ALMA MATER STUDIORUM – UNIVERSITÀ DI BOLOGNA

#### SCHOOL OF ENGINEERING AND ARCHITECTURE

MASTER'S DEGREE IN TELECOMMUNICATIONS ENGINEERING

## AI-BASED CHARACTERIZATION OF MICROSERVICE RESPONSE TIME IN CLOUD-NATIVE DEPLOYMENTS

Master Thesis

in

Laboratory of Advanced Networking M

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> SESSION III ACADEMIC YEAR 2021/2022

To my family, friends and everyone who has supported me on this journey. Special thanks to my supervisor and co-supervisor for their support and valuable advice.

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## Abstract

This thesis addresses the important topic of future networks and new technologies for cloud-native deployment and the exploitation of artificial intelligence (AI) for characterizing microservice response times. In particular, the characterization relies on the use mixture density networks (MDN), a feedforward neural network (FNN) architecture to approximate the distributions of a phenomenon, applied to generalize microservices response time distributions depending on system parameters.

The AI approach is motivated by the difficulty of using theoretical queueing models to characterize the full service. The thesis also explores the possibility of using the distributions obtained through MDN models to make simulations with a digital twin of the system.

The experiments show the approximation effectiveness of this approach for characterizing the response times and to make use of the results in a digital twin.

Finally the thesis makes considerations about possible improvements and future works in this direction.

ABSTRACT

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## Chapter 1

## Introduction

#### **1.1** Motivations

The advent of 5G technologies has opened the way for new applications and services, but telecommunications research does not stop there. At the same time cloud computing has become one of the most popular and requested technology services in the world, making possible the availability of computing resources on a large scale; this service has become possible thanks to virtualization, which allows computing resources to be created and managed flexibly and efficiently. Today, virtualization through containers has become a fundamental tool to manage and deploy applications in the cloud.

The networks of the future will have several key characteristics, including: higher speed, reliability, and sustainability. In fact, the networks of the future will be able to offer even higher connection speeds than today's networks, which will enable them to support data-intensive applications such as augmented and virtual reality, telemedicine, and more. Networks also will need to be highly reliable, capable of handling large amounts of data and traffic securely. Flexibility will be needed to adapt quickly to market changes and new technologies.

Other key point will be sustainability and security: the networks of the future will need to be sustainable, minimizing environmental impact and using renewable energy sources, and they will need to ensure a high level of security, protecting user data from cyber attacks, hacking and fraud.

With the evolution to 6G technology it is expected an increase in the amount of data and applications that need to be managed and distributed,

bringing to new challenges and opportunities to address.

In this context, the adoption of machine learning and implementation of digital twins could be the key to achieving superior performance and more sophisticated and customized services.

Machine learning can provide predictive models for network performance, while digital twin allows the network to be simulated virtually, providing a secure and cost-effective testing environment.

The integration of these technologies could lead to better network management and performance optimization, thereby increasing the efficiency and the offered quality of services .

This thesis aims to explore the potential of implementing machine learning and digital twin to telecommunications, in order to understand the challenges and opportunities that these technologies can offer and contribute to their implementation and development.

The remainder of this thesis is structured as follows. The next section 1.2 is a brief summary of inspirational articles for this thesis, then chapter 2 presents the main concepts and technologies related to the cloud-native deployment. Chapter 3 introduces the notions concerning artificial intelligence and the machine learning methodologies applied in the experimental part of this thesis while chapter 4 gives a description of the system under analysis and collects the experiments. Finally, chapter 5 summarizes the work and gives suggestions for possible improvements and new approaches to be evaluated.

### **1.2** Related works

The literature provides a valuable perspective for evaluating the current state of the art and identifying future opportunities.

In particular, [1] accounts for the importance of Kubernetes, an open-source orchestration platform for automatic deployment, scaling and management of containerized applications. Solutions for evaluating the behaviour of the systems with different configuration parameters in Kubernetes are needed to effectively deal with resources scheduling. In this direction [2] argues the performance assessment during the investigation of possible alternative deployments, requiring new service management tools providing *what-if scenario* analysis functions. The use of simulations tools to drive decisions is experimented as a solution to identify the most convenient options.

