

ALMA MATER STUDIORUM - UNIVERSITÀ DI BOLOGNA

DIPARTIMENTO di
INGEGNERIA DELL'ENERGIA ELETTRICA E DELL'INFORMAZIONE
"Guglielmo Marconi"
DEI

CORSO DI LAUREA IN
INGEGNERIA DELL'ENERGIA ELETTRICA

TESI DI LAUREA
in
Plasma engineering

Plasma diagnostics via microwave absorption

CANDIDATO

Jacopo Iurescia

RELATORE

Prof. Andrea Cristofolini

CORRELATORI

Dott. Arturo Popoli

Prof. Gabriele Neretti

Anno accademico 2024-2025

Sessione V

Abstract

This thesis presents an experimental investigation of cold plasma diagnostics via microwave absorption, with the primary objective of estimating the electron number density of a low-pressure laboratory plasma. The work serves as a preliminary assessment of the microwave absorption technique for characterising plasma parameters, particularly focusing on identifying the frequency range within which the plasma frequency lies.

The theoretical framework is established through a discussion of plasma fundamentals, including the criteria for an ionised gas to be classified as a plasma, Debye shielding, and electron plasma oscillations. The interaction between electromagnetic waves and cold plasmas is analysed using the cold plasma model, deriving the dispersion relations for both collisionless and collisional cases. Special attention is given to the condition of wave propagation, cutoff, and attenuation mechanisms.

The experimental part employs a microwave diagnostic setup consisting of horn antennas operating in the 2-18 GHz range, a vacuum chamber with electrode-based plasma generation, and appropriate signal acquisition systems. The transmission factor, defined as the ratio between received microwave power with and without plasma, is measured across multiple frequencies. Two sets of measurements are conducted at different pressure levels: 1.4 mbar and 0.5 mbar.

The data analysis uses a MATLAB script that implements a fitting algorithm based on the attenuation coefficient, allowing for the estimation of plasma frequency and thus electron density. Results yield electron number densities of $1.35 \times 10^{16} \text{ m}^{-3}$ at 1.4 mbar and $1.67 \times 10^{16} \text{ m}^{-3}$ at 0.5 mbar. The observed shift in transparency frequency from 2.5 GHz to 7.6 GHz upon pressure reduction is consistent with theoretical predictions, indicating higher electron densities at lower pressures due to reduced collisional losses and more efficient ionisation. Non-monotonic transmission behaviour observed in some measurements is discussed in terms of experimental limitations including wave interference and plasma instability.

Keywords: *plasma diagnostics, microwave absorption, electron number density, cold plasma, transmission factor, plasma frequency, wave propagation, cutoff, plasma-wave interaction*

Contents

1	Introduction	5
2	Plasma characteristics and waves interaction	9
2.1	The matter and its states	9
2.2	Plasma requirements	11
2.2.1	Collective behaviour	11
2.2.2	Macroscopic neutrality	12
2.2.3	Debye shielding	13
2.2.4	Plasma spatial requirements	15
2.2.5	Electron plasma oscillation and temporal requirement	15
2.3	Waves propagation	17
2.3.1	Waves basic principles	18
2.3.2	The dispersion relation in isotropic plasma	21
2.3.3	Propagation inside a collisionless plasma	23
2.3.4	Propagation in a collisional plasma	28
3	Measurements & diagnostic	33
3.1	Phenomenology and methodology of the measurement	34
3.1.1	Methodology of the diagnostic via microwave absorption	37
3.1.2	Diagnostic setup	38
3.2	Results	40
3.2.1	Test at 1.4 mbar	40
3.2.2	Test at 0.5 mbar	42
4	Conclusions	47
A	MATLAB script for data analysis	49

Chapter 1

Introduction

Plasma, often referred to as the fourth state of matter, constitutes the most abundant form of visible matter in the universe. Consequently, it exhibits a remarkable diversity of scales, temperatures, and densities. Despite this, all plasmas share fundamental characteristics: they are ionised gases comprising charged particles (electrons and ions) that exhibit collective behaviour. The study of plasma physics has gained increasing importance over the past decades due to its relevance to numerous scientific and technological applications. Controlled nuclear fusion relies on the confinement and heating of plasma to extremely high temperatures. Space propulsion systems, in particular electric thrusters, generate and accelerate plasma to produce thrust with exceptional fuel efficiency. In industrial settings, plasma processing is indispensable for semiconductor manufacturing, materials surface treatment, and the deposition of thin films. Atmospheric re-entry of spacecraft involves the formation of a plasma sheath around the vehicle, critically affecting radio communications, a phenomenon of direct relevance to aerospace engineering.

Paramount to all these applications is the ability to measure plasma parameters accurately. Among these parameters, the electron number density n_e holds particular significance. It determines the plasma frequency $\omega_{pe} = \sqrt{(n_e e^2 / m_e \epsilon_0)}$, which governs the propagation of electromagnetic waves through the plasma. The electron density many aspects of the ionised medium. Consequently, reliable diagnostic techniques for measuring n_e are essential for both fundamental plasma research and practical applications.

Plasma diagnostics includes a wide array of technique developed to extract information about plasma parameters, generating as little perturbation as possible to the medium. These technique can be categorized into intrusive (in-situ) methods and non intrusive methods; in the first group, probes are inserted directly into the plasma, while in the second group measurements are performed externally using electromagnetic radiation or particles beam.

Intrusive methods suffer one main disadvantage: the would be destroyed in high-temperature

environments such as fusion reactors or re-entry plasma. Furthermore, the insertion of a physical object inevitably perturbs the local plasma conditions, potentially compromising the measurement accuracy. Non-intrusive techniques often require sophisticated and expensive instrumentations, and their implementation can be challenging in confined geometries.

Microwave based diagnostics represent a solution to some of the problem mentioned above. They are non-intrusive, do not require sophisticated or expensive instrumentations, and due to the frequency range of the microwaves they result particularly well suited for investigating plasma with electron number density in the range of $10^{15} \div 10^{19} \text{ m}^{-3}$. Most of the laboratory plasma fall in this range.

The interaction between electromagnetic wave and cold plasma, the phenomenon studied in this thesis, is governed by the dielectric properties of the ionised gas. When a microwave propagates through a plasma, its behaviour depends on the relationship between the wave frequency ω and the plasma frequency ω_{pe} . For $\omega > \omega_{pe}$ the wave can propagate, although it experiences attenuation due to collisional processes. For $\omega < \omega_{pe}$ the wave becomes evanescent and is reflected, with the electromagnetic energy content of the wave converted mainly into kinetic energy of the plasma constituent particles.

In the collisional case, the transition between propagation and reflection is not abrupt but gradual. The electrons accelerated by the wave field may collide with neutral atoms or ions, breaking the wave coordinated motion and transferring energy from the wave to the particles, a dissipative process called wave damping. The extent of the damping depends on the wave frequency, the electron density and the electron-heavy collision frequency ν . By measuring the attenuation of microwaves at multiple frequencies, it is possible to infer ω_{pe} , and consequently the electron number density.

Wave damping can be considered the theoretical reason that makes the microwave diagnostics technique possible. The practical tool that makes it possible is the transmission factor $T_f = P_{on}/P_{off}$. It is defined as the ratio of received microwave power with plasma turned-on to that with plasma turned-off, basically quantifying the attenuation experienced by the wave. This quantity is easily measurable and is directly related to the attenuation coefficient χ through the relation $T_f = \exp(-2\chi\omega\Delta x/c)$, where Δx is the path travelled by the wave through the plasma. The attenuation coefficient is function of ω , ω_{pe} and ν , linking the attenuation process to the plasma density.

The present thesis is part of a broader research study aimed at investigating diagnostic capabilities for plasma relevant to aerospace applications. Such plasmas are characterised by low temperatures of the heavy particles, while the electrons reach higher energies, and moderate electron densities making the microwave diagnostic suited for their analysis. This work represents a preliminary assessment of microwave absorption as a diagnostic tool for the plasma mentioned previously. Rather than aiming for definitive measure-

ments with optimised instrumentation, the focus is on understanding the fundamental principles, testing experimental procedures and identifying the practical challenges that must be addressed in the following phases of the research study.

Chapter 2

Plasma characteristics and waves interaction

2.1 The matter and its states

Plasma is generally considered one of the four classical states of matter along with the solid, liquid, and gaseous states. Each state is characterised by a distinctive arrangement and by the level of mobility of the constitutive particles. The competition between the potential energy, coming from inter-particle forces, and the random kinetic energy of the particles, which translates into thermal energy, determines in which state a certain element is found. The inter-particle forces are relatively strong in solids, this results in their closely packed atomic structure which is organised in a regular lattice or in an irregular amorphous structure. By transferring heat to a solid the kinetic energy of its atoms or molecules is increased. When the kinetic energy is enough to exceed the inter-particle forces, the solid turns into a liquid; in this case the molecules are kept closely packed by moderate inter-particle forces, yet they possess enough kinetic energy to move around each other. By continuing to transfer heat, the liquid will eventually transition into a gas. The inter-particle forces inside a gas are very weak, if not almost absent, and thus molecules possess high mobility, allowing them to travel considerable distances on the molecular scale. This high mobility is also a feature of a plasma, where the atoms or molecules are not closely packed.

The process of a substance changing its state is called phase transition, it occurs when the external parameters of the medium, usually pressure or temperature, reach a specific point. Here, one of the main differences between plasma and the other states of matter appears: at constant pressure, the transition of the gaseous state into plasma does not have a well defined temperature, unlike other processes such as freezing, which take place at an exact temperature given a constant pressure. This is why the transition from a gas

to a plasma is not a phase transition in the thermodynamic sense [1]. The reason for this peculiar distinction lies in the gas to plasma transition process.

To generate a plasma, the gaseous state is a starting point. For a gas in thermal equilibrium, the possible velocities of its particles are given by a velocity distribution function, which specifies the fraction of particles inside any given velocity range. When heat is transferred to a gas, what is observed on a macroscopic scale is a rise of the average temperature. On a microscopic scale, the same process leads to an increase of particles located in high velocity ranges, which means the particles average velocity and average kinetic energy increase. As a consequence of this, molecules start to collide into each other at increasingly higher kinetic energy dissociating into single atoms, which in turn collide into other atoms or molecules. During such collisions, an atom can lose its outermost electron if the kinetic energy involved is sufficiently high, the process just described is known as ionisation. The ionisation mechanisms are different, but the requirement they all share is to overcome the energy binding the outermost electron to the atom nucleus, unsurprisingly called ionisation energy. Summarizing, a gas heated to a sufficiently high temperature shows that a certain fraction of its atoms become ionised, this behaviour is described by the Saha equation

$$\frac{n_i}{n_n} = 2.405 \times 10^{21} T^{\frac{3}{2}} \frac{1}{n_i} \exp\left(-\frac{U}{k_b T}\right)$$

which relates the degree of ionisation of a gas in thermal equilibrium and the temperature of the gas. Here n_i and n_n are the density of ionised atoms and neutral atoms respectively (number per m^3), T is the gas temperature (K), U is the ionisation energy of the gas (J) and k_b is the Boltzmann constant.

It should be clear now why the plasma transition temperature is not well defined. The ionisation of a gas is a gradual phenomenon that has an exponential dependency on the temperature of the gas. A temperature value of at least some thousands of kelvin is necessary to obtain a significant degree of ionisation. The ionisation energy of the gas and the total number density deeply affect the temperature at which ionisation becomes relevant, this can be seen in figure 2.1.

In figure 2.1 it can also be seen the higher amount of energy needed to ionise a more dense hydrogen gas. The reason is that a higher density of electrons implies a higher recombination rate, recombination is the process by which a free electron is caught by a positive ion [2].

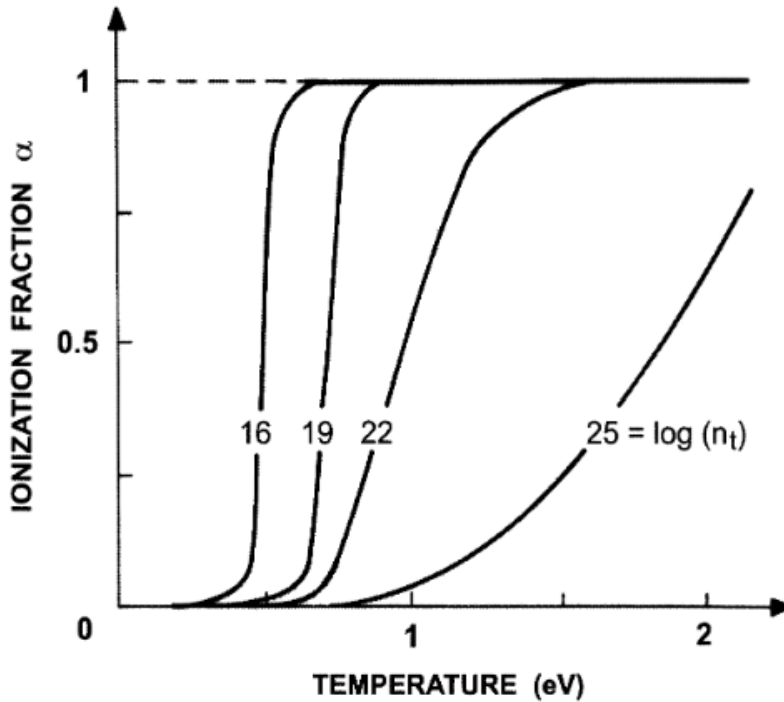


Figure 2.1: Degree of ionisation $\alpha = \frac{n_i}{n_i + n_n}$. The values are expressed for the atomic hydrogen ($U=13.6$ eV). Each curve refers to different number densities $n_t = n_i + n_n$ [1].

