ALMA MATER STUDIORUM – UNIVERSITÀ DI BOLOGNA
SEDE DI CESENA
SECONDA FACOLTÀ DI INGEGNERIA CON SEDE A CESENA
CORSO DI LAUREA MAGISTRALE IN INGEGNERIA BIOMEDICA

"TACTILE PERCEPTION – PERCEPTION OF TACTILE DISTANCE CHANGES WITH BODY SITE: A NEURAL NETWORK MODELLING STUDY."

Tesi in

Sistemi Neurali LM

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III SESSIONE
ANNO ACCADEMICO 2010/2011

To my family:
Mum, Dad, Erika, Massimo,
and my sweet Gaia….

KEY WORDS FOR THIS THESIS:

- Computational Model
- Synaptic Connections
- Tactile Perception
- Weber’s Illusion

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INTRODUCTION

Distortions in the perception of the distance between two punctual stimuli applied on the skin surface of different body regions are called **Weber's Illusion**. This illusion was confirmed by many experiments, in which subjects were asked to judge the distance between two stimuli applied on the skin surface of different body regions. Results have shown that the same distance between the stimuli was judged different for different body regions.

The concept that the distance on the skin is frequently misperceived is largely supported, but the neural mechanisms underlying this illusion are still far to be well understood. In particular, it is still unclear how the distance between two simultaneous tactile stimuli is codified at a neural level, and which brain areas are involved in this computation.

Weber's Illusion may be partly explained considering the differences in receptors density of various body regions, and the distorted image about the body that our brain has inside the Primary Somatic Sensory Cortex (homunculus). However, these mechanisms seem not to be sufficient to explain the observed phenomenon: indeed, according to 100 years-experimental results, the biases in distance judgement are much smaller than imbalances that primary representation would suggest. In other words, the observed illusion is much smaller compared to the effect that the differences in receptor density, or cortical extent would produce. This has lead to the hypothesis that judging tactile distance may require the recruitment of additional brain areas, and mechanisms that operate to rescale – at least partially – information from primary representation, in order to preserve tactile size constancy throughout the body surface

That is, it has been proposed the occurrence of sort of "**Rescaling Process**" that operates to reduce this illusion toward a more verisimilar perception.

The occurrence of this rescaling process is supported by many neuroscience researchers; in particular, Dr. *Matthew Longo* from the Department of Psychological Sciences, Birkbeck University of London is conducting research on tactile distance perception and body representation, and its research supports this hypothesis. However, the neural mechanisms and circuits at the basis of this potential rescaling process are still largely unknown.

The aim of this thesis was to clarify the possible network organization, and neural mechanisms explaining the Weber's Illusion and the rescaling process, by using a neural network model. Much of the work was conducted at the Department of Psychological Sciences, Birkbeck University of London, under the supervision of Dr. *Matthew Longo*.

In order to replicate Weber's Illusion and rescaling process, the neural network has been organized in two main layers of neurons that may correspond to two functionally different cortical areas:

- ***First Layer*** of neurons (that performs the first processing of the external stimuli): this layer might mimic the *Primary Somatic Sensory Cortex* affected by cortical magnification.
- ***Second Layer*** of neurons (that further elaborates information from the previous layer): this layer may represent a *Higher Cortical Areas* (e.g. in temporo-parietal region) involved in the implementation of the Rescaling Process.

Neural networks have been constructed including synapses connection within each neuron layers (lateral synapses), and between the two neural layers (feed-forward synapses), and assuming that the activity of each neurons depends on its input via a static sigmoidal relationship, and a first order dynamics. In particular, by using the previous structure, I have implemented two different neural networks, for two different body regions (for example, the Hand and the Arm), characterized by different tactile resolution and cortical magnification to replicate the Weber's Illusion and the Rescaling Process.

These models may help to understand the mechanism of the Weber's Illusion, and to give a possible explanation of the Rescaling Process. Moreover, the neural networks may help to understand how the brain can interpret the distance on the skin surface, and these models could be used to make new predictions to be verified later, in vivo, by tactile experiments on real subjects. It is important to underline that the developed models are mainly functional models and they are not intended replicate physiologic and anatomic details.

The main results achieved via the models are the reproduction of the Weber's Illusion for the two different considered body regions (Hand and Arm), as reported in many articles about the tactile illusions. Weber's Illusion was recorded from the output of the neural networks, and represented by graphics, trying to explain the reasons of these results.

The main part of this thesis was developed in the Department of Psychological Sciences, at Birkbeck, University of London under the supervision of Dr. *Matthew Longo*, for a period of 5 months. The contribute of *Dr. Longo* was very useful due to his great experience about experiments involved in the tactical perceptions. His help was directed especially toward the interpretation of the model outputs, giving suggestions about processing of network results, in order to obtain clearer information. Moreover, Dr. *Longo* has provided help in validation results, performing statistical test. Besides the benefit of interacting directly with an expert person as Dr. *Longo*, another benefit of may visit at the Birkbeck University was the improvement of my English, and the contact with a different university reality, as well as the experience of working in a team of researchers.

The present thesis is organized in four chapters.

The first chapter explains the theoretical aspects of the Weber's Illusion, and the tactile processing information that starts from the skin surface of a body region, and continues through different layers in the cortex.

The second chapter is entitled "Mathematical Model"; it explains the network structure, provides a quantitative description of the model containing all mathematical formulas, explanation of each parameter, and provides example of neural activation within the two network layers.

Chapter 3 shows the results of the simulations: this chapter provide deep analyses of the output of the neural network in response to the application of an input (two punctual stimuli).

Finally Chapter 4 concerns the “Parameters Sensitivity Analysis”: in that Chapter, simulation results have been analysed after changing the value of one parameter at a time, in order to assess which parameters mainly affect network behaviour.

Chapter 1

TACTILE INFORMATION PROCESSING AND TACTILE DISTANCE PERCEPTION

Introduction

The perception of distance for dual tactile pressures changes for different body regions. Over a century ago, Weber reported the concept that the distance on the skin is frequently misperceived. In the Weber's illusion, the perception of distance between two tactile punctual stimuli is different in different parts of the body, increasing as the spatial acuity of the stimulated area increases.

Probably, the main reasons of this Illusion, are relative to the different size of the Receptive Field (RF) on the skin surface bonded to each cortical neuron, and the fact that the Primary Somatic Sensory Cortex reserves different amounts of area for representation of different body regions: for example, the Area involved in the perception of the hand, is much bigger than the Area about the back or the arm (homunculus).

Different extensions of cortical surface implicate different resolutions on different body regions: there are much more neurons in the brain involved to the hand, than the arm. Thus, the area inside the cortex reserved to the hand is bigger compare with the arm's area. This means that the hand has a higher resolution in the comparison with the arm.

My thesis intends to implement a model able to simulate how the brain can interpret tactile stimuli on the skin surface in two different body regions. From the Weber's experiment the application of two simultaneous stimuli (on the skin surface with a **constant distance**, on two different regions (like the hand and the arm) produce two **different** perceptions of distance.

As I stated above, the classical explanation of the Weber's Illusion is the difference in the density of tactile receptors and cortical magnification across body parts. However, this explanation is unsatisfactory, because the illusion is much smaller than the differences in receptor density or cortical extent.

Therefore, there must be present a sort of rescaling process along the pathways of tactile information processing in the brain that might decrease this huge difference (relative to the homunculus dimension) and brings it to an acceptable proportion. It is clear that we feel a sort of distortion in the perceived distance, but basically it is kept down.

The neural mechanisms underlying this rescaling process are still largely unknown. Aim of this thesis is to contribute to clarify the neural mechanisms that may implement this rescaling, via a neural network modelling study. Of course, the model includes several simplifications and it doesn't aspire to reproduce exactly the reality, also because the number of variables that should be considered is massive. This is mainly a conceptual model that, with the help of some hypotheses, tries to reproduce and interpret some experimental evidences (Weber's illusion, distortions in perceive distance).

Before starting to explain my model, in this introduction I'm going to introduce few important concepts about the Receptive Field, the Primary Somatosensory Cortex, and the Homunculus.

1.1 Touch

Touch is mediated by mechanoreceptors in the skin and the tactile sensitivity is greatest on the hairless skin (glabrous), on the fingers, palm, sole of the foot and lips. When an object presses against the hand, the skin conforms to its contours. All mechanoreceptors sense these changes in skin contour, and they answer with a particular physiological function that will reach the Somatic Sensory Cortex inside the Central Nervous System.

All somatosensory information from the limbs and trunk is conveyed by **dorsal root ganglion neurons**. This kind of neuron is well suited to its two principal functions:

1. Stimulus transduction.
2. Transmission of encoded stimulus information to the central nervous system.

The cell body of these neurons lies in a ganglion on the dorsal root of spinal nerve. The axon has two branches, one direct to the periphery and the other one to the central nervous system.

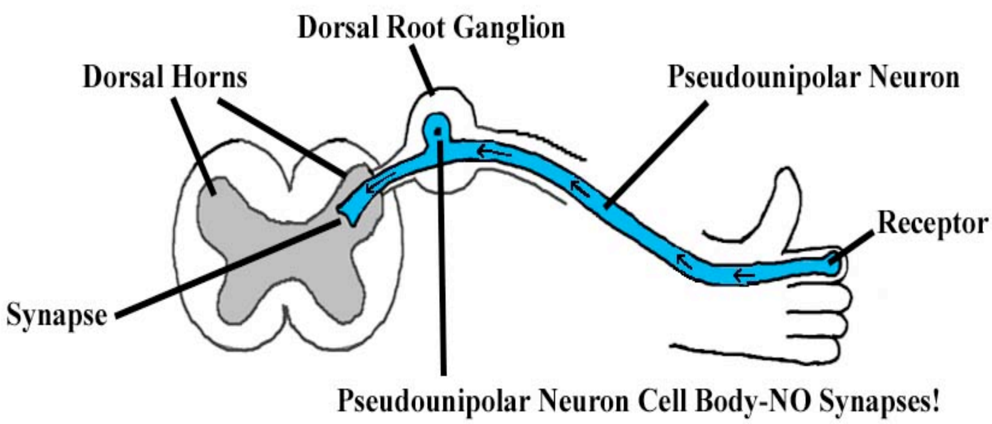


Figure 1. 1 *Position of the Dorsal Root Ganglion.*

Mechanoreceptors and proprioceptors are innervated by dorsal root ganglion neurons with large diameter and myelinated axons that conduct action potential quickly. **Thermal receptors** and **nocireceptors** have smaller diameter axons that are thinly myelinated; these nerves conduct impulses very slowly. ("*Principle of Neuron Science. Chapter 22: The Bodily Sense.*" Kandel)

Neurologists distinguish between two classes of somatic sensation: *epicritic* and *protopathic*. ***Epicritic sensations*** involve aspects of touch and are mediated by encapsulated receptor. Instead, ***Protopathic sensation*** involves pain and temperature senses, and are mediated by receptors with bare nerve endings. In this thesis we will concentrate on Epicritic Sensations.

Information transmitted to the brain from mechanoreceptors in the hand, enabled us to feel the shape and texture of objects, type on computer keyboards, play musical instruments, etc. The ability to recognize objects placed in the hand on the basis of touch alone, is one of the most important and complex function of the somatosensory system. Tactile information about an object is fragmented by peripheral sensors, and must be integrated by the brain. In fact, an object stimulates a large number of receptors and sensory nerve fibres, each of which scans a little part of the object. Spatial properties are processed by populations of receptors that form many parallel pathways to the brain. It is the job of the central nervous system to reconstruct the correct shape of the object from the received fragmented information.

1.2 Mechanoreceptors and Receptive Fields

Mechanoreceptors differ in morphology and skin location. Histological and physiological studies have identified four major types of mechanoreceptors on the glabrous skin. Two of these are located in the superficial layers of the skin (Meissner's corpuscle and Merkel disk receptor), and the other two in the subcutaneous tissue (Pacinian corpuscle and Ruffini ending). The small superficial receptors sense deformation of the papillary ridges in which they reside. The larger subcutaneous receptors sense deformation of a wider area of skin that extends beyond the overlying ridges ("*Principle of Neuron Science. Chapter 22: The Bodily Sense.*" Kandel).

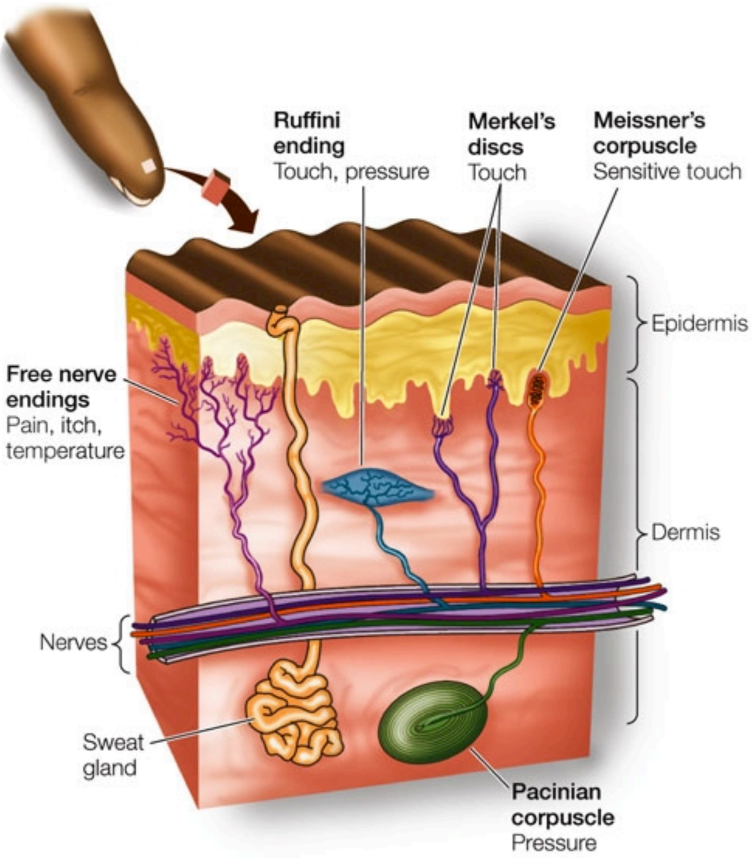


Figure 1. 2 *Position of the Dorsal Root Ganglion.*

Another important aspect is that mechanoreceptors on glabrus skin vary in the size and structure of their receptive field (RF).

The **receptive field** of a sensory neuron is a region of space on the skin in which the presence of a stimulus will alter the firing of that neuron. Receptive fields have been identified for neurons of the auditory system, the somatosensory system, and the visual system.In the somatosensory system, receptive fields are regions of the skin or of internal organs. Some types of mechanoreceptors have **large receptive fields**, while others have **smaller ones**.

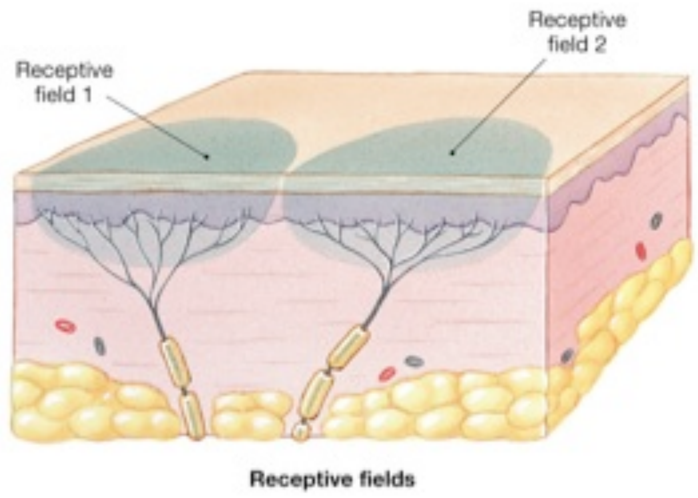
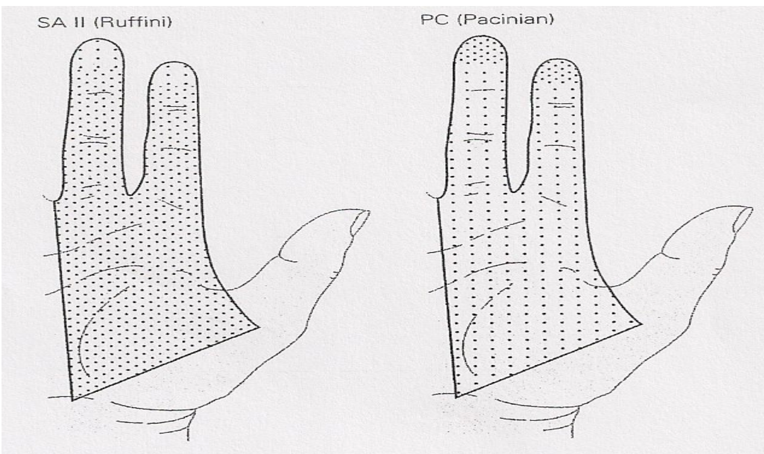


Figure 1. 3 Example of *Receptive Fields.*

Large receptive fields allow the cell to detect changes over a wider area, but lead to a less precise perception. Thus, the fingertips are the most densely innervated regions of the skin in the human body, receiving approximately 300 mechanoreceptors nerve fibres per square centimetre, with small receptive field, which enable the ability to detect fine details. While the back and legs, for example, have fewer receptors with large receptive fields. Receptors with large receptive fields usually have a "hot spot", an area within the receptive field (usually in the centre, directly over the receptor) where stimulation produces the most intense response. It has to be considered that the density of receptors changes along the entire body. So, there are regions with a high density receptors (like hands), and regions with low density receptors (like trunk, or arms).



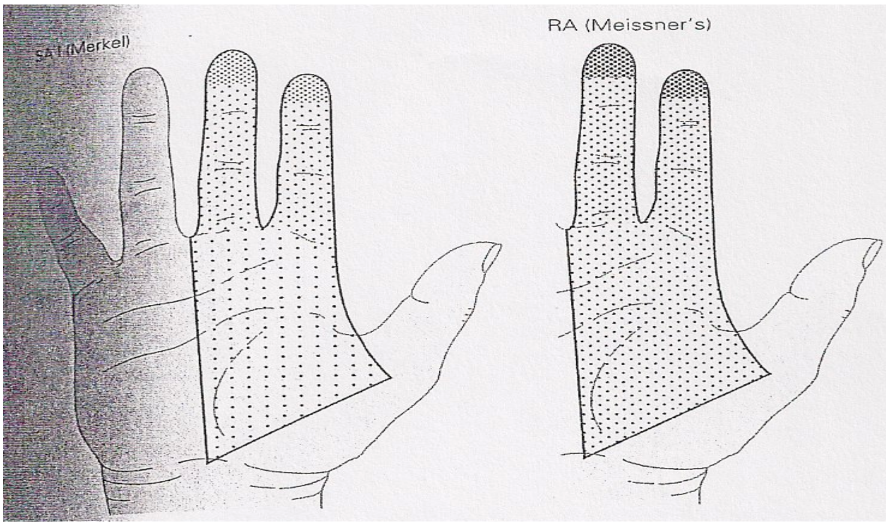


Figure 1. 4 *The distribution of receptor types in the human hand varies.*

Tactile-sense-related cortical neurons have receptive fields on the skin that can be modified by experience, or by injury to sensory nerves, resulting in changes in the field's size and position. In general these neurons have relatively large receptive fields (much larger than those of dorsal root ganglion cells). However, the neurons are able to discriminate fine details due to patterns of excitation and inhibition: I will explain this important concept in the next pages of this chapter.

1.3 Somatic Sensory Cortex

Sensory information is processed in a series of relay regions within the brain. There are three synaptic relay sites between sensory receptors in the skin, and the cerebral cortex. Mechanoreceptors in the skin send their axon to the **caudal medulla**, where they terminate in the **gracile nuclei**. These second order neurons project directly to the contralateral thalamus, terminating in the ventral posterior lateral nucleus. A parallel pathway from the principal **trigeminal nucleus**, which represents the face, ascends to the ventral posterior medial nucleus. The third order neurons in the thalamus send axons to the ***Primary Somatic Sensory Cortex***, located in the post-central gyrus of the parietal lobe.

The Primary Somatic Sensory Cortex (S-I) contains four cytoarchitectural areas: *Brodmann's areas 3a*, *3b*, *1*, and *2*. Most thalamic fibres terminate in area 3a and 3b, and the cells in area 3a and 3b project their axons to areas 1 and 2. Thalamic neurons also send a small projection directly to area 1 and 2. Area 3b and 1 receive information from receptor in the skin, while area 3a and 2 receive proprioceptive information from receptors in muscles and joints. However all these areas are interconnected, so that, both serial and parallel processing are involved in higher-order elaboration of sensory information.

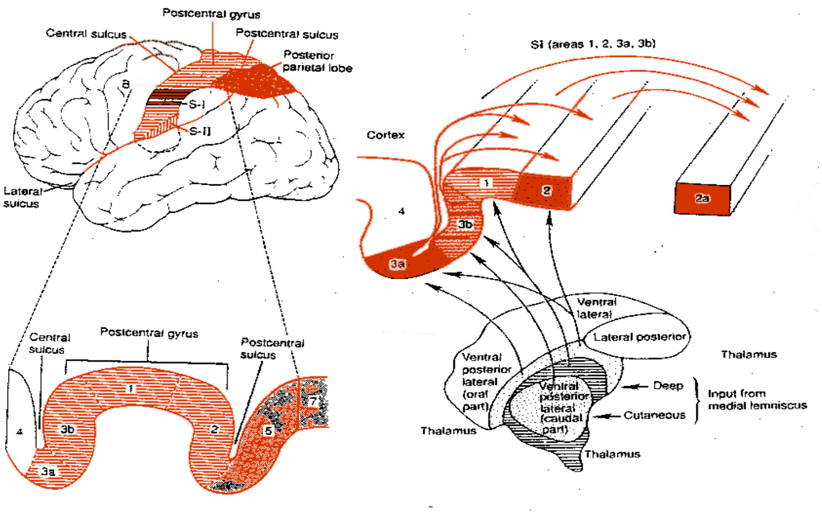


Figure 1. 5 *Primary Somatic Sensory Cortex.*

The ***Secondary Somatosensory Cortex (S-II)***, located on the superior bank of the lateral fissure, is innervated by neurons from each of the four areas of S-I. The fibres from S-I are required for the function of S-II. The S-II cortex projects to the insular cortex, which in turn innervates regions of the temporal lobe believed to be important for tactile memory.

Finally, other important somatosensory cortex areas are located in the *posterior parietal cortex*: Brodmann's area 5 and 7. These areas receive input from both **S-I** and **pulvinar** (associational function). Area 5 integrates tactile information from mechanoreceptors in the skin with proprioceptive inputs from muscles and joint. This region also encapsulates information about the hands. Area 7 receives visual, as well as tactile, and proprioceptive inputs. The posterior parietal cortex projects to the motor areas, and plays an important role in sensory initiation and guidance of movement.

1.4 Cortical Neuron RF

The neurons in the primary somatic sensory cortex receive, at least, three synaptic connections beyond the peripheral receptors. Thus, their responses reflect information processed in the dorsal column nuclei, the thalamus, and in the cortex itself. They receive information from the skin, and they can be slowly, or rapidly adapting neurons.

Since each cortical neuron receive inputs form receptors in a specific area on the skin, cortical neurons also have receptive fields. All the cortical neurons are identify by their RF, as well as by their sensory modality. Any point on the skin is represented in the cortex by a population of cortical cells, connected to the afferents fibres that innervate that point on the skin. When a point on the skin is stimulated, the population of cortical neurons linked to the receptors at that location is excited. We perceive contact at a particular region on the skin, because a specific population of neurons in the brain is activated.

The receptive fields of cortical neurons are much larger than those of peripheral neurons. For example, the RF of a neuron in area 3b represents a composite of inputs from about 300-400 mechanoreceptors afferents. Receptive fields in higher cortical areas are even larger. A cortical neuron responds best to excitation in the middle of its receptive field; as the stimulation site is moved toward the periphery of the field, response becomes weaker until eventually no spikes is recorded.

A more complex receptive field structure emerges when the skin is touched in two or more points simultaneously. Stimulation of regions of skin surrounding the excitatory region of the receptive field of a cortical neuron, can reduce the excitatory response of the neuron, because afferent inputs surrounding the excitatory region are inhibitory. These regions of the receptive field are called the *inhibitory surround*. This spatial distribution of excitation, and inhibition is necessary to sharpen the peak of activity inside the brain. Inhibitory interneurons are present in the dorsum column nuclei, in the thalamus, and in the cortex itself. Inhibitory interneurons are fundamental to limit the spatial spread of excitation, in order to obtain a distinct recognition of simultaneous stimuli on the skin.

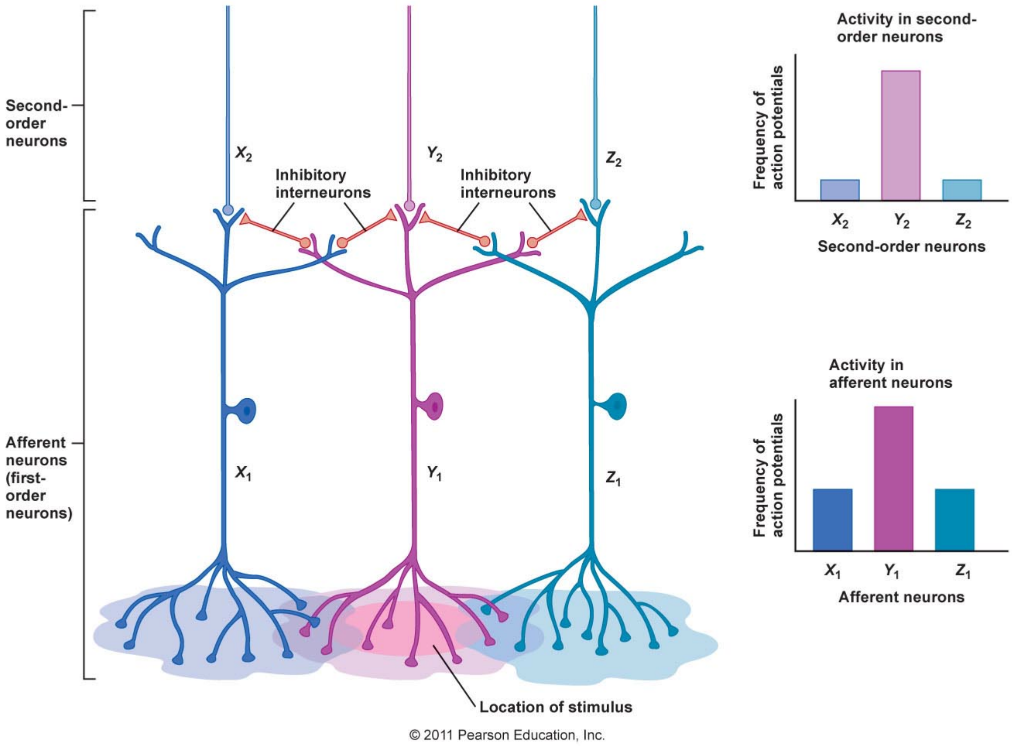


Figure 1. 6 *Example of Surround Inhibition.*

In this thesis project, we have implemented a neural network model that simulates the response to the application of two nearby tactile stimuli as input. Therefore is clear that the inhibitory process is a very important aspect to be considered. Especially, lateral inhibition can aid in two-point discrimination. We are able to perceive two points rather than one, because two distinct populations of neurons are activated in the cortex. If the stimuli are very close, the activity in the two populations tends to overlap, and the distinction between the two peaks might become blurred. However, the inhibition produced by each stimulus is summated in the overlap region. As a result of this more effective inhibition, the peaks of activity in the two corresponding populations become more sharpened, leading to a separation of the two populations.

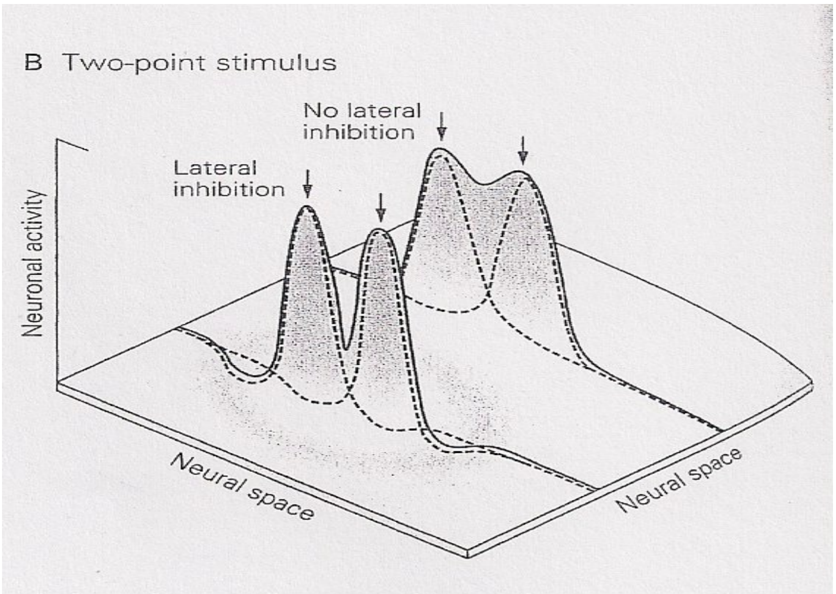


Figure 1. 7 *Stimulation of two adjacent points. Lateral Inhibitory networks suppress excitation of the neurons between the points, sharpening the central focus, and preserving the spatial clarity of the original stimulus (solid line).*

1.5 Tactile Illusion: Weber's Illusion

In 1834 E. H. Weber described a tactile illusion. Two points kept equidistant moved over the body surface are felt to converge or diverge. The two points are perceived as converging, when passing from a high resolution region (hand) to a less sensitivity region (arm), and are perceived as diverging when passing from a low-resolution region to a highest one. This means that the same distance between the two stimulated points is perceived in different way on different body regions.

A possible explanation of this illusion may be found in the structure (size) of the RF along the whole body, and also in the variance of receptor density on the skin surface.

The size of the RFs, in a particular region of skin, establishes the capacity to determine whether one or more points are stimulated. If two points within the same receptive field are stimulated, the neuron will signal only one detection. But if the points are located in the receptive field of two different nerve fibres, then information about both points of stimulation will be signalled. The contrast between active, and inactive fibre, seems to be fundamental for resolving spatial details, or to evaluate the distance between two points. Spatial resolution on various region of the skin can be quantified in humans by measuring their ability to perceive a pair of nearby stimuli as two different entities. The minimum distance between two detectable stimuli is called ***two point discrimination threshold***.

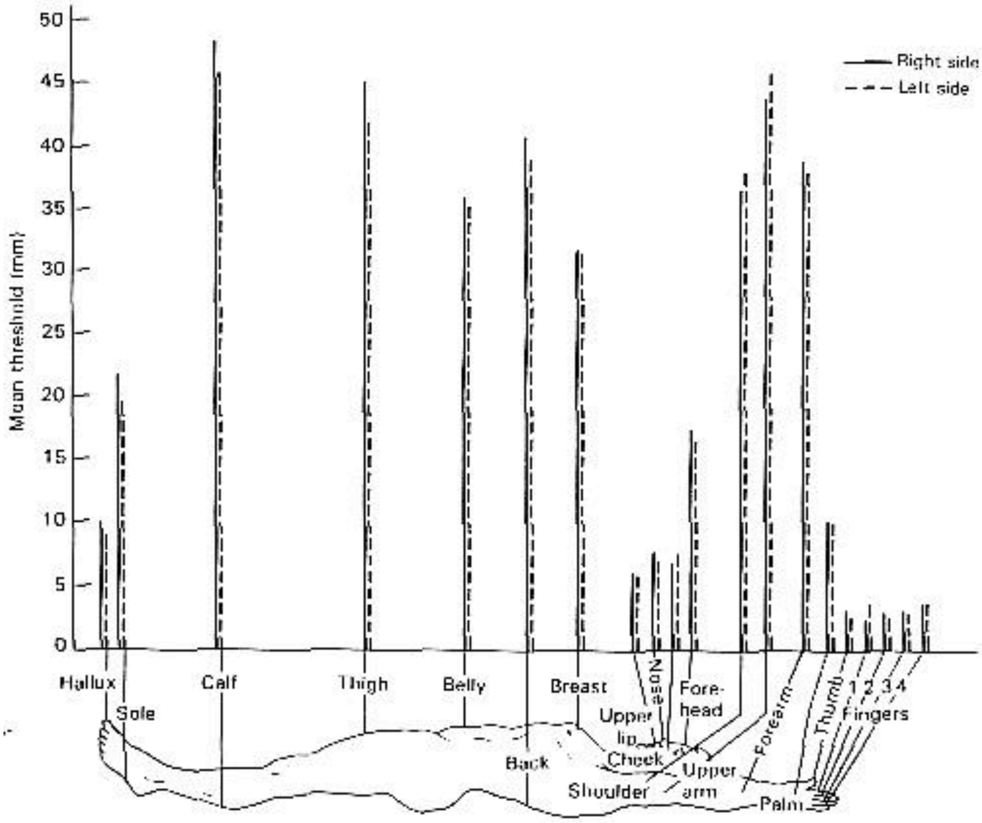


Figure 1. 8 *Two Point Discrimination Threshold along the body.*

These variations are correlated with the RF dimension and with the innervation density of mechanoreceptors on the skin surface.

1.5.1 Homunculus

However, it is licit think that the Weber's illusion is also linked to the distribution of space reserved for each body regions in the Primary Somatic Sensory Cortex. As it is well known, in this part of brain are present neurons that receive tactile and sensitive information. The space reserved for body regions doesn't reflect the spatial topography of these regions, rather reflects the density of peripheral innervation of the skin surface A good way to understand the Area reserved for each body parts inside the Primary Somatic Sensory Cortex, is to take a look at the somatosensory map in the figure:

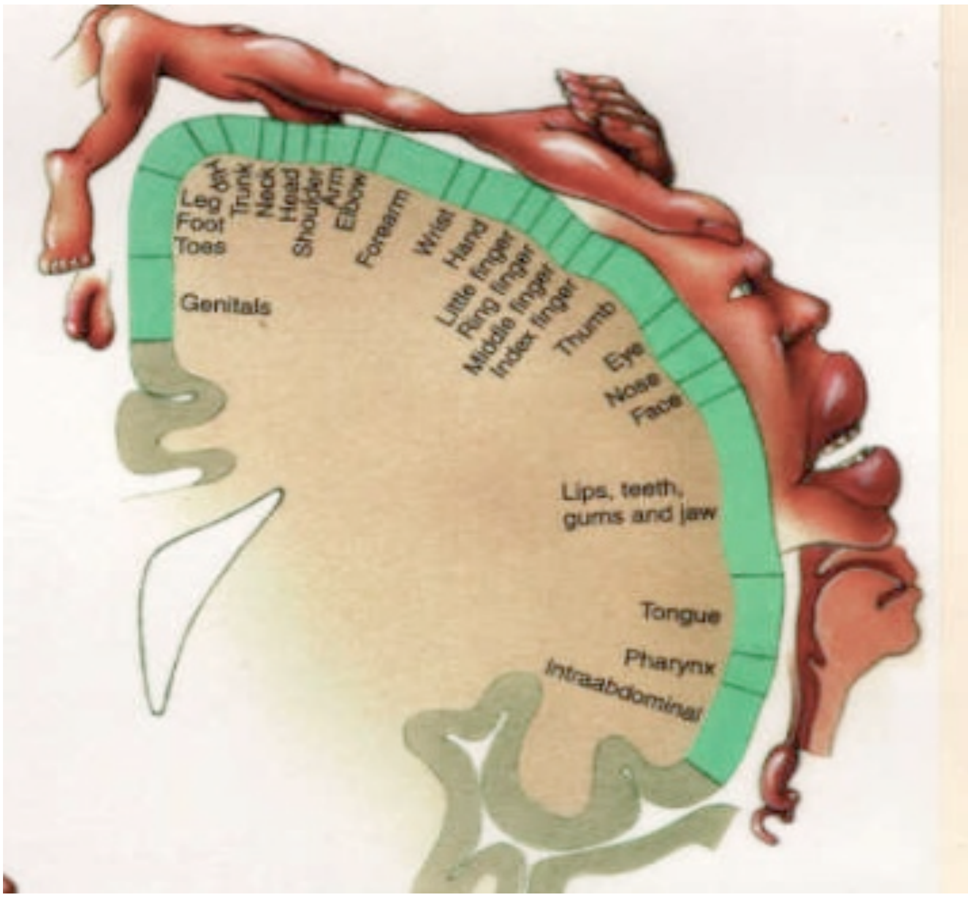


Figure 1. 9 *Body Regions in the Primary Somatic Sensory Cortex.*

The image of the body in the brain exaggerates certain body regions, particular the hand, foot and mouth, and compresses more proximal body parts. A drawing of somatosensory homunculus would be like this:

Figure 1. 10 Representation of the *Homunculus.*

The reason for the bizarre, distorted appearance of the homunculus is that the amount of cerebral tissue, or cortex, devoted to a given body region is proportional to how richly innervated and sensitive is it, not to its size. In fact, the map represents the innervation density of the skin rather than its total surface area. In humans, a lot of cortical columns receive input from the hands, especially from fingers. Similarly, a large number of cortical neurons receive input from the foot and the face. The proximal portions of the limbs and trunk, are much less densely innervated; so, fewer cortical neurons receive inputs from these regions.

It has to be considered that the somatotopic maps are not fixed, but can change by experience. The details of the map vary from subject to subject. A tennis player will develop a larger proportion of cortical neurons devoted to the arm than a pianist, who needs to increase sensitivity on different fingers.

An important consequence of the magnification of the hands representation in the cortex, is that the sizes of individual peripheral receptive fields on the hand cover a much smaller area of skin than receptive fields on the arm, which are smaller than receptive fields on the trunk. This is a very important aspect linked to the different resolution on different body parts. So, it is clear that the Weber's Illusion is associated with the concept of different resolution along the whole body.

1.5.2 Magnification concept

To a better understanding about the magnification effect mentioned before, I suggest to have a look at this article ["*Magnification, Receptive-Field Area, and Hypercolumn Size in Areas 3b and 1 of Somatosensory Cortex in Owl Monkeys*" MRIGANKA SUR, MICHAEL M. MERZENICH, AND JON H. KAAS], in which several features in cortical area 3b, and 1, of somatosensory cortex in monkeys, were quantitatively studied. In particular, some experiments performed in that work led to a quantification of the magnification of body regions inside the primary cortex.

The overall magnification curve is illustrated below:

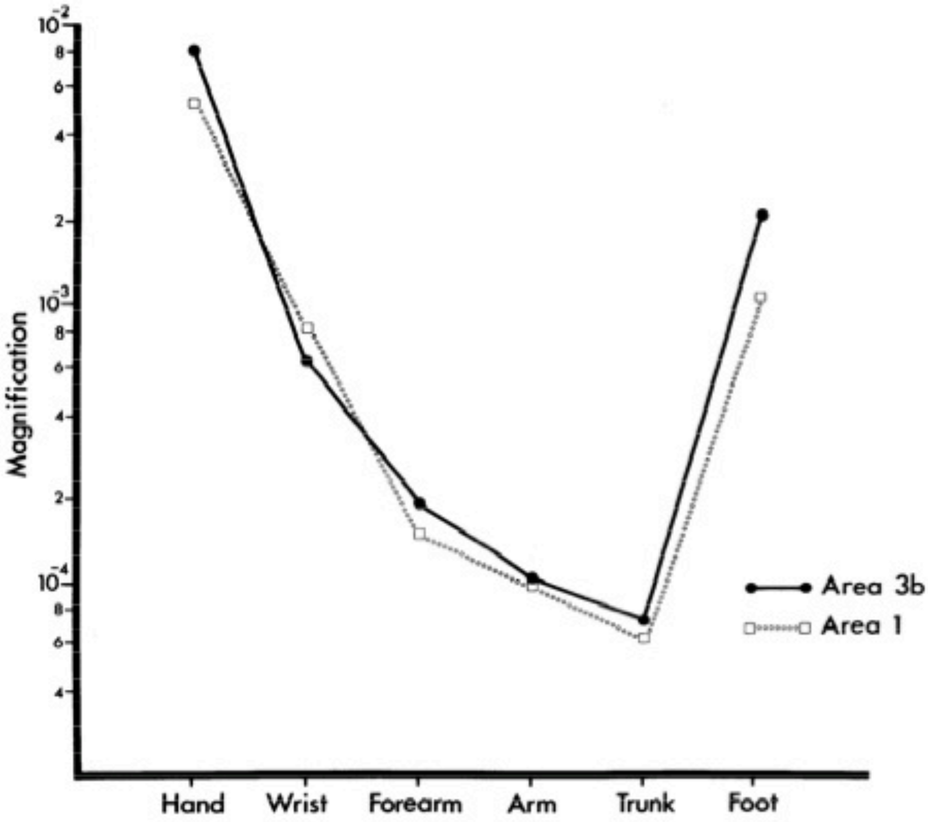


Figure 1. 11 *Recorded Magnification for different body regions in the MRIGANKA's experiment.*

Cortical Magnification varies greatly across different body regions. For example, the glabrous hand, or foot representation, occupies nearly 100 times more cortical tissue per unit body surface area than the trunk, or arm representations in both areas 3b, and 1. Even if these data comes from monkey, they can be also used to study the magnification in the human body, as there are several evidence about the similarity between the monkey's cortex, and human's cortex. This means that if we are going to consider a square surface area of **5x5 cm** on the **hand**, and **10x10 cm** on the **arm**, the equivalent regions in the primary somatic sensory cortex can be obtained with this simple equation:

$$\textbf{Skin} \times \textbf{Magnification} = \textbf{Cortex}$$

25 cm^2 = skin area (hand) → cortex area (hand) = 25 cm^2 × 0.01 = ***0,25 cm^2***

100 cm^2 = skin area (arm) → cortex area (arm) = 100 cm^2 × 0.0001 = ***0,01 cm^2***

The result is that the space reserved in the cortex for the hand is much bigger than that for the arm, even if the square Area considered on the skin of the arm is bigger (100 cm^2) compared with the hand (25 cm^2). That is consistent with the homunculus concept, in which some body regions are enlarged, and other regions are reduced within the cortex.

As a consequence, the application of two stimuli on the skin surface will produce a differences in the perceive distance, as the stimulated body region varies: basically, the distance on the hand should be perceived much bigger than on the arm, just because our brain has a distorted image of our body, and the magnification of each body regions is very different (like this case). After all, as we can see in the graphic, there are some body parts wherein the magnification's difference is low, like trunk, and thigh.

In addition, in the same study, it has been found a relationship between the receptive field size, and the amount of cortical magnification (figure 1.10). In particular, a linear relationship seems to exist between receptive field size and the inverse of cortical magnification.. Basically, regions with a high magnification (like the hands) have receptive field with a small size.

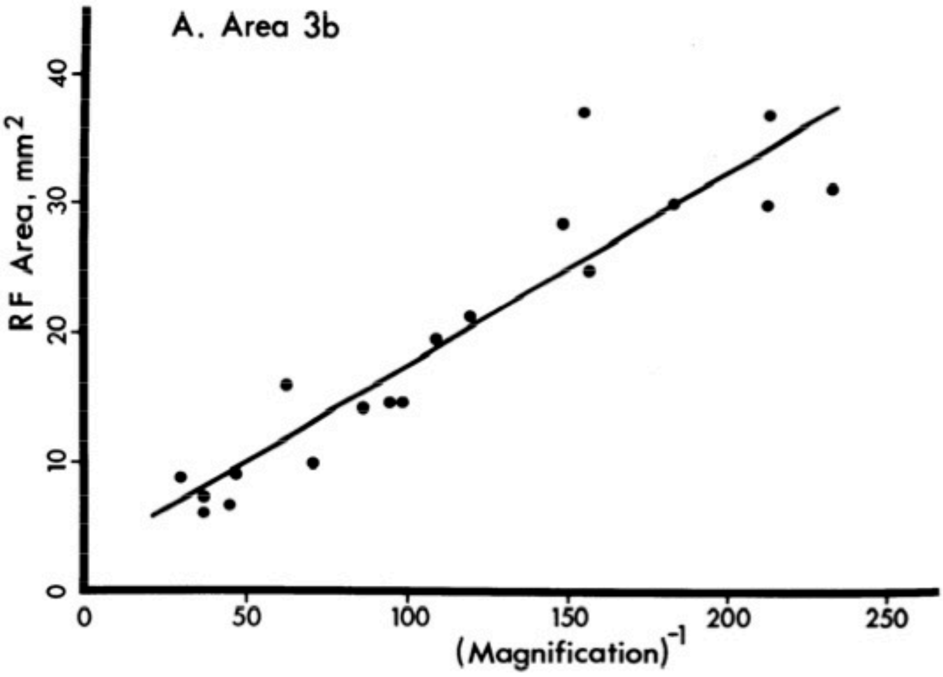


Figure 1. 12 *RF Area (mm^2) as a function of the inverse of the Magnification.*

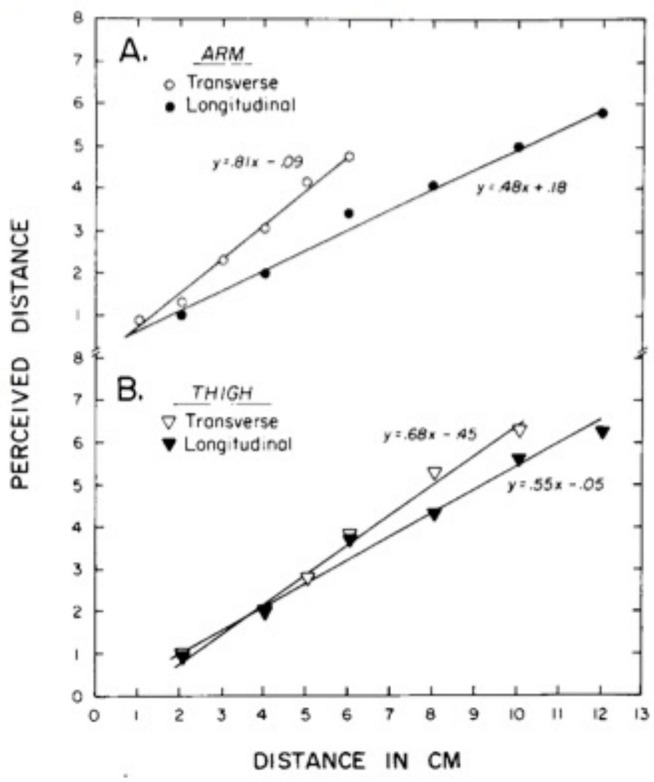
However, focusing to the hand and to the arm, a difference in magnification equivalents to 100 times is very high, and at the same time, this kind of value is not correct to define how the Weber's illusion could happen.

That is, the magnification concept cannot be, by itself, a complete and satisfying explanation of the Weber's illusion. Indeed, considering only the differences in receptor density and cortical magnification, the distortion of the perceived distance across the body regions would be much more higher compared with the actual extent of Weber's illusion. In particular, the gap between the perceived distance on the hand, and the perceived distance on the Arm, should be much more higher, if we considered only the different extension of cortical representations of the two body regions (referring to the monkey study, I should say 100 times different).

1.5.3 Green Experiment

Results of Green experiment entitled “*The perception of distance and location for dual tactile pressures*” support previous consideration. In one experiment Green has used the method of magnitude estimation to construct a psychophysical scale for tactile distance. In that experiment, subjects were asked to assign numbers to represent the apparent distance between two simultaneous punctual tactile stimuli. They were with eyes closed, and four area of the body exposed (palm, forearm, stomach and thigh). Each area was stimulated in both longitudinal and transversal direction. The four areas received stimulation in random sequence, and the precise location of stimulation on each area varied slightly from trial to trial. The stimuli were generated with two brass rods (contactor rods), capped with cylindrical plastic tips that were 4 mm in diameter. Stimulus distance ranged from 1 to 12 cm.

