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Relational Quantum Mechanics and Antistructuralism

Supervisor:
Prof. Elisa Ercolessi

Submitted by:
Pier Francesco Peperoni

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Abstract

Carlo Rovelli's relational quantum mechanics claims that the collapse of a quantum state is an update of information about some system relative to some observer. Unlike Hugh Everett's theory of the universal wave function it adopts a Heisenberg-type ontology about measurement outcomes, and states simply encode information about them. This point of view is analysed in particular with regard to its implications on the informational aspects which define the past of an observer, also assessing the claims about locality in the EPR debate: relational quantum mechanics is indeed local, the flaw in the EPR argument lying in not recognising the observer-dependent nature of the quantum state. EPR non-locality reduces to an arbitrary choice of mapping present information about measurement outcomes into causally disconnected events, while there is no notion of absolute evolution to the present state. Antistructuralism is thus introduced, as a paradigm which considers structures as different arbitrary frameworks in which to organise the same information. The structure of Minkowski spacetime is analysed, where past light cones can shrink to arbitrary observers, making every outcome causally connected, representing the evolution of measurement outcomes relative to a particular observer. The dissertation closes with some philosophical considerations to defend the consistency of the paradigm also from an ontic perspective which claims that there is no structure in reality.

Sommario

La meccanica quantistica relazionale di Carlo Rovelli sostiene che il collasso di uno stato quantistico è un aggiornamento dell'informazione su un sistema relativamente ad un osservatore. A differenza della teoria della funzione d'onda universale di Hugh Everett, adotta un'ontologia alla Heisenberg sui risultati delle misure, e gli stati semplicemente codificano informazioni su di essi. Questo punto di vista viene analizzato in particolare per quanto riguarda le sue implicazioni sugli aspetti informazionali che definiscono il passato di un osservatore, valutando anche le affermazioni sulla località nel dibattito EPR: la meccanica quantistica relazionale è effettivamente locale, e il difetto dell'argomentazione EPR risiede nel non riconoscere il carattere di dipendenza dall'osservatore dello stato quantistico. La non-località EPR si riduce ad una scelta arbitraria di mappare le informazioni presenti sui risultati delle misure in eventi causalmente disconnessi, mentre non esiste alcuna nozione di evoluzione assoluta verso lo stato presente. Viene quindi introdotto l'antistrutturalismo, come paradigma che considera le strutture come diverse rappresentazioni arbitrarie in cui organizzare la stessa informazione. Viene analizzata la struttura dello spaziotempo di Minkowski, dove i coni di luce del passato possono ridursi a osservatori arbitrari, rendendo ogni risultato causalmente connesso, rappresentando l'evoluzione dei risultati di misura rispetto ad un particolare osservatore. La tesi si conclude con alcune considerazioni filosofiche per difendere la coerenza del paradigma anche da una prospettiva ontica che sostiene l'assenza di struttura nella realtà.

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Introduction

Quantum mechanics tells you how to evaluate probabilities of future outcomes of an interaction, given that some outcomes have happened in the past of the interaction. However, it does not tell you when outcomes happen, and if they already happened before the interaction, or if the interaction in question will yield any outcome. This is known as the measurement problem: when should we update the quantum state? According to von Neumann one can arbitrarily choose the time of collapse of one system at any time between the interaction with the measurement device and the interaction with the observer. I consider this problem in chapter 1 from different perspectives about the time at which collapse happens, considering two consecutive measurements, which distinguish between sequence of interaction with the devices and sequence of actual realisation of outcomes. I then discuss this problem from the point of view of relative quantum states: such states describe outcomes as happening at interaction with an arbitrary observer. This means, that given an observer of reference, outcomes do not happen in an absolute spacetime, but rather they are local updates of the information that the reference observer has about the world.

This raises some questions about the nature of the past of systems and observers, which I discuss by considering the Wigner's friend thought experiment. I consider two relative state formulations which give different answers: Hugh Everett's Theory of the Universal Wave Function, and Carlo Rovelli's Relational Quantum Mechanics. While the former uses a Schroedinger-type ontology, the latter uses a Heisenberg-type ontology. The two ontologies are related to two equivalent pictures of quantum mechanics, the Schroedinger and the Heisenberg pictures, which are known to be empirically equivalent. In the former picture states evolve in time, and in the latter picture states are stationary, and are updated at collapse, so they simply carry the information about the outcome of collapse.

In chapter 2 I discuss casually disconnected and independent interactions, which give raise to non-locality. After introducing the EPR debate I discuss it in the relational perspective, which claims that there is no non-locality. The reason lies in the treatment, in the EPR argument, of the state evolution as something independent of the observer, which is wrong in relational quantum mechanics. In fact, relativity of states allows to reduce the past of a system to a present outcome for an other system.

In relational quantum mechanics we can choose the dynamics which brings to the present state, and different perspectives lead to the same probabilities for future outcomes, despite they describe different past evolutions to the present state, with different sequences of outcomes. This raises questions about the role of ordering structures in physical theories, and if we can have a paradigm in which they are not fundamental, but rather arbitrary ways to order present information. So, in chapter 3, I present my view of antistructuralism, which I discuss in relation to physical theories which want to make predictions given some informational content, whatever the structure, and therefore dynamics, used to make predictions. I apply this paradigm to Minkowski spacetime, and to relational quantum mechanics, in particular to the EPR argument. Then I conclude the chapter by introducing ontic antistructuralism, which claims that no structure exists in reality.

Chapter 1

Relative Quantum States

The quantum mechanics of a closed system (see for example [10]), a brief review of which is given in appendix A, is characterised by two types of dynamics[1]:

- First type is a continuous deterministic evolution in which information is conserved:

$$U(0)|\Psi\rangle \Rightarrow U(t)|\Psi\rangle \tag{1.1}$$

- Second type is the 'moment of measurement', in which the state randomly collapses to an eigenstate $|\Psi_1\rangle$ of the operator corresponding to the measured quantity:

$$U(t)|\Psi\rangle \rightarrow |\Psi_1\rangle \tag{1.2}$$

  This moment is a non-deterministic process in which the probabilities of the outcomes are determined by the Born rule:

$$P(1.2) = |\langle\Psi_1|U(t)|\Psi\rangle|^2. \tag{1.3}$$

The measurement is the moment at which new information is acquired. So, we have two types of evolution for a closed quantum system, one which is deterministic and thus preserves information, and one which provides new information. This formulation of quantum mechanics, built on the Dirac-von Neumann axioms (see appendix A), does not give clear indications in practice about which interactions count as measurement, and which don't. For example, according to Niels Bohr [3], the quantum world and the macroscopic world are to be considered fundamentally different, and we must distinguish interactions between quantum systems and interactions between a quantum system and

[1] In the present dissertation I use the double arrow ($\Rightarrow$) for an evolution following the first type of dynamics, and the single arrow ($\rightarrow$) for an evolution following the second type.

a macroscopic system. It is only interaction with macroscopic objects which causes the collapse of the quantum state.

We can't know from quantum theory how macroscopic systems emerge, and in practice it is a convention where to put the separation between the classical and quantum worlds. Usually, this cut is assumed to emerge at the time of interaction with the measurement apparatus. It can be argued though that an apparatus is just a very complex quantum system, and there is no reason in principle for which its interaction with a quantum particle should not be treated entirely as a unitary process. In fact, John von Neumann [9] argues that moment of collapse can be chosen arbitrarily at any moment before interaction with the observer. In any case it seems like the collapse must take over at some moment, and since we never experience quantum superposition this must happen before interaction with us. Eugene Wigner [16] pushes this moment to the limit, and argues that it might be interaction with a conscious observer which causes the collapse of the quantum state.

As of today there is still no evidence for any process of collapse, and the systems of which we are able to observe quantum properties are getting larger and larger. We wonder how further we must push the limits of the quantum world, and if it will ever reach beyond the conscious observers. It is the case to ask whether such limits exist at all, and if we should start making sense of ourselves as quantum systems. Hugh Everett has proposed [6, 7] that if we treat a device, or an observer, as a quantum system, we can assign to it an incomplete description of the world, in terms of relative quantum states. The Everettian approach, or the Theory of the Universal Wave Function, considers the existence of a global quantum state, which contains a complete description of the world, and every subsystem can only give an incomplete description. The ontology is in the unitary evolution. Complementary to this treatment is Carlo Rovelli's approach, or Relational Quantum Mechanics [12], for which all systems are equivalent, and no global quantum state is ever considered. The ontology of the theory is in the values taken by variables at the interactions, and it is only through interaction that knowledge of these variables is acquired. This chapter is dedicated to understanding relative quantum state formulations of quantum mechanics, which describe physical quantities not as a property of the systems, but of the interactions.

1.1 The Measurement Problem

In quantum mechanics, when two quantum systems interact, they get entangled. This process is a unitary evolution involving the Hamiltonians of the two systems and the interaction Hamiltonian. When they stop interacting, we can perform a measurement on one of the two systems, obtaining information about the result of a measurement on the second one. Since the interaction does not involve macroscopic systems until measurement, no measurement outcome happened during the interaction. Outcomes are created during the measurements, and are correlated even if the measurements are independent[2]. A measurement is therefore a very special type of interaction, which breaks the unitary evolution of the compound system, but it is not clear in general which interactions should count as measurement in this sense.

1.1.1 Von Neumann's Measurement Process

Von Neumann, treating also the device as a quantum system during measurement, describes the process in two moments [9]. Consider a quantum system, let's call it 'particle', whose state is described by $|\Psi\rangle$, and a measurement device whose state is described by $|d\rangle$. Moment 1 is described by the dynamics of the first type mentioned above, a continuous unitary interaction which entangles the possible outcomes $|d_i\rangle$ displayed on the device with the eigenstates $|\Psi_i\rangle$ of the measured observable[3]:

$$|\Psi\rangle \otimes |d\rangle = \sum_i a_i |\Psi_i\rangle \otimes |d\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |d_i\rangle \tag{1.4}$$

During this moment the device remains in a quantum superposition of states. At the beginning the two systems are not interacting, and their state is the tensor product between the state $|\Psi\rangle = \sum_i a_i |\Psi_i\rangle$ of the particle and the initial state $|d\rangle$ of the device. As they begin interacting, the unitary evolution, determined by the interaction Hamiltonian, entangles the states $|\Psi_i\rangle$ of the measured outcome with the reading $|d_i\rangle$ of that outcome $|\Psi_i\rangle$ from the device.

Then, at an unspecified time, moment 2, described by a dynamics of the second type, takes over, and the superposition randomly collapses to a definite outcome:

$$\sum_i a_i |\Psi_i\rangle \otimes |d_i\rangle \to |\Psi_1\rangle \otimes |d_1\rangle \tag{1.5}$$

The probability of collapse is given by the Born rule:

$$P(1.5) = |\sum_i a_i \langle d_1|\langle \Psi_1|\Psi_i\rangle|d_i\rangle|^2 = |a_1|^2. \tag{1.6}$$

1.1.2 Information in an Ideal Measurement

During moment 1 no new information is created in the compound system. The individual systems, though, are not closed during the interaction, and they get entangled: the

[2]This has led to the EPR argument [5], which will be discussed in chapter 2.

[3]State $|d_i\rangle$ means 'the device reads λ_i', where λ_i is the eigenvalue relative to $|\Psi_i\rangle$. Recall from appendix A that after collapse $|\Psi\rangle$ will result in an eigenstate corresponding to the measured eigenvalue. A functioning device by definition reads the correct eigenvalue, to which there corresponds an eigenvector into which the state evolves. For simplicity, we will often say that the device measures an eigenstate, without having to refer to the corresponding eigenvalue. This is consistent if there is no degeneracy.

device now stores information about one observable. This is an ideal process in which information inside the particle is not affected: as long as moment 2 has not taken over, the particle behaves as if nothing has happened. Suppose we have information about the evolution of the particle, but not about the compound system: if the measurement is ideal, our information about the particle is not affected. Suppose we have information about the device, but not about the compound system: after the interaction the device updates its information, so we can't predict its state after moment 1. It is only the device which stores new information.

Knowing the Hamiltonian of the particle means knowing the evolution of the probabilities of getting a particular result for any observable being measured, given an initial state of the particle. Knowing the Hamiltonian of the device means knowing how the probability of displaying a certain number varies when it is not measuring anything, given an initial state of the device. Knowing the interaction Hamiltonian between the device and that type of particle means knowing which observables are going to be measured for that type of particle. It is of course this latter Hamiltonian which tells if the measurement is ideal. If one knows this Hamiltonian it is irrelevant whether the measurement is ideal or not, since one can still work out the state of the particle just before interaction from the displayed result, and perhaps fix the reading of the device by keeping this into account. No information during moment 1 is ever lost.

During moment 1 information is stored in the device, but it is still in a superposition of states, until moment 2 takes over. It is moment 2 the reason for which it is in general impossible to measure some state without changing it. Moment 1 can, in principle, leave the individual state of the particle approximately unchanged, and entangle the systems just by changing the state of the device. It is however impossible for a unitary transformation to store information in the device without creating entanglement, thus leaving the two states independent, as it is well known by the no-cloning theorem. So, when the state on the device collapses, any further measurement on the particle must give the same outcome about its state after the interaction with the device.

At Moment 2 there is a change in the information stored in the compound system, information which becomes available at the particle and at the device. It is either arbitrary, or a matter of interpretation, to choose when moment 2 happens. Notice that it must happen at two events, one for the device and one for the particle, no matter how far apart they are. For this reason it makes no sense to talk about a unique time of collapse. Moment 2 as a change of the whole system is not a change localised in one of its components which spreads to the other parts, but it happens at many parts independently.

Determining when and how moment 2 happens is known as the measurement problem. If moment 2 is forced to happen before the entanglement was completely established, the device and the particle before collapse are only in partial correlation, and the measurement has some probability to fail. Of course, what is observed on the device are always definite outcomes, which however might not correspond the state of the particle. Every time we try to peep into the measurement process, moment 2 must have taken over. A more detailed analysis is given in [11].

