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Flavor Symmetries in the SMEFT: Theory and Phenomenology

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Ad Fatima,
Alla mia famiglia.

Abstract

The Standard Model Effective Field Theory (SMEFT) provides a powerful and systematic framework for studying physics beyond the Standard Model by incorporating higher-dimensional operators that respect symmetries. Within SMEFT, the flavor structure of these operators plays a crucial role in understanding fundamental aspects such as fermion masses, mixing, and possible new sources of flavor violation.

In this thesis, we analyze how the imposition of specific flavor symmetries, namely $U(3)^5$ and $U(2)^5$ flavor symmetries, constrain the SMEFT operators number and affect the flavor dynamics of the theory. The $U(3)^5$ symmetry, which corresponds to treating the three generations of fermions as indistinguishable at leading order, serves as a useful baseline for minimal flavor violation. On the other hand, the $U(2)^5$ distinguishes the third generation from the first two, providing a framework that is well-motivated by the hierarchical structure observed in fermion masses and mixing. The breaking patterns of these symmetries provide a rationale for the classification of new physics contributions to flavor observables.

A crucial aspect of our analysis involves renormalization group evolution , which governs the scale dependence of the SMEFT operators. The implementation of flavor symmetries in the SMEFT typically implies a truncation of the operator series in powers of the symmetry-breaking parameters. Therefore, to determine the RG evolution of the Wilson coefficients, it is necessary to derive RGEs that are truncated consistently at the chosen order in the symmetry-breaking insertions. To facilitate this, we introduce the "RunSMEFT" code.

The phenomenological implications of these flavor symmetries are then examined in a few experimental scenarios, with a particular focus on top physics. The top quark sector serves as a crucial testing ground for new physics effects.

Il modello efficace della teoria di campo del Modello Standard (SMEFT) fornisce un quadro potente e sistematico per studiare la fisica oltre il Modello Standard, incorporando operatori di dimensione superiore che rispettano le simmetrie. All'interno dello SMEFT, la struttura di sapore di questi operatori gioca un ruolo cruciale nella comprensione di aspetti fondamentali come le masse dei fermioni, la mescolanza e le possibili nuove fonti di violazione di sapore.

In questa tesi, analizziamo come l'imposizione di specifiche simmetrie di sapore, in particolare le simmetrie $U(3)^5$ e $U(2)^5$, vincoli il numero di operatori dello SMEFT e influenzi la dinamica di sapore della teoria. La simmetria $U(3)^5$, che corrisponde a trattare le tre generazioni di fermioni come indistinguibili al primo ordine, serve da riferimento utile per la violazione minima di sapore. D'altra parte, la simmetria $U(2)^5$ distingue la terza generazione dalle prime due, fornendo un quadro ben motivato dalla struttura gerarchica osservata nelle masse e nella mescolanza dei fermioni. I pattern di rottura di queste simmetrie offrono una base per la classificazione dei contributi della nuova fisica agli osservabili di sapore.

Un aspetto cruciale della nostra analisi riguarda l'evoluzione del gruppo di rinormalizzazione (RG), che governa la dipendenza in scala degli operatori dello SMEFT. L'implementazione delle simmetrie di sapore nello SMEFT implica tipicamente un troncamento della serie di operatori in potenze dei parametri di rottura della simmetria. Pertanto, per determinare l'evoluzione RG dei coefficienti di Wilson, è necessario derivare equazioni di evoluzione del gruppo di rinormalizzazione (RGE) troncate in modo coerente all'ordine scelto nelle inserzioni di rottura di simmetria. Per facilitare questo processo, introduciamo il codice "RunSMEFT".

Le implicazioni fenomenologiche di queste simmetrie di sapore vengono quindi esaminate in alcuni scenari sperimentali, con un'attenzione particolare alla fisica del top. Il settore del quark top rappresenta infatti un banco di prova cruciale per gli effetti della nuova fisica.

Contents

Chapter 1

Introduction

Since its conception in the early 70s, the Standard Model (SM) has been regarded as the low-energy limit of an extended theory that includes more degrees of freedom, around or above the electroweak scale, addressing some of its open issues. After years of running the Large Hardon Collider (LHC), we can state that there is a mass gap between the SM spectrum and these hypothetical new particles. How large this mass gap is, is probably the most interesting open question nowadays in high-energy physics.

The observation of a mass gap above the SM spectrum, and the need to describe it in possible general terms of physics beyond the SM, has motivated the systematic study under the Standard Model Effective Field Theory (SMEFT) [1–9]. Its formulation employs the degrees of freedom and gauge symmetries of the SM and it is structured as an infinite series of operators sorted by canonical dimension. At observables level, it reproduces a series expansion in (E/Λ), E being the energy exchanged in a process and Λ the mass or the cut-off scale that characterizes the Beyond Standard Model (BSM) dynamics.

With the situation of mass gap, SMEFT analysis had become standard across different sectors of high-pT phenomenology, including top quark [10–19], electroweak vector bosons [18–27], Higgs [18–20, 28–31] and jets [32–35], aiming at finding small deviation from the SM predictions.

The largest obstacle to such an analysis is the proliferation in the number of independent operators in the SMEFT. For instance, there are 2499 independent baryon and lepton number-conserving SMEFT operators that arise at the leading order (dimension 6) [6]. If the field content had only one single generation instead of three, this number would have been 76.

In this thesis, we will address the flavor structures of the baryon and lepton number conserving dimension-6 operators in the SMEFT. In the case of an anarchic flavor structure, where all real and imaginary coefficients are of order one, constraints from charged lepton flavor violation, neutral meson oscillations, and electric dipole moments push the new physics (NP) scale to many orders of magnitude above the TeV range. As a result, only a small subset of operators influencing these rare transitions would have any significant impact on phenomenology, making it unlikely that new effects could be observed in high-pT collider experiments.

Postulating a flavor symmetry and its breaking terms can help in reducing and ordering via an appropriate power counting such large number of independent operators. Also, it allows us to lower the overall cut-off scale of the EFT, ameliorating the fine-tuning problem of the Higgs mass.

The price to pay for this series of advantages is the choice of flavor symmetry. Imposing a flavor symmetry and its breaking pattern (via a set of spurions) means making a hypothesis about the UV physics. A flavor spurion can be viewed as a non-dynamical ,spurious, field that transforms under a nontrivial representation of the flavor group and whose background value breaks the flavor symmetry. In this thesis we will focus on two main cases, the $U(3)^5$ and $U(2)^5$ flavor symmetries.

The $U(3)^5$ flavor symmetry is the maximal symmetry allowed by the SM gauge group, while $U(2)^5$ is a subgroup from $U(3)^5$ that distinguishes the third generation from the first two generations. $U(3)^5$ allows us to implement the minimal flavor violation (MFV) hypothesis [36, 37], which is the most restrictive consistent hypothesis we can utilize in the SMEFT to suppress non-standard contributions to flavor-violating observables [37]. $U(2)^5$ is an excellent approximate symmetry of the SM [38–40], that allows us to have a much richer structure as far as third-generation dynamics is concerned.

In SMEFT, Renormalization Group Equations (RGEs) describe how the Wilson coefficients of higher-dimensional operators evolve with energy. From refs. [4–6] all the SMEFT RGEs were calculated. The RGEs are available and automated already, but only in the version with generic flavor indices [9, 41, 42]. In this thesis, we will present a truncation technique, to truncate the RGEs up to a specific order to remain consistent with the flavor symmetries. To do so we introduce the "RunSMEFT" code[1], that introduces a function that truncates and finds all the contributions to a specific order.

The remainder of this thesis is structured as follows:

- Chapter 2 provides an overview of Standard Model Theory (SM), outlining the theoretical framework and stating some of its struggles.

- Chapter 3 introduces Effective Field Theory (EFT), explaining its motivation, key concepts and technical tools.

- Chapter 4 presents the Standard Model Effective Field Theory (SMEFT), explaining how to remove redundancies in order to have a minimal non-redundant dimension-6 operator basis (Warsaw Basis), detailing the anomalous dimension matrices in SMEFT.

[1]The code is available on https://github.com/Mhmdkassir/RunSMEFT

- Chapter 5 is one of the core chapters in this thesis, where we implement flavor in Warsaw Basis structures, finding 2499 independent baryon and lepton number-conserving SMEFT operators.

  Then, we analyse how $U(3)^5$ and $U(2)^5$ flavor symmetries act on SMEFT, providing a reduction of the large number of dimension six operators involving fermion fields.

  Moreover, we establish a correspondence between Wilson coefficients in $U(3)^5$, considering only the third-generation Yukawa couplings, and those in $U(2)^5$ without the need for spurions.

- Chapter 6 is the second core chapter, that provides a general discussion of SMEFT RGEs, and the truncation techniques for both flavor symmetries up to a specific order.

  Then we introduce "Run SMEFT" code, that will calculate the truncated SMEFT RGEs in both flavor symmetries, and we showed someof the obtained results.

- Chapter 7 focuses on phenomenological analysis for U(2) breaking. And shows that the assumption of an approximate flavor symmetry helps lowering the bounds on the new physics scale.

Additional useful material is provided in the Appendices: List of contributions from Higgs self-coupling, Yukawa coupling and gauge couplings to the RGEs are respectively in app.A, app.B and app.C.

Chapter 2

Standard Model Theory

The Standard Model of particle physics is a quantum field theory that describes the fundamental particles and their interactions. It successfully unifies the electromagnetic, weak, and strong forces under a single framework [43–45].

The SM is based on a local symmetry:

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{2.1}$$

where each term represents the symmetry associated with the strong and electroweak forces.

2.1 Gauge Symmetries and Fields

- SU$(3)_C$: Quantum Chromodynamics:

  The $SU(3)_C$ group describes the strong interaction between quarks mediated by gluons. The quarks transform into triplets under $SU(3)_C$. The gauge field is denoted by G^A_μ, μ is the spacetime index (μ=0,1,2,3) and a is the color index (a=1,. . . ,8). The field strength tensor for the SU(3) is defined as:

  $$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_3 f^{ABC} G^B_\mu G^C_\nu \tag{2.2}$$

  where g_3 is the strong coupling constant and f^{ABC} is the structure constant of SU(3).

- SU$(2)_L \times U(1)_Y$: Electroweak Theory

  The $SU(2)_L \times U(1)_Y$ group defines the electroweak interaction. The gauge fields are W^I_μ (for $SU(2)_L$) and B_μ (for $U(1)_Y$).

  The field strength tensors are respectively :

  $$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g_2 \epsilon^{IJK} W^J_\mu W^K_\nu \tag{2.3}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.4}$$

where g_2 is the coupling constant for $\mathrm{SU}(2)_L$ and ϵ^{IJK} are the structural constant of SU(2). There are no non-linear terms in (2.4) because $\mathrm{U}(1)_Y$ is an Abelian group.

$\mathrm{SU}(2)_L$ indices are usually denoted as {i,j,k} and {I,J,K} in the fundamental and adjoint representations respectiviely. The $\mathrm{SU}(3)_C$ indices in the adjoint representation are instead denoted as {A,B,C}, each runs from {1... 8}.

2.2 Fermion Fields in SM

- Quarks are fundamental particles that carry a color charge under $\mathrm{SU}(3)_C$ and enter the weak interaction under $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$.

$$q_L, \quad u_R, \quad d_R \tag{2.5}$$

- Leptons do not carry color charge, they enter the weak interaction under $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$

$$\ell_L, \quad e_R \tag{2.6}$$

There is no evidence that an independent ν_R field exists.

Each of the five previously mentioned fermions can be described by the following representations, each consisting of three generations:

$$q_L(3,2)_{+1/6}, \quad u_R(3,1)_{+2/3}, \quad d_R(3,1)_{-1/3}, \quad \ell_L(1,2)_{-1/2}, \quad e_R(1,1)_{-1} \tag{2.7}$$

In the weak basis, we have $u_i = \{u_R, c_R, t_R\}, d_i = \{d_R, s_R, b_R\}$, and

$$q_1 = \begin{pmatrix} u_L \\ d'_L \end{pmatrix}, \quad q_2 = \begin{pmatrix} c_L \\ s'_L \end{pmatrix}, \quad q_3 = \begin{pmatrix} t_L \\ b'_L \end{pmatrix}, \quad \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} = V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}, \tag{2.8}$$

where V_{CKM} is the quark mixing matrix. We will discuss it later.

In the Leptonic sector, we have $e_i = \{e_R, \mu_R, \tau_R\}$, and

$$\ell_1 = \begin{pmatrix} \nu'_{eL} \\ e_L \end{pmatrix}, \quad \ell_2 = \begin{pmatrix} \nu'_{\mu L} \\ \mu_L \end{pmatrix}, \quad \ell_3 \begin{pmatrix} \nu'_{\tau L} \\ \tau_L \end{pmatrix}, \quad \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \tag{2.9}$$

where U_{PMNS} is the neutrino mixing matrix.

2.3 Scalar Field

The Standard Model contains one fundamental scalar field, the Higgs Field. This field is responsible for the electroweak symmetry breaking:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM} \tag{2.10}$$

The Higgs mechanism provides a way to generate masses for W and Z bosons and massless photon (γ). The Higgs field is introduced as a scalar doublet:

$$H(1,2)_{+1/2} \tag{2.11}$$

2.4 The Standard Model Lagrangian

The full Lagrangian of the Standard Model is [46]:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{Yukawa}}. \tag{2.12}$$

$\mathcal{L}_{\text{Gauge}}$ contains the kinetic terms for gauge fields, $\mathcal{L}_{\text{Higgs}}$ gives the scalar potential, $\mathcal{L}_{\psi}$ gives fermion mass terms,and $\mathcal{L}_{\text{Yukawa}}$ describes the Yukawa interactions.

I will use the same notation of [4–6].

2.4.1 $\mathcal{L}_{\textbf{Kinetic}}$

The Kinetic part is given by:

$$\begin{aligned}\mathcal{L}_{\text{Kinetic}} = & -\frac{1}{4}G_A^{\mu\nu}G_{A\mu\nu} - \frac{1}{4}W_I^{\mu\nu}W_{I\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + i\overline{q_L}\not{D}q_L + i\overline{u_R}\not{D}u_R + i\overline{d_R}\not{D}d_R \\ & + i\overline{\ell_L}\not{D}\ell_L + i\overline{e_R}\not{D}e_R + (D^\mu H)^\dagger(D_\mu H)\end{aligned} \tag{2.13}$$

The covariant derivative is

$$D_\mu = \partial_\mu + ig_3 G_\mu^A T^A + ig_2 W_\mu^I t_I + ig_1 y_i B_\mu \tag{2.14}$$

where $T^A = \lambda^A/2$ are the $SU(3)_c$ generators, with normalization $Tr(T^A T^B) = 2\delta^{AB}$ the Gell-Mann Matrices taken to be

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{2.15}$$

$t^I = \tau^I/2$ are the $SU(2)_L$ generators, and the Pauli matrices, taken to be

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.16}$$

and y_i are the $U(1)_Y$ charges.

Forinstance H has hypercharge y_H=1/2, it is $SU(3)_C$ singlet and a $SU(2)_L$ doublet so that D acting on H is given by the matrix equation $D_\mu H = (\partial_\mu + ig_2 t^I W^I_\mu + ig_1 B_\mu/2)$

$\mathcal{L}_{\text{Kinetic}}$ is flavor-universal and it conserves CP.

2.4.2 $\mathcal{L}_H$

The scalar part is defined by:

$$\mathcal{L}_H = -\mu^2 H^\dagger H - \lambda (H^\dagger H)^2 = -V(H) \tag{2.17}$$

$\mu^2 < 0$ is the mass term, it indicates spontaneous symmetry breaking (SSB) creating a Mexican hat. $\lambda > 0$ is the self-interaction coupling, it is needed in order to bound the potential from below, and have a stable system. This part is also CP conserving.

$V(H)$ is the Higgs potential which was postulated in 1964 by a group of physicists independently developing the idea of the spontaneous symmetry breaking, one of them is Peter Higgs [47–49].

Later Sheldon Glashow, Abdus Salam, and Steven Weinberg in 70s used the Higgs potential to break the symmetry and give the masses to W and Z while keeping the photon massless.

Without loss of generality, we can write 2.17:

$$\mathcal{L}_H = -\lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 \tag{2.18}$$

$v^2 = -\frac{\mu^2}{\lambda}$.

The scalar field H acquires a VEV, $|\langle H\rangle| = \frac{v}{\sqrt{2}}$. There is an arbitrary choice of the direction of $|\langle H\rangle|$. Having the real direction of the down component

$$\langle H\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \tag{2.19}$$

The residual symmetry after spontaneous symmetry breaking U(1)$_{\mathbf{EM}}$ has a generator that is associated to the electric charge that annihilates the vacuum. It is T_3+Y. The unbroken subgroup is identified with U(1)$_{EM}$, and then its generator Q, is identified as

$$Q = T_3 + Y \tag{2.20}$$

T_3 is the third generator of the SU(2)$_L$ group (the weak isospin), Y is the hypercharge of the field.

According to the non-Abelian spontaneous symmetry breaking and after introducing the Goldstone bosons and one physical degree of freedom, Higgs field can be written as the following:

$$H(x) = exp[\frac{i}{2}(\sigma^a\theta_a(x) - I\theta_3)]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{2.21}$$

The exponential represents the broken generators and their associated Goldstone bosons $\theta_a(x)$. *h(x)* is the physical scalar field that remains after symmetry breaking. The local $SU(2)_L \times U(1)_Y$ symmetry of the Lagrangian allows one to rotate away the explicit dependence on the three $\theta_a(x)$. In this gauge, Higgs field has only one degree of freedom:

$$H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{2.22}$$

Higgs boson mass can be obtained by substituting 2.22 in 2.18, and is given by

$$m_h^2 = 2\lambda v^2 \tag{2.23}$$

Experiment gives [50]

$$m_h = 125.35 \pm 0.15 GeV. \tag{2.24}$$

Via G_F measurement in the muon decay

$$v \approx 246 GeV \tag{2.25}$$

Defining an angle θ_W via

$$\tan\theta_W \equiv \frac{g'}{g} \tag{2.26}$$

Defining the following four boson gauge states [51]

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)_\mu, \quad Z_\mu^0 = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu, \quad A_\mu^0 = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu \tag{2.27}$$

W's are charged under electromagnetism, while A_μ^0 and Z_μ^0 are neutral [52].

Having $(D_\mu H)^\dagger (D^\mu H)$ in the following form

$$= \frac{1}{8}\begin{pmatrix} 0 & v \end{pmatrix}\begin{pmatrix} g_2 W_{3\mu} + g_1 B_\mu & g_2(W_1 - iW_2)_\mu \\ g_2(W_1 + iW_2)_\mu & -g_2 W_{3\mu} + g_1 B_\mu \end{pmatrix}\begin{pmatrix} g_2 W_3^\mu + g_1 B^\mu & g_2(W_1^\mu - iW_2^\mu) \\ g_2(W_1 + iW_2)^\mu & -g_2 W_3^\mu + g_1 B^\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.28}$$

and substituting the four gauge boson states, the vector boson becomes:

$$\frac{1}{4}g_2^2 v^2 W^{+\mu} W_\mu^- + \frac{1}{8}(g_2^2 + g_1^2) v^2 Z^{0\mu} Z_\mu^0 \tag{2.29}$$

It is easy to see that this a mass terms with

$$m_W^2 = \frac{1}{4}g_2^2 v^2, \quad m_Z^2 = \frac{1}{4}(g_2^2 + g_1^2) v^2, \quad m_A^2 = 0 \tag{2.30}$$

Then the gauge boson associated with the unbroken $U(1)_{\mathbf{EM}}$ symmetry, the photon A_μ, remains massless.

From this vector boson, some relations can be defined:

$$\frac{m_W^2}{m_Z^2} = \frac{g_2^2}{g_2^2 + g_1^2} \tag{2.31}$$

This relation is testable. The masses can be derived from the measured spectrum, and the other side from interaction rates. Using eq.2.26, relation 2.31 can be expressed in terms of θ_W [53]:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \tag{2.32}$$

This is a consequence of the way the symmetry is spontaneously broken through the Higgs mechanism with a single SU(2) doublet. And it is true only at tree level.

Experimentally [54, 55],

$$m_W = 80.377 \pm 0.012 GeV, \quad m_Z = 91.1876 \pm 0.0021 GeV \tag{2.33}$$

2.4.3 $\mathcal{L}_\Psi$

- In the SM there is no Dirac mass terms. For a generic Dirac form:

$$\mathcal{L}_{\mathbf{mass,Dirac}} = -m\overline{\Psi}\Psi = -m(\overline{\Psi_L}\Psi_R + \overline{\Psi_R}\Psi_L) \tag{2.34}$$

  The left-handed and right-handed fermions belong to different representations. Then the term $\overline{\Psi_L}\Psi_R$ is not gauge invariant. No direct coupling with mass term without the violation of gauge invariance.

- In SM there are no Majorana mas terms. For a generic Majorana form:

$$\mathcal{L}_{\textbf{mass,Majorana}} = -\frac{1}{2}m\overline{\Psi^c}\Psi + h.c \tag{2.35}$$

  where $\Psi^c = C\overline{\Psi}^T$ is the charge-conjugation of Ψ.

  The condition for Majorana mass term to exist is that the fermion must be neutral under $\text{U}(1)_Y$ so that Ψ^c has the same quantum numbers of Ψ. However, that is not right since all the fermions have $Y \neq 0$.

Thus,

$$\mathcal{L}_{\Psi} = 0 \tag{2.36}$$

2.4.4 $\mathcal{L}_{\textbf{Yukawa}}$

The Yukawa part is given by:

$$\mathcal{L}_{\text{Yukawa}} = -Y^d_{pr}\overline{d_{R_p}}H^{\dagger j}q_r - Y^u_{pr}\overline{u_{R_p}}\tilde{H}^{\dagger j}q_r - Y^e_{pr}\overline{\ell_{e_{R_p}}}H^{\dagger j}\ell_r + h.c \tag{2.37}$$

with p,r are the flavor indices. where $\tilde{H}$ is defined by $\tilde{H}_j = \epsilon_{jk}H^{\star k}$ where the $\text{SU}(2)_L$ invariant tensor ϵ_{jk} is defined by $\epsilon_{12} = 1$ and $\epsilon_{jk} = -\epsilon_{kj}, j, k = \{1, 2\}$.

After symmetry breaking,

$$\mathcal{L}_{\textbf{Yukawa}} = -\frac{v}{\sqrt{2}}Y^d_{pr}\overline{d_{R_p}}d_{L_r} - \frac{v}{\sqrt{2}}Y^u_{pr}\overline{u_{R_p}}u_{L_r} - \frac{v}{\sqrt{2}}Y^e_{pr}\overline{e_{R_p}}e_{L_r} + h.c \tag{2.38}$$

It seems as a mass term, but first some redundancy must be removed to get to the mass basis. Doing that diagonalizing Yukawas is needed.

Defining $M^u_{pr} = \frac{v}{\sqrt{2}}Y^u_{pr}$,$M^d_{pr} = \frac{v}{\sqrt{2}}Y^d_{pr}$, and $M^e_{pr} = \frac{v}{\sqrt{2}}Y^e_{pr}$, the mass matrices. Doing field redefinition by performing unitary transformation:

$$\begin{aligned} e_{L_p} &\to U_{eL_{pw}}e_{L_w}, \quad e_{R_p} \to U_{eR_{pw}}e_{R_w}, \\ u_{L_p} &\to U_{uL_{pw}}u_{L_w}, \quad u_{R_p} \to U_{uR_{pw}}u_{R_w}, \\ d_{L_p} &\to U_{dL_{pw}}d_{L_w}, \quad d_{R_p} \to U_{dR_{pw}}d_{R_w}. \end{aligned} \tag{2.39}$$

The mass matrices are diagonalized by:

$$\hat{M}_u = U^\dagger_{uL}M_uU_{uR}, \quad \hat{M}_d = U^\dagger_{dL}M_dU_{dR}, \quad \hat{M}_e = U^\dagger_{eL}M_eU_{eR}. \tag{2.40}$$

The flavour indices are suppressed here. Equations 2.40 are diagonal matrices containing the mass of the up-type, down-type quarks, and lepton, respectively.

Going back to the Kinetic terms it should be modified by these basis change. The Gauge boson interactions do not mix families in the flavour basis, and its Lagrangian is

$$\mathcal{L}_{\textbf{flavor-basis}} = \begin{pmatrix}\overline{u_L} & \overline{d_L}\end{pmatrix}^p \left[i\partial\!\!\!/ + \gamma_\mu \begin{pmatrix} \frac{g_1}{6}B_\mu + \frac{g}{2}W^3_\mu & \frac{g_2}{\sqrt{2}}W^+_\mu \\ \frac{g_2}{\sqrt{2}}W^-_\mu & \frac{g_1}{6}B_\mu - \frac{g_2}{2}W^3_\mu \end{pmatrix}\right] \begin{pmatrix} u_L \\ d_L \end{pmatrix}^p$$
$$+ \overline{u_R}^p \left(i\partial\!\!\!/ + g_1 \frac{2}{3} B\!\!\!/ \right) u^p_R + \overline{d_R}^p \left(i\partial\!\!\!/ - g_1 \frac{1}{3} B\!\!\!/ \right) d^p_R \tag{2.41}$$
$$- \frac{v}{\sqrt{2}} \left[\overline{d_L}^p M^d_{pr} d^r_R + \overline{u_L}^p M^u_{pr} u^r_R + h.c \right]$$

where p and r are the flavor indices. After field redefinition, there will be a part in the following form,

$$\frac{e}{\sin\theta_w} \left[W^+_\mu \overline{u_L}^i \gamma^\mu (U^\dagger_{uL} U_{dL})^{ij} d^j_L + W^-_\mu \overline{d_L}^i \gamma^\mu (U^\dagger_{dL} U^{ij}_{uL}) u^j_L \right] \tag{2.42}$$

If $U_{uL} = U_{dL}$ then the following transitions are forbidden:

- $K \to \pi$ decay ($s \to u$) [56].
- B-meson decay ($b \to u$ or $b \to c$) [57].

This contradicts experimental evidence of flavor-changing weak decays.

Then $U_{uL} \neq U_{dL}$. Therefore, all the interesting mixing effects are given by a single matrix,

$$V_{CKM} = U^\dagger_{uL} U_{dL} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2.43}$$

known as Cabbibo-Kobayashi-Maskawa (CKM) matrix [58,59]. V_{CKM} matrix is a unitary matrix with 9 independent parameters. Unitarity reduces the paramters from 18 to 9, leaving 3 angles and 6 phases. However, Yukawa interactions break the $\mathrm{U}(3)^3$ symmetry, preserving a residual global $\mathrm{U}(1)^6$ symmetry that acts independently on each quark flavor. Only one combination leaves physics invariant (it is the global phase, which is related to the conservation of baryon number). Thus, 5 of the phases can be eliminated leaving 1 phase, as a physical parameter. So, we remain with 3 angles and one phase.

Then the remaning are 3 angles, θ_{12}, θ_{23}, and θ_{13} correspond to rotations in the i-j flavor planes, and the phase δ, corresponds to the CP-violation.

The most general V_{CKM} can be written as

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s23 - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.44}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. This has become the standard parametrization.

Experimentally [60],

$$\theta_{12} = 13.02^{\circ} \pm 0.05^{\circ}, \quad \theta_{23} = 2.41^{\circ} \pm 0.03^{\circ}, \quad \theta_{13} = 0.20^{\circ} \pm 0.01^{\circ}, \quad \delta_{CKM} = 66^{\circ} \pm 3^{\circ} \tag{2.45}$$

Since the Standard model does not allow for bare mass terms for fermions, their masses can only arise from the Yukawa. After the SSB, the leptonic masses are,

$$m_e = \frac{y_e v}{\sqrt{2}}, \quad m_\mu = \frac{y_\mu v}{\sqrt{2}}, \quad m_\tau = \frac{y_\tau v}{\sqrt{2}} \tag{2.46}$$

the up-type quark masses are,

$$m_u = \frac{y_u v}{\sqrt{2}}, \quad m_c = \frac{y_c v}{\sqrt{2}}, \quad m_t = \frac{y_t v}{\sqrt{2}} \tag{2.47}$$

the down-type quark masses are,

$$m_d = \frac{y_d v}{\sqrt{2}}, \quad m_s = \frac{y_s v}{\sqrt{2}}, \quad m_b = \frac{y_b v}{\sqrt{2}} \tag{2.48}$$

While all the neutrinos remain massless:

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0 \tag{2.49}$$

Their masslessness is related to an accidental symmetry in the Standard Model.

Experimentally [61],

$$\begin{aligned}
&me = 0.510998946(3) MeV, \quad m_\mu = 105.6583745(24) MeV, \\
&m_\tau = 1776.86 \pm 0.12 MeV \\
&m_u = 2.2^{+0.6}_{-0.4}, \quad m_c = 1.27 \pm 0.03 GeV, m_t = 173.2 \pm 0.09 GeV \\
&m_d = 4.7^{+0.5}_{-0.4} MeV, \quad m_s = 96^{+8}_{-4} MeV, \quad m_b = 4.18^{+0.04}_{-0.03} GeV
\end{aligned} \tag{2.50}$$

where u-,d- and s-quark masses are given at scale $\mu = 2GeV$, c- and b-quark masses are the running masses in the $\overline{MS}$ scheme, and t-quark mass derived from direct measurements.

Standard Model particles can be summed up by the following tables:

2.5 Standard Model Equations of Motion

It is important to focus on the SM equations (EOM) of motion since it will play a useful role in the choice of basis in the SMEFT later.

- EOM for the Higgs field:

$$D^2 H_k - \lambda v^2 H_k + 2\lambda (H^\dagger H) H_k + \overline{q}^j Y_u^\dagger u \epsilon_{jk} + \overline{d} Y_d q_k + \overline{e} Y_e \ell_k = 0 \tag{2.51}$$

Particle	**Spin**	**Color**	**Q**	**Mass** $[v]$
$W^{\pm}$	1	(1)	± 1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$

Table 2.1: Bosonic particles

Particle	**Spin**	**Color**	**Q**	**Mass** $[v]$
e, μ, τ	$\frac{1}{2}$	(1)	-1	$\frac{y_{e,\mu,\tau}}{\sqrt{2}}$
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	(1)	0	0
u,c,t	$\frac{1}{2}$	(3)	$+\frac{2}{3}$	$\frac{y_{u,c,t}}{\sqrt{2}}$
d,s,b	$\frac{1}{2}$	(3)	$-\frac{1}{3}$	$\frac{y_{d,s,b}}{\sqrt{2}}$

Table 2.2: Fermionic particles

- EOM for the fermionic fields:

$$\begin{aligned} &i\not{D}q_j = Y_u^\dagger u\tilde{H}_j + Y_d^\dagger dH_j, \quad &&i\not{D}d = Y_d q_j H^{\dagger j}, \quad i\not{D}u = Y_u q_j \tilde{H}^{\dagger j}, \\ &i\not{D}\ell_j = Y_e^\dagger e H_j, &&i\not{D}e = Y_e \ell_j H^{\dagger j}. \end{aligned} \tag{2.52}$$

- EOM for the gauge fields:

$$[D^\alpha, G_{\alpha\beta}]^A = g_3 j_\beta^A, \quad [D^\alpha, W_{\alpha\beta}] = g_2 j_\beta^I, \quad D^\alpha B_{\alpha\beta} = g_1 j_\beta, \tag{2.53}$$

$[D^\alpha, F_{\alpha\beta}]$ is the covariant derivative in the adjoint representation.
And the gauge current are defined as:

$$\begin{aligned} &j_\beta = \sum_{\psi=u,d,Q,e,L} \overline{\psi} y_i \gamma_\beta \psi + \frac{1}{2} H^\dagger i \overleftrightarrow{D}_\beta H, \qquad j_\beta^I = \frac{1}{2}\overline{q}\tau^I\gamma_\beta q + \frac{1}{2}\overline{\ell}\tau^I\gamma_\beta\ell + \frac{1}{2}H^\dagger i \overleftrightarrow{D}_\beta^I H, \\ &j_\beta^A = \sum_{\psi'=u,d,Q} \overline{\psi} T^A \gamma_\beta \psi \end{aligned} \tag{2.54}$$

With

$$H^\dagger i \overleftrightarrow{D}_\beta H = iH^\dagger(D_\beta H) - i(D_\beta H)^\dagger H, \quad H^\dagger i \overleftrightarrow{D}_\beta^I H = iH^\dagger \tau^I (D_\beta H) - i(D_\beta H)^\dagger \tau^I H \tag{2.55}$$

τ are the Pauli matrices.

These equations of motion are not the final result which can be modified through corrections from the higher-order effective Lagrangian parts.

2.6 SM struggles

One of the greatest achievements of the Standard Model is being a renormalizable theory with dimension $d \leq 4$ for all operators. It unifies electromagnetism and weak interactions under the name of electroweak (EW) and incorporates Quantum Chromodynamics (QCD) for the description of strong interactions.

Beyond Standard Model Theories (BSM) emerged because SM lacks the description of some aspects of nature. For instance, SM does not describe gravity. As Quantum Gravity (QG) is non-renormalizable, when treating General Relativity as a quantum field theory, it diverges at high energies requiring an infinite number of counterterms, each having new parameters [62]. Moreover, SM describes only a little portion of the universe because recent discoveries have found that the universe is full of Dark matter and Dark energy that make $\sim 95\%$ of the universe, while ordinary matter $\sim 5\%$ [63–65]. Furthermore, SM treats neutrino as being massless, due to accidental symmetries, but recent experiments have found that there are neutrino oscillations [66–70], which indicates that they have masses. One way of describing neutrino mass is by going to the operator d = 5 with Majorana mass terms, described by the See-Saw mechanism [71–74]. SM holds CP violation in the weak force, but it is not enough to explain matter-antimatter asymmetry [75, 76]. Why is the universe matter dominant? How did the universe reach an out-of-equilibrium state?

Additionally, the Higgs sector of the SM is full of problems:

- Hierarchy problem [77]:

  In the presence of heavy BSM physics coupling to the Higgs, the Higgs boson receives large quantum corrections from loop diagrams, and without fine-tuning these corrections can drive the mass of the Higgs boson to be in the ultraviolet scale (UV). Supersymmetry (SUSY) [78] can be a possible solution, also Composite Higgs Models [79] can be a solution.

- Stability problem [80]:

  Assuming that there is only SM (no BSM), the Higgs quartic coupling runs to negative values at high energies, because of the running induced by the top loop. This means that at very high energies the EW vacuum could be metastable or not be a minimum at all.

- Triviality [81]:

  Using renormalization group equations (RGE) the running of λ can be calculated. Triviality implies that λ at some energy scale can approach zero making the theory non-interacting or trivial.

Flavor Puzzle is also a problem in the SM. It refers to the unexplained existence of three generations of quarks and leptons, as well as the pattern of their masses and mixing angles. SM describes the interactions of these particles successfully. However, it does not provide an explanation why there are exactly three generations or even why there is a several-orders-of-magnitude difference between their masses. Also, the CKM matrix appears to be arbitrary, it requires experimental input rather than emerging from the theory. The Yukawa couplings in SM that describe the masses and mixing are just free parameters. Various theories tried to explain these patterns, such as Froggatt-Nielsen mechanism [82], Grand unified theories (GUTs) [83], Pati-Salam model [84], Partial Compositeness [85, 86], Modular Flavor Symmetries [87] developed recently.

While the Standard Model has successfully described known particle interactions, several questions are still open. A powerful approach to explore such new physics while achieving the consistency with known experimental data is the framework of Effective Field Theories (EFTs). The next chapter will focus on the EFTs and their technicalities.

Chapter 3

Effective Field Theory

3.1 Why EFT's?

Effective Field Theory (EFT) is a very useful way to do calculations in multi-scale problems [88, 89]. Cases with separated scales are often in nature. We consider a system with two well-separated mass scales: a heavy sector with mass M and a light sector describing the observed degrees of freedom. Our focus is on describing physics at energies $E \ll M$, where the heavy states cannot be directly produced but induce low-energy effects via an effective field theory (EFT).[1].

In quantum field theory (QFT), only the degrees of freedom (DoF) relevant to the energy scale of interest are needed for calculations. Heavy particles with masses far above this scale do not disappear but instead influence low-energy physics indirectly through virtual effects. These particles momentarily "pop in and out" in quantum loops, modifying interaction strengths and other parameters. As a result, the operators describing physical observables involve only light degrees of freedom, while the effects of heavy particles are encoded in corrections to coupling constants and higher-dimensional operators. This leads to a scale-dependent description where interactions evolve with energy, a concept formalized by the renormalization group.

In the case of the hydrogen atom, you would only need to consider the electron and proton as dynamical degrees of freedom, since the top quark is decoupled at this scale. Only light interactions (such as electromagnetic interactions) matter for calculating the energy levels. In contrast, when measuring the muon g-2, the top quark's indirect effects (via loop corrections) must be included, but it doesn't change the fact that the muon is the primary dynamical degree of freedom in the system.

Moreover, as we will see later, it is sufficient to improve the accuracy of our prediction by introducing more operators in the theory that are allowed by the symmetries in

[1]For great review of EFT's see Refs. [90–96] . The original works that laid the groundwork for the modern EFT's include [97–103]

the theory. Symmetries are the guide of building new operators. The structure and coefficients in front of these operators, when fit to experimental data, might be able to tell something related to the integrated heavy particles.

As a result, EFT's are a very good tool to study new particles or new physics.

3.2 A theorem of Weinberg

Theorem 3.2.1 *For any given order in perturbation theory and for a given set of asymptotic states, the most general possible Lagrangian containing all terms allowed by the assumed symmetries will yield the most general S-matrix elements consistent with analyticity, perturbative unitarity, cluster decomposition, and assumed symmetry principles.*

Initially, this theorem was written in the domain of pion physics. But nothing in this theorem says that it is only applicable to this domain, and it is expected that this theorem should work in any EFT.

According to Weinberg, to construct the most general S-matrix consistent with symmetries, we must write the most general Lagrangian that includes all possible operators allowed by the symmetries, regardless of their dimension. However, this leads to an infinite number of interaction terms, making the theory non-renormalizable, meaning that an infinite number of counterterms would be required to absorb loop divergences. Renormalizable theories (with operators of dimension $d \leq 4$) are predictive because they allow us to relate a finite number of Lagrangian parameters to physical observables. However, they do not inherently tell us where they break down—e.g., QED is renormalizable, but its behavior at ultra-high energies remains unknown without a collider of unimaginable scale. The key to handling non-renormalizable interactions is Effective Field Theory (EFT): the infinite set of higher-dimensional operators can be systematically organized into a series by their increasing canonical dimension. Since higher-dimensional operators are suppressed by powers of some large energy scale Λ, we can truncate the series at a given order, making the theory predictive order by order in an expansion in $1/\Lambda$. This scale Λ represents the limit of validity of the EFT and signals the presence of new physics beyond it. Thus, while renormalizable theories are simpler, EFT provides a controlled way to include non-renormalizable effects while retaining predictivity.

Nonrenormalizable theories are more predictive theories. For example, Fermi theory for weak interaction [104] is a non-renormalizable theory that tells you that at 300 GeV it will loose unitarity and then break, then there is a UV-completion. For more discussion see Refs [105, 106] In addition, non-renormalizable theory are still renormalized.

3.3 Dimensional Analysis

In field theory of d dimensions, the action (S) can be expressed as:

$$S = \int \mathrm{d}^d x\, \mathcal{L}(x) \tag{3.1}$$

In natural units ($\hbar = c = 1$) , the action is dimensionless. Coordinates have the dimension [Length]=1/[Energy], therefore for d spacetime dimensions, the lagrangian density has mass dimension d,

$$[\mathcal{L}] = d \tag{3.2}$$

Lagrangian density has the following structure,

$$\mathcal{L}(x) = \sum_i c_i O_i(x) \tag{3.3}$$

O_i are Lorentz invariant operators with coefficients c_i. Then the sum if there dimensions is equal to d. Supposing, operator has the dimension of ζ, then its coefficient has dimension $d - \zeta$.

Doing this analysis for other lagrangian parts, we will find the following,

$$\begin{aligned} [\psi] &= \frac{1}{2}(d-1) & [H] &= \frac{1}{2}(d-2) \\ [D_\mu] &= 1 & [gA_\mu] &= 1 \\ [A_\mu] &= \frac{1}{2}(d-2) & [g] &= \frac{1}{2}(4-d) \end{aligned} \tag{3.4}$$

In d=4 spacetime dimensions,

$$[H] = 1, \quad [\psi] = 3/2, \quad [A_\mu] = 1, \quad [D_\mu] = 1, \quad [g] = 0 \tag{3.5}$$

In $d = 4 - 2\epsilon$ dimension [107], $[g] = \epsilon$.[2]

Renormalizable interactions have coefficients with mass dimension≥0. Any operator with dimension greater than 4 will be non-renormalizable and irrelevant. Operators with dimension less than 4 are super-renormalizable and relevant. Marginal operators have dimension equal to 4.

3.4 EFT Expansion

Following weinberg theorem in section (3.2), constructing the most general effective lagrangian is constructed from the allowed operators, respecting the symmetries and

[2]In dimensional regularization, using $\overline{MS}$ subtraction scheme, by introducing n power of $\hat{\mu}^{(4-d)/2} = (\mu\sqrt{e^\gamma/4\pi}^{(4-d)/2}$ for each power of the coupling present defining the amplitude [8], so that the renormalized coupling remains dimensionless. $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. This constant will always cancel in observable quantities [108]. This argument are formulated for a one loop example but they generalize to higher loop orders directly.

yielding the most general S-matrix elements, and has an expansion in powers of the operator dimension

$$\mathcal{L}_{\mathbf{EFT}} = \sum_{\zeta \geq 0, i} \frac{c_i^{(\zeta)} O_i^{(\zeta)}}{\Lambda^{\zeta - d}} = \sum_{\zeta \geq 0} \frac{\mathcal{L}_\zeta}{\Lambda^{\zeta - d}} \tag{3.6}$$

The main difference here that one does not stop at $\zeta = d$, but include operators with arbitrary high dimension. Λ is a short-distance scale at which new physics occur it is introduced such that the coefficients $c_i^{(\zeta)}$ are dimensionless.

In d=4,

$$\mathcal{L}_{\mathbf{EFT}} = \mathcal{L}_{\zeta \leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \dots \tag{3.7}$$

$\mathcal{L}_{\mathbf{EFT}}$ is an expansion in power of $1/\Lambda$.

3.5 Power Counting and Renormalizability

By dimensional analysis, the dimension of a scattering amplitude for a single insertion of an operator of dimension ζ with some typical momentum p,

$$\mathcal{M} \sim \left(\frac{p}{\Lambda}\right)^{\zeta - d} \tag{3.8}$$

The insertion of a set of higher dimension operators in a tree graph leads to an amplitude

$$\mathcal{M} \sim \left(\frac{p}{\Lambda}\right)^{n} \tag{3.9}$$

where

$$n = \sum_i (\zeta_i - d) \tag{3.10}$$

summing over the inserted operators. This is known as the EFT power counting formula. This formula holds for any graph, not just for tree graphs.

Looking back at 3.7, the first term is a renormalizable Lagrangian, and the the extra terms violate the condition of reneormalizability. However, its amplitude is 3.9. Thus, even though we require an infinite number of counterterms to get a finite results, we need only a finite subset of operators to get an answer accurate to a given order in an energy-momentum expansion. The higher terms are suppressed then we can truncate the series at a specific accuracy level. For most physical problems, only a finite number of terms are necessary, even if the theory requires an infinite set of operators.

3.6 Renormalization Group Equations

Renormalization Group is a great idea that shaped much our understanding of todays quantum field theory.

There are two versions of the renormalization group [109][3],

- The Wilsonian renormalization group:

  In a finite theory with a UV cutoff Λ, the physics at energies E$\ll$ Λ is independent of the precise value of Λ. Changing Λ changes the couplings in the theory so that observables remain the same.

- The continuum renormalization group:

  Observables are independent of the renormalization conditions, in particular, of the scales p_0 at which we choose to define our renormalized quantities. This invariance holds after the theory is renormalized and the cutoff is removed ($\Lambda = \infty, d = 4$). In dimensional regularization with $\overline{MS}$, the scales p_0 are replaced by μ, and the continuum renormalization group comes from μ independence

The two versions are closely related by technically different.

We will focus on the Continuum one, since for later SMEFT is designed for practical applications to low-energy processes where the scale dependence of operators coefficients matters. Also, it can be natural implemented using the $\overline{MS}$ scheme as we said.

Without going deep in calculations since it is not the point here.

- For a Wilsonian picture, the RGE will forces

$$\Lambda \frac{d}{d\Lambda} C_n = \beta_n[(C_m), \Lambda] \tag{3.11}$$

  where β_n are the beta function, and C_n are the Wilson coefficients of the operators.

- For continuum picture,

$$\mu \frac{d}{d\mu} C_n = \gamma_{nm} C_m \tag{3.12}$$

  where γ_{nm} is called the anomalous dimension. It is an important quantity that tell us about the deviation from the classical scaling behavior.

Any operator with dimension greater than 4 will be non-renormalizable and irrelevant. Operators with dimension less than 4 are super-renormalizable and relevant. Marginal operators have dimension equal to 4. However,if the If a marginal operator acquires an

[3]For further information about RG see Refs [110–116]

anomalous dimension, it can shift slightly to marginally relevant or marginally irrelevant. If it becomes marginally relevant, it behaves like a relevant operator. For marginally irrelevant, it behaves like an irrelevant operator. From the Wilsonian point of view, marginally relevant operators are the same as irrelevant ones – one cannot keep their couplings fixed at low energy and remove the cutoff.

3.7 Integrating out a Field

"Integrating out a field" as nomenclature follows from the path integral approach to defining an effective action as developed by Wilson [117,118]. Consider a physical system with lagrangian $\mathcal{L}_{UV}(L, H)$, L for light field and H for heavy one.

The full dynamics is inside $\mathcal{L}_{UV}$. For low energy scales $\ll$ the mass of the heavy, the external states are the light ones and the heavy could only be virtual.

Considering the following generating function,

$$Z_{UV}[J_L, J_H] = \int [\mathcal{D}L][\mathcal{D}H] exp\left[i \int d^4x (\mathcal{L}_{UV}(L, H) + J_L L + J_H H)\right] \tag{3.13}$$

All n-point correlation functions L's and H's can be obtained by differentiating Z_{UV} with respect to the current J:

$$(-i)^n \frac{1}{Z[0]} \frac{\partial^n Z[J]}{\partial J(x_1) \ldots \partial J(x_n)}\bigg|_{J=0} = \langle 0|T\{\phi_1\phi_2 \ldots \phi_n\}|0\rangle \tag{3.14}$$

In EFT we only need the correlators of L's.

By eliminating H, meaning that $J_H = 0$, we formally performed the integral:

$$e^{i \int d^4x \mathcal{L}_{EFT}} = \int \mathcal{D}H e^{i \int d^4x \mathcal{L}_{UV}} \tag{3.15}$$

Then we get the following:

$$Z_{\mathbf{EFT}}[J] = Z_{UV}[J_L, J_H]\Big|_{J_H=0} = \int [\mathcal{D}L] exp\left[i \int d^4x (L_{EFT}(L) + J_L L)\right] \tag{3.16}$$

$\mathcal{L}_{EFT}$ here is a complicated object, it has local terms which can appear from the expansion in local operators. Also, it could have local terms that emerge after integrating out degrees of freedom.

In an EFT, the interactions must be local, meaning that the Lagrangian is written as a sum of local operators involving fields and their derivatives at the same spacetime point. This follows from the fact that integrating out heavy fields with mass M (where

M≫p, with p being the relevant momentum scale) leads to a local expansion in powers of p/M.

Non-local interactions, characterized by terms with derivatives in the denominator (e.g., $1/\Box$ or $1/(p^2 - M^2)$), arise only when integrating out light or on-shell particles, which is inconsistent with the EFT approach. In EFT, we take the opposite limit: p→0, not M→0. This ensures that the non-local propagator structure collapses into a local power series.

Let's continue, the point is to find $\mathcal{L}_{EFT}$. The saddle-point approximation is useful here. The equation of motion of H in the presence of J_H and L at tree level is

$$\frac{\delta S}{\delta H} = 0 \quad \rightarrow \quad H = H_{cl}[L, J_H] \tag{3.17}$$

Since H is heavy, thus having a large mass M, this can be solved perturbatively in power of 1/M:

$$H_{cl} = f[L] + O(1/M) \tag{3.18}$$

Then equation 3.17 becomes

$$\left.\frac{\delta S}{\delta H}\right|_{H=H_{cl}(L)} = 0 \tag{3.19}$$

Then we get the following result:

$$Z_{UV}[J_L, 0] = \int [\mathcal{D}L] exp[i \int d^4x (\mathcal{L}_{UV}(L, H_{cl}(L)) + J_L L)] \tag{3.20}$$

Then at tree level,

$$\mathcal{L}_{EFT}(L) = \mathcal{L}_{UV}(L, H_{cl}(L)) \tag{3.21}$$

H is no more an independent degree of freedom. The saddle point approximation and the calculation done above are only valid on tree level. But integrating out with functional methods could be done at any loop order.

3.8 Matching

An EFT describes particle inetactions without the need of all the details in the UV completion. Since they both behave differently at high energies and they are treated separately each with its own renormalization and counterterms.

As we have seen in section 3.7, we did approximations. Sometimes, EFT can accurately reproduce the low-energy behavior of the UV theory, but to ensure the match, their parameters need to match. This process called matching, determining the Wilson coefficients that appears in the EFT as a function of the parameters in the UV model.

If the UV theory is weakly coupled, matching can be done perturbatively. That will include computing scattering amplitudes in both, then equating them in the low energy limit. This determines the EFT parameters in terms of the UV parameters.

Matching is typically done at tree level but it can be done at higher orders in perturbation theory. While both the UV and IR divergences appear in calculations, they cancel out when matching is done [106].

Matching is usually done at the energy cale of the heavy particles that were removed in the EFT. While this isn't required, but it makes the calculations more stable when combined with the RGE since it helps reducing the large logarithmic corrections, which track how parameters change with energy. Techniques like dimensional regularization and minimal subrtraction (MS) simplify these calculations. For excellent examples about matching see the Ref [119–128]

Defining the Effective Field Theory (EFT) framework, which allows us to describe physics beyond the Standard Model in a model-independent way, we now focus on a specific and well-motivated application: the Standard Model Effective Field Theory (SMEFT). In EFT, new physics effects are encoded in higher-dimensional operators built from SM fields, but without assuming a particular UV completion. SMEFT provides a systematic approach to extend the Standard Model (SM) while preserving its gauge symmetries, allowing us to parametrize potential deviations from the SM predictions. In the next chapter, we explore the construction of SMEFT, the classification of operators, and the implications of redundancies and symmetries in this effective description.

Chapter 4

The Standard Model Effective Field Theory (SMEFT)

4.1 Operator bases in SMEFT

The standard model theory has been striking successful in its experimental predictions. These experiments were successful and surprising as well. There were events that are difficult to understand within the context of standard model. Numerous suggestions have been made to understand the origin of such events, but we do not have enough evidence proving that any of these models are realized in nature.

Due to lack of evidences, it would be useful to have a model independent parametrization of the new physics. This can be done from the effective operator point of view, in the same spirit as the classification of baryon and lepton number violating processes [129,130].

If we assume that the standard model indeed describes physics well in the range up to the electroweak mass, but above, take it to be an effective low energy theory in which heavy particles are integrated out, then it is compelling to describe physics up to energies of order of Λ by an effective Lagrangian, that consist of higher (>4) dimensional effective operators which are scaled by appropriate inverse power of Λ. As we have said before these terms must respect the symmetries in the theory. Involving the scalar, fermions and gauge bosons of the standard model. The effective lagrangian is as follow:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \ldots, \quad \mathcal{L}^{(d)} = \sum_{i=1}^{nd} \frac{C^{(d)_i}}{\Lambda^{d-4}} Q_i^{(d)} \tag{4.1}$$

There are no dimension five operators consistent with the three requirements.

For dimension six operator, the first work or expansion of operator was in 1986 by Leung, Love and Rao [131]. They were able to find 120 operator, but it was full of redundances. Then in the same year Buchmuller and Wyler [1]were able using the equations of motion and Fierz identities to reduce the number of operators to 80. Furthermore, in 2010, a group of people in University of Warsaw were able to reduce the number more, until 59 independent operator that respect all the three requirements. These non-redundant operators and this work was named as Warsaw basis. We will not go for other operators, since our work will focus on operators of dimension 6 [1]. The Warsaw basis is widely used, but it is not the only possible option. Depending on the analysis done, other bases could be suitable. A commonly adopted subset of dimension-6 operators in phenomenological analysis is the so-called strongly interacting light Higgs (SILH) basis [2] is sometimes referred to as an operator basis in some literature,as is the HISZ subset of operators [148].

