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School of Science
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Perturbative Moduli Stabilization and String Inflation

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Abstract

This Master thesis is focused on the study of cosmological implications of type IIB string compactifications. In particular we will consider brane-antibrane inflation [1] which is one of the first and most-studied realizations of inflation in string theory. The model however suffers from two main issues: (*i*) the lack of a fully supersymmetric formulation of the effective field theory in terms of a Kähler potential and a superpotential; (*ii*) the infamous η-problem due to the non-perturbative stabilization of the volume modulus. We will follow the idea of Burgess and Quevedo [32] which proposed a supersymmetric formulation of the effective action of brane-antibrane inflation together with a mechanism to stabilize the volume mode at perturbative level based on RG-effects. In [32] these perturbative corrections are however just field theory-inspired without a proper string theory motivation. In this Master thesis we will put these perturbative corrections on more solid grounds by exploiting all known results in α' and string loop corrections to the effective action of type IIB string compactifications. In this way we will improve the results of Burgess and Quevedo finding a way to stabilize the moduli in brane-antibrane inflation just using known perturbative corrections. Our results open up the possibility to implement a hybrid inflation scenario where the volume mode can act as a waterfall field to end inflation, leading to a post-inflation minimum where the supersymmetry breaking scale is much lower than the inflationary scale.

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Chapter 1

Introduction

The goal of physics has always been to capture the workings of the universe starting from the largest structures of the universe to the smallest particles in the universe. The story of physics started when early hominids (according to current understanding of anthropology, an early Hominid species like Homo Naledi had culture and religion) looked up in the sky and wondered about the existence of the universe. Existence of culture and religion does indicate that "we" did start to think about "us". Modern humans did think about science and physics starting from the Greeks, Indians, Medieval Europeans and Arab counterparts. Nevertheless, what we call the scientific revolution started in the 16th and 17th century. Nicolas Copernicus, Galileo Galilei, Huygens, Newton and many others revolutionized our understanding of the world around us. Development of the theories on electricity and magnetism went in the hands of Faraday, Ohm, Gauss, Hertz, Marconi while being unified by James Clark Maxwell in the 19th century. This was a marvelous achievement as it was the first ever unification-merging of two fundamental forces of nature so as we thought.[1]We now know that in our 4 dimensional world, the force of electricity and magnetism are actually a single fundamental force of nature. Then came the first quantum revolution which was started off when Max Planck used quanta to explain black-body radiation which was put into a much more firm ground by Albert Einstein when he used it to explain photoelectric effect and who later became a great adversary of the theory. Bohr's quantized energy level of hydrogen atom, Heisenberg's matrix mechanics and uncertainty principle and the wave mechanics (thus the Schrödinger equation) by Schrödinger really put the theory of quantum mechanics in a very solid position. It also enjoyed various experimental success in a short amount of time in the early 20th century. In the early 1900 another theory also developed which challenged Newton's idea of space, time and gravity. It was the special theory of relativity in 1905 and the general theory of relativity in 1915 by Albert Einstein. Field theoretic version of quantum mechanics-QFT brought about unimaginable accuracy to

[1]Here in Bologna, Augusto Righi did fundamental research in the field of Electromagnetism as well.

measurements at the subatomic level. The revolutions in QFT were carried forward by Dirac, Dyson, Feynman, Schwinger, Weinberg, Salam, Glasshow, Coleman among many others. The three of the four fundamental forces of nature- Electromagnetism, Strong and Weak nuclear force are described pretty well by the framework of quantum field theory which we call the standard model. The observation of a particle in 2012 which we call the Higgs boson is one of the latest additions to the gigantic amount of success of standard model. On the other hand, Einstein relativity describes the universe at the cosmological scales up to a very good approximation. Observation of gravitational waves from black-hole mergers and its formation, perihelion of mercury, gravitational lensing are one of many success stories of relativity. Let us get back to the sub-atomic level again. The interactions in the standard model are measured by coupling constants. The unification of the three couplings corresponding to the three forces does tend to unify at a high energies. We already know that electromagnetism and weak interactions tend to unify in a single force which we call the electroweak force at energies of order 100 GeV. Does this mean, at much larger energies we have a unification of all the four fundamental forces of nature?

1.1 Why Quantum Gravity?

The dream carried forward from last section runs into major difficulty when we try to view gravity as a force and by describing it in the language of quantum field theory. Let us see what happens when we treat gravity as a quantum field i.e a spin 2 field.[2] For that, let us consider the d-dimensional version of the Einstein-Hilbert action

$$\mathrm{S}_{EH}^{(d)} = \frac{1}{16\pi G*}\int d^d x\sqrt{-detg}R \tag{1.1}$$

Here, $G*$ is the d dimensional version of the Newton's constant. Let us do some dimensional analysis before we proceed further. First, since the action should dimensionless we should be very careful with the dimensions. The dimension of the measure is

$$[d]$$

and the dimension of the Ricci scalar R is [-2]. Metric $g^{\mu\nu}$ and its determinant is dimensionless (obvious from the line element dimension analysis $ds^2 = g^{\mu\nu}dx^\mu dx^\nu$). The important takeaway from this dimensional analysis is that the dimension of the modified Newton's constant is

$$[G*] = 2 - d \tag{1.2}$$

[2]A theorem by Weinberg says that a theory of spin 2 particle must describe gravitation as it couples to stress-energy tensor in a way gravity does.

The above scaling might look harmless at first but we soon run into difficulties. Consider, the fluctuations of the metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Here, $\eta_{\mu\nu}$ is our flat Lorentzian metric(Classical saddles) and $h_{\mu\nu}$ captures the fluctuations around the classical saddles. The fluctuations of the metric produce some perturbations to the above Einstein-Hilbert term :

$$S_{EH}^{(d)} = S_{Classical} + S_{fluctuations/perturbations} \tag{1.3}$$

The fluctuating/perturbing part involves some derivatives of h and is written as:

$$S_{fluctuations/perturbations} = \frac{1}{16\pi G*} \int d^d x[(\partial h)^2 + (\partial h)^2 h + ...] \tag{1.4}$$

Notice that the dimension of $G*$ has to be changed in every term if we want our action to be dimensionless. Most importantly, as we know from renormalization in quantum field theory, $G*$ has got a bad mass dimension in d=4 and the theory is non-renormalizable. This is the first motivation of an urge for a quantum theory of gravity-a unification of all the known fundamental forces of nature. In the remainder of the section we will try to highlight the problems/shortcomings of the most successful theory of human history-the theory of standard model.

- **Quantum gravity and unification:** As we saw in the calculation above, having a quantum field theory of gravity is a hopeless case (maybe not as an effective field theory?). The standard model, thus, do not contain gravity and our hope of describing all of the four fundamental forces of nature using a single framework is not an enlightening one. Quantum effects of gravity are certainly there as using a classical (general relativity) theory of gravity does not explain (semi-)classical problems like black-hole information paradox, dark matter, dark energy and inflation.

- **Why questions:** Why is the gauge group of standard model $SU(3) \otimes SU(2) \otimes U(1)$? Why are there four interactions and why $3+1$ spacetime - dimensions? The last question also translates into the question of the $3+1$ spacetime dimensions "emerging" from something and the stability of them. This forces us to look for a quantum gravity theory which can explain them.

- **Parameter issue:** The standard model has around 20 parameters.[3] This is a quote from a biography of Fermi (Pope of Physics- Segre and Hoerlin)

  Even if we have a lot of parameters, it would be nice to have them arise naturally. This also motivates us to look beyond standard model.

[3] Although some particle physicists tell us that there are only one - the Higgs Mass

When Dyson met with him in 1953, Fermi welcomed him politely, but he quickly put aside the graphs he was being shown indicating agreement between theory and experiment. His verdict, as Dyson remembered, was "There are two ways of doing calculations in theoretical physics. One way, and this is the way I prefer, is to have a clear physical picture of the process you are calculating. The other way is to have a precise and self-consistent mathematical formalism. You have neither." When a stunned Dyson tried to counter by emphasizing the agreement between experiment and the calculations, Fermi asked him how many free parameters he had used to obtain the fit. Smiling after being told "Four," Fermi remarked, "I remember my old friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiggle his trunk." There was little to add.

Figure 1.1: Fermi's Quote

- **The hierarchy problem:** The hierarchy problem is the question of why the scale of the forces of nature so different i.e why is gravity so much weaker than the electroweak interactions

  $M_{\text{ew}} \approx 10^2\text{GeV}, \quad M_{\text{pl}}$[4] $\approx 10^{19}\text{GeV} \quad \Longrightarrow \frac{M_{\text{ew}}}{M_{\text{pl}}} \approx 10^{-15}.$

  A reason could be that gravity is weaker in some dimensions (in theories with extra-dimensions). This is another reason we should be looking beyond the standard model.

- **The cosmological constant problem:** We know that the expansion of the universe is accelerating and observations tell us that the cosmological constant is very small and positive. The current estimate is around $\Lambda \approx 10^{-120} M_{\text{pl}}^4$. If the universe is described by an effective local quantum field theory down to the Planck scale, then we would expect a cosmological constant of the order of M_{pl}^4. Thus we are off by an order of 120 magnitudes and this is arguably the biggest puzzle in all of physics.

These problems (along with a few others not listed above) motivate us to look for a more elegant theory where standard model and gravity can arise naturally. Of course, we can study gravitational interactions by viewing it as an effective quantum field theory which might make the situation a bit better. But, wouldn't it be better to have a UV-complete (no divergences) theory containing both standard model and gravity which attempts to answer all of the above reservations of the standard model that we posed?

1.2 Why String theory and What is String Theory?

Let us briefly highlight the history of string theory before going into the question - why string theory?

String theory was actually proposed as a model of strong interactions. In 1968, Gabriele Veneziano introduced a four particle scattering amplitude for the strong force

[4]In this section only we denote the Planck mass in 4 dimensions as M_{pl}. In all of the subsequent discussions we use M_4

and it was later interpreted as a one dimensional string in 1969-70 by Nambu, Neilson and Susskind. Later, the success of QCD and experimental issues put the theory outside the community's radar.

In 1974, John H. Schwarz and Joël Scherk, in separate work from Tamiaki Yoneya, investigated the characteristics of string vibrations resembling bosons. They made a significant discovery: these properties precisely matched those of the graviton.This put string theory back in business and thus the study of Bosonic string theory. As we know, bosonic string theory has a critical dimension of D=26, has tachyons in the spectrum and no fermions. The theory explained a lot of things but these issues were not to be ignored.

Considering supersymmetry in the theory gave birth to the first superstring revolution (1984–1994). The theory has fermions, no tachyons and lives in 10 spacetime dimensions. Michael Green, John H. Schwarz, Edward Witten, Philip Candelas, Gary Horowitz and the so called "Princeton String Quartet" (for the discovery of heterotic string) David Gross, Jeffrey Harvey, Emil Martinec, and Ryan Rohm. The second superstring revolution(1994–2003) started with the discovery of M-theory by Edward Witten which is an 11-dimensional supergravity theory and he found evidences that different superstring theory were different limits of this M-theory. Duality relationships between the string theories like S-duality, T-duality, U-duality, Mirror symmetry were also discovered. Polchinski's discovery of D-branes, Maldacena's AdS/CFT correspondence spiked interest in the theory. Notable discoveries and solutions to various problems were proposed by Edward Witten, Ashoke Sen, Cumrun Vafa, Maldacena, Joseph Polchinski and many others.

Let us now discuss what success does string theory has.

- **Unification:** The unification of gravity and standard model occurs in a consistent fashion in the UV complete theory.
- **Parameter Issue:** There is only one parameter in the theory which is the string length l_s. All the other couplings arise as vacuum expectation value of fields.
- **The hierarchy problem:** Roughly speaking, string theory, being an extra-dimensional theory, solves the hierarchy problem by the idea that gravity is not weaker in extra dimensions but rather a stronger force.

There are other success stories of string theory like the derivation of Bekenstein-Hawking entropy. The cosmological constant problem is also being addressed and there are proposed solutions like KKLT [34] and LVS [13],[14] which tells us about vacuum stabilization in string theory and a possible way to address the cosmological constant problem. This master thesis addresses this vacuum stabilization issue as well. Keeping this motivation aside, we will now try to understand string theory and move on to the work of this thesis.

Broadly speaking there are two kinds of string theory- Bosonic String theory and Superstring Theory. We will mainly focus on superstring theory so we roughly sketch the idea of bosonic string theory here. Bosonic String theory is described by the Polyakov action

$$S_{POLY} = \frac{-1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} h^{\alpha\beta} \partial_\alpha X . \partial_\beta X \tag{1.5}$$

The action has the following symmetries alongside Poincare symmetries :

- Reparametrization invariance (diffeomorphisms)
- Weyl invariance

Reparametrization invariance and Weyl invariance helps us to take the action in a convenient form

$$S_{POLY} = \frac{-1}{4\pi\alpha'} \int d^2\sigma \partial_\alpha X . \partial^\alpha X \tag{1.6}$$

The equations of motion is $\partial^\alpha \partial_\alpha X^\mu = 0$. Bosonic String theory has tachyons in the spectrum and no fermions. To include fermions and remove tachyons, we need Supersymmetry. One of the approaches to incorporate supersymmetry in string theory is called Ramond-Neveu-Schwarz (RNS) formalism. We introduce supersymmetry in the worldsheet by pairing bosonic fields $X^\mu(\sigma, \tau)$ with $\Psi^\mu(\sigma, \tau)$. The RNS action in light-cone gauge is

$$S_{RNS} = \frac{1}{\pi} \int d^2\sigma (2\partial_+ X \partial_- X + \Psi_+ \partial_- \Psi_+ + \Psi_- \partial_+ \Psi_-) \tag{1.7}$$

This theory is obviously invariant under supersymmetry and lives in 10 spacetime dimensions. The first massless spectrum contains a symmetric tensor $G_{\mu\nu}$, anti-symmetric tensor $B_{\mu\nu}$ (called the Kalb-Ramond field) and a scalar field called the dilaton ϕ. These come from a imposing boundary conditions on the closed string such as:

$$\psi^M_\pm(\tau, \sigma + \ell) = \begin{cases} \psi^M_\pm(\tau, \sigma) & \text{Ramond (R)} \\ -\psi^M_\pm(\tau, \sigma) & \text{Neveu-Schwarz (NS)} \end{cases} \tag{1.8}$$

Roughly speaking choice of these boundary conditions lead to different string theories.

1.3 Different String Theories And Their Low Energy Limits

There are five different consistent string theories which are: Type I, Type IIA, Type IIB, Heterotic SO(32) and Heterotic $E_8 \times E_8$. These string theories are related to each other via different dualities as displayed in the figure (1.2). The low energy limit of each of the five string theories or M-theory is governed by supergravity theories and we

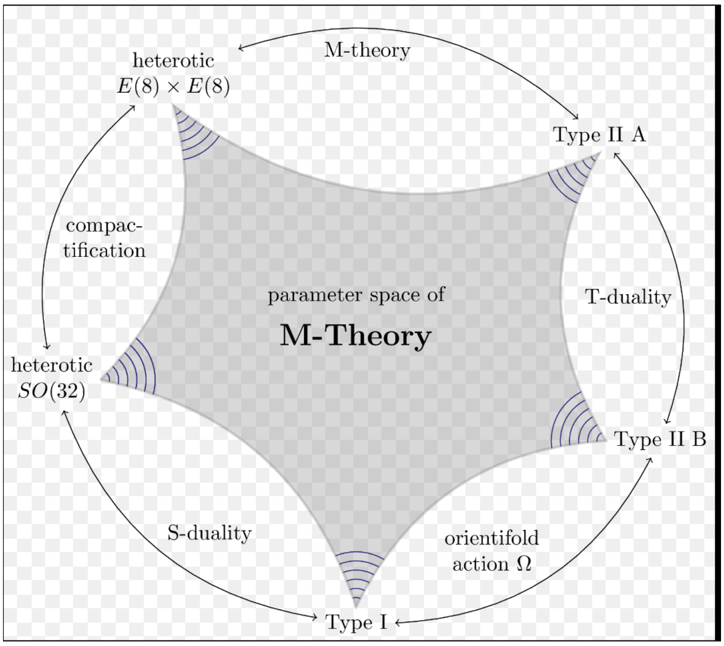

Figure 1.2: Different String Theories And Dualities

can extract interesting quantum gravity characteristics from these supergravity theories. In this section, we will look at the spectrum of each of the five string theories with particular emphasis to type IIB string theory which is the purpose of this thesis and which is phenomenologically very interesting.

1.3.1 Type IIA

The low-energy limit of Type IIA string theory is Type IIA supergravity. In this discussion, we will focus only on the bosonic part. As far as superstring theory is concerned, we can divide the fields in two sectors from which they arise. This comes from the choice of boundary conditions in the worldsheet. We have two choices for the fermion part on the worldsheet, one is a choice of periodic boundary condition which leads to Ramond-Ramond (RR) sector fields and other is a choice of anti-periodic boundary condition which leads to Neveu-Schwarz sector (NS-NS) fields. In the case of Type IIA,

NS-NS Sector: $G_{\mu\nu}, B^{(2)}_{\mu\nu}, \Phi$.

In the NS-NS sector, we have the metric $G_{\mu\nu}$, the anti-symmetric two-form field which is sometimes called the Kalb-Ramond field $B^{(2)}_{\mu\nu}$ and the dilaton Φ.

RR Sector: $C^{(1)}_{\mu}, C^{(3)}_{\mu\nu\rho}$.

In the RR sector, we have a one form field and a three form field $C^{(1)}_{\mu}$ and $C^{(3)}_{\mu\nu\rho}$

respectively. Now, we can write the field strength of these form fields as follows:

$$\begin{aligned} F^{(p)} &= dc^{(p-1)} && \text{for } p = 2, 4 \\ \tilde{F}^{(2)} &= F^{(2)} && ; H^{(3)} = dB^{(2)} \\ \tilde{F}^{(4)} &= F^{(4)} - c^{(1)} \wedge H^{(3)} && \end{aligned} \tag{1.9}$$

This will help us write the type IIA action. Let us separate out the NS-NS part and RR part. The NS-NS part is

$$\begin{aligned} S_{NS-NS} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G}e^{-2\Phi} \left(R_G + 4G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \right. \\ \left. -\frac{1}{2} \times \frac{1}{3!} G^{\mu\mu'}G^{\nu\nu'}G^{\rho\rho'}H^{(3)}_{\mu\nu\rho}H^{(3)}_{\mu'\nu'\rho'} \right) \end{aligned} \tag{1.10}$$

and R-R part is

$$\begin{aligned} S_{R-R} = \frac{-1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G} \left[\frac{1}{4} F^{(2)}_{\mu\nu} F^{(2)}_{\mu'\nu'} G^{\mu\mu'} G^{\nu\nu'} \right. \\ \left. + \frac{1}{2}\frac{1}{4!} \tilde{F}^{(4)}_{\mu'\nu'\rho'\sigma'} \tilde{F}^{(4)}_{\mu\nu\rho\sigma} G^{\mu\mu'} G^{\nu\nu'} G^{\rho\rho'} \sigma^{\sigma\sigma'} \right] \end{aligned} \tag{1.11}$$

In addition to the two pieces above, there is a third piece which we call the Chern-Simons term which can be written as:

$$S_{CS} = \frac{-1}{4\kappa_{10}^2} \int B^{(2)} \wedge F^{(4)} \wedge F^{(4)} \tag{1.12}$$

1.3.2 Type IIB

Let us now talk about type IIB supergravity which is the low-energy limit of type IIB string theory. The models we discuss in this thesis and the new model we construct in this thesis is in the framework of type IIB string theory. Similar to the previous analysis on type IIA, we discuss the bosonic part and discuss the field contents arising from two sectors i.e RR and NS-NS.

NS-NS Sector: $G_{\mu\nu}, B^{(2)}_{\mu\nu}, \Phi$.

In the NS-NS sector, we have the metric $G_{\mu\nu}$, the anti-symmetric two-form field which is sometimes called the Kalb-Ramond field $B^{(2)}_{\mu\nu}$ and the dilaton Φ. This is exactly the same as type IIA.

RR Sector: $C^{(0)}, C^{(2)}_{\mu\nu}, C^{(4)}_{\mu\nu\rho\sigma}$.

The R-R sector is different in this case. We have a 0-from field $C^{(0)}$, a 2-form field $C^{(2)}_{\mu\nu}$ and a 4-form field $C^{(4)}_{\mu\nu\rho\sigma}$. We define the field strengths, again, as follows:

$$
\begin{aligned}
&F^{(1)} = dC^{(0)}, F^{(3)} = dC^{(2)}, F^{(5)} = dC^{(4)}; H^{(3)} = dB^{(2)} \\
&\tilde{F}^{(1)} = F^{(1)}, \tilde{F}^{(3)} = F^{(3)} - C^{(0)} \wedge H^{(3)} \\
&\tilde{F}^{(5)} = F^{(5)} - \frac{1}{2}C^{(2)} \wedge H^{(3)} + \frac{1}{2}B^{(2)} \wedge F^{(3)}
\end{aligned}
\tag{1.13}
$$

The NS-NS part is identical to type IIA

$$
\begin{aligned}
S_{NS-NS} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G}e^{-2\Phi} \bigg(&R_G + 4G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \\
&-\frac{1}{2} \times \frac{1}{3!}G^{\mu\mu'}G^{\nu\nu'}G^{\rho\rho'}H^{(3)}_{\mu\nu\rho}H^{(3)}_{\mu'\nu'\rho'}\bigg)
\end{aligned}
\tag{1.14}
$$

and the R-R part is:

$$
\begin{aligned}
S_{R-R} =& \frac{-1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G} \bigg[\frac{1}{2}G^{\mu\nu}\tilde{F}^{(1)}_\mu \tilde{F}^{(1)}_2 + \frac{1}{2}\frac{1}{3!}G^{\mu\mu'}G^{\nu\nu'}G^{\rho\rho'}\tilde{F}^{(3)}_{\mu\nu\rho}\tilde{F}^{(3)}_{\mu'\nu'\rho'} \\
&+ \frac{1}{2} \times \frac{1}{5!}G^{\mu\mu'}G^{2\nu'}G^{\rho\rho'}G^{\sigma\sigma'}G^{\tau\tau\tau'}\tilde{F}^{(5)}_{\mu\nu\rho\sigma\tau}\tilde{F}^{(5)}_{\mu'\nu'\rho'\sigma'\tau'}
\end{aligned}
\tag{1.15}
$$

In addition to the two pieces above, there is a third piece which we call the Chern-Simons term which takes the following form in IIB:

$$
S_{CS} = -\frac{1}{4\kappa_{10}^2} \int C^{(4)} \wedge H^{(3)} \wedge F^{(3)}. \tag{1.16}
$$

There are two moduli fields in the action Φ and $C^{(0)}$(both have no potential term). We can combine these two into a single piece and observe something interesting. Let us introduce the following new definitions:

$$
\tau \equiv C^{(0)} + ie^{-\Phi}, \quad G^{(3)} \equiv F^{(3)} - \tau H^{(3)} \tag{1.17}
$$

Note that, $\operatorname{Im}\tau = e^{-\Phi}$. Let us now observe the kinetic term in this case,

$$
\partial_\mu\tau = \partial_\mu C^{(0)} + i\partial_\mu e^{-\Phi} = \partial_\mu C^{(0)} - ie^{-\Phi}\partial_\mu\Phi \tag{1.18}
$$

Similarly,

$$
\partial_\mu\bar{\tau} = \partial_\mu C^{(0)} + ie^{-\Phi}\partial_\mu\Phi. \tag{1.19}
$$

Thus,

$$
\begin{aligned}
\partial_\mu\tau\partial_\mu\bar{\tau} &= \left(\partial_\mu C^{(0)}\right)^2 + \left(e^{-\Phi}\partial_\mu\Phi\right)^2 \\
&= \partial_\mu C^{(0)}\partial^\mu C^{(0)} + e^{-2\Phi}\partial_\mu\Phi\partial^\mu\Phi
\end{aligned}
\tag{1.20}
$$

Using,

$$\begin{aligned}\tilde{F}^{(1)} &= dC^{(0)} = \partial_\mu C^{(0)} dx^\mu \\ \tilde{F}^{(1)}_\mu &= \partial_\mu C^{(0)},\end{aligned} \tag{1.21}$$

we get

$$\frac{1}{(Im\tau)^2}\partial_\mu\tau\partial^\mu\bar{\tau} = e^{2\Phi}\tilde{F}^{(1)}_\mu\tilde{F}^{\mu(1)} + \partial_\mu\Phi\partial^\mu\Phi. \tag{1.22}$$

Also,

$$\begin{aligned}G^{(3)}_{\mu\nu\rho}\bar{G}^{(3)}_{\mu'\nu'\rho'} &= \left(F^{(3)}_{\mu\nu\rho} - \tau H^{(3)}_{\mu\nu\rho}\right)\left(F^{(3)}_{\mu'\nu'\rho'} - \bar{\tau}H^{(3)}_{\mu\nu\rho'}\right) \\ &= F^{(3)}_{\mu\nu\rho}F^{(3)}_{\mu'\nu'\rho'} - \bar{\tau}F^{(3)}_{\mu\nu\rho}H^{(3)}_{\mu'\nu'\rho'} - \tau H^{(3)}_{\mu\nu\rho}F^{(3)}_{\mu'\nu'\rho} \\ &\quad + \tau\bar{\tau}H^{(3)}_{\mu\nu\rho}H^{(3)}_{\mu'\nu'\rho'}\end{aligned} \tag{1.23}$$

Thus, in terms of the τ field, we can write the type IIB action as

$$\begin{aligned}S_{\text{IIB}} =& \frac{1}{2\kappa^2_{10}}\int d^{10}x\sqrt{-g}\Bigg[R_g - \frac{1}{2}\frac{1}{(Im\tau)^2}g^{\mu\nu}\partial_\mu\tau\partial_\nu\bar{\tau} \\ &- \frac{1}{2}\frac{1}{3!}\frac{1}{(Im\tau)}g^{\mu\mu'}g^{\nu\nu'}g^{\rho\rho'}G^{(3)}_{\mu\nu\rho}\bar{G}^{(3)}_{\mu'\nu'\rho'} \\ &- \frac{1}{2}\frac{1}{5!}g^{\mu\mu'}g^{\nu\nu'}g^{\rho\rho'}g^{\sigma\sigma'}g^{\delta\delta'}\tilde{F}^{(5)}_{\mu\nu\rho\sigma\delta}\tilde{F}^{(5)}_{\mu'\nu'\rho'\sigma'\delta'}\Bigg] \\ &- \frac{i}{8\kappa^2_{10}}\int\frac{C^{(4)}\wedge G^{(3)}\wedge\bar{G}^{(3)}}{(Im\tau)}.\end{aligned} \tag{1.24}$$

Notice that we wrote the metric in a different form. This canonical formalism is achieved by

$$G_{\mu\nu} = e^{\Phi/2}g_{\mu\nu} \Rightarrow g_{\mu\nu} = e^{-\Phi/2}G_{\mu\nu} \tag{1.25}$$

The Riemann tensors are related by $\rightarrow$

$$R_G = e^{-\Phi/2}\left[\; R_g - \tfrac{2}{4}(d-1)\Box\Phi - \tfrac{(d-1)(d-2)}{16}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\;\right]$$

