ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

SCUOLA DI INGEGNERIA E ARCHITETTURA

CORSO DI LAUREA MAGISTRALE IN
INGEGNERIA PER L'AMBIENTE E IL TERRITORIO

WAVE CLIMATE IN THE SOUTHERN COAST OF MADEIRA ISLAND: ANNEX B

Tesi di laurea magistrale in Idraulica Marittima

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Sessione III Febbraio 2024

Anno Accademico 2022/2023

ANNEX B

Sommario

As it has been made for chapter 2 with Annex A, this paper presents the same analyses carried out in chapter 3, where the offshore boundary conditions were defined for North West point, for South West and South points.

The results will be showed mainly using graphics and tables.

In particular, the three main contents that will be illustrated are:

- Tables displaying the bivariate empirical distribution of Hs and Tp.
- Box plots illustrating the monthly variations.
- Analysing extreme values to determine the Hs for a given return period.

SOUTH WEST POINT

JOINT EMPIRICAL DISTRIBUTION OF Hs AND Tp

Figures 1 and 2 display the relationship that persists between Hs and Tp.

bin	tot
>0,5%	>5%
>1%	10%
>3%	20%
>5%	30%

Hs vs Tp

South West point

		Tp (s)															
		<4	4--5	5--6	6--7	7--8	8--9	9--10	10--11	11--12	12--13	13--14	14--15	15--16	16--17	>17	TOT
Hs (m)	0--0,5 m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%
	0,5--1 m	0,00%	0,01%	0,01%	0,10%	0,88%	1,39%	1,04%	0,61%	0,39%	0,33%	0,41%	0,26%	0,17%	0,17%	0,10%	5,86%
	1--1,5 m	0,00%	0,07%	0,17%	0,43%	1,77%	4,50%	6,11%	5,06%	3,76%	2,26%	1,49%	0,75%	0,43%	0,49%	0,28%	27,56%
	1,5--2 m	0,00%	0,00%	0,24%	0,56%	1,07%	2,22%	3,33%	4,46%	5,50%	4,75%	2,77%	1,01%	0,42%	0,44%	0,29%	27,06%
	2--2,5 m	0,00%	0,00%	0,02%	0,25%	0,48%	0,84%	1,28%	1,78%	2,84%	3,87%	3,52%	1,52%	0,53%	0,42%	0,25%	17,61%
	2,5--3 m	0,00%	0,00%	0,00%	0,04%	0,15%	0,26%	0,52%	0,75%	1,23%	1,73%	2,25%	1,60%	0,65%	0,43%	0,21%	9,84%
	3--3,5 m	0,00%	0,00%	0,00%	0,00%	0,05%	0,14%	0,21%	0,39%	0,58%	0,78%	1,07%	0,93%	0,56%	0,38%	0,18%	5,28%
	3,5--4 m	0,00%	0,00%	0,00%	0,00%	0,01%	0,04%	0,11%	0,17%	0,32%	0,43%	0,55%	0,47%	0,38%	0,30%	0,16%	2,95%
	4--4,5 m	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,05%	0,10%	0,18%	0,21%	0,27%	0,25%	0,19%	0,24%	0,14%	1,63%
	4,5--5 m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,02%	0,07%	0,09%	0,13%	0,17%	0,14%	0,11%	0,12%	0,10%	0,94%
	>5 m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,04%	0,09%	0,16%	0,21%	0,20%	0,17%	0,19%	0,18%	1,26%
	TOT	0,00%	0,08%	0,44%	1,38%	4,42%	9,41%	12,68%	13,43%	14,98%	14,65%	12,72%	7,14%	3,62%	3,18%	1,89%	100,00%

Figure 1, Joint distribution Tp and Hs, South West point.

