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Epistemology of the Inflation Theory

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Abstract

The present work discusses the core mechanics of inflationary cosmology from the viewpoint of theoretical physics, and shows how the inflation theory has evolved through the decades. Hundreds of models were proposed with increasing complexity and flexibility, and inflation can nowadays be adjusted to accommodate any possible precision data gathered by collaborations successive to WMAP and Planck. The present work proceeds performing an analysis from the viewpoint of philosophy of science on this epistemologically worrying scenario in which empirical evidence seems unable to falsify, nor consequently verify, inflation. Bayesian confirmation theory and meta-empirical confirmation theory are discussed and employed, then an original epistemological framework, inspired by the multiverse, is proposed to further investigate whether inflation is science or something beyond.

Contents

1 History and motivation

The story of the hot Big Bang theory is an interesting one to tell, closely related to that of the Cosmic Microwave Background radiation. The idea of a primordial fireball was first thought of by the Russian George Gamow at the end of the 1940s, elaborating on the model of an adiabatic gas that cools down when expanding and heats up when contracting. As the gas' particles bounce off the walls of an encapsulating box, they preserve their kinetic energy. If the box has retracting walls, however, the particles lose kinetic energy with every bounce, reducing the temperature of the gas. Something similar happens in the expanding universe, even in the absence of walls, as particles travelling from one galaxy to different ones are locally measured to have a lower speed, because galaxies recede from each other. Therefore, the matter filling the universe cools down with time, and following, this evolution backwards in time, the same matter must be hotter as it is temporally closer to the origins of the universe, until its temperature diverges at the Big Bang itself, the origin of time.

Following the time evolution of the universe again, this hot matter in the state of plasma made of dissociated elementary particles, upon reaching a temperature of $10^9 K$ could combine to form atomic nuclei in the process labelled nucleosynthesis. These nuclei, together with leptons, heavily scattered all the radiation filling the universe, until a temperature of about $3000K$ could be reached, roughly 380000 years after the Big Bang. After that, nuclei and electrons could combine into stable, electrically neutral atoms, mostly transparent to radiation, which in turn started propagating in a straight line across the universe. This event that left radiation free to propagate is called surface of last scattering, and the radiation that was unbound is nowadays called cosmic microwave background radiation (CMB). Gamow's team estimated the present temperature of the radiation at about only $5K$.

At Gamow's time, however, the scientific community didn't believe that a radiation at such a low temperature could be measured, and Gamow's prediction didn't experience any experimental follow up. More than a decade later, the American Robert Dicke independently reformulated the same idea of a primordial fireball leading to a permanent cosmic radiation. In 1965 his team in Princeton had prepared an experimental apparatus to measure the radiation, but the radiation had unexpectedly just been already measured at Bell Telephone Laboratories, less than fifty kilometres from there. Physicists Penzias and Wilson were developing an antenna to employ in the study of radio emission from the Milky Way and, after half a year of work, they could not get rid, nor make sense of a background noise radiation with a temperature of $2.7K$. It was exactly the relic radiation postulated by Gamow and later again by Dicke, and for this discovery they were awarded the Nobel prize in 1978. No prize has ever been assigned for the prediction of the CMB.

Following this experimental confirmation, the hot Big Bang theory that was developed upon this idea gained immense popularity among the scientific community and had success in explaining cosmological phenomena of crucial relevance. The hot Big Bang theory provided a framework to embed the expansion of the universe first observed by Edwin Hubble in 1929 and still lacking an explanation at the time, and the cosmological redshift together with it. Moreover, the theory prescribed nucleosynthesis ratios very close to the element abundances observed in the universe with about 75% hydrogen and 25% helium, with traces of deuterium and lithium and even fewer heavier elements. Most of these abundances have been experimentally measured and confirmed, even though some of them do not coincide with the predictions and are still open problems in astroparticle physics, such as the primordial Lithium problem [1]. The hot Big Bang theory also allowed to estimate the age of the universe to roughly 13.8 billions of years by extrapolating the current expansion rate backward in time, putting together measures of the current Hubble constant, H_0, with the measurement of the CMB, by itself considered an enormous achievement of the theory.

Among all this success, however, the hot Big Bang left many questions unanswered, a few of them arising precisely from subtleties in the problems considered solved with this theory.

First, the universe is enormous. Only its visible part is estimated to contain 10^{90} particles. The hot Big Bang theory doesn't address what banged, why it did it, or what existed before the bang. This theory, as a matter of fact, doesn't describe the bang at all, and only explains what happened afterwards. Similarly, the hot Big Bang theory doesn't explain where the matter in the universe came from. It only states that the matter, which was already there, was very compressed and at a stunningly high temperature, and might have been in a different, unknown state. It is very difficult to imagine how a stunning number such as 10^{90} could rise in a model involving only geometrical quantities such as natural numbers and elementary mathematical quantities such as π and e, unless such a model involves in some way an exponential computation [2].

Even though the regular hot Big Bang theory provides a framework for the Hubble expansion, it remains to be explained how this expansion began in first place. The repulsive gravity attributed to dark energy provides just enough repulsion to keep the universe expanding in such a way that each couple of points recede with a speed in first approximation proportional to their distance, according to Hubble law:

$$d_L = \frac{1}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + o(z^3)\right] \simeq \frac{z}{H_0}. \tag{1}$$

In the above equation, d_L represents the luminosity distance,

$$z = \frac{\lambda_{Observed} - \lambda_{Source}}{\lambda_{Source}} \tag{2}$$

the cosmological redshift and q_0 the current deceleration parameter (labelled *deceleration* for historical reasons, since the expansion is nowadays known to be accelerating). Such a fine tuning, providing just enough repulsion to have an approximately linear recession of any given two points in the universe, appears suspicious and requires deeper investigation.

When Penzias and Wilson measured the CMB in 1965, they did so for a single wavelength and, to determine whether this radiation was only a line in a broad spectrum, almost three decades were necessary. It was only with the launch of NASA's Cosmic Background Explorer (COBE) satellite in 1990 that the CMB spectrum was finally measured, and the subsequent data released in 1992 revealed the same intensity in all directions with a precision of one part per thousand [3]. This result may be further corrected, taking the motion of the Earth into account to the startling precision of one part over a hundred thousand. Such a precision in inexplicable by means of random causes, and such an homogeneity in the CMB temperature could only been explained if all the directions it comes from had been in causal contact long enough to reach thermal equilibrium. But for that to be possible by the time of the last scattering, causal interactions must have propagated at a speed a hundred times larger than the speed of light, obviously forbidden by special relativity. This matter, baptised *the horizon problem* suggests that the universe might have been in thermal equilibrium much before the last scattering, possibly a small fraction of a second after the Big Bang, prescribing an expansion unbelievably intense, much faster than the regular Hubble expansion.

Furthermore, in light of the horizon problem, the COBE measurements pointed at a universe definitely homogeneous and isotropic on a cosmic scale. But the COBE satellite's resolution was too large and subsequent measurements, performed by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite launched in 2001 [4] and later the Planck satellite launched in 2007 [5,6], were able to map with a much higher resolution the CMB anisotropies with Gaussian distributions and a fractal-like scale-invariant spectrum, and the regular hot Big Bang theory can't make sense of it.

Another fine-tuning scenario arises when taking into account the density parameter, Ω, defined as the fraction between the density of the universe ρ over the critical density, $\rho_{\text{Crit}} = \frac{3H^2}{8\pi G_N}$, that would make the universe perfectly flat. The range of values accepted by the scientific community for the density parameter falls around the unity with little deviation, and it is a remarkable observation because, even though a unitary density parameter is an unstable equilibrium point in the hot Big Bang cosmology and a flat universe would remain indefinitely flat, small deviations from unity in the early universe would be enormously amplified with time. Precisely,

$$\Omega - 1 \propto t \tag{3}$$

during the radiation-dominated era and

$$\Omega - 1 \propto t^{\frac{2}{3}} \tag{4}$$

during the matter-dominated era. In order to have the current closeness to unity, the density parameter in the first second of life of the universe must have been close to unity with a precision of one part in 10^15. Even the smallest deviation from unity at that time would have led to a universe that either quickly collapsed or expanded too rapidly for large-scale structures to form, and for us to be here inquiring its origins. The only way out of such a ridiculous fine-tuning problem, labelled *the flatness problem*, is to adopt a different early universe model that pushes the density parameter towards unity instead of away from it.

Moreover, all unified theory prescribe that extremely massive particles carrying a net magnetic charge, called monopoles, should have been produced during nucleosynthesis in such a quantity that they would be basically the dominant mass in the universe by a factor of 10^{12} [7]. The total absence of these monopoles, together with different Big Bang relics, in the observed universe is called *the monopole problem.* A possible solution is that these monopoles might have originated in such an early time in the universe to have missed most of the expansion and particle production dynamics, ending up to be only an infinitesimal part of the universe.

Finally, the destiny of the universe and of the human species according to the regular hot Big Bang cosmology is far from optimistic. Both a Big Crunch, in the case of a supercritical universe, and a thermal death, in the case of a subcritical universe, lead to our extinction and to the oblivion of all our accomplishments. It is thus natural to find a way out of this doom, even though not so impending, by investigating models that will allow us to secure ourselves a future.

To address many of these issues (the CMB anisotropy had not been discovered), the inflation theory was developed in the 1980s as a first order phase transition, with bubble formation, experienced the universe from a false vacuum state resulting from the symmetry breaking between electromagnetic, strong and weak interactions [15]. The models based on this assumption, however, were found to be incompatible with observational data and were soon abandoned, but the idea of inflation continued developing by shifting onto different assumptions and, strong of a great experimental success, gained increasing credibility among the scientific community.

Following Starobinsky's first model in 1980 [8], Alan Guth presented a much simpler inflationary model, now called *old inflation* [9]. Having an exponential expansion of the universe immediately after the hot Big Bang seemed to solve most of the major problems discussed above [10].

An exponential growth in the matter filling the universe is exactly the support needed to explain its inconceivable largeness. A growth described by

$$a(t) = e^{H_{\mathrm{Infl}} t} a_{\mathrm{Infl}}, \tag{5}$$

where H_{Infl} and a_{Infl} respectively represent the Hubble parameter and the cosmic scale at the beginning of inflation, translates the onerous task of justifying the stunning number of 10^{90} particles estimated to fill the universe today to the much more approachable problem of explaining sixty to seventy e-foldings. Astroparticle physics easily fulfils the task of producing much more than seventy e-foldings, in accordance to the prediction of inflationary cosmology dictating that the nowadays visible universe is only a small fraction of the entire cosmo.

It sounds by any means suspicious that energy (or mass) conservation may be so bluntly violated, but in the conservation principle there is an important loophole, known since the 1930s, that might actually allow matter to be created. To a gravitational field it is natural to associate a negative energy both in Newtonian gravity and in general relativity. During inflation, the total energy is conserved: as more and more positive energy appears in the form of matter expanding at constant density, more and more negative energy is stored in the gravitational field that permeates the whole space, balances the positive one.

Energy conservation aside, the dynamics that would allow some form of matter to expand at constant density need to be properly justified. The main idea is that there may be a different, exotic form of energy that produces a repulsive form of gravity. This is mathematically allowed because, according to general relativity, gravity is produced by both energy density and pressure, while in Newtonian mechanics pressure doesn't gravitate. Removing the energy density from the equation, there are convincing theoretical arguments in particle physics that would allow a state of matter to exhibit a negative pressure, even though this form of matter could only exist at energy scales far beyond the current experimental capabilities. A negative pressure brings about a repulsive gravity, theoretically allowing inflation to be realised.

The flatness problem is easily solved by inflation. Imagining a universe with the dimensions of the order of Planck's length ($10^{-35}m$) undergoing an expansion by a factor of at least 10^{26} (or e^{60}) in less than $10^{32}s$, like a quickly inflating balloon, exponentially drives the density parameter towards unity:

$$\Omega - 1 \propto e^{-2H_{\text{Infl}}t}. \tag{6}$$

An important remark is that, as long as inflation lasts long enough, the above result is basically independent of the value assumed by the density parameter at the beginning of inflation. This also explains the CMB isotropy: a very small universe at the beginning of inflation could easily have been in thermal equilibrium, even at a temperature unbelievably high, and inflation would have preserved such an equilibrium, with the CMB anisotropies (not known in the 1980s), explained by means of quantum fluctuations arising during the inflation process and being exponentially amplified by the process itself [11]. The same reasoning holds for density fluctuations that, amplified by inflation, would have led to the formation of great local inhomogeneities, corresponding to large-scale structures in the universe such as galaxies and clusters of galaxies.

The monopole problem is also addressed by inflation. In inflationary models, monopoles and different relics may be simply eliminated from the equation by adjusting parameters so that their synthesis happens before or in the earliest stages of inflation, ending up so diluted that it is impossible to observe them today.

Such an explanatory power made inflation the protagonist of modern cosmology and, even though it was far from free of issues, the community worked hard on improving the theory, testing all kinds of imaginative assumptions, instead of disregarding it altogether.

The greatest issue faced by Guth's old inflation, just as many other primitive models, was the *graceful exit problem*. It was related precisely to the first, fundamental assumption of the theory, that the universe experienced a first order phase transition from a false vacuum state, prescribing the phenomenon of bubble formation. In order for inflation to end, and provide an acceptable link to standard hot Big Bang cosmology, it is essential to have a controlled amount of inflation, where vacuum energy gradually diminishes and inflation decelerates instead of continuing indefinitely, and the remaining vacuum energy is converted into particles and radiation, reheating this cooled primordial universe. If the bubbles associated with the phase transition are formed too close to one another, their collisions would make the universe extremely inhomogeneous, as opposed to the observed homogeneity on a cosmic scale. If the bubbles are formed too far from each other, they would end up causally disconnected, each representing an open universe with vanishing density parameter and undergoing thermal death. As Guth himself claimed, both possibilities must be ruled out and the bubbles forming at a favourable distance from each other constitute one more fine-tuning problem, that in turn leads to a shift of assumptions and the formulation of more advanced inflationary models.

The graceful exit problem was solved with the introduction by Andrei Linde of a different inflation model, nowadays known as *new inflation* [12] as opposed to old inflation. According to this model, inflation doesn't need to begin in the false vacuum state, thus avoiding the fine-tuning problem in the bubble formation phenomenon, and it is described as being driven by a scalar field, in a quasi-stable state on top of an almost flat potential, that slowly rolls down the potential towards a true minimum. The main difference between old and new inflation is that the core component of inflation, responsible for solving the major problems discussed, only occurs while the scalar field is already evolving from the false vacuum state and its time derivative doesn't vanish.

After having introduced the historical motivations that led to the formulation of the inflation theory, nowadays central to the whole field of cosmology, the present work will discuss its main technical details and implications in science and philosophy, with the following structure. Section 2 will explain with quantitative details the dynamics of Linde's new inflation, and specifically what initiates, drives and stops inflation, how large scale structures are predisposed in such an early stage, how ordinary energy and matter are generated and how this links to the standard hot Big Bang cosmology. Section 3 will discuss chaotic inflation, probably the most successful model within the inflationary paradigm, and will present additional inflationary models that are currently under discussion from en epistemological point of view. Section 3 will also cover the extension of the inflation theory to the suggestive realisation of eternal inflation and the multiverse, with the natural implications in philosophy of science. Section 4 will discuss some notions of epistemology and in particular confirmation theory from a Bayesian and meta-empirical point of view, following the recent work by philosopher Richard Dawid. Section 4 will also reconstruct the epistemological debate of the models presented in Section 2 and 3, and will apply the discussed confirmation theories to the debate, then it will discuss the points of criticism that have been raised on meta-empirical assessment by various researchers. Finally, Section 5 will draw original conclusions on the debate and will propose an additional non-empirical criterion for science assessment.

2 Mechanics of inflation

As it is known from basic cosmology, inflation requires a source of energy density that remains nearly constant while the universe expands exponentially. This is obtained by requiring that some potential energy dominates over the correspondent kinetic energy during inflation, and over every other kind of energy and matter until inflation ends. Both potential and kinetic energies are associated with a field that is considered to drive this whole first phase of expansion of the universe.

The natural choice for a field is a scalar field, ϕ, due to multiple reason. Picking any field other than a scalar (such as a vector or tensor field) would break the Lorentz symmetries proper of a scalar field and would point towards a frame dependent inflation, which would deny the Copernican principle. A potential associated to the field that is hypothetically responsible for initiating and driving inflation needs to be invariant under coordinate transformations, otherwise, by performing a transformation, one would be able to obtain a different evolution of the universe (or even deny this evolution), which is physically simply unacceptable. Thus, constructing a Lorentz-invariant potential out of a non-scalar field would require some operation to contract the field's index(es) to solve the inconsistency, and this would of course make the description much more complicated. An additional reason for a scalar field is that it inherits the intrinsic simplicity of the scalar field dynamics. While Section 4 will argue that simplicity is not a feature towards which the fathers of the inflation theory have shown commitment to, since a scalar field is a plausible choice, there is no need to artificially increase the complexity of the work. Some advanced inflationary models do involve non-scalar fields, but, as a scalar field is sufficient to discuss all the core features of inflation, they will be only hinted to in the present work.

Having only one component (for each spacetime event), the scalar field is also more keen on describing a homogeneous and isotropic universe, in agreement with the Cosmological principle, while any different theory would require multiple constraints to enforce the symmetries of the observed universe. Moreover, a scalar field at the quantum level naturally experiences fluctuations that lead to density perturbations that are crucial to explain the CMB anisotropies and the large-scale structures of the universe.

This Section is entirely devoted to presenting from a quantitative point of view the main features of inflation that will be important to understand the object of the epistemological analysis in Section 4. Subsection 2.1 will employ techniques from quantum field theory in a curved spacetime to derive the cosmological Klein-Gordon equation, which rules the evolution for a scalar inflaton field, and Subsection 2.2 will derive the slow-roll approximation parameters that allow the cosmological Klein-Gordon equation to describe the desired situation in which the inflaton field drives the exponential expansion with almost constant energy density. Subsection 2.3 will derive an equation of motion in Fourier space for the quantum fluctuations experienced by the inflaton field, crucial for understanding the formation of large-scale structures and the CMB anisotropies that link the inflation theory to observational cosmology, and Subsection 2.4 will solve this equation to understand how a Fourier mode of a quantum fluctuation evolves with time. Subsection 2.5 and

2.6 will respectively discuss perturbative and non-perturbative reheating, which are the two main processes that rule the decay of the inflaton field into ordinary matter and radiation and mark the transition from the inflationary era to the radiation-dominated era, and Section 2.7 will employ techniques from statistical mechanics to compute the reheating temperature that characterises the newly generated matter, which sets the initial conditions for the subsequent evolution and a fundamental link to standard hot Big Bang cosmology.

2.1 Inflaton field dynamics

Since the field to be described is a scalar one, according to quantum field theory the fundamental equation that describes its dynamics is the Klein-Gordon equation,

$$(\Box - m^2)\phi = 0, \tag{7}$$

with signature choice $(-,+,+,+)$. $\Box$ represents the d'Alembertian operator, defined as

$$\Box = \partial^\mu \partial_\mu = -\partial_t^2 + \nabla^2. \tag{8}$$

The formalism employed is that of quantum field theory in a curved spacetime with no back reaction, meaning that the inflaton field is treated as a probe field that does not interfere directly with the structure of spacetime. The curvature of spacetime is embedded in the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, which is the standard one in modern cosmology because it was specifically designed to describe a homogeneous and isotropic (at the cosmic scale) universe. For the sake of simplicity, it is of great convenience to adopt the in conformal time.

Conformal time (τ) is related to cosmic time (t) by the differential equation

$$d\tau = \frac{dt}{a(t)}. \tag{9}$$

Performing the substitution in the FLRW metric provides

$$\begin{aligned} ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] = \\ -\left(a(\tau)\, d\tau\right)^2 + a^2(\tau)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] = \\ a^2(\tau)\left[-d\tau^2 + \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \end{aligned} \tag{10}$$

For a flat universe ($k = 0$), this metric simplifies to (with Cartesian coordinates)

$$\begin{aligned} ds^2 &= a^2(\tau)\left[-d\tau^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \\ &= a^2(\tau)\left[-d\tau^2 + dx^2 + dy^2 + dz^2\right]. \end{aligned} \tag{11}$$

The aim is now to generalise the Klein-Gordon equation to the curved spacetime described by this metric, starting from the d'Alembertian operator, which in curved spacetime is written as the covariant divergence of the covariant gradient:

$$\Box\phi = \nabla_\mu \nabla^\mu \phi. \tag{12}$$

From differential geometry it is known that the covariant gradient of a scalar field is given by the ordinary partial derivative (since no affine connections arise from the field's indexes, which are none):

$$\nabla_\mu \phi = \partial_\mu \phi. \tag{13}$$

Instead, the covariant divergence of a vector field V^μ is defined as

$$\nabla_\mu V^\mu = \partial_\mu V^\mu + \Gamma^\mu_{\mu\lambda} V^\lambda \tag{14}$$

and the trace of the affine connection simplifies to

$$\Gamma^\mu_{\mu\lambda} = \frac{1}{2} g^{\mu\nu} (\partial_\lambda g_{\mu\nu} + \partial_\mu g_{\nu\lambda} - \partial_\nu g_{\mu\lambda}) = \frac{1}{2} g^{\mu\nu} \partial_\lambda g_{\mu\nu}, \tag{15}$$

because the last two terms are antisymmetric in the indexes μ and ν while the inverse metric tensor multiplying such terms is symmetric in those same indexes.

This expression can be further simplified by considering the variation of the determinant of the metric g in relation to the metric tensor itself and its derivatives. The variation of the determinant can be expressed using Jacobi's formula for invertible matrices, known from linear algebra[1]:

$$\delta(\det A) = \det A \cdot \mathrm{Tr}(A^{-1}\delta A) \rightarrow \delta g = g \cdot g^{\mu\nu} \delta g_{\mu\nu}. \tag{19}$$

Therefore, the partial derivative with respect to x^λ is

$$\partial_\lambda g = g g^{\mu\nu} \partial_\lambda g_{\mu\nu} \rightarrow g^{\mu\nu} \partial_\lambda g_{\mu\nu} = \frac{\partial_\lambda g}{g}. \tag{20}$$

This last term can also be related to the square root of the trace of the metric tensor (written as $\sqrt{-g}$ because with the chosen signature the trace happens to be negative):

[1]This formula can be also checked by considering a first-order expansion of the determinant of a perturbed metric:

$$g' = \det(g_{\mu\nu} + \delta g_{\mu\nu}). \tag{16}$$

For small perturbations the determinant can be approximated as

$$\det(g_{\mu\nu} + \delta g_{\mu\nu}) \simeq g\,(1 + g^{\mu\nu}\delta g_{\mu\nu}) \rightarrow g' \simeq g\,(1 + g^{\mu\nu}\delta g_{\mu\nu}). \tag{17}$$

Taking the variations yields

$$\delta g = g g^{\mu\nu} \delta g_{\mu\nu}, \tag{18}$$

showing the correctness of the formula.

$$\partial_\lambda(\sqrt{-g}) = \frac{1}{2\sqrt{-g}}\partial_\lambda(-g). \tag{21}$$

Applying the obtained formula for the derivative of a determinant (14) yields

$$\begin{aligned} \partial_\lambda(\sqrt{-g}) = \frac{1}{2\sqrt{-g}}(-gg^{\mu\nu}\partial_\lambda g_{\mu\nu}) = \frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\lambda g_{\mu\nu} \\ \rightarrow \frac{\partial_\lambda(\sqrt{-g})}{\sqrt{-g}} = \frac{1}{2}g^{\mu\nu}\partial_\lambda g_{\mu\nu} = \Gamma^\mu_{\mu\lambda}. \end{aligned} \tag{22}$$

Henceforth, the covariant divergence of a vector field becomes

$$\nabla_\mu V^\mu = \partial_\mu V^\mu + \Gamma^\mu_{\mu\lambda}V^\lambda = \partial_\mu V^\mu + \frac{\partial_\mu(\sqrt{-g})}{\sqrt{-g}}V^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}V^\mu), \tag{23}$$

and applying this result to the gradient of a scalar field gives the covariant d'Alembertian operator included in the Klein-Gordon equation:

$$\Box\phi = \nabla_\mu\nabla^\mu\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi). \tag{24}$$

This is a general expression for the d'Alembertian operator in curved spacetime now to be adapted for the FLRW metric in conformal time, whose metric and inverse metric tensor are

$$g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}, \tag{25}$$

$$g^{\mu\nu} = \frac{1}{a^2(\tau)}\eta^{\mu\nu}. \tag{26}$$

It is known from linear algebra that the determinant of a product of matrices is the product of their determinants, therefore the determinant of the metric tensor can be written as

$$g = (a^2(\tau))^4\det(\eta_{\mu\nu}) = a^8(\tau)(-1) = -a^8(\tau), \tag{27}$$

and the volume element becomes

$$\sqrt{-g} = \sqrt{-(-a(\tau)^8)} = \sqrt{a^8(\tau)} = a^4(\tau). \tag{28}$$

Now, in order to compute the specific form of the d'Alembertian operator (24), it is useful to split the temporal and spatial parts. The inverse metric tensor components are

$$g^{\tau\tau} = -\frac{1}{a^2(\tau)} \tag{29}$$

and

$$g^{ij} = \frac{1}{a^2(\tau)}\delta^{ij}, \tag{30}$$

therefore

$$\begin{aligned}\Box\phi &= \frac{1}{a^4(\tau)}\left[\partial_\tau\left(a^4(\tau)\left(-\frac{1}{a^2(\tau)}\right)\partial_\tau\phi\right) + \sum_{i=1}^{3}\partial_i\left(a^4(\tau)\left(\frac{1}{a^2(\tau)}\right)\partial_i\phi\right)\right]\\ &= \frac{1}{a^4(\tau)}\left[-a^2(\tau)\partial_\tau\left(a^2(\tau)\partial_\tau\phi\right) + \sum_{i=1}^{3}a^2(\tau)\partial_i\left(a^2(\tau)\partial_i\phi\right)\right]\\ &= -\frac{1}{a^2(\tau)}\partial_\tau(a^2(\tau)\partial_\tau\phi) + \frac{1}{a^2(\tau)}\sum_{i=1}^{3}\partial_i(a^2(\tau)\partial_i\phi)\\ &= \frac{1}{a^2(\tau)}\left[-\partial_\tau(a^2\tau)\partial_\tau\phi) + \sum_{i=1}^{3}\partial_i(a^2(\tau)\partial_i\phi)\right],\end{aligned} \tag{31}$$

and this is the analytic expression for the covariant d'Alembertian operator in the flat FLRW metric.