For more details consult the paper: “*The perception of distance and location for dual tactile pressures*” BARRY G. GREEN. Anyway, results of that experiment are shown in the figures below:



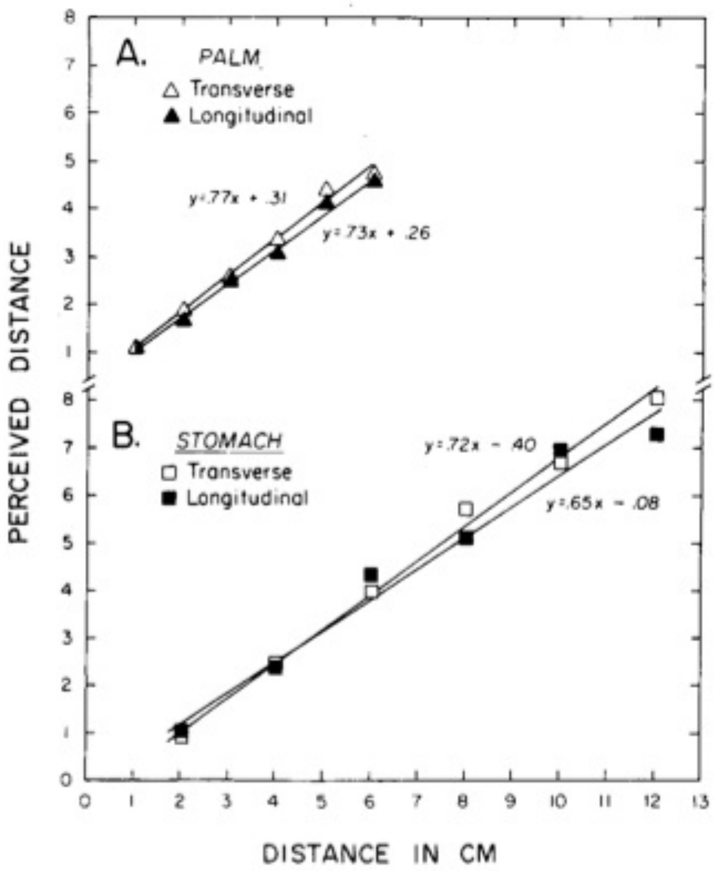


Figure 1. 1. 13 *Green's Experiment Results for four different body regions.*

Thanks to this graphics, it is clear that distortions of body shape produce anisotropy in perceived distance as function of orientation. In this study, that anisotropy appears more evident on the arm than on the palm. Given the importance of this topic, and for a deeper analysis of this aspect I suggest to consult Longo's article titled "*Weber's Illusion and Body Shape: Anisotropy of Tactile Size Perception on the Hand*".

Now, focusing on the graphics concerning the hand and the arm, we can assert that the perceived distances for the two body regions are different:

- 4 cm on the Arm = 2 perceive point.
- 4 cm on the Palm = 3 perceive point.

These results verify the existence of tactile spatial distortions that were first reported by Weber. But this difference is not as high as the difference in the cortical magnification of the two body regions. Considering only differences in cortical magnification, the difference between the two perceived distances should be much bigger.

So, we can confirm that the magnification plays a key role in the Weber's Illusion, because sets a distortion in terms of extension of cortical representation inside the primary cortex, but for sure is not the unique process. We hypothesize that there must be present other processing of tactile information that performs a sort of "rescaling"; this rescaling process, may act by decreasing the huge initial gap due to the different cortical magnification, still providing only a partial compensation (the final result is the Weber's illusion). The neural mechanisms underlying the Weber's illusion are still largely unknown.

1.6 The neural network and the simplifying assumptions

In this thesis, I developed a neural network model that aspires to contribute to clarify the neural mechanisms producing Weber's illusion. Some hypotheses have been considered in model implementation; *the main hypothesis is that the process of perceived distance involves two layers in the brain*. The first one may correspond to the Primary Somatic Sensory Cortex, which receives inputs from the skin surface, wherein we can observe the effect of "Cortical Magnification" . The second layer, linked with feed-forward synapses to the first one, is a higher-level area: it enables to implement a sort of rescaling about the perceived distance, that partially compensates the huge effect resulting from cortical magnification. See the scheme below:

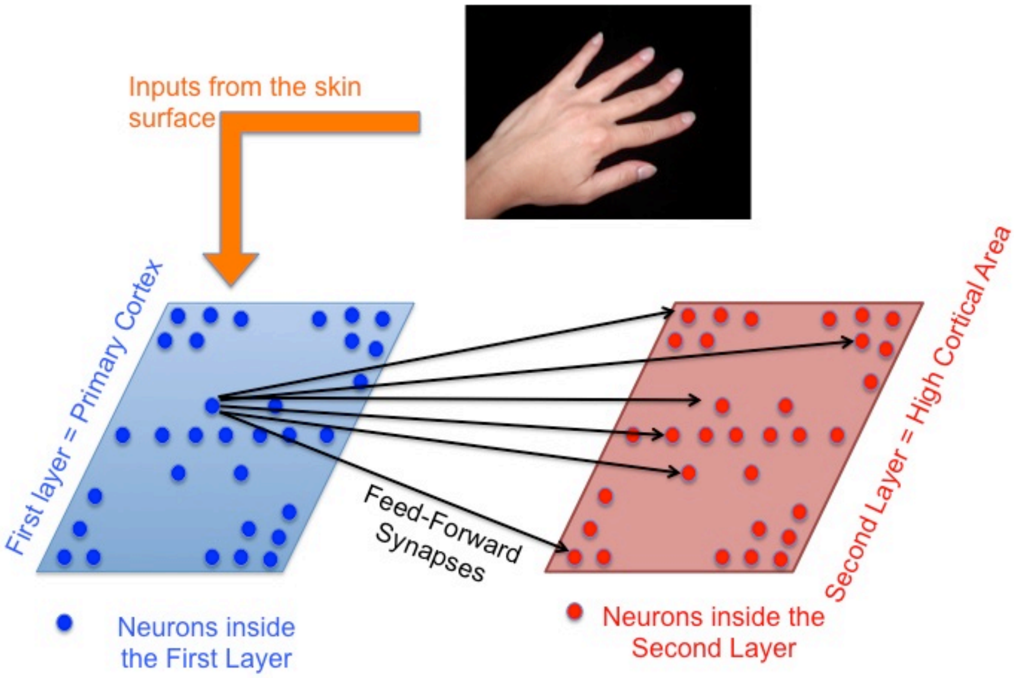


Figure 1. 14 *Neural Network Scheme.*

In particular, in this thesis I have developed a model (implemented in the MATLAB environment) that simulates how the brain can interpret the distance between two punctual stimuli on the skin surface of two different body regions, like hand and arm. Obviously, this solution encloses also the rescaling process mentioned before. The details of this model will be given in the next chapters.

Until now, I always talked about distance and perceived distance, but a question arises: how can the brain read out the distance between two different points on the skin surface, from neural activation in the cortex? This is still an open question. It is clear that this is a key point for the development of this thesis. In the model, we adopted a simplify solution to read distance from neural activation. Of course, we are used to think about the distance in terms of meter, centimetre, millimetre, etc., but these measure units are only a concept that we have learned with the school, and with the life experience. The real problem is to understand how the brain can associate these concepts of distance, with the effective process of elaboration distance that involves neurons inside our cortex. For the sake of simplicity, in the model, *the distance between two points is* *interpreted in terms of the number of inactivated neurons between two excited populations of cortical neurons* (activated by two punctual tactile stimuli).

This chapter is conceived as an introduction, entering the main problem faced by this thesis work. In the next Chapters, I will describe the mathematical model used to simulate Weber's illusion.

Chapter 2

MATHEMATICAL MODEL

2.1 Qualitative description of the model

2.1.1 The two main layers

The model aims to reproduce the main steps inside the cortex leading to the perception of distance between a pair of stimuli applied on the skin surface. To this end, two layers of artificial neurons (that may correspond to two different levels of somatosensory processing in the cerebral cortex) have been used.

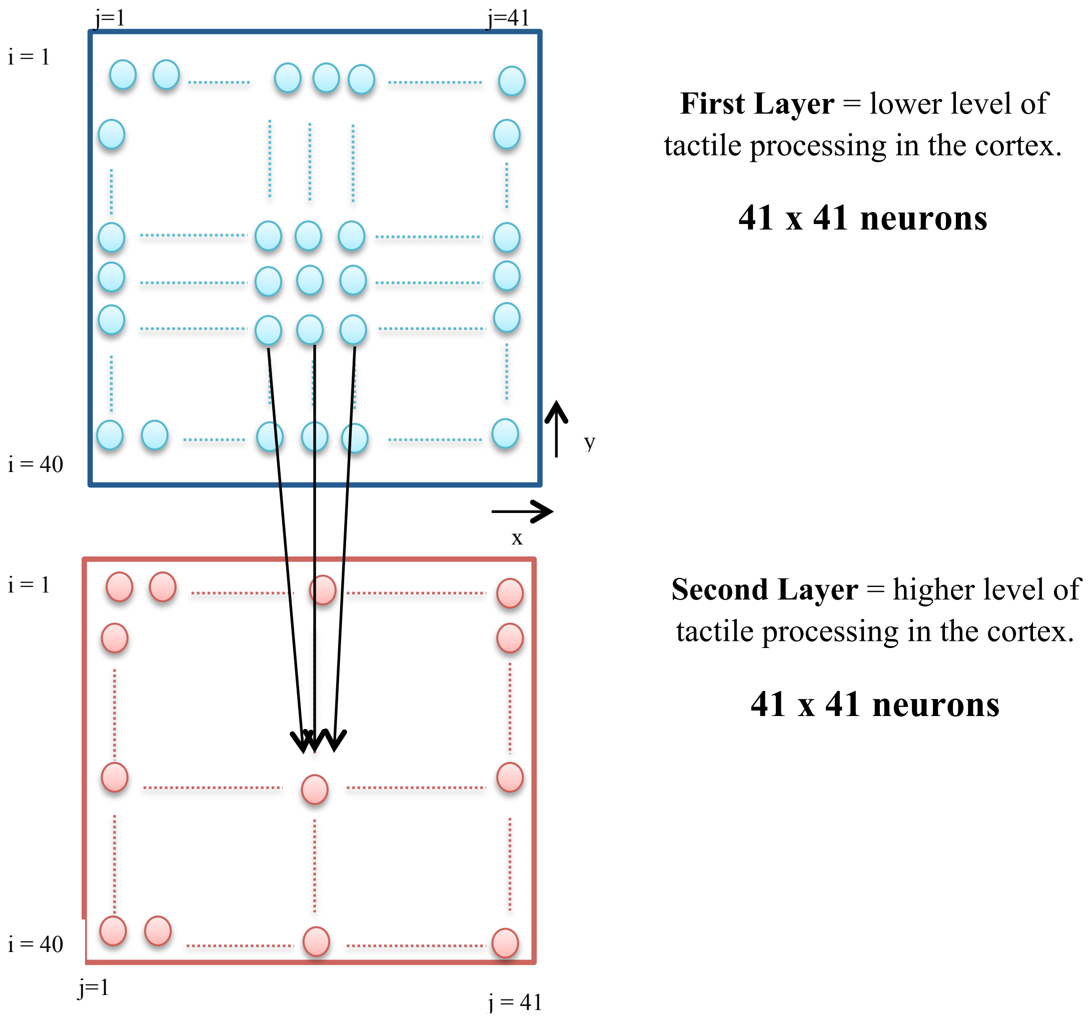
The main idea about this model is that the first layer could represent skin receptors plus primary somatosensory cortex neurons, and it may synthesize the property receptor density-cortical magnification of a skin region. An external tactile input directly stimulates this layer.

Suppose this first layer consists of 41 x 41 units (simulating cortical neurons), corresponding to a skin region of K x K cm (at the moment it does not matter the dimension of the represented skin region. This will be specified later). Each unit on this layer has a Receptive Field (RF) covering a specific portion on the skin surface. Since there are 41 neurons on each side of the matrix, the centres of the RFs are arranged at a distance of:

K cm / 41 neurons

A punctual stimulus, applied in a certain position, activates a "bubble" of neurons on the first layer, in particular all the neurons whose RFs cover that position.

The second layer may represent higher cortical area involved in tactile distance perception, receiving synapses from the first layer. I have assumed that this layer have the same number of units as the first layer (41 x 41 neurons). Activation of the neurons within this second layer in response to an external stimulus depends on the synapses connections from the first layer. A schematic diagram of network structure is represented below.



The model also includes the presence of lateral synapses within each layer (excitatory among proximal neurons, and inhibitory among distal neurons).The second layer may represent higher cortical areas that - starting from a distorted primary representation based on receptor density and cortical magnification - may partially rescale tactile information towards their true size.

The previous structure has been applied to represent two different body regions characterized by different density receptor, and cortical magnification.

- **Region** called **A**, with high-density receptors on the skin, and high cortical magnification. Supposed that this region has a dimension of **5 x 5**

cm on the skin surface, and it is mapped by a matrix of **41 x 41 neurons** in the cortex. In addition, each neuron has small **RF size**.

- **Region** called **B**, with low-density receptors on skin, and low cortical magnification. Supposed that this region has a dimension of **10 x 10 cm** on the skin surface, and it is mapped by a matrix of **41 x 41 neurons** in the cortex.. In addition, neurons in this area have **large RF size**.

Region A could be associated with the hand, whereas region B with the arm. Indeed the number of neurons devoted to each region is the same, but in the region A, 41 x 41 neurons correspond to a skin Area of 25 cm^2 , whereas in region B they cover an Area of 100 cm^2 Hence, Region A has a higher resolution compare with region B . In particular, the spatial resolution of neurons for the two regions is:

- ***REGION A (HAND)*:**

  5 cm / 41 neurons = 0,125 cm (the first neuron is centered in 0 cm and the last neuron is centered in 5 cm).

- ***REGION B (ARM):***

  10 cm / 41 neurons = 0,25 cm (the first neuron is centered in 0 and the last neuron is centered in 10 cm).

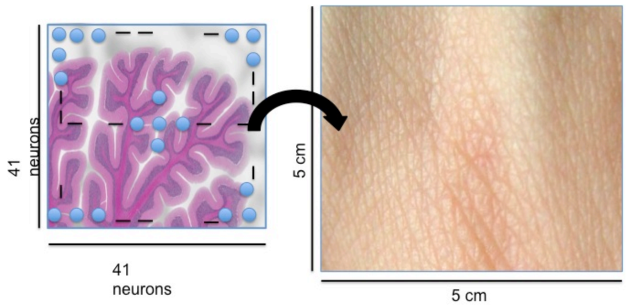


Figure 2. 1 ***Example: Cortex Area of 41 x 41 neurons codifies for a skin region of 5 x 5 cm.***

In Region A, the centers of neuron RFs are arranged at a distance of 0,12 cm whereas in Region B, the centers of neuron RFs are arranged at a distance of 0.24 cm. Moreover, RFs of neurons in the first layer have been assumed smaller in Region A than in Region B. Therefore, we can assert that Region A (Hand) has higher acuity in discriminating the presence of two nearby stimuli. Instead the arm acuity will be lower.

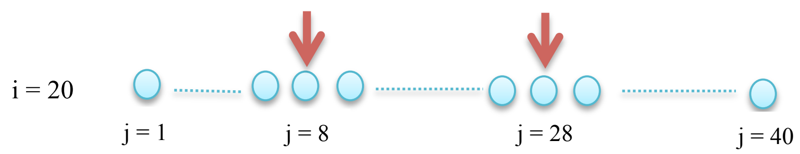
To summarize, the first layer of this model accounts for cortical magnification that occurs in the primary somatosensory cortex: indeed, in this first layer, Region A is represented by a higher number of neurons having smaller RF, whereas Region B is represented by a lower number of neurons having larger RFs. That is, this layer gives rise to a strong distorted representation. The second layer has been included to restore, at least, partially a more truthful representation

2.1.2 Application of two punctual stimuli : effects within the first layer

Once defined the main structure of the model, suppose to apply 2 punctual stimuli at the same distance (**2.5 cm**), on the skin surface of both regions. For simplicity, let's think along one dimension (for example assume that we stimulate units along a row of the matrix showed in figure 2.1).

Example of stimulation:

Region A:



The red arrows represent two punctual stimuli applied on the skin surface; these two stimuli stimulate neurons inside the first layer. In this case, the first stimulus is applied on the neuron in position 8 (*j=8, RF center = j*0.125 cm = 1 cm*), whereas the second one on the neuron in position 28 (*j=28, RF center = j*0.125 = 3.5 cm*). Each punctual stimulus creates a pattern of neurons activation (like a bubble), because there is overlap among Receptive Fields. Hence, the most excited neurons will be the eighth and the twenty-eighth, but also their neighbors will be activated, even if with a less strength.

Therefore, the pattern of neuron activation along the 20^{th} lines of the matrix (first layer), should be like this:

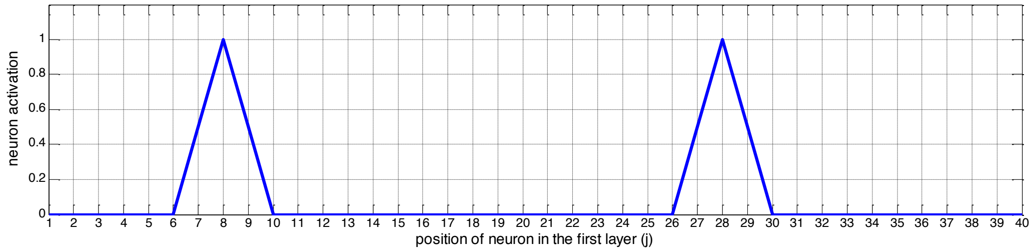
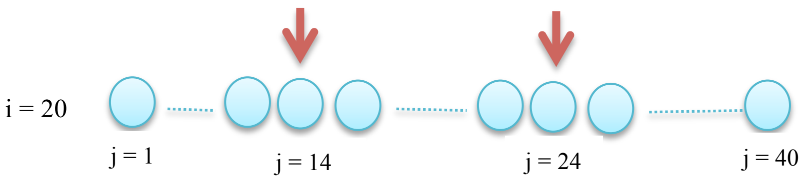


Figure 2. 2 ***Peaks of activation along the 20th lines of the neurons matrix (Area 1, HAND).***

Notice the distance between two peaks: 20 neurons.

Region B:



The units are design bigger just to remember that this region has less resolution. Hence, with the same distance of 2.5 cm the 2 most excited should be much closer. For example the first stimulus will hit the neurons number 14 (*j=14, RF center = j*0.25 =3.5 cm*), whereas the second one will hit the neuron number 24 (*j=24, RF center = j*0.25 = 6 cm*). The expected activation of neurons in that row is something like this:

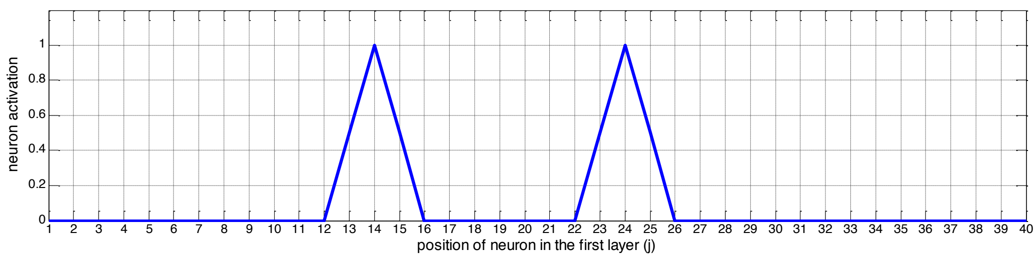


Figure 2. 3 ***Peaks of activation along the 20^{th} lines of the neurons matrix (Area 1, ARM).***

With the same pair of stimuli applied at the same distance of 2.5 cm , the distance between the 2 peaks is decreased (10 neurons) due to the different resolution associated with the two regions.

Notice that on this example we are considering only the first layer, which - as we have assumed -, it is involved in cortical magnification. In fact the difference of the distance between two peaks in Region A is higher than Region B:

- ***REAGION A (HAND)*** : Gap within First layer = 20 neurons ;
- ***REAGION B (ARM)*** : Gap within First layer = 10 neurons ;

The **ratio between the gaps** is:

20 neurons /10 neurons = 2

so, for the same pair of stimuli, the distance in Region A is represented in the first layer as double than the distance in Region B. This is consistent with the concept of cortical magnification, as we had considered two body regions with a different cortical magnification, like Hand and Arm.

2.1.3 Application of two Punctual Stimuli: Effects Within the Second Layer

Basically, in this model, the role of the second layer is to implement the rescaling process, in order to decrease the ratio calculated before.

The way to reach this target is to create a pattern of feed-forward synapses connections from Layer 1 to Layer 2 able to bring down the ratio with the activation and inhibition of neurons groups. The easiest solution would be decrease the gap between 2 peaks inside region A (Hand: high resolution), and increase the gap inside region B (Arm: low resolution). However, this solution seems to be not convenient, because it would lead to a resolution decrement in region A.

Therefore, it should be better a different implementation; as long as we want to maintain the high resolution of the region A (Hand) , it is much more convenient to work only by incrementing the resolution of the region B (Arm).

Hence, I have hypothesized that the brain implements the rescaling process by increasing the resolution of the lowest resolution region (region B in this case). At the same time, the high resolution of the region A presents in the first layer, will be kept also in the second layer too.

It is clear that to implement such kind of process, the synapses within the network concerning Region A will be different (in terms of parameters) from the network concerning Region B.

Continuing with the example, if we suppose that the rescaling process is going to increase the gap in the second layer of the region B, from 10 neurons to, for example, 14 neurons (and considering for region A that the second layer has the same gap of the first layer), the ratio becomes:

- ***REAGION A (HAND)*** : Gap within Second layer = 20 neurons ;
- ***REAGION B (ARM)*** : Gap within Second layer = 14 neurons ;

$$\boldsymbol{RATIO} = 20 \text{ neurons } / 14 \text{ neurons} = 1.42$$

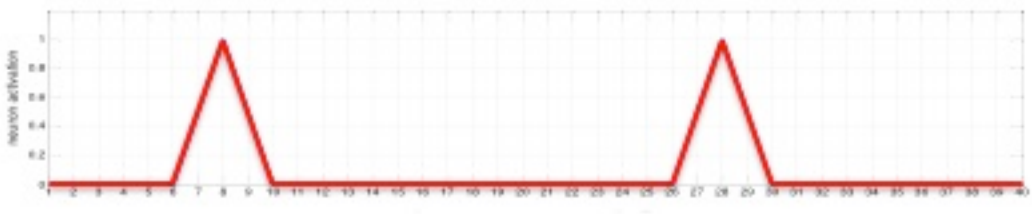


Figure 2. 4 ***Peaks of activation along the 20th line of the neurons matrix (Area 2, HAND)***

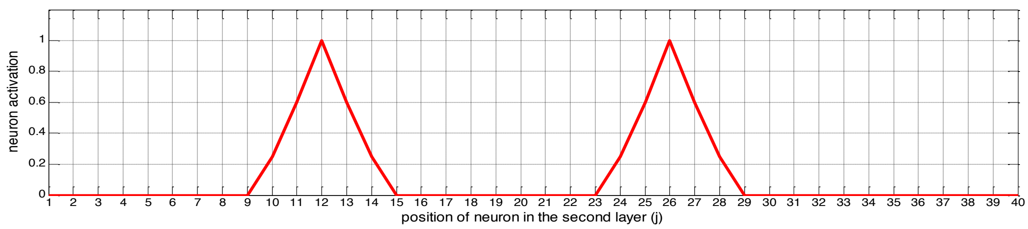


Figure 2. 5 ***Peaks of activation along the 20th lines of the neurons matrix (Area 2, ARM)***

Therefore, the illusion is still present, but now, on the second layer, the perceived distance is more similar across the two regions, with respect to what occurs in the first layer. That is, representation in the second layer is more truthful, since actually the distance applied externally is the same. That is, the second layer partially rescales the distortion occurring in the first layer, by reducing the difference in the perceived distance. So, in this model, the fundamental hypothesize is that the second layer (higher cortical level) of each body regions plays a key role in the perception of distance about two stimuli.

2.2 Mathematical description of the network

Since the networks representing Region A and Region B, has the same structure, only the equations for the Hand region will be presented. However, the two networks differ for the parameter values of some synapses. Hence, after description of network structure, I will emphasize the difference in parameter values between the two Regions.

The superscripts *f*, *s* will denote quantities concerning the first layer (Area 1) and the second layer (Area 2) respectively. The superscripts *H* and *A* will indicate the Hand (Region A) and the Arm (Region B). Finally the subscripts *ij*, *hk* will represent the spatial position of an individual neuron.

Each layer can be thought as a matrix of neurons. In the model, each layer is composed by N x N neurons with N = 41. The dimension of this matrix is the same for both regions. For Region A, this Area (matrix) corresponds to a skin region on the hand of 5 x 5 cm. Instead in Region B, this Area (matrix) is relative to a skin region on the arm of 10 x 10 cm.

To replicate the different resolution, RF's centers are arranged at a distance of ***0.125 cm*** on the Hand, and ***0.25 cm*** on the Arm,.

In the following, I will denote with x_i and y_i the center of the RF of a generic neuron *ij*. By considering a reference frame rigidly connected with the Hand, we can write:

x_i = -2.625 cm + i · 0.125 cm (i = 1, 2, …. , N) (2.1)

y_j = -2.625 cm +j · 0.125 cm (j = 1, 2, …. , N) (2.2)

The same formula can be used for the Arm:

x_i = -5.25 cm + i · 0.25 cm (i = 1, 2, …. , N) (2.3)

y_j = -5.25 cm + j ·.0.25cm (j = 1, 2, …. , N) (2.4)

Hereinafter, the RF will be denoted with the symbol Φ (receptive field). The RF of the cortical neurons in "Area 1" is described with a Gaussian Function. Therefore, for a neuron *ij* in "Area 1" the following equation holds:

$$\Phi_{ij}^{f,H}(x,y) = \Phi_0^{f,H} \cdot \exp\left(-\frac{(x - x_i^{f,H})^2 + (y - y_j^{f,H})^2}{2 \cdot (\sigma_\Phi^{f,H})^2} \right) \tag{2.5}$$

where x_i, y_i is the centre of the RF (on the skin), x and y are the spatial coordinates (still relative to the skin surface), and Φ_0^s and σ_Φ^s, represent the amplitude and the standard deviation of the Gaussian Function (three standard deviation approximately cover the overall RF). According to the equation, an external stimulus applied at the position (*x,y*) excites not only the neuron centred in that position, but also the proximal neurons with RFs covering that point.

2.2.1 First layer of neurons (Area 1)

The total input received by a generic neuron *ij* in "Area 1" is the sum of two contributes:

- The contribution due to the ***external stimulus*** applied on the skin (called $\varphi_{ij}(t)$).
- The contribution due to the ***lateral synapses,*** linking the neuron with other neurons within the same "Area 1" (called $\lambda_{ij}(t)$).

The input that reaches the neuron *ij* in presence of an external stimulus, is calculated as the product of the strength of the stimulus and the receptive field, according to this equation:

$$\varphi_{ij}^{f,H}(t) = \int_x \int_y \Phi_{ij}^{f,H}(x,y) \cdot I^{f,H}(x,y,t)dxdy$$

$$\cong \sum_x \sum_y \Phi_{ij}^{f,H}(x,y) \cdot I^{f,H}(x,y,t)\Delta x \Delta y \quad (2.6)$$

Where $I^{f,H}$ is the external stimulus applied on the skin (Hand or Arm) at the coordinates *(x,y)* at the time *t*. The right side of the equation (num. 2.6) means that the integral is computed with the **histogram rule** $\Delta x = \Delta y = 0.0312cm$ **.**

In this model the external stimulus is reproduced as a two dimensional Gaussian Function (like a circular point):

$$I^{f,H}(x,y,t) = \begin{cases} 0, & t < t_0 \\ I_0^{f,H} \cdot \exp\left(-\dfrac{(x - x_0^{f,H})^2 + (y - y_0^{f,H})^2}{2 \cdot (\sigma_I^f)^2}\right), & t > t_0 \end{cases}$$

(2.7)

Where t_0 is the instant of stimulus application, (x_0, y_0) is the central point of the stimulus, and $I_0^{f,H}$ and σ_I^f , are the amplitude, and the standard deviation of the stimulus, respectively. I have used a small standard deviation to simulate a punctual external stimulus (see Table).

In this model the application of 2 external stimuli is simulated, applied at the same time in 2 different positions. Hence, the application of 2 stimuli is represented by the following equation:

$$I^{f,H}(x,y,t)=\begin{cases} 0, & t<t_0 \\ I_1^{f,H}\cdot\exp\left(-\dfrac{(x-x_1^{f,H})^2+(y-y_1^{f,H})^2}{2\cdot(\sigma_{I1}^{f})^2}\right)+I_2^{s}\cdot\exp\left(-\dfrac{(x-x_2^{f,H})^2+(y-y_2^{f,H})^2}{2\cdot(\sigma_{I2}^{f})^2}\right), & t>t_0 \end{cases} \tag{2.8}$$

The input that a cortical neuron *ij* receives from other neurons within the same Area via lateral synapses, is computed as:

$$\lambda_{ij}^{l,H}(t)=\sum_{h=1}^{N^l}\sum_{k=1}^{N^l}L_{ij,hk}^{l,H}\cdot\chi_{hk}^{l,H}(t), \qquad l=f,\ s. \tag{2.9}$$

$\chi_{hk}^{l,H}(t)$ represents the activity of the neuron in position *(h,k)* inside "Area 1", and it is a variable state. $L_{ij,hk}^{l,H}$ is the strength of the synaptic connection from the pre-synaptic neuron *(h,k)*, to the postsynaptic neuron at the position *(i,j)*. These synapses are symmetrical and are organized as a Mexican Hat function (excitation among nearby neurons, and inhibition among distant neurons). The equation implementing Lateral Synapses is valid for the first layer, as well as for the second layer:

$$L_{ij,hk}^{l,H}=\begin{cases} L_{ex}^{l,H}\cdot\exp\left(-\dfrac{(x_i^{l,H}-x_h^{l,H})^2+(y_j^{l,H}-y_k^{l,H})^2}{2\cdot\left(\sigma_{L_{ex}}^{l,H}\right)^2}\right) & \\ \qquad -L_{in}^{l,H}\cdot\exp\left(-\dfrac{(x_i^{l,H}-x_h^{l,H})^2+(y_j^{l,H}-y_k^{l,H})^2}{2\cdot\left(\sigma_{L_{in}}^{l,H}\right)^2}\right), & ij\neq hk \\ 0, & ij=hk \end{cases} \tag{2.10}$$

$$l=f,\ s.$$

x_i and y_j represent the position of the post-synaptic neuron within the "Area 1" and x_h, y_k represent the position of the presynaptic neuron within Area1. $L_{ex}^{l,H}$ and $\sigma_{L_{ex}}^{l,H}$ define the Excitatory Gaussian function, whereas parameters $L_{in}^{l,H}$ and $\sigma_{L_{in}}^{l,H}$ the Inhibitory one. To implement a correct Mexican Hat function, some conditions have to be satisfied:

$$L_{ex}^{l,H}\ >\ L_{in}^{l,H} \qquad l=f,s. \tag{2.11}$$

$$\sigma_{Lex}^{l,H} < \sigma_{Lin}^{l,H} \qquad l = f, s \tag{2.12}$$

The null term in equation (num. 2.10), avoids the auto-excitation.

Finally, the total input, called $u_{ij}^{f,H}(t)$ received by a cortical neuron in "Area 1" (First Layer) is the sum of the two contributes:

$$u_{ij}^{f,H}(t) = \varphi_{ij}^{f,H}(t) + \lambda_{ij}^{f,H}(t)\,. \tag{2.13}$$

The <u>neuron activity</u> is computed from its input through a **first order dynamics** (simulation of the passage through the neuron's membrane), and a **static sigmoidal relationship** (simulation of the neuron answer):

$$\tau \frac{d\chi_{ij}^{f,H}(t)}{dt} = -\chi_{ij}^{f,H}(t) + F(u_{ij}^{f,H}(t)\,), \tag{2.14}$$

$$F\left(u_{ij}^{f,H}(t)\right) = \frac{G_{max}}{1+\exp\left(-k\cdot(u_{ij}^{f,H} - u_0^{f,H}\right)}\,. \tag{2.15}$$

Where $\boldsymbol{\chi_{ij}^{f,H}(t)}$ is the state variable representing neuron activity. Function $F\left(u_{ij}^{f,H}(t)\right)$ represents the **sigmoidal function of the neuron**.

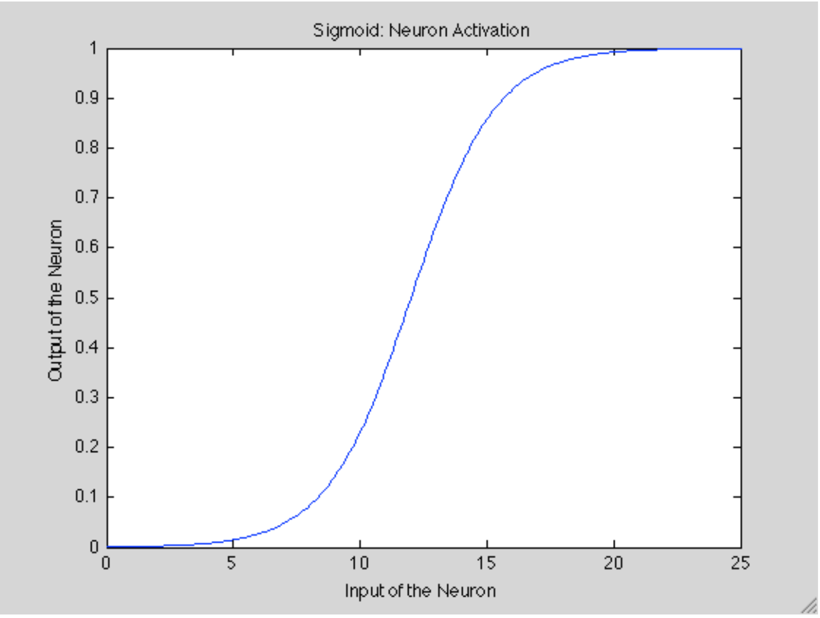


Figure 2. 6 ***Static Sigmoidal Relationship. .***

The parameter u_0^f is the value of the input at the central point (that is the value of the input at which activity is equal to *Gmax/2*; k is the slope of the sigmoid at the central point, and $G_{\max}$ is the upper saturation value of the sigmoid, that is the maximum activity value for a generic neuron. *Gmax* has been set equal to 1, so that neuron activity is normalized with respect to its maximum. According to previous equation (num. 2.15), the activity of a generic neuron inside "Area 1" is equal to zero until its total input is under a given threshold. τ is the time constant of the differential equation (num. 2.14).

Differential equation (num. 2.14) is implemented numerically with the Euler's method:

$$\chi_{n+1}^{ij,f} = \chi_n^{ij,f} + h \cdot f\left(t\,,\, \chi_n^{ij,f}\right), \qquad h = \left(\frac{T}{P}\right). \tag{2.16}$$

$$\chi_{n+1}^{ij,f} = \chi_n^{ij,f} + \frac{h}{\tau} \cdot \left(-\chi_n^{ij,f} + \frac{G_{max}}{1+\exp\left(-k\cdot(u_{ij}^{f,H} - u_0^{f,H}\right)}\right) \tag{2.17}$$

As long as T is the time length of the simulation, and P is the number of subdivisions of T, it is clear that h represents the sampling step of the Euler's method.

2.2.2 Second layer of neurons (Area 2)

The Second Layer (Area 2) is assumed to be associated with a high cortical layer. In this model, neurons inside this area, receive inputs from:

- Neurons in "Area 1" via Feed-Forward synapses, having a Mexican Hat distribution.
- Neurons of the same Area via Lateral Synapses, having a Mexican hat distribution.

The following equations hold:

$$u_{ij}^{s,H}(t) = \Psi_{ij}^{s,H}(t) + \lambda_{ij}^{s,H}(t) \tag{2.18}$$

$$\Psi_{ij}^{s,H}(t) = \sum_{h=1}^{N^f}\sum_{k=1}^{N^f} W_{ij,hk}^{f,H} \cdot \chi_{hk}^{f,H}(t), \tag{2.19}$$

$\chi_{hk}^{f,H}(t)$ represents the activity of the neuron *hk* in "Area 1". $W_{ij,hk}^{f,H}$ denotes the feed-forward synaptic strength from the pre-synaptic cortical neuron *hk* in "Area 1", to the post-synaptic neuron *ij* in "Area 2". These synapses can be described as follows:

$$\begin{aligned} W_{ij,hk}^{f,H} = {} & W_{ex}^{f,H} \cdot \exp\left(-\frac{(x_i^{s,H}-x_h^{f,H})^2+(y_j^{s,H}-y_k^{f,H})^2}{2\cdot\left(\sigma_{W_{ex}}^{f,H}\right)^2}\right) + \\ & -W_{in}^{l,H} \cdot \exp\left(-\frac{(x_i^{s,H}-x_h^{f,H})^2+(y_j^{s,H}-y_k^{f,H})^2}{2\cdot\left(\sigma_{W_{in}}^{f,H}\right)^2}\right) \end{aligned} \tag{2.20}$$

Where $x_i^{s,H}$, $y_j^{s,H}$, represents the position of the *ij* neuron in "Area 2", and $x_h^{f,H}$, $y_k^{f,H}$, the position of the neuron *hk* in "Area 1". Notice that when the coordinates of these two neurons are equals, the exponential term assumes an unitary value, and then the synapse connection between these two neurons has the strongest value.

The activity of a neuron in "Area 2" can be computed from its input with the same equation as before (num. 2.14):

$$\tau \frac{d\chi_{ij}^{s,H}(t)}{dt} = -\chi_{ij}^{s,H}(t) + F\left(u_{ij}^{s,H}(t)\right), \tag{2.21}$$

$$F\left(u_{ij}^{s,H}(t)\right) = \frac{G_{max}}{1+\exp\left(-k\cdot(u_{ij}^{s,H}-u_0^{s,H}\right)}. \tag{2.22}$$

k is the slope of the sigmoid at the central point, u_0^f is the value of the input at the central point, and $G_{\max}$ is the gain of the sigmoidal function.

It can be solved by Euler's method:

$$\chi_{n+1}^{ij,s} = \chi_n^{ij,s} + h \cdot f\left(t\,,\, \chi_n^{ij,s}\right) \qquad h = \left(\frac{T}{P}\right) \tag{2.23}$$

$$\chi_{n+1}^{ij,s} = \chi_n^{ij,s} + \frac{h}{\tau} \cdot \left(-\chi_n^{ij,s} + \frac{G_{max}}{1+\exp\left(-k\cdot(u_{ij}^{s,H}-u_0^{s,H}\right)}\right). \tag{2.24}$$

As long as T is the time length of the simulation, and P is the number of subdivisions of T, it is clear that h represents the sampling step of the Euler's method.

2.3 Parameters and their values

External Stimuli			
$I_1^{f,H} = 1.5$	$I_2^{f,H} = 1.5$	$\sigma_{I1}^{f,H} = 0.1\,cm$	$\sigma_{I2}^{f,H} = 0.1\,cm$
$I_1^{f,A} = 1.5$	$I_2^{f,A} = 1.5$	$\sigma_{I1}^{f,A} = 0.1\,cm$	$\sigma_{I2}^{f,A} = 0.1\,cm$
Receptive Fields			
$\Phi_0^{f,H} = 1$	$\sigma_{\Phi}^{f,H} = 0.125\,cm$		
$\Phi_0^{f,A} = 1$	$\sigma_{\Phi}^{f,A} = 0.35\,cm$		
Lateral Synapses in Area 1			
$L_{ex}^{f,H} = 1$	$L_{in}^{f,H} = 0.5$	$\sigma_{ex}^{f,H} = 2\,neurons$	$\sigma_{in}^{f,H} = 8\,neurons$
$L_{ex}^{f,A} = 1$	$L_{in}^{f,A} = 0.5$	$\sigma_{Lex}^{f,A} = 2\,neurons$	$\sigma_{Lin}^{f,A} = 8\,neurons$
Feed-Forward Synapses			
$W_{ex}^{f,H} = 4$	$W_{in}^{f,H} = 1$	$\sigma_{Wex}^{f,H} = 1\,neurons$	$\sigma_{Win}^{f,H} = 1.4\,neurons$
$W_{ex}^{f,A} = 4$	$W_{in}^{f,A} = 1$	$\sigma_{Wex}^{f,A} = 1\,neurons$	$\sigma_{Win}^{f,A} = 1.4\,neurons$
Lateral Synapses in Area 2			
$L_{ex}^{s,H} = 4.5$	$L_{in}^{s,H} = 2$	$\sigma_{ex}^{s,H} = 1.5\,neurons$	$\sigma_{in}^{s,H} = 2\,neurons$
$L_{ex}^{s,A} = 4.5$	$L_{in}^{s,A} = 2$	$\sigma_{ex}^{s,A} = 1\,neurons$	$\sigma_{in}^{s,A} = 2\,neurons$
Sigmoidal characteristic			
$G_{\max} = 1$	$k = 0.6$	$u_0^f = u_0^s = 12$	
Time constant			
$\tau = 3\,ms$			

Table 2. 1 *Reference Parameters and their values.*

As we can observe by the table, Hand and Arm differ just for two parameters:

- Standard Deviation of the Receptive Fields.
- Standard Deviation of the Excitatory component of the Lateral Synapses within “Area 2”.

All the others parameters are the same. The fact that there are differences in terms of parameters concerning the Receptive Fields and the Lateral Synapses is coherent with the nature of the neural network and its target.

Focusing on the Receptive Fields, we have already seen in the previous paragraphs that the Hand region has a different resolution, with respect the Arm region. In particular the Hand has a higher resolution than the Arm. That is the reason why I have chosen a standard deviation for Hand’s RFs smaller than the Arm’s RF. Just because the acuity of the hand in the discrimination of two nearby stimuli has to be higher than the Arm: so, small RFs on the Hand, have been needed to reproduce this situation.

The discussion is different about the Lateral Synapses. In fact, in this case, Standard Deviation of the Excitatory Component of the Arm is smaller than the Hand. The reason is just because the neural network of the Arm has been implemented in order to in crease the resolution of this region, incrementing the distance between the balls of activation inside “Area 2”. To achieve this target, was necessary reducing the excitatory component of the Mexican Hat function, in order to decrease the size of the balls of activated neurons, and therefore, increase the gap between them. In other words, reduce the excitatory component means excite less neurons, namely obtain small balls of activation.

2.4 Activation of neurons step by step

The two stimuli were applied on a skin surface area of 5x5cm on the hand, and 10 x10 cm on the arm. In the s below it is possible to see two punctual stimuli, with the same distance among the stimuli **equal to 2.5 cm**, and applied at the same time: one pair on the arm, and the other one on the hand.

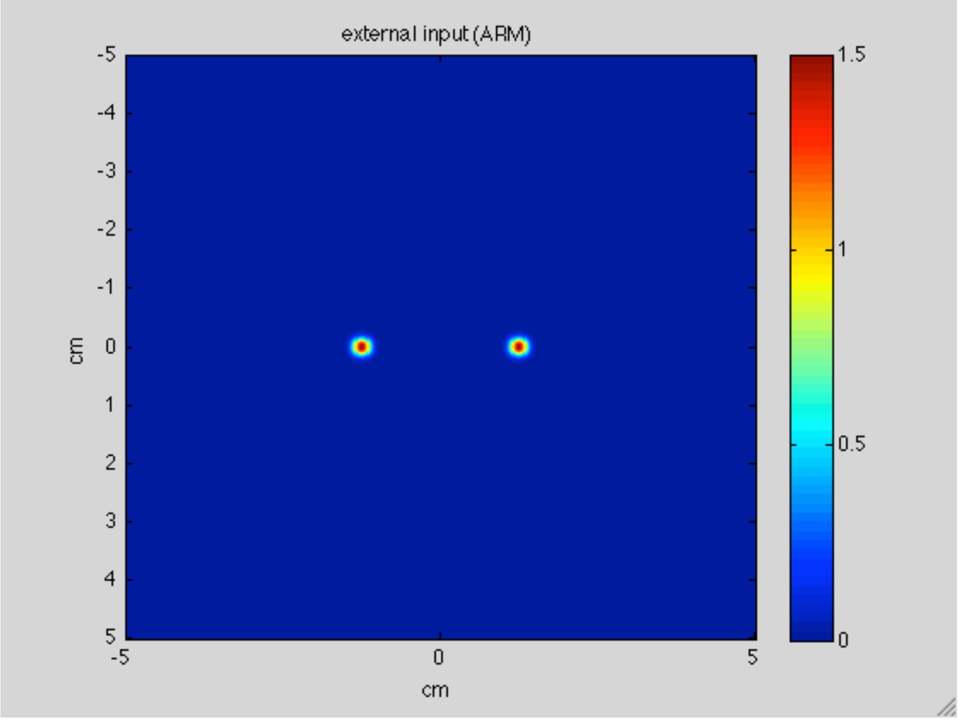


Figure 2. 7 ***Punctual Stimuli on the skin surface of the Arm (10 x 10 cm): distance between the two stimuli equal to 2.5 cm.***

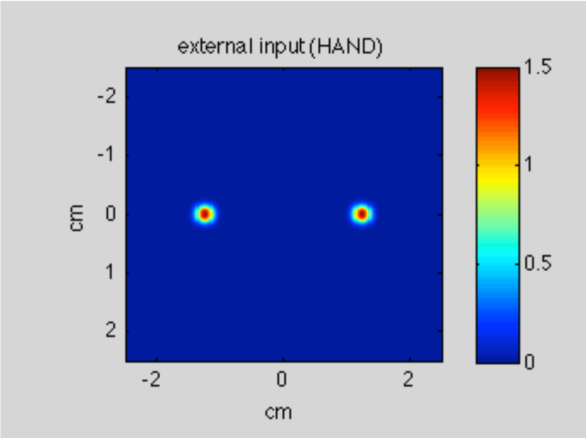


Figure 2. 8 ***Punctual Stimuli on the skin surface of the Hand (5 x 5 cm): distance between the two stimuli equal to 2.5 cm.***

The next figure shows the Receptive Field of a generic neuron positioned in the centre of "Area 1" (position 0,0). The first one is relative to the Arm, the second one to the Hand:

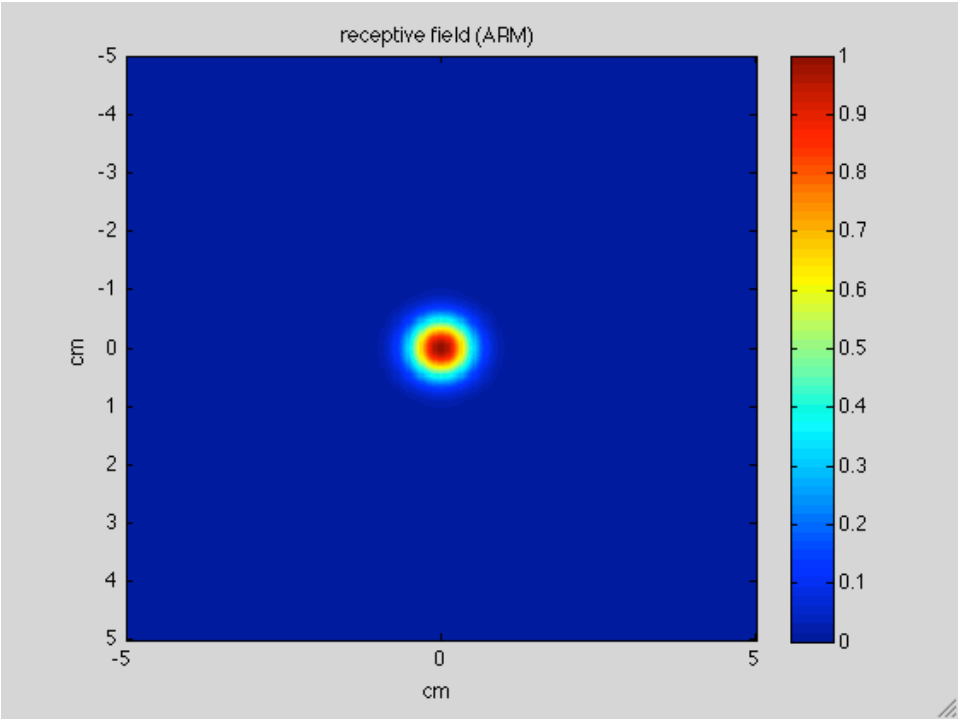


Figure 2. 9 *Example of Receptive Field on the skin surface of the Arm.*

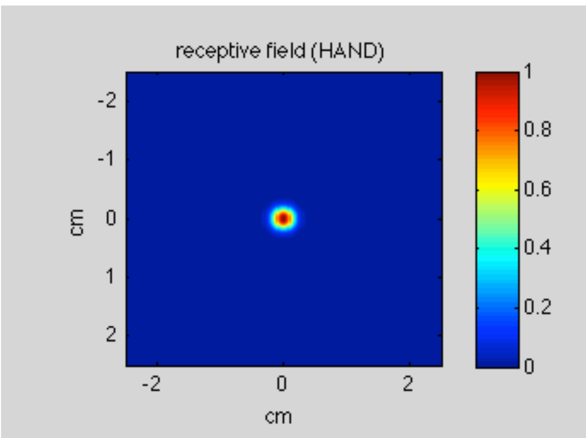


Figure 2. 10 *Example of Receptive Field on the skin surface of the Arm.*

It is evident the different size of the 2 Receptive Fields; the RF of a neuron codifying stimuli on the Hand (Region A) covers a smaller skin surface than neurons representing arm (Region B).