In d=10,

$$R_G = e^{-\Phi/2}\left[R_g - \frac{g}{2}\Box\Phi - \frac{g}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right]. \tag{1.26}$$

$$\begin{aligned}&R_G + 4G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \\ =& e^{-\Phi/2}\left[R_g - \frac{9}{2}\Box\Phi - \frac{9}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right] + 4e^{-\Phi/2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \\ =& e^{-\Phi/2}\left[R_g - \frac{9}{2}\Box\Phi + \left(4 - \frac{9}{2}\right)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right] \\ =& e^{-\Phi/2}\left[R_g - \frac{9}{2}\Box\Phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right]\end{aligned} \tag{1.27}$$

Thus, the NS-NS part of the action takes the form:

$$S_{NS-NS} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left[R_g - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{12}e^{-\Phi}g^{\mu\mu'}g^{\mu\nu'}g^{\rho\rho'}H^{(3)}_{\mu\nu\rho}H^{(3)}_{\mu'\nu'\rho'}\right] \tag{1.28}$$

And the R-R part of the action takes the form,

$$S_{RR} = \frac{-1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{5\Phi/2}\left\{\frac{1}{2}e^{-\Phi/2}g^{\mu\nu}\tilde{F}^{(1)}_\mu\tilde{F}^{(1)}_\nu + \frac{1}{2\cdot 3!}e^{-3\Phi/2}g^{\mu\mu'}g^{\nu\nu'}g^{\rho\rho'}\tilde{F}^{(3)}_{\mu\nu\rho}\tilde{F}^{(3)}_{\mu'\nu'\rho'} + \frac{1}{2\cdot 5!}e^{-5\Phi/2}g^{\mu\mu'}g^{\nu\nu'}g^{\rho\rho'}g^{\sigma\sigma'}g^{\tau\tau'}\tilde{F}^{(5)}_{\mu\nu\rho\sigma\tau}\tilde{F}^{(5)}_{\mu'\nu'\rho'\sigma'\tau'}\right\} \tag{1.29}$$

Type IIB action is invariant under SL$(2, R)$. SL$(2, R)$ is the group of real 2×2 matrices with determinant one. A matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, R) \quad ; \quad ad - bc = 1$$

This acts on fields as

$$\tau \to \frac{a\tau + b}{c\tau + d} \tag{1.30}$$

and keeps other fields except $G^{(3)}$ invariant:

$$G^{(3)} \to \frac{G^{(3)}}{c\tau + d}. \tag{1.31}$$

1.3.3 Type I

The action in the context of the Type I theory is derived by eliminating all the degrees of freedom from the Type IIB theory that exhibit an odd behavior under world-sheet parity. In other words, we retain only those states in the Type IIB theory that remain unchanged when subjected to world-sheet parity transformations. If Ω = world sheet parity then, Type I $\sim$ IIB/Ω. We cannot do the same projection exercise for type IIA since gravitinos in that theory has opposite chirality. The field contents in type I theory are

- NS-NS Sector : $G_{\mu\nu}, \Phi$
- R-R Sector : $C^{(2)}$
- 32 D-9 branes (this gives rise to SO(32) gauge fields which are coming from open strings)

A few comments are due. First, in the NS-NS sector, the Kalb-Ramond field is not invariant under the worldsheet parity operator and thus is not present in type I theory. Same occurs for other fields in R-R and we are left only with $C^{(2)}$. Second, we have added the 32 D9 branes, because by projecting out the odd parity states, we generate negative RR charge which is only cancelled by adding the D9 branes. The fields in NS-NS and R-R Sectors come only from the closed string spectrum whereas SO(32) gauge fields come from the open string spectrum. Type I theory is unique because in this, both open as well as closed strings state contribute whereas in the other 4 theories, only closed strings, state contribute. The NS-NS part of the action is

$$S_{NS-NS} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left[R_G + 4G^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi\right] \tag{1.32}$$

and the R-R part is

$$S_{R-R} = \frac{-1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[\frac{1}{2}\frac{1}{3!} G^{\mu\mu'} G^{\nu\nu'} G^{\rho\rho'} \tilde{F}^{(3)}_{\mu\nu\rho} \tilde{F}^{(3)}_{\mu'\nu'\rho'}\right] \tag{1.33}$$

and the gauge theory part is

$$S_{\text{gauge}} = \frac{-1}{2\kappa_{10}^2} \frac{C_I}{2!} \int d^{10}x \sqrt{-G} e^{-\Phi} \left[G^{\mu\mu'} G^{\nu\nu'} Tr_V \left(F_{\mu\nu} F_{\mu'\nu'}\right)\right]. \tag{1.34}$$

Here,

$$\tilde{F}^{(3)} = dC^{(2)} - C_I \omega^{(3)}(A)$$

$$\omega^{(3)}(A) = \mathrm{Tr}_V \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

$$C_I = \frac{\alpha'}{4}$$

$F_{\mu\nu} = F^a_{\mu\nu} T^a = F_{\mu\nu}$ is Yang-Mills field strength.

Also, the trace is over the vector representation of $SO(32)$ with $\mathrm{Tr}_V \left(T^a T^b\right) = \delta^{ab}$ Using, $G_{\mu\nu} = e^{\Phi/2 g_{\mu\nu}}$, we can also write the action in canonical form as follows:

$$\begin{aligned} S_I = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \Big[& R_g - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2}\frac{1}{3!} e^{\Phi} g^{\mu\mu'} g^{\nu\nu'} g^{\rho\rho'} \tilde{F}^{(3)}_{\mu\nu\rho} \tilde{F}^{(3)}_{\mu'\nu'\rho'} \\ & - \frac{C_I}{2!} e^{\Phi/2} g^{\mu\mu' g^{\nu\nu'}} Tr_V \left(F_{\mu\nu} F_{\mu'\nu}\right) \Big] \end{aligned} \tag{1.35}$$

1.3.4 Heterotic Theory

There are two heterotic supergravity theories based on two different groups: $SO(32)$, $E_8 \times E_8$. The number of generators of SO(32) is 496. The field contents in type I theory are

- NS-NS Sector : $G_{\mu\nu}, B^{(2)}_{\mu\nu}, \Phi$
- Gauge Fields : $A^a_u \quad a = 1, \ldots, 496$

The action can be written as:

$$
\begin{aligned}
S = & \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left[R_G + 4G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right. \\
& - \frac{1}{2} \frac{1}{3!} G^{\mu\mu'} G^{\nu\nu'} G^{\rho\rho'} \tilde{H}^{(3)}_{\mu\nu\rho} \tilde{H}^{(3)}_{\mu'\nu'\rho'} - \frac{C_H}{2!} G^{\mu\mu'} G^{\nu\nu'} Tr_V \left(F_{\mu\nu} F_{\mu'\nu'} \right)
\end{aligned} \tag{1.36}
$$

where, $F_{\mu\nu} = F^a_{\mu\nu} T^a$ is non-Abelian field strength. We have, $C_H = \frac{\alpha'}{4}$ for heterotic supergravity $H^{(3)} = dB^{(2)}$. Thus,

$$
\begin{aligned}
\tilde{H}^{(3)} &= H^{(3)} - C_H \omega^{(3)}(A) \\
\omega^{(3)}(A) &= \mathrm{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)
\end{aligned} \tag{1.37}
$$

As obvious, the two heterotic theories differ because of the generators T^a satisfying different algebra. In terms of canonical metric $g_{\mu\nu}$,

$$
\begin{aligned}
S = & \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[R_g - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} \frac{1}{3!} e^{-\Phi} g^{\mu\mu'} g^{\nu\nu'} g^{\rho\rho'} \tilde{H}^{(3)}_{\mu\nu\rho} \tilde{H}^{(3)}_{\mu'\nu'\rho'} \right. \\
& \left. - \frac{C_H}{2!} e^{-\Phi/2} g^{\mu\mu'} g^{\nu\nu'} Tr_V \left(F_{\mu\nu} F_{\mu'\nu'} \right) \right]
\end{aligned} \tag{1.38}
$$

Type I and SO(32) Heterotic theories are dual to each other, if we identify

$$
\begin{aligned}
\Phi_I &= -\Phi_H \\
C^{(2)}_I &= B^{(2)}_H \\
A_I &= A_H \\
g^I_{\mu\nu} &= g^H_{\mu\nu}
\end{aligned}
$$

The duality can be argued to hold as follows -

$$
\begin{aligned}
\Phi_I = -\Phi_H \Rightarrow e^{\Phi_I} &= e^{-\Phi_H} \\
\Rightarrow g^I_S &= \frac{1}{g^H_S}.
\end{aligned}
$$

This implies that when type I coupling is small, the Heterotic coupling is strong.

As an example, let us see how standard model can arise from the Heterotic theories. We will briefly overview this as a concluding remark. Let us start with SO(32) heterotic theory. It has a gauge group SO(32) and in needs to break in order to have our standard model.

$$\begin{aligned} \text{SO }(32) &\to \text{ SO }(12) \times \text{ SO }(20) \\ 496 &\to (66,1) + (12,20) + (1,190) \end{aligned}$$

and SO(12) can be broken further to achieve the standard model.

$$\begin{aligned} \text{SO}(12) \to &\text{SO}(8) \times \text{SO}(4) \\ \to &\text{SO}(8) \times \text{SU}(2) \times \text{U}(1)_1 \\ \to &\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_1 \times \text{U}(1)_2 \times \text{U}(1)_3 \end{aligned}$$

1.4 String Compactification

The main focus of this thesis is moduli stabilization in type IIB string and we will therefore focus mainly on compactification of type IIB and its moduli space.

As we saw in the previous discussions, superstring theory is a quantum theory of gravity living in 10 spacetime dimensions. Therefore, we need to compactify the extra dimensions (a short discussion on compactification is given in the appendix) to describe the four dimensional world around us. The 10 dimensional manifold is roughly compactified as follows:

$$\mathbb{R}^{1,3} \times M_6 \tag{1.39}$$

We call this compactification of type IIB on a background $\mathbb{R}^{1,3} \times M_6$ and M_6 is called the compactification manifold which is 6 dimensional. We usually compactify on a special kind of manifold called the Calabi-Yau (CY) manifold (discussed briefly in appendix A) as we are interested in Ricci flat manifolds. The reason for that is we want stable tachyon free solutions preserving some supersymmetry ([53] has interesting recent discussions). CY_3 manifolds admit SU(3) holonomy. As discussed in the appendix, they contain a closed $(1,1)$-form J called the Kähler form and a unique holomorphic $(3,0)$-form Ω_3.

Reducing 10D $\mathcal{N}=2$ Type IIB SUGRA with 32 supercharges on a CY threefold CY_3 we recover 4D$\mathcal{N}=2$ SUGRA with 8 supercharges. For the metric on $\mathbb{R}^{1,3} \times CY_3$,

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + g_{mn}\, \mathrm{d}y^m \otimes \mathrm{d}y^n \tag{1.40}$$

Here, the $x^\mu, \mu, \nu, \ldots = 0, \ldots, 3$, are the 4 dimensional space coordinates, while $y^m, m, n, \ldots = 4, \ldots, 9$, are internal coordinates on CY_3. In the Kaluza-Klein (KK) compactifications, we expand the 10D fields into eigenmodes on CY_3. Let us consider the simple example of a scalar field $\varphi(x,y)$. We assume that there exists a basis of eigenfunctions $\chi^{(k)}(y)$ of the 6D Laplacian $\Delta_y^{(6)}$ on the internal space so that we can write

$$\varphi(x,y) = \sum_{k=-\infty}^{\infty} \varphi^{(k)}(x)\chi^{(k)}(y), \quad \Delta_y^{(6)}\chi^{(k)}(y) = \lambda_k \chi^{(k)}(y)$$

The equation of motion for φ becomes

$$\Delta^{(10)}\varphi = \left(\Delta_x^{(4)} + \Delta_y^{(6)}\right)\varphi = \sum_{k=-\infty}^{\infty} \left(\Delta_x^{(4)} + \lambda_k\right)\varphi^{(k)}(x)\chi^{(k)}(y) = 0$$

If the eigenvalue is zero, we have a massless field or a moduli field. In case of form fields if we have a massless or moduli field if the "6 dimensional part" of the form is harmonic. Harmonic forms let us relate this to Dolbeult cohomology groups $H^{p,q}(X_3,\mathbb{C})$ which can be determined from the Hodge decomposition

$$H^k(X_3,\mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}(X_3,\mathbb{C}), \quad h^{p,q} = \dim\left(H^{p,q}(X_3,\mathbb{C})\right)$$

$h^{p,q}$ are called Hodge numbers and they tell us how many moduli fields are there. Let us write different sector fields in terms of the moduli fields.

- NSNS-sector: Expanding the Kähler form J, dilaton ϕ and Kalb-Ramond 2-form B_2:

$$J = t^a\omega_a, \quad \phi = \phi(x), \quad B_2 = B_2(x) + b^a\omega_a,$$

  where $\omega_a \in H^{1,1}(CY_3), a = 1,\ldots,h^{1,1}(CY_3)$, is a basis of harmonic $(1,1)$-forms. We therefore find the 4D scalars $\phi(x)$ and $b^\alpha(x)$ as well as a 4D 2-form $B_2(x)$. Further, there are $h^{1,2}(X_3)$ complex structure deformations parametrised by complex moduli $U^\alpha, \alpha = 1,\ldots,h^{1,2}$, as obtained from expanding the holomorphic 3-form Ω_3

- RR-sector: The p-form fields in this sector can be expanded as

$$\begin{aligned} C_0 &= C_0(x), \\ C_2 &= C_2(x) + c^a\omega_a, \\ C_4 &= Q_2^a \wedge \omega_\alpha + V^\alpha(x) \wedge \alpha_\alpha - V_\alpha(x) \wedge \beta^\alpha + \rho_a\tilde{\omega}^a, \end{aligned}$$

  where $a = 1,\ldots,h^{1,1}(CY_3)$ and $(\alpha_\alpha, \beta\alpha) \in H^3(CY_3), \alpha = 1,\ldots,h^{1,2}(CY_3)$. The $\tilde{\omega}^a \in H^{2,2}(CY_3)$ are harmonic forms Poincaré dual to the ω_a.

As we mentioned in the beginning of the section, we work with $N = 2$ supersymmetry in 4 dimensions but we need to truncate the number of supersymmetry to 1 in order to construct phenomenologically viable models. This is done with the help of orientifolds which is briefly mentioned in the beginning of section (2.1).

1.5 Cosmology and String Cosmology

We live in a universe whose expansion rate is accelerating. To discuss the physics of the universe, we resort to Einstein equations of general relativity. Solutions of the field equations of general relativity tells us the geometry of specific portion of space time we are dealing with. Some of the examples could be solving Einstein equation for a spherically symmetric body, a black hole, a free particle. The solutions tells us how these objects curve the space time manifold. To write a metric for the universe, we need to incorporate our experimental and theoretical observations in the solution. This is provided by the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric.

We thus start our journey of equations with the famous FLRW metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1.41}$$

A few comments about the metric

- Here, we have used symmetries, isotropy and homogeneity, to reduce 10 independent components of the metric tensor to a single function a(t) and constant curvature parameter k.
- We do not have a g_{0i} components as it would break isotropy.
- Using scaling symmetry of the FLRW, we can set the scale factor today as a≡ $a(t_0) \equiv 1$.
- We have the usual "big bang" singularity when a goes to 0.

Friedmann Equations

The famous Einstein equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ governs the dynamics of our universe. The requirements of homogeneity and isotropy also constraints the the energy momentum tensor and it constraints it to be that if a perfect fluid.

$$T^{\mu\nu} = (\rho + P)U^{\mu}U_{\nu} - P^{\mu\nu}_{\delta} \tag{1.42}$$

Here, ρ is the energy density of the fluid and P is the pressure of the fluid. We can write the energy momentum tensor conservation equation as :

$$\nabla_{\mu}T^{\mu}_{\nu} = 0$$

Plugging in the surviving Christoffel symbols of the FLRW and our energy momentum tensor components we obtain the continuity equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \tag{1.43}$$

We can use the continuity equation to find out how matter densities are effected due to the expansion. With a simple calculation we can conclude the following:

- For matter dominated universe, we have $\rho_{matter} \propto a^{-3}$.
- For radiation dominated universe, we have $\rho_{radiation} \propto a^{-3}$
- For dark energy dominated universe, $\rho_{vacuum} \propto a^{0}$

We are now in position to calculate the Einstein equations for our system.
Let us start with the G^0_0.

$$\begin{aligned} G^0{}_0 &= g^{00}G_{00} \\ 8\pi G(T^0{}_0) &= 3\left[(\frac{\dot{a}}{a})^2 + \frac{k}{a^2}\right] \\ 8\pi G\rho &= 3\left[(\frac{\dot{a}}{a})^2 + \frac{k}{a^2}\right] \end{aligned}$$

Finally, we arrive at the first Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{1.44}$$

Using a similar analysis for $G^i{}_j$, we arrive at the second Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \tag{1.45}$$

Using Hubble parameter i.e $H = \frac{\dot{a}}{a}$, we can rewrite the first Friedmann equation as

$$H^2 = \frac{8\pi G}{3} - \frac{k}{a^2} \tag{1.46}$$

1.5.1 Problems of Modern Big Bang Cosmology

This beautiful standard Big bang cosmology picture has a few problems. These include the flatness problem, the horizon problem, the baryon asymmetry problem, the issue of spacetime singularity and the problem of topological defects. The Baryon asymmetry problem is the problem of having a disproportionate matter and antimatter around us. Modern BB cosmology does not explain why we have such a large number of matter compared to antimatter. The problem of topological defects (monopoles, cosmic strings and domain walls) is that in the early universe these relics would be produced in very high amounts (1 monopole per nucleon or 1 monopole per 10^9 photons). Standard BB cosmology does not explain why we have not seen them (current estimation is ratio of monopole per nucleon is less than 10^{-30}). We will elaborately discuss the flatness problem and the horizon problem now.

1.5.2 Flatness problem

We can rewrite the Friedmann equation in terms of a dimensionless density parameter Ω where

$$\Omega = \frac{\rho}{\rho_c}. \tag{1.47}$$

The first Friedmann equation is

$$H^2 = \frac{8\pi G}{3} - \frac{k}{a^2}$$

Using the idea that scale factor a(t) is 1 now and for a flat universe (k=0), we get the critical density today:

$$\rho_c = \frac{3H^2}{8\pi G} \tag{1.48}$$

Dividing the first Friedmann equation by H^2 we can rewrite the the equation it terms of the density parameter:

$$\Omega - 1 = \frac{k}{a^2 H^2} \tag{1.49}$$

Using $\rho \propto t^{-2}$ and $a(t) \propto t^p$,

$$\frac{\Omega(t) - 1}{\Omega} \propto t^{2(1-p)}$$

As t $\longrightarrow$ 0, $\Omega \longrightarrow 1$ which implies that as time approaches the big bang singularity, we approach a k $\sim$ 0 regime. Standard BB cosmology does not have an explanation for this!

Horizon Problem

Another problem which pops out of our standard BB cosmology is the horizon problem. Before we go into the details, let us first make things convenient for us. We can write the FLRW metric in the following form:

$$ds^2 = a^2(\tau)\left[d\tau^2 - d\chi^2\right] \tag{1.50}$$

Here, we made the following redefinitions:

- We defined conformal time τ as $d\tau \equiv \dfrac{dt}{a(t)}$
- We redefined the radial coordinate as $d\chi \equiv \dfrac{dr}{\sqrt{1-kr^2}}$
- Using the freedom of our isotropic universe, we defined coordinates having only radial coordinates and thus we got rid of the angular dependence in the FLRW.

The horizon problem requires us to define the the concept of a particle horizon. Particle horizon is defined as the greatest comoving distance from which an observer at time t will be able to receive light signals from is

$$X_{PH}(\tau) = \tau - \tau_i = \int \frac{dt}{a(t)} \tag{1.51}$$

It can also be written as

$$\begin{aligned} X_{PH}(\tau) &= \int_{t_i}^{t_f} \frac{dt}{a(t)} \\ &= \int_{a_i}^{a} \frac{da}{\dot{a}a} \\ &= \int_{lna_i}^{lna} (aH)^{-1} d\ln a \end{aligned} \tag{1.52}$$

Here, $(aH)^{-1}$ is called the comoving Hubble radius (radius of a comoving sphere which increases during expansion) and it tells us if particles can communicate to each other now. The above relation for particle horizon relates the causal structure of space time with the comoving Hubble radius. Using constant equation of state $w \equiv \dfrac{\rho}{P}$, we get $(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}$. Plugging all these in the expression for physical horizon:

$$\begin{aligned} X_{PH} &= \frac{2H_0^{-1}}{1+3w} a^{\frac{1}{2}(1+3w)} \\ &= \frac{2}{1+3w} a H_0^{-1} \end{aligned} \tag{1.53}$$

Now, let us take a moment and look at the physics from the historical (universe's history) and experimental observations. Approximately 380,000 years after the big bang,

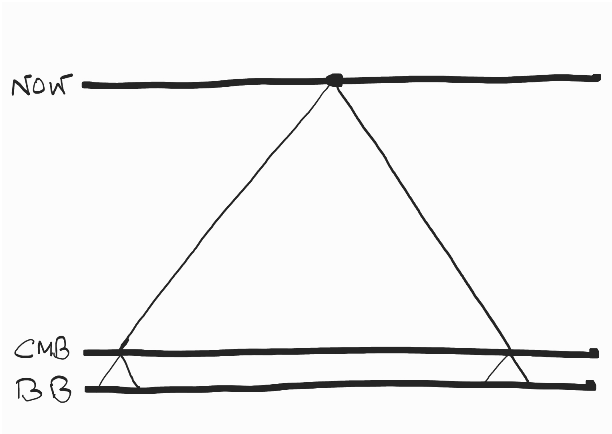

Figure 1.3: Horizon problem

the universe cooled down and allowed the very first atoms (of course hydrogen) to form and finally the photons decoupled from the hot dense plasma. The decoupling of the photons means we are finally able to look at the baby universe. This radiation is called the Cosmic Microwave Background Radiation (CMBR). Surprisingly, the radiation is almost perfectly isotropic which means if we look at two different parts of the sky they give almost exactly similar temperature. This put's an interesting and troublesome problem at our table. If the CMB fluctuations are this much correlated, they must have been in causal contact. If we draw the past light-cones of two distinct points (say two points separated by more than a degree) they do not have enough time to communicate causally (10^4 disconnected patches). But we just mentioned that they have the same temperature and this is not possible without being in causal contact. This is the famous horizon problem.

1.5.3 String Cosmology: Ideas and a toy model

As we discussed, the low energy limit of each of the five superstring theories is a 10 dimensional supergravity theory. In this discussion we will focus on type II superstring theory (Metric G_{MN}, dilaton ϕ and a two form B_{MN}). The Einstein-Hilbert action of General Relativity is complimented by a new contribution from the dilaton and the action is given by

$$S = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-\phi}(R + \partial^\alpha\phi\partial_\alpha\phi) \tag{1.54}$$

Where, S is the action given in the string frame in which the string length l_s as our fundamental unit. Ricci-Scalar R appears with dilaton (ϕ) dependent prefactor and of course it varies with dilaton in the string frame. We also have:

$$\kappa_{10} = \frac{1}{2}(2\pi)^4(\alpha')^4 \approx l_s^8$$
$$g_s = e^{\phi} \quad \text{(String coupling related to dilaton)}$$

We can also write the equation in Einstein frame with the Planck length as the fundamental unit. It is interesting and important to see how these two frames are related. In the discussion below we will use the subscript E to denote the Einstein frame.

$$\int d^Dx\sqrt{-g}e^{-\phi}R = \int d^Dx\sqrt{-g_E}(R_E + ...)$$
$$g_{\mu\nu} = e^{2w\phi}g_{E,\mu\nu}$$
$$\sqrt{-g} = e^{Dw\phi}\sqrt{g_E}$$

where we related the two metrics by using Weyl rescaling.

Scalar curvatures in the two frames are related by

$$R = e^{-2w\phi}(R_E - 2w(D-1)\nabla^2\phi - w^2(D-2)(D-1)\partial^\alpha\phi\partial_\alpha\phi$$
$$\sqrt{-g}e^{-\phi}R = e^{(Dw-1-2w)\phi}\sqrt{-g_E}(R_E - 2w(D-1)\nabla^2\phi - w^2(D-2)(D-1)\partial^\alpha\phi\partial_\alpha\phi)$$

Note,

$$Dw - 1 - 2w = 0$$
$$\implies w = \frac{1}{D-2}$$

and therefore,

$$S = -\frac{M_D^{D-2}}{2}\int d^Dx\sqrt{-g_E}(R_E - \frac{1}{D-2}\partial^\alpha\phi\partial_\alpha\phi) \tag{1.55}$$

FLRW-form can be generalized to D dimensions. In both the frames, they are related as follows

$$ds_E^2 = e^{2w\phi}ds^2 \tag{1.56}$$
$$= e^{2w\phi}(dt^2 - a^2d\vec{x}^2) \tag{1.57}$$
$$\equiv (dt_E^2 - a_E^2d\vec{x}^2) \tag{1.58}$$

Where,

$$a_E = e^{-w\phi}a$$
$$dt_E = e^{-w\phi}dt$$

Note that the two forms are physically equivalent. We will now perform the analysis in string frame. Let us assume a spatially homogeneous FLRW space time. Ricci scalar in this case is

$$R = -(D-1)(D-2)\frac{\dot{a}^2}{a^2} - 2(D-1)\frac{\ddot{a}}{a}$$

The action has a **remarkable symmetry** i.e the following transformation leaves my action invariant:

$$a(t) \rightarrow \frac{1}{a(t)}$$
$$\phi \rightarrow \phi(t) - 2(D-1)\ln a(t)$$

Let us verify this symmetry,

$$\begin{aligned}
\sqrt{g}e^{-\phi}(R+\dot{\phi}^2) &= a^{D-1}e^{-\phi}(-(D-1)(D-2)\frac{\dot{a}^2}{a^2}) - 2(D-1)\frac{\ddot{a}}{a} + \dot{\phi}^2 \\
&= a^{D-1}e^{-\phi}((D-1)(D-2)\frac{\dot{a}^2}{a^2} - 2(D-1)\frac{\dot{a}}{a}\dot{\phi} + \dot{\phi}^2) \\
&= a^{D-1}e^{-\phi}((D-1)(D-2)\frac{\dot{a}^2}{a^2} - 2(D-1)\frac{\dot{a}}{a}\dot{\phi} + \dot{\phi}^2) + \text{total derivatives} \\
&= a^{D-1}e^{-\phi}(-(D-1)\frac{\dot{a}^2}{a^2} + (\dot{\phi} - (D-1)\frac{\dot{a}}{a})^2 + \text{total derivatives}
\end{aligned}$$

Now, we have,

$$\begin{aligned}
a^{D-1}e^{-\phi} &\rightarrow a^{-(D-1)}e^{-\phi+2(D+1)\ln a} \\
&= a^{D-1}e^{-\phi} \\
\frac{\dot{a}}{a} &\rightarrow a\frac{d}{dt}(\frac{1}{a}) = -\frac{\dot{a}}{a} \\
\phi &\rightarrow \phi(t) - 2(D-1)\ln a(t) \\
\dot{\phi} &= \frac{2(D-1)}{a(t)\dot{a}}
\end{aligned}$$

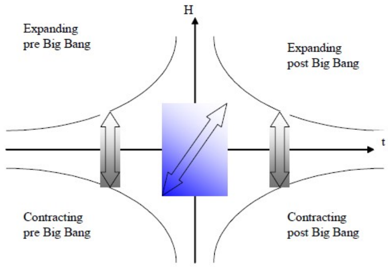

Figure 1.4: Relating Big Bang solutions [4]

Now plugging these in,

$$
\begin{aligned}
& a^{D-1}e^{-\phi}(-(D-1)\frac{\dot{a}^2}{a^2} + (\dot{\phi} - (D-1)\frac{\dot{a}}{a})^2 \\
\rightarrow\ & a^{D-1}e^{-\phi}(-(D-1)\frac{\dot{a}^2}{a^2} + \dot{\phi} - 2(D-1)\frac{\dot{a}}{a} + ((D-1)\frac{\dot{a}}{a})^2 \\
\rightarrow\ & a^{D-1}e^{-\phi}(-(D-1)\frac{\dot{a}^2}{a^2} + (\dot{\phi} - (D-1)\frac{\dot{a}}{a})^2
\end{aligned}
$$

We thus have an invariance of the action and thus if $a(t)$ and $\phi(t)$ solves the equation of motion so does $\frac{1}{a(t)}$ and $\phi(t) - 2(D-1)\ln a$. There is also another interesting symmetry

$$t \rightarrow -t$$

With these two symmetries at hand, we can relate two different solutions. The obvious thing to do would be to first focus on the scale factor then from a given solution $a(t)$, we can construct two new ones!

$$
\begin{aligned}
a(t) \rightarrow \frac{1}{a(t)} \quad H(t) \rightarrow -H(t) \\
a(t) \rightarrow \frac{1}{a(t)} \quad H(t) \rightarrow -H(-t)
\end{aligned}
$$

The basic foundation of string cosmology provides us with enough symmetries so that physics can be traced back in time through the Big Bang into the pre-Big Bang where many of the conditions for the post-Big Bang are determined naturally. This suggests that we should take on expanding pre-Big Bang theory and can match it with post Big-Bang theory. The figure 1.4 provides a clearer picture about relating different solutions.