Remembering that the need to derive such an operator was the description of the dynamics of the scalar inflaton field by means of the Klein-Gordon operator, substituting the newly derived expression inside the Klein-Gordon equation yields

$$\begin{aligned}(\Box - m^2)\phi = 0 &\rightarrow \frac{1}{a^2(\tau)}\left[-\partial_\tau\left(a^2(\tau)\partial_\tau\phi\right) + \sum_{i=1}^{3}\partial_i\left(a^2(\tau)\partial_i\phi\right)\right] - m^2\phi = 0\\ &\rightarrow -\partial_\tau\left(a^2(\tau)\partial_\tau\phi\right) + \sum_{i=1}^{3}\partial_i\left(a^2(\tau)\partial_i\phi\right) - a^2(\tau)m^2\phi = 0.\end{aligned} \tag{32}$$

Considering a homogeneous and isotropic universe, the spatial derivatives of the field are expected to give null contribution, simplifying the equation to

$$-\partial_\tau\left(a^2(\tau)\partial_\tau\phi\right) - a^2(\tau)m^2\phi = 0. \tag{33}$$

Reverting the temporal variable to cosmic time, through the coordinate transformation defined by $d\tau = \frac{dt}{a(t)}$, gives

$$\begin{aligned}-a(t)\partial_t\left(a^3(t)\partial_t\phi\right) - a^2(t)m^2\phi &= -a^4(t)\ddot{\phi} - 3a^2(t)\dot{a}(t)\dot{\phi} - a^2(t)m^2\phi = 0\\ &\rightarrow \ddot{\phi} + 3\frac{\dot{a}(t)}{a(t)}\dot{\phi} + \frac{m^2}{a^2(t)}\phi = \ddot{\phi} + 3H\dot{\phi} + \frac{m^2}{a^2(t)}\phi = 0,\end{aligned} \tag{34}$$

where $H(t) = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter.

Finally, this equation is generalised by substituting the mass term with a generic potential $V(\phi)$, the potential that drives the inflation process, in terms of its derivative with respect to the inflaton field:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{35}$$

This equation is known as the cosmological Klein-Gordon equation.

2.2 Slow-roll approximation

Having derived in the previous Subsection the equation of motion that describes the evolution over time of the inflaton field, the present Subsection will perform some approximations that greatly simplify the description of inflationary dynamics. This will be of much help in the following Subsection when discussing the behaviour of quantum fluctuations of the inflaton field during inflation, that lead to the formation of large scale structures and CMB anisotropies in the observable universe. Those constitute fundamental links that anchor the theoretical construct of inflation to the empirical cosmological observations, that allow to test the different models within the theory to confirm or disconfirm them. The meaning of such observations will be the object of the epistemological analysis in Section 4, since inflation has recently been accused of not being concretely testable and thus of lying outside of normal, empirical science. The model constructed in the previous Subsection assumes that, even during the inflation process, the dynamics of the universe is governed by the Friedmann equations, derived from Einstein's field equations with the hypothesis of a homogeneous and isotropic universe, therefore by imposing the FLRW metric (in cosmic time). The first Friedmann equation with the flat universe hypothesis ($k = 0$) becomes

$$3\left(H^2 + \frac{k}{a^2}\right) = 8\pi G\rho \rightarrow H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \tag{36}$$

where the spatial derivatives are expected to give a null contribution in the case of a homogeneous and isotropic universe. In the above equation, ρ represents the energy density, which during inflation receives a dominant contribution from that of the inflaton field, now to be computed.

Noether's theorem, applied to a general spacetime symmetry, provides a general formula for the energy-momentum tensor of an arbitrary system dependent on the Lagrangian density of that system:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L}. \tag{37}$$

Since the inflaton is a scalar field by hypothesis, the Klein-Gordon Lagrangian density for a real scalar field can be used, with the only adjustment to include a potential, $V(\phi)$, that, just as discussed with the cosmological Klein-Gordon equation (35), replaces the usual mass term:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi). \tag{38}$$

The flat FLRW metric tensor (in cosmic time) is

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2), \tag{39}$$

henceforth the energy density of the inflaton field is given by

$$\begin{aligned}\rho = T_{00} = \partial_0\phi\partial_0\phi - (-1)\left(\frac{1}{2}\partial_0\phi\partial^0\phi + V(\phi)\right) = \\ = \partial_0\phi\partial_0\phi + \left(-\frac{1}{2}\partial_0\phi\partial_0\phi + V(\phi)\right) = \frac{1}{2}\dot{\phi}^2 + V(\phi),\end{aligned} \tag{40}$$

which is a sum of the kinetic energy density of the inflaton and its potential energy density.

Substituting the derived energy density into the simplified first Friedmann equation yields

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{41}$$

This equation is often written in terms of the reduced Planck mass (M_{Pl}):

$$M_{\text{Pl}} = \sqrt{\frac{1}{8\pi G}} \to H^2 = \frac{1}{3M_{\text{Pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{42}$$

A crucial simplification in inflationary cosmology is that the inflaton field is hypothesized to evolve slowly. Quantitatively, this is imposed by requiring that in the cosmological Klein-Gordon equation the $\ddot{\phi}$ term is negligible with respect to the other two:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \to 3H\dot{\phi} \simeq -V'(\phi). \tag{43}$$

Furthermore, while the inflaton field slowly rolls down its almost flat potential, its energy density is hypothesized to stay almost constant, with the potential energy density dominating over the kinetic energy density:

$$H^2 = \frac{1}{3M_{\text{Pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \to H^2 \simeq \frac{V(\phi)}{3M_{\text{Pl}}^2}. \tag{44}$$

Equations (43) and (44) complete the announced approximations, but for a practical purpose they are usually cast in terms of two controllable parameters, ϵ and η, baptised slow-roll parameters. The rest of the present Section shows how these parameters are defined and constrained, one at a time.

Condition (34) relies on the kinetic energy density of the inflaton field being negligible when compared to the potential energy density, thus

$$\frac{\frac{1}{2}\dot{\phi}^2}{V(\phi)} \ll 1. \tag{45}$$

Equation (36) implies

$$3H\dot{\phi} \simeq -V'(\phi) \to \dot{\phi} = -\frac{V'(\phi)}{3H} \to \frac{\frac{1}{2}\dot{\phi}^2}{V(\phi)} \simeq \frac{\frac{1}{2}\left(\frac{V'(\phi)}{3H}\right)^2}{V(\phi)}, \tag{46}$$

while equation (37) yields

$$H^2 \simeq \frac{V(\phi)}{3M_{\mathrm{Pl}}^2} \rightarrow \frac{\frac{1}{2}\left(\frac{V'(\phi)}{3H}\right)^2}{V(\phi)} \simeq \frac{1}{2\cdot 3}M_{Pl}^2\frac{V'^2(\phi)}{V^2(\phi)} \equiv \frac{1}{3}\epsilon, \tag{47}$$

providing the first slow-roll parameter and condition:

$$\epsilon = \frac{1}{2}M_{Pl}^2\frac{V'^2(\phi)}{V^2(\phi)} \ll 1. \tag{48}$$

In order to derive the second slow-roll parameter, it is useful to derive both members of equation (36) with respect to time:

$$3H\dot{\phi} \simeq -V'(\phi) \rightarrow 3H\ddot{\phi} + 3\dot{H}\dot{\phi} \simeq -V''(\phi)\dot{\phi} \rightarrow \ddot{\phi} \simeq -\frac{V''(\phi)\dot{\phi} + 3\dot{H}\dot{\phi}}{3H}. \tag{49}$$

Since equation (36) itself relies on the second time derivative of the inflaton field being negligible with respect to the other two terms in the cosmological Klein-Gordon equation, the second time derivative of the inflaton shall be compared, for example, to the friction term $3H\dot{\phi}$ term:

$$\frac{\ddot{\phi}}{3H\dot{\phi}} = -\frac{V''(\phi)\dot{\phi}}{3H^2\dot{\phi}} - \frac{\dot{H}\dot{\phi}}{3H^2\dot{\phi}} \ll 1. \tag{50}$$

Substituting equation (37) into the first term in the right-hand side of the equation above gives

$$H^2 = \frac{V(\phi)}{3M_{Pl}^2} \rightarrow \frac{V''(\phi)\dot{\phi}}{3H^2\dot{\phi}} = \frac{3M_{Pl}^2V''(\phi)}{V(\phi)} \equiv \eta, \tag{51}$$

providing the second slow-roll parameter and condition:

$$\eta = 3M_{Pl}^2\frac{V''(\phi)}{V(\phi)} \ll 1. \tag{52}$$

Regarding the second term in the right hand side of equation (43), deriving with respect to time equation (37) yields

$$\begin{aligned} H^2 = \frac{V(\phi)}{3M_{Pl}^2} \rightarrow 2H\dot{H} = \frac{V'(\phi)\dot{\phi}}{3M_{Pl}^2} \rightarrow \dot{H} = \frac{V'(\phi)\dot{\phi}}{6HM_{Pl}^2} \rightarrow \\ \rightarrow -\frac{\dot{H}}{3H^2} = -\frac{V'(\phi)\dot{\phi}}{18H^3M_{Pl}^2}. \end{aligned} \tag{53}$$

Moreover, equation (36) gives

$$\dot{\phi} = -\frac{V'(\phi)}{3H} \rightarrow -\frac{V'(\phi)\dot{\phi}}{18H^3M_{Pl}^2} = \frac{V'^2(\phi)}{54H^4M_{Pl}^2}, \tag{54}$$

and since equation (37) provides

$$H^2 = \frac{V(\phi)}{3M_{Pl}^2} \to \frac{M_{Pl}^2 V'^2(\phi)}{2V^2(\phi)} = \epsilon, \tag{55}$$

that is the first slow-roll parameter already known to be much lesser than one, and providing thus no further conditions.

2.3 Quantum fluctuations

During the inflationary phase of the early universe, the quantum nature of the inflaton field brings about quantum fluctuations that play a crucial role in explaining the large scale structures and the CMB anisotropies of the universe. These fluctuations are hence very important from a phenomenological point of view since they act as the only link from the theory of the early universe to observable, quantities anchored to the present time. Moreover, they provide a unique probe for the energy scales of the early universe, far beyond reach of the nowadays particle accelerators.

Considering, as stated in Subsection 2.1, the framework of quantum field theory in a curved spacetime with no back reaction, the inflaton field, ϕ, which is in principle a quantum field, may be decomposed into two components. The first, its expectation value, ϕ_0, undergoes a classical evolution satisfying the cosmological Klein-Gordon equation (35) with the approximations described in the previous Subsection, and it is considered homogeneous in agreement with the observations of a homogeneous and isotropic universe at a cosmic scale. The second component arises from the uncertainty principle that, according to quantum field theory, does not allow the scalar field to be perfectly homogeneous, and dictates that it must experience quantum fluctuations, $\delta\phi$, around its classical trajectory. Such fluctuations, even though small compared to the classical component, are stretched by the exponential expansion provided by the inflationary process to much larger scales, and they entail the nature of the inhomogeneities of the observable universe. Decomposing the inflaton field into the classical component and the quantum fluctuations provides

$$\phi(t, x) = \phi_0(t) + \delta\phi(t, x). \tag{56}$$

To derive the equation of motion for the perturbation, the action is linearised around the background field, taking only first order perturbations into account.

As already mentioned, in a flat FLRW universe the action for a scalar field is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right], \tag{57}$$

where $\sqrt{-g} = a^3(t)$.

The kinetic term in the action is

$$\begin{aligned} \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = \frac{1}{2} \left[-(\partial_0 \phi)^2 + \frac{1}{a^2} \left[(\partial_1 \phi) + (\partial_2 \phi) + (\partial_3 \phi) \right] \right] = \\ \frac{1}{2} \left[-(\partial_0 \phi)^2 + \frac{1}{a^2} (\nabla \phi)^2 \right]. \end{aligned} \tag{58}$$

Expanding $\phi = \phi_0 + \delta\phi$ provides

$$(\partial_0\phi)^2 = (\dot{\phi_0} + \dot{\delta\phi})^2 = \dot{\phi_0}^2 + 2\dot{\phi_0}\dot{\delta\phi} + (\dot{\delta\phi})^2, \tag{59}$$

and

$$(\nabla\phi)^2 = (\nabla\phi_0 + \nabla\delta\phi)^2 = (\nabla\delta\phi)^2, \tag{60}$$

because the classical field is considered homogeneous and thus with vanishing spatial derivatives.

The perturbative part of the kinetic term is

$$\frac{1}{2}\left(-2\dot{\phi_0}\dot{\delta\phi} - (\dot{\delta\phi})^2 + \frac{1}{a^2}(\nabla\delta\phi)^2\right). \tag{61}$$

The potential term in the action can be expanded around the classical trajectory as well:

$$V(\phi) = V(\phi_0 + \delta\phi) \simeq V(\phi_0) + V'(\phi_0)\delta\phi + \frac{1}{2}V''(\phi_0)(\delta\phi)^2, \tag{62}$$

where $V'(\phi_0)$ and $V''(\phi_0)$ are the first and second derivative of the potential with respect to the field, and evaluated at ϕ_0.

The perturbative part of the potential term is

$$V'(\phi_0)\delta\phi + \frac{1}{2}V''(\phi_0)(\delta\phi)^2, \tag{63}$$

where the perturbation is actually kept at second order because it will soon be useful to perform a derivation of the perturbative Lagrangian density with respect to the perturbation itself, effectively lowering the perturbative order by one power of $(\delta\phi)^2$.

Combining the kinetic and potential terms, the first-order perturbative action becomes

$$\begin{aligned}\delta_{\delta\phi}S = \int d^4x\, a^3(t) \Big[&-\dot{\phi_0}\dot{\delta\phi} - \frac{1}{2}(\dot{\delta\phi})^2 + \frac{1}{2a^2}(\nabla\delta\phi)^2 \\ &+V'(\phi_0)\delta\phi + \frac{1}{2}V''(\phi_0)(\delta\phi)^2\ \Big],\end{aligned} \tag{64}$$

from which the perturbative Lagrangian density can be read:

$$\mathcal{L} = a^3(t)\left[-\dot{\phi_0}\dot{\delta\phi} - \frac{1}{2}(\dot{\delta\phi})^2 + \frac{1}{2a^2}(\nabla\delta\phi)^2 + V'(\phi_0)\delta\phi + \frac{1}{2}V''(\phi_0)(\delta\phi)^2\right]. \tag{65}$$

The aim is now to impose the Euler-Lagrange equation for the perturbation:

$$\frac{\partial\mathcal{L}}{\partial\delta\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\delta\phi\right)} = \frac{\partial\mathcal{L}}{\partial\delta\phi} - \frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\dot{\delta\phi}} - \nabla\left(\frac{\partial\mathcal{L}}{\partial(\nabla\delta\phi)}\right) = 0. \tag{66}$$

Computing the different terms in the equation yields

$$\frac{\partial\mathcal{L}}{\partial\delta\phi} = a^3(t)V'(\phi_0) + a^3(t)V''(\phi_0)\delta\phi; \tag{67}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\delta\phi}} = a^3(t)\left(-\dot{\phi}_0 - \dot{\delta\phi}\right) \rightarrow \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\delta\phi}} = \frac{\partial}{\partial t}\left[a^3(t)\left(-\dot{\phi}_0 - \dot{\delta\phi}\right)\right]$$
$$= a^3(t)\left(-\ddot{\phi}_0 - \ddot{\delta\phi}\right) + 3a^2(t)\dot{a}(t)\left(-\dot{\phi}_0 - \dot{\delta\phi}\right); \tag{68}$$

$$\frac{\partial \mathcal{L}}{\partial(\nabla\delta\phi)} = a^3(t)\frac{2\nabla\delta\phi}{2a^2(t)} = a(t)\nabla\delta\phi$$
$$\rightarrow \nabla\left(\frac{\partial \mathcal{L}}{\partial(\nabla\delta\phi)}\right) = \nabla\left(a(t)\nabla\delta\phi\right) = a(t)\nabla^2\delta\phi, \tag{69}$$

again under the assumption that the universe is homogeneous and isotropic, and thus the spatial derivatives of the cosmic scale, $a(t)$, vanish.

Plugging everything into the Euler-Lagrange equation gives

$$\frac{\partial \mathcal{L}}{\partial \delta\phi} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\delta\phi}} - \nabla\left(\frac{\partial \mathcal{L}}{\partial(\nabla\delta\phi)}\right) =$$
$$= a^3(t)V'(\phi_0) + a^3(t)V''(\phi_0)\delta\phi + a^3(t)\ddot{\phi}_0 + a^3(t)\ddot{\delta\phi}$$
$$+3a^2(t)\dot{a}(t)\dot{\phi}_0 + 3a^2(t)\dot{a}(t)\dot{\delta\phi} - a(t)\nabla^2\delta\phi = 0. \tag{70}$$

This equation can be massively simplified by considering that the background field, ϕ_0, satisfies the background equation of motion:

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi_0) = 0, \tag{71}$$

and all these three terms are included in equation (70).

In fact a brief algebraic computation shows

$$a^3(t)\ddot{\phi}_0 + 3a^2(t)\dot{a}(t)\dot{\phi}_0 + a^3(t)V'(\phi_0) = a^3(t)\left[\ddot{\phi}_0 + 3\frac{\dot{a}(t)}{a(t)}\dot{\phi}_0 + V'(\phi_0)\right] = 0, \tag{72}$$

since $\frac{\dot{a}(t)}{a(t)} = H(t)$ is just the usual Hubble parameter.

Equation (70) then becomes

$$a^3(t)V''(\phi_0)\delta\phi + a^3(t)\ddot{\delta\phi} + 3a^2(t)\dot{a}(t)\dot{\delta\phi} - a(t)\nabla^2\delta\phi = 0, \tag{73}$$

which, dividing everything by $a^3(t)$, gives

$$\ddot{\delta\phi} + 3H(t)\dot{a}(t)\dot{\delta\phi} - \frac{\nabla^2\delta\phi}{a^2(t)} + V''(\phi_0)\delta\phi = 0, \tag{74}$$

that is the equation that rules the evolution of a quantum fluctuation of the inflaton field.

It is now time to harvest what has been seeded in the previous Subsection, that is, the slow-roll approximation regulated by the slow-roll parameters introduced. It was argued that, to describe a physical situation with the potential energy of the inflaton field dominating its kinetic energy, a flat potential is needed, which means that the first derivative of the potential must be small (compared to the kinetic term). However, to sustain inflation long enough to solve the problems discussed in Section 1, such as the flatness problems, also the curvature of the potential in the surroundings of the first point considered must be flat, therefore, according to single variable calculus, the second derivative of the potential with respect to the inflaton field must be small as well. In accordance with the second slow-roll condition, described by equation (52), the second derivative term in equation (74) can be neglected.

To show this, it is useful to consider, as discussed in the previous Subsection, that the energy density contribution of the potential dominates over the contribution of the kinetic energy density:

$$\rho \simeq V(\phi). \tag{75}$$

The first Friedmann equation with the flat hypothesis may then be approximated to

$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}V(\phi), \tag{76}$$

and, remembering the definition of the Planck mass, $M_{\mathrm{Pl}} = \sqrt{\frac{1}{8\pi G}}$, this is reduced to

$$H^2 = \frac{V(\phi)}{3M_{\mathrm{Pl}}^2} \rightarrow V(\phi) = 3M_{\mathrm{Pl}}^2 H^2. \tag{77}$$

Moreover, the second slow roll parameter prescribes

$$\eta = \frac{3M_{Pl}^2 V''(\phi)}{V(\phi)} \ll 1 \rightarrow V''(\phi) = \eta\frac{V(\phi)}{3M_{Pl}^2} = \eta\frac{3M_{Pl}^2 V(\phi)H^2}{3M_{Pl}^2} = \eta H^2, \tag{78}$$

where the last member of the equality is suppressed by the parameter η already known to be much lesser than one.

Neglecting the term with the second derivative of the potential, the evolution equation for a fluctuation is now

$$\ddot{\delta\phi} + 3H(t)\dot{a}(t)\dot{\delta\phi} - \frac{\nabla^2\delta\phi}{a^2(t)} = 0. \tag{79}$$

To make sense of the evolution of a fluctuation, it is appropriate to decompose the fluctuation into Fourier modes:

$$\delta\phi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left[\delta\phi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}} + \delta\phi^*_{\vec{k}}(t)e^{-i\vec{k}\cdot\vec{x}}\right]. \tag{80}$$

This form of the Fourier transform ensures that the scalar field is real, because, for a complex number z, $Re[z]+Im[z] \in \mathcal{R}$, while the $3/2$ power for the 2π coefficient imposes normalisation in three spatial dimensions.

Substituting into equation (79), turns out that each mode independently satisfies the differential equation

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k = 0. \tag{81}$$

2.4 Fluctuations evolution

To understand the solutions to this equation, it is important to discuss the fluctuation's behaviour based on whether its wavelength, described by the comoving wavenumber of the Fourier mode, k, is larger or smaller than the Hubble radius considered at the same time and thus the fluctuation may or may not receive causal influences.

The mentioned Hubble radius, $R_H(t)$, is defined as the distance, generally dependent on time, over which causal interactions can occur due to the expansion of the universe, in the time span of one Hubble time, $H^{-1} = \frac{a(t)}{\dot{a}}$, constant during the inflation.

Thus,

$$R_H(t) = \frac{c}{H(t)} = \frac{1}{H(t)}, \tag{82}$$

with the employed units. Beyond this distance, any two points in space recede from each other faster than the speed of light and can't therefore influence each other[2].

When a Fourier mode starts its evolution within the Hubble radius, it is referred to as being on a sub-horizon scale, even though its wavelength is compared to the Hubble radius and not the Hubble horizon. When this is the case, writing the wavelength as a constant measure stripped off of the evolution enforced by the cosmological parameter gives

$$a(t)\lambda \ll R_H = \frac{1}{H} \rightarrow \frac{1}{a(t)\lambda} \propto \frac{k}{a(t)} \gg H, \tag{83}$$

and the evolution equation for the perturbative mode (81) becomes

$$\frac{k^2}{a^2} \gg 3H \rightarrow \delta\ddot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k = 0. \tag{84}$$

This is a standard differential equation for a harmonic oscillator with a time-dependent frequency. To solve it, a solution of the following form is assumed.

$$\delta\phi_k(t) = A_k e^{i\omega t} + B_k e^{-i\omega t}, \tag{85}$$

where $\omega = \frac{k}{a(t)}$.

This choice provides

$$-\omega^2\delta\phi_k + \frac{k^2}{a(t)^2}\delta\phi_k = 0, \tag{86}$$

validating the assumption.