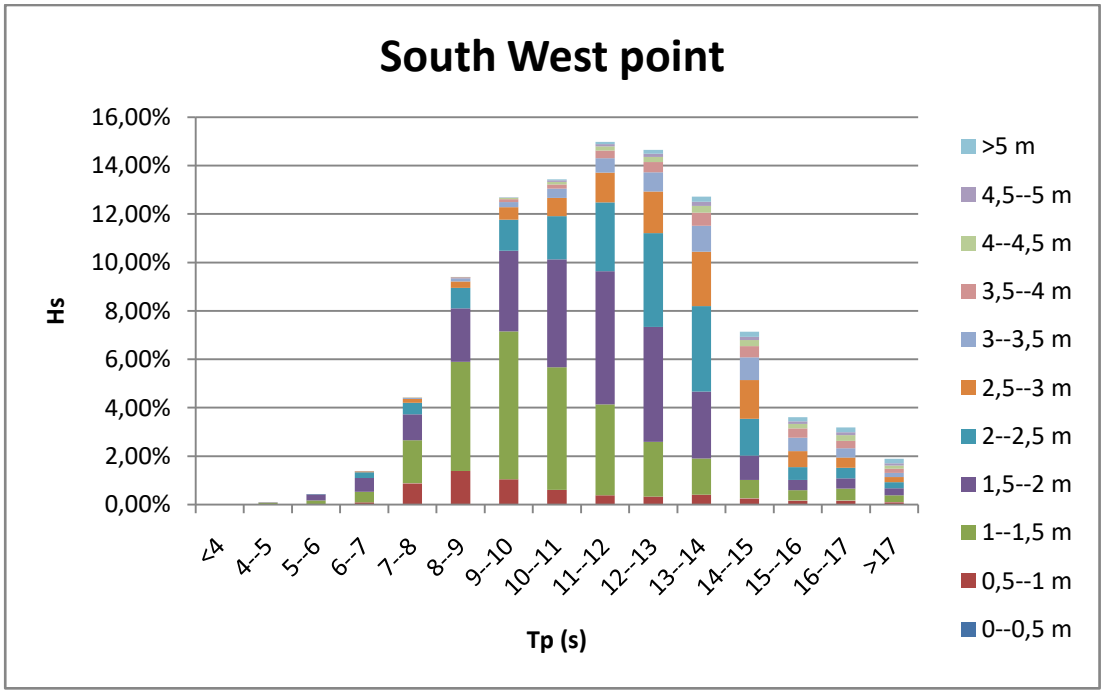

Figure 2, Histogram illustrating the relationship between Tp and Hs, South West point. For each Tp class, the percentage of waves with a period falling within that class is displayed, and for each bar, the colours represent the Hs classes that make up the respective bar.

Then, Figure 3 shows how data are distributed between the 16 classes of directions.

#Dir	Direction(°)	#Data	Data(%)
0	0	26434	20,953
1	22,5	7820	6,199
2	45	913	0,724
3	67,5	256	0,203
4	90	123	0,097
5	112,5	102	0,081
6	135	96	0,076
7	157,5	105	0,083
8	180	137	0,109
9	202,5	223	0,177
10	225	410	0,325
11	247,5	1140	0,904
12	270	3884	3,079
13	292,5	16877	13,378
14	315	31478	24,951
15	337,5	36109	28,622

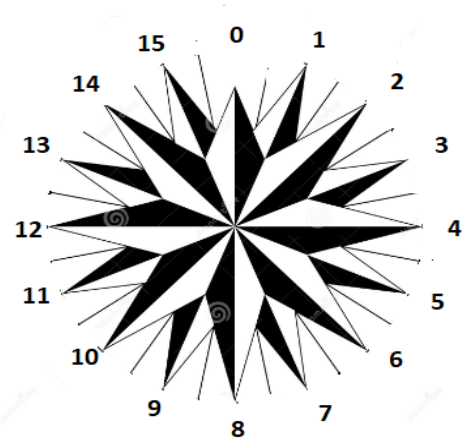

Figure 3, % of data per each direction, South West point.

MONTHLY VARIATION ANALYSES

Figure 4 shows a box plot displaying the monthly variation in terms of Hs.