4.2 Removing Redundancies

As it was mentioned the previous section, there was many redundancies. Operators are redundant if they yield to the same contribution to all physical observables. To reduce the redundant operators we will need some techniques, we will proceed in them. But this discussion is not meant as a complete description for reducing a given operator set to a basis, but instead it only highlights the most common mehtods used [149].

4.2.1 Integration by part

In quantum field theory, total derivative terms in the Lagrangian give no physical effects, as long as the field configurations go to zero fast enough at infinity. Keeping the action S unchanged under the integration by parts (IBP). Such property allows us to relate different operators by rewriting them using the IBP. For example in the SM, the Higgs kinetic term can be expressed in two equivalent forms:

$$(D_\mu H)^\dagger (D^\mu H) \quad and \quad -H^\dagger D^2 H \tag{4.2}$$

Similarly, for SMEFT, IBP can be used in order to help simplifying the operator basis and eliminate redundancies. For example:

$$(\overline{q}\gamma^\mu D^\nu q) W_{\mu\nu} \tag{4.3}$$

After doing integration by parts, we will have

$$\int d^4x (\overline{q}\gamma^\mu D^\nu q) W_{\mu nu} = -\int d^4x \overline{q}\gamma^\mu q (D^\nu W_{\mu\nu}) \tag{4.4}$$

Now, using equation of motion for the gauge field:

$$D^\nu W_{\mu\nu} = g_3 j_\mu \tag{4.5}$$

[1]For more details about higher dimensional operators see Ref [132–147]

where j_μ is the gauge current, after substituting

$$\int d^4x(\bar{q}\gamma^\mu D_\nu q)W_{\mu\nu} = -g\int d^4x\bar{q}\gamma^\mu q j_\mu \tag{4.6}$$

This means that operator in 4.3 is redundant and can be eliminated.

4.2.2 Field redefinitions

One of the most significant forms of equivalence among different effective operators arises from field redefinitions. According to Lehmann-Symanzik-Zimmermann (LSZ) reduction formula [150], we are free to choose any form for the interpolating quantum fields in our theory, provided that the fields we use, can generate all the relevant states from the vacuum, without impacting physical observables. Such flexibility enables us to perform field redefinitions in our effective lagrangian, which can modify the operators while keeping the physical observables unchanged [151–153]. The important field redefinitions that are useful for the SMEFT are perturbative transformations of the type

$$\phi \to \tilde{\phi}(\phi) = \phi + \alpha G(\phi) \tag{4.7}$$

where the new field is given by the original field ϕ plus a small ($\alpha \ll 1$) perturbation $G(\phi)$ that can depend on the field ϕ itself and also on all the SM fields, and their analytical derivatives. For the SMEFT the expansion parameter α is usually related to a power n of the EFT expansion parameter $(p/\Lambda)^n$, where p is the energy scale for the process of interest.

Next we give a concrete example of how field redefinition that preserves the local symmetries can eliminate the redundancy. According to [8], for the Bosonic field, we have the following set of $O(1/\Lambda^2)$ field redefinitions that preserve G_{SM}

$$H'_j \to H_j + h_1\frac{D^2H_j}{\Lambda^2} + h_2\frac{\bar{e}\ell_jY_e}{\lambda^2} + h_3\frac{\bar{d}q_jY_d}{\Lambda^2} + h_4\frac{(\bar{u}\epsilon q_j)^\star Y_u^\star}{\Lambda^2} + h_5\frac{H^\dagger HH_j}{\Lambda^2}, \tag{4.8}$$

$$B'_\mu \to B_\mu + b_1\frac{\bar{\psi}\gamma_\mu\psi}{\Lambda^2} + b_2\frac{H^\dagger i\overleftrightarrow{D}_\mu H}{\Lambda^2} + b_3\frac{D^\alpha B_{\alpha\mu}}{\Lambda^2} + b_4\frac{H^\dagger HB_\mu}{\Lambda^2}, \tag{4.9}$$

$$W_\mu^{I\prime} \to W_\mu^I + w_1\frac{\bar{q}\sigma^I\gamma_\mu q}{\Lambda^2} + w_2\frac{\bar{\ell}\sigma^I\gamma_\mu\ell}{\Lambda^2} + w_3\frac{H^\dagger\overleftrightarrow{D}_\mu H}{\Lambda^2} + w_4\frac{[D^\alpha, W_{\alpha\mu}]^I}{\Lambda^2} + w_5\frac{H^\dagger HW_\mu^I}{\Lambda^2} \tag{4.10}$$

$$G_\mu^{A\prime} \to G_\mu^A + g_1\frac{\bar{q}T^A\gamma_\mu q}{\Lambda^2} + g_2\frac{\bar{d}T^A\gamma_\mu d}{\Lambda^2} + g_3\frac{\bar{u}T^A\gamma_\mu u}{\Lambda^2} + g_4\frac{[D^\alpha, G_{\alpha\mu}]^A}{\Lambda^2} + g_5\frac{H^\dagger HG_\mu^A}{\Lambda^2} \tag{4.11}$$

Field redefinitions of the right-handed fermion fields are

$$e' \to e + e_1 \frac{\bar{\ell} i \not{D} H Y_e^\dagger}{\Lambda^2} + e_2 \frac{\bar{\ell} i \overleftarrow{\not{D}} H Y_e^\dagger}{\Lambda^2} + e_3 \frac{H^\dagger H e}{\Lambda^2} + e_4 \frac{D^2 e}{\Lambda^2} \tag{4.12}$$

$$d' \to d + d_1 \frac{\bar{q} i \not{D} H Y_d^\dagger}{\Lambda^2} + d_2 \frac{\bar{q} i \overleftarrow{\not{D}} H Y_d^\dagger}{\Lambda^2} + d_3 \frac{H^\dagger H d}{\Lambda^2} + d_4 \frac{D^2 d}{\Lambda^2} \tag{4.13}$$

$$u' \to d + d_1 \frac{\bar{q} i \not{D} H Y_u^\dagger}{\Lambda^2} + u_2 \frac{\bar{q} i \overleftarrow{\not{D}} \tilde{H} Y_u^\dagger}{\Lambda^2} + u_3 \frac{H^\dagger H u}{\Lambda^2} + u_4 \frac{D^2 u}{\Lambda^2} \tag{4.14}$$

Field redefinitions of the left-handed fermion fields are

$$q_j' \to q_j + q_1 \frac{u i \not{D} \tilde{H}_j Y_u^\dagger}{\Lambda^2} + q_2 \frac{\bar{q} i \overleftarrow{\not{D}} \tilde{H}_j Y_u^\dagger}{\Lambda^2} + q_3 \frac{d i \not{D} H_j Y_d^\dagger}{\Lambda^2} + q_4 \frac{d i \overleftarrow{\not{D}} H_j Y_d^\dagger}{\Lambda^2} + q_5 \frac{H^\dagger H Q_j}{\Lambda^2} + q_6 \frac{D^2 Q_j}{\Lambda^2} \tag{4.15}$$

$$\ell_j' \to \ell + l_1 \frac{e i \not{D} H_j Y_e^\dagger}{\Lambda^2} + l_2 \frac{e i \overleftarrow{\not{D}} H_j Y_e^\dagger}{\Lambda^2} + l_3 \frac{H^\dagger H L_j}{\Lambda^2} + l_4 \frac{D^2 L_j}{\Lambda^2} \tag{4.16}$$

Here $\{h_a, b_a, w_a, e_a, u_a, d_a, q_a, \ell_a\}$ are free variables. Performing field redefinitions with only a single $O(1/\Lambda^2)$ term on the right hand side of each equation one can choose to cancel an operator out of a full set of operators. For example, the B^μ dependent, flavor symmetric terms in an overcomplete $\mathcal{L}_{SMEFT}$ are

$$\begin{aligned}\mathcal{L}_{B'} = &-\frac{1}{4} B'_{\mu\nu} B'^{\mu\nu} - g_1 y_\psi \bar{\psi} \not{B}' \psi + (D^\mu H)^\dagger (D_\mu H) + C_B (H^\dagger \overleftrightarrow{D}^\mu H)(D^\nu B_{\mu\nu}) \\ &+ C_{BH} (D^\mu H)^\dagger (D^\nu H) B'_{\mu\nu} + C_{Hl_{tt}}^{(1)} Q_{Hl_{tt}}^{(1)} + C_{He_{tt}} Q_{He_{tt}} + C_{Hq_{tt}}^{(1)} Q_{Hq_{tt}}^{(1)} + C_{Hu_{tt}} Q_{Hu_{tt}} \\ &+ C_{Hd_{tt}} Q_{Hd_{tt}} + C_{HB} Q_{HB} + c_T (H^\dagger \overleftrightarrow{D}^\mu H)(H^\dagger \overleftrightarrow{D}^\mu H)\end{aligned} \tag{4.17}$$

Performing the small field redefinition

$$B'_\mu \to B_\mu + b_2 \frac{H^\dagger i \overleftrightarrow{D}_\mu H}{\Lambda^2} \tag{4.18}$$

yields the result $\mathcal{L}_{\mathcal{B}} - g_1 b_2 \Delta B$ where

$$\begin{aligned}\Delta B = &\, y_l Q_{Hl_{tt}}^{(1)} + y_e Q_{He_{tt}} + y_q Q_{Hq_{tt}}^{(1)} + y_u Q_{Hu_{tt}} + y_d Q_{Hd_{tt}} \\ &+ y_H (Q_{H\Box} + 4 Q_{HD}) + \frac{1}{g_1} B^{\mu\nu} \partial_\mu (H^\dagger i \overleftrightarrow{D}_\nu H).\end{aligned} \tag{4.19}$$

Choosing b_2 to cancel one of the $\mathcal{L}_{B'}$ operators introduces a shift in the Wilson coefficients of the remaining operators. Whne the full set of such field redefinitions has been performed, this corresponds to choosing a non-redundant basis.

These way of field redefinition is equivalent to using equation of motions in section 2.5. But if we work at the subleading power, like including the dimension-8 operators in the field redefinition. It is immediately clear that in this case the use of equations of motion is no longer equivalent to applying field redefinitions, as the former do not capture the subleading shift of the field . Then, when we have an effective lagrangian with different powers we must not use the equations of motion to remove redundancies, but it is more correct to apply field redefinitions to get correct results[2].

4.2.3 Fierz Identities

Fierz identities are fundamental relations among fermionic bilinears that arise from the algebra of gamma matrices in four-dimensional spacetime [156]. They allow for the re-ordering of spinor contractions in four-fermion interactions but do not account for the gauge structure of the Standard Model fermions, which carry additional SU(N) indices. The completeness relations of SU(N) generators, on the other hand, describe how gauge indices contract and must be treated separately from the Fierz transformation. While Fierz identities reorganize the spinor structure of interactions, SU(N) completeness relations ensure the proper handling of color and weak isospin indices. In effective field theories, such as SMEFT, both must be considered when reducing operators to a minimal basis, ensuring consistency between Lorentz and gauge symmetries. Next we will do the derivation following [157].

Without loss of generality, for any fundamental representation of SU(N) algebra $\{T_a\}$ satisfying Tr$[T_a T_b]$=Cδ_{ab}, we have the completeness relation

$$\frac{1}{C}(T_a)_{ij}(T_a)_{kl} + \frac{1}{N}\delta_{ij}\delta_{kl} = \delta_{il}\delta_{kj} \tag{4.20}$$

For example for SU$(2)_L$ this allows us to rewrite the Higgs operators as

$$(H^\dagger \tau^I H)(H^\dagger \tau^I H) = (H^\dagger H)^2 \tag{4.21}$$

For the O(N) groups there is no simple relation like 4.20 since the algebra is formed by $N \times N$ antisymmetric matrices.

Before starting the derivation, defining the following notation due to Takahashi [158], where it replaces the matrices by parentheses () and brackets []. For example for equation 4.20 it reads

$$\frac{1}{C}(T_a)[T_b] + \frac{1}{N}()[] = (][) \tag{4.22}$$

[2]For more details about this topic see Ref [154, 155]

where the blank entry means the identity matrix.

Since electroweak is chiral, we need to derive the chiral Fierz identities.Defining $P_{R/L}$ the chirality projector operators as $P_{R/L} = (1/2)(\mathbb{1} \pm \gamma_5)$ and treating chirally projected combinations such as

$$(P_R\gamma^\mu)[P_L\gamma_\mu] \tag{4.23}$$

It is more useful to derive the fierz identities in the chiral basis $\{\Gamma^n\}$

$$\{\Gamma^n\} = \{P_R, P_L, P_R\gamma^\mu, P_L\gamma^\mu, \sigma^{\mu\nu}\} \quad (\mu, \nu = 0, 1, 2, 3) \tag{4.24}$$

its respective dual basis

$$\{\tilde{\Gamma_n}\} = \{P_R, P_L, P_L\gamma_\mu, P_R\gamma_\mu, \frac{1}{2}\sigma_{\mu\nu}\} \tag{4.25}$$

where $\mu < \nu$. $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$.

Due to the dual basis, the orthogonality condition

$$Tr[\Gamma^n\tilde{\Gamma}_m] = 2\delta^n_m \tag{4.26}$$

is satisfied.

that implies the completeness relation

$$\begin{aligned}()[] &= \frac{1}{2}(\tilde{\Gamma}_n][\Gamma^n) \\ &= \frac{1}{2}\{(P_R][P_R) + (P_L][P_L) + (P_R\gamma^\mu][P_L\gamma_\mu) + (\frac{1}{2}\sigma^{\mu\nu}][\frac{1}{2}\sigma_{\mu\nu})\}\end{aligned} \tag{4.27}$$

That relation will directly leads to the chiral Fierz identities

$$(\Gamma^A)[\Gamma^B] = \frac{1}{4}Tr[\Gamma^A\Gamma_C\Gamma^B\Gamma_D](\Gamma^D][\Gamma^C) \tag{4.28}$$

Applying this on 4.23

$$\begin{aligned}(P_R\gamma^\mu)[P_L\gamma_\mu] &= \frac{1}{4}Tr[P_R\gamma^\mu\Gamma_C P_L\gamma_\mu\Gamma_D](\Gamma^D][\Gamma^C) \\ &= \frac{1}{4}Tr[P_R\gamma^\mu P_L P_L\gamma_\mu P_R](P_R][P_L) = 2(P_R][P_L)\end{aligned} \tag{4.29}$$

where we used $\gamma^\mu\gamma_\mu = 4 \times \mathbb{1}, \gamma^\mu\sigma^{\alpha\beta}\gamma_\mu = 0$ and the cyclic of the trace were all used.

When applying the identities to four-fermion operators we also anticommute two spinors, thus acquiring an additional minus sign with respect to the equations. For example, they will allow us to rewrite operator as

$$(\overline{\ell^i}\gamma^\mu q_i)(\bar{d}\gamma_\mu e) = -2(\overline{\ell^i}e)(\bar{d}q_i) \tag{4.30}$$

which has the quark and leptons in separate currents.

4.3 The Warsaw basis

$1: X^3$		$2: H^6$		$3: H^4D^2$		$5: \psi^2H^3 + \text{h.c.}$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$			Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$	Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$						

$4: X^2H^2$		$6: \psi^2XH + \text{h.c.}$		$7: \psi^2H^2D$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}\, W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$	Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$		$8: (\bar{R}R)(\bar{R}R)$		$8: (\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \text{h.c.}$		$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Table 4.1: List of all baryon- and leptonic- number- conserving SMEFT operator at dimension 6 in the Warsaw basis [3]. The division into classes 1-8 is taken from [6].

Warsaw basis represented in table 4.1 is the mostly used basis. It is a completely reduced basis without any redundant operator. The way used to remove redundancies and constructing the the Warsaw basis can be summarized as follow:

1. Use the IBP and equation of motion to remove operators with more derivatives.
2. Use Fierz identities .

The operators are divided into classes according to their field content and chirality in the manner taken by [3] and [6].

The purely bosonic operators are built out of combinations of the field-strength tensors $X_{\mu\nu} \in \{G_{\mu\nu}, W_{\mu\nu}, B_{\mu\nu}\}$, the Higgs doublet H, and the covariant derivatives D_μ. After removing the redundant operators, we are left with the following 4 classes:

- Four pure gauge operators containing three field-strength tensors (class 1:X^3).
- One pure scalar operator with six Higgs doublets (class 2:H^6).
- Two operators with four Higgs fields and two covariant derivatives (Class 3:H^4D^2).
- Eight mixed operators with two Higgs fields and two field-strength tensors (Class 4:X^2H^2).

Operators with two fermion fields. After removing the redundant operators, the remained operators are classified as:

- Three non-Hermitian Yukawa-like operators with a scalar fermion current and three Higgs fields (class 5:ψ^2H^3).
- Eight non-hermitian dipole operators with a tensor current, one Higgs field, and one field-strength tensor (class 6:ψ^2XH).
- Eight operators (all Hermitian except for Q_{Hud}) with a vector current, two Higgs fields, and a covariant derivative (class 7:ψ^2H^2D).

Last, we have 25 four-fermion operators in class 8 subdivided according to their chiral structure $(\overline{L}L)(L\overline{L})$, $(\overline{R}R)(R\overline{R})$, $(\overline{L}L)(\overline{R}R)$, $(\overline{L}R)(\overline{L}R)$ and $(\overline{L}R)(\overline{R}L)$.

4.4 The Anomalous Dimension Matrix

The complete list of dimension-six operators is given in table 4.1. The operators are divided into eight classes by field content and number of covariant derivatives. The dimension-six Lagrangian is

$$\mathcal{L}^{(6)} = \sum_i C_i Q_i \tag{4.31}$$

where Q_i are the operators in table 4.1 and C_i are the Wilson coefficients that have the dimension of $1/\Lambda^2$.

The SM at energies above the electroweak is a weakly coupled gauge theory, and SM gauge boson interaction are proportional to the gauge boson coupling g. For this reason, it is useful to use rescaled operators $\hat{Q}_i$ with coefficients $\hat{C}_i$. One can trivially convert between the two conventions. If $\hat{Q}_i = \eta_i Q_i$, then the rescaled coefficients and anomalous dimensions are

$$\hat{C}_i = \eta_i^{-1} C_i, \quad \hat{\gamma}_{ij} = \eta_i^{-1} \gamma_{ij} \eta_j. \tag{4.32}$$

Where the anomalous dimension in the rescale basis has the form shown in table 4.2, where there are the explicit operator rescaling.

The dimension-six lagrangian can be formed by the terms of the rescaled operators and their corresponding coefficients,

$$\mathcal{L}^{(6)} = \sum_i C_i Q_i = \sum_i \hat{C}_i \hat{Q}_i \tag{4.33}$$

The RG equations for the rescaled operator cofficients are given by

$$\dot{\hat{C}}_i = \hat{\gamma}_{ij} \hat{C}_j \tag{4.34}$$

The one-loop anomalous dimension matrix γ_{ij} is defined by the RG equation of the operator coefficients

$$\dot{C}_i \equiv 16\pi^2 \mu \frac{dC_i}{d\mu} = \gamma_{ij} C_j \tag{4.35}$$

γ_{ij} represent the 8×8 block form of the anomalous dimension matrix where i,j=1,. . . ,8. For example, γ_{15} is the 4×3 anomalous dimension submatrix which mixes the 3 independent class 5 operators coefficients into the 4 independent class 1 operators coefficients.

The difficulty of dealing with a large number of operators leads to searching of simplifying the calculation, or to look for a hidden structure in the anomalous dimension matrix to more easily understand the physics of the one-loop RGE flow. It was shown in [4] that the structure of the anomalous dimension matrix can be understood using Naive Dimensional Analysis (NDA) [159].

Recognizing Table 4.2, it was calculated in ref. [4] that the anomalous dimension matrix $\hat{\gamma}$ for the rescaled operators has entries proportional to

$$\hat{\gamma} \propto \left(\frac{\lambda}{16\pi^2}\right)^{n_\lambda} \left(\frac{y^2}{16\pi^2}\right)^{n_y} \left(\frac{g^2}{16\pi^2}\right)^{n_g}, \quad N = n_\lambda + n_y + n_g \tag{4.36}$$

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	0	0	0	1	0	0	0	0
H^6	2	$g^6\lambda$	0	$g^2\lambda, \lambda^2$	λg^4	λy^2	0	$\lambda g^2, \lambda y^2$	0
H^4D^2	3	g^6	0	g^2	g^4	0	g^2y^2	g^2	0
$g^2X^2H^2$	4	g^4	0	0	0	0	0	0	0
$y\psi^2H^3$	5	g^6	0	g^2, λ, y^2	g^4	y^2	$g^2\lambda, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	6	g^4	0	0	0	0	g^2, y^2	1	1
ψ^2H^2D	7	g^6	0	g^2	g^4	0	g^2y^2	g^2, y^2	g^2, y^2
ψ^4	8	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	g^2	0	0	1	0	0	0	0
H^6	2	0	λ, g^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	y^4	0	y^4	0
H^4D^2	3	0	0	g^2, λ	g^4	y^2	0	y^2	0
$g^2X^2H^2$	4	g^4	0	1	g^2, λ	0	y^2	1	0
$y\psi^2H^3$	5	0	0	g^2, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	y^2
$gy\psi^2XH$	6	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	7	0	0	y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	8	0	0	0	0	0	g^2y^2	y^2	g^2, y^2

Table 4.2: presents the structure of the one-loop anomalous dimension matrix for the Wilson coefficients of dimension-six operators in the rescaled basis. The table is divided into two sections, with rows and columns corresponding to the eight operator classes.

where N, the perturbative order of the anomalous dimension, is defined by the sum of the number of factors n_λ of the Higgs self-coupling λ, the number of factors n_y of y^2, and the number of factors n_g of g^2. For the rescaled dimension-six operators, N ranges from 0 to 4.

It was derived in ref. [7] a general formula for the perturbation order N of the anomalous dimension matrix $\hat{\gamma}_{ij}$,

$$N = 1 + w_i + w_j \tag{4.37}$$

where w_i is the NDA weight of the operators $\hat{Q}_i$ in the i^{th} class [160]. The class 2 operator $\hat{Q}_H$ has NDA weight $w_2 = 2$; the operator in classes {3,5,7,8} have NDA weight 1; the operators in class {4,6} have NDA weight 0; and the class 1 operators have NDA weight $w_1 = -1$. Using eq. 4.37, the possible coupling constant dependences of $\hat{\gamma_{ij}}$ are obtained.

As we have seen the anomalous dimension matrix is proportional to the Higgs self-coupling, Yukawa and Gauge coupling. The explicit RG equations are given in Appendix A, B and C as differential equations, rather as elements of the matrix γ.

Having established the SMEFT framework, including the structure of higher-dimensional operators, redundancies, the Warsaw basis, and the anomalous dimension matrix, we now turn our attention to one of its most crucial applications: flavor physics. In the Standard Model, flavor is encoded in the Yukawa couplings, but SMEFT allows us to systematically explore new sources of flavor violation beyond the SM. The renormalization group evolution (RGE) of SMEFT operators can generate additional flavor effects at low scales, making it essential to understand how SMEFT interacts with the flavor sector. In the next chapter, we will examine how flavor symmetries can be incorporated into SMEFT, their role in addressing the SM flavor puzzle, and the constraints imposed by experimental data.

Chapter 5

Adding Flavor to SMEFT

5.1 Flavor Assumptions

Standard Model is a great theory for understanding particle physics. It is a quantum field theory with Poincare spacetime symmetry and $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance. The field content includes five different gauge representations of Weyl fermions each with three flavors and a single scalar field that condensates at the electroweak scale, breaking the gauge symmetry down to $SU(3)_c \times U(1)_{EM}$. Its lagrangian is a local one consistence with the gauge symmetries up to mass dimension 4, know as a renormalizable operators. Thus SM is a renormalizable theory. A better way to deal with some struggles in SM, is to take it as a low energy property of an effective field theory.

The SM effective field theory is a framework that became so popular in recent years. SMEFT is an extension of the SM, using the same field content to construct terms that are also consistence with the gauge symmetry but with a mass dimension higher than 4. It was understood that the next higher order is 5 with only one operator that violates leptonic number, but it can give mass for neutrino. Then, the next higher order that we are focusing on is dimension 6. Having the Warsaw basis in table 4.1, the number of operators is 59 after removing all the redundancies.

One of the largest obstacles for doing analysis in SMEFT is after adding flavor. For instance, as will seen later, there are 2499 independent baryon and lepton number-conserving SMEFT operators that arise at leading order without having any flavor symmetries.

Postulating a flavor symmetry and its breaking pattern will help in decreasing the number of independent operators and can be useful to make hypotheses about the UV physics. In other words, we imagine that a UV theory will leave imprints on the flavor structure in the low-energy effective theory.

In nature, we do not observe an exact symmetry pattern, particularly in the case of flavor symmetries. The observed fermion masses and mixing angles in the Standard

Model exhibit hierarchical structures and deviations that indicate symmetry breaking effects. Consequently, a flavor symmetry serves as a guiding principle to approximate the underlying flavor structure of the SM, but it cannot be exact. Instead, it must be broken—either explicitly or spontaneously—to be consistent with experimental observations. This breaking accounts for the observed mass hierarchies and mixing patterns among quarks and leptons.

The minimal flavor violation [37], which is a flavor structure based on $U(3)^5$ Flavor symmetry broken by the SM Yukawa couplings Y_u, Y_d and Y_e promoted to spurions. While $U(3)^5$ flavor symmetry is the maximal flavor symmetry allowed by the SM gauge symmetry and field content. A flavor spurion can be viewed as a non-dynamical (spurious) field that transforms under a nontrivial representation of the flavor group and whose background value breaks the flavor symmetry.

Spurions act as "markers" of symmetry breaking in the sense that the symmetry is broken if and only if a spurion is inserted. If the symmetry is a good approximation, then the breaking effects-quantified by the spurions-must be small Y$\leq$ 1. Given this, we can assign numerical values to the Yukawa matrices from the measurements of fermion masses and mixings (ignoring the neutrino ones). An important consequence of this approach is that the flavor structure of the SMEFT can be systematically organized as a series expansion in powers of the spurions. The leading order is the exact U(3)^5 , while the largest deviations corresonds to a single insertion of spurions. Higher-order terms involve multiple insertions, leading to progressively smaller corrections.

A great competitor to MFV is the U(2) [39] flavor structures. $U(2)$ is the corresponding subgroup of $U(3)$ obtained by turning on one Yukawa coupling, typically the top quark Yukawa y_t, which is the largest in the SM with $y_t \sim 1$. This approach is motivated by the strong hierarchy in fermion masses, where the third generation are significantly heavier than the first two generations. It was pointed in [161–163], the $U(2)^5$ provides a very efficient EFT description of the recent flavor anomalies, which cannot be accommodated within a MFV framework.

It should be clear that the $U(3)^5$ and $U(2)^5$ symmetries are not the only options to efficiently suppress flavor-violating observables in the SMEFT [1]. In this chapter, we will start by adding flavor, then entering the flavor symmetries and doing the counting. We will follow [6, 165–167] for the counting.

5.2 Why 2499?

Flavors can be incorporated in SMEFT operators in table 4.1 and can be counted using refs. [6, 167] up to n generations.

Starting with each class of the 8 classes:

[1] For a detailed discussion see ref. [164]

1. Bosonic Operators:

   These classes are bosonic operators with no fermions in there structure, they will not get affected, then they will remain 15 operator.

   - Class 1:
     Four structure, 4 real Wilson coefficients, there are 2 CP-even and 2 CP-odd.
   - Class 2:
     One CP-even coefficient.
   - Class 3:
     Two CP-even coefficients.
   - Class 4:
     Eight structures, 4 CP-even and 4 CP-odd.

2. Fermion Bilinear Operators:

   - Class 5 & 6:
     These two classes are non-Hermitian structures containing two fermions. The coefficients are $n \times n$ complex matrix in the flavor space with n^2 complex entries, then for each n^2 CP-even coefficients and n^2 CP-odd. For class 5, $3n^2$ CP-even and $3n^2$ CP-odd. For class 6, $8n^2$ CP-even and $8n^2$ CP-odd.
   - Class 7:
     It consists of of 7 Hermitian structures and Q_{Hud} non-Hermitian. Q_{Hud} is the same as class 5 and 6, having n^2 complex entries.
     We need to understand some properties of Hermitian matrix.
     H is a 3×3 Hermitian matrix defined as,

$$H = \begin{pmatrix} Real & Complex & Complex \\ Conjugate & Real & Complex \\ Comjugate & Conjugate & Real \end{pmatrix} \tag{5.1}$$

     As we can see not all the entries are independent. The real entries are the upper triangle with diagonal being the hypotenuse. As we can see it starts by one entry at column 1 then 2 at second as so on. Then if we are dealing with $n \times n$ matrix it the real entries will 1+2+3+...+n, which is the summation of arithmetic series, the value will be $\frac{n(n+1)}{2}$ CP-even. For CP-odd, the same but without the diagonal then we need to subtract n from the CP-even value, giving $\frac{n(n-1)}{2}$.
     The total CP-even for class 7 is $n^2+\frac{7n(n+1)}{2}=\frac{1}{2}n(9n+7)$, the total CP-odd is $\frac{1}{2}n(9n-7)$.

3. Four Fermions Operators:

   We will not go in the known order as in table 4.1, but rather we will start with the simplest.

   - Class 8$(\overline{L}R)(\overline{L}R)$ & $(\overline{L}R)(\overline{R}L)$:

     The structures in that subclasses are all non-Hermitian, but instead of matrix it is now tensor. For a rank-4 n-dimension non hermitian tensor, we have n^4 complex entries, n^4 CP-even and n^4 CP-even. For $(\overline{L}R)(\overline{L}R)$ we have $4n^4$ CP-even and $4n^4$ CP-odd. And n^4 CP-even, n^4 CP-odd for $(\overline{L}R)(\overline{R}L)$.

   - Class 8$(\overline{L}L)(\overline{R}R)$:

     The structures in these subclass are Hermitian, but also here now we are dealing with tensors rather than matrix. We need to calculate the number of entries in the upper triangle in tensor, which is the sum of the following arithmetic series $1+2+\cdots+n^2$, that gives us $\frac{n^2(n^2+1)}{2}$ CP-even, to calculate the CP-odd it is the same as the CP-even minus n^2, that gives $\frac{n^2(n^2-1)}{2}$ CP-odd. The total for the whole subclass is $4n^2(n^2+1)$ CP-even and $4n^2(n^2-1)$ CP-odd.

   - Class 8$(\overline{L}L)(\overline{L}L)$:

     The structures in this subclass are Hermitian divided into two categories, different and identical current.

     The count of the different current is the same as the Hermitian one above, having $n^2(n^2+1)$ CP-even and $n^2(n^2-1)$ CP-odd.

     The identical currents are more interesting, since all four flavor indices transform under the same SU(n) flavor group. The structures transform as the 1+1+adj+adj+$\overline{a}$a+$\overline{s}s$ where adj is the adjoint representation, $\overline{a}a$ is the representation $T^{(ij)}_{[kl]}$ antisymmetric in the upper and lower indices, and $\overline{s}s$ is the representation $T^{(ij)}_{(kl)}$ symmetric in the upper and lower indices. These transformation can be understood from the Young tableaux point of view for 2-body operators.

     The singlet has one CP-even parameter, the adjoint has (n-1)(n+2)/2 CP-even and n(n-1)/2 CP-odd parameters, $\overline{a}a$ has n(n-3)(n^2+n+2)/8 CP-even and n(n-3)(n-1)(n+2)/8 CP-odd parameters, and $\overline{s}s$ has n(n-1)(n+1)(n+2)/8 CP-even and n(n-1)(n^2+3n-2)/8 CP-odd parameters. After substituting these values in the transformation we will have for the 3 operators of identical currents $\frac{3(n^4+3n^2)}{4}$ CP-even parameters and $\frac{3(n^4-n^2)}{4}$.

     Adding the values of the different currents and the identical currents, the total CP-even for the whole subclass is $\frac{7n^4+13n^2}{4}$ and the CP-odd is $\frac{7n^2(n^2-1)}{4}$ parameters.

Class		N_{op}	CP-even			CP-odd		
			n	1	3	n	1	3
1		4	2	2	2	2	2	2
2		1	1	1	1	0	0	0
3		2	2	2	2	0	0	0
4		8	4	4	4	4	4	4
5		3	$3n^2$	3	27	$3n^2$	3	27
6		8	$8n^2$	8	72	$8n^2$	8	72
7		8	$\frac{1}{2}n(9n+7)$	8	51	$\frac{1}{2}n(9n-7)$	1	30
8	: $(\overline{L}L)(\overline{L}L)$	5	$\frac{1}{4}n^2(7n^2+13)$	5	171	$\frac{7}{4}n^2(n-1)(n+1)$	0	126
8	: $(\overline{R}R)(\overline{R}R)$	7	$\frac{1}{8}n(21n^3+2n^2+31n+2)$	7	255	$\frac{1}{8}n(21n+2)(n-1)(n+1)$	0	195
8	: $(\overline{L}L)(\overline{R}R)$	8	$4n^2(n^2+1)$	8	360	$4n^2(n-1)(n+1)$	0	288
8	: $(\overline{L}R)(\overline{R}L)$	1	n^4	1	81	n^4	1	81
8	: $(\overline{L}R)(\overline{L}R)$	4	$4n^4$	4	324	$4n^4$	4	324
8	: All	25	$\frac{1}{8}n(107n^3+2n^2+89n+2)$	25	1191	$\frac{1}{8}n_g(107n^3+2n^2-67n-2)$	5	1014
Total		59	$\frac{1}{8}(107n^4+2n^3+213n^2+30n+72)$	53	1350	$\frac{1}{8}(107n^4+2n^3+57n^2-30n+48)$	23	1149

Table 5.1: Number of CP-even and CP-odd coefficients in $\mathcal{L}^{(6)}$ for n flavors. The total number of coefficients is $\frac{1}{4}(107n^4+2n^3+135n^2+60)$, which is 76 for $n=1$ and 2499 for $n=3$.

- Class $8(\overline{R}R)(\overline{R}R)$:

  Here we have 8 structures, 4 different currents, 2 identical current and one special which is the Q_{ee} that transforms as 1+adj+$\overline{s}s$.

  After doing the whole calculation and adding the them all, the total CP-even parameters for the whole subclass is $\frac{n(21n^3+2n^2+31n+2)}{8}$ parameters and $\frac{n(21n+2)(n-1)(n+1)}{8}$ parameters.

The total CP-even and CP-odd coefficients for the whole basis is the sum of all the values for every single class.

- Total CP-even coefficients: $\frac{107n^4+2n^3+213n^2+30n+72}{8}$

  For n=1 $\rightarrow$ 53 coefficient.

  For n=3 $\rightarrow$ 1350 coefficient.

- Total CP-odd coefficients:$\frac{107n^4+2n^3+57n^2-30n+48}{8}$

  For n=1$\rightarrow$ 23 coefficient.

  For n=3 $\rightarrow$ 1149 Coefficient.

The total number of coefficients is 76 for n=1 and 2499 for n=3, as table 5.1.

5.3 $U(3)^5$ Symmetry and Minimal Flavor Violation

The largest group of flavor-symmetry transformation compatible with the kinetic terms of the SM lagrangian is [36]:

$$G_{flavor} = U(3)^5 = U(3)_\ell \otimes U(3)_q \otimes U(3)_e \otimes U(3)_u \otimes U(3)_d = SU(3)^5 \otimes U(1)^5 \quad (5.2)$$

Each field is assigned to a **3** representation of the associated group: denoting a generic $U(3)_\psi$ transformation by Ω_ψ, the transformation rules are [6]

$$q \to \Omega_q q, \quad u \to \Omega_u u, \quad d \to \Omega_d d, \quad \ell \to \Omega_\ell \ell, \quad e \to \Omega_e e. \quad (5.3)$$

Vector currents $\overline{\psi}_p \gamma^\mu \psi_r$ are trivially made invariant by imposing δ_{pr} contraction, that corresponds to the singlet composition of $\overline{\mathbf{3}}$ and **3** representations. This is immediate to see applying the field transformation and using $\Omega_\psi \Omega_\psi^\dagger = \mathbb{1} = \Omega_\psi^\dagger \Omega_\psi$:

$$\overline{\psi}\gamma^\mu\psi \to \overline{\psi}\Omega_\psi^\dagger \gamma^\mu \Omega_\psi \psi = \overline{\psi}\gamma^\mu\psi, \quad \psi = \{q, u, d, \ell, e\}. \quad (5.4)$$

5.3.1 Exact $U(3)^5$ symmetry

Adopting the Warsaw basis in table 4.1, with the classification of classes as in [4], we start the counting.

1. Bosonic Operators:

   The structures of classes 1-4 do not contain fermions, then the counting is trivial: 9 independent CP-even coefficients and 6 CP-odd coefficients.

2. Fermion Bilinear Operators:

   - Class 5 & 6:
     The structure of these classes are forbidden in the exact $U(3)^5$ limit, since they contain a fermionic current of type $\overline{L}R$.
   - Class 7:
     In this class, there are 4 hermitian ($\overline{L}L$) fermionic current, 3 hermitian ($\overline{R}R$) fermionic current, that are allowed done as eq. 5.4 with each having CP-even coefficient, and Q_{Hud} non-hermitian and not allowed.

3. Four Fermion Operators:

   - Class $8(\overline{L}L)(\overline{L}L)$:
     In this subclass, there are 2 hermitian structures with different fermion currents, each has 2 CP-even coefficient. Also, there are 3 hermitian structures

with identical currents, each having 2 independent operators, since we can contract the flavor indices in two different ways. For example, for $Q_{qq}^{(1)}$ we have

$$(\overline{q}_p\gamma_\mu q_p)(\overline{q}_r\gamma_\mu q_r) \quad and \quad (\overline{q}_p\gamma_\mu q_r)(\overline{q}_r\gamma_\mu q_p), \tag{5.5}$$

where r and p denote the flavor indices. Each identical current has 2 CP-even coefficients.

- Class $8(\overline{R}R)(\overline{R}R)$:

  In this subclass, there are 4 hermitian structures with different fermion currents, and 3 identical currents, but Q_{ee} is special, since it corresponds to a single independent structure due to fierz identity

$$(\overline{e}_p\gamma_\mu e_r)(\overline{e}_s\gamma_\mu e_t) = (\overline{e}_s\gamma_\mu e_r)(\overline{e}_p\gamma_\mu e_t). \tag{5.6}$$

- Class $8(\overline{L}L)(\overline{R}R)$:

  The structures here are divided in a quite different way as: Leptonic-Leptonic current, Quark-Quark current, Leptonic-Quark current. They are all hermitian structures that are invariant under the exact $U(3)^5$. Each having 1 CP-even.

- Class $8(\overline{L}R)(\overline{R}L)$+h.c & $8(\overline{L}R)(\overline{L}R)$ +h.c:

  The strucutre of these subclasses are not allowed.

5.3.2 Minimal Flavor Violation

The minimal flavor violation (MFV) is the assumption that the SM Yukawa couplings are the only source of $U(3)^5$ breaking [37]. The exact $U(3)^5$ limit analyzed before is equivalent to employing the MFV hypothesis and working to the zeroth order in the symmetry breaking terms. To go beyond the leading order we promote the SM Yukawa to $U(3)^5$ spurion fields with the following transformation properties [166]:

$$Y_d \to \Omega_d Y_d \Omega_q^\dagger, \quad Y_u \to \Omega_u Y_u \Omega_q^\dagger, \quad Y_e \to \Omega_e Y_e \Omega_\ell^\dagger. \tag{5.7}$$

In this way the $(\overline{L}R)$ structure

$$\overline{d}Y_d q, \quad \overline{u}Y_u q, \quad \overline{e}Y_e \ell, \tag{5.8}$$

are formally invariant. For two insertions of Yukawa

$$\begin{aligned} Y_uY_u^\dagger &\to \Omega_u Y_u \Omega_q^\dagger \Omega_q Y_u^\dagger \Omega_u^\dagger = \Omega_u Y_u Y_u^\dagger \Omega_u^\dagger \\ Y_dY_d^\dagger &\to \Omega_d Y_d \Omega_q^\dagger \Omega_q Y_d^\dagger \Omega_d^\dagger = \Omega_d Y_d Y_d^\dagger \Omega_d^\dagger \\ Y_eY_e^\dagger &\to \Omega_e Y_e \Omega_\ell^\dagger \Omega_\ell Y_e^\dagger \Omega_e^\dagger = \Omega_e Y_e Y_e^\dagger \Omega_e^\dagger \end{aligned} \tag{5.9}$$

In this way the $(\overline{R}R)$ structure

$$\overline{d}Y_dY_d^\dagger d, \quad \overline{u}Y_uY_u^\dagger u, \quad \overline{e}Y_eY_e^\dagger e. \tag{5.10}$$

are invariant. We could also have different arrangement

$$\begin{aligned} Y_u^\dagger Y_u &\to \Omega_q^\dagger Y_u^\dagger \Omega_u \Omega_u Y_u \Omega_q^\dagger = \Omega_q Y_u^\dagger Y_u \Omega_q^\dagger \\ Y_d^\dagger Y_d &\to \Omega_q^\dagger Y_d^\dagger \Omega_d \Omega_d Y_d \Omega_q^\dagger = \Omega_q Y_d^\dagger Y_d \Omega_q^\dagger \\ Y_e^\dagger Y_e &\to \Omega_\ell^\dagger Y_e^\dagger \Omega_e \Omega_e Y_e \Omega_\ell^\dagger = \Omega_\ell Y_e^\dagger Y_e \Omega_\ell^\dagger \end{aligned} \tag{5.11}$$

Then $(\overline{L}L)$ structure

$$\overline{q}Y_u^\dagger Y_u q, \quad \overline{q}Y_d^\dagger Y_d q, \quad \overline{\ell}Y_e^\dagger Y_e \ell. \tag{5.12}$$

are invariant. In principle, the spurions can appear with arbitrary powers both in the dimension-6 effective operators. For example four insertions of Yukawa coupling will transforms as

$$Y_uY_u^\dagger Y_uY_u^\dagger \to \Omega_u Y_u \Omega_q^\dagger \Omega_q Y_u^\dagger \Omega_u^\dagger \Omega_u Y_u \Omega_q^\dagger \Omega_q Y_u^\dagger \Omega_u^\dagger = \Omega_u Y_u Y_u^\dagger \Omega_u^\dagger \tag{5.13}$$

The same could happen for any combination of the Yukawa, they will remain transforming well. So, that is a series expansion in powers of spurions (if Y<1 ,then $\delta > Y^2 > Y^4 \ldots$). However this series is finite, since there exist only a finite number of independent covariants that can be constructed out of the Yukawa spurions due to Cayley-Hamilton identity[2]

$$X^3 = (TrX)X^2 + \frac{1}{2}(TrX^2 - Tr^2X)X + \frac{1}{6}(2TrX^3 - 3TrX^2TrX + Tr^3X)I_3 \tag{5.14}$$

with I_3 the 3×3 identity matrix. This identity leads to many redundancy relations among polynomials of 3×3 matrices.

Typically people choose to truncate the series at a specific order, for us we will truncate at $\mathcal{O}(Y^2)$. Let's start with the counting of single and double insertions of spurions.

1. Fermion Bilinear Operators:

   - Class 5 & 6:

     The structure is non-hermitian transforms the same as eqs.5.8 for one insertion, each having one complex parameter, then 1 CP-even and 1 CP-odd.

     For two insertions they are not invariant.

[2] A useful reference for finding the required decomposition from a Hilbert series perspective is [168], though some further refinement is still needed.

- Class 7:

  For one insertion, the structures are not invariant.

  For two insertions, all the strucures are invariant even the Q_{Hud}. We obtain a $U(3)^5$ singlet contracting Y an $Y^\dagger$ to form an octet of SU(3), and then contracting this octet with the flavor indices

  $$\overline{q}_p \Gamma q_r (Y^\dagger_{u/d} Y_{u/d})_{pr}, \quad \overline{u}_p \Gamma u_r (Y_u Y_u^\dagger)_{pr}, \quad \overline{d}_p \Gamma d_r (Y_d Y_d^\dagger)_{pr}, \quad \overline{u}_p \Gamma d_r (Y_u Y_d^\dagger)_{pr}. \tag{5.15}$$

  where Γ denote a generic combination of Dirac matrices, color and $SU(2)_L$ generators, which play no role as far as the flavor structure is concerned. These are the same of what happen in eqs. 5.10 and 5.12, this could be generalized for the leptonic sector.

  Then 6 CP-even for $(\overline{L}L)$ parameters, 3 CP-even for $(\overline{R}R)$ parameters, and 1 complex for Q_{Hud}.

2. Four Fermion Operators:

- Class $8(\overline{L}L)(\overline{L}L)$:

  As we have seen in section 5.3.1, the structures are divided into two main categories: Identical- and different-currents.

  For one insertion, all the structures are not invariant.

  For two insertions, using eq.5.5 and 5.12 we will have 10 hermitian structures with 10 CP-even for the identical-currents. The different-currents will have 6 structures with 6 CP-even.

  In total we will have 16 CP-even parameter.

- Class $8(\overline{R}R)(\overline{R}R)$:

  Here also the structures are divided into the same categories: different- and identical-currents with a special operator Q_{ee} due to eq.5.6.

  For one insertion, all the structures are not invariant.

  For two insertions, using eq.5.5 and 5.10 we will have 5 hermitian structures with 5 CP-even for all the identical-currents including Q_{ee}. The different-currents are 8 hermitian structures with 8 CP-even.

  In total we have 13 CP-even.

- Class $8(\overline{L}L)(\overline{R}R)$:

  For one insertion of Yukawa, the strucutres are not invariant.

  For two insertions, the leptonic-Leptonic-current can be contracted by three contractions

  $$(\bar{\ell}_p \gamma_\mu \ell_r)(\overline{e}_s \gamma^\mu e_s)(Y_e^\dagger Y_e)_{pr}, \quad (\bar{\ell}_s \gamma_\mu \ell_s)(\overline{e}_p \gamma^\mu e_r)(Y_e Y_e^\dagger)_{pr}, \quad (\bar{\ell}_p \gamma_\mu \ell_r)(\overline{e}_s \gamma^\mu e_t)(Y_e^\dagger)_{pt}(Y_e)_{sr} \tag{5.16}$$

they are all hermitian, each has 1 CP-even.

The leptonic-Quark-currents are the same as the different-currents in the previous subclasses, they are 7 hermitian structures each with 1 CP-even.

The Quark-Quark-currents they can be contracted almost the same as the Leptonic-leptonic-currents but with a small different that the $(\overline{q}q)$ can be contracted by two combinations, the $(Y_u^\dagger Y_u)$ and $(Y_d^\dagger Y_d)$. However, we will have 16 hermitian structure each with 1 CP-even.

The total is 26 CP-even parameters.

- Class $8(\overline{L}R)(\overline{R}L)$+h.c:

  For single insertion, the structure is not invariant.

  For two insertions, the structure here can be contracted as

$$(\overline{\ell}_p^j e_r)(\overline{d}_s q_{tj})(Y_e^\dagger)_{pr}(Y_d)_{st} \tag{5.17}$$

  This is not hermitian with 1 complex parameter.

- Class $8(\overline{L}R)(\overline{L}R)$+h.c:

  For single insertion, the structures are not invariant.

  For two insertions,$Q_{lequ}^{(1,3)}$ structures can be contracted as eq.5.17, have each 1 complex parameter.

  Then we have $Q_{quqd}^{1,8}$ that can be contracted intwo different ways

$$(\overline{q}_p^j u_r)\epsilon_{jk}(\overline{q}_s^k d_t)(Y_u^\dagger)_{pr}(Y_d^\dagger)_{st}, \quad (\overline{q}_p^j u_r)\epsilon_{jk}(\overline{q}_s^k d_t)(Y_u^\dagger)_{sr}(Y_d^\dagger)_{pt}. \tag{5.18}$$

  Then we will have 4 non-hermitian strucutres, each with 1 complex parameter.

  In total we have 6 CP-even and 6 CP-odd.

The results thus obtained are reported in Table 5.2, the left (right) value in each entry indicates the number of CP-even (CP-odd) coefficients. Number of operators is up to $\mathcal{O}(Y^2)$ (summing $\mathcal{O}(Y^0)$+$\mathcal{O}(Y^1)$+$\mathcal{O}(Y^2)$.)

However, we have done a power counting for insertions of the Yukawa couplings up to $\mathcal{O}(Y^2)$ for which it totally agrees with ref. [165]. Instead, sometimes it can be chose to have the minimal insertions needed as done in ref. [166] to retain the leading invariant structure for each operator, corresponding to no Yukawa insertions in classes 7, $8(\overline{L}L)(\overline{L}L)$, $8(\overline{R}R)(\overline{R}R)$, and $8(\overline{L}L)(\overline{R}R)$. One insertion in classes 5 & 6 and two insertions in Q_{Hud} and classes $8(\overline{L}R)(\overline{R}L)$, $8(\overline{L}R)(\overline{L}R)$. Their corresponding counting is in Table 5.3.

Class	Operators	No symmetry				$U(3)^5$					
		3 Gen.		1 Gen.		Exact		$\mathcal{O}(Y^1)$		$\mathcal{O}(Y^2)$	
1–4	X^3, H^6, H^4D^2, X^2H^2	9	6	9	6	9	6	9	6	9	6
5	ψ^2H^3	27	27	3	3	–	–	3	3	3	3
6	ψ^2XH	72	72	8	8	–	–	8	8	8	8
7	ψ^2H^2D	51	30	8	1	7	–	7	–	16	1
	$(\bar{L}L)(\bar{L}L)$	171	126	5	–	8	–	8	–	24	–
	$(\bar{R}R)(\bar{R}R)$	255	195	7	–	9	–	9	–	22	–
8	$(\bar{L}L)(\bar{R}R)$	360	288	8	–	8	–	8	–	34	–
	$(\bar{L}R)(\bar{R}L)$	81	81	1	1	–	–	–	–	1	1
	$(\bar{L}R)(\bar{L}R)$	324	324	4	4	–	–	–	–	6	6
total:		1350	1149	53	23	41	6	52	17	123	25

Table 5.2: Number of independent operators in $U(3)^5$, MFV and without symmetry. In each column the left (right) number corresponds to the number of CP-even (CP-odd) coefficients. For the $U(3)^5$, they are summing from left to right.

Class	Operators	No symmetry				Minimal Needed					
		3 Gen.		1 Gen.		$\mathcal{O}(Y^0)$		$\mathcal{O}(Y^1)$		$\mathcal{O}(Y^2)$	
1–4	X^3, H^6, H^4D^2, X^2H^2	9	6	9	6	9	6	9	6	9	6
5	ψ^2H^3	27	27	3	3	–	–	3	3	3	3
6	ψ^2XH	72	72	8	8	–	–	8	8	8	8
7	ψ^2H^2D	51	30	8	1	7	–	7	–	8	1
	$(\bar{L}L)(\bar{L}L)$	171	126	5	–	8	–	8	–	8	–
	$(\bar{R}R)(\bar{R}R)$	255	195	7	–	9	–	9	–	9	–
8	$(\bar{L}L)(\bar{R}R)$	360	288	8	–	8	–	8	–	8	–
	$(\bar{L}R)(\bar{R}L)$	81	81	1	1	–	–	–	–	1	1
	$(\bar{L}R)(\bar{L}R)$	324	324	4	4	–	–	–	–	6	6
total:		1350	1149	53	23	41	6	52	17	60	25

Table 5.3: Number of independent operators in the leading invariant structures. In each column the left (right) number corresponds to the number of CP-even (CP-odd) coefficients. For the minimal needed, they are summing from left to right.