Therefore the general solution is

$$\delta\phi_k(t) = A_k e^{i\frac{k}{a(t)}t} + B_k e^{-i\frac{k}{a(t)}t}, \tag{87}$$

[2] A similar concept, the Hubble horizon, is different from the Hubble radius being not a measure local in time, but taking into account the evolution of the observable universe over its whole history: $H_H = c\int_0^t \frac{dt'}{a(t')}$.

but it is best expressed in terms of mode functions $u_k(t)$ to represent the amplitude of the perturbations, with a time-dependence related to the scale factor and the comoving wavenumber:

$$\delta\phi_k(t) = u_k(t)e^{i\frac{k}{a(t)}t} + u_k^*(t)e^{-i\frac{k}{a(t)}t}. \tag{88}$$

Here the mode functions encapsulate the slowly varying amplitude of the perturbation, while the exponential terms indicate the rapidly oscillating part. This separation of slow and fast degrees of freedom is useful to focus on the evolution of the amplitude, crucial for the second quantisation. It is therefore mandatory to properly normalise the mode functions.

To this end, it is better to return to position space and expand the scalar field $\phi(t,x)$ in terms of creation and annihilation operators, $(a_k, a_k^\dagger)$, as ordinarily provided by the framework of quantum field theory for a real scalar field.

$$\phi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[u_k(t)a_k e^{i\vec{k}\cdot\vec{x}} + u_k^*(t)a_k^\dagger e^{-i\vec{k}\cdot\vec{x}}\right]. \tag{89}$$

It is also important to compute the conjugate momentum, starting from the Lagrangian density defined by equation (65):

$$\pi(t,\vec{x}) = \frac{\partial\mathcal{L}}{\partial\delta\dot{\phi}} = a^3(t)\left(-\dot{\phi}_0 - \delta\dot{\phi}\right). \tag{90}$$

An important remark is that the time derivative of the fluctuation, when applied to equation (89) provides the time derivatives of the mode functions.

However, for the sake of the following computation's simplicity, it is useful to adopt a notation closer to equation (89):

$$\pi(t,\vec{x}) = a^3(t)\int \frac{d^3k}{(2\pi)^3}\left[\dot{u}_k(t)a_k e^{i\vec{k}\cdot\vec{x}} + \dot{u}_k^*(t)a_k^\dagger e^{-i\vec{k}\cdot\vec{x}}\right], \tag{91}$$

where the background dynamics, $\dot{\phi}_0$, is cut off from the notation and included in the derivatives of the mode functions.

As it is well known from quantum field theory, a scalar field and its conjugate momentum are required to satisfy the canonical commutation relation

$$[\phi(t,\vec{x}), \pi(t,\vec{y})] = i\,(2\pi)^3\,\delta^3(\vec{x}-\vec{y}), \tag{92}$$

which implies the following commutation relation for the creation and annihilation operators:

$$\left[a_{\vec{k}}, a_{\vec{k}'}^\dagger\right] = \delta^3(\vec{k}-\vec{k'}), \tag{93}$$

and a supplementary commutation relation for the mode functions, which can be uncovered by supposing true the commutation relation for the creation and annihilation operators and analytically checking relation (92).

$$[\phi(t,\vec{x}),\pi(t,\vec{y})] = \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}a^3(t)\left[u_{\vec{k}}(t)\dot{u}^*_{\vec{k}'}(t)e^{i(\vec{k}\cdot\vec{x}-\vec{k}'\cdot\vec{y})}[a_{\vec{k}},a^\dagger_{\vec{k}'}]\right.$$
$$\left.+u^*_{\vec{k}}(t)\dot{u}_{\vec{k}'}(t)e^{-i(\vec{k}\cdot\vec{x}-\vec{k}'\cdot\vec{y})}[a^\dagger_{\vec{k}},a_{\vec{k}'}]\right]. \quad (94)$$

Imposing the commutation relation for the creation and annihilation operators and performing the integration over the variable k' reduces this expression to

$$[\phi(t,\vec{x}),\pi(t,\vec{y})] = \int \frac{d^3k}{(2\pi)^3}a^3(t)\left[u_{\vec{k}}(t)\dot{u}^*_{\vec{k}}(t)-u^*_{\vec{k}}(t)\dot{u}_{\vec{k}}(t)\right]e^{i\vec{k}\cdot(\vec{x}-\vec{y})}$$
$$= i\delta^3(\vec{x}-\vec{y}). \quad (95)$$

Therefore, for relation (93) to hold, the integrand function, made up of mode functions and its time derivative, must satisfy the following, additional, generalised commutation relation:

$$u_{\vec{k}}(t)\dot{u}^*_{\vec{k}}(t)-u^*_{\vec{k}}(t)\dot{u}_{\vec{k}}(t) = \frac{i}{a^3(t)}, \quad (96)$$

where the inclusion of the $a^3(t)$ factor ensures that the mode functions are properly normalized in an expanding universe, while keeping the usual quantisation formalism for the scalar field.

A particular solution satisfying this condition is the Bunch-Davies vacuum state [13]:

$$u_{\vec{k}}(t) = \frac{1}{\sqrt{2k}}\frac{1}{a(t)}e^{-i\frac{k}{a(t)}t}. \quad (97)$$

In fact, together with the derived and conjugated fields,

$$u^*_{\vec{k}}(t) = \frac{1}{\sqrt{2k}}\frac{1}{a(t)}e^{+i\frac{k}{a(t)}t}, \quad (98)$$

$$\dot{u}_{\vec{k}}(t) = -\frac{1}{\sqrt{2k}}\frac{1}{a^2(t)}e^{-i\frac{k}{a(t)}t}\left(1+ik\right), \quad (99)$$

$$\dot{u}^*_{\vec{k}}(t) = -\frac{1}{\sqrt{2k}}\frac{1}{a^2(t)}e^{+i\frac{k}{a(t)}t}\left(1-ik\right), \quad (100)$$

it satisfies condition (96):

$$u_{\vec{k}}(t)\dot{u}^*_{\vec{k}}(t)-u^*_{\vec{k}}(t)\dot{u}_{\vec{k}}(t) = \frac{1}{2k}\left(-\frac{1}{a^3(t)}\right)(1-ik)-\frac{1}{2k}\left(-\frac{1}{a^3(t)}\right)(1+ik)$$
$$= \frac{ik}{2ka^3(t)}+\frac{ik}{2ka^3(t)} = \frac{2ik}{2ka^3(t)} = \frac{i}{a^3(t)}. \quad (101)$$

This vacuum state represents the ground state for a scalar field in a de Sitter spacetime. It is particularly important because, during the inflationary exponential expansion of the universe, any previous quantum excitation is expected to undergo an extreme redshift process, reducing it to a negligible perturbation over the lowest possible energy state, that is the Bunch-Davies vacuum state.

Therefore, at sub-horizon scales the perturbations $\delta\phi_k$ show a quantum behaviour, oscillating with a frequency $\omega = \frac{k}{a(t)}$. The amplitude of such oscillations decreases as the universe expands, due to the factor $\frac{1}{a(t)}$ in the mode functions, leading to slightly measurable fingerprints in the nowadays observable universe, such as the CMB anisotropies. However, if the amplitude kept decreasing in an inversely proportional way to the scale factor during the whole expansion process, it would get so small it wouldn't even be measured today. In fact, once the wavelength of the Fourier modes of a fluctuation becomes comparable to the Hubble radius, the fluctuation freezes and stops evolving according to equation (84). This is called the super-horizon regime.

While inflation progresses, the wavelength of a Fourier mode of a perturbation, $a(t)\lambda$, even when it starts within the horizon, is stretched until it catches up with the Hubble radius, that is constant during inflation, and it stops evolving according to equation (84). After a transient phase, described by the full equation (81), that is an extremely brief phase since the cosmological parameter grows exponentially, the wavelength becomes much larger than the Hubble radius:

$$a(t)\lambda \gg R_H = \frac{1}{H} \to \frac{1}{a(t)\lambda} \propto \frac{k}{a(t)} \ll H. \tag{102}$$

Now the friction term dominates over the gradient term in the evolution equation for the perturbative mode (81) which becomes

$$\frac{k^2}{a^2} \ll 3H \to \delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k = 0. \tag{103}$$

Even though this differential equation involves second order derivatives of the perturbation, it may be reduced to a first order linear differential equation, and the Hubble constant, H, being actually constant during inflation, makes it homogeneous. It can be solved by assuming an ansatz of the form

$$\delta\phi_k(t) = A_k e^{\lambda t}. \tag{104}$$

Substituting into the differential equation gives

$$\lambda^2 A_k e^{\lambda t} + 3H\lambda A_k e^{\lambda t} = 0 \to \lambda^2 + 3H\lambda = \lambda(\lambda + 3H) = 0. \tag{105}$$

The solutions to this algebraic equation are

$$\lambda = 0 \quad \text{and} \quad \lambda = -3H, \tag{106}$$

and the general solution to the differential equation is the overlapping of the two solutions with constant coefficients:

$$\delta\phi_k(t) = C_k + D_k e^{-3Ht}, \tag{107}$$

where C_k and D_k are constants of integration that depend on the initial conditions.

This solution consists of two distinct terms. The first one represents a constant amplitude, while the second one shows an exponentially decaying behaviour, with a rate determined by the factor $3H$, that soon becomes negligible when compared to the constant term. Henceforth, after a little time the perturbation coincides with the constant mode, that stops oscillating and its amplitude freezes until the expansion of the universe, well after inflation ends, allows it to re-enter the horizon when the horizon itself becomes large enough. Super-horizon perturbations, therefore, at the practical level stop evolving and thus show a classical (non-quantum) behaviour. The physical meaning of this freezing phenomenon is that, once a mode is outside the Hubble radius, all the physical causes that could influence its evolution can't causally propagate over such distances, and the perturbation stays frozen.

This freezing of the perturbations is a key feature of inflationary models that has profound phenomenological implications, allowing the nowadays observed universe to be compared with the evolution prescribed by the theory.

The constant solution, that is the frozen perturbation, sets the initial conditions for the primordial curvature perturbations, which later in the evolution can be traced into density fluctuations in the universe, leading to the formation of large-scale structures such as galaxies and clusters.

Moreover, the constant nature of super-horizon perturbations brings about the scale-invariant features observed today in the universe, such as the CMB anisotropies and the fractal-like distribution of large-scale structures. They are one of the few existing links with today's observational cosmological data, but, since inflation has evolved greatly from the new inflation presented in this Section, some authors have claimed that the way contemporary inflation makes predictions is epistemologically problematic, and testing such predictions cannot really confirm nor disconfirm the theory, making it a construct that lies outside of normal science. Different authors, including some of the historical fathers of inflation, defend its epistemology and claim that it is nothing different from regular science. Section 4 is devoted to reconstructing this debate and applying two different confirmation theories to understand whether contemporary inflation can be confirmed or not by comparing its predictions to experimental cosmological data. The spectrum of quantum fluctuations presented in this Subsection and the previous one is the main object of this analysis.

Before presenting modern inflationary models and performing the epistemological analysis, the rest of the present Section is devoted to describing why and how inflation comes to an end, and how this process links to the subsequent ordinary hot Big Bang cosmology.

2.5 Perturbative reheating

In relation to the dynamics introduced in Subsections 2.1 and 2.2, the inflationary phase of the universe comes to an end when the potential energy density of the inflaton field ceases to dominate over the kinetic energy density, and becomes of the same order of magnitude, leading to a violation of the slow-rolling conditions. The slow-roll parameters, ϵ and η, become of order unity and spacetime transitions from an exponential accelerated expansion to an extremely intense deceleration:

$$\epsilon \sim 1; \qquad \eta \sim 1. \tag{108}$$

Moreover, the Hubble parameter, H, experiences a substantial reduction in magnitude.

The inflaton field, no longer supported by the flatness of its potential, approaches the true minimum and begins to perform damped oscillations around this minimum, with the kinetic energy density, now comparable to the potential energy density,

$$V(\phi) \simeq \frac{1}{2}\dot{\phi}^2, \tag{109}$$

being converted into particles and radiation in a process called reheating.

Reheating accounts for the production of standard model particles, providing the missing link from inflationary cosmology to the standard Big Bang cosmology. Different inflationary models contemplate different reheating processes, and they may be divided into two main families: perturbative and non-perturbative reheating.

Perturbative reheating implies a perturbative decay of the inflaton field, according to which an inflaton particle is converted into lighter particles [14]. This process is characterized by a decay rate, Γ, a measure of the probability per unit time that an inflaton particle decays, that can be derived within the ordinary quantum field theory framework by means of the Fermi golden rule:

$$\Gamma = \frac{2\pi}{\hbar} \sum_f |\langle f|\mathcal{H}_{\text{int}}|i\rangle|^2 \delta(E_f - E_i), \tag{110}$$

where $\langle f|$ and $|i\rangle$ are the final and initial quantum states of the decay, $\mathcal{H}_{\text{int}}$ is the interacting part in the Hamiltonian describing the interaction and E_f and E_i inside the Kronecker's delta are the energy eigenvalues associated to the final and initial state, even though this last term only carries information about the conservation of energy and is often omitted.

The equation of motion for the inflaton field during reheating can be written by adding to the cosmological Klein-Gordon equation a term that accounts for the decay of the inflaton:

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0. \tag{111}$$

This equation is based on a key assumption: the decay rate is much smaller than the Hubble parameter during inflation, ensuring that the decay may effectively be treated perturbatively and that the inflaton field is long-lived compared to the timescale of the universe's expansion, in good accordance with the slow-roll conditions.

To give a description for the evolution of the energy density of the inflaton field, ρ, it is useful to take into account the conservation of energy applied to the total energy density in the expanding universe:

$$\frac{d}{dt}(a^3(t)\rho) = -\Gamma a^3(t)\rho, \tag{112}$$

where the energy density is stripped of the cosmic scale factor considered for the three spatial dimensions, and the Γ term represents a general loss in the total energy density due to the decay process.

Performing the derivation and dividing everything by the cosmic scale provides

$$3a^2(t)\dot{a}(t)\rho + a^3(t)\dot{\rho} = -\Gamma a^3(t)\rho \rightarrow \dot{\rho} + 3H\rho = -\Gamma\rho, \tag{113}$$

and this equation measures the total rate of change of the inflaton field's energy density, taking into account both the dilution due to the cosmic expansion, encapsulated by the Hubble parameter, and the decay of the inflaton into lighter particles.

As the deceleration of the expansion after inflation progresses, the Hubble parameter gets sensibly smaller until at the end of reheating the decay rate, that was before just a perturbative term, becomes comparable to it:

$$H \sim \Gamma. \tag{114}$$

This means that the rate at which the inflaton energy density is converted into radiation and particles now balances the rate at which the universe expands and cools down, as described by the Hubble parameter, and the universe is now ready to transition from an inflaton-dominated era to the radiation-dominated era.

2.6 Non-perturbative reheating

Perturbative reheating, described in the previous Subsection, provides a linear, individual decay of the inflaton quanta. As mentioned, however, a second family of reheating processes has been proposed, which involves non-perturbative effects that can't be described by the ordinary framework of perturbative quantum field theory. Non-perturbative reheating involves non-linear dynamics and provides a non-thermal distribution of the particles produced by the decay of the inflaton field, meaning that they do not follow the typical statical distribution expected for a generic hot plasma. This second family of reheating processes takes the name of non-perturbative reheating.

As opposed to perturbative reheating's individual particle decay (each inflaton quantum decaying independently of the others), non-perturbative reheating prescribes to a rapid, explosive, collective decay of the inflaton field, involving complex mechanisms that can't be framed by the Fermi golden rule, as its effect is non-perturbative and relies on non-linear differential equations that encompass the coupling of the inflaton to different fields.

To give a description for this family of decay processes, an additional scalar field, χ, is needed, with its own term in the global Lagrangian density of the system that also must include an interaction term through a coupling constant g that measures the intensity of the coupling. According to quantum field theory, the global Lagrangian density may be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}g^2\phi^2\chi^2, \tag{115}$$

where the last term is the one describing the interaction.

The equation of motion for the field χ may be obtained by imposing the Euler-Lagrange equation on the above Lagrangian density, considering an ordinary underlying Minkowski spacetime and then switching to the FLRW metric with the flatness hypothesis by means of the principle of general covariance provided by the framework of general relativity.

$$\frac{\partial \mathcal{L}}{\partial \chi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} \right) = 0, \tag{116}$$

in which the required terms are

$$\frac{\partial \mathcal{L}}{\partial \chi} = -m_\chi^2 \chi - g^2 \phi^2 \chi; \tag{117}$$

and

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} = \partial^\mu \chi \to \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} \right) = \partial_\mu \partial^\mu \chi = \Box \chi, \tag{118}$$

where $\Box$ is the usual d'Alembertian operator in a flat spacetime.

Putting these together, the equation of motion becomes

$$\Box \chi + \left(m_\chi^2 + g^2 \phi^2 \right) \chi = 0. \tag{119}$$

As already stated, this equation holds in flat (Minkowki) spacetime. In order to generalize it to an arbitrary curved spacetime, it is necessary to employ the principle of general covariance, according to which the Minkowski metric tensor shall be replaced with the metric tensor relative to the curved metric, while the ordinary partial derivatives shall be replaced with covariant derivatives.

The covariant d'Alembertian operator has already been computed in equation (31):

$$\Box = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \right). \tag{120}$$

In order to explicitate its analytic expression for the FLRW metric with the flatness hypothesis (with cosmic time), it is necessary to take into account the specific form of the metric tensor and inverse metric tensor,

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}, \tag{121}$$

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^{-2}(t) & 0 & 0 \\ 0 & 0 & a^{-2}(t) & 0 \\ 0 & 0 & 0 & a^{-2}(t) \end{pmatrix}; \tag{122}$$

from which the determinant of the metric tensor may be obtained by multiplying the eigenvalues, that may be directly read by its diagonal form:

$$g = \det(g_{\mu\nu}) = -a^6(t) \to \sqrt{-g} = a^3(t). \tag{123}$$

Substituting these results in equation (120), considering the d'Alembertian operator acting on the wanted field χ, provides the temporal component

$$\begin{aligned}\partial_t \left(a^3(t) g^{00} \partial_t \chi\right) = \partial_t \left(a^3(t)(-1)\partial_t \chi\right) = -\partial_t \left(a^3(t)\partial_t \chi\right) \\ = -3a^2(t)\dot{a}(t)\partial_t \chi - a^3(t)\partial_t^2 \chi;\end{aligned} \tag{124}$$

and for the spatial components

$$\begin{aligned}\partial_i \left(a^3(t) g^{ii} \partial_i \chi\right) = \partial_i \left(a^3(t) a^{-2}(t) \partial^i \chi\right) = \partial_i \left(a(t)\partial^i \chi\right) \\ = a(t)\partial_i \partial^i \chi = a(t)\nabla^2 \chi.\end{aligned} \tag{125}$$

Combining these results, and remembering the $\frac{1}{\sqrt{-g}}$ factor in front of the d'Alembertian operator, gives

$$\begin{aligned}\Box\chi = \frac{1}{\sqrt{a^3(t)}} \left[-3a^2(t)\dot{a}(t)\partial_t \chi - a^3(t)\partial_t^2 \chi + a(t)\nabla^2 \chi\right] \\ = -3\frac{\dot{a}(t)}{a(t)}\partial_t \chi - \partial_t^2 \chi + \frac{\nabla^2}{a^2(t)}\chi \\ = -3H\partial_t \chi - \partial_t^2 \chi + \frac{\nabla^2}{a^2(t)}\chi.\end{aligned} \tag{126}$$

Rearranging the terms gives the final form for the d'Alembertian operator applied to the field χ in the flat FLRW spacetime:

$$\Box\chi = \ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2}{a^2(t)}\chi. \tag{127}$$

Substituting this result in the equation of motion for the field χ (119) provides

$$\Box\chi + \left(m_\chi^2 + g^2\phi^2\right)\chi = 0 \to \ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2}{a^2}\chi + m_\chi^2 \chi + g^2\phi^2\chi = 0. \tag{128}$$

As usual, it is convenient to describe the evolution of the field χ after a decomposing it into Fourier modes:

$$\chi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \chi_k(t) e^{i\vec{k}\cdot\vec{x}}. \tag{129}$$

Substituting this into the equation of motion yields

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + g^2\phi^2\right)\chi_k = 0. \tag{130}$$

To describe a highly efficient, explosive decay, by means of an interaction Lagrangian, the coupling coefficient (g) in the interaction term is supposed to be much larger than the mass term, m_χ^2, and the term including the Hubble parameter, which in the discussion of perturbative reheating was hypothesized to be comparable to the decay rate, Γ, in principle considered perturbative. The equation of motion for a Fourier mode of the field χ thus gets simplified to

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2 \right) \chi_k = 0. \tag{131}$$

This equation, defining the dynamics of the Fourier mode χ_k coupled to the inflaton, resembles the Mathieu equation, a particular kind of differential equation encountered in different branches of physics when a periodical potential o time-dependent parameters are involved.

The general form of the Mathieu equation is

$$\frac{d^2y}{dx^2} + (a - 2q\cos(2x))\, y(x) = 0, \tag{132}$$

where A and q are parameters that determine the behavior of the solution.

The main feature of this equation is the periodic term $2q\cos(2x)$. This periodic dependence on the variable leads to a phenomenon called parametric resonace, a peculiar behaviour shown by the solution $y(x)$ that involves an exponential growth for certain values of the parameters A and q. The parameters sets that allow this exponential behavior are called instability regions, where a small initial fluctuation may grow exponentially and lead to a resonance phenomenon.

To expose the analogy between the equation of motion for the Fourier mode χ_k and the Mathieu equation, it is useful to impose in the former a periodic ansatz for the inflaton field dynamics, that after inflation it has already been discussed to oscillate around the minimum of the potential, thus it makes sense to suppose an ansatz of the following form:

$$\phi(t) = \phi_0 \cos(m_\phi t), \tag{133}$$

where ϕ_0 is the amplitude of the oscillations and m_ϕ the effective mass of the inflaton that appears in the full Lagrangian density of the coupled system. Such a mass synthesises the behavior of the inflaton under its potential. The space dependence of the inflaton is left implicit for the sake of simplicity.

Plugging this ansatz into the equation of motion for a Fourier mode of the field χ provides

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi_0^2\cos^2(m_\phi t) \right) \chi_k = 0. \tag{134}$$

To simplify the oscillatory term it is useful to derive a trigonometric formula by manipulating an elementary trigonometric identity:

$$\begin{aligned} \cos(2x) = \cos^2(x) - \sin^2(x) = \cos^2(x) - \left(1 - cos^2(x)\right) = 2\cos^2(x) - 1 \\ \rightarrow \cos^2(x) = \frac{1}{2}\left(\cos(2x) + 1\right), \end{aligned} \tag{135}$$

and identifying $x = m_\phi t$ yields

$$\cos^2(m_\phi t) = \frac{1}{2}\left(\cos(2m_\phi t) + 1\right). \tag{136}$$

The equation of motion for the Fourier mode then becomes

$$\begin{aligned} &\ddot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi_0^2\cos^2(m_\phi t)\right)\chi_k \\ &= \ddot{\chi}_k + \left(\frac{k^2}{a^2} + \frac{g^2\phi_0^2}{2}\left(1 + \cos(2m_\phi t)\right)\right)\chi_k = 0. \end{aligned} \tag{137}$$

Finally, identifying the parameters A_k and q_k, with the subscript referring to a dependence to the wave number of the Fourier mode, as

$$A_k = \frac{k^2}{a^2} + \frac{g^2\phi_0^2}{2}, \quad q_k = -\frac{g^2\Phi_0^2}{4}, \tag{138}$$

the equation above overlaps perfectly with the general Mathieu equation:

$$\begin{aligned} &\ddot{\chi}_k + \left(\frac{k^2}{a^2} + \frac{g^2\phi_0^2}{2}\left(1 + \cos(2m_\phi t)\right)\right)\chi_k \\ &= \ddot{\chi}_k + \left(A_k - 2q_k\cos(2m_\phi t)\right)\chi_k = 0. \end{aligned} \tag{139}$$

In this equation, the sinusoidal term plays the role of a periodic driving force, and with appropriate values for the coefficients brings about the parametric resonance phenomenon.

In resonance conditions, related to instability regions, the amplitude of a Fourier mode χ_k can grow exponentially with time and allow the rapid, very efficient production of χ particles, when compared to perturbative reheating. The stability of general solutions to the Mathieu equation is studied within the framework of the Floquet theory. The particles produced through parametric resonance are typically non thermal, as their distribution does not follow the thermal distribution expected in a generic hot plasma, and it shows important peaks aligned with the wavenumbers corresponding to the resonant modes.

A final remark is that not all the inflationary models prescribe both reheating processes, perturbative and non-perturbative. Depending on the inflationary model, if the couplings between the inflaton field and the other relevant fields in the theory are weak enough, non-perturbative reheating may not happen and the transition to the radiation-dominated era may be carried by perturbative reheating only. If, instead, the couplings are strong, non-perturbative reheating may be the phenomenon responsible for the decay of the inflaton field. When this is the case, perturbative reheating may still happen, but it is often considered a negligible phenomenon. Moreover, there are hybrid models in which a large fraction of the inflaton decay is carried out by non-perturbative reheating and the remaining, undecayed inflaton field undergoes perturbative reheating. It is from such scenarios that non-perturbative reheating takes the name of preheating. In some of these models, the two processes may even overlap towards the end of non-perturbative reheating.