The stimulation, and the presence of RFs on the skin are going to create an input for neurons in "Area 1", which can be calculated with the formula (num. 2.13). An example of input to the neurons, with a stimuli distance equal to 2.5 cm, is shown graphically below both for Region A (Hand), and Region B (Arm):

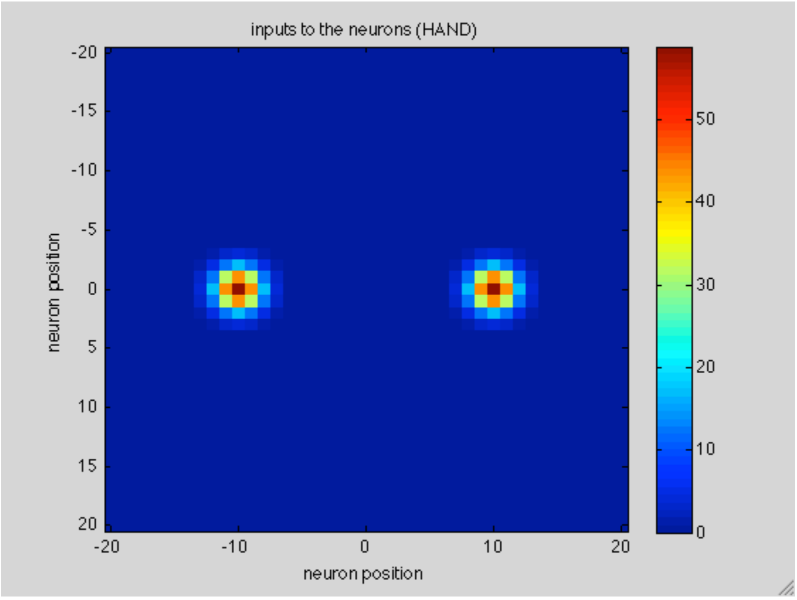


Figure 2. 11 Hand: ***inputs incoming to the neurons of the "First Layer".***

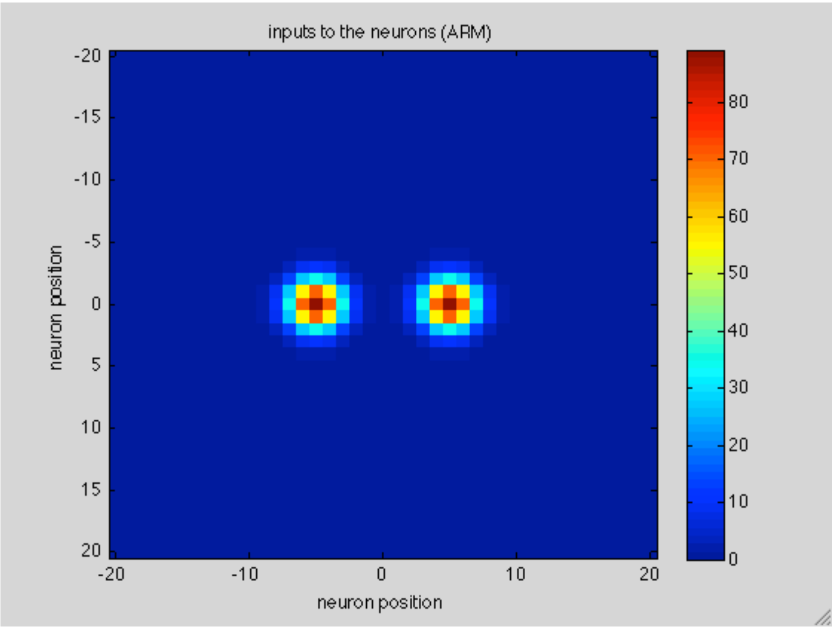


Figure 2. 12 Arm: ***inputs incoming to the neurons of the "First Layer".***

Remember that these graphics do not represent the state of activation of the neurons inside “Area 1”, but they just represent the effective neurons input. Each little square is a neuron in “Area 1” and the colour is its input value. The colour is associated with a value that can be consulted on the color bar. We can notice that with the same distance of the input stimuli (2.5 cm), in the Arm’s case, the bubbles are closer to each other, than the Hand’s case. Given that we have simulated two different body regions, with different resolution, these results are coherent with the reality. Remember that region A is an Area of 5 x 5 cm on the Hand, whereas Region B is an Area equal to 10 x 10 cm on the Arm. In addition, the cortical Areas linked with these two regions have the same dimension of 41 x 41 neurons; so, it is clear that Hand and Arm, have been interpreted by the model as two regions with different resolution. In particular, the spatial resolution of neurons for the two regions is:

- Hand 5 cm /41 neurons =
- Arm: 10 cm /41 neurons=

In Region A, the centers of neuron RFs are arranged at a distance of 0.12 cm, whereas, in Region B, the centers of neuron RFs are arranged at a distance of 0.24 cm. Therefore, it is clear that the same distance of the input is represented with different length, in terms of neurons, inside the cortical Areas.

This input will be added with inputs coming from lateral synapses, which have a Mexican Hat shape to excite proximal neurons, and inhibit distal neurons. Below, a graphical example of lateral synapses within “Area 1”, starting from a neuron in position (0,0) , relating to Region B (Arm):

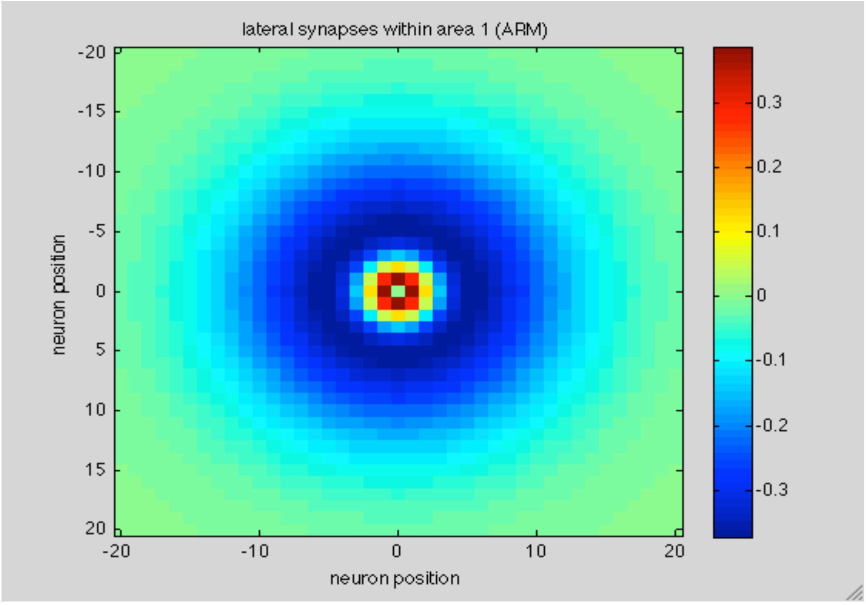


Figure 2. 13 ***Arm:*** **2D view of** ***Lateral Synapses within "Area 1", starting from neuron in position (0,0).***

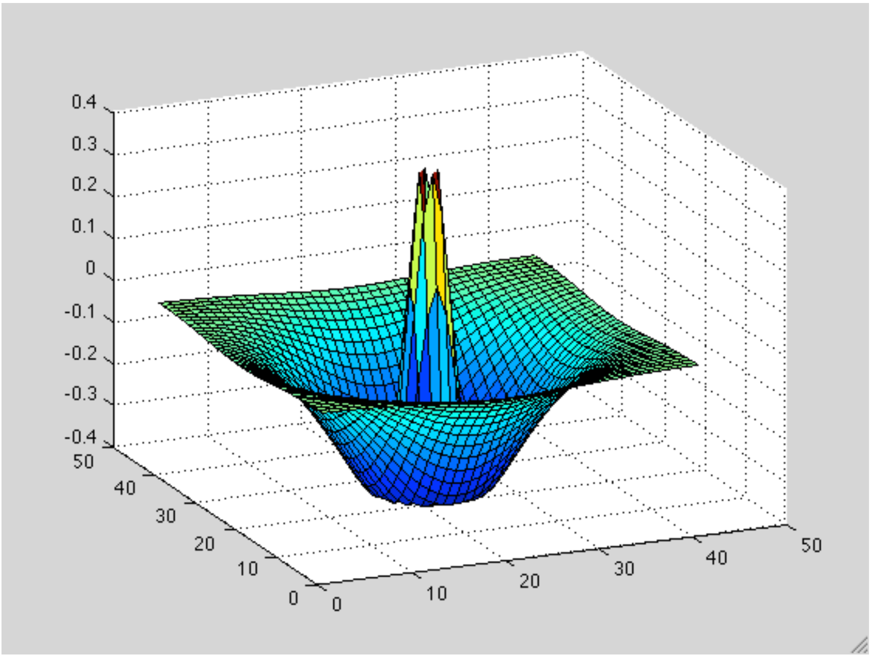


Figure 2. 14 ***Arm:*** **3D view of** ***Lateral Synapses within "Area 1", starting from neuron in position (0,0).***

In the first graphic (figure 2.13), a little coloured square represents the weight of the connection among this neuron and the neuron in position (0,0). As we can notice, the strongest connections are between close neurons to the neuron in position (0,0). Distant neurons provide inhibitory synapses. Figure 2.14 is a 3D representation of figure 2.13: in this last figure it is evident the Mexican Hat shape of the lateral synapses.

Since parameters about Lateral synapses within Area 1 are the same both for Region A, and Region B, also the graphics of Lateral Synapses within in "Area 1" relating to Region A (Hand) are the same of figure 2.13 and figure 2.14. These Lateral Synapses are reported in the next figure:

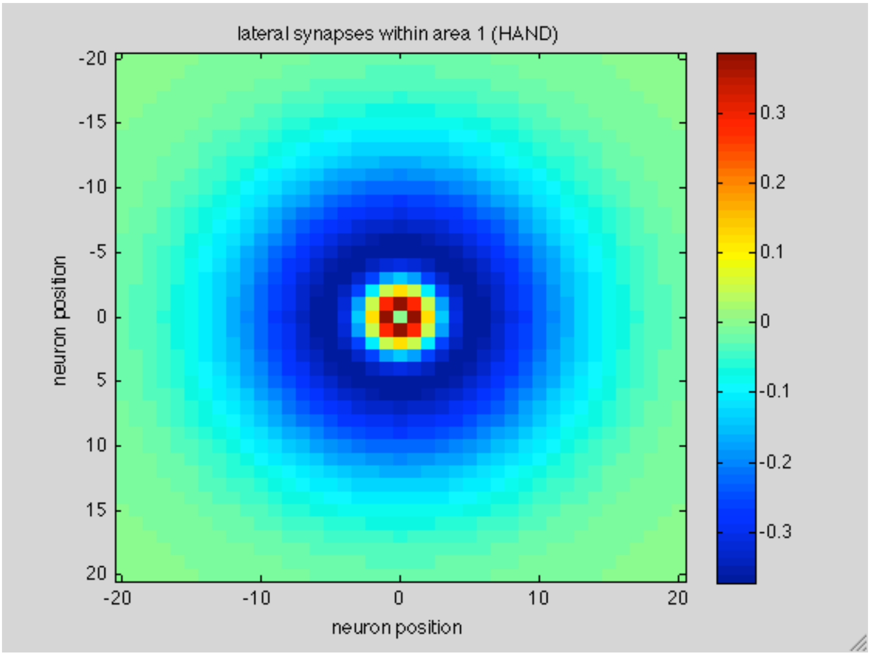


Figure 2. 15 ***Hand:*** **2D view of** ***Lateral Synapses within "Area 1", starting from neuron in position (0,0).***

The total input received by neurons in "Area 1" is used in the differential equation (num. 2.14). This equation is computed with a discrete method, as the Euler's method (num. 2.17), to obtain the activation state of each neuron presents in "Area 1".

Continuing the simulation example with a distance stimuli input equal to 2.5 cm, at time equal 200 (that is at the end of the simulation, when transient response has exhausted and the network is an a new steady state condition), the output of the activation pattern in "Area 1", concerning the Arm's case, is something likes two balls of activation:

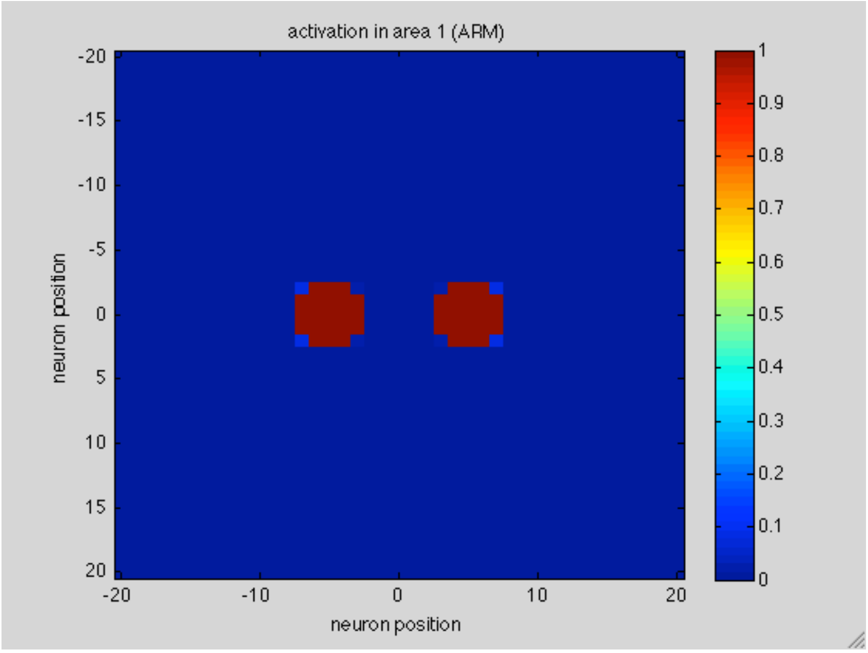


Figure 2. 16 Arm: activation of neurons within the First Layer.

Each little coloured square indicates the state of activation in a range scale from 0 to 1.The next figure is a different view of the activation bubbles. This figure gives the perfect idea of the activation peaks that I had mentioned at the beginning of this chapter:

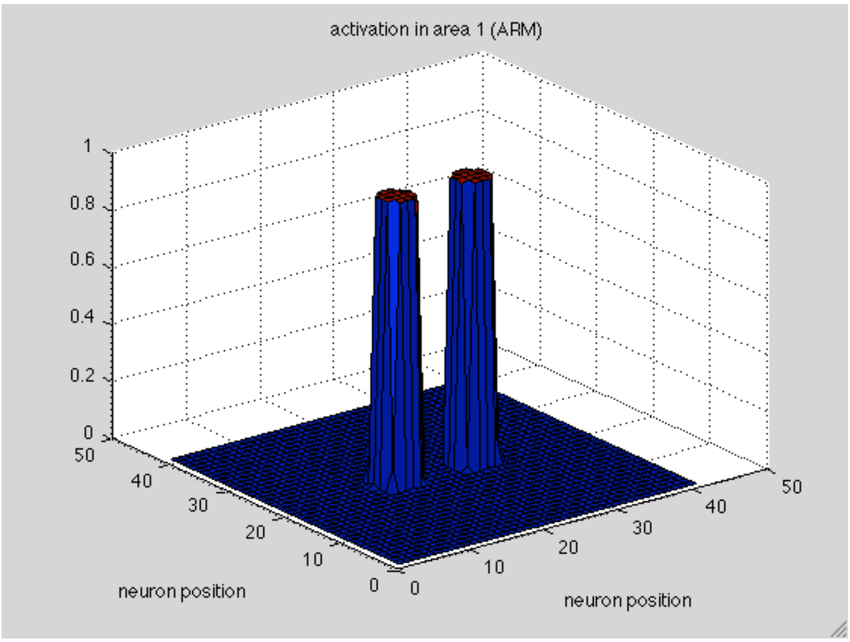


Figure 2. 17 Arm: Peaks of Activation within the First Layer (3D view).

As regards Region A (Hand), the activation pattern within "Area 1", for the same input stimuli distance of the Arm's example (2.5 cm), is showed in the next figures:

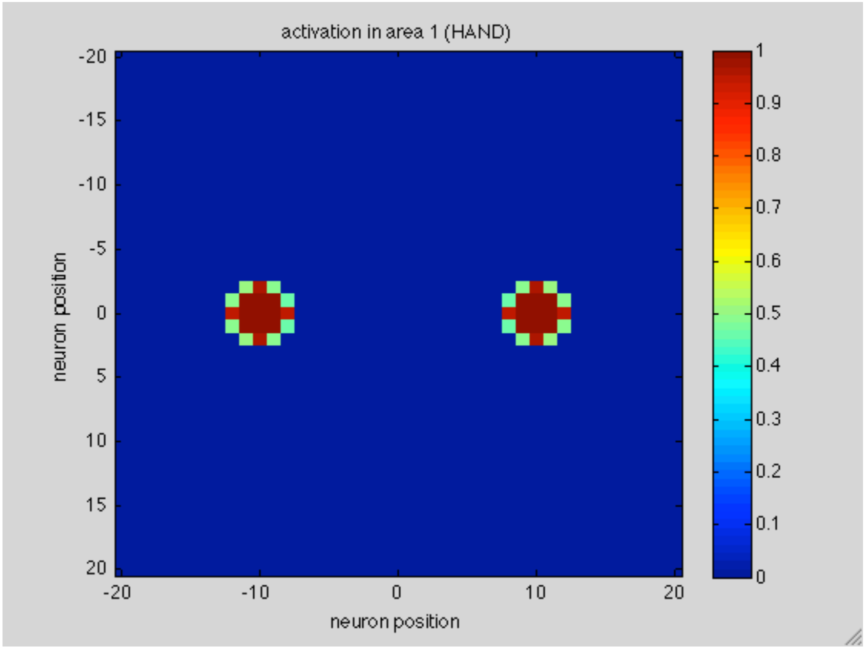


Figure 2. 18 Hand: activation of neurons within the First Layer.

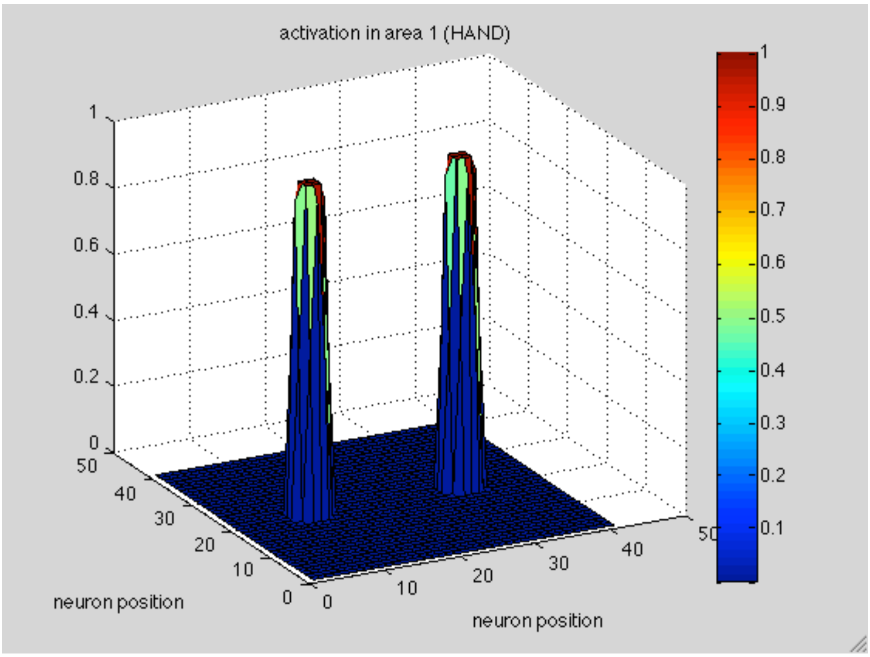


Figure 2. 19 ***Hand: Peaks of Activation within the First Layer (3D view).***

The activation of neurons in the First Layer (Area 1), is a part of input that will reach neurons within the Second Layer (Area 2) through feed-forward synapses. Feed-forward synapses were implemented with the same values of parameters, both for the Hand and the Arm (see table 2.1). These kind of synapses are still with a Mexican Hat shape, and, in the Arm' case (Region B), they were fundamental to achieve the increment of the gap between the two activation peaks (Rescaling Process). Even if, the negative part of this function is quite small (see figure 2.20), it plays an important role to implement the Rescaling Process, especially in the Arm's case.

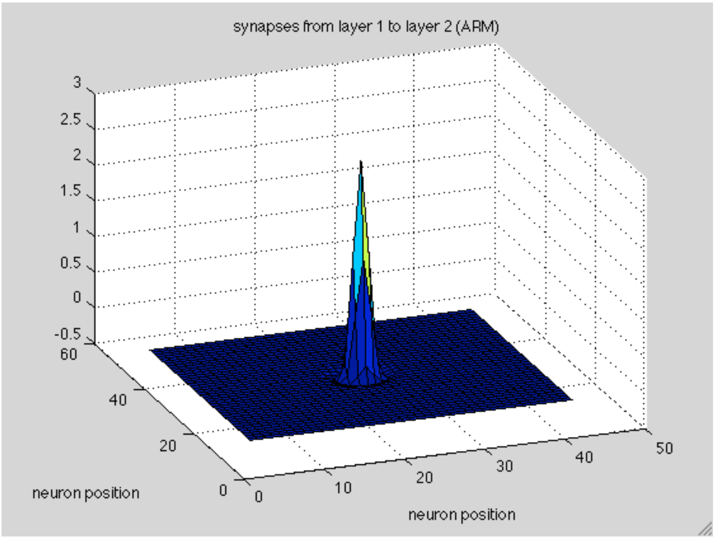


Figure 2. 20 ***Arm: Feed-Forward synapses from the First Layer to the Second Layer (3D view).***

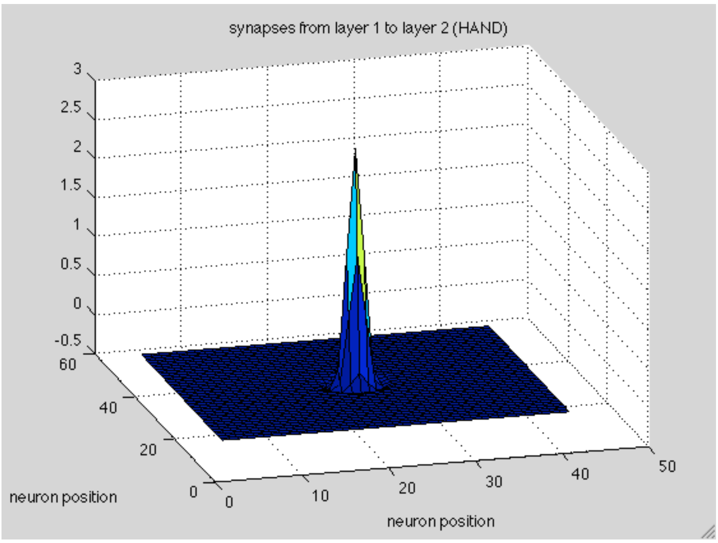


Figure 2. 21 ***Hand: Feed-Forward synapses from the First Layer to the Second Layer (3D view).***

Moreover, to rescale the gap inside the Second Layer of the Arm region, it was necessary consider the presence of Lateral Synapses also in “Area 2”. These synapses were constructed with the Mexican Hat function. I have implemented Lateral Synapses in order to enforce a strong inhibition (figure 2.22) on the previews activation balls, with the target to minimize their size, hence increasing the gap between the 2 balls.

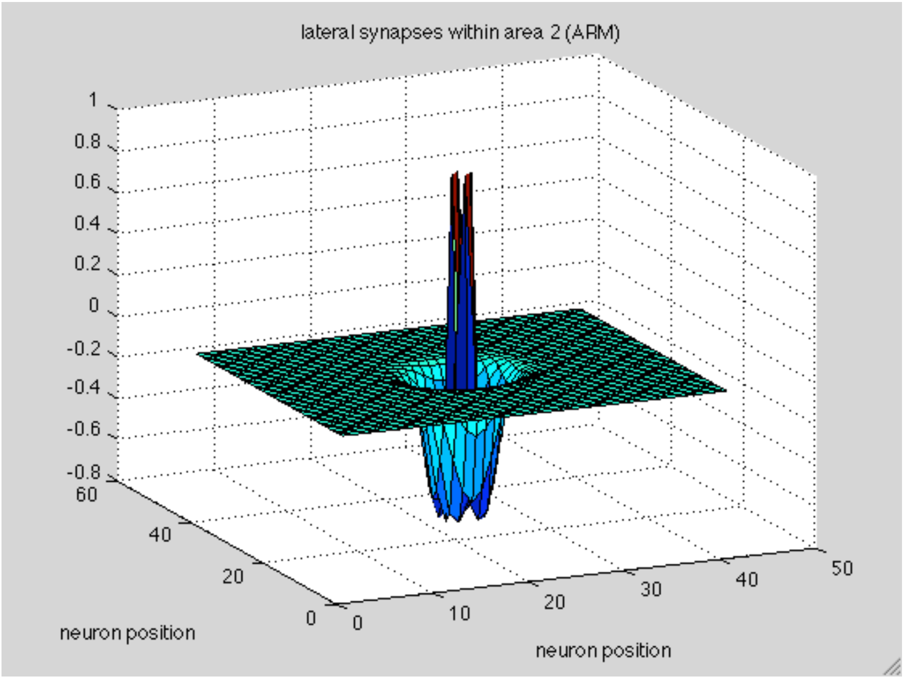


Figure 2. 22 ***Arm: 3D view Lateral Synapses within “Area 1”, starting from neuron in position (0,0).***

Instead, Lateral Synapses within “Area 2” concerning the Hand region were implemented with a smaller inhibitory component with respect the Arm. In fact, as we can observe by figure 2.23, the negative part of the Mexican Hat is much smaller than the one in figure 2.22. In the Hand’s case, this kind of Lateral Synapses are necessary to maintain the same size of the activation balls (and therefore, the same distance between them), during the passage from “Area 1” to “Area 2”. In fact, as I have already explained, the neural network of the Hand was implemented to maintain its already high resolution; to do this, lateral synapses of the Second Layer, have played a key role in the maintaining of the same size of the bubbles. At the same time, maintain the same size of the bubbles means keep constant the distance between the activation balls, that was the target of the Hand's neural network.

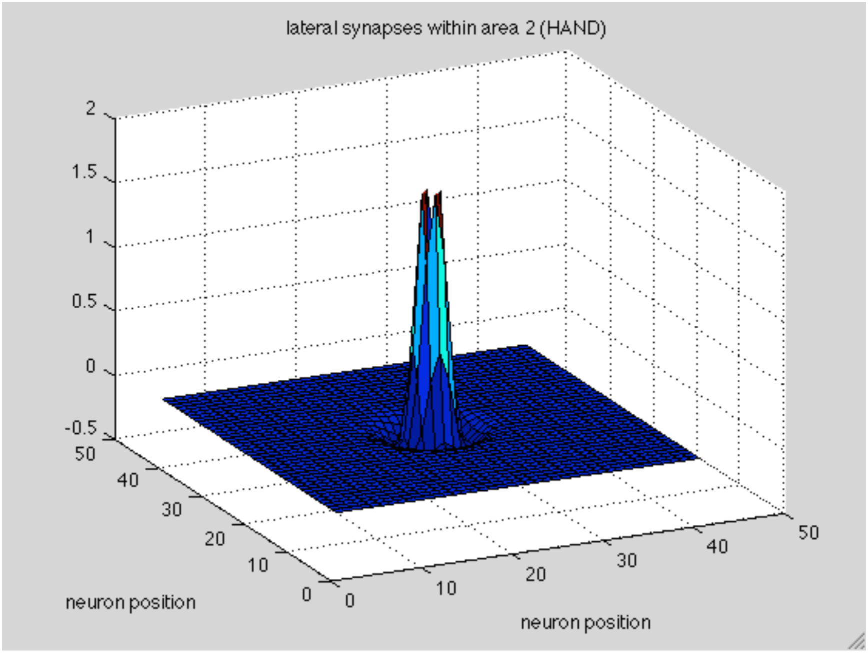


Figure 2. 23 ***Hand:*** **3D view** ***Lateral Synapses within "Area 1", starting from neuron in position (0,0).***

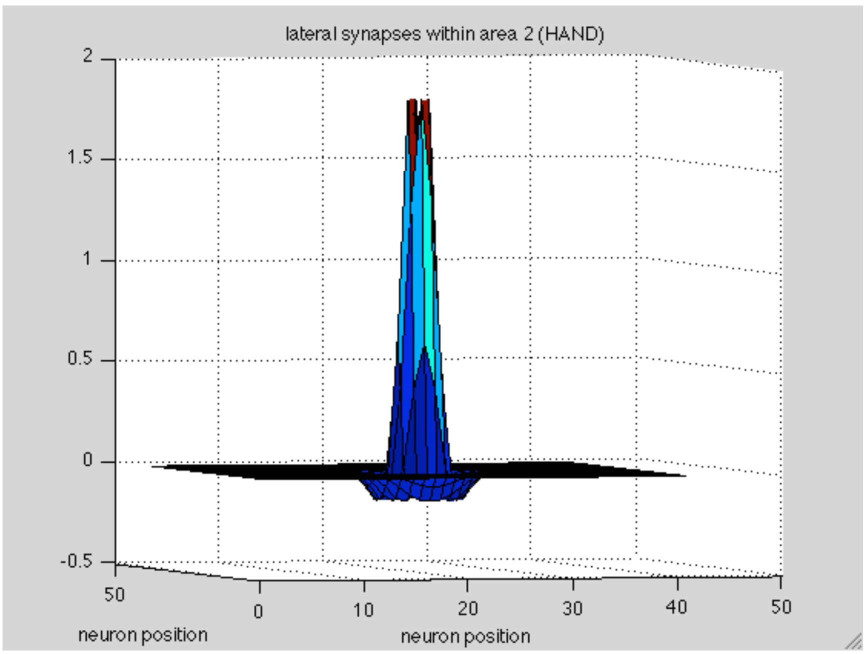


Figure 2. 24 ***Different point of view of figure 2.22.***

The final step is the activation of neuron inside "Area 2".

Considering the Arm's neural network, the application of a particular pattern of feed-forward synapses and lateral synapses (seen few pages before) lead to a resizing of the two activation bubbles inside the Second Layer. This means that the distance between the bubbles is decreased, and that represents a good result to achieve a rescaling in the perception distance. In fact, I have hypothesized that the second layer may be a higher cortical area that receives distortions information about the distance from the first layer (resembling primary somatic sensory cortex), and implements a sort of partial rescaling, in order to obtain a more truthful representation. Figures below represent the activation pattern inside "Area 2", caused by the activation pattern of "Area 1" that we have seen in figure 2.16.

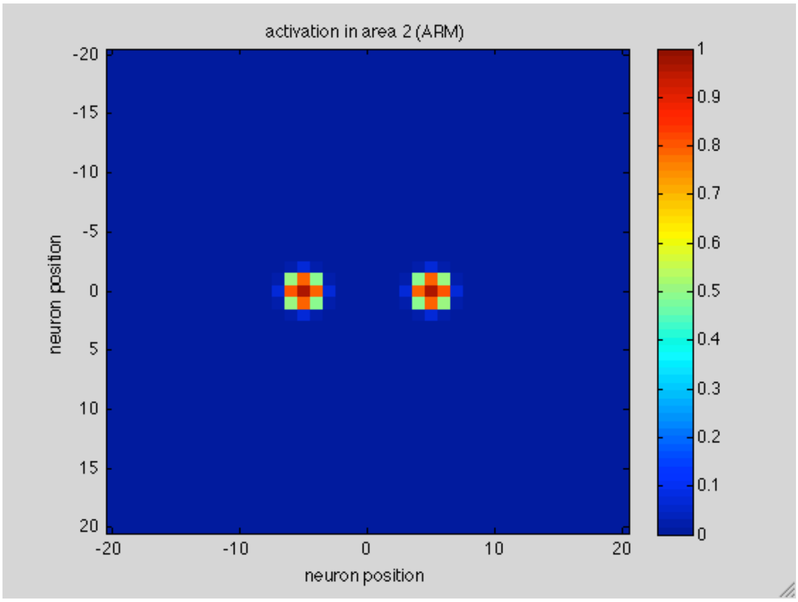


Figure 2. 25 ***Arm: activation of neurons within the Second Layer.***

Instead, the activation pattern inside "Area 2" concerning the Hand is shown in the next figure:

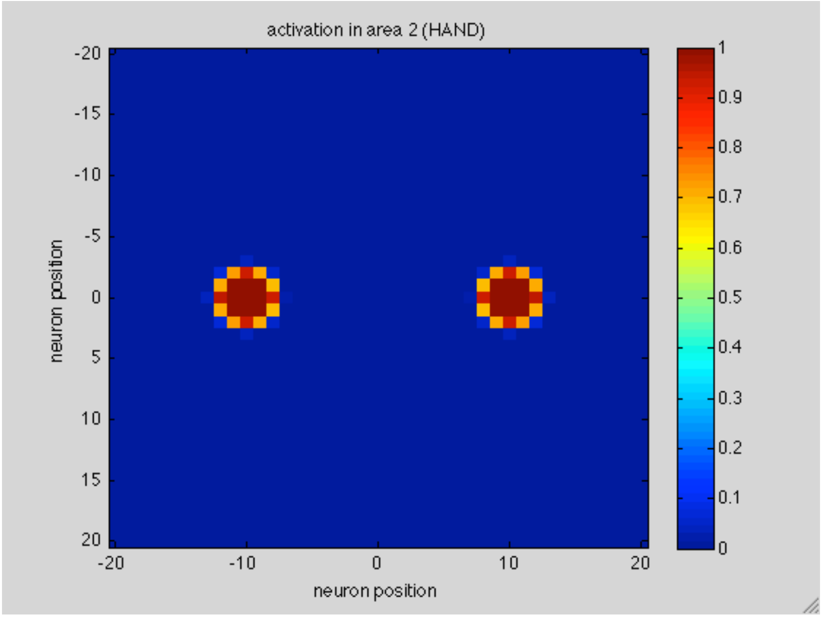


Figure 2. 26 ***Hand: activation of neurons within the Second Layer.***

We can see that the sizes of the bubbles are about the same of "Area 1" (figure 2.18) and therefore, the distance between them was kept constant in the passage from "Area 1" to "Area 2".

This was just an example to show how the Hand's Neural Network, and the Arm's Neural Network work with an input stimuli distance of 2.5 cm. In the next chapter I will show the results of multiple simulations, with different kinds of stimuli distances, to quantify the Weber's Illusion, the Rescaling Process and the Two Point Discrimination Threshold.

2.5 Periodic Domain

The implementation of the network has seen the introduction of a fundamental hypothesize concerning the **Domain** of each Area. Every Areas of the model are set up with a matrix of $N \, x \, N$ units (neurons) where N=41. It is clear that if we manage this matrix as it is, **border effects** problems arise. Focusing on the neurons positioned near the borders of this matrix, appear evident that these neurons are not in the same conditions of a generic neuron positioned in the centre of the matrix. Indeed, one neuron in position (21,21) (centre of the matrix) has got 8 close-set neurons, whereas a neuron in position (21,1) has got only 5 close-set neurons. Hence, the central neuron (21,21) is linked with much more close-set lateral synapses connections compare with the neuron (21,1): this leads to a substantial difference in terms of activity.

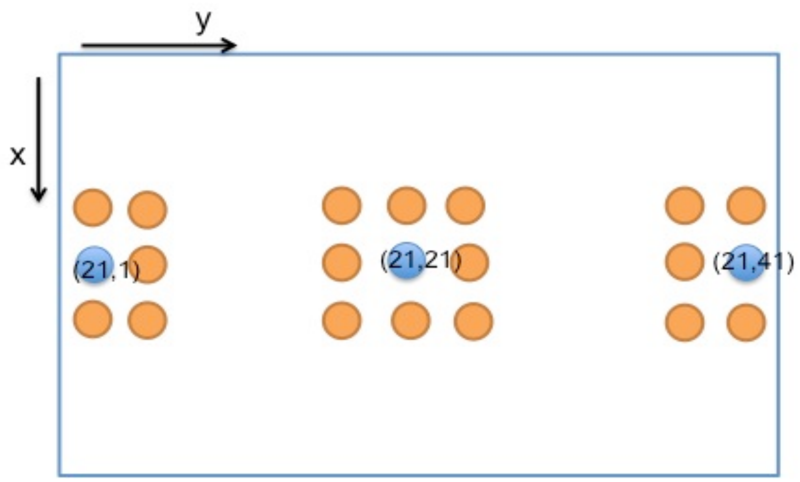


Figure 2. 27 ***Example of close-set neurons for two neurons on the border of the domain, and for one neuron in the centre of the domain.***

In the previous conditions (**close domain**), we would have a different stimulation effect on the generic neuron *ij* near the borders compared with another neuron *hk* near the centre.

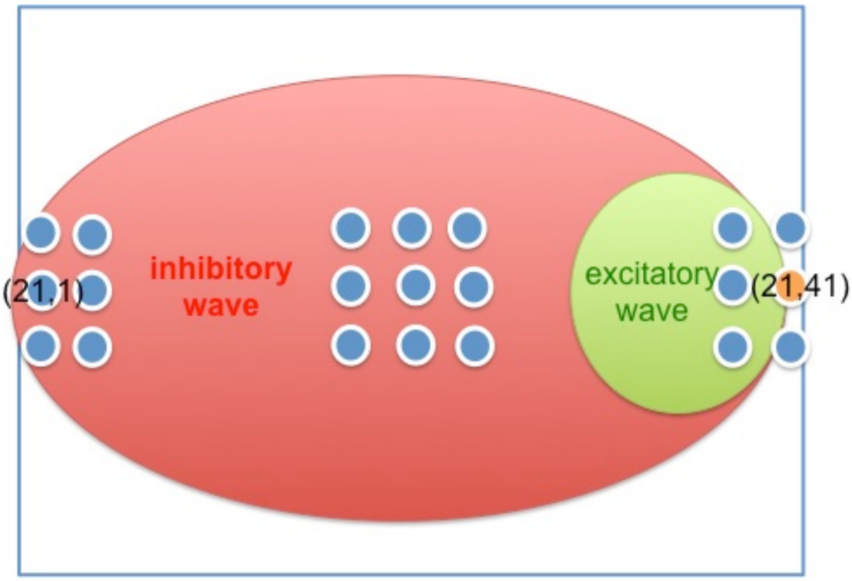


Figure 2. 28 *Excitatory Wave and Inhibitory Wave of the neuron* **(21,41)** *considering a close domain: neuron (21,1) receives only inhibitory wave from neuron (21,41).*

In particular, it is evident that the neuron (21,1) will receive less excitatory stimulation than the neuron (21,21), due to the different number of close-set neurons surrounding it. So, that configuration would lead to a different behaviour of the neurons depending on their position within the matrix, and it is not acceptable.

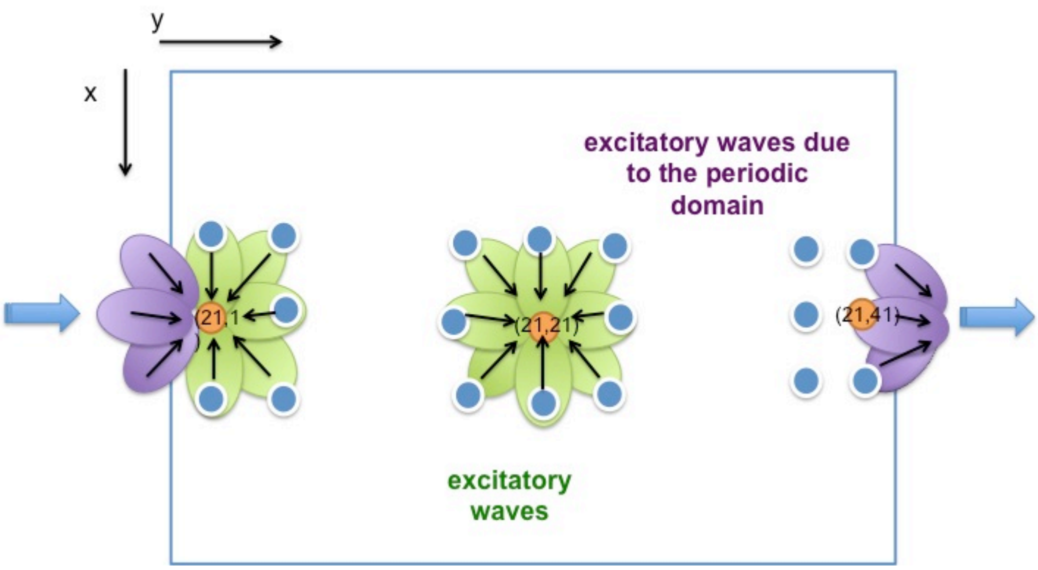


Figure 2. 29 In violet: *Excitatory Waves due to the periodic domain, able to excite neurons on the other side of the domain. Now, neuron (21,1) receives also excitatory wave from neuron (21,41). In green: "normal" Excitatory Waves .*

This is a classical problem concerning the close domain; in the model, these border effects have been avoided with the construction of a **periodic domain**.

In this kind of domain, each neuron is set to have the same number of neighbouring neurons. This is possible because with the periodic domain there is a sort of continuity among the left side of the matrix and the right side, as well as for the topside and the bottom side. In a nutshell, it is like having a **spherical domain**. Now the neuron in position (21,1) (on the left side) is managed as a neuron close to the neuron in position (21,2) (as before), and to the neuron in position (21,41) (on the right side). The same construction applies if we consider a neuron positioned on the top of the matrix: it will be managed as a neuron close with neurons on the top as well as neurons on the bottom.

In this way we avoid border effects. To ensure that in each layer all the neurons are inside a periodic domain, an algorithm has been introduced. This algorithm is present in every parts of the model concerning the construction of, ***External Stimuli***, ***Receptive Fields***, ***Laterals Synapses*** and ***Feed-Forward Synapses***.

Considering the neuron in position (*i,j*), a vector distance D_x^{ij} was constructed containing the distances $d_{ij,ik}$ among the neuron ij and all the others neurons in the same line (**x direction**). If the absolute distance between the neuron *ij* and the neuron *ik* was greater than N/2, 41 (=N) was subtracted from that distance ($d_{ij,ik}$). The same operations were performed along the **y direction** for the same neuron *ij*:

$$D_x^{ij} = \left\{ \dots d_{ij,ik} \dots \right\} \qquad k = 1, 2, \dots 41$$
$$\forall k = 1, 2, \dots 41 \quad if\left(\left| d_{ij,ik} \right| > \frac{N}{2} \right) \quad \Rightarrow \quad d_{ij,ik} = d_{ij,ik} - 41$$

$$D_y^{ij} = \left\{ \dots d_{ij,hj} \dots \right\}' \qquad h = 1, 2, \dots 41$$
$$\forall h = 1, 2, \dots 41 \quad if\left(\left| d_{ij,hj} \right| > \frac{N}{2} \right) \quad \Rightarrow \quad d_{ij,hj} = d_{ij,hj} - 41 \tag{2.25}$$

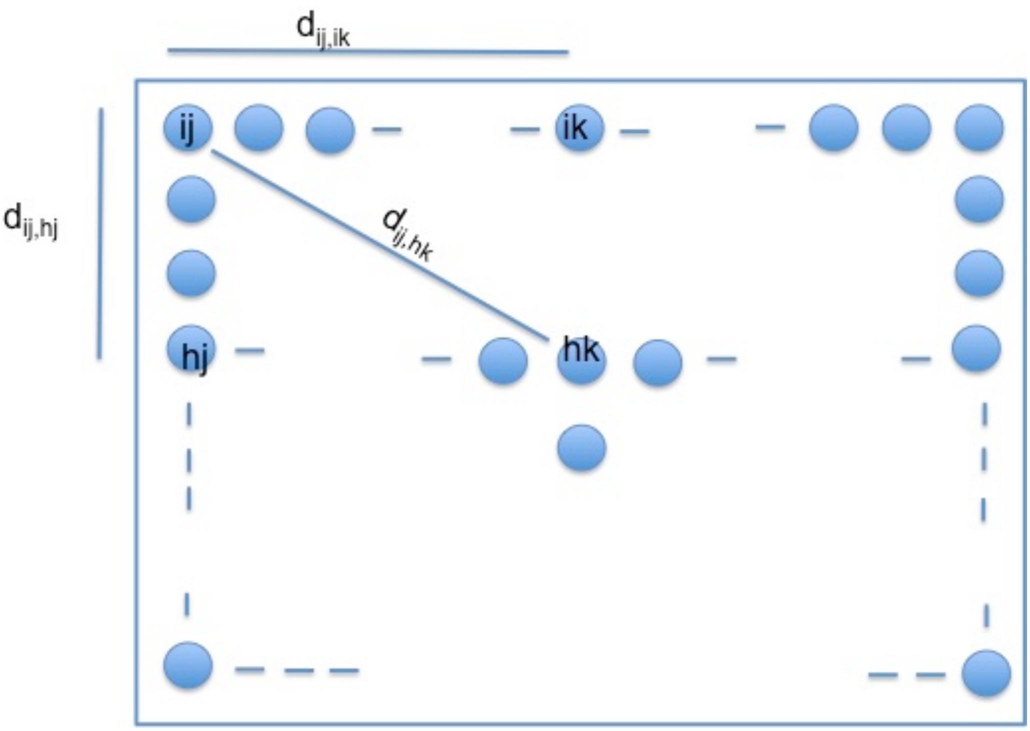


Figure 2. 30 ***Example of distances among neurons inside any "Area".***

A little example could help. Supposing the neuron (1,1) as reference, construct the vector Dx:

$$(i, j) = (1,1)$$
$$D_x^{11} = \{\ldots d_{11,1k} \ldots\} \qquad k = 1, 2, \ldots 41$$
$$D_x^{11} = \{0, 1, 2, \ldots 20, 21, \ldots\ldots 40\} \tag{2.26}$$

The last position of this vector, denotes the distance between the neuron (1,1) and neuron (1,41): this is equal to 40, so, these 2 neurons are far from each other. If we apply the condition number (num. 2.25), we will obtain a vector D_x^{11} like this:

$$D_x^{11} = \{0, 1, 2, \ldots 20, (21 - 41), (22 - 41), \ldots\ldots (40 - 41)\}$$
$$D_x^{11} = \{0, 1, 2, \ldots 20, -20, -21, \ldots\ldots -1\} \tag{2.27}$$

As we notice, the last distance in the vector, corresponding with the distance between the neuron (1,1) and the neuron (1,41) has become -1. Thus, now these two neurons are close. This implementation ensures continuity in the computation of the distance between each pair of neurons, linking the left side of the matrix with the right side, as well as for the topside and the bottom side.