Note that this picture is in String frame, things are a bit different in the Einstein frame. Let us see an explicit example. We start with matter in a definite equation of state $P = wp$. Our action has to account for the addition of matter and assume it becomes:

$$S = -\frac{1}{2R_4^2}\int d^4x\sqrt{-g}(e^{-\phi}(R + \partial^\alpha\phi\partial_\alpha\phi) + matter)$$

In Einstein frame, Friedmann equations takes the form:

$$H_E^2 = \frac{1}{3M_4^2}(\frac{M_4^2}{2}\frac{1}{2}(\frac{d\phi}{dt_E}^2) + P_E) \quad [D = 4]$$

Note that in the above equation we considered D=4. We are interested in the difference between the string frame and the Einstein frame. Let us now move to the string frame,

$$a_E = e^{-w\phi}a \quad [w = \frac{1}{D-2}]$$
$$dt_E = e^{-w\phi}dt$$

Incorporating the above changes,

$$H_E = e^{\frac{\phi}{2}}(H - \frac{1}{2}\dot{\phi}^2)$$
$$\frac{d\phi}{dt_E} = e^{\frac{\phi}{2}}\dot{\phi}$$
$$\sqrt{-g_E}\rho_E = e^{-2\phi}\sqrt{-g}\rho_E = \sqrt{-g}\rho$$

Thus we obtain the Friedmann equation in string frame.

$$H^2 = -\frac{1}{6}\dot{\phi}^2 + H\dot{\phi} + \frac{1}{3M_4^2}e^{\phi}\rho \tag{1.59}$$

And continuum equation for matter, $\rho = \rho_0 a^{-3(1+w)}$.
The lagrangian can be written in terms of the scale factor as

$$\mathcal{L} = -a^3e^{-\phi}(-6\frac{\dot{a}^2}{a^2} + 6\frac{\dot{a}}{a}\dot{\phi} - \dot{\phi}^2)$$
$$= 6a\dot{a}^2e^{-\phi} - 6\dot{a}a^2\dot{\phi}e^{-\phi} + a^3\dot{\phi}^2e^{-\phi}$$

We can use the ansatz $a \sim t^{\alpha}$ and $\phi = \beta \ln(t) + \text{constant}$. Let us first find the Euler-Lagrange equations and then use the ansatz to obtain an expression in terms of α and β.

$$
\begin{aligned}
\frac{\partial L}{\partial \phi} - \frac{d}{dt}(\frac{\partial L}{\partial \dot{\phi}}) = 0 \\
(6a\dot{a}^2 - 6\dot{a}a^2\dot{\phi} + a^3\dot{\phi}^2)(-e^{-\phi}) + \frac{d}{dt}(6\dot{a}a^2 e^{-\phi} - 2a^3\dot{\phi}e^{-\phi}) = 0 \\
-12\alpha^2 - \beta^2 + 6\alpha\beta - 2\beta + 6\alpha = 0 = 0
\end{aligned}
$$

To fully determine α and β we need one more equation. Luckily, we still have not use the continuity equation. From our continuity equation, we get

$$\beta - 3(1+w)\alpha = -2$$

The solution to α and β is

$$\alpha = \frac{2\omega}{1+3\omega^2} \qquad \& \ \beta = \frac{6\omega - 2}{1+3\omega^2}$$

As an example, let us assume that matter is only in the form of radiation $\omega = 1/3$. Thus,

$$
\begin{aligned}
\alpha &= \tfrac{1}{2} \quad &; \beta = 0 \\
a &\sim t^{1/2} \quad &; \phi = \text{constant}
\end{aligned}
$$

This implies we have a standard non-inflationary radiation dominated post big bang scenario. In particular, we have decreasing Hubble constant i.e

$$H = \dot{a}/a = 1/2t > 0 \tag{1.60}$$

$$\ddot{a} = -\frac{1}{4t^2} < 0 \quad \&\text{thus } \frac{\ddot{a}}{\dot{a}} < 0 \tag{1.61}$$

Applying our symmetry transformation,

$$a \sim (-t)^{1/2} \quad \phi = -6\ln(-t)^{1/2} + \text{ constant}$$

note that the above transformation is valid for $t < 0$. We get,

$H = \dot{a}/a = -1/2t > 0$

$\frac{\ddot{a}}{\dot{a}} = \frac{3}{2+2} > 0.$.

Here, $\dot{H} = \frac{1}{2t^2} > 0$ implies growing curvature. So, we have an inflating universe with growing curvature & coupling as $t \to O_{-}$ followed by constant radiation dominated cosmology which we considered as an example. The plot twist to this is that description a bit different in Einstein frame. There are no differences in post Bigbang scenario. In pre-bigbang phase,

$$dt_E \sim (-t)^{3/2} dt \Rightarrow t_E \sim -(-t)^{5/2}$$
$$a_E \sim (-t)^{3/2} \times (-t)^{1/2} = -t \sim (-t_E)^{2/5}$$

If we compute the derivatives,

$$a'_E \sim -(-t_E)^{-3/5} < 0$$
$$a''_E \sim -(-t_E)^{-8/5} < 0$$

This implies that the physical picture in Einstein frame is that of a contracting universe in the pre-Big Bang scenario whereas we had an expanding universe in the string frame [3].

1.6 Inflation and Brane-Antibrane Inflation

One of reasons of our long discussion of horizon problem is that it will help us define inflation and address the problem head on. A simple solution to the horizon problem is a phase of decreasing Hubble radius in the early universe. Shrinking Hubble sphere requires a strong energy condition violating liquid [5].

Inflation is basically a period which had a shrinking Hubble sphere and a mechanism to achieve $X_{PH} \gg (aH)^{-1}$. With this intuition, we are in position to define inflation as the phase with

$$\frac{d}{dt}(aH)^{-1} < 0. \tag{1.62}$$

With the definition at hand, let us analyze a few things. The decreasing Hubble sphere definition of inflation is related to another popular definition of inflation which commonly describes inflation as a period of rapid acceleration.

$$\frac{d}{dt}(aH)^{-1} = -\frac{\ddot{a}}{(\dot{a})^2}$$

Thus, we get

$$\ddot{a} > 0 \tag{1.63}$$

This is the reason why inflation is often described as a period of rapid acceleration but the important thing to ask here is what amount of rapid acceleration or inflation will

[5]SEC is a requirement $T_{\mu\nu}U^\mu U^\nu + \frac{1}{2}T \geq 0$. Any fluid with negative pressure violates it

solve our horizon problem? Put in another way, how long does inflation need to occur? At the very least, we need inflation to occur so that our entire observable universe fits into the comoving Hubble radius at the beginning of inflation that is

$$(a_0 H_0)^{-1} < (a_I H_I)^{-1} \tag{1.64}$$

Here, subscript I denotes the beginning of inflation. Let us assume the universe is radiation dominated (ignoring matter and dark energy domination eras for now) since the end of inflation to get a sense of what duration of inflation we are after. For radiation, we have $H \propto a^{-2}$. Therefore,

$$\begin{aligned}
\frac{a_0 H_0}{a_E H_E} &\sim \frac{a_0}{a_E}(\frac{a_E}{a_0})^2 \\
&= \frac{a_E}{a_0} \\
&\sim \frac{T_0}{T_E} \\
&\sim 10^{-28} \\
(a_I H_I)^{-1} > (a_0 H_0)^{-1} &\sim 10^{28}(a_E H_E)^{1}
\end{aligned}$$

Here, E denotes the end of inflation. Therefore, we need the Hubble radius to shrink by at least 10^{28} during inflation in order to get rid of the horizon problem. If H is constant during inflation i.e $H_I = H_E$, then

$$\begin{aligned}
\frac{a_E}{a_i} &> 10^{28} \\
\ln\left(\frac{a_E}{a_i}\right) &> 64
\end{aligned} \tag{1.65}$$

This is famously called the 60 e-folds of inflation. This is an enormously large number by which the scale factor at the beginning and end of inflation would increase.

Before going into discussion about the dynamics of the inflaton field, it is important to define the two parameters : the ϵ and η parameters. Let us get back into our definition of inflation $\frac{d}{dt}(aH)^{-1} < 0$. Now,

$$\begin{aligned}
\frac{d}{dt}(aH)^{-1} &= -\frac{\dot{a}H + a\dot{H}}{(aH)^2} \\
&= -\frac{1}{a}(1-\epsilon)
\end{aligned}$$

Here, we have defined $\epsilon \equiv -\frac{\dot{H}}{(H)^2}$. Our definition of inflation forces it to be

$$\epsilon = -\frac{\dot{H}}{(H)^2} < 1 \tag{1.66}$$

The parameter ϵ should remain small for sufficiently long time for inflation to solve the horizon problem. We define the η parameter which measures if ϵ remains small for enough Hubble times:

$$\eta \equiv \frac{d\ln\epsilon}{dN} = \frac{\dot{\epsilon}}{H\epsilon} \tag{1.67}$$

Here, $dN = d\ln a = Hdt$ and it measures the number of e-folds N of inflationary expansion. For, $\|\eta\| < 1$ fractional change of ϵ per Hubble time is small and inflation persists long enough. These two parameters will be of pivotal use throughout our upcoming discussion as they indicate if inflation occurs slowly enough and if this slow enough rate is there for a sufficient amount of time.

1.6.1 The dynamics of the Inflaton Field

Let us define a scalar field $\Phi(t,x)$ as our inflaton field which is minimally coupled to gravity. If our inflaton field dominates the universe i.e it drove inflation and is the driving/influencing the evolution of FLRW then we can write out the energy momentum tensor of this scalar field and equate it with the one we obtained before. The energy momentum tensor for this case is:

$$T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - V(\Phi)\right) \tag{1.68}$$

Symmetries of our FLRW metric require that the background value of our inflaton field should depend only on time $\Phi = \Phi(t)$. From our previous discussion, the symmetries of FLRW tells us that the energy momentum tensor for the universe should be that of a perfect fluid. Let us equate the two and plug them into the Friedmann equations.

For the case $T^0_{\ 0} = \rho_\Phi$, we obtain

$$\rho_\Phi = \frac{1}{2}(\dot{\Phi})^2 + V(\Phi) \tag{1.69}$$

So the total energy density is actually a sum over total potential and kinetic energy of the inflaton field.

For the case $T^i_{\ j} = -P_\Phi\delta^i_{\ j}$,

$$P_\Phi = \frac{1}{2}(\dot{\Phi})^2 - V(\Phi) \tag{1.70}$$

The first Friedmann equation $H^2 = \frac{\rho_\Phi}{3M_4^2}$ becomes

$$H^2 = \frac{1}{3M_4^2}\left(\frac{1}{2}(\dot{\Phi})^2 + V(\Phi)\right) \tag{1.71}$$

Taking the derivative of the above equation with respect to t,

$$2H\dot{H} = \frac{1}{3M_4^2}\left(\dot{\Phi}\ddot{\Phi} + V'(\Phi)\dot{\Phi}\right)$$

Now,

$$\begin{aligned}\dot{H} &= -\frac{\rho_\Phi + P_\Phi}{2M_4^2} \\ &= -\frac{1}{2}\frac{(\dot{\Phi})^2}{2M_4^2}\end{aligned}$$

Using this, we obtain the Klein Gordon equation for the Inflaton field:

$$\ddot{\Phi} + 3H\dot{\Phi} + V' = 0 \tag{1.72}$$

The second term acts like a friction term and the derivative of the potential obviously acts like a force term. A detail analysis would be done in the following section in which we discuss the famous slow roll inflation.

Slow-Roll Inflation

Let us first accumulate what we have at hand which will motivate our upcoming discussion, we got $\dot{H} = -\frac{1}{2}\frac{(\dot{\Phi})^2}{M_4^2}$ and we defined $\epsilon \equiv -\frac{\dot{H}}{(H)^2}$ which was must be less than 1. Plugging the value of the derivative of H here we get,

$$\epsilon = \frac{1}{2}\frac{(\dot{\Phi})^2}{M_4^2 H^2}$$

Thus, for $\epsilon < 1$ we need to have the kinetic term small and make small contribution to the total energy. This is called the Slow-Roll Inflation. In order for this condition to persist, the acceleration of the scalar field has to be small. We define another quantity

$$\delta \equiv -\frac{\ddot{\Phi}}{H\dot{\Phi}}$$

Let us now compute our second important parameter $\eta \equiv \frac{\dot{\epsilon}}{H\epsilon}$. Taking the derivative of ϵ, we find $\dot{\epsilon} = \frac{\dot{\Phi}\ddot{\Phi}}{M_4^2 H^2} - \frac{(\dot{\Phi})^2 \dot{H}}{M_4^2 H^3}$. Plugging all these,

$$\eta = 2(\epsilon - \delta) \tag{1.73}$$

Thus, the parameters $\epsilon \ll 1$ and $\|\delta\| \ll 1$ guarantee $\|\eta\| \ll 1$, and they altogether guarantee that inflation occurs and persists long enough.

Now, let us apply these "slow-roll" approximations in to our scalar field theory equations and study the dynamics. Let us apply them step by step.

- Firstly, $\epsilon \ll 1$ implies $\frac{(\dot{\Phi})^2}{2} \ll V$ and our Friedmann equation becomes :

$$H^2 \simeq \frac{1}{3M_4^2} V \tag{1.74}$$

- Secondly, our KG equation is modified as due to $\|\delta\| \ll 1$ i.e we can neglect the kinetic term due to the acceleration being small,

$$3H\dot{\Phi} \simeq -V' \tag{1.75}$$

Plugging these into our slow-roll parameters ϵ and η, we obtain

$$\epsilon = \frac{M_4^2}{2} \left(\frac{V'}{V} \right) \tag{1.76}$$

$$\|\eta\| = M_4^2 \left(\frac{\|V''\|}{V} \right) \tag{1.77}$$

Note that the expressions that we obtained are for generic potentials V. So, we can easily study if a specific potential V satisfies the criteria for a slow-roll inflation if they satisfy our slow-roll conditions which are $\epsilon \ll 1$ and $\|\eta\| \ll 1$.

1.6.2 Toy Models of Inflation

In this section we discuss two interesting toy models which give rise to our desired inflation and which will be relevant for our discussion on brane-antibrane inflation.

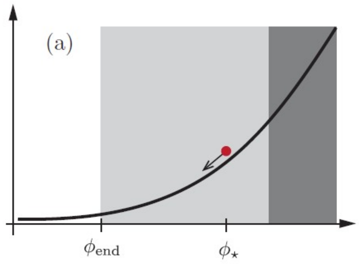

Figure 1.5: Chaotic Inflation from [2]

1.6.3 Chaotic Inflation

An interesting thing to note is that a potential like $V(\Phi) = \frac{1}{2}m^2\Phi^2$ can generate enough inflation. It makes us wonder if any power of Φ can generate required inflation. This brings us to an important class of models where Φ is just a simple monomial.

$$V(\Phi) = \mu^{4-p}\Phi^p \tag{1.78}$$

Here, $p > 0$ and μ is a parameter with mass dimension. It is interesting to see that a simple term like this solves such a big issue. We can thus calculate the slow-roll parameters

$$\begin{aligned}\epsilon &= \frac{M_4^2}{2}\left(\frac{V'}{V}\right)^2 \\ &= \frac{p^2}{2}\left(\frac{M_4}{\Phi}\right)^2\end{aligned}$$

Similarly, we can compute η

$$\eta = p(p-1)\left(\frac{M_4}{\Phi}\right)^2$$

It is very easy to see (from the above equations and figure 1.5) that slow-roll conditions can be easily satisfied.

1.6.4 Hybrid Inflation

Hybrid inflation provides an interesting natural way to end inflation alongside enjoying the features of a slow-roll potential. It provides a method to end inflation by coupling the Inflaton field Φ to an additional field Ψ (sometimes called a "waterfall field"). As a simple example, let us consider a simple potential of two fields of the form:

$$V(\Phi,\Psi) = V(\Phi) + V(\Psi) + \frac{1}{2}g\Phi^2\Psi^2$$

Here, $V(\Phi)$ is a slow-roll potential and the form of $V(\Psi)$ is

$$V(\Psi) \equiv \frac{1}{4\lambda}\left(M^2 - \lambda\Psi^2\right)^2$$

In the potential, we make the following assumption $V(\Phi) \ll (\frac{M^4}{4\lambda})$. This is because we want the dominant energy contribution to the inflationary potential to come from the false vacuum of the potential $V(\Psi)$. It is noteworthy that the interaction term between the two fields induces an effective mass for the value of the waterfall field in terms of the inflaton field which is

$$M_\Psi^2 = -M^2 + g\Phi^2$$

The mass of the waterfall field vanishes at the special point $\Phi_s \equiv \frac{M}{\sqrt{g}}$. For $\Phi > \Phi_s$, the field is stabilized at $\Psi = 0$ and we can reduce the effective theory into a potential of the form $V_{eff} \sim \frac{M^4}{4\lambda} + V(\Phi)$. But when Φ approaches Φ_c from above, the effective description must involve both fields. Lastly, when $\Phi < \Phi_s$, the field is tachyonic and we no longer have inflation as the potential no longer satisfies the slow-roll conditions.

1.6.5 End of Inflation and Reheating

Another interesting part of this inflation fairy tale is that of ending inflation and producing the universe as we see today. During inflation, most of the energy density of the universe is in the form of the inflaton potential and obviously inflation ends when the inflaton potential steepens and the inflaton field has a dominating kinetic term. Remember that when inflation ends, the inflaton field (by an unspecified mechanism) must transfer its energy to our Standard Model fields. This process is called reheating. Let us see an example below.

If we look at the graph of $V(\Phi)$, when inflation ends the kinetic term dominates over the potential term i.e the field Φ oscillates at the bottom. We can approximate $V(\Phi)$ as

$$V(\Phi) \simeq \frac{1}{2}m^2(\Phi)^2$$

Using this, we can write our KG equation as

$$(\ddot{\Phi}) + 3H\dot{\Phi} + V'(\Phi) = 0$$
$$(\ddot{\Phi}) + 3H\dot{\Phi} = -m^2\Phi$$

Next, we can make the approximation that $H^{-1} \gg m^{-1}$ because the expansion time scale soon becomes larger than the oscillation time scale. This let us ignore the friction term. Moreover, if we remember our continuity equation

$$\begin{aligned}\dot{\rho_\Phi} + 3H\rho_\Phi &= -3HP_\Phi \\ &= -\frac{3}{2}H(m^2(\Phi)^2 - (\dot{\Phi})^2)\end{aligned}$$

The right-hand side averages to zero over one oscillation period, and writing out the Hubble parameter in terms of the scale factor gives us the following $\rho_\Phi \propto a^{-3}$. Besides the scalar field oscillations and energy density evolution becoming similar to that of matter, there are other ways of reheating as well. Another way of inflaton transferring energy to Standard Model particles is through the inflaton fields decaying into other particles.

1.6.6 Brane Dynamics and the DBI action

D-branes are dynamical objects in String theory. They usually arise when we fix the boundary conditions for open strings. Similar to a string action (Polyakov action), we can also have an action for branes. Defining coordinates ζ^a, where a=0,1,.....,p on the brane. The following action will be in terms of the dynamics of the action in the brane "worldvolume". Fields on the branes are basically embeddings on the spacetime X^μ and gauge fields A_a. If we just focus on the embedding part for now, we can write the action as follows:

$$S_P = -T_P \int d^{p+1}\zeta e^{-\phi}\sqrt{G}$$

where T_P is the tension of the D_p brane. The exponential dependence $e^{-\phi} = g_s^{-1}$ due to the action being an open string tree level action. The determinant of the induced metric on brane, G_{ab}, is the pullback of the spacetime metric, $G_{\mu\nu}$, to the brane.

T-duality rules mixes components of the embeddings into worldvolume gauge fields so the above action is not sufficient. We need to take this into account the kinetic term of transverse fields (fields oscillating transverse to the brane) $\partial_a X^m$ where m=p+1,....,D-1 will become derivative of gauge fields $2\pi\alpha'\partial_a A_m$. For the gauge field part, a combination of $B_a b + 2\pi\alpha' F_{ab}$ works (in the sense that it preserves gauge symmetry). It is easy to check it.

$$\frac{1}{2\pi\alpha'}\int_\mu B + \int_{\partial M} A$$

Where we have written the gauge field in forms $\int A = \int A_a d\zeta^a$. It is evident that the action is invariant under the gauge transformation $\delta A = \delta\lambda$ but $\delta B = \delta\zeta$ gives a surface term which is only cancelled by the following transformation.

$$\delta A = -\frac{\zeta}{2\pi\alpha'}$$

There are many ways to deduce the complete worldvolume action. One of them is to use a technique like the string sigma model. The easy one out that we follow here is using the rules of T-duality.

Let us start with a simple example and build our case. Consider D_2 branes extended in x^1 and x^2 direction and let there be a constant gauge field F_{12}. Let us choose a gauge $A_2 = X^1 F_{12}$ and T-dualize along x^2 direction i.e $X'^2 = 2\pi\alpha' X^1 F_{12}$. Thus, the resulting D-brane is tilted by $\theta = \tan^{-1}(2\pi\alpha' F_{12})$ to the x^2 axis. This gives an extra geometric factor in D_1 brane worldvolume action.

$$S \sim \int_{D1} ds = \int dX^1 \sqrt{1 + (\partial_1 X'^2)^2} = -\int dX^1 \sqrt{(1 + 2\pi\alpha' F_{12})^2}$$

We are always boost the D-brane to be aligned with coordinate axes and then rotate to bring it into a block diagonal form. This brings us to the Born-Infield action.

$$S \sim \int d^P X (\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}))^{\frac{1}{2}} \tag{1.79}$$

We are now in position to derive the Dirac-Born-Infield (DBI) action. We do a compactification and label the compactified directions as X^m where m=p+1,....,D-1. We label the non-compact directions as X^a where a=0,....,p. We neglect the derivatives with respect to the compactified directions because we compactified on "small circles". Now, the matrix whose determinant is in the Born-Infield action has the matrix form

$$\begin{pmatrix} W & -A^T \\ A & M \end{pmatrix}$$

Here, N = $\eta_{ab} + 2\pi\alpha' F_{ab}$, $M = \delta_{mn}$ and $A = 2\pi\alpha' \partial_a A_M$. We can write the determinant as $|M||N + A^T M^{-1} A|$ which takes our action into the form

$$S \sim \int d^{P+1} X (\det(\eta_{ab} + \partial_a X^m \partial_b X_m + 2\pi\alpha' F_{ab}))^{\frac{1}{2}}$$

This is up to a numerical factor which usually comes from the volume of the torus we compactified on. The differential term comes from using T-duality rules $2\pi\alpha' A_M = X^M$. Plugging generic metric, anti-symmetric two form and our dilaton and using a general coordinate ζ^a, we can write the DBI action as

$$S_{DBI} = -T_P \int d^{P+1}\zeta e^{-\phi}(\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}))^{\frac{1}{2}}$$

The exponential comes up since all of this physics arises at open string tree level.

One of the last things that we need to discuss is about D-branes being "charged objects". This is important because in the brane-antibrane picture presented in [1], the D-branes are R-R charged. Let us briefly discuss them here. Scalar field Φ, metric $G_{\mu\nu}$ and anti-symmetric tensor $B_{\mu\nu}$ are common to IIA and IIB string theories (they arise from NS-NS sector). Closed strings couple "electrically" to $B_{\mu\nu}$. It can be written in the following way

$$v_1 \int_{M_2} B(2)$$

where $v_1 = (2\pi\alpha')^{-1}$, μ_2 is the worldsheet and ζ^a are the worldsheet coordinates. Also, $B(2) = B_{ab}\, d\zeta^a d\zeta^b$, where B_{ab} is the pullback of $B_{\mu\nu}$. By Hodge duality, we can form its magnetic dual $dB_{(6)} = *dB_{(2)}$. Similar to the electric case, we can couple the magnetic dual to a 5 dimensional extended object (sometimes called NS5 brane which is an NS-NS charged object) by

$$v_5 \int_{M_6} B(6)$$

Here, M_6 is the world volume of the 5d extended object. Since, we are interested in R-R charged object, let us move on to there. Potentials in the R-R sector are

$$\begin{aligned} \text{type IIA} &: C_1, C_3, C_5, C_7 \\ \text{type IIB} &: C_0, C_2, C_4, C_6, C_8 \end{aligned}$$

D_p branes are p dimensional extended objects which couple to all these via an electric coupling of the form (thus R-R charged)

$$\mu_P \int_{M_P+1} C_{P+1} \tag{1.80}$$

In type IIA string theory there are stable D_p branes with p even and in type IIB string theory there are stable D_p branes with p odd. The above action is also called Chern-Simons action and it is the higher dimensional generalization of a charged particle coupled to a gauge potential. Lastly

$$S_{DP} = S_{DBI} + S_{CS}$$

This completes bosonic action for D-branes.