The following Subsection describes how particles produced through perturbative and non-perturbative reheating processes, described in the present Subsection and the previous one, come to thermal equilibrium. The following Subsection also shows how to compute the temperature of such equilibrium, which constitutes a fundamental initial condition that links inflationary cosmology, described in this Section, to the standard hot Big Bang cosmology that describes the subsequent evolution of the universe from the radiation-dominated era to the present day and the future.

2.7 Thermalisation

Subsections 2.5 and 2.6 have described how inflation comes to an end and how standard model's particles are produced by the decay of the inflaton field. After the decay processes, the present model assumes that the decay products constitute a hot plasma that, even though non-perturbative reheating provides a non-thermal distribution of such particles, they come in contact and quickly thermalise, following a statistical distribution (Bose-Einstein or Fermi-Dirac) depending on their bosonic or fermionic nature. The present Section computes the average temperature at the end of the thermalisation process, referred to as *reheating temperature*, which constitutes the mentioned, fundamental initial condition that sets off the standard evolution prescribed by the hot Big Bang cosmology.

The reheating temperature may be computed starting from the first Friedmann equation with the flatness ($k = 1$) hypothesis:

$$3\left(H^2 + \frac{k}{a^2}\right) = 8\pi G\rho \rightarrow H^2 = \frac{8\pi G}{3}\rho = \frac{\rho}{3M_{Pl}^2} \rightarrow \rho = 3M_{Pl}^2 H^2. \tag{140}$$

Having discussed that the Hubble parameter during this phase becomes comparable to the decay rate of the inflaton field, the above equation may be rewritten in terms of it.

$$\rho = 3M_{Pl}^2\Gamma^2. \tag{141}$$

In order to proceed in the reheating temperature computation, it is necessary to take into account the dependence on the temperature of the energy density of a particle species, and for the sake of simplicity an ultrarelativistic gas of a mixture of bosons and fermions may be taken into account.

According to quantum statistical mechanics, the energy density of a generic particle species in the ultrarelativistic limit is given by

$$\rho = \frac{g}{(2\pi)^3}\int f(p)\epsilon(p)d^3p, \tag{142}$$

where g is the number of internal degrees of freedom, $f(p)$ is the distribution function of the particles, p is the linear momentum and $\epsilon(p)$ is the energy associated with that momentum.

In the ultrarelativistic limit the particle energy may be approximated with the very simple relation $\epsilon(p) = p$, helping with the following computation.

Again from the framework of quantum statistical mechanics, the distribution functions for bosons and fermions are given by the Bose-Einstein and Fermi-Dirac distributions, that with the chosen units may be written as

$$f_B(p) = \frac{1}{e^{p/T} - 1}, \qquad f_F(p) = \frac{1}{e^{p/T} + 1}, \tag{143}$$

where T is the temperature.

The energy density is then expressed according to the Bose-Einstein or the Fermi Dirac statistics, that may be written together as

$$\rho = \frac{g}{(2\pi)^3} \int \frac{p^2 \epsilon(p)}{e^{p/T} \mp 1} d^3p, \tag{144}$$

with the $-$ sign is for bosons and the $+$ sign is for fermions, and in the ultrarelativistic limit it becomes

$$\rho = \frac{g}{(2\pi)^3} \int \frac{p^3}{e^{p/T} \mp 1} d^3p. \tag{145}$$

It is now useful to switch to spherical coordinates in momentum space as follows:

$$\rho = \frac{g}{2\pi^2} \int_0^\infty \frac{p^3}{e^{p/T} \mp 1} dp, \tag{146}$$

and the well-known integral may be evaluated as

$$\int_0^\infty \frac{x^3}{e^x \mp 1} dx = \begin{cases} \frac{\pi^4}{15} & \text{for bosons;} \\ \frac{7\pi^4}{120} & \text{for fermions.} \end{cases} \tag{147}$$

Therefore, the energy density becomes

$$\rho = \frac{gT^4}{2\pi^2} \begin{cases} \frac{\pi^4}{15} & \text{for bosons;} \\ \frac{7\pi^4}{120} & \text{for fermions.} \end{cases} \tag{148}$$

Given this result, it is then customary to give a definition for the effective number of relativistic degrees of freedom, g_X, encompassing those of both species, as follows:

$$g_X = \sum_b g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8} \sum_f g_f \left(\frac{T_f}{T}\right)^4, \tag{149}$$

where the first sum is referred to the ensemble of all the boson species included in the gas and the second sum to the ensemble of the fermion species, while T_b and T_f represent the average temperature of each species and g_b and g_f indicate their internal degrees of freedom. The $\frac{7}{8}$ factor turns the $\frac{1}{15}$ factor in the above row of equation (148) into the $\frac{7}{120}$ factor in the below row and allows to take them both into account in a single equation that holds for both bosons and fermions.

Then the total energy density of the universe, considering all particle species in the ultrarelativistic limit, may be written as

$$\rho = \frac{\pi^2}{30} g_X T^4. \tag{150}$$

This results only holds with the hypothesis that all particles belonging to some particle species are at thermal equilibrium with each other, making the variables T_b and T_f well-defined, and moreover at thermal equilibrium with each other, hence the particles produced by the decay of the inflaton field, even though generated in a random, non-thermal state, verge towards a thermalized state and behave according to the spin-statistics appropriate to their nature.

Taking a step back into the computation of the reheating temperature, combining equations (141) and (150) yields

$$\frac{\pi^2}{30} g_X T_\mathrm{R}^4 = 3M_\mathrm{Pl}^2 \Gamma_\phi^2 \to T_\mathrm{R}^4 = \frac{90}{\pi^2 g_X} \Gamma_\phi^2 M_\mathrm{Pl}^2, \tag{151}$$

and taking the fourth root and expliciting one power of the Planck mass provides

$$T_\mathrm{R} = \left(\frac{90}{\pi^2 g_X}\right)^{1/4} \left(\Gamma_\phi M_\mathrm{Pl}^2\right)^{1/2} = \left(\frac{90}{8\pi^3 g_X}\right)^{1/4} \left(\Gamma_\phi M_\mathrm{Pl}\right)^{1/2}. \tag{152}$$

This result for the reheating temperature is very important in inflationary cosmology as it determines the whole thermal evolution of the universe from this point onwards, described by the standard hot Big Bang cosmology, and sets the initial consitions for the primordial nucleosynthesis at the core of astroparticle physics.

The present Section has quantitatively described the mechanics of the inflation process according to the theory named new inflation: it has shown how the inflaton field evolves and how its equation of motion is approximated by means of the slow-roll conditions, how quantum fluctuations arise and evolve and how they link to the present day's observational cosmological data, and it has shown how inflation comes to an end, with the inflaton field decaying into ordinary radiation and matter, and how the results of this decay is a fundamental initial condition for the subsequent evolution of the universe. Before performing an epistemological analysis of how contemporary inflationary models make predictions and how they are checked against experimental data in Section 4, the following Section shows how the inflation theory has evolved after the formulation of new inflation and presents more advanced inflationary models, with their implications in philosophy of science, that will be the object of the announced epistemological analysis.

3 Further inflationary models

The previous Section has discussed the core mechanics of the inflation theory, taking new inflation as a model. Historically, following old and new inflation, numerous models have been proposed, while experimental data to constrain them has been relatively limited. The current Section will present and briefly discuss a sample of models that embed the evolution of inflation as a theory from the early eighties to the present day. In particular, models that involve plateau-like potentials, like new inflation, and models that rely on hill-top potentials will be particularly relevant for the epistemological discussion in Section 4. Regardless of the form of the potential involved, inflationary models can be classified in three independent ways.

The first criterion is about initial conditions, whether they are specific, like old and new inflation, beginning in a state of thermal equilibrium at a very high temperature, or generic, like chaotic inflation (discussed in the following Subsection), prescribing a study of all possible initial conditions, not requiring an initial thermal equilibrium state.

The second criterion classifies models based on the regimes ruling the inflationary epoch, such as quasi exponential, like old and new inflation, or power law-driven, independently of the initial conditions (criterion one). Chaotic inflation, for example, can be realised in both ways according to different models.

The third, final criterion is based on the end of inflation, which may be due to a slow rolling that provides fast oscillations in proximity of the true minimum of the potential, like old inflation, or a first order phase transition with bubble formation.

Section 3.1 will discuss, among other features of inflation, how all the initial hypotheses that characterised Guth's original old inflation were gradually abandoned. Contemporary inflationary models are still related to the original idea of inflation by a few features: the extremely quick expansion of the universe after the Planckian era, that solves the problems discussed in Section 1, quantum fluctuations that seed large-scale structures and CMB anisotropies, that provide a link between theoretical predictions and cosmological observations, and the reheating mechanism that sets the initial condition for the primordial nucleosynthesis and the subsequent evolution of the universe, described by standard hot Big Bang cosmology.

Before discussing the epistemological problems related to modern inflation in Section 4, the present Section will discuss the probably most successful model within the inflation theory, chaotic inflation, in Subsection 3.1. Then it will discuss the further evolution of the theory towards contemporary inflation that brings about the mentioned epistemological problems in Subsection 3.2, and finally it will discuss a suggestive consequence of most inflationary models, eternal inflation and the multiverse, that bring about further conceptual problems in Subsection 3.3.

3.1 Chaotic inflation

As mentioned in Section 1, Linde's new inflation model was very successful, but it wasn't free from its own problems. The inflaton field rolling down its potential towards a global minimum, even though an appealing idea, would typically take too much time, so that an acceptable amount of inflation happens before the minimum is reached, or doesn't occur at all. Even if the inflaton field happens to be close enough to the minimum of its potential in the first place, inflation would start so late that a realistic, closed universe would collapse before inflation has a chance to begin. Moreover, density perturbations happened to be predicted excessively large. This problem could be solved by considerably reducing the coupling constant for the inflaton, but then it wouldn't be able to get into a state of thermal equilibrium with other matter fields. Moreover, the theory of cosmological phase transitions, that was the basis for Guth's original old inflation, couldn't be applied to new inflation [15].

To evolve, the inflation theory had to take the step back and remove the assumption that the inflaton field is trapped from the beginning in a local minimum of its potential. This one was the last assumptions relating inflation to its origins with Guth's old inflation, and removing it meant that from the first inflation model only the idea that the universe experienced an exponential expansion was taken, but inflation was becoming a different theory. Instead of the removed assumption, it turned out useful to investigate the sets of conditions that favour the inflationary regime. This led to the formulation of a wide class of models encompassed by the newborn inflation theory with the signature of Andrei Linde, the same author of new inflation, who baptised it *chaotic inflation* [16].

Thus, differently from new inflation (Subsections 2.1 and 2.2), chaotic inflation does not require tunnelling from a false vacuum state or rolling down an artificial flat top of a potential, like new inflation did. Chaotic inflation is based on the generic, flexible idea that any potential, $V(\phi)$, may be taken into account, and initial conditions can be inquired to check whether they bring about an inflationary epoch. The universe doesn't need to be in a state of thermal equilibrium before inflation, while new inflation did to explain the CMB apparent isotropy, and the inflaton field is in the global minimum of its potential from the very beginning. In fact, an incredibly simple model for the potential can be assumed, such as the simple polynomials

$$V(\phi) = \frac{1}{2}m^2\phi^2, \tag{153}$$

or

$$V(\phi) = \frac{1}{4}\lambda\phi^4. \tag{154}$$

As a matter of fact, it is nowadays preferred not to assume even that the early universe, before inflation, is homogeneous and isotropic, and to rely on the evidence behind the cosmological no-hair conjecture according to which initial conditions play a little role in the theory and a very wide class of primordial states supports the outcome of chaotic inflation [17]. This flexibility has been then further increased by successive models, briefly presented in the next Subsection, and it is precisely the main object of the epistemological debate in Section 4.

Following Linde's original reasoning, the main qualitative idea behind chaotic inflation is easily understood by considering a toy model involving a scalar field with an unrealistic, degenerate potential:

$$V(\phi) = V_0 = constant > 0. \tag{155}$$

In order to have such a potential, there is no reason to constrain the scalar field to any specific value in the whole universe. It is perfectly plausible that, across the universe, the scalar field may assume all possible values across different regions, with equal probabilities and thus a random distribution. The only legitimate constraint is that the energy density associated with the spacetime distribution of the field doesn't exceed the Planck energy (with a reasonable power to enforce dimensional consistency):

$$\partial_\mu \partial^\mu \phi < M_{Pl}^4, \tag{156}$$

otherwise the universe would be in the pre-Planckian era, and it would require a quantum gravity framework to be described.

Supposing that a set of the scalar field's values satisfy some condition that may trigger a spacetime expansion, during such an expansion the energy density associated with the distribution of the scalar field is bound to decrease, while the total matter energy density reduces to the potential described in equation (155). Having a constant energy density prescribes a de Sitter expansion described by

$$a(t) = a_0 e^{Ht}, \tag{157}$$

with

$$H = \left(\frac{8\pi V_0}{3M_{Pl}^2}\right)^{\frac{1}{2}}. \tag{158}$$

As a result, the universe becomes divided into multiple inflationary regions with a quasi-homogeneous scalar field. In such a way, inflation doesn't require a fine-tuning of the initial conditions of the universe, and it is a natural and possibly unavoidable consequence of the chaotic distribution of some scalar field. The result is not a globally homogeneous and isotropic universe, as the first inflationary models had the aim to describe, but a set of multiple domains which inflate until becoming larger than the nowadays observable universe. This very suggestive idea of multiple, inflationary universes will be expanded on in the next Subsection, when introducing the theory of *eternal inflation* [18].

Considering now a much more realistic model, but still with a very simple potential density such as

$$V(\phi) = \frac{1}{4}\lambda\phi^4, \quad \text{with } \lambda \ll 1, \tag{159}$$

the condition of starting the classical description after the Planckian era translates into

$$V(\phi) < M_{Pl}^4 \to -\frac{M_{Pl}}{\lambda^{\frac{1}{4}}} \lesssim \phi \lesssim +\frac{M_{Pl}}{\lambda^{\frac{1}{4}}}. \tag{160}$$

With a coupling parameter, λ, sufficiently small, this is enough to trigger inflation[3].

Once inflation begins, a region with an adequate value for the scalar field expands with the scale factor (locally) satisfying

$$a(t) = e^{Ht} a_0, \tag{161}$$

with the Hubble parameter given by

$$H = \left(\frac{8}{3}\pi\frac{V(\phi)}{M_{Pl}^2}\right)^{\frac{1}{2}} = \left(\frac{2\pi\lambda}{3}\right)^{\frac{1}{2}} \frac{\phi^2}{M_{Pl}}. \tag{162}$$

[3] An important note is that, at a Planckian timescale, the energy-time uncertainty principle allows the energy to be known only at accuracy grater than M_{Pl}.

The equation of motion for the scalar field, as justified in Subsection 2.1, will follow the relation

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial}{\partial \phi}V(\phi) = -\lambda\phi^3. \tag{163}$$

The above differential equation is satisfied with some approximation by a solution which may be written in the following form:

$$\phi = \phi_0 exp\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right). \tag{164}$$

Such a solution, pugged into the above differential equation, with the substitution of the Hubble parameter as given by equation (162), provides the following first two terms:

$$\begin{gathered}\ddot{\phi} + 3H\dot{\phi} = \phi_0\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}\right)^2 \exp\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right) \\ +3\left(\frac{2\pi\lambda}{3}\right)^{\frac{1}{2}}\frac{1}{M_{Pl}}\phi_0^3\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}\right)\exp^3\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right) \\ = \lambda\phi_0 \exp\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right)\left[\frac{M_{Pl}^2}{6\pi} - \phi_0^2\exp^2\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right)\right].\end{gathered} \tag{165}$$

The approximating assumption

$$\phi^2 = \phi_0^2\exp^2\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right) \gg \frac{M_{Pl}}{6\pi} \tag{166}$$

simplifies the right-hand side of equation (165) to

$$\begin{aligned}\lambda\phi_0 \exp\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right)\left[\frac{M_{Pl}^2}{6\pi} - \phi_0^2\exp^2\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right)\right] \\ \simeq -\lambda\exp^3\left(-\sqrt{\frac{\lambda}{6\pi}}M_{Pl}t\right) = -\lambda\phi^3,\end{aligned} \tag{167}$$

which coincides with the right-hand side of the differential equation (163), proving therefore the equality and the validity of the solution.

The characteristic time of the process is

$$\sqrt{\frac{\lambda}{6\pi}}M_{Pl}\Delta t \sim 1 \rightarrow \Delta t \sim \sqrt{\frac{6\pi}{\lambda}}\frac{1}{M_{Pl}}, \tag{168}$$

during which the scalar field loses most of its value and the universe experiences an exponential expansion.

The scale factor grows according to the following equation:

$$a(\Delta t) = e^{H\Delta t} a_0 = exp\left(\sqrt{\frac{2\pi\lambda}{3}}\frac{\phi^2}{M_{Pl}}\sqrt{\frac{6\pi}{\lambda}}\frac{1}{M_{Pl}}\right) a_0 = e^{\frac{2\pi\phi^2}{M_{Pl}^2}} a_0. \tag{169}$$

But at this characteristic time the scalar field assumes a value

$$\phi(\Delta t) = exp\left(-\sqrt{\frac{\lambda}{6\pi}} M_{Pl}\sqrt{\frac{6\pi}{\lambda}}\frac{1}{M_{Pl}}\right) \phi_0 = \frac{\phi_0}{e}. \tag{170}$$

This implies

$$\phi^2(\Delta t) = \frac{\phi_0^2}{e^2}, \tag{171}$$

that, plugged into the previous equation for the cosmic scale at the characteristic time, provides

$$a(\Delta t) \sim exp\left(\frac{2\pi\phi_0^2}{M_{Pl}^2}\right) a_0. \tag{172}$$

In this equation, an initial value for the scalar field such as

$$\phi_0 = 3M_{Pl} \tag{173}$$

is enough to reach the desired amount of inflation of about sixty e-foldings:

$$a(\Delta t, \phi_0 = 3M_{Pl}) \sim exp\left(\frac{2\pi 9 M_{Pl}^2}{M_{Pl}^2}\right) a_0 = e^{18\pi} a_0 \simeq e^{60} a_0. \tag{174}$$

Such a simple choice for the initial value of the scalar field doesn't violate the condition of not exceeding the Planckian scale, as can be shown by substituting the chosen value for the scalar field and plugging it into the condition itself:

$$V(\phi) = \frac{1}{4}\lambda\phi^4 < M_{Pl}^4 \to \frac{81}{4}\lambda M_{Pl}^4 < M_{Pl}^4, \tag{175}$$

as long as the coupling constant is lesser than about the hundredth part of the unity:

$$\lambda < \frac{4}{81}. \tag{176}$$

Due to the chaotic initial conditions on the scalar field, the primordial universe reasonably includes regions in which the scalar field assumes values such that

$$\phi_0 > 3M_{Pl}, \tag{177}$$

and these regions are bound to experience inflation.

Inevitably in the primordial universe there are many other regions not meeting the conditions for inflation, but, as long as there are a few regions suitable for inflation, or, if in the limit case there is even only one, the universe can undergo inflation and the conditions for life can be realised. This point of view strongly supports the weak anthropic principle, according to which the universe is not bound to exist in a state that allows life to exist, but such a region in the universe is inevitably the only one we have experience of because we could not have risen anywhere else.

The main challenge faced by chaotic inflation is that, just like with old inflation, density perturbations on a scale such as

$$\frac{\delta\rho}{\rho} \sim 10^{-4}, \tag{178}$$

which would give rise to the large-scale structures nowadays observed in the universe, are not easily obtained. At $\phi \sim 3M_{Pl}$ the coupling constant would need to be unrealistically small as $\lambda \sim 10^{-10}$. However, even though such a requirement would disqualify a different model like new inflation, chaotic inflation is definitely more flexible as long as there are not too restrictive requirements on the potentials, such as the slow-roll conditions proper to new inflation, and models involving very weak interactions of the scalar field with different matter fields, with the addition of self-interactions, may be suggested.

The final remark is that, with the adoption of chaotic inflation, the inflationary epoch can be realised in regions of the universe in an almost model-independent way, and with chaotic initial conditions it would be in fact difficult to manipulate the parameters in order to prevent, and not realise, inflation.

3.2 Towards postmodern inflation

Section 1 and Subsection 2.1 have respectively presented new and chaotic inflation, very different models related by the same underlying idea of an extremely quick expansion of the universe right after the singularity. However, new and chaotic inflation were only the beginning of a massive proliferation of inflationary models that followed during the years and the decades, presenting an increasing level of complexity and versatility. The present Subsection has the aim to quickly present some of them without delving into the quantitative details, as those discussed in Section 2 will suffice to understand the object of the epistemological analysis in Section 4.

A few relevant models from the '90s are *hybrid inflation*, according to which inflation ends and the reheating process is triggered due the instability related to a second, auxiliary scalar field [19]; *natural inflation*, prescribing a naturally flat potential without fine-tuning needs arising from the study of the pseudo-Nambu-Goldstone boson and its potential in particle physics [20]; *warm inflation*, as opposed to regular, cold inflation, hypothesising that fluctuations in the early universe might be classical, thermal fluctuations and not of quantum origins [21].

The 2000s were just as fertile for inflationary models, that kept increasing in complexity and becoming more and more exotic. The Dirac-Born-Infeld (DBI) model, derived from string theory, allows giving an interpretation to the inflaton as the position of a brane that moves in a warped additional dimension [22]. Non-minimal Higgs inflation assumes that the particle physics standard model's Higgs Boson, under the assumption that it is non-minimally coupled to gravity, acts as the inflaton field and drives inflation [23]. Hilltop inflation, which will be important in Section 4, proposes that the inflaton rolls down a hilltop potential instead of a plateau, and requires lower energy scales to realise inflation [24].

A very large number of different models may be explored in the Encyclopaedia Inflationaris [25]. The overall trend across the 90s and the 2000s is that the proposals of inflationary models became more and more complex, often inspired by advancements in string theory and supergravity, and start involving potentials depending on multiple fields with numerous local minima. Correlated with this trend, models also increase in flexibility, gaining the ability to easily fit WMAP and Planck's precision data, without sacrificing the adjustability to account for future data. This situation is very far from new inflation presented in Section 2 and chaotic inflation presented in Subsection 3.1. It is precisely the enhanced flexibility of contemporary models the object of the epistemological attack that will be discussed in Section 4. Models that are so flexible can be adjusted *a posteriori* to account for any newly acquired data. It has been proven that a potential involving a single field and as few as three free parameters can be adjusted to fit any desired data for the measurable cosmological variables [53]. Because of this, according to some authors, the theory of inflation cannot be empirically disconfirmed, nor consequently confirmed, any more. It has hence been argued that inflation has evolved from its origins into a different theory, evocatively named *postmodern inflation*, with the term used in literature, art and philosophy to indicate ideas related to the denial of absolutes and the commitment to deconstructed positions and relative truths.

Section 4 will discuss this problem in detail, but, before that, the following Subsection will first discuss eternal inflation and the multiverse, which will raise an additional problem related to how predictions are defined, to be discussed in Section 4 as well.

3.3 Eternal inflation

A common consequence to both new and chaotic inflation and most of the models presented in Subsection 3.2 is that inflation may be a never-ending process. As first proposed by Alexander Vilenkin in 1983, *eternal inflation* is an addition, more than a variation, to regular inflationary cosmology, and it is a simple, logical consequence of the exponential expansion and decay processes of the false vacuum, even though it may appear paradoxical [26]. Every successful inflationary model prescribes that the growth rate of the false vacuum must exceed its decay rate, implying a growth with time of the universe, in terms of both mass and volume. The decay of the false vacuum, however, is a quantum, probabilistic process, and it may be triggered at random locations inside the false vacuum at different times.

In inflationary models that prescribe a flat top for the inflation field potential such as new inflation, while the inflaton field is rolling down its potential towards the true minimum, the quantum fluctuations it experiences are small compared to its driving force, thus the field may reach the minimum roughly simultaneously in every region of space, with small density perturbations. While the inflaton is still close to the flat top of the potential, however, under slow-roll conditions, the same quantum fluctuations are not small any more when compared to the regular field dynamics. The field therefore experiences a random walk until it reaches a configuration onto the potential which is steep enough to trigger the down-rolling associated with inflation.

It is important to note that quantum fluctuations may be coherent, in the sense of happening in the same direction for every point in space, at most in a region of size lesser than the Hubble distance,

$$d_H = \frac{c}{H} = \frac{1}{H} \tag{179}$$

with the employed units.