Combining the two vectors distance D_x^{ij} , D_y^{ij} with the Pitagora's rule we can obtain the "**distance square matrix**", containing the square distances among the neuron *ij* and all other neurons present in the layer.

PITAGORA'S RULE: $$d_{ij,hk} = \sqrt{(d_{ij,ik})^2 + (d_{ij,hj})^2}$$

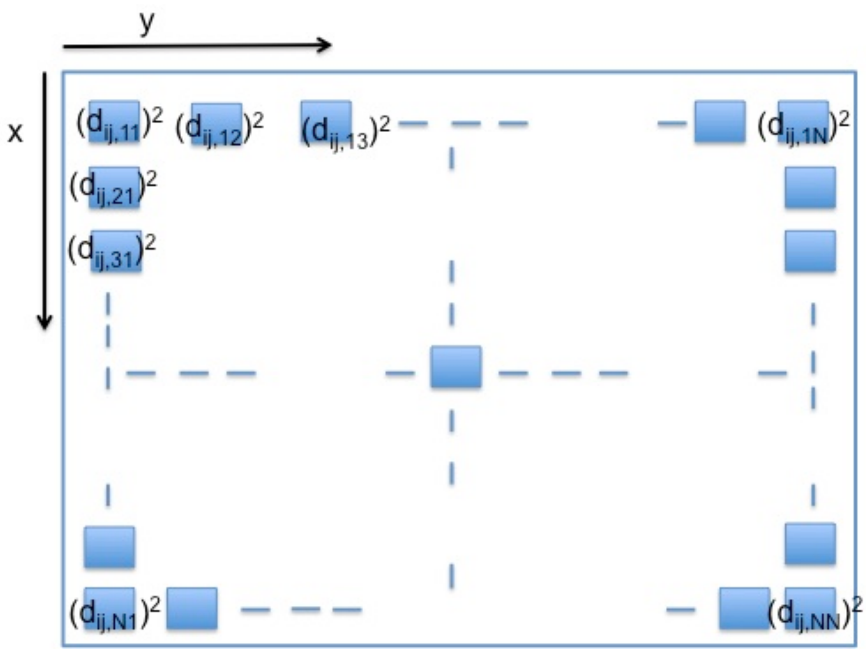


Figure 2. 31 ***Example of "distance square matrix" .***

Obviously in this matrix the position (i,j) will be equal to 0, because it represents the distance between the neuron *ij* and itself ($d_{ij,ij} = 0$).

Chapter 3

SIMULATIONS AND RESULTS

Introduction

In this chapter, all results of the simulations will be shown and they will be analysed making tables, graphics and comparisons among them. Before starting with the simulation results, a short panoramic view about the developed environment of these simulations will be shown.

The neural network was implemented in MATLAB *7.9.0* environment. Different programs in order to simulate the mechanisms of the perception of the distances on Region A (hand), and Region B (Arm) have been implemented. Below, a quick file list names with the relatives features:

- *RegionA.m* : allows to create a simulation concerning the mechanism of perception of the distance between two punctual stimuli applied on the hand. In this model, the skin region area is equal to 5 x 5 cm =25 cm^2. This is a **deterministic neural network,** because it does not simulate the presence of noise in the parameters.
- *RegionB.m* : allows to create a simulation concerning the mechanism of perception of the distance between 2 punctual stimuli applied on the arm. The skin region simulated is equal to 10 x 10 cm =100 cm^2. This is a **deterministic model because** it does not simulate the presence of noise in the parameters.
- *Simulation.m*: includes in a unique file *RegionA.m* and *RegionB.m* programs, but it adds noise in some parameters, changing the network in a **stochastic model**. This program repeats automatically the simulations on both regions for 100 times, and for different distances between the two stimuli, in order to record results in terms of perception distance and make comparison among the two regions (hand and arm).

- *ThresholdofPerceptionHand.m*: is a program that computes the **two points discrimination threshold** about the hand**.**
- *ThresholdofPerceptionArm.m*: is a program that computes the **two points discrimination threshold** about the arm**.**
- *CompareHandArm.m*: loads data from *ThresholdofPerceptionHand.m*, *ThresholdofPerceptionArm.m* and constructs graphics confronting the different relations between distances in centimetres, and distances in terms of number of neurons between the activation balls.

These were the main files used to perform simulations and to produce results.

Other important concepts, that have to be mentioned before starting with the explanation of the neural network outputs, are linked with the **noise** that afflicts some parameters, and the neuron **activation threshold** adopted to read out perceived distance from the network activation.

The Noise

Focusing on the hand region, the activations inside the two layers could be different in base on the distance between the two stimuli, and the intensity of the two stimuli on the skin. So, if we change this kind of parameters we will obtain different activation patterns within these layers. This is a very significant aspect and it has to be managed. Just as in the reality, the application of two stimuli on the skin of a subject, in two different trials, won't be the same. That is because in the reality it is almost impossible to replicate exactly the same experimental conditions in different trials. For example the position of each stimulus could change of few mm, as well as the intensity could change of few measure units. Moreover, also neural activation is affected by noise.

To replicate these conditions, and reproduce external noise, and internal noise, it was added **noise** in the parameters of the model. In particular, the noise was superimposed to the position of each stimulus and to its amplitude (intensity of the Gaussian function). The noise utilized in every simulation is a **Gaussian**

Noise, and it was generated by a MATLAB's function called *randn( )*. For example, using this formula:

$$x = \mu + \sigma \cdot randn(1{,}100). \tag{3.1}$$

we can create a vector x (in this case with a length equal to 100) of random values, with a Normal Distribution that is defined by a mean value μ and variance σ^2. With this equation, Gaussian noise in the position of the input and in its amplitude (intensity) has been added. MATLAB's instructions for the construction of the position and the intensity of two stimuli are reported here:

```
inp1=inh1+0.3/2*randn(1,1);
posizione_in_1=[inp1 0];
forza_in_1 =1.5+0.15*randn(1,1);

inp2=inh2+0.3/2*randn(1,1);
posizione_in_1=[inp1 0];
forza_in_2 =1.5+0.15*randn(1,1);
```

inp1 and *inp2* are the x coordinates for positions of the first stimulus and the second stimulus respectively. Hence, if we want to simulate two stimuli at a distance equal to 2.5 cm along the x axis, we have to impose:

- ***inp1*** = -1.25 cm
- ***inp2*** = 1.25 cm

since the position of the stimuli is referred to the center of the reference system. This means that if we simulate a distance between the two stimuli of 2.5 cm for 100 times, the effective distance will not be always equal to 2.5 cm for every trail, but sometimes it will be a bit greater and other times it will be a bit less than 2.5 cm (it depends on the variance σ^2 of the noise added to the position). In particular, as indicated in the MATLAB instructions, the noise superimposed on the stimulus position has 0 mean value, and 0.15 standard deviation Hence, collecting all the 100 distances, we will notice that these distances are going to spread as a Normal Distribution with mean μ and variance σ^2. However, also the random intensity of the stimuli plays a key role in the patterns of activation inside the first and the second layer. The noise superimposed on stimulus intensity has 0 mean value and 0.15 standard deviation Therefore, the use of noise helps in the reproduction of the real experiment condition, and changes the model from a deterministic model to a ***stochastic model***. “Stochastic" means having a random variable. A stochastic model is a tool for estimating probability distributions of potential outputs by allowing for random variation in one or more inputs. Distributions of potential outputs are derived from a large number of simulations (stochastic projections), which reflect the random variation in the inputs.

Threshold of activation

In this chapter, when we will speak of distance between the two activation balls in any layer, will consider the number of inactivated neurons between the two activation balls (obviously the activation balls are composed by activated neurons). The distinction among activated and inactivated neurons depends on a **threshold of activation**. Given that the **state variable** (activation index of a generic neuron) ranges **from 0 to 1,** in every simulation the threshold of activation is always imposed equal **0.9**. So, neurons with a value of the state variable greater than 0.9 will be marked as activated neurons, otherwise as inactivated neurons.

3.1 First Experiment

3.1.1 Tactile Size Perception on the Hand vs Arm

The main idea to simulate the Weber's Illusion mechanism was to simulate the application of two stimuli separated by different distances on different body regions (Hand, Arm), and "ask" to the simulator which stimuli was larger. In each trial the skin is touched with two stimuli **on the Hand,** and with two other stimuli **on the Arm**. Inside *Simulation.m* there are 10 pairs of stimuli, according to the size of the Hand and Arm stimuli (Hand/Arm):

Distance (cm) Hand/Arm	Ratio
2.4 / 6	*0.4*
2.7 / 6	*0.45*
2/ 4	*0.5*
2.2 / 4	*0.55*
3 / 5	*0.6*
3.25 / 5	*0.65*
2.8 / 4	*0.7*
3.5 / 4.5	*0.8*
2.7 / 3	*0.9*
3 /3	*1*

Table 3. 1 ***Input Stimuli Distances.***

Each pair is applied 100 times, for a total of 1000 trials. **For simplicity, in every simulation, we have implemented two stimuli along the x direction and on the same line of neurons. As in the figure below :**

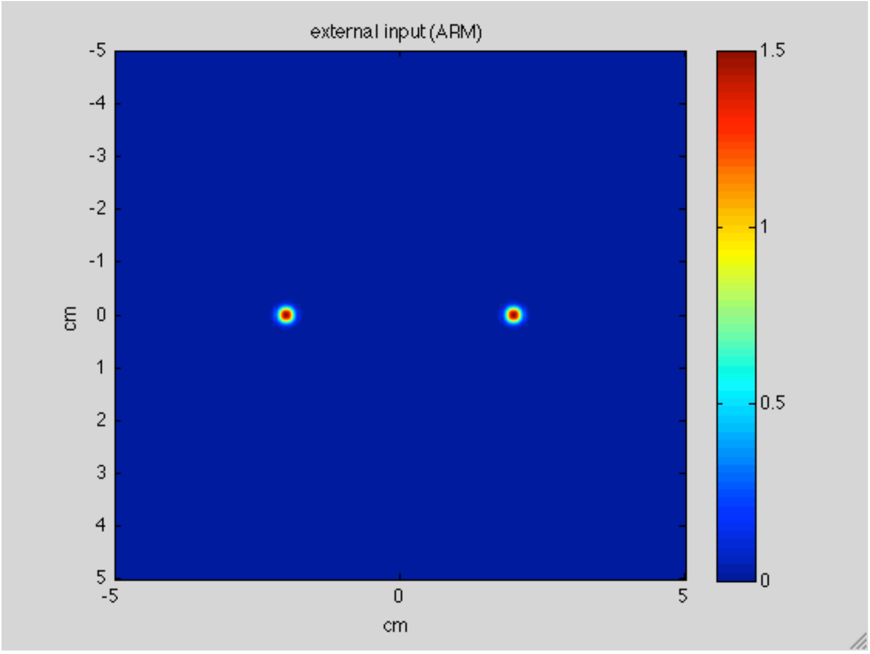


Figure 3. 1 ***Example of external punctual stimuli.***

Obviously the model is not a real person, and the “answer” mentioned above is just the reading out of the output model. In fact, given the association of the second layer of the model as a “High Cortical Area” able to reconstruct information deriving from low levels, it seems opportune to use this layer as the one involved in the perception distance. So, considering a pair of stimuli, like 3/3 cm (Hand/Arm), the distance between the two activation balls (in terms of number of inactivated neurons) in the second layer of the Hand, as well as of the Arm, was computed. Then these two distances were compared, and the distance resulting bigger was recorded. Hence, we can think that “Area 2” is the cortical area directly involved in the perception of the distance, namely the layer that produce a “judgment” of the virtual subject about the distance dimension of the two stimuli. At the end, the first output of the model is ***the proportion of trials in which the stimuli on the hand were “judged” larger as a function of the ratio of the length of the Hand and Arm stimuli (length on the Hand / length on the Arm).*** Moreover, also the gap between the balls inside the first layer was computed, as well as the same proportion as above (second output). In short, the neural network provide as output two proportions, as we can see in the *table 3.2*. Even if the “Second Layer” is the layer involved in the perception of the distance, the results of the first layer (involved in the cortical magnification) has been recorded too, with the target to analyse the main differences between the first and second layer.

The next table show the results that comes from the neural network at the end of the simulation:

distance hand/arm	ratio	Proportion hand stimuli judge bigger in Area 1	Proportion hand stimuli judge bigger in Area 2
2.4 / 6 cm	0.4	0.05	0
2.7 / 6 cm	0.45	0.14	0.01
2/ 4 cm	0.5	0.53	0.02
2.2 / 4 cm	0.55	0.83	0.14
3 / 5 cm	0.6	0.98	0.65
3.25 / 5 cm	0.65	1	0.9
2.8 / 4 cm	0.7	1	0.94
3.5 / 4.5 cm	0.8	1	0.99
2.7 / 3 cm	0.9	1	1
3 / 3 cm	1	1	1

Table 3. 2 ***Simulation results.***

These data was fitted by a Cumulative Gaussian function with least-squares regression using R 2.8.0 (a software for the statistical analysis of data). The results of the fitting are shown in the graphic below:

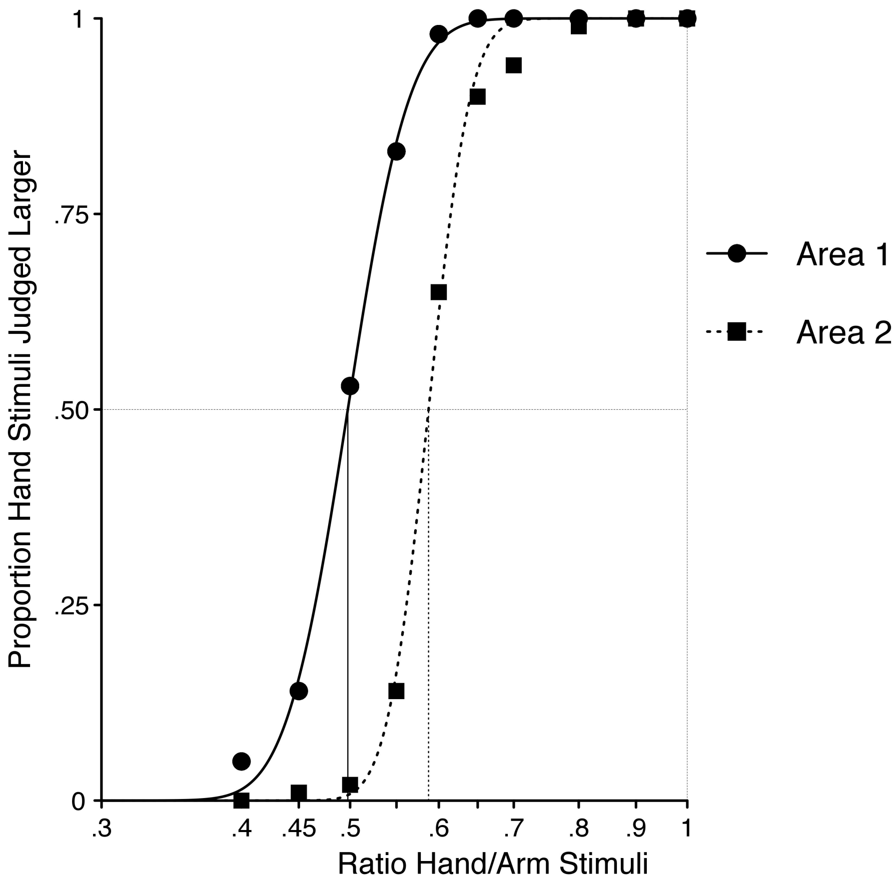


Figure 3. 2 ***Results of the simulation.***

The two Cumulative Gaussian functions represent the proportion of trials for every ratio (**length stimuli Hand/length stimuli Arm**) in which the distance between the two stimuli on the hand was "perceived" larger than the distance between the two stimuli on the Arm.

- The **solid line** represents the proportion considering the **first layer**.
- The **dotted line** represents the proportion considering the **second layer**.

The graphic is in a semi-logarithmic scale (a logarithmic scale has been used for the x-axis), and the two sigmoidal curves are characterized by two parameters:

- **Point** of **subjective equality** (PSE) : is defined as the point at which the function crossed the 50%.
- **Inter-Quartile Ranges** (IQR): as a measure of the slope of the function. IQR is the range between the 25% of the function and the 75%. It is common to use this index as an indicator of the slope.

The solid curve shows how the perception distance would be, if there was only the first layer, or better, if there was only the cortical magnification's effect (first layer = primary somatic sensory cortex). When the second layer (such as a High Cortical Area) has effect too, it is able to partially adjust the distorted information coming from the first layer; as a consequence the sigmoidal curve tends to shift on the right (dotted curve).

Hence, dotted curve just represents the proportion when the information about the perception distance comes from the second layer. This is the effect of the rescaling process. For example, with a ratio = 0.5, we have this kind of length stimuli:

- 2 cm on the Hand
- 4 cm on the Arm

If we take a look at the table 3.2 and at the graphic, considering the curve of the first layer, the simulator judged 2 cm stimulus on the Hand larger than 4 cm stimulus on the Arm, 53% of the time. Whereas, after the "help" of the second layer, we can notice that this value decreases down to 2%. This is the effect of the rescaling implemented by Layer 2. Moreover, it is interesting the comparison of the results between ratio 0.5 and 0.6:

distance hand/arm	ratio	Proportion hand stimuli judge bigger in Area 1	Proportion hand stimuli judge bigger in Area 2
2/ 4 cm	0.5	0.53	0.02
3 / 5 cm	0.6	0.98	0.65

Table 3. 3 ***Ratio 0.5 and 0.6 .***

As we can observe, the ratio is different but the absolute difference of the two couples of distances is the same, equal to 2 cm. The differences of the outputs for these 2 kind of ratio is evident, so we can assert that the network is independent by the absolute difference distance of the two stimuli, but is strongly dependent on the ratio.

3.1.2 PSE and IQR of the First Experiment

The main aspect of simulation results concerns the PSE. If there were no differences in the representations of the Arm and Hand, PSE would be a ratio equal to 1, indicating that stimulus localization along the body does not bias (is independent from the) perceived distance. For example, the **Ideal Case** about the proportion of trials in which the stimuli on the hand are "judged" larger as a function of the ratio (Hand/Arm) , should appear a function like the red one (step):

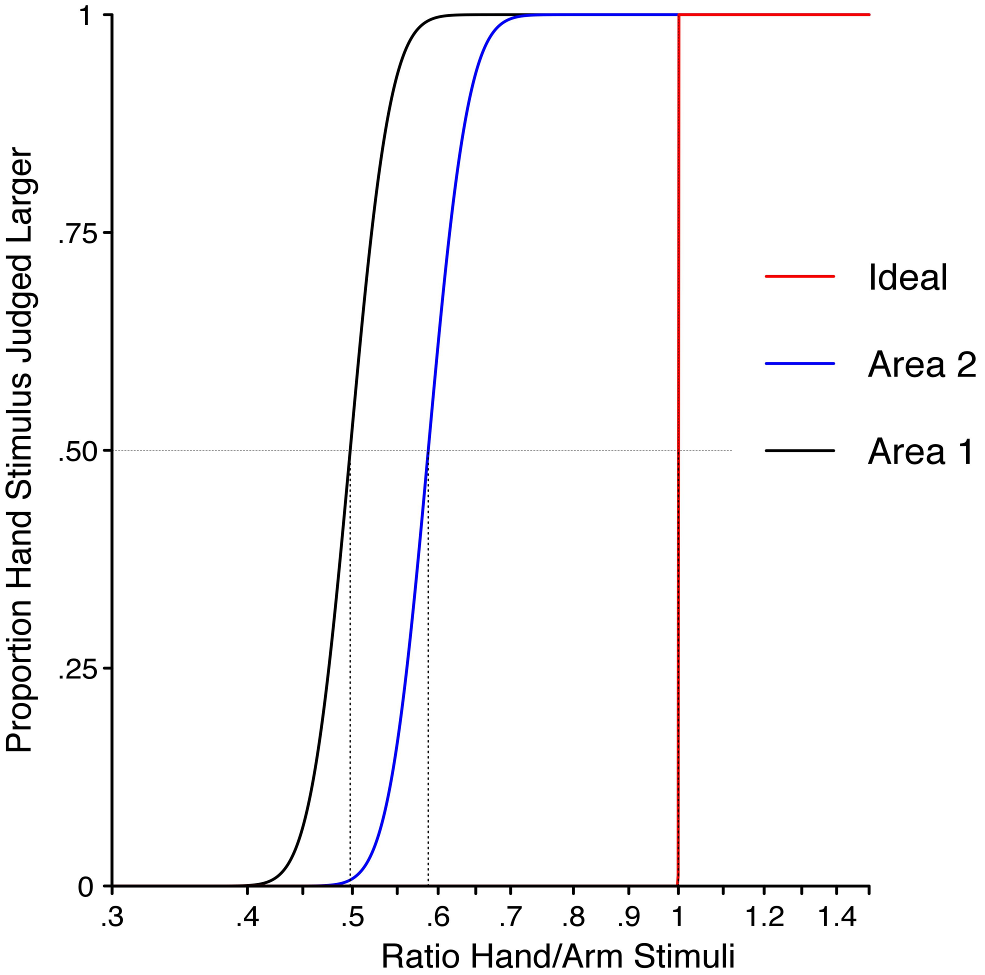


Figure 3. 3 ***Comparison of the simulation results with the "Ideal Case".***

In the Ideal curve, PSE is equal to 1 and the slope of its curve is infinity. PSE is equal 1 because, if we think to apply the same pair of distances, for example 3/3 cm (Hand/Arm) , 100 times on the skin surface of the subject, the subject will judge larger the stimuli on the hand approximately half of the times, whereas in the other half of the times he will judge larger the stimuli on the arm. This because the two stimuli are equals, and the subject is not able to discriminate between them (that is, he will answer at chance). Instead, the slope of the ideal curve is infinity (like a step) simply because in an ideal condition, the subject is able to classify correctly each couple of distances, with an extremely high precision. Namely, in this condition the subject can understand correctly if the stimulus on the hand is bigger than the stimulus on the arm. In fact, for every ratio smaller than 1 (that means stimuli on the hand smaller than stimuli on the arm), the curve is equal 0, whereas, for every ratio bigger than 1 the curve is constant equal to 1. However, the Weber's Illusion comes from a distortion of the body shape, resulting in a sort of alteration in the judgment of which stimulus is larger. This alteration tends to shift the curve toward smaller ratios (black and blue curve), and the PSE of this curve is going to be smaller than the ideal case. In addition, the slope of these curves won't be equal to infinity, because the subject tends to misperceive the real dimension of the stimuli.

So, **we can assert that the Weber's illusion is directly related with the shift of the PSE toward smaller ratio**, whereas **the precision in the judgment of which stimulus (on the hand or on the Arm) is bigger, is linked with the slope of the sigmoidal curve**. The shift of the PSE (or the curve in general) is also called **bias**.

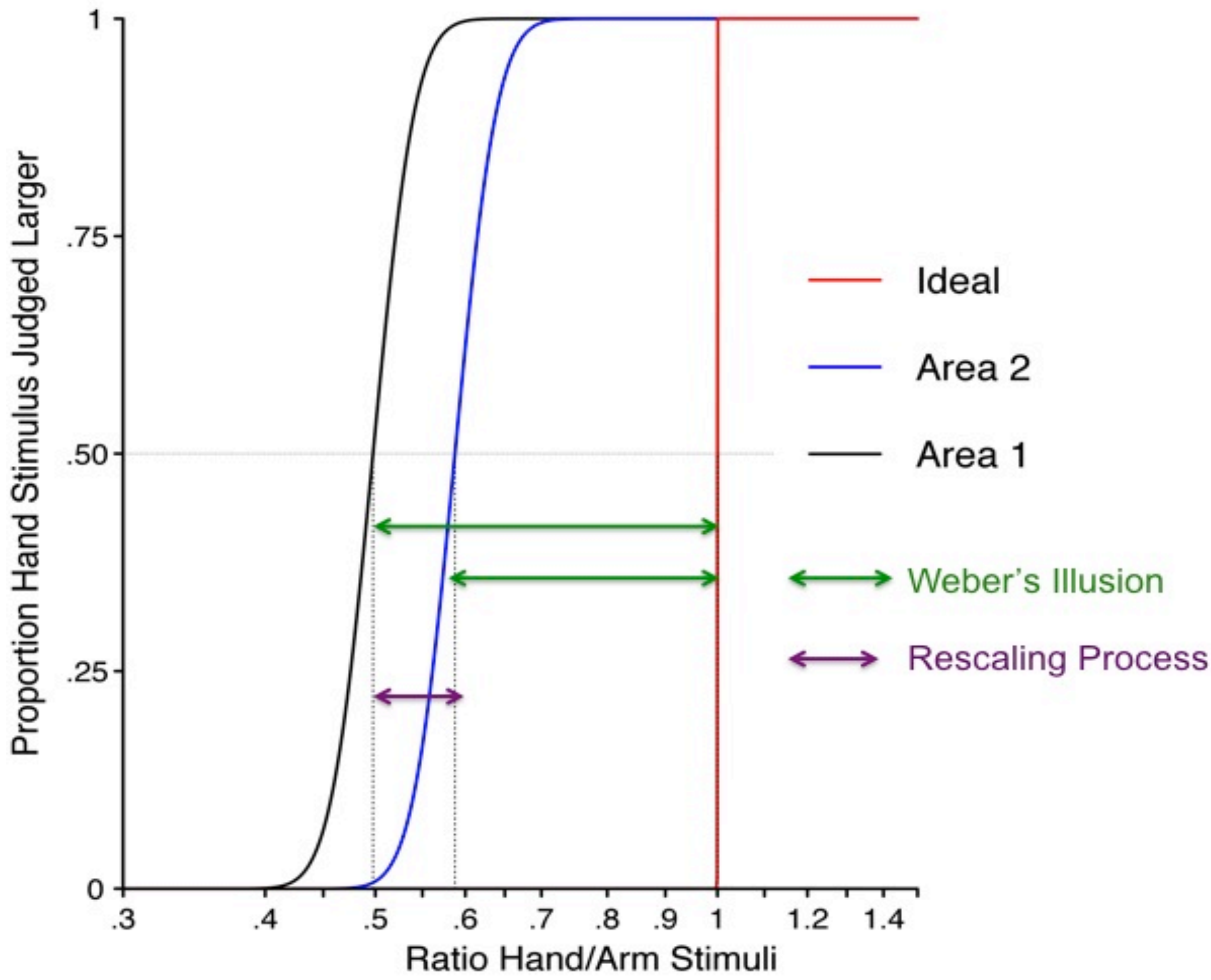


Figure 3. 4 *Weber's Illsuion and Rescaling Process in the simulation results.*

Observing the figure (3.4), we can see that the Weber's Illusion is marked in green as the distance between the PSE of the curve and the PSE of the Ideal Case (PSE =1). In addition, the entity of Rescaling Process has been marked in violet. It is clear that the "Rescaling Process" is quantified as the absolute difference between "Area 1"'s PSE and "Area 2"'s PSE. Going back to the simulation results, the PSE and IQR values are reported below:

	PSE	25%	75%	IQR
First Layer	0.5	0.47	0.53	0.06
Second Layer	0.59	0.56	0.61	0.05
Green's Result	0.62	/	/	/
Ideal	1	1	1	0

Table 3. 4 ***Point of Subjective Equality, IQR, and Weber's Illsuion entity about the simulation results.***

As I told before PSE of the second layer is greater than the first layer but it is still lower than the PSE of an ideal curve (no distortions). *This means that there is still a distortion in the perception, but it is attenuated with respect to the first layer.* Hence, the Area 1's output has higher bias from the ideal curve compared with the output of the second layer. At the same time, the slope of the second layer is lightly higher than the first one (IQR second layer < IQR first layer), therefore it is licit to assert that the second layer shows a lightly higher precision in the judgment of distance.

Moreover, the difference between the Second Layer PSE and the PSE computed by Green's Results is very small. This is a very important result, because we can interpret this similarity as a validation of the simulation results (since Green's data come from in vivo experiment on real subject).

Given that results the rescaling process implemented by the neural network covers the 20% of the bias of layer 1. Unfortunately, do not exist data obtained from real experiments to compare the effect of this process, but an increment of the 20 % looks verisimilar.

3.1.3 Activation of neurons in the Hand and in the Arm

The neural network is defined with parameters reported in table (2.1). With these values the simulator can create a pattern of activation in "Area 1" and "Area 2". As I have explained in Chapter 2, the network works in order to:

- maintain the same pattern of activation inside both Area 1 and Area 2 concerning the Hand's region (maintaining the high resolution).
- increase the distance between the activation balls in the passage from Area 1 to Area 2 concerning the Arm's region (increasing the resolution of this region).

A visual example is better to understand this process. So, consider the next figures as just an example of what can happen in a simulation (subsequently I will show the real results of the simulations):

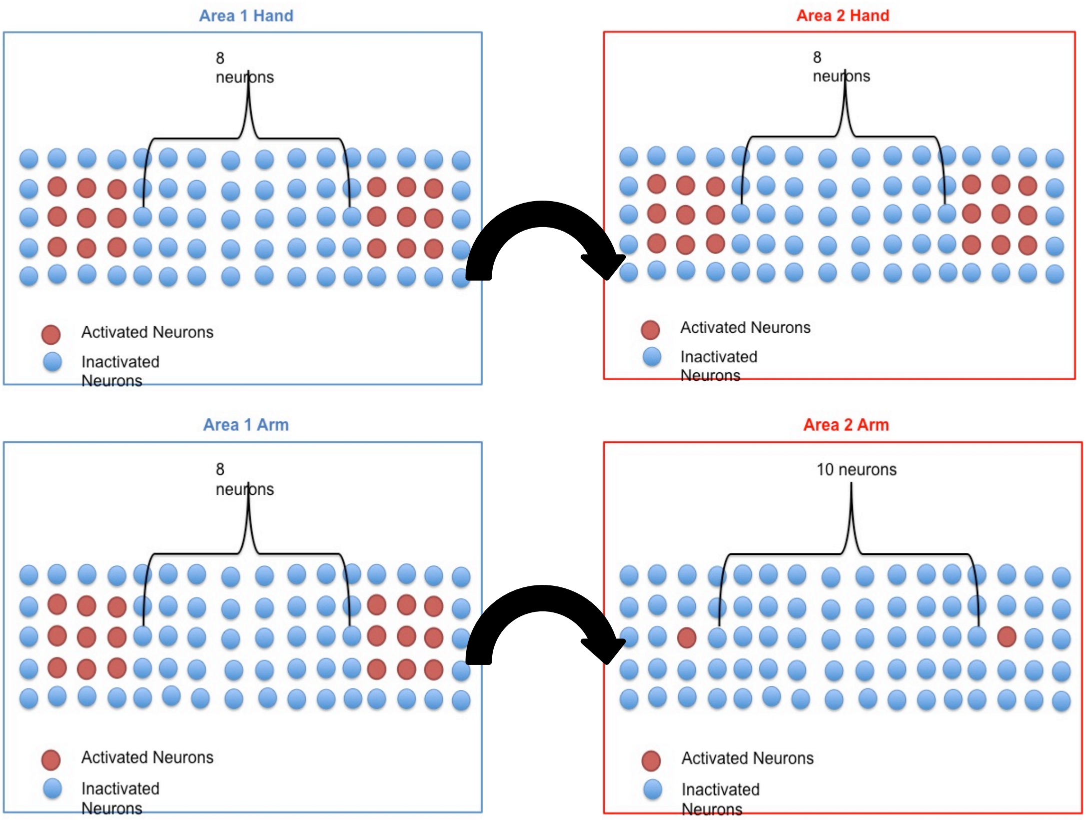


Figure 3. 5 *Example of neural activation inside the two layers for the Hand, and for the Arm.*

In the Arm's case, shifting from "Area 1" to "Area 2" produces an increment equal to 2 neurons in the gap between the bubbles, due to a reduction of the dimensions of the two activations balls. Whereas in the Hand's case, the number of neurons in the separation gap remains the same. The main aspect that has to be considered concerns the **difference in terms of number of neurons between the two balls of activation when shifting from "Area1" to "Area 2"**. In this example we can conclude:

- **Hand**'s Case: difference = 0 neurons
- **Arm**'s Case: difference = 2 neurons

This was just an example, but **the real results of the first simulation about the Average Difference in terms of number of neurons between the two balls of activation, concerning the passage from Area1 to Area 2 are** :

	Average Difference between Area 1 & Area 2
Hand	*0.02 neurons*
Arm	*4.01 neurons*

Table 3. 5 *Average Difference in terms of number of neurons.*

These results were calculated from the average of the all (10) couples of distances (ratios). Hence, focusing on the Arm's case, in the passage from layer 1 to layer 2 the gap between the activation balls increase in average of about 4 neurons. I have said increase, and not decrease, because the network works in the way to increase the resolution of the Arm region. Instead, in the Hand's case the gap length is almost the same. The conclusion is that, the neural network of the Arm implements the rescaling process losing about 2 neurons per each activation ball. Whereas the Hand's neural network tries to maintain the same pattern of activation shifting from "Area 1" to "Area 2".

3.2 Second Experiment

3.2.1 Student t-Test

Data and results shown until now come from a single simulation. So, this was not enough to validate the results and assert that there is a relevant statistical difference between the output of the first layer, and the output of the second. Thereby I have decided to implement 10 simulations on 10 different virtual subjects, and performed the t-Test. Each simulation was conducted as the simulation of the first experiment; hence, every subject was submitted at the same procedure, with the same pairs of distances (ratios). The unique difference was in the number of trials per ratio, that it was 10 times and not 100 times as in the first experiment

The outputs of the 10 simulations are shown below: the first table is relative to the proportion of stimuli on the hand judge larger concerning the SECOND layer, whereas the second table is related with the FIRST layer. Tables are divided in the middle, for space reasons:

Proportion Hand Stimuli judge large on the SECOND layer (Area 2)

Distance hand/ Arm	Ratio	*Subject 1*	*Subject 2*	*Subject 3*	*Subject 4*	*Subject 5*
2.4 / 6 cm	0.4	0	0	0	0	0
2.7 / 6 cm	0.45	0	0	0	0	0
2/ 4 cm	0.5	0.1	0	0	0	0
2.2 / 4 cm	0.55	0.2	0.2	0	0.2	0.3
3 / 5 cm	0.6	0.8	0.4	0.6	0.5	0.5
3.25 / 5 cm	0.65	0.9	1	1	0.9	0.9
2.8 / 4 cm	0.7	1	1	1	1	1
3.5 / 4.5 cm	0.8	1	1	1	1	1
2.7 / 3 cm	0.9	1	1	1	1	1
3 /3 cm	1	1	1	1	1	1

Distance hand/ Arm	Ratio	Subject 6	Subject 7	Subject 8	Subject 9	Subject 10
2.4 / 6 cm	0.4	0	0	0	0	0
2.7 / 6 cm	0.45	0	0	0	0	0
2/ 4 cm	0.5	0.1	0	0	0	0
2.2 / 4 cm	0.55	0	0.1	0.2	0.2	0
3 / 5 cm	0.6	0.5	0.6	0.8	0.5	0.7
3.25 / 5 cm	0.65	0.8	0.9	0.8	0.9	1
2.8 / 4 cm	0.7	0.9	1	1	1	0.9
3.5 / 4.5 cm	0.8	1	1	1	1	1
2.7 / 3 cm	0.9	1	1	1	0.9	1
3 /3 cm	1	1	1	1	1	1

Table 3. 6 *Results about the Second Layer.*

Proportion Hand Stimuli judge large on the FIRST layer

(Area 1)

Distance hand/ Arm	Ratio	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5
2.4 / 6 cm	0.4	0	0	0.1	0	0
2.7 / 6 cm	0.45	0.1	0.2	0.1	0.3	0.3
2/ 4 cm	0.5	0.6	0.7	0.8	0.5	0.4
2.2 / 4 cm	0.55	0.8	0.9	0.8	0.8	0.8
3 / 5 cm	0.6	1	1	1	1	1
3.25 / 5 cm	0.65	1	1	1	1	1
2.8 / 4 cm	0.7	1	1	1	1	1
3.5 / 4.5 cm	0.8	1	1	1	1	1
2.7 / 3 cm	0.9	1	1	1	1	1
3 /3 cm	1	1	1	1	1	1

Distance hand/ Arm	Ratio	Subject 6	Subject 7	Subject 8	Subject 9	Subject 10
2.4 / 6 cm	0.4	0.1	0	0	0	0
2.7 / 6 cm	0.45	0.4	0.1	0.1	0.4	0.1
2/ 4 cm	0.5	0.5	0.7	0.6	0.5	0.9
2.2 / 4 cm	0.55	1	0.6	1	0.9	1
3 / 5 cm	0.6	1	1	1	1	1
3.25 / 5 cm	0.65	1	1	1	1	1
2.8 / 4 cm	0.7	1	1	1	1	1
3.5 / 4.5 cm	0.8	1	1	1	1	1
2.7 / 3 cm	0.9	1	1	1	1	1
3 /3 cm	1	1	1	1	1	1

Table 3. 7 *Results about the First Layer.*

As in the first experiment, I have fit the data with program R, using Gaussian Cumulative Functions. The PSE values of the functions of each subject are:

Subject	*PSE Second Layer*	*PSE First Layer*
1	0.57	0.5
2	0.6	0.48
3	0.6	0.48
4	0.59	0.49
5	0.59	0.5
6	0.6	0.48
7	0.59	0.5
8	0.58	0.49
9	0.59	0.48
10	0.59	0.47

Table 3. *8 Points of Subjective Equality.*

The mean and standard deviations of these 2 populations are:

	PSE Second Layer	PSE First Layer
Mean (μ)	0.59	0.49
Std (σ)	0.00998	0.00952

Table 3. 9 *Mean and Standard Deviation.*

With the Standard Deviation, we can compute the **Standard Errors** with this formula:

$$Ste = \frac{\sigma}{\sqrt{n}} \qquad n = 10 = population\ dimension \tag{3.2}$$

- **Ste Second Layer = *0.00315***
- **Ste First Layer = *0.00301***

With these kinds of values we can understand that the two populations are significantly different. We can see in the next histogram (figure 3.6), the PSE mean value for both the first layer and the second one; moreover, there are presents two vertical black bars that represent the Standard Error (Ste) for “Area 1” and for “Area 2”: they are very shorts, and they do not overlap at all to each other.

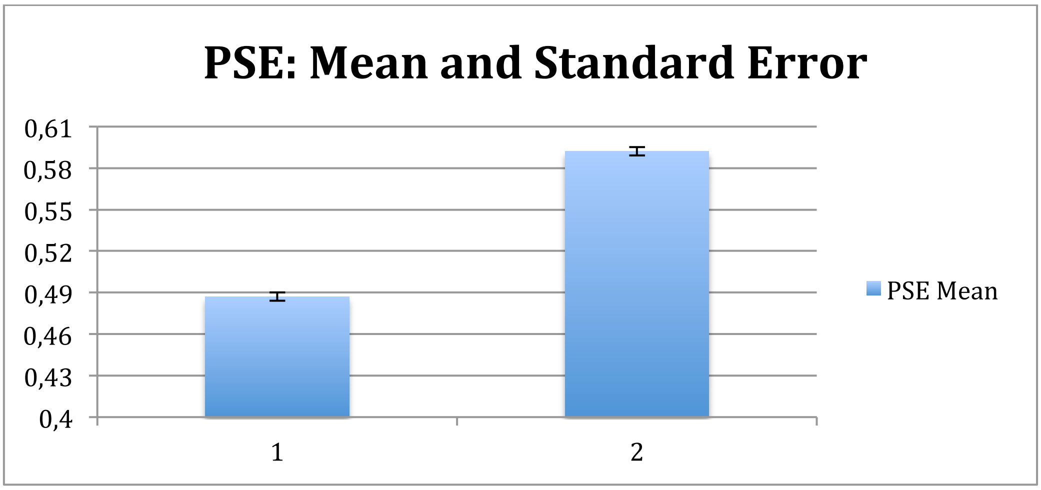


Figure 3. 6 ***Mean and Standard Error of the PSE.***

To confirm the difference between the two populations, “paired t-Test” with MATLAB’s function has been implemented. In short, the paired samples t-Test is used to test the null hypothesis that the average of the differences between a series of paired observations is zero. Observations are paired when, for example, they are performed on the same samples or subjects (as in our case). The t-test is part of the class of hypothesis tests also called significant tests, or the most important methods of statistical inference. To compute this special test we have to define two conflicting hypotheses:

- H0: null hypothesis, or "nothing out of the ordinary".
- H1: alternative hypothesis.

In this study the null hypothesis was “ The average of the Second layer’s mean values is the same of the average of the First layer’s mean values”. MATLAB function called **“ttest(population1, population2)”** gave as output the following values:

- **p-value** = 1.4707e-08
- **t** = 18.9346
- **degrees of freedom** = 9

t is the characteristic value of this statistical test, and it comes from this formula:

$$t = \frac{\frac{\sum_{i=1}^{n} d_i}{n}}{\sqrt{\frac{\sum_{i=1}^{n} {d_i}^2 - \frac{(\sum_{i=1}^{n} d_i)^2}{n}}{n \cdot (n-1)}}} \qquad d_i = PSE_i^{Area2} - PSE_i^{Area1} \qquad (3.3)$$

The degrees of freedom can be extracted by the dimension of the two populations under examination:

$$df \ = n - 1 = 9 \qquad (3.4)$$

This value should be compared with those tabulated in special tables, found in all the books on statistics. The comparison between the value obtained, and the printout leads to obtain the *p-value* (for more information consult: *"Introduction to statistics for psychology, third edition. London: Prentice Hall "*).

p-value is a measure of the credibility of the null hypothesis: thereby if p is small, it means that the difference between the 2 means is not due to chance, but there is a statistical difference. For this study the p-value is very low, equal to **1.47*e-08.** Writing in a compact manner the results of the t-test:

$$t\ (9) = \ 18.9346, \qquad p < 0.0001$$

The result is that the two populations (PSE Second Layer – PSE First layer) are really different; thereby, there is a substantial statistical difference between the two layers. We can think this substantial difference as the effect of the rescaling process.

In addition, t-Test between **PSEs of Area 2** and the **PSEs of Ideal Case** (without distortion, represented by a step with PSE =1) has been performed in order to ensure the validity of the results. Result of the t-test was:

$$t\ (9) = 129.25, \qquad p < 0.001$$

It is clear that even the difference between **Area2's** output and the **Ideal Case** is significant; therefore we can reject the null hypothesis due to the lower value of the *p-value*.

3.3 Third Experiment

3.3.1 Two Point Discrimination Threshold

Spatial resolution on various region of the skin can be quantified in humans by measuring their ability to perceive a pair of nearby stimuli as two different entities. The minimum distance between 2 detectable stimuli is called ***two point discrimination threshold***. In this third experiment we have implemented a simulation to investigate the value of this kind of threshold concerning the Hand and the Arm. We have used a deterministic model, hence, without the presence of noise in the position and in the intensity of the stimuli. In the graphic below it is possible to observe the relation between the distance in cm of the two stimuli applied on the skin surface, and its corresponding distance in terms of number of neurons between the two activations balls, within the second layer.

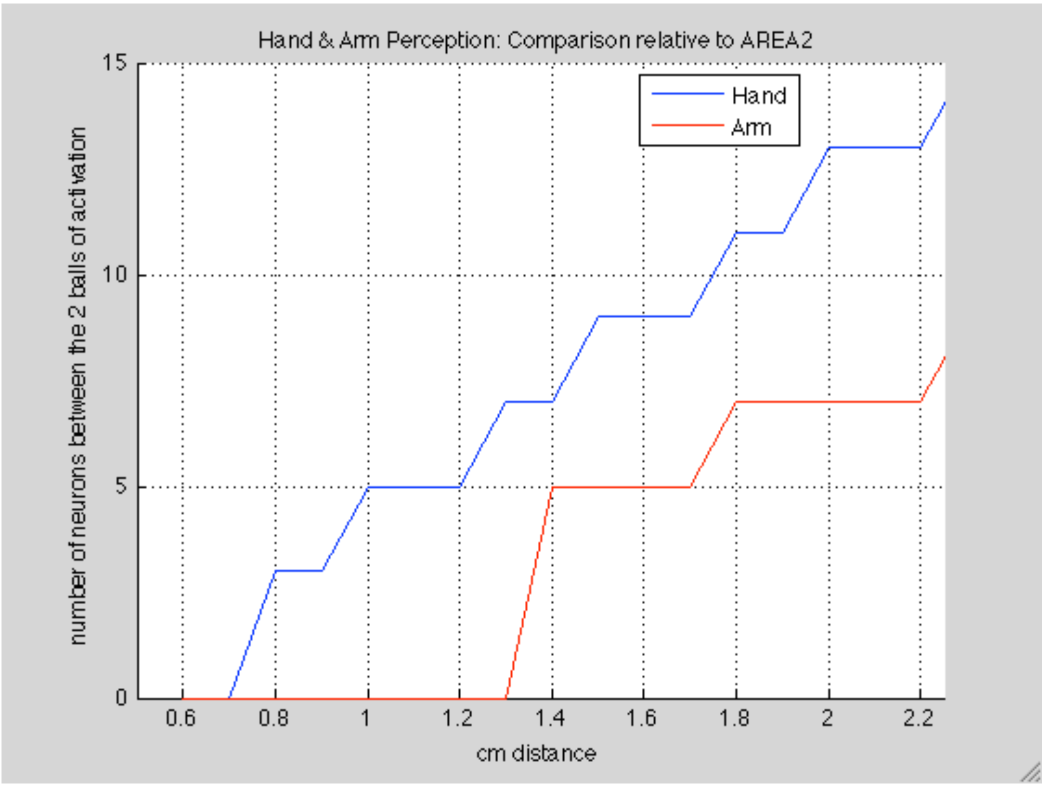


Figure 3. 7 ***Hand and Arm Perception.***

Obviously the ***two points discrimination threshold*** corresponds to the first distance giving an output different from 0. In this network these thresholds are:

- **0.8 cm** for the **Hand**.
- **1.4 cm** for the **Arm**.