We lastly turn our focus on the brane-antibrane inflation. It is a string theory derived model (probably the first rigorously derived one) and most importantly has all the nice properties of inflationary dynamics. It also has a geometric interpretation for the inflaton field and also provides a mechanism to end inflation. The model presented by Majumdar et al in [1] is done for general branes and in a brane cosmology set up (although they discuss the idea in a string theory set up as well). In the following discussion, we will restrict ourselves to a string theory (type IIA and IIB) point of view i.e we will reproduce their calculation keeping 10 space-time dimensions in mind. The basic understanding is that our universe in one of the D_3 branes where the extra dimensions are compactified (in a fashion $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{X}_6$). Standard model physics (gauge interactions) are localized in this brane. Let us see how inflation fits into the picture. The idea is that standard BB cosmology is reproduced after two brane collisions. We have a net attractive interaction if the two branes are oppositely R-R charged i.e. brane and anti-brane interaction. Our inflaton field here is basically a scalar function of inter-brane distance. We can thus compute the effective potential and see if it reproduced the required amount of inflation as discussed in the previous sections. We start from the following action

$$S = S_{bulk} + S_{D_p} + S_{\bar{D}_p}$$

Here, S_{bulk} is the 10 dimensional bulk action specified by the choice of superstring theory we are interested in. The other two actions are (p+1) dimensional D_p brane and $\bar{D}_p$ brane action. The exact form can be obtained by expanding the DBI action

$$S_{DBI} = -\int_{\Sigma} d^{P+1}\zeta\sqrt{|\gamma|}(T_p + ..) \tag{1.81}$$

Here, $T_P \sim \lambda_s^{-(P+1)} e^{-\frac{\phi}{2}}$ is the D-brane tension and the induced metric $\gamma^i_{\mu\nu} = \partial_\mu X^A_i \partial_\nu X^B_i g_{AB}$ where the embedding is $X^A_i(\zeta)$. Here, the indices μ and ν runs from 0 to P. Lastly, index 'i' specifies the two branes and takes values 1 and 2. Let us now assume that the two branes are parallel. We can write the embeddings X^A_i as $X^A_i = (\zeta^\mu, X^m_i)$ where we have separated out the coordinates of the transverse position X^m_i where m goes from p+1 to 9. We are interested in the relative motion Y^m (our speculated inflaton) which is

$$Y^m = (X^m_1 - X^m_2)$$

Motion of center of mass is $\bar{X}^m \equiv (X^m_1 + X^m_2)$ then in terms of X_1 and X_2, we can write $X^m_1 = \frac{1}{2}(\bar{X}^m + X^m)$ and $X^m_2 = \frac{1}{2}(\bar{X}^m - Y^m)$.

Now, our induced metric $\gamma^i_{\mu\nu} = \partial_\mu X^A_i \partial_\nu X^B_i g_{AB}$ can be written in terms of brane separation and the center of mass

$$\gamma^1_{\mu\nu} = h_{\mu\nu} + \frac{1}{4}\partial_\mu Y^m \partial_\nu Y^n g_{mn} + \frac{1}{4}\partial_\mu \bar{X}^m \partial_\nu \bar{X}^n g_{mn} + \frac{1}{2}\partial_\mu \bar{X}^m \partial_\nu Y^n g_{mn}$$

$$\gamma^2_{\mu\nu} = h_{\mu\nu} + \frac{1}{4}\partial_\mu Y^m \partial_\nu Y^n g_{mn} + \frac{1}{4}\partial_\mu \bar{X}^m \partial_\nu \bar{X}^n g_{mn} - \frac{1}{2}\partial_\mu \bar{X}^m \partial_\nu Y^n g_{mn}$$

Our main interest lies in inter-brane separation so expanding $\sqrt{\gamma}$ in terms of $\partial_\mu Y^m$ will make the action in a convenient form for studying inflation

$$S_{D_p} + S_{\bar{D}_p} = -\int d^{P+1}\zeta\sqrt{h}T_P(2 + \frac{1}{4}h^{\mu\nu}\partial_\mu Y^m \partial_\nu Y^\nu g_{mn} + ...) \quad (1.82)$$

We have a kinetic term at hand and a weird potential term contribution $\zeta\sqrt{h}T_P 2$. We need to include the inter-brane potential energy. The dominant interaction between two branes at large distances is due to the exchange of massless bulk modes which includes the metric, dilaton and any massless bosons which couple to the brane. The potential decreases like a Newtonian potential, i.e. $V \sim \frac{1}{Y^{d_\perp - 2}}$ where $d_\perp = d - P + 3$ is the number of spatial dimensions transverse to the brane. The potential between two branes is thus in a form similar to the gravitational potential. The energy per area is given by

$$\frac{E}{A_p} = -\beta(\frac{1}{M_{10}^2})\frac{T_P^2}{Y^{d_\perp - 2}}$$

Here, M_{10} is the 10 dimensional Planck's constant. The 10 dimensional Planck's constant is related to the 4 dimensional one via $M_4^2 = M_{10}^2 V_\perp V_\parallel$. $V_\perp$ and $V_\parallel$ are the volumes of transverse and parallel directions/dimensions to the brane respectively. The volumes can be written as

$$V_\perp \equiv r_\perp^{d_\perp}$$
$$V_\parallel \equiv r_\parallel^{p-3}$$

The constant β in the potential expression above is an order unity constant from string theory which has the following form

$$\beta = \pi^{-\frac{d_\perp}{2}}\Gamma^{\frac{d_\perp - 2}{2}}$$

Before proceeding with the compactification, a few comments are due. The first is that Majumdar et al. [1] considered the transverse volume $V_\perp$ as an adjustable parameter so that its size can give us valuable insights to study cases where and if inflation occurs. Second, the compactification done below will change give an expression for Planck mass in different dimensions that we used above. As an example the following equation provides a sneak peak into the compactification method again:

$$\frac{M_{10}^2}{2}\int d^4x d^6y\sqrt{-g}(R + ...) \sim \frac{M_4^2}{2}\int d^4x(R + ...)$$

The action we will be compactifying is

$$S_{D+\bar{D}} = -\int d^4x d^{P-3}y\sqrt{-\gamma}T_P(2 + \frac{1}{4}g_{mn}\gamma^{ab}\partial_a Y^m \partial_b Y^n + ...)$$

This gives rise to the long desired expression of the potential.

$$V \equiv 2T_P V_{\parallel} - \frac{\beta}{M_{10}^2} \frac{T_P^2 V_{\parallel}}{Y^{d_\perp - 2}} \tag{1.83}$$

Let us canonically normalize our scalar field (i.e the separation) so that we have a canonical expression for the kinetic term. We redefine as follows:

$$\phi = \sqrt{\frac{T_P V_{\parallel}}{2}} Y$$

Also, we define the constants A and B to aid our calculation.

$$V = 2T_P V_{\parallel} - \frac{\beta}{M_{10}^2} \frac{T_P^2 V_{\parallel}}{Y^{d_\perp - 2}} \equiv A - \frac{B}{Y^{d_\perp} - 2}$$

We are now in position to compute the slow-roll parameters and see if they give rise to inflation. The first and second derivative of the potential needs to be computed. The first derivative is

$$V' = \frac{\partial V}{\partial \phi} = \frac{2}{\sqrt{T_P V_{\parallel}}} (d_\perp - 2) \frac{\beta}{y^{d_\perp - 1}}$$

Now, let us compute the ϵ parameter.

$$\begin{aligned}
\epsilon &= \frac{M_4^2}{2} \left(\frac{V'}{V} \right)^2 \\
&= \frac{M_4^2}{2} \left(\frac{\frac{2}{\sqrt{T_P V_{\parallel}}} \frac{\beta}{Y^{d_\perp - 1}}}{2T_P V_{\parallel} - \frac{\beta}{M_{10}^2} T_P^2 \frac{V_{\parallel}}{Y_\perp^d - 2}} \right)^2 \\
&= \frac{M_4^2}{T_P V_{\parallel}} \left(\frac{B}{A} (d_\perp - 2) \frac{1}{Y^{d_\perp - 1}} \right)^2 \\
&= \frac{M_{10}^2 V_\perp V_{\parallel}}{T_P V_{\parallel}} \frac{T_P^2}{4} \frac{\beta^2}{M_{10}^4} (d_\perp - 2) \left(\frac{1}{Y^{d_\perp - 1}} \right)^2 \\
&= \frac{\beta^2}{4} (d_\perp - 2)^2 \frac{T_P}{M_{10}^2} \frac{1}{Y^{d_\perp - 2}} \frac{V_\perp}{Y^{d_\perp}}
\end{aligned}$$

Thus, it can be approximated as

$$\epsilon \sim g_s \left(\frac{l_s}{Y} \right)^{d_\perp - 2} \left(\frac{r_\perp}{Y} \right)^{d_\perp} \tag{1.84}$$

Let us now compute the second slow-roll parameter η:

$$\begin{aligned}\eta &= M_4^2 \frac{V''}{V} \\ &\sim -2M_4^2 \frac{1}{T_p V_\parallel} \frac{B}{A} (d_\perp - 2)(d_\perp - 1) \frac{1}{Y^{d_\perp}} \\ &= -\beta (d_\perp - 2)(d_\perp - 1) \left(\frac{r_\perp}{Y}\right)^{d_\perp} \\ &\sim \left(\frac{r_\perp}{Y}\right)^{d_\perp}\end{aligned} \tag{1.85}$$

For inflation to occur, we need $\epsilon \ll 1$ and $\|\eta| \ll 1$. $\epsilon \ll 1$ is easy to hold because even if $Y \leq r_\perp$ because there are other factor like l_s and g_s which keep ϵ very small. Problem arises when we want $\|\eta| \ll 1$ because it would imply $Y \gg r_\perp$ but the brane separation cannot be larger than the size of the compactified dimensions! Therefore, inflation cannot occur in this way. In [1], they did there calculation for general branes but for branes in string theory, they identified a case (of course when the separation size is less than the size of the compactified dimensions) where inflation will occur and provides a mechanism to end inflation and give standard BB evolution.

The inflationary scenario

In this section, we will consider the case when the separation is comparable the radius of the compactified dimension i.e $Y \sim r_\perp$. We assume the special case when the compactified transverse manifold is a $d_\perp$ dimensional torus. So, when the brane-antibrane separation Y is comparable to $r_\perp$, the potential receives contribution from the p-brane (or more clearly the antibrane) images. We study the potential in the covering space of the torus. The potential becomes of the form

$$V(\bar{r}) = A - \sum_i \frac{B}{|\bar{r} - \bar{r}_i|^{d_\perp - 2}}$$

$\bar{r}$ and $\bar{r}_i$ denote the positions of the p-branes and anti-branes in our $d_\perp$ dimensional coordinate space. The above sum is over the lattice sites occupied by the brane images and we have labelled them by i. It is easy to see that the first and second derivative of the potential vanishes when the antibrane is at the centre of a hyper-cubic cell (denoted by $\bar{r}_0$.

$$\left.\frac{\partial V}{\partial r_a}\right|_{\bar{r}=\bar{r}_0} = (d_\perp - 2) \sum_i \frac{B\,(r_0 - r_i)_a}{(\bar{r}_0 - \bar{r}_i)^{d_\perp}}$$

The first derivative vanishes due to the reflection symmetry of the lattice and this implied the net force on the antibrane at this point also vanishes. It is unusual and remarkable that the second derivative also vanishes.

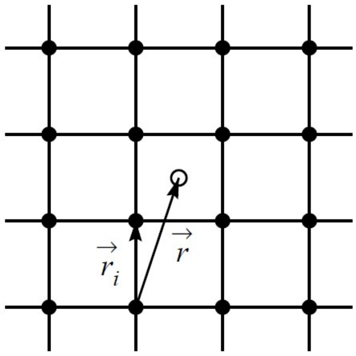

Figure 1.6: Square torus depicting the brane-antibrane positions. Filled up circle represents branes and unfilled one antibrane

$$\left.\frac{\partial^2 V}{\partial r_a \partial r_b}\right|_{\bar{r}=r_0} = (d_\perp - 2) \sum_i \frac{B}{(r_i - r_i)\, d_\perp} \left[\delta ab - d_\perp \, (r_0 - r_i)_a \right.$$
$$\left. \frac{(r_0 - r_i)_b}{(\bar{r}_0 - \bar{r}_i)^2} \right]$$

In [1], it is explicitly shown that the second derivative also vanishes for both the cases when a and b are equal and when they are not.

Let us now consider the motion of the antibrane when it is at the centre of a hypercubic cell (the figure below clarifies it).

We can expand the potential in terms of a power series of the displacement from the centre. Thus from the arguments presented above, we make the following considerations before expanding our potential in terms of small antibrane displacement z.

- Symmetry arguments makes all odd powers vanish.
- Quadratic term also vanishes as discussed above.
- Leading order contribution is thus from the quartic term.

Expanding out the potential in powers of z

$$V(z) = A - \frac{1}{4} C z^4 \tag{1.86}$$

where, A is the same as before and $C = \gamma M_{10}^{-2} T_p^2 V_\parallel r_\perp^{-(2+d_\perp)}$. Here, γ is an order 1 constant and $M_{10}^{-2} = \dfrac{V_\perp V_\parallel}{M_4^2}$. We can thus compute the slow-roll parameters again.

$$
\begin{aligned}
\eta &= M_4^2 \frac{V''}{V} \\
&= M_4^2 \frac{-3cz^2}{A - \frac{Cz^4}{4}} \\
&\sim \frac{-3CM_4^2 z^2}{2T_p V_{\|}} \\
&= -3\gamma (r_\perp)^{-2} z^2 \\
&= -3\gamma \left(\frac{z}{r_\perp}\right)^2
\end{aligned}
$$

Thus, the requirement of slow-roll parameter $\|\eta| << 1$ is guaranteed for sufficiently small z. Now, we address the question of colliding branes in the next section.

1.6.7 Brane Collisions and End of Inflation

In the final stages, when the branes and anti-branes are on the annihilation track, we need to make a few more additions other than just the separation. The amplitude for brane-antibrane annihilation develops a negative mode when distance between them shrinks to M_S^{-1} i.e Y$\sim$ M_S^{-1} signals the appearance of a tachyonic open string connecting the brane and the antibrane. Initially, the open string mode was massive, became massless at smaller separations and ultimately became tachyonic. Complete form of the potential is not known but limiting values are. It is conjectured to be of the approximate form

$$
V(Y,T) = \frac{1}{4\alpha'} \left(\frac{Y^2}{2\pi^2 \alpha'} - 1 \right) T^2 + cT^4 + \cdots . \tag{1.87}
$$

Few comments about the above potential:

- When $Y = Y_c = \sqrt{2\alpha'}\pi$, then mass term vanishes and when $Y < Y_c$, the modes become tachyonic.
- In this system of branes, at some point, a pair of branes will be at a separation close to the equilibrium point and gives rise to inflation by dominating the expansion of the universe and giving a naturally beautiful geometric picture for initial conditions for inflation.
- It is important to note that most of the times D_p and $\bar{D_p}$ branes will interact and annihilation will occur giving rise to p-2 branes and the process goes on until the Brandenberger-Vafa mechanism (discussed in appendix) permits. So, the BV mechanism leaves only D_3 in the picture and thus gives a natural explanation of our universe confined to a D_3 and inflation.

- Lastly and most importantly, the potential has two fields Y and T and thus has all the beautiful features of hybrid inflation built in. So, when branes are far apart, potential is parabolic and tachyon field is localized at the bottom of the potential at T=0. This is when inflation occurs. Once the separation reaches Y_c, the tachyon field is in the picture and it provides the necessary ingredients to end inflation.

So the story goes as follows. Large dimensional branes annihilate quite quickly leading to a population of D_3 branes only. Since due to the argument presented in the appendix, D_3 branes cannot be annihilated further in a superstring theory. We thus have 4 large dimensions and 6 compactified ones. Ultimately, we have the particular brane-antibrane collision which causes inflation. Our universe is that particular brane and our standard model lives in that brane.

1.6.8 Warped Brane-Antibrane Inflation

In this section, we briefly discuss the warped brane-antibrane inflationary scenario. In the original paper [1], they didn't consider moduli stabilization. Moduli are basically massless fields and would create huge problems if we do not produce a potential term for them. The problem with that is, in the moduli stabilization mechanism we have to turn on fluxes and fluxes can backreact on the metric and give us a deviation from the usual Calabi-Yau. Thus, the resulting compactification deviates from the usual Calabi-Yau (some torsion classes do not vanish) and becomes a "warped" Calabi-Yau

$$\mathrm{d}s^2 = \left(1 + \frac{e^{4\mathcal{A}}}{\mathcal{V}^{2/3}}\right)^{-1/2} \mathrm{d}s_4^2 + \left(1 + \frac{e^{4\mathcal{A}}}{\mathcal{V}^{2/3}}\right)^{1/2} \mathrm{d}s_{CY}^2 \tag{1.88}$$

This warped inflationary scenario was originally proposed in [35] (popularly called the KKLMMT proposal) to evade the problem of the eta parameter being order 1 after flux compactification. Let us denote $\mathcal{W} := \left(1 + e^{4\mathcal{A}\mathcal{V}}\mathcal{V}^{-2/3}\right)^{-1/2}$ which is called the warp factor. In the KKLMMT proposal, they suggested that the anti-D3 brane in this inflationary scenario which energetically prefers to sit at the tip of the warped throat and the D3 brane being BPS is free to move and experiences no position dependent forces. Combining brane tension term with the Coulomb interaction between a mobile D3 brane and the anti-D3 brane sitting in a warped environment gives the candidate brane-antibrane inflationary potential

$$V = 2T_3\left(e^{-4\rho}\mathcal{V}^{2/3}\right)\left(1 - \frac{27}{64\pi^2}\frac{2T_3\left(e^{-4\rho}\mathcal{V}^{2/3}\right)}{|\varphi|^4}\right) =: \Omega\left(1 - \frac{\mathfrak{b}\Omega}{|\varphi|^4}\right) \tag{1.89}$$

.

Here,

$$\Omega = \frac{\mathfrak{c} e^{-4\rho} M_4^4}{\mathcal{V}^{4/3}}, \quad \mathfrak{c} = \frac{(2\pi)^{11} g_s^3}{2} \quad \text{and} \quad \mathfrak{b} = \frac{27}{64\pi^2} \tag{1.90}$$

The slow-roll parameters for this motion in the regime $\mathfrak{b}\Omega \ll |\varphi|^4$ are

$$\varepsilon = \frac{M_4^2}{2}\left(\frac{V_\varphi}{V}\right)^2 \simeq 8\mathfrak{b}^2 \left(\frac{\Omega M_4}{|\varphi|^5}\right)^2 \quad \text{and} \quad \eta = \frac{M_4^2 V_{\varphi\varphi}}{V} \simeq -\frac{20\mathfrak{b}\Omega M_4^2}{|\varphi|^6} \tag{1.91}$$

Ideally, we can make the warp factor buried in Ω as small as we want and produce the required e-foldings. This is what makes this KKLMMT proposal much more attractive.

Chapter 2

Moduli Stabilization in Type IIB

As we saw in the section on string compactifications, the compactification on Calabi-Yau or in general compactifications give rise to a lot of moduli or scalar fields. These scalar fields in general need to have a potential or it would create unobserved modifications of gravity and fifth force problems. We call this process of generating potential for the moduli fields- moduli stabilization.

In section 1.3.2, we saw the action and fields in type IIB string theory. The type IIB compactification in CY_3 folds results in $N = 2$ supergravity in 4 spacetime dimensions. We then use orientifolds to reduce the amount of supersymmetry to $N = 1$. The number of Kähler moduli is $h^{11}(CY_3)$ where h is the Hodge number. But introducing orientifolds mods out half the amount so the Kähler moduli is τ_a where $a = 1, 2, 3...., h^{11}_+$. Also, the complex structure moduli U^α counted by $h^{2,1}_-(CY_3)$. Let us first focus on the moduli fields arising from the RR sector. The RR sector gives rise to fields C_p where $p = 0, 2, 4$. The C_0 piece is combined with the dilaton to form the axio-dilaton multiplet:

$$S = e^{-\phi} + \mathrm{i}C_0 \equiv \frac{1}{g_s} + \mathrm{i}C_0 \equiv s + \mathrm{i}C_0. \tag{2.1}$$

Other R-R and NS-NS p-form potentials can be expanded as follows:

$$B_2 = b^i v_i, \quad C_2 = c^i v_i, \quad C_4 = \rho_a \tilde{\mu}^a + \dots \tag{2.2}$$

Here, b^i, c^i and ρ_a are various axions and i=0,1,2,... h^{11}_-. We can describe the dynamics of the $\mathcal{N} = 1$ type IIB 4D effective supergravity theory can be described by using the following chiral variables (U^α, S, G^a, T_i) as:

$$\begin{aligned} &U^\alpha = v^\alpha - iu^\alpha, \quad S = e^{-\phi} + \mathrm{i}C_0 = s + \mathrm{i}C_0, \quad G^i = c^i + Sb^i, \\ &T_a = \left(\tau_a - \frac{s}{2}\hat{k}_{aij}b^i b^j\right) + \mathrm{i}\left(\rho_a + \hat{k}_{aij}c^i b^j + \frac{1}{2}C_0\hat{k}_{aij}b^i b^j\right), \end{aligned} \tag{2.3}$$

where $\tau_a = \frac{1}{2}k_{abc}t^bt^c$ is an Einstein frame 4-cycle volume. The tree-level Kähler potential K_{tree} can be expressed in the following manner which depends logarithmically on the various moduli fields as follows:

$$K_{tree} = -\ln\left(-i\int_X \Omega\wedge\bar{\Omega}\right) - \ln\big(S+\bar{S}\big) - 2\ln(\mathcal{V}), \tag{2.4}$$

Here, Ω is the holomorphic 3-form which gives rise to the complex structure moduli. Also, the internal volume $\mathcal{V}$ of the CY threefold can be given in terms of the two-cycle volume moduli as follows:

$$\mathcal{V} = \frac{1}{6}k_{abc}t^at^bt^c$$

where k_{abc} denotes the classical triple intersection numbers on X.

With the field strengths as discussed in section 1.3.2 and fluxes, we have the tree-level superpotential which was written by Gukov, Vafa and Witten (hence the name GVW) [38] as follows:

$$W_0 = \int_{CY_3} G_3\wedge\Omega\left(U^\alpha\right).$$

Combining all of them we can write the F-term superpotential into the familiar supergravity form as

$$V = e^K\left(\sum_{A,B}\left(D_AW\right)K^{A\bar{B}}\left(D_{\bar{B}}\bar{W}\right) - 3|W|^2\right), \tag{2.5}$$

where D_A is the Kähler covariant derivative which is $D_A = \partial_A + K_A$. The sum above runs over all the moduli fields in the theory. With all of these at hand, let us study the interesting moduli stabilization in type IIB string theory. This chapter is organized as follows. First, we talk about the challenges we face when stabilizing the moduli fields in type IIB string theory. We will then move on to the proposed solutions in literature including the KKLT construction, the Large Volume Scenario and a few possible perturbative stabilization proposals. We then built on this and present our perturbative stabilization mechanism in the following chapter.

2.1 Issues in Moduli Stabilization

As we see from GVW superpotential, it depends on the complex structure moduli U^{alpha} and the axio-dilaton S. Using the tree-level Kähler potential and superpotential, through the standard two-step procedure, one initially fixes the axio-dilaton S and the complex structure moduli U^α via the supersymmetric (flatness) conditions imposed to preserve the supersymmetry,

$$D_SW_0 = 0\,, \qquad D_{U^\alpha}W_0 = 0\,.$$

.

On the contrary, the Kähler moduli or the volume modulus remain unfixed as the superpotential does not depend on them. Thus, at the classical level the effective potential for Kähler moduli remain unfixed. This is the so called no-scale structure. To fix this issue and to generate the potential for the Kähler moduli we need to consider quantum corrections on top of the tree-level ones. As we know from supersymmetric renormalization theorems, the Kähler potential can receive both perturbative and non-perturbative corrections whereas the superpotential can only receive non-perturbative ones. The perturbative corrections are perturbative expansion of α' (higher derivative ones) and in string coupling g_s which are like the loop corrections. Perturbative analysis is done in this master thesis. Non-perturbative effects are used in models like KKLT and LVS and it is briefly discussed in the appendix as well.

2.2 KKLT Construction

The KKLT (Kachru, Kallosh, Linde and Trivedi) [34] proposal is one of the widely accepted solution of dS construction. In this section, I will briefly overview their main idea. We know that the F-term potential is

$$V = e^K \left(\sum_{A,B} K^{A\bar{B}} D_A W \overline{D_B W} - 3|W|^2 \right) \tag{2.6}$$

Here, as usual, $K^{A\bar{B}}$ is the inverse Kähler metric and D_A is the Kähler covariant derivative which is $D_A = \partial_A + K_A$. The sum runs over all moduli in the system. W is the familiar Gukov-Vafa-Witten (GVW) superpotential [38] which is of the form:

$$W = \int_M G_3 \wedge \Omega \tag{2.7}$$

G_3 is the imaginary self-dual flux and Ω is the nowhere vanishing unique holomorphic 3-form present in Calabi-Yau compactifications. The tree-level Kähler potential is of the form

$$K_{tree} = -\ln\left(-i \int_X \Omega \wedge \bar{\Omega}\right) - \ln\big(S + \bar{S}\big) - 2\ln(\mathcal{V}), \tag{2.8}$$

We consider only one Kähler moduli in our system with $T = \tau + i\rho$ and $\mathcal{V} \sim \tau^{3/2}$

$$K = -3\ln\tau \tag{2.9}$$

The complex-structure moduli and the dilaton is fixed by the tree-level results but the Kähler potential is not due to the no-scale cancellation

$$K^{\tau\bar{\tau}} D_\tau W \overline{D_\tau W} = 3|W|^2 \tag{2.10}$$

and the potential becomes of the form

$$V = e^K \left(\sum_{i,j} K^{i\bar{j}} D_i W \overline{D_j W} \right) \tag{2.11}$$

The non-perturbative correction to the super-potential is

$$W = W_0 + A\, e^{-aT} \tag{2.12}$$

For a supersymmetric minima, $D_T W = 0$.

$$\begin{aligned} 0 &= D_\tau \left(W_0 + Ae^{-a\tau} \right) \\ &= \partial_\tau \left(W_0 + Ae^{-a\tau} \right) + (\partial_\tau K) \left(W_0 + Ae^{-a\tau} \right) \\ &= aAe^{-a} + \frac{3}{2}\frac{1}{\tau} \left(W_0 + Ae^{-a} \right) \end{aligned} \tag{2.13}$$

Thus,

$$W_0 = -Ae^{-a\tau} \left(\frac{2}{3} a\tau + 1 \right) \tag{2.14}$$

Since, we are looking at a supersymmetric vacuum, the potential has an AdS minima at

$$\begin{aligned} V_{\text{AdS}} &= -3e^K W^2 \\ &= -\frac{a^2 A^2 e^{-2a\tau}}{6\tau} \end{aligned} \tag{2.15}$$

Since, the vacuum is AdS, we need to uplift the vacuum to dS. In their famous paper by KKLT, the uplifting of the vacuum is done via adding anti-branes. The antibrane $\bar{D}3$ breaks supersymmetry explicitly and give a positive contribution to the scalar potential. The anti-brane term is proportional to $\frac{1}{\tau^3}$ (this is what they did in their initial paper. In recent papers, terms proportional to $\frac{1}{\tau^2}$ were added). The potential is finally of the following form and is vacuum is dS.

$$V = \frac{aAe^{-a\tau}}{3\tau} \left(aA\tau e^{-a\tau} + 2W_0 + Ae^{-a\tau} \right) + \frac{D}{\tau^3} \tag{2.16}$$

One of the criticisms of the KKLT proposal is that whether the exponentially small W_0 is a reasonable assumption. Even though this seems unnatural but given the large number of flux vacua, some of them might accidentally realize small values. Another possible argument against KKLT comes in the form of the stability of the anti-branes (see [39]). Nevertheless, the KKLT proposal should be regarded as a road-map for a fully moduli stabilized scenario. Below we show a plot of AdS and uplifted dS vacua obtained from KKLT analysis. Here, real part of T is our τ.