At larger scales, quantum fluctuations must be independent of each other in order not to violate causality. Henceforth, once two points become far apart by a distance greater than the Hubble distance, their evolutions must diverge from each other. The consequence is that, while the small density fluctuations measured in the observable universe prescribe that the whole observable region was well within the Hubble radius while the inflaton potential rolled downhill, regions beyond the horizon certainly have undergone a different evolution, and some of them might very well still be experiencing inflation.

Moreover, supposing that the bubbles of true vacuum are separated from the true vacuum by a potential barrier with the role of some sort of surface tension, the true vacuum undergoes an accelerated expansion pushing the barrier away with a speed that may exceed the speed of light, but they are also driven far away from each other by the expansion of the false vacuum that separates them, never meeting. This stochastic process brings about a random pattern of coexisting, expanding false and true vacuum regions, with the false vacuum decaying eternally. Even though inflation has ended in the region including the observable universe, and we supposedly live in a true vacuum bubble, inflation may continue in different, causally disconnected regions, giving birth every second to new inflationary universes. These universes, often called mini-universes, bubble universes, pocket universes or baby universes, can't have any interaction with each other, therefore from a global point of view they are all effectively independent universes.

As it has been discussed, the nature of eternal new inflation relies on the inflaton field experiencing fluctuations at the flat top of its potential. Chaotic inflation, however, does not rely on a potential with a flat top, so it is more tricky to reproduce eternal inflation with its premises. Still, in 1986 it was shown that eternal chaotic inflation is indeed possible even for non-flat potentials such as many of those contemplated by chaotic inflation, with the random walk pushing the inflaton towards the steep, far from its stable equilibrium configuration, where it triggers inflation by rolling down with a driving force much larger than the fluctuations [27].

Considering again an even polynomial potential in a chaotic inflation scenario, the inflation-triggering condition may be synthesised with the inflaton field surpassing a threshold:

$$\phi > \phi^*. \tag{180}$$

When this condition is satisfied due to large enough quantum fluctuations in a space domain, in an e-folding time H^{-1} the size of the domain increases of a factor

e and its volume of a factor $e^3 \simeq 20$. This means that, after an e-folding, there are about twenty regions the size of the original domain.

Standard statistics prescribes that a random-distributed inflaton field at this point surpasses the threshold with a 15.9% probability, implying that in roughly three of the twenty regions inflation may proceed further. Therefore, even though inflation stops (at least temporarily) in most of the regions, the effective volume of the inflationary domain increases exponentially with a factor of three. The universe becomes thus divided into an exponentially increasing number of self-reproducing regions with a fractal-like behaviour.

While this peculiar evolution brings about many universes, possibly an infinite number of them, they are not bound to look alike. It is in fact sufficient that the inflaton potential, both in the case of new inflation-like and chaotic inflation-like models, has more than one minimum that can be reached by the inflaton, and the possible stable equilibrium states multiply when considering models involving more than one scalar field, each contributing to the global description of the inflation dynamics. The universe we live in is determined by one minimum of the potentials, but different universes may be very different even in terms of physical laws and constants. This scenario is evocatively named *multiverse hypothesis*.

Of course the exponentially growing domains filling the inflationary universe, and the same holds for the multiverse, are argued to be causally disconnected from each other, and therefore unobservable from one another. However, eternal inflation might leave observable traces that would lead to an experimental confirmation to this theory.

Due to quantum fluctuations, inflation is a stochastic process and the probability of it happening is never zero. A phase transition might in fact happen within the Hubble radius of the expanding boundary of the observable universe. In that case, the boundaries of the two bubbles would collide, producing a detectable radiation in the form of a very localised hot spot in the cosmic microwave background radiation. Of course, this is not a reliable experiment, as there is no reason to believe that such an event is bound to happen.

Furthermore, while our universe was in the slow-rolling phase, different bubbles, associated with other local minima of the potential, may have formed inside and started expanding. At the end of inflation, with the phase transition complete, these bubbles may have been surrounded by a (true) vacuum at a lower energy, therefore may have reversed their behaviour and started contracting instead. Without a mechanism to stop the contraction, the process may have continued indefinitely, giving rise to black holes. Specifically, bubbles formed early in the history of inflation were larger and may have given rise to larger, monstrous black holes up to millions of solar masses, that may still be somewhere in the universe as relics of the multiverse. Specifically, the mass distribution of black holes may be checked against the theoretical predictions following from inflationary models, providing experimental evidence for eternal inflation.

Even though eternal inflation is not free from its own problems, there are a few important consequences that it brings about in the framework of regular inflation and even the whole cosmology.

Event probabilities tend to be ill-defined in a scenario in which the number of universes diverges, because everything that can happen is bound to happen and repeat an infinite amount of times. The *a priori* probability for any event to happen is then a degenerate fraction representing infinity over infinity, an undetermined result that brings very little information, and the same holds for the fraction of universes that may show a specific feature. This problem will be relevant in the epistemological discussion in Section 4, where experimental inconsistencies, instead of be interpreted as disconfirming the theory, will be dealt with by *a posteriori* redefining the metric.

A regularisation technique is needed to make sense of undetermined probabilities and frequencies, with the additional complication that the order of birth of different universes, or of nucleation of different bubbles, is of no help at all since different universes are space-like separated, outside the light cone of each other. According to special relativity, given two space-like separated events, it is always possible to determine inertial reference frames in which one event precedes the other, they are simultaneous, and their order is inverted. The chronological order of the universes' evolution is therefore frame-dependent.

Also the most natural kind of measure, the volume-weighting measure, is unsuccessful, as it brings about the youngness paradox and the Boltzmann brains paradox. According to the youngness paradox, due to eternal inflation the universe is still undergoing an exponential expansion, and this makes the universe we live in almost impossible to observe, and we should be observing a much, much younger horizon instead [28,29]. According to the Boltzmann brains paradox, first proposed by Boltzmann in the late nineteenth century, the universe's evolution with the formation of intelligent life is much less probable than individual, self-conscious living beings popping out of the vacuum due to quantum fluctuations with false memories and illusionary sensorial activities [30].

One more problem is that, to track the evolution of a single universe, a time coordinate is needed. The choice of a reference time, however, is far from easy. Inside a universe it makes sense to count the time starting from the Big Bang, but in an infinitely expanding multiverse, made of separated bubble universes receding at a speed grater than the speed of light, it might be impossible to synchronise the clocks and thus have a global view of the multiverse history.

Ignoring for the time being these fundamental problems, eternal inflation still implies important consequences for the framework of cosmology.

First, even though probabilities are ill-defined, there is common consent in supporting the idea that a single region, inside a universe infinitely inflating, loses all the information about the initial state and reaches its steady state regardless of the initial conditions. Furthermore, while eternal inflation certainly prescribes a universe that is future-eternal, in this framework the questions about the origin of the multiverse, that is arguably the Big Bang, lose some of their capital relevance. In the optics of eternal chaotic inflation, it has even been proposed that there might have been no Big Bang at all and the universe might be a self-replicating structure eternally evolving, both in the past and in the future. It was later proven that a future-eternal spacetime must be past geodesically incomplete, therefore it must have a singularity [31]. But the proof requires the weak energy condition (WEC), stating that for every timelike observer, u^α, the matter density measured in the reference frame of such observer must be non-negative:

$$T_{\alpha\beta}u^{\alpha}u^{\beta} \geq 0 \to \rho \geq 0 \wedge \rho + p_i \geq 0, \tag{181}$$

where $T_{\alpha\beta}$ represents the energy-momentum tensor, u^{α} the four-velocity of the geodesic observer, ρ the energy density and p_i the i-th component of the pressure, with $i = 1, 2, 3$ denoting the spatial index. The weak energy condition can be violated by quantum fluctuations, limiting the predictive value of the proof.

Another main consequence of eternal inflation is that, in the hypothesis that more than one inflation model is possible, such as new inflation happening simultaneously as chaotic inflation, it is irrelevant which model is more likely to drive inflation. Even if a model has an infinitesimal probability to trigger inflation, and would appear negligible with respect to the most probable one, given an infinite evolution time, as long as it provides an even slightly faster inflation, it will still be the main mechanism. It will in fact be responsible for the vast majority of the bubble universes created in such a multiverse, overshadowing the most probable but slower mechanism by an infinite amount. This greatly reinforces models that prescribe an exponential inflation, such as new inflation.

One more consequence of eternal inflation relates inflationary cosmology to M theory. This theory doesn't clearly provide a unique vacuum state, and its predictive power heavily relies on the features of such vacuum. Taking eternal inflation into account, even if there are more possible vacuum states, one of them may provide a faster inflation mechanism, ending up dominating the universe with an infinite ratio over the other possible vacua. This could allow M theory to provide unique predictions despite the ambiguity of the vacua.

It was argued in Section 1 that the destiny of life in the universe is far from optimistic according to standard hot Big Bang cosmology, as life supposedly shares the destiny of the universe, either thermally dying or collapsing back into the singularity, respectively in the case of an open or closed Friedmann universe. Taking eternal inflation into account, instead, life might be able to exist eternally [32]. Life may in fact be able to evolve all over again in the bubble universes, continuously reproducing. More and more forms of life may as well proliferate in the new universes, and, with the multiverse hypothesis, different forms of life inconceivable to us might be possible. The important question for us, however, is whether life can exist eternally in our part of the universe, or at least if we may be able to send information about our evolution and discoveries to the new forms of life arising in new bubble universes, preventing our legacy from extinguishing together with our universe.

Right now there is evidence that we live in a domain featuring a positive density fluctuation, $\delta\rho > 0$, on the Hubble scale, and, even if it is locally negative, the density fluctuation may still be positive on a larger scale, meaning that our domain will eventually collapse into a black hole of the size of the Hubble horizon. But, according to eternal inflation, such a black hole contains self-reproducing bubble universes that from a point of view within look like inflationary domains connected to ours by wormholes. Therefore, there are regions inside the black hole that never actually collapse and, even though we may be unable to escape the black hole before its genesis, we have a chance to survive by moving into the new inflationary domains that are being born inside, or at transmitting information there. This idea is extremely speculative and there is no reason at the present time to believe whether it would be possible or not, but it is estimated that 10^{10000} years may be needed for such a black hole to form, giving us enough time to eventually elaborate a strategy for the survival of our legacy.

There is one more idea following the multiverse theory that, even though somewhat disturbing, might solve, or at least bypass, the survival of our legacy problem. As extensively discussed, the multiverse is future-eternal and thus the number of bubble universes is unbound. The number of possible configurations and evolution histories of such bubble universes, however, is limited, even though enormous. This is because every feature of a bubble universe ultimately depends on the quantum states of its elementary particles, and quantum states are, as ordinary quantum mechanics prescribes, in general discrete and not continuous. The number of possible states in a given volume is therefore finite, and this means that, while evolving, an infinite number of bubble universes extract their configuration from the same, finite pool. Every possible configuration is thus realised and repeated an infinite amount of times. Our universe is one of the possible configurations, so it is identically repeated in clone universes an infinite amount of times, with its galaxies and forms of lives, including our undistinguishable clones. More importantly, universes extremely similar to ours, with very small differences, are bound to exist as long as there is no *a priori* reason to exclude them for the pool of possible worlds. This means that, for every event in our past and future history, which does not have a bound, mathematically certain outcome, all of its possible outcomes will be realised, and repeated an infinite amount of times, in some of the bubble universes. Supposing that we may or may not succeed in surviving our bubble universe collapsing into a black hole, the specific outcome for us doesn't matter. Even if we don't succeed, there is somewhere a bubble universe with its every feature identical to ours, with the only difference that in such bubble universe we succeed in the survival problem. And, even though we can't win in the survival problem, every accomplishment of our race will still be reproduced in other bubble universes, not being lost in the end. This idea naturally supports a nihilistic point of view. If everything that can happen will happen, it makes no sense to put an effort in our success: whatever the trial, across the multiverse we will both succeed and fail an infinite amount of times.

The statement that everything that can happen will happen, however, is ambiguous. A question naturally arising is what exactly can happen, and what cannot happen. This question translates the problem from theoretical cosmology to modal logic and philosophy of science. An intuitive idea on the nature of science, developed in fact not too much before of eternal inflation, is that science is what is true in every possible world [33], and what is not possible is what can't happen in any of the possible worlds. Eternal inflation provides strong support for an ontology of the possible worlds, already hinted by the many worlds interpretation of quantum mechanics [34]. This definition of science looks circular. Since the other possible worlds, whether they exist or not, cannot be observed, what is possible and impossible in them is *a priori* judged on the basis of science itself. Other than circular, the definition is in fact dialectical: the competence to imagine different worlds with a different reality content comes from science and evolves with it. What can now be considered possible in different worlds, such as a different symmetry breaking scenario, couldn't have been thought of a century ago. In the same way, a century from now science will have evolved to cast light onto some different scenarios, possible in different worlds, that couldn't nowadays be even thought of. While science evolves, with it evolves the idea of science itself, and maybe one day this idea will be obsolescent and science will appear with an altogether different face.

A final remark is that, while eternal inflation strongly supports the ontology of the possible worlds, it doesn't alter the ability to imagine them in thought experiments with the aim of understanding the boundaries of science. Again, letting contemporary science guide the thinking flow, an ergodically dual situation may be explored. Instead of thinking of possible worlds, distinct from the one we live in, we can think of our own world, rewinding its history and letting it evolve again. The overall reality content may become very different, while only science is destined to live again identical to itself. Julius Caesar's death is not a metaphysical necessity, in fact he could not have been assassinated if one of the senators denounced the conspiracy. The Mona Lisa could have not been painted, if Leonardo da Vinci did not meet Lisa Gherardini. But, as long as intelligent life is allowed, we can't imagine a world in which the expression $\vec{F} = m\vec{a}$, emblem of science, is not sooner or later written. Possibly not by a British *sir*, and maybe not with our vectorial notation, but it is unthinkable that history makes its course without classical physics being formulated, and then surpassed. This is a rationalisation of the intuitive idea that physics is science, while art and history are not.

In of the considerations in this Subsection, science may be seen as what is invariant across the possible worlds, or, equivalently, what is destined to repeat itself imagining letting the world we live in evolve again and again from its beginning. This view will inform the original conclusions drawn in Section 5 regarding the epistemological frameworks presented in Section 4, and on the epistemology of the inflation theory itself.

4 Inflation and Epistemology

Section 2 has discussed the core mechanics of the inflation theory, taking new inflation as a model, then Section 3 has presented chaotic inflation and further inflationary models, and has introduced the excessive flexibility of postmodern inflation and the measure problem related to eternal inflation. The present Section will now discuss these problems from an epistemological point of view.

Subsection 4.1 will introduce the concept of underdetermination in philosophy of science, that will be crucial when introducing meta-empirical confirmation theory; Subsection 4.2 will introduce the framework of Bayesian confirmation theory that will be used to reconstruct the debate on the epistemology of inflation; Subsection 4.3 will present the first act of such debate, while Subsection 4.4 will present the second act. Subsection 4.5 will discuss the general framework of non-empirical confirmation theory, especially contextualised in the realisation of the meta-empirical confirmation framework, that Subsection 4.6 will apply to the case of inflation; finally, Subsection 4.7 will discuss the points of criticisms arisen by the scientific and philosophical community towards meta-empirical confirmation. It will be then time for the conclusions, to be drawn with the idea of science presented in Subsection 3.3 in mind.

4.1 Underdetermination

At the dawn of science, the humankind started formulating theories with arguably two main objectives in mind: explaining phenomena in the past and predicting phenomena in the future. From a mathematical point of view, this translates into developing models that fit all the available data and that will fit newly discovered data in the future. This second part, of course, produces an amount of uncertainty, since, at the relative present time, future data is not yet available. When new data becomes available, scientists compare it with the old theory's prediction.

Naturally, different theories, including unknown, not formulated theories, may fit all the available data and predict different future phenomena. Until assessment comes, every scientific theory is therefore in a situation of *underdetermination.* With this word, philosophers of science actually refer to two different situations, which are now to be distinguished to the purpose of the present study.

Underdetermination as historically known since its promulgation by the eighteenth century's Scottish philosopher David Hume, dictates the relativism on currently assessed scientific theories, all of which may be discredited by future data, and they are consequently future-undetermined. This situation is referred to as *underdetermination in practice.*

Underdetermination in a more modern sense, as brought about by the American analytic philosopher Willard Van Orman Quine, refers to the absolute situation in which all possible empirical evidence (both known and unknown at the present time) may be explained by more than one empirically equivalent, but logically incompatible scientific theories [35]. This second situation is referred to as *underdetermination in principle.*

For the purpose of the present work, only Humean underdetermination (in practice) will be taken into account, with the difference that scientists, as intended in the modern meaning of the word, presuppose correct the inductive-deductive method of acquiring knowledge, and they do not consider the possibility that the generalisation of scientific theories to future phenomena may be just a mental construction unrelated to reality. Underdetermination in principle would entail this situation because, if no evidence allows to decisively distinguish among a set of competing theories, the choice of a particular theory can be driven by practical or aesthetical features, resembling a mental construct instead of a step that gets science closer to reality. To this sense, scientific progress discriminates between theories and reduces underdetermination, proceeding towards new theories that fit present data as much as old theories, and in addition they also fit new data [36]. A great example is general relativity, which reduces to Newtonian gravitation in the classical approximation, hence these two theories live in a situation of underdetermination in practice with respect to the astrophysical data available in the nineteenth century. This underdetermination is broken by newly available data such as the precession of Mercury, in light of which general relativity has been preferred over Newtonian mechanics.

Underdetermination in practice allows discriminating between confirmed theories, that have been well-tested experimentally on their distinctive predictions, and speculative theories, that have not been experimentally confirmed. A speculative theory fits all the present data and, in addition, makes new predictions that are not yet accessible with the available experimental technologies (and there is no guarantee that they will ever be). Moreover, in the last century of progress, scientific theories have surpassed experimental capabilities so much that they have to wait in the speculative limbo a large amount of time before being confirmed or rejected in an experimental environment. The Higgs boson [37] and gravitational waves [38] exemplify this well.

Other theories, such as the Hawking radiation [39], already received consensus among the scientific community despite not having been experimentally confirmed and probably not having a possibility to be tested in the following decades, hinting that the way scientists give consensus to a theory is shifting in some ways. One other scientific theory that is definitely speculative, and might never have the chance to be experimentally tested, is string theory [40].

String theory is still today incomplete. It doesn't make clear physical predictions that may be tested against future data, and it might never be tested. Not only that; it isn't even able to reproduce some past data available from particle physics. It is not the only candidate for a unified field theory, as it competes with loop quantum gravity [41]. Despite all this, many efforts have been and are being devoted to theoretical research on string theory, with the scientific community working on it being one of the largest in all physics. String theory is related to many of the most-cited research papers in the field, and even different unconfirmed theories, such as supersymmetry, get significant credence attributed from being implied by string theory.

In light of all these considerations, string theory puts the philosopher of science in an awkward situation. While on the one hand it exuberates a great deal from ordinary scientific theory assessment, on the other hand it has gained so much authority that waiting for it to be fully developed, let alone tested, would mean for the philosopher to shoot themselves in the foot while theorising over science and neglecting one of its most influential theories.

On this opposite side of the table there is the case of quantum mechanics, which in the twentieth century received an enormous amount of experimental confirmation, but due to its intrinsically paradoxical nature raised scepticism to most physicists, even the founders of the theory itself, with Bell's inequality possibly marking the acceptance of theory for most of the scientific community. This is a proof that in theory assessment there is something more than just empirical evidence, and, when the right conditions align, the non-empirical side can get as strong as to compete with the actual empirical evidence.

The most logical conclusion is that science assessment has been changing in the last century, and string theory especially marks an important shift from traditional theory assessment, greatly devaluating the role played by underdetermination awareness in the cutting-edge scientific research [42]. It is important to keep this scenario in mind when discussing the assessment of cosmological inflation, since it is another theory with very limited experimental confirmation potential, but it is nonetheless being assessed in different, non-traditional ways. While the authors of the mentioned epistemological attack to inflation argue that inflationary models nowadays suffer from a huge situation of underdetermination in principle, underdetermination in practice will play a leading role when discussing meta-empirical confirmation theory in Subsection 4.5. The hypothesis of having a situation of bounded underdetermination, that needs indeed to be extensively and properly justified, will allow inferring that an untestable theory, such as string theory and postmodern inflationary cosmology, is more likely to be viable. What it means for a theory to be "more likely to be viable" will be clarified in the following Subsection within the framework of Bayesian confirmation theory.

4.2 Bayesian confirmation theory

Traditional theory assessment in philosophy of science, including in a non-exhaustive way Hempel's hypothetico-deductive model, the Vienna Circle's logical positivism and Popper's falsifiability, do not contemplate ways to assess theories that exuberate from empirical confirmation, that is a-posteriori fitting data. The pragmatic difficulty of accessing the higher energy scale and exotic physical systems such as black holes has convinced the scientific community to commit to developing alternative theory assessment scenarios.

This approach encountered high scepticism in the points of view of many researchers. Nonetheless, while it is more than ever important to clarify the argumentative strategies involved and the logical assumptions, Bayesian confirmation theory started developing since the 1950s on the basis of Thomas Bayes' work on probabilities from two centuries before, as a systematic, quantitative framework to describe how belief in a scientific theory evolves [43]. Bayesian confirmation theory indeed includes methods that do not coincide with traditional, rigorous empirical confirmation, and it can hence be employed to analyse indirect confirmation of scientific theories.

When indirect confirmation is experienced, Bayesian confirmation theory can help to expose logical holes in the arguments chain. Naturally, it may be especially useful within far-fetched scientific paradigms, such as string theory and cosmic inflation, for which direct, empirical confirmation is not available yet. This is important because, other than verifying whether a theory is correct or not, precociously assessing the theory can help the scientific community to decide whether time, efforts and resources are well-spent on such a theory, or they should be redirected elsewhere for maximum efficiency.

According to Bayesian confirmation theory, the belief of a scientist, called rational agent, in a theory (T), or simply in a statement subordinate to a theory, is represented by a subjective probability, $P(T)$ assigned to the theory to be empirically adequate. A unitary probability is assigned to a sure, absolutely reliable statement, such as the impossibility to send superluminal signals, while an absurd statement, a paradox, receives a null probability. All the probabilities in this range satisfy the axioms of probability theory in statistical mathematics.

In order to clarify the above definition, two important notes must be made. First, having a subjective probability might seem scientifically weak and unrigorous; however, because of how probabilities will be shown to evolve in Bayesian confirmation theory, even a subjective probability can have an intrinsic rigour that will eventually lead to global consensus for an affirmed theory. Second, empirically adequate means for a theory to give a likely description of reality, to fit the data in quantitative terms, and doesn't necessarily imply the theory to be true. Indeed, any true theory must of course be empirically adequate, but only one theory in a set of rivalling empirically adequate theories is expected to be true. This is a consequence of scientific underdetermination, in the Humean sense described in the previous Subsection.

The main agent that influences belief in a theory is empirical evidence, represented in Bayesian confirmation theory by a statement, E, that synthesises some available data. When the evidence becomes available, the belief in the evidence shifts from some value $P(E) < 1$ to $P^*(E) = 1$. It would be absurd in a scientific context to doubt of a manifest evidence. Here, P^* represents the posterior probability assigned by the rational agent after becoming aware of the evidence.

In general, in Bayesian confirmation theory the posterior probability associated to a theory (T) influenced by some evidence (E) is given by the conditional probability for T given E:

$$P^*(T) = P(T|E). \tag{182}$$

It is of course important to clarify what *influenced by some evidence* precisely means in Bayesian confirmation theory. This statement simply indicates that the subjective probability assigned by a rational agent to a theory changes after becoming aware of the evidence:

$$P^*(T) = P(T|E) \neq P(T). \tag{183}$$

In particular, if $P^*(T) = P(T|E) > P(T)$, the evidence is said to confirm the theory. Otherwise, if $P^*(T) = P(T|E) < P(T)$, the evidence is said to disconfirm the theory. The evidence is said to be irrelevant for the theory in the neutral case. Here it is important to note that the theory doesn't become empirically adequate, nor of course true, once it's confirmed by some evidence. In Bayesian confirmation theory, a theory is said to be confirmed or disconfirmed even when the probabilities change minimally, reflecting a gradual, continuous process of updating the credence in a theory based on newly acquired evidence. This is different from the natural language, which gives attributes a meaning to confirmation associated with overwhelming, conclusive evidence. Bayesian confirmation theory rejects the idea of a crucial experiment to entirely confirm or disconfirm a theory, as opposed to Popper's perspective, according to which science evolves thanks to decisive experiments designed to provide definitive falsification.