1.1.3 Relational Interpretation

In Relational Quantum Mechanics, moment 2 reveals information about the device-particle interaction. Such information can be stored by an observer (e.g. another device) after interacting with either the device or the particle, and this is when moment 2 happens relative to such observer. Moment 2 is acquisition of new information about the outcome of an interaction, and the quantum state of the world is updated only at this moment. Quantum states represent relative dynamics of information about the same outcomes, and different observers need not agree on past dynamics, as long as when they meet they make the same probabilistic predictions for future outcomes. When they meet, i.e. when they share the same information, they can project the present event into different arbitrary pasts, each one representing the point of view of some observer sharing their information at the present event. If a device measures a particle, collapse happens when the observer reads the device. Relative to the device, the collapse happens as soon as it interacts with the particle. At least, this is what the device tells you, based on the information created when you interact with it. While we have no reason to think that the outcome did not exist before our interaction with the device, there is no way physics can guarantee this reasonable metaphysical assumption. The idea of Relational Quantum Mechanics is that physics can only be formulated about relative information. It is such information which is relevant for interactions, not some absolute information stored inside one system independently of our interaction with it. This point will be further discussed later in the dissertation. Collapse is an update of relative information (it is a property shared only by the interacting systems) about the outcome of the interactions between systems. Information is about interactions, and it is stored at future interactions with other systems. There are no absolute states in this picture.

1.2 Consecutive Measurements

What is remarkable in von Neumann's treatment is the description of a macroscopic system, the device, as being, at least for some moment in time, in a quantum superposition. Now, let's keep the superposition for a little longer, and let's consider a second device, d', measuring the particle after it has interacted with the first device. We don't introduce moment 2, so the particle is still in a superposition, but it has memory of the interaction with the first device d. It is thus described by the entangled state $\sum_i a_i|\Psi_i\rangle \otimes |d_i\rangle$.

1.2.1 Same Observable

First, we consider a measurement of the same observable:

$$\sum_i a_i|\Psi_i\rangle \otimes |d_i\rangle \otimes |d'\rangle \Rightarrow \sum_i a_i|\Psi_i\rangle \otimes |d_i\rangle \otimes |d'_i\rangle. \tag{1.7}$$

Now the same information is in both devices. When moment 2 happens, the system will collapse:

$$\sum_i a_i|\Psi_i\rangle \otimes |d_i\rangle \otimes |d'_i\rangle \to |\Psi_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \tag{1.8}$$

with probability:

$$P(1.8) = |\sum_i a_i\langle d'_1|\langle d_1|\langle \Psi_1|\Psi_i\rangle|d_i\rangle|d'_i\rangle|^2 = |a_1|^2 = P(1.5)$$

.

It is plausible that, unless it is in a stationary state, the particle has quite evolved between the two measurements. Then the state of the particle at device 2 only changes for a global factor $U(t)$, where t is the proper time of the particle between the two measurements. We can always give device 2 information about the transformation $U(t)$, so that it performs $U^\dagger(t)$ on the particle, and we don't need to change the expressions above. In fact, unitary transformations conserve information.

In the Copenhagen interpretation, if the devices are not 'macroscopic enough' (whatever that means) to induce a collapse, the two devices remain in an entangled superposition of outcomes until either one of the two or the particle interacts with a proper macroscopic system. In the case we just showed there is no practical difference in the choice of the moment of collapse.

What if device 1 had already induced the collapse? Then moment 1 at device 2 would be:

$$|\Psi_1\rangle \otimes |d_1\rangle \otimes |d'\rangle \Rightarrow |\Psi_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \tag{1.9}$$

and the collapse is trivial:

$$|\Psi_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \to |\Psi_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \tag{1.10}$$

with probability:

$$P(1.10) = 1. \tag{1.11}$$

Even if the probability of collapse is 1, an observer B who has not interacted with device 1 after the measurement does not know the outcome of the collapse, which they have to guess. So, according to their own information, the probability of observing outcome (1.10) is still $P(1.5) = P(1.8)$, as if the collapse didn't occur. This is true also for some observer A guessing the outcome they will observe at device 1, or for device 2 (treated like an observer) guessing the outcome it will observe at the interaction with the particle.

It is useful to define the relative state of the system (particle, device 1) with respect to device 2. It is simply the state of (particle, device 1) as if the collapse occurs only at the interaction with device 2. So, its dynamics is analogous to that of the previous section:

$$|\Psi\rangle \otimes |d\rangle = \sum_i a_i |\Psi_i\rangle \otimes |d\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |d_i\rangle \rightarrow |\Psi_1\rangle \otimes |d_1\rangle, \tag{1.12}$$

where the collapse of the last passage occurs at interaction with device 2 and with probability $P(1.5)$. The dynamics of the same system relative to an observer A interacting with device 1, rather than with the particle, is identical, the only difference being the event of the collapse, this time happening at interaction with A.

In general: the relative state of S with respect to O is the state of S whose collapse is taken to happen only at interaction with O.

Analogously, the dynamics of the relative state (particle, device 1, device 2) with respect to an observer B observing the outcome from device 2 is given by:

$$\sum_i a_i |\Psi_i\rangle \otimes |d_i\rangle \otimes |d'\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |d_i\rangle \otimes |d'_i\rangle \rightarrow |\Psi_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle, \tag{1.13}$$

with collapse occurring at interaction with B with probability $P(1.5) = P(1.8)$.

The left hand side of (1.10) is accessible only to whom has interacted with device 1, and therefore has already observed the outcome of the collapse with probability $P(1.5) = P(1.8)$. Observer B, in order to guess the outcome from device 2, uses the probability computed from his own relative quantum state. This might not work if the particle interacts with another quantum system before reaching device 2: the relative state will consider the particle still in superposition, and different outcomes of the interaction might interfere. This does not happen if the collapse has occurred. So, in the Copenhagen interpretation, relative quantum states are generally incomplete and don't give the most accurate information. Relational Quantum Mechanics, instead, suggests that the dynamics of any system S, which is not interacting with observer O, is unitary for O, until interaction with O. After that, every past interaction involving S must be updated. So, in this interpretation, relative quantum states do offer the most complete description of the world.

1.2.2 Different Observables

Now, let's suppose device 2 measures a different observable. We can write, for each eigenstate $|\Psi_i\rangle$ of the first observable, $|\Psi_i\rangle = \sum_j b_{ij} |\Phi_j\rangle$, where $|\Phi_j\rangle$ are the eigenstates of the second observable. Similarly, $|\Phi_j\rangle = \sum_i b^{-1}_{ji}|\Psi_i\rangle$ for the inverse transformation.

Let's consider a measurement on the first observable by device 1, followed by a measurement on the second observable by device 2.

The state evolution, if the collapse happens after measurement 1, is:

$$|\Psi\rangle \otimes |d\rangle \otimes |d'\rangle = \sum_i a_i|\Psi_i\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{1.14}$$

$$\sum_i a_i|\Psi_i\rangle \otimes |d_i\rangle \otimes |d'\rangle \rightarrow |\Psi_1\rangle \otimes |d_1\rangle \otimes |d'\rangle = \tag{1.15}$$

$$\sum_j b_{1j}|\Phi_j\rangle \otimes |d_1\rangle \otimes |d'\rangle \Rightarrow \tag{1.16}$$

$$\sum_j b_{1j}|\Phi_j\rangle \otimes |d_1\rangle \otimes |d'_j\rangle \rightarrow |\Phi_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle. \tag{1.17}$$

The probabilities of collapse are: $P(1.15) = |a_1|^2$, $P(1.17) = |b_{12}|^2$. $P(1.17)$ is the probability of obtaining $|d'_2\rangle$ given the outcome $|d_1\rangle$. The overall probability of obtaining the two outcomes is then given by: $P(1.15) \times P(1.17) = |a_1 b_{12}|^2$.

It is arbitrary to choose when outcome 1 happens, since the two devices do not interact with each other, and the dynamics of one is independent of the state of the other. If we ignore the collapse, the dynamics is completely unitary, and it's given by:

$$|\Psi\rangle \otimes |d\rangle \otimes |d'\rangle = \sum_i a_i|\Psi_i\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{1.18}$$

$$\sum_i a_i|\Psi_i\rangle \otimes |d_i\rangle \otimes |d'\rangle = \sum_{i,j} a_i b_{ij}|\Phi_j\rangle \otimes |d_i\rangle \otimes |d'\rangle \Rightarrow \tag{1.19}$$

$$\sum_{i,j} a_i b_{ij}|\Phi_j\rangle \otimes |d_i\rangle \otimes |d'_j\rangle \tag{1.20}$$

At this point, an observer can induce the first collapse by interacting first with device 1, obtaining:

$$\sum_{i,j} a_i b_{ij}|\Phi_j\rangle \otimes |d_i\rangle \otimes |d'_j\rangle \rightarrow \sum_j b_{1j}|\Phi_j\rangle \otimes |d_1\rangle \otimes |d'_j\rangle \tag{1.21}$$

with probability $P(1.21) = \sum_j |a_1 b_{1j}|^2 = |a_1|^2 = P(1.15)$, and ignoring the global phase $\frac{a_1}{|a_1|}$. Then, the observer can induce the second collapse by interacting with device 2:

$$\sum_j b_{1j}|\Phi_j\rangle \otimes |d_1\rangle \otimes |d'_j\rangle \rightarrow |\Phi_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle \tag{1.22}$$

with probability: $P(1.22) = |b_{12}|^2 = P(1.17)$.

Or vice versa, first interacting with 2, and then with 1:

$$\sum_{i,j} a_i b_{ij} |\Phi_j\rangle \otimes |d_i\rangle \otimes |d'_j\rangle \to \frac{1}{\sqrt{\sum_i |a_i b_{i2}|^2}} \sum_i a_i b_{i2} |\Phi_2\rangle \otimes |d_i\rangle \otimes |d'_2\rangle \tag{1.23}$$

with probability $P(1.23) = \sum_i |a_i b_{i2}|^2$. Then

$$\frac{1}{\sqrt{\sum_i |a_i b_{i2}|^2}} \sum_i a_i b_{i2} |\Phi_2\rangle \otimes |d_i\rangle \otimes |d'_2\rangle \to |\Phi_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle \tag{1.24}$$

with probability: $P(1.24) = \frac{|a_1 b_{12}|^2}{\sum_i |a_i b_{i2}|^2}$

The order does not matter, the final result will be $|\Phi_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle$ with probability $|a_1 b_{12}|^2$.

If the first observer does not induce the collapse, it might be a second observer to induce the collapse by interacting with the first one, who communicates the results. The first observer is in this case treated like a device which stores interaction information through entanglement, but they gain a definite memory only at interaction with the second observer. The two outcomes in this case happen simultaneously in one collapse:

$$\sum_{i,j} a_i b_{ij} |\Phi_j\rangle \otimes |d_i\rangle \otimes |d'_j\rangle \to |\Phi_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle \tag{1.25}$$

with probability $P(1.25) = |a_1 b_{12}|^2$.

Even in this case you can keep asking any question you want to observer 1, he will always behave as if the collapse had occurred when he himself had observed the outcome. He behaves based on his memory, independently on when it was created. He claims that he can deduce when the collapse happened, based on his own relative quantum states, which work as well as yours.

The order of the outcomes (moment 2) does not matter. It is only the order of the interactions (moment 1) which counts. If one is interested only on the probability of the final result, it might be more convenient to consider the collapse as happening at once, which is the point of view of the second observer. In the considered cases, there is no right answer to when the collapse happens, it is an arbitrary choice of frame of reference. There is no absolute causality of collapse. The causality of the interactions, however, matters.

1.2.3 Inverting the Order of the Interactions

Let's now invert the order of the interactions. First we have entanglement with device 2, then with device 1. If collapse happens after measurement we have:

$$|\Psi\rangle \otimes |d\rangle \otimes |d'\rangle = \sum_{i,j} a_i b_{ij} |\Phi_j\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{1.26}$$

$$\sum_{i,j} a_i b_{ij} |\Phi_j\rangle \otimes |d\rangle \otimes |d'_j\rangle \rightarrow |\Phi_2\rangle \otimes |d\rangle \otimes |d'_2\rangle = \tag{1.27}$$

$$\sum_i b^{-1}_{2i} |\Psi_i\rangle \otimes |d\rangle \otimes |d'_2\rangle \Rightarrow \tag{1.28}$$

$$\sum_i b^{-1}_{2i} |\Psi_i\rangle \otimes |d_i\rangle \otimes |d'_2\rangle \rightarrow |\Psi_1\rangle \otimes |d_1\rangle \otimes |d'_2\rangle \tag{1.29}$$

with probabilities $P(1.27) = |\sum_i a_i b_{i2}|^2$, $P(1.29) = |b^{-1}_{12}|^2$. The overall probability is: $P(1.27) \times P(1.29) = |\sum_i a_i b_{i2} b^{-1}_{12}|^2$.

If collapse happens only after interaction with device 2 we have the evolution:

$$|\Psi\rangle \otimes |d\rangle \otimes |d'\rangle = \sum_{i,j} a_i b_{ij} |\Phi_j\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{1.30}$$

$$\sum_{i,j} a_i b_{ij} |\Phi_j\rangle \otimes |d\rangle \otimes |d'_j\rangle = \sum_{i,j,k} a_i b_{ij} b^{-1}_{jk} |\Psi_k\rangle \otimes |d\rangle \otimes |d'_j\rangle \Rightarrow \tag{1.31}$$

$$\sum_{i,j,k} a_i b_{ij} b^{-1}_{jk} |\Psi_k\rangle \otimes |d_k\rangle \otimes |d'_j\rangle \rightarrow |\Psi_1\rangle \otimes |d_1\rangle \otimes |d'_2\rangle. \tag{1.32}$$

where the final outcome has probability $P(1.32) = |\sum_i a_i b_{i2} b^{-1}_{12}|^2$. As above, it does not matter whether the collapse happens in two stages or at once.

Let's compare these results with the original ordering of the interactions. If interaction with device 1 happens first, the final state is:

$$\sum_{i,j} a_i b_{ij} |\Phi_j\rangle \otimes |d_i\rangle \otimes |d'_j\rangle. \tag{1.33}$$

Inverting the order of the interactions we get:

$$\sum_{i,j,k} a_i b_{ij} b^{-1}_{jk} |\Psi_k\rangle \otimes |d_k\rangle \otimes |d'_j\rangle \tag{1.34}$$

We can rewrite (1.33) as:

$$\sum_{i,j,k} a_i b_{ij} b^{-1}_{jk} |\Psi_k\rangle \otimes |d_i\rangle \otimes |d'_j\rangle, \tag{1.35}$$

from which we see that in (1.33) the information about observable 1 is now uncorrelated with device 1: a third device measuring observable 1 will yield a different result from device 1. The particle has lost connection with the first device. Only the information of the interaction with device 2 is carried. In (1.34) we have the opposite.