5.4 $U(2)^5$ Symmetry

The $U(2)^5$ symmetry is a subgroup of $U(3)^5$, specifically designed to differentiate the first two generations of fermions from the third one [38–40]. The $U(2)^5$ symmetry refers to five copies of the unitary group U(2), acting on the first two generations of fermions. This means treating the first and second generations as a doublet under these symmetries while the third generation is treated separately. This symmetry helps explain the smallness of first- and second-generation masses and why the third generation dominates flavor-changing processes. The five independent flavor doublets are denoted q,ℓ,e,u,d in addition to five independent singlets denoted by $q_3, u_3, d_3, e_3, \ell_3$ and the flavor symmetry is decomposed as

$$U(2)^5 = U(2)_\ell \otimes U(2)_q \otimes U(2)_e \otimes U(2)_u \otimes U(2)_d \tag{5.19}$$

under which only the doublets transforms:

$$\begin{aligned} &q \to \xi_q q, \quad u \to \xi_u u, \quad d \to \xi_d d, \quad \ell \to \xi_\ell \ell, \quad e \to \xi_e e, \\ &q_3 \to q_3, \quad u_3 \to u_3, \quad d_3 \to d_3, \quad \ell_3 \to \ell_3, \quad e_3 \to e_3. \end{aligned} \tag{5.20}$$

Furthermore in analogy with the MFV case, we assume that this $U(2)^5$ is broken by spurions transforming as [39]

$$\Delta_u \to \xi_q \Delta_u \xi_u^\dagger, \quad \Delta_d \to \xi_q \Delta_d \xi_d^\dagger, \quad \Delta_e \to \xi_\ell \Delta_e \xi_e^\dagger. \tag{5.21}$$

In fact, if these bi-doublets were the only breaking terms, the third generation, made of singlets under $U(2)^5$, would not be able to communicate with the first two generations at all. For this to happen, one needs two doublets. Inserting these two is a minimal choice that allows to reinstate the mixing between the light and heavy generations, and they transform as

$$V_q \to \xi_q V_q, \quad V_\ell \to \xi_\ell V_\ell. \tag{5.22}$$

In terms of these spurions, we can express the Yukawa matrices as

$$Y_e^\dagger = y_\tau \begin{pmatrix} \Delta_e & x_\tau V_\ell \\ 0 & 1 \end{pmatrix}, \quad Y_u^\dagger = y_t \begin{pmatrix} \Delta_u & x_t V_q \\ 0 & 1 \end{pmatrix}, \quad Y_d^\dagger = y_b \begin{pmatrix} \Delta_d & x_b V_q \\ 0 & 1 \end{pmatrix}. \tag{5.23}$$

where $y_{\tau,t,b}$ and $x_{\tau,t,b}$ are free complex parameters expected to be of order O(1)[3].

Using the residual $U(2)^5$ invariance, we can transform the spurions to the following explicit form

$$V_{q(\ell)} = e^{i\bar{\phi}_{q(\ell)}} \begin{pmatrix} 0 \\ \epsilon_{q(\ell)} \end{pmatrix}, \quad \Delta_e = O_e^T \begin{pmatrix} \beta_e' & 0 \\ 0 & \beta_e \end{pmatrix}, \quad \Delta_u = U_u^\dagger \begin{pmatrix} \beta_u' & 0 \\ 0 & \beta_u \end{pmatrix}, \quad \Delta_d = U_d^\dagger \begin{pmatrix} \beta_d' & 0 \\ 0 & \beta_d \end{pmatrix} \tag{5.24}$$

[3]According to [39], due to the holomorphicity of the Superpotential, in supersymmetric framework we are not able to add term on the lower-left sector of the Yukawa matrices.

Here O and U represent 2×2 orthogonal and complex unitary matrices, respectively

$$O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}, \quad U_q = \begin{pmatrix} c_q & s_q e^{i\alpha_q} \\ -s_q e^{-i\alpha_q} & c_q \end{pmatrix} \tag{5.25}$$

with $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$. The ϵ_i and $\beta_i^{(\prime)}$ are small position real parameters controlling the overall size of the spurions. From the observed hierarchies of the Yukawa couplings, we deduce

$$1 \gg \epsilon_i \gg \beta_i \gg \beta_i' > 0 \tag{5.26}$$

or, more precisely,

$$\epsilon_i = \mathcal{O}(10^{-1}), \quad \beta_i = \mathcal{O}(10^{-2}), \quad \beta_i' = \mathcal{O}(10^{-3}). \tag{5.27}$$

As in MFV, spurions can appear with arbitrary powers in the lagrangian but still transforms in a proper way. Due to eq.5.27, we can have a series expansion in power of spurions. However this series is finite, since there exist only a finite number of independent covariants that can be constructed out of the U(2) spurions due to Cayley-Hamilton identity

$$X^2 = Tr(X)X - \det(X)I_2 \tag{5.28}$$

with I_2 the 2×2 identity matrix.

We will choose to truncate at $\mathcal{O}(V^3, \Delta V)$. We will use alphabetic letters for real coefficients and Greek letters for complex coefficients. Let's start the counting.

1. Fermion Bilinears :

   We will start the analysis from the operators of classes 5, 6 and 7, which contains a fermion bilinear. We will terminate the expansion up to O(ΔV), we will divide the analysis to 4 different categories, and we will discuss in detail the leptonic case (the translation to the quark case being trivial).

   - $(\overline{L}L)$ structure :
     The terms generated up to $\mathcal{O}(\Delta V)$ are

$$\begin{aligned} V^0 &: [a_1 \overline{\ell}\ell + a_2 \overline{\ell_3}\ell_3], \\ V^1 &: [\kappa_1 \overline{\ell} V_\ell \ell_3 + h.c], \\ V^2 &: [b_1 \overline{\ell} V_\ell V_\ell^\dagger \ell], \\ \Delta^1, \Delta^1 V^1 &: - \end{aligned} \tag{5.29}$$

     The results can be summarized as follows in terms of the flavor tensor Λ_{LL}:

$$\overline{\ell}_p \Gamma \Lambda_{LL}^{pr} \ell_r, \quad \Lambda_{LL} = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_1 + b_1\epsilon_\ell^2 & \kappa_1 \epsilon_\ell \\ 0 & \kappa_1^\star \epsilon_\ell & a_2 \end{pmatrix} + \mathcal{O}(\beta_e^2). \tag{5.30}$$

Λ_{LL}, in the absence of any flavor symmetry is parametrized by 6 real and 3 imaginary coefficients has only 4 real $(a_{1,2}, b_1, Re\kappa_1)$ and 1 imaginary $(Im\kappa_1)$ coefficients.

- $(\overline{R}R)$ Structure:
The terms generated up to $\mathcal{O}(\Delta V)$ are

$$\begin{aligned} V^0 &: [a_1\overline{e}e + a_2\overline{e_3}e_3], \\ V^1, V^2, \Delta^1 &: -, \\ \Delta^1 V^1 &: [\sigma_1\overline{e_3}V_\ell^\dagger\Delta_e e + h.c]. \end{aligned} \tag{5.31}$$

The results can be summarized as follows in terms of the flavor tensor Λ_{RR}:

$$\overline{e}_p\Gamma\Lambda_{RR}^{pr}e_r, \quad \Lambda_{RR} = \begin{pmatrix} a_1 & 0 & \sigma_1^\star\epsilon_\ell s_e\beta_e' \\ 0 & a_1 & \sigma_1^\star\epsilon_\ell\beta_e \\ \sigma_1\epsilon_\ell s_e\beta_e' & \sigma_1\epsilon_\ell\beta_e & a_2 \end{pmatrix} + \mathcal{O}(\beta_e^2). \tag{5.32}$$

Λ_{RR} contains 3 real and 1 imaginary.

- Q_{Hud}:
The terms generated up to $\mathcal{O}(\Delta V)$ are

$$\begin{aligned} V^0 &: [\alpha_1\overline{u}_3d_3 + h.c], \\ V^1, V^2, \Delta^1 &: -, \\ \Delta^1 V^1 &: [\sigma_1\overline{u}\Delta_u^\dagger V_q d_3 + \sigma_2\overline{u}_3V_q^\dagger\Delta_d d + h.c]. \end{aligned} \tag{5.33}$$

In this case one finds 3 real 3 imaginary coefficients.

- $(\overline{L}R)$ strucutre:
The terms generated up to $\mathcal{O}(\Delta V)$ are

$$\begin{aligned} V^0 &: [\alpha_1\overline{\ell}_3e_3 + h.c], \\ V^1 &: [\kappa_1\overline{\ell}V_\ell e_3 + h.c], \\ V^2 &: - \\ \Delta^1 &: [\rho_1\overline{\ell}\Delta_e e + h.c], \\ \Delta^1 V^1 &: [\sigma_1\overline{\ell}_3V_\ell^\dagger\Delta_e e + h.c]. \end{aligned} \tag{5.34}$$

The results can be summarized as follows in terms of the flavor tensor Λ_{LR}

$$\overline{\ell}_p\Gamma\Lambda_{LR}^{pr}e_r, \quad \Lambda_{LR} = \begin{pmatrix} \rho_1\beta_e' & -\rho_1 s_e\beta_e & 0 \\ \rho_1 s_e\beta_e' & \rho_1\beta_e & \kappa_1\epsilon_\ell \\ \sigma_1\epsilon_\ell s_e\beta_e' & \sigma_1\epsilon_\ell\beta_e & \alpha_1 \end{pmatrix} + \mathcal{O}(\beta_e\epsilon_\ell^2). \tag{5.35}$$

Λ_{LR} contains 4 real and 4 imaginary coefficients.

Class	N. indep. structures	$U(2)^5$ breaking terms									
		V^0		V^1		V^2		Δ^1		$\Delta^1 V^1$	
5 & 6: $\left(\bar{L}R\right)$	11	11	11	11	11	–	–	11	11	11	11
7: $\left(\bar{L}L\right)$	4	8	–	4	4	4	–	–	–	–	–
7: $\left(\bar{R}R\right)$	3	6	–	–	–	–	–	–	–	3	3
7: Q_{Hud}	1	1	1	–	–	–	–	–	–	2	2
total:	19	26	12	15	15	4	–	11	11	16	16

Table 5.4: Number of independent operators with fermion bilinears in $U(2)^5$. Notation as in Table 5.2; however, here each column denotes the operators with a precise power of spurions, as indicated in the first row.

The total number of CP-even and CP-odd coefficients for all the operators with fermion bilinears constructed with spurions up to $\mathcal{O}(\Delta^1 V^1)$ are reported in Table 5.4.

2. Four Fermion Operators

- $(\overline{L}L)(\overline{L}L)$ structures:
  For the identical currents:
  The terms generated up to $\mathcal{O}(V^3)$ are

$$\begin{aligned}
V^0 &: [a_1(\overline{\ell}^p\ell^p)(\overline{\ell}^r\ell^r) + a_2(\overline{\ell}^p\ell^r)(\overline{\ell}^r\ell^p) + a_3(\overline{\ell}\ell)(\overline{\ell}_3\ell_3) + a_4(\overline{\ell}\ell_3)(\overline{\ell}_3\ell) \\
&\quad + a_5(\overline{\ell}_3\ell_3)(\overline{\ell}_3\ell_3)], \\
V^1 &: [\kappa_1(\overline{\ell}^p V_\ell^p \ell_3)(\overline{\ell}^r\ell^r) + \kappa_2(\overline{\ell}V_\ell\ell_3)(\overline{\ell}_3\ell_3) + \kappa_3(\overline{\ell}^p V_\ell^p\ell^r)(\overline{\ell}^r\ell_3) + h.c], \\
V^2 &: [b_1(\overline{\ell}^p V_\ell^p V_\ell^{\dagger r}\ell^r)(\overline{\ell}^s\ell^s) + b_2(\overline{\ell}^p V_\ell^p V_\ell^{\dagger r}\ell^r)(\overline{\ell}_3\ell_3) + b_3(\overline{\ell}^p V_\ell^p\ell_3)(\overline{\ell}_3 V_\ell^{\dagger r}\ell^r) \\
&\quad + b_4(\overline{\ell}^p V_\ell^p\ell^r)(\overline{\ell}^r V^{\dagger s}\ell^s) + (\eta_1(\overline{\ell}^p V_\ell^p\ell_3)(\overline{\ell}^r V_\ell^r\ell_3) + h.c)], \\
V^3 &: [\iota_1(\overline{\ell}^p V_\ell^p V_\ell^{\dagger r}\ell^r)(\overline{\ell}^s V_\ell^s\ell_3) + h.c].
\end{aligned} \tag{5.36}$$

  For the different currents:
  The terms generated up to $\mathcal{O}(V^3)$ are

$$\begin{aligned}
V^0 &: [a_1(\overline{\ell}\ell)(\overline{q}q) + a_2(\overline{\ell}\ell)(\overline{q}_3q_3) + a_3(\overline{\ell}_3\ell_3)(\overline{q}q) + a_4(\overline{\ell_3}\ell_3)(\overline{q}_3)q_3], \\
V^1 &: [\kappa_1(\overline{\ell}V_\ell\ell_3)(\overline{q}q) + \kappa_2(\overline{\ell}V_\ell\ell_3)(\overline{q}_3q_3) + \kappa_3(\overline{\ell}\ell)(\overline{q}V_qq_3) + \kappa_4(\overline{\ell}_3\ell_3)(\overline{q}V_qq_3) + h.c], \\
V^2 &: [b_1(\overline{\ell}^p V_\ell^p V_\ell^{\dagger r}\ell^r)(\overline{q}q) + b_2(\overline{\ell}^p V_\ell^p V_\ell^{\dagger r}\ell^r)(\overline{q}_3q_3) + b_3(\overline{\ell}\ell)(\overline{q}^p V_q^p V_q^{\dagger r}q^r) \\
&\quad + c_4(\overline{\ell}_3\ell_3)(\overline{q}^p V_q^p V_q^{\dagger r}q^r) + (\eta_1(\overline{\ell}V_\ell\ell_3)(\overline{q}V_qq_3) + \eta_2(\overline{\ell}V_\ell\ell_3)(\overline{q}_3V_q^\dagger q) + h.c)].
\end{aligned} \tag{5.37}$$

For the $(\overline{L}L)(\overline{L}L)$ structure we find the following number of real and imaginary coefficients at a given order in the spurion expansion

	V^0		V^1		V^2		Δ^1		$\Delta^1 V^1$		V^3	
$Identical\ [Q_{\ell\ell}, Q_{qq}^{(1,3)}]$:	5	–	3	3	5	1	–	–	–	–	1	1
$Different\ [Q_{\ell q}^{(1)}, Q_{\ell q}^{(3)}]$:	4	–	4	4	6	2	–	–	–	–	2	2

- $(\overline{R}R)(\overline{R}R)$ structure:
  For the identical currents:
  The terms generated up to $\mathcal{O}(V^3)$ are

$$\begin{aligned} V^0 &: [a_1(\overline{u}^p u^p)(\overline{u}^r u^r) + a_2(\overline{u}^p u^r)(\overline{u}^r u^p) + a_3(\overline{u}u)(\overline{u}_3 u_3) \\ &\quad + a_4(\overline{u}u_3)(\overline{u}_3 u) + a_5(\overline{u}_3 u_3)(\overline{u}_3 u_3)], \\ \Delta^1 V^1 &: [\sigma_1(\overline{u}_3 V_q^{\dagger s}\Delta_u^{sr} u^r)(\overline{u}^p u^p) + \sigma_2(\overline{u}_3 V_q^{\dagger}\Delta_u u)(\overline{u}_3 u_3) \\ &\quad + \sigma_3(\overline{u}^p V_q^{\dagger s}\Delta_u^{sr} u^r)(\overline{u}_3 u^p) + h.c]. \end{aligned} \tag{5.38}$$

For the Q_{ee} structure:

$$\begin{aligned} V^0 &: [a_1(\overline{e}^p e^p)(\overline{e}^r e^r) + a_2(\overline{e}e)(\overline{e}_3 e_3) + a_3(\overline{e}_3 e_3)(\overline{e}_3 e_3)], \\ \Delta^1 V^1 &: [\sigma_1(\overline{e}^p e^p)(\overline{e}_3 V_\ell^{\dagger t}\Delta_e^{ts} e^s) + \sigma_2(\overline{e}_3 e_3)(\overline{e}_3 V_\ell^{\dagger t}\Delta_e^{tp} e^p) + h.c]. \end{aligned} \tag{5.39}$$

For the different currents:

$$\begin{aligned} V^0 &: [a_1(\overline{e}e)(\overline{u}u) + a_2(\overline{e}e)(\overline{u}_3 u_3) + a_3(\overline{e}_3 e_3)(\overline{u}u) + a_4(\overline{e}_3 e_3)(\overline{u}_3 u_3)], \\ \Delta^1 V^1 &: [\sigma_1(\overline{e}_3 V_\ell^{\dagger}\Delta_e e)(\overline{u}u) + \sigma_2(\overline{e}_3 V_\ell^{\dagger}\Delta_e e)(\overline{u}_3 u_3) + \sigma_3(\overline{e}e)(\overline{u}_3 V_q^{\dagger}\Delta_u u) \\ &\quad + \sigma_4(\overline{e}_3 e_3)(\overline{u}_3 V_q^{\dagger}\Delta_u u) + h.c]. \end{aligned} \tag{5.40}$$

For the $(\overline{R}R)(\overline{R}R)$ structure, we find the following number of real and imaginary coefficients at a given order in the spurion expansion

	V^0		V^1		V^2		Δ^1		$\Delta^1 V^1$		V^3	
$Identical\ [Q_{uu(dd)}]$:	5	–	–	–	–	–	–	–	3	3	–	–
$Identical\ [Q_{ee}]$:	3	–	–	–	–	–	–	–	2	2	–	–
$Different\ [Q_{eu}, Q_{ed}, Q_{ud}^{(1,8)}]$:	4	–	–	–	–	–	–	–	4	4	–	–

- $(\overline{L}L)(\overline{R}R)$ structure:
  Leptonic-Leptonic and Quark-Quark operators:
  The terms generated up to $\mathcal{O}(V^3)$ are

$$
\begin{aligned}
V^0 &: [a_1(\bar{\ell}\ell)(\bar{e}e) + a_2(\bar{\ell}\ell)(\bar{e}_3e_3) + a_3(\bar{\ell}_3\ell_3)(\bar{e}e) + a_4(\bar{\ell}_3\ell_3)(\bar{e}_3e_3)],\\
V^1 &: [\kappa_1(\bar{\ell}V_\ell\ell_3)(\bar{e}e) + \kappa_2(\bar{\ell}V_\ell\ell_3)(\bar{e}_3e_3) + h.c],\\
V^2 &: [b_1(\bar{\ell}^pV_\ell^pV_\ell^{\dagger r}\ell^r)(\bar{e}e) + b_1(\bar{\ell}^pV_\ell^pV_\ell^{\dagger r}\ell^r)(\bar{e}_3e_3)],\\
\Delta^1V^0 &: [\rho_1(\bar{\ell}\ell_3)\Delta_e(\bar{e}_3e) + h.c],\\
\Delta^1V^1 &: [\sigma_1(\bar{\ell}^pV_\ell^{\dagger r}\ell^r)\Delta_e^{pt}(\bar{e}_3e^t) + \sigma_2(\bar{\ell}^p\ell^p)V_\ell^{\dagger r}\Delta_e^{rt}(\bar{e}_3e^t)\\
&\quad + \sigma_3(\bar{\ell}_3\ell_3)V_\ell^\dagger\Delta_e(\bar{e}_3e) + h.c].
\end{aligned}
\tag{5.41}
$$

For the Leptonic-Quark structure:
The terms generated up to $\mathcal{O}(V^3)$ are

$$
\begin{aligned}
V^0 &: [a_1(\bar{\ell}\ell)(\bar{u}u) + a_2(\bar{\ell}\ell)(\bar{u}_3u_3) + a_3(\bar{\ell}_3\ell_3)(\bar{u}u) + a_4(\bar{\ell}_3\ell_3)(\bar{u}_3u_3)],\\
V^1 &: [\kappa_1(\bar{\ell}V_\ell\ell_3)(\bar{u}u) + \kappa_2(\bar{\ell}V_\ell\ell_3)(\bar{u}_3u_3) + h.c],\\
V^2 &: [b_1(\bar{\ell}^pV_\ell^pV_\ell^{\dagger r}\ell^r)(\bar{u}u) + b_2(\bar{\ell}^pV_\ell^pV_\ell^{\dagger r}\ell^r)(\bar{u}_3u_3)],\\
\Delta^1V^1 &: [\sigma_1(\bar{\ell}\ell)V_q^\dagger\Delta_u(\bar{u}_3u) + \sigma_2(\bar{\ell}_3\ell_3)V_q^\dagger\Delta_u(\bar{u}_3u) + h.c].
\end{aligned}
\tag{5.42}
$$

For the $(\bar{L}L)(\bar{R}R)$ strucutre, we find the following number of real and imaginary coefficients at a given order in the spurion expansion

	V^0		V^1		V^2		Δ^1		Δ^1V^1		V^3	
$L/Q - L/Q$ $[Q_{le}, Q_{qu}^{(1,8)}, Q_{qd}^{(1,8)}]$:	4	–	2	2	2	–	1	1	3	3	–	–
$L - Q$ $[Q_{lu}, Q_{ld}, Q_{qe}]$:	4	–	2	2	2	–	–	–	2	2	–	–

- $(\bar{L}R)(\bar{R}L)$+h.c strucutre:
  The single operator Q_{ledq}, for which the spurion decomposition up to $\mathcal{O}(V^3, \Delta^1V^1)$:

$$
\begin{aligned}
V^0 &: [\alpha_1(\bar{\ell}_3e_3)(\bar{d}_3q_3) + h.c],\\
V^1 &: [\kappa_1(\bar{\ell}V_\ell e_3)(\bar{d}_3V_q^\dagger q_3) + \kappa_2(\bar{\ell}_3e_3)(\bar{d}_3V_q^\dagger q) + h.c],\\
V^2 &: [\eta_1(\bar{\ell}V_\ell e_3)(\bar{d}_3V_q^\dagger q) + h.c],\\
\Delta^1V^0 &: [\iota_1(\bar{\ell}\Delta_e e)(\bar{d}_3q_3) + \iota_2(\bar{\ell}_3e_3)(\bar{d}\Delta_d^\dagger q) + h.c],\\
\Delta^1V^1 &: [\sigma_1(\bar{\ell}\Delta_e e)(\bar{d}_3V_q^\dagger q) + \sigma_2(\bar{\ell}_3V_\ell^\dagger\Delta_e e)(\bar{d}_3q_3)\\
&\quad + \sigma_3(\bar{\ell}V_\ell e_3)(\bar{d}\Delta_d^\dagger q) + \sigma_4(\bar{\ell}_3e_3)(\bar{d}\Delta_d^\dagger V_q q_3) + h.c]
\end{aligned}
\tag{5.43}
$$

Operators	$U(2)^5$ Exact		$\mathcal{O}(V^1)$		$\mathcal{O}(V^2)$		$\mathcal{O}(V^1,\Delta^1)$		$\mathcal{O}(V^2,\Delta^1)$		$\mathcal{O}(V^2,\Delta^1 V^1)$		$\mathcal{O}(V^3,\Delta^1 V^1)$	
	[terms summed up to different orders]													
Class 1–4	9	6	9	6	9	6	9	6	9	6	9	6	9	6
$\psi^2 H^3$	3	3	6	6	6	6	9	9	9	9	12	12	12	12
$\psi^2 XH$	8	8	16	16	16	16	24	24	24	24	32	32	32	32
$\psi^2 H^2 D$	15	1	19	5	23	5	19	5	23	5	28	10	28	10
$(\bar{L}L)(\bar{L}L)$	23	–	40	17	67	24	40	17	67	24	67	24	74	31
$(\bar{R}R)(\bar{R}R)$	29	–	29	–	29	–	29	–	29	–	53	24	53	24
$(\bar{L}L)(\bar{R}R)$	32	–	48	16	64	16	53	21	69	21	90	42	90	42
$(\bar{L}R)(\bar{R}L)$	1	1	3	3	4	4	5	5	6	6	10	10	10	10
$(\bar{L}R)(\bar{L}R)$	4	4	12	12	16	16	24	24	28	28	48	48	48	48
total:	124	23	182	81	234	93	212	111	264	123	349	208	356	215

Table 5.5: Number of independent operators in the SMEFT assuming a minimally broken $U(2)^5$ symmetry, including breaking terms up to $\mathcal{O}(V^3,\Delta^1 V^1)$. Notations as in Table 5.2.

- $(\overline{L}R)(\overline{L}R)$ structures:
  $Q^{(1,3)}_{lequ}$ have the same decomposition as for Q_{ledq}, while $Q^{(1,8)}_{quqd}$ decomposed as follow

$$
\begin{aligned}
V^0 &: [\alpha_1(\overline{q}_3 u_3)(\overline{q}_3 d_3) + h.c],\\
V^1 &: [\kappa_1(\overline{q}V_q u_3)(\overline{q}_3 d_3) + \kappa_2(\overline{q}_3 u_3)(\overline{q}V_q d) + h.c],\\
\Delta^1 V^0 &: [\iota_1(\overline{q}\Delta_u u)(\overline{q}_3 d_3) + \iota_2(\overline{q}_3 u_3)(\overline{q}\Delta_d d)\\
&\quad + \iota_3(\overline{q}_3\Delta_u u)(\overline{q}d_3) + \iota_4(\overline{q}u_3)(\overline{q}_3\Delta_d d) + h.c],\\
\Delta^1 V^1 &: [\sigma_1(\overline{q}_3 V_q^\dagger \Delta_u u)(\overline{q}_3 d_3) + \sigma_2(\overline{q}^p \Delta_u^{pr} u^r)(\overline{q}^s V_q^s d_3) + \sigma_3(\overline{q}_3 u_3)(\overline{q}_3 V_q^\dagger \Delta_d d)\\
&\quad + \sigma_4(\overline{q}^p V_q^p u_3)(\overline{q}^r \Delta_d^{rs} d^s) + \sigma_5(\overline{q}^p V_q^p u^r)\Delta_u^{sr}(\overline{q}^s d_3) + \sigma_6(\overline{q}^p u_3)\Delta_d^{pr}(\overline{q}^s V_q^s d^r) + h.c]
\end{aligned}
\tag{5.44}
$$

For the $(\overline{L}R)(\overline{R}L)$ and $(\overline{L}R)(\overline{L}R)$ strucutre, we find the following number of real and imaginary coefficients at a given order in the spurion expansion

	V^0		V^1		V^2		Δ^1		$\Delta^1 V^1$		V^3	
$l-q$ $[Q_{ledq}, Q^{(1,3)}_{lequ}]$:	1	1	2	2	1	1	2	2	4	4	–	–
$q-q$ $[Q^{(1,8)}_{quqd}]$:	1	1	2	2	1	1	4	4	6	6	–	–

The total number of counting is summarized in Table 5.5. Using the same notation of Table 5.2. The counting is done up to $\mathcal{O}(V^3, \Delta^1 V^1)$ which is in total agreement with [165].

5.5 Matching with Other Countings

Ref. [166] considers flavor symmetry scenarios based on U(3) and U(2) but with some differences in the spurion power counting compared to what we have done in the thesis: in particular it keeps the minimum spurion insertion required for each operator, and for U(2) it neglects fermionic mixing, ie $V_q = V_\ell = 0$ and $V_{CKM} = 1$. Also, for the leptons it doesn't use U(2) but either U(3) or $U(1)^3$.

Let's try to do the explicity check for the verion with $U(2)^3$ for quarks and $U(3)^2$ for leptons :

1. Fermionic Bilinear Operators:

   - Class 5:
     For Q_{eH}, is a non-hermitian operator with 1 complex parameter.
     For Q_{uH} and Q_{dH} taking the remaining in eq.5.34, that are 4: 2 real and 2 complex parameters.
     For the whole classs we have 5 real and 5 imaginary coefficients.
   - Class 6:
     Same for class 5, that leads to total of 14 real and 14 imaginary coefficients.
   - Class 7:
     The operators that are $(\overline{L}L)$ will have 2 real coefficients each coming from eq.5.29.
     The operators that are $(\overline{R}R)$ will have 2 real coefficients each coming from eq.5.31.
     For Q_{Hud} will has 1 real and 1 imaginary coefficients coming from eq.5.33.
     Then, in total for class 7 we will have 12 real and 1 imaginary.

2. Four Fermion Operators:

   - $8(\overline{L}L)(\overline{L}L)$ structures:
     For the identical currents, we will have 5 real coefficient each, coming from eq.5.36. And for the different currents we will have 4 real coefficeints each coming from eq.5.37.
     For the leptonic structure $Q_{\ell\ell}$, we will have 2 real coefficients coming from the cross as in eq.5.5.
     The total number for the whole subclass $8(\bar{L}L)(\bar{L}L)$ is 16 real coefficients.

- $8(\bar{R}R)(\bar{R}R)$ structure:

  For the identical currents , we will have 5 real coefficients each from eq.5.38. And for the different currents we will have two subsets and referring to eq.5.40, one with lepton-quark having 2 real coefficients each and the quark-quark having 4 CP-even from $\mathcal{O}(V^0)$ and then we have interaction contracted in the following form $(\bar{u}^p u_3)\Delta_u^{\dagger pt}\Delta_d^{ts}(\bar{d}_3 d^s)$ which has a complex parameter, we have this contraction in $Q_{ud}^{(1,8)}$. For Q_{ee} due to eq.5.6, there is only one real coefficient.

  The total number for the whole subclass $8(\bar{R}R)(\bar{R}R)$ is 25 real and 2 imaginary coefficients.

- $8(\bar{L}L)(\bar{R}R)$ structure:

  For the Leptonic-Leptonic each has one real coefficient. For the Quark-Quark, we have 5 real and 1 imaginary for each from eq.5.41. For the Leptonic-Quark we have 2 real coefficient.

  The total number for the whole subclass $8(\bar{L}L)(\bar{R}R)$ is 27 real and 4 imaginary coefficients.

- $8(\bar{L}R)(\bar{R}L)$ structure:

  From eq.5.43 we have 2 real and 2 imaginary coefficients.

- $8(\bar{L}R)(\bar{L}R)$ structure:

  $Q_{lequ}^{(1,3)}$ are the same as the Q_{leqd}, then each has 2 real and 2 imaginary coefficients.

  For the $Q_{ququd}^{(1,8)}$ we have 5 real and 5 imaginary coefficient for each from eq.5.44. But we have also contraction in the following form $(\bar{q}\Delta_u u)(\bar{q}\Delta_d d)$ should be counted also. Such structure has 1 complex coefficient. The total number for the subclass $8(\bar{L}R)(\bar{L}R)$ is 18 real and 18 imaginary coefficients.

The operators that we have counted here can be written explicitly in table 5.7 done in ref. [166], using her own notations, we will not go through it and we will not use it, but just a general overview.

We can have a different assumption in the leptonic sector, one can be to consider a $U(1)^3_{l+e} = U(1)_e \times U(1)_\mu \times U(1)_\tau$ symmetry under which the fields transform as

$$\ell_1 \to e^{i\alpha_e}\ell_1, \quad \ell_2 \to e^{i\alpha_\mu}\ell_2, \quad \ell_3 \to e^{i\alpha_\tau}\ell_3. \tag{5.45}$$

$$e_1 \to e^{i\alpha_e}e_1, \quad e_2 \to e^{i\alpha_\mu}e_2, \quad e_3 \to e^{i\alpha_\tau}e_3, \tag{5.46}$$

Here we can do also the counting, based on the previous counting but in addition to the leptonic sector.

1. Fermionic Bilinear Operators:

	$U(3)^5$ up to $\mathcal{O}(Y^2)$		$U(3)^5$ needed		$U(2)^5$		$U(2)^3$ without V		$U(2)^3 \times U(1)^3_{l+e}$	
	all	$\not{CP}$	all	$\not{CP}$	all	$\not{CP}$	all	$\not{CP}$	all	$\not{CP}$
$\mathcal{L}_6^{(1)}$	4	2	4	2	4	2	4	2	4	2
$\mathcal{L}_6^{(2,3)}$	3	-	3	-	3	-	3	-	3	-
$\mathcal{L}_6^{(4)}$	8	4	8	4	8	4	8	4	8	4
$\mathcal{L}_6^{(5)}$	6	3	6	3	24	12	14	7	10	5
$\mathcal{L}_6^{(6)}$	16	8	16	8	64	32	36	18	28	14
$\mathcal{L}_6^{(7)}$	17	1	9	1	38	10	21	2	15	2
$\mathcal{L}_6^{(8\bar{L}L)(\bar{L}L)}$	24	-	8	-	105	31	31	-	16	-
$\mathcal{L}_6^{8(\bar{R}R)(\bar{R}R)}$	22	-	9	-	77	24	40	2	27	2
$\mathcal{L}_6^{8(\bar{L}L)(\bar{R}R)}$	34	-	8	-	132	42	54	4	31	4
$\mathcal{L}_6^{8(\bar{L}R)(\bar{R}L)}$	2	1	1	1	20	10	64	32	40	20
$\mathcal{L}_6^{8(\bar{L}R)(\bar{L}R)}$	12	6	12	6	96	48	64	32	40	20
tot	148	25	85	25	571	215	275	71	182	53

Table 5.6: Number of independent real parameters in each class of dimension 6 operators, for the 5 flavor structures done.

- Class 5:

  Q_{eH} now has 3 real and 3 imaginary coefficients. Other remains as counted before.

  The total become 7 real and 7 imaginary coefficients.

- Class 6:

  Q_{eW} and Q_{eB} each has 3 real and 3 imaginary coefficients. While all the others remain the same.

  The total become 18 real and 18 imaginary coefficients.

- Class 7:

  Q_{He} and $Q_{H\ell}^{(1,3)}$ are hermitian having 3 real coefficients each.

  The total become 19 real and 2 imaginary coefficients.

2. Four Fermion Operators:

- $8(\bar{L}L)(\bar{L}L)$ structure:

  $Q_{\ell\ell}$ has 9 real coefficients.

$Q_{\ell q}^{(1,3)}$ each has 6 real coefficient.
The total become 31 real coefficient.

- $8(\bar{R}R)(\bar{R}R)$:
  Q_{ee} has 6 real coefficients.
  Q_{eu} and Q_{ed} each has 6 coefficients.
  The total become 38 real and 2 imaginary coefficients.
- $8(\bar{L}L)(\bar{R}R)$ strucutres:
  $Q_{\ell e}$ has 12 real coefficients.
  $Q_{\ell u}$, $Q_{\ell d}$ and Q_{qe} each has 6 real coefficients.
  The total become 50 real and 4 imaginary coefficients.
- $8(\bar{L}R)(\bar{R}L)$ strucutre:
  Q_{leqd} has now 6 real and 6 imaginary coefficients.
- $8(\bar{L}R)(\bar{L}R)$ strucutre:
  $Q_{lequ}^{(1,3)}$ each has 6 real and 6 imaginary coefficients.
  The total become 26 real and 26 imaginary coefficients.

The final counting of all the flavor assumptions done so far is summarized in table 5.6.

5.6 Mapping between $\mathbf{U(3)^5}$ and $\mathbf{U(2)^5}$

From a symmetry point of view, the insertion of 3rd generation Yukawas breaks the $U(3)^5$ symmetry leaving a residual $U(2)^5$. The conditions will be to turn off all the Yukawas $U(3)^5$ but keeping the third generation on (y_t, y_b and y_τ). And for $U(2)^5$ we will not have any spurion. So the two conditions should be the same, and therefore the number of independent SMEFT parameters should be the same. We will need to show this explicitly. To do this, we need to define the Wilson coefficient in each flavor symmetric limit.

In the case of $U(3)^5$ up to $\mathcal{O}(Y^2)$, the coefficients can be written as follow
Operators starts with $\mathcal{O}(Y^0)$:

$$C_{Hq,pr}^{(3)} = C_{Hq}^{(3,0)}\delta_{pr} + C_{Hq}^{(3,1)}(Y_u^\dagger Y_u)_{pr} + C_{Hq}^{(3,2)}(Y_d^\dagger Y_d)_{pr} \tag{5.47}$$

$$\begin{aligned} C_{qq,prst}^{(3)} = {} & C_{qq}^{(3,0)}\delta_{pr}\delta_{st} + C_{qq}^{(3,1)}\delta_{pt}\delta_{sr} + C_{qq}^{(3,2)}(Y_u^\dagger Y_u)_{pr}\delta_{st} + C_{qq}^{(3,3)}(Y_u^\dagger Y_u)_{pt}\delta_{sr} \\ & + C_{qq}^{(3,4)}(Y_d^\dagger Y_d)_{pr}\delta_{st} + C_{qq}^{(3,5)}(Y_d^\dagger Y_d)_{pt}\delta_{sr} \end{aligned} \tag{5.48}$$

To break $U(3)^5$ explicitly we need to make sure to include all the independent spurion insertions. An insertion is independent if it breaks U(3) in a way that was not already

$\mathcal{L}_6^{(5)} - \psi^2 H^3$							
Q_{uH}	$(H^\dagger H)(\bar{q}\, Y_u^\dagger\, u \tilde{H})$	Q_{dH}	$(H^\dagger H)(\bar{q}\, Y_d^\dagger\, dH)$	Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$		
Q_{tH}	$(H^\dagger H)(\bar{Q}\tilde{H}t)$	Q_{bH}	$(H^\dagger H)(\bar{Q}Hb)$				
$\mathcal{L}_6^{(6)} - \psi^2 XH$							
Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\sigma^i H W^i_{\mu\nu}$	Q_{uW}	$(\bar{q}\, Y_u^\dagger\, \sigma^{\mu\nu}u)\sigma^i \tilde{H} W^i_{\mu\nu}$	Q_{uB}	$(\bar{q}\, Y_u^\dagger\, \sigma^{\mu\nu}u)\tilde{H} B_{\mu\nu}$	Q_{uG}	$(\bar{q}\, Y_u^\dagger\, \sigma^{\mu\nu}T^a u)\tilde{H} G^a_{\mu\nu}$
Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	Q_{tW}	$(\bar{Q}\sigma^{\mu\nu}t)\sigma^i \tilde{H} W^i_{\mu\nu}$	Q_{tB}	$(\bar{Q}\sigma^{\mu\nu}t)\tilde{H} B_{\mu\nu}$	Q_{tG}	$(\bar{Q}\sigma^{\mu\nu}T^a t)\tilde{H} G^a_{\mu\nu}$
Q_{dW}	$(\bar{q}\, Y_d^\dagger\, \sigma^{\mu\nu}d)\sigma^i H W^i_{\mu\nu}$	Q_{dB}	$(\bar{q}\, Y_d^\dagger\, \sigma^{\mu\nu}d)H B_{\mu\nu}$	Q_{dG}	$(\bar{q}\, Y_d^\dagger\, \sigma^{\mu\nu}T^a d)H G^a_{\mu\nu}$		
Q_{bW}	$(\bar{Q}\sigma^{\mu\nu}b)\sigma^i H W^i_{\mu\nu}$	Q_{bB}	$(\bar{Q}\sigma^{\mu\nu}b)H B_{\mu\nu}$	Q_{bG}	$(\bar{Q}\sigma^{\mu\nu}T^a b)H G^a_{\mu\nu}$		
$\mathcal{L}_6^{(7)} - \psi^2 H^2 D$							
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^i_\mu H)(\bar{l}_p\sigma^i\gamma^\mu l_r)$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$		
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^i_\mu H)(\bar{q}\sigma^i\gamma^\mu q)$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$
$Q_{HQ}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{Q}\gamma^\mu Q)$	$Q_{HQ}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^i_\mu H)(\bar{Q}\sigma^i\gamma^\mu Q)$	Q_{Ht}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{t}\gamma^\mu t)$	Q_{Hb}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{b}\gamma^\mu b)$
Q_{Hud}	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}\, Y_u Y_d^\dagger\, \gamma^\mu d)$	Q_{Htb}	$i(\tilde{H}^\dagger D_\mu H)(\bar{t}\gamma^\mu b)$				
$\mathcal{L}_6^{(8a)} - (\bar{L}L)(\bar{L}L)$							
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}\gamma^\mu q)$	$Q_{lq}^{(3)}$	$(\bar{l}_p\sigma^i\gamma_\mu l_r)(\bar{q}\sigma^i\gamma^\mu q)$	Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$		
$Q_{lQ}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{Q}\gamma^\mu Q)$	$Q_{lQ}^{(3)}$	$(\bar{l}_p\sigma^i\gamma_\mu l_r)(\bar{Q}\sigma^i\gamma^\mu Q)$	$Q_{QQ}^{(1)}$	$(\bar{Q}\gamma_\mu Q)(\bar{Q}\gamma^\mu Q)$	$Q_{QQ}^{(8)}$	$(\bar{Q}T^a\gamma_\mu Q)(\bar{Q}T^a\gamma^\mu Q)$
$Q_{qq}^{(1,1)}$	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$	$Q_{qq}^{(1,8)}$	$(\bar{q}T^a\gamma_\mu q)(\bar{q}T^a\gamma^\mu q)$	$Q_{qq}^{(3,1)}$	$(\bar{q}\sigma^i\gamma_\mu q)(\bar{q}\sigma^i\gamma^\mu q)$	$Q_{qq}^{(3,8)}$	$(\bar{q}\sigma^i T^a\gamma_\mu q)(\bar{q}\sigma^i T^a\gamma^\mu q)$
$Q_{Qq}^{(1,1)}$	$(\bar{Q}\gamma_\mu Q)(\bar{q}\gamma^\mu q)$	$Q_{Qq}^{(1,8)}$	$(\bar{Q}T^a\gamma_\mu Q)(\bar{q}T^a\gamma^\mu q)$	$Q_{Qq}^{(3,1)}$	$(\bar{Q}\sigma^i\gamma_\mu Q)(\bar{q}\sigma^i\gamma^\mu q)$	$Q_{Qq}^{(3,8)}$	$(\bar{Q}\sigma^i T^a\gamma_\mu Q)(\bar{q}\sigma^i T^a\gamma^\mu q)$
$\mathcal{L}_6^{(8b)} - (\bar{R}R)(\bar{R}R)$							
Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}\gamma^\mu u)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}\gamma^\mu d)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$		
Q_{et}	$(\bar{e}_p\gamma_\mu e_r)(\bar{t}\gamma^\mu t)$	Q_{eb}	$(\bar{e}_p\gamma_\mu e_r)(\bar{b}\gamma^\mu b)$	Q_{tt}	$(\bar{t}\gamma_\mu t)(\bar{t}\gamma^\mu t)$	Q_{bb}	$(\bar{b}\gamma_\mu b)(\bar{b}\gamma^\mu b)$
$Q_{uu}^{(1)}$	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$	$Q_{uu}^{(8)}$	$(\bar{u}T^a\gamma_\mu u)(\bar{u}T^a\gamma^\mu u)$	$Q_{tu}^{(1)}$	$(\bar{t}\gamma_\mu t)(\bar{u}\gamma^\mu u)$	$Q_{tu}^{(8)}$	$(\bar{t}T^a\gamma_\mu t)(\bar{u}T^a\gamma^\mu u)$
$Q_{dd}^{(1)}$	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$	$Q_{dd}^{(8)}$	$(\bar{d}T^a\gamma_\mu d)(\bar{d}T^a\gamma^\mu d)$	$Q_{bd}^{(1)}$	$(\bar{b}\gamma_\mu b)(\bar{d}\gamma^\mu d)$	$Q_{bd}^{(8)}$	$(\bar{b}T^a\gamma_\mu b)(\bar{d}T^a\gamma^\mu d)$
$Q_{ud}^{(1)}$	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$	$Q_{ud}^{(8)}$	$(\bar{u}T^a\gamma_\mu u)(\bar{d}T^a\gamma^\mu d)$	$Q_{td}^{(1)}$	$(\bar{t}\gamma_\mu t)(\bar{d}\gamma^\mu d)$	$Q_{td}^{(8)}$	$(\bar{t}T^a\gamma_\mu t)(\bar{d}T^a\gamma^\mu d)$
$Q_{ub}^{(1)}$	$(\bar{u}\gamma_\mu u)(\bar{b}\gamma^\mu b)$	$Q_{ub}^{(8)}$	$(\bar{u}T^a\gamma_\mu u)(\bar{b}T^a\gamma^\mu b)$	$Q_{tb}^{(1)}$	$(\bar{t}\gamma_\mu t)(\bar{b}\gamma^\mu b)$	$Q_{tb}^{(8)}$	$(\bar{t}T^a\gamma_\mu t)(\bar{b}T^a\gamma^\mu b)$
$Q_{utbd}^{(1)}$	$(Y_u Y_d^\dagger)_{pr}(\bar{u}_p\gamma_\mu t)(\bar{b}\gamma^\mu d_r)$	$Q_{utbd}^{(8)}$	$(Y_u Y_d^\dagger)_{pr}(\bar{u}_p T^a\gamma_\mu t)(\bar{b}T^a\gamma^\mu d_r)$				
$\mathcal{L}_6^{(8c)} - (\bar{L}L)(\bar{R}R)$							
Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}\gamma^\mu u)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}\gamma^\mu d)$	Q_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}_p\gamma^\mu e_r)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{lt}	$(\bar{l}_p\gamma_\mu l_r)(\bar{t}\gamma^\mu t)$	Q_{lb}	$(\bar{l}_p\gamma_\mu l_r)(\bar{b}\gamma^\mu b)$	Q_{Qe}	$(\bar{Q}\gamma_\mu Q)(\bar{e}_p\gamma^\mu e_r)$		
$Q_{qu}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma^\mu u)$	$Q_{Qu}^{(1)}$	$(\bar{Q}\gamma_\mu Q)(\bar{u}\gamma^\mu u)$	$Q_{qt}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{t}\gamma^\mu t)$	$Q_{Qt}^{(1)}$	$(\bar{Q}\gamma_\mu Q)(\bar{t}\gamma^\mu t)$
$Q_{qu}^{(8)}$	$(\bar{q}T^a\gamma_\mu q)(\bar{u}T^a\gamma^\mu u)$	$Q_{Qu}^{(8)}$	$(\bar{Q}T^a\gamma_\mu Q)(\bar{u}T^a\gamma^\mu u)$	$Q_{qt}^{(8)}$	$(\bar{q}T^a\gamma_\mu q)(\bar{t}T^a\gamma^\mu t)$	$Q_{Qt}^{(8)}$	$(\bar{Q}T^a\gamma_\mu Q)(\bar{t}T^a\gamma^\mu t)$
$Q_{qd}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma^\mu d)$	$Q_{Qd}^{(1)}$	$(\bar{Q}\gamma_\mu Q)(\bar{d}\gamma^\mu d)$	$Q_{qb}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{b}\gamma^\mu b)$	$Q_{Qb}^{(1)}$	$(\bar{Q}\gamma_\mu Q)(\bar{b}\gamma^\mu b)$
$Q_{qd}^{(8)}$	$(\bar{q}T^a\gamma_\mu q)(\bar{d}T^a\gamma^\mu d)$	$Q_{Qd}^{(8)}$	$(\bar{Q}T^a\gamma_\mu Q)(\bar{d}T^a\gamma^\mu d)$	$Q_{qb}^{(8)}$	$(\bar{q}T^a\gamma_\mu q)(\bar{b}T^a\gamma^\mu b)$	$Q_{Qb}^{(8)}$	$(\bar{Q}T^a\gamma_\mu Q)(\bar{b}T^a\gamma^\mu b)$
$Q_{qQtu}^{(1)}$	$(Y_u^\dagger)_{pr}(\bar{q}_p\gamma_\mu Q)(\bar{t}\gamma^\mu u_r)$	$Q_{qQtu}^{(8)}$	$(Y_u^\dagger)_{pr}(\bar{q}_p T^a\gamma_\mu Q)(\bar{t}T^a\gamma^\mu u_r)$	$Q_{qQbd}^{(1)}$	$(Y_d^\dagger)_{pr}(\bar{q}_p\gamma_\mu Q)(\bar{b}\gamma^\mu d_r)$	$Q_{qQbd}^{(8)}$	$(Y_d^\dagger)_{pr}(\bar{q}_p T^a\gamma_\mu Q)(\bar{b}T^a\gamma^\mu d_r)$
$\mathcal{L}_6^{(8d)} - (\bar{L}R)(\bar{R}L), (\bar{L}R)(\bar{L}R)$							
Q_{ledq}	$(\bar{l}^j_p e_r)(\bar{d}\, Y_d\, q_j)$	Q_{lebQ}	$(\bar{l}^j_p e_r)(\bar{b}Q_j)$	$Q_{leQt}^{(1)}$	$(\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{Q}^k\, t)$	$Q_{leQt}^{(3)}$	$(\bar{l}^j_p\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{Q}^k\sigma^{\mu\nu}t)$
$Q_{lequ}^{(1)}$	$(\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k\, Y_u^\dagger\, u)$	$Q_{lequ}^{(3)}$	$(\bar{l}^j_p\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}^k\, Y_u^\dagger\, \sigma^{\mu\nu}u)$	$Q_{QtQb}^{(1)}$	$(\bar{Q}^j\, t)\varepsilon_{jk}(\bar{Q}^k\, b)$	$Q_{QtQb}^{(8)}$	$(\bar{Q}^j\, T^a t)\varepsilon_{jk}(\bar{Q}^k\, T^a b)$
$Q_{quqd}^{(1)}$	$(\bar{q}^j\, Y_u^\dagger\, u)\varepsilon_{jk}(\bar{q}^k\, Y_d^\dagger\, d)$	$Q_{quqd}^{(8)}$	$(\bar{q}^j\, Y_u^\dagger\, T^a u)\varepsilon_{jk}(\bar{q}^k\, Y_d^\dagger\, T^a d)$	$Q_{quqd}^{(1)\prime}$	$(Y_u^\dagger)_{sr}(Y_d^\dagger)_{pt}(\bar{q}^j_p\, u_r)\varepsilon_{jk}(\bar{q}^k_s\, d_t)$	$Q_{quqd}^{(8)\prime}$	$(Y_u^\dagger)_{sr}(Y_d^\dagger)_{pt}(\bar{q}^j_p\, T^a u_r)\varepsilon_{jk}(\bar{q}^k_s\, T^a d_t)$
$Q_{Qtqd}^{(1)}$	$(\bar{Q}^j\, t)\varepsilon_{jk}(\bar{q}^k\, Y_d^\dagger\, d)$	$Q_{Qtqd}^{(8)}$	$(\bar{Q}^j\, T^a t)\varepsilon_{jk}(\bar{q}^k\, Y_d^\dagger\, T^a d)$	$Q_{quQb}^{(1)}$	$(\bar{q}^j\, Y_u^\dagger\, u)\varepsilon_{jk}(\bar{Q}^k\, b)$	$Q_{quQb}^{(8)}$	$(\bar{q}^j\, Y_u^\dagger\, T^a u)\varepsilon_{jk}(\bar{Q}^k\, T^a b)$
$Q_{Quqb}^{(1)}$	$(Y_u^\dagger)_{pr}\,(\bar{Q}^j\, u_r)\varepsilon_{jk}(\bar{q}^k_p\, b)$	$Q_{Quqb}^{(8)}$	$(Y_u^\dagger)_{pr}\,(\bar{Q}^j\, T^a u_r)\varepsilon_{jk}(\bar{q}^k_p\, T^a b)$	$Q_{qtQd}^{(1)}$	$(Y_d^\dagger)_{pr}\,(\bar{q}^j_p\, t)\varepsilon_{jk}(\bar{Q}^k\, d_r)$	$Q_{qtQd}^{(8)}$	$(Y_d^\dagger)_{pr}\,(\bar{q}^j_p\, T^a t)\varepsilon_{jk}(\bar{Q}^k\, T^a d_r)$

Table 5.7: Basis of fermionic operators for $U(2)^3$ flavor assumptions without V and $U(2)^3 \times U(1)^3_{l+e}$ flavor assumptions. Here (q, u, d), Y_u, Y_d denote quarks of the first 2 generations and their 2×2 Yukawa matrices. Quark fields of the 3rd generation are (Q, t, b). Flavor indices p, r, s, t run over $\{1, 2\}$ for light quarks and $\{1, 2, 3\}$ for leptons. Whenever flavor indices are not specified, they are implicitly contracted within each current. This table was done by ref. [166] notations and we will not use it, it just a general overview in this flavor symmetry limit.

presented. So for the four fermion operators we will need to go up $\mathcal{O}(Y_u^4)$ for Quark-Quark currents, in order to have all the breakings. For the two fermions structures, $\mathcal{O}(Y_t^4)$ is not independent from the $\mathcal{O}(Y_t^1)$ and $\mathcal{O}(Y_t^2)$ insertions.