2.2 Plasma requirements

While ionisation constitutes the fundamental necessary condition for the plasma state, it is not a sufficient one. It is possible to create a gas with a low degree of ionisation which does not show any plasma behaviour. To be classified as a plasma, an ionised medium must exhibit two principal features: macroscopic electrical neutrality and collective behaviour, the meaning of both and their implication in plasma physics is going to be investigated in the current section.

2.2.1 Collective behaviour

The particles constituting an ionised gas drift freely in virtue of their kinetic energy. The random motion can result in a local concentration of electric charge, this can be described as the conversion of the thermal kinetic energy into electrostatic potential energy. It should also be considered that a moving ion or electron is a charge in motion producing a magnetic field. These fields exert an influence over the other free charges in the plasma, so that their random motion is not governed by the mechanical collision with other particles only, as is reflected in the equations of motion for particles inside a plasma. Considering this, the essence of collective behaviour emerges: the trajectory of

any charged particle depends on both local collisions and the long-range electromagnetic fields generated by the plasma and its charged particles as a whole [2].

Of course, the behaviour of a plasma and the phenomena characterising it are not described following each single particle's trajectory; average plasma densities can span from 10^{15} to 10^{25} particles per cubic meter, it appears obvious that following each single particle's trajectory to calculate the electric and magnetic field distribution generated and then, in turn, consider how these fields affect the trajectories of the said particles is not an efficient approach. To describe most of the plasma phenomena the single particle is not considered, on the contrary, the fluid element approach is employed. More specifically, the plasma is thought as made of two or more superimposed fluids, each for the species of particle present, and each fluid is characterised by its own distribution function regarding particles density and velocity (the most simple case involves just singly charged ions, electrons and no neutral particles) [2].

This natural tendency of a plasma to behave collectively in its components results in many phenomena, some of them are fundamental to an understanding of plasma physics and therefore will be discussed in the following sections.

2.2.2 Macroscopic neutrality

The concepts of electrostatic potential and local concentration of charge may seem, apparently, not compatible with the second requirement of a plasma, that of macroscopic neutrality. Previously, the conversion of thermal kinetic energy into electrostatic potential energy was mentioned, this process spontaneously and continuously occurs in the plasma. Given the free movement of charged particles and their capacity to neutralise any local charge imbalance, a straightforward calculation demonstrates the conditions required to sustain even a slight disparity: a plasma with a 1% imbalance between electron and positive ion number densities would require a temperature on the order of several millions of kelvin to provide particles with sufficient thermal kinetic energy to overcome the resulting electric potential energy [1].

In the absence of external forces, it can be observed that inside a plasma under equilibrium condition the creation of a charge density implies a clash between two forces opposing each other: the thermal kinetic energy of the particles, which randomly alters charge neutrality, and the electrostatic potential resulting from the created charge density, which tries to restore electrical neutrality. Considering the substantial magnitude of the coulombic forces resulting even from small charge densities, and thus the predominant performance of the electrostatic potential over the thermal kinetic energy, a balance between the two opposing forms of energy can be obtained only over a certain distance. In other words,

the plasma can support charge densities not larger than the said distance, this distance is known as Debye length, and it is one of the main features of any plasma.

To summarise, inside a plasma under equilibrium condition with no external forces applied, deviation from charge neutrality spontaneously occurs due to the random motion of the charges, the maximum dimension of this deviation is described by the Debye length. The magnitude of the Debye length and the size of the medium are key aspects to consider for the condition of macroscopic neutrality and for the definition of a plasma itself, as it will be clarified in the following paragraphs. This condition is characterised by localised and transient charge densities, constantly forming and exerting electromagnetic influence over their surroundings; nevertheless, the plasma presents a net charge of zero because of the limited spatial extent of the charge densities. The following approximation is valid in the presence of a plasma made of singly charged ions only, and it is an expression of the quasi-neutral nature of the plasma in its state of rest: $n_e \approx n_i \approx n$ and n is the plasma density [1], [2].

The notion of Debye length is not limited to what has been described so far, it is a fundamental block of plasma physics and it is useful to describe another phenomenon plasma must necessarily feature.

2.2.3 Debye shielding

Another expression of the collective behaviour of the plasma constituent particles is represented by the Debye shielding. The classical example used to describe this phenomenon involves a spherical charge distribution introduced inside the plasma, while it is at equilibrium; the charge distribution is assumed to have a time-invariant spatial position and electrostatic potential, not affected by eventual charge recombination events. In light of the high mobility of the particles, an amount of charge of sign opposite to the charge density introduced will surround the sphere; if the absence of thermal agitation of the plasma particles is also assumed, the amount of charge surrounding the sphere would be equal to the amount of charge inside the sphere. Complete shielding of the electrostatic potential produced by the spherical charge density is achieved, and no net electric field is observed in the space outside of the two distinct charge distributions [1].

For a more realistic scenario thermal agitation has to be considered. Due to the spherical symmetry of the charge density, the electrostatic potential is only a function of the radial distance r from the centre of the sphere $\phi = \phi(\mathbf{r})$. Consequently, there must exist a spherical layer where the particles possess an amount of thermal kinetic energy sufficient to overcome the electrostatic attraction, this causes a non perfect shielding of the charge distribution and an electric field in the surrounding of the charge distribution which quickly decays [2].

When any charge distribution is present inside a plasma, the number densities of the charged species present are altered in the proximity of the distribution considered, leading to the formation of a secondary charge distribution which surrounds the first. The overall electrostatic potential is the result of the combined effect of the first and second charge distributions, this electrostatic potential is known with the name of Debye potential, in honour of Peter Debye who derived, together with Erich Hückel, an expression for the potential carrying his name in the Debye-Hückel theory of electrolytic solutions [1]. The expression of the Debye potential is the following:

$$\phi(r) = \frac{Q}{4\pi\epsilon_0 r} \exp\left(-\frac{\sqrt{2}r}{\lambda_D}\right)$$

where Q is the total charge of the spherical charge distribution. According to the Debye shielding, inside a plasma the charges arrange themselves in a spatial disposition that shields the effects of any electric perturbation. The Debye length characterises this phenomenon through an exponential term. This distance gives a measure about the range of action of the electrostatic potential resulting from any charge and, more generally, the distance over which a charged particle inside a plasma can electrically interact with other charges. Due to the exponential term, after a distance of a few Debye lengths, the potential produced by a charge distribution goes to zero.

The Debye sphere can now be introduced. The radial nature of the Debye potential, generated by a certain charge, allows the definition of a volume with a radius equal to λ_D and centred on that charge. This concept is useful to understand the Debye shielding, in fact, a charge interacts collectively only with the charges located inside its own Debye sphere; moreover, an electrostatic field whose origin lies outside of a Debye sphere does not contribute significantly to the electric field at the centre [1]. In other words, the Debye sphere can be seen as the locus of space where a certain charge distribution can exert its electrostatic influence.

The two phenomena described so far, quasi neutrality and Debye shielding, are the basis for understanding the behaviour of a plasma; both are described by the Debye length, whose physical meaning can be interpreted differently according to what is being described, as explained in the two previous paragraphs.

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_b T}{n_e e^2}}$$

where n_e is the electron number density, it is noteworthy that the temperature T in this expression is the electron temperature. This attribution is given by the significantly higher electron mobility; the shielding process, which relies on the rearrangement of charged particles to screen a potential, is dominated by the more responsive electron population.

2.2.4 Plasma spatial requirements

Any ionised gas must present the phenomena of macroscopic neutrality and Debye shielding to be classified as a plasma, but not every medium containing charged particles is affected by them. It is possible to define some quantitative characteristics that indicate whether the conditions for the existence of a plasma are met. The conditions for measuring the spatial extension of the plasma are correlated with the Debye length because, as previously shown, this parameter characterises both macroscopic neutrality and Debye shielding.

The first requirement is consequence of the charges recombination occurring during the Debye shielding. A charge distribution is formed as a result of the recombination around an initial space charge, thus the shielding process requires an amount of space, and the Debye length is a measure of the amount required. Consequently, the medium should be large enough to host the shielding phenomenon. Considering the characteristic dimension of the plasma L , the first requirement an ionised gas must show to be defined as a plasma can be written as:

$$L \gg \lambda_D$$

If this condition holds, then also the macroscopic neutrality of the plasma is verified. Since the deviation from charge neutrality, spontaneously formed, can reach at most a size equal to the Debye length, and the space charges introduced in the bulk of the plasma are shielded after a distance equal to λ_D , it is simple to understand how an ionised gas can be considered macroscopically neutral when its dimensions are much larger than the Debye length.

The second requirement is again linked to the Debye shielding. In fact, the shielding is a collective phenomenon which requires a large number of particles to occur successfully. Therefore, inside a Debye sphere, the volume where the shielding takes place, there must be a sufficiently large number of electrons; this requirement can be expressed with the following mathematical expression:

$$n_e \lambda_D^3 \gg 1$$

2.2.5 Electron plasma oscillation and temporal requirement

The starting point for the description of the Debye shielding was a spherical charge distribution introduced inside a plasma under equilibrium condition. The charge distribution acts as a disturbance of the plasma charge neutrality, which leads to the radial mass displacement of electrons towards or outward to the introduced charge distribution, depending on its polarity. In paragraph 1.2.3 the focus was on the shielding effect produced

by the displaced charges, another important effect stems from the displacement of charges and their collective motion, it is the electron plasma oscillation.

To describe the electron plasma oscillation the ions are considered as a uniform background, not able to follow the electrons due to the large mass difference. Instead, the electrons are supposed to remain rigidly with respect to each other while being able to move freely through the ions. If the charge distribution is negative and spherical, then in order to restore charge neutrality all electrons will be displaced radially outward. They move until an equilibrium position is reached, while travelling the negative particles acquire kinetic energy and because of inertia, the electrons overshoot their equilibrium position. The kinetic energy is converted into electrostatic potential energy and a new charge imbalance arises, between the heavy ions and the moving electrons. Consequently, an electric field trying to restore the charge neutrality is established. The combined action of the restoring electric field and electrons inertia produces an oscillation of the electrons around their equilibrium position.

The electrons oscillate in the direction of the electric field, thus creating a longitudinal wave. Furthermore, the electron plasma oscillation is a stationary and electrostatic wave, in fact, it can be demonstrated that the oscillation produces no magnetic field. The frequency of the electron plasma oscillation is called plasma frequency and is given by:

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m_e \epsilon_0}}$$

Plasma frequency is the natural frequency of oscillation of the free electrons when they are disturbed from their position of equilibrium, it depends only on the electron density and it is characterised by high values; only in highly rarefied plasma, like interstellar gas, the plasma frequency is below the megahertz.

The assumption of electrons moving freely between ions does not hold in a real case scenario, where electron-ion or electron-neutral collisions occur. In particular, collisions between electrons and neutral particles act as a dissipative effect, and they randomise the electron's momentum; as a result, the phase relationship between the single electron and the wave is disrupted, and the motion of the electrons is not collective neither governed by electromagnetic forces any more. The rate at which collisions between neutrals and electrons occur is measured by the electron-neutral collision frequency ν_{en} ; if the latter is higher than the plasma frequency, the dissipative effect prevails over the collective motion and the electrons will be forced by collisions to be in equilibrium with the neutral particles. By defining $\nu_{pe} = \frac{\omega_{pe}}{2\pi}$, the third requirement that a plasma must satisfy can be written as:

$$\nu_{pe} > \nu_{en}$$

A more intuitive interpretation can be obtained if the average time between two consecutive collisions is considered $\tau = \frac{1}{\nu_{en}}$. In this case, when the period of the plasma wave is longer than τ , then the average electron does not have time to complete a whole plasma wave oscillation before it collides with a neutral, disrupting its motion. Thus, a plasma requires the average time between two consecutive electron-neutral collisions to be large compared to the period of the plasma wave. This concept, equivalent to the previous requirement, is expressed by:

$$\omega\tau > 1$$

where ω is the angular frequency of the typical plasma oscillation [1], [2], [3].