1.3 Interpretation

The particle of the previous section, in the end, will be either in an eigenstate of the last observable being measured (if collapse happens), or it will be entangled with the last device it interacted with (if collapse does not happen). In the relational interpretation, the entanglement is information about an interaction having occurred. It is not additional information, since entanglement is part of the unitary evolution of the compound system. For the individual systems, instead, it's not entanglement which occurs, but a definite outcome, thus providing additional information for each of them. For the compound system the outcome of the interaction is not definite. This is equivalent to how the Copenhagen interpretation treats interactions between quantum systems. A relative state formulation, instead, does not distinguish between microscopic and macroscopic systems: everything is a quantum system, and the unitary evolution is broken only at the interaction with the observer who is describing the evolution. This is the case in Rovelli's Relational Quantum Mechanics [12] and in Everett's Theory of the Universal Wave Function [7], although in the latter case the breaking of the evolution is not real, but a description given by a subsystem which only has partial information about the universal wave function. For Everett the universal wave function is physical reality, and collapse is emergent: he adopts a Schroedinger type ontology. For Rovelli, any wave function description is emergent, and reality is in the interactions, each of which is a collapse relative to the interacting systems: he adopts a Heisenberg type ontology [13].

1.3.1 Wigner's Friend

Let's go back to von Neumann's analysis of measurement. Let's say that the measurement device remains in a superposition even after the interaction is completed. Von Neumann suggested that the collapse can happen at any point between the interaction and the observation of the device performed by an observer. Wigner proposed a thought experiment [16], in which even the observer, Wigner's friend, behaves like Von Neumann's device: he can store and communicate information, but cannot collapse the system. The scenario is the following. A device measures a quantum particle. Wigner asks his friend to observe the outcome from the device, and then communicate the result. His friend, acting like a second device, interacts with the system (particle, device), and gets entangled with a unitary process, but remains in a superposition until observation by Wigner. As soon as Wigner interacts with his friend, the system (particle, device, friend) collapses to a definite outcome, and his friend tells about observing one definite outcome on the device. Before interaction with Wigner, not only the device, but also his friend has no reality, in the sense of Heisenberg's notion of reality corresponding to definite outcomes created at the collapse (see appendix B and [13]): it is Wigner who, by measuring the system, generates memory in his friend and the device. This apparent solipsism of a special observer is why we are uncomfortable to treat observers as quantum systems. For example, Wigner does not suggest that his friend's experience may come from the content of his present memory which is created at once by Wigner, but as an evidence of collapse having already occurred independently of Wigner's observation. But in principle, the observer can be thought of as a very complex device, whose screen displays one particular outcome, which can be read in how he reacts to questions, how he behaves, and so on. Exactly like in (1.4) the compound system evolves to a superposition of the possible realities 'The particle is in an eigenstate with eigenvalue λ_i and the device reads λ_i', here the possible realities are 'The particle is in an eigenstate with eigenvalue λ_i, the device reads λ_i, and the observer reads λ_i on the device (and behaves accordingly)', without any reality, in the sense of Heisenberg, existing yet.

The way out of this problem, without invoking any mechanism of collapse, is the introduction, as already argued in this chapter, of the notion of relative quantum states. This possibility has been developed in two notably different ways, one which shifts to a continuous Schroedinger-type ontology (the quantum state, evolving in time between measurements, is the real content of the theory), and one which maintains a discrete Heisenberg-type ontology (the results of measurements are the real content of the theory, the quantum state being just a mathematical tool). The former is Everett's theory of the Universal Wave Function, which came to be known as the Many Worlds Interpretation, and the latter is Rovelli's Relational Quantum Mechanics. These two complementary types of ontology are briefly reviewed in Appendix B.

1.3.2 Everett's Approach Vs the Relational Approach

Before delving into relational quantum mechanics in the next chapters, I'll give a brief review of Everett's interpretation, which is at the origin of the notion of relative quantum state. His solution to the measurement problem is simple: the universal wave function never collapses. The collapse arises from the entanglement between the observer and the system, and it is relative only to the observer and the system. Wigner describes his friend in a superposition of states, and when he asks for the outcome his state gets entangled with the state of his friend. According to Heisenberg's interpretation, which renounces to a physical description of the system before measurement [8], there would be no physical reality without collapse, the wave function being only a mathematical object for the calculation of probabilities. Everett, on the contrary, claims that the reality is not at the interaction between systems, but during the unitary evolution. The interactions between subsystems appear non-unitary because they are described from the point of view of one of the interacting subsystems, which cannot give a complete description of its own state. But all the outcomes actually exist, even if they are not interacting with each other, so one Wigner has no knowledge of the other Wigner. So, while there exists a universal, never-collapsing state, our description is constrained to only relative quantum states, which are only a partial description of reality.

'Everett's lasting contribution to the understanding of quantum theory', according to Rovelli [13], is understanding that we describe reality in terms of relative quantum states. Wigner friend's reality is not in the hands of Wigner, as Copenhagen would imply if we dare describe his friend as a quantum system. The interaction with the measurement device collapses the state relatively to Wigner's friend, who experiences a definite outcome. But Wigner has no knowledge of the definite outcome until interaction with his friend, and therefore the state (particle, device, friend) didn't collapse from his point of view. The two observers will agree on the final outcome, because they get entangled, but they disagree about the moment of collapse. Collapse of B is relative to A, and it happens when A and B interact. So, variables take values at interactions, but the interaction is described as a unitary process in which all the interacting systems agree about the value taken at the interaction, but all possible agreements, one for every possible value, exist in superposition, until we interact with the systems. Some observer O who hasn't interacted with us describes our dynamics as unitary, involving all possible paths interfering with each other, until we interact with O, who gains information about the outcome we observed. O's description is also a valid physical point of view.

Relational Quantum Mechanics claims that this relativity of dynamics is a consistent view within a Heisenberg-type ontology, without requiring the ontology of the unitary evolution. A hypothetical super-observer outside our universe, who interacted with it in the past, describes us with a universal wave function, and can predict that interactions happen, without being able to predict the outcome. From the point of view of Everett's interpretation he must conclude that every outcome happens, because the wave function, which is real, describes them all simultaneously. When he interacts with our universe, he will obtain different non-interacting outcomes, all parts of a super-universal wave function, which is the actual universal wave function. No one has access to the actual universal wave function, which cannot be known, and our description is bound only to relative states. From the point of view of Relational Quantum Mechanics, instead, the super-observer must conclude that some outcomes happen for some observers, but he does not know the result, until he interacts with them, at which point he inevitably gets entangled, from the point of view of a super-super-observer, and can update his description of us, while the super-super observer cannot yet.

In Relational Quantum Mechanics it does not matter when outcomes are created. Every point of view is equivalent. The quantum state of S relative to O represents the best O can know about the values of the outcomes created at the interactions between the subsystems of S. There are no multiple outcomes. Although it is not possible to say whether only one outcome exists, physics always involves only one outcome, so the possibility of the existence of different outcomes is a metaphysical problem. Relational Quantum Mechanics cannot say anything about the existence of alternative realities in which different outcomes have occurred. In Everett's interpretation we can't have knowledge of other outcomes either, but we have to accept their existence because we claim that the universal function is real, and it is such a claim which solves the measurement problem, which is a physical problem. Relational Quantum Mechanics solves the measurement problem without needing to invoke the reality of the wave function, and thus without needing the existence of different outcomes. Which might of course exist, but only to solve one's eventual metaphysical problems. They need not appear in the physics.

Chapter 2

Independent Measurements

In the previous chapter we analysed consecutive measurements, and concluded that interaction with the second device which measures a different observable erases the memory of interaction with device 1, so that if we use device 1 again we will find a different result from before.

In this chapter we consider independent measurements on two particles which have previously interacted. Once the interaction happens, the two particles get entangled. They have information about each other, so, in relational language, an outcome is realised for them. For an external observer, however, the interaction is unitary, and no outcome is realised, until they interact with one of the two particles. Only at that point, a definite outcome is found on the entangled state. This process is an exchange of information between the observer and the particle, which contains information about the interaction with the other particle. So, now, the observer is also entangled with the system, and they all agree on the first interaction, which is in the past of all of them. The measurement on the whole entangled system is completed only when we interact with a system which measured also the other particle. Depending on the observable measured on the distant particle, we can evaluate the probability of what we will get at interaction with such system. Such interaction is a quantum mechanical process, and represents a moment of collapse for the interacting systems.

2.1 Entangled State

Let two particles, A and B, interact at O. Then, particle A travels to Alice at event A, and particle B travels to Bob at event B. A performs a measurement with device A, in the state $|d\rangle$, on particle A, in the state $|\Psi\rangle$, and Bob performs a measurement with device B, in the state $|d'\rangle$, on particle B, in the state $|\Psi'\rangle$. Alice and Bob are far enough, and perform their measurements quickly enough, so that they are independent. Then they communicate their results to Charles at C (or, equivalently, they meet at C where they interact to communicate their results). Like in the previous chapter we can ignore the unitary evolution of the isolated particles between measurements, since it commutes with the other interactions and can be added at any time after the experiment, or erased by the devices themselves with an inverse transformation.

2.1.1 Same Observable

Let's consider a measurement of the same observable. Let Alice perform the measurement on the basis $\{|\Psi_i\rangle\}$, and Bob on the basis $\{|\Psi'_i\rangle\}$. Let $|d_i\rangle$ correspond to the outcome 'device A reads $|\Psi_i\rangle$', and $|d'_i\rangle$ correspond to the outcome 'device B reads $|\Psi'_i\rangle$'.

From the point of view of Alice, the particles first interact and get entangled at O, then particle A interacts with device A, and information about the interaction at O is acquired:

$$|\Psi\rangle \otimes |\Psi'\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{2.1}$$

$$\sum_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d_i\rangle \otimes |d'\rangle \rightarrow |\Psi_1\rangle \otimes |\Psi'_1\rangle \otimes |d_1\rangle \otimes |d'\rangle \Rightarrow \tag{2.2}$$

$$|\Psi_1\rangle \otimes |\Psi'_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \rightarrow |\Psi_1\rangle \otimes |\Psi'_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \tag{2.3}$$

where $P(2.2) = |a|^2$. The state at A, after interaction with Alice, is relative to both Alice and particle A. Particle B does not share the same quantum state in general, since it has not interacted with Alice, and the common information it shares is only about what happened at O. If we accounted for the quantum properties of Alice, they would hold some value for particle A, but no definite value for particle B. Values are realised at interactions, and are a property of the interaction, affecting only the systems which have interacted. Bob and particle B are not yet affected by the interaction between Alice and particle A, so they describe the interaction unitarily, with no additional information, until event C. The quantum state will be shared only at C. Alice meeting Bob, or Charles receiving a signal from them, at C, is a quantum event. In this case, though, since Alice and Bob measured the same observable, the probability of the second collapse is $P(2.3) = 1$. This is the evolution of the quantum state relative to the path OAC. The probability of the full evolution is $P(A) = P(2.2) \times P(2.3) = |a|^2$.

Analogously, for the path OBC we have:

$$|\Psi\rangle \otimes |\Psi'\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{2.4}$$

$$\sum_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d\rangle \otimes |d'_i\rangle \rightarrow |\Psi_1\rangle \otimes |\Psi'_1\rangle \otimes |d\rangle \otimes |d'_1\rangle \Rightarrow \tag{2.5}$$

$$|\Psi_1\rangle \otimes |\Psi'_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \rightarrow |\Psi_1\rangle \otimes |\Psi'_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \tag{2.6}$$

with probabilities $P(2.5) = |a|^2$, $P(2.6) = 1$, and overall probability $P(B) = |a|^2$.

For Charles, who receives the information about the independent paths simultaneously, there is only one collapse:

$$|\Psi\rangle \otimes |\Psi'\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{2.7}$$

$$\sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d_i\rangle \otimes |d'_i\rangle \to |\Psi_1\rangle \otimes |\Psi'_1\rangle \otimes |d_1\rangle \otimes |d'_1\rangle \tag{2.8}$$

with probability $P(C) = P(2.8) = |a_i|^2$

2.1.2 Different Observables

Let's now consider Alice and Bob measuring different observables. Let Alice do the same measurements as before, while Bob performs a measurement on the basis $\{|\Phi'_i\rangle\}$, and let now $|d'_i\rangle$ denote 'device B reads $|\Phi'_i\rangle$'.

Alice describes the evolution as:

$$|\Psi\rangle\otimes|\Psi'\rangle\otimes|d\rangle\otimes|d'\rangle \Rightarrow \sum_i a_i|\Psi_i\rangle\otimes|\Psi'_i\rangle\otimes|d\rangle\otimes|d'\rangle = \sum_{i,j} a_i b'_{ij}|\Psi_i\rangle\otimes|\Phi'_j\rangle\otimes|d\rangle\otimes|d'\rangle \Rightarrow \tag{2.9}$$

$$\sum_{i,j} a_i b'_{ij} |\Psi_i\rangle \otimes |\Phi'_j\rangle \otimes |d_i\rangle \otimes |d'\rangle \to \sum_j a_1 b'_{1j} |\Psi_1\rangle \otimes |\Phi'_j\rangle \otimes |d_1\rangle \otimes |d'\rangle \Rightarrow \tag{2.10}$$

$$\sum_j a_1 b'_{1j} |\Psi_1\rangle \otimes |\Phi'_j\rangle \otimes |d_1\rangle \otimes |d'_j\rangle \to |\Psi_1\rangle \otimes |\Phi'_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle \tag{2.11}$$

where $P(2.10) = \sum_j |a_1 b'_{1j}|^2 = |a|^2$, $P(2.11) = |b'_{12}|^2$, and overall probability $P(A) = |a_1 b'_{12}|^2$.

Bob describes the evolution as:

$$|\Psi\rangle \otimes |\Psi'\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d\rangle \otimes |d'\rangle = \sum_{i,j} a_i b'_{ij} |\Psi_i\rangle \otimes |\Phi'_j\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{2.12}$$

$$\sum_{i,j} a_i b'_{ij} |\Psi_i\rangle \otimes |\Phi'_j\rangle \otimes |d\rangle \otimes |d'_j\rangle \to \frac{1}{\sqrt{\sum_i |a_i b'_{i2}|^2}} \sum_i a_1 b'_{i2} |\Psi_i\rangle \otimes |\Phi'_2\rangle \otimes |d\rangle \otimes |d'_2\rangle \Rightarrow \tag{2.13}$$

$$\frac{1}{\sqrt{\sum_i |a_i b'_{i2}|^2}} \sum_i a_1 b'_{i2} |\Psi_i\rangle \otimes |\Phi'_2\rangle \otimes |d_i\rangle \otimes |d'_2\rangle \to |\Psi_1\rangle \otimes |\Phi'_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle \tag{2.14}$$

where $P(2.13) = \sum_i |a_i b'_{i2}|^2$, $P(2.14) = \frac{|a_1 b'_{12}|}{\sum_i |a_i b'_{i2}|^2}$, and overall probability $P(B) = |a_1 b'_{12}|^2$.

