)\ (1+\nu)} + \frac{q Y\ (3-4\nu)\ \nu\ (\mathrm{BesselI}[n, q R] + \mathrm{BesselI}[2+n, q R])}{2\ (1-2\nu)\ (1+\nu)} - \frac{1}{2\ (1+\nu)}$$

$$q R Y \left(\frac{1}{2} q\ (\mathrm{BesselI}[-1+n, q R] + \mathrm{BesselI}[1+n, q R]) + \frac{1}{2} q\ (\mathrm{BesselI}[1+n, q R] + \mathrm{BesselI}[3+n, q R])\right) - \frac{1}{2\ (1-2\nu)\ (1+\nu)}$$

$$q\,R\,Y\,\nu\left(\frac{1}{2}q\left(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R]\right)+\frac{1}{2}q\left(\text{BesselI}[1+n,\,q\,R]+\text{BesselI}[3+n,\,q\,R]\right)\right)\Bigg)\Bigg)\left(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\right.$$

$$\left.\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)-$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\left(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]\right)}{2\,(1+\nu)}\right)$$

$$\left(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\right.$$

$$\frac{q\,Y\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$

$$q\,R\,Y\left(\frac{1}{2}q\left(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R]\right)\right)-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}$$

$$q\,R\,Y\,\nu\left(\frac{1}{2}q\left(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R]\right)\right)-$$

$$\left.\frac{Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}\right)\Bigg)+$$

$$\Bigg(\Bigg(\frac{i\,n\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\left(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]\right)}{2\,(1+\nu)}-$$

$$\frac{i\,q\,Y\,(1-\nu)\left(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]\right)}{1+\nu}\Bigg)\left(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\right.$$

$$\left.\frac{i\,q^2\,R\,Y\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)-$$

$$\left(-\frac{i\,q\,Y\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\left(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]\right)}{2\,(1+\nu)}\right)$$

$$\left(\frac{i\,n\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}+\right.$$

$$\left.\frac{i\,q\,Y\,(1-\nu)\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}\right)\Bigg)$$

$$\Bigg(\left(-\frac{i\,q\,Y\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\text{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\text{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)$$

$$\left(\frac{q^2\,Y\,\nu\,\text{BesselI}[n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{n^2\,Y\,\nu\,\text{BesselI}[n,\,q\,R]}{R^2\,(1-2\,\nu)\,(1+\nu)}-\frac{q\,Y\,\nu\left(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R]\right)}{2\,R\,(1-2\,\nu)\,(1+\nu)}-\right.$$

$$\frac{q\,Y\left(\frac{1}{2}q\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)+\frac{1}{2}q\left(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]\right)\right)}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\left(\frac{1}{2}q\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)+\frac{1}{2}q\left(\text{BesselI}[n,\,q\,R]+\text{BesselI}[2+n,\,q\,R]\right)\right)}{2\,(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,\nu\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\left.\frac{n\,q\,Y\,\nu\,\text{KroneckerDelta}[1,\,n]}{2\,R\,(1-2\,\nu)\,(1+\nu)}\right)-\left(-\frac{i\,q^2\,Y\left(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R]\right)}{2\,(1+\nu)}+\frac{i\,q^2\,Y\,\text{KroneckerDelta}[1,\,n]}{4\,(1+\nu)}\right)$$

$$\left(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\right.$$

$$\frac{q\,Y\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\left(\text{BesselI}[-2+n,\,q\,R]+\text{BesselI}[n,\,q\,R]\right)}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$

$$q\,R\,Y\left(\frac{1}{2}q\left(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R]\right)\right)-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}$$

$$q\,R\,Y\,\nu\left(\frac{1}{2}q\left(\text{BesselI}[-3+n,\,q\,R]+\text{BesselI}[-1+n,\,q\,R]\right)+\frac{1}{2}q\left(\text{BesselI}[-1+n,\,q\,R]+\text{BesselI}[1+n,\,q\,R]\right)\right)-$$

```
Y (3 - 4 ν) ν KroneckerDelta[1, n]   n Y (3 - 4 ν) ν KroneckerDelta[1, n]
---------------------------------- + ------------------------------------ ))))
      R (1 - 2 ν) (1 + ν)                   R (1 - 2 ν) (1 + ν)
```