2.3 Waves propagation

In the second part of this chapter the propagation of waves in cold plasma is investigated. The cold plasma model is an approximation introduced to simplify the system of transport equations of a plasma. The main implications of this approximation are to neglect the effect of the thermal motion of the particles (thus plasma temperature is equal to zero $T_i = T_e = 0$) and to neglect the pressure gradient force in the momentum equation. By using this model, it is possible to obtain satisfactory results only if the thermal velocity of the particles is small compared to the wave phase velocity. To put it simple, when the motion imposed by the wave on the electron is much faster than the thermal motion, the latter is not important.

Plasma, because of its peculiar electromagnetic nature, is capable of supporting a varied array of wave phenomena. Thus, it is necessary to narrow down the theory of interest for this thesis. The first step has been the adoption of the cold plasma model, a wave propagating in a cold homogeneous plasma immersed in a magnetic field is the subject of study of the magneto-ionic theory. The magneto-ionic medium is assumed to be an infinitely extended cold plasma (without boundary conditions), with a uniform and constant magnetic field applied to it. The waves analysed are plane waves, to ensure mathematical simplicity and generality. When propagating through a plasma, the wave produces a disturbance to the equilibrium state of its parameters. It is assumed that the variations of the plasma parameters are small, with respect to their equilibrium values, this allows to use the linear perturbation theory. Obviously, the wave amplitude has to be small, according to the small disturbance induced in the plasma. The last wave characteristic discussed is the frequency. The classic approach to magneto-ionic theory is to neglect the ions motion, a condition that requires to assume a wave frequency much higher than the ion cyclotron frequency; therefore, considering only the electron motion, the plasma appears as a cold electron gas.

The study of the properties of small-amplitude high-frequency waves propagating in a cold plasma is particularly useful for plasma diagnostic. In fact, by studying how the wave behaves as it travels through the plasma, it is possible to extract information on the parameters of the plasma, such as electron temperature and electron density. It is also possible to investigate oscillation around the equilibrium value of the plasma parameters. The last aspect that needs to be clarified regards the method to study the wave propagation in the context of magneto-ionic theory. There are two possible approaches: the first employs the conductivity or the dielectric constant of the plasma medium together with the Maxwell's equation to derive the wave equation; the second method uses the Maxwell's equation along with the fluid equation of motion to derive the dispersion relation. The dispersion relation is a function that relates the angular frequency ω of a wave to the wave number k , in this way it is possible to model the wave behaviour without explicitly deriving the wave equation. Before starting with the derivation of the dispersion relation an introduction to basic wave concepts is provided, notions such as the representation of a wave, the wave number, phase and group velocity are all fundamental to grasp properly the wave propagation phenomena [1].

2.3.1 Waves basic principles

Electromagnetic waves are the result of the combined effect of the electric and magnetic field. In the free space, where $\rho = 0$, $\mathbf{J} = 0$, they satisfy the following pair of vectorial equations obtained by manipulating Maxwell's equations

$$\begin{aligned}\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} &= 0 \\ \nabla^2 \mathbf{B} - \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} &= 0\end{aligned}$$

where c is the speed of light in vacuum and also the speed of propagation of such waves. The solutions of interest are those of the transverse plane wave. For a transverse plane wave, the electric field \mathbf{E} and magnetic field \mathbf{B} are orthogonal to each other and both are transverse to the propagation direction defined by the unit vector $\hat{\mathbf{k}}$. The surfaces of constant phase, known as wavefronts, are planes perpendicular to $\hat{\mathbf{k}}$. The field vectors \mathbf{E} and \mathbf{B} are constant along the wave front but vary with time and with the distance r , which is the distance from the origin to the wave front and is perpendicular to it. The distance r should not be confused with \mathbf{r} , the vector identifying a generic point on the wave front plane.

$$\hat{\mathbf{k}} \cdot \mathbf{r} = r$$

If a wave can be described by a sinusoidal function, it is called a harmonic wave and its mathematical formulation is:

$$w(t, r) = A \cos(kr - \omega t)$$

The term ω represents the angular frequency of the oscillation, k is the wave number (or propagation constant) and A is the amplitude. From these quantities the wavelength λ and the period T of the wave derive:

$$\lambda = \frac{2\pi}{k} \quad T = \frac{2\pi}{\omega}$$

Multiplying the unit vector $\hat{\mathbf{k}}$ with the wave number (the former represents direction, the latter magnitude), the propagation vector is obtained $\hat{\mathbf{k}} \cdot \mathbf{r} = \frac{\mathbf{k}}{k} \cdot \mathbf{r} = r$. At this point, the definition of a harmonic plane wave, with an amplitude equal to A , propagating towards a direction given by $\hat{\mathbf{k}}$ is made possible:

$$w(t, \mathbf{r}) = A \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$

The scalar function $w(t, \mathbf{r})$ could represent any of the three component of the vectorial fields \mathbf{E} and \mathbf{B} . Representing the solutions of \mathbf{E} and \mathbf{B} with the exponential notation is particularly practical when a linear differential equation is calculated. The nabla operator and the time derivative can be written as $\nabla = i\mathbf{k}$ and $\frac{\partial}{\partial t} = -i\omega$, consequently, linear differential equations become algebraic equations. The physical and measurable information of the vector fields \mathbf{E} and \mathbf{B} is given by the real parts of their complex representations [1], [2].

$$w(t, \mathbf{r}) = A \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

Phase velocity, group velocity & dispersive medium

Keeping in mind the argument of the sinusoidal function, a point of constant phase is defined with the following condition:

$$\mathbf{k} \cdot \mathbf{r} - \omega t = kr - \omega t = \text{constant}$$

The velocity of propagation of a point of constant phase is called phase velocity and is obtained by deriving with respect to time the previous equation:

$$\frac{d}{dt}(kr - \omega t) = 0 \quad \Rightarrow \quad v_{ph} \equiv \frac{dr}{dt} = \frac{\omega}{k}$$

Disturbances in the real case are not simple harmonic waves with only one value of k and ω . The Fourier analysis explains that any periodic signal is the result of the superposition of harmonic waves with different values of k and ω ; from this concept stems the notion of wave packet. The wave packet is the result of the sum of virtually infinite set of sinusoidal waves with different wave numbers, the phases and amplitude of the different sinusoidal waves produce a constructive interference in a limited region of space while the interference is destructive everywhere else. As a result, the wave packet is characterised by a small spread of wave number centred around a central value. Also the frequencies of the wave packet will show a similar spread, because for any wave it exists a relationship between k and ω . This relationship is the dispersion relation mentioned in the introduction and it depends on the medium the wave travel through. When a carrier wave is modulated a wave packet is created.

The envelope of the wave packet is what carries the information, and it travels at a speed called group velocity and is defined as:

$$v_g = \frac{d\omega}{dk}$$

Using the definition of phase velocity $v_{ph} = \omega/k$ it is possible to express the group velocity in terms of the phase velocity.

$$v_g = \left. \frac{\partial\omega}{\partial k} \right|_{k_0} = \left. \frac{\partial(v_{ph}k)}{\partial k} \right|_{k_0} = v_{ph_0} + k_0 \left. \frac{\partial v_{ph}}{\partial k} \right|_{k_0}$$

The group velocity is always lower than the speed of light in vacuum, contrary to the phase velocity whose value can exceed c . The last detail is not in contradiction with the theory of relativity because the phase velocity is the velocity of propagation of a specific point on a wave with constant amplitude everywhere in space and time, thus such a wave is identical in every point of space and time not carrying any type of information.

Whether the phase velocity depends on the frequency of the propagating wave is particularly important. In empty space, for example, the dispersion relation is $\omega = ck$; it can be generalized to $\omega(k) \propto k$. Therefore, applying the phase velocity formula is straightforward $v_{ph} = \frac{\omega}{k} = c$, a constant phase velocity has been obtained, unrelated to the wave frequency. According to the formulation of the group velocity in terms of the phase velocity, the two quantities are equal ($v_{ph} = v_g$). This type of medium is called a non dispersive medium, a wave packet propagating through it does not experience dispersion. In fact, the different frequencies composing the wave packet travel at the same phase velocity, which is equal to the group velocity.

Conversely, a dispersive medium is a material in which the phase velocity of a wave propagating through it depends on the frequency. Plasma belongs to this category, where the

different frequencies of a wave packet have different velocities, depending on their value. A wave packet travelling in a dispersive medium experiences the dispersion of its frequency components. The group velocity differs from the phase velocity, and whether phase velocity increases or decreases with increasing wave number ($\frac{\partial v_{ph}}{\partial k} > 0$ or $\frac{\partial v_{ph}}{\partial k} < 0$) the medium is called, respectively, anomalously dispersive or normally dispersive [1], [2].

2.3.2 The dispersion relation in isotropic plasma

In light of the hypothesis discussed in the introductory paragraph of section 1.3, the dispersion relation of a small-amplitude plane wave propagating inside a cold plasma is derived by working on the Maxwell's equations and the Langevin equation of motion for the electrons:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \\ m \frac{D\mathbf{u}}{Dt} &= q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - m\nu\mathbf{u}\end{aligned}$$

The hypothesis of a very high wave frequency allows to neglect the ions motion, it translates in $\mathbf{u}_i = 0$, therefore, the current density can be expressed as $\mathbf{J} = -en\mathbf{u}$. The quantities $n(\mathbf{r}, t)$ and $\mathbf{u}(\mathbf{r}, t)$ are two hydrodynamic variables involved and are the electron number density and the average electron velocity, respectively. In the Langevin equation, the leftmost term accounts for the rate of change of the average electron momentum caused by electron-neutral collisions $\mathbf{F}_{coll} = -m\nu\mathbf{u}$, the ν is the electron-neutral collision frequency and m is the electron mass.

The first step is to rewrite the magnetic induction field and the electron number density as a sum of two components. In this way, it is possible to decouple the undisturbed and uniform (background) part of the parameter from the part representing the perturbation introduced by the wave.

$$\begin{aligned}\mathbf{B}(\mathbf{r}, t) &= \mathbf{B}_0 + \mathbf{B}_1(\mathbf{r}, t) \\ n(\mathbf{r}, t) &= n_0 + r_1(\mathbf{r}, t)\end{aligned}$$

The Langevin equation contains two non-linear terms, a linearisation process is performed to tackle the difficulty represented by $(\mathbf{u} \cdot \nabla)\mathbf{u}$ and $\mathbf{u} \times \mathbf{B}_1$, the terms accounting for the convective acceleration and the magnetic component of the Lorentz force. The convective term is neglected due to the cold plasma approximation, while the two-component form

adopted for the magnetic induction simplifies the Lorentz force as

$$q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}_0 + \mathbf{u} \times \mathbf{B}_1)$$

The term $\mathbf{u} \times \mathbf{B}_1$ can be neglected if the phase velocity of the wave is much higher than the average electron velocity.

$$|\mathbf{u}| \ll \left| \frac{\omega}{k} \right|$$

Generally, this condition is valid for electromagnetic waves, unless the propagating wave approaches the condition of resonance, in which case the fraction ω/k is small. Finally, the solutions of plane wave allows to substitute the nabla and time derivative operator with $i\mathbf{k}$ and $-i\omega$.

By performing all the aforementioned simplifications, the equation of motion, the Faraday-Maxwell and the Ampere-Maxwell equations become:

$$-i\omega m\mathbf{u} = -e(\mathbf{E} + \mathbf{u} \times \mathbf{B}_0) - m\nu\mathbf{u} \quad (2.1)$$

$$\mathbf{k} \times \mathbf{E} = \omega\mathbf{B}_1 \quad (2.2)$$

$$i\mathbf{k} \times \mathbf{B}_1 = \mu_0(-en_0\mathbf{u} - i\omega\epsilon_0\mathbf{E}) \quad (2.3)$$

The current density vector has been written in terms of the average electron velocity. The case study analysed in this thesis involves a non-magnetised plasma, condition that translates to a background magnetic induction equal to zero $\mathbf{B}_0 = 0$. This condition simplifies the analysis of the wave propagation mathematically and conceptually, because the vector product disappears from the equation of motion. The presence of a magnetic field introduces a space anisotropy in the plasma medium. The calculation of the dispersion relation for high frequency wave propagating inside a cold magnetised plasma leads to the Appleton-Hartree equation. It is beyond the scope of this thesis.