Charles describes the evolution with only one collapse:

$$|\Psi\rangle \otimes |\Psi'\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \otimes |d\rangle \otimes |d'\rangle = \sum_{i,j} a_i b'_{ij} |\Psi_i\rangle \otimes |\Phi'_j\rangle \otimes |d\rangle \otimes |d'\rangle \Rightarrow \tag{2.15}$$

$$\sum_{i,j} a_i b'_{ij} |\Psi_i\rangle \otimes |\Phi'_j\rangle \otimes |d_i\rangle \otimes |d'_j\rangle \to |\Psi_1\rangle \otimes |\Phi'_2\rangle \otimes |d_1\rangle \otimes |d'_2\rangle \tag{2.16}$$

with probability $P(C) = P(2.16) = |a_1 b'_{12}|^2$. The order of the interactions does not matter, because the measurements are independent, not consecutive. Charles is the product of two independent paths, Alice's and Bob's.

2.1.3 Relativity of Evolution

When did the collapse happen? It depends on the state you choose to represent the evolution which brings to C. Either at A and C, or at B and C, or at once at C. Or at O, if you treat the particles as observers: Relational Quantum Mechanics does not make any distinction between systems and observers. The sequence of collapse for independent measurements is a convention, like it is any sequence of causally disconnected events. Of course, while A and B are independent measurements, C depends on A and B (it is consecutive to them).

The two paths are independent and do not influence each other, until they meet at C. Interacting at C is a quantum event which creates an outcome whose probability is given by the rules of quantum mechanics. Both observers can consistently use quantum mechanics to make predictions. From Alice's perspective a definite past (or memory) for Bob is created at C, and from Bob's perspective a definite past for Alice is created at C. This by itself is not just something of quantum mechanics, but of the relativistic paradigm, since it is due to the relativity of simultaneity: it is meaningless to ask whether Alice and Bob exist simultaneously, or when Bob exists for Alice, since the notion of simultaneity is not physical. You come to knowledge of the past of one system only by interacting with it, and it is meaningless to locate the past of the system somewhere in your time axis. Its past will lie on independent time axes.

What is instead remarkable in quantum mechanics is that the past measurements of Bob are correlated with the past measurements of Alice, with such a correlation not being present at O in the common past of Alice and Bob. We can say that the same information originates at two spatially disconnected events. The knowledge of Alice and Bob is updated independently, but they arrive to the same result at C. This is what happens if we want to believe to both perspectives and put them on the same map. But at this point, collapse does not coincide with an update of information of one observer, since it the state of Bob had already changed in the past independently from Alice, and vice versa for Bob.

2.2 EPR Argument and non-Locality

Let's try to interpret the results of the previous section. At O there is no information about the result at A, and at A there can be information about O, but no information about the result at C (unless B is known to perform a measurement of the same observable, in which case the entanglement gives complete information about the outcome at B given the outcome at A). Only C can have complete information about O and A. This makes sense, since OAC is a causal path, i.e. it's time oriented. But this is true also for the path OBC. If standard quantum mechanics is complete, information is created at the time of measurement, or else it would be carried by hidden variables not present in the theory. In particular, there is at least one observable whose measurement is guaranteed to yield a particular result with certainty. We can say that we have complete information about this observable even before measurement. In the words of Albert Einstein, Boris Podolsky, and Nathan Rosen [5], such observables represent 'elements of reality'. We can say that elements of reality correspond to information carried by the particle. Since the state of a quantum system is an eigenvector, in the Hilbert space, of such an observable, we can compute the probabilities of outcomes for other observables with the Born rule. If quantum mechanics is complete, complete information of such an observable, i.e. it being an element of reality, precludes definite knowledge of observables for which the state vector is not an eigenstate. This is the heart of Heisenberg's uncertainty principle.

2.2.1 EPR Argument

The EPR argument was proposed in [5] by Einstein, Podolski, and Rosen. They argue that entanglement leads to correlations which are not explainable with the local dynamics of quantum information: more information (more elements of reality) is needed to obtain such correlations. In fact, consider the quantum states of the previous section, which I now write, as they did, ignoring the entanglement with the devices. Suppose compatible observables are being measured. We can write the dynamics, like we did for Charles' point of view, as:

$$|\Psi\rangle \otimes |\Psi'\rangle \Rightarrow \sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle, \tag{2.17}$$

$$\sum_i a_i |\Psi_i\rangle \otimes |\Psi'_i\rangle \to |\Psi_1\rangle \otimes |\Psi'_1\rangle \tag{2.18}$$

Passage (2.18) can be taken to happen at A, at B, or as well at C. The original authors, though, don't consider the relativity of the moment of collapse, and exclude the perspective for which any outcome is created at C. For them the first system collapses at A, and the second one at B. Alice must conclude, at A, that observable 1 is an element of reality at B, since Alice's measurement can't have the effect of influencing the reality at B. But Quantum Mechanics does not describe such an element of reality prior to Alice's measurement. A measurement on entangled observables is enough to show the incompleteness of quantum mechanics: additional information, having origin in the common past, is needed to correlate the two distant outcomes. But their paper proceeds to directly show that, by admitting the entangled observable to be an element of reality, it is in fact Heisenberg's uncertainty principle which is contradicted. There are states, like the 'isotropic states', which are simultaneously entanglement of incompatible quantities (quantities whose operators do not commute). Let $|\Psi_i\rangle$ represent the eigenstates of some operator P, and $|\Phi_i\rangle$ of some operator Q, incompatible with P (P and Q do not commute). We have seen above that the entangled quantity must be an element of reality before the end of the interaction. We can write:

$$\sum_i a_i|\Psi_i\rangle \otimes |\Psi_i'\rangle = \sum_{i,j,k} a_i b_{ij} b_{ik} |\Phi_j\rangle \otimes |\Phi_k'\rangle,$$

and if $\sum_i a_i b_{ij} b_{ik} = c_j \delta_{jk}$, then

$$\sum_i a_i|\Psi_i\rangle \otimes |\Psi_i'\rangle = \sum_i c_i |\Phi_i\rangle \otimes |\Phi_i'\rangle,$$

an example in $C^2 \otimes C^2$ being the isotropic state $\frac{|00\rangle+|11\rangle}{\sqrt{2}} = \frac{|++\rangle+|--\rangle}{\sqrt{2}}$, where we used the common notation for the 2-state quantum systems. Then both P and Q are simultaneously elements of reality, which contradicts the uncertainty principle.

2.2.2 Non-Locality and Relative States

Of course, another possibility is that quantum mechanics is indeed complete, but information does not have a common origin. Two independent measurements will be found to have created the same information, no matter how far apart and isolated they are. In the literature, this is commonly referred to as a type of non-locality. It is a very particular type, since it has nothing to do with communication or action at a distance: Alice cannot control the result of her measurement to communicate with Bob, and her performing the measurement has no influence on the local dynamics of Bob. What about the original conclusion of the EPR paper, on incompleteness of quantum mechanics? Maybe there is some completion of it which explains the correlations without non-locality. This is the question to which John Stewart Bell gave a definite answer in [1]. It turns out that the correlations are too strong to be carried locally: not only quantum mechanics, but any completion of it is EPR non-local.

The EPR has been further debated from a relational perspective in [15], in which the authors argue that the EPR correlations do not entail any form of non-locality in a relational context: 'From the relational perspective, the apparent "quantum non-locality" is a mistaken illusion caused by the error of disregarding the quantum nature of all physical systems.' By locality they mean 'the principle demanding that two spatially separated events cannot have instantaneous mutual influence.' The term 'instantaneous' is redundant, as the paper can be fully understood in a relativistic paradigm in which the word has no meaning outside of coordinate conventions, and the events in question can only be said to be 'spatially separated'. On the other hand, such word can be meaningfully used to denote an influence which is discontinuous in spacetime, connecting only the two events but no other event in the middle. But even this notion is not needed. This principle of locality, though, is valid also in the Copenhagen interpretation in which the two outcomes originate at causally disconnected events, as argued above. To avoid confusion I will refer to the type of non-locality found in the EPR correlations as EPR non-locality. The remarkable aspect of relational quantum mechanics is that exchanging information is itself a quantum event, so the past of a system must be considered an outcome for another system. Such outcome, being an update of information relative to some observer, happens at causally connected events, because every other event is treated like a present outcome: 'Observer A can of course measure the state of B, but only when A is back into causal contact with B. This is, needless to say, in the future light-cone of A, and therefore poses no challenge for locality. In other words, Einstein's reasoning requires the existence of a hypothetical super-observer that can instantaneously measure the state of A and B. It is the hypothetical existence of such nonlocal super-being, and not QM, that violates locality.' It is therefore the premise of a shared state of the universe which is wrong.

Due to the relativity of simultaneity there is no absolute notion of 'time of collapse' for a quantum state describing two distant particles. Relativity only allows to talk about 'events of collapse', and there is at most one for each subsystem. If the events of collapse are Alice's and Bob's causally disconnected measurements, they happen in no particular order, and if one insists that there is some action at a distance going on, it is impossible to tell which one acts on the other: as shown in the previous section, the sequence of collapse used for calculations, whether Bob's outcome given Alice's, or Alice's outcome given Bob's, is completely arbitrary. Relational Quantum Mechanics further suggests that not only there is no absolute sequence of collapse, which is simply a requirement of relativity for causally disconnected events: it also suggests, as discussed in the previous chapter, that even the events themselves of collapse are relative, and they can be made to happen in sequence by choosing causally connected events of collapse: you can choose the path to follow, whether Alice (collapse at A, and then at C) or Bob (collapse at B, and then at C), who give two different descriptions of the evolution, as we have seen above, but none of them is privileged. In one case, Alice interacting with Bob creates Bob's memory, according to the rules of quantum mechanics. In the other case, Bob interacting with Alice creates Alice's memory according to the rules of quantum mechanics. Both descriptions are compatible with observation, and they don't require to think of entangled states as exchanging information. Such information can always be thought of as being created at interaction, as it is the whole past of a system which is created. Non-locality arises when you chose two causally disconnected events of collapse, A and B, choice which is totally arbitrary and does not correspond to any observer[1]. Collapse is just an update of information.

From a Relational Perspective, entanglement correlations reinforce the relativistic paradigm: not only the sequence of events is arbitrary, but even their content is. In classical physics, what Bob measures at B is absolute: its location is relative to a choice of coordinates, but the event B is the same in every frame, and everyone agrees that Bob observed such outcome at such event. Now, relational quantum mechanics can't even guarantee the existence of B, nor the ontology of events[2]: outcomes are not properties of the events, and the events can only be instrumental entities in the quantum theory, like it is their 'location' in classical physics. Coordinates are names for the events, and events are sets for the outcomes. Two events may or may not happen at the same time, depending on the coordinate frame, and two outcomes may or may not happen at the same event, depending on the dynamical frame. Quantum mechanics seems to allow different relative dynamical descriptions. In relational quantum mechanics outcomes are the ontology, and they can be arbitrarily ordered in a map of events.

O interacting with S is an event which may or may not correspond to any outcome, depending on the map you choose. An observer P who interacts with S and O gains knowledge of an outcome, which can be interpreted as the outcome of the interaction between O and S, in which case he is describing the frame of O or S, or as the outcome of the interaction between P and S, or O and P (depending on which of the two happens first), in which case he is describing the frame of reference of P. In particular, in an observer's perspective, outcomes can be chosen to happen on their own time axis, the rest of spacetime being a projection of outcomes. We conclude with this consideration: the space of outcomes can be further restricted, by passing from an observer, as made of many causally connected events, to a single event: the past itself of an observer can be thought of as a projection of present outcomes, all happening at once in a single event. If P interacts with O twice, he can't tell whether the first interaction with O, present in his own memory during the second interaction, already existed or was created at the second interaction. It has no meaning for Alice to ask whether her own past exists independently of her present memory, exactly like it has no meaning to ask whether Bob's past, now fully accessible after she has interacted with him, exists independently of her present memory. Wigner's friend, whom I introduced in the previous chapter, can't trust his own memory because it might be a property of his interaction with Wigner: all the outcomes of his past happen now. All the ontology, i.e. the measurement outcomes, can

[1]In the next chapter I will define an observer as a set of causally connected events.

[2]Given that the ontology is in the outcomes.

be thought of as happening at a single event, from which all the structure emerges as a map of outcomes, following some arbitrary criteria of ordering. This is the heart of Antistructuralism, which I will present in the following chapter.

Chapter 3

Antistructuralism

The past can be seen as a projection of present information. Whether it has some ontology, which I don't question here[1], has no physical relevance. When Alice interacts, she updates her memory. Even her own past is now just a projection of her present. She projects herself, and the systems she interacts with, onto the past. When she interacts with Bob, she and Bob project each other's present memory into the past. In the Copenhagen interpretation, a quantum system has no definite history until interaction with a macroscopic system. In the Relational Interpretation, there is no definite history relative to A, until interaction with A. Bob has no past until he interacts with Alice. Or, in 'Bob's frame', Alice has no past until she interacts with Bob. Both perspectives are equally valid. You can choose the evolution you prefer, the order of outcomes does not matter. In one case interaction generates Bob's memory, in the other case it generates Alice's memory. Asking who has ontology before interaction is physically meaningless. Of course, in the relativistic paradigm, there is not only one 'before', but one for Alice and one for Bob, and it is meaningless to ask whether they exist simultaneously. Non-interacting systems can be seen as a generalisation of causally disconnected systems. It makes no sense to ask whether they exist simultaneously before interaction, and the present can project all its information into one observer of its choice, rather than in a whole light cone. It is quantum mechanics which suggests this 'shrinking'of the causal structure into an arbitrary world line.