The probabilities assigned to a theory to be true, of course, are still subjective, and different rational agents may assign different probabilities, but, in order for Bayesian confirmation theory to be effective, it is mandatory to hypothesise that different rational agents shall agree on whether some evidence confirms or disconfirms the theory, regardless of how much it does so. This is a bold hypothesis and an objection against Bayesian confirmation theory can be that sometimes, even in a rigorous scientific environment such as physics, different authors (rational agents) happen to hold opposite positions on whether some evidence confirms or disconfirms a theory. This is exactly the case for the current work, in fact in the following Subsections it will be shown that some authors believe that the Planck satellite's 2013 data supports cosmic inflation, while different authors believe that the same data negates inflation. This case doesn't break the rationality hypothesis mentioned at the core of Bayesian confirmation theory, because it can happen that the empirical evidence is not straightforward to link to theory, especially when such a theory is still under development. The critical discussion on how the evidence links to the theory, once mature, constitutes a further element, even though not empirical evidence, on which rational agents supposedly agree *a-posteriori*, and they will change the probabilities they assign to the theory accordingly, in a coherent way. This rational debate constitutes one method employed in theory assessment in a non-empirical way.

For practical applications to scientific theories, probabilities in Bayesian confirmation theory are in general written in the form dictated by Bayes' theorem (named after Thomas Bayes even though it was not actually him to prove, nor state this theorem in a recognisable form, and it was Pierre Simon Laplace to write it in a way relatable to that employed nowadays):

$$P^*(T) = P(T|E) = \frac{P(E|T)P(T)}{P(E)}. \tag{184}$$

In the above equation, $P(T)$ represents the subjective probability assigned *a priori* to the theory by some rational agent; $P(E|T)$, the conditional probability for the evidence to appear, in the hypothesis that the theory is true, is called the likelihood of the evidence according to the theory; $P(E)$ represents the expectancy of the evidence, and it refers to the prior probability for the evidence to show itself from the pool of the outcomes of an apposite test or the subjective probability to encounter the said evidence in a more general environment.

Bayes' theorem can be proven for discrete events by taking into account the definition of conditional probability:

$$P^*(T) = P(T|E) = \frac{P(T \wedge E)}{P(E)}, \tag{185}$$

with the condition $P(E) \neq 0$.

Symmetrising the above relation with respect to T and E gives

$$P(E|T) = \frac{P(E \wedge T)}{P(T)} = \frac{P(T \wedge E)}{P(T)} \rightarrow P(T \wedge E) = P(E|T)P(T), \tag{186}$$

with the condition $P(T) \neq 0$.

Putting the two previous equations together proves Bayes' theorem:

$$P(T|E) = \frac{P(E|T)P(T)}{P(E)}. \tag{187}$$

The two conditions $P(E) \neq 0$ and $P(T) \neq 0$ respectively mean that Bayesian confirmation theory breaks down respectively if the evidence taken into account is impossible and if the rational agent estimates the considered theory to be absurd.

A great advantage of Bayesian confirmation theory over the approaches mentioned at the beginning of this Subsection (logical positivism, falsifiability) is that, by assigning and manipulating probabilities, it allows quantifying how much some evidence confirms or disconfirms a theory. In particular, as long as rational agents behave coherently in assigning probabilities, repeated evidence (E_1, E_2,...E_k for any natural k), confirms more and more the prescribing theory:

$$P^*(T) = P(T|E_1 \wedge E_2) = \frac{P(E_2|T)P(T|E_1)}{P(E_2)} \frac{P(E_1|T)P(T)}{P(E_1)} > P(T|E_1), \tag{188}$$

and

$$\begin{aligned} P^*(T) = P(T|E_1 \wedge E_2 \wedge ... \wedge E_k) &= \prod_{i=1}^{k} \frac{P(E_i|T)P(T|E_{i-1})}{P(E_i)} \\ &= \frac{P(E_k|T)P(T|E_{k-1})}{P(E_k)} \prod_{i=1}^{k-1} \frac{P(E_i|T)P(T|E_{i-1})}{P(E_i)} \\ &= \frac{P(E_k|T)P(T|E_{k-1})}{P(E_k)} P(T|E_1 \wedge E_2 \wedge ... \wedge E_{k-1}) \\ &> P(T|E_1 \wedge E_2 \wedge ... \wedge E_{k-1}), \end{aligned} \tag{189}$$

with the auxiliary definition for the prior probability $P(T|E_0) = P(T)$.

However, this does not guarantee that the series for the posterior probability, $P^*(T)$, converges to unity as evidence is repeated. This is because, as the same evidence is repeated, it becomes less and less surprising and, in agreement with the natural intuition, confirms less and less the theory. This happens because the term $P(E)$ in the denominator of the right-hand side of Bayes' theorem becomes very close to unity when the corresponding evidence is surely expected, therefore it loses confirming power. Heterogeneous evidence from different sources, still in agreement with the scientist's natural intuition, is needed to strongly confirm the theory and drive its corresponding probability towards unity.

The here introduced Bayesian confirmation theory constitutes a useful framework for representing and tackling a situation of underdetermination in practice, discussed in the previous Subsection. Such a situation arises when different theories show the same value of confirmation or disconfirmation with respect to the same evidence. Bayes' theorem, however, casts in clear light the elements that make up this confirmation and allow a critical reasoning of the priors assigned by rational agents to the competing theories, on which the posteriors heavily depend. A revaluation of the priors in light of past evidence might help to discriminate between competing theories when empirical evidence cannot, even relying on non-empirical factors of pragmatic power or aesthetics. However, Bayesian confirmation theory is powerless in situations of underdetermination in principle, where all the possible discrimination is relegated to those arbitrary non-empirical factors.

One more great advantage of Bayesian confirmation theory is that, while its application in situations where there is a direct dependency between the theory and the evidence, it also allows accounting for situations in which the evidence is not a deductive consequence of the theory, and it is related to the theory by means of another element called common cause. Constructing the logical dependency of theories and evidence is what constitutes Bayesian networks, which will be crucial in defining the meta-level hypothesis in the discussion of meta-empirical assessment of scientific theories in Subsection 4.5. In particular, when the common cause variable is unknown, theory and evidence show some indirect dependency, but, when the common cause variable is understood, theory and evidence become independent and their dependency is regulated by the common cause, which is said to screen off the evidence from the theory. By means of a common cause, Bayesian confirmation theory thus allows to model cases of indirect confirmation that are outside the range of deductivist confirmation theories and constitute their weakest point.

Bayesian confirmation theory, due to its dual nature of essentially trying to quantify a philosophical topic, can be attacked in many ways [44]. However, it is defendable and it is still employed today by philosophers of science, while not so much by scientists.

First, it might be observed that Bayesian confirmation theory is too flexible, as it makes possible to always construct a Bayesian network that supports the claimer's intentions. However, the assumptions of a model cannot be just invented out of the blue, and they need to be justified in a rational way. In the construction of a Bayesian network, in order for the network to hold, it is important to make sure that the common cause actually screens the evidence from the theory, and there can be more problems arising in its construction, such as variables other than the common cause representing features of the specific problem at hand and interacting with the evidence and the theory. Moreover, in cases where the logical reasoning supporting a theory is foggy, casting the assumptions and the motivations in Bayesian terms can make them clearer and more accessible by the scientific community.

Another objection can claim that science is based on consensus among the scientific community, which is based on very strong, indisputable evidence, and only empiric evidence is strong enough. The counter-objection is that confirmation of a theory and acceptance of such theory do not coincide. While on the one hand theory confirmation is, in Bayesian terms, an epistemic notion, and it is based on an increase of subjective probabilities assigned to the theory by a rational agent, on the other hand acceptance of the theory by the scientific community has a pragmatic flavour that in Bayesian confirmation theory is related to a very high posterior probability, ideally close to unity. It is indeed true that indirect evidence does not necessarily lead to an acceptance of the theory, but Bayesian confirmation theory's domain does not extend as far as to cover acceptance, and it only rules confirmation and disconfirmation of a theory based on a rational agent's thinking.

One more objection can claim that indirect confirmation is negligible when compared to empiric evidence. A supporter of Bayesian confirmation theory can agree that this is arguably the case, but it doesn't confute the theory, since indirect confirmation can still play an important role before empiric evidence is discovered, especially in directing the scientific community's efforts where they are most epistemically profitable. Indeed, it can also play a different role in accordance or discordance with empirical evidence: it is again, for example, the interesting case of quantum mechanics, strong of uncountable empirical evidence but still subject to half a century of scepticism by researchers.

Furthermore, while there is an important subjective component to Bayesian confirmation theory, objective paradigms suffered from problems that Bayesian analysis tries to overcome. It is also possible that in the end there is an intrinsic subjective component to science itself, and the Bayesian approach simply casts it in a clear light. Identifying what role the subjective component plays in the logical reasoning can also help the scientific community rationalising it and discussing it explicitly.

Having introduced the main concepts of Bayesian confirmation theory, it is now time to apply them to the contemporary debate on the epistemology of inflationary cosmology. As will be discussed in Subsections 4.3 and 4.4, in the case of inflation there are numerous models to be compared, and absolute probabilities, in the form of priors for different models, are hard to define. The theoretical construct most suited for this situation is arguably the comparative Bayesian framework [45]. It differs from ordinary Bayesianism as it avoids the problem of defining priors and focuses on relative likelihood ratios for the posteriors of two distinct theories given the same evidence, such as

$$\frac{P(T_1|E)}{P(T_2|E)} = \frac{P(E|T_1)P(T_1)}{P(E|T_2)P(T_2)}, \tag{190}$$

in light of Bayes' theorem. While the prior probabilities for both theories are involved in the above equation, according to the comparative Bayesian framework only their ratio needs to be specified $\left(\frac{P(T_1)}{P(T_2)}\right)$ and updated in light of the new evidence according to the likelihood ratio $\left(\frac{P(E|T_1)}{P(E|T_2)}\right)$.

4.3 Debate on 2013 Planck satellite's data

Section 2 discussed new inflation, based on a plateau-like potential, and its core mechanics, while Section 3 showed how the inflation theory evolved through the decades by presenting many other inflationary models, including the hilltop model. In particular, Subsection 3.3 discussed the measure problem associated with eternal inflation. The present Subsection is devoted to presenting the first act of the epistemological debate on the inflation theory, with the formalism of Bayesian confirmation theory introduced in the last Subsection. The following Subsection (4.4) will present the second act of the debate.

Following the publication in 2013 of the Planck satellite's data about the CMB anisotropies, which, according to the authors involved in the collaboration, supports the simplest inflationary models, Anna Ijjas, Paul J. Steinhardt and Abraham Loeb started attacking the inflation theory, questioning explicitly its scientific nature [46]. In their paper, they claim that the presumed simplicity of the inflationary models supported by Planck's data only refers to the inflaton potential depending on one single scalar field in order to be able to fit the data. However, when observed from different angles, these simplest models suffer from complexities that are hard to tackle, as they are affected by necessary fine-tuning of the initial conditions and bring about the multiverse as a consequence, with all the related problems already discussed at the end of the previous Section. Moreover, according to the general features of the inflation theory as it evolved since the 2010s, the favoured simple potentials are extremely unlikely to be responsible for driving inflation. Ijjas et al. baptised this the *unlikeliness problem*.

Specifically, in modern inflationary cosmology the likeliness of different models is compared by means of $r - n_s$ plots. Such plots describe how different tensor to scalar ratios of the fluctuations (r), representing the relative contribution of tensor fluctuations with respect to scalar fluctuations in the overall spectrum, generate scalar spectral indices (n_s), representing the distributions of density fluctuations during the inflationary epoch and quantifying the scale-invariance of predicted large-scale structures[4]. Confirming WMAP's data and integrating it with further evidence, Planck's data show an unlikeliness in the range of 1.5σ for the simplest models such as power-law and chaotic inflation, discussed in Subsection 3.1, while they show a strong likeliness for plateau-like potentials such as new inflation, presented in Section 2. Ijjas at al. claim that this puts the inflation theory into troubles, since the simplest, unlikely models easily obtain the desired amount of inflation (more than 60 e-folds) by only requiring one parameter with no tuning at all on the initial conditions, while the favoured plateau-like models need three or more parameters with fine-tuned initial conditions to reach the same outcome.

Taking into account a classic plateau-like potential such as

$$V(\phi) = \lambda(\phi^2 - \phi_0^2)^2, \tag{191}$$

[4] $n_s = 1$ is associated with a perfectly scale-invariant universe; a greater value indicates that smaller scales have more power and the spectrum is said to be *red-tilted*; a smaller value indicates that larger scales have more power and the spectrum is said to be *blue-tilted*. Planck satellite's data show $n_s \simeq 0.965$, indicating a slightly red-tilted spectrum.

which features two power-law sides and a plateau in the middle, not only inflation from the power-law sides requires no fine-tuning, but it is also much more probable, as it covers a much wider range of values for the inflaton field, and it doesn't suffer from the long-standing graceful exit problem. According to Ijjas et al., by disfavouring power-law inflation, Planck satellite's data look as though they closed the door to the inflation theory and only left the window open for inflation to enter the picture. The lesson to be learned here is that, even if empirical evidence confirms a theory, because Planck's data, confirming previous WMAP's data, do actually confirm inflation, such evidence can represent a failure for the theory as long as the model or class of models it requires happen to be unlikely according to the theory. This is arguably a *unicum* for the history of science.

The final argument in Ijjas et al.'s paper is related to the multiverse, predicted by the models favoured by Planck's data (and by most of the other models in the inflationary context), which they claim is a deal-breaker for the theory itself since the measurement problem, despite the many attempts, has not yet been solved (see for example [47]).

Ijjas et al. conclude that the three independent problems (unlikeliness, fine-tuning and multiverse becoming inevitable) discussed above, brought about by a single set of measurement, that is Planck satellite's, constitute both a confirmation of the quality of the experiment performed and a huge issue that puts the theory of inflation in a seriously difficult position in the epistemological landscape.

According to Richard Dawid's reconstruction of the debate [48], Ijjas et al.'s reasoning may be synthesised in two main points. First, they answer the question "*are the observations what is expected, given a specific inflaton potential?*" in a negative way, since plateau-like potentials do not reproduce Planck's data with the important hypothesis of having natural initial conditions; such potentials do in fact need fine-tuning of the initial conditions in order to reproduce the data. In relation to this first question, it was remarked that Ijjas et al.'s arguments apply to hilltop-like potentials (presented among the others in Subsection 3.2) and not to genuine plateau-like potentials (like new inflation), which in turn are in fact favoured by Planck's data and do not suffer from the same fine-tuning of the initial conditions' problem. Despite the confusion among the potentials on the strictly physical side, Dawid highlights that this does not undercut the philosophical importance of the debate.

The second main question, *"is the inflaton potential X that fits the data what is expected according to the internal logic of the paradigm[5]?"*, even not considering the fine-tuning problem, finds a negative answer as well: plateau-like potentials are extremely unlikely according to the inflation theory.

In his Bayesian reconstruction, with reference to the comparative Bayesian framework introduced at the end of the previous Subsection, Dawid notes that Ijjas et al.'s arguments broadly follow a likelihoodist scheme, involving the law of likelihood: the evidence (E) favours one theory (T_1) over another theory (T_2) when the probability to observe the evidence while considering true the first theory is estimated to be greater than the probability to observe the same evidence considering

[5]Dawid points out that Ijjas et al. use the words *paradigm* and *theory* interchangeably. The present work will adopt the same convention.

true the second theory. Considering the assumption of Bayesian confirmation theory to write such likelihoods in terms of conditional probabilities, the mathematical condition is the following:

$$P(E|T_1) > P(E|T_2). \tag{192}$$

Ijjas et al., however, employ a further likelihoodist scheme by assessing at the same time the likelihood of the data according to the models, M, and the likelihood of the models according to the paradigm of reference, with the hypothesis that the models are partitions of (i.e. subsets that united realise the) whole paradigm and that models embody all the causal effects of the theory (screen off the evidence from the theory):

$$P(E|M) = P(E|M \wedge T). \tag{193}$$

Then, after labelling with an index i, with $i \in 1, 2$, two distinct theories, T_i, and with one more index j, $k \in 1, 2, ...n$ different models of the same theory, M^i_j, the likelihood of a theory given the evidence, may be written as

$$P(E|T_i) = \sum_{j=1}^{n} P(E|M^i_j) \cdot P(M^i_j|T_i). \tag{194}$$

This expression may be proven by expanding the right-hand side:

$$\begin{aligned} &\sum_{j=1}^{n} P(E|M^i_j) \cdot P(M^i_j|T_i) = \sum_{j=1}^{n} P(E|M^i_j \wedge T_i) \cdot P(M^i_j|T_i) \\ &= \sum_{j=1}^{n} \frac{P(E \wedge M^i_j \wedge T_i)}{P(M^i_j \wedge T_i)} \cdot \frac{P(M^i_j \wedge T_i)}{P(T_i)} = \frac{P(E \wedge T_i)}{P(T_i)} = P(E|T_i), \end{aligned} \tag{195}$$

by the definition of conditional probability.

For the purpose of the current work it is important to note that, while the likelihood of a model according to the data is a strictly empirical assessment, the likelihood of the model according to the theory constitutes an assessment of the non-empirical kind.

The answer to this critique of inflation by some supporters of the theory, including some of its fathers, arrived promptly in less than one year and assumed quite an oppositive tone.

Guth et al. [49] negate Ijjas et al.'s claim that inflation is only obtained by means of a plateau-like potentials and state that their claim is based on deprecated assumptions no longer at the core of the inflation theory. Departing from the original view developed in the 1980s and 90s, inflation is now an extremely versatile theory that does not need specific assumptions on the shape of the potential, and it may be very well adjusted so to fit the data. Instead of bringing rigid physical predictions, inflation has multiple free parameters that may be set after the observations. This does not mean trouble for the inflation theory, since also most of the parameters of the standard model in particle physics are set by observations, and there is no question today about the validity of the standard model.

Regarding the inflation theory, the simplicity assumption of the potential, brought *in auge* by the rise of chaotic inflation, is now long abandoned, and, as it was discussed in Subsection 3.2, a complicated multiple-field potential with distinct local minima is just as likely to drive inflation as a simple single-field potential. Moreover, there is nowadays no need to expect, nor of course require, that inflation happened in a single, uninterrupted phase, as many models now include multiple phases of inflation. The measurable features of the universe, such as CMB anisotropies, would only show features from the last chronological phase, and they would be unsensitive to whatever happened before. Considering a multiple-phase inflation model, most of Ijjas et al.'s assumptions do not hold.

Similarly, Guth et al. admit that the multiverse together with the measure problem is indeed an open problem, but they think of it simply as a yet-unanswered question, and it does not undermine the validity of the inflation theory:

> [...] the mere fact that we have not solved this problem is no reason to believe that nature would avoid eternal inflation. Nature does not care whether we understand it or not. (A. H. Guth et al., Inflationary paradigm after Planck 2013 (2014), Physics letters B 733, pp.115)

In his 2023 paper, Dawid agrees that Guth et al. that the measure problem certainly does not preclude the predictive power of inflation, otherwise Planck's data wouldn't have predictions to show agreement with.

In the conclusions to their paper, Guth et al. agree with Ijjas et al. that important questions remain, but they claim that inflationary cosmology rests on very firm foundations and, given the recent progress on both the observational and theoretical fronts, is in better shape than ever before.

Linde joined the debate with a second answer to Ijjas et al. in aggressive tones, fully agreeing with Guth et al.'s defence [50]. He claims that the problems related to some measures associated to the multiverse are long contained [51], and that it is incredibly hard to propose an alternative theory to inflation. Such a theory would need to explain why only one vacuum of string theory exists, and why the other countless vacua can't, and would need to find alternative explanations to many of the problems solved by inflation. Moreover, inflation has the great advantage to be almost independent of the singularity problem: it can describe the evolution after the singularity, after the big bounce or after the primordial universe quantum tunnelling from vacuum into existence, and the multiverse scenario makes the evolution completely insensitive to what was before inflation.

4.4 Inflationary schism

Following Guth et al.'s and Linde's responses, Ijjas et al. published another paper attacking inflation on a different side [52]. They expose what they call the *inflationary schism*, meaning that, while they discussed the original theory of inflation, named *classic inflation*, the authors defending it brought arguments from a different, modern version of inflation. This is the postmodern inflation referred to in Subsection 3.2.

According to Ijjas et al., the now deprecated classic inflation was based onto three main foundations. To determine the prediction of a classic inflationary model, initial conditions must be specified at a time comparable to Planck's time, when general relativity starts describing approximatively well the evolution of the universe. The shape of the potential for the inflaton field must be specified to determine all possible classical evolutions constrained to the considered model, with the initial conditions discriminating between subsets of them, and the measure must be specified to compute the probability of the distinct predictions within the picked subset of evolutions. Classic inflation had pretty naive features including simple initial conditions, simple potentials and a common-sense volume-weighting measure that couldn't make sense of eternal inflation and the multiverse due to the many problems already discussed. The authors claim that "classic inflation is a catastrophic failure by this measure; numerically, it is one of the worst failures in the history of science" and they ask the question of why such a catastrophic failure survived the scientific debate for all these decades. They claim that it is because, when ignoring the multiverse and the measure problem, classic inflation *seems* to fit the relevant observations, while their previous paper show that this is no longer supportable. Ijjas et al. highlight that Guth et al. do not dispute that classic inflation is observationally disfavoured by Planck's data, even though the confusion between plateau-like and hilltop-like potential had not been exposed yet at the time of their claims.

Postmodern inflation, on the other hand, following Guth et al.'s response, does not consider realistic assuming simple potentials for the inflaton field providing a single phase of inflation any more, and it favours instead complicated potentials with numerous parameters and local minima leading to multiple stages of inflation. Initial conditions do not play a central role in postmodern inflation, whose core is in fact shifted towards a highly flexible measure. Even if someday exact initial conditions will be discovered, the measure will simply be adjusted *a posteriori* so that future data becomes likely to emerge from a chosen model. It was discussed in Subsection 3.2 that a potential involving a single field and as few as three free parameters can be adjusted to fit any precision data. Ijjas et al. claim that this is highly problematic from an epistemological point of view, since with this premise no empirical observation can falsify, nor consequently confirm, the theory. Any cosmological measurement becomes simply an exercise more than an epistemic verification, thus postmodern inflation lies outside the boundaries of regular science. Future data is meaningless to the inflation theory because the features of the theory are chosen afterwards to match the evidence. Failing to fit the data doesn't lead to the rejection of the theory, but only to a readjustment of the measure. Furthermore, many different models become able to fit any data, bringing inflation into a serious situation of underdetermination in the modern sense identified by Quine (already discussed in the first Subsection of the present Section), with the scientific community becoming powerless to discriminate among different models except for a possible application of Ockham's razor.

Another problem associated with postmodern inflation is that, historically, initial conditions were useful to relate inflationary cosmology to the singularity that happened before. One of the main reasons for developing the inflation theory was to explain how the observed universe came to evolve starting from a wide set of possible initial conditions. With postmodern inflation, the inflationary epoch(s) become totally independent of the initial conditions just after the singularity. Inflation doesn't try to explain how the universe emerged from them, and relegates initial conditions to just a problem of adjusting the measure accordingly, thus it comes to lose even this ounce of explanatory power. Ijjas et al. stress that, as admitted by the same Guth et al., a persuasive measure to account for the probabilities in the multiverse doesn't exist yet. Since classic inflation has failed and is now deprecated, the important question is whether the scientific community can accept postmodern inflation instead, even if it is something different from usual science, or it is better to reject inflation altogether and look for a different cosmological theory.

Guth et al.'s response, signed by thirty-three authors, stresses that not only inflation is testable, but it has been so far subjected to numerous tests and has passed all of them [54]. The authors don't directly contest the event of inflationary schism, even though Dawid highlights that, among the physicist working in inflationary cosmology, probably most of them would not admit the schism, but they state that inflation is not a unique theory, and it is better viewed as a collection of models, all connected by the underlying idea that the universe inflated very soon after the singularity. In order to validate inflation, there is non need for all of the models to be correct, in fact nobody would support such a claim, and the situation is similar to the early development of particle physics, in which a great number of models associated with quantum field theory were explored before applying eliminative reasoning and identifying one to fit all the data.

Guth et al., opposing Ijjas et al.'s claim, state that there is no reason to suppose that the testability of a theory must require that all predictions be computed before the testing, independently of the parameters choice. Again, if that was the case, there would be reasons to question the epistemology of the standard model in particle physics as well. The standard model was in fact modified *a posteriori* to account for the discovery of the second and third generations of quarks and leptons. According to Guth et al., it is common practice in science to modify a theory to accommodate newly discovered data. Empirical, scientific theories do not get proven like mathematical theories, but, as tests are performed and evidence is collected, the successful ones gain more and more consensus within the scientific community, and this has happened with inflation.