Then eq.5.47 can be modifed as follow:

$$C^{(3)}_{Hq,pr} = C^{(3,0)}_{Hq}\delta_{pr} + C^{(3,1)}_{Hq}[(Y_u^\dagger Y_u)_{pr} + (Y_d^\dagger Y_d)_{pr}] \tag{5.49}$$

So eq.5.48 we will need to include the remaining

$$\begin{aligned} C^{(3)}_{qq,prst} &= C^{(3,0)}_{qq}\delta_{pr}\delta_{st} + C^{(3,1)}_{qq}\delta_{pt}\delta_{sr} + C^{(3,2)}_{qq}(Y_u^\dagger Y_u)_{pr}\delta_{st} + C^{(3,3)}_{qq}(Y_u^\dagger Y_u)_{pt}\delta_{sr} \\ &+ C^{(3,4)}_{qq}(Y_d^\dagger Y_d)_{pr}\delta_{st} + C^{(3,5)}_{qq}(Y_d^\dagger Y_d)_{pt}\delta_{sr} + C^{(3,6)}_{qq}(Y_u^\dagger Y_u)_{pr}(Y_u^\dagger Y_u)_{st} \end{aligned} \tag{5.50}$$

Operators starts with $\mathcal{O}(Y^1)$:

$$C_{uG,pr} = C^{(0)}_{uG}(Y_u^\dagger)_{pr} \tag{5.51}$$

Operators starts with $\mathcal{O}(Y^2)$:

$$C_{Hud,pr} = C^{(0)}_{Hud}(Y_u Y_d^\dagger)_{pr} \tag{5.52}$$

These are some general examples of how the coefficients look like in $U(3)^5$.

For $U(2)^5$, this case has a different way of representation for the coefficients. We will need to name the alphabetic letters and the complex letters used in section 5.4.

We will use our own notations depending on the inner fermionic structure, if we have two q's then the coefficient will be with 2 q's, and in the case of 4 q's, it will has 4 q's and so on. p,r,s & t runs over 1 and 2 here, for 1 it is the light and 2 is the heavy.They will be denoted as in eq.5.20. ANd the higher upper indices the first will be from the operator and the second is the number of contracting if there are more than one. For example, $C^{(3,1)}_{qq}$ this is the first contraction of the light-light currents of $C^{(3)}_{Hq}$.

For the case of $(\bar{L}L)$, from eq.5.29 the coefficients of $C^{(3)}_{Hq}$ will be:

$$a_1 = C^{(3,1)}_{qq}, \quad a_2 = C^{(3)}_{q_3q_3}, \quad \kappa_1 = C^{(3)}_{qq_3}, \quad b_1 = C^{(3,2)}_{qq} \tag{5.53}$$

The coefficients denoted with Greek letters are complex and also have the h.c terms will the alphabetic letters are real.

as we can see that a_1 and b_1 will contribute to the same block coefficient. They both contribute to the light-light curents.

For the $C^{(3)}_{qq,prst}$, from eq.5.36 we will have the following

$$\begin{aligned} &a_1 = C^{(3,1)}_{qqqq}, \quad a_2 = C^{(3,2)}_{qqqq}, \quad a_3 = C^{(3,1)}_{qq_3q_3}, \quad a_4 = C^{(3,1)}_{qq_3q_3q}, \\ &a_5 = C^{(3)}_{q_3q_3q_3q_3}, \quad \kappa_1 = C^{(3,1)}_{qqqq_3}, \quad \kappa_2 = C^{(3)}_{qq_3q_3q_3}, \quad \kappa_3 = C^{(3,2)}_{qqqq_3}, \\ &b_1 = C^{(3,3)}_{qqqq}, \quad b_2 = C^{(3,3)}_{qqq_3q_3}, \quad b_3 = C^{(3,3)}_{qq_3q_3q}, \\ &b_4 = C^{(3,6)}_{qqqq}, \quad \eta_1 = C^{(3,1)}_{qq_3qq_3}, \quad \iota_1 = C^{(3,4)}_{qqqq_3}. \end{aligned} \tag{5.54}$$

Also we have the complex conjugate of each of the complex numbers.
For the case of $(\bar{L}R)$, from eq.5.34, the coefficients of C_{uG} will be

$$\alpha_1 = C_{q_3u_3}, \quad \kappa_1 = C_{qu_3}, \quad \rho_1 = C_{qu}, \quad \sigma_1 = C_{q_3u}. \tag{5.55}$$

For the case of Q_{Hud}, form eq.5.33, we will have

$$\alpha_1 = C_{u_3d_3}, \quad \sigma_1 = C_{ud_3}, \quad \sigma_1 = C_{u_3d}. \tag{5.56}$$

We can do the same procedure for all the operators, and we have done that in our program that will be linked later in the next chapter.

Now we have defined the Wilson coefficients in the symmetric SMEFT, we want to do a mapping between the $U(3)^5$ with only the third generation Yukawas and $U(2)^5$ without spurions.

After imposing the conditions, let's see the mapping.
Starting with $Q_{Hq}^{(3)}$, we will have:

$$\begin{aligned} C_{qq}^{(3,1)} &= C_{Hq}^{(3,0)}, \\ C_{q_3q_3}^{(3)} &= C_{Hq}^{(3,0)} + C_{Hq}^{(3,1)} y_t^2 + C_{Hq}^{(3,2)} y_b^2. \end{aligned} \tag{5.57}$$

But as we have said, the y_b^2 is redundant, then we can have the following:

$$\begin{aligned} C_{qq}^{(3,1)} &= C_{Hq}^{(3,0)}, \\ C_{q_3q_3}^{(3)} &= C_{Hq}^{(3,0)} + C_{Hq}^{(3,1)} (y_t^2 + y_b^2). \end{aligned} \tag{5.58}$$

Then we have the same number of parameters, and it is solvable.
Now for $Q_{qq}^{(3)}$, we will have:

$$\begin{aligned} C_{qqqq}^{(3,1)} &= C_{qq}^{(3,0)}, \\ C_{qqqq}^{(3,2)} &= C_{qq}^{(3,1)}, \\ C_{qqq_3q_3}^{(3,1)} &= C_{qq}^{(3,0)} + C_{qq}^{(3,2)} (y_t^2 + y_b^2), \\ C_{qq_3q_3q}^{(3,1)} &= C_{qq}^{(3,1)} + C_{qq}^{(3,3)} (y_t^2 + y_b^2), \\ C_{q_3q_3q_3q_3}^{(3)} &= C_{qq}^{(3,0)} + C_{qq}^{(3,1)} + C_{qq}^{(3,2)} (y_t^2 + y_b^2) + C_{qq}^{(3,3)} (y_t^2 + y_b^2) + C_{qq}^{(3,6)} y_t^4. \end{aligned} \tag{5.59}$$

Here also number of parameters are the same, and it is solvable.
Now for Q_{uG} we will have the following

$$C_{q_3u_3} = C_{uG} y_t \tag{5.60}$$

For Q_{Hud} we have

$$C_{u_3d_3} = C_{Hud} y_t y_b \tag{5.61}$$

And of course both are solvable.

We have done all the mapping for all the different structures in our program. Each independent Wilson coefficient in the SMEFT Lagrangian under $U(3)^5$ finds a well-defined counterpart in the $U(2)^5$ framework, with clear relationships dictated by the Yukawa structure. This means that any constraints or correlations found in a $U(3)^5$-based SMEFT analysis can be consistently translated into the $U(2)^5$ language. In practical applications, this is particularly relevant when analyzing collider data or indirect precision measurements, as it enables a meaningful comparison between different symmetry assumptions.

The $U(3)^5$ flavor symmetry is the maximal flavor symmetry allowed by the fermionic field content of the Standard Model, while $U(2)^5$ is the corresponding subgroup acting only on the first two generations. The $U(3)^5$ symmetry allows us to implement the Minimal Flavor Violation hypothesis, which is the most restrictive consistent hypothesis we can utilize in the SMEFT to suppress non-standard contributions to flavor-violating observables. The $U(2)^5$ symmetry with minimal breaking is quite interesting since it retains most of the MFV virtues, but it allows us to have a much richer structure as far as third-generation dynamics is concerned. We have seen in this chapter different counting of these flavor symmetries. And we have done the mapping between the $U(3)^5$ with only the Yukawa of the third generation (y_t, y_b and y_τ) and $U(2)^5$ without any spurions. In the next chapter we will take about the RGE's of the Wilson coefficients in these symmetries and introduce our code.

Chapter 6

1-Loop RGE in Flavor-Symmetric SMEFT

6.1 Introduction

In the Standard Model Effective Field Theory (SMEFT), Renormalization Group Equations (RGEs) describe how the Wilson coefficients of higher-dimensional operators evolve with energy. As in section 4.4, these equations take the form

$$\dot{C}_i \equiv 16\pi^2 \mu \frac{dC_i}{d\mu} = \gamma_{ij} C_j \tag{6.1}$$

where γ_{ij} is the anomalous dimension matrix that governs how different operators mix under renormalization. At dimension-6, one-loop RGEs have been extensively studied, revealing significant contributions from gauge couplings, Yukawa interactions (especially the top quark), and operator mixing effects. This mixing means that even if certain operators are absent at a high scale, they can be induced at lower scales through running.

From refs. [4–6] we had all the RGE's calculated in the appendices A,B and C. We will not work with all the the terms in these appendices, but we will focus on terms with Yukawas. The RGEs are available and automated already, but only in the version with generic flavor indices [9, 41, 42]. So, we want here to calculate the RGEs for the symmetric SMEFT that were derived before. We will apply a truncation technique to the RGEs to ensure a consistent treatment of the flavor symmetry. This technique systematically removes terms beyond a specific order in the RGEs. In our case, we will truncate the $U(3)^5$ RGEs up to $\mathcal{O}(Y^2)$, retaining only terms up to second order in the Yukawa couplings. After truncation, we will analyze the contributions from other operators to determine their significance in the evolution of the Wilson coefficients.

6.2 RunSMEFT

In this section, we will introduce the Mathematica code. We will talk about its usage and its results.

RunSMEFT is the code done by me to find the RGEs in the $U(3)^5$ and in $U(2)^5$.

We started the code with the $U(2)^5$ symmetry, we defined the Yukawas as in eq.5.23, but we assumed no mixing in the leptonic sector, the $V_\ell = 0$ and also inside the Δ_e we don't have the mixing between the first two generations. However, we defined the coefficients structures as in sections 5.4 and 5.6, for the Bosonic structures the coefficients are simply a numerical values. In the Fermion Bilinear structure, splitting the light and the heavy indices, the coefficients are presented as 2×2 matrices since the indices (p,r) run over 1 and 2 . Moreover, the four Fermion structure are defined as $2 \times 2 \times 2 \times 2$ tensor, and (p,r,s,t) indices runs over 1 and 2. We picked up all the terms the depend explicitly on Yukawa couplings form the appendices A,B,C, and added them together to have the most general Yukawa dependent RGEs. We defined a function that truncate the series on a specific order since we are only looking up to $\mathcal{O}(V^3, \Delta^1 V^1)$ and calculated the running of the coefficients with blocks.

For $U(3)^5$, we defined the coefficients as in sections 5.3 and 5.6. Also, we had the same general Yukawa RGEs. We did truncation up to $\mathcal{O}(Y^2)$ to this RGEs.

At the end, in the same code we did the correspondence between $U(3)^5$ with only the third generation Yukawa (y_t, y_b and y_τ) and $U(2)^5$ without any spurions, in the same way done in section 5.6.

By implementing these calculations, we derived a systematic framework for computing RGEs in different symmetry scenarios, facilitating a deeper understanding of their behavior.

6.3 Truncated RGEs

The truncation of the RGEs is necessary to treat consistently the flavor symmetry. Given that SMEFT operates as an effective field theory, higher-order terms in spurion or Yukawa expansions often contribute subleading corrections, which may be negligible within the precision of our analysis. In the $U(2)^5$ framework, we truncate at $\mathcal{O}(V^3, \Delta^1 V^1)$ since higher-order terms involve additional suppressions from small mixing parameters and masses, making them less significant. Similarly, in the $U(3)^5$ case, truncation at $\mathcal{O}(Y^2)$ ensures that we capture the dominant Yukawa-dependent contributions while avoiding excessive computational complexity. This approach balances accuracy and feasibility, ensuring that our results remain analytically tractable and numerically efficient while preserving the key physical insights of the SMEFT RGE evolution.

6.3.1 $U(3)^5$ truncated RGEs

Having in mind the following key points, Class 7 except Q_{Hud}, $8(\bar{L}L)(\bar{L}L)$, $8(\bar{R}R)(\bar{R}R)$ and $8(\bar{L}L)(\bar{R}R)$ starts contracting with no Yukawa insertion, Class 5 & 6 with one insertion, Q_{Hud}, $8(\bar{L}R)(\bar{R}L)$ and $8(\bar{L}R)(\bar{L}R)$ with two insertions. We will list some of the truncated RGEs in the $U(3)^5$ assumption up to $\mathcal{O}(Y^2)$.

$$\begin{aligned}\dot{C}^{(3)}_{Hq,pr} = &-\frac{1}{2}(Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{pr} C_{H\Box} + 2Tr[N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C^{(3)}_{Hq,pr} \\ &- (2N_c C^{(3)}_{qq,prst} + 2N_c C^{(3)}_{qq,stpr} + C^{(1)}_{qq,ptsr} + C^{(1)}_{qq,srpt} - C^{(3)}_{qq,ptsr} - C^{(3)}_{qq,srpt})(Y_d^\dagger Y_d + Y_u^\dagger Y_u)_{ts} \\ &- 2C^{(3)}_{lq,stpr}(Y_e^\dagger Y_e)_{ts} + \frac{3}{2}(Y_d^\dagger Y_d - Y_u^\dagger Y_u)_{pt} C^{(1)}_{Hq,tr} + \frac{3}{2} C^{(1)}_{Hq,pt}(Y_d^\dagger Y_d - Y_u^\dagger Y_u)_{tr} \\ &+ \frac{1}{2}(Y_d^\dagger Y_d + Y_u^\dagger Y_u)_{pt} C^{(3)}_{Hq,tr} + C^{(3)}_{Hq,pt}(Y_d^\dagger Y_d + Y_u^\dagger Y_u)_{tr} + (Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{pt} C^{(3)}_{Hq,tr}\end{aligned} \tag{6.2}$$

$$\begin{aligned}\dot{C}^{(3,0)}_{Hq} &= 2Tr[N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C^{(3,0)}_{Hq} - 2C^{(3,0)}_{lq} Tr(Y_e^\dagger Y_e) + 2(C^{(1,1)}_{qq} \\ &- C^{(3,1)}_{qq} + 2N_c C^{(3,0)}_{qq}) Tr(Y_u^\dagger Y_u + Y_d^\dagger Y_d)\end{aligned} \tag{6.3}$$

$$\begin{aligned}\dot{C}^{(3,1)}_{Hq} &= -\frac{1}{2} C_{H\Box} + 2Tr[N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C^{(3,1)}_{Hq} \\ &+ 2(C^{(1,0)}_{qq} - C^{(3,0)}_{qq} + 2N_c C^{(3,1)}_{qq}) - 2C^{(3,2)}_{lq} Tr(Y_e^\dagger Y_e) - \frac{3}{2} C^{(1,0)}_{Hq} + \frac{5}{2} C^{(3,0)}_{Hq}\end{aligned} \tag{6.4}$$

$$\begin{aligned}\dot{C}^{(3,2)}_{Hq} &= -\frac{1}{2} C_{H\Box} + 2Tr[N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C^{(3,2)}_{Hq} \\ &+ 2(C^{(1,0)}_{qq} - C^{(3,0)}_{qq} + 2N_c C^{(3,1)}_{qq}) - 2C^{(3,3)}_{lq} Tr(Y_e^\dagger Y_e) - \frac{3}{2} C^{(1,0)}_{Hq} + \frac{5}{2} C^{(3,0)}_{Hq}\end{aligned} \tag{6.5}$$

Now for C_{uG} structure:

$$\begin{aligned}\dot{C}_{uG,pr} &= Tr[N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C_{uG,pr} - 4(Y_u^\dagger)_{pr} g_3 (C_{HG} + iC_{H\tilde{G}}) \\ &- 3g_3^2 c_{a,3} (Y_u^\dagger)_{pr} (C_G + iC_{\tilde{G}})\end{aligned} \tag{6.6}$$

$$\begin{aligned}\dot{C}^{(0)}_{uG} &= Tr[N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C^{(0)}_{uG} - g_3 (C_{HG} + iC_{H\tilde{G}}) \\ &- 3g_3^2 c_{a,3} (C_G + iC_{\tilde{G}})\end{aligned} \tag{6.7}$$

Now for $C^{(3)}_{qq}$ structure:

$$\begin{aligned}\dot{C}^{(3)}_{qq,prst} &= -\frac{1}{2}(Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{pr} C^{(3)}_{Hq,st} - \frac{1}{2}(Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{st} C^{(3)}_{Hq,pr} \\ &- \frac{1}{8}((Y_u^\dagger)_{pv}(Y_u)_{wt} C^{(8)}_{qu,srvw} + (Y_u^\dagger)_{sv}(Y_u)_{wr} C^{(8)}_{qu,ptvw}) \\ &- \frac{1}{8}((Y_d^\dagger)_{pv}(Y_d)_{wt} C^{(8)}_{qd,srvw} + (Y_d^\dagger)_{sv}(Y_d)_{wr} C^{(8)}_{qd,ptvw}) \\ &+ \frac{1}{2}(Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{pv} C^{(3)}_{qq,vrst} + \frac{1}{2}(Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{sv} C^{(3)}_{qq,prvt} \\ &+ \frac{1}{2} C^{(3)}_{qq,pvst}(Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{vr} + \frac{1}{2} C^{(3)}_{qq,prsv}(Y_u^\dagger Y_u + Y_d^\dagger Y_d)_{vt}\end{aligned} \tag{6.8}$$

$$\dot{C}^{(3,0)}_{qq} = 0 \tag{6.9}$$

$$\dot{C}^{(3,2)}_{qq} = 0 \tag{6.10}$$

$$\dot{C}^{(3,2)}_{qq} = 2C^{(3,0)}_{qq} + C^{(3,0)}_{Hq} \tag{6.11}$$

$$\dot{C}^{(3,3)}_{qq} = 2C^{(3,0)}_{qq} + C^{(3,0)}_{Hq} \tag{6.12}$$

$$\dot{C}^{(3,4)}_{qq} = \frac{5}{2} C^{(3,1)}_{qq} - \frac{1}{4} C^{(8,0)}_{qu} - \frac{1}{4} C^{(8,0)}_{qd} \tag{6.13}$$

$$\dot{C}^{(3,4)}_{qq} = \frac{5}{2} C^{(3,1)}_{qq} - \frac{1}{4} C^{(8,0)}_{qu} - \frac{1}{4} C^{(8,0)}_{qd} \tag{6.14}$$

For $C_{Hud,pr}$

$$\begin{aligned}\dot{C}_{Hud,pr} &= (Y_u Y_d^\dagger)_{pr}(2C_{H\Box} - C_{HD}) + 2Tr[N_c Y_u Y_u^\dagger + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C_{Hud,pr} \\ &+ 4(C^{(1)}_{ud,ptsr} + c_{f,3} C^{(8)}_{ud,ptsr})(Y_u Y_d^\dagger)_{ts} - 2(Y_u Y_d^\dagger)_{pt} C_{Hd,tr} + 2C_{Hu,pt}(Y_u Y_d^\dagger)\end{aligned} \tag{6.15}$$

$$\begin{aligned}\dot{C}^{(0)}_{Hud} &= 2C_{H\Box} - C_{HD} + 2Tr[N_c Y_u Y_u^\dagger + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e] C^{(0)}_{Hud} \\ &+ 4(C^{(1,0)}_{ud} + c_{f,3} C^{(8,0)}_{ud}) - 2C^{(0)}_{Hd} + 2C^{(0)}_{Hu}\end{aligned} \tag{6.16}$$

where $c_{a,3} = \frac{(N_c^2-1)}{2N_c}$ and $c_{f,3} = N_c$ are SU(3) quadratic Casimirs in adjoint and fundamental representation, respectively, and N_c is the number of colors.

The equations above only include contributions to the RGE that contain Yukawa couplings. To obtain the complete expressions one would need to add all the terms that are independent of the Yukawas, that are reported in the Appendices.

6.3.2 $U(2)^5$ truncated RGEs

We will list also the truncated RGEs in $U(2)^5$. They will be in blocks, then we will give an example of a block that was divided. Using our notations, we will have the following:

Starting with $C_{Hq}^{(3)}$ structures:

$$\begin{aligned}
\dot{C}_{qq}^{(3)} &= -2(C_{qq_3q_3q}^{(1,1)} - C_{qq_3q_3q}^{(1,3)} + 2N_cC_{qqq_3q_3}^{(1)})(y_b^2+y_t^2) + \frac{3}{2}C_{qq_3}^{(1)}V_qV_q^\dagger(x_by_b^2 - x_ty_t^2) \\
&+ C_{qq_3}^{(3)}V_qV_q^\dagger(x_by_b^2+x_ty_t^2) - 2(C_{qqqq_3}^{(1,1)} + C_{qqqq_3}^{(1,2)} - C_{qqqq_3}^{(3,2)} + 2N_cC_{qqqq_3}^{(3,2)} \\
&+ C_{qqqq_3}^{(3,1)}(-1+2N_c))V_qV_q^\dagger(x_by_b^2+x_ty_t^2) + 3C_{qq}^{(1,1)}V_qV_q^\dagger(x_b^2y_b^2 - x_t^2y_t^2) \\
&+ 2C_{qq}^{(3,1)}V_qV_q^\dagger(x_b^2y_b^2+x_t^2y_t^2) - \frac{1}{2}C_{H\Box}V_qV_q^\dagger(x_b^2y_b^2+x_t^2y_t^2) \\
&+ 2C_{qq}^{(3,1)}N_cV_qV_q^\dagger(x_b^2y_b^2+x_t^2y_t^2) - 2(C_{qqqq}^{(1,1)} + C_{qqqq}^{(1,2)} - C_{qqqq}^{(3,2)} + 2C_{qqqq}^{(3,2)}N_c \\
&+ C_{qqqq}^{(3,1)}(-1+2N_c))V_qV_q^\dagger(x_b^2y_b^2+x_t^2y_t^2) - 2C_{\ell_3\ell_3qq}^{(3,1)}\tau^2 + 2C_{qq}^{(3,1)}(N_c(y_b^2+y_t^2)+y_\tau^2) \\
&+ \frac{3}{2}V_qV_q^\dagger(x_by_b^2 - x_ty_t^2)C_{qq_3}^{\star(1)} - 2V_qV_q^\dagger(x_by_b^2+x_ty_t^2)(C_{qqqq_3}^{\star(1,1)} + (-1+2N_c)C_{qqqq_3}^{\star(1,1)} \\
&+ C_{qqqq_3}^{\star(1,2)} - C_{qqqq_3}^{\star(3,2)} + 2N_cC_{qqqq_3}^{\star(3,2)}) + V_qV_q^\dagger(x_by_b^2+x_ty_t^2)C_{qq_3}^{\star(3)} - 2C_{\ell_3\ell_3qq}^{(3,2)}y_\tau^2V_qV_q^\dagger \\
&+ 2C_{qq}^{(3,2)}(N_c(y_b^2+y_t^2)+y_\tau^2)V_qV_q^\dagger + (y_b^2+y_t^2)(-2C_{qq_3q_3q}^{(1,2)} + 2C_{qq_3q_3q}^{(3,2)} \\
&- 4C_{qqq_3q_3}^{(3,2)}N_cV_q.V_q^\dagger)
\end{aligned} \tag{6.17}$$

$$\begin{aligned}
\dot{C}_{qq_3}^{(3)} &= \frac{1}{2}(3C_{qq_3}^{(1)}(y_b^2-y_t^2) + 2C_{qq_3}^{(3)}(y_b^2+y_t^2) - 4(C_{qq_3q_3q_3}^{(1)} + C_{qq_3q_3q_3}^{(3)}(-1+2N_c))(y_b^2+y_t^2) \\
&+ 3C_{qq}^{(1,1)}(x_by_b^2 - x_ty_t^2) + 3C_{q_3q_3}^{(1)}(x_by_b^2-x_ty_t^2) + 2C_{q_3q_3}^{(3,1)}(x_by_b^2+x_ty_t^2) \\
&+ 2C_{q_3q_3}^{(3)}(x_by_b^2+x_ty_t^2) - C_{H\Box}(x_by_b^2+x_ty_t^2) - 4(C_{qqq_3q_3}^{(1,1)} - C_{qqq_3q_3}^{(3,1)} + 2C_{qq_3q_3q}^{(3,1)}N_c)(x_by_b^2+x_ty_t^2) \\
&+ 3C_{qq_3}^{(1)}V_qV_q^\dagger(x_b^2y_b^2 - x_t^2y_t^2) + 2C_{qq_3}^{(3)}V_qV_q^\dagger(x_b^2y_b^2+x_t^2y_t^2) + 4C_{qq_3}^{(3)}N_cV_qV_q^\dagger(x_b^2y_b^2+x_t^2y_t^2) \\
&+ 4(C_{qqqq_3}^{(1,1)} + C_{qqqq_3}^{(1,2)}(-1+2N_c))V_qV_q^\dagger(x_b^2y_b^2+x_t^2y_t^2) - 4C_{\ell_3\ell_3qq_3}^{(3,1)}y_\tau^2 + 4C_{qq_3}^{(3)}(N_c(y_b^2+y_t^2)+y_\tau^2) \\
&- 4(C_{qq_3qq_3}^{(1)} + C_{qq_3qq_3}^{(3)}(-1+2N_c))V_qV_q^\dagger(x_by_b^2+x_ty_t^2) + 3C_{qq}^{(1,2)}(x_by_b^2-x_ty_t^2)V_qV_q^\dagger \\
&+ C_{qq}^{(3,2)}(x_by_b^2+x_ty_t^2)V_qV_q^\dagger - 4(x_by_b^2+x_ty_t^2)(C_{qqq_3q_3}^{(1,2)} - C_{qqq_3q_3}^{(3,2)} + 2C_{qq_3q_3q}^{(3,2)}N_c)V_qV_q^\dagger)
\end{aligned} \tag{6.18}$$

$$\dot{C}_{q_3q}^{(3)} = \dot{C}_{qq_3}^{\star(3)} \tag{6.19}$$

$$
\begin{aligned}
\dot{C}^{(3)}_{q_3q_3} &= 3C^{(1)}_{q_3q_3}(y_b^2 - y_t^2) + 2C^{(3)}_{q_3q_3}(y_b^2 + y_t^2) - \frac{1}{2}C_{H\Box}(y_b^2 + y_t^2) - 2(C^{(1)}_{q_3q_3q_3q_3} + C^{(3)}_{q_3q_3q_3q_3} \\
&\quad (-1 + 2N_c))(y_b^2 + y_t^2) + \frac{3}{2}C^{(1)}_{qq_3}V_qV_q^\dagger(x_by_b62 - x_ty_t^2) + C^{(3)}_{qq_3}V_qV_q^\dagger(x_by_b^2 + x_ty_t^2) \\
&\quad - 2(C^{(1)}_{qq_3q_3q_3} + C^{(3)}_{qq_3q_3q_3}(-1 + 2N_c))V_qV_q^\dagger(x_by_b^2 + x_ty_t^2) + 2C^{(3)}_{q_3q_3}N_cV_qV_q^\dagger(x_b^2y_b^2 + x_t^2y_t^2) \\
&\quad - 2(C^{(1,1)}_{qq_3q_3q} - C^{(3,1)}_{qq_3q_3q} + 2C^{(3,1)}_{qqq_3q_3}N_c)V_qV_q^\dagger(x_b62y_b^2 + x_t^2y_t^2) - 2C^{(3)}_{\ell_3\ell_3q_3q_3}y_\tau^2 \\
&\quad + 2C^{(3)}_{q_3q_3}(N_c(y_b^2 + y_t^2) + y_\tau^2) + \frac{3}{2}V_qV_q^\dagger(x_by_b^2 - x_ty_t^2)C^{\star(1)}_{qq_3} + V_qV_q^\dagger(x_by_b^2 + x_ty_t^2)C^{\star(3)}_{qq_3} \\
&\quad + V_qV_q^\dagger(x_by_b^2 + x_ty_t^2)(-2C^{\star(1)}_{qq_3q_3q_3} - 2(-1 + 2N_c)C^{\star(3)}_{qq_3q_3q_3})
\end{aligned}
\tag{6.20}
$$

Now for the $C^{(3)}_{qq}$ structure: Also here there are blocks,

$$
\begin{aligned}
\dot{C}^{(3)}_{qqqq} &= \frac{1}{8}V_q(-2C^{(8,1)}_{qqd_3d_3}V_q^\dagger x_b^2y_b^2 - 2C^{(8,1)}_{qqu_3u_3}V_q^\dagger x_t^2y_t^2 + 8(C^{(3,1)}_{qqqq_3} + C^{(3,2)}_{qqqq_3})V_q^\dagger(x_by_b^2 + x_ty_t^2) \\
&\quad - 8C^{(3,1)}_{qq}V_q^\dagger(x_b^2y_b^2 + x_t^2y_t^2) + 16(C^{(3,1)}_{qqqq} + C^{(3,2)}_{qqqq})V_q^\dagger(x_b^2y_b^2 + x_t^2y_t^2) \\
&\quad + 8V_q^\dagger(x_by_b^2 + x_ty_t^2)(C^{\star(3,1)}_{qqqq_3} + C^{\star(3,2)}_{qqqq_3}))
\end{aligned}
\tag{6.21}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{qqqq_3} &= \frac{1}{16}(-2C^{(8,1)}_{qqd_3d_3}V_qx_by_b^2 - 2C^{(8,1)}_{qqu_3u_3}V_qx_ty_t^2 + 8(C^{(3,2)}_{qqqq_3} + C^{(3,1)}_{qqqq_3})V_q(y_b^2 + y_t^2) \\
&\quad - 8C^{(3,1)}_{qq}V_q(x_by_b^2 + x_ty_t^2) + 8C^{(3,1)}_{qq_3q_3q}V_q(x_by_b^2 + x_ty_t^2) + 8C^{(3,1)}_{qqq_3q_3}(x_by_b^2 + x_ty_t^2) \\
&\quad + 8(C^{(3,1)}_{qqqq} + C^{(3,2)}_{qqqq})V_q(x_by_b^2 + x_ty_t^2) - 8C^{(3)}_{qq_3}V_q^2V_q^\dagger(x_b^2y_b^2 + x_t^2y_t^2) \\
&\quad + 24(C^{(3,2)}_{qqqq_3} + C^{(3,1)}_{qqqq_3})V_q^2V_q^\dagger(x_b^2y_b^2 + x_t^2y_t^2) + 2V_q^2V_q^\dagger x_bx_ty_by_tC^{\star(1)}_{q_3u_3qd_3} \\
&\quad + 2V_q^2V_q^\dagger x_bx_ty_by_tC^{\star(1)}_{qu_3q_3d_3} + V_q^2V_q^\dagger x_bx_ty_by_tC^{(8)}_{q_3u_3qd_3} - V_q^2V_q^\dagger x_bx_ty_by_tC^{\star(8)}_{qu_3q_3d_3} \\
&\quad - \frac{1}{N_c}V_q^2V_q^\dagger x_bx_ty_by_t(C^{\star(8)}_{q_3u_3qd_3} + C^{\star(8)}_{qu_3q_3d_3}) + 2C^{(1)}_{qu_3qd_3}V_q^\dagger V_q^2x_by_by_t \\
&\quad - C^{(8)}_{qu_3qd_3}V_q^\dagger V_q^2x_by_by_t + 2C^{(1)}_{qu_3qd_3}V_q^\dagger V_q^2x_ty_by_t - C^{(8)}_{qu_3qd_3}V_q^\dagger V_q^2x_ty_by_t \\
&\quad - \frac{1}{N_c}C^{(8)}_{qu_3qd_3}V_q^\dagger V_q^2(x_b + x_t)y_by_t + 8C^{(3)}_{qq_3qq_3}V_q^\dagger V_q^2(x_by_b^2 + x_ty_t^2) \\
&\quad - 8C^{(3,2)}_{qq}V_q^2V_q^\dagger(x - by_b^2 + x_ty_t^2) + 8C^{(3,2)}_{qqq_3q_3}V_q^2V_q^\dagger(x_by_b^2 + x_ty_t^2) \\
&\quad + 8(C^{(3,3)}_{qqqq} + C^{(3,4)}_{qqqq})V_q^12V_q^\dagger(x_by_b^2 + x_ty_t^2) + 2V_q^2V_q^\dagger(C^{(8)}_{qq_3d_3d_3}x_b + C^{(8,2)}_{qqu_3u_3}) \\
&\quad + 2V_q^2V_q^\dagger(C^{(8)}_{qq_3u_3u_3}x_t + C^{(8,2)}_{qqu_3u_3}) + 8C^{(3,2)}_{qq_3q_3q}V_q^2V_q^\dagger(x_by_b^2 + x_ty_t^2) \\
&\quad + 8C^{(3,3)}_{qqqq_3}(y_b^2 + y_t^2)V_q^2V_q^\dagger)
\end{aligned}
\tag{6.22}
$$

$$\dot{C}^{(3)}_{qqq_3q} = \dot{C}^{\star(3)}_{qqqq_3} \tag{6.23}$$

$$\begin{aligned}
\dot{C}^{(3)}_{qqq_3q_3} = &\frac{1}{16}(2C^{(1)}_{q_3u_3qd_3}V_qV_q^\dagger x_by_by_t - C^{(8)}_{qu_3q_3d_3}V_qV_q^\dagger x_by_by_t + 2C^{(1)}_{qu_3q_3d_3}V_qV_q^\dagger x_ty_by_t \\
&- \frac{1}{N_c}V_qV_q^\dagger(C^{(8)}_{q_3u_3qd_3}x_b + C^{(8)}_{qu_3q_3d_3}y_by_t) - 8C^{(3,1)}_{qq}(y_b^2 + y_t^2) + 16C^{(3,1)}_{qqq_3q_3}(y_b^2 + y_t^2) \\
&+ 8C^{(3)}_{qq_3q_3q_3}V_qV_q^\dagger(x_by_b^2 + x_ty_t^2) + 8(C^{(3,2)}_{qqqq_3} + C^{(3,1)}_{qqqq_3})V_qV_q^\dagger(x_by_b^2 + x_ty_t^2) \\
&+ 16C^{(3,1)}_{qqq_3q_3}V_qV_q^\dagger(x_b^2y_b^2 + x_t^2y_t^2) - 8C^{(3)}_{q_3q_3}V_qV_q^\dagger(x_b^2y_b^2 + x_t^2y_t^2) \\
&+ 8V_qV_q^\dagger(x_by_b62 + x_ty_t^2)(C^{\star(3,1)}_{qqqq_3} + C^{\star(3,2)}_{qqqq_3}) - 2V_qV_q^\dagger x_by_b^2(C^{(8)}_{qq_3d_3d_3} + C^{\star(8)}_{qq_3d_3d_3}) \\
&+ 2V_qV_q^\dagger x_ty_t^2(C^{(8)}_{qq_3u_3u_3} + C^{\star(8)}_{qq_3u_3u_3}) + 2V_qV_q^\dagger x_by_by_tC^{\star(1)}_{q_3u_3qd_3} + 2V_qV_q^\dagger x_ty_by_tC^{(1)}_{qu_3q_3d_3} \\
&- V_qV_q^\dagger x_ty_by_tC^{(8)}_{q_3u_3qd_3} - V_qV_q^\dagger x_ty_by_tC^{(8)}_{qu_3q_3d_3} - \frac{1}{N_c}V_qV_q^\dagger y_by_t(x_bC^{\star(8)}_{q_3u_3qd_3} + x_tC^{\star(8)}_{qu_3q_3d_3}) \\
&+ 8V_qV_q^\dagger(x_by_b^2 + x_ty_t^2) + 8V_qV_q^\dagger(x_by_b^2 + x_ty_t^2)C^{\star(3)}_{qq_3q_3q_3} - 8C^{(3,2)}_{qq}(y_b^2 + y_t^2)V_qV_q^\dagger \\
&+ 16C^{(3,2)}_{qqq_3q_3}(y_b^2 + y_t^2)V_qV_q^\dagger)
\end{aligned} \tag{6.24}$$

$$\dot{C}^{(3)}_{qq_3qq} = \dot{C}^{(3)}_{qqqq_3} \tag{6.25}$$

$$\begin{aligned}
\dot{C}^{(3)}_{qq_3qq_3} = &\frac{1}{8}(-2C^{(8)}_{qq_3d_3d_3}x_by_b^2 - 2C^{(8)}_{qq_3u_3u_3}x_ty_t^2 + 4(C^{(3,1)}_{qqqq_3} + C^{(3,2)}_{qqqq_3})(x_by_b^2 + x_ty_t^2) \\
&- 8C^{(3)}_{qq_3}(x_by_b^2 + x_ty_t^2) + 8C^{(3)}_{qq_3q_3q_3}(x_by_b^2 + x_ty_t^2) + 4(C^{(3,2)}_{qqqq_3} + C^{(3,1)}_{qqqq_3})(x_by_b^2 + x_ty_t^2) \\
&+ 2x_bx_ty_by_tC^{\star(1)}_{q_3u_3q_3d_3} - x_bx_ty_by_tC^{\star(1)}_{q_3u_3q_3d_3} - \frac{1}{N_c}x_bx_ty_by_tC^{\star(8)}_{q_3u_3q_3d_3} + 2C^{(1)}_{qu_3qd_3}y_by_t \\
&- C^{(8)}_{qu_3qd_3}y_by_t - \frac{1}{N_c}C^{(8)}_{qu_3qd_3}y_t + 8C^{(3)}_{qq_3qq_3}(y_b^2 + y_t^2))
\end{aligned} \tag{6.26}$$

$$
\begin{aligned}
\dot{C}^{(3)}_{qq_3q_3q} = \frac{1}{16}(&-2C^{(8,1)}_{qqd_3d_3}y_b^2 + 2C^{(1)}_{qu_3q_3d_3}V_qV_q^\dagger x_by_by_t - C^{(8)}_{q_3u_3qd_3}V_qV_q^\dagger x_by_by_t \\
&+ 2C^{(1)}_{q_3u_3qd_3}V_qV_q^\dagger x_ty_by_t - C^{(8)}_{qu_3q_3d_3}V_qV_q^\dagger x_ty_by_t - \frac{1}{N_c}V_qV_q^\dagger(C^{(8)}_{qu_3q_3d_3}x_b \\
&+ C^{(8)}_{q_3u_3qd_3}x_t)y_by_t - 2C^{(8,1)}_{qqu_3u_3}y_t^2 + 16C^{(3,1)}_{qq_3q_3q}(y_b^2 + y_t^2) + 8(C^{(3,1)}_{qqqq_3} \\
&+ C^{(3,2)}_{qqqq_3})V_qV_q^\dagger(x_by_b^2 + x_ty_t^2) - 8C^{(3)}_{qq_3}V_qV_q^\dagger(x_by_b^2 + x_tty_t^2) + 8C^{(3)}_{qq_3q_3q_3}V_qV_q^\dagger(x_by_b^2 \\
&+ x_ty_t^2) + 16C^{(3,1)}_{qq_3q_3q}V_qV_q^\dagger(x_b^2y_b^2 + x_t62y_t^2) + 8V_qv_q^\dagger(x_by_b^2 + x_ty_t^2)(C^{\star(3,1)}_{qqqq_3} \\
&+ C^{\star(3,2)}_{qqqq_3}) - 8V_qV_q^\dagger(x_by_b^2 + x_ty_t^2)C^{(3)}_{qq_3} + 8V_qV_q^\dagger(x_by_b^2 + x_ty_t^2)C^{\star(3)}_{qq_3q_3q_3} \\
&+ 2V_qV_q^\dagger x_ty_by_tC^{\star(1)}_{q_3u_3qd_3} + 2V_qV_q^\dagger C^{\star(1)}_{qu_3q_3d_3} - V_qV_q^\dagger x_by_by_tC^{\star(8)}_{q_3u_3qd_3} - V_qV_q^\dagger x_by_by_tC^{\star(8)}_{qu_3q_3d_3} \\
&- \frac{1}{N_c}V_qV_q^\dagger y_by_t(x_tC^{\star(8)}_{q_3u_3qd_3} + x_bC^{\star(8)}_{qu_3q_3d_3}) + 16C^{(3,2)}_{qq_3q_3q}(y_b^2 + y_t^2)V_qV_q^\dagger \\
&- 2y_b^2(C^{(8)}_{q_3q_3d_3d_3}V_qV_q^\dagger x_b^2 + C^{(8,2)}_{qqd_3d_3}V_qV_q^\dagger) - 2y_t^2(C^{(8)}_{q_3q_3u_3u_3}V_qV_q^\dagger x_t^2 + C^{(8,2)}_{qqu_3u_3}V_qV_q^\dagger))
\end{aligned}
\tag{6.27}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{qq_3q_3q_3} = \frac{1}{16N_c}(&-8C^{(3)}_{qq_3}N_cy_b^2 + 24C^{(3)}_{qq_3q_3q_3}N_cy_b^2 - 2C^{(8)}_{qq_3d_3d_3}N_cy_b^2 + 8C^{(3,1)}_{qq_3q_3q}N_cx_by_b^2 \\
&+ 8C^{(3,1)}_{qqq_3q_3}N_cx_by_b^2 - 8C^{(3)}_{q_3q_3}N_cx_by_b^2 - 8C^{(3)}_{q_3q_3q_3q_3}N_cx_by_b^2 - 2C^{(8)}_{q_3q_3d_3d_3}N_cx_by_b^2 \\
&+ 8C^{(3)}_{qq_3q_3q_3}N_cV_qV_q^\dagger x_b^2y_b^2 - C^{(8)}_{q_3u_3qd_3}y_by_t - C^{(8)}_{qu_3q_3d_3}y_by_t + 2C^{(1)}_{q_3u_3qd_3}N_cy_by_t \\
&+ 2C^{(1)}_{qu_3q_3d_3}N_cy_by_t - C^{(8)}_{q_3u_3qd_3}N_cy_by_t - C^{(8)}_{qu_3q_3d_3}N_cy_by_t + 8C^{(3)}_{qq_3}N_cy_t^2 \\
&+ 24C^{(3)}_{qq_3q_3q_3}N_cy_t^2 + 2C^{(8)}_{qq_3u_3u_3}N_cy_t^2 + 8C^{(3,1)}_{qq_3q_3q}N_cx_ty_t^2 - 8C^{(3,1)}_{qqq_3q_3}N_cx_ty_t^2 \\
&- 8C^{(3)}_{q_3q_3}N_cx_ty_t^2 + 8C^{(3)}_{q_3q_3q_3q_3}N_cx_ty_t^2 - 2C^{(8)}_{q_3q_3u_3u_3}N_cx_ty_t^2 + 8C^{(3)}_{qq_3q_3q_3}N_cV_qv_q^\dagger x_t^2y_t^2 \\
&+ 2N_c(x_b + x_t)y_by_tC^{\star(1)}_{q_3u_3q_3d_3} - (1 + N_c)(x_b + x_t)y_by_tC^{\star(8)}_{q_3u_3q_3d_3} \\
&+ 8C^{(3)}_{qq_3qq_3}N_cV_qV_q^\dagger x_by_b^2 + 8C^{(3)}_{qq_3qq_3}N_cV_qV_q^\dagger x_ty_t^2 + 8C^{(3,2)}_{qq_3q_3q}N_cV_qV_q^\dagger x_by_b^2 \\
&+ 8C^{(3,2)}_{qqq_3q_3}N_cV_qV_q^\dagger x_by_b^2 + 8C^{(3,2)}_{qq_3q_3q}N_cV_qV_q^\dagger x_ty_t^2 + 8C^{(3,2)}_{qqq_3q_3}N_cV_qV_q^\dagger x_ty_t^2)
\end{aligned}
\tag{6.28}
$$

$$\dot{C}^{(3)}_{q_3qqq} = \dot{C}^{\star(3)}_{qqqq_3} \tag{6.29}$$

$$\dot{C}^{(3)}_{q_3qqq_3} = \dot{C}^{(3)}_{qq_3q_3q} \tag{6.30}$$

$$\dot{C}^{(3)}_{q_3qq_3q} = \dot{C}^{\star(3)}_{qq_3qq} \tag{6.31}$$

$$\dot{C}^{(3)}_{q_3qq_3q_3} = \dot{C}^{(3)}_{qq_3q_3q_3} \tag{6.32}$$

$$\dot{C}^{(3)}_{q_3q_3qq} = \dot{C}^{(3)}_{qqq_3q_3} \tag{6.33}$$

$$\dot{C}^{(3)}_{q_3q_3qq_3} = \dot{C}^{(3)}_{qq_3q_3q_3} \tag{6.34}$$

$$\dot{C}^{(3)}_{q_3q_3q_3q} = \dot{C}^{\star(3)}_{qq_3q_3q_3} \tag{6.35}$$

$$\begin{aligned}
\dot{C}^{(3)}_{q_3q_3q_3q_3} &= \frac{1}{8}(-8C^{(3)}_{q_3q_3}y_b^2 + 16C^{(3)}_{q_3q_3q_3q_3}y_b^2 - 2C^{(8)}_{q_3q_3d_3d_3}y_b^2 + 8C^{(3)}_{qq_3q_3q_3}V_qV_q^\dagger x_by_b^2 \\
&+ 2C^{(1)}_{q_3u_3q_3d_3}y_by_t - C^{(8)}_{q_3u_3q_3d_3}y_by_t - \frac{1}{N_c}C^{(8)}_{q_3u_3q_3d_3}y_by_t - 8C^{(3)}_{q_3q_3}y_t^2 \\
&+ 16C^{(3)}_{q_3q_3q_3q_3}y_t^2 - 2C^{(8)}_{q_3q_3u_3u_3}y_t^2 + 8C^{(3)}_{qq_3q_3q_3}V_qV_q^\dagger x_ty_t^2 + 2y_by_tC^{\star(1)}_{q_3u_3q_3d_3} \\
&+ 4V_qV_q^\dagger(x_by_b^2 + x_ty_t^2)C^{\star(3)}_{qq_3q_3q_3} - y_by_tC^{\star(8)}_{q_3u_3q_3d_3} - \frac{1}{N_c}y_by_tC^{\star(8)}_{q_3u_3q_3d_3} \\
&+ 4V_qV_q^\dagger x_by_b^2C^{\star(3)}_{qq_3q_3q_3} + 4V_qV_q^\dagger x_ty_t^2C^{\star(3)}_{qq_3q_3q_3})
\end{aligned} \tag{6.36}$$

Now for C_{uG} structure:

$$\begin{aligned}
\dot{C}_{quG} &= -C^{(1)}_{q_3uqd_3}g_3y_b + \frac{1}{N_c}C^{(8)}_{q_3uqd_3}g_3y_b - g_3(4C_{HG} + 4iC_{H\tilde{G}} + 3C_{a,3}(C_G + iC_{\tilde{G}})g_3)y_t \\
&+ C_{quG}(N_c(y_b^2 + y_t^2) + y_\tau^2)
\end{aligned} \tag{6.37}$$

$$\begin{aligned}
\dot{C}_{qu_3G} &= \frac{1}{2N_c}(2C^{(8)}_{q_3u_3qd_3}g_3y_b + N_c(-2C^{(1)}_{q_3u_3qd_3}g_3y_b - 3C_{q_3u_3G}x_By_b^2 - 8C_{HG}g_3x_ty_t \\
&- 8iC_{H\tilde{G}}g_3x_ty_t - 6c_{a,3}C_Gg_3^2x_ty_t - 6ic_{a,2}C_{\tilde{G}}g_3^2x_ty_t - 2C_{qd_3G}y_by_t \\
&- 2C_{qd_3G}V_qV_q^\dagger x_bx_ty_by_t + 5C_{q_3u_3G}x_ty_t^2 + C_{qu_3G}(-3V_qV_q^\dagger x_b^2y_b^2 + 4y_t^2 + 9V_qV_q^\dagger x_t^2y_t^2 \\
&+ 2N_c(y_b^2 + V_qV_q^\dagger x_b^2y_b^2 + y_t^2 + V_qV_q^\dagger x_t^2y_t^2) + 2y_\tau^2)) + 2g_3(C^{(8)}_{qu_3qd_3} - C^{(1)}_{qu_3qd_3}N_c)V_q^\dagger x_by_bV_q)
\end{aligned} \tag{6.38}$$

$$\begin{aligned}
\dot{C}_{q_3uG} &= \frac{1}{2N_c}(2C^{(8)}_{quq_3d_3}g_3x_by_b + N_c(-2C^{(1)}_{quq_3d_3}g_3x_by_b - 3C_{quG}x_by_b^2 - 2C_{q_3d_3G}x_bY_by_t \\
&+ 4C_{q_3u_3}x_ty_t^2 + 5C_{quG}x_ty_t^2) + 2C^{(8)}_{q_3uq_3d_3}g_3y_b - 2C^{(1)}_{q_3uq_3d_3}g_3N_cy_b \\
&+ C_{q_3uG}N_c((-3 + 2N_c)y_b^2 + (5 + 2N_c)y_t^2 + 2y_\tau^2))
\end{aligned} \tag{6.39}$$

$$\begin{aligned}
\dot{C}_{q_3u_3G} &= -C^{(1)}_{q_3u_3q_3d_3}g_3y_b + \frac{1}{N_c}C^{(8)}_{q_3u_3q_3d_3} - C^{(1)}_{qu_3q_3d_3}g_3V_qV_q^\dagger x_by_b + \frac{1}{N_c}C^{(8)}_{qu_3q_3d_3}g_3V_qV_q^\dagger x_by_b \\
&- \frac{3}{2}C_{q_3u_3G}y_b^2 + C_{q_3u_3G}N_cy_b^2 - \frac{3}{2}C_{qu_3G}V_qV_q^\dagger x_by_b^2 + C_{q_3u_3G}N_cV - qV_q^\dagger x_b^2y_b^2 \\
&- 4C_{HG}g_3y_t - 4iC_{H\tilde{G}}g_3y_t - 3c_{a,3}C_Gg_3^2y_t - 3ic_{a,3}C_{\tilde{G}}g_3^2y_t - C_{q_3d_3G}y_by_t \\
&- C_{q_3d_3G}V_qV_q^\dagger x_bx_ty_by_t + \frac{9}{2}C_{q_3u_3G}y_t^2 + C_{q_3u_3}N_cy_t^2 + \frac{5}{2}C_{qu_3G}V_qV_q^\dagger x_ty_t^2 \\
&+ 2C_{q_3u_3G}V_qV_q^\dagger x_t^2y_t^2 + C_{q_3u_3G}N_cV_qV_q^\dagger x_t^2y_t^2 + C_{q_3u_3G}y_\tau^2
\end{aligned} \tag{6.40}$$

Now for C_{Hud} strucutre:

$$\dot{C}_{ud} = 0 \tag{6.41}$$

$$\begin{aligned}\dot{C}_{ud_3} &= y_t(4C^{(1)}_{uud_3d_3}x_by_b - 2C_{d_3d_3}x_by_b + 4C^{(8)}_{uud_3d_3}c_{f,3}x_by_b + 2C_{H\Box}x_by_b - C_{HD}x_by_b \\ &+ 2C_{uu}x_by_b + 3C_{u_3d_3}x_ty_t) + C_{ud_3}(3+2N_c)y_b^2 + 2(2C^{(1)}_{uu_3d_3d_3} + 2C^{(8)}_{uu_3d_3d_3}c_{f,3} \\ &+ C_{uu_3})y_by_t + 2C_{ud_3}(N_cy_t^2 + y_\tau^2)\end{aligned} \tag{6.42}$$

$$\begin{aligned}\dot{C}_{u_3d} &= y_b(3C_{u_3d_3}x_by_b + (4C^{(1)}_{u_3u_3dd} - 2C_{dd} + 4C^{(8)}_{u_3u_3dd}c_{f,3} + 2C_{H\Box} - C_{HD} + 2C_{u_3u_3})x_ty_t) \\ &+ 2C_{u_3d}N_cy_b^2 + 3C_{u_3d}y_t^2 + 2C_{u_3d}N_cy_t^2 + 2C_{u_3d}y_\tau^2 + 4y_by_tC^{\star(1)}_{u_3u_3dd_3} + 4c_{f,3}y_by_tC^{(8)}_{u_3u_3dd_3} \\ &- 2y_by_tC^{\star}_{dd_3}\end{aligned} \tag{6.43}$$

$$\begin{aligned}\dot{C}_{u_3d_3} &= (4C^{(1)}_{u_3u_3d_3d_3} - 2C_{d_3d_3} + 4C^{(8)}_{u_3u_3d_3d_3}c_{f,3} + 2C_{H\Box} - C_{HD} + 2C_{u_3u_3})(1 + V_qV_q^\dagger x_bx_t)y_by_t \\ &+ C_{u_3d_3}((3+2N_c)(1+V_qV_q^\dagger)x_b^2y_b^2 + (3+2N_c)(1+V_qV_q^\dagger x_t^2)y_t^2 + 2y_\tau^2)\end{aligned} \tag{6.44}$$

All these calculations were done with blocks. However, for example eq.6.17, is the running of two coefficients. In the code we did an example, and it was on this exact block in order to separate them, and we were able to find the following results

$$\begin{aligned}\dot{C}^{(3,1)}_{qq} &= -2((C^{(1,1)}_{qq_3q_3q} - C^{(3,1)}_{qq_3q_3q} + 2C^{(3,1)}_{qqq_3q_3}N_c - C^{(3,1)}_{qq}N_c)(y_b^2 + y_t^2) + (C^{(1,2)}_{qqqq_3}x_by_b^2 \\ &- C^{(3,2)}_{qqqq_3}x_by_b^2 + 2C^{(3,1)}_{qqqq_3}N_cx_by_b^2 + C^{(1,2)}_{qqqq}x_b^2y_b^2 - C^{(3,2)}_{qqqq}x_b^2y_b^2 + 2C^{(3,1)}_{qqqq}N_cx_b^2y_b^2 \\ &- C^{(3,1)}_{qq}N_cx_b^2y_b^2 + C^{(1,2)}_{qqqq_3}x_ty_t^2 + C^{(3,2)}_{qqqq_3}x_ty_t^2 + 2C^{(3,1)}_{qqqq_3}N_cx_ty_t^2 + C^{(1,2)}_{qqqq}x_t^2y_t^2 \\ &- C^{(3,2)}_{qqqq}x_t^2y_t^2 + 2C^{(3,1)}_{qqqq}N_cx_t^2y_t^2 - C^{(3,1)}_{qq}N_cx_t^2y_t^2 + 2N_c(x_by_b^2 + x_ty_t^2)C^{\star(3,1)}_{qqqq_3} \\ &+ x_by_b^2C^{\star(1,2)}_{qqqq_3} + x_ty_t^2C^{\star(1,2)}_{qqqq_3} - x_by_b^2C^{\star(3,2)}_{qqqq_3} - x_ty_t^2C^{\star(3,2)}_{qqqq_3})V_q^\dagger V_q)\end{aligned} \tag{6.45}$$

$$\begin{aligned}\dot{C}^{3,2}_{qq} &= \frac{1}{2}(4C^{(3,2)}_{qq}(y_b^2 + y_t^2) - 4(C^{(1,2)}_{qq_3q_3q} - C^{(3,2)}_{qq_3q_3q} + 2C^{(3,2)}_{qqq_3q_3}N_c)(y_b^2 + y_t^2) \\ &+ 3C^{(1)}_{qq_3}(x_by_b^2 - x_ty_t^2) + 2C^{(3)}_{qq_3}(x_by_b^2 + x_ty_t^2) - 4(C^{(1,1)}_{qqqq_3} - C^{(3,1)}_{qqqq_3} \\ &+ 2C^{(3,2)}_{qqqq_3}N_c)(x_by_b^2 + x_ty_t^2) + 6C^{(1,1)}_{qq}(x_b^2y_b^2 - x_t^2y_t^2) - C_{H\Box}(x_b^2y_b^2 + x_t^2y_t^2) \\ &+ 4C^{(3,1)}_{qq}(x_b^2y_b^2 + x_t^2y_t^2) - 4(C^{(1,1)}_{qqqq} - C^{(3,1)}_{qqqq} + 2C^{(3,2)}_{qqqq}N_c)(x_b^2y_b^2 + x_t^2y_t^2) \\ &- 4(x_by_b^2 + x_ty_t^2)(C^{\star(1,1)}_{qqqq_3} - C^{\star(3,1)}_{qqqq_3} + 2N_cC^{\star(3,2)}_{qqqq_3}) + 3(x_by_b^2 - x_ty_t^2)C^{\star(1)}_{qq_3} + 2(x_by_b^2 + x_ty_T^2)C^{\star(3)}_{qq_3})\end{aligned} \tag{6.46}$$

These are just some examples of our work, we have done the calculation of all the Wilson coefficients of all classes truncated to a some orders.