In this chapter I present my view of 'antistructuralism', which treats structure as

[1] I personally can't see why realising the impossibility of proving that systems exist independently of their interactions with you would lead to solipsism. It's like saying that the impossibility of measuring your motion in the aether would lead everyone into thinking they are stationary in the aether. In fact, exactly like the latter impossibility leads to the relativistic paradigm, in which every observer is equivalent, the former impossibility should lead to the relational paradigm, in which multiple evolutions of variables are equivalent. Relationality, in my view, leads to the loss of individuation, which is incompatible with solipsism, which assumes an individuation, even if the only possible one. Why shouldn't Alice treat the ontology of Bob like she treat hers, if he acts and makes exactly the same points as her? We will come back to individuation in the section about Ontic Antistructuralism.

emergent and not fundamental. In antistructuralism one needs not preserve agreement on past events. The past of a systems can also be considered as emerging at the interaction with the observer. Events are an arbitrary choice of structure, and an absolute description of the past of a system in terms of absolute events is meaningless. The information acquired at interaction can't be distinguished from information existing in the other system 'before' interaction with it, or as being created in that moment like in a quantum measurement. Structures are arbitrary choices of ordering such information, so there is no absolute notion of 'independent existence' of information into given events. Sets and relations are chosen arbitrarily, and different choices of frame can disagree on the distribution of information in spacetime. To the question whether there must be a unique choice of distribution of information, i.e. whether every frame must be made to agree on the events, antistructuralism gives a negative answer, and Relational quantum mechanics is an example of theory which can be formulated for frames which disagree on past events. Information about one outcome can be chosen to emerge in two distinct events, or it can be chosen to emerge at one event.

While in classical physics you can both make a map of events about which every observer agrees and preserve EPR locality, this is not the case in quantum mechanics, as shown in the previous chapters: you can either choose Alice's map or Bob's map. Both maps lead to the same present state, and this is the only thing which needs to be consistent. So, in an antistructuralist paradigm, the past has no ontology, and it serves just as an arbitrary map of present information. Of course there are also interesting philosophical considerations, to which I dedicate the last section.

3.1 Antistructuralist Picture of Causality

Antistructuralism treats past events as sets of informational content present at some event. I will treat informational content as a general primitive concept, whose meaning depends on the theory which is being described. I think of it as the ontology of the theory, as the structure is meant to represent the instrumental elements. So, in the case of an Heisenberg picture of quantum mechanics, it is information about measurement outcomes. In classical physics it can be the events themselves, since, unlike in relational quantum mechanics, any observer agrees on what happens at each event.

We can schematise a measurement by a series of events, each one corresponding to a particular counting. Each one of these events, though, must 'contain' memory about the previous counting, or else it would not be possible to keep counting. Measurement automatically comes with an arrow of time, towards the direction of increasing information about counting. Each event only knows about the events describable by its own informational content alone, and these events represent the 'informational past'. The events need not lie in an absolute structure, and if they do, we wouldn't be able to tell anything about their absolute order. We can only *decide* an arbitrary order for them, based on some given convention, for instance according to causality relations.

3.1.1 Causal Connection

Let's give an approximate description of measurement in terms of events, focusing on their informational content. We will schematise a measurement as some generic counting for simplicity.

Event A: I count 1. (I have no memory of having counted 2).

Event B: I count 2. (I have memory of having counted 1).

The information contained in A is just about A, while the information contained in B is about both B and A: the information which we focus on during a measurement is not symmetric, fact which allows one to order the events, defining an order of time. At B I can describe A, but I cannot say whether the content of A also exists 'outside' of B: if A is the event of interaction between Wigner's friend and the particle, and B is the event of interaction between Wigner and his friend, then A exists in the past ('outside of the present') of Wigner friend's frame, but it does not exist in the past of Wigner. And Wigner cannot make in general any consistent map of events outside of his past light cone. At B we are forced to choose one of the two observers. So, being able to reconstruct a past, doesn't mean that it must exist for every observer: in fact, the relational EPR discussed in the previous chapter suggests that only a common agreement on the outcomes (the informational content) can be saved in quantum mechanics, but there can be no common agreement on when they occur, which is, on their separation into distinct events. All the knowable content is in the event itself, and it has no meaning to talk about the events existing independently in some given structure 'outside'. At B I place A in the past because A lacks some information. If there was a universal time direction in which B happened before A, my system would be losing information, so I couldn't measure such direction: in fact I would measure the wrong one! Causality is, in an antistructuralist picture, a specific way of ordering present information. Antistructuralism aims to describe events in terms of their informational content, and structures play an instrumental role. And even the way of gathering of information in events is not unique.

We shall not start from defining events as points at which information is stored, since we want to start from information itself. We define an event, relative to a particular type of information, as identifying a particular content of such information. We define a clock, or observer, relative to such information, as a set of events which can all be ordered in terms of their informational content, with the relation $A \subseteq B$, meaning 'all the information of A is contained in B'. A is called the past of B, and B is called the future of A. The informational content is a primitive concept in a general formulation, and must be associated to concrete data via experience.

Events which define a clock are said to be causally connected, their connection being realised through the causal relation $\subseteq$. Formally, regarding information as a primitive concept, an event is a set of information, and a clock, or observer, is a set of events all related by the set inclusion relation.

We sometimes use the adjective ‘informational’, e.g. informational event, informational clock, informational past, and so on, when we want to differentiate the definitions just given from the usual ones referring for instance to events in a Minkowski spacetime.

3.1.2 Relating Different Observers

After relating causally connected events, we now want to see how we can relate causally disconnected events. This relates to the problem of relating different observers.

Event A_1: Alice counts 1. Events in memory: A_1.

Event A_2: Alice counts 2. Events in memory: A_1, A_2.

Event B_1: Bob counts 1. Events in memory: B_1.

Event B_2: Bob counts 2. Events in memory: B_1, B_2.

Event C: Charles receives signals from both A_2 and B_2 (Or, equivalently, Alice and Bob meet at C to communicate their results). Events in memory: A_1, A_2, B_1, B_2.

Formally, writing informational content in cursive capital letters, we have the 5 events:

$A_1 = \{\mathcal{A}_1\}$

$A_2 = \{\mathcal{A}_1, \mathcal{A}_2\}$

$B_1 = \{\mathcal{B}_1\}$

$B_2 = \{\mathcal{B}_1, \mathcal{B}_2\}$

$C = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2\}$

and the 2 observers who meet at C are defined by:

$\{A_1, A_2, C\}$

$\{B_1, B_2, C\}$

Recalling from what has been said in the previous paragraph, the mere existence of these unordered events is enough for Alice to believe that time flows from A_1 to A_2, even though she can't prove that without self-reference. Similarly, Bob believes that time flows from B_1 to B_2. We identified the direction of time at one event based on its informational content alone, not in relation to a given structure outside the event. The two ways time flows, for distant A and B, are completely independent. We see that there is no notion of simultaneity. There is no order between the events of A and B, as they are disconnected. Alice and Bob only interact at C: we can say, in our informational picture, that two observers interact at the events at which their information converge. If Alice and Bob meet at C, all their future measurements, even when they separate again, will be in the future of all A_1, A_2, B_1, B_2.

Let now Alice and Bob separate again after having met at C, or equivalently let them receive a signal from Charles at C, so that they update their information about each other (We could also omit Charles from our discussion and let Alice and Bob communicate directly, in which case we simply ignore C in the following discussion). Since they are

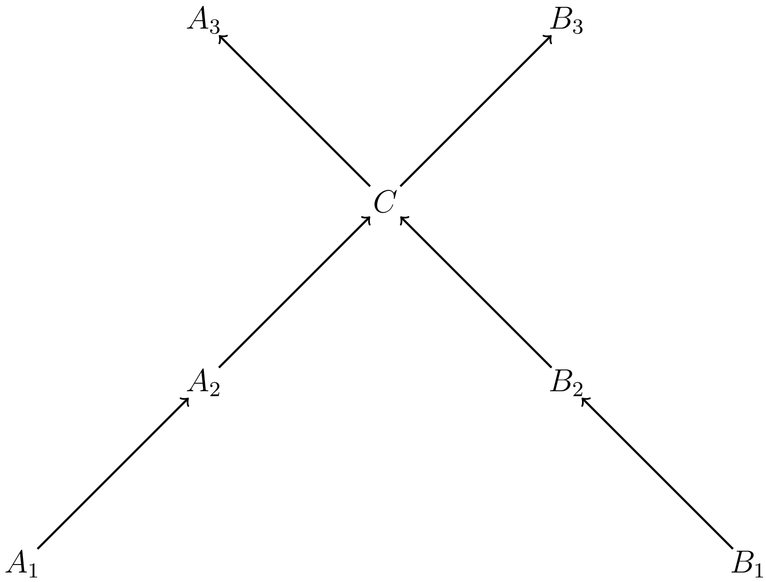

Figure 3.1: Example of causal diagram.

clocks, let them keep counting (or else they would simply end up with the same content as C). We have 2 additional events:

$A_3 = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, \mathcal{A}_3\}$

$B_3 = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\}$

We have 2 observers at A_3: (C can be omitted from the sets since in this example Charles doesn't add new information):

$\{A_1, A_2, C, A_3\}$

$\{B_1, B_2, C, A_3\}$

and 2 observers at B_3:

$\{B_1, B_2, C, B_3\}$

$\{A_1, A_2, C, B_3\}$

The situation can be represented in the diagram in figure 3.1, where the arrows point towards increase of information, identifying the flow of time. The observers are obtained by following the arrows[2].

From a mathematical point of view, the framework we obtained offers a description in terms of sets and relations, and it is, therefore, a structure. The antistructuralist element of our formulation is in emphasising that the structure we created is an arbitrary, not absolute criterion of ordering informational content.

[2]Quick remark: I said observers 'at' a particular event, but notice that the 'locations' are already identified by the events in the set, and they don't come from any additional structure. An observer 'at' E is simply any observer containing the event E.

3.1.3 Spatial Connection

While a time measurement, as already discussed, is a counting of events on the same clock, spatial distance is a relation between causally disconnected events. At a particular time we have some information, and we define our past by progressively removing information, as discussed in the first section. We can also remove information in a non-progressive way, so to obtain causally disconnected events.

Consider a set of events which are all causally disconnected. If, *in a given spacetime*, their union gives an event which is in the given spacetime, it is a 'spatially connected region'. In this perspective, spatial connection can only be defined as a projection of the information contained at one particular event. If we are given an arbitrary set of causally disconnected events, in general their union may not represent any physical event.

In Minkowski's spacetime, the definition just given defines three-dimensional regions as bases of 4-D light-cones in which the information of the events at the tip is projected. A discrete example is $\{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$, for which we have 5 distinct spatial regions[3]:

$\{\{\mathcal{A}\}, \{\mathcal{B}\}, \{\mathcal{C}\}\}$
$\{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{C}\}\}$
$\{\{\mathcal{A}\}, \{\mathcal{B}, \mathcal{C}\}\}$
$\{\{\mathcal{A}, \mathcal{C}\}, \{\mathcal{B}\}\}$
$\{\{\mathcal{A}, \mathcal{B}, \mathcal{C}\}\}$ (the trivial region at the event itself)

Any two elements of a spatially connected region are said to be 'spatially connected'. Informationally, a spacetime is 'completely connected' if any pair of causally disconnected events is spatially connected (e.g. Minkowski's spacetime). Intuitively, if any pair of events always lies in the past of some future event, i.e. if two observers can always potentially meet each other in their future: no information is irremediably lost.

E.g., in the given spacetime $\{A = \{\mathcal{A}\}, B = \{\mathcal{B}\}, C = \{\{\mathcal{C}\}, \{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}\}$, A and C are not spatially connected. Intuitively they belong to separate worlds, each one accessible from B. Observer at B ends up in only one of the two worlds, and can't go back. There is no event at which you can have knowledge of both worlds as they separate: spacetime is informationally disconnected. To restore connection we must add the event $\{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$, so that now the two worlds are spatially connected.

If two events are causally connected or spatially connected, they are spacetime connected. A spacetime is said to be completely connected if and only if all the events are spacetime connected.

We can think of a particular type of connected spacetime, in which there exists an event which has information of all the other events (intuitively, every observer converges at a common point in the future). In this case, we say that such a spacetime is informationally closed. Otherwise it is open. E.g., Minkowski is open, but completely connected.

[3]Recall that the events are sets of informational content, denoted by cursive capital letters, so regions are sets of sets. For instance $\{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{C}\}\}$ means the region containing the two events $\{\mathcal{A}, \mathcal{B}\}$ and $\{\mathcal{C}\}$.

3.2 Structure as a Projection of Present Information

Information is stored into systems. The evolution of closed systems can be fully predicted from one of its states: every state contains the same information, so information is conserved. Open systems can gain or lose information, so the entire evolution can't be projected onto a single state. In classical mechanics, if we focus on every bit of information, there is no direction of time in a closed system, and the region of spacetime it occupies thus represents the same informational event. If our system is a physical device, though, it interacts with the environment in a particular way, and most of information is ignored by it. A clock, for example, can be modelled by an open system, in which new information enters the device, and no information exits, so that it keeps counting, in the way showed in the previous section. So, we are concerned now with modelling open systems in an antistructuralist picture. Maps can be defined with Local Determinism.

3.2.1 Local Determinism

Laplacian Determinism considers the universe as a closed system, for which it is possible to know the full evolution given a complete set of initial conditions. Local determinism does not focus on knowing the entire universe or a full closed system, but on determining single events. The initial conditions which determine one event are information encoded in a spacetime surface (we consider 3D spacetime for simplicity). Such surfaces can be taken to define space. Space, in Newtonian spacetime, is often described as the whole universe at one instant of time. We must adapt determinism to the language of relativity, namely without separating space and time. There are no such things as instants of time to define extended regions of space: every instant of time is perfectly localised in one single event. So, our target is not the state of the whole system, but the state of one event. To know that state, what we can do is to identify a set of causally disconnected events and call it 'spatial region'. We can identify these regions by starting at a particular event E, and tracing back, continuously, *all* the causal trajectories, or clocks, or 'time axes', which meet at E. Considering all such trajectories we are guaranteed to obtain a continuous region. If we miss a trajectory, we might miss a cause, and our region will be incomplete, and does not fully determine the event E. Moreover, how far back we trace those trajectories is arbitrary, but the further back we go, the more causal branches we must consider, hence the larger our 'initial space'. In a 3D Minkowski spacetime, such initial spaces are represented by any surface which fully closes the past light-cone of E. Each one of these surfaces is isolated from any trajectory which is not causally connected with E, and meets all the trajectories which are causally connected with E. So, each one of them contains the exact same information, which coincides precisely with the information contained in E.