Ijjas et al.'s answer, published in the same article, stresses that postmodern inflation cannot benefit from the early success in terms of experimental confirmation of classic inflation, even though postmodern inflation actually includes classic inflation as a collection of models in its framework, because eternal inflation was not understood at the time, and all the papers hinting towards inflation's predictions ignore this problem. Indeed, if anything that can happen will happen, it makes no sense to even talk about predictions. Guth et al.'s take on this is that in a multiverse scenario it is true that anything that can happen will happen, but not with equal probability, thus it still makes sense to talk about predictions and probabilities, and the measure problem must be solved to account for this.

According to Dawid's reconstruction of the debate, already introduced in the previous Subsection, it looks indeed that accepting postmodern inflation requires the costly sacrifice of empirical testability in the inflationary framework, and Guth et al. seems most in difficulty in contesting this point. However, their reasoning is better understood considering that empirical testability only succeeds at confirming models, while the theory as a whole (considered as a collection of models just as Guth et al. suggest) lies outside empirical testability. Henceforth, while Planck satellite's data does disfavour inflationary models based on simple single-field potentials, they don't confute the inflationary paradigm itself. As claimed by McCoy, eliminative reasoning will have a central role in the future of inflationary cosmology [55]. With this interpretation in mind, it is possible to defend inflation against Ijjas et al.'s attack by substituting accommodation of the measure with simple eliminative reasoning.

The authors do not seem to be able to find common ground. While Ijjas et al. adopt a traditional view of theory testing according to which a theory's predictions must be uniquely determined before empirical testing can begin, Guth et al. relax the tradition stating that it would be counterproductive to reject a fertile scientific theory such as inflation only because of a conservative approach to science assessment. At this point, according to Dawid, the important question in philosophy of science becomes what conditions legitimise the relaxing of the consolidated, traditional approach to theory testing in perspective of future successful developments in the predictive power of a theory at the boundary of ordinary science. It is also important to understand how much time, and how many resources, is the scientific community willing to invest in a theory that might turn out to be empirically not verifiable. Here is where non-empiric theory assessment comes into play.

In his 2023 paper, Dawid claims that the polemical tone of the debate on the epistemology of the inflation theory doesn't do justice to this important, and possibly fertile, matter in philosophy of science. It can be useful to take into account the credence subjectively put in a theory and cast Ijjas et al.'s reasoning under a Bayesian spotlight. In Dawid's point of view, valuing credence in scientific thinking is not simply a metaphysical addition to the debate. It is a natural element that comes into play when discussing the scientific status of theories, and it is already implicitly, strongly present in the claims of the defenders of inflation, even though Guth et al. do claim that inflation can be and has been tested multiple times without breaking the rules of regular science. Moreover, Ijjas et al. do not explicitly consider credence in their reasoning as well, but credence may still be an important factor to clarify their attack to inflation and understand why, under a Bayesian light, the flexibility of cosmology does leas to a serious problem in theory assessment.

Dawid's reconstruction of Ijjas et al.'s position assumes that inflation as a theory altogether does lead to unique predictions such as the flatness of the universe, the isotropy of the CMB and the absence of magnetic monopoles, which under regular scientific circumstances would generate significant credence directly in the theory and not in the individual models, as they all share this predictive feature. However, it is not clear whether deviations from precise, quantitative predictions, such as CMB anisotropies, indicate that more complex inflaton potential models are needed for eliminative reasoning, or simply show empirical evidence against inflation, and this fogginess prevents credence to be attributed to the theory. Furthermore, at the current stage, there are no substantial limits to the models that can be developed within the inflationary framework, and this poses strong constraints on the effectiveness of any eliminative reasoning, until the high energy physics the models depend on is understood and experimental technologies advance at a point in which they can discriminate if the evolution of the early universe underwent inflationary dynamics. Of course, there is no guarantee of success on the theoretical high energy side, nor on the experimental side. In the absence of both, it is looks legitimate to attribute low priors, in the Bayesian sense, to inflation.

If there are no limits to the models, no significant credence can then grow onto a single model, since the source of such a credence would be shared within a wide class of competing models. Therefore, with no significant credence being acquired by the models, no significant credence can be acquired by the theory either, especially when, as Guth at al. themselves suggested, it is interpreted merely by a collection of those models. The consequence is that, in the scenario in which inflation cannot gain significant credence directly nor via its models, it cannot gain significant credence altogether. Here, the keyword is *significant*, as in Bayesian confirmation theory the increase in credence in a theory due to an event does not need to be significant in order for the event to confirm the theory, hence not gaining significant credence does not automatically disqualify the theory as unscientific. It is still important to understand, however, how much time and how many resources are legitimately employed in a theory that can't gain significant credence despite the progress. Ijjas et al. have the merit to have raised this doubt.

Bayesian confirmation theory nowadays plays a relevant role in model selection within scientific theories, as it is employed to extract information about in what ways the empiric data available relatively favours or disfavours a model with respect to another. Comparing two models, M_1 and M_2, within the same scientific theory, the comparison regarding the same evidence, E, can be set up in the following way:

$$R_{12} = \frac{P(M_1|E)}{P(M_2|E)}. \tag{196}$$

Plugging in Bayes' theorem,

$$P^*(M) = P(M|E) = \frac{P(E|M)P(M)}{P(E)}, \tag{197}$$

for both the terms in the ration, yields:

$$R_{12} = \frac{P(E|M_1)P(M_1)}{P(E)} \cdot \frac{P(E)}{P(E|M_2)P(M_2)} = \frac{P(E|M_1)P(M_1)}{P(E|M_2)P(M_2)}. \tag{198}$$

In the above equation, the general theory, T, doesn't play a role, thus it is epistemologically inert. This is a representation of Ijjas et al.'s claim that, while future data may help to select models within the inflationary paradigm, models comparison at the current state doesn't have a way of generating credence in the theory of inflation. Therefore, an exclusive application of Bayesian confirmation theory to inflation seems to align with Ijjas et al.'s reasoning. This all rests on the shoulders of a reasonable choice of priors, towards which a meta-empirical analysis will be introduced in the following Subsections. Before deepening the topic, it is important to note that this is not the only conclusion that Bayesian confirmation theory brings to the dispute on inflation.

As already mentioned, classic inflation puts constraints on the potentials that it contemplates, while postmodern inflation, in addition to simple potential models, also involves complex potentials with multiple fields and distinct local minima, therefore postmodern inflation includes classic inflation as a subset. For the purpose of this discussion, it is useful to label with T_C classic inflation, with T_I postmodern inflation, and with T_+ the complementation of classic inflation with respect to postmodern inflation, that represents a theory including only the complex models included by postmodern inflation and excluded by classic inflation. Classic inflation and this auxiliary-defined complementary inflation partition postmodern inflation:

$$T_I = T_C \cup T_+. \tag{199}$$

For some evidence, E, the law of total probability prescribes

$$\begin{aligned} P(E) &= P(E|T_I)P(T_I) + P(E|\neg T_I)P(_I) \\ &= P(E|T_C)P(T_C) + P(E|T_+)P(T_+) + P(E|\neg T_I)P(\neg T_I). \end{aligned} \tag{200}$$

An important, legitimate hypothesis is that the expectancy $P(E|\neg T_I)$ is very small, which means that the evidence that supposedly confirms inflation is not prescribed by "non-inflation", that is any theory other than inflation. If this is not the case, it means that an unfortunate choice has been made for the evidence and, if it is predicted both by inflation and non-inflation, it cannot be expected to actually confirm inflation.

The posterior probability for classic inflation can then be expressed as

$$\begin{aligned} P^*(T_C) =& P(T_C|E) = \frac{P(E|T_C)P(T_C)}{P(E)} \\ &= \frac{P(E|T_C)P(T_C)}{P(E|T_C)P(T_C) + P(E|T_+)P(T_+) + P(E|\neg T_I)P(\neg T_I)}, \end{aligned} \tag{201}$$

while the posterior probability for postmodern inflation consequently is

$$\begin{aligned} P^*(H_I) =& P(T_I|E) = \frac{P(E|T_I)P(T_I)}{P(E)} \\ &= \frac{P(E|T_I)P(T_I)}{P(E|T_I)P(T_I) + P(E|\neg T_I)P(_I)}. \end{aligned} \tag{202}$$

Having discussed that $P(E|\neg T_I)$ is naturally small, in the above equation the prior probability $P(H_I)$ can also be considered small in order to follow Ijjas et al.'s logic. If, as they claim, no future evidence can provide significant confirmation for postmodern inflation, this means that the postmodern inflation has very limited predictive power, and the conditional probability $P(E|T_I)$ both in the numerator and the denominator should be negligibly larger than $P(E|\neg T_I)$, hence of the expectancy $P(E)$ as well. Consequently, unless there is some other way to assess the theory and grant it higher priors, it starts with low priors, and it is destined to end up with low posteriors, with any evidence failing to provide significant confirmation for it.

In their second paper from 2014, however, Ijjas et al. claim that, while postmodern inflation is epistemologically problematic for the reasons exposed, classic inflation does not share the same problems. In a Bayesian framework, this assertion is unsustainable. This is because it would imply that, after some empirical evidence, E, the posterior, $P(T_C|E)$, has the chance to be sensibly higher than the prior, $P(T_C)$. As it was assumed, postmodern inflation is composed of classic inflation and the complementary, auxiliary-defined inflation, T_+: $T_I = T_C \cup T_+$. But, if it is true that no evidence can significantly confirm postmodern inflation, then classic inflation can't receive significant confirmation as well, because any confirmation towards it would be immediately inherited by postmodern inflation, which includes it. This statement is based on the assumption that the only difference between the two theories is that postmodern inflation contemplates more models, but it is a very sound assumption since postmodern inflation historically evolved from classical inflation by considering increasingly complex potential models for the inflation field, and, as mentioned at the beginning of this Subsection, most cosmologists would not agree that there is an inflationary schism at all.

According to Dawid, Ijjas et al.'s position to favour on an epistemological basis classic inflation over postmodern inflation can still be defended with a strong commitment to simple models. However, the history of science shows that in the evolution of scientific theories simple, preferred models were often discarded to commit to advanced, more complex models in light of newly acquired discriminating empirical evidence. Moreover, the fathers of classic inflation, among which there is Guth himself, never made explicit such a commitment to simplicity, even though another father of classic inflation, Linde, noted the simplicity of the new models as one very convincing feature of the newborn chaotic inflation (discussed in Subsection 3.1).

In light of the reasons discussed in the present Subsection, it looks that Ijjas et al., despite the strong defences by Guth et al. and Linde, are not completely wrong in raising a concern on the epistemological state of inflation. In order to analyse the topic further, it is necessary to leave empirical evidence behind and discuss the role of meta-empirical confirmation in scientific assessment. The next Subsection discusses meta-empirical assessment in general, as Dawid originally developed it [56], while the Subsection that follows it discussed its outcomes when applied to the inflation theory.

4.5 Meta-empirical confirmation theory

As discussed in Subsection 4.1, inflationary cosmology is not the only theory that raised concerns about traditional, empirical theory confirmation, with string theory arguably being the other most important protagonist of the debate. In 2013, Dawid claimed that, while for contemporary cutting-edge theories empirical assessment is becoming more and more out of reach, such theories can still acquire - and are in fact acquiring - significant credence by purely theoretical means of non-empirical evidence, in a process called non-empirical confirmation.

The basis of non-empirical confirmation, similarly to ordinary empirical confirmation, is still observational evidence. The main difference is that observational evidence for non-empirical confirmation lies outside the theory's intended domain, meaning that it is not made up of deductive or inductive consequences of the theory itself. If it is, the discussion involves ordinary empirical confirmation and not a non-empirical confirmation at all.

Dawid argues that such a confirmation has always been important in scientific research, even though its relevance has often been unrecognised by researchers and philosophers of science. Even Einstein thought that research in theoretical physics is generally guided by a researcher's metaphysical prejudice, that is a pre-constituted idea of how the world is or must work, which guides their cognitive process of selecting and discussing significant data among countless other. Hence, there should be no controversy in stating that scientists assess their theories much before they can perform empirical tests, at least to decide whether it is worth to invest time and resources to keep working on such theories.

While it is important to understand how much strength can be attributed to non-empirical confirmation, Dawid argues that its reliability, and by consequence a measure of how much the scientific community is willing to accept it, is based on the past successes of such confirmation. Arguably, in a hypothetical world in which non-empirical assessment has been employed multiple times, and it always happened to confirm actually viable theories, there would be little reason to doubt about its reliability. However, the conclusive strength that non-empirical confirmation can hold in principle is something distinct from its actual reliability in the current context of theoretical physics. In the world we live in, non-empirical assessment has no history of providing conclusive evidence for viable theories in fields related to science, therefore any reasoning on its strength should take this historical element into account. However, even if that would be the desired outcome, there is no reason nowadays to push non-empirical confirmation to provide *conclusive* evidence, and it can still play an important, even leading, role when paired together with ordinary empirical assessment, and in the relevant, discussed cases in which empirical assessment is simply out of reach.

In the ideal scenario, then, non-empirical confirmation can significantly increase the credence (higher posterior) in a theory even without the ambition to provide conclusive evidence. Dawid argues that such a scenario is hardly reached by a single issue of non-empirical confirmation, which can be expected in general to provide little confirmation, and significant confirmation can sometimes be achieved by the combined application of different (at least two) non-empirical confirmation arguments.

The problem is now to build a reliable way in which rational agents coherently agree that some non-empirical evidence confirms the discussed theory (in Bayesian terms, increases the probability for the theory to be viable). The most reliable way looks that to build a system involving a mechanism that resembles empirical confirmation, which is certainly well-accepted, but relies on a non-empirical observation at its core.

To build such a system, Dawid proposes a generic hypothesis, named the *meta-*

level hypothesis, with the specific aim to predict, in the sense of to have as a logical consequence, some non-empirical observable fact, similarly to how scientific theories predict ordinary empirical data. Even though the meta-level hypothesis might not have the features of a sound scientific theory and might not be able to undergo the ordinary empirical tests, if it shows a positive correlation with the theory under discussion, than it behaves as an intermediary for the non-empirical evidence to indirectly confirm the theory, not differently than in a Bayesian network. The vagueness of the mechanism in which the non-empirical evidence may confirm the meta-level hypothesis strongly supports the claim that non-empirical confirmation is not to be intended as conclusive evidence.

The meta-level hypothesis proposed by Dawid in his realisation of non-empirical confirmation is the assumption that the context of local underdetermination, in which the theory under consideration is encompassed, is bounded. This means that it is not possible to formulate many competitor theories to the one under consideration, that is, theories that share the same empirical domain (they predict the same type of data in the same scientific context) but are logically incompatible, hence if one is viable the others must be not.

If, instead, local underdetermination is unbounded, empirical and non-empirical evidence equally confirm both the theory under discussion and the whole empirically equivalent set. It would be then irrational to assume that, among the variety of confirmed theories, the one under discussion should be the viable one. It is very hard to justify credence in a theory contextualised in an unbounded underdetermination situation. In Bayesian terms this means that, in front of any empirical evidence, probabilities to be viable shift coherently for a large number of theories, hence they all have the same probability to be viable and there is no scientific reason to favour one instead of the others. It is thus legitimate to expect that a bound to local underdetermination positively correlates with the viability of a theory involved. Vice versa, it is also legitimate to expect that emphasised credence in a theory is supported by the scarcity or absence of rival theories whose domain overlaps with the discussed theory's one, or, in other words, that local underdetermination is limited.

Having assumed that the meta-level hypothesis correlates with a theory's viability, to complete the non-empirical confirmation framework it is important that such a hypothesis is supported by multiple observable arguments. Dawid highlights that empirical evidence is ineffective at this point, since any empirically-confirming evidence would support equally the theory under discussion and any variety of empirically equivalent theories allowed by scientific underdetermination. Hence, the meta-level hypothesis shall be supported by multiple non-empirical arguments. Dawid suggests three distinct (but not necessarily independent) arguments that, when combined, can supposedly provide a significant confirmation for the theory.

The first non-empirical argument comes into play in confirming a theory after researchers have put considerable efforts in trying to build alternative theories with the same domain as the discussed theory, and have obtained little or no results. The confirmation of the meta-level hypothesis is in this case very straightforward, and there is proof under the Bayesian framework that this argument can in fact provide a form of confirmation [58]. When this situation arises, efforts and resources are intuitively expected to converge towards the discussed theory, and this is a non-empirical indicator, named the *no-alternatives argument*, towards the viability of the theory. In the asymptotic situation, when, despite many efforts, no alternative theories to the one under discussion were formulated, high credence is naturally attributed to the theory even while incomplete. Naturally, the meta-level hypothesis is not the only assumption that can predict the no-alternatives argument, as the latter finds justification also in the alternative hypothesis that researchers have not devoted enough efforts in looking for competing theories, or simply they were not able to find any. As a consequence, the amount of confirmation that the non-empirical argument provides to the meta-level hypothesis depends on the priors subjectively attributed to the meta-level hypothesis and the alternative hypothesis now formulated. While it was argued that Bayesian theory is based on the assumption that distinct rational agents agree on whether some evidence confirms or disconfirms a theory, rational agents might still attribute very different priors to the two hypothesis, setting up troubles for the no-alternatives argument.

The second argument is brought about from the history of the research field that encapsulates the discussed theory. In such a research field, when some set of conditions satisfied by scientific theories have shown positive correlation with the viability of those theories, another theory satisfying the same set of conditions has increased chances of turning out viable. This is directly predicted by the meta-level hypothesis, which may be considered as the condition to satisfy by the theories: when in the scientific fields under scrutiny there are no or few competing theories, such theories show increased chances of turning out viable. This reasoning constitutes the *meta-inductive argument*. A problem associated to the meta-inductive argument is that, as long as there are no other reasons to believe in the potential viability of the theory under consideration, taking into account other successful theories that share with it a set of conditions can be expected to produce very limited credence. Again, rational agents might disagree on those further reasons to believe in the viability of the theory. One other problem is that, in a given research field, a theory can benefit from a meta-inductive argument only as long as other theories have experienced issues of regular, empirical confirmation. If this is not the case, there is no conclusive proof that such theories are viable in the first place (non-empirical confirmation is never intended to become conclusive).

The third and final argument consists in the observation that, when a theory is formulated in order to solve a specific problem, sometimes it happens that it is also able to solve different problems within or outside the intended domain of the theory. This argument is predicted by the meta-level hypothesis because, when the number of theories in a scientific context exceeds the number of problems raised in the same context, there is no expectation that a single theory, designed to solve a specific problem, also solves different problems already tackled by many other theories in the same context. If, however, only a few theories were formulated in the given scientific horizon, with an exceeding number of problems to solve, a single theory is expected to, and statistically happens to, solve also problems other than the intended one. In the asymptotic case, a single theory in a scientific context is expected to solve all the open problems. When this is the case, the hint towards the viability of the theory is evident, and it constitutes the *unexpected explanatory argument*. However, the unexpected explanatory power of the theory under consideration might be due to logical connections between the problems tackled by the theory that researchers have not discovered yet.

This specific realisation of non-empirical confirmation was later named by Dawid *meta-empirical confirmation*, to distinguish it among the in principle broader set of non-empirical confirmation scenarios [57]. The three arguments (no-alternatives, meta-inductive, unexpected explanatory) are referred to as meta-empirical arguments.

As mentioned, Dawid stresses that the three meta-empirical arguments can't be expected to provide substantial confirmation individually, which might in turn be provided by the combined application of two or all three of such arguments. Specifically, the meta-inductive argument supports the no-alternatives argument, covering its weakest point. While the meta-level hypothesis can predict theories with a tendency of viability in a research field also tend to have fewer possible competitors, the alternative hypothesis that not enough efforts were invested in the research is completely uncorrelated with such a prediction. Therefore, an application of the meta-inductive argument can support the meta-level hypothesis against this alternative hypothesis in deciding which one the no-alternatives argument provides confirmation to. Vice versa, the no-alternatives argument can support the meta-inductive argument by providing the precise condition to look for in theories that turned out viable, that is, the feature of having limited competitors. Finally, more than simply having its own weight in confirming the meta-level hypothesis, the unexpected explanatory argument further supports the meta-inductive hypothesis by adding to the set of conditions to look for in successful theories the feature of solving problems they were not designed to tackle.

Dawid formulated this framework of interconnected arguments that support each other with the aim of formalising the significant non-empirical confirmation experienced by string theory (discussed in Subsection 4.1), but it naturally applies to an epistemological analysis of inflationary cosmology as well, with the possibly more important role to side with the existent empirical evidence that confirms inflationary cosmology but is not strong enough to be considered conclusive yet.

4.6 Meta-empirical assessment of inflationary cosmology

When applying meta-empirical confirmation to inflationary cosmology, Dawid underlines that what fits the case is a form of *meta-empirical assessment*, to be distinguished from the pure meta-empirical confirmation developed for the case of string theory, and discussed in the previous Subsection, by the non-absence of empirical evidence. What Dawid calls meta-empirical assessment is a framework that mostly overlaps with meta-empirical confirmation, but, instead of building credence in a theory by non-empirical means alone, it is flexible enough to account for newly acquired empirical evidence to influence the credence in such theory as well.

Dawid claims that, while a discussion on the lines of meta-empirical assessment is expectedly at the core of private reasoning and discussions among researchers, it is also already present in some important published works. Specifically, no-alternatives arguments are brought by Linde precisely in inflationary cosmology [59, 60], while a meta-inductive argument supported by a further no-alternatives reasoning is exposed by Peebles in the role played by general relativity in the evolution of cosmology [61]. Furthermore, as discussed in Section 1, the theory of inflation provided a natural explanation for the CMB anisotropies, even though they had not yet been observed when the inflation theory was formulated, and the evolution of inflation brought about the multiverse, which in turn provided a legitimate explanation for cosmological constant problem in light of the weak anthropic principle. This is clear non-empirical evidence of the kind prescribed by the unexpected explanatory argument.

While Guth et al. and the majority of the scientific community involved in the research on inflationary cosmology relies on collecting empirical evidence and employing it in confirming the theory, Ijjas et al. oppose such a confirmation in the case of postmodern inflation stating that the evidence has no confirmatory power towards the theory. A pure Bayesian analysis, performed in Subsection 2.4, shows that they may be right, but the non-empirical evidence exposed in the previous paragraph suggests that deviating from pure Bayesianism and relying on meta-empirical assessment might prove a better confirmatory framework. It turns out that meta-empirical assessment is exactly the missing ring to show that newly acquired empirical evidence can in fact confirm the inflation theory. Bayesian confirmation theory still plays the larger role of defining the prior and posterior probabilities assigned by rational agents, while meta-empirical assessment has the fundamental role of comparing the probabilities put in play by the Bayesian analysis and drive the search for conclusions.

At the core of Bayesian confirmation theory there is the tautology affirming that, if a theory, T, such as inflation, predicts some strong enough evidence, E, the posterior probability for the theory will be higher than the posterior probability for the negation of the theory, $\neg T$, namely non-inflation:

$$P(T|E) > P(\neg T|E). \tag{203}$$

Considering that all the probabilities are positive definite, applying Bayes' theorem to both sides gives

$$\begin{aligned} P(T|E) = \frac{P(E|T)P(T)}{P(E)} \rightarrow \frac{P(E|T)P(T)}{P(E)} > \frac{P(E|\neg T)P(\neg T)}{P(E)} \\ \rightarrow \frac{P(E|\neg T)}{P(E|T)} < \frac{P(T)}{P(\neg T)}, \end{aligned} \tag{204}$$

which is an inequality whose plausibility can be evaluated by means of meta-empirical assessment.

Representing Ijjas et al.'s claim according to which postmodern inflation, due to its unbounded variety of models, does not make clear physical predictions, the term $P(E|T)$ may be assumed to be small. However, since the evidence E is by definition in favour of inflation, it is just as legitimate to assume that the priors $P(E|\neg T)$ is small as well, otherwise the evidence would be more strongly predicted by non-inflation and it would end up disconfirming inflation, which is not the situation to be represented in this context.

It is now necessary to assume the meta-level hypothesis, stating that local underdetermination is bounded. The validity of such hypothesis ultimately depends on how sustainers of inflation will be able to defend their theory against competing models such as the ekpyrotic universe[6] [62]. All three meta-empirical arguments have been discussed at the beginning of the current Subsection to provide coordinate evidence for a sound meta-level hypothesis. This hypothesis prescribes that the priors of a theory competing to inflation shall be small as well.

The last term to evaluate is the prior probability for inflation itself, $P(T)$. This term, as prescribed by Bayesian confirmation theory, is in general arbitrary. However, inflationary cosmology, as discussed in Section 1, has proven valuable in solving multiple problems left open by hot Big Bang cosmology, and it is nowadays an important component of the standard cosmological model. Dawid claims that, even for supporters of alternative theories, it would be irrational to state that inflation is not a serious contender over its domain, and Ijjas et al. themselves raise concerns on the testability of the theory, without denying its ultimate viability. Henceforth, it is legitimate to assume that the priors $P(T)$ are not excessively small, and most cosmologist would consider them considerably high.