In[48]:= **g**

```
Out[48]= -(2 (1 + ν) (2 m q R^3 BesselI[n, q R] - m n q R^3 BesselI[n, q R] + 3 m n q R^3 ν BesselI[n, q R] - 6 m q R^3 ν^2 BesselI[n, q R] - 2 m n q R^3 ν^2 BesselI[n, q R] +
      4 m q R^3 ν^3 BesselI[n, q R] - 2 m q R ν ρ^2 BesselI[n, q R] - m n q R ν ρ^2 BesselI[n, q R] + 6 m q R ν^2 ρ^2 BesselI[n, q R] + 2 m n q R ν^2 ρ^2 BesselI[n, q R] -
      4 m q R ν^3 ρ^2 BesselI[n, q R] - 4 m R^2 BesselI[1 + n, q R] - 12 m n R^2 ν BesselI[1 + n, q R] + 12 m R^2 ν^2 BesselI[1 + n, q R] + 20 m n R^2 ν^2 BesselI[1 + n, q R] -
      8 m R^2 ν^3 BesselI[1 + n, q R] - 8 m n R^2 ν^3 BesselI[1 + n, q R] + 4 m ν ρ^2 BesselI[1 + n, q R] + 4 m n ν ρ^2 BesselI[1 + n, q R] - 12 m ν^2 ρ^2 BesselI[1 + n, q R] -
      12 m n ν^2 ρ^2 BesselI[1 + n, q R] + 8 m ν^3 ρ^2 BesselI[1 + n, q R] + 8 m n ν^3 ρ^2 BesselI[1 + n, q R] + 2 m q R^3 BesselI[2 + n, q R] - m n q R^3 BesselI[2 + n, q R] +
      3 m n q R^3 ν BesselI[2 + n, q R] - 6 m q R^3 ν^2 BesselI[2 + n, q R] - 2 m n q R^3 ν^2 BesselI[2 + n, q R] + 4 m q R^3 ν^3 BesselI[2 + n, q R] - 2 m q R ν ρ^2 BesselI[2 + n, q R] -
      m n q R ν ρ^2 BesselI[2 + n, q R] + 6 m q R ν^2 ρ^2 BesselI[2 + n, q R] + 2 m n q R ν^2 ρ^2 BesselI[2 + n, q R] - 4 m q R ν^3 ρ^2 BesselI[2 + n, q R])) /
    (R^2 (-1 + ν) (-1 + 2 ν) (-4 q^2 R^2 BesselI[-2 + n, q R] BesselI[n, q R] + 4 q^2 R^2 ν BesselI[-2 + n, q R] BesselI[n, q R] +
      8 q R BesselI[-1 + n, q R] BesselI[n, q R] - 2 n q R BesselI[-1 + n, q R] BesselI[n, q R] - 16 q R ν BesselI[-1 + n, q R] BesselI[n, q R] +
      8 q R ν^2 BesselI[-1 + n, q R] BesselI[n, q R] - 4 q^2 R^2 BesselI[n, q R]^2 + 4 q^2 R^2 ν BesselI[n, q R]^2 + 8 q R BesselI[-2 + n, q R] BesselI[1 + n, q R] +
      2 n q R BesselI[-2 + n, q R] BesselI[1 + n, q R] - 16 q R ν BesselI[-2 + n, q R] BesselI[1 + n, q R] + 8 q R ν^2 BesselI[-2 + n, q R] BesselI[1 + n, q R] -
      16 BesselI[-1 + n, q R] BesselI[1 + n, q R] + 48 ν BesselI[-1 + n, q R] BesselI[1 + n, q R] - 32 ν^2 BesselI[-1 + n, q R] BesselI[1 + n, q R] +
      8 q R BesselI[n, q R] BesselI[1 + n, q R] + 2 n q R BesselI[n, q R] BesselI[1 + n, q R] - 16 q R ν BesselI[n, q R] BesselI[1 + n, q R] +
      8 q R ν^2 BesselI[n, q R] BesselI[1 + n, q R] - 4 q^2 R^2 BesselI[-2 + n, q R] BesselI[2 + n, q R] + 4 q^2 R^2 ν BesselI[-2 + n, q R] BesselI[2 + n, q R] +