These results don't replicate the real results of the hand and the arm as they are reported in the next figure:

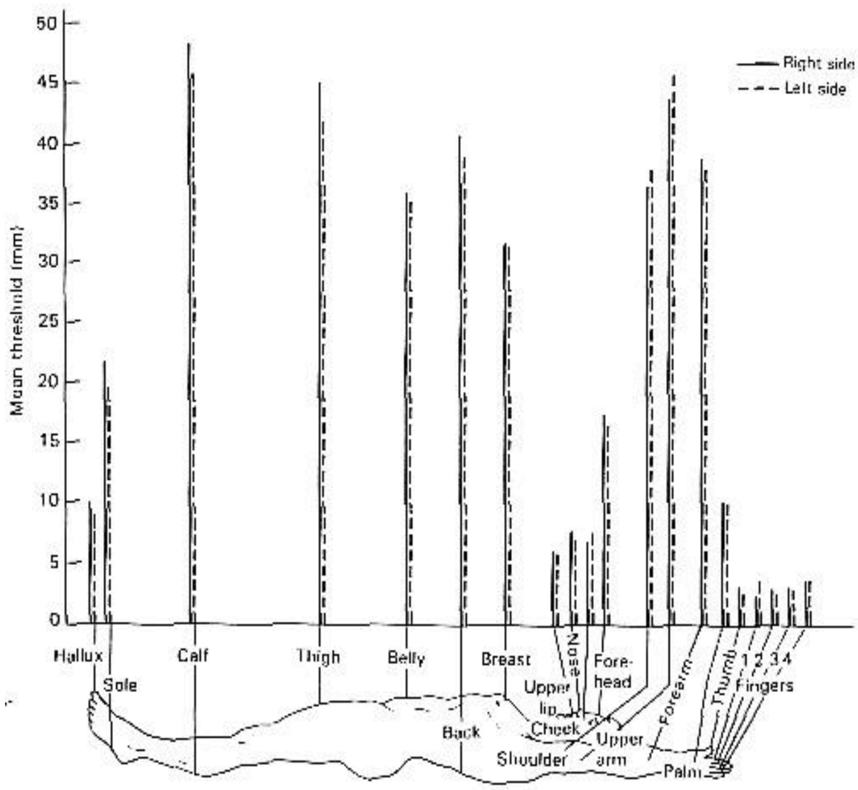


Figure 3. 8 ***Two Points Discrimination Threshold.***

But the important aspect is that there is a significant difference between the two thresholds. In addition, the results are coherent with the nature of the neural network, given that the model was structured with a high resolution on the hand and less resolution on the arm. The result that the threshold of the Hand is **lower** than the threshold of the Arm is correct, because, even in the reality, closer stimuli on the hand are perceived as separated.

Moreover, analysing the graphics of the activated neurons within the second layer, for a distance stimuli equal to the two points threshold, we have found an interest aspect. The next two figures will show the activation pattern within "Area 2" about the hand and about the arm, that is, it was produced by a distance between the two input stimuli equal to the *two point discrimination threshold* recorded before:

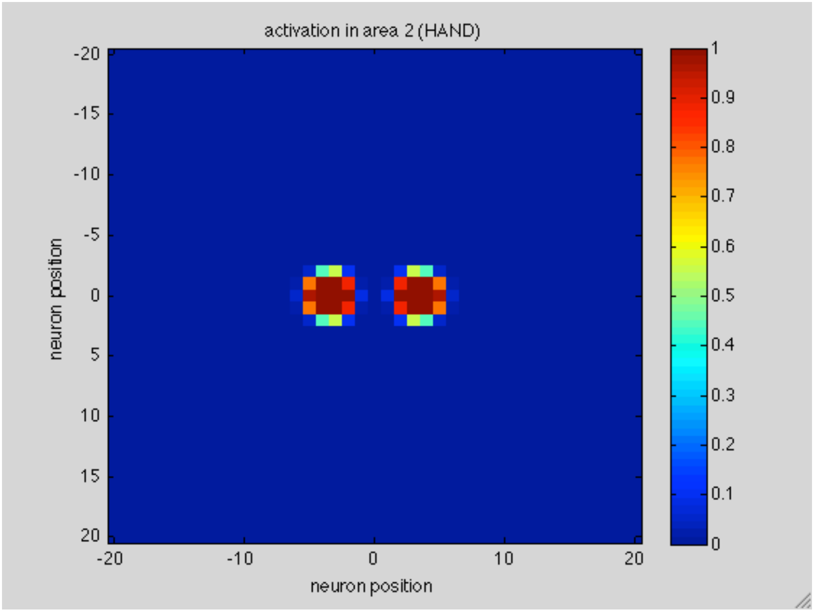


Figure 3. 9 ***Hand: distance stimuli on the skin equal to 0.8 cm.***

In figure 3.10 we can see the activation in Area 2 concerning the Arm for a distance stimuli equal to 0.8 cm (Hand's threshold): the two bubbles are attached, hence the stimuli should be perceived as an unique stimulus.

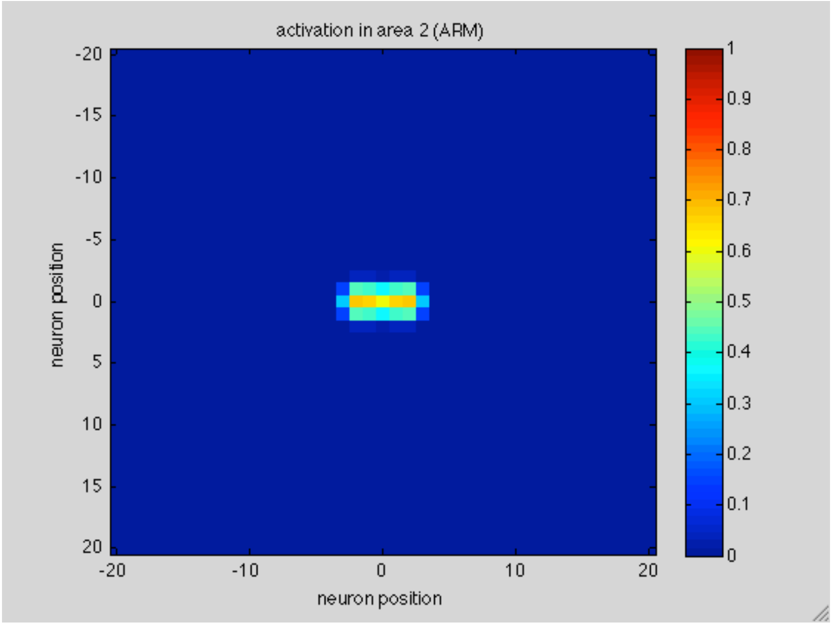


Figure 3. 10 *Arm: distance stimuli on the skin equal to 0.8 cm.*

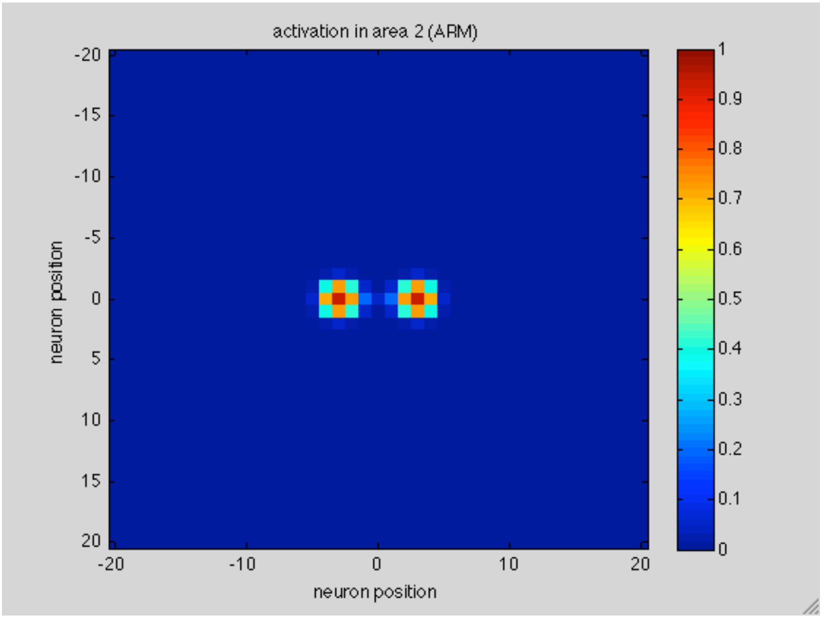


Figure 3. 11 *Arm: distance stimuli on the skin equal to 1.4 cm.*

The neural activation on the Areas shown before can be represented with a 3D view, in order to visualize the peaks of activation (graphics will present in the same order of 2D graphics):

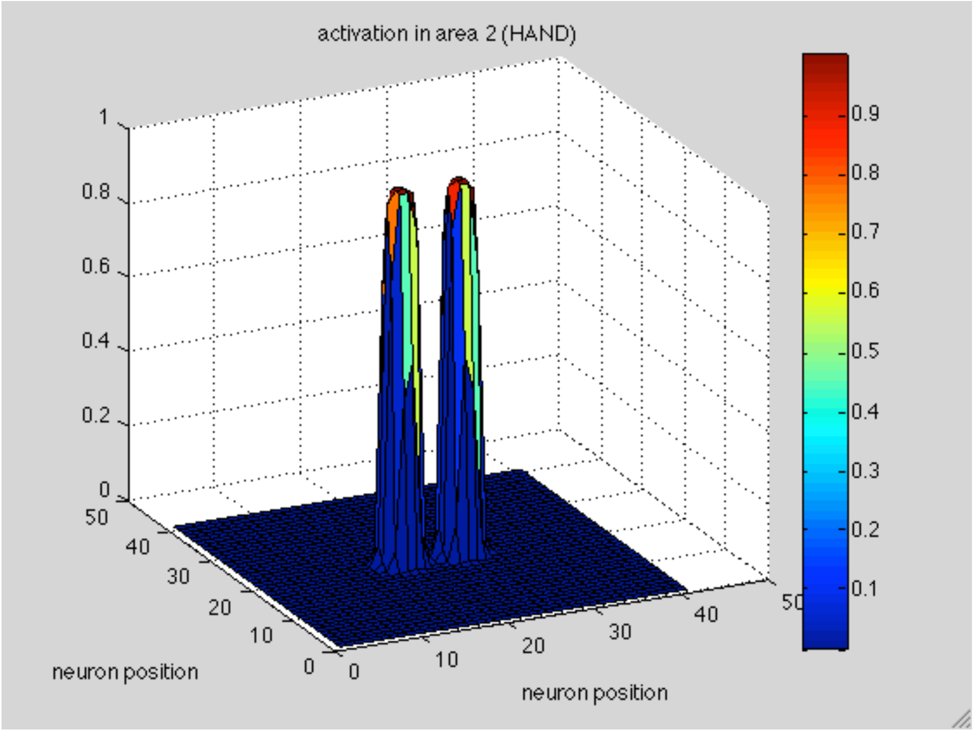


Figure 3. 12 *Hand: distance stimuli on the skin equal to 0.8 cm.*

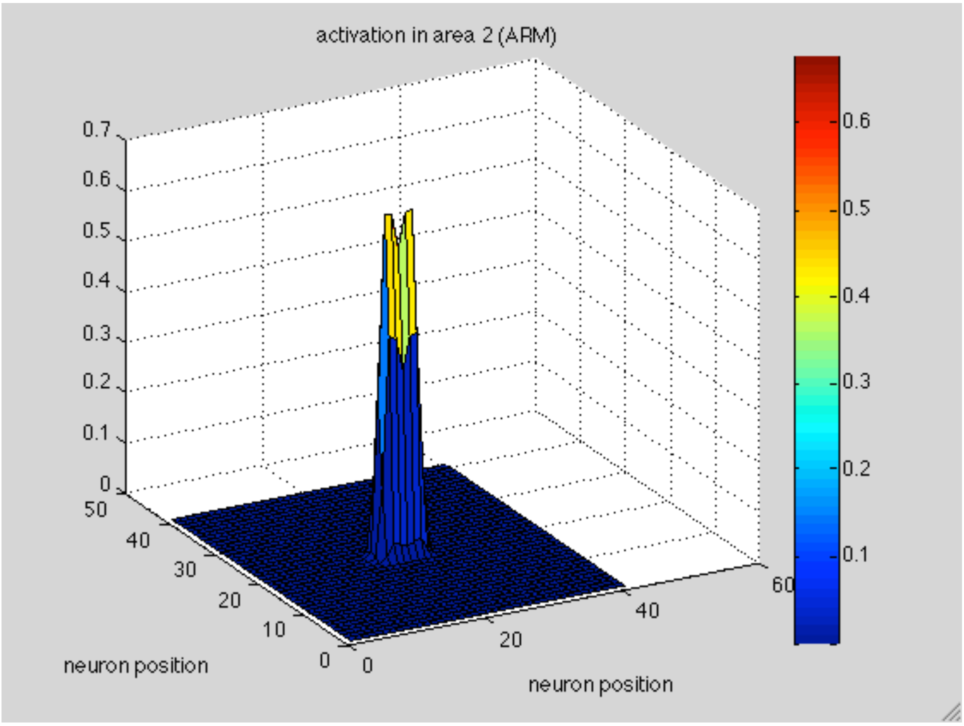


Figure 3. 13 *Arm: distance stimuli on the skin equal to 0.8 cm.*

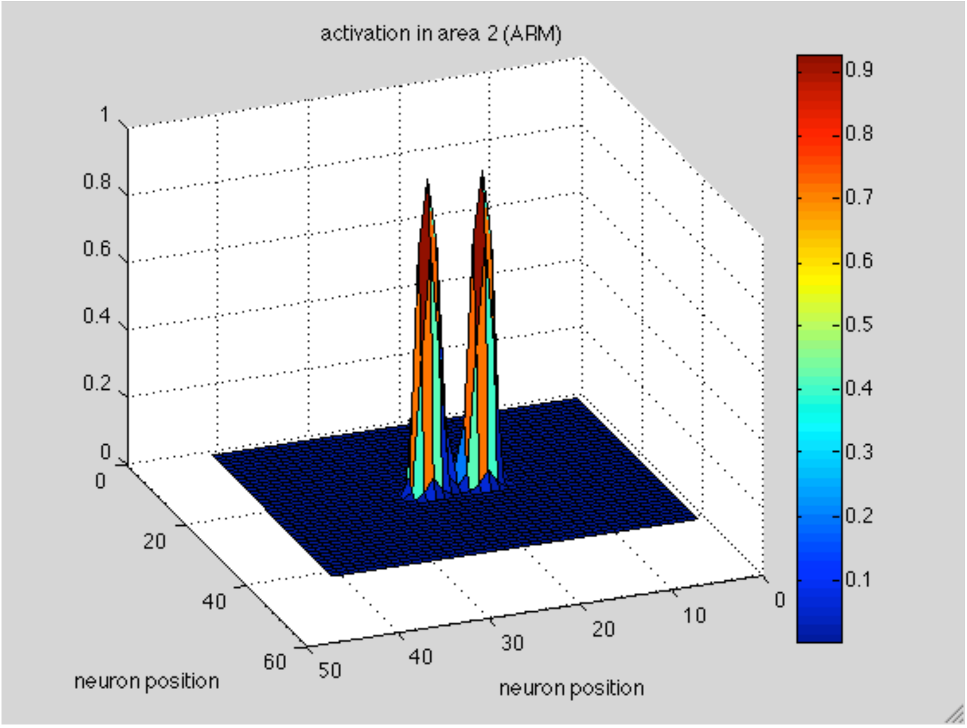


Figure 3. 14 ***Arm: distance stimuli on the skin equal to 1.4 cm.***

These three 3D graphics show the same results of the three preview graphics (figure 3.9, figure 3.10 and figure 3.11); it is just a different point of view of the second layer.

We can notice a significant difference between Hand and Arm, in terms of activation patterns. In the Hand's case, the two balls of activation are quite big, and they have a high number of activated neurons excited over the activation threshold (0.9). Whereas, in the Arm's case, the situation is different: the two activation balls are much more smaller than the hand's case, and as we can notice by the colour, only one neuron (the central one) is excited over the activation threshold. So, a spontaneous question arises: why, is it present such kind of difference? The answer is in the pattern of the synapses used in the two body regions. In fact, the arm is characterized by a strong presence of inhibitory synapses attempting to increase the resolution of this area. These strong inhibitory synapses are not present in the model simulating Hand region. Instead, these kind of strong inhibitory synapses in the Arm region lead to minimize the dimension of the bubbles, resulting in a smaller activation pattern in Area 2.

However, in this project we did not give importance to the dimensions of the activation balls, considering only the number of inactivated neurons between the two balls. As reported in figure 3.7 the number of neurons between the bubbles, are:

- **5 in the Hand's case**.
- **3 in the Arm's case**.

I have conducted the same study on the **first layer**, and the results are shown below:

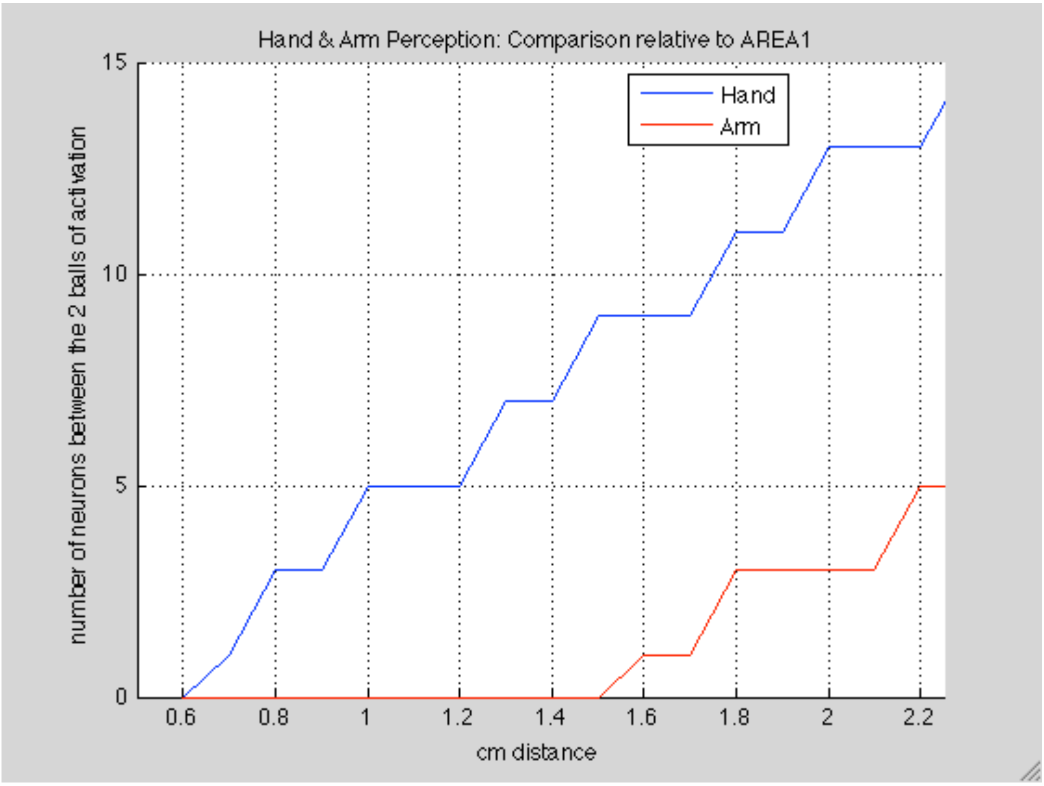


Figure 3. 15 ***Hand and Arm perception within "Area 2".***

Thereby, results about the *two points discrimination threshold* are:

- **0.7 cm** on the skin surface of the Hand.
- **1.6 cm** on the skin surface of the Arm.

The next graphics will show the same results collected per Arm and Hand:

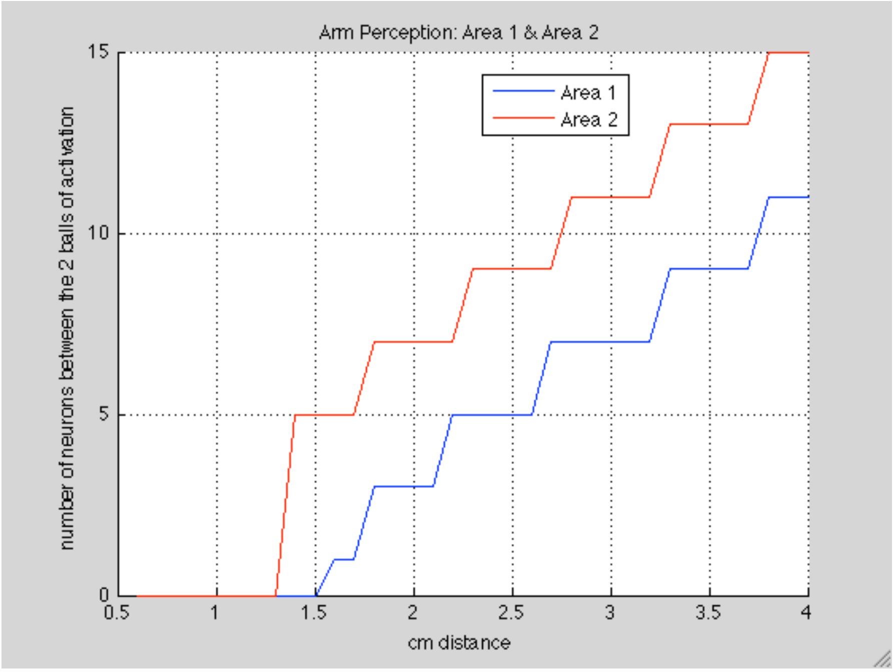


Figure 3. 16 *Arm perception for "Area 1" and "Area 2".*

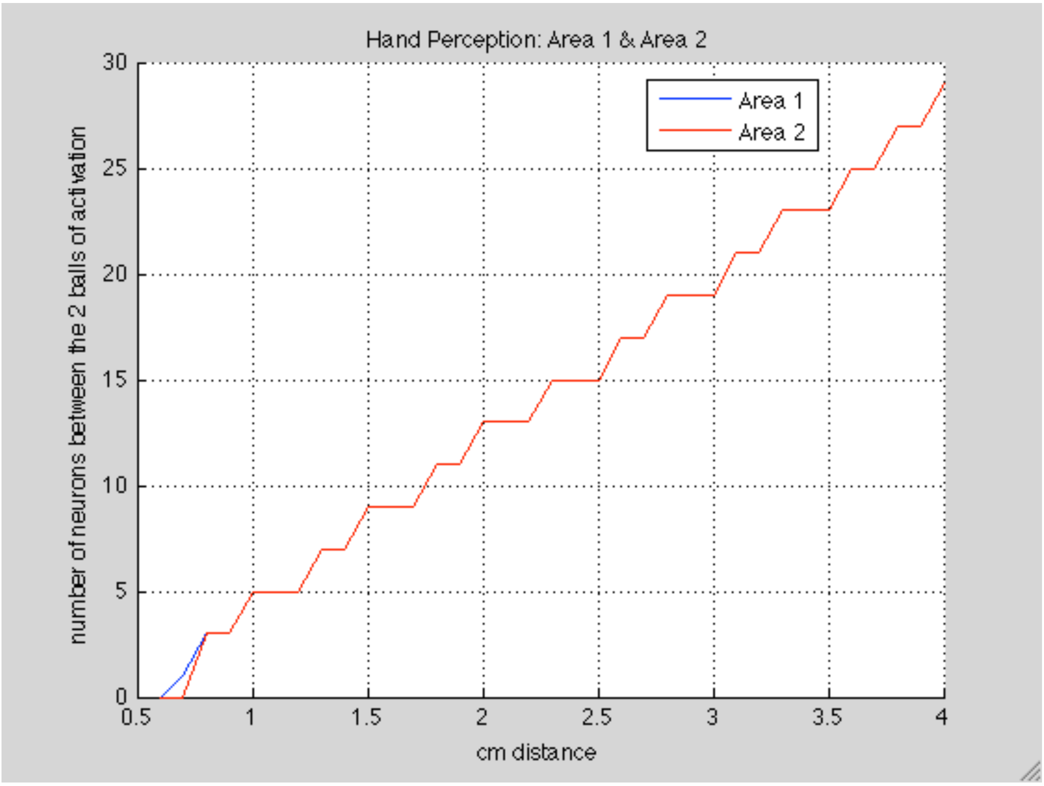


Figure 3. 17 *Hand perception for "Area 1" and "Area 2".*

The trend of the Hand is the same in both the first layer, and the second layer, whereas the trend of the Arm changes. This changing is due to the rescaling process effect. For example, in the Arm's case we can see that a distance of 4 cm is equivalent to 11 neurons in the "First Layer" and 15 neurons in the "Second Layer". In addition, the "*two points discrimination threshold*" of the Arm decrease from Area 1 (1.6 cm) to Area 2 (1.4 cm): so, this is another result confirming that the rescaling process of the network works in order to increase the resolution on the arm.

It is interesting to observe that the trend of the first layer of the hand is about the same of the second layer (Area 2), but the threshold of perception is 0.1 cm smaller in "Area 1" with respect to "Area 2". This is acceptable even if they should be equal, because we have constructed the network in order to maintain the high resolution of the "First Layer" of the hand even in its "Second Layer". But due to the nature of the feed-forward synapses (remember that they have a wake inhibitory component), a sort of lightly enlargement of the balls of activation is always present, leading in the attachment of the balls for very close stimuli (like 0.7 cm) within "Area 2". Therefore, this variance of the two thresholds concerning the hand is acceptable.

It is clear that to construct these kinds of graphics we have implemented a simulation with stimuli of different distances, ranging from 0.3 cm to 4 cm, and then we have recorded the outputs. In the next figures (figure 3.18 and figure 3.19) we can see the interpolation of the output of the neural network about the second layer (Area 2) and the first layer (Area 1). The slope of each line was computed by MATLAB's function called *polyval.m*, and the results of the slopes are reported in the table below:

	Slope Area 2	**Slope Area 1**
Hand	*8.01*	*7.97*
Arm	*4.81*	*3.67*

Table 3. 10 ***Slope for the Hand and slope for the Arm.***

The interpolation in the first layer is shown below:

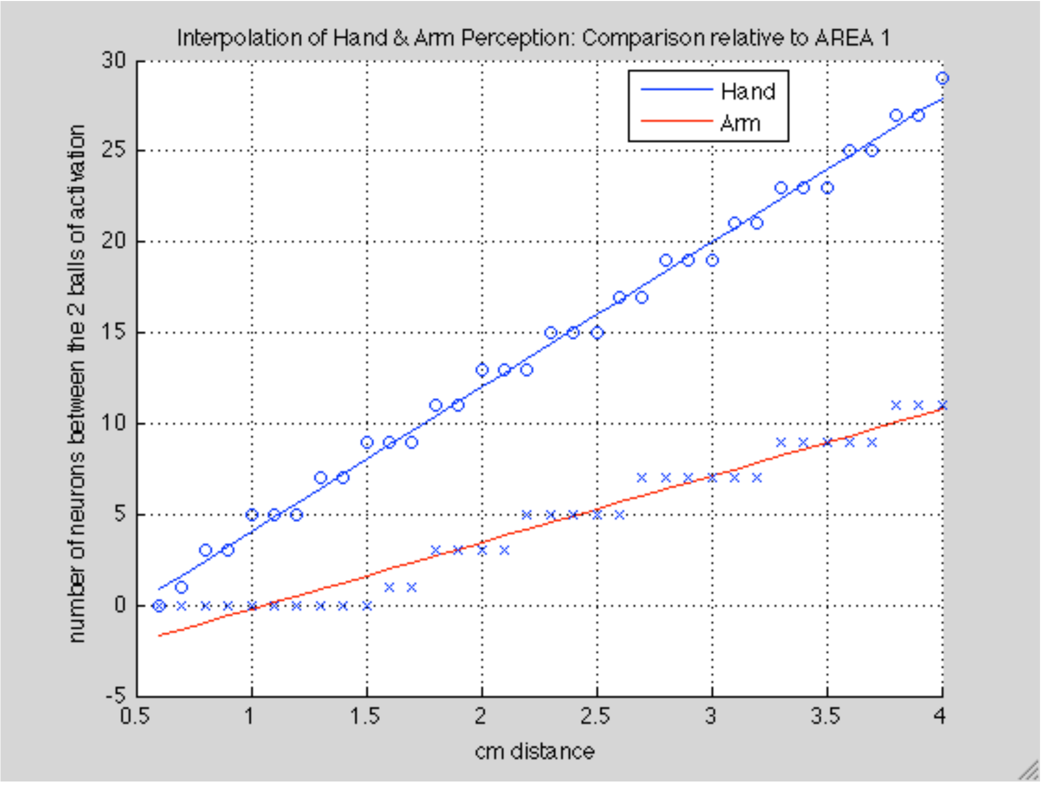


Figure 3. 18 ***Interpolation for "Area 1".***

Whereas for the second layer the interpolation is this one:

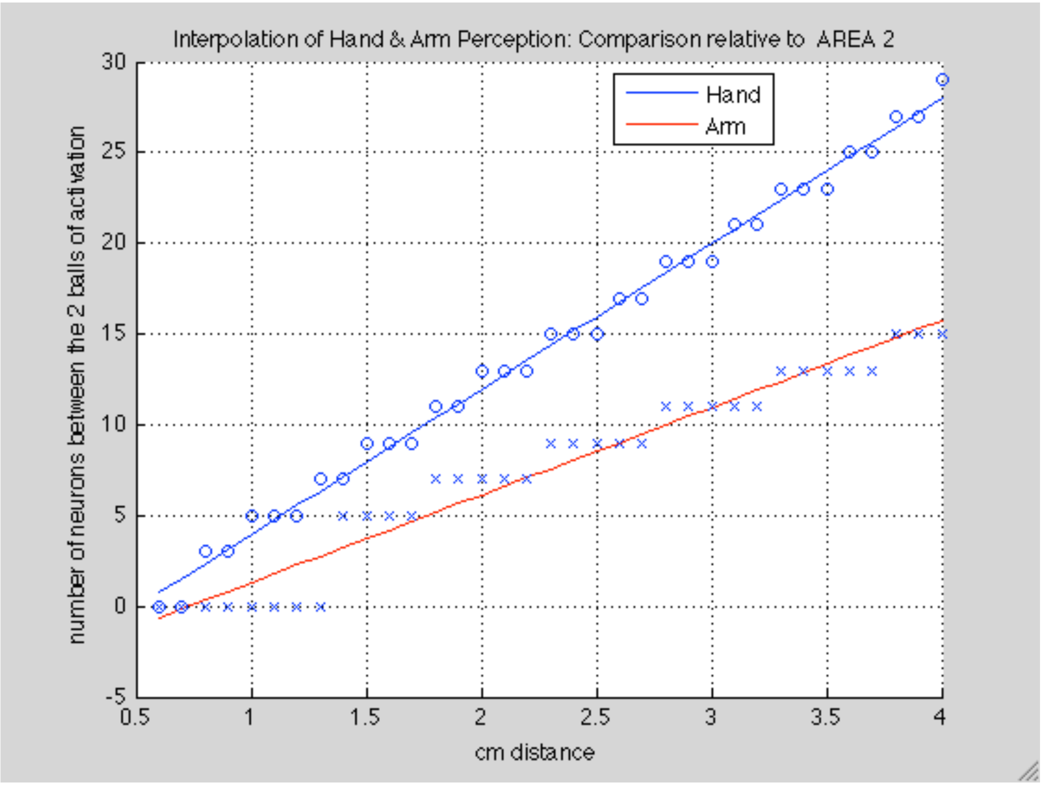


Figure 3. 19 ***Interpolation for "Area 2".***

It is evident that the red line (Arm) tends to increase its slope in the passage from Area 1 to Area 2..

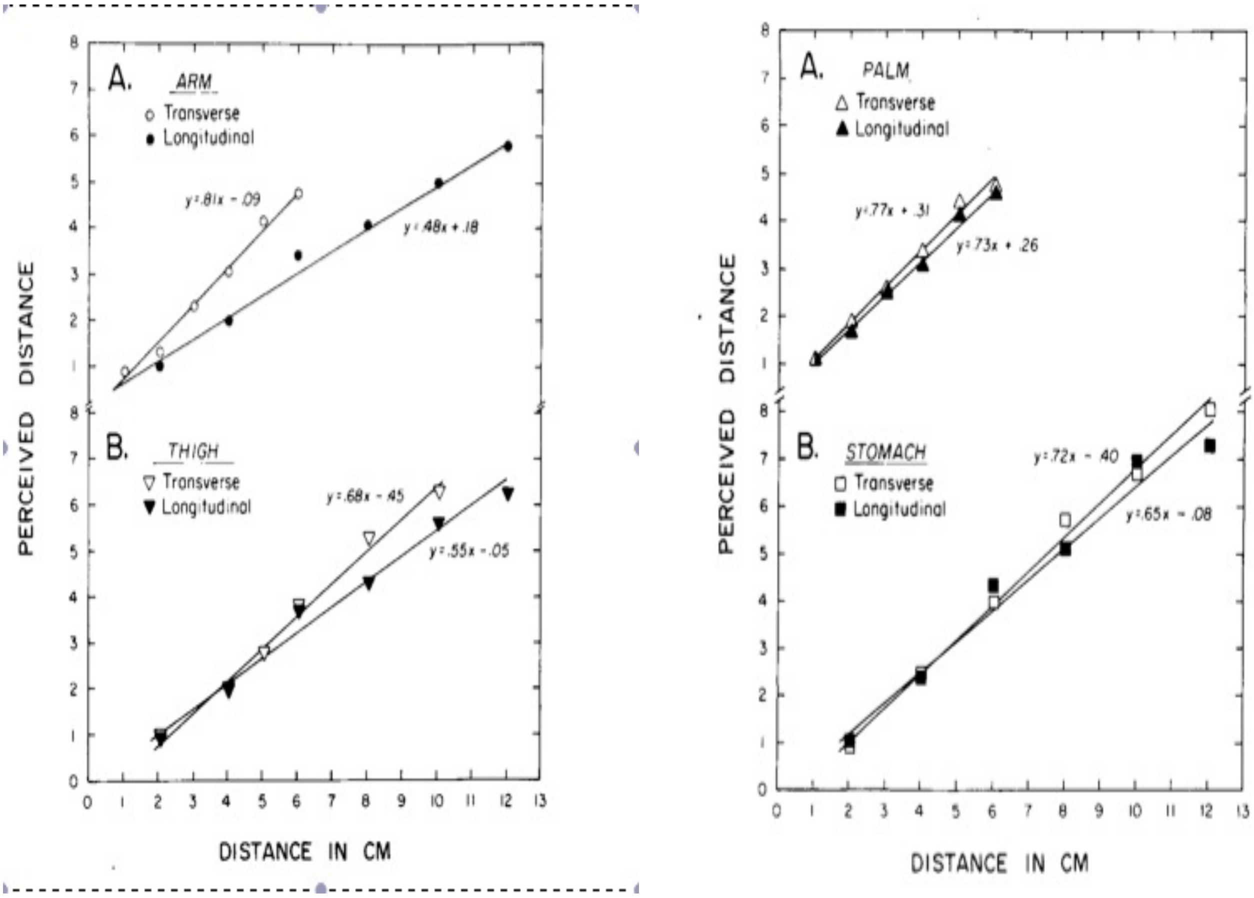


Figure 3. 20 *Green's Experiment results about the perceived distance.*

3.3.2 Comparison with Green's results

In the following, results of the Green's Experiment will be shown again (I have already talked about this experiment in chapter 1). In the Green's experiment subjects were asked to assign numbers to represent the apparent distance between two simultaneous tactile stimuli. This was an experiment conducted on real subjects, so we have decided to compare **outputs concerning the second layers of the network**, with **Green's experiment results**.

The legend "*Transvers*" and "*Longitudinal*" indicates that the stimuli were oriented in the transversal direction in one experiment, and in the longitudinal direction in another experiment. This means that apart distortions involved on different body regions, there are also distortions in the perception of stimuli distances based on their orientation (called **anisotropy**). However, anisotropy was not considered in the construction of the neural network. An interesting comparison with these data, and the simulation data, is about the slope of the lines. Regarding Green's data, an average slope between the transversal and longitudinal direction, both for Hand and Arm, have been computed. Therefore:

$$GEslope_{hand} = \frac{0.77 + 0.73}{2} = \mathbf{0.75} \qquad GEslope_{arm} = \frac{0.81 + 048}{2} = \mathbf{0.645}$$

Instead, the slopes of the neural network concerning the second layer:

$$NNslope_{hand} = \mathbf{8.01} \qquad NNlope_{arm} = \mathbf{4.81}$$

At the end:

$$\boldsymbol{NNslope_{hand}} > \boldsymbol{NNlope_{arm}} \qquad \boldsymbol{GEslope_{hand}} > \boldsymbol{GEslope_{arm}}$$

So, in the Green's experiment the Hand's slope is greater than the Arm's slope, as it happens for the slopes of the neural network. Hence, there is a qualitative agreement between model results and experimental results. This experimental results reproduced by the model indicate that the Weber's illusion increases as the stimulus becomes larger

It should be clear the reason why we have compared only the Second Layer's results, and not also outputs from the "First Layer": clearly, the key point is that the second layer of the neural network is the one involved in the perception and in the judgment of the distance between two punctual stimuli. So, if we want to associate the network to a virtual subject, his virtual "judgment" should come from the "Second Layer", because we have hypnotized "Area 2" as a **High Cortical Area**. Instead, we have thought to "Area 1" as the **primary somatic sensory cortex,** whose output is further elaborated by higher-level areas to produce the final perceptual judgment of distance. .

3.3.3 Rescaling Process results

The effect of the rescaling process can be appreciated in a better way with the next two graphics:

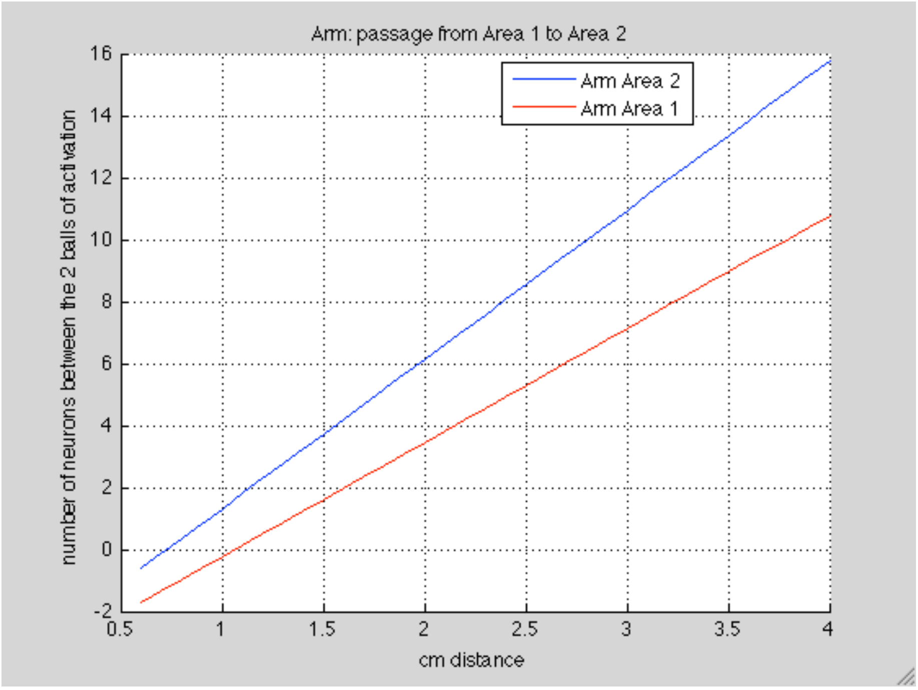


Figure 3. 21 ***Arm: passage from "Area 1" to " Area 2".***

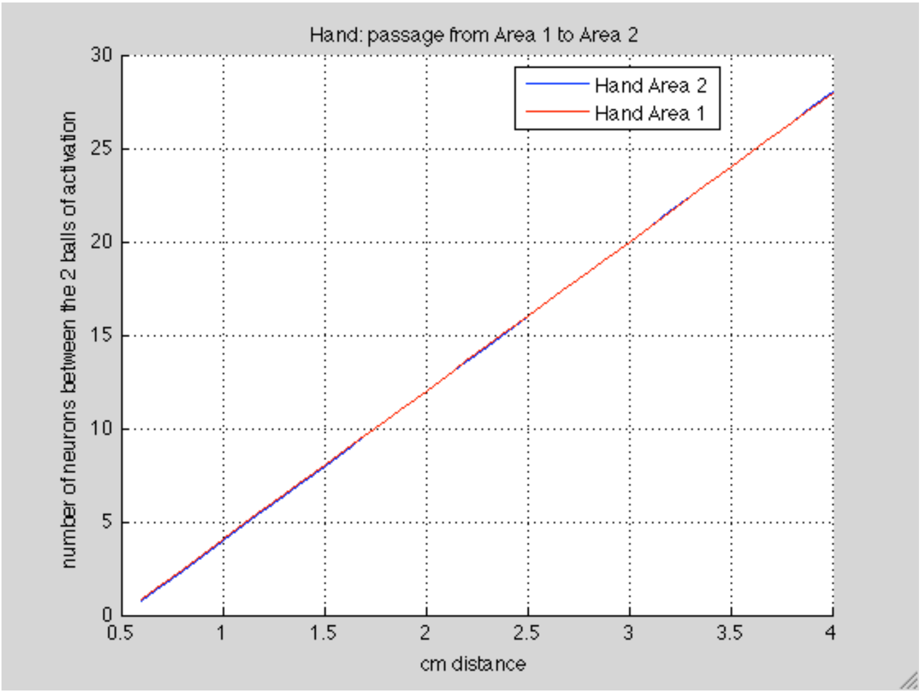


Figure 3. 21 ***Hand: passage from "Area 1" to " Area 2".***

In the Hand's case there is not a changing in the passage from the first layer to the second layer, due to the nature of Hand's neural network to maintain its high resolution; in fact the two lines are superimposed. Whereas, in the Arm's case, the passage from "Area 1" to "Area 2" is marked with an evident change of the slope. For example, 4 cm on the Arm are equivalents to 11 neurons in the first layer, and about 16 neurons in the second; whereas at 1.5 cm the situation is different: 2 neurons in the first layer and 4 neurons in the second one. In short:

- **Absolute difference at 4 cm = 5 neurons**;
- **Absolute difference at 1.5 cm = 2 neurons;**

Now a crucial aspect is to understand if the rescaling process is the same for every distance or it depends on the dimension of the distances in input. Focusing on **4 cm** and **2 cm**, we had collected these data in terms of numbers of neurons:

Hand		
	2 cm	4 cm
Area 1	*13*	*29*
Area 2	*13*	*29*

Arm		
	2 cm	4 cm
Area 1	*3*	*11*
Area 2	*7*	*15*

Table 3. 11 ***Distancse in terms of number of neurons for stimuli distances equals to 2 cm, and 4 cm.***

Computing the ratios to understand the entity of the rescaling process:

<u>Case 2 cm:</u>

Area 1 → $\frac{2cm\ Hand}{2cm\ Arm} = \frac{13}{3} = 4.3$ $\qquad \log_{10}(4.3) = 1.46$

Area 2 → $\frac{2cm\ Hand}{2cm\ Arm} = \frac{13}{7} = 1.85$ $\qquad \log_{10}(1.85) = 0.62$

Case 4cm:

$$\text{Area 1} \rightarrow \frac{4cm\ Hand}{4cm\ Arm} = \frac{29}{11} = 2.63 \qquad \log_{10}(2.63) = 0.97$$

$$\text{Area 2} \rightarrow \frac{4cm\ Hand}{4cm\ Arm} = \frac{29}{15} = 1.93 \qquad \log_{10}(1.93) = 0.65$$

	Case 2cm	Case 4cm
Absolute difference between “ratio Area 1” and “ratio Area 2”	**2.98**	**0.7**
Absolute difference between log “ratio Area 1” and “log ratio Area” 2	**0.84**	**0.32**

Table 3. 12 ***Entity of the Rescaling.***

In each case the ratio tends to decrease due to the presence of the rescaling process, but the entity of the decrement is different: it is greater in case of 2 cm than case of 4 cm. Thereby, in base on the neural network’s results, we can assert that the rescaling process is strongest for small distances, and it is weak for big distances. So there is an **inverse proportion** between the **distances** and the entity of the **Rescaling**. At the same time, going back to the Green’s Experiment results (figure 3.20), we can see that for big distances the Weber’s Illusion tends to increase due to the different slope of the Hand and the Arm: in particular the Hand’s slope is higher than the Arm’s slope, hence the two lines will diverge for big distances.

A possible explanation of the *Illusion Increment* might be given by these results, because, the point is that for big distances the “Rescaling Process” is going to become weak, allowing an increment of the Weber’s Illusion. Hereby, we do not want to assert that this is the real reason for the increment of the Weber’s Illusion, even because there are not experimental evidences to validate this theory. However, the neural network gives in output these kinds of results, and this theory seems congruent to explaining the *Illusion Increment*.

3.4 Fourth Experiment

3.4.1 Dependency on the stimuli dimension

Distances of the stimuli utilized in each simulation and trial until now, were selected in a random way, just paying attention to the ratio (distance stimuli hand/distance stimuli arm). But during the simulation we have noticed that the results were afflicted by a dependence on the distance stimuli. In particular, consider the case below, in which we have two different types of stimuli distances in cm:

Mean 3 (cm)			Mean 4 (cm)	
Distance Hand	Distance Arm	ratio	Distance Hand	Distance Arm
1.71	4.29	0.4	2.29	5.71
1.86	4.14	0.45	2.48	5.52
2.00	4.00	0.5	2.67	5.33
2.13	3.87	0.55	2.84	5.16
2.25	3.75	0.6	3.00	5.00
2.36	3.64	0.65	3.15	4.85
2.47	3.53	0.7	3.29	4.71
2.67	3.33	0.8	3.56	4.44
2.84	3.16	0.9	3.79	4.21
3.00	3.00	1	4.00	4.00

Table 3. 13 ***Inputs Stimuli Distances utilzied.***

The first column is called "Mean 3" because for every ratio, the average between the stimuli distance on the Hand and the stimuli distance on the Arm is always 3 cm. The second column is called "Mean 4" since the average value is equal 4 cm. In short, the distances were selected in order to obtain different ratios, but keeping the same average distance between each couple of stimuli. We have created these two different kinds of inputs, in order to understand, how different distances could affect the results obtained until now. Running the simulation, first with "Mean 3" as input we have obtained these results: (remember that every ratio Hand/Arm was applied 100 times, with the presence of Gaussian noise):

Mean 3 cm					
Distance Hand (cm)	Distance Arm (cm)	ratio	average (cm)	Proportion hand stimuli judge bigger in Area 1	Proportion hand stimuli judge bigger in Area 2
1.71	4.29	0.4	3.00	0.02	0
1.86	4.14	0.45	3.00	0.3	0
2.00	4.00	0.5	3.00	0.6	0.07
2.13	3.87	0.55	3.00	0.82	0.15
2.25	3.75	0.6	3.00	0.97	0.38
2.36	3.64	0.65	3.00	1	0.67
2.47	3.53	0.7	3.00	1	0.85
2.67	3.33	0.8	3.00	1	0.98
2.84	3.16	0.9	3.00	1	0.99
3.00	3.00	1	3.00	1	1
PSE				*0.48*	*0.62*

Table 3. 14 ***Results of simulation "Mean 3".***

Whereas the results for "Mean 4" are in table below:

Mean 4 cm					
Distance Hand (cm)	Distance Arm (cm)	ratio	average (cm)	Proportion hand stimuli judge bigger in Area 1	Proportion hand stimuli judge bigger in Area 2
2.29	5.71	0.4	4.00	0	0
2.48	5.52	0.45	4.00	0.25	0
2.67	5.33	0.5	4.00	0.61	0.05
2.84	5.16	0.55	4.00	0.9	0.27
3.00	5.00	0.6	4.00	0.97	0.53
3.15	4.85	0.65	4.00	1	0.94
3.29	4.71	0.7	4.00	1	0.97
3.56	4.44	0.8	4.00	0.99	1
3.79	4.21	0.9	4.00	0.98	1
4.00	4.00	1	4.00	0.97	1
PSE				*0.48*	*0.59*

Table 3. 15 ***Results of simulation "Mean 4".***

To validate these results, a "**paired samples t-Test**" was implemented, simulating the presence of 10 subjects, and applying on each subject the distances reported in table 3.13. Anyway, the implementation of this t-Test will be examined apart, inside paragraph 3.4.4.