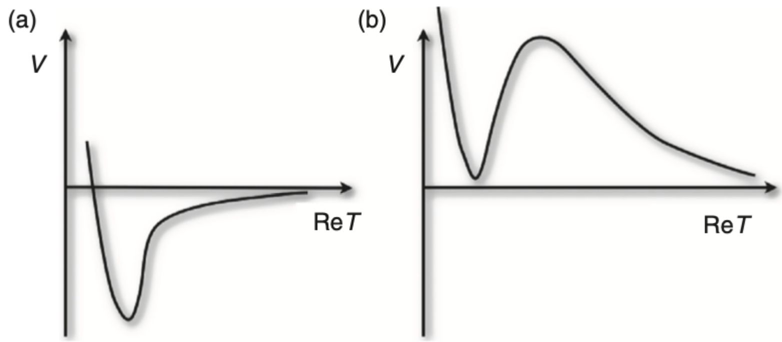

Figure 2.1: (a) The supersymmetric AdS vacuum, and (b) the uplifted dS vacuum from [40]

2.3 Large Volume Scenario

The large volume scenario or LVS ([13],[14]) does better in this regard which includes an α' correction to describe the moduli stabilization for generic W_0. It has been argued that in this type IIB setting the leading correction to Kähler potential can compete with non-perturbative corrections to the superpotential and give us very large volume moduli stabilization.

A toy model of LVS can be realized with two Kähler moduli τ_b and τ_s where τ_b controls the overall volume and τ_s describe geometric holes corresponding to blow-up modes. This is sometimes called a "Swiss-Cheese" CY. The Kähler potential for such a model takes the form

$$K = -2\ln\left(\mathcal{V} + \frac{\zeta}{2g_s^{3/2}}\right) \tag{2.17}$$

with the leading correction (at order α'^3) $\zeta > 0$ or positive. This forces the CY to have a negative Euler-characteristic. This correction is discussed in further detail in the next chapter. As for the superpotential, it includes a nonzero positive contribution W_0 in addition to the the non-perturbative corrections. Since, τ_b is much bigger than τ_s we can neglect the non-perturbative correction to it (or not even consider them at all). The superpotential takes the form

$$W = W_0 + A_s\, e^{-a_s T_s} \tag{2.18}$$

where the co-efficient a_s takes the value $\frac{2\pi}{N}$ for gaugino condensation and 2π for Euclidean D3 branes. After stabilizing the axion at $\mathrm{Im}(T_s) = \pi/a_s$, we obtain the following scalar potential considering $\mathcal{V} \simeq \tau_b^{3/2}$

$$V \simeq \frac{\lambda a_s^2 |A_s|^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{\mu a_s W_0 A_s \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\nu\xi |W_0|^2}{g_s^{3/2}\mathcal{V}^3} \tag{2.19}$$

Instead of going into a detailed quantitative analysis, let us look at the potential qualitatively and analyze where the potential might have a minima. For $\tau_b \leftarrow \infty$, the second term dominates and we have the potential approach zero. Moreover for smaller values of ν and τ_s the other terms dominate and thus we have a minima at negative value corresponding to an AdS minima. Like KKLT, we have to uplift this to get a dS minima as well. We make the following comments regarding the case:

- The LVS minima occurs between a competition between the α' correction and the non-perturbative effect.
- LVS does better than KKLT in two specific areas- the need for very fine tuning of W_0 and the fact that the α' correction is needed to be included which is expected from supergravity limit.
- The uplifting mechanisms in case of LVS are anti-branes, magnetized branes, additional α' corrections, T-branes [18] and etc. Ref. [23] lists all the cases where and how uplifting can be achieved.
- LVS is robust against further α' and g_s corrections ([16], [24], [25], [26], [29], [30]).
- Loop corrections were exploited for inflationary purposed by Cicoli et al. in [15] where they considered a K3-fibred CY and used the loop corrections to generate an inflationary potential.

2.4 Perturbative Stabilization

Perturbative stabilization is based on the idea that the Kähler moduli can be stabilized using perturbative effects only. As seen in the previous section, the KKLT and LVS scenarios use non-perturbative effects to stabilize the Kähler moduli. An interesting method of perturbative stabilization based on RG group re-summation technique was introduced by Burgess and Quevedo [32]. We will briefly present their idea here and finally comment about another perturbative stabilization model done in Type IIB/F-theory framework by Antoniadis et al. [27].

2.4.1 RG Induced Modulus Stabilization

The perturbative analysis by Burgess and Quevedo starts off by considering the accidental symmetries present in all of the ten-dimensional string derived supergravities

[33]. According to analysis presented in the paper by Burgess, Cicoli et al. [33], all ten-dimensional supergravities have two accidental scaling symmetries which are broken order by order in perturbation theory and 11-dimensional supergravity has one accidental symmetry. They used this accidental scale invariance to write the Kähler potential as follows

$$e^{-K/3} = \tau - k + \frac{h}{\tau} + \cdots \tag{2.20}$$

and the ellipses denote higher order terms in $1/\tau$. The analysis by Burgess and Quevedo rests on the idea that the factors k and h have logarithmic dependence on τ i.e $k(\ln\tau)$ and $h(\ln\tau)$. The logarithmic dependence were initially inspired by the results obtained in [37], [24], [25]. We will justify this later using renormalization group technique, but let us first see what happens if we do such an approximation. We know that the scalar potential is given as

$$V = e^K \left[K^{\bar{T}T}\overline{D_T W} D_T W - 3|W|^2\right] \tag{2.21}$$

. The Kähler covariant derivative becomes

$$D_T W = W_T + K_T W \simeq \left(-\frac{3}{\tau} + \cdots\right) w_0. \tag{2.22}$$

Plugging all of them into the scalar potential, we get

$$V \simeq -\frac{3k_{T\bar{T}}}{\mathcal{P}^2}|w_0|^2 + \cdots = \frac{3(k' - k'')}{\tau^4}|w_0|^2 + \mathcal{O}\left(\tau^{-5}\right) \tag{2.23}$$

where, $\mathcal{P} := e^{-K/3} = \tau - k + \cdots$ and the primes are derivatives with respect to the logarithm of τ. The leading contribution in the large τ regime comes from the $\frac{1}{\tau^4}$ term and let us consider that term for now.

$$V(\tau) \simeq \frac{U(\ln\tau)}{\tau^4} \tag{2.24}$$

where, $U(\ln\tau) = -3\tau^2 k_{T\bar{T}}|w_0|^2 = 3(k' - k'')|w_0|^2$.

Now, coming back to logarithmic dependence of k. They put forward the point that k can acquire the logarithmic dependence on τ through the running of some dimensionless coupling α_g with perturbative expansion of the form

$$k \simeq k_0 + k_1\alpha_g + \frac{k_2}{2}\alpha_g^2 + \cdots$$

which can be expressed with an renormalization group equation

$$\tau\frac{\mathrm{d}\alpha_g}{\mathrm{d}\tau} = \beta(\alpha_g) = b_1\alpha_g^2 + b_2\alpha_g^3 + \cdots \tag{2.25}$$

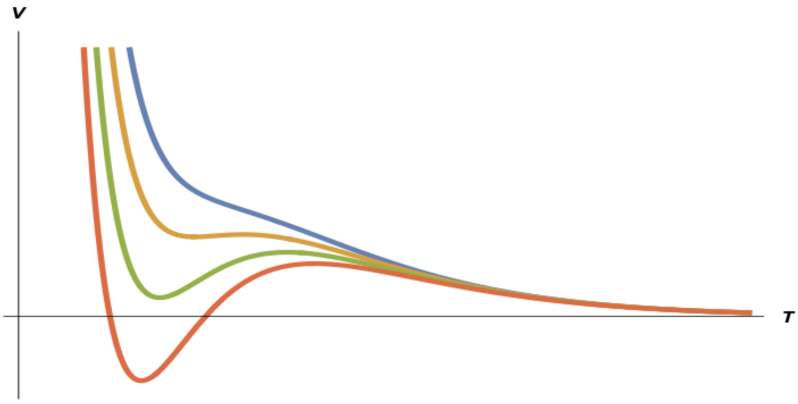

Figure 2.2: $V(\tau)$ vs τ plot as found in([32])

. For small α_g,

$$\alpha_g(\tau) = \frac{\alpha_{g0}}{1 - b_1 \alpha_{g0} \ln \tau} \tag{2.26}$$

For a particular integration constant, denoted as $\alpha_{g0} = \alpha_g(\tau = 1)$, we need to focus on the $\ln \tau$ dependence. It is essential to note that the derivation of this dependence only disregards additional terms involving powers of α_g present in equation. As a result, for large values of τ, this relationship remains valid without any restrictions imposed by higher-order terms involving $\alpha_g^2 \ln \tau$. This renormalization-group re-summation is crucial because it allows us to have confidence in the minima of the potential that occur when $\ln \tau$ is on the order of $1/\alpha_g$.

Now, let us get to the main point. To compute the derivatives of k with respect to τ, we obtain expressions like $k' = (k_1 + k_2\alpha_g + \cdots)\,\beta(\alpha_g)$. Similarly, we can calculate k'' in a similar manner. Employing these expressions results in the following expression for U:

$$U \simeq U_1\alpha_g^2 - U_2\alpha_g^3 + U_3\alpha_g^4 + \cdots \tag{2.27}$$

This equation represents an approximation for U with terms involving different powers of the parameter α_g. The coefficients take the values $U_1 = 3k_1 b_1 |w_0|^2$, $U_2 = 3(k_1 b_2 + k_2 b_1 - k_1 b_1^2)$ and so on. They showed that if these coefficients satisfy a mild hierarchy i.e

$$\left|\frac{U_1}{U_2}\right| \sim \left|\frac{U_2}{U_3}\right| \sim \mathcal{O}(\epsilon), \tag{2.28}$$

we can expect a minima. Here, ϵ is much less than 1. They obtained dS and AdS minima for different values of k_1, k_2 and k_3.

2.4.2 Perturbative Stabilization based on Logarithmic Corrections

Now, we will present a model of perturbative stabilization as put forward by Antoniadis et al. in [25] based on their previous work [26],[27] and [28]. Consider a correction to the Kähler potential as follows

$$K = -2\ln\left(\tau^{\frac{3}{2}} + \beta f[\tau]\right) \tag{2.29}$$

where, β is some co-efficient and $f[\tau]$ is some function of the Kähler modulus. $\beta f[\tau]$ is assumed to be much smaller than the $\tau^{3/2}$. The superpotential with no non-perturbative corrections is again denoted by W_0 and thus our scalar potential stands:

$$V_F(\tau) = \frac{\beta W_0^2}{2\tau^{9/2}}\left(3f[\tau] - 4\tau f'[\tau] + 4\tau^2 f''[\tau]\right) + O\left(\eta^2\right) \tag{2.30}$$

Antoniadis et al. obtained logarithmic corrections at $\alpha'3$ but next order in g_s whose results we will summarise in the next chapter. For now, let us go forward with the assumption that:

$$f[\tau] = \ln(\tau) \tag{2.31}$$

Inserting a small scale parameter μ inside the log correction term gives $f[\tau] = \ln\left(\mu^4\tau\right)$. This is similar to adding a constant to the Kähler potential. Now,

$$K = -2\ln\left(\mathcal{V} + \xi + \beta\ln(\mathcal{V}) + O\left(\frac{1}{\mathcal{V}}\right)\right) = -2\ln\left(\mathcal{V} + \beta\ln\left(\mu^6\mathcal{V}\right) + O\left(\frac{1}{\mathcal{V}}\right)\right) \tag{2.32}$$

where the parameter $\mu \equiv e^{\xi/6\beta}$. In the case of $\beta < 0$, we can find a minima of the potential in terms of $\mathcal{V}$, which in the large volume limit is:

$$\mathcal{V}_{\text{min}} = e^{13/3}/\mu^6 \quad ; \quad V_F^{\text{min}} = \frac{\beta \mathcal{W}_0^2}{3\mathcal{V}_{\text{min}}^3} \tag{2.33}$$

Since, the minima exists for $\beta < 0$ and from the potential we can see that it assumes a negative value at the minima hence it is an AdS minima. Antoniadis and collaborators then used D-term contributions coming from magnetized fluxes which is proportional to $1/(\tau^3)$ to uplift the potential to dS. The contribution is as follows

$$V_{D_i} = \frac{d_i}{\tau_i^3} \tag{2.34}$$

The sum is over three Kähler moduli. The correction term arise from three intersecting D7 branes and thus τ_i is the corresponding world-volume modulus with the constants d_i being positive. With $\mathcal{V} = (\tau_1\tau_2\tau_3)^{1/2}$, the scalar potential is as follow

$$V_{\text{tot}} = \frac{3\beta W_0^2}{\mathcal{V}^3}\left(\ln\left(\mathcal{V}\mu^6\right) - 4\right) + \frac{d_1}{\tau_1^3} + \frac{d_2}{\tau_2^3} + \frac{d_3\tau_1^3\tau_2^3}{\mathcal{V}^6} \tag{2.35}$$

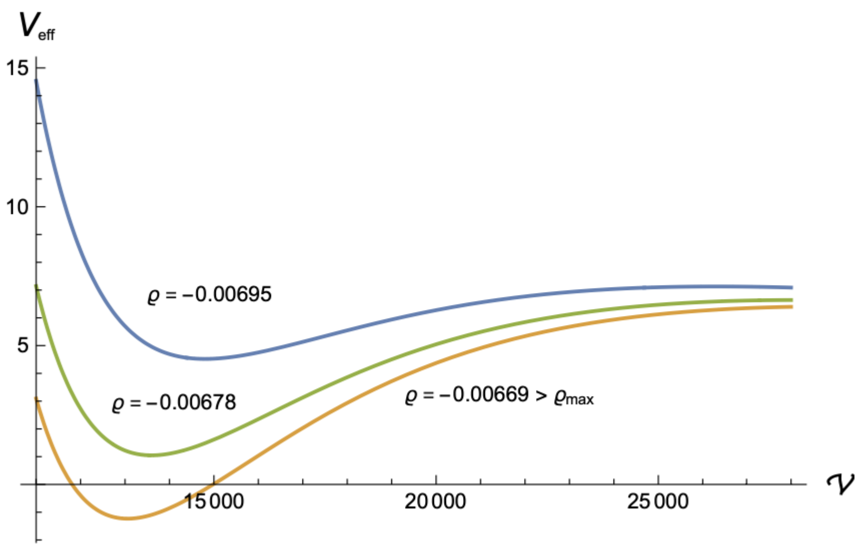

Figure 2.3: Plot of V vs $\mathcal{V}$

Minimising with respect to τ_1 and τ_2 and fixing:

$$\frac{\tau_i}{\tau_j} = \left(\frac{d_i}{d_j}\right)^{1/3},$$

and the scalar potential becomes:

$$V_{\text{tot}} = \frac{3\beta\mathcal{W}_0^2}{\mathcal{V}^3}\left(\ln\left(\mathcal{V}\mu^6\right) - 4\right) + 3\frac{d}{\mathcal{V}^2} \quad ; \quad d = (d_1 d_2 d_3)^{1/3}. \tag{2.36}$$

Now, minimizing with respect to volume with the requirement,

$$V_{\text{tot}}^{\text{min}} = \frac{3\eta\mathcal{W}_0^2}{\mathcal{V}_{\text{min}}^3} + \frac{3d}{\mathcal{V}_{\text{min}}^2} > 0 \tag{2.37}$$

The plot gives us a better look at their calculations. Here, $\varrho = \frac{d}{\eta\mu^6\mathcal{W}_0^2}$ and only the lowest curve corresponds to an AdS minima.

This model proposed by Antoniadis and collaborators does better than non-perturbative proposals as we naively expect perturbative corrections to be stronger than non-perturbative ones. As we will see in our perturbative model, the need to include D-term or even non-perturbative corrections is not necessary at all.

2.4.3 Perturbative Stabilization based on Loop Corrections

Based on the loop corrections [44] (which will be discussed in detail in the following section), the authors proposed a volume stabilization method in [45]. We will shortly discuss this in this section. First, the consider the tree-level no scale Kähler potential:

$$K(T,\bar{T}) = -3\ln\big(T+\bar{T}\big) \tag{2.38}$$

In the realm of string theory's perturbation theory, the emergence of the no-scale Kähler potential linked to the volume modulus T can be traced back to the examination of sphere diagrams. These diagrams are expanded to their primary order in α'. This Kähler potential, however, undergoes multiple adjustments, some of which become evident even at the disk level, corresponding to the contributions at the tree-level originating from open strings. More specifically, at the disk level, it becomes evident that modifications occur within the logarithmic argument. These modifications are driven by a function reliant on the collective open string scalars, denoted as A and the complex structure moduli identified as U.

$$\begin{aligned} K(T,\bar{T}) &= -\ln[T+\bar{T}] \\ \xrightarrow{\text{disk}} K(T,\bar{T},A,\bar{A},U,\bar{U}) &= -\ln[T+\bar{T}+f(A,\bar{A},U,\bar{U})], \end{aligned}$$

This can be considered the classical tree-level Kähler potential at leading order in α'.

Beyond leading order in α', there is a correction known at order α'^3 (which is the BBHL piece). Beyond leading order in the string coupling, the corrections in are one-loop corrections from Klein bottle, annulus, and Möbius strip diagrams, that are typically suppressed by $(T+\bar{T})^{-1}$ and $(T+\bar{T})^{-2}$, with coefficients that depend on the complex structure and axio-dilaton as well as on open string moduli. These corrections ruin the no-scale structure and "hopefully" generate a potential for the volume moduli. Putting everything together,

$$\begin{aligned} K =& -3\ln\left[T+\bar{T}+f_1(A,\bar{A},U,\bar{U})\right] + \frac{1}{T+\bar{T}}\left[\frac{f_2(A,A,U,U)}{S+\bar{S}}+\dots\right] \\ &+ \frac{1}{(T+\bar{T})^{3/2}}\left[\alpha(S+\bar{S})^{3/2}+\dots\right] + \frac{1}{(T+\bar{T})^2}\left[f_3(A,\bar{A},U,\bar{U})+\dots\right]+\dots, \end{aligned} \tag{2.39}$$

The potential for U and S is generated by GVW superpotential, so writing the F-term superpotential only in terms of A and T,

$$V = e^K\left(K^{\bar{T}T}K_{\bar{T}}K_T + K^{\bar{T}A}K_{\bar{T}}K_A + K^{\bar{A}T}K_{\bar{A}}K_T + K^{\bar{A}A}K_{\bar{A}}K_A - 3\right)|W|^2 \tag{2.40}$$

Taking these points into consideration, the scalar potential that arises when we establish $D_S W = D_U W = 0$, while simultaneously having $D_\rho W \neq 0 \neq D_A W$ (resulting in spontaneous supersymmetry breaking),

$$V = \frac{1}{(T+\bar{T})^3}\left[\frac{c_1}{(T+\bar{T})^{3/2}} + \frac{c_2}{(T+\bar{T})^2}+\dots\right]|W|^2$$

The potential is then minimized with respect to A and T, and to obtain a minima at large volumes, we require

$$\left|\frac{c_2}{c_1}\right| \gg 1.$$

In [45], they then considered the specific case of $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Assuming $f(A, \bar{A}, U, \bar{U}) = 0$ at the minimum, they found at perturbative minima at

$$\mathrm{Re}(T)|_{\min} \sim \frac{\left(\mathcal{E}_2^{\mathrm{D7}}(0,U)\right)^2}{(\mathrm{Re}(S))^3} \tag{2.41}$$

The value of the potential at the Minima (which they checked for certain numerical values) is negative corresponding to a AdS minima.

Chapter 3

A Model of Perturbative Stabilization

This chapter contains the original work of this master thesis. In the first chapter we will discuss all the perturbative corrections that have been computed in literature. In the following section, we incorporate them into our Kähler potential and F-term scalar potential and find the corresponding minima. We present two cases, one in which we do not consider moduli redefinitions [24] and in the one we do. We find the minima and in the final section we consider the warped brane-antibrane scenario and see if the slow-roll conditions are satisfied. In this chapter, we also follow the notations and identities put forward in the beginning of chapter 2. We will tend to use everything in terms of Kähler moduli τ, inverse string coupling s instead of their chiral superfields multiplets. This was already done when we discussed KKLT, LVS and the perturbative models.

3.1 All Known Perturbative Corrections

3.1.1 BBHL α'^3 Correction

The prepotential for the Kähler deformations receives worldsheet corrections as computed by Candelas et al. [47]. The leading correction to the Kähler potential comes from the $\mathcal{O}(\alpha'^3)$ R^4 term and can be captured by a shift ζ in the volume $\mathcal{V}$. This was shown by Becker, Becker, Hack and Louis [42] and hence the name BBHL. We can derive the equation of motion for the dilaton to order α'^3 from the 10d action:

$$S = \int d^{10}x\sqrt{-g^{10}}e^{2\phi}\left(R + 4(\partial\phi)^2 + \alpha'^3 J_0\right) \tag{3.1}$$

which has a solution $\phi = \phi_0 + \zeta(3)Q/16$. Here, Q is defined through the 6d Euler integrand $\chi = \int_{CY_3} d^6x\sqrt{g}Q$. When we compactify with 6 compact dimensions, α'^3-

corrections are incorporated in the redefinition of 4D dilaton:

$$e^{-2\phi_4} = e^{-\frac{1}{2}\phi_{10}}(\mathcal{V} + \hat{\zeta}/2).$$

Here, $\mathcal{V}$ is the Calabi-Yau volume in the Einstein frame and the correction $\hat{\zeta}$ thus appears as a shift in the overall volume. The volume and the correction term is given by:

$$\mathcal{V} = \frac{1}{3!}\kappa_{ijk}t^i t^j t^k, \quad \hat{\zeta} = -\frac{\zeta(3)\chi(X)}{2(2\pi)^3 g_s^{3/2}}. \tag{3.2}$$

Note that we can relate the Einstein-frame volumes with corresponding string-frame quantities as $\mathcal{V} = g_s^{-3/2}\mathcal{V}_s$ and $t^i = g_s^{-1/2}t^i_s$, where subscript s denotes the string frame quantities. The tree-level Kähler potential is

$$K = -2\ln\mathcal{V} \tag{3.3}$$

The volume along with α' correction shifts as follows:

$$\mathcal{V} \to \mathcal{V} + \frac{\hat{\zeta}}{2} \equiv \mathcal{V} + \frac{\zeta}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2} \equiv \mathcal{V} + \frac{\zeta}{2g_s^{3/2}} \tag{3.4}$$

Thus, our Kähler potential with α'^3 correction is

$$K = -2\ln(\mathcal{V} + \frac{\zeta}{2g_s^{3/2}}) \tag{3.5}$$

In terms of the 4-cycles τ, the Kähler potential is ($\mathcal{V} \sim \tau^{3/2}$)

$$K = -3\ln\tau - \frac{\zeta s^{3/2}}{\tau^{3/2}} \tag{3.6}$$

Lastly, this is the correction which was used in LVS models [13],[14] to generate a minima for the τ moduli.

3.1.2 Higher-Derivative F^4-corrections

The BBHL correction appears at two-derivative level by entering the Kähler potential. There are additional α'^3 corrections coming from higher derivative $\mathcal{O}(F^4)$ effects. It was computed by Westphal et al. in [46].

They are argued to be generic for a given CY orientifold model (although for certain CY with specific topologies, they might be absent). The scalar potential can be expressed in the following simple form:

$$V_{F^4} = -\frac{g_s^2}{4}\frac{\lambda|W_0|^4}{g_s^{3/2}\mathcal{V}^4}\Pi_i t^i, \tag{3.7}$$

where the t^i 's are 2-cycle volumes, λ is a combinatorial factor and the Π_i 's are topological numbers, also called second Chern numbers, defined as:

$$\Pi_i = \int_{D_i} c_2(X)$$

We can sandwich all of these combinatorial factors and topological numbers into a single constant and write the scalar potential as

$$V_{F^4} = -\frac{g_s^2}{4}\frac{\lambda\,|W_0|^4}{g_s^{3/2}\mathcal{V}^4}\Pi_i t^i \equiv \frac{C}{s^2\tau^{11/2}}. \tag{3.8}$$

These corrections were exploited for inflation by Cicoli et al. in [48] and by Pramod and Leontaris for moduli stabilization in [29].

3.1.3 Perturbative String Loop Corrections

String loop corrections (corrections in g_s^2) were computed by Berg, Haack and Körs in [44] for toroidal orientifolds. They computed two corrections which goes as:

$$K = -2\ln(\mathcal{V}) - \frac{\zeta s^{3/2}}{\mathcal{V}} + \sum_{i=1}^{3}\frac{\mathcal{E}_i^{(KK)}(U,\bar{U})}{4\tau_i s} + \sum_{i\neq j\neq k}^{3}\frac{\mathcal{E}_k^{(W)}(U,\bar{U})}{4\tau_i\tau_j}. \tag{3.9}$$

Here, alongside the tree-level term we have written the α'^3 corrections as well. Here, U denotes the complex structure moduli, s is the real part of the axio-dilaton, and the two coefficients are both functions of the complex structure moduli. Let us explain how both corrections arise for toroidal case and then we make a conjecture about the CY case based on reasonable assumptions.

The first loop correction with superscript (KK) occurs at α'^2 and it is due to the exchange of Kaluza-Klein (KK) modes between D7-branes (or O7-planes) and D3-branes (or O3-planes, both localized in the internal space). The toroidal orientifold case computed in [44] and presented for moduli stabilization purposes in [43] , the correction is suppressed by the dilaton ($s = g_s^{-1}$ in our analysis) and a Kähler modulus τ_i which is related to the volume of the 4-cycle wrapped by the D7-branes (or O7-planes, respectively). Figure 3.1 gives us a look at how these terms might arise.

The second loop correction is due to the exchange of winding modes (or winding strings) between intersecting stacks of D7-branes (or between intersecting D7-branes and O7-planes). The figure 3.2 is an example of winding corrections being present for two Kähler moduli. These corrections arise from intersection between small 4 -cycle τ_s and the large 4-cycle τ_b. In the case of toroidal orientifolds, the term is suppressed by two Kähler moduli.

Now, let us motivate the case for CY. In the toroidal case, the KK correction term is suppressed by one Kähler modulus and it might not be the case for CY. This is because,

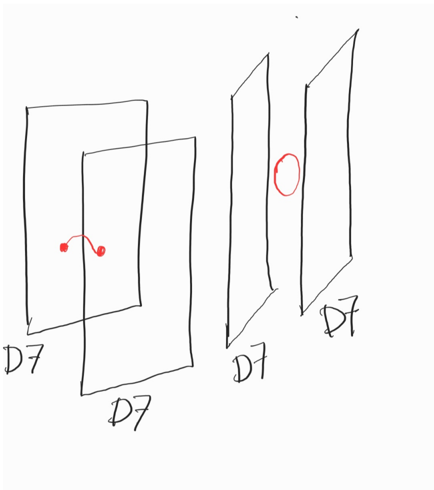

Figure 3.1: Cartoon diagram for the exchange of KK modes between two D7-branes which gives us the KK correction term.