      8 q R BesselI[-1 + n, q R] BesselI[2 + n, q R] - 2 n q R BesselI[-1 + n, q R] BesselI[2 + n, q R] - 16 q R ν BesselI[-1 + n, q R] BesselI[2 + n, q R] +
      8 q R ν^2 BesselI[-1 + n, q R] BesselI[2 + n, q R] - 4 q^2 R^2 BesselI[n, q R] BesselI[2 + n, q R] + 4 q^2 R^2 ν BesselI[n, q R] BesselI[2 + n, q R] -
      9 q R BesselI[n, q R] KroneckerDelta[1, n] + 18 q R ν BesselI[n, q R] KroneckerDelta[1, n] - 8 q R ν^2 BesselI[n, q R] KroneckerDelta[1, n] +
      18 BesselI[1 + n, q R] KroneckerDelta[1, n] + 6 n BesselI[1 + n, q R] KroneckerDelta[1, n] - 48 ν BesselI[1 + n, q R] KroneckerDelta[1, n] -
      8 n ν BesselI[1 + n, q R] KroneckerDelta[1, n] + 32 ν^2 BesselI[1 + n, q R] KroneckerDelta[1, n] - 9 q R BesselI[2 + n, q R] KroneckerDelta[1, n] +
      18 q R ν BesselI[2 + n, q R] KroneckerDelta[1, n] - 8 q R ν^2 BesselI[2 + n, q R] KroneckerDelta[1, n])) +

 ((-4 q^2 R^2 BesselI[-1 + n, q R] BesselI[n, q R] + 4 q^2 R^2 ν BesselI[-1 + n, q R] BesselI[n, q R] - 4 n q R BesselI[n, q R]^2 +
      8 q R BesselI[-1 + n, q R] BesselI[1 + n, q R] + 4 n q R BesselI[-1 + n, q R] BesselI[1 + n, q R] - 8 q R ν BesselI[-1 + n, q R] BesselI[1 + n, q R] +
      8 n BesselI[n, q R] BesselI[1 + n, q R] - 4 q^2 R^2 BesselI[n, q R] BesselI[1 + n, q R] - 16 n ν BesselI[n, q R] BesselI[1 + n, q R] +
      4 q^2 R^2 ν BesselI[n, q R] BesselI[1 + n, q R] + 8 q R BesselI[1 + n, q R]^2 + 4 n q R BesselI[1 + n, q R]^2 - 8 q R ν BesselI[1 + n, q R]^2 -
      4 q^2 R^2 BesselI[-1 + n, q R] BesselI[2 + n, q R] + 4 q^2 R^2 ν BesselI[-1 + n, q R] BesselI[2 + n, q R] - 4 n q R BesselI[n, q R] BesselI[2 + n, q R] -
      4 q^2 R^2 BesselI[1 + n, q R] BesselI[2 + n, q R] + 4 q^2 R^2 ν BesselI[1 + n, q R] BesselI[2 + n, q R] + 3 q^2 R^2 BesselI[n, q R] KroneckerDelta[1, n] -
      2 q^2 R^2 ν BesselI[n, q R] KroneckerDelta[1, n] - 6 q R BesselI[1 + n, q R] KroneckerDelta[1, n] - 2 n q R BesselI[1 + n, q R] KroneckerDelta[1, n] +
      8 q R ν BesselI[1 + n, q R] KroneckerDelta[1, n] + 3 q^2 R^2 BesselI[2 + n, q R] KroneckerDelta[1, n] - 2 q^2 R^2 ν BesselI[2 + n, q R] KroneckerDelta[1, n])