In light of all the assumptions discussed in the current Subsection, it is legitimate to conclude that newly acquired empirical evidence has the chance to provide significant confirmation to the theory (and not only the models) of inflation, even in the postmodern scenario. This, of course, does not mean that newly acquired evidence will be expected to certainly do so, but it can be expected to support the no-alternatives argument by making it harder and harder for competing theories to reproduce the success of inflation.

In the closing line to his 2023 paper, Dawid stresses that his work has not weighted the strength of specific meta-empirical arguments related to inflation, which shall be carefully evaluated by the careful combined efforts of the scientific community instead, but he has provided the theoretical framework to help them assess the state and evolution of the theory of inflation employing arguments outside its empirical domain.

4.7 Criticism of meta-empirical confirmation

While receiving some support among the scientific and philosophical community, Dawid's meta-empirical confirmation also raised strong points of criticism [63].

C. Rovelli, one of the fathers of loop quantum gravity, the most quoted rival theory of string theory, criticises especially the no-alternatives argument, which he recognises it is often employed by researcher in string theory [64]. A no-alternatives argument, however, only holds under some set of assumptions, which are in general arbitrary and may turn out to be false. Contenders of string theory indeed exist, and they are sometimes considered to have no alternatives by their supporters under their own set of assumptions. Any theory, both right or wrong, can be said to have no alternatives under a set of assumptions suitably defined.

[6]Linde in [50] highlights that one of the authors of [46, 52], P. Steinhardt, is also one of the authors of one of the ekpyrotic universe, theory competing with inflation, but less successful.

While Rovelli agrees that researchers employ non-empirical arguments for driving their research, otherwise the historical success of theoretical physics would be incomprehensible, but this is far from a novelty in science. Even Kepler had credence in Copernicus' astronomy before empirical evidence proved him right. However, this non-empirical confirmation is best left to the preliminary logical tests of theories in early development, and it should not interfere with the consolidated empirical confirmation of such theories. No scientist, Rovelli argues, regardless of their credence in string theory, would trust a prediction of string theory with their life at stake, even though everybody entrust their lives to classical mechanics and thermodynamics every day. This marks a clear distinction between non-empirically and empirically generated credence. The former is simply inadequate to raise the credence in a theory as much as to make it established in the scientific landscape.

Non-empirical confirmation is dangerous because it is a human feature to be blinded by personal beliefs, and beliefs are most of the time wrong. The scientific community has nourished their beliefs in supersymmetry for sixty years, and have been surprisingly disconfirmed by the 13TeV experiments at LHC [65]. Moreover, Rovelli brings the example of western medicine, which was developed based on statistical analysis of the employed remedies, substantially meaning by directly distrusting non-empirical confirmation. The result is that the life expectancy of humans has more than doubled thanks to western medicine.

String theory in particular has experienced the dangers of non-empirical confirmation, promising commendable achievements such as the reproduction of Standard model's cross-sections and the SU(5) group from first principles, and many others, and has failed to do so. Rovelli states that, from a Popperian point of view, these huge failures do not directly falsify the theory, as it is flexible enough to be adjusted accordingly, just like postmodern inflation under the accuses of Ijjas et al. However, from a Bayesian point of view, every one of the theory's failures should accordingly diminish the credence posed in it, just as any confirming evidence (empirical or not) would increase it. Deciding to adopt a Bayesian point of view for the successes of a theory, and a safe, Popperian point of view for the failures, would inevitably mean to have a strong confirmation bias towards the theory.

Another criticism to meta-empirical assessment comes from D. Oriti, who recognises the scientific and philosophical relevance of Dawid's work, but accuses him to have been excessively generous with his evaluation of string theory and raises doubts on the meta-empirical confirmation proposed by him [66]. Oriti agrees that non-empirical confirmation is an important component in the work of researchers, who very often rely on it at least on an unconscious level, but claims that it is legitimately always subordinate to traditional empirical confirmation, which has no power to replace. With the advent of theories outside the current testing capabilities, such as string theory and inflationary cosmology, non-empirical confirmation has a larger share of theory assessment than ever before, but this does not represent an actual shift in perspective on the methodology of science.

Given the importance of non-empirical confirmation in general, Oriti proceeds criticising Dawid's meta-empirical confirmation in particular. He claims that the meta-level hypothesis, stating that local underdetermination in the domain of a reference theory must be bounded, is unrealistic in the field of science. While some limitations to underdetermination are necessary to develop credence in scientific theories at all, such limitations are never as strong as to reduce to a linear path the space of possibilities on which scientific theories are constructed. Scientific underdetermination is unavoidable, and Oriti claims that it is good that it is so, since the proliferation of theories plays an important role in non-empirical confirmation.

In regard to the three meta-empirical arguments, Oriti admits that the no-alternatives argument (in a weaker form) and the unexpected explanatory argument are often employed by researchers defending their supported theories, while the meta-inductive argument is rarely employed, and not in an explicit way.

Oriti's largest criticism attacks the no-alternatives argument. According to Oriti, having a single theory trying to explain a specific problem is an idealised situation that never happens in the actual scientific landscape, and, in order to employ a no-alternatives argument, it is necessary to first make sure that there are really no alternatives to a given theory. This condition almost always fails. When a single theory remains in a given domain, it is so as a result of traditional empirical confirmation such as eliminative reasoning in light of newly acquired discriminating data. This is the definition of confirmed science.

On a purely theoretical level, if there is only one theory on a given domain before any empirical test is performed, it can be because of two reasons. First, the alternative theories may have been excluded because they did not satisfy a pre-chosen set of desired features or conditions, that moreover may have been chosen to adhere with the theory under scrutiny, looping the problem. A set of exclusive conditions can always be defined and expanded until only one theory is left in the set. This arbitrary process, already exposed by Rovelli, tells nothing about the viability of that theory. Vice versa, if there is a situation in which a single theory is trying to solve a given problem, it is often sufficient to lift a few conditions or prejudices about the problem to be able to formulate alternative theories, which Oriti stresses to be a very healthy habit in scientific research.

The second reason that might theoretically leave no alternatives is a comparison between competing theories. If the excluded theories are in early stages of development with respect to the defended theory, such as most proposed paradigms of quantum gravity, it is illegitimate to remove them simply because not enough efforts and resources have been invested in them. For a theory in a more advanced state of development, there is nothing to infer about its viability from having, perhaps accidentally, received more contributions. If the theories compared are on a similar level of development, the competing theories may have been excluded because of theoretical failures or shortcoming in the development, which might be overcome with more efforts invested by the scientific community.

In general, whatever the case considered, if competing theories are missing on a domain before empirical tests are performed, it is because some form of non-empirical confirmation has already been performed, and Dawid himself highlights that such confirmation can't be conclusive. Having no alternatives for a theory on a domain inevitably involves the scientific community's consensus, and it may be a situation influenced by flawed human preconceptions and sociological dynamics, far from the rigour of science.

The no-alternatives argument, then, acts as a reinforcement for the already-given consensus, which is the opposite of what an effective non-empirical confirmation should do, that is providing a critical analysis contesting and eventually disconfirming such consensus. Therefore, even if there were actual situations in which a single theory has no alternatives over its domain, satisfying the hypothesis of the no-alternatives, argument, still there would be nothing to infer from this argument, which is simply flawed.

According to Oriti, the meta-inductive argument is flawed as well. While not necessarily illogical, its intrinsic structure makes it unreliable in science confirmation. Having at its core the history of a research field, it heavily depends on the accuracy of the historical research carried out, which may turn out very complex. Moreover, relying on a statistical analysis, its efficacy depends on the number of cases analysed, and the variety of theories in a research field might not amount to a large number.

In Subsection 3.5 it was discussed that the meta-inductive argument relies on inductive reasoning based on different non-empirical arguments shared by affine theories that turned out to be viable after empirical confirmation. Oriti claims that, for the meta-empirical argument to have some power, it is mandatory to show that the related non-empirical arguments were primarily responsible for the theory to reach consensus, and not only that they contributed to it. Such non-empirical arguments must also have a proven record with a very high success rate in carrying towards consensus theories that later turned out viable, while the number of successes has little meaning.

Such a historical analysis is indeed very complex and might put in serious trouble even historians of science, hence it is hard to imagine scientists and philosophers of science being able to perform it. This rules the meta-inductive argument out of the framework.

In regard to the unexpected explanatory argument, Oriti agrees that it relies on a property common to many successful theories and its value is unanimously accepted. Just as Dawid theorised, it has confirmatory power for a theory against its competitors when employed in combination with different non-empirical arguments. Dawid, however, pairs the unexpected explanatory argument with the no-alternatives argument, so it inherits all the discussed problems, and this is the only feature that Oriti criticises.

To support the unexpected explanatory argument in place of the no-alternatives argument, Oriti proposes the *principle of proliferation*, and he claims that it is at the very core of science methodology. Notoriously supported by Feyerabend, and by him attributed back to nineteenth century's philosopher J. S. Mill, the principle of proliferation states that, before performing empirical testing, as many alternatives as possible to the dominant paradigm shall be formulated, and the object of empirical testing shall be the whole set of competing alternatives [67].

The principle of proliferation increases the depth of comprehension of scientific theories, which is still the core of science methodology, for multiple reasons. It allows putting a given theory in the perspective imposed by the set of alternatives, casting in clear light what is denied in the dominant theory in the hypothesis that one alternative, closer or farther to it, is proven correct. Alternative theories may happen to suggest novel empirical tests involving also the dominant theory, and such testability features might have been obscured by some features of the dominant theory until the application of the principle of proliferation. Alternative theories also help to relate theoretical predictions to experimental discrepancies, contributing to discriminating whether they are negligible features or essential characteristics. Moreover, alternative theories help in overcoming psychological constraints that might have been accidentally generated in association to the dominant theory, preventing the researchers to see the weaknesses of the theory. Linde in [15] stated that it was particularly difficult to come up with chaotic inflation, discussed in Subsection 3.1, specifically because of a psychological barrier that prevented the researchers to abandon all of the core assumptions of the original old inflation.

In particular, the principle of proliferation speaks against abandoning theories and models simply because they have become disconfirmed or disfavoured by means of empirical or non-empirical assessment, since the events that led to this transition may have been accidental, and new observations may still bring them *in auge*, and, even if this does not happen, disconfirmed paradigms still enrich the scientific landscape over their domain because of all the reasons discussed above. As already noted by Feyeraband, a theory may win a competition because it received more attention or by chance, before the competitors have had the time and efforts to show their strengths. Oriti claims that the principle of proliferation should be at the core of any proposal of non-empirical assessment, which in turn should be always subordinate to ordinary empirical assessment and should aim to produce scepticism towards dominant theories with the aim of challenging them instead of further promoting them.

Oriti proceeds in proposing a revisitation of Dawid's meta-empirical confirmation theory. He claims that, if the meta empirical criteria have any confirming power at all, it is so in light of the principle of proliferation, and this is probably agreed on by Dawid himself, even though he did not make it explicit. The principle of proliferation does bring a solid basis to the no-alternatives argument, because the natural selection of theories can only be fruitful after such theories have been supported to proliferate, but this does not mean that the no-alternatives argument has confirmatory power. When a situation arises in which a no-alternatives argument could be properly employed, the rational reaction by the researchers should be to support more proliferation instead of non-empirically concluding that the dominant theory is probably the viable one.

While disregarding the meta-inductive argument entirely, but not disagreeing with its core ideas, Oriti accepts the unexpected explanatory argument as a useful contribution to non-empirical assessment when employed in combination with other criteria, and he proposes the principle of proliferation itself as a support for it. The only way to rule out the alternative explanations for the unexpected explanatory power of a theory, he claims, is to proliferate the alternative paradigms that pose as their domain the problem unexpectedly explained by the theory under scrutiny. When the possibility that the unexpected explanation depends on some underlying more general feature can't be rule out, the way to expose it is to formulate more theories that significantly differ from the one under scrutiny but embed the said general feature in some way, and evaluate whether they provide an acceptable explanation for the problem unexpectedly explained.

Accepting the value of non-empirical confirmation in the current scientific landscape, Oriti concludes that, when empirical assessment becomes hard to come by, it is the proliferation of theories and the careful employment of non-empirical confirmation criteria to constrain and at the same time facilitate the proliferation that helps to navigate this harder scenario. Science progress is driven by an open mindset, which is naturally associated with a high theoretical pluralism.

Many more points of criticism were issued towards meta-empirical confirmation [68–70], and Dawid, while addressing them, stated that they contributed raising relevant issues in his framework [71, 72].

The present Subsection, while not providing an exhaustive list of the discussions about non-empirical confirmation, has shown that confirmation beyond the empirical level is nowadays significant and debated, and, with the proliferation of theories outside the current experimental reach, it can only acquire more relevance in the scientific landscape.

5 Conclusions: is inflation scientific?

The current work has presented the historical motivations for the formulation of inflationary cosmology, the technical details of its general features and some of its implications in philosophy of science, then has shown why it is so problematic to discuss the assessment of inflationary cosmology from a purely scientific point of view. This conclusive Section will harvest everything that has been seeded and will try to draw original conclusions on the assessment of the inflation theory, and more generally on philosophy of science. The statements presented in this final section are not to be intended as conclusive claims, but only as a set of speculative ideas that, once deeply elaborated and tested in future works, *might* provide the basis for a viable epistemological framework.

As it has been discussed in Subsection 3.3, eternal inflation and the multiverse provide the ontological basis to discuss what is science in terms of the possible worlds. Science is what must be true in every possible world, and this definition is ergodically equivalent to what is destined to repeat itself imagining rewinding the world's evolution to a pre-civilisation state. This discussion opens a different point of view to assess the validity of the meta-empirical arguments: a necessary requisite for them to provide scientific confirmation to a theory is that they can be soundly formulated in every possible world. Such a line of reasoning aims to extract the real scientific value of the arguments from a pool of accidental (in the metaphysical sense of contingent, as opposed to necessary) features that might characterise them, or, in other words, features that belong to the specific world we live in and not to every possible world.

To this end, it is important to note that this analysis will not affect the meta-level hypothesis, because it is in fact a hypothesis and not a conclusive statement. A hypothesis can always be made, more or less confidently, in every possible world. It is the facts that confirm or disconfirm the hypothesis that may change in different possible worlds. Such facts in meta-empirical confirmation are embodied by the three meta-empirical arguments, therefore it is those arguments that the present discussion will direct its attention to. As it will be shown, all three of those arguments are based, with more or less weight, on historical features. It was already discussed in Subsection 3.3 that historical facts are per se accidental, in the sense of not metaphysically necessary, since it is generally easy to imagine different possible worlds in which such events have had a different outcome, or, in the majority of the possibilities, in which they have not occurred at all (for Julius Caesar to be assassinated, it is necessary that Julius Caesar is born in the first place, which is not a metaphysical necessity).

Exposing some of the meta-empirical arguments' features as science hermeneutics, the present line of reasoning can hence be expected to undermine the meta-empirical arguments' confirming power, but this view on science is deeper than simply stating that history is unscientific and therefore can't provide any argument in favour of (or against) a scientific theory. Just as scientific laws are considered a metaphysical necessity, thus they are expected to be formulated in every possible world with some unsubstantial variation in form, it is hard to imagine a civilised world without art or literature, and, for example, a world with art and literature in which sooner or later a realist movement is not realised. With the same line of reasoning in the field of history, most of the laws and hierarchies in ancient empires and kingdoms can't be expected to repeat identically in different possible worlds (they are contingent), but it is hard to imagine a world in which sooner or later a form of democracy is not realised, or some alliance such as the organisation of the United Nations is founded. This means that, even in a notoriously unscientific field such as history, there is sometimes a scientific core that with the right precautions can be extracted to understand something more about both the epistemology of such field and the nature of science.

Before proceeding with the analysis, two cautionary notes. First, different people may disagree on what is true in different possible worlds. In order for the current discussion to have meaning, however, it is important to postulate the opposite: observers agree on what is true in different worlds. While this hypothesis might seem far-fetched, the very same hypothesis is at the core of the whole Bayesian confirmation theory. Rational agents must agree on whether some evidence confirms or disconfirms a reference theory. The reader that rejects the current premise because of this hypothesis, arguably should disagree with Bayesian confirmation theory as well. Naturally, different rational agents do not need to agree immediately over a confirming evidence or a possible worlds' statement (or a statement that some evidence confirms a given theory), but they need to be able to convince each other. The whole field of science, after all, is driven by different perspectives reaching consensus. If this was not the case, the epistemological debate presented in the current work would be meaningless.

Second, eternal inflation and the multiverse, even though attracting much credence nowadays, are not confirmed, and might one day be disconfirmed. Such an event would not discredit the current analysis for two reasons. First, this analysis does not depend on the ontology of the possible worlds, in fact the employed definition of science started developing in the seventies, much before eternal inflation was formulated (even though it might have been inspired by the many worlds interpretation of quantum mechanics). Second, the analysis can be cast in light of imagining rewinding the world's evolution and letting it repeat a great number of times, which is a solution independent of the multiverse, and was in fact independently formulated. Everything is now on the table to analyse the meta-empirical arguments in light of this view on science.

The no-alternatives argument states that, if a theory has no or few alternatives over its domain, then it is more likely to be viable. The absence of alternatives over an empirical horizon, however, might be contingent (not a metaphysical necessity). It might be that the research field is not mature enough to produce valuable alternatives, or not enough time and efforts have been devolved to the research for alternatives, or the alternatives are hidden somewhere and the scientific community has simply not been able to formulate them, possibly because of the absence of a suitable mathematical formalism. A research project, after all, can succeed or fail in formulating a theory. Neither success nor failure is metaphysical necessity, and, if a no-alternative situation is not due to a metaphysical necessity, it may be contaminated by contingent features that have no reliable confirmatory power (as long as confirmation is associated to an idea of science as what is true in every possible world). This supports Dawid's alternative hypothesis, competing with the meta-level hypothesis in justifying the no-alternatives arguments, that researchers have simply not been able to come up with alternatives to the theory under scrutiny.

The meta-inductive argument states that, if a theory shares (non-empirical) features with theories that have been proven viable, then it is more likely to be viable. But again, as long as the features under consideration are non-empirical, they might be contingent. For example, both Planck and Einstein, respectively during the early works in old quantum mechanics and special relativity, have drawn ideas from macroscopic thermodynamics. Planck employed the thermodynamical concept of entropy while trying to solve the black body's emission curves, while Einstein strongly desired for his theory to have the same simplicity, generalisability and applicability of thermodynamics. Naively, the meta-inductive argument would imply that a theory connected in its origins to macroscopic thermodynamics is more likely to be viable, but this is a difficult claim to defend. As long as special relativity is considered true, it must be true in every possible world, and it is hard to imagine that in any other possible world it could not be formulated without ideas drawn from thermodynamics. This implies that the relation between quantum mechanics and special relativity is contingent, and, moreover, that a meta-inductive argument based on such relation is stronger in some possible worlds than in others. This line of reasoning shows that in some situations the meta-inductive argument may be unscientific. Theories may have non-empirical features in common, but if such features are contingent there is little to infer on the viability of the theories.

Moreover, quantum mechanics and special relativity happened to be formulated at about the same time, at the very beginning of the twentieth century, and this may be another contingent feature. Non having an easily identifiable common cause, quantum mechanics and special relativity could have been formulated one century apart from each other[7]. Therefore, while the meta-inductive argument relies on theories formulated in the past (which, as Dawid stresses, are not necessarily predecessor theories), it might be contingent that theories with the desirable, shared features have been formulated in the past and not in the future of the theory under scrutiny. Moreover, many other theories might be there waiting to be formulated and then proven viable. If such theories share the features object of meta-induction, they can't be employed in the argument simply because of some chronological accident. In light of these observations, the meta-inductive argument does not look really sound with its confirmatory power.

Finally, the unexpected explanatory argument states that, if a theory provides an explanation for different problems than the one it was designed to solve, it is more likely to be viable. The criticising authors cited in Subsection 4.7 have agreed the most with this idea that giving unexpected solutions to problems is a feature shared by many relevant, viable theories. The issue with this is that an explanation to a problem might be expected or unexpected contingently. In different possible worlds, a theory mostly overlapping with inflation could be designed to solve the flatness problem and end up solving the monopole problem, but in other possible worlds the

[7]A historical study may prove or disprove the existence of a common cause, but this is only an example and countless others can be made to support this work's claim on the meta-inductive argument.

opposite could happen, or it could happen that the same theory is designated with both problems to solve in mind. If the unexpected explanatory argument is in many cases consequently reduced to a "killing many birds with one stone" argument, it certainly loses some confirmatory power. Even so, what are different problems in the present world might differ in other possible worlds. The same problems could be cast in a unified way, possibly because of the contingent scientific path that leads to the formulation of such problems. The critics have already exposed that apparently different problems might be related by some more fundamental theory, and the present analysis supports this view.

The line of reasoning here presented does not only behave as a *pars destruens*, though, as it can show support for the principle of proliferation presented in Subsection 4.7, according to which the set of all possible alternatives to a dominant theory over its domain shall be object of empirical testing. If this principle is employed in every possible world (and if it is, as some authors claim, at the core of science methodology, it is expected to be sooner or later formulated in every possible world), then empirical and non-empirical selection looks similar in all of them, with the exception of contingent features, reinforcing the view of science as an invariant across the possible worlds. Only with a proliferating variety of theories the pool of scientific and unscientific notions can undergo a natural selection that converges towards real science. And, even if the principle of proliferation is not consciously applied, the pool of notions in due time will still converge towards science. If it is destined to emerge as an invariant across the possible worlds, science cannot be stopped, not even by a bad methodology. After his abjuration, Galileo seemed to have realised this when he pronounced "And yet it moves".

The line of reasoning presented in this conclusive chapter might be cast as an additional non-empirical argument to analyse scientific theories and models to understand which and how many of them show scientific features. This *possible worlds argument* could then be employed to give a contribution to the question that entitles Dawid's 2023 paper "Is inflationary Cosmology Scientific?"

To this end, classic inflation solved so many problems, discussed in Section 1, that it is hard to imagine it not being formulated at some point in different possible worlds. For this to happen, there would be the need for too many problems not to be formulated, or to be formulated in a different, unrecognisable way. It is safe to claim that this is unimaginable nowadays, and there is a small chance that it will be imaginable when the research field will be much more mature than it is now, possibly due to great advances in particle and high energy physics, and maybe quantum gravity. Therefore, even though such an analysis on science is better performed *a posteriori* to understand more deeply the nature, the meaning and the structure of science, the framework proposed still hints that classic inflation is in fact scientific, but it is important to note that this is a form of non-empirical evidence that, as Dawid highlighted many times, is not supposed to become conclusive.

On the other hand, postmodern inflation has led to the proposal of an enormous number of models, and it is hard to imagine all of them being formulated in every possible world, as they sometimes look like they are born motivated only by the momentary inspiration of the researchers, and perhaps the urge to publish. At a first glance, then, the framework here proposed seems to agree with Ijjas et al.'s view discussed in Subsection 4.4, that classic inflation is not epistemologically troublesome while postmodern inflation is, and that there is indeed an epistemological distinction between classic and postmodern inflation, even though the boundary between the two would need to be investigated further.

However, if an individual postmodern inflationary model is some day empirically confirmed (in a non-Bayesian, absolute sense), then such a model will be conclusively scientific, and it will be so in every possible world. Would this event confirm as scientific the whole theory of postmodern inflation as well? To answer, it is important to note that in this scenario only the eventually confirmed model is necessary to the picture (thus strictly scientific), but, from what is the current state of postmodern inflation, it is hard to imagine that in any other possible world it is the only model formulated and immediately tested and confirmed, without a transient phase in which a multitude of models are formulated and later rejected. While the many other models proposed would be cast as not metaphysically necessary, it is still easy to imagine that the methodology of postmodern inflationary cosmology of formulating numerous models before being able to empirically testing them, in agreement with the principle of proliferation, is employed in most of the possible worlds. Thus, while individual models different from the eventually confirmed one are not strictly scientific, the set that encompasses the multitude of them is expected to be present in most of the possible worlds, elevating them (or reducing them, depending on the original point of view) to a form of *soft science*.

While this conclusion does not completely agree with Ijjas et al.'s statement that postmodern inflation is unscientific, it does support their view of an inflationary schism. There is an analogy between this conclusion and the claim that, for example, important novels in literature cannot be expected to be identically produced in all possible worlds, but a realist movement is probably there to be found in all the worlds in which literature exists, confirming that this scientific core, that can be extracted from notoriously unscientific fields, may show the features of soft science.

Finally, it can be noted the speculative framework presented in this chapter may be also beneficial to different epistemologically problematic fields such as the no-go theorems in quantum mechanics [73], which, waiting for a clarification on the ontology of quantum mechanics itself, still nowadays produces negative-stating theorems that try to constrain and direct the search for a deep meaning of the theory [74].

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