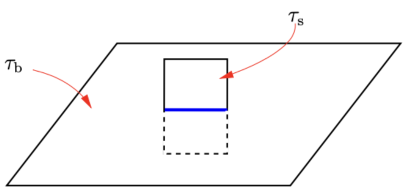

Figure 3.2: Winding correction for two moduli [43].

toroidal orientifolds are special in the sense that their volume is simply $\mathcal{V} \sim t_i \tau_i$ with no sum over i. This motivates us to conjecture that in the CY case, the term might be suppressed by a factor of $\mathcal{V}$ instead. Thus, following [44], we can conjecture

$$\begin{aligned} \text{Calabi-Yau:} \quad \Delta K_{g_s} &\overset{!}{\sim} \frac{\sum_{\mathrm{KK}} m_{\mathrm{KK}}^{-2}}{s\mathcal{V}} \sim \sum_{\mathfrak{a}} \frac{g_{\mathrm{K}}^{\mathfrak{a}}(t,s)\,\mathcal{E}_{\mathfrak{a}}^{(K)}}{S_1\mathcal{V}} \\ \text{and} \quad & \frac{\sum_{\mathrm{W}} m_{\mathrm{W}}^{-2}}{\mathcal{V}} \sim \sum_{\mathfrak{q}} \frac{g_{\mathrm{W}}^{\mathfrak{q}}(t,s)\,\mathcal{E}_{\mathfrak{q}}^{(W)}}{\mathcal{V}} \end{aligned} \tag{3.10}$$

Here, the sums run over KK and winding states, respectively. $\mathcal{E}^{(K)}$ and $\mathcal{E}^{(W)}$ are some unknown functions of the complex structure and open string moduli, t stands for the 2-cycle volumes and the functions $g_{\mathrm{K}}(t,s)$ and $g_{\mathrm{W}}(t,s)$ determine the scaling of the KK and winding mode masses with the Kähler moduli and the dilaton. In the paper by Cicoli et al. [16], they considered these corrections in the context of CYs and its implications for LVS. Let us now the form of the corrections they used and make it relevant for our case.

$$\delta K_{(g_s)}^{KK} \sim \sum_{i=1}^{h_{1,1}} g(U) \frac{\left(a_l t^l\right) e^{\varphi}}{\mathcal{V}} = \sum_{i=1}^{h_{1,1}} \frac{\mathcal{C}_i^{KK}(U,\bar{U})\left(a_{il} t^l\right)}{\mathrm{Re}(S)\mathcal{V}} \tag{3.11}$$

$$\delta K_{(g_s)}^{KK} \sim \frac{A t^1}{s\mathcal{V}} \tag{3.12}$$

where we have re-defined all the constant terms as A. Now, we can write the volume as $\mathcal{V} = (1/6)k_{ijk}t_i t_j t_k$ and 4-cycle as $\tau_i = \frac{\partial \mathcal{V}}{\partial t_i}$. This tells us that $t \sim \tau^{1/2}$ and $\mathcal{V} \sim \tau^{3/2}$. Plugging all this above we get,

$$\delta K_{(g_s)}^{KK} \sim \frac{A}{s\tau} \tag{3.13}$$

Since we only consider one Kähler modulus, we will use the above form in our Kähler potential.

Similarly, the winding corrections can be written as

$$\delta K_{(g_s)}^{W} \sim \sum_{i=1}^{h_{1,1}} \frac{\mathcal{C}_i^{W}(U,\bar{U})}{\left(a_{il} t^l\right)\mathcal{V}} \tag{3.14}$$

and redefining all constants as B, we write the form which we use for our calculations

$$\delta K_{(g_s)}^{W} \sim \frac{B}{\tau^2} \tag{3.15}$$

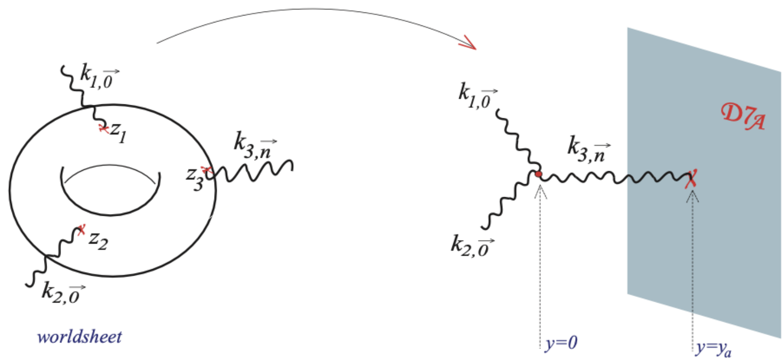

Figure 3.3: Logarithmic corrections from [30]

3.1.4 Logarithmic Corrections

In this section, we present another one loop at $\mathcal{O}(\alpha'^3, g_s^2)$ correction which occurs at strong gravity and which we will be incorporating in our Kähler potential. These corrections arise when we consider higher order curvature terms arising at the fourth power of R into our 10 dimensional action. Including this R^4 term in the action,

$$\mathcal{S}_{10D}^{IIB} \supset \frac{1}{(2\pi)^7\alpha'^4}\int_{M_{10}} e^{-2\phi_{10}} R_{(10)} - \frac{6}{(2\pi)^7\alpha'}\int_{M_{10}} \left(-2\zeta(3)e^{-2\phi_{10}} - 4\zeta(2)\right) R^4_{(10)} \wedge e^2. \tag{3.16}$$

Here, $R_{10}$is the 10 dimensional Ricci scalar and ϕ_{10} is the 10 dimensional dilaton. Compactifying on CY_3, we get a tree-level and one loop generated Einstein-Hilbert action

$$\mathcal{S}_{\text{grav}} = \frac{1}{(2\pi)^7\alpha'^4}\int_{M_4\times \mathrm{CY}_3} e^{-2\phi_{10}} R_{(10)} + \frac{\chi}{(2\pi)^4\alpha'}\int_{M_4} \left(2\zeta(3)e^{-2\phi_{10}} + 4\zeta(2)\right) R_{(4)} \tag{3.17}$$

Here, M_4 are the four non-compact dimensions and the Euler character depends on three powers of R as follows:

$$\chi = \frac{3}{4\pi^3}\int_{\mathrm{CY}_3} R\wedge R\wedge R \tag{3.18}$$

From this analysis, ref. [25] computed a non-zero contribution at 1 loop from three graviton scattering. Figure 3.3 depicts the corrections at 1 loop induced by three graviton scattering on a stack of three D7-branes.

As discussed in the previous chapter on perturbative stabilization, this logarithmic correction was used to stabilize Kähler moduli perturbatively. Including this correction alongside BBHL, the Kähler potential stands:

$$K = -2\ln\left(\mathcal{V} + \frac{\zeta s^{3/2}}{2} + D s^{-1/2}\ln\mathcal{V}\right) \tag{3.19}$$

In terms of τ,

$$K = -3\ln(\tau) - \frac{1}{\tau^{3/2}}\left[\zeta + \frac{3D\ln\tau}{\sqrt{s}}\right] \tag{3.20}$$

Here, D is the co-efficient of the log correction term.

3.1.5 Moduli Redefinitions

Until now, we studied the loop and α' corrections. In this section, we present the work by Conlon and Pedro [24] where the considered field redefinitions that can occur at the loop level and which can modify the moduli and thus the Kähler potential. They studied how this might effect LVS and we will briefly discuss that in this section as well.

In string compactifications, it is a well-established characteristic that the definitions of moduli undergo modifications at the 1-loop level. However, it is important to note that these redefinitions do not take place universally across all scenarios.

$$\tau_{\text{new}} = \tau_{\text{old}} - \alpha\ln(\mathcal{V}) \tag{3.21}$$

The redefined moduli depends on the old moduli and some logarithmic correction. This idea stems from the running of gauge couplings in string theory [41]. With this moduli redefinition, let us see what happens in LVS when we consider them. This discussion is heavily based on the paper [24]. Let us choose our familiar Swiss-cheese CY with two moduli τ_b and τ_s and do a redefinition of τ_s.

$$K = -2\ln\left[\frac{1}{\lambda}\left(\tau_b^{3/2} - \left[\tau_s - \alpha\ln(\mathcal{V})\right]^{3/2}\right) + \frac{\zeta s^{3/2}}{2}\right] \tag{3.22}$$

The F-term scalar potential is,

$$\begin{aligned} V =& \frac{3}{4}|W_0|^2\frac{\zeta\lambda^3}{g_s^{3/2}}\left(1 - \frac{6\alpha\sqrt{\tau_s}}{\zeta\lambda}\right)\frac{1}{\tau_b^{9/2}} + \frac{8}{3}\frac{a^2|A|^2\lambda^2 e^{-2a\tau_s}}{\tau_b^{3/2}}\sqrt{\tau_s}\left(1 - \frac{\alpha}{2\tau_s}\ln(\mathcal{V})\right) - \\ & - \frac{\lambda^2}{\tau_b^3}2a\tau_s\left(\bar{A}W_0 + A\bar{W}_0\right)e^{-a\tau_s}\left(1 - \frac{\alpha}{\tau_s}\ln(\mathcal{V})\right) \end{aligned} \tag{3.23}$$

Note that, W_0 is the superpotential and the exponential suppression term of the moduli is the non-perturbative correction term present in LVS. Conlon and Pedro found out that for small values of $\alpha\,(0 < \alpha < 3\times10^{-3})$ the LVS minima holds. We can do the same exercise of redefining the τ_b and the Kähler potential would as follows:

$$K = -2\ln\left[\frac{1}{\lambda}\left(\left[\tau_b - \beta\ln(\mathcal{V})\right]^{3/2} - \tau_s^{3/2}\right) + \frac{\zeta s^{3/2}}{2}\right] \tag{3.24}$$

In this case, the scalar potential becomes:

$$\begin{aligned} V =& \frac{3\left|W_0\right|^2\lambda^3\zeta}{4g_s^{3/2}\tau_b^{9/2}} + \frac{9\left|W_0\right|^2\beta\lambda^2}{2\tau_b^4} + \frac{8a^2|A|^2\lambda^2\sqrt{\tau_s}e^{-2a\tau_s}}{3\tau_b^{3/2}} \\ &- \frac{a\lambda^2\tau_s e^{-2a\tau_s}\left(6\beta\ln\mathcal{V} + 3\beta + 2\tau_b\right)\left(A\left(\bar{A} + e^{a\tau_s}\bar{W}_0\right) + \text{ c.c. }\right)}{\tau_b^4} \end{aligned} \tag{3.25}$$

In this case, the allowed range of of the redefining constant β ($\tau_b^{new} = \tau_b^{old} - \beta\ln\mathcal{V}$) is much smaller. LVS minima do not survive if the coefficient β takes on larger values.

3.2 Perturbative Stabilization of Kähler Moduli

Let us incorporate all the corrections to the Kähler potential as discussed in the previous section. The Kähler potential becomes

$$K = -3\ln(\tau) + \frac{A}{s\tau} - \frac{1}{\tau^{3/2}}\left[\zeta + \frac{3D\ln\tau}{\sqrt{s}}\right] + \frac{B}{\tau^2} \tag{3.26}$$

Here,

- A is the coefficient of the string loop correction coming from KK modes.
- B is the coefficient of the string loop correction coming from winding modes.
- D is the coefficient of logarithmic loop corrections at order $\mathcal{O}(\alpha'^3 g_s^2)$ which are present only when there is high curvature localised in the extra dimensions. D is proportional to the D7-brane tension, hence $D = \hat{D}s$.
- We will denote the coefficient of the order F^4 term in the scalar potential as C.
- Note that ζ scales as $\zeta = \hat{\zeta}s^{3/2}$

We will consider two scenarios in the subsequent discussion. In the first, we will not consider modulus redefinition but in the next we will consider modulus redefinition.

3.2.1 No Modulus Redefinition

We take the Kähler potential above and compute the scalar potential.

$$K_\tau = \frac{1}{2}\left(-\frac{A}{s\tau^2} - \frac{2B}{\tau^3} + \frac{3\zeta}{2\tau^{5/2}} - \frac{3D}{\sqrt{s}\tau^{5/2}} + \frac{9D\ln(\tau)}{2\sqrt{s}\tau^{5/2}} - \frac{3}{\tau}\right) \tag{3.27}$$

The Kähler metric is

$$K_{\tau\bar{\tau}} = \frac{1}{4}\left(\frac{2A}{s\tau^3} + \frac{6B}{\tau^4} - \frac{15\zeta}{4\tau^{7/2}} + \frac{12D}{\sqrt{s}\tau^{7/2}} - \frac{45D\ln(\tau)}{4\sqrt{s}\tau^{7/2}} + \frac{3}{\tau^2}\right) \tag{3.28}$$

The inverse Kähler metric is

$$K^{\tau\bar{\tau}} = \frac{4}{\frac{2A}{s\tau^3} + \frac{6B}{\tau^4} - \frac{15\zeta}{4\tau^{7/2}} + \frac{12}{\sqrt{s}\tau^{7/2}} - \frac{45\ln(\tau)}{4\sqrt{s}\tau^{7/2}} + \frac{3}{\tau^2}} \tag{3.29}$$

Incorporating the order F^4 term which is

$$V_{F^4} = -\frac{g_s^2}{4}\frac{\lambda\,|W_0|^4}{g_s^{3/2}V^4}\Pi_i t^i \equiv \frac{C}{s^2\tau^{11/2}} \tag{3.30}$$

The F-term scalar potential is

$$\begin{aligned} V &= \frac{3\,(\zeta\sqrt{s}+3D\ln(\tau)-8D)}{4\sqrt{s}(\tau)^{9/2}} + \frac{A^2-6Bs^2}{3s^2\tau^5} - \frac{3A\,(\zeta s+9D\ln(\tau)\sqrt{s})+4C}{4s^2\tau^{11/2}} \\ &+ \frac{16A^3+96ABs^2+810\zeta s^{5/2}\ln(\tau)-1080\zeta s^{5/2}+135\zeta^2 s^3+1215 s^2\ln^2(\tau)}{144s^3\tau^6} \\ &+ \frac{-3240s^2\ln(\tau)+3888s^2}{144s^3\tau^6} + O\left(\left(\frac{1}{\tau}\right)^{13/2}\right) \end{aligned} \tag{3.31}$$

If we set all other coefficients corresponding to perturbative corrections A, B, C, D and ζ to zero, we recover our usual no scale result. In the large τ case, the leading contribution to the scalar potential is

$$V \simeq \frac{3\,(\zeta\sqrt{s}+3D\ln(\tau)-8D)}{4\sqrt{s}(\tau)^{9/2}} \tag{3.32}$$

3.2.2 With Modulus redefinition

Let us again consider the Kähler potential as before but label our Kähler modulus as τ_{old} as we will do a modulus redefinition. LVS with moduli redefinition was considered in Conlon and Pedro in [24] and it was proved to be robust with moduli redefinition as well.

$$K = -3\ln(\tau_{\text{old}}) + \frac{A}{s\tau_{\text{old}}} - \frac{\zeta}{\tau_{\text{old}}^{3/2}} - \frac{3D}{\sqrt{s}\tau_{\text{old}}^{3/2}}\ln(\tau_{\text{old}}) + \frac{B}{\tau_{\text{old}}^2} \tag{3.33}$$

Now, let us do a modulus redefinition $\tau_{old} = \tau + \alpha\ln\tau$. Here, τ is the redefined modulus and we will base our calculations based on this. Our Kähler potential attains the following form:

$$\begin{aligned} K &= -3\ln(\alpha\ln(\tau)+\tau) - \frac{\zeta}{(\alpha\ln(\tau)+\tau)^{3/2}} + \frac{A}{s(\alpha\ln(\tau)+\tau)} \\ &- \frac{3D\ln(\alpha\ln(\tau)+\tau)}{\sqrt{s}(\alpha\ln(\tau)+\tau)^{3/2}} + \frac{B}{(\alpha\ln(\tau)+\tau)^2} \end{aligned} \tag{3.34}$$

After taking derivatives with respect to our redefined Kähler modulus τ,

$$K_\tau = \frac{1}{2}\left(\frac{3\zeta\left(\frac{\alpha}{\tau}+1\right)}{2(\alpha\ln(\tau)+\tau)^{5/2}} - \frac{3\left(\frac{\alpha}{\tau}+1\right)}{\alpha\ln(\tau)+\tau} - \frac{A\left(\frac{\alpha}{\tau}+1\right)}{s(\alpha\ln(\tau)+\tau)^2} - \frac{2B\left(\frac{\alpha}{\tau}+1\right)}{(\alpha\ln(\tau)+\tau)^3} + \frac{9\left(\frac{\alpha}{\tau}+1\right)\ln(\alpha\ln(\tau)+\tau)}{2\sqrt{s}(\alpha\ln(\tau)+\tau)^{5/2}} - \frac{3\left(\frac{\alpha}{\tau}+1\right)}{\sqrt{s}(\alpha\ln(\tau)+\tau)^{5/2}}\right) \tag{3.35}$$

The Kähler metric becomes

$$\begin{aligned}K_{\tau\bar\tau} = \frac{1}{4}\Bigg(&-\frac{3\alpha\zeta}{2\tau^2(\alpha\ln(\tau)+\tau)^{5/2}} - \frac{15\zeta\left(\frac{\alpha}{\tau}+1\right)^2}{4(\alpha\ln(\tau)+\tau)^{7/2}} + \frac{3\alpha}{\tau^2(\alpha\ln(\tau)+\tau)} \\ &+ \frac{3\left(\frac{\alpha}{\tau}+1\right)^2}{(\alpha\ln(\tau)+\tau)^2} + \frac{\alpha A}{s\tau^2(\alpha\ln(\tau)+\tau)^2} + \frac{2A\left(\frac{\alpha}{\tau}+1\right)^2}{s(\alpha\ln(\tau)+\tau)^3} \\ &+ \frac{2\alpha B}{\tau^2(\alpha\ln(\tau)+\tau)^3} + \frac{6B\left(\frac{\alpha}{\tau}+1\right)^2}{(\alpha\ln(\tau)+\tau)^4} + \frac{3\alpha}{\sqrt{s}\tau^2(\alpha\ln(\tau)+\tau)^{5/2}} \\ &- \frac{9\alpha\ln(\alpha\ln(\tau)+\tau)}{2\sqrt{s}\tau^2(\alpha\ln(\tau)+\tau)^{5/2}} + \frac{12\left(\frac{\alpha}{\tau}+1\right)^2}{\sqrt{s}(\alpha\ln(\tau)+\tau)^{7/2}} \\ &- \frac{45\left(\frac{\alpha}{\tau}+1\right)^2\ln(\alpha\ln(\tau)+\tau)}{4\sqrt{s}(\alpha\ln(\tau)+\tau)^{7/2}}\Bigg)\end{aligned} \tag{3.36}$$

The inverse Kähler metric is

$$\begin{aligned}K^{\tau\bar\tau} &= \frac{4}{-\frac{3\alpha\zeta}{2\tau^2(\alpha\ln(\tau)+\tau)^{5/2}} - \frac{15\zeta\left(\frac{\alpha}{\tau}+1\right)^2}{4(\alpha\ln(\tau)+\tau)^{7/2}} + \frac{3\alpha}{\tau^2(\alpha\ln(\tau)+\tau)} + \frac{3\left(\frac{\alpha}{\tau}+1\right)^2}{(\alpha\ln(\tau)+\tau)^2}} \\ &+ \frac{\alpha A}{s\tau^2(\alpha\ln(\tau)+\tau)^2} + \frac{2A\left(\frac{\alpha}{\tau}+1\right)^2}{s(\alpha\ln(\tau)+\tau)^3} + \frac{2\alpha B}{\tau^2(\alpha\ln(\tau)+\tau)^3} + \frac{6B\left(\frac{\alpha}{\tau}+1\right)^2}{(\alpha\ln(\tau)+\tau)^4} \\ &+ \frac{3\alpha}{\sqrt{s}\tau^2(\alpha\ln(\tau)+\tau)^{5/2}} - \frac{9\alpha\ln(\alpha\ln(\tau)+\tau)}{2\sqrt{s}\tau^2(\alpha\ln(\tau)+\tau)^{5/2}} \\ &+ \frac{12\left(\frac{\alpha}{\tau}+1\right)^2}{\sqrt{s}(\alpha\ln(\tau)+\tau)^{7/2}} - \frac{45\left(\frac{\alpha}{\tau}+1\right)^2\ln(\alpha\ln(\tau)+\tau)}{4\sqrt{s}(\alpha\ln(\tau)+\tau)^{7/2}}\end{aligned} \tag{3.37}$$

Finally we are in position to calculate the F-term scalar potential. Incorporating the order F_4 term

$$V_{F^4} = -\frac{g_s^2}{4}\frac{\lambda\,|W_0|^4}{g_s^{3/2}V^4}\Pi_i t^i \equiv \frac{C}{s^2\tau^{11/2}} \tag{3.38}$$

and finally the scalar potential is

$$
\begin{aligned}
V &= -\frac{3\alpha}{\tau^4}+\frac{3(-8D+\sqrt{s}\zeta+3D\ln[\tau])}{4\sqrt{s}\tau^{9/2}} \\
&+ \frac{A^2-6Bs^2-6As\alpha+27s^2\alpha^2+18s^2\alpha^2\ln[\tau]}{3s^2\tau^5}+O\left(\tau^{-11/2}\right)
\end{aligned}
\tag{3.39}
$$

In the large volume or large τ limit, the leading contribution to the scalar potential comes from

$$
V \simeq -\frac{3\alpha}{\tau^4}+\frac{3(s^2\zeta+3D\ln[\tau])}{4\sqrt{s}\tau^{9/2}} \tag{3.40}
$$

Note that, we have obtained a leading correction which occur at $\mathcal{O}(\tau^4)$ in the scalar potential. This has an interesting implication. First, this correction was not present in LVS like models or and is certainly stronger than the BBHL piece unless α and g_s takes on extremely small values alongside ζ taking on larger values. Second, we have two leading terms at $\mathcal{O}(\tau^{9/2})$ and they can compete in the minima. Moreover, we have done better in the aspect that no non-perturbative correction is included here. In the following section, we analyze the behaviour of this function and see which values of these coefficients lets us find minima.

3.3 Vacua: dS, AdS, Runaway?

Let us now consider the leading corrections to the F-term scalar potential based on a large volume approximation which we can write as

$$
V = -\frac{3\alpha}{\tau^4}+\frac{3(s^2\zeta+3D\ln[\tau])}{4\sqrt{s}\tau^{9/2}} \tag{3.41}
$$

Let us calculate the minima by following a few steps. First, let us set α to zero (no moduli redefinitions). The motivation behind this is that the term acts like an uplifting term when we want to obtain dS vacua. We obtain

$$
V_{\alpha=0} = \frac{3(s^2\zeta+3D\ln[\tau])}{4\sqrt{s}\tau^{9/2}} \tag{3.42}
$$

For ease of analysis and looking for a large volume like case, let us redefine the constants above

$$
V_{\alpha=0} \equiv \frac{\delta-\gamma\ln[\tau])}{\tau^{9/2}} \tag{3.43}
$$

where, $\gamma \equiv \frac{9D}{4\sqrt{s}}$ and $\delta \equiv \frac{3s^2\zeta}{4\sqrt{s}}$. Setting $V'_{\alpha=0}(\tau) = 0$, gives us the extrema which is

$$\tau = \exp\left\{\frac{\delta}{\gamma}\right\}\exp\{(2/9)\} \tag{3.44}$$

Furthermore, for $V''_{\alpha=0}(\tau) > 0$ gives us the conditions that $\gamma > 0$ which forces D to be positive. If we evaluate $V_{\alpha=0}$ at the extrema, we get:

$$\begin{aligned} < V >_{\alpha=0} &= \frac{-2\gamma}{9\tau^{9/2}} \\ &= \frac{-D}{2\sqrt{s}\tau^{9/2}} \end{aligned}$$

Now, let us reinstate the modulus redefinition term that is $\alpha \neq 0$. This can act like an uplifting term and thus $V_{up} = -\frac{3\alpha}{\tau^4}$. To act as an uplifting term, it should compete with $V_{\alpha=0}$. Therefore, $V_{up} \sim V_{\alpha=0}$ gives $\alpha \sim \frac{D}{6\tau^{1/2}\sqrt{s}}$. Thus, to avoid runaway our parameter α should roughly scale as state above. Which of these values of α gives us dS or AdS is given as examples in the following section. The dS vacua we obtain has an interestingly small value. Unlike KKLT proposal where another uplifting term is added whose coefficient is finely tuned so that the vacuum energy is sufficiently small, our analysis gives a very small value naturally for both the dS and AdS case.

Before going into that analysis, we will roughly point out why the values of our constants α, s, ζ, D are not unreasonable and what restrictions they put in the theory.

- Considering α to be negative or positive is fine as moduli redefinitions occur up to an overall factor of $\ln\tau$ and the sign of the constant is irrelevant in the original analysis as well.

- The range of values of s that we pick is also fine as the if it is less than 1 it would signal the breaking of perturbative analysis. We should always consider it to be more than 1 as $s = 1/g_s$ would otherwise imply g_s is more than 1 and the breakdown of perturbation theory.

- The positivity(and range of values) of ζ puts some topological restrictions on the CY. It forces the CY to have negative Euler-characteristic. This is due to the relation:

  $$\zeta = \frac{-\chi(X)\zeta(3)}{2(2\pi)^3}$$

  where X is the compactification manifold.

- D is the co-efficient of the logarithmic loop corrections proposed by Antoniadis et al. and it is fine to have it be negative.

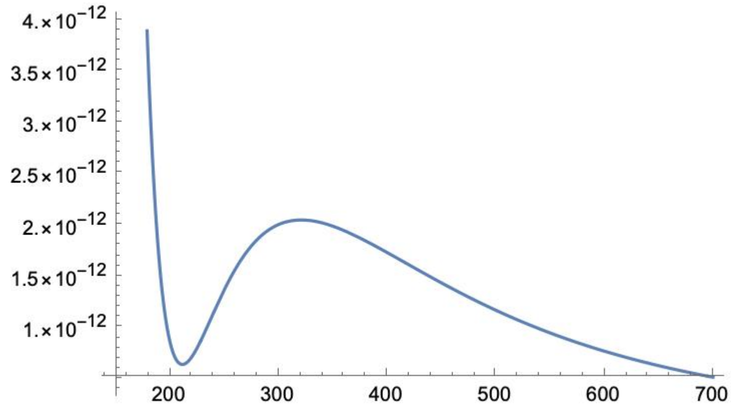

Figure 3.4: $V(\tau)$ vs τ plot

3.3.1 Example 1: dS Case

We can find a dS minima and maxima at large volume. As an example, we choose the following set of values and see what the extrema looks like

$$\begin{aligned} &\alpha = -0.33; \\ &s = 10; \\ &\zeta = 1; \\ &D = -10; \end{aligned} \tag{3.45}$$

We find the following set of values

$$\begin{aligned} &\text{Local minimum value: } 6.43048 \times 10^{-13}; \\ &\text{Value of } \tau \text{ at local minimum: } 211.535; \\ &\text{Local maximum value: } 2.04672 \times 10^{-12}; \\ &\text{Value of } \tau \text{ at local maximum: } 315.434; \end{aligned} \tag{3.46}$$

3.3.2 Example 2: AdS Case

Keeping our other values fixed as before, the AdS minima occurs for the following:

$$\begin{aligned} &|\alpha| < 0.33; \\ &s = 10; \\ &\zeta = 1; \\ &D = -10; \end{aligned} \tag{3.47}$$

As an example, we take the following set of constants and plot the potential.