   (1 / ((1 - 2 ν) (1 + ν)) i q Y (-m (1 - 2 ν) + ν (-3 m + 1/2 (-(m (1 + ν) ρ^2)/(R^2 (-1 + ν)) - (m (1 + ν) (2 R^2 (-1 + ν) + (1 - 2 ν) ρ^2))/(R^2 (-1 + ν))) +
      ((m (1 + ν) ρ^2)/(2 R (-1 + ν)) - (m (1 + ν) (2 R^2 (-1 + ν) + (1 - 2 ν) ρ^2))/(2 R (-1 + ν)))/R)))

    ((n^2 Y ν BesselI[-1 + n, q R])/(R (1 - 2 ν) (1 + ν)) + (q^2 R Y ν BesselI[-1 + n, q R])/((1 - 2 ν) (1 + ν)) + (Y (3 - 4 ν) ν BesselI[-1 + n, q R])/(R (1 - 2 ν) (1 + ν)) - (4 n Y (1 - ν) ν BesselI[-1 + n, q R])/(R (1 - 2 ν) (1 + ν)) -
     (q Y (BesselI[-2 + n, q R] + BesselI[n, q R]))/(2 (1 + ν)) + (q Y (3 - 4 ν) (BesselI[-2 + n, q R] + BesselI[n, q R]))/(2 (1 + ν)) -
     (q Y ν (BesselI[-2 + n, q R] + BesselI[n, q R]))/((1 - 2 ν) (1 + ν)) + (q Y (3 - 4 ν) ν (BesselI[-2 + n, q R] + BesselI[n, q R]))/(2 (1 - 2 ν) (1 + ν)) - 1/(2 (1 + ν))
     q R Y (1/2 q (BesselI[-3 + n, q R] + BesselI[-1 + n, q R]) + 1/2 q (BesselI[-1 + n, q R] + BesselI[1 + n, q R])) - 1/(2 (1 - 2 ν) (1 + ν))
     q R Y ν (1/2 q (BesselI[-3 + n, q R] + BesselI[-1 + n, q R]) + 1/2 q (BesselI[-1 + n, q R] + BesselI[1 + n, q R])) - (Y (3 - 4 ν) ν KroneckerDelta[1, n])/(R (1 - 2 ν) (1 + ν)) +
     (n Y (3 - 4 ν) ν KroneckerDelta[1, n])/(R (1 - 2 ν) (1 + ν))) (((i n Y BesselI[1 + n, q R])/(2 R (1 + ν)) + (i n Y (3 - 4 ν) BesselI[1 + n, q R])/(2 R (1 + ν)) + (2 i Y (1 - ν) BesselI[1 + n, q R])/(R (1 + ν)) -
      (i n q Y (BesselI[n, q R] + BesselI[2 + n, q R]))/(2 (1 + ν)) - (i q Y (1 - ν) (BesselI[n, q R] + BesselI[2 + n, q R]))/(1 + ν)) (-(i q Y BesselI[-1 + n, q R])/(2 (1 + ν)) +
      (i q Y (3 - 4 ν) BesselI[-1 + n, q R])/(2 (1 + ν)) - (i q^2 R Y (BesselI[-2 + n, q R] + BesselI[n, q R]))/(2 (1 + ν)) - (i q Y (3 - 4 ν) KroneckerDelta[1, n])/(2 (1 + ν))) -
    (-(i q Y BesselI[1 + n, q R])/(2 (1 + ν)) + (i q Y (3 - 4 ν) BesselI[1 + n, q R])/(2 (1 + ν)) - (i q^2 R Y (BesselI[n, q R] + BesselI[2 + n, q R]))/(2 (1 + ν)))
     ((i n Y BesselI[-1 + n, q R])/(2 R (1 + ν)) + (i n Y (3 - 4 ν) BesselI[-1 + n, q R])/(2 R (1 + ν)) - (2 i Y (1 - ν) BesselI[-1 + n, q R])/(R (1 + ν)) - (i n q Y (BesselI[-2 + n, q R] + BesselI[n, q R]))/(2 (1 + ν)) +
      (i q Y (1 - ν) (BesselI[-2 + n, q R] + BesselI[n, q R]))/(1 + ν) + (i Y (3 - 4 ν) KroneckerDelta[1, n])/(2 R (1 + ν)) - (i n Y (3 - 4 ν) KroneckerDelta[1, n])/(2 R (1 + ν)))) -
```