We notice that such surfaces can have any shape, and even contain some causally connected events, in which case they would contain some events on a same trajectory, which is redundant, since the successive event on such a trajectory is already determined by other events on the surface. We can consider the surfaces with only causally disconnected events, fully closing the light-cone, and call them minimal surfaces. These minimal surfaces encapsulate the idea of 'spatial regions', since every time axis inside the cone shares one and only one instant with them, and are compatible with the informational definition given in the previous chapter. Some spatial surfaces can close the light-cone at infinity, an example being the hyperbolic surfaces obtained by tracing back each causal path for the same amount of proper time.

Bell introduces Local Determinism in [2], in reference to Fig. 3.2: every 'beable' (every element of reality which the theory regards as independent of observation) in the region 1 is determined by any localised region V which fully closes the past light cone of 1. We can define Local Determinism making explicit reference to the informational content of the event E (which approximates the region 1), which is the only beable in our formulation: 'The information contained in one event E is the same information contained in any spatial surface closing the past light cone of E'. In this informational picture, local determinism is equivalent to the definition of informational space. Spacetime here is defined by informational relations, and it is not a separate structure in which information travels while obeying some principle of locality. Space here is definable only as a projection of the information contained at some event, and in Minkowski it is always confined inside one light cone, and never contains information about the whole spacetime. If I consider an infinitely extended region of space, in the sense that it closes every light cone (e.g. I can consider the familiar Euclidean planes), it defines information of one event in the infinite future, and it does not define any single event in the future. Any spatial region I take there will always be events which are not determined by it. Similarly, starting from any event, if I consider a space region in its infinite past, no matter how far back I go, such a region will never determine causally disconnected events. No region in the infinite past can be regarded as the 'initial state of the universe', as it will never define every event in the universe. This is why Minkowski's space is informationally open, and in this model there is no initial nor final state of the universe.

While in Laplacian Determinism one can know all of spacetime, their past as well as their future, this is not the case in Local Determinism, where one can fully know only their own past light cone. Bell defined determinism from an extrinsic point of view, concerning events in a known spacetime. Realistically, such known spacetime can only be the past light cone of some observer. Like quantum events contain new information, the surface of the light cone contains new information which is unknown to any other event inside the cone, so that no event can totally know its own future.

We described local determinism as the possibility to project the information about one event onto a finite region of space. Bell's formulates it backwards, as the possibility to determine an event given a surface which closes its past light-cone. They are equivalent formulations, reflecting two different paradigms. Local determinism can be used to

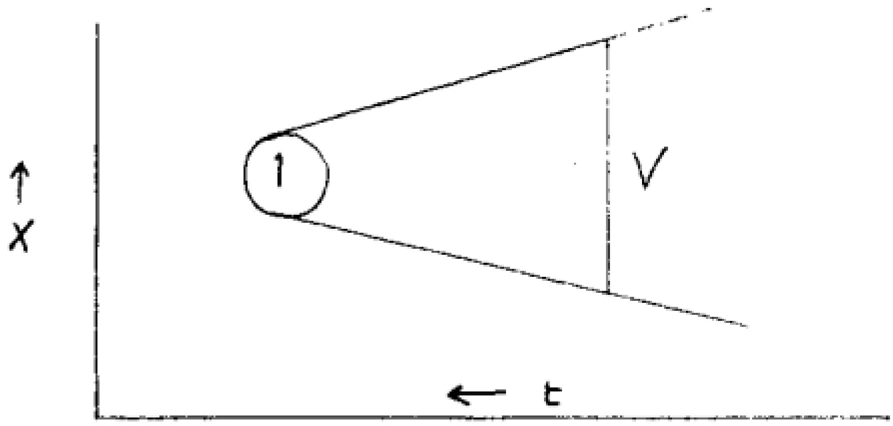

Figure 3.2: A space region according to Bell. Source: [2]

determine one event from a localised surface rather than directly, useful when reading the information from a surface is easier than reading the information directly from the event which identifies it (See fig. 3.3), and it can be used to make future predictions only in sufficiently closed systems.

3.2.2 Local Causality

If you are in an open system, you can't fully determine the future. In Minkowski's informational structure you get new information at every instant, but this new information can be traced back to the infinite past. This does not hold anymore if unpredictable events, e.g. quantum events, are present in the past light-cone of the target event, between the surface and the tip: in this case you must consider a region which not only closes the past light cone of the target event, but also closes its intersection (of such past light cone) with the future light cone of the quantum event (fig. 3.4). The informational content of such event can't be fully traced back further in the past. Some information has origin in it.

So, one can generalise local determinism for indeterministic models. The new information produced still travels locally in this model, meaning that only the future light cone of the unpredictable event can have complete information about it: the model is 'locally causal'. While unpredictable events in classical mechanics can be conceived as emerging from our ignorance of the underlying dynamics, they are fundamental in quantum mechanics. Is quantum mechanics a locally causal theory? We have seen in chapter 2, how the EPR argument shows that this is not the case if we consider causally disconnected events of collapse, which is the case Bell considers in his own theorem [1], which shows that any completion of standard quantum mechanics, deterministic or indeterministic, is not locally causal.

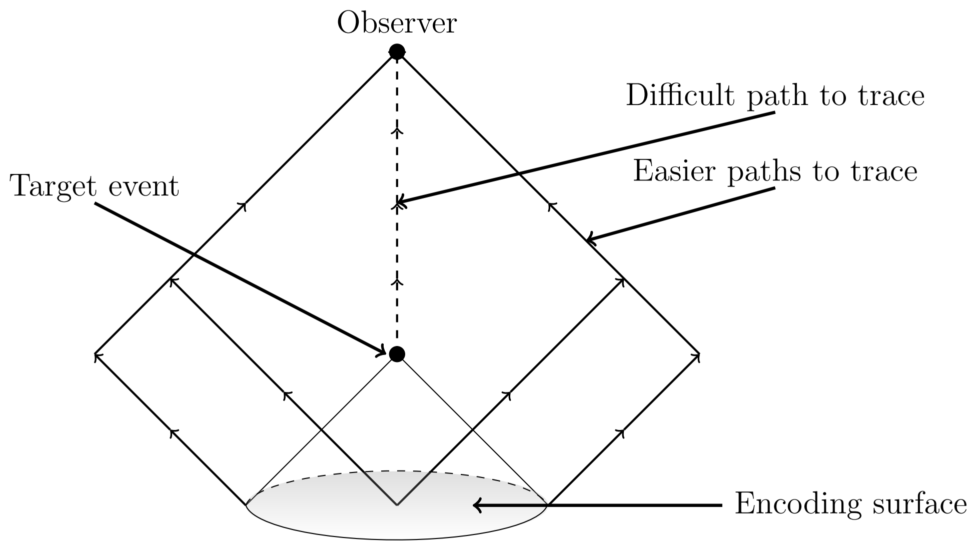

Figure 3.3: Encoding information of an event in a surface closing its past light cone.

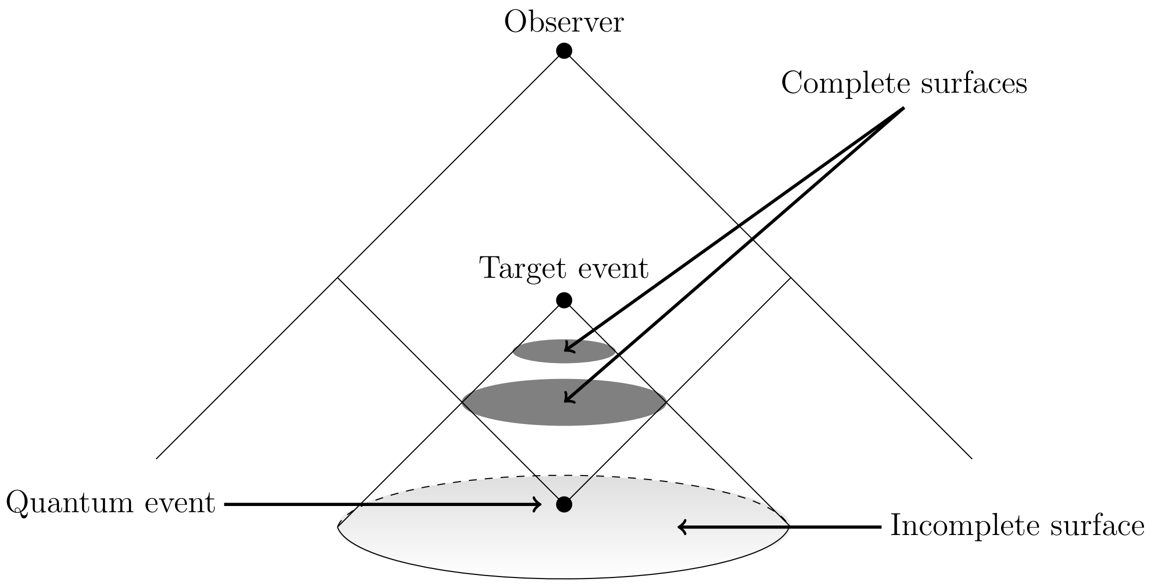

Figure 3.4: The presence of a quantum event, source of new information which can't be further projected into the past.

John Bell defines local causality in [2]. While in local determinism information represents knowledge of definite outcomes, in local causality it represents in general probabilistic knowledge of outcomes. The dynamics of these probability distributions is the same of a locally deterministic theory, and in the absence of unpredictable events, like measurement, it represents unitary evolution in quantum mechanics. The unpredictable events are quantum measurements. Of course, unitary evolution in standard quantum theory can't fit this model, since collapse is non-local. But one can assume a completion of quantum mechanics in which this is not the case. But Bell shows that EPR correlations are too strong to be carried locally. Thus quantum mechanics is not locally causal.

3.2.3 Information in Relational Quantum Mechanics

In Relational Quantum Mechanics, the Minkowski informational structure of outcomes can't apply to unitary evolution, since every interaction, which happens at every event in a Minkowski structure, is a quantum event. Unitary evolution is not something which happens in the spacetime of outcomes. EPR correlations are not the result of something which travels non-locally, but they appear as soon as you interact with a system, since it is both your system and the other system which emerge as memory of the interaction, and whether it's your system which created the other, or it's the other which created yours, are equally valid perspectives in physics. The physical information is about interactions, not about systems alone, and systems emerge from the interaction.

The map is about informational content, which are the outcomes. If you put the same outcomes at causally disconnected events, your map is EPR non-local. Rather than modifying the structure of Copenhagen (collapse at causally disconnected events) by adding new causal processes to complete the theory, we should rethink the very nature of the structure, and recognise its instrumental role. Relational Quantum Mechanics seems to suggest an antistructuralist paradigm, where the same information is free to be projected onto any structure, each structure identifying a particular dynamics.

Let's consider the experiment in the previous chapter, with Alice measuring at A, Bob measuring at B, and them meeting at C. They measure the state of two particles. Alice obtains outcome $\mathcal{A}$, and Bob outcome $\mathcal{B}$. In the Copenhagen interpretation we must choose the collapse at A and B. If the particles are not entangled, then we draw the diagram in fig. 3.5. In relational quantum mechanics Alice can draw diagram A in fig. 3.6, while Bob can draw diagram B.

Let's now consider entangled particles: some information $\mathcal{C}$ is generated at both A and B. The outcome at A is represented by $\{\mathcal{A},\mathcal{C}\}$, while the outcome at B by $\{\mathcal{B},\mathcal{C}\}$. Copenhagen draws the diagram in fig. 3.7. In relational quantum mechanics, Alice draws diagram A in fig. 3.8, and Bob diagram B. There is no non-locality if spacetime is reduced to one observer, either Alice or Bob. These two diagrams are physically equivalent. Any future state will depend on the same information. In Alice's frame this information is

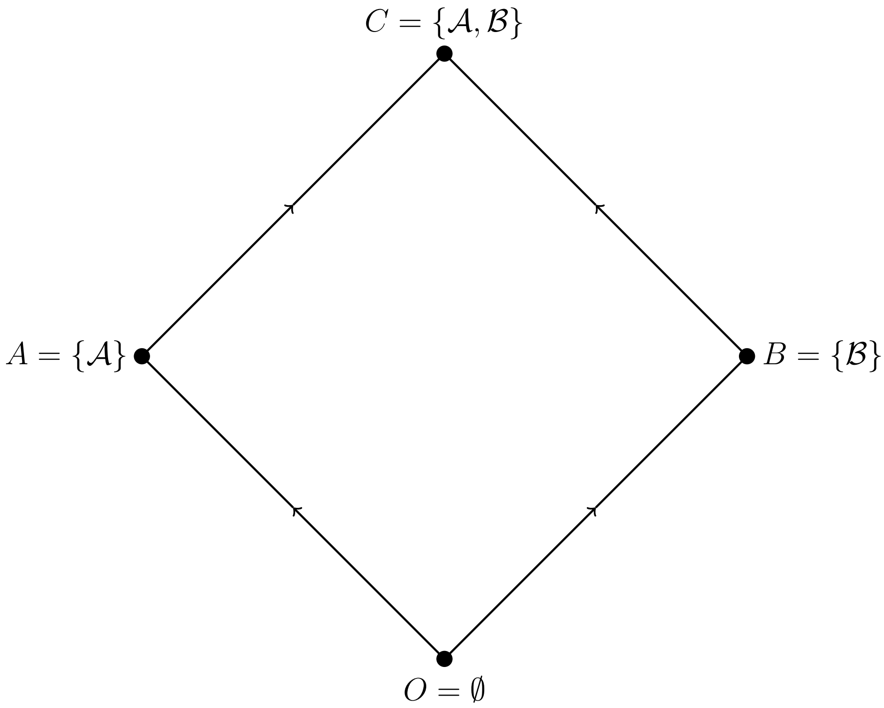

Figure 3.5: Collapse at causally disconnected events for non-entangled particles.