The analysis of the $U(3)^5$ and $U(2)^5$ symmetry and symmetry-breaking in the flavor sector are quite clear. However, it is worth to stress that the flavor symmetry is useful to simplify and organise analysis of hightpT observables at the LHC. In the next chapter, we will discuss some phenomenological application of what we have already did so far.

Chapter 7

Phenomenology in the domain of Flavor symmetric SMEFT

7.1 Introduction

The flavor symmetries and the spurion series expansion give a rationale for phenomenology because they tell us which effects are expected to be larger compared to others, based on the known fermion masses and mixings. Concretely, the yukawa of the top is the largest spurion $y_t \sim 1$, and it breaks U(3) to U(2), which is the main motivation to use U(2) instead of U(3).

U(2) is currently used in state-of-the-art LHC searches for SMEFT signals, especially in top quark processes, but also in global fits. Many of the Monte Carlo codes that allow to simulate SMEFT signals at LHC implement a U(2) symmetry (e.g. SMEFTsim [166], SMEFT@NLO [169]). It's also used in global analyses, by the ATLAS/CMS experiments [170] and by theorists [171]. These tools implement an exact U(2) symmetry in the quark sector, simplifying the treatment of flavor (or quark type) interactions. However, this simplification can overlook important flavor-dependent effects that arise during the Renormalization Group Equation (RGE) evolution of the theory and could become relevant, for instance, in the context of a combined analysis of high-pT data and low-energy flavor observables.

To enhance the accuracy of these simulations, it is crucial to account for the flavor structure of the RG equations, ensuring that flavor-changing interactions are appropriately accounted for [9,41]. By doing so, one can capture the complete set of flavor effects, leading to more precise predictions for LHC observables.

U(2) is used in all the examples above as an exact symmetry, ie. typically neglecting all spurions. So, by asking what is the largest flavor effect neglected, we expect the next breaking in the flavor symmetry hierarchy typically to come from from the insertion of the Vq spurion, whose size is mainly determined by the Cabibbo-Kobayashi-

Maskawa (V_{CKM}) matrix element V_{cb}, which is associated with the second-generation quarks (specifically the b- and c-quark). The magnitude of V_{cb} is approximately $\mathcal{O}(10^{-2})$, much smaller than y_t. This hierarchy in the magnitudes of the flavor parameters is important because it sets the scale for which effects are expected to be subleading in new physics models.

At this stage, we can identify operators that involve only one instance of the spurion V_q, and no other spurions. These operators can be constructed by contracting flavor indices in a way that ensures they are invariant under the U(2) symmetry and that no additional spurions contribute. For example, one such operator would be the current ($\bar{q}V_q q_3$), where the quark fields q and q_3 belong to the first and third generations, respectively.

7.2 Operators of Interest

Using what we did in section 5.4, and having the notations used in section 5.6. These are the list of contractions with single V_q:

- Class 5:

$$C_{qu_3H}V_q, \quad C_{qd_3H}V_q. \tag{7.1}$$

- Class 6:

$$C_{qu_3G}V_q, \quad C_{qu_3W}V_q, \quad C_{qu_3B}V_q, \quad C_{qd_3G}V_q, \quad C_{qd_3W}V_q, \quad C_{qd_3B}V_q. \tag{7.2}$$

- Class 7:

$$C^{(1)}_{qq_3}V_q, \quad C^{(3)}_{qq_3}V_q. \tag{7.3}$$

- Class $8(\bar{L}L)(\bar{L}L)$:

$$\begin{aligned} &C^{(1,1)}_{qqqq_3}V_q, \quad C^{(1,2)}_{qqqq_3}V_q, \quad C^{(1)}_{qq_3q_3q_3}V_q, \quad C^{(3,1)}_{qqqq_3}V_q, \quad C^{(3,2)}_{qqqq_3}V_q, \quad C^{(3)}_{qq_3q_3q_3}V_q \\ &C^{(1)}_{\ell\ell qq_3}V_q, \quad C^{(1)}_{\ell_3\ell_3qq_3}V_q, \quad C^{(3)}_{\ell\ell qq_3}V_q, \quad C^{(3)}_{\ell_3\ell_3qq_3}V_q \end{aligned} \tag{7.4}$$

- Class $8(\bar{L}L)(\bar{R}R)$:

$$\begin{aligned} &C_{qq_3ee}V_q, \quad C_{qq_3e_3e_3}V_q, \quad C^{(1)}_{qq_3uu}V_q, \quad C^{(1)}_{qq_3u_3u_3}V_q, \quad C^{(8)}_{qq_3uu}V_q, \quad C^{(8)}_{qq_3u_3u_3}V_q, \\ &C^{(1)}_{qq_3dd}V_q, \quad C^{(1)}_{qq_3d_3d_3}V_q, \quad C^{(8)}_{qq_3dd}V_q, \quad C^{(8)}_{qq_3d_3d_3}V_q, \end{aligned} \tag{7.5}$$

- Class $8(\bar{L}R)(\bar{R}L)$:

$$C_{\ell_3 e_3 d_3 q} V_q^\dagger \tag{7.6}$$

- Class $8(\bar{L}R)(\bar{L}R)$:

$$C^{(1)}_{qu_3q_3d_3}V_q, \quad C^{(1)}_{q_3u_3qd_3}V_q, \quad C^{(8)}_{qu_3q_3d_3}V_q, \quad C^{(8)}_{q_3u_3qd_3}V_q, \quad C^{(1)}_{\ell_3e_3qu_3}V_q, \quad C^{(3)}_{\ell_3e_3qu_3}V_q^{\cdot} \tag{7.7}$$

These operators could induce many interactions that are flavor changing interactions, $\Delta F = 1$. Since q and q_3 are doublets, and some operators have Pauli matrices inside, then we could have $\bar{t}cX$, $\bar{t}sX$, $\bar{b}sX$ and $\bar{b}cX$ where X are outgoing particles.

We will focus on interactions with top quark. Using ref. [172] we can know the induced interactions.

Using unitary gauge, and up to 4-point interactions we have the follow:

$$\begin{gathered}\bar{t}cg, \quad \bar{t}c\gamma, \quad \bar{t}cZ, \quad \bar{t}ch, \quad \bar{t}cgh, \quad \bar{t}c\gamma h, \quad \bar{t}cZh, \quad \bar{t}chh, \quad \bar{t}cW^+W^-, \quad \bar{t}cgg, \quad \bar{t}c\bar{\psi}\psi, \\ \bar{t}sW^+, \quad \bar{t}sW^+h, \quad \bar{t}s\bar{\psi}\psi'\end{gathered} \tag{7.8}$$

were $\bar{\psi}\psi$ is a generic fermion-antifermion pair of any flavor, while $\bar{\psi}\psi'$ is a flavor-conserving fermion pair with positive electric charge (eg $\bar{u}d, e^+\nu_e$).

Identifying the list of leading breakings of U(2) that can be relevant for LHC processes (especially involving the top quark) and also provided the RGEs, we can produce such effects via running. For instance, from eq.6.18 we have the truncated RGEs for the $C^{(3)}_{qq_3}$ up to $\mathcal{O}(V^3)$. To do thing more consistent here, we will need to do a truncation by hand up to $\mathcal{O}(V^1)$:

$$\begin{aligned}\dot{C}^{(3)}_{qq_3} = \frac{1}{2}(&3C^{(1)}_{qq_3}(y_b^2 - y_t^2) + 2C^{(3)}_{qq_3}(y_b^2 + y_t^2) - 4(C^{(1)}_{qq_3q_3q_3} + C^{(3)}_{qq_3q_3q_3}(-1 + 2N_c))(y_b^2 + y_t^2) \\ &+ 3C^{(1,1)}_{qq}(x_b y_b^2 - x_t y_t^2) + 3C^{(1)}_{q_3q_3}(x_b y_b^2 - x_t y_t^2) + 2C^{(3,1)}_{q_3q_3}(x_b y_b^2 + x_t y_t^2) \\ &+ 2C^{(3)}_{q_3q_3}(x_b y_b^2 + x_t y_t^2) - C_{H\Box}(x_b y_b^2 + x_t y_t^2) - 4(C^{(1,1)}_{qqq_3q_3} - C^{(3,1)}_{qqq_3q_3} + 2C^{(3,1)}_{qq_3q_3q}N_c)(x_b y_b^2 + x_t y_t^2) \\ &- 4C^{(3,1)}_{\ell_3\ell_3qq_3}y_\tau^2 + 4C^{(3)}_{qq_3}(N_c(y_b^2 + y_t^2) + y_\tau^2)\end{aligned} \tag{7.9}$$

The operators found in eq.7.9 generate $C^{(3)}_{qq_3}$ via the running at high energy scales. There are exact symmetry terms showing in the running, then even if we sart from flavor-preseving operators ($C^{(1,1)}_{qq}$ and $C^{(1)}_{q_3q_3}$) they generate flavor-violating operators. Flavor-violating operators produces interaction as $\bar{t}cZ$, $\bar{t}cZh$, $\bar{t}sW^+$ and $\bar{t}sW^+h$. The running of these operators at higher scales modifies the strength of the interaction, which is essential for understanding the behavior of the system at these energy levels. This running effect allows for the operators to influence low-energy observables, linking high-energy physics with phenomena observable at lower scales.

7.3 Possible phenomenological consequences of U(2) breaking

Top quark decay refers to the process by which a top quark, the heaviest known fundamental particle, decays into other particles. The top quark is unstable and decays almost immediately after it is produced. Its dominant decay mode is through the weak interaction, where it decays into a W boson and a bottom quark (b quark), typically represented as $t \to W + b$. The W boson itself quickly decays into various lighter particles, such as a quark-antiquark pair or a lepton and neutrino. The relatively short lifetime of the top quark means that it does not hadronize like lighter quarks, and its decay products provide valuable insight into the fundamental forces and particle interactions.

In section 7.2, we have listed some of the interactions of $\mathcal{O}(V^1)$ which is the next highest breaking in size. And we have the following 2-body decays that can be induced:

$$\begin{aligned} &t \to cZ, \quad t \to ch, \\ &t \to cg, \quad t \to c\gamma. \end{aligned} \tag{7.10}$$

In eq. 7.10, we have the most important flavor changing-neutral current (FCNC) processes. If U(2)is broken hierarchically, it can lead to controlled flavor mixing while still preserving the smallness of FCNCs. Then eq.7.10 can be observed when U(2) is softly broken.

Based on refs. [173–177], we can have the following branching ratio:

$$\begin{aligned} &\mathcal{BR}(t \to ch) < 3.7 \times 10^{-4}, \quad \mathcal{BR}(t \to cZ) < 1.3 \times 10^{-4}, \\ &\mathcal{BR}(t \to cg) < 3.7 \times 10^{-4}, \quad \mathcal{BR}(t \to c\gamma) < 4.2 \times 10^{-5}. \end{aligned} \tag{7.11}$$

These bounds on the branching ratio, can impose bound on the Wilson coefficients. We will Follow ref. [178] for such derivation[1].

The tree-level prediction for the leading decay mode $t \to bW^+$:

$$\Gamma(t \to bW^+) = \frac{\alpha}{16 s_w^2} |V_{tb}|^2 \frac{m_W^3}{m_w^2} \left[1 - 3\frac{m_W^2}{m_t^2} + 2\frac{m_w^6}{m_t^6} \right] \tag{7.12}$$

The partial widths for FCNC decays are given by

[1]Reference [178] uses different notations in the lagrangian that need to be careful while dealing with

$$\Gamma(t \to cZ)_\gamma = \frac{\alpha}{32 s_w^2 c_w^2} |C^{(3)}_{qq3} V_q \frac{v^2}{\Lambda^2}|^2 \frac{m_t^3}{m_Z^2} \left[1 - \frac{m_Z^2}{m_t^2}\right]^2 \left[1 + 2\frac{m_Z^2}{m_t^2}\right] \tag{7.13}$$

$$\Gamma(t \to cZ)_\sigma = \frac{\alpha}{16 s_w^2 c_w^2} |Re(C_{qu_3W}) V_q \frac{2\sqrt{2} v c_w^2 m_t}{g\Lambda^2}|^2 m_t \left[1 - \frac{m_Z^2}{m_t^2}\right]^2 \left[2 + \frac{m_Z^2}{m_t^2}\right] \tag{7.14}$$

$$\Gamma(t \to c\gamma) = \frac{\alpha}{2} |Re(C_{qu_3W}) V_q \frac{\sqrt{2} v s_w m_t}{e\Lambda^2}|^2 m_t \tag{7.15}$$

$$\Gamma(t \to cg) = \frac{2\alpha_s}{3} |Re(C_{qu_3G}) V_q \frac{\sqrt{2} v m_t}{g_3\Lambda^2}|^2 m_t \tag{7.16}$$

$$\Gamma(t \to ch) = \frac{\alpha}{32 s_w^2} |Re(C_{qu_3H}) V_q \frac{3 v^2 s_w^2}{g\Lambda^2}|^2 m_t \left[1 - \frac{m_h^2}{m_t^2}\right]^2 \tag{7.17}$$

We will consider one SMEFT operator at a time. Assuming that each of them introduces only one exotic decay channel, we can calculate the corresponding branching ratio as:

$$\mathcal{BR}(t \to cX) = \frac{\Gamma_X}{\Gamma_X + \Gamma(t \to bW^+)} \tag{7.18}$$

With V_{tb}=1,$m_h = 125.35 Gev$, $m_t = 173.2 GeV$, $m_Z = 91.1876 GeV$, $m_W = 90.377 GeV$, $s_w = \sqrt{0.2229}$, $c_w = \sqrt{1 - 0.2229}$, $v = 246.22 GeV$, V_q=0.04182, $e = 0.302$, $g = 0.652$, $\alpha_s(m_Z) = 0.118$, $\alpha = 1/137$, $g_3(m_Z) = 1.22$ and $1 GeV = 10^{-3} TeV$

Having an upper bounds on the branching ratio from eq.7.11, and having the theoretical eq.7.18, an upper bound on the Wilson coefficients in the above decay width can be found.

From eq.7.13, we can find a bound on $C^{(3)}_{qq3}$ as

$$C^{(3)}_{qq3} < 6.62477 \frac{\Lambda^2}{TeV^2} \tag{7.19}$$

From eq.7.14, we can have a bound on $Re(C_{qu_3W})$ as

$$Re(C_{qu_3W}) < 3.0999 \frac{\Lambda^2}{TeV^2} \tag{7.20}$$

From eq.7.15, we can have a more restricitve upper bound than eq.7.20 on $Re(C_{qu_3W})$,

$$Re(C_{qu_3W}) < 2.48931 \frac{\Lambda^2}{TeV^2} \tag{7.21}$$

From eq.7.17, we can have an upper bound for $Re(C_{qu_3H})$ as

$$Re(C_{qu_3H}) < 9.90495\frac{\Lambda^2}{TeV^2} \tag{7.22}$$

From eq.7.16, e can have an upper bound for $Re(C_{qu_3G})$ as

$$Re(C_{qu_3G}) < 3.03573\frac{\Lambda^2}{TeV^2} \tag{7.23}$$

In conclusion, the bounds on the Wilson coefficients are not particularly stringent. The constraints are significantly milder—roughly by a factor of $V_q \sim 0.04$ compared to a scenario where no flavor symmetry is assumed.

This leaves room for some flavor-violating contributions being detected by future experiments. Detecting FCNC decays of the top quark would provide compelling evidence for physics beyond the Standard Model. Such observations could shed light on the underlying mechanisms of flavor symmetry breaking and offer insights into the structure of the fundamental interactions. Therefore, continued searches and precise measurements at the LHC are crucial for exploring these potential signals of new physics.

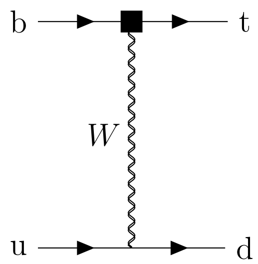

Figure 7.1: U(2) preserving diagram, representing $C^{(3)}_{q_3q_3}$ contribution to the single top production. The box is the SMEFT vertix while the other is the SM vertix.

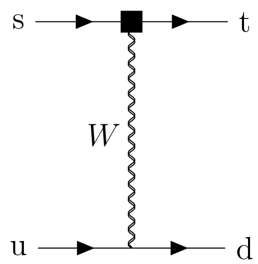

Figure 7.2: U(2) breaking diagram, representing $C^{(3)}_{qq_3}$ contribution to the single top production. The box is the SMEFT vertix while the other is the SM vertix.

Another example process where U(2) breaking operators could potentially be relevant is single top production at the LHC. Diagram 7.1, represent a U(2) preserving (usual) diagram with the relevant operator entering can be $C_{Hq}^{(3)}$, while diagram 7.2 represent a U(2) breaking diagram which is suppressed by V_q. But diagram 7.2 has a strange quark in the initial state instead of bottom, which have larger proton pdfs. It would take a dedicated study to quantify the impact of the U(2)-breaking operator or determine the sensitivity of single-top measurements to it, but for sure they will benefit from some enhancement, that could potentially make them non-negligible.

In conclusion, the phenomenological analysis conducted in this chapter has provided valuable insights into the constraints on new physics within the SMEFT framework. While the bounds on the Wilson coefficients are milder than initially expected, they still contribute to narrowing down the parameter space and enhancing our understanding of possible deviations from the Standard Model. These more moderate constraints [179], though not yet restrictive enough to rule out certain new physics scenarios, offer guidance for future experiments and theoretical developments. Further refinements in both experimental measurements and theoretical calculations could potentially lead to stronger constraints or even uncover new signals of beyond-the-Standard-Model physics. As such, while the findings may not immediately imply dramatic new discoveries, they represent an important step toward a more comprehensive understanding of the fundamental interactions in the universe.

Conclusion

In this thesis, we have systematically explored the Standard Model Effective Field Theory (SMEFT) within the context of flavor symmetries, particularly focusing on the $U(3)^5$ and $U(2)^5$ groups. By implementing flavor structures in the Warsaw Basis, we identified 2,499 independent baryon and lepton number-conserving SMEFT operators. Our analysis of the action of $U(3)^5$ and $U(2)^5$ flavor symmetries on these operators facilitated a reduction in their number, enhancing the theoretical framework's predictive power. Furthermore, we established a correspondence between Wilson coefficients in $U(3)^5$, considering only the third-generation Yukawa couplings, and those in $U(2)^5$, eliminating the need for spurions.

In our examination of SMEFT Renormalization Group Equations (RGEs) and the truncation technique within both flavor symmetries, we introduced the "RunSMEFT" code. This computational tool calculates truncated SMEFT RGEs,enabling a consistent treatment of RG equations for the SMEFT lagrangians implementing approximate flavor symmetries. The truncated RGEs could be interfaced for instance with existing Monte Carlo tools for SMEFT simulations at the LHC, which typically assume a $U(3)^5$ or $U(2)^5$ symmetric flavor structure.

The phenomenological analysis of $U(2)$ breaking presented in Chapter 7 offers an understanding of flavor dynamics and their implications for top-quark observables. We identified flavor-changing top decays and single-top productions as observables which could be sensitive to U(2)-breaking phenomena. For the former processes, we have also shown that the assumption of an approximate flavor symmetry helps lowering the bounds on the new physics scale.

Collectively, the methodologies and findings discussed in this thesis contribute to refining the SMEFT as a tool for probing new physics. The comprehensive operator list, insights into flavor symmetries, and the development of computational tools like "RunSMEFT" enhance our capability to analyze and predict phenomena that may reveal the presence of new physics beyond the Standard Model.

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Appendix A

The Higgs self-coupling contributions to the one loop RGE

We list here the full one-loop contributions to the SM RGE from $\mathcal{L}^{(6)}$ [4]. These terms are *in addition* to the usual SM anomalous dimensions.

$$\begin{aligned}\mu\frac{d}{d\mu}\lambda = \frac{m_H^2}{16\pi^2}\Bigg[&12C_H + \left(-32\lambda + \frac{10}{3}g_2^2\right)C_{H\Box} + \left(12\lambda - \frac{3}{2}g_2^2 + 6g_1^2\mathsf{y}_H^2\right)C_{HD}\\&+ 2\eta_1 + 2\eta_2 + 12g_2^2c_{F,2}C_{HW} + 12g_1^2\mathsf{y}_H^2C_{HB} + 6g_1g_2\mathsf{y}_HC_{HWB} + \frac{4}{3}g_2^2C^{(3)}_{\underset{tt}{Hl}}\\&+ \frac{4}{3}g_2^2N_cC^{(3)}_{\underset{tt}{Hq}}\Bigg]\end{aligned} \tag{A.1}$$

$$\mu\frac{d}{d\mu}m_H^2 = \frac{m_H^4}{16\pi^2}\left[-4C_{H\Box} + 2C_{HD}\right] \tag{A.2}$$

$$\begin{aligned}\mu\frac{d}{d\mu}[Y_u]_{rs} = \frac{m_H^2}{16\pi^2}\Bigg[&3C^*_{\underset{sr}{uH}} - C_{H\Box}[Y_u]_{rs} + \frac{1}{2}C_{HD}[Y_u]_{rs} - [Y_u]_{rt}\left(C^{(1)}_{\underset{ts}{Hq}} - 3C^{(3)}_{\underset{ts}{Hq}}\right) + C_{\underset{rt}{Hu}}[Y_u]_{ts}\\&- C_{\underset{rt}{Hud}}[Y_d]_{ts} - 2\left(C^{(1)}_{\underset{psrt}{qu}} + c_{F,3}C^{(8)}_{\underset{psrt}{qu}}\right)[Y_u]_{tp} - C^{(1)*}_{\underset{ptsr}{lequ}}[Y_e^\dagger]_{pt} + N_cC^{(1)*}_{\underset{srpt}{quqd}}[Y_d^\dagger]_{pt}\\&+ \frac{1}{2}\left(C^{(1)*}_{\underset{prst}{quqd}} + c_{F,3}C^{(8)*}_{\underset{prst}{quqd}}\right)[Y_d^\dagger]_{pt}\Bigg]\end{aligned} \tag{A.3}$$

$$\mu\frac{d}{d\mu}[Y_d]_{rs} = \frac{m_H^2}{16\pi^2}\Bigg[3C^*_{\underset{sr}{dH}} - C_{H\Box}[Y_d]_{rs} + \frac{1}{2}C_{HD}[Y_d]_{rs} + [Y_d]_{rt}\left(C^{(1)}_{\underset{ts}{Hq}} + 3C^{(3)}_{\underset{ts}{Hq}}\right) - C_{\underset{rt}{Hd}}[Y_d]_{ts}$$

$$- [Y_u]_{ts}C^*_{\underset{tr}{Hud}} - 2\left(C^{(1)}_{\underset{psrt}{qd}} + c_{F,3}C^{(8)}_{\underset{psrt}{qd}}\right)[Y_d]_{tp} + C_{\underset{ptrs}{ledq}}[Y_e]_{tp} + N_c C^{(1)*}_{\underset{ptsr}{quqd}}[Y_u^\dagger]_{pt}$$

$$+ \frac{1}{2}\left(C^{(1)*}_{\underset{sptr}{quqd}} + c_{F,3}C^{(8)*}_{\underset{sptr}{quqd}}\right)[Y_u^\dagger]_{tp}\Bigg] \tag{A.4}$$

$$\mu\frac{d}{d\mu}[Y_e]_{rs} = \frac{m_H^2}{16\pi^2}\Bigg[3C^*_{\underset{sr}{eH}} - C_{H\Box}[Y_e]_{rs} + \frac{1}{2}C_{HD}[Y_e]_{rs} + [Y_e]_{rt}\left(C^{(1)}_{\underset{ts}{Hl}} + 3C^{(3)}_{\underset{ts}{Hl}}\right) - C_{\underset{rt}{He}}[Y_e]_{ts}$$

$$- 2C_{\underset{psrt}{le}}[Y_e]_{tp} + N_c C^*_{\underset{srpt}{ledq}}[Y_d]_{pt} - N_c C^{(1)*}_{\underset{srpt}{lequ}}[Y_u^\dagger]_{pt}\Bigg], \tag{A.5}$$

$$\mu\frac{dg_3}{d\mu} = -4\frac{m_H^2}{16\pi^2}g_3C_{HG}\,, \qquad \mu\frac{dg_2}{d\mu} = -4\frac{m_H^2}{16\pi^2}g_2C_{HW}\,, \qquad \mu\frac{dg_1}{d\mu} = -4\frac{m_H^2}{16\pi^2}g_1C_{HB}\,,$$
$$\mu\frac{d}{d\mu}\theta_3 = -\frac{4m_H^2}{g_3^2}C_{H\widetilde{G}}\,, \qquad \mu\frac{d}{d\mu}\theta_2 = -\frac{4m_H^2}{g_2^2}C_{H\widetilde{W}}\,, \qquad \mu\frac{d}{d\mu}\theta_1 = -\frac{4m_H^2}{g_1^2}C_{H\widetilde{B}}\,, \tag{A.6}$$

where

$$\eta_1 = \left(\frac{1}{2}N_c C_{\underset{rs}{dH}}[Y_d]_{sr} + \frac{1}{2}N_c C_{\underset{rs}{uH}}[Y_u]_{sr} + \frac{1}{2}C_{\underset{rs}{eH}}[Y_e]_{sr}\right) + h.c.\,,$$
$$\eta_2 = -2N_c C^{(3)}_{\underset{rs}{Hq}}[Y_u^\dagger Y_u]_{sr} - 2N_c C^{(3)}_{\underset{rs}{Hq}}[Y_d^\dagger Y_d]_{sr} + N_c C_{\underset{rs}{Hud}}[Y_d Y_u^\dagger]_{sr} + N_c C^*_{\underset{rs}{Hud}}[Y_u Y_d^\dagger]_{rs}$$
$$- 2C^{(3)}_{\underset{rs}{Hl}}[Y_e^\dagger Y_e]_{sr}\,, \tag{A.7}$$

and $N_c = 3$ is the number of colors, $c_{F,3} = 4/3$, and $c_{A,2} = 2$.

Class H^6:

$$\mu\frac{d}{d\mu}C_H = \frac{1}{16\pi^2}\left[108\,\lambda\,C_H - 160\,\lambda^2\,C_{H\Box} + 48\,\lambda^2\,C_{HD}\right] + \frac{8\lambda}{16\pi^2}\eta_1 + \frac{8\lambda}{16\pi^2}\eta_2 \tag{A.8}$$

Class X^2H^2 :

$$
\begin{aligned}
\mu\frac{d}{d\mu}C_{HG} &= \frac{12\lambda}{16\pi^2}\,C_{HG}\,, & \mu\frac{d}{d\mu}C_{H\widetilde{G}} &= \frac{12\lambda}{16\pi^2}\,C_{H\widetilde{G}}\,,\\
\mu\frac{d}{d\mu}C_{HW} &= \frac{12\lambda}{16\pi^2}\,C_{HW}\,, & \mu\frac{d}{d\mu}C_{H\widetilde{W}} &= \frac{12\lambda}{16\pi^2}\,C_{H\widetilde{W}}\,,\\
\mu\frac{d}{d\mu}C_{HB} &= \frac{12\lambda}{16\pi^2}\,C_{HB}\,, & \mu\frac{d}{d\mu}C_{H\widetilde{B}} &= \frac{12\lambda}{16\pi^2}\,C_{H\widetilde{B}}\,,\\
\mu\frac{d}{d\mu}C_{HWB} &= \frac{4\lambda}{16\pi^2}\,C_{HWB}\,, & \mu\frac{d}{d\mu}C_{H\widetilde{W}B} &= \frac{4\lambda}{16\pi^2}\,C_{H\widetilde{W}B}\,,
\end{aligned}
\tag{A.9}
$$

Class H^4D^2 :

$$
\mu\frac{d}{d\mu}C_{H\Box} = \frac{24\lambda}{16\pi^2}\,C_{H\Box}\,, \qquad \mu\frac{d}{d\mu}C_{HD} = \frac{12\lambda}{16\pi^2}\,C_{HD}\,,
\tag{A.10}
$$

Class ψ^2H^4:

$$
\begin{aligned}
\mu\frac{d}{d\mu}\underset{rs}{C_{uH}} &= \frac{\lambda}{16\pi^2}\Bigg[24\,\underset{rs}{C_{uH}} - 4\underset{rt}{C^{(1)}_{Hq}}\,[Y_u^\dagger]_{ts} + 12\underset{rt}{C^{(3)}_{Hq}}\,[Y_u^\dagger]_{ts} + 4[Y_u^\dagger]_{rt}\,\underset{ts}{C_{Hu}} - 4[Y_d^\dagger]_{rt}\,\underset{st}{C^*_{Hud}}\\
&\quad - 4\,[Y_u^\dagger]_{rs}C_{H\Box} + 2[Y_u^\dagger]_{rs}C_{HD} - 8\underset{rpts}{C^{(1)}_{qu}}\,[Y_u^\dagger]_{pt} - 8c_{F,3}\underset{rpts}{C^{(8)}_{qu}}\,[Y_u^\dagger]_{pt} - 4\underset{ptrs}{C^{(1)}_{lequ}}[Y_e]_{tp}\\
&\quad + 4N_c\underset{rspt}{C^{(1)}_{quqd}}[Y_d]_{tp} + 2\underset{psrt}{C^{(1)}_{quqd}}[Y_d]_{tp} + 2c_{F,3}\underset{psrt}{C^{(8)}_{quqd}}[Y_d]_{tp}\Bigg]\,,\\
\mu\frac{d}{d\mu}\underset{rs}{C_{dH}} &= \frac{\lambda}{16\pi^2}\Bigg[24\,\underset{rs}{C_{dH}} + 4\underset{rt}{C^{(1)}_{Hq}}\,[Y_d^\dagger]_{ts} + 12\underset{rt}{C^{(3)}_{Hq}}\,[Y_d^\dagger]_{ts} - 4[Y_d^\dagger]_{rt}\,\underset{ts}{C_{Hd}} - 4[Y_u^\dagger]_{rt}\,\underset{ts}{C_{Hud}}\\
&\quad - 4\,[Y_d^\dagger]_{rs}C_{H\Box} + 2[Y_d^\dagger]_{rs}C_{HD} - 8\underset{rpts}{C^{(1)}_{qd}}\,[Y_d^\dagger]_{pt} - 8c_{F,3}\underset{rpts}{C^{(8)}_{qd}}\,[Y_d^\dagger]_{pt} + 4\underset{ptsr}{C^*_{ledq}}[Y_e^\dagger]_{pt}\\
&\quad + 4N_c\underset{ptrs}{C^{(1)}_{quqd}}[Y_u]_{tp} + 2\underset{rtps}{C^{(1)}_{quqd}}[Y_u]_{tp} + 2c_{F,3}\underset{rtps}{C^{(8)}_{quqd}}[Y_u]_{tp}\Bigg]\,,\\
\mu\frac{d}{d\mu}\underset{rs}{C_{eH}} &= \frac{\lambda}{16\pi^2}\Bigg[24\,\underset{rs}{C_{eH}} + 4\underset{rt}{C^{(1)}_{Hl}}\,[Y_e^\dagger]_{ts} + 12\underset{rt}{C^{(3)}_{Hl}}\,[Y_e^\dagger]_{ts} - 4[Y_e^\dagger]_{rt}\,\underset{ts}{C_{He}}\\
&\quad - 4\,[Y_e^\dagger]_{rs}C_{H\Box} + 2[Y_e^\dagger]_{rs}C_{HD} - 8\underset{rpts}{C_{le}}\,[Y_e^\dagger]_{pt} + 4N_c\underset{rspt}{C_{ledq}}[Y_d^\dagger]_{tp} - 4N_c\underset{rspt}{C^{(1)}_{lequ}}[Y_u]_{tp}\Bigg]\,,
\end{aligned}
\tag{A.11}
$$

There are no other one-loop λ, λ^2 and λy^2 terms.

Appendix B

The Yukawa contributions to the one loop RGE

The invariants $c_{F,3} = (N_c^2 - 1)/(2N_c)$ and $c_{A,3} = N_c$ are the $SU(3)$ quadratic Casimirs in the fundamental and adjoint representation, respectively; $N_c = 3$ is the number of colors, and $\mathsf{y}_{q,l,u,d,e}$ denotes the $U(1)$ hypercharges of the fermions. We use the notation

$$\dot{C} \equiv 16\pi^2 \mu \frac{\mathrm{d}}{\mathrm{d}\mu} C \tag{B.1}$$

in the renormalization group equations given below. The wavefunction renormalization contributions proportional to Yukawa couplings are written in terms of

$$\begin{aligned}
&\gamma^{(Y)}_{\substack{l\\rs}} = \frac{1}{2}[Y_e^\dagger Y_e]_{rs}, && \gamma^{(Y)}_{\substack{e\\rs}} = [Y_e Y_e^\dagger]_{rs}, && \gamma^{(Y)}_H = \mathrm{Tr}\left[N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d + Y_e^\dagger Y_e\right], \\
&\gamma^{(Y)}_{\substack{q\\rs}} = \frac{1}{2}[Y_u^\dagger Y_u + Y_d^\dagger Y_d]_{rs}, && \gamma^{(Y)}_{\substack{u\\rs}} = [Y_u Y_u^\dagger]_{rs}, && \gamma^{(Y)}_{\substack{d\\rs}} = [Y_d Y_d^\dagger]_{rs},
\end{aligned} \tag{B.2}$$

which are $16\pi^2$ times the field anomalous dimensions. The gauge contributions to wavefunction renormalization will be included with the gauge terms for the anomalous dimension matrix, since only the total combination is gauge invariant.

To simplify later expressions, it is useful to define the constants η_{1-5} and $\xi_{e,d,u}$. $\eta_{1,2}$

$$
\begin{aligned}
\eta_1 =& \frac{1}{2}N_c C_{\underset{rs}{dH}}[Y_d]_{sr} + \frac{1}{2}N_c C^*_{\underset{rs}{dH}}[Y_d^\dagger]_{rs} + \frac{1}{2}N_c C_{\underset{rs}{uH}}[Y_u]_{sr} + \frac{1}{2}N_c C^*_{\underset{rs}{uH}}[Y_u^\dagger]_{rs} + \frac{1}{2}C_{\underset{rs}{eH}}[Y_e]_{sr} \\
&+ \frac{1}{2}C^*_{\underset{rs}{eH}}[Y_e^\dagger]_{rs}\,, \\
\eta_2 =& -2N_c C^{(3)}_{\underset{rs}{Hq}}[Y_u^\dagger Y_u + Y_d^\dagger Y_d]_{sr} + N_c C_{\underset{rs}{Hud}}[Y_d Y_u^\dagger]_{sr} + N_c C^*_{\underset{sr}{Hud}}[Y_u Y_d^\dagger]_{sr} - 2C^{(3)}_{\underset{rs}{Hl}}[Y_e^\dagger Y_e]_{sr}\,, \\
\eta_3 =& N_c C^{(1)}_{\underset{rs}{Hq}}[Y_d^\dagger Y_d - Y_u^\dagger Y_u]_{sr} + 3N_c C^{(3)}_{\underset{rs}{Hq}}[Y_d^\dagger Y_d + Y_u^\dagger Y_u]_{sr} + N_c C_{\underset{rs}{Hu}}[Y_u Y_u^\dagger]_{sr} \\
&- N_c C_{\underset{rs}{Hd}}[Y_d Y_d^\dagger]_{sr} - N_c C_{\underset{rs}{Hud}}[Y_d Y_u^\dagger]_{sr} - N_c C^*_{\underset{sr}{Hud}}[Y_u Y_d^\dagger]_{sr} + \Big(3C^{(3)}_{\underset{rs}{Hl}} + C^{(1)}_{\underset{rs}{Hl}}\Big)[Y_e^\dagger Y_e]_{sr} \\
&- C_{\underset{rs}{He}}[Y_e Y_e^\dagger]_{sr} \\
\eta_4 =& 4N_c C^{(1)}_{\underset{rs}{Hq}}[Y_d^\dagger Y_d - Y_u^\dagger Y_u]_{sr} + 4N_c C_{\underset{rs}{Hu}}[Y_u Y_u^\dagger]_{sr} - 4N_c C_{\underset{rs}{Hd}}[Y_d Y_d^\dagger]_{sr} + 2N_c C_{\underset{rs}{Hud}}[Y_d Y_u^\dagger]_{sr} \\
&+ 2N_c C^*_{\underset{sr}{Hud}}[Y_u Y_d^\dagger]_{sr} + 4C^{(1)}_{\underset{rs}{Hl}}[Y_e^\dagger Y_e]_{sr} - 4C_{\underset{rs}{He}}[Y_e Y_e^\dagger]_{sr} \\
\eta_5 =& -\frac{1}{2}iN_c C_{\underset{rs}{dH}}[Y_d]_{sr} + \frac{1}{2}iN_c C^*_{\underset{rs}{dH}}[Y_d^\dagger]_{rs} + \frac{1}{2}iN_c C_{\underset{rs}{uH}}[Y_u]_{sr} - \frac{1}{2}iN_c C^*_{\underset{rs}{uH}}[Y_u^\dagger]_{rs} \\
&- \frac{1}{2}iC_{\underset{rs}{eH}}[Y_e]_{sr} + \frac{1}{2}iC^*_{\underset{rs}{eH}}[Y_e^\dagger]_{rs}
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
\xi_{\underset{pt}{e}} &= 2C_{\underset{prst}{le}}[Y_e^\dagger]_{rs} - N_c C_{\underset{ptsr}{ledq}}[Y_d^\dagger]_{rs} + C^{(1)}_{\underset{ptsr}{lequ}}N_c[Y_u]_{rs} \\
\xi_{\underset{pt}{d}} &= 2\Big(C^{(1)}_{\underset{prst}{qd}} + c_{F,3}C^{(8)}_{\underset{prst}{qd}}\Big)[Y_d^\dagger]_{rs} - \Big(N_c C^{(1)}_{\underset{srpt}{quqd}} + \frac{1}{2}C^{(1)}_{\underset{prst}{quqd}} + \frac{1}{2}c_{F,3}C^{(8)}_{\underset{prst}{quqd}}\Big)[Y_u]_{rs} - C^*_{\underset{srtp}{ledq}}[Y_e^\dagger]_{sr} \\
\xi_{\underset{pt}{u}} &= 2\Big(C^{(1)}_{\underset{prst}{qu}} + c_{F,3}C^{(8)}_{\underset{prst}{qu}}\Big)[Y_u^\dagger]_{rs} - \Big(N_c C^{(1)}_{\underset{ptsr}{quqd}} + \frac{1}{2}C^{(1)}_{\underset{stpr}{quqd}} + \frac{1}{2}c_{F,3}C^{(8)}_{\underset{stpr}{quqd}}\Big)[Y_d]_{rs} + C^{(1)}_{\underset{srpt}{lequ}}[Y_e]_{rs}
\end{aligned}
\tag{B.4}
$$

The Yukawa contributions to the one-loop renormalization group equations of the 59 dimension-six operator coefficients are listed by operator class in the following eight subsections [5].

Class X^3 :

$$
\dot{C}_G = 0, \qquad \dot{C}_{\widetilde{G}} = 0, \qquad \dot{C}_W = 0, \qquad \dot{C}_{\widetilde{W}} = 0\,. \tag{B.5}
$$

Class H^6 :

$$
\begin{aligned}
\dot{C}_H = &-4N_c C_{\underset{rs}{uH}}[Y_u Y_u^\dagger Y_u]_{sr} - 4N_c C^*_{\underset{rs}{uH}}[Y_u^\dagger Y_u Y_u^\dagger]_{rs} - 4N_c C_{\underset{rs}{dH}}[Y_d Y_d^\dagger Y_d]_{sr} \\
&- 4N_c C^*_{\underset{rs}{dH}}[Y_d^\dagger Y_d Y_d^\dagger]_{rs} - 4C_{\underset{rs}{eH}}[Y_e Y_e^\dagger Y_e]_{sr} - 4C^*_{\underset{rs}{eH}}[Y_e^\dagger Y_e Y_e^\dagger]_{rs} + 6\gamma_H^{(Y)} C_H
\end{aligned} \tag{B.6}
$$

Class H^4D^2 :

$$
\dot{C}_{H\Box} = -2\eta_3 + 4\gamma_H^{(Y)} C_{H\Box} \qquad \dot{C}_{HD} = -2\eta_4 + 4\gamma_H^{(Y)} C_{HD} \tag{B.7}
$$

Class X^2H^2:

$$
\dot{C}_{HG} = -2g_3(C_{\underset{rs}{uG}}[Y_u]_{sr} + [Y_u^\dagger]_{rs} C^*_{\underset{rs}{uG}}) - 2g_3(C_{\underset{rs}{dG}}[Y_d]_{sr} + [Y_d^\dagger]_{rs} C^*_{\underset{rs}{dG}}) + 2\gamma_H^{(Y)} C_{HG} \tag{B.8}
$$

$$
\dot{C}_{H\widetilde{G}} = 2g_3(iC_{\underset{rs}{uG}}[Y_u]_{sr} - i[Y_u^\dagger]_{rs} C^*_{\underset{rs}{uG}}) + 2g_3(iC_{\underset{rs}{dG}}[Y_d]_{sr} - i[Y_d^\dagger]_{rs} C^*_{\underset{rs}{dG}}) + 2\gamma_H^{(Y)} C_{H\widetilde{G}} \tag{B.9}
$$

$$
\begin{aligned}
\dot{C}_{HW} = &-g_2(C_{\underset{rs}{eW}}[Y_e]_{sr} + [Y_e^\dagger]_{rs} C^*_{\underset{rs}{eW}}) - g_2 N_c(C_{\underset{rs}{uW}}[Y_u]_{sr} + [Y_u^\dagger]_{rs} C^*_{\underset{rs}{uW}}) \\
&- g_2 N_c(C_{\underset{rs}{dW}}[Y_d]_{sr} + [Y_d^\dagger]_{rs} C^*_{\underset{rs}{dW}}) + 2\gamma_H^{(Y)} C_{HW}
\end{aligned} \tag{B.10}
$$

$$
\begin{aligned}
\dot{C}_{H\widetilde{W}} = &\, g_2(iC_{\underset{rs}{eW}}[Y_e]_{sr} - i[Y_e^\dagger]_{rs} C^*_{\underset{rs}{eW}}) + g_2 N_c(iC_{\underset{rs}{uW}}[Y_u]_{sr} - i[Y_u^\dagger]_{rs} C^*_{\underset{rs}{uW}}) \\
&+ g_2 N_c(iC_{\underset{rs}{dW}}[Y_d]_{sr} - i[Y_d^\dagger]_{rs} C^*_{\underset{rs}{dW}}) + 2\gamma_H^{(Y)} C_{H\widetilde{W}}
\end{aligned} \tag{B.11}
$$

$$
\begin{aligned}
\dot{C}_{HB} = &-2g_1(\mathsf{y}_l + \mathsf{y}_e)(C_{\underset{rs}{eB}}[Y_e]_{sr} + [Y_e^\dagger]_{rs} C^*_{\underset{rs}{eB}}) - 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_u)(C_{\underset{rs}{uB}}[Y_u]_{sr} + [Y_u^\dagger]_{rs} C^*_{\underset{rs}{uB}}) \\
&- 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_d)(C_{\underset{rs}{dB}}[Y_d]_{sr} + [Y_d^\dagger]_{rs} C^*_{\underset{rs}{dB}}) + 2\gamma_H^{(Y)} C_{HB}
\end{aligned} \tag{B.12}
$$

$$
\begin{aligned}
\dot{C}_{H\widetilde{B}} = &\, 2g_1(\mathsf{y}_l + \mathsf{y}_e)(iC_{\underset{rs}{eB}}[Y_e]_{sr} - i[Y_e^\dagger]_{rs} C^*_{\underset{rs}{eB}}) + 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_u)(iC_{\underset{rs}{uB}}[Y_u]_{sr} - i[Y_u^\dagger]_{rs} C^*_{\underset{rs}{uB}}) \\
&+ 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_d)(iC_{\underset{rs}{dB}}[Y_d]_{sr} - i[Y_d^\dagger]_{rs} C^*_{\underset{rs}{dB}}) + 2\gamma_H^{(Y)} C_{H\widetilde{B}}
\end{aligned} \tag{B.13}
$$

$$
\begin{aligned}
\dot{C}_{HWB} &= -g_2(\underset{rs}{C_{eB}}[Y_e]_{sr} + [Y_e^\dagger]_{rs}\underset{rs}{C^*_{eB}}) + g_2 N_c(\underset{rs}{C_{uB}}[Y_u]_{sr} + [Y_u^\dagger]_{rs}\underset{rs}{C^*_{uB}}) - g_2 N_c(\underset{rs}{C_{dB}}[Y_d]_{sr} \\
&\quad + [Y_d^\dagger]_{rs}\underset{rs}{C^*_{dB}}) - 2g_1(\mathsf{y}_l + \mathsf{y}_e)(\underset{rs}{C_{eW}}[Y_e]_{sr} + [Y_e^\dagger]_{rs}\underset{rs}{C^*_{eW}}) + 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_u)(\underset{rs}{C_{uW}}[Y_u]_{sr} \\
&\quad + [Y_u^\dagger]_{rs}\underset{rs}{C^*_{uW}}) - 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_d)(\underset{rs}{C_{dW}}[Y_d]_{sr} + [Y_d^\dagger]_{rs}\underset{rs}{C^*_{dW}}) + 2\gamma_H^{(Y)} C_{HWB} \qquad \text{(B.14)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{H\widetilde{W}B} &= g_2(i\underset{rs}{C_{eB}}[Y_e]_{sr} - i[Y_e^\dagger]_{rs}\underset{rs}{C^*_{eB}}) - g_2 N_c(i\underset{rs}{C_{uB}}[Y_u]_{sr} \\
&\quad - i[Y_u^\dagger]_{rs}\underset{rs}{C^*_{uB}}) + g_2 N_c(i\underset{rs}{C_{dB}}[Y_d]_{sr} - i[Y_d^\dagger]_{rs}\underset{rs}{C^*_{dB}}) + 2g_1(\mathsf{y}_l + \mathsf{y}_e)(i\underset{rs}{C_{eW}}[Y_e]_{sr} \\
&\quad - i[Y_e^\dagger]_{rs}\underset{rs}{C^*_{eW}}) - 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_u)(i\underset{rs}{C_{uW}}[Y_u]_{sr} - i[Y_u^\dagger]_{rs}\underset{rs}{C^*_{uW}}) + 2g_1 N_c(\mathsf{y}_q + \mathsf{y}_d) \\
&\quad (i\underset{rs}{C_{dW}}[Y_d]_{sr} - i[Y_d^\dagger]_{rs}\underset{rs}{C^*_{dW}}) + 2\gamma_H^{(Y)} C_{H\widetilde{W}B} \qquad \text{(B.15)}
\end{aligned}
$$