$$\left(-1\Big/\left(R\,(1-2\nu)\,(1+\nu)\right)\,i\,n\,Y\left((1-2\nu)\left(-m+\frac{\frac{m\,(1+\nu)\,\rho^2}{2R\,(-1+\nu)}-\frac{m\,(1+\nu)\,\left(2R^2\,(-1+\nu)+(1-2\nu)\,\rho^2\right)}{2R\,(-1+\nu)}}{R}\right)+\nu\left(-3m+\frac{1}{2}\left(-\frac{m\,(1+\nu)\,\rho^2}{R^2\,(-1+\nu)}-\frac{m\,(1+\nu)\,\left(2R^2\,(-1+\nu)+(1-2\nu)\,\rho^2\right)}{R^2\,(-1+\nu)}\right)+\right.\right.\right.$$

$$\left.\left.\frac{\frac{m\,(1+\nu)\,\rho^2}{2R\,(-1+\nu)}-\frac{m\,(1+\nu)\,\left(2R^2\,(-1+\nu)+(1-2\nu)\,\rho^2\right)}{2R\,(-1+\nu)}}{R}\right)\right)\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\right.$$

$$\left.\frac{i\,q\,Y\,(3-4\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)+$$

$$1\Big/\left((1-2\nu)\,(1+\nu)\right)i\,q\,Y\left(-m\,(1-2\nu)+\nu\left(-3m+\frac{1}{2}\left(-\frac{m\,(1+\nu)\,\rho^2}{R^2\,(-1+\nu)}-\frac{m\,(1+\nu)\,\left(2R^2\,(-1+\nu)+(1-2\nu)\,\rho^2\right)}{R^2\,(-1+\nu)}\right)+\frac{\frac{m\,(1+\nu)\,\rho^2}{2R\,(-1+\nu)}-\frac{m\,(1+\nu)\,\left(2R^2\,(-1+\nu)+(1-2\nu)\,\rho^2\right)}{2R\,(-1+\nu)}}{R}\right)\right)$$

$$\left(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\right.$$

$$\left.\left.\frac{i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2R\,(1+\nu)}\right)\right)$$

$$\left(\left(\left(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{(1-2\nu)\,(1+\nu)}+\frac{Y\,(3-4\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\right.\right.\right.$$

$$\frac{q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{(1-2\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\nu)\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1-2\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$

$$q\,R\,Y\left(\frac{1}{2}q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R])\right)-\frac{1}{2\,(1-2\nu)\,(1+\nu)}$$

$$\left.\left.q\,R\,Y\,\nu\left(\frac{1}{2}q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R])\right)\right)\right)\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\right.$$

$$\left.\frac{i\,q\,Y\,(3-4\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)}\right)-$$

$$\left(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}\right)$$

$$\left(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\nu)\,(1+\nu)}+\frac{Y\,(3-4\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\nu)\,(1+\nu)}-\right.$$

$$\frac{q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{(1-2\nu)\,(1+\nu)}+$$

$$\frac{q\,Y\,(3-4\nu)\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1-2\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}q\,R\,Y\left(\frac{1}{2}q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}q\right.$$

$$\left.(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\right)-\frac{1}{2\,(1-2\nu)\,(1+\nu)}q\,R\,Y\,\nu\left(\frac{1}{2}q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}q\right.$$

$$\left.\left.\left.(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])\right)-\frac{Y\,(3-4\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\nu)\,(1+\nu)}\right)\right)\Big/$$

$$\Big(2R\,\big(-4q^2R^2\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]+4q^2R^2\,\nu\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]+8\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]-$$

$$2\,n\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]-16\,q\,R\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]+$$
$$8\,q\,R\,\nu^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[n,\,q\,R]-4q^2R^2\,\mathrm{BesselI}[n,\,q\,R]^2+4q^2R^2\,\nu\,\mathrm{BesselI}[n,\,q\,R]^2+$$
$$8\,q\,R\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+2\,n\,q\,R\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-$$
$$16\,q\,R\,\nu\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+8\,q\,R\,\nu^2\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-$$
$$16\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+48\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-$$
$$32\,\nu^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+8\,q\,R\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+$$
$$2\,n\,q\,R\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-16\,q\,R\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]+$$
$$8\,q\,R\,\nu^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[1+n,\,q\,R]-4q^2R^2\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+$$
$$4q^2R^2\,\nu\,\mathrm{BesselI}[-2+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+8\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-$$
$$2\,n\,q\,R\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-16\,q\,R\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+$$
$$8\,q\,R\,\nu^2\,\mathrm{BesselI}[-1+n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-4q^2R^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]+$$
$$4q^2R^2\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{BesselI}[2+n,\,q\,R]-9\,q\,R\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+$$
$$18\,q\,R\,\nu\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-8\,q\,R\,\nu^2\,\mathrm{BesselI}[n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+$$