As for the first simulation in the first experiment, these data were fitted with program ***R*** by Cumulative Gaussian Functions, and then they were validated performing a "paired t-Test". The results of this fitting are represented in the next two figures.

Mean 3:

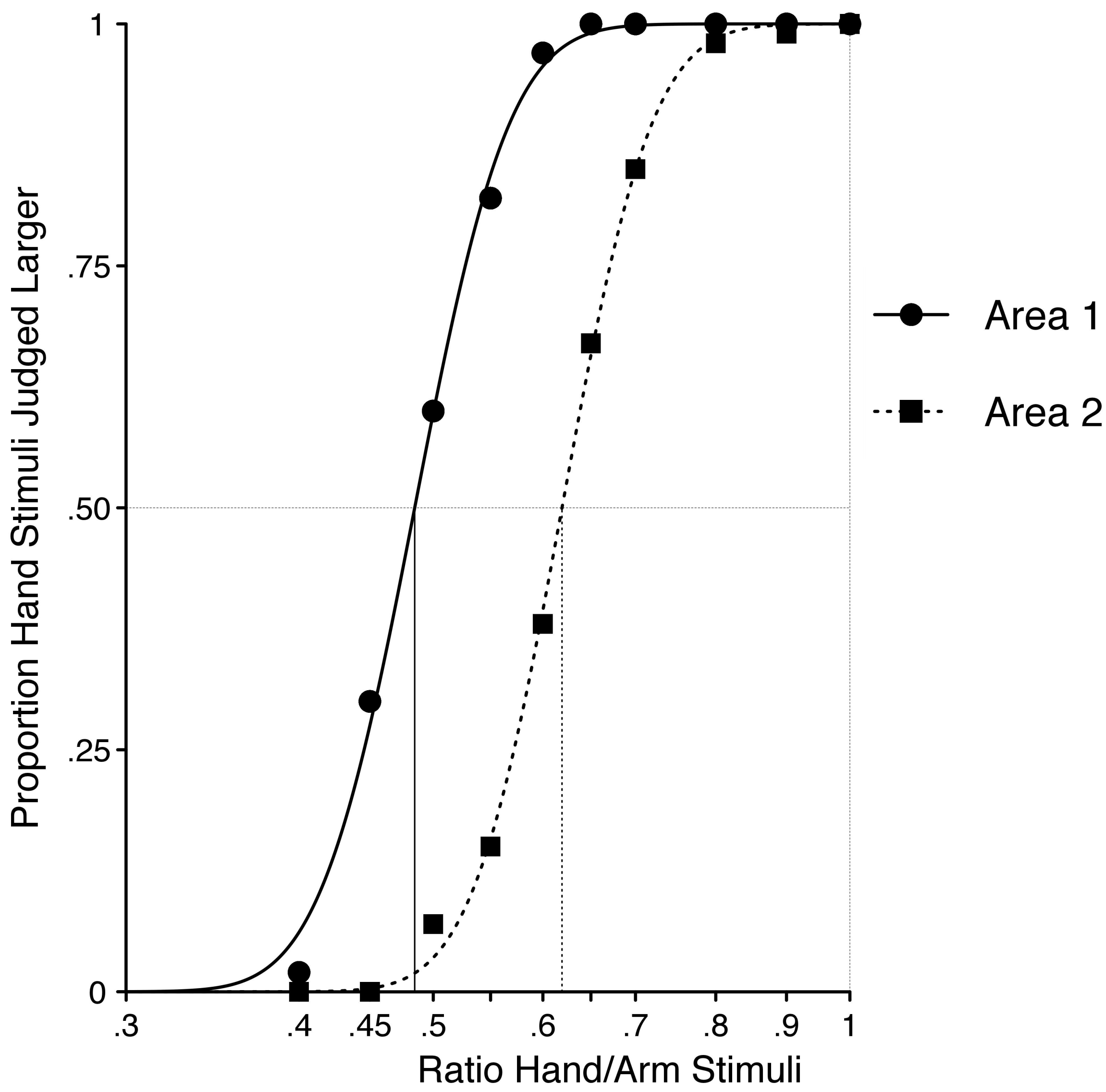


Figure 3. 22 ***Result of the "Mean 3" simulation.***

Mean 4:

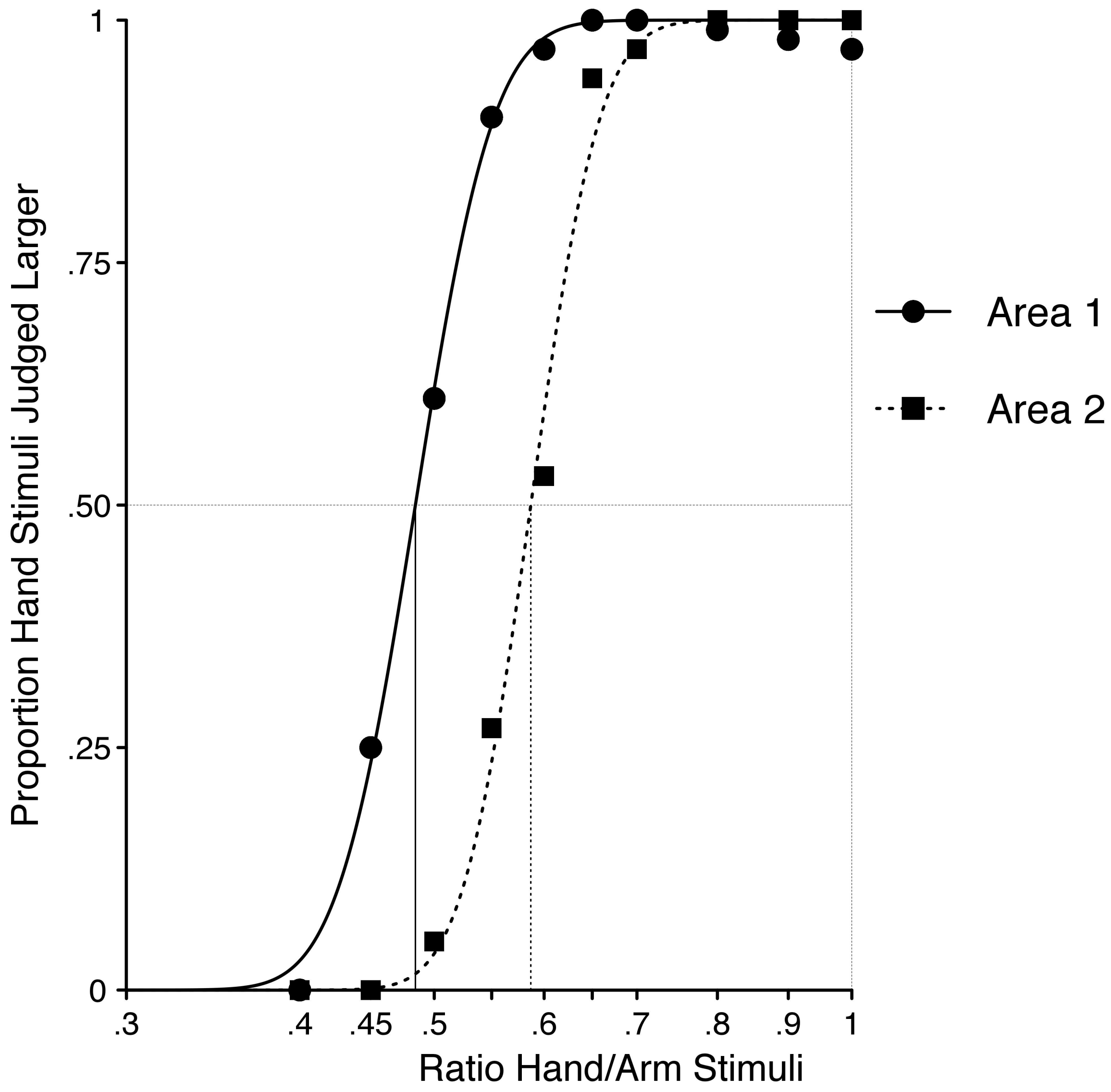


Figure 3. 23 ***Result of the "Mean 4" simulation.***

The main conclusion is about the entity of the Rescaling. Given that the curves relative to "Area 1" are the same, these graphics confirm the previously results about the inverse proportion between the dimensions of the stimuli, and the entity of the rescaling. In fact, as we can observe by the PSEs value of the second layers, in "Mean 3"'s case the PSE is positioned beyond ratio of 0.6, whereas for "Mean 4" is under 0.6., In other words, for biggest input (distances of "Mean 4") the rescaling is smaller, and for small input (distances of "Mean 3") the rescaling is grater.

In addition, in this simulation the **Average Absolute Difference (Avg) in terms of number of neurons between the two activation balls, concerning the passage from Area 1 to Area 2** have been recorded for each ratio:

Mean 3					
Distance Hand (cm)	Distance Arm (cm)	ratio	average (cm)	Avg Arm	Avg Hand
1.71	4.29	0.4	3.00	3.96	0.04
1.86	4.14	0.45	3.00	4	0.08
2.00	4.00	0.5	3.00	4.04	0.06
2.13	3.87	0.55	3.00	3.96	0.02
2.25	3.75	0.6	3.00	3.98	0.03
2.36	3.64	0.65	3.00	3.94	0.04
2.47	3.53	0.7	3.00	4.01	0.05
2.67	3.33	0.8	3.00	3.88	0
2.84	3.16	0.9	3.00	3.87	0.03
3.00	3.00	1	3.00	3.88	0.02

Table 3. 16 *Average Difference for the simulation "Mean 3".*

Mean 4					
Distance Hand (cm)	Distance Arm (cm)	ratio	average (cm)	Avg Arm	Avg Hand
2.29	5.71	0.4	4.00	4.01	0.01
2.48	5.52	0.45	4.00	4.05	0.01
2.67	5.33	0.5	4.00	4.06	0.04
2.84	5.16	0.55	4.00	4.06	0.03
3.00	5.00	0.6	4.00	4.07	0.01
3.15	4.85	0.65	4.00	4.05	0.04
3.29	4.71	0.7	4.00	4	0.03
3.56	4.44	0.8	4.00	3.99	0.3
3.79	4.21	0.9	4.00	3.98	0.63
4.00	4.00	1	4.00	3.92	0.86

Table 3. 17 *Average Difference for the simulation "Mean 3".*

The average is computed on 100 trials (remember that there are 100 trial per ratio). The average difference about the Hand is similar both for "Mean 4" and for "Mean 3".. Instead, the trend about the Arm is lightly different from "Mean 3" to "Mean 4", but always close to 4 neurons.

Anyway, after the collection of these data, I have implemented a comparison of the results between "Mean 3" and "Mean 4". The first comparison was between "Area 1" of "Mean 3" and "Area 1" of "Mean 4". I have fitted the data with a Cumulative Gaussian function, obtaining two sigmoidal curves that represent the outputs of "Area 1", for different kinds of stimuli distances (figure 3.25). I have done the same comparison among "Area 2"'s outputs (figure 3.26), and the all graphics results are reported in the next two figures:

- **Solid** curves represent the output with input average stimuli equal to **3 cm** (Mean 3).
- **Dotted** curves represent the output with input average stimuli equal to **4 cm** (Mean 4).

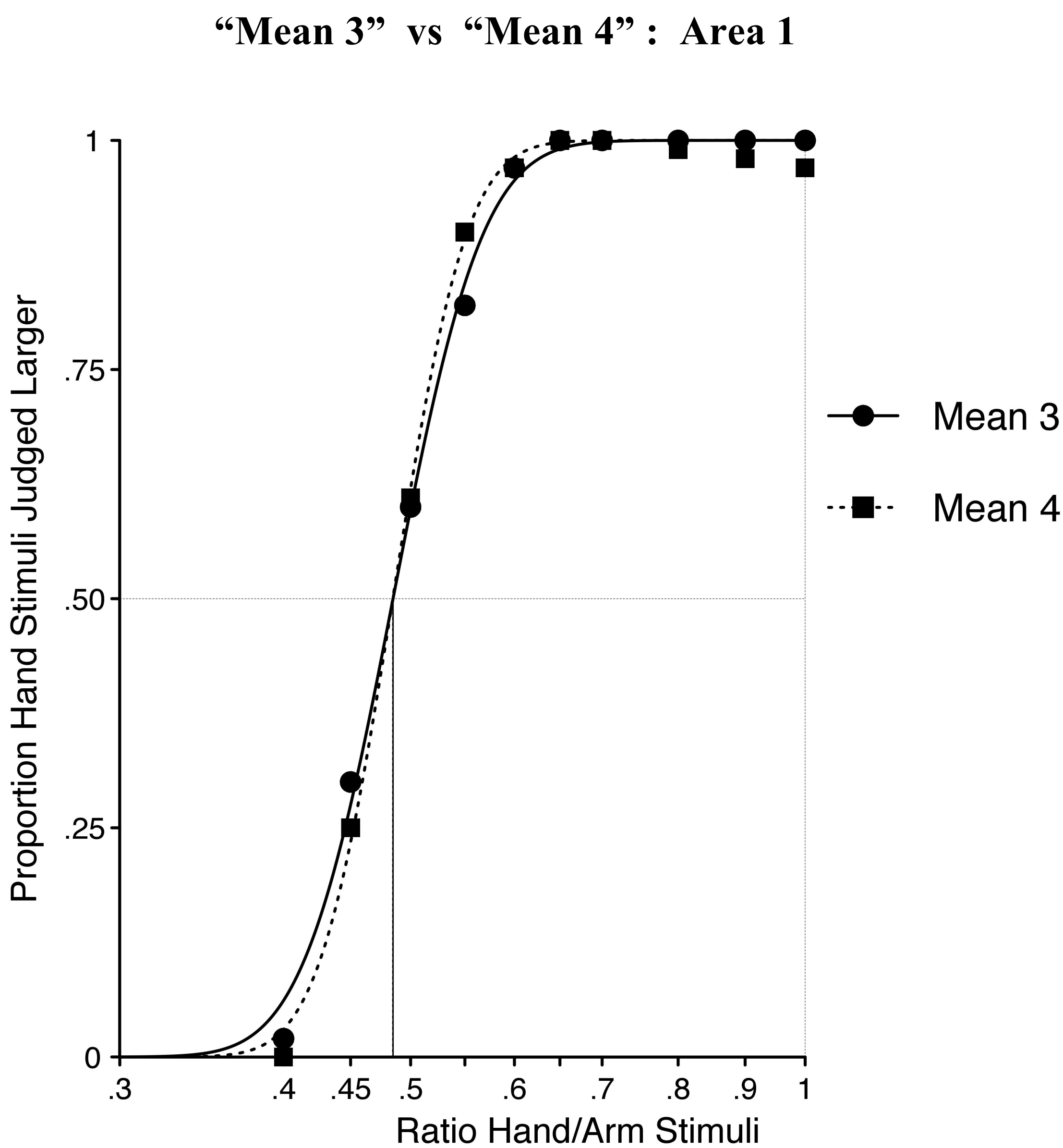


Figure 3. 24 ***Proportion Hand Stimuli judjed larger within Area 1.***

“Mean 3” vs “Mean 4” : Area 2

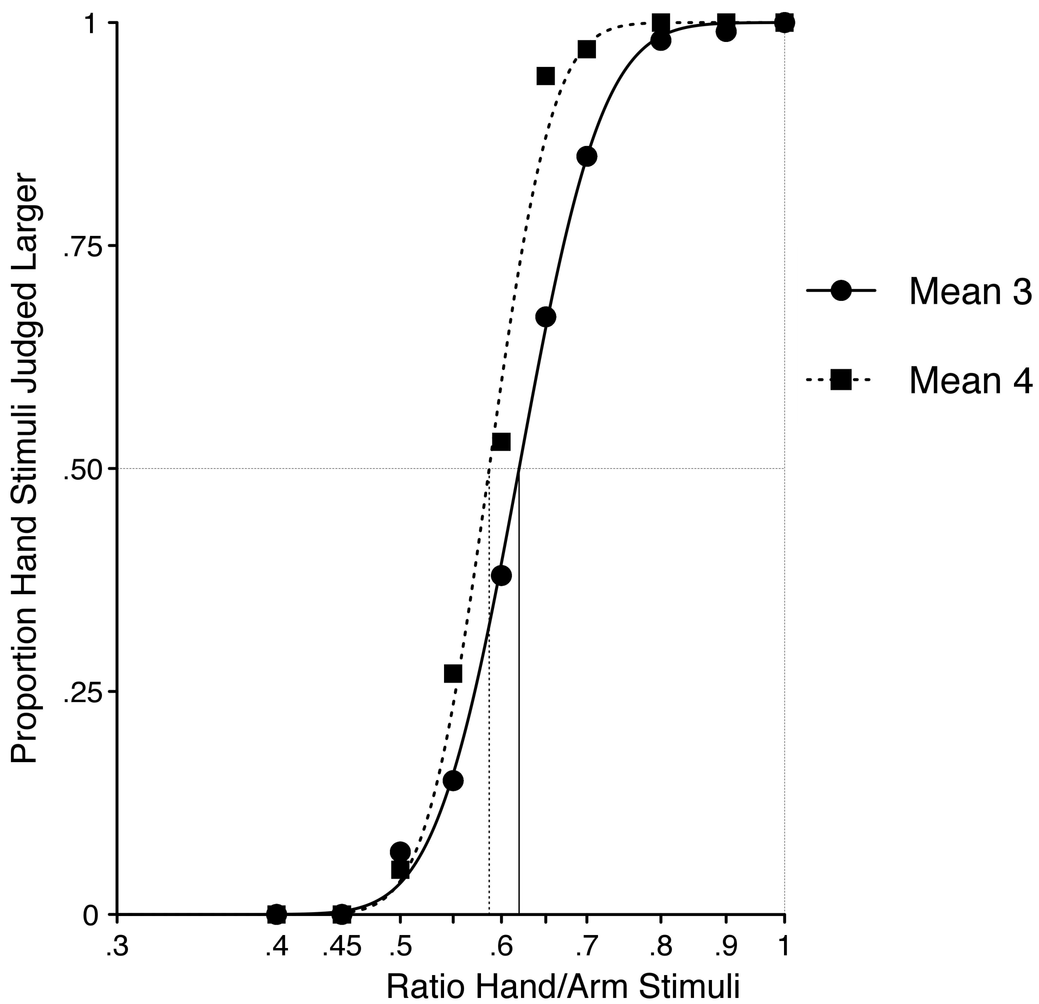


Figure 3. 25 ***Proportion Hand Stimuli judjed larger within Area 2.***

3.4.2 Analysing of Area1's output

	PSE	25%	75%
Mean 3: Area 1	0.48	0.45	0.53
Mean 4: Area 1	0.48	0.45	0,52

Table 3. 18 ***"Area 1" output about the PSE and slope.***

PSEs are equals as we can notice from the table (and also from figure 3.25), as well as for the slope that is very similar. We cannot conclude which curve has more bias, because they are completely superimposed in the centre of the sigmoid.

3.4.3 Analysing of Area2's output

	PSE	25%	75%
Mean 3: Area 2	0.62	0.57	0.67
Mean 4: Area 2	0.59	0.55	0.62

Table 3. 19 ***"Area 2" output about the PSE and slope.***

Figure 3.26 represents "**Area 2"'s output**, and we can see a light discrepancy among the two curves. In particular, the PSE of the curve "Mean 4" cm is lightly shifted on the left than the PSE of the curve "Mean 3". So the bias of the curve "Mean 4" is higher than curve "Mean 3". This is an important result, because it means that for big distances of the stimuli, the Weber's Illusion is bigger. Notice that this kind of trend was recorded by Green. Observing his results, it is clear that increasing the stimuli distance, the two curves (lines) going to diverge. Therefore, if the diverging increase, it means that the distortion is grater step-by-step and the Weber's Illusion increase as well.

Finally, we can assert that the results of the neural network, and the results of the Green's experiment are in qualitative agreement.

3.4.4 Validation with t-Test

Results about "Mean 3" and "Mean 4" have been submitted to a t-Test with the target to validate them. As in the second experiment, we have supposed to conduct the experiment on 10 virtual subjects, applying "Mean 3"'s stimuli and "Mean 4"'s stimuli. Results of these simulations are reported in the tables below:

Mean 3: Proportion Hand Stimuli judge larger within the SECOND layer.

Distance (cm) Hand/ Arm	Ratio	*Subject 1*	*Subject 2*	*Subject 3*	*Subject 4*	*Subject 5*
1.71/4.29	0.4	0	0	0	0	0
1.86/4.14	0.45	0	0	0	0	0
2/4	0.5	0	0.2	0.1	0.1	0.1
2.13/3.87	0.55	0	0.2	0.1	0.2	0
2.25/3.75	0.6	0.4	0.3	0.4	0.3	0.8
2.36/3.64	0.65	0.6	0.5	0.7	0.8	0.7
2.47/3.53	0.7	0.9	0.8	0.9	0.9	0.6
2.67/3.33	0.8	1	1	0.9	1	1
2.84/3.16	0.9	0.9	1	1	1	1
3/3	1	1	1	1	1	1

Distance (cm) Hand/ Arm	Ratio	*Subject 6*	*Subject 7*	*Subject 8*	*Subject 9*	*Subject 10*
1.71/4.29	0.4	0	0	0	0	0
1.86/4.14	0.45	0	0	0	0	0
2/4	0.5	0	0.1	0	0.1	0
2.13/3.87	0.55	0.2	0.1	0.1	0.3	0.3
2.25/3.75	0.6	0.3	0.4	0.4	0.2	0.3
2.36/3.64	0.65	0.8	0.6	0.6	0.7	0.7
2.47/3.53	0.7	1	0.9	0.7	1	0.8
2.67/3.33	0.8	1	0.9	1	1	1
2.84/3.16	0.9	1	1	1	1	1
3/3	1	1	1	1	1	1

Table 3. 20 *Simulation on ten virtual subjects with "Mean 3" input stimuli: results about "Area2".*

Mean 3: Proportion Hand Stimuli judge larger within the FIRST layer.

Distance (cm) Hand/ Arm	Ratio	*Subject 1*	*Subject 2*	*Subject 3*	*Subject 4*	*Subject 5*
1.71/4.29	0.4	0	0	0	0.1	0
1.86/4.14	0.45	0.3	0.3	0.2	0.2	0.6
2/4	0.5	0.6	0.5	0.6	0.6	0.8
2.13/3.87	0.55	0.8	0.8	0.6	0.9	0.5
2.25/3.75	0.6	1	1	1	0.9	1
2.36/3.64	0.65	1	1	1	1	1
2.47/3.53	0.7	1	1	1	1	1
2.67/3.33	0.8	1	1	1	1	1
2.84/3.16	0.9	1	1	1	1	1
3/3	1	1	1	1	1	1

Distance (cm) Hand/ Arm	Ratio	*Subject 6*	*Subject 7*	*Subject 8*	*Subject 9*	*Subject 10*
1.71/4.29	0.4	0.1	0	0	0	0
1.86/4.14	0.45	0.5	0.3	0.3	0.1	0.2
2/4	0.5	0.6	0.6	0.7	0.7	0.3
2.13/3.87	0.55	1	1	0.9	0.8	0.9
2.25/3.75	0.6	0.9	1	0.9	1	1
2.36/3.64	0.65	1	1	1	1	1
2.47/3.53	0.7	1	1	1	1	1
2.67/3.33	0.8	1	1	1	1	1
2.84/3.16	0.9	1	1	1	1	1
3/3	1	1	1	1	1	1

Table 3. 21 *Simulation on ten virtual subjects with "Mean 3" input stimuli: results about "Area1".*

Mean 4: Proportion Hand Stimuli judge larger within the SECOND layer

Distance (cm) Hand/ Arm	Ratio	*Subject 1*	*Subject 2*	*Subject 3*	*Subject 4*	*Subject 5*
2.29/5.71	0.4	0	0	0	0	0
2.48/5.52	0.45	0	0	0	0	0
2.67/5.33	0.5	0.1	0	0.1	0	0.1
2.84/5.16	0.55	0.2	0.2	0.6	0.4	0.3
3/5	0.6	0.5	0.6	0.5	0.6	0.6
3.15/4.85	0.65	0.9	1	0.9	1	0.9
3.29/4.71	0.7	0.9	1	0.9	1	1
3.56/4.44	0.8	1	1	1	1	1
3.79/4.21	0.9	1	1	1	1	1
4/4	1	1	1	1	1	1

Distance (cm) Hand/ Arm	Ratio	Subject 6	Subject 7	Subject 8	Subject 9	Subject 10
2.29/5.71	0.4	0	0	0	0	0
2.48/5.52	0.45	0	0	0	0	0
2.67/5.33	0.5	0	0.1	0	0.1	0
2.84/5.16	0.55	0.3	0.2	0.1	0.2	0.2
3/5	0.6	0.4	0.4	0.6	0.5	0.6
3.15/4.85	0.65	1	1	0.9	1	0.8
3.29/4.71	0.7	1	1	1	0.9	1
3.56/4.44	0.8	1	1	1	1	1
3.79/4.21	0.9	1	1	1	1	1
4/4	1	1	1	1	1	1

Table 3. 22 *Simulation on ten virtual subjects with "Mean 4" input stimuli: results about "Area 2".*

Mean 4: Proportion Hand Stimuli judge larger within the FIRST layer.

Distance (cm) Hand/ Arm	Ratio	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5
2.29/5.71	0.4	0	0	0	0	0
2.48/5.52	0.45	0.4	0.1	0.3	0.2	0.3
2.67/5.33	0.5	0.4	0.6	0.5	0.7	0.6
2.84/5.16	0.55	0.9	0.9	0.9	0.8	1
3/5	0.6	1	1	1	1	1
3.15/4.85	0.65	1	1	1	1	1
3.29/4.71	0.7	1	1	1	1	1
3.56/4.44	0.8	1	1	1	1	0.9
3.79/4.21	0.9	1	0.9	1	1	1
4/4	1	1	1	0.9	1	1

Distance (cm) Hand/ Arm	Ratio	Subject 6	Subject 7	Subject 8	Subject 9	Subject 10
2.29/5.71	0.4	0	0	0	0	0
2.48/5.52	0.45	0.3	0.3	0.2	0.1	0.3
2.67/5.33	0.5	0.6	0.6	0.7	0.8	0.6
2.84/5.16	0.55	1	0.9	0.8	0.9	0.9
3/5	0.6	1	0.8	1	0.9	1
3.15/4.85	0.65	1	1	1	1	1
3.29/4.71	0.7	1	1	1	1	1
3.56/4.44	0.8	1	1	1	1	1
3.79/4.21	0.9	1	0.9	1	1	1
4/4	1	0.9	1	1	1	0.9

Table 3. 23 *Simulation on ten virtual subjects with "Mean 4" input stimuli: results about "Area 1"*

Using **R**, we have extrapolated PSEs values both for “Mean 3” and “Mean 4” stimuli:

Mean 3		
Subject	*PSE Second Layer*	*PSE First Layer*
1	0.63	0.49
2	0.63	0.49
3	0.62	0.5
4	0.61	0.49
5	0.6	0.47
6	0.61	0.46
7	0.62	0.48
8	0.63	0.48
9	0.62	0.49
10	0.62	0.51

Mean 4		
Subject	*PSE Second Layer*	*PSE First Layer*
1	0.59	0.49
2	0.59	0.49
3	0.57	0.49
4	0.57	0.48
5	0.58	0.48
6	0.59	0.48
7	0.6	0.48
8	0.59	0.48
9	0.59	0.48
10	0.59	0.48

Table 3. 24 *PSE values for “Mean 3” and “Mean 4” input stimuli.*

	Average PSE second layer	Average PSE first layer
Mean 3	0.62	0.48
Mean 4	0.59	0.48

Table 3. 25 Average *PSE values for “Mean 3” and “Mean 4” input stimuli.*

The first "***paired samples t-Test***" was computed among the PSE of the "Second Layer" and the "First Layer" of the "Mean 3"'s case:

Mean 3: t(9) = 27.6864, p < 0.0001

Then, t-Test between the two layer concerning "Mean 4" has been implemented:

Mean 4: t(9) = 26.9123, p < 0.0001

These kinds of values, confirm and replicate what we have seen in paragraph 3.2.1: there is a significant statistical difference among the first layer and the second layer; even if the average dimension of the input stimuli is different (like in this case).

3.4.4.1 First layer: Mean 3 vs Mean 4

However, more interesting results have been obtained implementing t-Test between "Mean 3"'s PSEs and "Mean 4"'s PSEs. So, to investigate if there was a significant statistical difference among first layer's PSEs of Mean 3 and first layer's PSEs of Mean 4, a "***two samples t-Test***" has been implemented. Notice that this kind of t-Test is different with respect the "*paired samples t-Test*" seen previously, because in this case data were independent, and they didn't come from "judgment" of the same subject; the key concept is that the simulation about "Mean 3" was different and independent from the simulation concerned "Mean 4". In fact, now the degrees of freedom are different than the previous case (df = 9) , due to the presence of two independent populations:

$$df = n1 - 1 + n2 - 1 = 18 \tag{3.5}$$

Formula to obtain **t-value** for the "*two samples t-test*" is shown below:

$$t = \frac{\mu_{Area1Mean3} - \mu_{Area1Mean4}}{\sqrt{\frac{\sigma^2_{Area1Mean3} + \sigma^2_{Area1Mean4}}{n}}} \qquad n = pop\ dimension \tag{3.6}$$

And results of the comparison between "Mean3" and "Mean4", considering the first layer are:

First Layer		
Subject	PSE Mean 3	PSE Mean 4
1	0.49	0.49
2	0.49	0.49
3	0.5	0.49
4	0.49	0.48
5	0.47	0.48
6	0.46	0.48
7	0.48	0.48
8	0.48	0.48
9	0.49	0.48
10	0.51	0.48

Table 3. 26 ***Comparison of the PSE values related to "Area 1" between "Mean 3" and " Mean 4".***

$t(18) = 0.1466$, $p = 0.8851$

On the basis of this results, there is not a significant difference between the means of the two populations This means that the two sigmoidal curves of *figure 3.25* are not significantly different, and the dimension of the input stimuli does not affect the construction of the sigmoidal curve (proportion of hand stimuli judged larger) in Area 1.

3.4.4.2 Second layer: Mean 3 vs Mean 4

In this last t-Test we have investigated if there was a statistical difference between second layers' PSEs of "Mean 3", and second layer's PSEs of "Mean 4". The two populations examined are:

Second Layer		
Subject	**PSE Mean 3**	**PSE Mean 4**
1	0.63	0.59
2	0.63	0.59
3	0.62	0.57
4	0.61	0.57
5	0.6	0.58
6	0.61	0.59
7	0.62	0.6
8	0.63	0.59
9	0.62	0.59
10	0.62	0.59

Table 3. 27 ***Comparison of the PSE values related to "Area 2" between "Mean 3" and " Mean 4".***

$$\mathbf{t(18) = 7.8588, \ p < 0.0001}$$

The result confirms that there is a significant statistical difference between the two populations, and this difference is not due to chance. In this conditions, PSEs relating to "Mean 4" are more bias than PSEs relating to "Mean 3", namely "Mean 3"'s PSEs are closer to the ideal curve (see figure 3.4) than "Mean 4"'s PSEs. . This validates the theory that the rescaling is greater for small dimensions of the input stimuli, as we have asserted in paragraph 3.3.2.

Chapter 4

PARAMETER SENSITIVITY ANALYSIS

4.1 Parameter Sensitivity Analysis and Reference Results

This kind of analysis has the purpose of investigating the behaviour of the neural network changing parameters one by one. In short, we have conducted different simulations, in each simulation changing the value of just <u>one parameter at a time with respect to the reference value</u> reported in table 4.1.

The changing of some kinds of parameters might leads to a dramatically change of the network outputs, whereas, other parameters could not change substantially the behaviour of the network. Hence, it is clear that this study was very important to understand which parameters influence more network behaviour. .

The table below shows the **parameters** of the neural network and their reference (or basal) values:

External Stimuli			
$I_1^{f,H}=1.5$	$I_2^{f,H}=1.5$	$\sigma_{I1}^{f,H}=0.1\,cm$	$\sigma_{I2}^{f,H}=0.1\,cm$
$I_1^{f,A}=1.5$	$I_2^{f,A}=1.5$	$\sigma_{I1}^{f,A}=0.1\,cm$	$\sigma_{I2}^{f,A}=0.1\,cm$
Receptive Fields			
$\Phi_0^{f,H}=1$	$\sigma_{\Phi}^{f,H}=0.125\,cm$		
$\Phi_0^{f,A}=1$	$\sigma_{\Phi}^{f,A}=0.35\,cm$		
Lateral Synapses in Area 1			
$L_{ex}^{f,H}=1$	$L_{in}^{f,H}=0.5$	$\sigma_{ex}^{f,H}=2\,neurons$	$\sigma_{in}^{f,H}=8\,neurons$
$L_{ex}^{f,A}=1$	$L_{in}^{f,A}=0.5$	$\sigma_{Lex}^{f,A}=2\,neurons$	$\sigma_{Lin}^{f,A}=8\,neurons$
Feed-Forward Synapses			
$W_{ex}^{f,H}=4$	$W_{in}^{f,H}=1$	$\sigma_{Wex}^{f,H}=1\,neurons$	$\sigma_{Win}^{f,H}=1.4\,neurons$
$W_{ex}^{f,A}=4$	$W_{in}^{f,A}=1$	$\sigma_{Wex}^{f,A}=1\,neurons$	$\sigma_{Win}^{f,A}=1.4\,neurons$
Lateral Synapses in Area 2			
$L_{ex}^{s,H}=4.5$	$L_{in}^{s,H}=2$	$\sigma_{ex}^{s,H}=1.5\,neurons$	$\sigma_{in}^{s,H}=2\,neurons$
$L_{ex}^{s,A}=4.5$	$L_{in}^{s,A}=2$	$\sigma_{ex}^{s,A}=1\,neurons$	$\sigma_{in}^{s,A}=2\,neurons$
Sigmoidal characteristic			
$G_{\max}=1$	$k=0.6$	$u_0^f=u_0^s=12$	
Activation Threshold			
thr = 0.9			

Table 4. 1 ***Reference parameters.***

The superscripts *f*, *s* will denote quantities concerning the first layer (Area 1) and the second layer (Area 2) respectively. The superscripts *H* and *A* will indicate the Hand (Region A) and the Arm (Region B). Finally the subscripts *ij*, *hk* will represent the spatial position of an individual neuron.

We have run simulations with input stimuli distances listed in the next table (these simulations are the same as in Chapter 3):

Stimuli Distance hand(cm) / arm (cm)	Ratio
2.4 / 6	*0.4*
2.7 / 6	*0.45*
2/ 4	*0.5*
2.2 / 4	*0.55*
3 / 5	*0.6*
3.25 / 5	*0.65*
2.8 / 4	*0.7*
3.5 / 4.5	*0.8*
2.7 / 3	*0.9*
3 /3	*1*

Table 4. 2 ***Input Stimuli Distances.***

We have run simulations consisting of 25 trials per stimuli distances, for a total of 250 trials per simulation. That is, in one simulation, each pair of stimuli distances (hand/arm) was applied 25 times on the "virtual subject".

Simulation results obtained with basal parameters and input stimuli as reported in table 4.2, are shown in the next graphic, and we will call them as "Reference Results" .

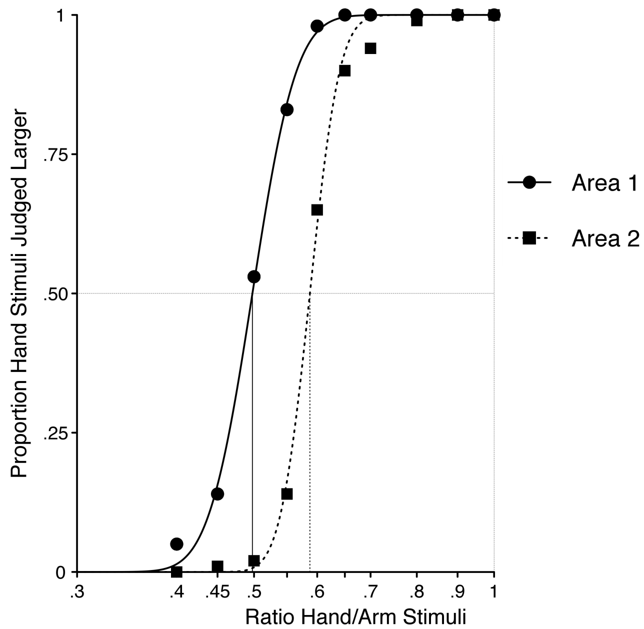


Figure 4. 1 ***Results of the simulation with Reference Parameters.***

The graphic of the Lateral Synapses, entering central neuron within "Area 2" (Arm region) is reported below:

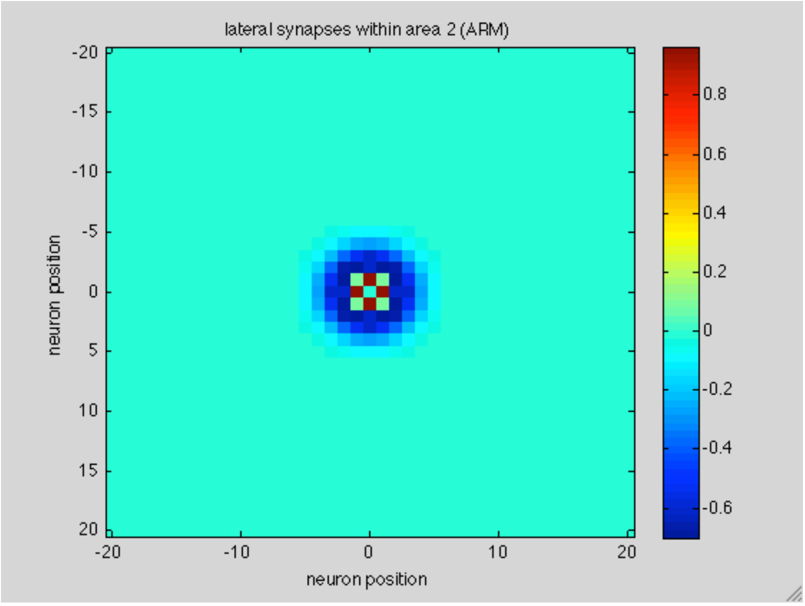


Figure 4. 2 ***Lateral Synapses linked to the central neuron inside the Second Layer of the Arm (Reference Parameters).***

The graphic of the Feed-Forward Synapses from the central neuron inside "Area 1", to neurons in "Area 2" (all related to the Arm region) is shown below:

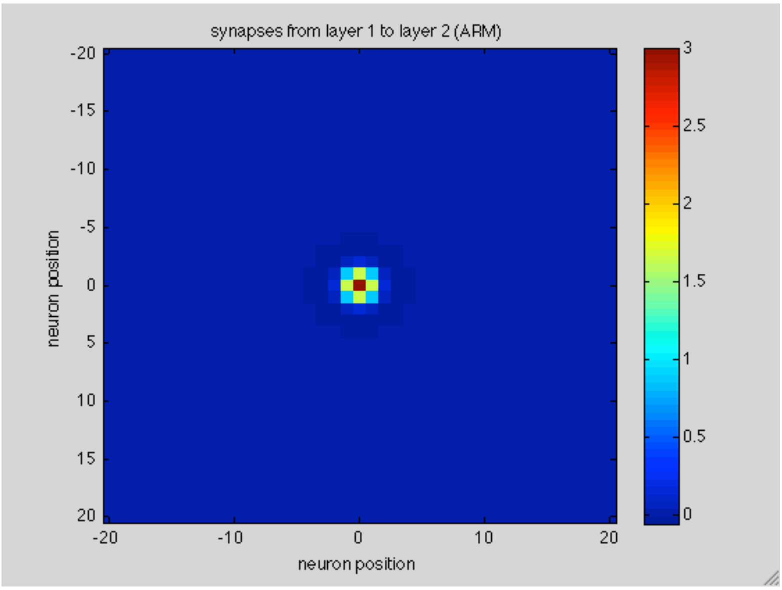


Figure 4. 3 ***Feed-Forward Synapses from the "First Layer" to the "Second Layer" related to the central neuron of the "First Layer".***

Remember that each little coloured square in position (i,j) represents the intensity of the feed forward synapse that reaches the neuron inside Area 2 in position (i,j).

Finally, the next table shows the entity of the Weber's Illusion and the Rescaling Process, still related to the "Reference Results":

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0,5	0.09
Second Layer	0.59	0,41	

Table 4. 3 *Reference Results.*

4.2 The Changed Parameters

In this study we have focused the attention on the **neural network of the Arm**, changing parameters concerning feed forward-synapses and lateral synapses within Area 2. Below, a list of the parameters that have been changed:

- $\sigma_{ex}^{s,A}$ (Lateral Synapses within Area 2: standard deviation of the excitatory component)
- $\sigma_{in}^{s,A}$ (Lateral Synapses within Area 2: standard deviation of the inhibitory component)
- $L_{ex}^{s,A}$ (Lateral Synapses within Area 2: intensity of the excitatory component)
- $L_{in}^{s,A}$ (Lateral Synapses within Area 2: intensity of the inhibitory component)
- $\sigma_{Wex}^{f,A}$ (Feed-Forward Synapses: standard deviation of the excitatory component)
- $\sigma_{Win}^{f,A}$ (Feed-Forward Synapses: standard deviation of the inhibitory component)
- $W_{ex}^{f,A}$ (Feed-Forward Synapses: intensity of the excitatory component)
- $W_{in}^{f,A}$ (Feed-Forward Synapses: intensity of the inhibitory component)

4.2.1 Parameters Alteration

In this analyse only parameters relative to the Arm have been changed. The reasons of this choose lies in the role of the Arm network. In fact, in the Arm's neural network we have implemented a sort of rescaling of the distance between the two activation balls, in order to increase the resolution of this region. On the contrary, on the Hand, the shift from "Area 1" to "Area 2" is characterized by the maintenance of its already high resolution, without changing the distance between the balls. Therefore, the Arm region seemed to be the most sensible one, because the effect of the rescaling process occurs in this region (thanks to the alteration of the distance between the activation balls within Area 2). So we have decided to change the values of parameters listed before, one a time and then we have studied the output of the network, trying to explain what happened.

It is important to remember that in this study the parameters were changed in a specific way: parameters concerning the inhibitory components were changed in order to decrease their values, whereas, excitatory parameters were changed to increase their values (quindi in tutti e due i casi, il risultato è un increment dell'eccitazione laterale!!!) . The reason is clear: **increasing** the **inhibitory component** (in terms of intensity, or in terms of standard deviation) leaded to a situation in which neurons inhibited reciprocally to a greater extent. Especially, with small distance stimuli, no balls of activation were recorded inside "Area 2" with an **activation threshold equal to 0.9,** making it impossible to obtain a network output. In fact, if the activation balls (both within "Area 1" or "Area 2") were very close, activated neurons composing one ball were able to strongly inhibit neurons of the other ball, and vice versa, resulting in a very weak activation of neurons, under the value of the activation threshold (0.9) Thereby, in paragraph 4.3, <u>parameters about the inhibitory components were changed in order to decrease their values, whereas parameters concerned the excitatory component have been increased.</u>

However, an activation threshold of 0.9 is very high, and so to investigate the behaviour of the network after an increment of the inhibitory component, a decrement of the activation threshold was needed. Hence, in paragraph 4.7 results of the simulations in these conditions will be presented.

4.3 Alteration of the Parameters and Neural Network Behaviour

Lateral Synapses Area 2 : $\boldsymbol{L_{ex}^{s,A} = 5.5}$ ***(reference :*** $\boldsymbol{L_{ex}^{s,A} = 4.5}$***)***

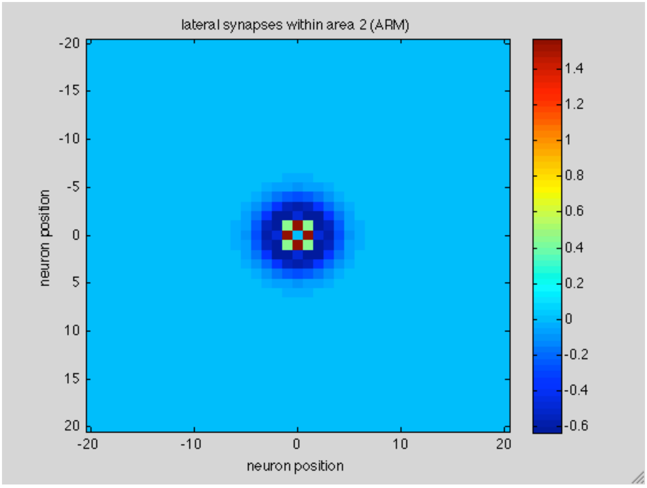


Figure 4. 4 ***Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with*** $\boldsymbol{L_{ex}^{s,A} = 5.5}$

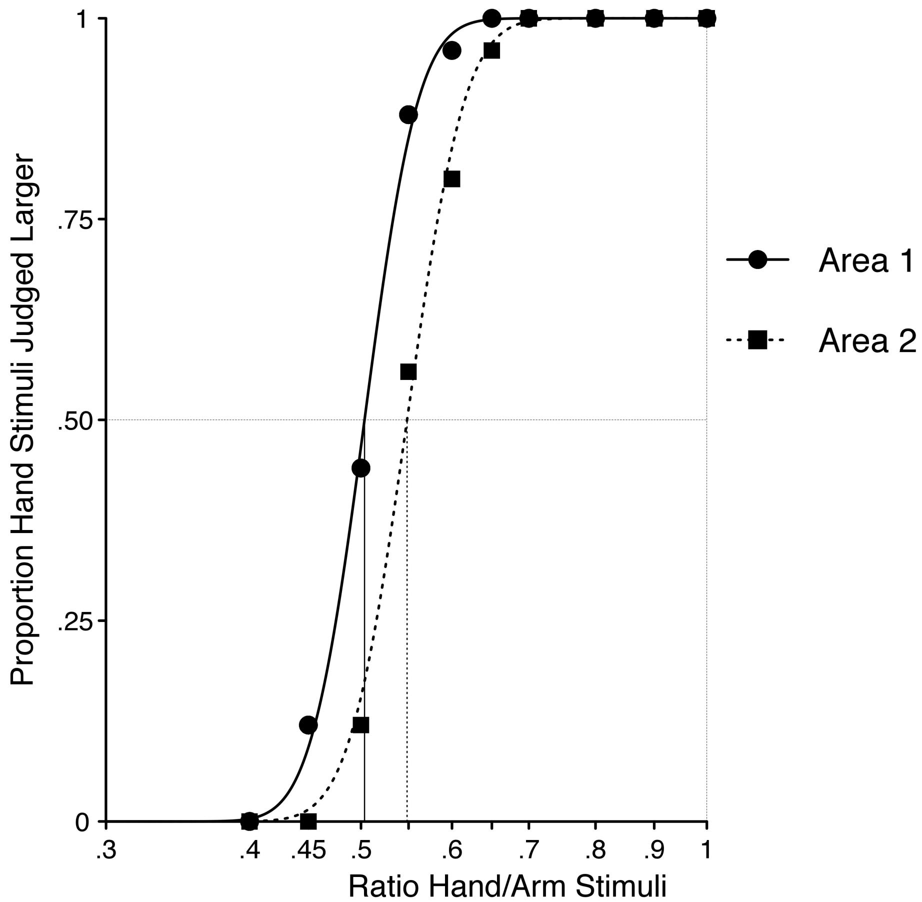


Figure 4. 5 ***Results of the simulation with*** $\boldsymbol{L_{ex}^{s,A} = 5.5}$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0,5	0.05
Second Layer	0.55	0,45	

Table 4. 4 ***PSE, Weber's Illsuion and Rescaling Proces.***

The intensity of the excitatory component of the Lateral Synapses has been increased of 1 unit, from the original value. With this new value, Lateral Synapses changed only in the intensity; the shape and the width of the Mexican Hat (figure 4.4) remained unchanged.