Class $\psi^2 H^3$:

$$
\begin{aligned}
\underset{rs}{\dot{C}_{uH}} &= 2(\eta_1 + \eta_2 - i\eta_5)[Y_u^\dagger]_{rs} + [Y_u^\dagger Y_u Y_u^\dagger]_{rs}(C_{HD} - 6C_{H\Box}) - 2\underset{rt}{C^{(1)}_{Hq}}[Y_u^\dagger Y_u Y_u^\dagger]_{ts} \\
&\quad + 6\underset{rt}{C^{(3)}_{Hq}}[Y_d^\dagger Y_d Y_u^\dagger]_{ts} + 2[Y_u^\dagger Y_u Y_u^\dagger]_{rt}\underset{ts}{C_{Hu}} - 2[Y_d^\dagger Y_d Y_d^\dagger]_{rt}\underset{st}{C^*_{Hud}} + 8\Big(\underset{rpts}{C^{(1)}_{qu}} + c_{F,3}\underset{rpts}{C^{(8)}_{qu}}\Big) \\
&\quad [Y_u^\dagger Y_u Y_u^\dagger]_{pt} - 2\Big(2N_c\underset{rstp}{C^{(1)}_{quqd}} + \underset{tsrp}{C^{(1)}_{quqd}} + c_{F,3}\underset{tsrp}{C^{(8)}_{quqd}}\Big)[Y_d Y_d^\dagger Y_d]_{pt} + 4\underset{tprs}{C^{(1)}_{lequ}}[Y_e Y_e^\dagger Y_e]_{pt} \\
&\quad + 4\underset{rt}{C_{uH}}[Y_u Y_u^\dagger]_{ts} + 5[Y_u^\dagger Y_u]_{rt}\underset{ts}{C_{uH}} - 2[Y_d^\dagger]_{rt}\underset{ut}{C^*_{dH}}[Y_u^\dagger]_{us} - \underset{rt}{C_{dH}}[Y_d Y_u^\dagger]_{ts} - 2[Y_d^\dagger Y_d]_{rt}\underset{ts}{C_{uH}} \\
&\quad + 3\gamma_H^{(Y)}\underset{rs}{C_{uH}} + \underset{rv}{\gamma_q^{(Y)}}\underset{vs}{C_{uH}} + \underset{rv}{C_{uH}}\underset{vs}{\gamma_u^{(Y)}} \qquad \text{(B.16)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{rs}{\dot{C}_{dH}} &= 2(\eta_1 + \eta_2 + i\eta_5)[Y_d^\dagger]_{rs} + [Y_d^\dagger Y_d Y_d^\dagger]_{rs}(C_{HD} - 6C_{H\Box}) + 2\underset{rt}{C^{(1)}_{Hq}}[Y_d^\dagger Y_d Y_d^\dagger]_{ts} \\
&\quad + 6\underset{rt}{C^{(3)}_{Hq}}[Y_u^\dagger Y_u Y_d^\dagger]_{ts} - 2[Y_d^\dagger Y_d Y_d^\dagger]_{rt}\underset{ts}{C_{Hd}} - 2[Y_u^\dagger Y_u Y_u^\dagger]_{rt}\underset{ts}{C_{Hud}} + 8\Big(\underset{rpts}{C^{(1)}_{qd}} + c_{F,3}\underset{rpts}{C^{(8)}_{qd}}\Big) \\
&\quad [Y_d^\dagger Y_d Y_d^\dagger]_{pt} - 4\underset{ptsr}{C^*_{ledq}}[Y_e^\dagger Y_e Y_e^\dagger]_{pt} - 2\Big(2N_c\underset{tprs}{C^{(1)}_{quqd}} + \underset{rpts}{C^{(1)}_{quqd}} + c_{F,3}\underset{rpts}{C^{(8)}_{quqd}}\Big)[Y_u Y_u^\dagger Y_u]_{pt} \\
&\quad + 4\underset{rt}{C_{dH}}[Y_d Y_d^\dagger]_{ts} + 5[Y_d^\dagger Y_d]_{rt}\underset{ts}{C_{dH}} - 2[Y_u^\dagger]_{rt}\underset{ut}{C^*_{uH}}[Y_d^\dagger]_{us} - \underset{rt}{C_{uH}}[Y_u Y_d^\dagger]_{ts} - 2[Y_u^\dagger Y_u]_{rt}\underset{ts}{C_{dH}} \\
&\quad + 3\gamma_H^{(Y)}\underset{rs}{C_{dH}} + \underset{rv}{\gamma_q^{(Y)}}\underset{vs}{C_{dH}} + \underset{rv}{C_{dH}}\underset{vs}{\gamma_d^{(Y)}} \qquad \text{(B.17)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{\substack{eH\\rs}} &= 2(\eta_1+\eta_2+i\eta_5)[Y_e^\dagger]_{rs} + [Y_e^\dagger Y_e Y_e^\dagger]_{rs}(C_{HD}-6C_{H\Box}) + 2C^{(1)}_{\substack{Hl\\rt}}[Y_e^\dagger Y_e Y_e^\dagger]_{ts} \\
&- 2[Y_e^\dagger Y_e Y_e^\dagger]_{rt} C_{\substack{He\\ts}} + 8C_{\substack{le\\rpts}}[Y_e^\dagger Y_e Y_e^\dagger]_{pt} - 4C_{\substack{ledq\\rspt}} N_c [Y_d^\dagger Y_d Y_d^\dagger]_{tp} \\
&+ 4C^{(1)}_{\substack{lequ\\rstp}} N_c [Y_u Y_u^\dagger Y_u]_{pt} + 4C_{\substack{eH\\rt}}[Y_e Y_e^\dagger]_{ts} + 5[Y_e^\dagger Y_e]_{rt} C_{\substack{eH\\ts}} + 3\gamma_H^{(Y)} C_{\substack{eH\\rs}} + \gamma^{(Y)}_{\substack{l\\rv}} C_{\substack{eH\\vs}} \\
&+ C_{\substack{eH\\rv}} \gamma^{(Y)}_{\substack{e\\vs}} \qquad \text{(B.18)}
\end{aligned}
$$

Class $\psi^2 XH$:

$$
\dot{C}_{\substack{eW\\rs}} = -2g_2 N_c C^{(3)}_{\substack{lequ\\rspt}}[Y_u]_{tp} + C_{\substack{eW\\rt}}[Y_e Y_e^\dagger]_{ts} + \gamma_H^{(Y)} C_{\substack{eW\\rs}} + \gamma^{(Y)}_{\substack{l\\rv}} C_{\substack{eW\\vs}} + C_{\substack{eW\\rv}} \gamma^{(Y)}_{\substack{e\\vs}} \qquad \text{(B.19)}
$$

$$
\begin{aligned}
\dot{C}_{\substack{eB\\rs}} &= 4g_1 N_c(\mathsf{y}_u+\mathsf{y}_q) C^{(3)}_{\substack{lequ\\rspt}}[Y_u]_{tp} + C_{\substack{eB\\rt}}[Y_e Y_e^\dagger]_{ts} + 2[Y_e^\dagger Y_e]_{rt} C_{\substack{eB\\ts}} + \gamma_H^{(Y)} C_{\substack{eB\\rs}} + \gamma^{(Y)}_{\substack{l\\rv}} C_{\substack{eB\\vs}} \\
&+ C_{\substack{eB\\rv}} \gamma^{(Y)}_{\substack{e\\vs}} \qquad \text{(B.20)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{\substack{uG\\rs}} &= -g_3 \left(C^{(1)}_{\substack{quqd\\psrt}} - \frac{1}{2N_c} C^{(8)}_{\substack{quqd\\psrt}} \right) [Y_d]_{tp} + 2[Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{rt} C_{\substack{uG\\ts}} - C_{\substack{dG\\rt}}[Y_d Y_u^\dagger]_{ts} \\
&+ C_{\substack{uG\\rt}}[Y_u Y_u^\dagger]_{ts} + \gamma_H^{(Y)} C_{\substack{uG\\rs}} + \gamma^{(Y)}_{\substack{q\\rv}} C_{\substack{uG\\vs}} + C_{\substack{uG\\rv}} \gamma^{(Y)}_{\substack{u\\vs}} \qquad \text{(B.21)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{\substack{uW\\rs}} &= -2g_2 C^{(3)}_{\substack{lequ\\ptrs}}[Y_e]_{tp} + \frac{1}{4} g_2 \left(C^{(1)}_{\substack{quqd\\psrt}} + c_{F,3} C^{(8)}_{\substack{quqd\\psrt}} \right) [Y_d]_{tp} + 2[Y_d^\dagger Y_d]_{rt} C_{\substack{uW\\ts}} - C_{\substack{dW\\rt}}[Y_d Y_u^\dagger]_{ts} \\
&+ C_{\substack{uW\\rt}}[Y_u Y_u^\dagger]_{ts} + \gamma_H^{(Y)} C_{\substack{uW\\rs}} + \gamma^{(Y)}_{\substack{q\\rv}} C_{\substack{uW\\vs}} + C_{\substack{uW\\rv}} \gamma^{(Y)}_{\substack{u\\vs}} \qquad \text{(B.22)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{\substack{uB\\rs}} &= 4g_1(\mathsf{y}_e+\mathsf{y}_l) C^{(3)}_{\substack{lequ\\ptrs}}[Y_e]_{tp} - \frac{1}{2} g_1 (\mathsf{y}_d+\mathsf{y}_q) \left(C^{(1)}_{\substack{quqd\\psrt}} + c_{F,3} C^{(8)}_{\substack{quqd\\psrt}} \right) [Y_d]_{tp} + 2[Y_u^\dagger Y_u \\
&- Y_d^\dagger Y_d]_{rt} C_{\substack{uB\\ts}} - C_{\substack{dB\\rt}}[Y_d Y_u^\dagger]_{ts} + C_{\substack{uB\\rt}}[Y_u Y_u^\dagger]_{ts} + \gamma_H^{(Y)} C_{\substack{uB\\rs}} + \gamma^{(Y)}_{\substack{q\\rv}} C_{\substack{uB\\vs}} + C_{\substack{uB\\rv}} \gamma^{(Y)}_{\substack{u\\vs}} \\
&\qquad \text{(B.23)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{\substack{dG\\rs}} &= -g_3 \left(C^{(1)}_{\substack{quqd\\rtps}} - \frac{1}{2N_c} C^{(8)}_{\substack{quqd\\rtps}} \right) [Y_u]_{tp} - 2[Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{rt} C_{\substack{dG\\ts}} - C_{\substack{uG\\rt}}[Y_u Y_d^\dagger]_{ts} \\
&+ C_{\substack{dG\\rt}}[Y_d Y_d^\dagger]_{ts} + \gamma_H^{(Y)} C_{\substack{dG\\rs}} + \gamma^{(Y)}_{\substack{q\\rv}} C_{\substack{dG\\vs}} + C_{\substack{dG\\rv}} \gamma^{(Y)}_{\substack{d\\vs}} \qquad \text{(B.24)}
\end{aligned}
$$

$$\dot{C}_{\substack{dW\\rs}} = 2[Y_u^\dagger Y_u]_{rt} C_{\substack{dW\\ts}} + \frac{1}{4} g_2 \left(C^{(1)}_{\substack{quqd\\rtps}} + c_{F,3} C^{(8)}_{\substack{quqd\\rtps}} \right) [Y_u]_{tp} - C_{\substack{uW\\rt}} [Y_u Y_d^\dagger]_{ts} + C_{\substack{dW\\rt}} [Y_d Y_d^\dagger]_{ts}$$
$$+ \gamma_H^{(Y)} C_{\substack{dW\\rs}} + \gamma^{(Y)}_{\substack{q\\rv}} C_{\substack{dW\\vs}} + C_{\substack{dW\\rv}} \gamma^{(Y)}_{\substack{d\\vs}} \tag{B.25}$$

$$\dot{C}_{\substack{dB\\rs}} = -\frac{1}{2} g_1 \left(\mathsf{y}_u + \mathsf{y}_q\right) \left(C^{(1)}_{\substack{quqd\\rtps}} + c_{F,3} C^{(8)}_{\substack{quqd\\rtps}} \right) [Y_u]_{tp} - 2[Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{rt} C_{\substack{dB\\ts}} - C_{\substack{uB\\rt}} [Y_u Y_d^\dagger]_{ts}$$
$$+ C_{\substack{dB\\rt}} [Y_d Y_d^\dagger]_{ts} + \gamma_H^{(Y)} C_{\substack{dB\\rs}} + \gamma^{(Y)}_{\substack{q\\rv}} C_{\substack{dB\\vs}} + C_{\substack{dB\\rv}} \gamma^{(Y)}_{\substack{d\\vs}} \tag{B.26}$$

Class $\psi^2 H^2 D$

$$\dot{C}^{(1)}_{\substack{Hq\\pr}} = \frac{1}{2} [Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{pr} \left(C_{H\Box} + C_{HD}\right) - [Y_u^\dagger]_{ps} C_{\substack{Hu\\st}} [Y_u]_{tr} - [Y_d^\dagger]_{ps} C_{\substack{Hd\\st}} [Y_d]_{tr}$$
$$+ 2 C_{\substack{qe\\prst}} [Y_e Y_e^\dagger]_{ts} 2 C^{(1)}_{\substack{lq\\stpr}} [Y_e{}^\dagger Y_e]_{ts} + \frac{3}{2} [Y_d^\dagger Y_d + Y_u^\dagger Y_u]_{pt} C^{(1)}_{\substack{Hq\\tr}} + \frac{3}{2} C^{(1)}_{\substack{Hq\\pt}} [Y_d^\dagger Y_d + Y_u^\dagger Y_u]_{tr}$$
$$+ \frac{9}{2} [Y_d^\dagger Y_d - Y_u^\dagger Y_u]_{pt} C^{(3)}_{\substack{Hq\\tr}} + \frac{9}{2} C^{(3)}_{\substack{Hq\\pt}} [Y_d^\dagger Y_d - Y_u^\dagger Y_u]_{tr} - \Big(2N_c C^{(1)}_{\substack{qq\\prst}} + 2N_c C^{(1)}_{\substack{qq\\stpr}} + C^{(1)}_{\substack{qq\\ptsr}}$$
$$+ C^{(1)}_{\substack{qq\\srpt}} + 3 C^{(3)}_{\substack{qq\\ptsr}} + 3 C^{(3)}_{\substack{qq\\srpt}} \Big) [Y_d{}^\dagger Y_d - Y_u{}^\dagger Y_u]_{ts} - 2N_c C^{(1)}_{\substack{qu\\prst}} [Y_u Y_u{}^\dagger]_{ts} + 2N_c C^{(1)}_{\substack{qd\\prst}} [Y_d Y_d{}^\dagger]_{ts}$$
$$+ 2\gamma_H^{(Y)} C^{(1)}_{\substack{Hq\\pr}} + \gamma^{(Y)}_{\substack{q\\pt}} C^{(1)}_{\substack{Hq\\tr}} + C^{(1)}_{\substack{Hq\\pt}} \gamma^{(Y)}_{\substack{q\\tr}} \tag{B.27}$$

$$\dot{C}^{(3)}_{\substack{Hq\\pr}} = -\frac{1}{2} [Y_u^\dagger Y_u + Y_d^\dagger Y_d]_{pr} C_{H\Box} + \frac{3}{2} [Y_d^\dagger Y_d - Y_u^\dagger Y_u]_{pt} C^{(1)}_{\substack{Hq\\tr}} + \frac{3}{2} C^{(1)}_{\substack{Hq\\pt}} [Y_d^\dagger Y_d - Y_u^\dagger Y_u]_{tr}$$
$$+ \frac{1}{2} [Y_d^\dagger Y_d + Y_u^\dagger Y_u]_{pt} C^{(3)}_{\substack{Hq\\tr}} + \frac{1}{2} C^{(3)}_{\substack{Hq\\pt}} [Y_d^\dagger Y_d + Y_u^\dagger Y_u]_{tr}$$
$$- \Big(2N_c C^{(3)}_{\substack{qq\\prst}} + 2N_c C^{(3)}_{\substack{qq\\stpr}} + C^{(1)}_{\substack{qq\\ptsr}} + C^{(1)}_{\substack{qq\\srpt}} - C^{(3)}_{\substack{qq\\ptsr}} - C^{(3)}_{\substack{qq\\srpt}} \Big) [Y_d{}^\dagger Y_d + Y_u{}^\dagger Y_u]_{ts}$$
$$- 2 C^{(3)}_{\substack{lq\\stpr}} [Y_e^\dagger Y_e]_{ts} + 2\gamma_H^{(Y)} C^{(3)}_{\substack{Hq\\pr}} + \gamma^{(Y)}_{\substack{q\\pt}} C^{(3)}_{\substack{Hq\\tr}} + C^{(3)}_{\substack{Hq\\pt}} \gamma^{(Y)}_{\substack{q\\tr}} \tag{B.28}$$

$$\dot{C}_{\substack{Hd\\pr}} = [Y_d Y_d^\dagger]_{pr} \left(C_{H\Box} + C_{HD}\right) - 2[Y_d]_{ps} C^{(1)}_{\substack{Hq\\st}} [Y_d^\dagger]_{tr} + 3[Y_d Y_d^\dagger]_{pt} C_{\substack{Hd\\tr}} + 3 C_{\substack{Hd\\pt}} [Y_d Y_d^\dagger]_{tr}$$
$$- [Y_d Y_u^\dagger]_{pt} C_{\substack{Hud\\tr}} - C^*_{\substack{Hud\\tp}} [Y_u Y_d^\dagger]_{tr} + 2\Big(N_c C_{\substack{dd\\prst}} + N_c C_{\substack{dd\\stpr}} + C_{\substack{dd\\ptsr}} + C_{\substack{dd\\srpt}} \Big) [Y_d Y_d{}^\dagger]_{ts}$$
$$+ 2 C_{\substack{ed\\stpr}} [Y_e Y_e{}^\dagger]_{ts} - 2 C_{\substack{ld\\stpr}} [Y_e^\dagger Y_e]_{ts} - 2N_c C^{(1)}_{\substack{ud\\stpr}} [Y_u Y_u{}^\dagger]_{ts} - 2N_c C^{(1)}_{\substack{qd\\stpr}} [Y_d{}^\dagger Y_d - Y_u{}^\dagger Y_u]_{ts}$$
$$+ 2\gamma_H^{(Y)} C_{\substack{Hd\\pr}} + \gamma^{(Y)}_{\substack{d\\pt}} C_{\substack{Hd\\tr}} + C_{\substack{Hd\\pt}} \gamma^{(Y)}_{\substack{d\\tr}} \tag{B.29}$$

$$\dot{C}_{\substack{Hu\\pr}} = -[Y_u Y_u^\dagger]_{pr}\left(C_{H\Box}+C_{HD}\right) - 2[Y_u]_{ps} C^{(1)}_{\substack{Hq\\st}}[Y_u^\dagger]_{tr} + 3[Y_u Y_u^\dagger]_{pt} C_{\substack{Hu\\tr}} + 3C_{\substack{Hu\\pt}}[Y_u Y_u^\dagger]_{tr}$$
$$+ [Y_u Y_d^\dagger]_{pt} C^*_{\substack{Hud\\rt}} + C_{\substack{Hud\\pt}}[Y_d Y_u^\dagger]_{tr} - 2\Big(N_c C_{\substack{uu\\prst}} + N_c C_{\substack{uu\\stpr}} + C_{\substack{uu\\ptsr}} + C_{\substack{uu\\srpt}}\Big)[Y_u {Y_u}^\dagger]_{ts}$$
$$+ 2C_{\substack{eu\\stpr}}[Y_e {Y_e}^\dagger]_{ts} - 2C_{\substack{lu\\stpr}}[Y_e^\dagger Y_e]_{ts} + 2N_c C^{(1)}_{\substack{ud\\prst}}[Y_d {Y_d}^\dagger]_{ts} - 2N_c C^{(1)}_{\substack{qu\\stpr}}[{Y_d}^\dagger Y_d - {Y_u}^\dagger Y_u]_{ts}$$
$$+ 2\gamma_H^{(Y)} C_{\substack{Hu\\pr}} + \gamma^{(Y)}_{\substack{u\\pt}} C_{\substack{Hu\\tr}} + C_{\substack{Hu\\pt}}\gamma^{(Y)}_{\substack{u\\tr}} \tag{B.30}$$

$$\dot{C}_{\substack{Hud\\pr}} = [Y_u Y_d^\dagger]_{pr}\left(2C_{H\Box}-C_{HD}\right) - 2[Y_u Y_d^\dagger]_{pt} C_{\substack{Hd\\tr}} + 2C_{\substack{Hu\\pt}}[Y_u Y_d^\dagger]_{tr}$$
$$+ 4\Big(C^{(1)}_{\substack{ud\\ptsr}} + c_{F,3} C^{(8)}_{\substack{ud\\ptsr}}\Big)[Y_u {Y_d}^\dagger]_{ts} + 2[Y_u Y_u^\dagger]_{pt} C_{\substack{Hud\\tr}} + 2C_{\substack{Hud\\pt}}[Y_d Y_d^\dagger]_{tr}$$
$$+ 2\gamma_H^{(Y)} C_{\substack{Hud\\pr}} + \gamma^{(Y)}_{\substack{u\\pt}} C_{\substack{Hud\\tr}} + C_{\substack{Hud\\pt}}\gamma^{(Y)}_{\substack{d\\tr}} \tag{B.31}$$

$$\dot{C}^{(1)}_{\substack{Hl\\pr}} = -\frac{1}{2}[Y_e^\dagger Y_e]_{pr}\left(C_{H\Box}+C_{HD}\right) - [Y_e^\dagger]_{ps} C_{\substack{He\\st}}[Y_e]_{tr} + \frac{3}{2}[Y_e^\dagger Y_e]_{pt}\Big(C^{(1)}_{\substack{Hl\\tr}} + 3C^{(3)}_{\substack{Hl\\tr}}\Big)$$
$$+ \frac{3}{2}\Big(C^{(1)}_{\substack{Hl\\pt}} + 3C^{(3)}_{\substack{Hl\\pt}}\Big)[Y_e^\dagger Y_e]_{tr} + 2C_{\substack{le\\prst}}[Y_e Y_e^\dagger]_{ts} + \Big(-2C_{\substack{ll\\prst}} - 2C_{\substack{ll\\stpr}} - C_{\substack{ll\\ptsr}} - C_{\substack{ll\\srpt}}\Big)$$
$$[{Y_e}^\dagger Y_e]_{ts} - 2N_c C^{(1)}_{\substack{lq\\prst}}[{Y_d}^\dagger Y_d - {Y_u}^\dagger Y_u]_{ts} - 2N_c C_{\substack{lu\\prst}}[Y_u {Y_u}^\dagger]_{ts} + 2N_c C_{\substack{ld\\prst}}[Y_d {Y_d}^\dagger]_{ts}$$
$$+ 2\gamma_H^{(Y)} C^{(1)}_{\substack{Hl\\pr}} + \gamma^{(Y)}_{\substack{l\\pt}} C^{(1)}_{\substack{Hl\\tr}} + C^{(1)}_{\substack{Hl\\pt}}\gamma^{(Y)}_{\substack{l\\tr}} \tag{B.32}$$

$$\dot{C}^{(3)}_{\substack{Hl\\pr}} = -\frac{1}{2}[Y_e^\dagger Y_e]_{pr} C_{H\Box} + \frac{1}{2}[Y_e^\dagger Y_e]_{pt}\Big(3C^{(1)}_{\substack{Hl\\tr}} + C^{(3)}_{\substack{Hl\\tr}}\Big) + \frac{1}{2}\Big(3C^{(1)}_{\substack{Hl\\pt}} + C^{(3)}_{\substack{Hl\\pt}}\Big)[Y_e^\dagger Y_e]_{tr}$$
$$- \Big(C_{\substack{ll\\ptsr}} + C_{\substack{ll\\srpt}}\Big)[{Y_e}^\dagger Y_e]_{ts} - 2N_c C^{(3)}_{\substack{lq\\prst}}[Y_d^\dagger Y_d + Y_u^\dagger Y_u]_{ts} + 2\gamma_H^{(Y)} C^{(3)}_{\substack{Hl\\pr}} + \gamma^{(Y)}_{\substack{l\\pt}} C^{(3)}_{\substack{Hl\\tr}}$$
$$+ C^{(3)}_{\substack{Hl\\pt}}\gamma^{(Y)}_{\substack{l\\tr}} \tag{B.33}$$

$$\dot{C}_{\substack{He\\pr}} = [Y_e Y_e^\dagger]_{pr}\left(C_{H\Box}+C_{HD}\right) - 2[Y_e]_{ps} C^{(1)}_{\substack{Hl\\st}}[Y_e^\dagger]_{tr} + 3[Y_e Y_e^\dagger]_{pt} C_{\substack{He\\tr}} + 3C_{\substack{He\\pt}}[Y_e Y_e^\dagger]_{tr}$$
$$- 2C_{\substack{le\\stpr}}[{Y_e}^\dagger Y_e]_{ts} + 2\Big(C_{\substack{ee\\prst}} + C_{\substack{ee\\stpr}} + C_{\substack{ee\\ptsr}} + C_{\substack{ee\\srpt}}\Big)[Y_e {Y_e}^\dagger]_{ts} - 2N_c C_{\substack{eu\\prst}}[Y_u {Y_u}^\dagger]_{ts}$$
$$+ 2N_c C_{\substack{ed\\prst}}[Y_d {Y_d}^\dagger]_{ts} - 2N_c C_{\substack{qe\\stpr}}[{Y_d}^\dagger Y_d - {Y_u}^\dagger Y_u]_{ts} + 2\gamma_H^{(Y)} C_{\substack{He\\pr}} + \gamma^{(Y)}_{\substack{e\\pt}} C_{\substack{He\\tr}} + C_{\substack{He\\pt}}\gamma^{(Y)}_{\substack{e\\tr}} \tag{B.34}$$

Class ψ^4 $(\overline{L}L)(\overline{L}L)$:

$$\dot{C}_{\underset{prst}{ll}} = -\frac{1}{2}[Y_e^\dagger Y_e]_{pr} C^{(1)}_{\underset{st}{Hl}} - \frac{1}{2}[Y_e^\dagger Y_e]_{st} C^{(1)}_{\underset{pr}{Hl}} + \frac{1}{2}[Y_e^\dagger Y_e]_{pr} C^{(3)}_{\underset{st}{Hl}} + \frac{1}{2}[Y_e^\dagger Y_e]_{st} C^{(3)}_{\underset{pr}{Hl}}$$
$$- [Y_e^\dagger Y_e]_{sr} C^{(3)}_{\underset{pt}{Hl}} - [Y_e^\dagger Y_e]_{pt} C^{(3)}_{\underset{sr}{Hl}} - \frac{1}{2}[Y_e^\dagger]_{sv}[Y_e]_{wt} C_{\underset{prvw}{le}} - \frac{1}{2}[Y_e^\dagger]_{pv}[Y_e]_{wr} C_{\underset{stvw}{le}}$$
$$+ \gamma^{(Y)}_{\underset{pv}{l}} C_{\underset{vrst}{ll}} + \gamma^{(Y)}_{\underset{sv}{l}} C_{\underset{prvt}{ll}} + C_{\underset{pvst}{ll}} \gamma^{(Y)}_{\underset{vr}{l}} + C_{\underset{prsv}{ll}} \gamma^{(Y)}_{\underset{vt}{l}} \tag{B.35}$$

$$\dot{C}^{(1)}_{\underset{prst}{qq}} = \frac{1}{2}[Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{pr} C^{(1)}_{\underset{st}{Hq}} + \frac{1}{2}[Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{st} C^{(1)}_{\underset{pr}{Hq}}$$
$$+ \frac{1}{4N_c}\left([Y_u^\dagger]_{pv}[Y_u]_{wr} C^{(8)}_{\underset{stvw}{qu}} + [Y_u^\dagger]_{sv}[Y_u]_{wt} C^{(8)}_{\underset{prvw}{qu}}\right) + \frac{1}{4N_c}\left([Y_d^\dagger]_{pv}[Y_d]_{wr} C^{(8)}_{\underset{stvw}{qd}}\right.$$
$$\left. + [Y_d^\dagger]_{sv}[Y_d]_{wt} C^{(8)}_{\underset{prvw}{qd}}\right) - \frac{1}{8}\left([Y_u^\dagger]_{pv}[Y_u]_{wt} C^{(8)}_{\underset{srvw}{qu}} + [Y_u^\dagger]_{sv}[Y_u]_{wr} C^{(8)}_{\underset{ptvw}{qu}}\right)$$
$$- \frac{1}{8}\left([Y_d^\dagger]_{pv}[Y_d]_{wt} C^{(8)}_{\underset{srvw}{qd}} + [Y_d^\dagger]_{sv}[Y_d]_{wr} C^{(8)}_{\underset{ptvw}{qd}}\right) + \frac{1}{16N_c}\left([Y_d]_{wt}[Y_u]_{vr} C^{(8)}_{\underset{pvsw}{quqd}}\right.$$
$$\left. + [Y_d]_{wr}[Y_u]_{vt} C^{(8)}_{\underset{svpw}{quqd}}\right) + \frac{1}{16N_c}\left([Y_d^\dagger]_{sw}[Y_u^\dagger]_{pv} C^{(8)*}_{\underset{rvtw}{quqd}} + [Y_d^\dagger]_{pw}[Y_u^\dagger]_{sv} C^{(8)*}_{\underset{tvrw}{quqd}}\right)$$
$$+ \frac{1}{16}\left([Y_d]_{wt}[Y_u]_{vr} C^{(8)}_{\underset{svpw}{quqd}} + [Y_d]_{wr}[Y_u]_{vt} C^{(8)}_{\underset{pvsw}{quqd}}\right) + \frac{1}{16}\left([Y_d^\dagger]_{sw}[Y_u^\dagger]_{pv} C^{(8)*}_{\underset{tvrw}{quqd}}\right.$$
$$\left. + [Y_d^\dagger]_{pw}[Y_u^\dagger]_{sv} C^{(8)*}_{\underset{rvtw}{quqd}}\right) - \frac{1}{2}[Y_u^\dagger]_{pv}[Y_u]_{wr} C^{(1)}_{\underset{stvw}{qu}} - \frac{1}{2}[Y_d^\dagger]_{pv}[Y_d]_{wr} C^{(1)}_{\underset{stvw}{qd}}$$
$$- \frac{1}{2}[Y_u^\dagger]_{sv}[Y_u]_{wt} C^{(1)}_{\underset{prvw}{qu}} - \frac{1}{2}[Y_d^\dagger]_{sv}[Y_d]_{wt} C^{(1)}_{\underset{prvw}{qd}} - \frac{1}{8}[Y_d]_{wt}[Y_u]_{vr} C^{(1)}_{\underset{pvsw}{quqd}}$$
$$- \frac{1}{8}[Y_d^\dagger]_{sw}[Y_u^\dagger]_{pv} C^{(1)*}_{\underset{rvtw}{quqd}} - \frac{1}{8}[Y_d]_{wr}[Y_u]_{vt} C^{(1)}_{\underset{svpw}{quqd}} - \frac{1}{8}[Y_d^\dagger]_{pw}[Y_u^\dagger]_{sv} C^{(1)*}_{\underset{tvrw}{quqd}}$$
$$+ \gamma^{(Y)}_{\underset{pv}{q}} C^{(1)}_{\underset{vrst}{qq}} + \gamma^{(Y)}_{\underset{sv}{q}} C^{(1)}_{\underset{prvt}{qq}} + C^{(1)}_{\underset{pvst}{qq}} \gamma^{(Y)}_{\underset{vr}{q}} + C^{(1)}_{\underset{prsv}{qq}} \gamma^{(Y)}_{\underset{vt}{q}} \tag{B.36}$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\substack{qq\\prst}} &= -\frac{1}{2}[Y_u^\dagger Y_u + Y_d^\dagger Y_d]_{pr} C^{(3)}_{\substack{Hq\\st}} - \frac{1}{2}[Y_u^\dagger Y_u + Y_d^\dagger Y_d]_{st} C^{(3)}_{\substack{Hq\\pr}} \\
&\quad - \frac{1}{8}\left([Y_u^\dagger]_{pv}\,[Y_u]_{wt}\,C^{(8)}_{\substack{qu\\srvw}} + [Y_u^\dagger]_{sv}\,[Y_u]_{wr}\,C^{(8)}_{\substack{qu\\ptvw}}\right) - \frac{1}{8}\left([Y_d^\dagger]_{pv}\,[Y_d]_{wt}\,C^{(8)}_{\substack{qd\\srvw}}\right. \\
&\qquad \left. + [Y_d^\dagger]_{sv}\,[Y_d]_{wr}\,C^{(8)}_{\substack{qd\\ptvw}}\right) - \frac{1}{16N_c}\left([Y_d]_{wt}\,[Y_u]_{vr}\,C^{(8)}_{\substack{quqd\\pvsw}} + [Y_d]_{wr}\,[Y_u]_{vt}\,C^{(8)}_{\substack{quqd\\svpw}}\right) \\
&\quad - \frac{1}{16N_c}\left([Y_d^\dagger]_{sw}\,[Y_u^\dagger]_{pv}\,C^{(8)*}_{\substack{quqd\\rvtw}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(8)*}_{\substack{quqd\\tvrw}}\right) \\
&\quad - \frac{1}{16}\left([Y_d]_{wt}\,[Y_u]_{vr}\,C^{(8)}_{\substack{quqd\\svpw}} + [Y_d]_{wr}\,[Y_u]_{vt}\,C^{(8)}_{\substack{quqd\\pvsw}}\right) - \frac{1}{16}\left([Y_d^\dagger]_{sw}\,[Y_u^\dagger]_{pv}\,C^{(8)*}_{\substack{quqd\\tvrw}}\right. \\
&\qquad \left. + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(8)*}_{\substack{quqd\\rvtw}}\right) + \frac{1}{8}[Y_d]_{wt}\,[Y_u]_{vr}\,C^{(1)}_{\substack{quqd\\pvsw}} + \frac{1}{8}[Y_d^\dagger]_{sw}\,[Y_u^\dagger]_{pv}\,C^{(1)*}_{\substack{quqd\\rvtw}} \\
&\quad + \frac{1}{8}[Y_d]_{wr}\,[Y_u]_{vt}\,C^{(1)}_{\substack{quqd\\svpw}} + \frac{1}{8}[Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(1)*}_{\substack{quqd\\tvrw}} + \gamma^{(Y)}_{\substack{q\\pv}} C^{(3)}_{\substack{qq\\vrst}} + \gamma^{(Y)}_{\substack{q\\sv}} C^{(3)}_{\substack{qq\\prvt}} + C^{(3)}_{\substack{qq\\pvst}}\,\gamma^{(Y)}_{\substack{q\\vr}} \\
&\quad + C^{(3)}_{\substack{qq\\prsv}}\,\gamma^{(Y)}_{\substack{q\\vt}} \qquad \text{(B.37)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{lq\\prst}} &= -[Y_e^\dagger Y_e]_{pr} C^{(1)}_{\substack{Hq\\st}} + [Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{st} C^{(1)}_{\substack{Hl\\pr}} + \frac{1}{4}\,[Y_u]_{wt}\,[Y_e]_{vr}\,C^{(1)}_{\substack{lequ\\pvsw}} \\
&\quad + \frac{1}{4}\,[Y_u^\dagger]_{sw}\,[Y_e^\dagger]_{pv}\,C^{(1)*}_{\substack{lequ\\rvtw}} - [Y_u^\dagger]_{sv}\,[Y_u]_{wt}\,C_{\substack{lu\\prvw}} - [Y_d^\dagger]_{sv}\,[Y_d]_{wt}\,C_{\substack{ld\\prvw}} \\
&\quad - [Y_e^\dagger]_{pv}\,[Y_e]_{wr}\,C_{\substack{qe\\stvw}} + \frac{1}{4}\left([Y_d^\dagger]_{sw}\,[Y_e]_{vr}\,C_{\substack{ledq\\pvwt}} + [Y_e^\dagger]_{pv}\,[Y_d]_{wt}\,C^*_{\substack{ledq\\rvws}}\right) \\
&\quad - 3\left([Y_e]_{vr}\,[Y_u]_{wt}\,C^{(3)}_{\substack{lequ\\pvsw}} + [Y_e^\dagger]_{pv}\,[Y_u^\dagger]_{sw}\,C^{(3)*}_{\substack{lequ\\rvtw}}\right) + \gamma^{(Y)}_{\substack{l\\pv}} C^{(1)}_{\substack{lq\\vrst}} + \gamma^{(Y)}_{\substack{q\\sv}} C^{(1)}_{\substack{lq\\prvt}} + C^{(1)}_{\substack{lq\\pvst}}\,\gamma^{(Y)}_{\substack{l\\vr}} \\
&\quad + C^{(1)}_{\substack{lq\\prsv}}\,\gamma^{(Y)}_{\substack{q\\vt}} \qquad \text{(B.38)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\substack{lq\\prst}} &= -[Y_e^\dagger Y_e]_{pr} C^{(3)}_{\substack{Hq\\st}} - [Y_u^\dagger Y_u + Y_d^\dagger Y_d]_{st} C^{(3)}_{\substack{Hl\\pr}} - \frac{1}{4}\,[Y_u]_{wt}\,[Y_e]_{vr}\,C^{(1)}_{\substack{lequ\\pvsw}} \\
&\quad - \frac{1}{4}\,[Y_u^\dagger]_{sw}\,[Y_e^\dagger]_{pv}\,C^{(1)*}_{\substack{lequ\\rvtw}} + \frac{1}{4}\left([Y_d^\dagger]_{sw}\,[Y_e]_{vr}\,C_{\substack{ledq\\pvwt}} + [Y_e^\dagger]_{pv}\,[Y_d]_{wt}\,C^*_{\substack{ledq\\rvws}}\right) \\
&\quad + 3\left([Y_e]_{vr}\,[Y_u]_{wt}\,C^{(3)}_{\substack{lequ\\pvsw}} + [Y_e^\dagger]_{pv}\,[Y_u^\dagger]_{sw}\,C^{(3)*}_{\substack{lequ\\rvtw}}\right) + \gamma^{(Y)}_{\substack{l\\pv}} C^{(3)}_{\substack{lq\\vrst}} + \gamma^{(Y)}_{\substack{q\\sv}} C^{(3)}_{\substack{lq\\prvt}} + C^{(3)}_{\substack{lq\\pvst}}\,\gamma^{(Y)}_{\substack{l\\vr}} \\
&\quad + C^{(3)}_{\substack{lq\\prsv}}\,\gamma^{(Y)}_{\substack{q\\vt}} \qquad \text{(B.39)}
\end{aligned}
$$

$(\overline{R}R)(\overline{R}R)$:

$$\dot{C}_{\substack{ee\\prst}} = [Y_e Y_e^\dagger]_{pr} C_{\substack{He\\st}} + [Y_e Y_e^\dagger]_{st} C_{\substack{He\\pr}} - [Y_e^\dagger]_{wr}\,[Y_e]_{pv}\,C_{\substack{le\\vwst}} - [Y_e^\dagger]_{wt}\,[Y_e]_{sv}\,C_{\substack{le\\vwpr}}$$
$$+ \gamma^{(Y)}_{\substack{e\\pv}} C_{\substack{ee\\vrst}} + \gamma^{(Y)}_{\substack{e\\sv}} C_{\substack{ee\\prvt}} + C_{\substack{ee\\pvst}} \gamma^{(Y)}_{\substack{e\\vr}} + C_{\substack{ee\\prsv}} \gamma^{(Y)}_{\substack{e\\vt}} \tag{B.40}$$

$$\dot{C}_{\substack{uu\\prst}} = -[Y_u Y_u^\dagger]_{pr} C_{\substack{Hu\\st}} - [Y_u Y_u^\dagger]_{st} C_{\substack{Hu\\pr}} - [Y_u^\dagger]_{wr}\,[Y_u]_{pv}\,C^{(1)}_{\substack{qu\\vwst}} - [Y_u^\dagger]_{wt}\,[Y_u]_{sv}\,C^{(1)}_{\substack{qu\\vwpr}}$$
$$+ \frac{1}{2\,N_c}[Y_u]_{pv}\,[Y_u^\dagger]_{wr}\,C^{(8)}_{\substack{qu\\vwst}} + \frac{1}{2\,N_c}[Y_u]_{sv}\,[Y_u^\dagger]_{wt}\,C^{(8)}_{\substack{qu\\vwpr}} - \frac{1}{2}[Y_u^\dagger]_{wr}\,[Y_u]_{sv}\,C^{(8)}_{\substack{qu\\vwpt}}$$
$$- \frac{1}{2}[Y_u^\dagger]_{wt}\,[Y_u]_{pv}\,C^{(8)}_{\substack{qu\\vwsr}} + \gamma^{(Y)}_{\substack{u\\pv}} C_{\substack{uu\\vrst}} + \gamma^{(Y)}_{\substack{u\\sv}} C_{\substack{uu\\prvt}} + C_{\substack{uu\\pvst}} \gamma^{(Y)}_{\substack{u\\vr}} + C_{\substack{uu\\prsv}} \gamma^{(Y)}_{\substack{u\\vt}} \tag{B.41}$$

$$\dot{C}_{\substack{dd\\prst}} = [Y_d Y_d^\dagger]_{pr} C_{\substack{Hd\\st}} + [Y_d Y_d^\dagger]_{st} C_{\substack{Hd\\pr}} - [Y_d^\dagger]_{wr}\,[Y_d]_{pv}\,C^{(1)}_{\substack{qd\\vwst}} - [Y_d^\dagger]_{wt}\,[Y_d]_{sv}\,C^{(1)}_{\substack{qd\\vwpr}}$$
$$+ \frac{1}{2\,N_c}[Y_d]_{pv}\,[Y_d^\dagger]_{wr}\,C^{(8)}_{\substack{qd\\vwst}} + \frac{1}{2\,N_c}[Y_d]_{sv}\,[Y_d^\dagger]_{wt}\,C^{(8)}_{\substack{qd\\vwpr}} - \frac{1}{2}[Y_d^\dagger]_{wr}\,[Y_d]_{sv}\,C^{(8)}_{\substack{qd\\vwpt}}$$
$$- \frac{1}{2}[Y_d^\dagger]_{wt}\,[Y_d]_{pv}\,C^{(8)}_{\substack{qd\\vwsr}} + \gamma^{(Y)}_{\substack{d\\pv}} C_{\substack{dd\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}} C_{\substack{dd\\prvt}} + C_{\substack{dd\\pvst}} \gamma^{(Y)}_{\substack{d\\vr}} + C_{\substack{dd\\prsv}} \gamma^{(Y)}_{\substack{d\\vt}} \tag{B.42}$$

$$\dot{C}_{\substack{eu\\prst}} = 2[Y_e Y_e^\dagger]_{pr} C_{\substack{Hu\\st}} - 2[Y_u Y_u^\dagger]_{st} C_{\substack{He\\pr}} + \left([Y_e]_{pv}\,[Y_u]_{sw}\,C^{(1)}_{\substack{lequ\\vrwt}} + [Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{wt}\,C^{(1)*}_{\substack{lequ\\vpws}}\right)$$
$$- 2\,[Y_e]_{pv}\,[Y_e^\dagger]_{wr}\,C_{\substack{lu\\vwst}} - 12\,\left([Y_e]_{pv}\,[Y_u]_{sw}\,C^{(3)}_{\substack{lequ\\vrwt}} + [Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{wt}\,C^{(3)*}_{\substack{lequ\\vpws}}\right)$$
$$- 2\,[Y_u]_{sv}\,[Y_u^\dagger]_{wt}\,C_{\substack{qe\\vwpr}} + \gamma^{(Y)}_{\substack{e\\pv}} C_{\substack{eu\\vrst}} + \gamma^{(Y)}_{\substack{u\\sv}} C_{\substack{eu\\prvt}} + C_{\substack{eu\\pvst}} \gamma^{(Y)}_{\substack{e\\vr}} + C_{\substack{eu\\prsv}} \gamma^{(Y)}_{\substack{u\\vt}} \tag{B.43}$$

$$\dot{C}_{\substack{ed\\prst}} = 2[Y_e Y_e^\dagger]_{pr} C_{\substack{Hd\\st}} + 2[Y_d Y_d^\dagger]_{st} C_{\substack{He\\pr}} - 2[Y_e]_{pv}\,[Y_e^\dagger]_{wr}\,C_{\substack{ld\\vwst}} - 2[Y_d]_{sv}\,[Y_d^\dagger]_{wt}\,C_{\substack{qe\\vwpr}}$$
$$+ [Y_e]_{pv}\,[Y_d^\dagger]_{wt}\,C_{\substack{ledq\\vrsw}} + [Y_e^\dagger]_{vr}\,[Y_d]_{sw}\,C^{*}_{\substack{ledq\\vptw}} + \gamma^{(Y)}_{\substack{e\\pv}} C_{\substack{ed\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}} C_{\substack{ed\\prvt}} + C_{\substack{ed\\pvst}} \gamma^{(Y)}_{\substack{e\\vr}}$$
$$+ C_{\substack{ed\\prsv}} \gamma^{(Y)}_{\substack{d\\vt}} \tag{B.44}$$

$$\dot{C}^{(1)}_{\substack{ud\\prst}} = -2[Y_uY_u^\dagger]_{pr}C_{\substack{Hd\\st}} + 2[Y_dY_d^\dagger]_{st}C_{\substack{Hu\\pr}} + \frac{2}{N_c}[Y_dY_u^\dagger]_{sr}C_{\substack{Hud\\pt}} + \frac{2}{N_c}[Y_uY_d^\dagger]_{pt}C^*_{\substack{Hud\\rs}}$$
$$+ \frac{1}{N_c}\left([Y_d]_{sv}\,[Y_u]_{pw}\,C^{(1)}_{\substack{quqd\\vrwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(1)*}_{\substack{quqd\\vpws}}\right) - [Y_d]_{sw}\,[Y_u]_{pv}\,C^{(1)}_{\substack{quqd\\vrwt}}$$
$$- [Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(1)*}_{\substack{quqd\\vpws}} + \frac{N_c^2-1}{2\,N_c^2}\left([Y_d]_{sv}\,[Y_u]_{pw}\,C^{(8)}_{\substack{quqd\\vrwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(8)*}_{\substack{quqd\\vpws}}\right)$$
$$- 2[Y_u]_{pv}\,[Y_u^\dagger]_{wr}\,C^{(1)}_{\substack{qd\\vwst}} - 2[Y_d]_{sv}\,[Y_d^\dagger]_{wt}\,C^{(1)}_{\substack{qu\\vwpr}} + \gamma^{(Y)}_{\substack{u\\pv}}C^{(1)}_{\substack{ud\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}}C^{(1)}_{\substack{ud\\prvt}} + C^{(1)}_{\substack{ud\\pvst}}\,\gamma^{(Y)}_{\substack{u\\vr}}$$
$$+ C^{(1)}_{\substack{ud\\prsv}}\,\gamma^{(Y)}_{\substack{d\\vt}} \tag{B.45}$$

$$\dot{C}^{(8)}_{\substack{ud\\prst}} = 4[Y_dY_u^\dagger]_{sr}C_{\substack{Hud\\pt}} + 4[Y_uY_d^\dagger]_{pt}C^*_{\substack{Hud\\rs}} + 2\left([Y_d]_{sv}\,[Y_u]_{pw}\,C^{(1)}_{\substack{quqd\\vrwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(1)*}_{\substack{quqd\\vpws}}\right)$$
$$- 2[Y_u]_{pv}\,[Y_u^\dagger]_{wr}\,C^{(8)}_{\substack{qd\\vwst}} - 2[Y_d]_{sv}\,[Y_d^\dagger]_{wt}\,C^{(8)}_{\substack{qu\\vwpr}} - \frac{1}{N_c}\left([Y_d]_{sv}\,[Y_u]_{pw}\,C^{(8)}_{\substack{quqd\\vrwt}}\right.$$
$$\left.+[Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(8)*}_{\substack{quqd\\vpws}}\right) - \left([Y_d]_{sw}\,[Y_u]_{pv}\,C^{(8)}_{\substack{quqd\\vrwt}} + [Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(8)*}_{\substack{quqd\\vpws}}\right)$$
$$+ \gamma^{(Y)}_{\substack{u\\pv}}C^{(8)}_{\substack{ud\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}}C^{(8)}_{\substack{ud\\prvt}} + C^{(8)}_{\substack{ud\\pvst}}\,\gamma^{(Y)}_{\substack{u\\vr}} + C^{(8)}_{\substack{ud\\prsv}}\,\gamma^{(Y)}_{\substack{d\\vt}} \tag{B.46}$$