$$18\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+6\,n\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-$$
$$48\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-8\,n\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+$$
$$32\,\nu^2\,\mathrm{BesselI}[1+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-9\,q\,R\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]+$$
$$18\,q\,R\,\nu\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n]-8\,q\,R\,\nu^2\,\mathrm{BesselI}[2+n,\,q\,R]\,\mathrm{KroneckerDelta}[1,\,n])$$

$$((-(\frac{i\,n\,Y\,\mathrm{BesselI}[n,\,q\,R]}{R^2\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])}{2\,R\,(1+\nu)}-\frac{i\,q\,Y\,\mathrm{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)}+\frac{i\,n\,q\,Y\,\mathrm{KroneckerDelta}[1,\,n]}{4\,R\,(1+\nu)})$$

$$(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-$$
$$\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)})+(-\frac{i\,q^2\,Y\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])}{2\,(1+\nu)}+\frac{i\,q^2\,Y\,\mathrm{KroneckerDelta}[1,\,n]}{4\,(1+\nu)})$$

$$(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$
$$\frac{i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}))$$

$$((\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$
$$\frac{q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$
$$\frac{q\,Y\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$
$$q\,R\,Y\,(\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R]))-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}$$
$$q\,R\,Y\,\nu\,(\frac{1}{2}\,q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])+\frac{1}{2}\,q\,(\mathrm{BesselI}[1+n,\,q\,R]+\mathrm{BesselI}[3+n,\,q\,R])))\,(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+$$
$$\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)})-$$

$$(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)})$$

$$(\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-$$
$$\frac{q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{(1-2\,\nu)\,(1+\nu)}+$$
$$\frac{q\,Y\,(3-4\,\nu)\,\nu\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}q\,R\,Y\,(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}\,q$$
$$(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]))-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}q\,R\,Y\,\nu\,(\frac{1}{2}\,q\,(\mathrm{BesselI}[-3+n,\,q\,R]+\mathrm{BesselI}[-1+n,\,q\,R])+\frac{1}{2}$$
$$q\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R]))-\frac{Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\mathrm{KroneckerDelta}[1,\,n]}{R\,(1-2\,\nu)\,(1+\nu)}))+$$

$$((\frac{i\,n\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)}-$$
$$\frac{i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{1+\nu})\,(-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-$$
$$\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)})-$$
$$(-\frac{i\,q\,Y\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[n,\,q\,R]+\mathrm{BesselI}[2+n,\,q\,R])}{2\,(1+\nu)})$$
$$(\frac{i\,n\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}+\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,R\,(1+\nu)}-\frac{2\,i\,Y\,(1-\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{R\,(1+\nu)}-\frac{i\,n\,q\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}+$$
$$\frac{i\,q\,Y\,(1-\nu)\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{1+\nu}+\frac{i\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}-\frac{i\,n\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,R\,(1+\nu)}))$$

$$((-\frac{i\,q\,Y\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}+\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{BesselI}[-1+n,\,q\,R]}{2\,(1+\nu)}-\frac{i\,q^2\,R\,Y\,(\mathrm{BesselI}[-2+n,\,q\,R]+\mathrm{BesselI}[n,\,q\,R])}{2\,(1+\nu)}-\frac{i\,q\,Y\,(3-4\,\nu)\,\mathrm{KroneckerDelta}[1,\,n]}{2\,(1+\nu)})$$