However, we can notice the PSE of "Area 2" is smaller than the Reference PSE = 0.59. So, in the comparison with data of the "Reference Results", we can assert that an increment of the intensity of the excitatory component produces a weakening of the Rescaling Process effect, and at the same time, an increasing of the Weber's Illusion . The reason of this effect is very simply: the Rescaling Process starts inside "Area 2" of the Arm, trying to decrease the dimension of the activation balls, in order to achieve an increment of the distance between the two balls. Obviously, to reduce the sizes of the activation balls, an increment of the inhibitory component, and a reduction of the excitatory component is necessary. Increasing the excitatory component of the lateral synapses within "Area 2", produces a higher excitation of neurons in Area 2(above reference results), so that the distance between the two balls did not increase as much as in the reference case.

This means that the rescaling process was partially reduced, due to the big value of the excitatory component compared with the inhibitory component. This kind of Lateral Synapses pattern was not strong enough to minimize the dimensions of the bubbles, resulting in a shift of the PSE toward smaller ratio.

Lateral Synapses Area 2 : $L_{ex}^{s,A} = 5$ *(reference :* $L_{ex}^{s,A} = 4.5$)

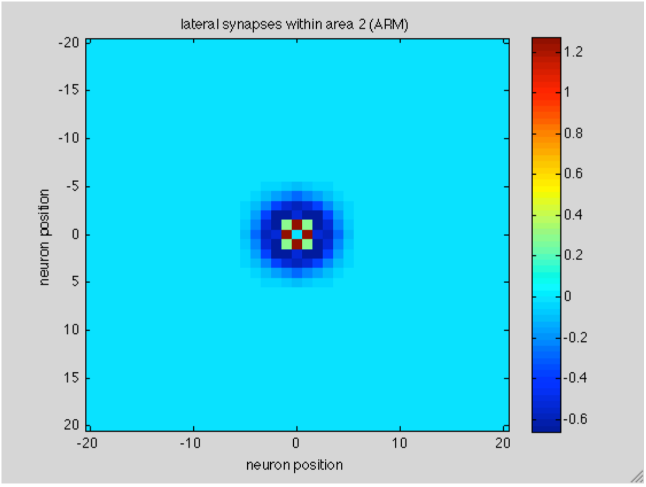


Figure 4. 6 *Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with* $L_{ex}^{s,A} = 5$

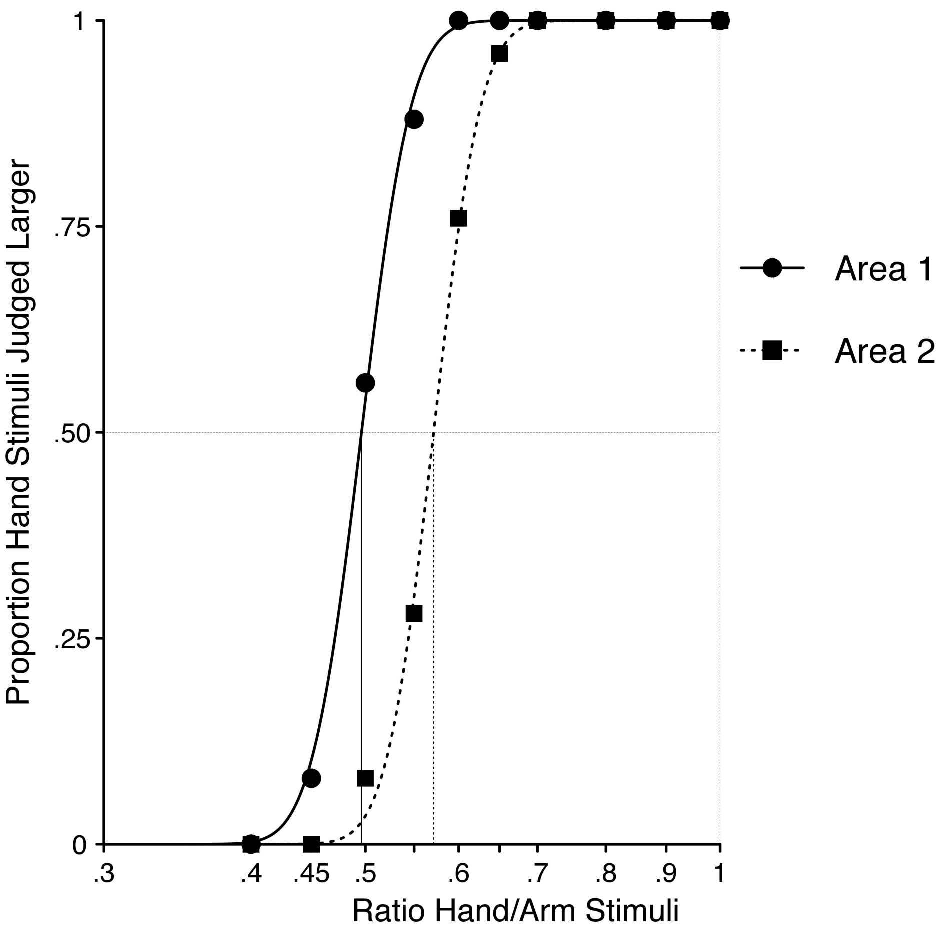


Figure 4. 7 *Results of the simulation with* $L_{ex}^{s,A} = 5$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0,5	0.07
Second Layer	0.57	0,43	

Table 4. 5 *PSE, Weber's Illsuion and Rescaling Process.*

The strength of lateral excitations was increased by 0.5. With this new value, results differ less from the "“Reference Results”". In fact, (see table 4.5) , the entity of the "Rescaling Process" is very similar to the "“Reference Results”". That is well expected as the parameter was subjected to a smaller increase. In other words, the Weber's Illusion of "Area 2" (0.43) is very similar to the reference Weber's Illusion. Therefore, the effect of increasing this parameter by just 0.5 is negligible.

Lateral Synapses Area 2 : $L_{in}^{s,A} = 1.5$ ***(reference :*** $L_{in}^{s,A} = 2$)

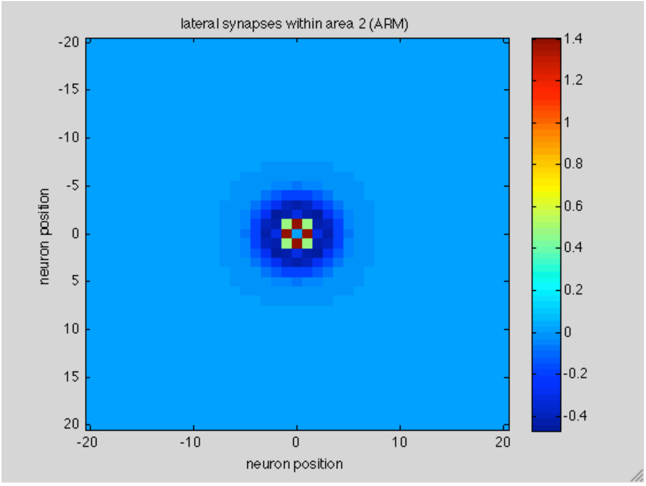


Figure 4. 8 ***Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with*** $L_{in}^{s,A} = 1.5$

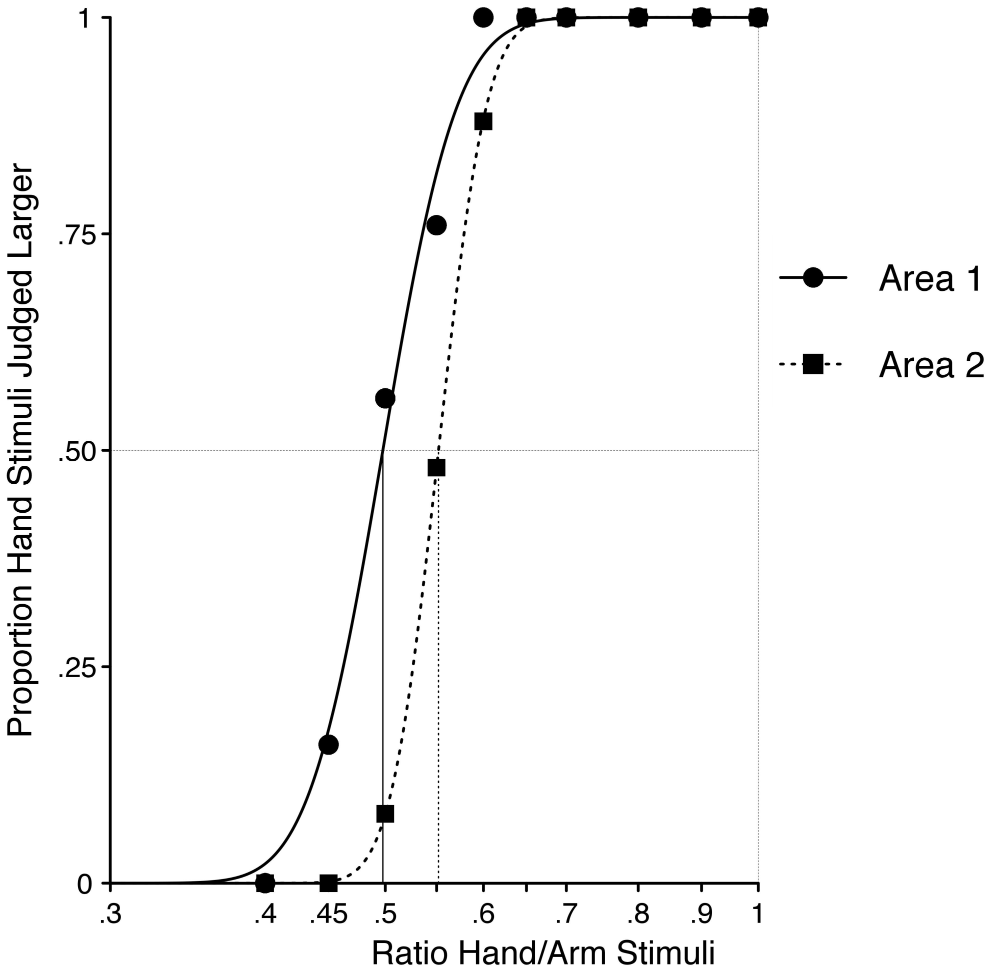


Figure 4. 9 ***Results of the simulation with*** $L_{in}^{s,A} = 1.5$

<table>
<tr><th></th><th>PSE</th><th>Weber’s Illusion =
1 – PSE</th><th>Rescaling Process =
PSE Area2 – PSE Area1</th></tr>
<tr><td>First Layer</td><td>0.5</td><td>0.5</td><td rowspan="2">0.05</td></tr>
<tr><td>Second Layer</td><td>0.55</td><td>0.45</td></tr>
</table>

Table 4. 6 ***PSE, Weber’s Illsuion and Rescaling Process.***

In this case we have modify the intensity of the inhibitory component concerning Lateral Synapses within “Area 2”. Results are similar as those obtained by increasing $\boldsymbol{L_{ex}^{s,A} = 5.5}$. In this case too, we have a reduction of the Second layer’s PSE. Indeed, reducing the intensity of the inhibitory component equals an increase in the overall excitation, resulting in the shifting of the PSE toward smaller ratio.

Lateral Synapses Area 2 : $\sigma_{ex}^{s,A} = 1.5$ *(reference :* $\sigma_{ex}^{s,A} = 1$)

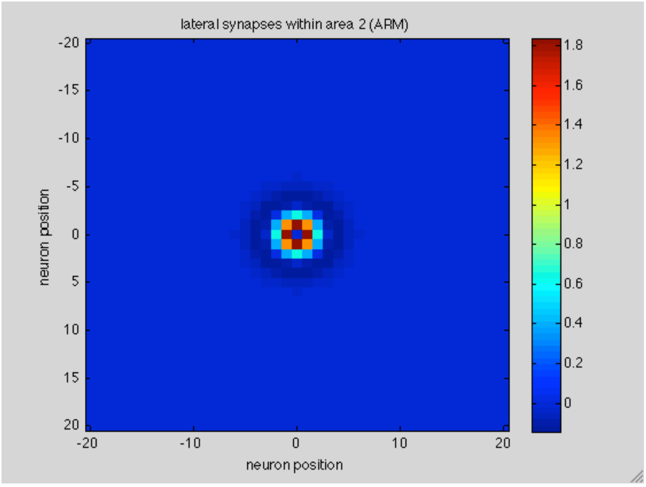


Figure 4. 10 *Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with* $\sigma_{ex}^{s,A} = 1.5$

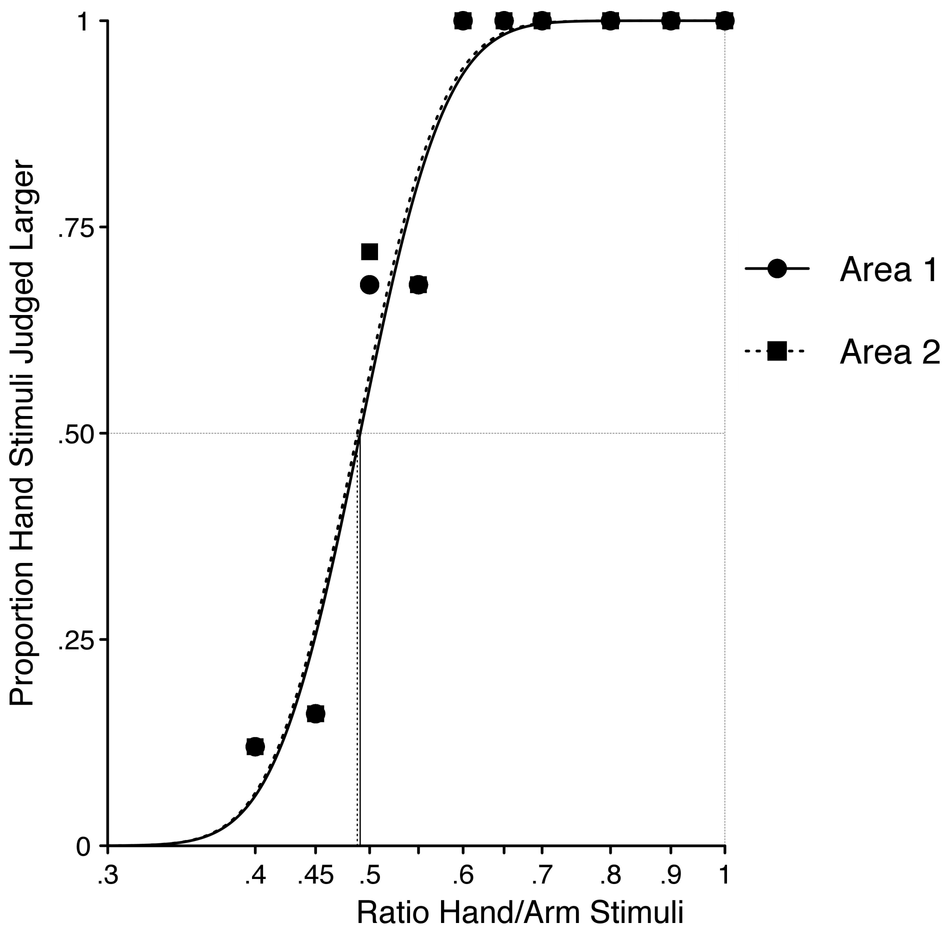


Figure 4. 11 *Results of the simulation with* $\sigma_{ex}^{s,A} = 1.5$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.49	0.51	0
Second Layer	0.49	0.51	

Table 4. 7 *PSE, Weber's Illusion and Rescaling Process.*

This is a very interesting case, because with a slight increment of the standard deviation of the excitatory component, the two curves tend to overlap. In fact, the PSE value in Area 2 is about the same both as in Area 1. That is, the same Weber's Illusion occurs for "Area 1" and "Area 2", and the "Rescaling Process" is nulled. The key point is that a small change in the parameter produces a dramatic shift of the "Second Layer" curve toward smaller ratio. Increasing $\boldsymbol{L_{ex}^{s,A}}$ (intensity of the excitatory component of the lateral synapses) produced a decrease in PSE to 0.55, whereas in the present case, the PSE was reduced to 0.49 (the same PSE asArea 1). Hence, network behaviour is much more sensible to parameter $\sigma_{ex}^{s,A}$ $\boldsymbol{L_{ex}^{s,A}}$ or $\boldsymbol{L_{in}^{s,A}}$, since a small change of its value is able to delete completely the effect of the Rescaling Process. With the next table, the situation just explained should be much more understandable:

	Second Layer PSE	Changing Entity (neurons)
$\boldsymbol{L_{ex}^{s,A}}$	0,55	1
$\boldsymbol{L_{in}^{s,A}}$	0,55	0.5
$\boldsymbol{\sigma_{ex}^{s,A}}$	0,49	0.5

Table 4. 8 *Second Layer PSEs and Entity of the Changing.*

Another important aspect that we have to report is about the shape, and width of the Lateral Synapses. In this case we have raised the standard deviation of the excitatory component, and (see figure 4.10), the positive part of the Mexican Hat is much more bigger than the previous cases. This means that neurons inside "Area 2" were under a big excitatory wave, without the possibility for the inhibitory component to decrease their activation. In this situation, the distance between the two activation balls within "Area 2" remained the same as "Area 1". That is the reason why the PSE of the second layer is the same as the first one.

Lateral Synapses Area 2 : $\sigma_{in}^{s,A} = 1.5$ *(reference :* $\sigma_{in}^{s,A} = 2$)

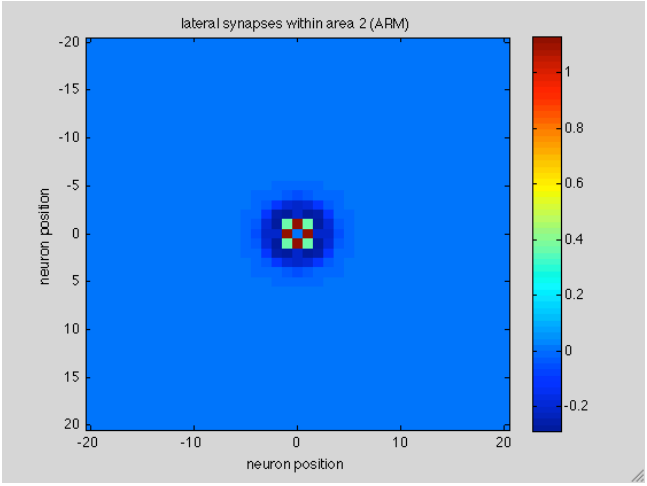


Figure 4. 12 *Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with* $\sigma_{in}^{s,A} = 1.5$

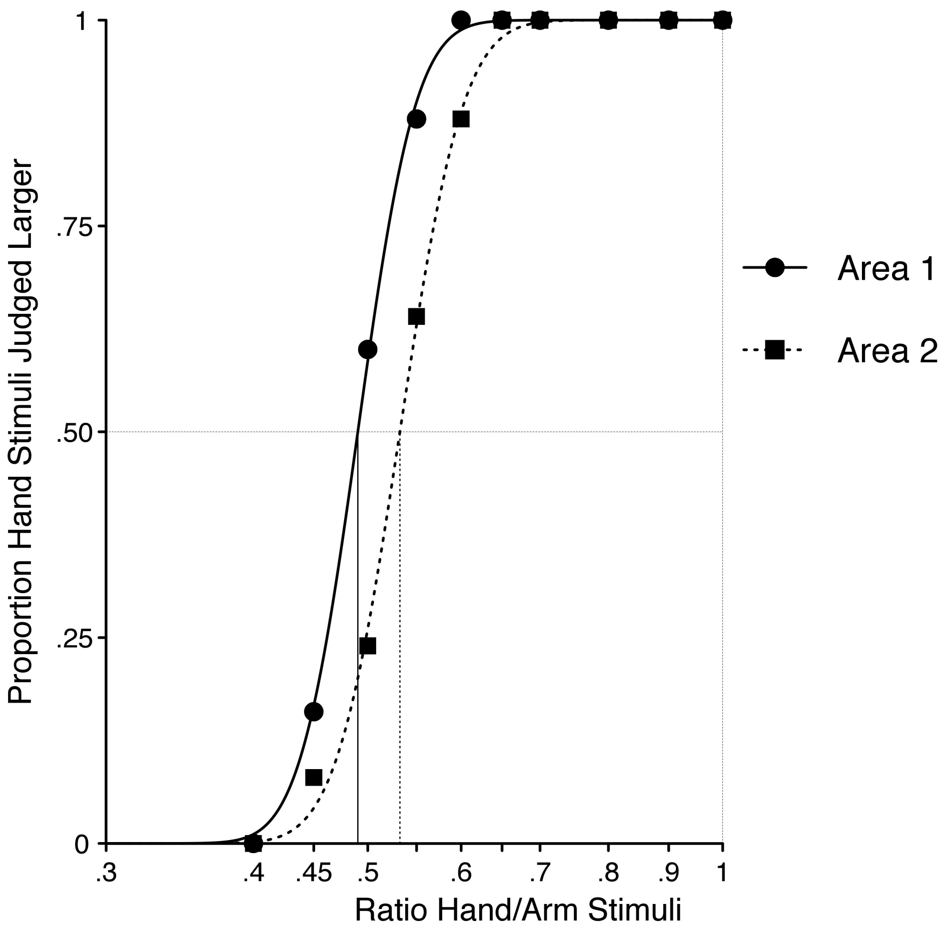


Figure 4. 13 *Results of the simulation with* $\sigma_{in}^{s,A} = 1.5$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.49	0.51	0.04
Second Layer	0.53	0.47	

Table 4. 9 *PSE, Weber's Illusion and Rescaling Process.*

Reducing the standard deviation of the inhibitory component (Lateral Synapses within Area 2) produced the following results. The decrement of the PSE of the second layer is quite relevant, but not as high as in the previous case, in which PSE of "Area 1" and PSE of "Area 2" had the same value. That is, the Weber's Illusion of the "Second Layer" is smaller than the "First Layer"; it is bigger than the Weber's Illusion of the second layer that appears in the ""Reference Results"".

Reducing the value of this parameter has the effect of decreasing the width of the inhibitory wave, leaving space for the excitatory wave (figure 4.12).

Feed-Forward Synapses: $W_{ex}^{s,A} = 5$ ***(reference :*** $W_{ex}^{s,A} = 4$)

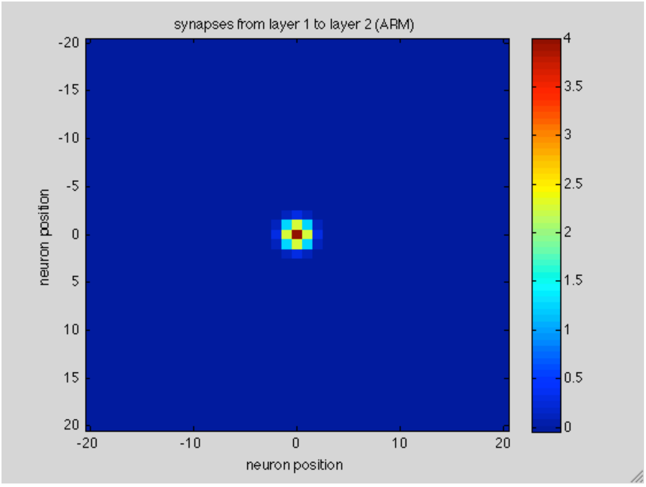


Figure 4. 14 ***Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with*** $W_{ex}^{s,A} = 5$

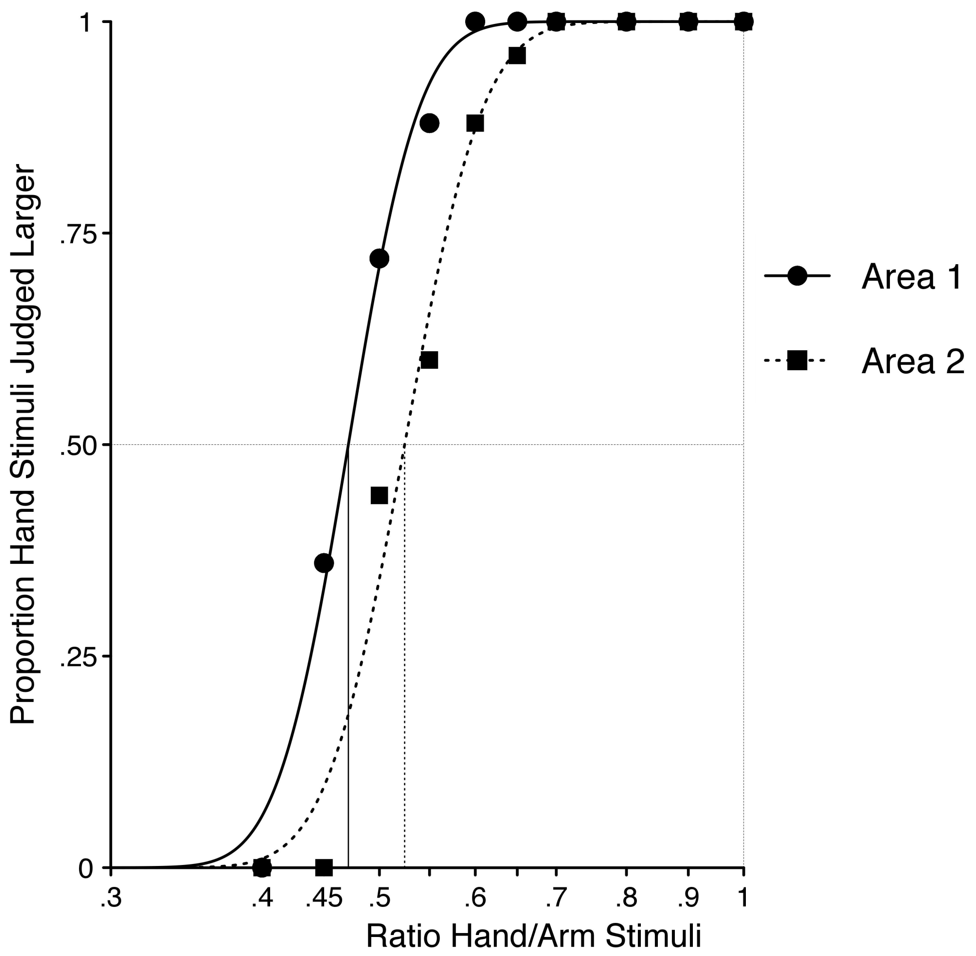


Figure 4. 15 ***Results of the simulation with*** $W_{ex}^{s,A} = 5$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.47	0.53	0.05
Second Layer	0.52	0.48	

Table 4. 10 *PSE, Weber's Illsuion and Rescaling Process.*

In this study we have increased the intensity of the excitatory component about the Feed-Forward synapses. The results are similar as before, namely the curve of the Second Layer was shifted toward smaller ratio (compared with the second layer curve of the ""Reference Results""). The entity of the shifting is not the same as the previous case ($\boldsymbol{\sigma_{ex}^{s,A}}$), because PSE of "Area 1" and PSE of "Area 2" are different. As in the other cases, the increment of the intensity $W_{ex}^{s,A}$ has determined a reduction of the Rescaling Process.

Feed-Forward Synapses : $W_{in}^{f,A} = 0.01$ (*reference* : $W_{in}^{f,A} = 1$)

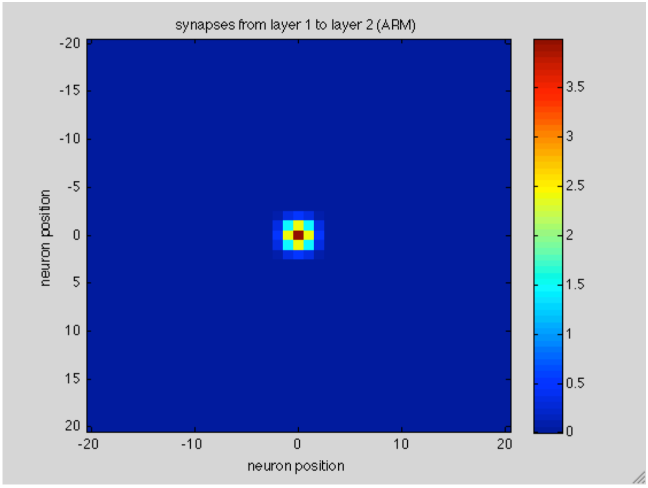


Figure 4. 16 *Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with* $W_{in}^{s,A} = 0.01$

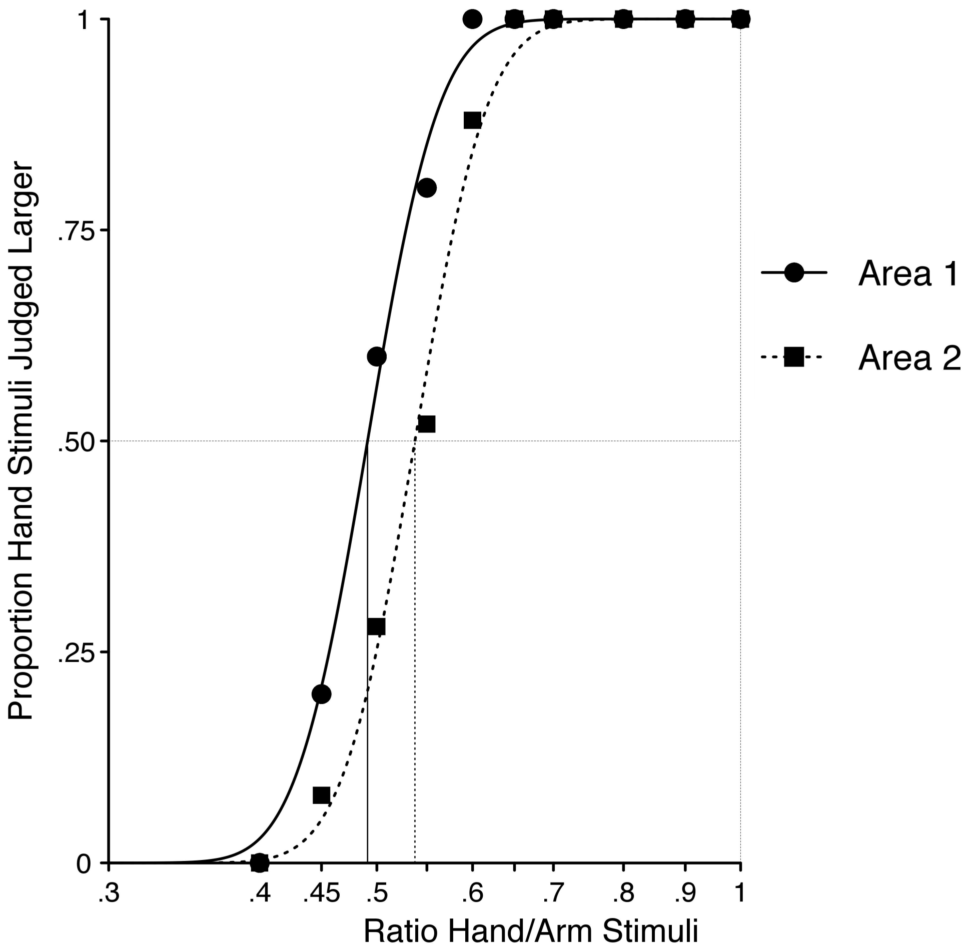


Figure 4. 17 *Results of the simulation with* $W_{in}^{s,A} = 0.01$

	PSE	Weber's Illusion= 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.49	0.51	0.05
Second Layer	0.54	0.46	

Table 4. 11 *PSE, Weber's Illsuion and Rescaling Process.*

This is another example in which the decrement of the intensity of the inhibitory component, (in this case the intensity of the inhibitory component concerning Feed-Forward synapses), leaded to a shift of the Second Layer's PSE toward smaller ratio. Notice that that the entity of the recalling process is still equal to 0.05.

Feed-Forward Synapses *:* $\sigma_{Wex}^{f,A} = 1.5$ *(reference :* $\sigma_{Wex}^{f,A} = 1$)

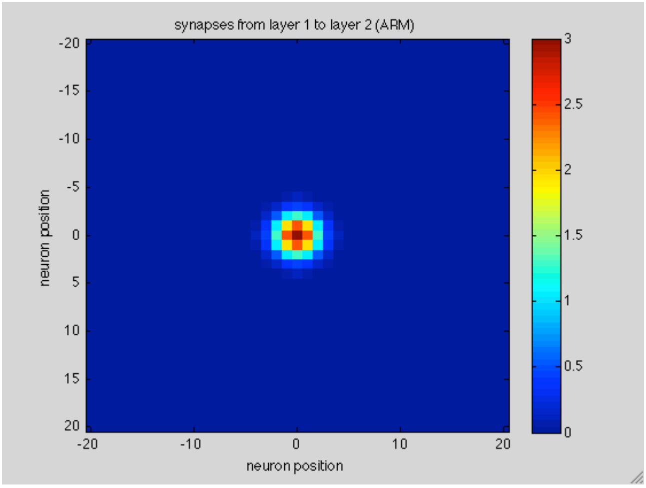


Figure 4. 18 ***Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with*** $\sigma_{Wex}^{f,A} = 1.5$

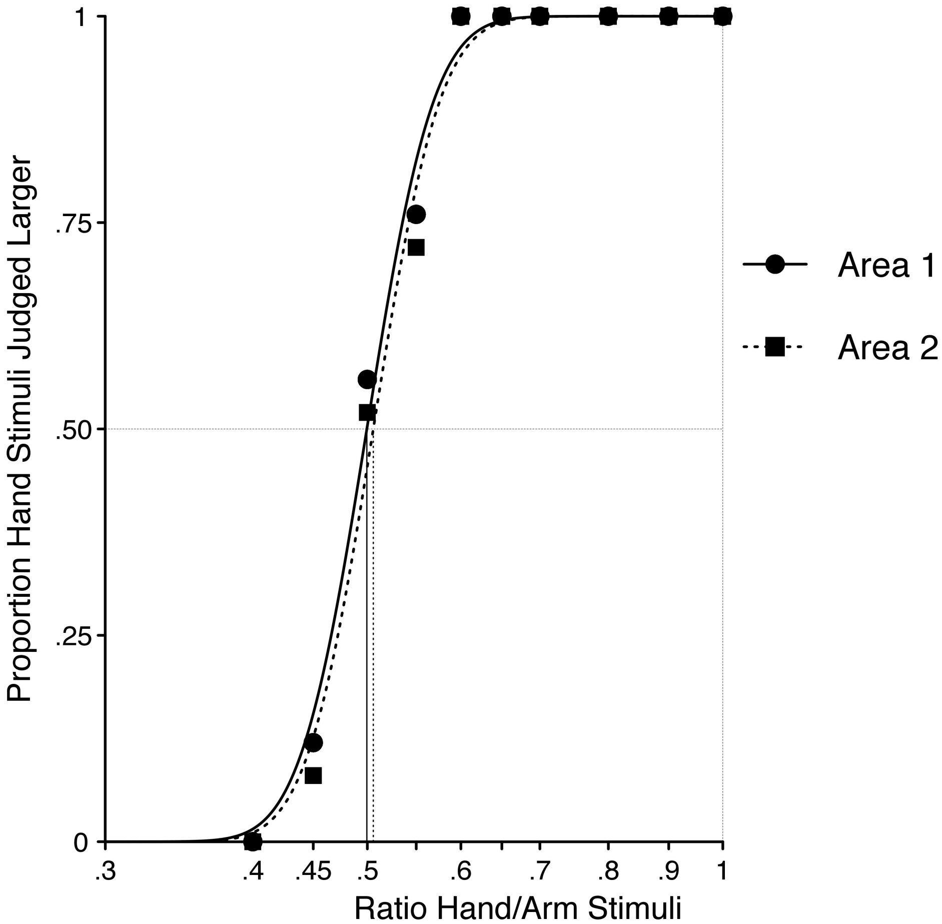


Figure 4. 19 ***Results of the simulation with*** $\sigma_{Wex}^{f,A} = 1.5$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	0.01
Second Layer	0.51	0.49	

Table 4. 12 ***PSE, Weber's Illusion and Rescaling Process:***

In this case we have increased the standard deviation of the excitatory component (feed-forward synapses) $\boldsymbol{\sigma_{Wex}^{f,A}}$ of 0.5 neurons. <u>The results are similar as case number 3</u> ($\boldsymbol{\sigma_{ex}^{s,A}}$): namely the Weber's Illusion is the same, both for "Area 1" and "Area 2". This means an elimination of the "Rescaling Process", in fact its value is negligible.

The shape and the dimension of the Feed-Forward synapse changed: in particular, the width of the excitatory component increases considerably.

Feed-Forward Synapses **:** $\boldsymbol{\sigma_{Wex}^{f,A} = 2}$ ***(reference*** **:** $\boldsymbol{\sigma_{Wex}^{f,A} = 1}$**)**

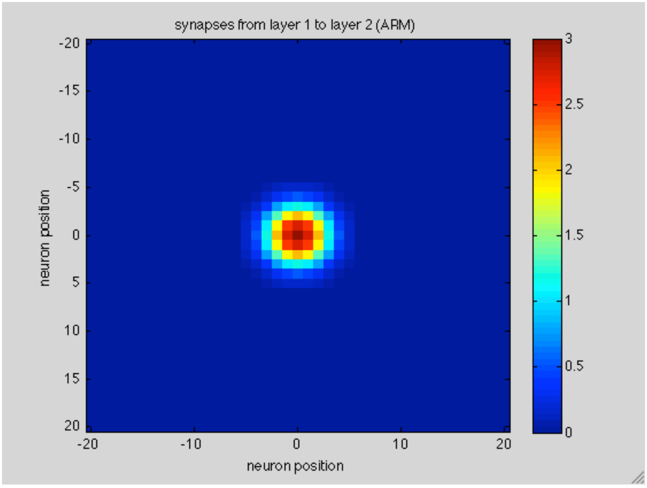


Figure 4. 20 ***Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with*** $\boldsymbol{\sigma_{Wex}^{f,A} = 2}$

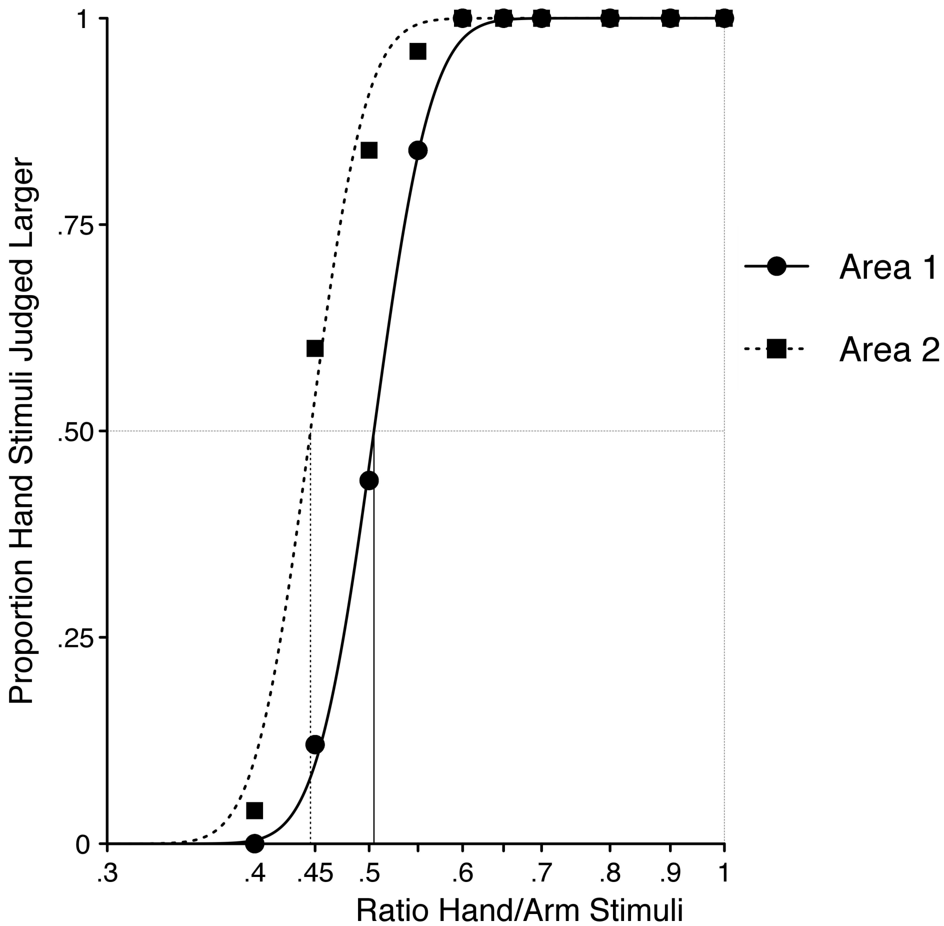


Figure 4. 21 ***Results of the simulation with*** $\boldsymbol{\sigma_{Wex}^{f,A} = 2}$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	- 0.05
Second Layer	0.45	0.55	

Table 4. 13 *PSE, Weber's Illsuion and Rescaling Process.*

In this case we have further increased the parameter $\sigma_{Wex}^{f,A}$.

This is an interesting case, since we obtained a graphic with the PSE of "Area 2" **smaller** than "Area 1". In fact, the sigmoidal curve of the Second Layer was moved toward smaller ratio and beyond the curve of the first layer: a higher Weber's Illusion occurs for the "Second Layer" than the first one.

With this new value, Feed-Forward synapses had a very large excitatory wave (figure 4.20), and they were able to excite much more neurons than the previous case ($\boldsymbol{\sigma_{Wex}^{f,A}}$ = 1.5), resulting in an enlargement of the activation balls. We have obtained the opposite effect compared the "Reference Results", i.e,, the "Rescaling Process" assumes a negative value.. In the "Reference Results", the reference values of the parameters were set in order to obtain a decreasing of the dimension of the activation balls inside "Area 2". On the contrary, the new value of the standard deviation increased the size of the bubbles, resulting in a decrement of the distance between the two balls. Moreover, **this means a reduction of the arm resolution**. Thereby, the proportion of stimuli on the hand judged larger was altered, because large stimuli on the arm are "judged" (from the virtual subject) much smaller than they are, due to an enlargement of the activation balls inside "Area 2".

Feed-Forward Synapses $: \sigma_{Win}^{f,A} = 0.4$ *(reference* $: \sigma_{Win}^{f,A} = 1.4$*)*

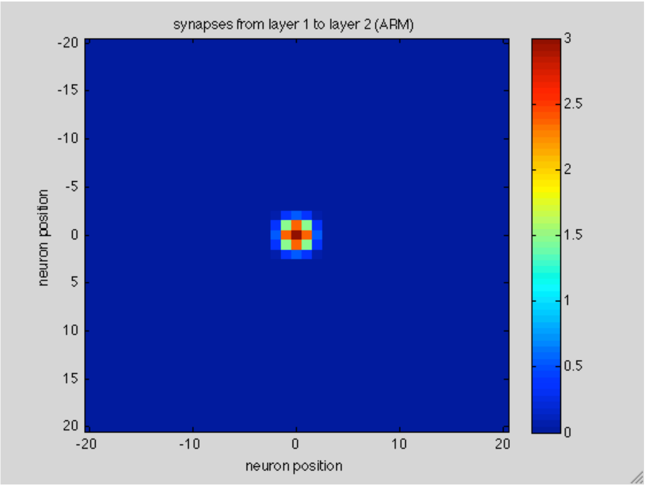


Figure 4. 22 ***Lateral Synapses linked to the central neuron inside the Second Layer of the Arm, with*** $\sigma_{Win}^{f,A} = 0.4$

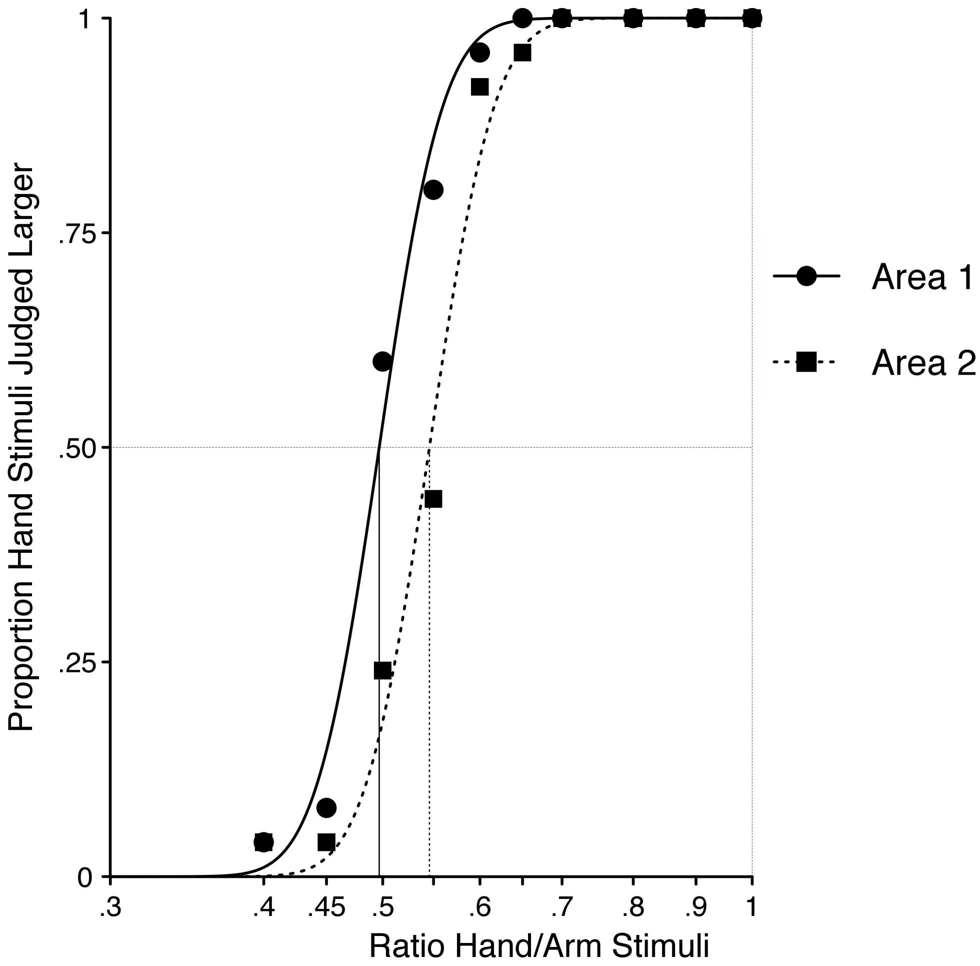


Figure 4. 23 ***Results of the simulation with*** $\sigma_{Win}^{f,A} = 0.4$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	0.05
Second Layer	0.55	0.45	

Table 4. 14 *PSE, Weber's Illusion and Rescaling Process.*

The standard deviation of the inhibitory component (Feed-Forward synapses) was the last parameter under study. We have decreased its value of about 1 neuron. In this case too, we have recorded a lightly reduction of the Second layer's PSE. The curve of the second layer moved toward smaller ratio after the reduction of the inhibitory standard deviation. That happened due to the reduction of the width (std) of the inhibitory component: the results is a slight increase of the overall excitation from Area 1, resulting in a weaken of the Rescaling Process.