$(\overline{L}L)(\overline{R}R)$:

$$\dot{C}_{\substack{le\\prst}} = [Y_e]_{sr}\xi_{\substack{e\\pt}} + [Y_e^\dagger]_{pt}\xi^*_{\substack{e\\rs}} - [Y_e^\dagger Y_e]_{pr}C_{\substack{He\\st}} + 2[Y_eY_e^\dagger]_{st}C^{(1)}_{\substack{Hl\\pr}} - [Y_e^\dagger]_{pv}\,[Y_e]_{wr}\,C_{\substack{ee\\vtsw}}$$
$$- [Y_e^\dagger]_{pw}\,[Y_e]_{vr}\,C_{\substack{ee\\wtsv}} - 2[Y_e^\dagger]_{pv}\,[Y_e]_{wr}\,C_{\substack{ee\\vwst}} + [Y_e^\dagger]_{pw}\,[Y_e]_{sv}\,C_{\substack{le\\vrwt}} - [Y_e^\dagger]_{wt}\,[Y_e]_{sv}\,C_{\substack{ll\\pwvr}}$$
$$- [Y_e^\dagger]_{vt}\,[Y_e]_{sw}\,C_{\substack{ll\\pvwr}} - 4[Y_e^\dagger]_{wt}\,[Y_e]_{sv}\,C_{\substack{ll\\prvw}} + [Y_e^\dagger]_{vt}\,[Y_e]_{wr}\,C_{\substack{le\\pvsw}} + \gamma^{(Y)}_{\substack{l\\pv}}C_{\substack{le\\vrst}}$$
$$+ \gamma^{(Y)}_{\substack{e\\sv}}C_{\substack{le\\prvt}} + C_{\substack{le\\pvst}}\,\gamma^{(Y)}_{\substack{l\\vr}} + C_{\substack{le\\prsv}}\,\gamma^{(Y)}_{\substack{e\\vt}} \tag{B.47}$$

$$\dot{C}_{\substack{lu\\prst}} = -[Y_e^\dagger Y_e]_{pr}C_{\substack{Hu\\st}} - 2[Y_uY_u^\dagger]_{st}C^{(1)}_{\substack{Hl\\pr}} - \frac{1}{2}\left([Y_e]_{vr}\,[Y_u]_{sw}\,C^{(1)}_{\substack{lequ\\pvwt}} + [Y_e^\dagger]_{pv}\,[Y_u^\dagger]_{wt}\,C^{(1)*}_{\substack{lequ\\rvws}}\right)$$
$$- 2\,[Y_u]_{sv}\,[Y_u^\dagger]_{wt}\,C^{(1)}_{\substack{lq\\prvw}} - 6\left([Y_e]_{vr}\,[Y_u]_{sw}\,C^{(3)}_{\substack{lequ\\pvwt}} + [Y_e^\dagger]_{pv}\,[Y_u^\dagger]_{wt}\,C^{(3)*}_{\substack{lequ\\rvws}}\right)$$
$$- [Y_e]_{wr}\,[Y_e^\dagger]_{pv}\,C_{\substack{eu\\vwst}} + \gamma^{(Y)}_{\substack{l\\pv}}C_{\substack{lu\\vrst}} + \gamma^{(Y)}_{\substack{u\\sv}}C_{\substack{lu\\prvt}} + C_{\substack{lu\\pvst}}\,\gamma^{(Y)}_{\substack{l\\vr}} + C_{\substack{lu\\prsv}}\,\gamma^{(Y)}_{\substack{u\\vt}} \tag{B.48}$$

$$
\begin{aligned}
\dot{C}_{\substack{ld\\prst}} &= -[Y_e^\dagger Y_e]_{pr} C_{\substack{Hd\\st}} + 2[Y_d Y_d^\dagger]_{st} C^{(1)}_{\substack{Hl\\pr}} - \frac{1}{2}\left([Y_e]_{vr}\,[Y_d^\dagger]_{wt}\,C_{\substack{ledq\\pvsw}} + [Y_e^\dagger]_{pv}\,[Y_d]_{sw}\,C^*_{\substack{ledq\\rvtw}}\right)\\
&\quad - 2\,[Y_d]_{sv}\,[Y_d^\dagger]_{wt}\,C^{(1)}_{\substack{lq\\prvw}} - [Y_e]_{wr}\,[Y_e^\dagger]_{pv}\,C_{\substack{ed\\vwst}} + \gamma^{(Y)}_{\substack{l\\pv}} C_{\substack{ld\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}} C_{\substack{ld\\prvt}} + C_{\substack{ld\\pvst}}\gamma^{(Y)}_{\substack{l\\vr}}\\
&\quad + C_{\substack{ld\\prsv}}\gamma^{(Y)}_{\substack{d\\vt}}
\end{aligned}
\tag{B.49}
$$

$$
\begin{aligned}
\dot{C}_{\substack{qe\\prst}} &= [Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{pr} C_{\substack{He\\st}} + 2[Y_e Y_e^\dagger]_{st} C^{(1)}_{\substack{Hq\\pr}} - \frac{1}{2}\left([Y_d^\dagger]_{pw}\,[Y_e]_{sv}\,C_{\substack{ledq\\vtwr}}\right.\\
&\qquad \left.+[Y_e^\dagger]_{vt}\,[Y_d]_{wr}\,C^*_{\substack{ledq\\vswp}}\right) - 2\,[Y_e]_{sv}\,[Y_e^\dagger]_{wt}\,C^{(1)}_{\substack{lq\\vwpr}} - \frac{1}{2}\left([Y_u]_{wr}\,[Y_e]_{sv}\,C^{(1)}_{\substack{lequ\\vtpw}}\right.\\
&\qquad \left.+[Y_e^\dagger]_{vt}\,[Y_u^\dagger]_{pw}\,C^{(1)*}_{\substack{lequ\\vsrw}}\right) - [Y_d]_{wr}\,[Y_d^\dagger]_{pv}\,C_{\substack{ed\\stvw}}\\
&\quad - 6\left([Y_e]_{sv}\,[Y_u]_{wr}\,C^{(3)}_{\substack{lequ\\vtpw}} + [Y_e^\dagger]_{vt}\,[Y_u^\dagger]_{pw}\,C^{(3)*}_{\substack{lequ\\vsrw}}\right) - [Y_u]_{wr}\,[Y_u^\dagger]_{pv}\,C_{\substack{eu\\stvw}}\\
&\quad + \gamma^{(Y)}_{\substack{q\\pv}} C_{\substack{qe\\vrst}} + \gamma^{(Y)}_{\substack{e\\sv}} C_{\substack{qe\\prvt}} + C_{\substack{qe\\pvst}}\gamma^{(Y)}_{\substack{q\\vr}} + C_{\substack{qe\\prsv}}\gamma^{(Y)}_{\substack{e\\vt}}
\end{aligned}
\tag{B.50}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{qu\\prst}} &= \frac{1}{N_c}[Y_u]_{sr}\xi_{\substack{u\\pt}} + \frac{1}{N_c}[Y_u^\dagger]_{pt}\xi^*_{\substack{u\\rs}} + [Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{pr} C_{\substack{Hu\\st}} - 2[Y_u Y_u^\dagger]_{st} C^{(1)}_{\substack{Hq\\pr}}\\
&\quad + \frac{1}{N_c}\left([Y_u^\dagger]_{pw}\,[Y_u]_{sv}\,C^{(1)}_{\substack{qu\\vrwt}} + [Y_u^\dagger]_{vt}\,[Y_u]_{wr}\,C^{(1)}_{\substack{qu\\pvsw}} + [Y_d]_{wr}\,[Y_u]_{sv}\,C^{(1)}_{\substack{quqd\\ptvw}}\right.\\
&\qquad \left.+[Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vt}\,C^{(1)*}_{\substack{quqd\\rsvw}}\right) - \frac{1}{2\,N_c^2}\left([Y_u^\dagger]_{pw}\,[Y_u]_{sv}\,C^{(8)}_{\substack{qu\\vrwt}} + [Y_u^\dagger]_{vt}\,[Y_u]_{wr}\,C^{(8)}_{\substack{qu\\pvsw}}\right.\\
&\qquad \left.+[Y_d]_{wr}\,[Y_u]_{sv}\,C^{(8)}_{\substack{quqd\\ptvw}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vt}\,C^{(8)*}_{\substack{quqd\\rsvw}}\right)\\
&\quad - \frac{2}{N_c}\left([Y_u^\dagger]_{vt}\,[Y_u]_{sw}\,C^{(1)}_{\substack{qq\\pvwr}} + [Y_u^\dagger]_{pv}\,[Y_u]_{wr}\,C_{\substack{uu\\vtsw}}\right)\\
&\quad - \frac{6}{N_c}[Y_u^\dagger]_{vt}\,[Y_u]_{sw}\,C^{(3)}_{\substack{qq\\pvwr}} + \frac{1}{2}\left([Y_u^\dagger]_{pw}\,[Y_u]_{sv}\,C^{(8)}_{\substack{qu\\vrwt}} + [Y_u^\dagger]_{vt}\,[Y_u]_{wr}\,C^{(8)}_{\substack{qu\\pvsw}}\right)\\
&\quad + \frac{1}{2}\left([Y_u]_{sv}\,[Y_d]_{wr}\,C^{(1)}_{\substack{quqd\\vtpw}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vt}\,C^{(1)*}_{\substack{quqd\\vsrw}} + [Y_u]_{sv}\,[Y_d]_{wr}\,C^{(8)}_{\substack{quqd\\ptvw}}\right.\\
&\qquad \left.+[Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vt}\,C^{(8)*}_{\substack{quqd\\rsvw}}\right) - 4[Y_u^\dagger]_{wt}\,[Y_u]_{sv}\,C^{(1)}_{\substack{qq\\prvw}} - 2[Y_u^\dagger]_{pv}\,[Y_u]_{wr}\,C_{\substack{uu\\vwst}}\\
&\quad - [Y_d^\dagger]_{pv}\,[Y_d]_{wr}\,C^{(1)}_{\substack{ud\\stvw}} + \gamma^{(Y)}_{\substack{q\\pv}} C^{(1)}_{\substack{qu\\vrst}} + \gamma^{(Y)}_{\substack{u\\sv}} C^{(1)}_{\substack{qu\\prvt}} + C^{(1)}_{\substack{qu\\pvst}}\gamma^{(Y)}_{\substack{q\\vr}} + C^{(1)}_{\substack{qu\\prsv}}\gamma^{(Y)}_{\substack{u\\vt}}
\end{aligned}
\tag{B.51}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{qd\\prst}} &= \frac{1}{N_c}[Y_d]_{sr}\xi_{\substack{d\\pt}} + \frac{1}{N_c}[Y_d^\dagger]_{pt}\xi^*_{\substack{d\\rs}} + [Y_u^\dagger Y_u - Y_d^\dagger Y_d]_{pr} C_{\substack{Hd\\st}} + 2[Y_d Y_d^\dagger]_{st} C^{(1)}_{\substack{Hq\\pr}} \\
&+ \frac{1}{N_c}\Bigg([Y_d^\dagger]_{pw}\,[Y_d]_{sv}\,C^{(1)}_{\substack{qd\\vrwt}} + [Y_d^\dagger]_{vt}\,[Y_d]_{wr}\,C^{(1)}_{\substack{qd\\pvsw}} + [Y_u]_{wr}\,[Y_d]_{sv}\,C^{(1)}_{\substack{quqd\\vwpt}} \\
&\quad + [Y_u^\dagger]_{pw}\,[Y_d^\dagger]_{vt}\,C^{(1)*}_{\substack{quqd\\vwrs}}\Bigg) \\
&- \frac{1}{2\,N_c^2}\Bigg([Y_d^\dagger]_{pw}\,[Y_d]_{sv}\,C^{(8)}_{\substack{qd\\vrwt}} + [Y_d^\dagger]_{vt}\,[Y_d]_{wr}\,C^{(8)}_{\substack{qd\\pvsw}} + [Y_u]_{wr}\,[Y_d]_{sv}\,C^{(8)}_{\substack{quqd\\vwpt}} \\
&\quad + [Y_u^\dagger]_{pw}\,[Y_d^\dagger]_{vt}\,C^{(8)*}_{\substack{quqd\\vwrs}}\Bigg) - \frac{2}{N_c}\Bigg([Y_d^\dagger]_{vt}\,[Y_d]_{sw}\,C^{(1)}_{\substack{qq\\pvwr}} + [Y_d^\dagger]_{pv}\,[Y_d]_{wr}\,C_{\substack{dd\\vtsw}}\Bigg) \\
&- \frac{6}{N_c}[Y_d^\dagger]_{vt}\,[Y_d]_{sw}\,C^{(3)}_{\substack{qq\\pvwr}} + \frac{1}{2}\Bigg([Y_d^\dagger]_{pw}\,[Y_d]_{sv}\,C^{(8)}_{\substack{qd\\vrwt}} + [Y_d^\dagger]_{vt}\,[Y_d]_{wr}\,C^{(8)}_{\substack{qd\\pvsw}}\Bigg) \\
&+ \frac{1}{2}\Bigg([Y_d]_{sw}\,[Y_u]_{vr}\,C^{(1)}_{\substack{quqd\\pvwt}} + [Y_u^\dagger]_{pv}\,[Y_d^\dagger]_{wt}\,C^{(1)*}_{\substack{quqd\\rvws}} + [Y_d]_{sv}\,[Y_u]_{wr}\,C^{(8)}_{\substack{quqd\\vwpt}} \\
&\quad + [Y_u^\dagger]_{pw}\,[Y_d^\dagger]_{vt}\,C^{(8)*}_{\substack{quqd\\vwrs}}\Bigg) - 4[Y_d^\dagger]_{wt}\,[Y_d]_{sv}\,C^{(1)}_{\substack{qq\\prvw}} - 2[Y_d^\dagger]_{pv}\,[Y_d]_{wr}\,C_{\substack{dd\\vwst}} \\
&- [Y_u^\dagger]_{pv}\,[Y_u]_{wr}\,C^{(1)}_{\substack{ud\\vwst}} + \gamma^{(Y)}_{\substack{q\\pv}} C^{(1)}_{\substack{qd\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}} C^{(1)}_{\substack{qd\\prvt}} + C^{(1)}_{\substack{qd\\pvst}}\gamma^{(Y)}_{\substack{q\\vr}} + C^{(1)}_{\substack{qd\\prsv}}\gamma^{(Y)}_{\substack{d\\vt}}
\end{aligned}
\tag{B.52}
$$

$$
\begin{aligned}
\dot{C}^{(8)}_{\substack{qu\\prst}} &= 2[Y_u]_{sr}\xi_{\substack{u\\pt}} + 2[Y_u^\dagger]_{pt}\xi^*_{\substack{u\\rs}} \\
&- \frac{1}{N_c}\Bigg([Y_u^\dagger]_{pw}\,[Y_u]_{sv}\,C^{(8)}_{\substack{qu\\vrwt}} + [Y_u^\dagger]_{vt}\,[Y_u]_{wr}\,C^{(8)}_{\substack{qu\\pvsw}} + [Y_d]_{wr}\,[Y_u]_{sv}\,C^{(8)}_{\substack{quqd\\ptvw}} \\
&\quad + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vt}\,C^{(8)*}_{\substack{quqd\\rsvw}}\Bigg) \\
&+ 2\Bigg([Y_u]_{sv}\,[Y_d]_{wr}\,C^{(1)}_{\substack{quqd\\ptvw}} + [Y_u^\dagger]_{vt}\,[Y_d^\dagger]_{pw}\,C^{(1)*}_{\substack{quqd\\rsvw}} + \frac{1}{4}\,[Y_u]_{sv}\,[Y_d]_{wr}\,C^{(8)}_{\substack{quqd\\vtpw}} \\
&\quad + \frac{1}{4}\,[Y_u^\dagger]_{vt}\,[Y_d^\dagger]_{pw}\,C^{(8)*}_{\substack{quqd\\vsrw}}\Bigg) \\
&- 2\Bigg(2[Y_u^\dagger]_{vt}\,[Y_u]_{sw}\,C^{(1)}_{\substack{qq\\pvwr}} - [Y_u^\dagger]_{pw}\,[Y_u]_{sv}\,C^{(1)}_{\substack{qu\\vrwt}} - [Y_u^\dagger]_{vt}\,[Y_u]_{wr}\,C^{(1)}_{\substack{qu\\pvsw}}\Bigg) \\
&- 4[Y_u^\dagger]_{pv}\,[Y_u]_{wr}\,C_{\substack{uu\\vtsw}} - 12[Y_u^\dagger]_{vt}\,[Y_u]_{sw}\,C^{(3)}_{\substack{qq\\pvwr}} \\
&- [Y_d^\dagger]_{pv}\,[Y_d]_{wr}\,C^{(8)}_{\substack{ud\\stvw}} + \gamma^{(Y)}_{\substack{q\\pv}} C^{(8)}_{\substack{qu\\vrst}} + \gamma^{(Y)}_{\substack{u\\sv}} C^{(8)}_{\substack{qu\\prvt}} + C^{(8)}_{\substack{qu\\pvst}}\gamma^{(Y)}_{\substack{q\\vr}} + C^{(8)}_{\substack{qu\\prsv}}\gamma^{(Y)}_{\substack{u\\vt}}
\end{aligned}
\tag{B.53}
$$

$$
\begin{aligned}
\dot{C}^{(8)}_{\substack{qd\\prst}} &= 2[Y_d]_{sr}\xi_{\substack{d\\pt}} + 2[Y_d^\dagger]_{pt}\xi^*_{\substack{d\\rs}} \\
&- \frac{1}{N_c}\Bigg([Y_d^\dagger]_{pw}\,[Y_d]_{sv}\,C^{(8)}_{\substack{qd\\vrwt}} + [Y_d^\dagger]_{vt}\,[Y_d]_{wr}\,C^{(8)}_{\substack{qd\\pvsw}} + [Y_u]_{wr}\,[Y_d]_{sv}\,C^{(8)}_{\substack{quqd\\vwpt}} \\
&\quad +[Y_u^\dagger]_{pw}\,[Y_d^\dagger]_{vt}\,C^{(8)*}_{\substack{quqd\\vwrs}}\Bigg) \\
&+ 2\Bigg([Y_d]_{sv}\,[Y_u]_{wr}\,C^{(1)}_{\substack{quqd\\vwpt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{pw}\,C^{(1)*}_{\substack{quqd\\vwrs}} + \frac{1}{4}\,[Y_u]_{vr}\,[Y_d]_{sw}\,C^{(8)}_{\substack{quqd\\pvwt}} \\
&\quad +\frac{1}{4}\,[Y_u^\dagger]_{pv}\,[Y_d^\dagger]_{wt}\,C^{(8)*}_{\substack{quqd\\rvws}}\Bigg) \\
&- 2\Bigg(2[Y_d^\dagger]_{vt}\,[Y_d]_{sw}\,C^{(1)}_{\substack{qq\\pvwr}} - [Y_d^\dagger]_{pw}\,[Y_d]_{sv}\,C^{(1)}_{\substack{qd\\vrwt}} - [Y_d^\dagger]_{vt}\,[Y_d]_{wr}\,C^{(1)}_{\substack{qd\\pvsw}}\Bigg) \\
&- 4[Y_d^\dagger]_{pv}\,[Y_d]_{wr}\,C_{\substack{dd\\vtsw}} - 12[Y_d^\dagger]_{vt}\,[Y_d]_{sw}\,C^{(3)}_{\substack{qq\\pvwr}} \\
&- [Y_u^\dagger]_{pv}\,[Y_u]_{wr}\,C^{(8)}_{\substack{ud\\vwst}} + \gamma^{(Y)}_{\substack{q\\pv}}C^{(8)}_{\substack{qd\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}}C^{(8)}_{\substack{qd\\prvt}} + C^{(8)}_{\substack{qd\\pvst}}\gamma^{(Y)}_{\substack{q\\vr}} + C^{(8)}_{\substack{qd\\prsv}}\gamma^{(Y)}_{\substack{d\\vt}}
\end{aligned}
\tag{B.54}
$$

$(\overline{L}R)(\overline{R}L)$

$$
\begin{aligned}
\dot{C}_{\substack{ledq\\prst}} &= -2[Y_d]_{st}\xi_{\substack{e\\pr}} - 2[Y_e^\dagger]_{pr}\xi^*_{\substack{d\\ts}} + 2[Y_e^\dagger]_{pv}\,[Y_d]_{wt}\,C_{\substack{ed\\vrsw}} - 2[Y_e^\dagger]_{vr}\,[Y_d]_{wt}\,C_{\substack{ld\\pvsw}} \\
&+ 2[Y_e^\dagger]_{vr}\,[Y_d]_{sw}\,C^{(1)}_{\substack{lq\\pvwt}} + 6[Y_e^\dagger]_{vr}\,[Y_d]_{sw}\,C^{(3)}_{\substack{lq\\pvwt}} - 2[Y_e^\dagger]_{pw}\,[Y_d]_{sv}\,C_{\substack{qe\\vtwr}} \\
&+ 2[Y_d]_{sv}\,[Y_u]_{wt}\,C^{(1)}_{\substack{lequ\\prvw}} + \gamma^{(Y)}_{\substack{l\\pv}}C_{\substack{ledq\\vrst}} + \gamma^{(Y)}_{\substack{d\\sv}}C_{\substack{ledq\\prvt}} + C_{\substack{ledq\\pvst}}\gamma^{(Y)}_{\substack{e\\vr}} + C_{\substack{ledq\\prsv}}\gamma^{(Y)}_{\substack{q\\vt}}
\end{aligned}
\tag{B.55}
$$

$(\overline{L}R)(\overline{L}R)$:

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{lequ\\prst}} &= 2[Y_u^\dagger]_{st}\xi_{\substack{e\\pr}} + 2[Y_e^\dagger]_{pr}\xi_{\substack{u\\st}} + 2[Y_d^\dagger]_{sv}\,[Y_u^\dagger]_{wt}\,C_{\substack{ledq\\prvw}} + 2[Y_e^\dagger]_{pv}\,[Y_u^\dagger]_{sw}\,C_{\substack{eu\\vrwt}} \\
&+ 2[Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{wt}\,C^{(1)}_{\substack{lq\\pvsw}} - 6[Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{wt}\,C^{(3)}_{\substack{lq\\pvsw}} - 2[Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{sw}\,C_{\substack{lu\\pvwt}} \\
&- 2[Y_e^\dagger]_{pw}\,[Y_u^\dagger]_{vt}\,C_{\substack{qe\\svwr}} + \gamma^{(Y)}_{\substack{l\\pv}}C^{(1)}_{\substack{lequ\\vrst}} + \gamma^{(Y)}_{\substack{q\\sv}}C^{(1)}_{\substack{lequ\\prvt}} + C^{(1)}_{\substack{lequ\\pvst}}\gamma^{(Y)}_{\substack{e\\vr}} + C^{(1)}_{\substack{lequ\\prsv}}\gamma^{(Y)}_{\substack{u\\vt}}
\end{aligned}
\tag{B.56}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\substack{lequ\\prst}} &= -\frac{1}{2}[Y_u^\dagger]_{sw}\,[Y_e^\dagger]_{pv}\,C_{\substack{eu\\vrwt}} - \frac{1}{2}[Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{wt}\,C^{(1)}_{\substack{lq\\pvsw}} + \frac{3}{2}[Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{wt}\,C^{(3)}_{\substack{lq\\pvsw}} \\
&- \frac{1}{2}[Y_e^\dagger]_{vr}\,[Y_u^\dagger]_{sw}\,C_{\substack{lu\\pvwt}} - \frac{1}{2}[Y_e^\dagger]_{pw}\,[Y_u^\dagger]_{vt}\,C_{\substack{qe\\svwr}} \\
&+ \gamma^{(Y)}_{\substack{l\\pv}}C^{(3)}_{\substack{lequ\\vrst}} + \gamma^{(Y)}_{\substack{q\\sv}}C^{(3)}_{\substack{lequ\\prvt}} + C^{(3)}_{\substack{lequ\\pvst}}\gamma^{(Y)}_{\substack{e\\vr}} + C^{(3)}_{\substack{lequ\\prsv}}\gamma^{(Y)}_{\substack{u\\vt}}
\end{aligned}
\tag{B.57}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{quqd\\prst}} &= -2[Y_u^\dagger]_{pr}\xi_{\substack{d\\st}} - 2[Y_d^\dagger]_{st}\xi_{\substack{u\\pr}} \\
&- \frac{2}{N_c^2}\left([Y_u^\dagger]_{vr}\,[Y_d^\dagger]_{pw}\,C^{(8)}_{\substack{qd\\svwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{sw}\,C^{(8)}_{\substack{qu\\pvwr}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(8)}_{\substack{ud\\vrwt}}\right) \\
&+ \frac{4}{N_c}\left([Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(1)}_{\substack{qq\\svpw}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(1)}_{\substack{qq\\pvsw}} - 3[Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(3)}_{\substack{qq\\svpw}}\right. \\
&\quad \left. -3[Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(3)}_{\substack{qq\\pvsw}}\right) \\
&+ \frac{4}{N_c}\left([Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vr}\,C^{(1)}_{\substack{qd\\svwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{sw}\,C^{(1)}_{\substack{qu\\pvwr}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(1)}_{\substack{ud\\vrwt}}\right) \\
&- 4\left([Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(1)}_{\substack{qq\\pvsw}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(1)}_{\substack{qq\\svpw}}\right) + 12\left([Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(3)}_{\substack{qq\\pvsw}}\right. \\
&\quad \left. +[Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(3)}_{\substack{qq\\svpw}}\right) \\
&+ 2\left([Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vr}\,C^{(8)}_{\substack{qd\\svwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{sw}\,C^{(8)}_{\substack{qu\\pvwr}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(8)}_{\substack{ud\\vrwt}}\right) \\
&- 4[Y_d^\dagger]_{sw}\,[Y_u^\dagger]_{pv}\,C^{(1)}_{\substack{ud\\vrwt}} + \gamma^{(Y)}_{\substack{q\\pv}} C^{(1)}_{\substack{quqd\\vrst}} + \gamma^{(Y)}_{\substack{q\\sv}} C^{(1)}_{\substack{quqd\\prvt}} + C^{(1)}_{\substack{quqd\\pvst}}\gamma^{(Y)}_{\substack{u\\vr}} + C^{(1)}_{\substack{quqd\\prsv}}\gamma^{(Y)}_{\substack{d\\vt}}
\end{aligned} \tag{B.58}
$$

$$
\begin{aligned}
\dot{C}^{(8)}_{\substack{quqd\\prst}} &= -\frac{4}{N_c}\left([Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vr}\,C^{(8)}_{\substack{qd\\svwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{sw}\,C^{(8)}_{\substack{qu\\pvwr}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(8)}_{\substack{ud\\vrwt}}\right) \\
&+ 8\left([Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(1)}_{\substack{qq\\svpw}}\right. \\
&\quad \left. +[Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(1)}_{\substack{qq\\pvsw}}\right) - 24\left([Y_d^\dagger]_{wt}\,[Y_u^\dagger]_{vr}\,C^{(3)}_{\substack{qq\\svpw}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{wr}\,C^{(3)}_{\substack{qq\\pvsw}}\right) \\
&+ 8\left([Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{vr}\,C^{(1)}_{\substack{qd\\svwt}} + [Y_d^\dagger]_{vt}\,[Y_u^\dagger]_{sw}\,C^{(1)}_{\substack{qu\\pvwr}} + [Y_d^\dagger]_{pw}\,[Y_u^\dagger]_{sv}\,C^{(1)}_{\substack{ud\\vrwt}}\right) \\
&- 4[Y_d^\dagger]_{sw}\,[Y_u^\dagger]_{pv}\,C^{(8)}_{\substack{ud\\vrwt}} + \gamma^{(Y)}_{\substack{q\\pv}} C^{(8)}_{\substack{quqd\\vrst}} + \gamma^{(Y)}_{\substack{q\\sv}} C^{(8)}_{\substack{quqd\\prvt}} + C^{(8)}_{\substack{quqd\\pvst}}\gamma^{(Y)}_{\substack{u\\vr}} + C^{(8)}_{\substack{quqd\\prsv}}\gamma^{(Y)}_{\substack{d\\vt}}
\end{aligned} \tag{B.59}
$$

Appendix C

The Gauge Coupling contributions to one loop RGE

The parameters η_{1-5} are defined . Some equations use ξ_B, defined by

$$\xi_B = \frac{4}{3}\mathsf{y}_h\left(C_{H\Box} + C_{HD}\right) + \frac{8}{3}\left[2\mathsf{y}_l C^{(1)}_{\underset{tt}{Hl}} + 2\mathsf{y}_q N_c C^{(1)}_{\underset{tt}{Hq}} + \mathsf{y}_e C_{\underset{tt}{He}} + \mathsf{y}_u N_c C_{\underset{tt}{Hu}} + \mathsf{y}_d N_c C_{\underset{tt}{Hd}}\right] \tag{C.1}$$

The other parameters are $c_{A,2} = 2$, $c_{F,2} = 3/4$, $c_{A,3} = N_c$, $c_{F,3} = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, $b_{0,1} = -1/6 - 20n_g/9$, $b_{0,2} = 43/6 - 4n_g/3$ and $b_{0,3} = 11 - 4n_g/3$.

The gauge contributions to the one-loop renormalization group equations of the 59 dimension-six operator coefficients are listed by operator class in the following eight subsections [6].

Class X^3 :

$$\begin{aligned} \dot{C}_G &= \left(12c_{A,3} - 3b_{0,3}\right) g_3^2 C_G & \dot{C}_{\widetilde{G}} &= \left(12c_{A,3} - 3b_{0,3}\right) g_3^2 C_{\widetilde{G}} \\ \dot{C}_W &= \left(12c_{A,2} - 3b_{0,2}\right) g_2^2 C_W & \dot{C}_{\widetilde{W}} &= \left(12c_{A,2} - 3b_{0,2}\right) g_2^2 C_{\widetilde{W}} \end{aligned} \tag{C.2}$$

Class H^6 :

$$\dot{C}_H = \left(-\frac{27}{2}g_2^2 - \frac{9}{2}g_1^2\right) C_H + \lambda \left[\frac{40}{3}g_2^2 C_{H\Box} + \left(-6g_2^2 + 24g_1^2 \mathsf{y}_h^2\right) C_{HD}\right]$$
$$- \frac{3}{4}\left(4\mathsf{y}_h^2 g_1^2 + g_2^2\right)^2 C_{HD} + 12\lambda\left(3g_2^2 C_{HW} + 4g_1^2 \mathsf{y}_h^2 C_{HB} + 2g_1 g_2 \mathsf{y}_h C_{HWB}\right)$$
$$- \left(12g_1^2 g_2^2 \mathsf{y}_h^2 + 9g_2^4\right) C_{HW} - \left(48g_1^4 \mathsf{y}_h^4 + 12g_1^2 g_2^2 \mathsf{y}_h^2\right) C_{HB} - \left(24g_1^3 g_2 \mathsf{y}_h^3 + 6g_1 g_2^3 \mathsf{y}_h\right) C_{HWB}$$
$$+ \frac{16}{3}\lambda g_2^2 \left(C^{(3)}_{\substack{Hl\\tt}} + N_c C^{(3)}_{\substack{Hq\\tt}}\right) \tag{C.3}$$

Class H^4D^2 :

$$\dot{C}_{H\Box} = -\left(4g_2^2 + \frac{16}{3}g_1^2 \mathsf{y}_h^2\right) C_{H\Box} + \frac{20}{3}g_1^2 \mathsf{y}_h^2 C_{HD} + 2g_2^2\left(C^{(3)}_{\substack{Hl\\tt}} + N_c C^{(3)}_{\substack{Hq\\tt}}\right)$$
$$+ \frac{4}{3}g_1^2 \mathsf{y}_h \left(N_c \mathsf{y}_u C_{\substack{Hu\\tt}} + N_c \mathsf{y}_d C_{\substack{Hd\\tt}} + \mathsf{y}_e C_{\substack{He\\tt}} + 2N_c \mathsf{y}_q C^{(1)}_{\substack{Hq\\tt}} + 2\mathsf{y}_l C^{(1)}_{\substack{Hl\\tt}}\right) \tag{C.4}$$

$$\dot{C}_{HD} = \frac{80}{3}g_1^2 \mathsf{y}_h^2 C_{H\Box} + \left(\frac{9}{2}g_2^2 - \frac{10}{3}g_1^2 \mathsf{y}_h^2\right) C_{HD}$$
$$+ \frac{16}{3}g_1^2 \mathsf{y}_h \left(N_c \mathsf{y}_u C_{\substack{Hu\\tt}} + N_c \mathsf{y}_d C_{\substack{Hd\\tt}} + \mathsf{y}_e C_{\substack{He\\tt}} + 2N_c \mathsf{y}_q C^{(1)}_{\substack{Hq\\tt}} + 2\mathsf{y}_l C^{(1)}_{\substack{Hl\\tt}}\right) \tag{C.5}$$

Class X^2H^2 :

$$\dot{C}_{HG} = \left(-6\mathsf{y}_h^2 g_1^2 - \frac{9}{2}g_2^2 - 2b_{0,3}g_3^2\right) C_{HG} \tag{C.6}$$

$$\dot{C}_{HB} = \left(2\mathsf{y}_h^2 g_1^2 - \frac{9}{2}g_2^2 - 2b_{0,1}g_1^2\right) C_{HB} + 6g_1 g_2 \mathsf{y}_h C_{HWB} \tag{C.7}$$

$$\dot{C}_{HW} = -15g_2^3 C_W + \left(-6\mathsf{y}_h^2 g_1^2 - \frac{5}{2}g_2^2 - 2b_{0,2}g_2^2\right) C_{HW} + 2g_1 g_2 \mathsf{y}_h C_{HWB} \tag{C.8}$$

$$\dot{C}_{HWB} = 6g_1 g_2^2 \mathsf{y}_h C_W + \Big(-2\mathsf{y}_h^2 g_1^2 + \frac{9}{2} g_2^2 - b_{0,1} g_1^2 - b_{0,2} g_2^2\Big) C_{HWB} + 4g_1 g_2 \mathsf{y}_h C_{HB} \\ + 4g_1 g_2 \mathsf{y}_h C_{HW} \tag{C.9}$$

$$\dot{C}_{H\widetilde{G}} = \Big(-6\mathsf{y}_h^2 g_1^2 - \frac{9}{2} g_2^2 - 2b_{0,3} g_3^2\Big) C_{H\widetilde{G}} \tag{C.10}$$

$$\dot{C}_{H\widetilde{B}} = \Big(2\mathsf{y}_h^2 g_1^2 - \frac{9}{2} g_2^2 - 2b_{0,1} g_1^2\Big) C_{H\widetilde{B}} + 6g_1 g_2 \mathsf{y}_h C_{H\widetilde{W}B} \tag{C.11}$$

$$\dot{C}_{H\widetilde{W}} = -15 g_2^3 C_{\widetilde{W}} + \Big(-6\mathsf{y}_h^2 g_1^2 - \frac{5}{2} g_2^2 - 2b_{0,2} g_2^2\Big) C_{H\widetilde{W}} + 2g_1 g_2 \mathsf{y}_h C_{H\widetilde{W}B} \tag{C.12}$$

$$\dot{C}_{H\widetilde{W}B} = 6g_1 g_2^2 \mathsf{y}_h C_{\widetilde{W}} + \Big(-2\mathsf{y}_h^2 g_1^2 + \frac{9}{2} g_2^2 - b_{0,1} g_1^2 - b_{0,2} g_2^2\Big) C_{H\widetilde{W}B} + 4g_1 g_2 \mathsf{y}_h C_{H\widetilde{B}} \\ + 4g_1 g_2 \mathsf{y}_h C_{H\widetilde{W}} \tag{C.13}$$

Class $\psi^2 H^3$:

$$\begin{aligned}
\dot{C}_{\substack{eH\\rs}} &= [Y_e^\dagger]_{rs} \left[\frac{10}{3} g_2^2 C_{H\Box} + \left(-\frac{3}{2} g_2^2 + 6 g_1^2 \mathsf{y}_h^2\right) C_{HD}\right] \\
&- \left[3(3\mathsf{y}_l^2 + 3\mathsf{y}_e^2 - 4\mathsf{y}_l \mathsf{y}_e) g_1^2 + \frac{27}{4} g_2^2\right] C_{\substack{eH\\rs}} + 3[Y_e^\dagger]_{rs}\Big(3g_2^2 (C_{HW} + iC_{H\widetilde{W}}) \\
&+ 4(\mathsf{y}_h^2 + 2\mathsf{y}_l \mathsf{y}_e) g_1^2 (C_{HB} + iC_{H\widetilde{B}}) + 2g_1 g_2 \mathsf{y}_l (C_{HWB} + iC_{H\widetilde{W}B})\Big) \\
&- 3\Big(3g_1 \mathsf{y}_e C_{\substack{eB\\rt}} + g_2 C_{\substack{eW\\rt}}\Big)[Y_e Y_e^\dagger]_{ts} - 3[Y_e^\dagger Y_e]_{rv}\Big(2g_1(\mathsf{y}_l + \mathsf{y}_e) C_{\substack{eB\\vs}} - g_2 C_{\substack{eW\\vs}}\Big) \\
&- 6\left(4g_1^3 \mathsf{y}_h^2 \mathsf{y}_e + 4g_1^3 \mathsf{y}_h^2 \mathsf{y}_l + g_2^2 g_1 \mathsf{y}_h\right) C_{\substack{eB\\rs}} \\
&- 3\left(4g_1^2 g_2 \mathsf{y}_h \mathsf{y}_e + 4g_1^2 g_2 \mathsf{y}_h \mathsf{y}_l + 3g_2^3\right) C_{\substack{eW\\rs}} + \Big(3g_2^2 + 12 g_1^2 \mathsf{y}_l \mathsf{y}_h\Big)[Y_e^\dagger]_{rt} C_{\substack{He\\ts}} \\
&+ 12 g_1^2 \mathsf{y}_e \mathsf{y}_h C^{(1)}_{\substack{Hl\\rt}} [Y_e^\dagger]_{ts} + 12 g_1^2 \mathsf{y}_e \mathsf{y}_h C^{(3)}_{\substack{Hl\\rt}} [Y_e^\dagger]_{ts} + \frac{4}{3} g_2^2 [Y_e^\dagger]_{rs} \left(C^{(3)}_{\substack{Hl\\tt}} + N_c C^{(3)}_{\substack{Hq\\tt}}\right)
\end{aligned} \tag{C.14}$$

$$
\begin{aligned}
\dot{C}_{\substack{uH\\rs}} =& [Y_u^\dagger]_{rs}\left[\frac{10}{3}g_2^2 C_{H\Box} + \left(-\frac{3}{2}g_2^2 + 6g_1^2\mathsf{y}_h^2\right)C_{HD}\right] \\
&- \left[3(3\mathsf{y}_q^2 + 3\mathsf{y}_u^2 - 4\mathsf{y}_q\mathsf{y}_u)g_1^2 + \frac{27}{4}g_2^2 + 6c_{F,3}g_3^2\right]C_{\substack{uH\\rs}} \\
&+ 3[Y_u^\dagger]_{rs}\Big(8g_3^2 c_{F,3}(C_{HG} + iC_{H\widetilde{G}}) + 3g_2^2(C_{HW} + iC_{H\widetilde{W}}) \\
&+ 4(\mathsf{y}_h^2 + 2\mathsf{y}_q\mathsf{y}_u)g_1^2(C_{HB} + iC_{H\widetilde{B}}) - 2\mathsf{y}_q g_1 g_2(C_{HWB} + iC_{H\widetilde{W}B})\Big) \\
&- 12[Y_d^\dagger Y_d]_{rt} g_2 C_{\substack{uW\\ts}} - 6g_2 C_{\substack{dW\\rt}}[Y_d Y_u^\dagger]_{ts} - 3\Big(4g_3 c_{F,3}C_{\substack{uG\\rt}} + g_2 C_{\substack{uW\\rt}} \\
&+ (3\mathsf{y}_u + \mathsf{y}_d)g_1 C_{\substack{uB\\rt}}\Big)[Y_u Y_u^\dagger]_{ts} - 3[Y_u^\dagger Y_u]_{rv}\Big(4c_{F,3}g_3 C_{\substack{uG\\vs}} - g_2 C_{\substack{uW\\vs}} + 2(\mathsf{y}_q + \mathsf{y}_u)g_1 C_{\substack{uB\\vs}}\Big) \\
&- 6\left(4g_1^3\mathsf{y}_h^2\mathsf{y}_u + 4g_1^3\mathsf{y}_h^2\mathsf{y}_q - g_2^2 g_1\mathsf{y}_h\right)C_{\substack{uB\\rs}} + 3\left(4g_1^2 g_2\mathsf{y}_h\mathsf{y}_u + 4g_1^2 g_2\mathsf{y}_h\mathsf{y}_q - 3g_2^3\right)C_{\substack{uW\\rs}} \\
&- \left(3g_2^2 - 12g_1^2\mathsf{y}_q\mathsf{y}_h\right)[Y_u^\dagger]_{rt}C_{\substack{Hu\\ts}} + 3g_2^2[Y_d^\dagger]_{rt}C^*_{\substack{Hud\\st}} + 12g_1^2\mathsf{y}_u\mathsf{y}_h C^{(1)}_{\substack{Hq\\rt}}[Y_u^\dagger]_{ts} \\
&- 12g_1^2\mathsf{y}_u\mathsf{y}_h C^{(3)}_{\substack{Hq\\rt}}[Y_u^\dagger]_{ts} + \frac{4}{3}g_2^2[Y_u^\dagger]_{rs}\left(C^{(3)}_{\substack{Hl\\tt}} + N_c C^{(3)}_{\substack{Hq\\tt}}\right)
\end{aligned} \tag{C.15}
$$

$$
\begin{aligned}
\dot{C}_{\substack{dH\\rs}} =& [Y_d^\dagger]_{rs}\left[\frac{10}{3}g_2^2 C_{H\Box} + \left(-\frac{3}{2}g_2^2 + 6g_1^2\mathsf{y}_h^2\right)C_{HD}\right] \\
&- \left[3(3\mathsf{y}_q^2 + 3\mathsf{y}_d^2 - 4\mathsf{y}_q\mathsf{y}_d)g_1^2 + \frac{27}{4}g_2^2 + 6c_{F,3}g_3^2\right]C_{\substack{dH\\rs}} \\
&+ 3[Y_d^\dagger]_{rs}\Big(8c_{F,3}g_3^2(C_{HG} + iC_{H\widetilde{G}}) + 3g_2^2(C_{HW} + iC_{H\widetilde{W}}) \\
&+ 4(\mathsf{y}_h^2 + 2\mathsf{y}_q\mathsf{y}_d)g_1^2(C_{HB} + iC_{H\widetilde{B}}) + 2\mathsf{y}_q g_1 g_2(C_{HWB} + iC_{H\widetilde{W}B})\Big) - 12[Y_u^\dagger Y_u]_{rt} g_2 C_{\substack{dW\\ts}} \\
&- 6g_2 C_{\substack{uW\\rt}}[Y_u Y_d^\dagger]_{ts} - 3\Big(4c_{F,3}g_3 C_{\substack{dG\\rt}} + g_2 C_{\substack{dW\\rt}} + (3\mathsf{y}_d + \mathsf{y}_u)g_1 C_{\substack{dB\\rt}}\Big)[Y_d Y_d^\dagger]_{ts} \\
&- 3[Y_d^\dagger Y_d]_{rt}\Big(4c_{F,3}g_3 C_{\substack{dG\\ts}} - g_2 C_{\substack{dW\\ts}} + 2\left(\mathsf{y}_q + \mathsf{y}_d\right)g_1 C_{\substack{dB\\ts}}\Big) \\
&- 6\left(4g_1^3\mathsf{y}_h^2\mathsf{y}_d + 4g_1^3\mathsf{y}_h^2\mathsf{y}_q + g_2^2 g_1\mathsf{y}_h\right)C_{\substack{dB\\rs}} - 3\left(4g_1^2 g_2\mathsf{y}_h\mathsf{y}_d + 4g_1^2 g_2\mathsf{y}_h\mathsf{y}_q + 3g_2^3\right)C_{\substack{dW\\rs}} \\
&+ \left(3g_2^2 + 12g_1^2\mathsf{y}_q\mathsf{y}_h\right)[Y_d^\dagger]_{rt}C_{\substack{Hd\\ts}} + 3g_2^2[Y_u^\dagger]_{rt}C_{\substack{Hud\\ts}} + 12g_1^2\mathsf{y}_d\mathsf{y}_h C^{(1)}_{\substack{Hq\\rt}}[Y_d^\dagger]_{ts} \\
&+ 12g_1^2\mathsf{y}_d\mathsf{y}_h C^{(3)}_{\substack{Hq\\rt}}[Y_d^\dagger]_{ts} + \frac{4}{3}g_2^2[Y_d^\dagger]_{rs}\left(C^{(3)}_{\substack{Hl\\tt}} + N_c C^{(3)}_{\substack{Hq\\tt}}\right)
\end{aligned} \tag{C.16}
$$

Class $\psi^2 XH$:

$$\dot{C}_{\underset{rs}{eW}} = \Big[(3c_{F,2} - b_{0,2})\, g_2^2 + \Big(-3\mathsf{y}_e^2 + 8\mathsf{y}_e\mathsf{y}_l - 3\mathsf{y}_l^2\Big)\, g_1^2\Big] C_{\underset{rs}{eW}} + g_1 g_2 (3\mathsf{y}_l - \mathsf{y}_e) C_{\underset{rs}{eB}}$$
$$- [Y_e^\dagger]_{rs}\Big(g_2(C_{HW} + iC_{H\widetilde{W}}) + g_1(\mathsf{y}_l + \mathsf{y}_e)(C_{HWB} + iC_{H\widetilde{W}B})\Big) \qquad \text{(C.17)}$$

$$\dot{C}_{\underset{rs}{eB}} = \Big[-3c_{F,2} g_2^2 + \Big(3\mathsf{y}_e^2 + 4\mathsf{y}_e\mathsf{y}_l + 3\mathsf{y}_l^2 - b_{0,1}\Big)\, g_1^2\Big] C_{\underset{rs}{eB}} + 4c_{F,2} g_1 g_2 (3\mathsf{y}_l - \mathsf{y}_e) C_{\underset{rs}{eW}}$$
$$- [Y_e^\dagger]_{rs}\Big(2g_1(\mathsf{y}_l + \mathsf{y}_e)(C_{HB} + iC_{H\widetilde{B}}) + \frac{3}{2} g_2 (C_{HWB} + iC_{H\widetilde{W}B})\Big) \qquad \text{(C.18)}$$

$$\dot{C}_{\underset{rs}{uG}} = \Big[(10c_{F,3} - 4c_{A,3} - b_{0,3})\, g_3^2 - 3c_{F,2} g_2^2 + \Big(-3\mathsf{y}_u^2 + 8\mathsf{y}_u\mathsf{y}_q - 3\mathsf{y}_q^2\Big)\, g_1^2\Big] C_{\underset{rs}{uG}}$$
$$+ 8c_{F,2} g_2 g_3 C_{\underset{rs}{uW}} + 4g_1 g_3 (\mathsf{y}_u + \mathsf{y}_q) C_{\underset{rs}{uB}} - 4[Y_u^\dagger]_{rs} g_3 (C_{HG} + iC_{H\widetilde{G}})$$
$$+ 3g_3^2 c_{A,3} [Y_u^\dagger]_{rs} \Big(C_G + iC_{\widetilde{G}}\Big) \qquad \text{(C.19)}$$

$$\dot{C}_{\underset{rs}{uW}} = \Big[2c_{F,3} g_3^2 + (3c_{F,2} - b_{0,2})\, g_2^2 + \Big(-3\mathsf{y}_u^2 + 8\mathsf{y}_u\mathsf{y}_q - 3\mathsf{y}_q^2\Big)\, g_1^2\Big] C_{\underset{rs}{uW}} + 2c_{F,3} g_2 g_3 C_{\underset{rs}{uG}}$$
$$+ g_1 g_2 (3\mathsf{y}_q - \mathsf{y}_u) C_{\underset{rs}{uB}} - [Y_u^\dagger]_{rs}\Big(g_2(C_{HW} + iC_{H\widetilde{W}}) - g_1(\mathsf{y}_q + \mathsf{y}_u)(C_{HWB} + iC_{H\widetilde{W}B})\Big)$$
$$\text{(C.20)}$$

$$\dot{C}_{\underset{rs}{uB}} = \Big[2c_{F,3} g_3^2 - 3c_{F,2} g_2^2 + \Big(3\mathsf{y}_u^2 + 4\mathsf{y}_u\mathsf{y}_q + 3\mathsf{y}_q^2 - b_{0,1}\Big)\, g_1^2\Big] C_{\underset{rs}{uB}} + 4c_{F,3} g_1 g_3 \,(\mathsf{y}_u + \mathsf{y}_q)\, C_{\underset{rs}{uG}}$$
$$+ 4c_{F,2} g_1 g_2 (3\mathsf{y}_q - \mathsf{y}_u) C_{\underset{rs}{uW}} - [Y_u^\dagger]_{rs}\Big(2g_1(\mathsf{y}_q + \mathsf{y}_u)(C_{HB} + iC_{H\widetilde{B}})$$
$$- \frac{3}{2} g_2 (C_{HWB} + iC_{H\widetilde{W}B})\Big) \qquad \text{(C.21)}$$

$$\dot{C}_{\underset{rs}{dG}} = \Big[(10c_{F,3} - 4c_{A,3} - b_{0,3})\, g_3^2 - 3c_{F,2} g_2^2 + \Big(-3\mathsf{y}_d^2 + 8\mathsf{y}_d\mathsf{y}_q - 3\mathsf{y}_q^2\Big)\, g_1^2\Big] C_{\underset{rs}{dG}}$$
$$+ 8c_{F,2} g_2 g_3 C_{\underset{rs}{dW}} + 4g_1 g_3 (\mathsf{y}_d + \mathsf{y}_q) C_{\underset{rs}{dB}} - 4[Y_d^\dagger]_{rs} g_3 (C_{HG} + iC_{H\widetilde{G}})$$
$$+ 3g_3^2 c_{A,3} [Y_d^\dagger]_{rs} \Big(C_G + iC_{\widetilde{G}}\Big) \qquad \text{(C.22)}$$

$$\dot{C}_{\underset{rs}{dW}} = \Big[2c_{F,3} g_3^2 + (3c_{F,2} - b_{0,2})\, g_2^2 + \Big(-3\mathsf{y}_d^2 + 8\mathsf{y}_d\mathsf{y}_q - 3\mathsf{y}_q^2\Big)\, g_1^2\Big] C_{\underset{rs}{dW}} + 2c_{F,3} g_2 g_3 C_{\underset{rs}{dG}}$$
$$+ g_1 g_2 (3\mathsf{y}_q - \mathsf{y}_d) C_{\underset{rs}{dB}} - [Y_d^\dagger]_{rs}\Big(g_2(C_{HW} + iC_{H\widetilde{W}}) + g_1(\mathsf{y}_q + \mathsf{y}_d)(C_{HWB} + iC_{H\widetilde{W}B})\Big)$$
$$\text{(C.23)}$$

$$
\begin{aligned}
\dot{C}_{\substack{dB\\rs}} &= \Big[2c_{F,3}g_3^2 - 3c_{F,2}g_2^2 + \Big(3\mathsf{y}_d^2 + 4\mathsf{y}_d\mathsf{y}_q + 3\mathsf{y}_q^2 - b_{0,1}\Big)\, g_1^2\Big] C_{\substack{dB\\rs}} + 4c_{F,3}g_1g_3\left(\mathsf{y}_d + \mathsf{y}_q\right) C_{\substack{dG\\rs}} \\
&\quad + 4c_{F,2}g_1g_2(3\mathsf{y}_q - \mathsf{y}_d)C_{\substack{dW\\rs}} - [Y_d^\dagger]_{rs}\Big(2g_1(\mathsf{y}_q + \mathsf{y}_d)(C_{HB} + iC_{H\widetilde{B}}) \\
&\quad + \frac{3}{2}g_2(C_{HWB} + iC_{H\widetilde{W}B})\Big) \qquad \text{(C.24)}
\end{aligned}
$$

Class ψ^2H^2D :

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{Hl\\rs}} &= \frac{1}{2}\xi_B g_1^2\delta_{rs}\mathsf{y}_l + \frac{4}{3}g_1^2\mathsf{y}_h^2 C^{(1)}_{\substack{Hl\\rs}} + \frac{4}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_h C_{\substack{ld\\rsww}} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C_{\substack{le\\rsww}} + \frac{8}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\substack{ll\\rsww}} \\
&\quad + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\substack{ll\\rwws}} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\substack{ll\\wsrw}} + \frac{8}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\substack{ll\\wwrs}} + \frac{8}{3}g_1^2N_c\mathsf{y}_h\mathsf{y}_q C^{(1)}_{\substack{lq\\rsww}} \\
&\quad + \frac{4}{3}g_1^2N_c\mathsf{y}_h\mathsf{y}_u C_{\substack{lu\\rsww}} \qquad \text{(C.25)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\substack{Hl\\rs}} &= \frac{1}{6}g_2^2C_{H\Box}\delta_{rs} + \frac{2}{3}g_2^2C^{(3)}_{\substack{Hl\\tt}}\delta_{rs} + \frac{2}{3}g_2^2N_cC^{(3)}_{\substack{Hq\\tt}}\delta_{rs} + \frac{1}{3}g_2^2C^{(3)}_{\substack{Hl\\rs}} + \frac{1}{3}g_2^2C_{\substack{ll\\rwws}} + \frac{1}{3}g_2^2C_{\substack{ll\\wsrw}} \\
&\quad + \frac{2}{3}g_2^2N_cC^{(3)}_{\substack{lq\\rsww}} - 6g_2^2C^{(3)}_{\substack{Hl\\rs}} \qquad \text{(C.26)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{\substack{He\\rs}} &= \frac{1}{2}\xi_B g_1^2\delta_{rs}\mathsf{y}_e + \frac{4}{3}g_1^2\mathsf{y}_h^2C_{\substack{He\\rs}} + \frac{4}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_hC_{\substack{ed\\rsww}} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_hC_{\substack{ee\\rsww}} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_hC_{\substack{ee\\rwws}} \\
&\quad + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_hC_{\substack{ee\\wsrw}} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_hC_{\substack{ee\\wwrs}} + \frac{4}{3}g_1^2N_c\mathsf{y}_h\mathsf{y}_uC_{\substack{eu\\rsww}} + \frac{8}{3}g_1^2\mathsf{y}_h\mathsf{y}_lC_{\substack{le\\wwrs}} \\
&\quad + \frac{8}{3}g_1^2N_c\mathsf{y}_h\mathsf{y}_qC_{\substack{qe\\wwrs}} \qquad \text{(C.27)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{Hq\\rs}} &= \frac{1}{2}\xi_B g_1^2\delta_{rs}\mathsf{y}_q + \frac{4}{3}g_1^2\mathsf{y}_h^2C^{(1)}_{\substack{Hq\\rs}} + \frac{8}{3}g_1^2\mathsf{y}_h\mathsf{y}_lC^{(1)}_{\substack{lq\\wwrs}} + \frac{4}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_hC^{(1)}_{\substack{qd\\rsww}} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_hC_{\substack{qe\\rsww}} \\
&\quad + \frac{8}{3}g_1^2N_c\mathsf{y}_h\mathsf{y}_qC^{(1)}_{\substack{qq\\rsww}} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_qC^{(1)}_{\substack{qq\\rwws}} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_qC^{(1)}_{\substack{qq\\wsrw}} + \frac{8}{3}g_1^2N_c\mathsf{y}_h\mathsf{y}_qC^{(1)}_{\substack{qq\\wwrs}} \\
&\quad + 4g_1^2\mathsf{y}_h\mathsf{y}_qC^{(3)}_{\substack{qq\\rwws}} + 4g_1^2\mathsf{y}_h\mathsf{y}_qC^{(3)}_{\substack{qq\\wsrw}} + \frac{4}{3}g_1^2N_c\mathsf{y}_h\mathsf{y}_uC^{(1)}_{\substack{qu\\rsww}} \qquad \text{(C.28)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\substack{Hq\\rs}} &= \frac{1}{6}g_2^2C_{H\Box}\delta_{rs} + \frac{2}{3}g_2^2C^{(3)}_{\substack{Hl\\tt}}\delta_{rs} + \frac{2}{3}g_2^2N_cC^{(3)}_{\substack{Hq\\tt}}\delta_{rs} + \frac{1}{3}g_2^2C^{(3)}_{\substack{Hq\\rs}} + \frac{2}{3}g_2^2C^{(3)}_{\substack{lq\\wwrs}} + \frac{1}{3}g_2^2C^{(1)}_{\substack{qq\\rwws}} \\
&\quad + \frac{1}{3}g_2^2C^{(1)}_{\substack{qq\\wsrw}} + \frac{2}{3}g_2^2N_cC^{(3)}_{\substack{qq\\rsww}} - \frac{1}{3}g_2^2C^{(3)}_{\substack{qq\\rwws}} - \frac{1}{3}g_2^2C^{(3)}_{\substack{qq\\wsrw}} + \frac{2}{3}g_2^2N_cC^{(3)}_{\substack{qq\\wwrs}} - 6g_2^2C^{(3)}_{\substack{Hq\\rs}} \\
&\qquad \text{(C.29)}
\end{aligned}
$$

$$\begin{aligned}\dot{C}_{\underset{rs}{Hu}} =& \frac{1}{2}\xi_B g_1^2 \delta_{rs}\mathsf{y}_u + \frac{4}{3}g_1^2\mathsf{y}_h^2 C_{\underset{rs}{Hu}} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C_{\underset{wwrs}{eu}} + \frac{8}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\underset{wwrs}{lu}} + \frac{8}{3}g_1^2 N_c\mathsf{y}_h\mathsf{y}_q C^{(1)}_{\underset{wwrs}{qu}} \\ &+ \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_h C^{(1)}_{\underset{rsww}{ud}} + \frac{4}{3}g_1^2 N_c\mathsf{y}_h\mathsf{y}_u C_{\underset{rsww}{uu}} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_u C_{\underset{rwws}{uu}} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_u C_{\underset{wsrw}{uu}} \\ &+ \frac{4}{3}g_1^2 N_c\mathsf{y}_h\mathsf{y}_u C_{\underset{wwrs}{uu}} \end{aligned} \tag{C.30}$$

$$\begin{aligned}\dot{C}_{\underset{rs}{Hd}} =& \frac{1}{2}\xi_B g_1^2 \delta_{rs}\mathsf{y}_d + \frac{4}{3}g_1^2\mathsf{y}_h^2 C_{\underset{rs}{Hd}} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_h C_{\underset{rsww}{dd}} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_h C_{\underset{rwws}{dd}} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_h C_{\underset{wsrw}{dd}} \\ &+ \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_h C_{\underset{wwrs}{dd}} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C_{\underset{wwrs}{ed}} + \frac{8}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\underset{wwrs}{ld}} + \frac{8}{3}g_1^2 N_c\mathsf{y}_h\mathsf{y}_q C^{(1)}_{\underset{wwrs}{qd}} \\ &+ \frac{4}{3}g_1^2 N_c\mathsf{y}_h\mathsf{y}_u C^{(1)}_{\underset{wwrs}{ud}} \end{aligned} \tag{C.31}$$

$$\dot{C}_{\underset{rs}{Hud}} = -3g_1^2(\mathsf{y}_u - \mathsf{y}_d)^2 C_{\underset{rs}{Hud}} \tag{C.32}$$