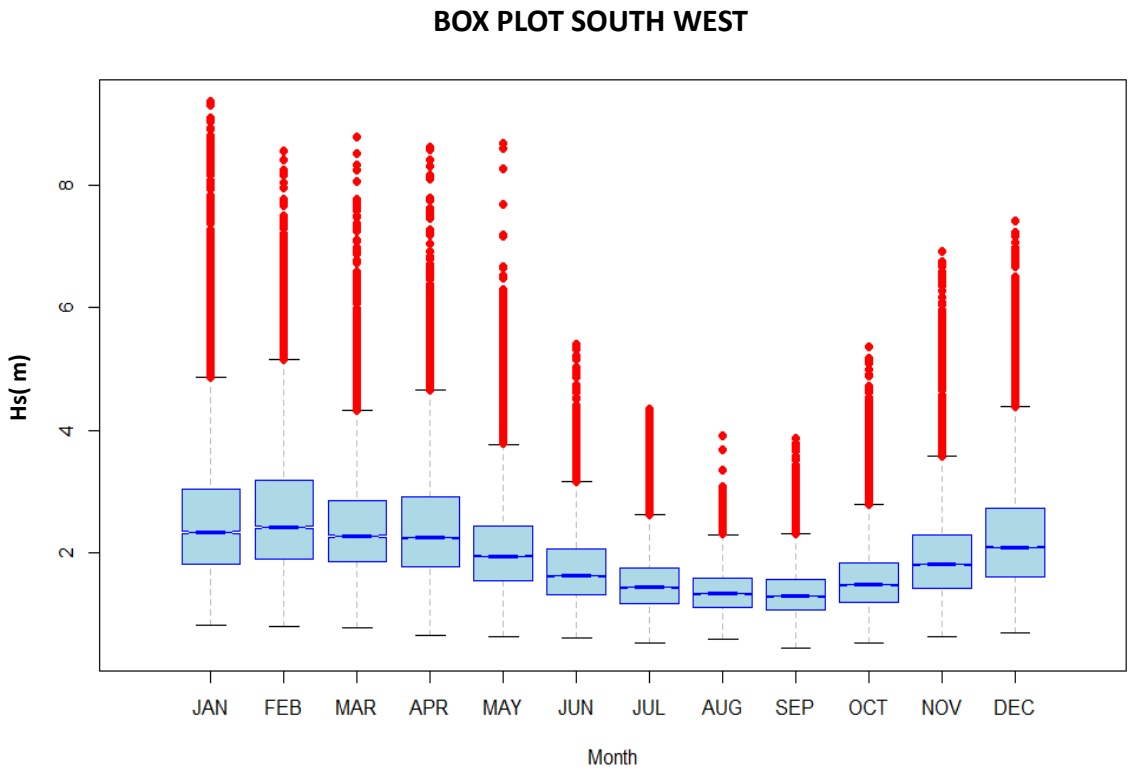

Figure 4, Figure 3.6,Box plot showing the monthly variability in terms of Hs. Red points: outliers; Black lines: "whiskers", Blue lines: median; upper side of the box: 75th percentile; lower side of the box: 25th percentile. South West point.

EXTREME VALUES ANALYSIS

Following the steps described in paragraph 3.2.3 in the main paper, similar results obtained for point North West has been reached for point South West, which are now illustrated.

As for the North West point, the CDF that best fit the data sample has been the Gumbel distribution, and its parameters where established with the MLE method.

This is showed in Figures 5 and 6.

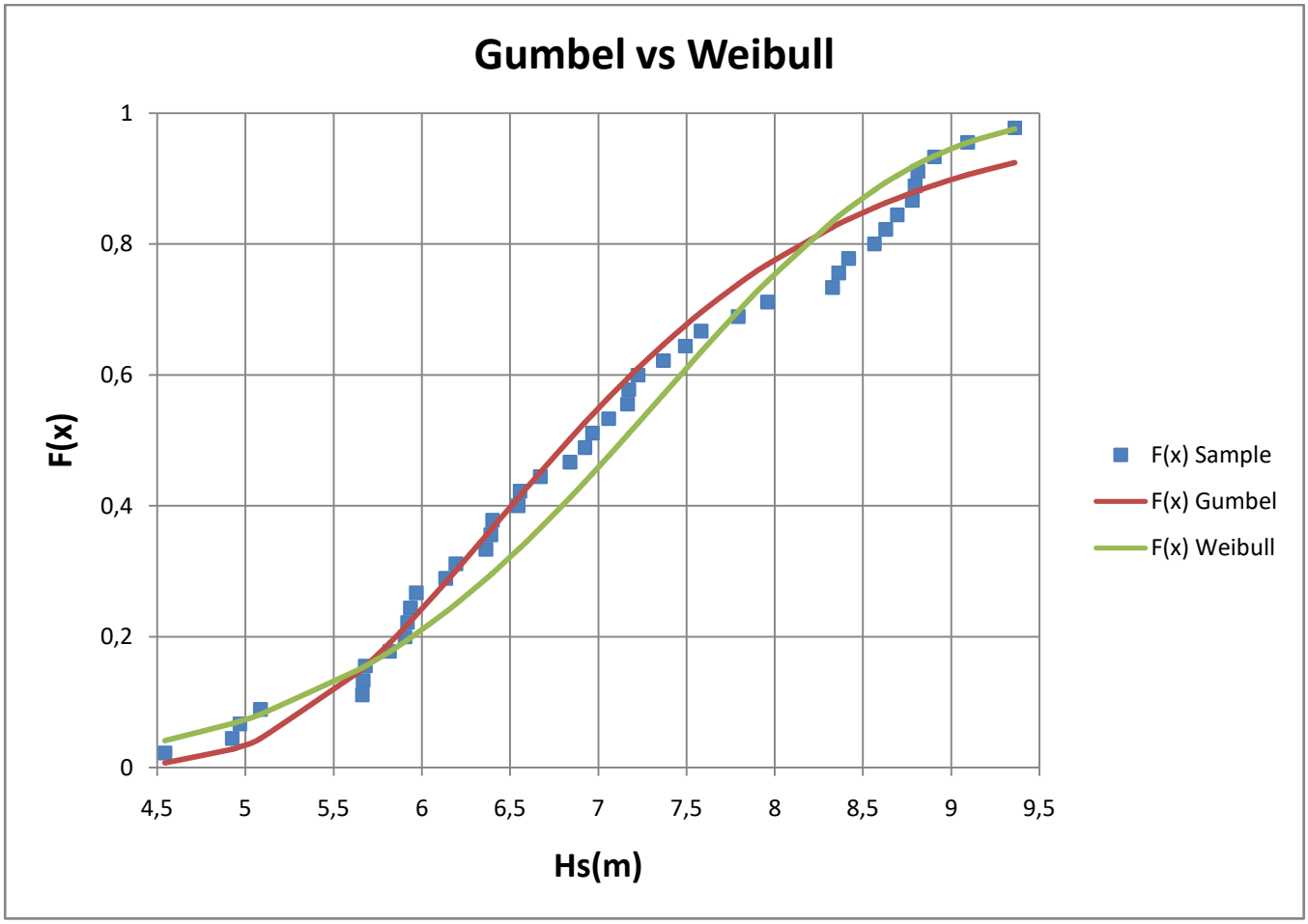

Figure 5, Plot comparing frequency distribution relative to the sample of data under consideration with both the Gumbel and Weibull curves to determine which curve provides a better fit to the data, South West point.