In the absence of an externally applied magnetic induction field, the average electron velocity is parallel to the electrostatic field. This is expressed by rewriting equation (2.1) as

$$\mathbf{u} = -\frac{e}{m(\nu - i\omega)}\mathbf{E} \quad (2.4)$$

Then, by expressing the field \mathbf{B}_1 as

$$\mathbf{B}_1 = \frac{\mathbf{k} \times \mathbf{E}}{\omega} \quad (2.5)$$

and by substituting equation (2.5) and (2.4) inside equation (2.3) it is possible to obtain:

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = - \left[\frac{i\omega\mu_0 e^2 n_0}{m(\nu - i\omega)} + \frac{\omega^2}{c^2} \right] \mathbf{E} \quad (2.6)$$

The last step is to formulate the electric field as the sum of a component parallel to \mathbf{k} and a component transversal to \mathbf{k} (respectively \mathbf{E}_ℓ and \mathbf{E}_t), then by writing equation 2.6 in terms of these two components it is possible to obtain the dispersion relation for a longitudinal and transverse wave propagation modes.

$$\mathbf{E} = \mathbf{E}_\ell + \mathbf{E}_t$$

The vector product $\mathbf{k} \times (\mathbf{k} \times (\mathbf{E}_\ell + \mathbf{E}_t))$ gives

$$\mathbf{k} \times \mathbf{E}_\ell = 0$$

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}_t) = -k^2 \mathbf{E}_t$$

By collecting the previous results, it is possible to write equation 2.6 in a more compact form. The definition of the speed of light in vacuum $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, the plasma frequency $\omega_{pe} = \sqrt{\frac{n_0 e^2}{m\epsilon_0}}$ and the formulation of the electric field allow to write

$$-k^2 \mathbf{E}_t = - \left[- \frac{\omega_{pe}^2}{c^2(1 + i\nu/\omega)} + \frac{\omega^2}{c^2} \right] (\mathbf{E}_\ell + \mathbf{E}_t) \quad (2.7)$$

Equation 2.7 is divided in the longitudinal and transverse components, then the meaningful solutions ($\mathbf{E} \neq 0$) are considered. The dispersion relations for the longitudinal and transverse wave propagation modes are:

$$\textit{longitudinal} \quad \omega^2 \left(1 + i \frac{\nu}{\omega} \right) - \omega_{pe}^2 = 0 \quad (2.8)$$

$$\textit{transverse} \quad \left(\omega^2 - k^2 c^2 \right) \left(1 + \frac{i\nu}{\omega} \right) - \omega_{pe}^2 = 0 \quad (2.9)$$

This pair of equations regulates the propagation of the transverse or longitudinal wave mode. All the information regarding a wave mode propagation can be obtained from the analysis of its appropriate dispersion relation [1].

2.3.3 Propagation inside a collisionless plasma

The electron-neutral collision frequency ν accounts for the effect of collisions, the case of a collisionless plasma can be studied by setting to zero the term ν . A propagating

wave characterised by an angular frequency much higher than the collision frequency ($\frac{\nu}{\omega} \ll 1$) is suitably described by a collisionless dispersion relation. The study of wave propagation in a collisionless plasma simplifies the dispersion relation, representing an ideal and more intuitive approach. The longitudinal wave mode is particularly simple, equation 2.8 becomes:

$$\omega^2 = \omega_{pe}^2$$

In fact, The only frequency allowed for longitudinal oscillations is the plasma frequency. The average electron velocity is obtained by using 2.4 and imposing $\nu = 0$ and $\mathbf{E} = \mathbf{E}_\ell$, the result is:

$$\mathbf{u} = -\frac{ie}{\omega m} \mathbf{E}_\ell$$

This equation shows that the motion of the electrons is parallel to the electric field \mathbf{E}_ℓ and the propagation direction given by \mathbf{k} . Considering that the rotor of \mathbf{E}_ℓ is equal to zero and considering equation 2.2, the magnetic field \mathbf{B}_1 related to the longitudinal wave mode is zero, and thus the resulting wave is electrostatic. A longitudinal electrostatic wave oscillating at the plasma frequency corresponds to the electron plasma oscillation previously discussed in section 1.2.5.

The dispersion relation for the transverse wave mode in a collisionless plasma differs slightly from that of transverse electromagnetic waves in a vacuum. The only difference lies in the natural plasma frequency ω_{pe}^2 , which is missing in the vacuum case. Equation 2.9 adapted to the collisionless case could be expressed in two formulations, one with omega as a function of k or vice-versa.

$$\omega^2 = \omega_{pe}^2 + k^2 c^2 \tag{2.10}$$

$$k^2 c^2 = \omega^2 - \omega_{pe}^2 \tag{2.11}$$

Expressing k as a function of ω is useful for understanding the cutoff phenomenon. The dispersion relation binds omega to the wave number; if the frequency of a wave is above the plasma frequency, $\omega > \omega_{pe}$, the value of k^2 is positive and a real number. The transverse wave that satisfies the condition $\omega > \omega_{pe}$ propagates through the plasma with a phase and a group velocity that are easily obtained by equation 2.10:

$$v_{ph} = \frac{\omega}{k} = \sqrt{\frac{\omega_{pe}^2}{k^2} + c^2} \quad (\omega > \omega_{pe}) \tag{2.12}$$

$$v_g = \frac{\partial \omega}{\partial k} = \frac{c^2}{v_{ph}} \quad (\omega > \omega_{pe}) \tag{2.13}$$

Regarding phase velocity, two important aspects deserve discussion. First, the value of v_{ph} is always higher than the speed of light, as can be seen from 2.12. Second, as the frequency of the propagating wave tends to infinity, the phase velocity approaches the speed of light; this is obtained by taking the limit $\omega \rightarrow \infty$ in equation 2.10. Physically, this limit corresponds to a regime where the wave oscillates so rapidly that even the small inertia of the free electrons prevents them from responding to the field. Their inertia renders them practically immobile, and the wave propagates as in free space. Lastly, the phase and group velocities can also be obtained from the graph of the dispersion relation. If a point P is considered on the curve of $\omega = \omega(k)$, the phase velocity is given by the slope of the line passing in the origin and in point P; the group velocity, due to its definition $v_g = \frac{\partial \omega}{\partial k}$, is provided by the slope of the line tangent to the curve in the point P.

On the other hand, when the frequency of the propagating wave is lower than the plasma frequency, the value acquired by k^2 is negative and the wave number becomes imaginary. The cutoff condition occurs at $\omega = \omega_{pe}$, this frequency is also called a reflection point. Given that the plasma frequency depends solely on the electron density, a wave propagating through a plasma with a non-uniform density profile will encounter a spatially varying cutoff frequency. More precisely, when a wave with a certain frequency ω propagates through a plasma where the density gradually increases, thus also ω_{pe} , the wave number will decrease as described by equation 2.11. A decreasing wave number means an elongation of the wavelength ($\lambda = 2\pi/k$), when the density is such that the cutoff condition is met k becomes zero and the wavelength is equal to infinite. Thanks to the plasma frequency equation it is possible to identify the critical density corresponding to the cutoff condition:

$$n_c = \frac{\epsilon_0 m \omega^2}{e^2}$$

For any ω lower than the plasma frequency the wave number is imaginary and the wave cannot propagate, these waves are called evanescent and do not carry any time averaged power [1], [2].

Once the cutoff has been clarified, the representation of the wave number as $k = \beta + i\alpha$ acquires a precise meaning. For $\omega > \omega_{pe}$ the wave number is real $k = \beta, \alpha = 0$, the wave can propagate without damping and its phase velocity exceeds that of light in vacuum. Instead, for $\omega < \omega_{pe}$ the wave number becomes purely imaginary $k = i\alpha, \beta = 0$, by substituting this formulation of k inside the transverse wave exponential representation, the reason why evanescent waves do not propagate emerges:

$$E_t \propto \exp(ikr - i\omega t) = \exp(-\alpha r) \exp(-i\omega t)$$

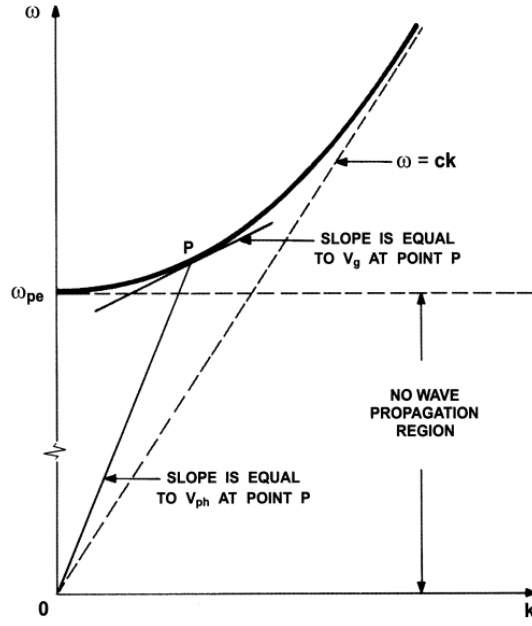


Figure 2.2: *Dispersion relation for transverse wave mode propagating in an isotropic cold electron plasma [1].*

The wave undergoes exponential damping with increasing distance, the decay is ruled by $e^{-\alpha x}$. Its penetration depth into the plasma is quantified by the parameter called skin depth (or also attenuation length), and it is defined as the distance over which the wave field amplitude decreases by a factor of $1/e$. The skin depth is provided by the following formula [4]:

$$\delta = \frac{1}{\text{Im}(k)} = \frac{c}{\sqrt{\omega_{pe}^2 - \omega^2}}$$

Power flow

At the base of any electromagnetic wave there are the vector fields \mathbf{E} and \mathbf{H} . As Poynting's theorem states, these two vector fields carry a flow of energy equal to:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

Due to the harmonic waves considered in this thesis, the Poynting vector can be expressed as the vector product of the complex vector fields \mathbf{E} and \mathbf{H} . As mentioned above, the physical information of the wave is stored in the real part of the complex representation; thus, only the real part of \mathbf{E} and \mathbf{H} is used to calculate the Poynting vector. Furthermore, the Poynting vector of harmonic waves is averaged over one period of oscillation.

$$\langle \mathbf{S} \rangle = \langle \text{Re}(\mathbf{E}) \times \text{Re}(\mathbf{H}) \rangle \quad (2.14)$$

It is possible to demonstrate that the equation 2.14 is equal to one half of the real part of the vector product of the complex vector fields \mathbf{E} and \mathbf{H} .

$$\langle \mathbf{S} \rangle = \frac{1}{2} \text{Re}\{\mathbf{E} \times \mathbf{H}^*\} \quad (2.15)$$

According to the previous expression, the time averaged power flow associated with the transverse wave can be calculated without performing the time average. In the presence of a non magnetised plasma ($\mathbf{B}_0 = 0$) we consider the oscillating magnetic field $\mathbf{B}_1 = \mu_0 \mathbf{H}_1$, and by using equation 2.5, \mathbf{H}_1 is expressed in terms of the electric field:

$$\begin{aligned} \mathbf{H}_1 &= \frac{1}{\mu_0 \omega} (\mathbf{k} \times \mathbf{E}) \\ \langle \mathbf{S} \rangle &= \frac{1}{2} \text{Re}\{\mathbf{E} \times \mathbf{H}_1^*\} = \frac{1}{2\mu_0 \omega} \text{Re}\{\mathbf{E} \times (\mathbf{k}^* \times \mathbf{E}^*)\} = \\ &= \frac{1}{2\mu_0 \omega} \text{Re}\{k^* E(\mathbf{r}, t) E^*(\mathbf{r}, t)\} \hat{\mathbf{n}} \end{aligned}$$

The direction of $\mathbf{E} \times \mathbf{H}_1^*$ aligns with the unit vector $\hat{\mathbf{n}}$. The final formulation is derived employing the exponential form of the electric field:

$$\langle \mathbf{S} \rangle = \left(\frac{E^2}{2\mu_0 \omega} \right) \text{Re}\{k^* \exp[i(k - k^*)r]\} \hat{\mathbf{n}} \quad (2.16)$$

As clarified above, the wave number k is real or imaginary depending on the value of the angular frequency of the wave with respect to the plasma frequency. For $\omega > \omega_{pe}$ the wave number is real $k = \beta$, consequently the argument of the exponential term in equation 2.16 becomes zero. Thus, the expression of the averaged Poynting vector becomes:

$$\langle \mathbf{S} \rangle = \left(\frac{E^2}{2\mu_0 \omega} \right) \text{Re}\{k\} \hat{\mathbf{n}} = \frac{\epsilon_0 k E^2}{2\mu_0 \epsilon_0 \omega} \hat{\mathbf{n}} = \frac{1}{2} \epsilon_0 E^2 v_g \hat{\mathbf{n}} \quad (\omega > \omega_{pe})$$

To obtain the rightmost formulations, the relation $c^2 = \frac{1}{\mu_0 \epsilon_0}$ and $v_g = c^2 k / \omega$ have been employed (the latter derive from equation 2.13).

Conversely, for $\omega < \omega_{pe}$ the wave number becomes imaginary ($k = i\alpha$), leading to a purely imaginary argument of the real part operator, resulting in a time-averaged Poynting vector of zero. This mathematically confirms that an evanescent wave transports no net power [1].

$$\langle \mathbf{S} \rangle = 0 \quad (\omega < \omega_{pe})$$

2.3.4 Propagation in a collisional plasma

The collisionless case can be regarded as the ideal case, in which if the wave is able to propagate ($\omega > \omega_{pe}$) then it does not experience any damping. This is reflected by the wave number being purely real or purely imaginary, depending on whether the wave frequency is above or below the cutoff condition, respectively. When the effect of collision is considered, all of the above does not hold any longer.