Class ψ^4 :
$(\overline{L}L)(\overline{L}L)$:

$$
\begin{aligned}
\dot{C}_{\substack{ll\\prst}} &= \frac{2}{3}g_1^2 \mathsf{y}_h \mathsf{y}_l C^{(1)}_{\substack{Hl\\st}}\delta_{pr} - \frac{1}{6}g_2^2 C^{(3)}_{\substack{Hl\\st}}\delta_{pr} + \frac{1}{3}g_2^2 C^{(3)}_{\substack{Hl\\sr}}\delta_{pt} + \frac{1}{3}g_2^2 C^{(3)}_{\substack{Hl\\pt}}\delta_{rs} \\
&+ \frac{2}{3}g_1^2 \mathsf{y}_h \mathsf{y}_l C^{(1)}_{\substack{Hl\\pr}}\delta_{st} - \frac{1}{6}g_2^2 C^{(3)}_{\substack{Hl\\pr}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\prww}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\pwwr}}\delta_{st} + \frac{2}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\swwt}}\delta_{pr} + \frac{2}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\wrpw}}\delta_{st} \\
&+ \frac{2}{3}g_1^2 \mathsf{y}_l^2 C_{\substack{ll\\wtsw}}\delta_{pr} - \frac{1}{6}g_2^2 C_{\substack{ll\\pwwr}}\delta_{st} - \frac{1}{6}g_2^2 C_{\substack{ll\\swwt}}\delta_{pr} - \frac{1}{6}g_2^2 C_{\substack{ll\\wrpw}}\delta_{st} - \frac{1}{6}g_2^2 C_{\substack{ll\\wtsw}}\delta_{pr} \\
&+ \frac{1}{3}g_2^2 C_{\substack{ll\\swwr}}\delta_{pt} + \frac{1}{3}g_2^2 C_{\substack{ll\\pwwt}}\delta_{rs} + \frac{1}{3}g_2^2 C_{\substack{ll\\wrsw}}\delta_{pt} + \frac{1}{3}g_2^2 C_{\substack{ll\\wtpw}}\delta_{rs} + \frac{4}{3}g_1^2 N_c \mathsf{y}_l \mathsf{y}_q C^{(1)}_{\substack{lq\\prww}}\delta_{st} \\
&+ \frac{4}{3}g_1^2 N_c \mathsf{y}_l \mathsf{y}_q C^{(1)}_{\substack{lq\\stww}}\delta_{pr} - \frac{1}{3}g_2^2 N_c C^{(3)}_{\substack{lq\\prww}}\delta_{st} - \frac{1}{3}g_2^2 N_c C^{(3)}_{\substack{lq\\stww}}\delta_{pr} + \frac{2}{3}g_2^2 N_c C^{(3)}_{\substack{lq\\srww}}\delta_{pt} \\
&+ \frac{2}{3}g_2^2 N_c C^{(3)}_{\substack{lq\\ptww}}\delta_{rs} + \frac{2}{3}g_1^2 N_c \mathsf{y}_l \mathsf{y}_u C_{\substack{lu\\prww}}\delta_{st} + \frac{2}{3}g_1^2 N_c \mathsf{y}_l \mathsf{y}_u C_{\substack{lu\\stww}}\delta_{pr} + \frac{2}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_l C_{\substack{ld\\prww}}\delta_{st} \\
&+ \frac{2}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_l C_{\substack{ld\\stww}}\delta_{pr} + \frac{2}{3}g_1^2 \mathsf{y}_e \mathsf{y}_l C_{\substack{le\\prww}}\delta_{st} + \frac{2}{3}g_1^2 \mathsf{y}_e \mathsf{y}_l C_{\substack{le\\stww}}\delta_{pr} + 6g_2^2 C_{\substack{ll\\ptsr}} \\
&- 3\left(g_2^2 - 4\mathsf{y}_l^2 g_1^2\right) C_{\substack{ll\\prst}} \qquad \text{(C.33)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\underset{prst}{qq}} &= \frac{2}{3}g_1^2 \mathsf{y}_h \mathsf{y}_q C^{(1)}_{\underset{st}{Hq}}\delta_{pr} + \frac{2}{3}g_1^2 \mathsf{y}_h \mathsf{y}_q C^{(1)}_{\underset{pr}{Hq}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_l \mathsf{y}_q C^{(1)}_{\underset{wwst}{lq}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_l \mathsf{y}_q C^{(1)}_{\underset{wwpr}{lq}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_q^2 C^{(1)}_{\underset{prww}{qq}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_q^2 C^{(1)}_{\underset{stww}{qq}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_q^2 C^{(1)}_{\underset{wwst}{qq}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 N_c \mathsf{y}_q^2 C^{(1)}_{\underset{wwpr}{qq}}\delta_{st} + \frac{2}{3}g_1^2 \mathsf{y}_q^2 C^{(1)}_{\underset{pwwr}{qq}}\delta_{st} + \frac{2}{3}g_1^2 \mathsf{y}_q^2 C^{(1)}_{\underset{swwt}{qq}}\delta_{pr} + \frac{2}{3}g_1^2 \mathsf{y}_q^2 C^{(1)}_{\underset{wrpw}{qq}}\delta_{st} \\
&+ \frac{2}{3}g_1^2 \mathsf{y}_q^2 C^{(1)}_{\underset{wtsw}{qq}}\delta_{pr} + \frac{1}{6}g_3^2 C^{(1)}_{\underset{swwr}{qq}}\delta_{pt} + \frac{1}{6}g_3^2 C^{(1)}_{\underset{pwwt}{qq}}\delta_{rs} + \frac{1}{6}g_3^2 C^{(1)}_{\underset{wrsw}{qq}}\delta_{pt} + \frac{1}{6}g_3^2 C^{(1)}_{\underset{wtpw}{qq}}\delta_{rs} \\
&- \frac{1}{3N_c}g_3^2 C^{(1)}_{\underset{pwwr}{qq}}\delta_{st} - \frac{1}{3N_c}g_3^2 C^{(1)}_{\underset{swwt}{qq}}\delta_{pr} - \frac{1}{3N_c}g_3^2 C^{(1)}_{\underset{wrpw}{qq}}\delta_{st} - \frac{1}{3N_c}g_3^2 C^{(1)}_{\underset{wtsw}{qq}}\delta_{pr} \\
&+ 2g_1^2 \mathsf{y}_q^2 C^{(3)}_{\underset{pwwr}{qq}}\delta_{st} + 2g_1^2 \mathsf{y}_q^2 C^{(3)}_{\underset{swwt}{qq}}\delta_{pr} + 2g_1^2 \mathsf{y}_q^2 C^{(3)}_{\underset{wrpw}{qq}}\delta_{st} + 2g_1^2 \mathsf{y}_q^2 C^{(3)}_{\underset{wtsw}{qq}}\delta_{pr} + \frac{1}{2}g_3^2 C^{(3)}_{\underset{swwr}{qq}}\delta_{pt} \\
&+ \frac{1}{2}g_3^2 C^{(3)}_{\underset{pwwt}{qq}}\delta_{rs} + \frac{1}{2}g_3^2 C^{(3)}_{\underset{wrsw}{qq}}\delta_{pt} + \frac{1}{2}g_3^2 C^{(3)}_{\underset{wtpw}{qq}}\delta_{rs} - \frac{1}{N_c}g_3^2 C^{(3)}_{\underset{pwwr}{qq}}\delta_{st} - \frac{1}{N_c}g_3^2 C^{(3)}_{\underset{swwt}{qq}}\delta_{pr} \\
&- \frac{1}{N_c}g_3^2 C^{(3)}_{\underset{wrpw}{qq}}\delta_{st} - \frac{1}{N_c}g_3^2 C^{(3)}_{\underset{wtsw}{qq}}\delta_{pr} + \frac{2}{3}g_1^2 N_c \mathsf{y}_q \mathsf{y}_u C^{(1)}_{\underset{prww}{qu}}\delta_{st} + \frac{2}{3}g_1^2 N_c \mathsf{y}_q \mathsf{y}_u C^{(1)}_{\underset{stww}{qu}}\delta_{pr} \\
&+ \frac{2}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_q C^{(1)}_{\underset{prww}{qd}}\delta_{st} + \frac{2}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_q C^{(1)}_{\underset{stww}{qd}}\delta_{pr} + \frac{1}{12}g_3^2 C^{(8)}_{\underset{srww}{qu}}\delta_{pt} + \frac{1}{12}g_3^2 C^{(8)}_{\underset{ptww}{qu}}\delta_{rs} \\
&- \frac{1}{6N_c}g_3^2 C^{(8)}_{\underset{prww}{qu}}\delta_{st} - \frac{1}{6N_c}g_3^2 C^{(8)}_{\underset{stww}{qu}}\delta_{pr} + \frac{1}{12}g_3^2 C^{(8)}_{\underset{srww}{qd}}\delta_{pt} + \frac{1}{12}g_3^2 C^{(8)}_{\underset{ptww}{qd}}\delta_{rs} \\
&- \frac{1}{6N_c}g_3^2 C^{(8)}_{\underset{prww}{qd}}\delta_{st} - \frac{1}{6N_c}g_3^2 C^{(8)}_{\underset{stww}{qd}}\delta_{pr} + \frac{2}{3}g_1^2 \mathsf{y}_e \mathsf{y}_q C_{\underset{prww}{qe}}\delta_{st} + \frac{2}{3}g_1^2 \mathsf{y}_e \mathsf{y}_q C_{\underset{stww}{qe}}\delta_{pr} \\
&+ 3g_3^2 C^{(1)}_{\underset{ptsr}{qq}} + 9g_3^2 C^{(3)}_{\underset{ptsr}{qq}} + 9g_2^2 C^{(3)}_{\underset{prst}{qq}} - \frac{6}{N_c}\left(g_3^2 - 2N_c \mathsf{y}_q^2 g_1^2\right) C^{(1)}_{\underset{prst}{qq}}
\end{aligned}
\tag{C.34}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\underset{prst}{qq}} &= \frac{1}{6}g_2^2 C^{(3)}_{\underset{st}{Hq}}\delta_{pr} + \frac{1}{6}g_2^2 C^{(3)}_{\underset{pr}{Hq}}\delta_{st} + \frac{1}{3}g_2^2 C^{(3)}_{\underset{wwst}{lq}}\delta_{pr} + \frac{1}{3}g_2^2 C^{(3)}_{\underset{wwpr}{lq}}\delta_{st} \\
&+ \frac{1}{6}g_2^2 C^{(1)}_{\underset{swwt}{qq}}\delta_{pr} + \frac{1}{6}g_2^2 C^{(1)}_{\underset{wtsw}{qq}}\delta_{pr} + \frac{1}{6}g_2^2 C^{(1)}_{\underset{pwwr}{qq}}\delta_{st} + \frac{1}{6}g_2^2 C^{(1)}_{\underset{wrpw}{qq}}\delta_{st} + \frac{1}{6}g_3^2 C^{(1)}_{\underset{pwwt}{qq}}\delta_{rs} \\
&+ \frac{1}{6}g_3^2 C^{(1)}_{\underset{swwr}{qq}}\delta_{pt} + \frac{1}{6}g_3^2 C^{(1)}_{\underset{wtpw}{qq}}\delta_{rs} + \frac{1}{6}g_3^2 C^{(1)}_{\underset{wrsw}{qq}}\delta_{pt} + \frac{1}{3}g_2^2 N_c C^{(3)}_{\underset{prww}{qq}}\delta_{st} + \frac{1}{3}g_2^2 N_c C^{(3)}_{\underset{stww}{qq}}\delta_{pr} \\
&+ \frac{1}{3}g_2^2 N_c C^{(3)}_{\underset{wwst}{qq}}\delta_{pr} + \frac{1}{3}g_2^2 N_c C^{(3)}_{\underset{wwpr}{qq}}\delta_{st} - \frac{1}{6}g_2^2 C^{(3)}_{\underset{pwwr}{qq}}\delta_{st} - \frac{1}{6}g_2^2 C^{(3)}_{\underset{swwt}{qq}}\delta_{pr} - \frac{1}{6}g_2^2 C^{(3)}_{\underset{wrpw}{qq}}\delta_{st} \\
&- \frac{1}{6}g_2^2 C^{(3)}_{\underset{wtsw}{qq}}\delta_{pr} + \frac{1}{2}g_3^2 C^{(3)}_{\underset{pwwt}{qq}}\delta_{rs} + \frac{1}{2}g_3^2 C^{(3)}_{\underset{swwr}{qq}}\delta_{pt} + \frac{1}{2}g_3^2 C^{(3)}_{\underset{wtpw}{qq}}\delta_{rs} + \frac{1}{2}g_3^2 C^{(3)}_{\underset{wrsw}{qq}}\delta_{pt} \\
&+ \frac{1}{12}g_3^2 C^{(8)}_{\underset{ptww}{qu}}\delta_{rs} + \frac{1}{12}g_3^2 C^{(8)}_{\underset{srww}{qu}}\delta_{pt} + \frac{1}{12}g_3^2 C^{(8)}_{\underset{ptww}{qd}}\delta_{rs} + \frac{1}{12}g_3^2 C^{(8)}_{\underset{srww}{qd}}\delta_{pt} \\
&- 3g_3^2 C^{(3)}_{\underset{ptsr}{qq}} - \frac{6}{N_c}g_3^2 C^{(3)}_{\underset{prst}{qq}} - 6g_2^2 C^{(3)}_{\underset{prst}{qq}} + 12\mathsf{y}_q^2 g_1^2 C^{(3)}_{\underset{prst}{qq}} + 3g_3^2 C^{(1)}_{\underset{ptsr}{qq}} + 3g_2^2 C^{(1)}_{\underset{prst}{qq}}
\end{aligned}
\tag{C.35}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{lq\\prst}} &= \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C^{(1)}_{\substack{Hq\\st}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_q C^{(1)}_{\substack{Hl\\pr}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C_{\substack{ll\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C_{\substack{ll\\wwpr}}\delta_{st} \\
&+ \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C_{\substack{ll\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C_{\substack{ll\\wrpw}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_q^2 C^{(1)}_{\substack{lq\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l^2 C^{(1)}_{\substack{lq\\wwst}}\delta_{pr} \\
&+ \frac{8}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_q C^{(1)}_{\substack{qq\\stww}}\delta_{pr} + \frac{8}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_q C^{(1)}_{\substack{qq\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C^{(1)}_{\substack{qq\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C^{(1)}_{\substack{qq\\wtsw}}\delta_{pr} \\
&+ 4g_1^2\mathsf{y}_l\mathsf{y}_q C^{(3)}_{\substack{qq\\swwt}}\delta_{pr} + 4g_1^2\mathsf{y}_l\mathsf{y}_q C^{(3)}_{\substack{qq\\wtsw}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_u C^{(1)}_{\substack{qu\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_l C^{(1)}_{\substack{qd\\stww}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{qe\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C_{\substack{lu\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C_{\substack{ld\\prww}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C_{\substack{le\\prww}}\delta_{st} \\
&+ 12\mathsf{y}_l\mathsf{y}_q g_1^2 C^{(1)}_{\substack{lq\\prst}} + 9g_2^2 C^{(3)}_{\substack{lq\\prst}}
\end{aligned}
\tag{C.36}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\substack{lq\\prst}} &= \frac{1}{3}g_2^2 C^{(3)}_{\substack{Hq\\st}}\delta_{pr} + \frac{1}{3}g_2^2 C^{(3)}_{\substack{Hl\\pr}}\delta_{st} + \frac{2}{3}g_2^2 N_c C^{(3)}_{\substack{lq\\prww}}\delta_{st} + \frac{2}{3}g_2^2 C^{(3)}_{\substack{lq\\wwst}}\delta_{pr} + \frac{1}{3}g_2^2 C^{(1)}_{\substack{qq\\swwt}}\delta_{pr} \\
&+ \frac{1}{3}g_2^2 C^{(1)}_{\substack{qq\\wtsw}}\delta_{pr} + \frac{2}{3}g_2^2 N_c C^{(3)}_{\substack{qq\\stww}}\delta_{pr} + \frac{2}{3}g_2^2 N_c C^{(3)}_{\substack{qq\\wwst}}\delta_{pr} - \frac{1}{3}g_2^2 C^{(3)}_{\substack{qq\\swwt}}\delta_{pr} - \frac{1}{3}g_2^2 C^{(3)}_{\substack{qq\\wtsw}}\delta_{pr} \\
&+ \frac{1}{3}g_2^2 C_{\substack{ll\\pwwr}}\delta_{st} + \frac{1}{3}g_2^2 C_{\substack{ll\\wrpw}}\delta_{st} + 3g_2^2 C^{(1)}_{\substack{lq\\prst}} - 6\left(g_2^2 - 2\mathsf{y}_l\mathsf{y}_q g_1^2\right) C^{(3)}_{\substack{lq\\prst}}
\end{aligned}
\tag{C.37}
$$

$(\overline{R}R)(\overline{R}R)$:

$$\begin{aligned}
\dot{C}_{\substack{ee\\prst}} &= \frac{1}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C_{\substack{He\\st}}\delta_{pr} + \frac{1}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C_{\substack{He\\pr}}\delta_{st} + \frac{1}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C_{\substack{He\\ss}}\delta_{pt} + \frac{1}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C_{\substack{He\\pt}}\delta_{sr} \\
&+ \frac{2}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{le\\wwst}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{le\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{le\\wwsr}}\delta_{pt} + \frac{2}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{le\\wwpt}}\delta_{sr} \\
&+ \frac{2}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_q C_{\substack{qe\\wwst}}\delta_{pr} + \frac{2}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_q C_{\substack{qe\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_q C_{\substack{qe\\wwsr}}\delta_{pt} \\
&+ \frac{2}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_q C_{\substack{qe\\wwpt}}\delta_{sr} + \frac{1}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_u C_{\substack{eu\\stww}}\delta_{pr} + \frac{1}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_u C_{\substack{eu\\prww}}\delta_{st} \\
&+ \frac{1}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_u C_{\substack{eu\\srww}}\delta_{pt} + \frac{1}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_u C_{\substack{eu\\ptww}}\delta_{sr} + \frac{1}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_e C_{\substack{ed\\stww}}\delta_{pr} \\
&+ \frac{1}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_e C_{\substack{ed\\prww}}\delta_{st} + \frac{1}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_e C_{\substack{ed\\srww}}\delta_{pt} + \frac{1}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_e C_{\substack{ed\\ptww}}\delta_{sr} \\
&+ \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\stww}}\delta_{pr} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\prww}}\delta_{st} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\srww}}\delta_{pt} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\ptww}}\delta_{sr} \\
&+ \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wwst}}\delta_{pr} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wwpr}}\delta_{st} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wwsr}}\delta_{pt} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wwpt}}\delta_{sr} \\
&+ \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\swwt}}\delta_{pr} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\pwwr}}\delta_{st} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\swwr}}\delta_{pt} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\pwwt}}\delta_{sr} \\
&+ \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wtsw}}\delta_{pr} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wrpw}}\delta_{st} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wrsw}}\delta_{pt} + \frac{1}{3}g_1^2\mathsf{y}_e^2 C_{\substack{ee\\wtpw}}\delta_{sr} \\
&+ 12\mathsf{y}_e^2 g_1^2 C_{\substack{ee\\prst}}
\end{aligned} \tag{C.38}$$

$$
\begin{aligned}
\dot{C}_{\substack{uu\\prst}} &= \frac{2}{3}g_1^2\mathsf{y}_h\mathsf{y}_u C_{\substack{Hu\\st}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_h\mathsf{y}_u C_{\substack{Hu\\pr}}\delta_{st} \\
&+ \frac{2}{3}g_1^2\mathsf{y}_e\mathsf{y}_u C_{\substack{eu\\wwst}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_e\mathsf{y}_u C_{\substack{eu\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{lu\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{lu\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2N_c\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{qu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2N_c\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{qu\\wwpr}}\delta_{st} + \frac{1}{3}g_3^2C^{(8)}_{\substack{qu\\wwpt}}\delta_{rs} + \frac{1}{3}g_3^2C^{(8)}_{\substack{qu\\wwsr}}\delta_{pt} \\
&- \frac{1}{3N_c}g_3^2C^{(8)}_{\substack{qu\\wwst}}\delta_{pr} - \frac{1}{3N_c}g_3^2C^{(8)}_{\substack{qu\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2N_c\mathsf{y}_u^2C_{\substack{uu\\prww}}\delta_{st} + \frac{2}{3}g_1^2N_c\mathsf{y}_u^2C_{\substack{uu\\stww}}\delta_{pr} \\
&+ \frac{2}{3}g_1^2N_c\mathsf{y}_u^2C_{\substack{uu\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2N_c\mathsf{y}_u^2C_{\substack{uu\\wwst}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_u^2C_{\substack{uu\\pwwr}}\delta_{st} + \frac{2}{3}g_1^2\mathsf{y}_u^2C_{\substack{uu\\swwt}}\delta_{pr} \\
&+ \frac{2}{3}g_1^2\mathsf{y}_u^2C_{\substack{uu\\wrpw}}\delta_{st} + \frac{2}{3}g_1^2\mathsf{y}_u^2C_{\substack{uu\\wtsw}}\delta_{pr} + \frac{1}{3}g_3^2C_{\substack{uu\\pwwt}}\delta_{rs} + \frac{1}{3}g_3^2C_{\substack{uu\\swwr}}\delta_{pt} + \frac{1}{3}g_3^2C_{\substack{uu\\wtpw}}\delta_{rs} \\
&+ \frac{1}{3}g_3^2C_{\substack{uu\\wrsw}}\delta_{pt} - \frac{1}{3N_c}g_3^2C_{\substack{uu\\pwwr}}\delta_{st} - \frac{1}{3N_c}g_3^2C_{\substack{uu\\swwt}}\delta_{pr} - \frac{1}{3N_c}g_3^2C_{\substack{uu\\wrpw}}\delta_{st} \\
&- \frac{1}{3N_c}g_3^2C_{\substack{uu\\wtsw}}\delta_{pr} + \frac{2}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_u C^{(1)}_{\substack{ud\\prww}}\delta_{st} + \frac{2}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_u C^{(1)}_{\substack{ud\\stww}}\delta_{pr} + \frac{1}{6}g_3^2C^{(8)}_{\substack{ud\\ptww}}\delta_{rs} \\
&+ \frac{1}{6}g_3^2C^{(8)}_{\substack{ud\\srww}}\delta_{pt} - \frac{1}{6N_c}g_3^2C^{(8)}_{\substack{ud\\prww}}\delta_{st} - \frac{1}{6N_c}g_3^2C^{(8)}_{\substack{ud\\stww}}\delta_{pr} + 6g_3^2C_{\substack{uu\\ptsr}} \\
&- 6\left(\frac{1}{N_c}g_3^2 - 2\mathsf{y}_u^2g_1^2\right)C_{\substack{uu\\prst}}
\end{aligned}
\tag{C.39}
$$

$$
\begin{aligned}
\dot{C}_{\substack{dd\\prst}} &= \frac{2}{3}g_1^2\mathsf{y}_d\mathsf{y}_h C_{\substack{Hd\\st}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_d\mathsf{y}_h C_{\substack{Hd\\pr}}\delta_{st} \\
&+ \frac{2}{3}g_1^2N_c\mathsf{y}_d^2C_{\substack{dd\\prww}}\delta_{st} + \frac{2}{3}g_1^2N_c\mathsf{y}_d^2C_{\substack{dd\\stww}}\delta_{pr} + \frac{2}{3}g_1^2N_c\mathsf{y}_d^2C_{\substack{dd\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2N_c\mathsf{y}_d^2C_{\substack{dd\\wwst}}\delta_{pr} \\
&+ \frac{2}{3}g_1^2\mathsf{y}_d^2C_{\substack{dd\\pwwr}}\delta_{st} + \frac{2}{3}g_1^2\mathsf{y}_d^2C_{\substack{dd\\swwt}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_d^2C_{\substack{dd\\wtsw}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_d^2C_{\substack{dd\\wrpw}}\delta_{st} + \frac{1}{3}g_3^2C_{\substack{dd\\pwwt}}\delta_{rs} \\
&+ \frac{1}{3}g_3^2C_{\substack{dd\\swwr}}\delta_{pt} + \frac{1}{3}g_3^2C_{\substack{dd\\wtpw}}\delta_{rs} + \frac{1}{3}g_3^2C_{\substack{dd\\wrsw}}\delta_{pt} - \frac{1}{3N_c}g_3^2C_{\substack{dd\\pwwr}}\delta_{st} - \frac{1}{3N_c}g_3^2C_{\substack{dd\\swwt}}\delta_{pr} \\
&- \frac{1}{3N_c}g_3^2C_{\substack{dd\\wtsw}}\delta_{pr} - \frac{1}{3N_c}g_3^2C_{\substack{dd\\wrpw}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{ld\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{ld\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{qd\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{qd\\wwst}}\delta_{pr} + \frac{1}{3}g_3^2C^{(8)}_{\substack{qd\\wwsr}}\delta_{pt} + \frac{1}{3}g_3^2C^{(8)}_{\substack{qd\\wwpt}}\delta_{rs} \\
&- \frac{1}{3N_c}g_3^2C^{(8)}_{\substack{qd\\wwpr}}\delta_{st} - \frac{1}{3N_c}g_3^2C^{(8)}_{\substack{qd\\wwst}}\delta_{pr} + \frac{2}{3}g_1^2\mathsf{y}_d\mathsf{y}_e C_{\substack{ed\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2\mathsf{y}_d\mathsf{y}_e C_{\substack{ed\\wwst}}\delta_{pr} \\
&+ \frac{2}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_u C^{(1)}_{\substack{ud\\wwpr}}\delta_{st} + \frac{2}{3}g_1^2N_c\mathsf{y}_d\mathsf{y}_u C^{(1)}_{\substack{ud\\wwst}}\delta_{pr} + \frac{1}{6}g_3^2C^{(8)}_{\substack{ud\\wwpt}}\delta_{rs} + \frac{1}{6}g_3^2C^{(8)}_{\substack{ud\\wwsr}}\delta_{pt} \\
&- \frac{1}{6N_c}g_3^2C^{(8)}_{\substack{ud\\wwpr}}\delta_{st} - \frac{1}{6N_c}g_3^2C^{(8)}_{\substack{ud\\wwst}}\delta_{pr} + 6g_3^2C_{\substack{dd\\ptsr}} - 6\left(\frac{1}{N_c}g_3^2 - 2\mathsf{y}_d^2g_1^2\right)C_{\substack{dd\\prst}}
\end{aligned}
\tag{C.40}
$$

$$
\begin{aligned}
\dot{C}_{\substack{eu\\prst}} &= \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_h C_{\substack{Hu\\st}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_h \mathsf{y}_u C_{\substack{He\\pr}}\delta_{st} + \frac{8}{3}g_1^2 \mathsf{y}_l \mathsf{y}_u C_{\substack{le\\wwpr}}\delta_{st} + \frac{8}{3}g_1^2 \mathsf{y}_e \mathsf{y}_l C_{\substack{lu\\wwst}}\delta_{pr} \\
&+ \frac{8}{3}g_1^2 N_c \mathsf{y}_q \mathsf{y}_u C_{\substack{qe\\wwpr}}\delta_{st} + \frac{8}{3}g_1^2 N_c \mathsf{y}_e \mathsf{y}_q C^{(1)}_{\substack{qu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_e \mathsf{y}_u C_{\substack{uu\\stww}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 N_c \mathsf{y}_e \mathsf{y}_u C_{\substack{uu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_u C_{\substack{uu\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_u C_{\substack{uu\\wtsw}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_u C_{\substack{ee\\prww}}\delta_{st} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_u C_{\substack{ee\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_u C_{\substack{ee\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_u C_{\substack{ee\\wrpw}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_u^2 C_{\substack{eu\\prww}}\delta_{st} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_e^2 C_{\substack{eu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_e C^{(1)}_{\substack{ud\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_u C_{\substack{ed\\prww}}\delta_{st} + 12 \mathsf{y}_e \mathsf{y}_u g_1^2 C_{\substack{eu\\prst}}
\end{aligned}
\tag{C.41}
$$

$$
\begin{aligned}
\dot{C}_{\substack{ed\\prst}} &= \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_h C_{\substack{Hd\\st}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_h C_{\substack{He\\pr}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_e C_{\substack{dd\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_e C_{\substack{dd\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_e C_{\substack{dd\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_e C_{\substack{dd\\wtsw}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_e C_{\substack{ee\\prww}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_e C_{\substack{ee\\wwpr}}\delta_{st} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_e C_{\substack{ee\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_e C_{\substack{ee\\wrpw}}\delta_{st} + \frac{8}{3}g_1^2 \mathsf{y}_e \mathsf{y}_l C_{\substack{ld\\wwst}}\delta_{pr} + \frac{8}{3}g_1^2 \mathsf{y}_d \mathsf{y}_l C_{\substack{le\\wwpr}}\delta_{st} \\
&+ \frac{8}{3}g_1^2 N_c \mathsf{y}_e \mathsf{y}_q C^{(1)}_{\substack{qd\\wwst}}\delta_{pr} + \frac{8}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_q C_{\substack{qe\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_u C_{\substack{eu\\prww}}\delta_{st} \\
&+ \frac{4}{3}g_1^2 N_c \mathsf{y}_e \mathsf{y}_u C^{(1)}_{\substack{ud\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d^2 C_{\substack{ed\\prww}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_e^2 C_{\substack{ed\\wwst}}\delta_{pr} + 12 \mathsf{y}_e \mathsf{y}_d g_1^2 C_{\substack{ed\\prst}}
\end{aligned}
\tag{C.42}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{ud\\prst}} &= \frac{4}{3}g_1^2 \mathsf{y}_h \mathsf{y}_u C_{\substack{Hd\\st}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_h C_{\substack{Hu\\pr}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_u C_{\substack{uu\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_u C_{\substack{uu\\wwpr}}\delta_{st} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_u C_{\substack{uu\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_u C_{\substack{uu\\wrpw}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_u C_{\substack{dd\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_u C_{\substack{dd\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_u C_{\substack{dd\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_u C_{\substack{dd\\wtsw}}\delta_{pr} + \frac{8}{3}g_1^2 N_c \mathsf{y}_d \mathsf{y}_q C^{(1)}_{\substack{qu\\wwpr}}\delta_{st} + \frac{8}{3}g_1^2 N_c \mathsf{y}_q \mathsf{y}_u C^{(1)}_{\substack{qd\\wwst}}\delta_{pr} \\
&+ \frac{8}{3}g_1^2 \mathsf{y}_d \mathsf{y}_l C_{\substack{lu\\wwpr}}\delta_{st} + \frac{8}{3}g_1^2 \mathsf{y}_l \mathsf{y}_u C_{\substack{ld\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c \mathsf{y}_d^2 C^{(1)}_{\substack{ud\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c \mathsf{y}_u^2 C^{(1)}_{\substack{ud\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 \mathsf{y}_d \mathsf{y}_e C_{\substack{eu\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2 \mathsf{y}_e \mathsf{y}_u C_{\substack{ed\\wwst}}\delta_{pr} + 12 \mathsf{y}_u \mathsf{y}_d g_1^2 C^{(1)}_{\substack{ud\\prst}} + 3\left(\frac{N_c^2-1}{N_c^2}\right) g_3^2 C^{(8)}_{\substack{ud\\prst}}
\end{aligned}
\tag{C.43}
$$

$$
\begin{aligned}
\dot{C}^{(8)}_{\substack{ud\\prst}} &= \frac{4}{3}g_3^2 C_{\substack{uu\\pwwr}}\delta_{st} + \frac{4}{3}g_3^2 C_{\substack{uu\\wrpw}}\delta_{st} + \frac{4}{3}g_3^2 C_{\substack{dd\\swwt}}\delta_{pr} + \frac{4}{3}g_3^2 C_{\substack{dd\\wtsw}}\delta_{pr} + \frac{4}{3}g_3^2 C^{(8)}_{\substack{qu\\wwpr}}\delta_{st} \\
&+ \frac{4}{3}g_3^2 C^{(8)}_{\substack{qd\\wwst}}\delta_{pr} + \frac{2}{3}g_3^2 C^{(8)}_{\substack{ud\\prww}}\delta_{st} + \frac{2}{3}g_3^2 C^{(8)}_{\substack{ud\\wwst}}\delta_{pr} + 12\left(\mathsf{y}_u \mathsf{y}_d g_1^2 - \frac{1}{N_c}g_3^2\right) C^{(8)}_{\substack{ud\\prst}} \\
&+ 12 g_3^2 C^{(1)}_{\substack{ud\\prst}}
\end{aligned}
\tag{C.44}
$$

$(\overline{L}L)(\overline{R}R)$:

$$
\begin{aligned}
\dot{C}_{\substack{le\\prst}} = {} & \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\substack{He\\st}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C^{(1)}_{\substack{Hl\\pr}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ll\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ll\\wwpr}}\delta_{st} \\
& + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ll\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ll\\wrpw}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_q C^{(1)}_{\substack{lq\\prww}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_q C_{\substack{qe\\wwst}}\delta_{pr} \\
& + \frac{4}{3}g_1^2\mathsf{y}_e^2 C_{\substack{le\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l^2 C_{\substack{le\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_u C_{\substack{lu\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_e C_{\substack{ld\\prww}}\delta_{st} \\
& + \frac{4}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_u C_{\substack{eu\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_l C_{\substack{ed\\stww}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ee\\stww}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ee\\swwt}}\delta_{pr} \\
& + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ee\\wtsw}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ee\\wwst}}\delta_{pr} - 12\mathsf{y}_l\mathsf{y}_e g_1^2 C_{\substack{le\\prst}}
\end{aligned}
\tag{C.45}
$$

$$
\begin{aligned}
\dot{C}_{\substack{lu\\prst}} = {} & \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\substack{Hu\\st}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_u C^{(1)}_{\substack{Hl\\pr}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{ll\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{ll\\wwpr}}\delta_{st} \\
& + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{ll\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{ll\\wrpw}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{lq\\prww}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_q C^{(1)}_{\substack{qu\\wwst}}\delta_{pr} \\
& + \frac{4}{3}g_1^2 N_c\mathsf{y}_u^2 C_{\substack{lu\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l^2 C_{\substack{lu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_u C_{\substack{ld\\prww}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_u C_{\substack{le\\prww}}\delta_{st} \\
& + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_l C^{(1)}_{\substack{ud\\stww}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{eu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_u C_{\substack{uu\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_u C_{\substack{uu\\wwst}}\delta_{pr} \\
& + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{uu\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C_{\substack{uu\\wtsw}}\delta_{pr} - 12\mathsf{y}_l\mathsf{y}_u g_1^2 C_{\substack{lu\\prst}}
\end{aligned}
\tag{C.46}
$$

$$
\begin{aligned}
\dot{C}_{\substack{ld\\prst}} = {} & \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_l C_{\substack{Hd\\st}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_h C^{(1)}_{\substack{Hl\\pr}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{ll\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{ll\\wwpr}}\delta_{st} \\
& + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{ll\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{ll\\wrpw}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{lq\\prww}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_q C^{(1)}_{\substack{qd\\wwst}}\delta_{pr} \\
& + \frac{4}{3}g_1^2 N_c\mathsf{y}_d^2 C_{\substack{ld\\prww}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l^2 C_{\substack{ld\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_u C_{\substack{lu\\prww}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_e C_{\substack{le\\prww}}\delta_{st} \\
& + \frac{4}{3}g_1^2 N_c\mathsf{y}_l\mathsf{y}_u C^{(1)}_{\substack{ud\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C_{\substack{ed\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_l C_{\substack{dd\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_l C_{\substack{dd\\wwst}}\delta_{pr} \\
& + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{dd\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C_{\substack{dd\\wtsw}}\delta_{pr} - 12\mathsf{y}_l\mathsf{y}_d g_1^2 C_{\substack{ld\\prst}}
\end{aligned}
\tag{C.47}
$$

$$
\begin{aligned}
\dot{C}_{\substack{qe\\prst}} &= \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_q C_{\substack{He\\st}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_h C^{(1)}_{\substack{Hq\\pr}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_q C^{(1)}_{\substack{qq\\prww}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_q C^{(1)}_{\substack{qq\\wwpr}}\delta_{st} \\
&+ \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C^{(1)}_{\substack{qq\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C^{(1)}_{\substack{qq\\wrpw}}\delta_{st} + 4g_1^2\mathsf{y}_e\mathsf{y}_q C^{(3)}_{\substack{qq\\pwwr}}\delta_{st} + 4g_1^2\mathsf{y}_e\mathsf{y}_q C^{(3)}_{\substack{qq\\wrpw}}\delta_{st} \\
&+ \frac{8}{3}g_1^2\mathsf{y}_e\mathsf{y}_l C^{(1)}_{\substack{lq\\wwpr}}\delta_{st} + \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C_{\substack{le\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e^2 C_{\substack{qe\\prww}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_q^2 C_{\substack{qe\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 N_c\mathsf{y}_e\mathsf{y}_u C^{(1)}_{\substack{qu\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_e C^{(1)}_{\substack{qd\\prww}}\delta_{st} \\
&+ \frac{4}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C_{\substack{eu\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C_{\substack{ed\\stww}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C_{\substack{ee\\stww}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C_{\substack{ee\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C_{\substack{ee\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C_{\substack{ee\\wtsw}}\delta_{pr} - 12\mathsf{y}_q\mathsf{y}_e g_1^2 C_{\substack{qe\\prst}}
\end{aligned}
\tag{C.48}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{qu\\prst}} &= \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_q C_{\substack{Hu\\st}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_u C^{(1)}_{\substack{Hq\\pr}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{qq\\prww}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{qq\\wwpr}}\delta_{st} \\
&+ \frac{4}{3}g_1^2\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{qq\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{qq\\wrpw}}\delta_{st} + 4g_1^2\mathsf{y}_q\mathsf{y}_u C^{(3)}_{\substack{qq\\pwwr}}\delta_{st} + 4g_1^2\mathsf{y}_q\mathsf{y}_u C^{(3)}_{\substack{qq\\wrpw}}\delta_{st} \\
&+ \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_u C^{(1)}_{\substack{lq\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_u C_{\substack{qe\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_u C^{(1)}_{\substack{qd\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c\mathsf{y}_u^2 C^{(1)}_{\substack{qu\\prww}}\delta_{st} \\
&+ \frac{8}{3}g_1^2 N_c\mathsf{y}_q^2 C^{(1)}_{\substack{qu\\wwst}}\delta_{pr} + \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C_{\substack{lu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C_{\substack{eu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{ud\\stww}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C_{\substack{uu\\stww}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C_{\substack{uu\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_q\mathsf{y}_u C_{\substack{uu\\swwt}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2\mathsf{y}_q\mathsf{y}_u C_{\substack{uu\\wtsw}}\delta_{pr} - 12\mathsf{y}_q\mathsf{y}_u g_1^2 C^{(1)}_{\substack{qu\\prst}} - 3\left(\frac{N_c^2-1}{N_c^2}\right) g_3^2 C^{(8)}_{\substack{qu\\prst}}
\end{aligned}
\tag{C.49}
$$

$$
\begin{aligned}
\dot{C}^{(8)}_{\substack{qu\\prst}} &= \frac{4}{3}g_3^2 C^{(1)}_{\substack{qq\\pwwr}}\delta_{st} + \frac{4}{3}g_3^2 C^{(1)}_{\substack{qq\\wrpw}}\delta_{st} + 4g_3^2 C^{(3)}_{\substack{qq\\pwwr}}\delta_{st} + 4g_3^2 C^{(3)}_{\substack{qq\\wrpw}}\delta_{st} \\
&+ \frac{2}{3}g_3^2 C^{(8)}_{\substack{qu\\prww}}\delta_{st} + \frac{2}{3}g_3^2 C^{(8)}_{\substack{qd\\prww}}\delta_{st} + \frac{4}{3}g_3^2 C^{(8)}_{\substack{qu\\wwst}}\delta_{pr} + \frac{2}{3}g_3^2 C^{(8)}_{\substack{ud\\stww}}\delta_{pr} + \frac{4}{3}g_3^2 C_{\substack{uu\\swwt}}\delta_{pr} \\
&+ \frac{4}{3}g_3^2 C_{\substack{uu\\wtsw}}\delta_{pr} - \left(12\mathsf{y}_q\mathsf{y}_u g_1^2 + 6\left(N_c - \frac{2}{N_c}\right) g_3^2\right) C^{(8)}_{\substack{qu\\prst}} - 12 g_3^2 C^{(1)}_{\substack{qu\\prst}}
\end{aligned}
\tag{C.50}
$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{qd\\prst}} &= \frac{4}{3}g_1^2\mathsf{y}_h\mathsf{y}_q C_{\substack{Hd\\st}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_h C^{(1)}_{\substack{Hq\\pr}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{qq\\prww}}\delta_{st} + \frac{8}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{qq\\wwpr}}\delta_{st} \\
&+ \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{qq\\pwwr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_q C^{(1)}_{\substack{qq\\wrpw}}\delta_{st} + 4g_1^2\mathsf{y}_d\mathsf{y}_q C^{(3)}_{\substack{qq\\pwwr}}\delta_{st} + 4g_1^2\mathsf{y}_d\mathsf{y}_q C^{(3)}_{\substack{qq\\wrpw}}\delta_{st} \\
&+ \frac{8}{3}g_1^2\mathsf{y}_d\mathsf{y}_l C^{(1)}_{\substack{lq\\wwpr}}\delta_{st} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_e C_{\substack{qe\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_u C^{(1)}_{\substack{qu\\prww}}\delta_{st} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d^2 C^{(1)}_{\substack{qd\\prww}}\delta_{st} \\
&+ \frac{8}{3}g_1^2 N_c\mathsf{y}_q^2 C^{(1)}_{\substack{qd\\wwst}}\delta_{pr} + \frac{8}{3}g_1^2\mathsf{y}_l\mathsf{y}_q C_{\substack{ld\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_e\mathsf{y}_q C_{\substack{ed\\wwst}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 N_c\mathsf{y}_q\mathsf{y}_u C^{(1)}_{\substack{ud\\wwst}}\delta_{pr} + \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C_{\substack{dd\\stww}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_q C_{\substack{dd\\swwt}}\delta_{pr} + \frac{4}{3}g_1^2\mathsf{y}_d\mathsf{y}_q C_{\substack{dd\\wtsw}}\delta_{pr} \\
&+ \frac{4}{3}g_1^2 N_c\mathsf{y}_d\mathsf{y}_q C_{\substack{dd\\wwst}}\delta_{pr} - 12\mathsf{y}_q\mathsf{y}_d g_1^2 C^{(1)}_{\substack{qd\\prst}} - 3\left(\frac{N_c^2-1}{N_c^2}\right)g_3^2 C^{(8)}_{\substack{qd\\prst}} \qquad \text{(C.51)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}^{(8)}_{\substack{qd\\prst}} &= \frac{4}{3}g_3^2 C^{(1)}_{\substack{qq\\pwwr}}\delta_{st} + \frac{4}{3}g_3^2 C^{(1)}_{\substack{qq\\wrpw}}\delta_{st} + 4g_3^2 C^{(3)}_{\substack{qq\\pwwr}}\delta_{st} + 4g_3^2 C^{(3)}_{\substack{qq\\wrpw}}\delta_{st} \\
&+ \frac{2}{3}g_3^2 C^{(8)}_{\substack{qu\\prww}}\delta_{st} + \frac{2}{3}g_3^2 C^{(8)}_{\substack{qd\\prww}}\delta_{st} + \frac{4}{3}g_3^2 C^{(8)}_{\substack{qd\\wwst}}\delta_{pr} + \frac{2}{3}g_3^2 C^{(8)}_{\substack{ud\\wwst}}\delta_{pr} + \frac{4}{3}g_3^2 C_{\substack{dd\\swwt}}\delta_{pr} \\
&+ \frac{4}{3}g_3^2 C_{\substack{dd\\wtsw}}\delta_{pr} - \left(12\mathsf{y}_q\mathsf{y}_d g_1^2 + 6\left(N_c - \frac{2}{N_c}\right)g_3^2\right)C^{(8)}_{\substack{qd\\prst}} - 12g_3^2 C^{(1)}_{\substack{qd\\prst}} \qquad \text{(C.52)}
\end{aligned}
$$

$(\overline{L}R)(\overline{R}L)$:

$$\dot{C}_{\underset{prst}{ledq}} = -\left(6\left(\mathsf{y}_d\left(\mathsf{y}_q - \mathsf{y}_e\right) + \mathsf{y}_e\left(\mathsf{y}_e + \mathsf{y}_q\right)\right)g_1^2 + 3\left(N_c - \frac{1}{N_c}\right)g_3^2\right)C_{\underset{prst}{ledq}} \tag{C.53}$$

Class $(\overline{L}R)(\overline{L}R)$:

$$\begin{aligned}
\dot{C}^{(1)}_{\underset{prst}{quqd}} =\ & 4g_1(\mathsf{y}_q + \mathsf{y}_u)C_{\underset{st}{dB}}[Y_u^\dagger]_{pr} - 6g_2 C_{\underset{st}{dW}}[Y_u^\dagger]_{pr} - \frac{8}{N_c}g_1(\mathsf{y}_q + \mathsf{y}_u)C_{\underset{pt}{dB}}[Y_u^\dagger]_{sr} \\
& + \frac{12}{N_c}g_2 C_{\underset{pt}{dW}}[Y_u^\dagger]_{sr} - 8\frac{N_c^2 - 1}{N_c^2}g_3 C_{\underset{pt}{dG}}[Y_u^\dagger]_{sr} + 4g_1(\mathsf{y}_q + \mathsf{y}_d)C_{\underset{pr}{uB}}[Y_d^\dagger]_{st} - 6g_2 C_{\underset{pr}{uW}}[Y_d^\dagger]_{st} \\
& - \frac{8}{N_c}g_1(\mathsf{y}_q + \mathsf{y}_d)C_{\underset{sr}{uB}}[Y_d^\dagger]_{pt} + \frac{12}{N_c}g_2 C_{\underset{sr}{uW}}[Y_d^\dagger]_{pt} - 8\frac{N_c^2 - 1}{N_c^2}g_3 C_{\underset{sr}{uG}}[Y_d^\dagger]_{pt} \\
& - \frac{1}{2}\left(\left(3\mathsf{y}_d^2 + 2\mathsf{y}_d\mathsf{y}_u + 3\mathsf{y}_u^2\right)g_1^2 + 3g_2^2 + 12\left(N_c - \frac{1}{N_c}\right)g_3^2\right)C^{(1)}_{\underset{prst}{quqd}} \\
& - \frac{1}{N_c}\left(\left(\left(3\mathsf{y}_d^2 + 10\mathsf{y}_d\mathsf{y}_u + 3\mathsf{y}_u^2\right)g_1^2 - 3g_2^2\right) + 8\left(N_c - \frac{1}{N_c}\right)g_3^2\right)C^{(1)}_{\underset{srpt}{quqd}} \\
& - \frac{1}{2}\left(1 - \frac{1}{N_c^2}\right)\left(\left(3\mathsf{y}_d^2 + 10\mathsf{y}_d\mathsf{y}_u + 3\mathsf{y}_u^2\right)g_1^2 - 3g_2^2 + 4\left(N_c - \frac{2}{N_c}\right)g_3^2\right)C^{(8)}_{\underset{srpt}{quqd}} \\
& + 2\left(1 - \frac{1}{N_c^2}\right)g_3^2 C^{(8)}_{\underset{prst}{quqd}}
\end{aligned} \tag{C.54}$$

$$\begin{aligned}
\dot{C}^{(8)}_{\underset{prst}{quqd}} =\ & 8g_3 C_{\underset{st}{dG}}[Y_u^\dagger]_{pr} - 16g_1(\mathsf{y}_q + \mathsf{y}_u)C_{\underset{pt}{dB}}[Y_u^\dagger]_{sr} + 24g_2 C_{\underset{pt}{dW}}[Y_u^\dagger]_{sr} + \frac{16}{N_c}g_3 C_{\underset{pt}{dG}}[Y_u^\dagger]_{sr} \\
& + 8g_3 C_{\underset{pr}{uG}}[Y_d^\dagger]_{st} - 16g_1(\mathsf{y}_q + \mathsf{y}_d)C_{\underset{sr}{uB}}[Y_d^\dagger]_{pt} + 24g_2 C_{\underset{sr}{uW}}[Y_d^\dagger]_{pt} + \frac{16}{N_c}g_3 C_{\underset{sr}{uG}}[Y_d^\dagger]_{pt} \\
& + 8g_3^2 C^{(1)}_{\underset{prst}{quqd}} + \left(-2\left(3\mathsf{y}_d^2 + 10\mathsf{y}_d\mathsf{y}_u + 3\mathsf{y}_u^2\right)g_1^2 + 6g_2^2 + 16\frac{1}{N_c}g_3^2\right)C^{(1)}_{\underset{srpt}{quqd}} \\
& + \left(\left(-\frac{3}{2}\mathsf{y}_d^2 - \mathsf{y}_d\mathsf{y}_u - \frac{3}{2}\mathsf{y}_u^2\right)g_1^2 - \frac{3}{2}g_2^2 + 2\left(N_c - \frac{1}{N_c}\right)g_3^2\right)C^{(8)}_{\underset{prst}{quqd}} \\
& + \frac{1}{N_c}\left(\left(3\mathsf{y}_d^2 + 10\mathsf{y}_d\mathsf{y}_u + 3\mathsf{y}_u^2\right)g_1^2 - 3g_2^2 + 4\left(-N_c - \frac{2}{N_c}\right)g_3^2\right)C^{(8)}_{\underset{srpt}{quqd}}
\end{aligned} \tag{C.55}$$

$$
\begin{aligned}
\dot{C}^{(1)}_{\substack{lequ\\prst}} = &-\left(6\left(\mathsf{y}_e^2+\mathsf{y}_e\left(\mathsf{y}_u-\mathsf{y}_q\right)+\mathsf{y}_q\mathsf{y}_u\right)g_1^2+3\left(N_c-\frac{1}{N_c}\right)g_3^2\right)C^{(1)}_{\substack{lequ\\prst}} \\
&-\left(24\left(\mathsf{y}_q+\mathsf{y}_u\right)\left(2\mathsf{y}_e-\mathsf{y}_q+\mathsf{y}_u\right)g_1^2-18g_2^2\right)C^{(3)}_{\substack{lequ\\prst}}
\end{aligned}
\tag{C.56}
$$

$$
\begin{aligned}
\dot{C}^{(3)}_{\substack{lequ\\prst}} = &\, g_1(\mathsf{y}_q+\mathsf{y}_u)C_{\substack{eB\\pr}}[Y_u^\dagger]_{st}-\frac{3}{2}g_2C_{\substack{uW\\st}}[Y_e^\dagger]_{pr}+g_1(\mathsf{y}_l+\mathsf{y}_e)C_{\substack{uB\\st}}[Y_e^\dagger]_{pr}-\frac{3}{2}g_2C_{\substack{eW\\pr}}[Y_u^\dagger]_{st} \\
&+\left(\left(2\left(\mathsf{y}_e^2-\mathsf{y}_e\mathsf{y}_q+\mathsf{y}_e\mathsf{y}_u-2\mathsf{y}_q^2+5\mathsf{y}_q\mathsf{y}_u-2\mathsf{y}_u^2\right)g_1^2-3g_2^2\right)+\left(N_c-\frac{1}{N_c}\right)g_3^2\right)C^{(3)}_{\substack{lequ\\prst}} \\
&+\frac{1}{8}\left(-4\left(\mathsf{y}_q+\mathsf{y}_u\right)\left(2\mathsf{y}_e-\mathsf{y}_q+\mathsf{y}_u\right)g_1^2+3g_2^2\right)C^{(1)}_{\substack{lequ\\prst}}
\end{aligned}
\tag{C.57}
$$

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