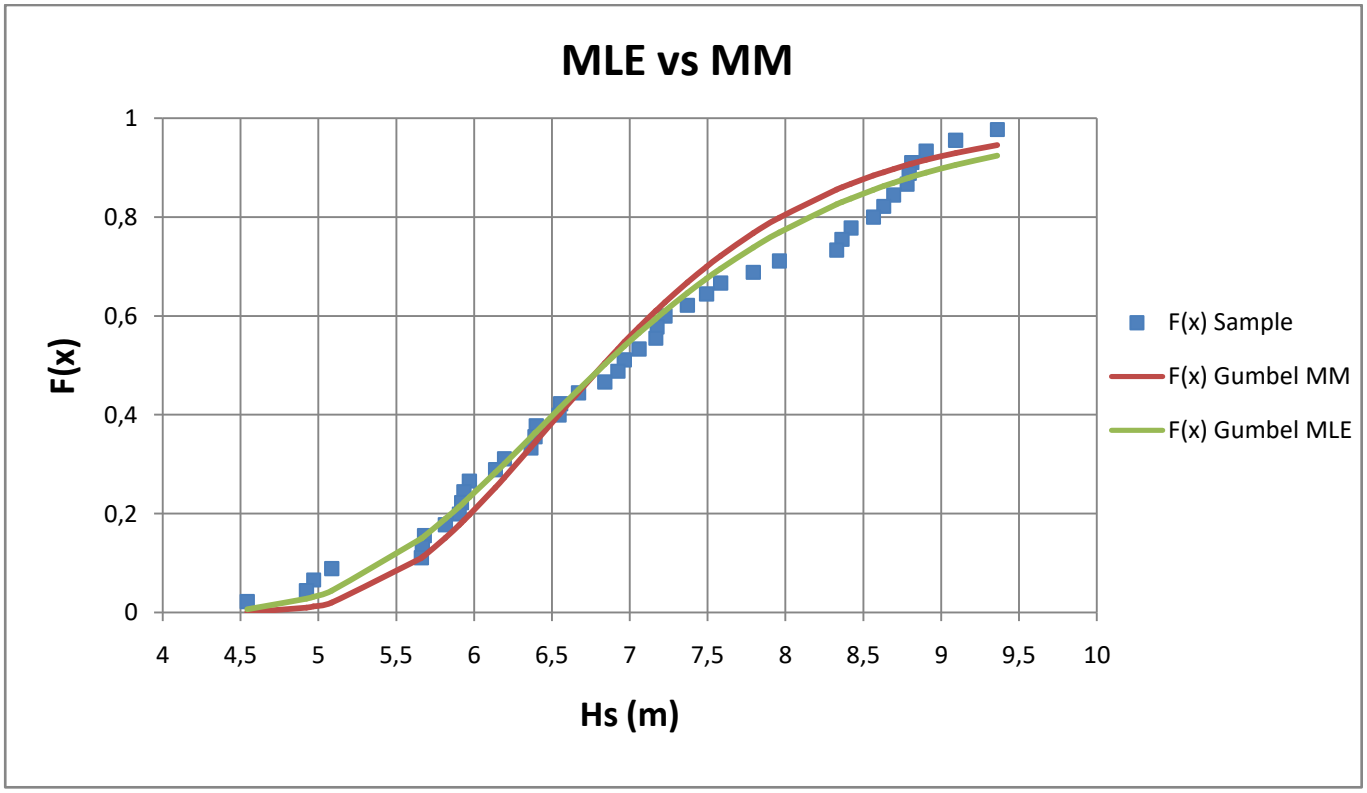

Figure 6, Plot comparing the CDFs fitted to the data sample using both MLE method and MM , in order to determine which method provides better parameters for fitting the data, South West point.

Furthermore, Table 1 displays the parameters found for each CDF with both MLE and MM.

CDF	GUMBEL		WEIBULL	
PARAMETERS	μ (location)	α (shape)	β(form)	A(shape)
MLE	6.40644	1.162391	6.19495	7.57425
MM	6.45623	1.006657	4.77843	7.70093

Table 1, Gumbel and Weibull parameters obtained using MLE and MM, South West point

Table 2 sums up the CDF, the parameters and the equation used to calculate the Hs related to a given return period.

CDF	GUMBEL
CDF equation	$F(x) = \exp\left(-\exp\left(-\left(\frac{x-\mu}{\alpha}\right)\right)\right)$
μ (location parameter)	6.40644
α (shape parameter)	1.162391
Hs equation	$Hs_{Rp} = \mu + \alpha * -\ln\left(\ln\left(\frac{R_p}{R_p - 1}\right)\right)$

Table 2, CDF used with relative parameters and equation to calculate the Hs values corresponding to a given Return Period, South West point.