An electron accelerated by the wave can randomly collide with an ion or neutral particle, interrupting the ordered wave motion. Collisions cause the wave energy to be gradually absorbed by the plasma particles. This results in an exponential decay of the wave amplitude with the propagation distance, the dissipative process just described is also known as damping of the wave. In the plasma layer where the frequency of the incoming wave approaches the plasma frequency ω_{pe} the phenomenon of resonant absorption happens. In a collisional plasma this phenomenon implies a transfer of energy from the electromagnetic wave to the plasma particles, whose kinetic energy rises [5].

Equations 2.8 and 2.9 account for the effect of collision, and in the case of a longitudinal wave, the equation is as follows:

$$\omega^2 + i\nu\omega - \omega_{pe} = 0$$

The solution of a second order equation is easily found.

$$\omega_{1,2} = \frac{1}{2} \left[-i\nu \pm \sqrt{(4\omega_{pe} - \nu^2)} \right]$$

The imaginary part of this solution is $-\nu/2$, a negative quantity. The longitudinal oscillation is characterised by a harmonic variation in time proportional to $\exp(-i\omega t)$. By substituting the solution inside the exponential, the longitudinal waves propagation can be clarified.

$$\exp(-i\omega t) = \exp(-\frac{\nu}{2}t) \exp[\pm \frac{i}{2}(4\omega_{pe} - \nu^2)^{\frac{1}{2}}t]$$

Longitudinal oscillations are always damped according to the exponential term $\exp(-\frac{\nu}{2}t)$. The analysis of transverse waves is similar, with the difference of a more complex dispersion relation. Equation 2.9 is rewritten to express k as a function of ω . Multiplying the resulting expression by the complex conjugate of $(1 + i\nu/\omega)$ yields the following result:

$$k^2 c^2 = \omega^2 - \frac{\omega_{pe}^2}{1 + (\nu/\omega)^2} + i \frac{\omega_{pe}^2 (\nu/\omega)}{1 + (\nu/\omega)^2} \quad (2.17)$$

It is easy to verify that when the condition of $\omega \gg \nu$ is satisfied, equations 2.17 become equal to the collisionless form. Equation 2.17 highlights the real and imaginary parts of

the wave number squared. The analysis of the imaginary part is particularly useful to study the behaviour of the propagating wave; to accomplish this task the wave number expressions are shown:

$$\begin{aligned}
 k &= \beta + i\alpha \\
 k^2 &= A + iB \\
 A = \text{Re}(k^2) &= \beta^2 - \alpha^2 \\
 B = \text{Im}(k^2) &= 2\beta\alpha
 \end{aligned}$$

Similarly to the examination of longitudinal propagation, the transverse wave exponential and sinusoidal representations are analysed.

$$\exp(ikr - i\omega t) = \exp(-\alpha r) \exp[i(\beta r - \omega t)] = \exp(-\alpha r) \cos(\beta r - \omega t) \quad (2.18)$$

Being β the coefficient of the spatial variable, its sign determines the direction of propagation of the wave. Namely, for $\beta > 0$ the propagation occurs in a positive spatial direction, while for $\beta < 0$ the wave propagates toward the negative spatial direction. The damping of the wave is ruled by the sign of the attenuation factor α that appears in the real exponential. From equation 2.18, it is evident how a $\alpha > 0$ implies an attenuation of the wave as the distance r increases, while a $\alpha < 0$ implies a growing wave when the distance r increases. The pictures 2.3 and 2.4 visualise the trend of the propagating wave as a function of the distance.

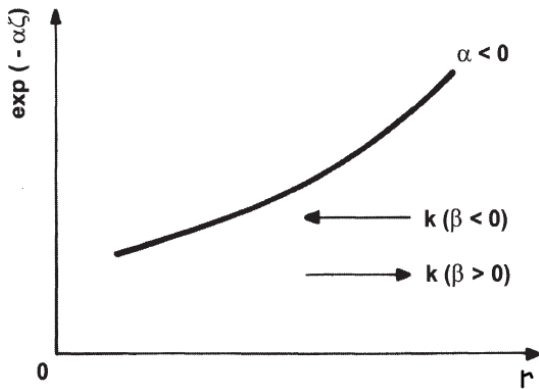


Figure 2.3: *Trend of the wave amplitude as a function of distance and with a negative α [1]*

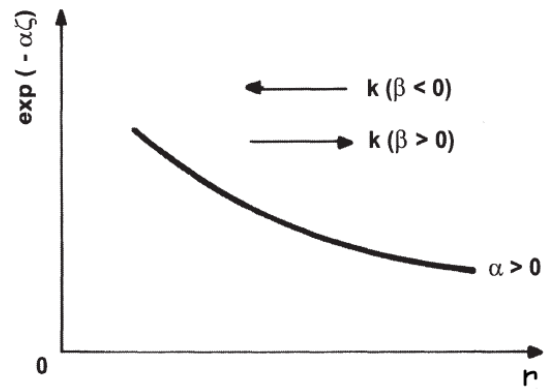


Figure 2.4: *Trend of the wave amplitude as a function of distance and with a positive α [1]*

From the combined analysis of the signs of α and β the propagation behaviour of the wave can be deduced; more specifically, its evolution with increasing distance either in the positive or negative direction. In the case α and β share the same sign then the

wave is damped as it propagates further from the origin. On the contrary, when they show opposite sign the wave propagates in one direction of the distance axis (for instance positive) and it is damped in the other direction (negative), implying a wave growing exponentially with distance.

The expression of B in Equation 2.17 is composed of positive quantities only, and a positive B requires α and β to share the same sign. It follows that the only regime allowed for a wave propagating in a plasma is to experience attenuation while travelling [1].

$$B = 2\alpha\beta = \frac{\omega_{pe}^2(\nu/\omega)}{1 + (\nu/\omega)^2}$$

So far, the dispersion relation for a collisional plasma has been investigated in the form of $k = k(\omega)$ only. To study the behaviour of the frequency as a function of the wave number, $\omega = \omega(k)$, another approach to the analysis should be adopted. Equation 2.17 serves as the starting point, after an appropriate algebraic rearrangement and a multiplication by the complex conjugate of $(\omega - i\nu)$ it can be cast into the desired form as shown in the following:

$$\begin{aligned} -\omega^2 + k^2c^2 &= -\frac{\omega_{pe}^2}{1 + (\nu/\omega)^2} + i\frac{\omega_{pe}^2(\nu/\omega)}{1 + (\nu/\omega)^2} \\ -\omega^2 + k^2c^2 &= \omega_{pe}^2 \frac{-\omega + i\nu}{\omega^2 + \nu^2} \omega \\ -\omega^2 + k^2c^2 &= -\omega_{pe}^2 \frac{\omega - i\nu}{\omega^2 + \nu^2} \omega \frac{\omega + i\nu}{\omega + i\nu} \\ -\omega^2 + k^2c^2 &= -\omega_{pe}^2 \frac{\omega}{\omega + i\nu} \end{aligned}$$

The latter equation is in the form suited for the study of the frequency spectrum as a function of the wave number.

$$\omega^2 - \omega_{pe}^2 \frac{\omega}{\omega + i\nu} - k^2c^2 = 0 \quad (2.19)$$

Equation 2.19 is a third degree equation and as such presents three distinct solutions; under the hypothesis of $\nu \ll \sqrt{k^2c^2 + \omega_{pe}^2}$ the spectrum of weakly damped transverse electromagnetic wave in a plasma is obtained for the dispersion relation 2.19. These solutions are:

$$\omega_{1,2} = \pm \sqrt{k^2c^2 + \omega_{pe}^2} - i\frac{\nu}{2} \frac{\omega_{pe}^2}{k^2c^2 + \omega_{pe}^2} \quad (2.20)$$

$$\omega_3 = i\nu \frac{k^2c^2}{k^2c^2 + \omega_{pe}^2} \quad (2.21)$$

The solutions $\omega_{1,2}$ are different from the solution ω_3 both in amplitude and in nature. Due to the assumed inequality $\nu \ll \sqrt{k^2 c^2 + \omega_{pe}^2}$, the absolute value of ω_3 differs substantially from that of the other two solutions. As a consequence, the spectra of frequencies described by ω_3 diverge from those described by $\omega_{1,2}$. Furthermore, solution 2.20 is related to weakly damped, high frequency, transverse electromagnetic wave as mentioned above; thus, a period wave phenomenon. On the contrary, solution 2.21 describes an aperiodic process which is not wave phenomenon. To conclude, in the study of propagation of electromagnetic waves inside a collisional plasma solution 2.21 is not a matter of interest [6].

Chapter 3

Measurements & diagnostic

The nature of plasma physics is particularly complex as introduced in the previous chapter. Aspects as collective behaviour, the wave-particle interaction with its electromagnetic implications and the chemistry of an ionised gas complicate the study of the evolution of a plasma; the measurement of its parameters, thus the experimental validation of theoretical model's prediction, is therefore particularly helpful to study and investigate plasma related phenomena. In general terms, diagnostic refers to the process of collecting data from a system for the purpose of characterising and understanding a problem that affects its behaviour. In the context of plasma physics, the diagnostic main focus is the collection of information to investigate about the state of the plasma. Thus, any technique aimed at measuring the parameters of an ionised gas can be regarded as a plasma diagnostic technique. The techniques used are diverse and exploit different phenomena to measure one or more parameters. Some important parameters to characterise a plasma are: electron temperature, electron density, ion temperature, electron energy distribution functions, electron-heavy collision frequency, pressure, electric and magnetic field distribution.

A fundamental distinction among the different diagnostic techniques regards the intrusiveness of the measurement; in fact, a measurement performed from the inside of the plasma is an intrusive, or in-situ, technique. This type of technique usually feature good spatial resolution but have the drawback of exposing the measurement instrument, usually a probe, to the harsh environment of the plasma. Clearly, an intrusive measurement is not suited for a hot plasma or a re-entry type of plasma, since the high temperature would compromise the integrity and accuracy of the instrument. The second type of measurement is non-intrusive, which means that the measurement is performed from outside the plasma using, for example, an electromagnetic wave or a laser beam. In this way, it is possible to measure the parameters of interest without exposing the instrument and also without perturbing the plasma with the measurement [7].

To correctly choose the diagnostic technique, the parameters of interest should be clearly

identified. Each technique exploits a certain physical phenomenon to perform the measurement, and every such phenomenon can provide information on a limited number of plasma parameters. Moreover, each technique is characterised by a particular level of accuracy and resolution when measuring a given parameter. Obviously, the parameters to be measured are not the only criterion to consider, since a technique may not be compatible with certain types of plasma. It follows that it is of primary importance to understand the typology of the plasma under investigation.

The study from which this thesis is derived requires a non intrusive technique to estimate the electron number density of a re-entry type of plasma. Based on the criteria outlined above as the type of plasma, the desired parameter, the instrumentation available at the dipartimento DEI laboratory of the university of Bologna and the technical experience of the research team dedicated to the study, the microwave absorption phenomenon was identified as the most suitable technique.

3.1 Phenomenology and methodology of the measurement

The technique employed to estimate the electron number density of the plasma is the microwave absorption. Generally speaking, microwaves represent a powerful tool to investigate the properties of a plasma. Microwave radiations are a non intrusive mean, causing little to none perturbation to the plasma studied if their intensity is not too great. The values of ω_{pe} associated to electron number densities in the order of $10^{16} \div 10^{19} \text{m}^{-3}$, typical values of the plasmas studied in this thesis, fall into the range of frequency of microwave radiations ($\sim 1 \div 300 \text{ GHz}$). This makes them particularly suitable to exploit the resonant absorption of the wave, a phenomenon described in the sub-section about the propagation in a collisional plasma. Furthermore, this technique is robust from the theoretical point of view, since it has no dependency on the electron energy distribution. Consequently, it does not require to assume anything about the thermodynamic equilibrium regime of the system.

Considering the direct relation between electron number density and plasma frequency, it is evident that n_e is a fundamental parameter for the propagation or absorption of microwaves inside a plasma, and thus it can be estimated by sensing whether the wave passes or is absorbed by the plasma. In the presence of a collisionless plasma, the wave is absorbed when the condition $\omega = \omega_{pe}$ is met. When the wave is not detected by the receiving antenna, the previous condition is verified and the number density can be derived using the formulation of the plasma frequency.

$$n_c = \frac{m_e \omega^2 \epsilon_0}{e^2} \quad (3.1)$$

The real case scenario of a collisional plasma adds a second parameter, the electron-heavy collision frequency ν . The effect of collisions between electrons and heavier particles leads to a dissipative effect which dampens the wave, and turns the absorption of the wave in a gradual transition, in contrast to the collision-less case of total reflection of the wave or propagation without attenuation. The wave propagating in the plasma is not completely absorbed when $\omega = \omega_{pe}$, but its intensity is gradually reduced. Measuring the power of the wave at the receiving antenna (and comparing it to the initial power) allows to quantify the reduction the wave experienced. This reduction is related to a parameter known as the attenuation coefficient (χ), which is used to estimate the plasma frequency and thus the electron number density.