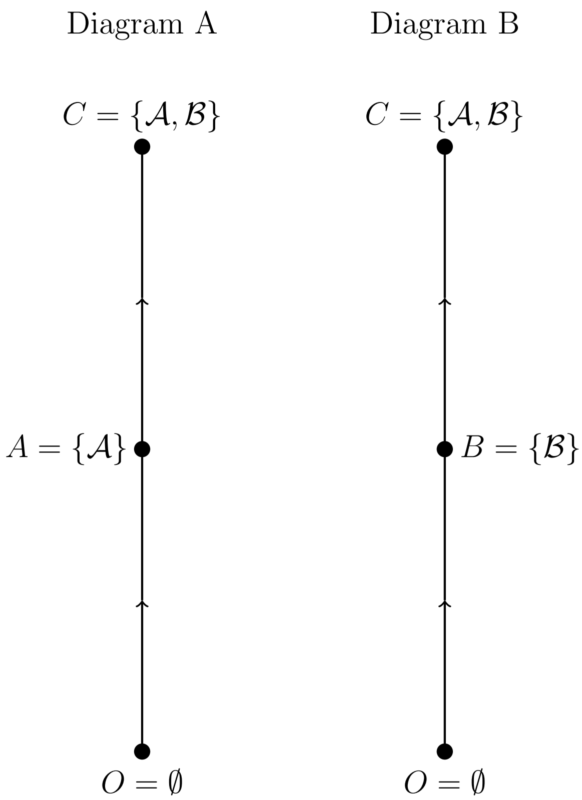

Figure 3.6: Collapse at causally connected events for non-entangled particles. Alice's frame on the left, Bob's frame on the right.

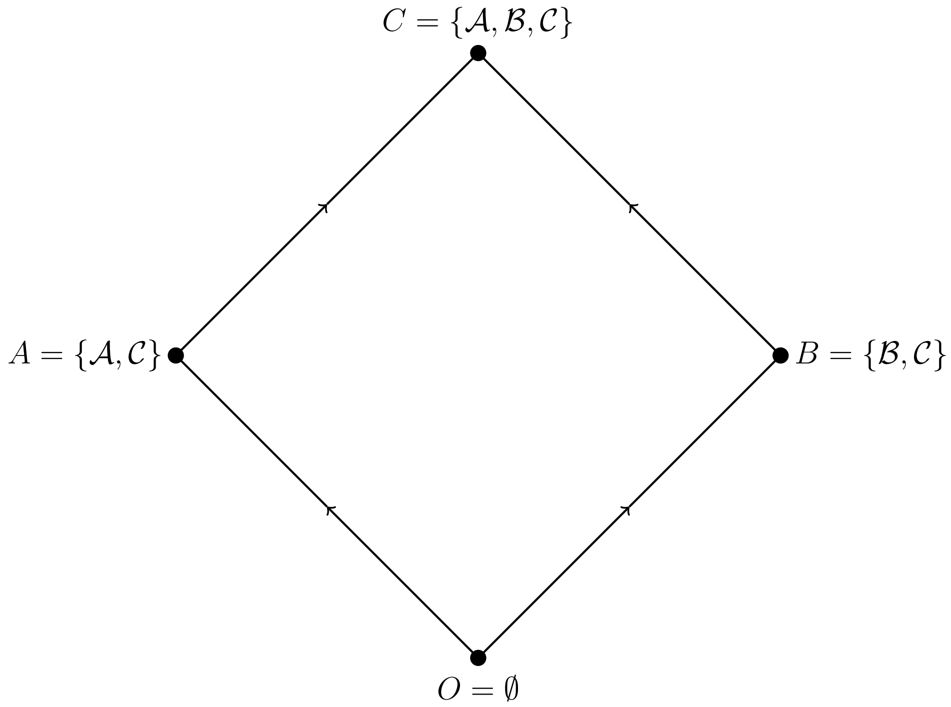

Figure 3.7: Collapse at causally disconnected events for entangled particles.

projected onto Alice's time axis, while in Bob's frame it is projected onto Bob's time axis, but it is only the present state which counts. Diagram in fig. 3.7 forces you to a perspective which does not pertain to any individual observer, but rather treats every observer equally as a projection of the present event. This misses the point that from a quantum mechanical point of view the whole past of a system is a quantum outcome for another system at C. You can chose every outcome to happen at once. This of course does not mean that fig. 3.7 is wrong: it is a perfectly valid ordering of information, if one wants to further project the information at C, but it is an arbitrary choice of structure.

Alice, after confronting with Bob, can conclude 'Ok, you have your own past, but it shares some information with my own past, despite they are independent stories', which is accepting EPR non-locality, and drawing the diagram in fig. 3.7. But Alice can also says 'You have no past, I created your memory', or, equivalently, Bob can say the same. This of course does not mean that some of them has absolutely no past: it just reinforces the instrumentalist nature of the quantum state. In the following section I will defend this antistructuralist paradigm from a metaphysical point of view.

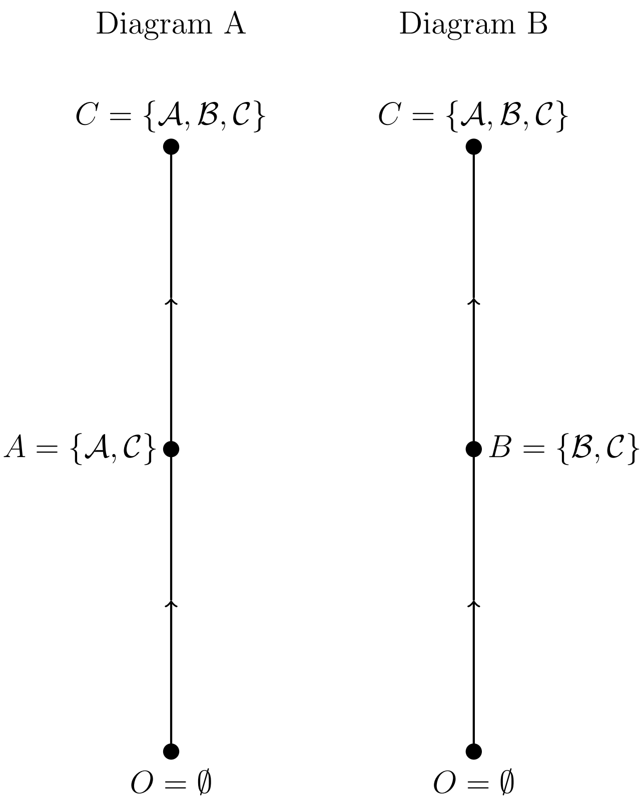

Figure 3.8: Collapse at causally connected events for entangled particles. Alice's frame on the left, Bob's frame on the right.

3.3 Ontic Antistructuralism

Relational Quantum Mechanics, despite its self-consistency, clashes with the common notions of time and identity: if Alice creates the past of Bob, or Bob creates the past of Alice, or Charles creates both pasts, who is right? Physically it does not matter, since every point of view is instrumental, and brings to the same empirical results, and everyone can have their own past. But if one has a strong metaphysical notion of self, or flow of time, this may play an important role in their acceptance of the theory. Why shouldn't physics care about structure, if there is one? In this chapter I show that an antistructuralist metaphysics, which claims that there is no structure, is possible and self-consistent. Whether or not one accepts antistructuralism in an ontological sense, this chapter serves to defend the whole paradigm from any potential accusation of solipsism, and to show that common sense objections to antistructuralism lie in the metaphysical domain, rather than in the physical one discussed above, and that also in an ontic perspective it can be applied with self-consistency. In particular it is in the metaphysics, with less constraints from experience, that the idea behind this paradigm can be better understood. I'll argue that Ontic Antistructuralism is efficiently represented by the ideas of Eternalism and Open Individualism.

3.3.1 Eternalism

What we said about time poses a question on whether time actually flows in the perceived direction. But what is time flow first of all? Metaphysically it is the idea of change, that what exists today will not exist tomorrow, or at least not in the same way. Eternalism argues that such time flow does not exist: every event will always be there. Time exists only as a relation between events, not as real change. The perception of flow of time, like the measurement of time that we discussed at the beginning of this chapter, depends on the informational content of events, regardless of any given order nor flow of them in the existence, that thus need not exist. Eternalism is an antistructuralist interpretation of time, since it interprets experience of time as arising from the mere existence of events, and not from an actual order of them in a given structure. Existence does not change at all. What you are living now exists, has always existed, and will always exist. You live the events of your life simultaneously and forever. At everyone of them there is a particular memory which gives the illusion of time flowing towards the unknown events with additional memory, the 'future' events. At each event you only have knowledge of certain events which you call 'past events', and all this ordering is done spontaneously only with the information available at each event.

Eternalism not only is consistent with the experience of one observer, but also with the experience of multiple observers, which would be otherwise difficult to interpret without a given notion of simultaneity. If existence changes, and so we can talk about what exists 'now', then we are automatically thinking in terms of simultaneity. Physically it is believed to be impossible to determine planes of simultaneity, which is why we shifted to the relativistic paradigm, in which simultaneity has no fundamental role. In eternalism simultaneity does not exist, or, in a sense, the entire existence occurs simultaneously. When we discussed non-interacting observers, we saw that there is no way, informationally, to relate their events before interaction, and we cannot know which one occurs first. Either we believe that there is an actual order which, like the Aether's frame, can't be known, or we get rid of the concept of time order for causally disconnected events, like in the physics of the relativistic paradigm. Eternalism is not constrained in relating every event in terms of time, since time order is an abstract relation which two events might or might not satisfy, and has nothing to do with real change.

One can argue, nonetheless, that Alice and Bob are two separate entities, each one with its own 'personal existence', so that time flow might exist, but there is one for every 'person', and it has no meaning to relate the existence of non-interacting observers, since it has no consequence on personal experience: every observer only perceives their own story, and flow of time is 'personal'. This view seems consistent, even in a relational framework, but it relies on another structural prejudice, besides the flow of time. Namely the identification in one particular observer. Antistructuralism argues against personal identity and individuation, showing that it is completely arbitrary.

3.3.2 Open Individualism

Depending on the evolution you choose to follow, it's either Bob whose past gets created at the interaction, or Alice's. But we have seen that, from Alice's point of view, even her own past emerges from her present information. So, we can stop thinking in terms of evolution of a system, and think in terms of events: at the event of interaction, both systems, Alice and Bob, emerge. We can schematise Alice and Bob in such a way that at their interaction they have complete knowledge of each other, and they merge to the same event, at which point they are indistinguishable[4].

Consider again Alice and Bob directly communicating at a distance, and let's omit the intermediary Charles, since unnecessary in the following discussion:

$A_1 = \{\mathcal{A}_1\}$

$A_2 = \{\mathcal{A}_1, \mathcal{A}_2\}$

$A_3 = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, \mathcal{A}_3\}$

$B_1 = \{\mathcal{B}_1\}$

$B_2 = \{\mathcal{B}_1, \mathcal{B}_2\}$

$B_3 = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\}$

Recall that the observers are:

$O_1 = \{A_1, A_2, A_3\}$

[4]Of course no observer intersects in reality, but we can think to approximate very similar knowledge to the same knowledge, so to the same informational event in a simplified map.

$O_2 = \{B_1, B_2, A_3\}$
$O_3 = \{B_1, B_2, B_3\}$
$O_4 = \{A_1, A_2, B_3\}$

Intuitively, an observer, or clock, would correspond to a measuring device, even to a person. If we were to name these observers, we wouldn't have any doubt in naming O_1 'Alice', and O_3 'Bob'. We can also name $O_5 = \{A_1, A_2\}$ 'past Alice', and $O_6 = \{B_1, B_2\}$ 'past Bob'. But how to name O_2 and O_4? They respectively contain the same present information of the observers Alice and Bob, but they are related to a different past. In such a schematic situation, in which two people (or devices) cannot be distinguished in a particular instant, since they share exactly the same memory, we have a problem of identity. Of course, this is not necessarily a problem, since two persons never merge in reality, and their 'world lines' never actually intersect. But this simplification is useful to amplify the structural problem about individuation.

Antistructuralism recognises that identifying in a O_1 rather than O_2 is completely arbitrary. In general, when we identify with an observer, we do it because of continuity in the informational content: identity is an abstract order of experience, exactly like time direction is an abstract direction of experience. So, who are you? The simplest answer, consistent with antistructuralism, is that you are both observers, O_1, and O_2: you live the existence of both of them. And in general, since different entities don't not exist, you are all of O_1, O_2, O_3, O_4. You are everyone. You are all the existing events. The idea that there is only one being for every person is known as Open Individualism, as opposed for example to closed individualism for which there is one being for every organism. A useful way to visualise the absence of identity is this: the world produces sensations, and sensations don't belong to any being. They just are there, and you *are* them all. No one owns sensations[5].

Ultimately, Ontic Antistructuralism is the view that there are no different entities at all: there is only you, the existence itself.

Intuitively, our description of experience in terms of pure informational content at the events, could also allow for an antistructuralist ontology of the events, in which they are simply there with no particular order. We have seen, though, that already in quantum mechanics we are led to distinguish the events from the informational content about the outcomes, and events lose their ontology, becoming containers of outcomes. So, relativity gives a first hint to antistructuralism, by relating every event by its own informational content, with no external structure in which they are ordered. The only thing which seems to exist is the event. But then relational quantum mechanics shows us how something which relativity managed to make fundamental and unbreakable, the event, can be completely and consistently shattered into pieces by introducing relative

[5] This model has been presented here with very particular language, using words such as sensations, and individuation of you in the sensations. Whether or not this is an accurate language to use is not important, as long as the antistructuralist idea of identity is expressed clearly.

quantum states. Ontic antistructuralism further claims that not only there is no structure between entities, but that entities are not ontologically separated, and it is only our language which creates the separation. Even if there is no structure in which to understand a reality with no structure, we managed to grasp an important aspect of the absence of structure: you are everything.

Conclusions

I have discussed the measurement problem from a relational perspective, in which the evolution of the realisation of outcomes chosen to represent the measurement process is arbitrary. I have first considered a case in which only the order of interaction with the device matters, but it does not matter which outcome collapses first. Copenhagen interpretation claims that collapse happens at particular events (interactions) regardless of the status of interaction with the observer. For consecutive (causally connected) measurements, also the order of the two outcomes in time is given. In the Relational Interpretation, measurements, as interaction with the device, don't induce collapse. Collapse happens only for the interacting systems. This means that, even if the interactions follow a particular order, for an external observer the outcomes may happen at once, depending on which device he first interacted with (as long as the interaction of the particle with the first device is ideal, i.e. it does not affect the state of the particle). For causally disconnected measurements, Copenhagen still claims that the collapse happens at given events regardless of the observer. This time, though, the measurements don't have any particular order. So, Copenhagen, to be consistent with the causal structure of relativity, cannot claim an absolute sequence of collapse for the entangled pair. Copenhagen can only claim the events of collapse. In relational quantum mechanics collapse does not happen globally in a shared spacetime, but it represents the update of the state of a system relative to some observer.