Table 3 shows the Hs for 10, 20, 50 and 100 years of Return Period.

Return Period(years)	**Hs(m)**
10	9.0
20	9.9
50	10.9
100	11.8

Table 3, Hs values corresponding to return periods of 10, 20, 50, and 100 years, calculated using the Gumbel distribution, with parameters determined using MLE method, South West point.

SOUTH POINT

JOINT EMPIRICAL DISTRIBUTION OF Hs AND Tp

Figures 7 and 8 display the relationship that persists between Hs and Tp.

Hs vs Tp

South point

bin	tot
>0,5%	>5%
>1%	10%
>3%	20%
>5%	30%

		Tp (s)															
		<4	4--5	5--6	6--7	7--8	8--9	9--10	10--11	11--12	12--13	13--14	14--15	15--16	16--17	>17	TOT
Hs (m)	0--0,5 m	0,00%	0,00%	0,00%	0,02%	0,15%	0,26%	0,28%	0,16%	0,17%	0,19%	0,16%	0,07%	0,06%	0,08%	0,09%	1,69%
	0,5--1 m	0,15%	0,31%	0,11%	0,48%	1,49%	3,89%	6,53%	6,21%	4,79%	3,82%	3,31%	1,97%	1,12%	1,33%	0,86%	36,38%
	1--1,5 m	0,00%	1,05%	0,58%	0,50%	0,90%	1,48%	2,67%	4,70%	6,66%	6,36%	4,30%	1,97%	0,83%	0,82%	0,55%	33,39%
	1,5--2 m	0,00%	0,02%	0,66%	0,38%	0,33%	0,51%	0,64%	1,01%	2,05%	3,41%	3,67%	1,90%	0,79%	0,61%	0,30%	16,26%
	2--2,5 m	0,00%	0,00%	0,05%	0,22%	0,22%	0,22%	0,30%	0,38%	0,59%	0,90%	1,36%	1,22%	0,56%	0,40%	0,24%	6,65%
	2,5--3 m	0,00%	0,00%	0,00%	0,04%	0,11%	0,12%	0,13%	0,14%	0,23%	0,39%	0,46%	0,44%	0,31%	0,27%	0,13%	2,78%
	3--3,5 m	0,00%	0,00%	0,00%	0,00%	0,03%	0,07%	0,07%	0,06%	0,14%	0,24%	0,23%	0,15%	0,12%	0,12%	0,07%	1,32%
	3,5--4 m	0,00%	0,00%	0,00%	0,00%	0,01%	0,02%	0,04%	0,06%	0,07%	0,08%	0,11%	0,08%	0,07%	0,09%	0,06%	0,68%
	4--4,5 m	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,02%	0,04%	0,03%	0,06%	0,08%	0,07%	0,04%	0,04%	0,05%	0,44%
	4,5--5 m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,02%	0,04%	0,03%	0,02%	0,04%	0,02%	0,03%	0,01%	0,22%
	>5 m	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,02%	0,02%	0,02%	0,04%	0,03%	0,04%	0,02%	0,20%
	TOT	0,16%	1,37%	1,39%	1,64%	3,25%	6,58%	10,69%	12,81%	14,77%	15,51%	13,73%	7,94%	3,95%	3,82%	2,37%	100,00%

Figure 7, Joint distribution Tp and Hs, South point.