Two new parameters must be introduced to proceed with the description of the study methodology. The first is the transmission factor T_f , defined as the ratio between the power of the microwave radiation at the moment when it is fired (P_1) and the power of the same microwave when received (P_2) [8].

$$T_f = \frac{P_2}{P_1} = \exp(-2\chi \frac{\omega}{c} \Delta x) \quad (3.2)$$

The variables appearing in the exponential of Equation 3.2 are: ω , the angular frequency of the microwave; c , the speed of light; Δx , the length of the path travelled by the wave within the plasma; and χ , the attenuation coefficient associated with wave propagation in the plasma. Regarding Δx , any portion of the path lying in air is neglected, as the wave experiences attenuation almost exclusively within the plasma; the contribution of air to the overall damping is negligible. Equation 3.2 states that the reduction experienced by the wave has an exponential trend proportional to χ and to the distance travelled inside the plasma.

The attenuation coefficient corresponds to the imaginary part of the complex refractive index $\bar{\mu} = \mu + i\chi$. The following two formulae define χ and link it to the real part of the wave number, respectively [8]:

$$\alpha = \chi \frac{\omega}{c} \quad (3.3)$$

$$\chi = \left\{ -\frac{1}{2} \left(1 - \frac{\omega_{pe}^2}{\omega^2 + \nu^2} \right) + \frac{1}{2} \left[\left(1 - \frac{\omega_{pe}^2}{\omega^2 - \nu^2} \right)^2 + \left(\frac{\omega_{pe}^2 \nu}{\omega^2 + \nu^2 \omega} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \quad (3.4)$$

Substituting equation 3.3 into equation 3.2 highlights that wave damping is also described by the real part of the wave number, in accordance with what has been described in the

section on wave propagation (see equation 2.18). In equation 3.4 the unknown variables are two, the plasma frequency and the electron-heavy collision frequency. At constant pressure, ν can be considered constant. Consequently, the attenuation coefficient depends solely on the number density, allowing this parameter to be estimated.

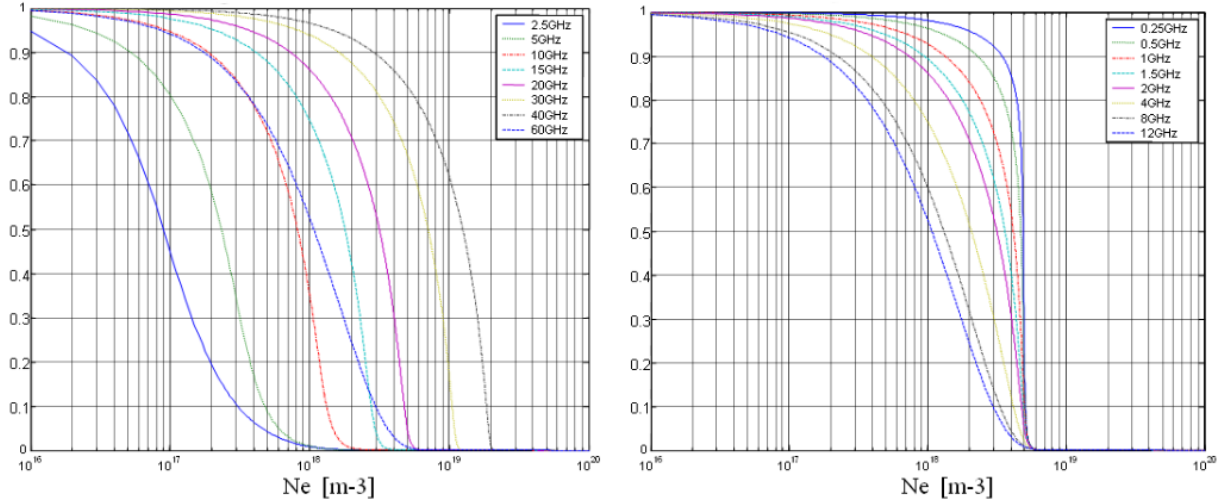


Figure 3.1: *Transmission factor with $\nu = 2GHz$ and variable microwave frequency (left-hand side), and with a microwave frequency of 20 GHz and variable electron-heavy collision frequency (right-hand side) [8]*

Plasma generation

A vacuum chamber and a pump are used to create a highly rarefied atmosphere, with a pressure below 3 mbar. Two antennas are positioned on opposite sides of the chamber, inside which two electrodes are located. One electrode is grounded, while the other is connected to a generator and supplied with an alternating voltage. The applied voltage is not particularly high, with its peak value under 600 V with the discharge on, but varies with a frequency in the range of 40 kHz. A strong electric field is established between the electrodes as a result of the voltage. The effect of this field is to inject electrons directly from the electrode into the low pressure region, a process known as field emission. These free electrons generate a region of cold plasma between the two electrodes. Although the electrons accelerate to very high kinetic energies, the plasma remains cold. This is due to two factors: the small mass of the electrons (compared to the gas molecules) and the low particle number density characteristic of the rarefied atmosphere within the chamber. Indeed, when the particle number per unit volume is sufficiently low, the mean free path of the electrons becomes long and collisions are infrequent. The combination of low electron mass and scarce collisional events results in inefficient heat transfer to the neutral gas, which consequently remains cold.

3.1.1 Methodology of the diagnostic via microwave absorption

The quantity directly measured is the power of the microwave after it passed through the plasma. Knowing the power of the wave when it is emitted, the transmission factor is easily computed by performing the ratio of the two quantities previously mentioned. Once the left hand side of equation 3.2 is known, the value of χ can be easily calculated since the distance travelled by the wave inside the plasma and its frequency ω are quantities set to perform the measure. The value of the plasma frequency can be obtained exploiting equation 3.4.

Inside this formula the unknown parameters are two, the plasma density and the electron-heavy collision frequency. The latter has to be estimated to proceed with the measurement; to do this a lookup table obtained from the BOLSIG+ software was used. The peak voltage and the distance between electrodes are used to compute the peak value of the electric field; the reduced electric field is computed as the ratio of the peak electric field and the gas number density, obtained in turn from the ideal gas law. The lookup table provides a set of plasma parameters, each tabulated as a function of the reduced electric field, by knowing the reduced electric field we can retrieve the electron-heavy collision frequency.

The procedure described entails practical complications. In fact, the frequency at which the microwave starts to be absorbed is not known in advance, hence a microwave generator with variable frequency is necessary. Scanning a range of frequencies is possible to detect at which value of ω the received wave power is reduced. A further complication is that for laboratory plasma generated in a highly rarefied atmosphere the electron number density can reach low values, so low that the corresponding plasma frequency is around 1 GHz; this frequency value is below the bandwidth of the antennas used for the experiment (2 to 18 GHz) and in general is very close to the lower limit of the microwave setup sensitivity. The low values of the electron number density implies that the microwave beam is far from the condition of total absorption, a necessary condition for the determination of the electron-heavy collision frequency. To overcome the problems associated with the low values of n_e , the measurement is performed in the following manner:

1. Several frequencies inside the antenna bandwidth range are selected and set as the microwave frequency ω_k (*with* $k=1,2,\dots,k$), for each of these values two measures are performed.
2. The power of the microwave is measured in the absence of the discharge, this value corresponds to P_1 in the formula 3.2.
3. Once the first measure is completed, the plasma is turned on and the attenuated value of the microwave power can be measured (P_2).

4. For each incident microwave frequency ω_k a different value of the transmission factor is computed T_{fk} ; The formula 3.2 is used to calculate the corresponding attenuation coefficient χ_k .
5. The k values of the attenuation coefficient can be substituted into equation 3.4, together with the corresponding microwave angular frequency and the estimated electron-heavy collision frequency. Since the plasma frequency ω_{pe} remains constant across all equations, determining the value that best fits the system of equations allows for the estimation of the plasma frequency and, consequently, the electron number density.

Indeed, the electron number density and the electron-heavy collision frequency remain the same if all the measurements are performed under the same external condition, hence in a relatively short time interval at constant pressure. The complexity and non-linearity of the equation 3.4 require the use of an algorithm to determine the values of ω_{pe} that best fit the constructed set of equations of χ . To determine the values of the plasma frequency, a script has been realised. It employs the function "fminsearch" to minimise the residual between the measured value of χ and the curve of χ , as a function of ω_{pe} , provided by the theoretical model (equation 3.4) given ν and ω as known input parameters. In particular, the residuals are computed through a cost function defined as the sum of the squared values of the difference between the measured vectors of χ and theoretic χ ; an initial value of the plasma frequency is set to start the function "fminsearch", which then performs iterative calculations to minimise the cost function. When the desired accuracy is reached, the function stops and provides the values of ω_{pe} . The code used is reported in appendix A.

3.1.2 Diagnostic setup

The scheme of the setup adopted for the microwave diagnostic is illustrated in figure 3.2. The plasma is generated inside a vacuum chamber, as briefly mentioned before. A pump connected to the chamber ensures that a highly rarefied atmosphere is achieved, the pressure is measured through a digital pressure sensor. Inside the chamber two flat electrodes are positioned, the radius of their circular surface is equal to 5.6 cm, and the distance between the two surfaces is 11.5 cm. A high voltage generator is connected to one electrode providing a high frequency voltage able to generate the discharge; the other electrode is grounded.

A Yokogawa DL1640 oscilloscope was employed, with an analogue bandwidth of 200 MHz, to measure the three signals collected. The voltage signal is provided to the oscilloscope through a Tektronix p6015a high voltage probe connected to the high voltage wire. The

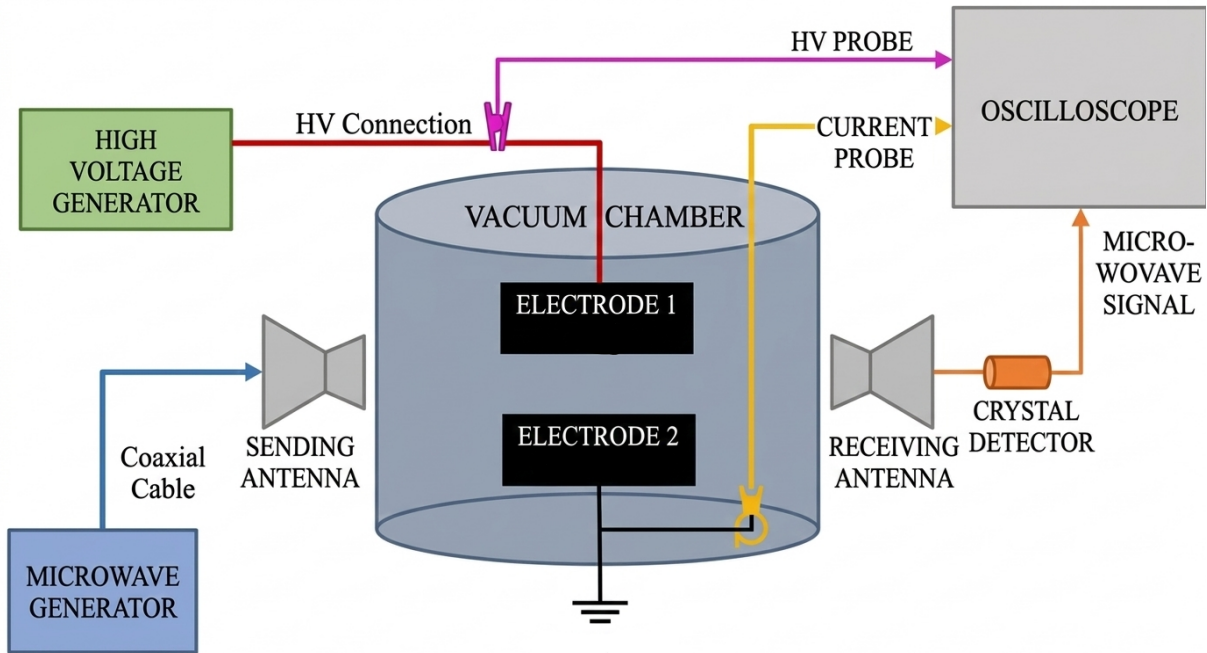


Figure 3.2: *Microwave diagnostic setup*

current probe model is the Tektronix TCPA300, it is attached to the grounding wire of the low voltage electrode and provides a noise filtering of the signal.