4.4 Average Difference between the two activation balls

The average difference, in terms of inactivated neurons, among the distance between the balls of activation in "Area 2", and the distance between the balls of activation in "Area 1", has been computed for each previous cases. In short, the average difference represents the average increment of the distance between the activation balls shifting from "Area 1" to "Area 2". Results are shown below:

		Average Difference between Area 2 - Area 1	
Changed Parameter with new value	**Entity of the changing from the original value**	**Arm**	**Hand**
$L_{ex}^{s,A} = 5.5$	**+ 1**	*+2.43*	*+0.06*
$L_{ex}^{s,A} = 5$	**+ 0.5**	*+3.35*	*+0.03*
$L_{in}^{s,A} = 1.5$	**- 0.5**	*+2.31*	*+0.02*
$\sigma_{ex}^{s,A} = 1.5$	**+ 0.5**	*+0.02*	*+0.004*
$\sigma_{in}^{s,A} = 1.5$	**- 0.5**	*+ 2.36*	*+0.02*
$W_{ex}^{s,A} = 5$	**+ 1**	*+2*	*+0.03*
$W_{in}^{f,A} = 0.01$	**- 0.99**	*+1.7*	*+0.02*
$\sigma_{Wex}^{f,A} = 1.5$	**+ 0.5**	*+0.24*	*+0.03*
$\sigma_{Wex}^{f,A} = 2$	**+ 1**	*-1.6*	*+0.02*
$\sigma_{Win}^{f,A} = 0.4$	**- 1**	*+1.79*	*+0.04*
Reference Parameters		+ 4.01	+ 0.02

Table 4. 15 *Average Differences between the activation balls.*

The results highlighted in blue concern the simulation with reference parameters:

In this kind of analyse we have modified only parameters concerning the Arm region, so the Hand region was not affected by this changing: we can notice that the average difference of the **Hand** is always equal zero in each case, just because its parameters were kept constant in each case.

Positive signs denoted an increment of the distance between the activation balls within "Area 2", whereas, negative signs denoted a decrement of the distance between the activation balls within "Area 2".

The first consideration was about the comparison among the value of the average difference obtained with "Reference Parameter" (table 4.1), and the average difference obtained by changing of one parameter per time. Obviously, in the **Arm**'s case, the average difference changes every time. However, the average difference of the Arm is always abundantly under the value recorded in the Reference Situation *(+ 4.01*). Hence, the increment of the resolution on the Arm was reduced, or completely absent (see cases in which the average difference is equal 0), when changing one parameter; indeed, the distance between the activation balls in "Area 2" is always under 2.5 neurons. This value might be not sufficient to implement a proper Rescaling Process.

Cases $\boldsymbol{\sigma_{ex}^{f,A} = 1.5}$ and $\boldsymbol{\sigma_{Wex}^{f,A} = 1.5}$ are the most interesting cases, because the average difference is very close to zero, both for the Hand and Arm; hence, the distance between the activation balls is the same both for "Area 1" and "Area 2". Figure 4.11 And Figure 4.19 show that in these cases the two sigmoidal curves are superimposed.

However with $\boldsymbol{\sigma_{Wex}^{f,A} = 2}$ the average value decrease until *-1.6*: this means that the two activation balls within "Area 2" have increased their size, and the distance between them has been reduced of about 1.6 neurons. So, this is the worst case in absolute, because with this value we have achieved a decrement in the resolution of the Arm, namely a total opposite effect against the target of the neural network.

4.5 Conclusion about the Sensitivity Analysis

Concerning the sensitivity analysis, we can conclude that **the most sensible parameters** seemed to be the standard deviation of the excitatory component, that appears in the Lateral Synapses ($\boldsymbol{\sigma_{ex}^{s,A}}$), and the standard deviation of the excitatory component, defining Feed-Forward synapses ($\boldsymbol{\sigma_{Wex}^{f,A}}$). Indeed (see table 4.15, figure 4.11 and figure 4.19), alteration of these parameters produced a dramatically change of the network, that affects the position of the PSE of the second layer. Remember that the entity of the increase for these two standard deviations was equal to 0.5. The same increase applied to the other parameters still produced a shifting of the second layer curve toward smaller ratio, but the entity of the shifting was much smaller compared with cases $\boldsymbol{\sigma_{ex}^{s,A}}$ and $\boldsymbol{\sigma_{Wex}^{f,A}}$. Thereby, the network is extremely sensible to these parameters, since as light alteration of their value null the "Rescaling Process": in others words, the Weber's Illusion associated with "Area 2" remained the same as "Area 1".

4.6 Sensitivity Analysis about the Activation Threshold

In addition to the sensitivity analysis conducted on the network parameters (in particular, synapses parameters), we have investigated the behaviour of the neural network after the alteration of a fundamental parameter: the ***activation threshold*** for computation of perceived distance among two punctual stimuli. This threshold separates between active and inactive neurons, and is fundamental in the computation of the perceived distance as only inactivated neurons participate in establishing the perceived distance. Until now, we have hypothesized an activation threshold equal to *0.9*. But, how do network output change when this value is changed?

Activation balls mentioned in this thesis are groups of neurons in which every neuron has a state of activation equal, or greater than the *activation threshold*. Obviously, changing of the activation threshold affects the dimension of the activation balls, because with a low threshold, the number of neurons considered active increase (increasing the dimension of the ball). Vice versa, if the threshold is high, balls will appear of smaller dimension.

In this paragraph I will show the output of the neural network for different values of the activation threshold:

- ***Activation threshold = 0.8***
- ***Activation threshold = 0.7***
- ***Activation threshold = 0.5***
- ***Activation threshold = 0.3***
- ***Activation threshold = 0.1***

Activation Threshold: $\boldsymbol{thr = 0.8}$ ***(reference :*** $\boldsymbol{thr = 0.9}$***)***

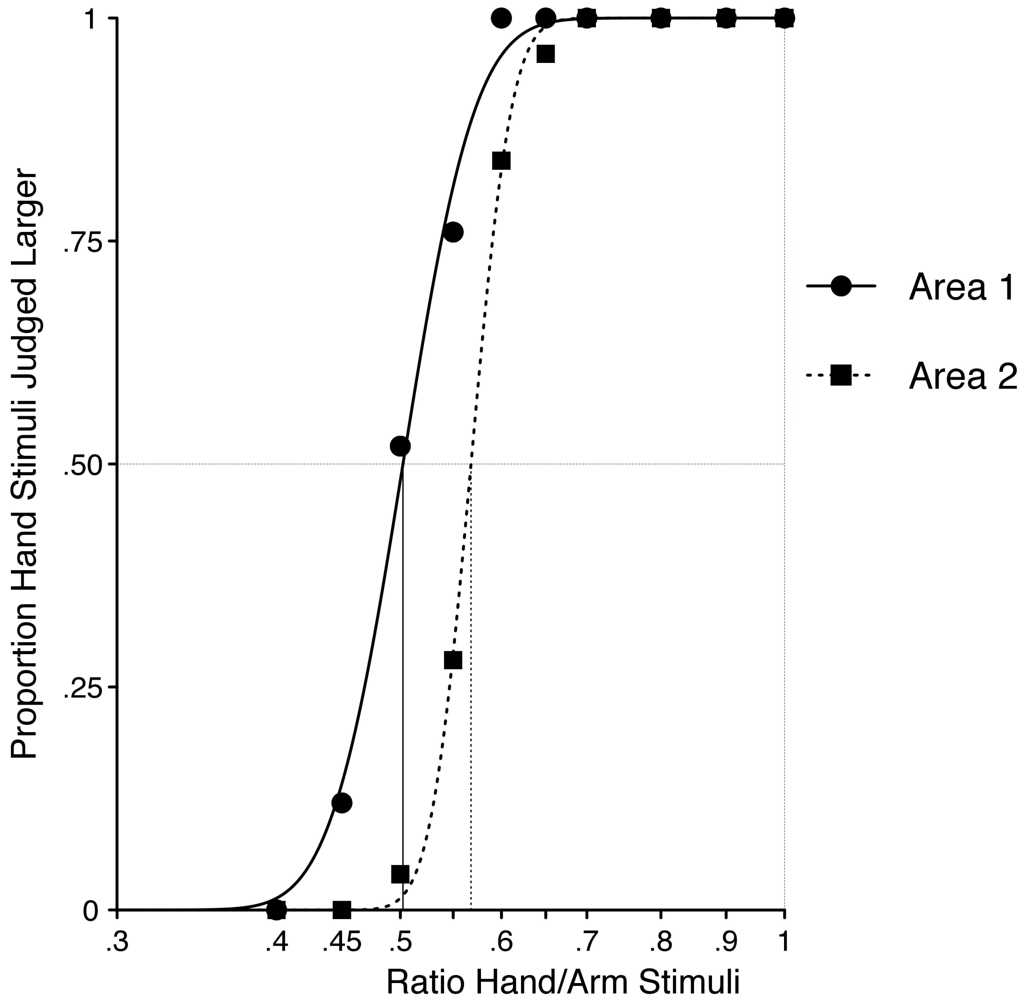


Figure 4. 24 ***Results of the simulation with*** $\boldsymbol{thr = 0.8}$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	0.07
Second Layer	0.57	0.43	

Table 4. 16 ***PSE, Weber's Illsuion and Rescaling Process.***

Activation Threshold: $\boldsymbol{thr = 0.7}$ ***(reference:*** $\boldsymbol{thr = 0.9}$***)***

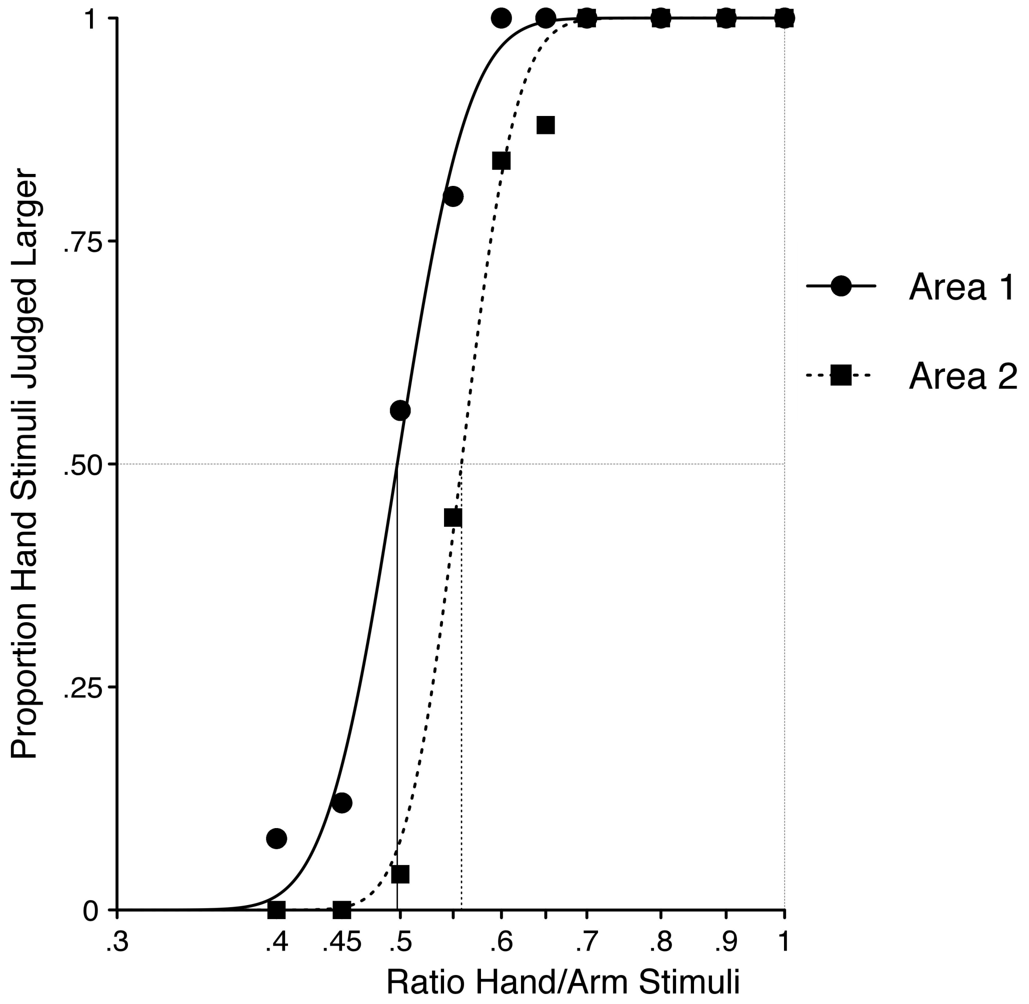


Figure 4. 25 ***Results of the simulation with*** $\boldsymbol{thr = 0.7}$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	0.06
Second Layer	0.56	0.44	

Table 4. 17 ***PSE, Weber's Illsuion and Rescaling Process.***

Activation Threshold: $\boldsymbol{thr = 0.5}$ ***(reference:*** $\boldsymbol{thr = 0.9}$)

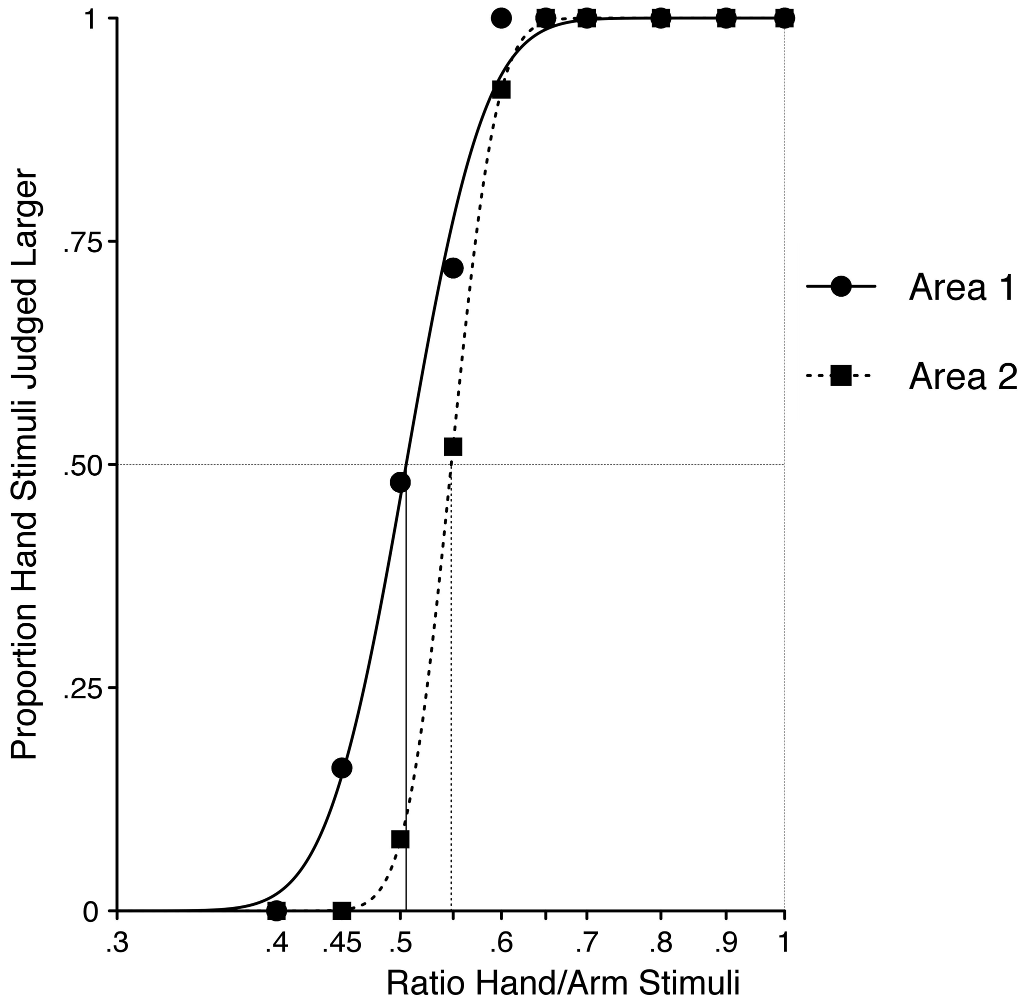


Figure 4. 26 ***Results of the simulation with*** $\boldsymbol{thr = 0.5}$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	0.05
Second Layer	0.55	0.45	

Table 4. 18 ***PSE, Weber's Illsuion and Rescaling Process.***

Activation Threshold: $\boldsymbol{thr} = \mathbf{0.3}$ ***(reference:*** $\boldsymbol{thr} = \mathbf{0.9}$)

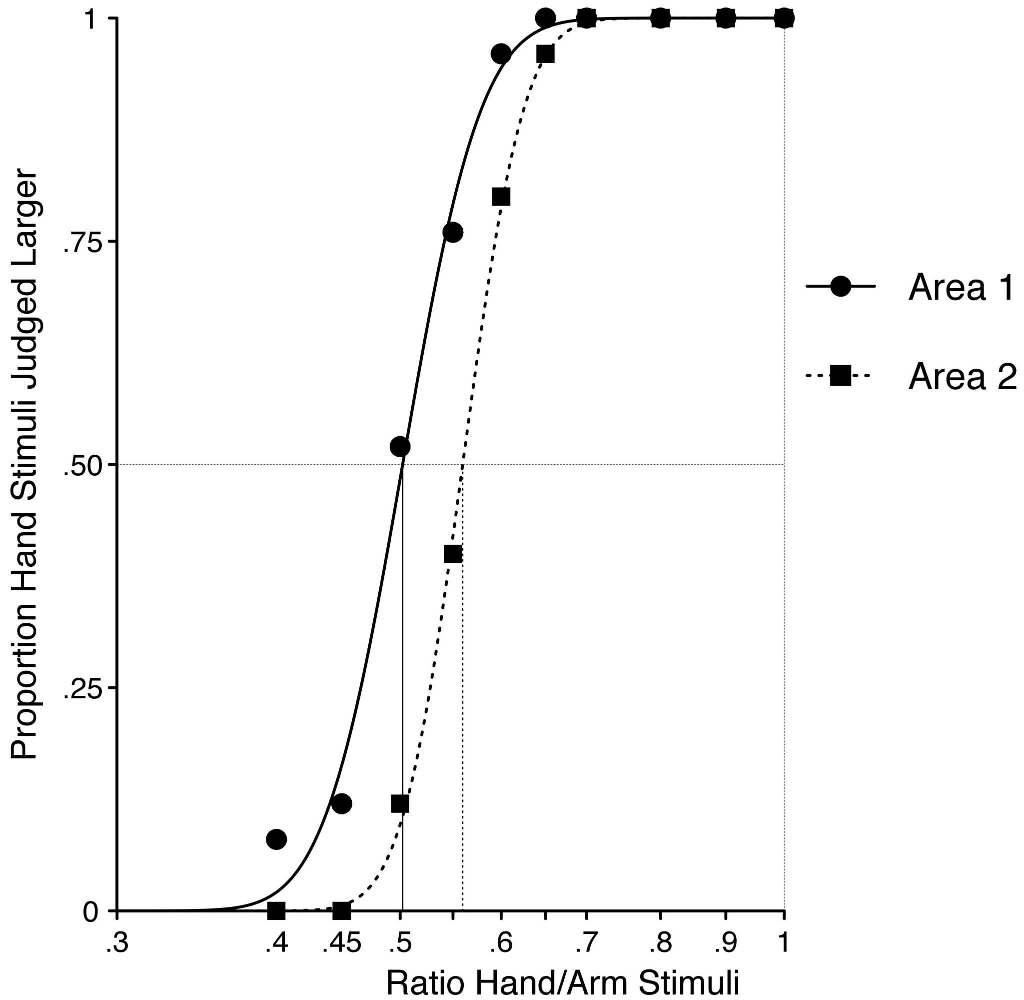


Figure 4. 27 ***Results of the simulation with*** $\boldsymbol{thr} = \mathbf{0.3}$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	0.06
Second Layer	0.56	0.44	

Table 4. 19 ***PSE, Weber's Illsuion and Rescaling Process.***

Activation Threshold: $\boldsymbol{thr = 0.1}$ ***(reference:*** $\boldsymbol{thr = 0.9}$***)***

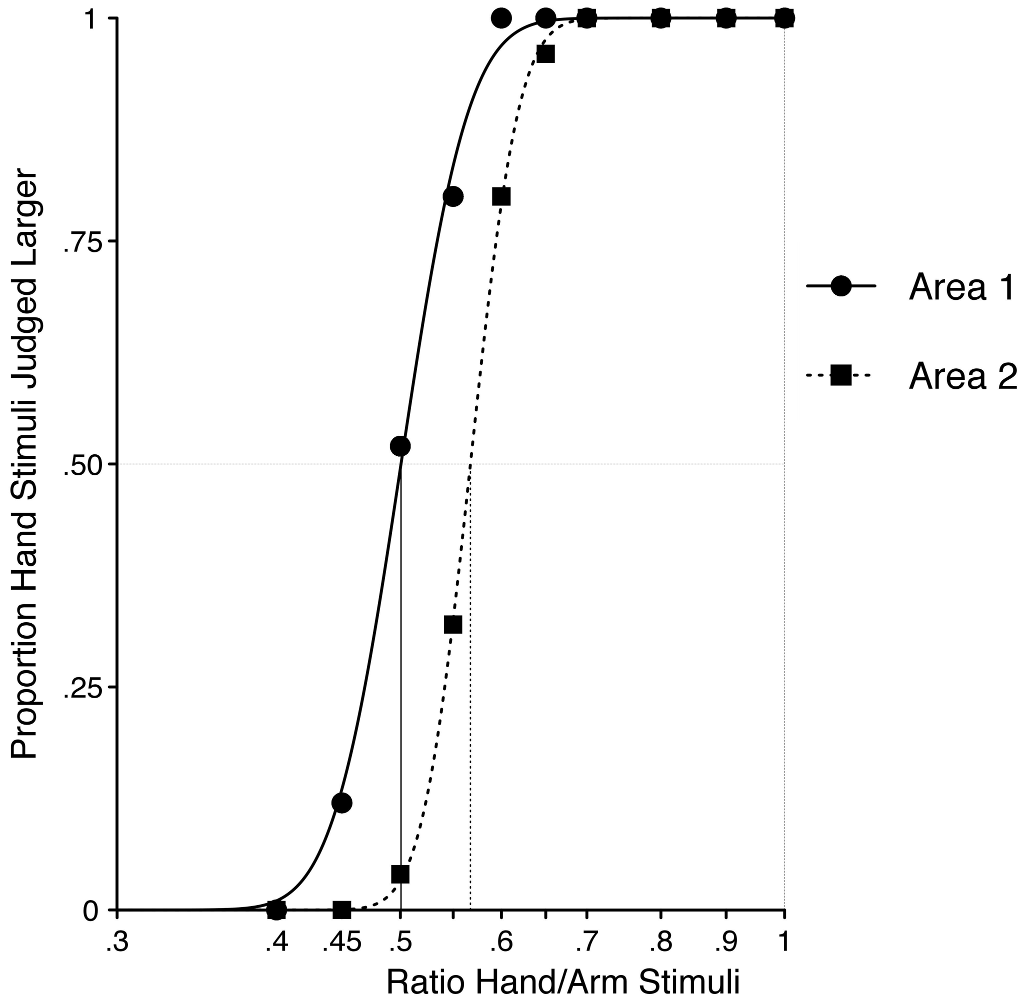


Figure 4. 28 ***Results of the simulation with*** $\boldsymbol{thr = 0.1}$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.5	0.5	0.07
Second Layer	0.57	0.43	

Table 4. 20 ***PSE, Weber's Illsuion and Rescaling Proces.***

Also in this analysis, the average difference, in terms of inactivated neurons, among the distance between the activation balls in “Area 2”, and the distance between the activation balls in “Area 1”, has been computed for each examined case:

Activation Threshold	Entity of the changing from the original value	Average Difference between Area 2 - Area 1	
		Arm	Hand
$thr = 0.8$	**- 0.1**	*+3.33*	*+0.04*
$thr = 0.7$	**- 0.2**	*+2.7*	*+0.04*
$thr = 0.5$	**- 0.4**	*+2.4*	*+0.1*
$thr = 0.3$	**- 0.6**	*+2.3*	*+0.1*
$thr = 0.1$	**- 0.8**	*+ 2.1*	*-0.75*
$thr = 0.9$	**0**	*+ 4.01*	*+0.02*

Table 4. 22 *Average Differences between the activation balls.*

The results highlighted in blue correspond to simulation performed with reference value of the threshold.

Decreasing the activation threshold down to 0.5 produces a small decrement of “Area 2”’s PSE. By further decreasing of the threshold value (thr = 0.3 and thr= 0.1), PSE shows a slight increase.. However, a continuous decrement is present in the ***average difference*** about the Arm, as the threshold decreases (table 4.22).

Reasons have to search in the activation value of the neurons positioned in the gap between the activation balls. Focusing on the Arm’s case (figure 4.29), in “Area 1” the activation balls are composed by neurons strongly activated (almost every one equal to 1), whereas neurons between the bubbles are inactivated, with value of the activation close to zero. So, in this situation there is a strong boundary between activation balls and inactivated neurons. Therefore, the changing of the threshold has not affected the dimension of the activation balls.

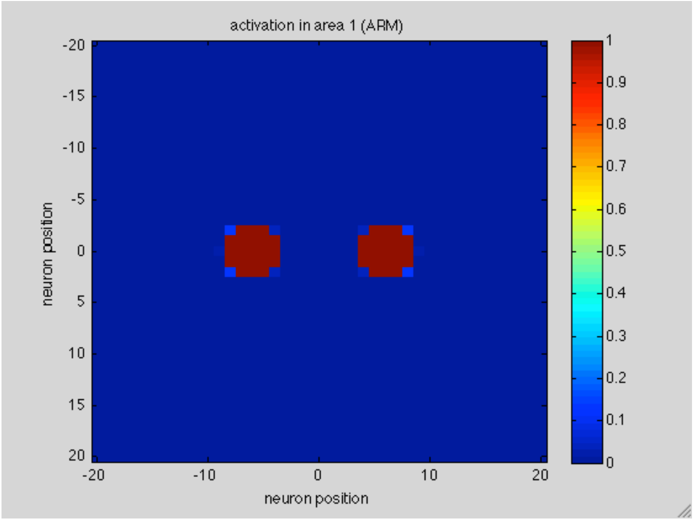


Figure 4. 29 *Example of neural activation within "Area 1" (Arm): input stimuli distances = 3 cm.*

However, the situation is different for "Area 2": here, the boundary between balls of activation and inactivated neurons is more blurred with respect to "Area 1" (see figure 4.30). Therefore, a drop of the activation threshold produced a decrease in the number of neurons interpreted as "inactivated" between the two bubbles, because those neurons that in the previous case (thr = 0.9) were under the threshold, now they are interpreted as belonging to the activated balls. This corresponds to a reduction of the distance between the activation balls within "Area 2.

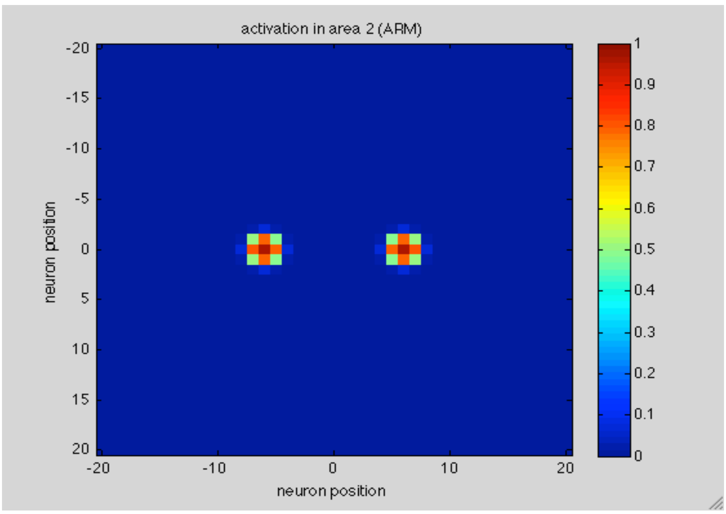


Figure 4. 30 *Example of neural activation within "Area 2" (Arm): input stimuli distances = 3 cm*

Focusing on the Hand's case, both "Area 1" and " Area 2" are characterized by a strong boundary between activation balls, and inactivated neurons, as we can observe in the next figures:

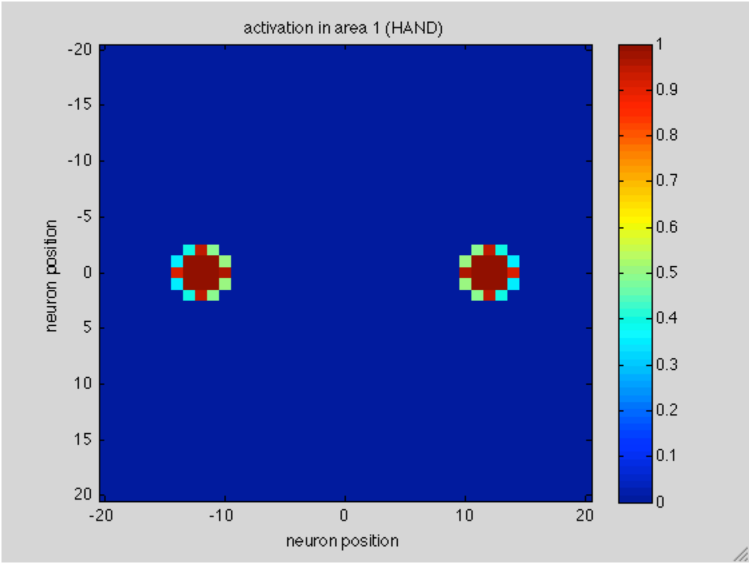


Figure 4. 31 ***Example of neural activation within "Area 1" (Hand): input stimuli distances = 3 cm.***

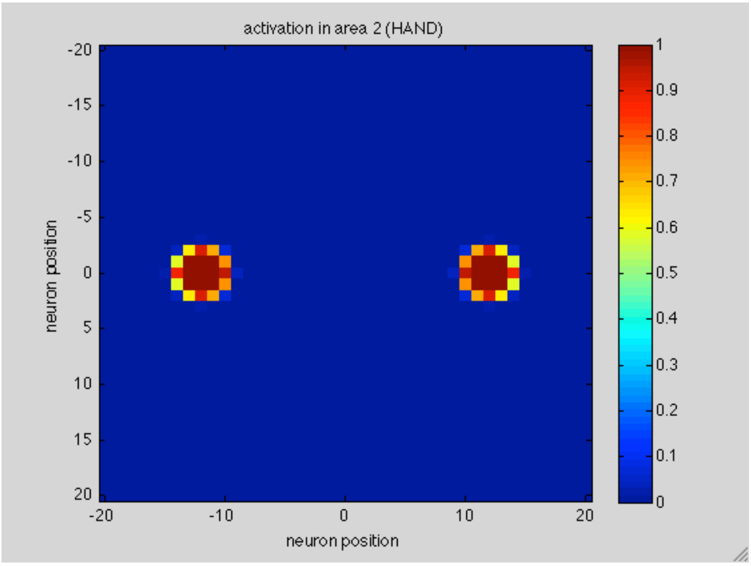


Figure 4. 32 ***Example of neural activation within "Area 2" (Hand): input stimuli distances = 3 cm***

In fact, the average difference about the Hand in table 4.22 is always close to zero. Therefore, the changing of the activation threshold were quite negligible. However, we can notice that only with a very low threshold = 0.1, the average difference become negative: namely, the distance between the activation balls inside “Area 2” is become quite smaller than “Area 1”, just because with ***thr* = *0.1***, the number of neurons considered active increases, that is the balls are read out as bigger,

Hence, in the Arm, the drop of the activation threshold coincides with an enlargement of the activation balls inside “Area 2” (and therefore the perceived distance between the balls decreases); on the contrary, in the Hand, the effect in “Area 2” is almost negligible. As a consequence, PSE of “Area 2” tends to move toward smaller ratio. However, the entity of the shifting due to a decrease of the threshold remains small: just because, the gap between the activation balls is a real “deep” gap, composed by strongly inactivated neurons. <u>This is a good aspect of the neural network, since we can assert that model behaviour exhibits only a mild dependence on the activation threshold.</u>

4.7 Increment of the Inhibitory Component

In every simulation conducted until now of the “Sensitivity Analysis” we have changed values of the parameters always in order to increase the excitatory wave, or decrease the inhibitory component. That because increasing the inhibitory component will lead to a situation in which neurons are less activated, and at the same time they are not activated enough over the reference Activation Threshold (0.9). Hence we will not obtain any activation balls, nulling every possibility to study the network. However, to investigate the behaviour of the model with an inhibitory increment of the Lateral Synapses within “Area 2”, a decrement of the activation threshold is needed for the activation balls formation. For example, decreasing the Activation Threshold until 0.6, neurons activated with a value of 0.7 will be considered as activated neuron,s forming part of the activation balls.

Indeed we have conducted few simulations increasing the inhibitory component of the Lateral Synapses within "Area 2" and contemporaneously decreasing the Activation Threshold. Simulation results are shown in the next pages:

Lateral Synapses "Area 2" : $\sigma_{in}^{s,A} = 6$ (reference : $\sigma_{in}^{s,A} = 2$) and Activation Threshold: $thr = 0.6$ (reference: $thr = 0.9$)

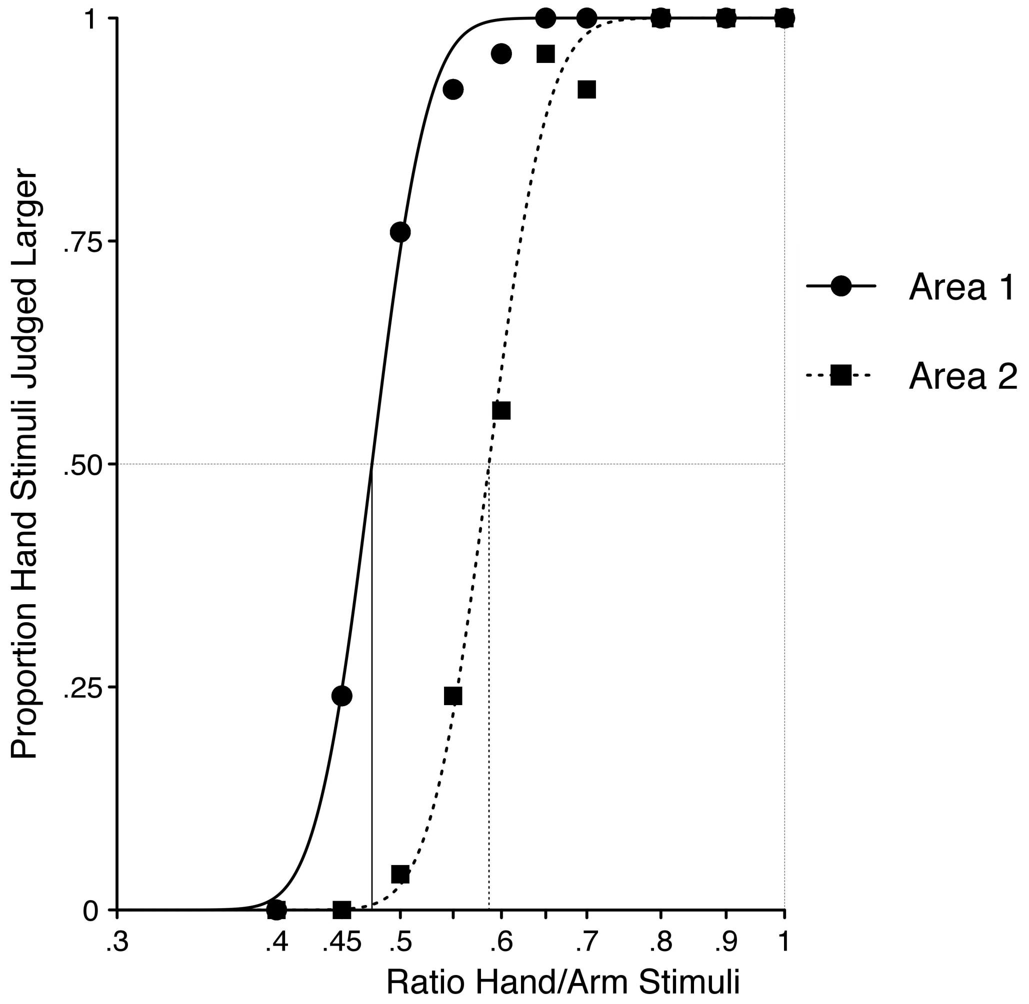


Figure 4. 33 *Results of the simulation with $thr = 0.6$ and $\sigma_{in}^{s,A} = 6$*

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.48	0.52	0.11
Second Layer	0.59	0.41	

Table 4. 21 *PSE, Weber's Illsuion and Rescaling Proces.*

As we can observe, in this simulation we have strongly increased the standard deviation of the inhibitory component of the Lateral Synapses within “Area 2”, and, at the same time we have decreased the Activation Threshold until 0.6. After this kind of alteration results suggest that the Rescaling process seems to be a bit strong then the reference simulation, even if the general result seems to be equivalent to the reference one. Continuing with further alteration of the inhibitory parameters, and the activation threshold:

Lateral Synapses “Area 2” : $\boldsymbol{\sigma_{in}^{s,A} = 8}$ ***(reference :*** $\boldsymbol{\sigma_{in}^{s,A} = 2}$***) and***

Activation Threshold: $\boldsymbol{thr = 0.5}$ ***(reference:*** $\boldsymbol{thr = 0.9}$***)***

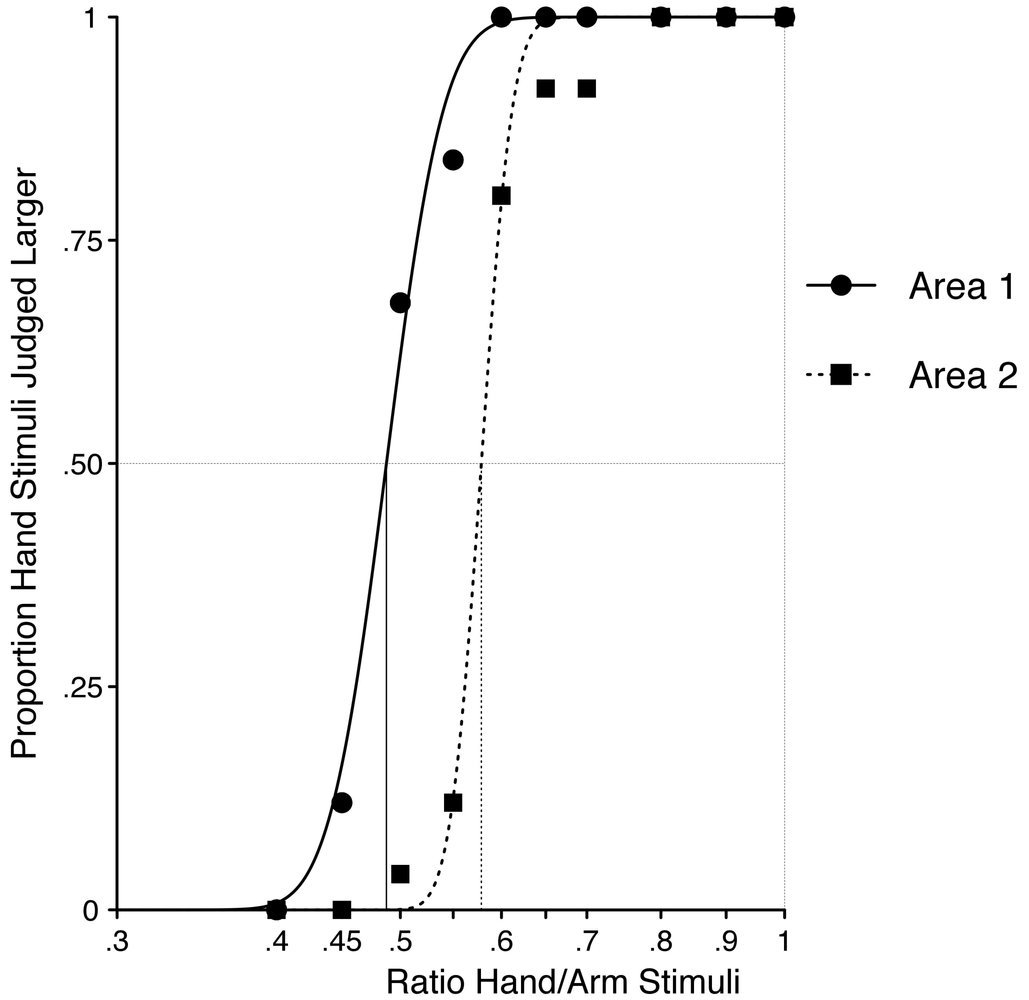


Figure 4. 34 *Results of the simulation with* $thr = 0.5$ *and* $\sigma_{in}^{s,A} = 8$

	PSE	Weber's Illusion = 1 – PSE	Rescaling Process = PSE Area2 – PSE Area1
First Layer	0.48	0.51	0.1
Second Layer	0.58	0.42	

Table 4. 22 ***PSE, Weber's Illsuion and Rescaling Proces.***

A further increment of the $\sigma_{in}^{s,A}$ until 8, and a further decrement of the Activation Threshold until 0.5 produce results of figure 4.34. Results obtained are coherent with the nature of the model: the strong increment of $\sigma_{in}^{s,A}$ has created a situation in which the most excited neurons have an activation state of about 0.6, thereby to record activation balls a threshold imposed to 0.5 is needed.

PSE of the second layer and PSE of the first layer do not bias so much from the "reference results". In other words, these results (I mean even the results of figure 4.33) assert that the model appears to be strong, and it can resist well to an alteration of some parameters, giving in output the same "Reference Results" of the first simulation (paragraph 4.1).

CONCLUSIONS

The same distance between two punctual stimuli applied on the body surface is perceived different across the body surfaces, an illusion known as **Weber's Illusion** (from the name of the researcher who first described such effect in the scientific literature). The differences in receptors density across body regions, and the distorted body image inside the Primary Somatic Sensory Cortex (homunculus) are good starting points to investigate the Weber's Illusion. However, it seems not sufficient to consider only these aspects. Indeed experimental results conducting along the last 100 years about this illusion have led to the idea that higher order cortical areas, and other mechanisms are involved in the perception of tactile distance. This idea has mainly arisen from the observations that the illusion is much smaller than the differences in receptor density, or cortical extent. Hence, perception of tactile distance might involve cortical areas that operate a sort of "Rescaling Process" able to reduce this illusion toward a verisimilar perception.

In the present thesis, we have tried to reproduce this illusion, by means of a neural network model. The model is composed of two layers of neurons, able to simulate the perception of the tactile distance between two punctual stimuli applied on a virtual skin surface. In particular, we have simulated two different body regions characterized by different receptor density and cortical magnification (as the Hand and the Arm), in order to reproduce the Weber's Illusion as it was descripted in literature. This project wishes to provide insight into the functional mechanisms that lie under the tactile perception, especially tactile distance perception, and not to replicate physiologic and anatomic details.

The development of the neural network (in MATLAB environment) was based on some simplification hypothesis. First, the presence of just two layers of neurons: the "First Layer" considered as representing a part of the Primary

Somatic Sensory Cortex, receiving directly inputs from the stimuli applied on the skin; and the “Second Layer”, considering as representing as higher cortical areas. The second hypothesis concerns the role of these two layers: the first layer was interpreted as the one affected by the Cortical Magnification as it occurs in the primary somatosensory cortex, whereas the second layer as the area providing the rescaling process, that is where distortions in tactile distance perception occurring in the first layer are party compensated. The two layers are connected by Feed-Forward synapses; moreover Lateral Synapses are present inside each layer. Each neuron in the first layer is characterized by a tactile Receptive Field; the RFs size on the simulated hand, was implemented smaller than on the simulated Arm, in order to reproduce the higher tactile resolution in the Hand compared to the Arm. Another important hypothesis introduced in the model concerns how we read out network output. Indeed, in order to assess network behaviour in terms of tactile distance perception, we need a quantity that represents distance perception starting from the neuron population activity. The simulation of two punctual stimuli on the skin surface produces a typical pattern of neural activation inside the two layers, with the formation of two balls of activated neurons within each layer. We have hypothesized that the perceived distance might be read out as the number of inactivated neurons between the two activation balls.

The so developed model is able to reproduce the Weber’s Illusion in which for example, the same input stimuli distance applied first on the Hand, and then on the Arm, was perceived different. Conducting many simulations with different pairs of distances applied on the two body regions, we have computed the perceived distance considering only the “First Layer”, and then the perceived distance at the “Second Layer” (resulting from the interaction between the tow layers). Results have shown that considering only the processing in the “First Layer”, the ***Point of Subjective Equality (PSE)*** (i.e. the point at which the applied distance on the hand is judged equal to the applied distance on the arm) was quite small, equal to 0.48 (this means that, to perceive the same distance on the two body regions, the actual distance on the arm must be more than double than the actual distance on the hand). Considering also the processing in the “Second Layer”, the illusion is reduced and the PSE is increased up to 0.6.

Notice that in the ideal case (no Illusion) the PSE should be equal to 1. Therefore, the illusion (or distortion) was reduced of about the 20% within the "Second Layer". Value of PSE obtained in the "Second Layer" of the model (0.6) is similar to the PSE obtained in experiments conducted of real subject, (see Green's paper in which PSE is equal 0.62); hence, the model reproduces quite well the experimental data.
Furthermore, the neural network is able to reproduce another results of experimental literature. In fact, real data about the hand and arm, show that there is an almost linear function (with a positive slope) between real applied distance and perceived distance, and that these functions on the two body parts tend to diverge as applied distance increases. This means that as the applied distance increases, the distance perception on the two body regions tends to become more and more different. These results can be replicated with the neural network model; this may further validate model architecture and assumptions. In fact the output of the model suggest that for big input stimuli distances the Weber's Illusion increase, and at the same time we have recorded a decrement of the Rescaling Process. This suggests us a possible explanation for the phenomena in which the Weber's Illusion increases as the stimuli dimension increase.

Moreover, model results about the ***two point discrimination thresholds*** on the simulated Hand and Arm are coherent with the in vivo experimental results: indeed, the neural network gave a smaller threshold for the hand with respect to the arm. In fact, acuity of the hand in the perception of two nearby stimuli is higher than on the arm, hence the *two point discrimination threshold* of the hand is smaller than on arm.

Finally, the neural network has been demonstrated to be robust against variations in some model parameters: in particular in the changing of the intensity of the synapses connections (for both Feed-Forward synapses and Lateral Synapses) and even the variation of the Activation Threshold. In particular, in the simulation in which we have increased the inhibitory component of the lateral synapses within "Area 2", and contemporaneously decreased the Activation Threshold, we have recorded about the same results obtained with the

“Reference Parameters”: this means that the dependence of the model from the Activation Threshold is not absolute. Conversely, the alteration of the *std* (standard deviation) of the excitatory component (or even the inhibitory component) in the Lateral synapses within the Second Layer has produced a dramatic changing in the mechanism of the Rescaling Process, nulling the rescale effect. For example, std increment of the excitatory component in the Lateral Synapses within “Area 2”, put the network in conditions to compute a PSE of the second Layer, equal to the “First Layer”: it is clear that in this situation, the Rescaling Process is completely off. The same result could be appreciated decreasing the *std* of the inhibitory component.

The present model, besides providing insights into the mechanisms of tactile distance perception and Weber’s illusion, in perspective it might be of value to make some predictions, which can be verified later, in vivo, by tactile experiments on real subjects.

Furthermore, in future works, this model could be associated, and unified, with the neural network implemented by my university mate Monti Luca, about the effect of stimulus orientation on tactile distance perception, in order to create a unique, more complete model to investigate the tactile distance illusion.

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Acknowledgments

I want to say thank you to my professor Elisa Magosso since she gave me the possibility to make this thesis project at the Birkbeck, University of London, giving me the possibility to make a wonderful, and different study experience that I have never tried.

Another important acknowledgment is direct to Dr. Matthew Longo due to his huge contribute in the analyse of the model results, as well as for his big availability to follow us even outside the university, doing also a fantastic guide around London.

A very big thank you to my family, always ready to support me in every moment, and to help me every time that I needed. I love you!

Finally, I have to say thank you to my Italian University mates (especially Luca Monti) whose I have shared 5 years of joys and sorrows, as well as to my new International friends that I have met in London during this long experience: even if it will be difficult to meet you guys anymore, I will remember you guys forever, because this experience in London was especial and unique even due to your presence…

…Enrico