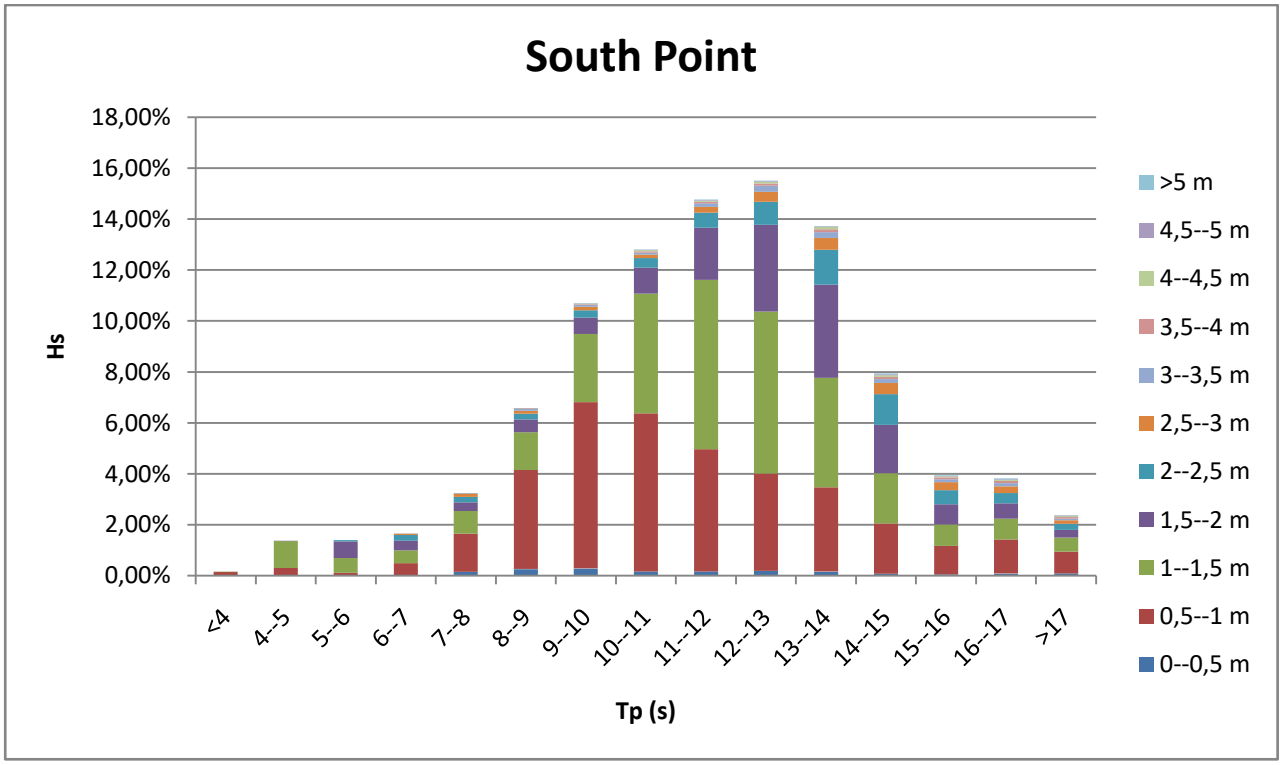

Figure 8, Histogram illustrating the relationship between Tp and Hs, South point. For each Tp class, the percentage of waves with a period falling within that class is displayed, and for each bar, the colours represent the Hs classes that make up the respective bar.

Then, Figure 9 shows how data are distributed between the 16 classes of directions.

#Dir	Direction(°)	#Data	Data(%)
0	0	9723	7,707
1	22,5	8742	6,929
2	45	6574	5,211
3	67,5	2541	2,014
4	90	1001	0,793
5	112,5	485	0,384
6	135	386	0,306
7	157,5	340	0,27
8	180	432	0,342
9	202,5	768	0,609
10	225	1668	1,322
11	247,5	3617	2,867
12	270	10822	8,578
13	292,5	31621	25,064
14	315	31783	25,193
15	337,5	15656	12,41

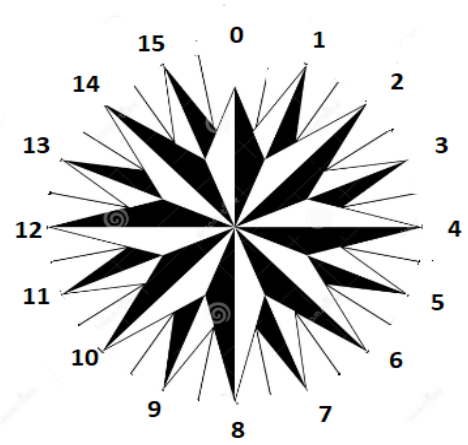

Figure 9, % of data per each direction, South point.

MONTHLY VARIATION ANALYSES

Figure 10 shows a box plot displaying the monthly variation in terms of Hs.