$$(\frac{q^2\,Y\,\nu\,\mathrm{BesselI}[n,\,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{n^2\,Y\,\nu\,\mathrm{BesselI}[n,\,q\,R]}{R^2\,(1-2\,\nu)\,(1+\nu)}-\frac{q\,Y\,\nu\,(\mathrm{BesselI}[-1+n,\,q\,R]+\mathrm{BesselI}[1+n,\,q\,R])}{2\,R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\frac{q\,Y\left(\frac{1}{2}q\,(\text{BesselI}[-2+n,q\,R]+\text{BesselI}[n,q\,R])+\frac{1}{2}q\,(\text{BesselI}[n,q\,R]+\text{BesselI}[2+n,q\,R])\right)}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\left(\frac{1}{2}q\,(\text{BesselI}[-2+n,q\,R]+\text{BesselI}[n,q\,R])+\frac{1}{2}q\,(\text{BesselI}[n,q\,R]+\text{BesselI}[2+n,q\,R])\right)}{2\,(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,\nu\,\text{KroneckerDelta}[1,n]}{2\,R\,(1-2\,\nu)\,(1+\nu)}-$$

$$\left.\frac{n\,q\,Y\,\nu\,\text{KroneckerDelta}[1,n]}{2\,R\,(1-2\,\nu)\,(1+\nu)}\right)-\left(-\frac{i\,q^2\,Y\,(\text{BesselI}[-1+n,q\,R]+\text{BesselI}[1+n,q\,R])}{2\,(1+\nu)}+\frac{i\,q^2\,Y\,\text{KroneckerDelta}[1,n]}{4\,(1+\nu)}\right)$$

$$\left(\frac{n^2\,Y\,\nu\,\text{BesselI}[-1+n,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{q^2\,R\,Y\,\nu\,\text{BesselI}[-1+n,q\,R]}{(1-2\,\nu)\,(1+\nu)}+\frac{Y\,(3-4\,\nu)\,\nu\,\text{BesselI}[-1+n,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\frac{4\,n\,Y\,(1-\nu)\,\nu\,\text{BesselI}[-1+n,q\,R]}{R\,(1-2\,\nu)\,(1+\nu)}-\right.$$

$$\frac{q\,Y\,(\text{BesselI}[-2+n,q\,R]+\text{BesselI}[n,q\,R])}{2\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,(\text{BesselI}[-2+n,q\,R]+\text{BesselI}[n,q\,R])}{2\,(1+\nu)}-$$

$$\frac{q\,Y\,\nu\,(\text{BesselI}[-2+n,q\,R]+\text{BesselI}[n,q\,R])}{(1-2\,\nu)\,(1+\nu)}+\frac{q\,Y\,(3-4\,\nu)\,\nu\,(\text{BesselI}[-2+n,q\,R]+\text{BesselI}[n,q\,R])}{2\,(1-2\,\nu)\,(1+\nu)}-\frac{1}{2\,(1+\nu)}$$

$$q\,R\,Y\left(\frac{1}{2}q\,(\text{BesselI}[-3+n,q\,R]+\text{BesselI}[-1+n,q\,R])+\frac{1}{2}q\,(\text{BesselI}[-1+n,q\,R]+\text{BesselI}[1+n,q\,R])\right)-\frac{1}{2\,(1-2\,\nu)\,(1+\nu)}$$

$$q\,R\,Y\,\nu\left(\frac{1}{2}q\,(\text{BesselI}[-3+n,q\,R]+\text{BesselI}[-1+n,q\,R])+\frac{1}{2}q\,(\text{BesselI}[-1+n,q\,R]+\text{BesselI}[1+n,q\,R])\right)-$$

$$\left.\frac{Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R\,(1-2\,\nu)\,(1+\nu)}+\frac{n\,Y\,(3-4\,\nu)\,\nu\,\text{KroneckerDelta}[1,n]}{R\,(1-2\,\nu)\,(1+\nu)}\right)))$$

# Thanks

Firstly, I would like to thank my supervisor Prof Ortolani for his kindness and help.

I will be forever grateful to Prof Aqua of Université Pierre et Marie Curie of Paris: thanks to him I knew a new scientific field, but most of all, I clarified which will be my future career.

I want to thank the guys of "apartment 6" for the great experience in Bologna.

I want to "thank" Paris (yes, the city), *elle* (Paris, a beautiful elegant old lady) *sera toujours dans mon coeur*.

But most of all, I want to thank Valeria, *la petite parisienne*: our pursuit of happiness is going on.

At the end, but not for importance, I want to say thank you, to my beautiful family: bad moments are useful to appreciate happy ends.