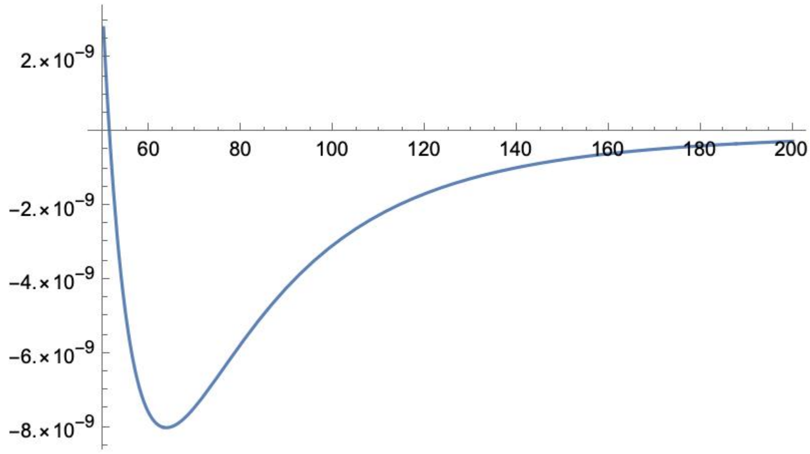

Figure 3.5: $V(\tau)$ vs τ plot (AdS Minima)

$$\begin{aligned} &\alpha = -0.2; \\ &s = 10; \\ &\zeta = 1; \\ &D = -10; \end{aligned} \tag{3.48}$$

$$\begin{aligned} &\text{Local minimum value: } -8.00341 \times 10^{-9}; \\ &\text{Value of } \tau \text{ at local minimum: } 63.666; \end{aligned} \tag{3.49}$$

3.3.3 Example 3: Runaway

Keeping our other values fixed as before, the runaway part occurs for the following:

$$\begin{aligned} &|\alpha| > 0.34; \\ &s = 10; \\ &\zeta = 1; \\ &D = -10; \end{aligned} \tag{3.50}$$

As an example we take the following set of values and observe the runaway or no-minima case

$$\begin{aligned} &\alpha = -2; \\ &s = 10; \\ &\zeta = 1; \\ &D = -10; \end{aligned} \tag{3.51}$$

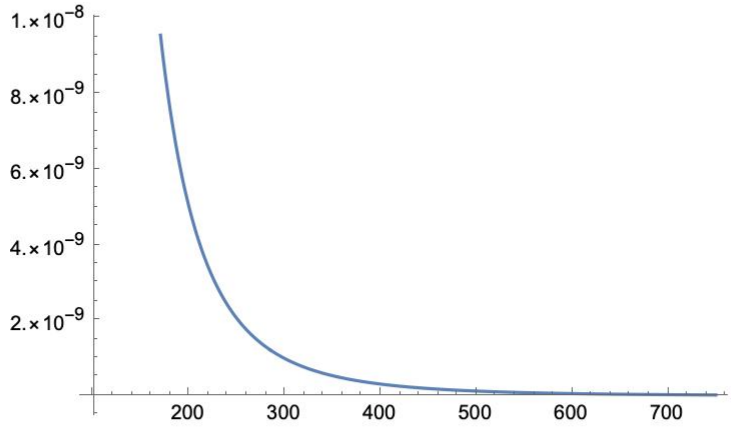

Figure 3.6: $V(\tau)$ vs τ plot

3.4 Application to Inflationary Dynamics

In this section, we will discuss how to embed inflation in our model. We will try to study the brane-antibrane inflationary scenario as discussed at length in the first chapter. In section 1.6.8 we discussed the warped brane-antibrane inflation. We will use this warped brane-antibrane inflation which suffers the infamous eta problem when we try to do a moduli stabilization using non-perturbative effects. We will try to see if brane-antibrane inflation occurring in our model suffers the same fate. We will use the nilpotent superfield formalism (as an example, we presented the nilpotent superfield to uplift the KKLT AdS vacua in the appendix) in this analysis. We imagine that this superfield X satisfies the nilpotency condition:

$$X^2 = 0 \tag{3.52}$$

and is chiral

$$\bar{D}X = 0. \tag{3.53}$$

Presence of this nilpotent superfield modifies our superpotential and Kähler potential. Let us highlight the modification of our superpotential. In the following analysis, the inflaton field, which is the brane-separation for brane-antibrane inflation, is denoted by Φ.

$$W = W_0 + XW_X(\Phi, \bar{\Phi}) \quad \text{with} \quad W_X(\Phi, \bar{\Phi}) = \mathfrak{t} - \frac{\mathfrak{g}}{|\Phi|^4} + \cdots \tag{3.54}$$

Here, the term W_X captures the antibrane tension and Coulomb interaction between the branes. Let us now consider the following Kähler potential with the the nilpotent superfield in play:

$$K = -3\ln(\tau_{old}) + \frac{X\bar{X}}{\tau_{old}} + \frac{\Phi\bar{\Phi}}{\tau_{old}} + \frac{A}{s\tau_{old}} - \frac{1}{\tau_{old}^{3/2}}\left[\zeta s^{3/2} + 3D\ln(\tau_{old})\sqrt{s}\right] + \frac{B}{\tau_{old}^2} \tag{3.55}$$

We do a moduli redefinition as before $\tau_{old} = \tau - \alpha \ln \tau$. Let us proceed in finding the F-term scalar potential.

$$V = e^K (K^{T\bar{T}} D_T W D_{\bar{T}} \bar{W} + K^{X\bar{X}} D_X W D_{\bar{X}} \bar{W}) + \frac{C}{s^2 \tau_{old}^{11/2}} \tag{3.56}$$

In many inflationary models, the nilpotent superfield vanishes during inflation and we can set X=0. Using this and the nilpotency condition, in the large-volume limit, we find the leading terms as follows:

$$V = \frac{|W_X|^2}{\tau^2} + \frac{2\alpha |W_X|^2 \ln \tau}{\tau^3} + \frac{A|W_X|^2}{\tau^3} \tag{3.57}$$

Using the explicit form of W_X, we find

$$V(\Phi, \tau) = \left(|\mathfrak{t}|^2 - \frac{2Re(\bar{\mathfrak{t}}\mathfrak{g})}{|\Phi|^4} \right) \left(\frac{1}{\tau^2} + \frac{2\alpha \ln \tau}{\tau^3} + \frac{A}{\tau^3} \right) \tag{3.58}$$

We checked below if τ part of the potential is at the minima and for certain values of the potential it does reside in a minima. We will denote the minima for τ as $V_{min}(\tau)$ from now on. Thus,

$$V(\Phi, \tau) = \left(|\mathfrak{t}|^2 - \frac{2Re(\bar{\mathfrak{t}}\mathfrak{g})}{|\Phi|^4} \right) V_{min}(\tau) \tag{3.59}$$

Here, the factors $\mathfrak{t}$ and $\mathfrak{g}$ depends on the warping of the Calabi-Yau and goes as $\mathfrak{t} \sim e^{-2\rho}$ and $\mathfrak{g} \sim e^{-6\rho}$ where ρ is the warp factor. Let us now evaluate the slow-roll conditions.

$$V_\Phi = \frac{8Re(\bar{\mathfrak{t}}\mathfrak{g})}{|\Phi|^5} V_{min}(\tau) \tag{3.60}$$

$$V_{\Phi\Phi} = \frac{-40Re(\bar{\mathfrak{t}}\mathfrak{g})}{|\Phi|^6} V_{min}(\tau) \tag{3.61}$$

$$\begin{aligned}
\epsilon &= \frac{M_p^2}{2} \left(\frac{V_\Phi}{V} \right)^2 \\
&= \frac{M_p^2}{2|\Phi|^2} \left(\frac{8Re(\bar{\mathfrak{t}}\mathfrak{g})}{|\Phi|^4 |\mathfrak{t}|^2 - 2Re(\bar{\mathfrak{t}}\mathfrak{g})} \right) \\
&\sim \frac{M_p^2}{2|\Phi|^2} \left(\frac{8e^{-8\rho}}{|\Phi|^4 e^{-4\rho} - e^{-8\rho}} \right)^2 \\
&= \frac{M_p^2}{2|\Phi|^2} \left(\frac{8e^{-2\rho}}{1 - e^{-2\rho}|\Phi|^4} \right)^2 \\
&= \frac{M_p^2}{2|\Phi|^{10}} \left(8e^{-2\rho}(1 + e^{-4\rho}|\Phi|^4 + ...) \right)^2
\end{aligned}$$

Thus, the warping factors make it possible to have $\epsilon \ll 1$. Similarly,
It looks like the code you provided has a few issues. Here is the corrected version:

$$\begin{aligned}
\eta &= M_p^2 \left(\frac{V_{\Phi\Phi}}{V}\right) \\
&= -40 M_p^2 \frac{1}{|\Phi|^2} \left(\frac{\mathrm{Re}(\bar{\mathfrak{t}}\mathfrak{g})}{|\Phi|^4|\mathfrak{t}|^2 - 2\mathrm{Re}(\bar{\mathfrak{t}}\mathfrak{g})}\right) \\
&\sim -40 M_p^2 \frac{1}{|\Phi|^2} \left(e^{-2\rho}(1 + e^{-4\rho}|\Phi|^4 + \ldots)\right)
\end{aligned}$$

Thus, $|\eta| \ll 1$.

One of major problems in string inflationary scenarios is that it is futile to arrange a potential in a possible slow-roll inflaton direction if the potential is much steeper in other directions in field space. A slowly rolling field would prefer to evolve in the steepest possible direction. We would thus need to check if τ is actually at the minima. Let us write the scalar potential as follows

$$V(\Phi, \tau) \equiv V(\Phi)V(\tau) \tag{3.62}$$

We will now check if $V(\tau)$ is stuck at the minima as inflation occurs. We write $V(\tau)$ as follows:

$$V(\tau) \equiv \left(\frac{c}{\tau^2} + \frac{2\alpha \ln \tau}{\tau^3} + \frac{A}{\tau^3}\right) \tag{3.63}$$

We have added a coefficient c to track which values of it would give would give us a minima corresponding to a positive value at the minima. Note that, this is fine as we can rescale other coefficients as well and bring out a constant in front of $\frac{1}{\tau^2}$ term. We found a positive value of the minima when the coefficients take the following values:

$$\begin{aligned}
\alpha &= -0.3; \\
A &= 0.1; \\
\text{c} &= 0.043;
\end{aligned} \tag{3.64}$$

Value at the minima : 5.90668×10^{-6}
The graph of the potential looks as follows:

3.4.1 Comments on eta problem

In this section, we will check if our model has an eta problem (eta problem is discussed in the appendix). In [32], the brane-antibrane inflationary setup of Burgess and Quevedo

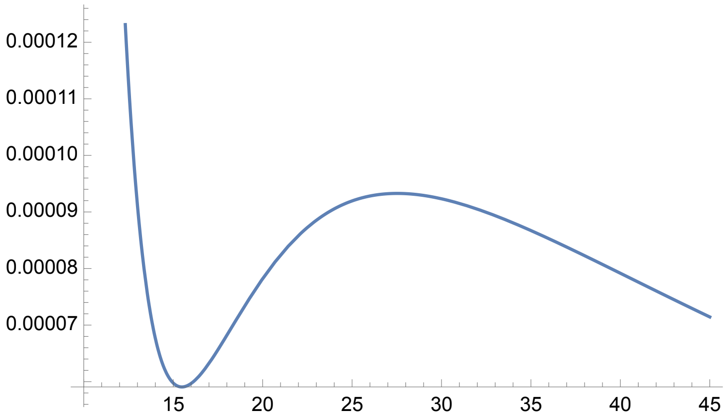

Figure 3.7: $V(\tau)$ vs τ plot (Inflationary Minima)

did not contain any term proportional to the mass of the inflaton. In our setup, we do generate a mass term but it is very much suppressed. Aligning our calculation with equation (B.14) and (B.15),

$$K = -3\ln(\tau - k) \qquad k = \mathfrak{K} + (X + \overline{X})\mathfrak{K}_X + \overline{X}X\mathfrak{K}_{X\overline{X}} \tag{3.65}$$

$$\mathfrak{K} = A + \alpha \ln\tau + \overline{\phi}\phi \qquad \mathfrak{K}_X = 0 \qquad \mathfrak{K}_{X\overline{X}} = 1 \tag{3.66}$$

where $\mathfrak{K}$ contains terms proportional to $\frac{1}{\tau}$. For checking the eta problem, other corrections in subleading order of τ are neglected.

$$e^K = \frac{1}{(\tau - k)^3} = \frac{1}{\left(\tau - A - \alpha\ln\tau - \overline{\phi}\phi - \overline{X}X\right)^3} \tag{3.67}$$

Let us now define $\mathcal{P} \equiv \tau - \overline{\phi}\phi$

$$e^K = \frac{1}{\mathcal{P}^3}\left(1 - \frac{\left(A + \alpha\ln\tau + \overline{X}X\right)}{\mathcal{P}}\right)^{-3} \simeq \frac{1}{\mathcal{P}^3} + 3\frac{\left(A + \alpha\ln\tau + \overline{X}X\right)}{\mathcal{P}^4} \tag{3.68}$$

Evaluating this expression at $X = 0$ and rewriting τ as $\tau = \mathcal{P} + \overline{\phi}\phi$ we get:

$$e^K \simeq \frac{1}{\mathcal{P}^3} + 3\frac{(A + \alpha\ln\mathcal{P})}{\mathcal{P}^4} + 3\alpha\frac{\overline{\phi}\phi}{\mathcal{P}^5} \tag{3.69}$$

When inserted into the scalar potential, this would generate corrections to η that can be small enough to evade the η-problem since they would scale as (note that the canonically normalized field is $\Phi_c = \frac{\Phi}{\sqrt{\tau_{old}}}$):

$$\delta\eta \sim \frac{\alpha}{\mathcal{P}} \ll 1 \qquad \text{for} \qquad \alpha \ll 1 \quad \text{and} \quad \mathcal{P} \gg 1 \tag{3.70}$$

Naively plugging in the numerics of the previous section also settles the case.

Chapter 4

Concluding Remarks and Future Directions

This master thesis started off by discussing the need for a quantum gravity theory and why string theory is still a leading candidate. We also discussed the story of inflation, its necessity and implication. Discussion then continued on the first string derived inflationary scenario, the brane-antibrane inflation, followed by its implications. We then moved on to discuss the moduli stabilization in type IIB scenario citing various proposals like KKLT, LVS, RG induced moduli stabilization and so on.

The main work of this thesis is contained in chapter 3. Let us summarise the new results of the master thesis:

- We consider all the known perturbative corrections in literature and try to stabilized the Kähler moduli using these correction. The leading term of the potential arises at $\mathcal{O}(\tau^2)$ similar to what Burgess and Quevedo proposed in [32] (and hence stabilized at that order in their proposal) but we do have other subsequent leading terms at $\mathcal{O}(\tau^3)$.

- We also studied different values of the coefficient which makes the value of the potential at the minima positive or negative or a scenario where there is no minima at all. These corresponds to an AdS and dS minima. We also briefly mention the consequence of taking the coefficients as we do.

- Finally, we study the brane-antibrane inflation in our perturbatively moduli stabilized scenario. We implement this using the nilpotent superfield formalism. The volume modulus remains at a minima while inflation occurs by the help of the inflaton field which is the brane-antibrane separation.

- Lastly, we check if the slow-roll conditions are satisfied and check for the eta problem. Both results are fortunately positive.

In this work, we find an interesting result in the sense that the Kähler modulus is stabilized perturbatively and we can obtain a dS vacuum using only perturbative corrections. Let us now highlight some possible future directions of this work:

- It would be interesting to study more in detail the post-inflationary evolution of our brane-antibrane inflationary model, paying particular attention to the reheating process. This should involve the study of potential epochs of kinetic and matter domination driven by the evolution of the volume modulus from its inflationary minimum to the post-inflationary one.

- We worked with only one Kähler modulus in this thesis. The next line of action could be to consider two Kähler moduli and study their stabilization.

- It will be interesting to see if we can consider a K3-fibred Calabi-Yau manifold with two moduli and see if there can be a possibility to fix both moduli using just perturbative corrections which can lead to a new version of the fibre inflation scenario as implemented by Cicoli et al. in [15].

- Another possible direction is studying type IIA or Heterotic string theory frameworks using known loop and α' corrections and seek for a dS minima in a similar fashion.

- Cosmological scenarios which lead to primordial black-hole formation (analogous to the one done by Cicoli et al. in [49] and [50]) can be addressed in our model with all the EFT constraints in play.

It will be interesting to study the future directions and what implications they might have.

Appendix A

Mathematical Background

A.1 Holonomy

Let M be a real manifold. Take some point P and a curve C which passes through P and let $\vec{a}$ be some tangent vector at point P, i.e. a vector in tangent space at P. Now, if we parallel transport the vector $\vec{a}$ along the curve C, it will come back as a different vector, say $\vec{a}'$, where

$$\vec{a}' = R(C)\vec{a}$$

where $R(C)$ is a linear operator (matrix of same dimension as the tangent vector). The operator $R(C)$ is called the holonomy along C.

For a $2n$-dimensional Riemannian manifold, the norm of the tangent vector is preserved under the parallel transport. Thus, the holonomy is

$$R(C) \subset SO(2n), \text{ C is contractible}$$
$$R(C) \subset O(2n), \text{ C is non-contractible}$$

If we take the collection of all holonomy, i.e. $\{R(C)\}$ for all possible curves C through P, then they form a group under multiplication. The group composition would be

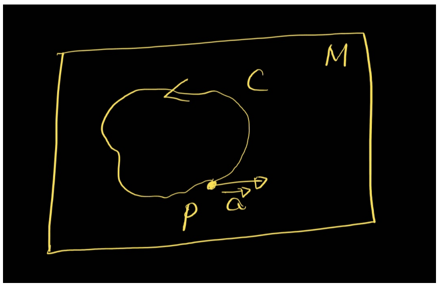

Figure A.1: Holonomy

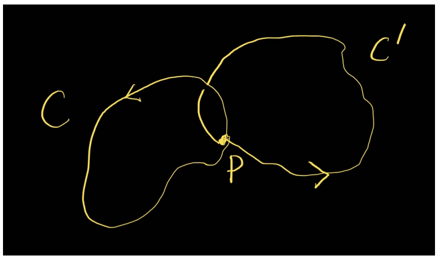

Figure A.2: Group Multiplication Law

$R(C)R(C') = R(C \cdot C')$ where $C \cdot C'$ means that curve which is obtained by first traversing C' and then traversing C.

A helpful way to look at holonomy group relevant for physics is the Berger classification. Berger's classification of the holonomy group is as follows: If M simply connected Riemannian manifold, then either M is a product of lower dimensional manifold or M is a symmetric space (a coset space G/H) or one of the following:

1. $\mathrm{Hol}(M) = SO(n)$
2. $\mathrm{Hol}(M) = U(k) \subset SO(2k)$ for $n = 2k$
3. $\mathrm{Hol}(m) = \mathrm{Sp}(k) \subset \mathrm{SO}(4k)$ fer $n = 4k$
4. $\mathrm{Hol}(M) = S_p(k) \subset S_0(4k)$ for $n = 4K$
5. $\mathrm{Hol}(M) = S_p(k)S_p(1) \subset S_0(4k)$ for $n = 4K$
6. $\mathrm{Hol}(M) = G_2 \subset SO(7)$ for $n = 7$
7. $\mathrm{Hol(M)} = \mathrm{Spin}(7) \subset 50(8)$ for $n = 8$

Let us make a few comments about relevant cases of holonomy in string compactifications. Manifold with holonomy $U(k)$ is a Kähler manifold. Christoffels in a Kähler only have holomorphic indices, so parallel transport does not transform holomorphic into anti-holomorphic vectors. Manifolds with $SU(k)$ holonomy are Calabi-Yau manifolds which is special type of Kähler manifold.

A.2 Homology

A k-dimensional subspace C_k of a manifold M is called a k-cycle if it has no boundary i.e

$$\partial C_k = 0 \tag{A.1}$$

where ∂ is called the boundary operator. The 0 on the right hand side stands for an empty set. For example, a 1 cycle will be a closed curve on the manifold. It may be contractible or non-contractible. Roughly speaking, a contractible curve can be continuously shrunk to a point in the manifold. Boundary of a boundary is always zero, i.e $\partial\left(\partial C_k\right)=0$ (even if $\partial C_k \neq 0$). Thus, every boundary can be viewed as a cycle. Though, the reverse is not true in general.

k-dimensional subspace C_k is called exact if it is a boundary of some higher subspace B_{K+1} i.e

$$C_k=\partial B_{k+1} \tag{A.2}$$

Using the idea that the boundary of a boundary vanishes, we can define an equivalence relation as follows:

$$C_k \cong \tilde{C}_k \text{ are equivalent if } C_k=\tilde{C}_k+\partial B_{k+1} \tag{A.3}$$

Thus, if two k-cycles differ only by a boundary, then they are equivalent. This defines a Homology class. Each element in a class k are equivalent. We can also construct a group with it like the kth-homology group $H_k(M)$ on a manifold M. Let us construct an intuitive example below to help us

The region B_2 has two distinct curves as boundary: C_1 and $\tilde{C}_1$ ($L_1 \& L_2$ are identified, Hence, their contribution cancels), i. e.

$$\partial B_2=\tilde{C}_1+c_1 \Rightarrow \quad \tilde{C}_1=C_1+\partial B_2$$

A.3 Cohomology

If $A^{(K)}$ is a k-form, then we can write it as

$$A^{(k)}=\frac{1}{k!}A^{(k)}_{m_1}\cdots m_k dx^{m_1}\wedge\cdots\wedge dx^{m_k} \tag{A.4}$$

An exterior derivative d takes a k form and turns it into a $(k+1)$- form. Taking the exterior derivative, we obtain

$$dA^{(k)}=\frac{1}{k!}\partial_{m_0}A^{(k)}_{m_1}\cdots m_k dx^{m_0}\wedge dx^{m_1}\wedge\cdots\wedge dx^{m_k} \tag{A.5}$$

Since the exterior derivative satisfies the nilpotency condition i.e $d^2=0$, we have $d\left(dA^{(k)}\right)=0$. Let us now motivate the idea of cohomology.

A k-form $\omega^{(k)}$ is closed if $d\omega^{(k)}=0$. A k - form $\omega^{(k)}$ is exact if $\omega^{(k)}=d\omega^{(k-1)}$ for some $(k-1)$ - form $\omega^{(k-1)}$. Therefore, we can form an equivalence class as follows:

$$\omega^{(k)} \cong \tilde{\omega}^{(k)} \text{ if } \omega^{(k)}=\tilde{\omega}^{(k)}+d\omega^{(k-1)} \tag{A.6}$$

Thus, we say that two k-forms belong to the same cohomology class if they differ by an exact k-form. The set of equivalence classes of 'k - forms' form the cohomology group $H^k(\mathbb{R})$ or $H^k(\mathbb{C})$. Let us now briefly talk about the complex cohomology group Dolbeault cohomology. Before that, let us see what complex differential forms are:

If α_r and β_r are two real r-forms, the sum

$$\gamma_r \equiv \alpha_r + i\beta_r$$

is termed a complex $\boldsymbol{r}$-form. We denote the vector space of complex $\boldsymbol{r}$-forms by $\Omega^r_{\mathbb{C}}(M)$. The conjugate of γ_r is $\bar{\gamma}_r \equiv \alpha_r - i\beta_r$.An $(\boldsymbol{r}, \boldsymbol{s})$-form is a complex-valued differential form with r holomorphic indices and s anti-holomorphic indices. In local coordinates, a basis for (r, s) forms is

$$\mathrm{d}z_{\mu_1} \wedge \ldots \wedge \mathrm{d}z_{\mu_r} \wedge \mathrm{d}\bar{z}_{\bar{\nu}_1} \wedge \ldots \wedge \mathrm{d}\bar{z}_{\bar{\nu}_s} \equiv \mathrm{d}z_M \wedge \mathrm{d}\bar{z}_{\bar{N}}$$

where we have defined the multi-indices $\boldsymbol{M} = (\mu_1 \ldots \mu_r)$ and $\boldsymbol{N} = (\nu_1 \ldots \nu_s)$. We denote the vector space of (r, s)-forms on a manifold M by $\Omega^{r,s}(M)$. An element $\gamma_{r,s}$ of $\Omega^{r,s}(M)$ can be written as

$$\gamma_{r,s} = \frac{1}{r!s!}\gamma_{\boldsymbol{M}\overline{\boldsymbol{N}}}\mathrm{d}z^{\boldsymbol{M}} \wedge \mathrm{d}\bar{z}^{\overline{\mathbf{N}}}$$

The Dolbeault operators are maps $\partial : \Omega^{r,s} \to \Omega^{r+1,s}$ and $\bar{\partial} : \Omega^{r,s} \to \Omega^{r,s+1}$ whose actions on $\gamma_{r,s}$ are

$$\partial\gamma_{r,s} = \left(\frac{\partial}{\partial z^{\kappa}}\gamma_{\boldsymbol{M}\overline{\mathbf{N}}}\right)\mathrm{d}z^{\kappa} \wedge \mathrm{d}z^{M} \wedge \mathrm{d}\bar{z}^{\bar{N}}$$
$$\bar{\partial}\gamma_{r,s} = \left(\frac{\partial}{\partial \bar{z}^{\bar{\kappa}}}\gamma_{\boldsymbol{M}\overline{\mathbf{N}}}\right)\mathrm{d}\bar{z}^{\bar{\kappa}} \wedge \mathrm{d}z^{M} \wedge \mathrm{d}\bar{z}^{\bar{N}}.$$

An $(r, 0)$-form $\gamma_{r,0}$ is said to be holomorphic if and only if

$$\bar{\partial}\gamma_{r,0} = 0$$

Let $Z^{r,s}_{\bar{\partial}}(M)$ denote the set of $\bar{\partial}$-closed (r, s)-forms and $B^{r,s}_{\bar{\partial}}(M)$ the set of $\bar{\partial}$-exact (r, s)-forms. The Dolbeault cohomology group is then defined as

$$H^{r,s}_{\bar{\partial}}(M, \mathbb{C}) \equiv Z^{r,s}_{\bar{\partial}}(M)/B^{r,s}_{\bar{\partial}}(M)$$

We can then define two types of Laplacians,

$$\Delta_{\partial} = \partial\partial^{\dagger} + \partial^{\dagger}\partial, \quad \Delta_{\bar{\partial}} = \bar{\partial}\bar{\partial}^{\dagger} + \bar{\partial}^{\dagger}\bar{\partial}.$$

We denote by $\mathcal{H}^{r,s}_{\bar{\partial}}(M)$ the set of $\bar{\partial}$-harmonic (r, s)-forms, i.e. (r, s)-forms annihilated by $\Delta_{\bar{\partial}}$. Hodge's theorem then states that $H^{r,s}_{\bar{\partial}}(M, \mathbb{C}) \cong \mathcal{H}^{r,s}_{\bar{\partial}}(M)$. The Hodge numbers are the complex dimensions of the Dolbeault cohomology groups:

$$h^{r,s} = \dim H^{r,s}_{\bar{\partial}}(M, \mathbb{C}).$$

By using the Hodge star, one finds that $h^{r,s} = h^{k-r,k-s}$ on a manifold of complex dimension k.