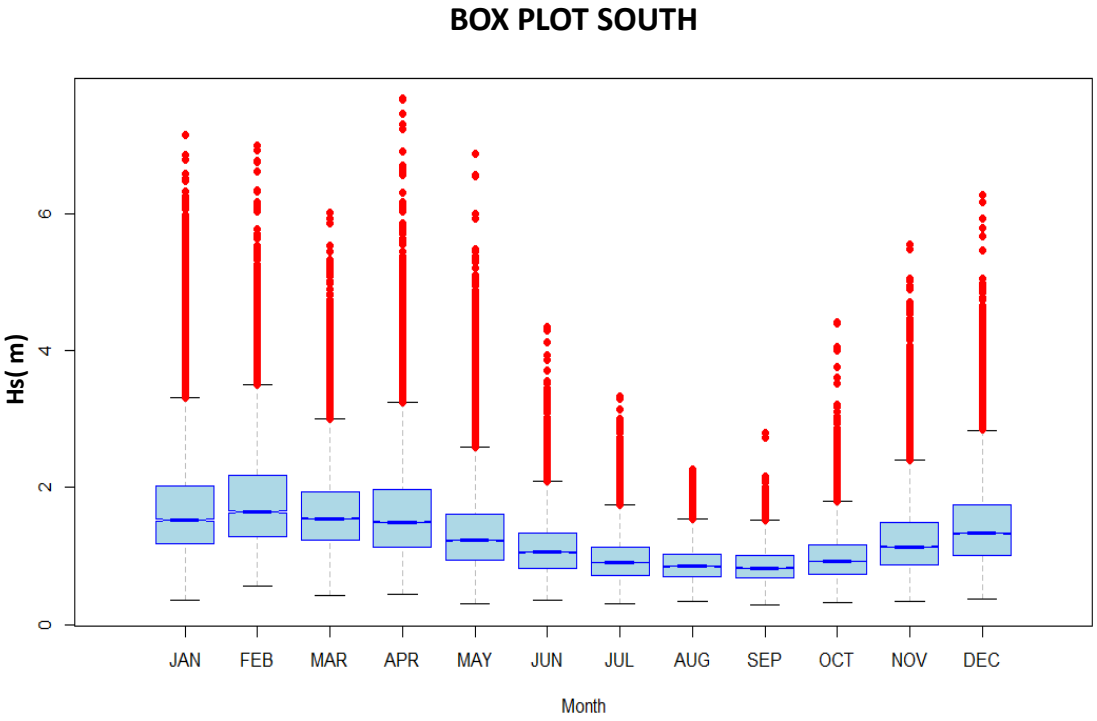

Figure 10, Box plot showing the monthly variability in terms of Hs. Red points: outliers; Black lines: "whiskers", Blue lines: median; upper side of the box: 75th percentile; lower side of the box: 25th percentile. South point.

EXTREME VALUES

As for the North West and the South West points, the CDF that best fit the data sample has been the Gumbel distribution, and its parameters where established with the MLE method.

This is showed in Figures 11 and 12.

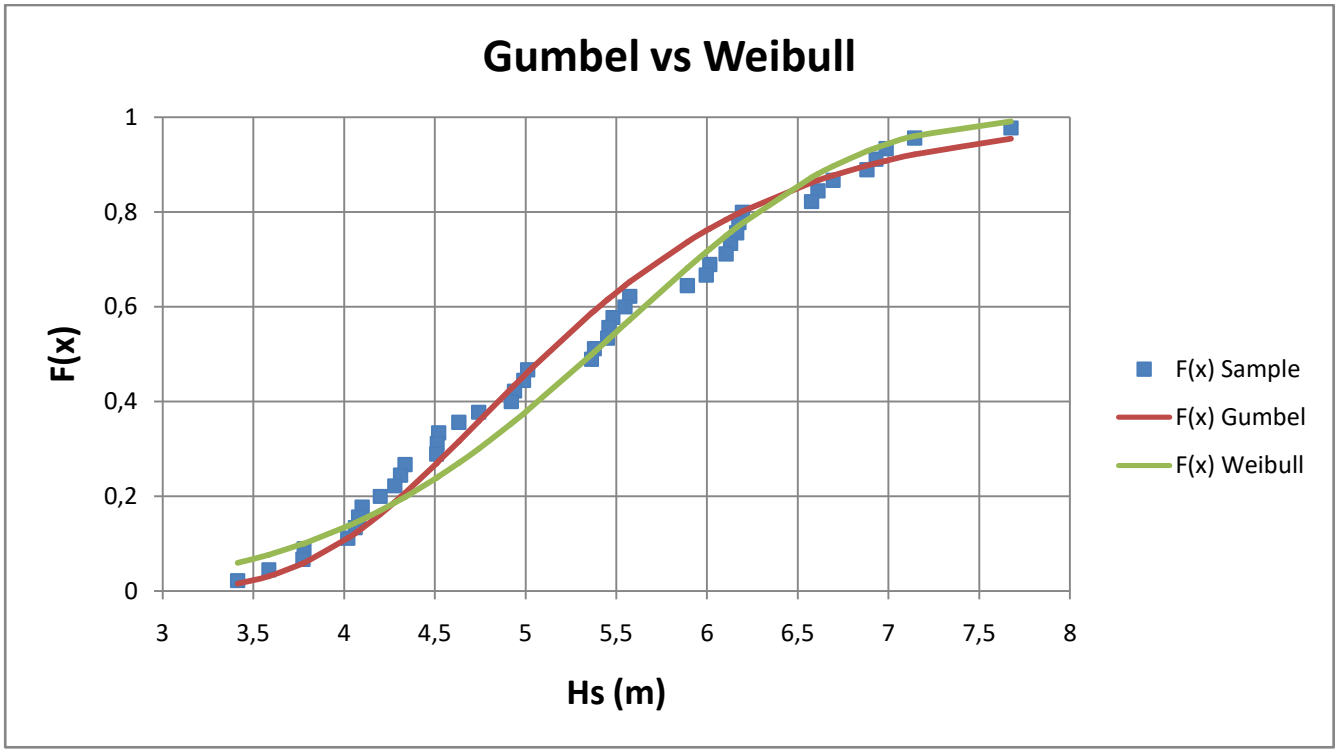

Figure 11, Plot comparing frequency distribution relative to the sample of data under consideration with both the Gumbel and Weibull curves to determine which curve provides a better fit to the data, South point.