A Gigatronics 1018-009 microwave generator is used to produce the radiation, the generator is able to provide an microwave signal in the range of 50 MHz to 18 GHz, at constant amplitude and with a 10 dBm output power, a level low enough to ensure a non-intrusive measurement. A coaxial cable connects the generator to the sending antenna. One pair of Q-par Anhgus ltd. horn antennas are used to transmit and receive the microwave signal; they are characterised by an aperture of 119x96 mm, a bandwidth of 2 to 18 GHz and a $7\div 13$ dBi gain. The two antennas are placed on the sides of the vacuum chamber, perpendicularly to the axis passing by the two electrodes and at a distance of 41 cm from each other. The antennas must irradiate towards the plasma, hence the choice of two directional antennas. The limited aperture of the antennas prevents an excessive dispersion of the microwave beam, which should pass through the plasma volume. For this purpose a layer of aluminium foil was applied on the external wall of the vacuum chamber, in order to focus the microwave on the plasma volume only. The receiving antenna detects the microwave signal, which is then converted into an electrical signal by a crystal detector connected to the antenna. The transducer used is an HP8472A model, featuring a bandwidth of 10 MHz to 18.5 GHz. The crystal detector output signal is a voltage proportional to the power of the microwave signal, this is the third and last signal sent to the oscilloscope.

3.2 Results

The aim of the present measurement is the estimation of the plasma electron number density. To this end, two sets of measurements were performed and the data elaborated according to the methodology described in section 3.1.1. The first set is characterised by a higher pressure value (1.4 mbar) and only four chosen frequencies. As a result of the experience acquired with the initial measurements, the discharge was performed at a lower pressure (0.5 mbar), and the number of sampled frequencies has been increased to ten. The microwave frequency values at which the measure was performed have been chosen considering the bandwidth of the antenna, which is 2 to 18 GHz.

The parameter directly measured is the microwave power. The experimental procedure begins by bringing the chamber to the desired pressure. The measurement campaign is then divided into two phases: a plasma-off measurement and a plasma-on measurement. During the plasma-off phase, the supply voltage is set to zero. The frequency sweep is performed stepwise across the range of interest, with each frequency maintained for roughly three seconds. This results in the acquisition of a single waveform that captures the microwave signal corresponding to the entire frequency ramp. Once the sweep is completed, the waveform is saved. For the plasma on phase, the voltage is increased to the peak value of 600 V. As the discharge is stably localised between the two electrodes, the microwave generator is turned on and the same continuous frequency sweep, with identical three second steps, is repeated. A single waveform from the crystal detector is acquired, containing the signal with all the frequencies in succession. This waveform is saved together with the corresponding voltage and current traces.

The two waveforms of the microwave signal allow to compute the transmission factor T_f . This parameter along with the electron-heavy collision frequency and the geometrical dimension of the discharge constitute all it is required for the code to compute the attenuation factor and, in turn, the electron number density. In the following paragraphs, the two sets of measurements are described.

3.2.1 Test at 1.4 mbar

A first attempt to measure microwave attenuation was made at a pressure of 1.4 mbar. This value was selected because, at lower pressures, the plasma becomes increasingly diffuse and less confined to the region between the electrodes. The chosen pressure therefore represents a compromise between the requirement of a sufficiently low pressure to sustain the discharge and the need for a pressure high enough to ensure adequate plasma localisation between the electrodes. In table 3.1 the microwave frequency values, the crystal detector signal with and without plasma and the computed transmission factor are listed.

The results are coherent with the consideration done previously in different parts of the

f [GHz]	P_{OFF} [mV]	P_{ON} [mV]	$T_f = P_{ON}/P_{OFF}$
2.0	-0.0480	-0.0303	0.6311
2.5	-0.1318	-0.1226	0.9305
3.2	-0.0612	-0.0592	0.9669
4.7	-0.0352	-0.0334	0.9484

Table 3.1: *Values of the wave frequency, crystal detector signal and transmission factor.*

thesis. The frequency of total absorption is below the lower limit of the antenna bandwidth (2 GHz); thus, the complete absorption of the wave cannot be achieved, as a matter of fact T_f at 2 GHz is not 0. The small dimension of the plasma also contributes to a low absorption, as the wave travels in a region of attenuation only for a short distance. The transmission factor rises quickly over 90%, reflecting the exponential trend that rules the phenomenon (see formulae 2.18 and 3.2), as shown by the plot 3.3. Here, the transmission factor measured for a collision frequency $\nu = 3$ GHz and a plasma depth of 5.6 cm is plotted as a function of microwave frequency. The dimension of 5.6 cm was chosen because it is the diameter of the electrodes generating the plasma. At $f = 2.5$ GHz, 93% of the wave energy is able to go through the plasma, this frequency can be considered the transparency frequency; that means the frequency at which the microwave passes through the plasma without significant absorption, as if the plasma was transparent to the microwave. These data and considerations put together suggest that the plasma frequency is below the value of 2 GHz.

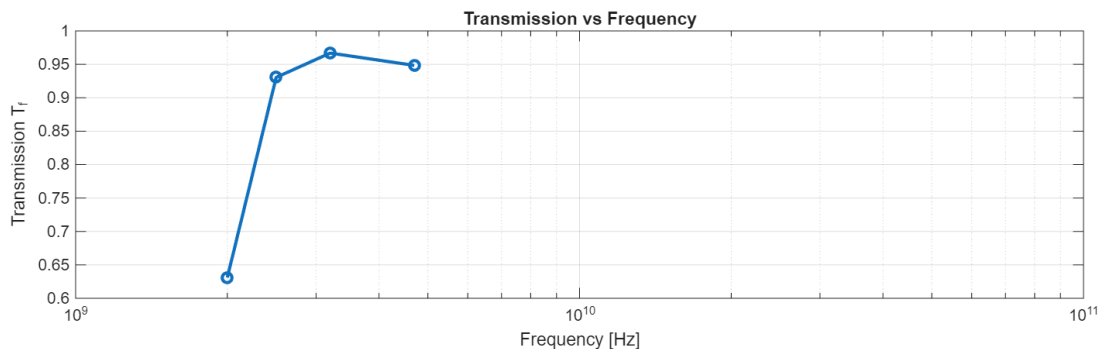


Figure 3.3: *Transmission factor as a function of the microwave frequency*

The acquired waveforms of the transmission factor are displayed in the figures 3.4 and 3.5, for the measurement phases of the plasma turned off and turned on respectively. In these two images, the regions with a different microwave frequency can be clearly distinguished, particularly in the plasma-off case. When the discharge is inactive, the transmission factor T_f exhibits little dispersion over each constant-frequency time interval. In contrast,

when the plasma is turned on, the same intervals are characterized by a significantly greater dispersion of T_f values. To address this issue, the transmission factor for each frequency was computed as the average of all T_f values recorded within the corresponding constant-frequency interval. Images 3.4 and 3.5 show the presence of 5 intervals with constant frequency, starting from the left each interval corresponds to 2, 2.5, 3.2, 4 and 4.7 GHz respectively. The presence of an additional interval is due to the fact that the transmission factor at 4 GHz was not considered in the evaluation of the data. In fact, at this frequency value the power of the microwave with plasma-ON resulted higher than with plasma-OFF.

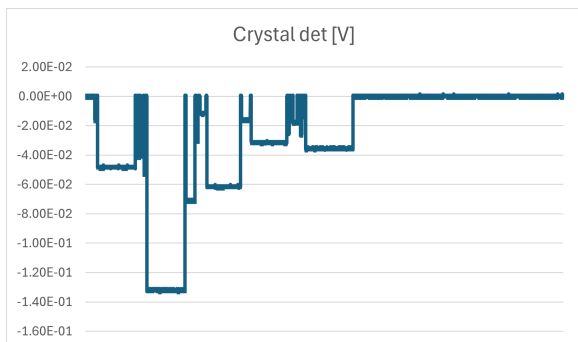


Figure 3.4: *Waveform of the transmission factor with plasma turned-off.*

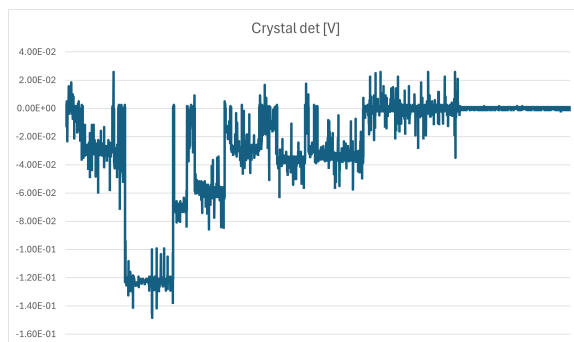


Figure 3.5: *Waveform of the transmission factor with plasma turned-on.*

The code reported in appendix A was used to compute the electron number density. The input data set are a plasma depth of 0.056 m, an electron-heavy collision frequency of 3 GHz, and the frequencies with their relative transmission factor presented in this paragraph. The results of the computation are:

$$f_{pe} = 1.0452 \times 10^9 \text{ Hz}$$

$$n_e = 1.3552 \times 10^{16} \text{ m}^{-3}$$

The computed plasma frequency confirms the statement of a f_{pe} lower than the lowest frequency scanned during the measurements. The number density is in the order of magnitude of a low-pressure cold plasma.

3.2.2 Test at 0.5 mbar

The second set of measurements was aimed at generating the plasma at a lower pressure, compared to the previous value of 1.4 mbar, and at obtaining a higher number of transmission factors to ensure a stronger fitting. The plasma at 0.5 mbar is more dispersed and is not entirely localised between the electrodes, this should be avoided since compromises the accuracy of the measurements. Layers of insulating tape and were applied to the metallic edges on the inside of the vacuum chamber in order to confine the discharge, as

much as possible, to the region between the electrodes. The results achieved were good enough to proceed with the measurements. In table 3.2 the microwave frequencies, the crystal detector signal with plasma-on and plasma-off, and the computed transmission factor are listed.

f [GHz]	P_{OFF} [mV]	P_{ON} [mV]	$T_f = P_{ON}/P_{OFF}$
2.0	-0.0789	-0.0495	0.6273
2.5	-0.1229	-0.1064	0.8658
3.2	-0.0596	-0.0409	0.6861
4.7	-0.0312	-0.0255	0.8174
5.2	-0.0195	-0.0153	0.7824
7.6	-0.0308	-0.0285	0.9257
8.1	-0.0308	-0.0264	0.8575
9.1	-0.0226	-0.0199	0.8799
10	-0.0082	-0.0072	0.8820
11.1	-0.0032	-0.0032	0.9367

Table 3.2: Values of the wave frequency, crystal detector signal and transmission factor.

Similarly to the previous test, the lowest frequency shows the lowest transmission factor, thus the microwave frequency at which the plasma absorbs the most. The transmission factor, although tends to increase with increasing frequency, does not show a strictly monotone trend, as is described by the theory of wave propagation in cold plasma. Specifically, the increase to $T_f = 0.86$ at 2.5 GHz followed by a sharp decrease to $T_f = 0.68$ at 3.2 GHz. The possible explanations for this counter-intuitive result are multiple; they are

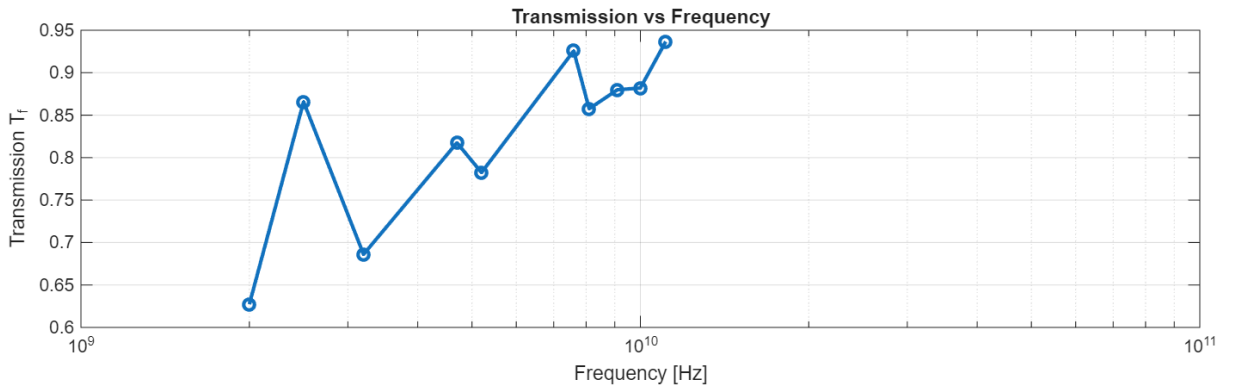


Figure 3.6: Transmission factor as a function of the microwave frequency

mainly connected to the interaction of the microwave with the laboratory environment and the variable nature of a plasma. Wave reflection and interference are often phenomena that alter the measurements when the setup and the surrounding environment are not adequately shielded, like in this test. In such a case, the wave could reflect on a metallic

surface causing an interference with the signal. The instability of plasma conditions can also justify the anomalous result. It cannot be guaranteed that parameters like the electron number density or collision frequency remain constant throughout the duration of the measurement. Furthermore, density gradients inside the plasma lead to critical layers located at different depths for different frequencies, thus resulting in wave reflection point that change according to the microwave frequency. More analysis and studies are required to explain the reason for this result. The available data do not provide a clear explanation.