A.4 Duality Between Homology and Cohomology

A theorem in mathematics called de-Rham's theorem says that rth-homology group $H_r(M)$ and the rth-cohomology group $H^r(M)$ are dual vector spaces. We can motivate this by defining an inner product $(\Sigma, \omega) : H_r(M) \times H^r(M) \to \mathbb{R}$

$$(\Sigma, \omega) \equiv \int_\Sigma \omega \tag{A.7}$$

The above integral does not depend on the particular element of a homology or cohomology class rather any element or every element of the specific class suffices. We can use Stokes' theorem, which states that for ω being an r-form and C_r is an r-chain, then

$$\int_{C_r} \mathrm{d}\omega = \int_{\partial C_r} \omega, \tag{A.8}$$

where ∂C_r is the boundary of C_r. An r-chain is a formal sum of r-cycles with real or complex coefficients. Let us do the following exercise by adding an exact r-form $\xi = \mathrm{d}\lambda$ to ω, and an r-boundary $\Pi = \partial\Lambda$ to Σ.

$$\int_{\Sigma+\Pi} (\omega + \xi) = \int_\Sigma \omega + \int_{\partial\Sigma} \lambda + \int_\Lambda \mathrm{d}\omega + \int_\Lambda \mathrm{d}^2\lambda = \int_\Sigma \omega \tag{A.9}$$

Thus, the inner product we defined is independent of the choice of elements of either of the cohomology or homology classes.

In a manifold of dimension D, a typical r-cycle and a typical $(D-r)$-cycle intersect at a finite number of points. This intersection allows us to establish a pairing between two homology groups as follows:

$$H_r(M) \times H_{D-r}(M) \to \mathbb{R}. \tag{A.10}$$

This pairing leads to a vector space isomorphism called Poincaré duality:

$$H_r(M) \cong H_{D-r}(M). \tag{A.11}$$

Poincaré duality can also be understood in terms of cohomology as well. Given an element ω in $H^r(M)$ and an element η in $H^{D-r}(M)$, we can define a mapping $H^r(M) \times H^{D-r}(M) \to \mathbb{R}$ as follows:

$$(\omega, \eta) \equiv \int_M \omega \wedge \eta. \tag{A.12}$$

This leads to the conclusion that

$$H^r(M) \cong H^{D-r}(M). \tag{A.13}$$

When combined with de Rham's theorem, which relates cohomology and homology, we can deduce that

$$H^{D-r}(M) \cong H^r(M) \cong H_r(M) \cong H_{D-r}(M). \tag{A.14}$$

The rth Betti number,

$$b^r \equiv \dim H^r(M, \mathbb{R}), \tag{A.15}$$

represents the count of nontrivial cohomology classes of r-forms on M or, equivalently, the count of nontrivial homology classes of r-cycles. Poincaré duality implies that

$$b^r = b^{D-r}. \tag{A.16}$$

The Euler characteristic of M is calculated as the alternating sum:

$$\chi(M) \equiv \sum_{r=0}^{D} (-1)^r b^r. \tag{A.17}$$

A.5 Kähler and Calabi-Yau Manifolds

Kähler manifolds are special kind of hermitian manifolds whose metric we can write locally as

$$g_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} K(z, \bar{z}) \tag{A.18}$$

Note that, the function K does not need to be a globally defined function. Roughly speaking, we can stitch different K's defined on some patches via some transformation (which is clearly invariant)

$$K \mapsto K + f\left(z^i\right) + \bar{f}\left(\bar{z}^{\bar{\imath}}\right) \tag{A.19}$$

.

With this metric, we can define a $(1,1)$-form called a Kähler form as follows:

$$J = g_{\alpha\bar{\beta}} dz^\alpha \wedge d\bar{z}^{\bar{\beta}} \tag{A.20}$$

In Kähler manifolds, we can prove that the Kähler form is closed:

$$\begin{aligned}
\bar{\partial} J &= -\partial_{\bar{y}} g_{\alpha\bar{\beta}} dz^\alpha \wedge d\bar{z}^{\bar{\gamma}} \wedge d\bar{z}\bar{\beta} \\
&= -\partial_{\bar{\gamma}} \partial_\alpha \partial_{\bar{\beta}} K dz^\alpha \wedge d\bar{z}^{\bar{\gamma}} \wedge d\bar{z}^{\bar{\beta}} \\
&= 0
\end{aligned} \tag{A.21}$$

Thus, J is $\bar{\partial}$ closed and must be an element of $H^{(1,1)}$ cohomology. Thus, these characterize the Kähler moduli space which is the reason of the number of Kähler moduli being $h^{1,1}$.

The Ricci form is defined by

$$\mathcal{R} = iR_{\mu\bar{\nu}}\mathrm{d}z^{\mu} \wedge \mathrm{d}\bar{z}^{\bar{\nu}}$$

The Ricci form of a Kähler manifold is then given by

$$\mathcal{R} = i\partial\bar{\partial}\ln g$$

Using $\mathrm{d}(\bar{\partial} - \partial) = 2\partial\bar{\partial}$, it is easy to see that $\mathcal{R}$ is closed with respect to the ordinary exterior derivative d, i.e. $\mathrm{d}\mathcal{R} = 0$. Because g is not a scalar, the equation above does not imply that Ricci form $\mathcal{R}$ is exact. We can say that the Ricci form of a Kähler manifold defines a de Rham cohomology class known as the first Chern class:

$$c_1 \equiv \frac{1}{2\pi}[\mathcal{R}] \in H^2(M, \mathbb{R})$$

This lets us define the long awaited Calabi-Yau manifold. A Calabi-Yau ***k***-fold is a compact Kähler manifold M of complex dimension k that is simply connected and satisfies the following equivalent conditions:

- M admits a Kähler metric with holonomy in $SU(k)$.
- There exists a nowhere-vanishing $(k, 0)$-form Ω on M.
- M admits a Kähler metric with vanishing Ricci curvature.
- The first Chern class $c_1(M)$ vanishes.

In the master thesis, we compactified our type IIB string theory on Calabi-Yau 3-fold which has $SU(3)$ holonomy, has a nowhere vanishing holomorphic three form Ω_3 and is a Ricci flat manifold which implies that the first Chern class vanishes.

Appendix B

String Theory Extras

B.1 Kaluza Klein Compactification

Consider a massless scalar field in 5 dimensions, the action is of the form

$$S_{5D} = \int d^5x \partial^M \phi \partial_M \phi \tag{B.1}$$

Here, $M = 0, 1, 2, 3, 4$. Notice that in addition to our usual 4 space time dimensions, we considered one extra spatial dimension. Let us set $x^4 = y$ and consider the extra spatial dimension to be a circle of radius r, i.e. $y = y + 2\pi r$. Our space time is thus $\mathcal{M}_4 \times S^1$. Since our extra-dimension is period in y, we can do a discrete Fourier transformation and our Fourier series is:

$$\phi\left(x^\mu, x\right) = \sum_{n=-\infty}^{\infty} \phi_n\left(x^\mu\right) \exp\left(\frac{iny}{r}\right) \tag{B.2}$$

Here, the Fourier expansion coefficients $\phi_n\left(x^\mu\right)$ are functions of our 4 dimensional coordinates and we thus have an infinite number of scalar fields. The equation of motion is

$$\begin{aligned}
&\partial M \partial^M \phi = 0 \\
&\sum_{n=-\infty}^{\infty} \left(\partial \mu \partial^\mu - \frac{n^2}{r^2}\right) \varphi_n\left(x^u\right) e^{iny/n} = 0 \\
&\left(\partial u \partial^\mu - n^2/r^2\right) \phi_n\left(x^4\right) = 0
\end{aligned} \tag{B.3}$$

We thus have an infinite number of Klein-Gordon equations for massive 4 dimensional scalar fields. Each of them have mass $m = \dfrac{n^2}{r^2}$ and only the zero mode is massless. We have a tower (called the Kaluza-Klein tower) of massive states due to extra dimension

S^1. They are also called KK momentum states. The effective 4 dimensional action can be obtained from the 5 dimensional action:

$$
\begin{aligned}
S_{5D} &= \int d^4x dy \sum_n \left(\partial^\mu \varphi_n \partial_\mu \phi_n^* - \frac{n^2}{r^2} |\phi_n|^2 \right) \\
&= 2\pi r \int d^4x \left(\partial^\mu \phi_0 \partial_\mu \phi_0^* + \cdots \right) = S_{4D} + \cdots
\end{aligned}
\tag{B.4}
$$

We have thus reduced the 5D action to one 4D action for massless scalar fields plus an infinite sum of massive scalar actions. If we focus on the zero modes and truncate the higher momentum states, this is dimensional reduction. If we keep all the higher momentum modes then we call this compactification.

B.2 Calabi-Yau Compactification

For Superstring theories, the 10 dimensional space M_{10} decomposes as M_4 & M_6 i.e

$$M_{10} = M_4 \times M_6$$

where, M_4 is the non-compact 4d spacetime and M_6 is internal space. Maximally symmetric solutions for M_4 are as we know Minkowski, AdS and dS ($R = 0, R < 0, \& R > 0$). We want to preserve some supersymmetry which was the point of using Calabi-Yau manifolds in the first place. Conditions for unbroken supersymmetry ($N = 1$) are as follows

- Invariant under supersymmetry means that the vacuum is annihilated by the charges Q_α (quantum mechanically) or classically speaking Q_α leaves a particular background invariant.
- Only nontrivial transformation from fermionic variations

$$\delta_\epsilon(\text{ fermion fields }) = 0$$

- If the vacuum expectation value of fermionic fields still vanish after performing a supersymmetry variation than one obtains a bosonic equation of motion that preserves supersymmetry.
- In order to obtain unbroken $N = 1$ local supersymmetry, the equation above needs to hold for 4 linearly independent $\varepsilon(x)$ forming a 4 component Majorana spinor.
- Supersymmetry transformation of the gravitino is proportional to the covariant derivative of the supersymmetry parameter ϵ

$$\delta_\varepsilon \psi_M = \nabla_M \varepsilon.$$

Non-vanishing $\nabla_M \varepsilon$. means broken supersymmetry that's why this must vanish. $\nabla_M \epsilon = 0$ is also called killing spinor equation.

- This implies unbroken $N = 1$ local supersymmetry needs a covariantly constant spinor $\epsilon(x)$ into a product structure .

$$\varepsilon(x, y) = \xi(x) \otimes \eta(y)$$

where,$\xi(x)$ is spacetime components which are Grassmann even and $\eta(y)$ is CY/internal ones are Grassmann odd.

Let us see what happens in the external space,

- The existence of covariantly constant spinor $\xi(x)$ implies the vanishing of curvature scalar.

- $[\nabla_\mu, \nabla_\nu]\, \xi = \frac{1}{4}\{\mathrm{Re}_{\mu\nu\rho\sigma}\, \Gamma^{\rho\sigma}\xi = 0$ and maximal summary ensures it.

- Supersymmetry contains the external space to be a 4d Minkowski spacetime. (AdS can be supersymmetric but it still does not solve the cosmological constant problem).

Lastly for internal manifold

- $\nabla_m \eta(x) = 0$ is covariantly constant for M.

- This leads to the integrability conditions

$$[\nabla_m, \nabla_n]\, \eta = 1/4 R_{mnpq} \Gamma^{pq} \eta = 0$$

- This implies the internal manifold is Ricci-flat i.e $R_{mn} = 0$.

This is the main point of using Ricci flat manifolds. CY are these certain kind of Ricci flat manifolds. Now, the 10 dimensional Lorentz group $SO(1, 9)$ decomposes into

$$SO(1, 9) \to SO(1, 3) \times SO(6)$$

Spinor representation $16 \in SO(1, 9)$ looks like $16 \to (2/4) \oplus (\bar{2}, \bar{4})$ where $(2)\&(\bar{2})$ are Weyl spinors of $SO(1, 3)$ which transform under $SL(2, \mathbb{C})$ and $(4)\&(\bar{4}) \to$ Weyl spinors of $SO(6)$ Thus, this compactification is non-supersymmetry as the spinors are not invariant.

In order to preserve some supersymmetry, we need to select a particular manifold with a reduced structure group i.e some sub-group of $S0(6)$. $SU(3) \subset SO(6) \cong SU(4)$.where $SO(6) \cong SU(4)$under $SU(3)$, Thus spinor representation 4 is decomposed as $4 \to 3 \oplus 1$. where 1 is singlet nowhere vanishing and globally well-defined invariant spinor.
$\to SU(3)$ holonomy (CY_3) preserves 1 supersymmetry in spacetime & $SU(2)$ holonomy preserves 2 supersymmetry in spacetime.

$$(4 \to 2 \oplus 1 \oplus 1)$$

B.3 T-duality

In appendix A, we saw compactification of extra-dimensions. In this section, we deal with bosonic string theory to aid our purpose. As discussed, bosonic string theory lives in 26 dimensional spacetime. When we compactify (on a circle) say the 25th dimension, our compact direction X^{25} has radius R and period $2\pi R$. The momentum p^{25} takes discrete values $\frac{n}{R}$ where n is an integer. Note that strings, being one dimensional objects, can wind around the compact dimension. First consider the case of closed strings. Under $\sigma \sim \sigma + 2\pi$, X^{25} does not stick to a single value, it can change by $2\pi wR$ where w is an integer and called the winding number. The level matching is modified and becomes:

$$M^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2) \tag{B.5}$$

$$nw + N - \tilde{N} = 0. \tag{B.6}$$

In addition to the oscillator modes, the spectrum receives contribution from the usual Kaluza-Klein momentum states and the winding states. Let us analyse the behaviour of the spectrum. For larger and larger values of R, momentum states become lighter (in the sense that it is easier to excite them) and winding states become heavier i.e. as $R \to \infty$ all of the winding states become infinity massive. For smaller values, the reverse is true. As $R \to 0$, all of the momentum states (with $n \neq 0$) become infinitely heavy. In a QFT setup, we have done a dimensional reduction. Lastly, notice that the mass spectrum is invariant under

$$n \leftrightarrow w \quad \text{and } R \leftrightarrow R' \equiv \frac{\alpha'}{R}$$

Therefore string theory compactified on a circle of radius R is the T-dual theory of a string theory compactified on a circle of radius R'. They basically describe the same physics. The process of going from one string theory to another is called T-dualising. Studying string dualities are a field of its own and irrelevant for our purposes. Let us now intuitively modified the results that we need. Consider an open string case. Going back into the $R \to 0$ limit, non-zero momentum states go to infinite mass but there are no winding states since open string theory does not have conserved winding around periodic dimension. So, in this limit close strings live in D dimensions and open strings live in $D-1$ dimensions. This is not a problem since open strings can be confined into a $D-1$ hyperplane called D-branes. If we again compactify the 25th dimension, the T-dualised direction X^{25} develop a Dirichlet boundary condition that is its endpoints are fixed to a hyperplane.

$$X'^{25}(\pi) - X'^{25}(0) = \frac{2\pi\alpha' n}{R} = 2\pi n R'$$

The T-dualised coordinate at the endpoints are equal up to an integral multiple of the periodicity of the dual dimension. Another beautiful artifact of the T-duality rules is that for a compactified dimension say X^M, we can switch between the embeddings X^M in the following way : $X^M = 2\pi\alpha' A^M$.

B.4 Brandenberger-Vafa argument and the one presented in [1]

Branderberger and Vafa in [5] commented on the dimensionality of spacetime and they argued that the worldsheet of two strings can meet in 4 spacetime dimensions but not in larger ones. They usually miss each other in dimensions more than 4. Similar argument can be extended to the brane-antibrane system. The argument can be generalised to p-branes in D dimensions and we find the following critical dimension:

$$D_{critical} = 2p + 2 \tag{B.7}$$

Interestingly, D_3 branes will meet and annihilate each other in dimensions smaller than or equal to 8 but miss each other in higher dimensions. So, in superstring theory with 10 space time dimensions, D_3 branes are on a stable footing. D_5 branes will meet and annihilate each other in dimensions smaller than or equal to 12. Thus, in type IIB string theory, branes with $p > 3$ will meet and annihilate very quickly and leave us with a system of D_3 branes.

B.5 Anti-brane Uplifting of KKLT Using Nilpotent Superfiled

The KKLT idea of using an antibrane was originally proposed in the paper [34] to uplift the vacua to dS. Several developments occurred after the initial proposal and in [20] they used a nilpotent chiral superfield in the antibrane for uplifting. Let us briefly sketch the idea here.

Let us denote the nilpotent chiral superfield as X (nilpotency requires $X^2 = 0$) and it can be written as:

$$X = X_0(y) + \sqrt{2}\Psi(y)\theta + \theta\bar{\theta}F(y), \tag{B.8}$$

where the only propagating degree of freedom is $\Psi(y)$. This is ensured by the nilpotency and chirality conditions. These has been exploited for inflationary purposes as highlighted in [20]. For us, in this section, the exact form of X is not strictly necessary.

In this case, the Kähler potential becomes:

$$K = -3\ln(\tau + X\bar{X}) = -3\ln\tau + \frac{\bar{X}X}{\tau} \tag{B.9}$$

The superpotential with non-perturbative correction and the superfield X is

$$W = W_0 + Ae^{-a\tau} + \rho X \tag{B.10}$$

and

$$\bar{W} = \bar{W_0} + Ae^{-a\tau} + \bar{\rho}\bar{X} \tag{B.11}$$

Using this, we can calculate the scalar potential and obtain and uplifting term similar to the analysis presented in section 2.2 on the KKLT proposal. In this case, we obtain a term as follows:

$$V = V_{KKLT} + V_{uplift} \tag{B.12}$$

where,

$$V_{uplift} = \frac{|\rho|^2}{\tau^2} \tag{B.13}$$

This nilpotent superfield formalism is used in this thesis for inflationary purposes.

B.6 Application of RG-Induced Moduli Stabilization to Inflation

The inflationary case of this RG induced perturbative stabilization mechanism follows the similar route to the one described in [32] and as discussed in chapter 2. As before, we can expand the Kähler potential as:

$$e^{-K/3} = \tau - k + \frac{h}{\tau} + \cdots , \tag{B.14}$$

In this inflationary case,

$$k = \mathfrak{K}(\Phi, \bar{\Phi}, \ln\tau) + (X + \bar{X})\mathfrak{K}_X(\Phi, \bar{\Phi}, \ln\tau) + \bar{X}X\mathfrak{K}_{X\bar{X}}(\Phi, \bar{\Phi}, \ln\tau) \tag{B.15}$$

Similar to our analysis in chapter 3, X is a nilpotent chiral superfield and Φ is a chiral superfield where ϕ acts as a potential inflaton candidate. Most general form of superpotential is

$$W \simeq w_0(\Phi) + Xw_X(\Phi, \bar{\Phi}) \tag{B.16}$$

Denoting $z^A := \{T, X\}$, the scalar potential is of the familiar supergravity form

$$V = e^K \left[K^{\bar{A}B}\overline{D_AW}D_BW - 3|W|^2 \right].$$

Leading terms in the scalar potential are:

$$V = \frac{A\,|w_x|^2}{\mathcal{P}^2} - \frac{2\,\mathrm{Re}\,(B\overline{w_X}w_0)}{\mathcal{P}^3} + \frac{C\,|w_0|^2}{\mathcal{P}^4}, \tag{B.17}$$

where (as before) $\mathcal{P} := \tau - k + \cdots$ and we keep in mind that each T derivative of k costs a power of $1/\mathcal{P}$ because k is a function of $\ln \mathcal{P}$ rather than just $\ln \tau$. The coefficients above are

$$A \simeq \frac{1}{3}\mathfrak{K}^{\bar{X}X}, \quad \frac{B}{\mathcal{P}} \simeq \mathfrak{K}^{\bar{X}X}\mathfrak{K}_{X\bar{T}} \quad \text{and} \quad \frac{C}{\mathcal{P}^2} \simeq -\frac{3\left(\mathfrak{K}_{T\bar{T}} - \mathfrak{K}^{\bar{X}X}\mathfrak{K}_{T\bar{X}}\mathfrak{K}_{X\bar{T}}\right)}{1 + 2\mathfrak{K}^{X\bar{X}}\mathfrak{K}_X\mathfrak{K}_{\bar{X}}},$$

Also, we assume $A > 0$ so the leading $|w_X|^2$ term is positive. Notice that the above expression for V in the limit $w_X = k_X = k_{X\bar{T}} = 0$ reduces to their standard RG procedure F-term potential sans inflation. There are two stages of the above potential (a function of two variables τ and ϕ): a late time dS stage and an early time inflationary stage.

In the late time stage, there should exist a configuration ϕ_0 for which $w_X(\phi_0) = 0$. Thus, the leading term is proportional to $\mathcal{P}^4$ and the analysis of section 2.4.1 holds.

In the early time or inflationary regime, the w_x is not zero and ϕ is away from ϕ_0. In general, the motion along τ is not slow-roll and it is stuck at a local minima during inflation. The effective potential for ϕ is thus,

$$V_{\text{eff}}(\phi) \simeq \frac{(A - BD_+)}{2\mathcal{P}_+^2}|w_X(\phi)|^2 = \frac{(A - BD_+)D_+^2}{2|w_0|^2}|w_X(\phi)|^4. \tag{B.18}$$

The slow-roll parameters for evolution in the ϕ direction is

$$\varepsilon = \frac{1}{2}\left(\frac{M_4 \partial V/\partial\varphi}{V}\right)^2 \sim \left(\frac{BM_4^3\tau_+}{C|w_0|}\frac{\partial w_X}{\partial\varphi}\right)^2 \sim \tau_+^3\left(\frac{BM_4^3}{C|w_0|}\frac{\partial w_X}{\partial\phi}\right)^2 \tag{B.19}$$

Thus, slow roll requires $\partial w_X/\partial\phi$ to be much smaller than order $C|w_0|/\left(BM_4^3\tau_+^{3/2}\right)$. Evaluating η,

$$\begin{aligned}\eta = \frac{M_4^2}{V}\frac{\partial^2 V}{\partial\varphi^2} &\sim \frac{1}{H_I^2}\left[\frac{B|w_0|}{M_4^2\tau_+^3}\left(\frac{\partial^2 w_X}{\partial\varphi^2}\right) + \frac{A}{\tau_+^2}\left(\frac{\partial w_X}{\partial\varphi}\right)^2\right] \\ &\sim \tau_+\left[\frac{BM_4^4\tau_+}{C|w_0|}\left(\frac{\partial^2 w_X}{\partial\phi^2}\right) + \frac{AM_4^6\tau_+^2}{C|w_0|^2}\left(\frac{\partial w_X}{\partial\phi}\right)^2\right].\end{aligned} \tag{B.20}$$

The above equation implies that slow roll requires the derivatives of w_X to satisfy

$$\left|\frac{\partial w_X}{\partial\phi}\right| \ll \frac{C|w_0|}{BM_4^3\tau_+^{3/2}} \sim \frac{\epsilon^2 M_4}{\tau_+^{3/2}} \quad \text{and} \quad \left|\frac{\partial^2 w_X}{\partial\phi^2}\right| \ll \frac{C|w_0|}{BM_4^4\tau_+^2} \sim \frac{\epsilon^2}{\tau_+^2}$$

where we used $w_0 \sim M_4^3$ and $(B/A)^2 \sim C/A \sim \epsilon^4 M_4^2$. Burgess and Quevedo in [32] then went on to study the specific case of Brane-Antibrane inflation [1] in light of the proposed stabilization. They found that their analysis had no η problem and satisfied the EFT constraints as well.

B.7 Eta Problem in String Inflation

In the initial brane-antibrane proposal of [1], the slow-roll potential got ruined after stabilization of the Kähler moduli using the superpotential. This is a common problem in string inflationary scenarios where the stabilization of the Kähler moduli using superpotential (like KKLT or LVS), the slow-roll parameter η becomes order 1. This arises due to the mechanism to fix the Kähler modulus also generates a mass term of the inflaton field ϕ and it comes about from the Kähler potential. Let us see a sample case below.

The Kähler potential very generically depends on both τ and ϕ. Considering the case, where

$$K \simeq -3\ln\left(\tau - \phi\bar{\phi}\right)$$

Constructing the F-term scalar potential,

$$\begin{aligned} V &= e^K V_0 \\ &= \frac{V_0}{(\tau - \phi\bar{\phi})^3} \\ &\simeq \frac{V_0}{\tau^3}\left[1 + \frac{3\bar{\phi}\phi}{\tau} + \cdots\right] \\ &\simeq \frac{V_0}{\tau^3}[1 + \bar{\varphi}\varphi + \cdots] \end{aligned}$$

where we have canonically normalized the inflaton field as: $\varphi \equiv \dfrac{\phi}{\sqrt{3\tau}}$. The terms inside V_0 (coming from the superpotential) has warp factors that allows small ϕ dependence which in turn allows inflation to happen. The Hubble scale is fixed by V_0 i.e $H_I^2 \simeq V/M_4^2 \simeq V_0/\left(\tau^3 M_4^2\right)$. Reinstating the M_4 factors, that ϕ inevitably has a mass contribution that is of order $m_\phi^2 \sim \widehat{V}_0/\left(\tau^3 M_4^2\right) \sim H_I^2$ which therefore contributes a factor of order unity to the second slow-roll parameter $\eta = M_4^2 V_{\varphi\varphi}/V \simeq m_\phi^2/H_I^2$. This is the famous η problem. Obviously, we do and can get rid of it by a particular finely tuned term proportional to unwarped $\varphi\bar{\varphi}$ which cancels the mass term and there is no η problem.

In our construction presented in this thesis, we do not encounter any eta problems as discussed in penultimate section of chapter 3.

B.8 Non-perturbative Corrections to the Superpotential

Non-perturbative effects to the superpotential were used in the KKLT and LVS constructions. Non-perturbative superpotential in M-theory and type IIB and heterotic string theory were calculated by Witten [31]. A class of non-perturbative effects can arise from

E3 instanton effects. Another non-perturbative effect can come from gaugino condensation through D7 branes wrapping a suitable 4 cycle in the compactification manifold. We can write both types of the non-perturbative terms as follows:

$$W_{\rm np} = \sum_b A_b \, e^{-a_b T_b} \,, \tag{B.21}$$

where T is the volume multiplet. In general, the coefficients A_b can depend on the the complex structure moduli U or the axio-dilaton S. But in most moduli stabilization scenarios we consider them to be constants since U and S are already stabilized by GVW superpotential. Coefficients in the exponent a_b are equal to $a_b = 2\pi$ for E3-instanton and $a_b = 2\pi/N$ for gaugino codensation where N being the rank of the corresponding gauge group.

Moreover, subleading corrections to the Kähler moduli can be generated by something called the poly-instanton contributions (used for inflationary purposes by Cicoli et al. in [22]). These go like:

$$W_{\rm np}^{\rm poly} = \sum_b A_b \, e^{-a_b \left(T_b + \sum_c A_c \, e^{-a_c T_c}\right)} \tag{B.22}$$

Lastly, there are also non-perturbative corrections to the Kähler potential but it outside the scope of this discussion.

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