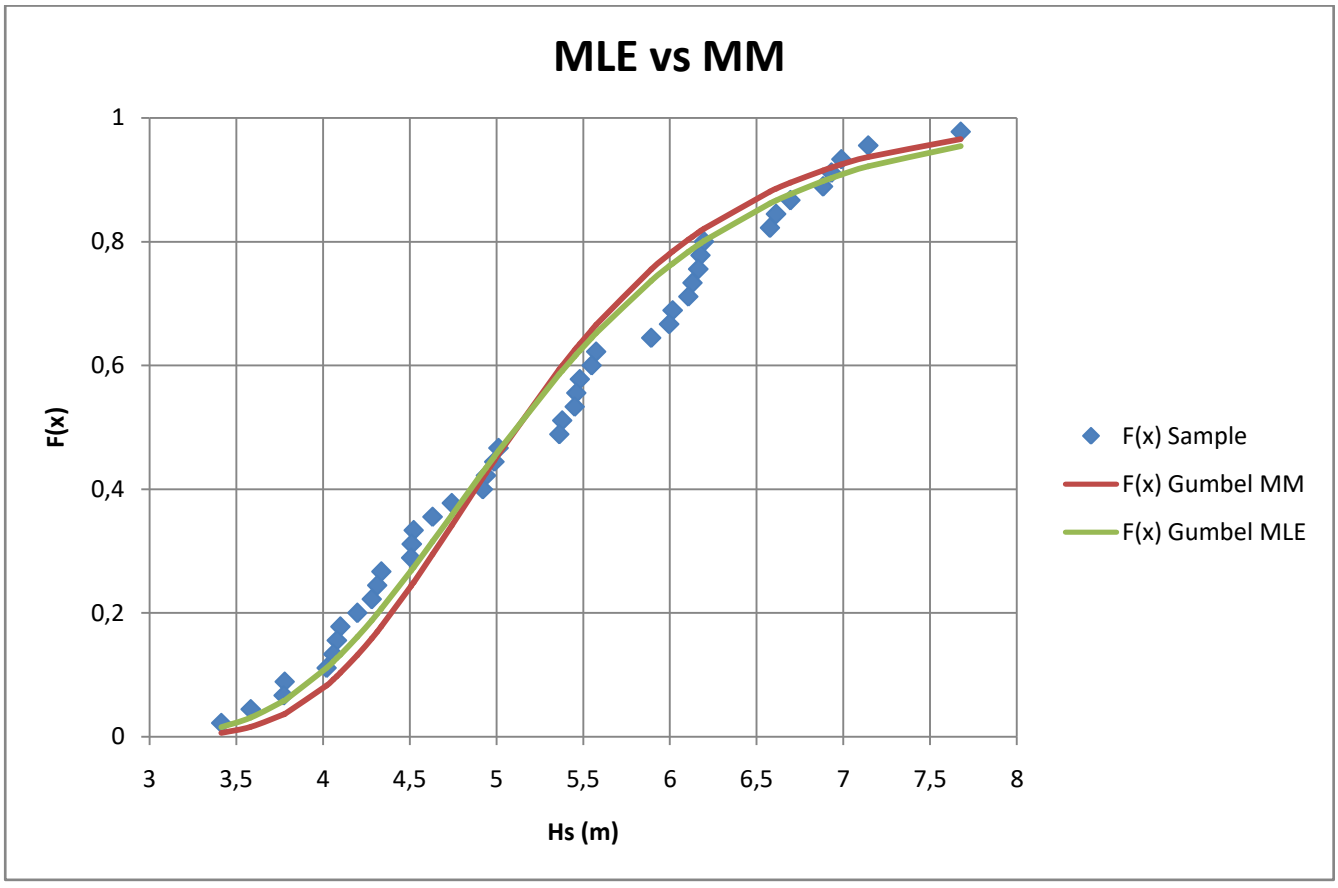

Figure 12, Plot comparing the CDFs fitted to the data sample using both MLE method and MM , in order to determine which method provides better parameters for fitting the data, South point.

Furthermore, Table 4 displays the parameters found for each CDF with both MLE and MM.

CDF	GUMBEL		WEIBULL	
PARAMETERS	μ (location)	α (shape)	β(form)	α (shape)
MLE	4.767663	0.948869	5.365629	5.46418
MM	4.803575673	0.8578515	4.969835531	5.7984311

Table 4, Gumbel and Weibull parameters obtained using MLE and MM, South point.

Table 5 sums up the CDF, the parameters and the equation used to calculate the Hs related to a given return period.

CDF	GUMBEL
CDF equation	$F(x) = \exp\left(-\exp\left(-\left(\frac{x-\mu}{\alpha}\right)\right)\right)$
μ (location parameter)	4,767663
α (shape parameter)	0,948869
Hs equation	$Hs_{Rp} = \mu + \alpha * -\ln\left(\ln\left(\frac{R_p}{R_p - 1}\right)\right)$

Table 5, CDF used with relative parameters and equation to calculate the Hs values corresponding to a given Return Period, South point.

Table 6 shows the Hs for 10, 20, 50 and 100 years of Return Period.

Return Period(years)	**Hs(m)**
10	**6.9**
20	**7.6**
50	**8.5**
100	**9.1**

Table 6, Hs values corresponding to return periods of 10, 20, 50, and 100 years, calculated using the Gumbel distribution, with parameters determined using MLE method, South point.