So, the characteristic of Copenhagen is 1) the existence of collapse which breaks the unitary evolution, and 2) an absolute set of events at which the collapse happens (but not a definite order in general). Rovelli's relational interpretation only keeps point 1), and rejects point 2): the events of collapse are arbitrary, and they identify an observer who updates their description of the state of the world. We have mentioned the difference with Everett's universal wave function interpretation (which introduced the relative states), which also rejects point 1). The drawback of the Copenhagen interpretation over the relative state ones is that, despite claiming that some interactions are the events of collapse regardless of the frame of reference, it can't tell which interactions count as such: this is the measurement problem. The advantage of a relative state formulation (Everett's and Rovelli's) is that there is no such a problem. Everett's interpretation, though, requires the abandonment of Heisenberg's ontology of values taken by variables at collapse, in favour of a Schroedinger's ontology of the wave function. This type of ontology is necessary in order to have only the unitary evolution, and solve in this way the measurement problem, without requiring any additional unknown dynamics of which we still have no evidence. Relational Quantum Mechanics instead shows that it is not necessary to abandon collapse in order to solve the measurement problem without adding new physics.

Relational Quantum Mechanics gives a great importance to interaction: without interaction with S, no definite history of outcomes exists. When O interacts with S, all the history of S assumes a definite value. An observer is defined by a set of interactions and their outcomes. All the history of S is, for O, an outcome created at that instant. This raises the question of whether it makes physical sense to describe the past of a system, and the structure of its interactions, as something external with an independent reality. So I introduced my view of Antistructuralism, as a paradigm in which every knowable structure is a projection of present information. I applied this paradigm to classical physics, where new information is given by interaction with another system, a process which mirrors the quantum event of collapse in relational quantum mechanics. I applied this idea to the informational structure of Minkowski spacetime, and then in quantum mechanics, in both the Copenhagen and in the relational interpretations. I applied it to the EPR argument, showing that EPR-non-locality arises from a particular choice of structure which considers outcomes at causally disconnected events. But not for this reason it is a wrong choice of map: antistructuralism suggests that physical theories are just tools which project the same information onto different structures, and that the ontology of the theory should not lie in the structure, but in the informational content of the theory. You can choose either locality or a shared spacetime. There is no right nor wrong answer, they just reflect different consistent choices of structure describing the same outcomes. While antistructuralism can be regarded just as a physical framework, it can have metaphysical implications. I concluded by introducing Ontic Antistructuralism, which gives a consistent description of the real world removing structural elements, such as time and identity, and which claims that structure is not real.

Appendix A

Quantum Mechanics of a Closed System

The Quantum Mechanics of a closed system can be formulated with a Dirac-von Neumann type of postulates, following the works of Paul Dirac [4] and John von Neumann [9] as:

1) Quantum state postulate. States are unitary vectors $|\Psi\rangle$ in a Hilbert space $\mathcal{H}$. The state of a compound system made of the subsystems $|\Psi_1\rangle$ in $\mathcal{H}_1$ and $|\Psi_2\rangle$ in $\mathcal{H}_2$ lies in the product space $\mathcal{H}_1 \otimes \mathcal{H}_2$. If the subsystems are not interacting, their joint state is given by the tensor product $|\Psi_1\rangle \otimes |\Psi_2\rangle$.

2) Observable postulate. Observables are linear Hermitian operators A acting on the Hilbert space $\mathcal{H}$. Their expectation value $\langle A\rangle_\Psi$ on a state $|\Psi\rangle$ is given by $\langle A\rangle_\Psi = \langle\Psi|A|\Psi\rangle$.

3) Result of a Measurement. Measuring an observable described by operator A yields as result one of the eigenvalues λ_i of A. The probability of measuring one particular eigenvalue λ_i for operator A acting on state $|\Psi\rangle$ is given by the Born Rule: $P(\lambda_i) = |\langle\Psi_i|A|\Psi\rangle|^2 = |\lambda_i|^2$, where $|\Psi_i\rangle$ is the eigenstate associated with the eigenvalue λ_i. If λ_i is degenerate, with n eigenvectors $|\Psi_{ij}\rangle$ for $j = 1, ..., n$, then $P(\lambda_i) = \sum_{j=1}^{n} |\langle\Psi_{ij}|A|\Psi\rangle|^2 = n|\lambda_i|^2$

4) Collapse postulate. After measurement, the state $|\Psi\rangle$ collapses to the eigenstate $|\Psi_i\rangle$ relative to the eigenvalue λ_i being measured. If λ_i is degenerate, the state collapses to a vector which is a linear combination of the eigenstates $|\Psi_{ij}\rangle$ of λ_i.

5) Unitary Evolution. An isolated quantum system in the state $|\Psi\rangle$ evolves, while not being measured, following the Schroedinger Equation:

$$i\hbar\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle, \tag{A.1}$$

where H is the Hamiltonian operator characterising the evolution. This means that given an initial state $|\Psi_0\rangle$ at time t_0, it evolves continuously and deterministically to a state

$|\Psi(t)\rangle = U(t)|\Psi_0\rangle$, where $U(t)$ is the evolution operator determined by the Schroedinger equation.

Postulate 5 describes a continuous and deterministic dynamics, while postulates 3 and 4 describe a discrete and probabilistic dynamics. The postulates tell to consider the continuous evolution when the system is not being measured, and the discrete evolution when a measurement is performed on the system, but no information is given about when an interaction between two systems counts as one system measuring the other, and when it is part of the continuous evolution of the compound system.

The Copenhagen Interpretation introduces the concept of macroscopic systems (see for instance [3]), which induce the collapse on quantum systems by interacting with them: an interaction with a macroscopic system breaks unitary evolution, while an interaction between two quantum systems does not induce any collapse. The problem of giving a unified description of macroscopic and quantum systems, and explaining how this ability of collapsing the state of microscopic systems emerges, lies outside of the above formalisation, and it remains an open question of the interpretation.

Appendix B

The Ontologies of Quantum Theory

In this dissertation I distinguish between two classes of ontologies for quantum theory. I call them the Heisenberg-type ontology, and the Schroedinger-type ontology. By ontology for a physical theory I mean the set of entities which are considered to be the aim of the description, as opposed to the purely instrumental ones. I may call the ontological entities 'physical reality', not to be confused with the reality of phenomena: different ontologies for the same theory don't necessarily lead to different empirical predictions, rather they define different attitudes towards its development and its applications. I think that this definition of physical reality with emphasis on the attitude allows one to better appreciate the different motivations and prejudices behind different formulations of quantum mechanics, characteristics which the definition of reality based on simple agreement of observation would not distinguish. Different physical ontologies can describe the same world of phenomena, while projecting it to different pictures in which experience is understood.

Quantum Mechanics was developed in parallel directions, following two different approaches in particular, the Schroedinger Picture and the Heisenberg Picture. Despite their equivalence in terms of predictions, they reflect two complementary attitudes: Schroedinger focuses on the description of the quantum state and its evolution, while Heisenberg is interested on the results of measurements and on the sequence of outcomes.

In the Dirac-von Neumann formalism of Appendix A I presented postulate 5 in what is commonly referred to as the 'Schroedinger Picture', in which the unitary evolution is applied to the quantum state. We can write the state in the form $|\Psi(t)\rangle = U(t)|\Psi_0\rangle$, where $|\Psi_0\rangle = |\Psi(0)\rangle$, and $U(t)$ is the time evolution operator. Plugging in this expression into the Schroedinger equation (A.1), one gets the equation for the evolution operator:

$$i\hbar\frac{d}{dt}U(t) = HU(t). \tag{B.1}$$

The state is treated like an object which evolves continuously through time. If we aim to describe the state, i.e. we consider it to be the ontology of the theory, we get a physical reality which is reflected by the continuous and deterministic dynamics of postulate 5 and the discrete probabilistic jumps of postulate 3 and 4. One of the works which gave origin to this picture is Erwin Schroedinger's paper 'Quantisierung als Eigenwertproblem' [14]. The wave function here is treated like the fundamental physical object, from whose dynamics emerge quantum phenomena. Hugh Everett further developed this picture by removing the discrete dynamics from the ontology, regarding it as a tool to make predictions for subsystems [6, 7]: the only state with some ontology is the wave function of the whole universe.

The Heisenberg picture can be obtained by replacing postulate 5 with: any operator A evolves in time, while nothing is being being measured, following the Heisenberg's equation:

$$\frac{d}{dt}A = \frac{i}{\hbar}[H, A] + \frac{\partial A}{\partial t}, \tag{B.2}$$

where $[H, A] = HA - AH$ is the commutator of the Hamiltonian H and the operator A. In this picture quantum states are stationary in between measurement, and only follow the discrete evolution of postulates 3) and 4). The expectation of A is still found with postulate 2, but its evolution in time is carried by the observable rather than the state: $\langle A\rangle_{\Psi_0} = \langle\Psi_0|A(t)|\Psi_0\rangle$. A can be any operator, not necessarily Hermitian. We get to the Schroedinger picture by transforming the operators with a unitary transformation $U(t)$, obtaining $A_S(t) = U(t)A(t)U^\dagger(t)$ such that $\frac{d}{dt}A_S(t) = \frac{\partial_S}{\partial t}A_S(t)$, where $\frac{\partial_S}{\partial t}$ keeps constant all the other transformed operators, e.g. $P_S = UPU^\dagger$ and $Q_S = UQU^\dagger$. Operators in the Schroedinger picture only admit an explicit time dependence: their dynamics is not determined by the structure of the theory, since the equation of evolution in this picture only exists for quantum states, and it must be specified at each moment if the operator is changing. So we can write $\langle A\rangle_{\Psi_H}(t) = \langle\Psi_H|U^\dagger(t)A_S(t)U(t)|\Psi_H\rangle = \langle\Psi(t)|A_S(t)|\Psi(t)\rangle = \langle A_S\rangle_{\Psi(t)}(t)$, where the last passage is true if the state $|\Psi(t)\rangle = U(t)|\Psi_H\rangle$ evolves with the Schroedinger equation with initial condition $|\Psi_0\rangle = |\Psi_H\rangle$. In which case we would have empirical equivalence between the Heisenberg and Schroedinger pictures, since all we never measure states and observables in reality, but only their expectation value on a given state, and the two pictures would agree on such number at any time: $\langle A\rangle_{\Psi_H}(t) = \langle A_S\rangle_{\Psi(t)}(t)$.

To show that $U(t)$ is indeed the evolution operator in the Schroedinger picture, one can plug in $A = U^\dagger A_S U$ into the Heisenberg equation, and show that U follows the Schroedinger equation for the evolution operators.

The left hand side of (B.2) gives:

$$\frac{d}{dt}A = \frac{d}{dt}(U^\dagger A_S U) = \frac{dU^\dagger}{dt}A_S U + U^\dagger A_S\frac{dU}{dt} + U^\dagger\frac{dA_S}{dt}U, \tag{B.3}$$

while the right hand side gives:

$$\frac{i}{\hbar}[H, A] + \frac{\partial A}{\partial t} = \frac{i}{\hbar}[H, U^\dagger A_S U] + \frac{\partial (U^\dagger A_S U)}{\partial t}. \tag{B.4}$$

Now let's focus on the terms which contain a derivative of A_S. The last term in (B.3) can be written as $U^\dagger \frac{dA_S}{dt} U = U^\dagger \frac{\partial_S A_S}{\partial t} U$. By expanding A_S in powers of the other observables, e.g. P_S and Q_S, and time, we have a summation of terms of the type:

$$U^\dagger \frac{\partial_S}{\partial t}(a_n P_S^a Q_S^b t^c) U = a_n c t^{c-1} U^\dagger P_S^a Q_S^b U = a_n c t^{c-1} (U^\dagger P_S U)^a (U^\dagger Q_S U)^b = a_n c P^a Q^b t^{c-1} \tag{B.5}$$

Let's compare to the corresponding term in (B.4): by expanding A_S the same way in $\frac{\partial (U^\dagger A_S U)}{\partial t}$ we get terms of the type:

$$\frac{\partial}{\partial t}(a_n U^\dagger P_S^a Q_S^b t^c U) = a_n \frac{\partial}{\partial t}[(U^\dagger P_S U)^a (U^\dagger Q_S U)^b t^c] = a_n \frac{\partial}{\partial t}(P^a Q^b t^c) = a_n c P^a Q^b t^{c-1}, \tag{B.6}$$

which are the same as before.

So it is makes no difference to take the partial derivative first or to change picture first:

$$U^\dagger \frac{\partial_S A_S}{\partial t} U = \frac{\partial U^\dagger A_S U}{\partial t}, \tag{B.7}$$

and (B.2) gives:

$$\frac{dU^\dagger}{dt} A_S U + U^\dagger A_S \frac{dU}{dt} = \frac{i}{\hbar}[H, U^\dagger A_S U] \tag{B.8}$$

The right hand can be written as:

$$\frac{i}{\hbar}[H, U^\dagger A_S U] = \frac{i}{\hbar}(H U^\dagger A_S U - U^\dagger A_S U H) = \frac{i}{\hbar}(U^\dagger H_S A_S U - U^\dagger A_S H_S U), \tag{B.9}$$

where $H_S = U H U^\dagger$ is the Hamiltonian operator in the new picture.

So we get for U the differential equation:

$$\frac{dU^\dagger}{dt} A_S U + U^\dagger A_S \frac{dU}{dt} = \frac{i}{\hbar} U^\dagger H_S A_S U - \frac{i}{\hbar} U^\dagger A_S H_S U, \tag{B.10}$$

for any A_S, and we obtain (B.1) and its complex conjugate by comparison of the terms.

If we consider the states of the Schroedinger Picture to be the ontology, we get a discrete reality, and each element of it corresponds to a measurement outcome: the state contains information about a certain outcome, and it is updated at each measurement.

The ontology of states in this picture reflects the discrete ontology of measurement outcomes. The continuous evolution is in the observable, which is a mere mathematical tool, and it is not part of the quantum ontology. This view has origin in Werner Heisenberg's foundational work [8], in which no concept of quantum state nor wave function is present. The paper begins with the statement: 'the aim of this work is to set the basis for a theory of quantum mechanics based exclusively on relations between quantities that are in principle observable', entailing a renounce to the physical description of certain processes, such as the orbit of the electron, of which there appeared to be no evidence, and whose description in terms of electromagnetism seemed to contradict experience. So, what is physical in this view is only the value observed at measurement. This type of ontology is thoroughly discussed in [13], where it is developed in the context of the relational interpretation: what is physical are the values created at the interaction, and the description in terms of the dynamics of quantum states is relative.

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