The plot 3.6 is related to a plasma characterised by a depth of 5.6 cm and a collision frequency estimated at $\nu = 1.6$ GHz, a ν lower than the previous test due to the lower pressure. The frequency value beyond which more than 90% of the wave passes is 7.6 GHz, marking a clear shift upward for the transparency frequency and suggesting an increase for the plasma frequency (the electron number density follows). Such a result is not surprising, and it is explicable by the effect of the pressure on the plasma parameters. As the pressure lowers, the influence of the neutral gas is reduced (thus the neutral number density too); this leads to a growth in the mean free path of the electrons which reach higher kinetic energy levels. All of these effects combined result in more efficient ionisation processes, and less recombination processes due to the lower gas density, a set of conditions that explain why the plasma is able to reach higher electron number density values. It is straightforward that a higher number density means a higher plasma frequency and thus a higher transparency frequency.

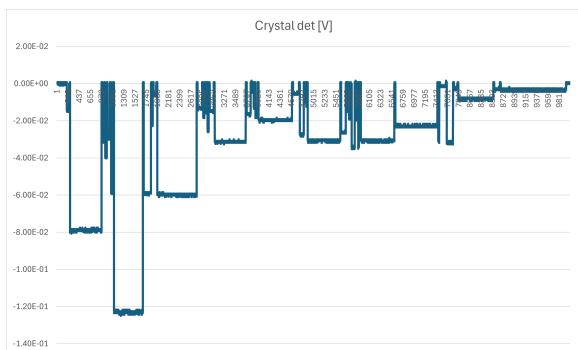


Figure 3.7: *Waveform of the transmission factor with plasma turned-off.*

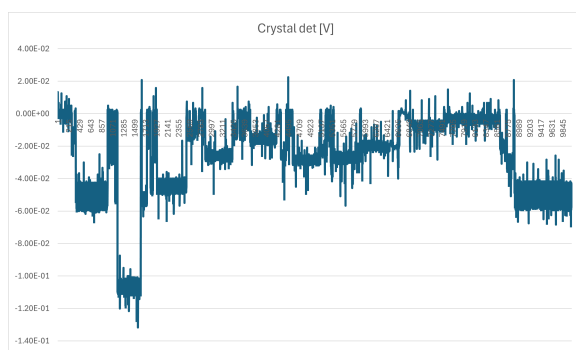


Figure 3.8: *Waveform of the transmission factor with plasma turned-on.*

The trend of the microwave signal is shown above. The description of pictures 3.7 and 3.8 is similar to the one given in paragraph 3.2.1, with the exception of an additional constant-frequency interval in picture 3.8. This 11th interval is caused by a final microwave with a frequency of 2 GHz, it was emitted to assess the plasma condition at the end of the measurement; more precisely, if they remained similar to the condition at the beginning. The signal intensity for this last measurement is equal to -0.0488 mV, particularly close

to the initial value at 2 GHz. For this test too, the transfer factor values were computed performing the average value over a constant-frequency interval.

In the end, using the code in the appendix A the plasma frequency and the electron number density were computed. Like for the previous test, the plasma depth has been set to 0.056 m while the electron-heavy collision frequency has been lowered to 1.64 GHz because of the reasons explained previously. The results of the computation are:

$$f_{pe} = 1.1611 \times 10^9 \text{ Hz}$$

$$n_e = 1.6724 \times 10^{16} \text{ m}^{-3}$$

The plasma frequency and the electron number density values grew as inferred before, although the increase is not significant enough to justify a supposed transparency frequency between 5.2 GHz and 7.6 GHz.

Chapter 4

Conclusions

This thesis summarises a study about the interaction of cold plasma with electromagnetic waves and the related experimental part, the latter aimed at estimating the electron number density of a low pressure plasma. The fundamental characteristics that define a plasma were discussed in the first half of chapter 2, phenomena like the Debye shielding or the electron plasma oscillation, which are fundamental for the understanding of plasma physics. A more focused approach to the theoretical model describing the interaction between a cold plasma and an electromagnetic wave has been presented in the second half of the same chapter. The theory and the various parameters discussed here are necessary for the comprehension and interpretation of the experimental results presented in the chapter 3. Here, a basic introduction to plasma diagnostics is proposed and the subject of the study is explained. Thanks to the solid knowledge on wave absorption built in the section 2.3 the logic behind microwave absorption has been discussed. It followed a phenomenology paragraph, where the laws governing the wave-plasma interaction were linked to practical applications in the field of diagnostics, together with the setup adopted for the measurements. At last, the methodology of the measurements elaboration and the data obtained were illustrated and interpreted in the section 3.2.

The power signal of a microwave propagating in a cold, low pressure plasma was recorded. This measurement was performed under two conditions, a first one with plasma-ON and a second with plasma-OFF. In this manner, the transmission factor for various wave frequencies was obtained as a simple ratio between the signals. The transmission factor as a function of wave frequency is a powerful parameter, that expresses the attenuation a wave experiences. The attenuation phenomenon is mainly related to three variables, the wave frequency, the electron-heavy collision frequency and the plasma frequency which in turn provides the electron number density. The latter is the parameter of interest for the current thesis. Strictly speaking, it represents the objective of the plasma diagnosis reported here; but with a broader view of the whole study project, in terms of time-frame,

technique employed and instrumentation used, this diagnosis can be regarded as a preliminary assessment aimed at understanding whether the microwave absorption diagnostic can provide reliable results for the case study of interest.

To this end, the data obtained are in line with the typical values of a cold low pressure plasma, the computed electron number densities for the two pressure levels are $n_e = 1.67 \times 10^{16} \text{ m}^{-3}$ and $n_e = 1.35 \times 10^{16} \text{ m}^{-3}$, for 0.5 and 1.4 mbar respectively; the increase reflects the role of pressure in the ionisation mechanism and the consequent electron number density. The measurements showed a clear and expected pattern for the transmission factor; in fact, the lowest frequency available, 2 GHz, yielded the most substantial absorption in both pressure tests. After the value of 2 GHz, the transmission factor increased allowing to infer a plasma frequency below the lower limit of the setup sensitivity; in this way narrowing down experimentally the possible value of the plasma frequency, in addition to the data computation which provided a more accurate estimation of the desired parameter.

Although the investigated parameter lies in the expected order of magnitude, the data are not free of outliers. As shown in paragraph 3.2.2, the transmission factor did not show a monotone trend like the theory prescribe. Furthermore, a transmission factor greater than 1 was briefly mentioned in 3.2.1, even if it was not reported among the processed data. These aspects suggest the presence of practical limitations in the diagnosis. The possible presence of wave reflections and interferences, plasma parameters instabilities and the low intensity of the microwave signal registered for frequencies beyond 10 GHz are all issues that require a further and accurate analysis to be addressed. Identifying limitations related to the technique and instrumentation, together with finding inconsistencies in the data respect to the theory is beneficial to the study project, in light of the preliminary assessment consideration.

The future developments of this research project will deal with the issues mentioned previously. In particular, the use of more sophisticated model of waves propagation and algorithms for data processing, along with the employment of instrumentation tailored for the generation of plasma and plasma diagnostics, can provide a considerable improvement in the interpretation of the results.

Appendix A

MATLAB script for data analysis

The code used for the analysis of the experimental data is here reported:

```
1 clear; clc; close all;
2
3 % --- Physical constants for chi calculation---
4 c = 2.998e8; % Speed of light [m/s]
5
6 % --- Physical constants for n_e calculation ---
7 e = 1.602e-19; % Electron charge [C]
8 m_e = 9.109e-31; % Electron mass [kg]
9 eps0 = 8.854e-12; % Vacuum permittivity [F/m]
10
11 % --- Input: plasma path length ---
12 Delta_x = input('Enter plasma path length Delta_x [m]: ');
13
14 % --- Input: frequencies and transmission coefficients ---
15 f_Hz = input('Enter frequency vector [Hz] (e.g., [2e9 3e9 4e9 5e9 6e9]):
16 ');
17 Tf = input('Enter transmission coefficient Tf vector (P1/P2): ');
18
19 % --- Calculate angular frequency ---
20 omega = 2 * pi * f_Hz; % [rad/s]
21
22 % --- Calculate attenuation coefficient chi ---
23 % Formula: chi = -ln(Tf) / (2*Delta_x) * (c / omega)
24 chi = -(log(Tf)) ./ (2*Delta_x) .* (c ./ omega); % [m^-1]
25
26 % --- Display results ---
27 fprintf('\n===== RESULTS =====\n');
28 fprintf('Frequency [GHz]\tTf\t\tchi [m^-1]\n');
29 for i = 1:length(f_Hz)
30     fprintf('%.2f\t\t%.4f\t\t%.4e\n', f_Hz(i)/1e9, Tf(i), chi(i));
31 end
```

```

30 end
31
32 % --- quick plot ---
33 figure;
34 subplot(2,1,1);
35 semilogx(f_Hz, Tf, 'o-', 'LineWidth', 2);
36 xlabel('Frequency [Hz]');
37 ylabel('Transmission T_f');
38 title('Transmission vs Frequency');
39 xlim([1e8 1e11]);          % Set x-axis range from 10^8 to 10^11 Hz
40 grid on;
41
42 % --- Input: collision frequency ni ---
43 nu = input('Enter electron-heavy collision frequency ni [rad/s]: ');
44
45 % --- Define the corrected theoretical chi function ---
46 chi_theory = @(wp, w, nu) ...
47     sqrt( ...
48         -0.5 * (1 - wp^2 ./ (w.^2 + nu^2)) ...
49         + 0.5 * sqrt( ...
50             (1 - wp^2 ./ (w.^2 + nu^2)).^2 ...
51             + ( (wp^2 ./ (w.^2 + nu^2)) .* (nu ./ w) ).^2 ...
52         ) ...
53     );
54 % --- Cost function to minimize (sum of squared errors) ---
55 cost = @(wp) sum( (chi - chi_theory(wp, omega, nu)).^2 );
56
57 % --- Initial guess for omega_pe ---
58 wp_guess = 1e10;    % rad/s (adjust if needed)
59
60 % --- Minimize to find best omega_pe ---
61 options = optimset('Display', 'iter');
62 wp_opt = fminsearch(cost, wp_guess, options);
63
64 % --- Calculate electron density n_e from omega_pe ---
65 n_e = (wp_opt^2 * eps0 * m_e) / e^2;    % [m^-3]
66
67 % --- Display results ---
68 fprintf('\n===== PLASMA PARAMETERS =====\n');
69 fprintf('Optimal omega_pe = %.4e rad/s\n', wp_opt);
70 fprintf('Optimal f_pe = %.4e Hz\n', wp_opt/(2*pi));
71 fprintf('Electron density n_e = %.4e m^{-3}\n', n_e);
72
73 % --- Plot: measured chi vs fitted chi ---
74 w_fit = linspace(min(omega), max(omega), 200);

```

```
75 chi_fit = chi_theory(wp_opt, w_fit, nu);
76
77 figure;
78 plot(omega/(2*pi*1e9), chi, 'o', 'LineWidth', 2); hold on;
79 plot(w_fit/(2*pi*1e9), chi_fit, '-', 'LineWidth', 1.5);
80 xlabel('Frequency [GHz]');
81 ylabel('Attenuation chi [m^{-1}]');
82 legend('Measured chi', 'Fitted chi');
83 title('Fit of chi(omega) to determine omega_{pe}');
84 grid on;
```


Bibliography

- [1] J. A. Bittencourt, *Fundamentals of plasma physics*. Springer, 2004.
- [2] F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion*. Springer, 2016.
- [3] D. R. Nicholson, *Introduction to Plasma Theory*. John Wiley Sons, 1983.
- [4] K. Wiesemann, “A short introduction to plasma physics,” *CAS-CERN Accelerator School, Ion Sources*, pages 85–122., CERN, 2013.
- [5] B. Li, Q. Nie, X. Wang, Z. Wang, A. Mao, and P. Chen, “Resonant absorption of incident electromagnetic waves in collisional inhomogeneous plasma slabs,” *AIP Advances*, vol. 9, no. 9, p. 095 020, Sep. 2019. [Online]. Available: <https://doi.org/10.1063/1.5113689>.
- [6] M. Kuzelev, A. Rukhadze, and W. S. (Firm), *Methods of Wave Theory in Dispersive Media*. World Scientific Publishing Company, 2010, ISBN: 9789814261708.
- [7] I. H. Hutchinson, *Principles of Plasma Diagnostics*, 2nd ed. Cambridge University Press, 2002.
- [8] G. Neretti, “Diagnostic techniques for ehd and mhd interaction,” Ph.D. dissertation, Alma Mater Studiorum - Universita di Bologna, 2009.