#### 1.2. RELATED WORKS

Finally, [3] underlines the importance of dynamic and intelligent managing of heterogeneous resources such as communication, computing, energy, and storage to improve resource utilization efficiency and satisfy quality of service requirements. Digital twin technology is proposed as a solution to achieve this, indeed many components need to be emulated (e.g., processing nodes and communication network devices) with requirements on latency and accuracy to maintain real-time consistency with physical systems.

### Chapter 2

## Overview

This chapter presents the main concepts and technologies related to cloudnative computing, such as virtualization, virtualization with containers and service composition.

### 2.1 Cloud-native deployment

Cloud-native service deployment refers to the practice of deploying applications and services within a cloud environment using architectures and methodologies that take advantage of cloud computing features. Specifically, cloud-native deployment focuses on creating highly scalable, reliable and flexible services that can be easily managed and updated in a cloud environment.

This requires the adoption of a number of specific development and management practices, including the use of microservices, service-oriented design, virtualization, containerization, automation, error handling and self-healing. In addition, the use of cloud-native technologies and platforms, such as Kubernetes, enables greater agility and scalability, as well as the ability to deliver services quickly and reliably.

The concept of *DevOps* is strictly related to the cloud-native context. Indeed DevOps is an organizational and development culture that involves collaboration between development and operations teams to improve the efficiency and quality of application development and deployment processes. DevOps focuses on continuous integration (CI) and continuous release (CR) to ensure the rapid and reliable release of applications.



Figure 2.1: Cloud-native pillars. [4]

### 2.2 Virtualization

The virtualization is a technique aiming to create abstract versions of computer hardware and network resources to make them available to a software as *virtual resources*. This technique allows the implementation of *virtual machines* that act like a real computer. As a matter of fact, a virtual machine is equipped with its operating system and applications. The virtualization of hardware requires an *hypervisor*, a software which manages the physical resources and isolates them from the virtual environment; this task is reached by sharing the resources.

In traditional virtualization there are two kind of hypervisors:

- Type 1 (native): in this paradigm the hypervisor directly schedules the virtual machines (VM) resources to the hardware, no OS is present on the host machine. An example of this kind of hypervisor is the KVM (Kernel-based virtual machine).
- Type 2 (hosted): in this paradigm the hypervisor is a process executing on a OS.

The benefits introduced by the use of the virtualization are:

- Cost reduction: virtualization allows to reduce the number of physical computer by executing many virtual machines on a single physical one. This reduces the capital expenditures, and the operational costs related to cooling and maintenance.
- Flexibility: any virtual machine can be executed on any available server

on the network, it is possible to move a VM from a server to another in case of need. Moreover, it is possible to scale VM quickly.

- Reliability: it is possible to make checkpoints and backups of VM in such a way any VM can be easily restored. This makes virtual system fault resistant.
- Control independence and DevOps: developers can easily start a VM without having an impact on the production environment since in virtualization the VMs are independent.

### 2.3 Virtualization with containers

In the virtualization context, the use of *containers* is a effective technique to manage applications and services in a shared environment. The definition of container can be summarized in "a single executable package of software". Differently to traditional virtualization, where every VMs have their own OS, the use of containers is based on the host kernel to run applications and so no hypervisor is needed. To make containers work, the host kernel has installed a runtime engine to enable containers to interact with it [5].

The use of host kernel makes the containers "lightweight", meaning the image of a container is typically in the order of megabyte, much smaller than the image of a VM that is in the order of gigabyte. Moreover a container does not embed anything bigger than an application and its running environment, for this reason they are often used to implement only a single function performing specific task, known as *microservice*.

The "lightweight" architecture of containers enables the possibility to easily move and run an application in any environment or infrastructure's operating system. This property turns out to be very useful to deploy cloud-native apps (i.e., collection of microservices whose composition provides a more complete service).

An example of containerization technology is *Docker*, enabling the creation and use of Linux containers.

The virtualization with containers extends the benefits introduced by the traditional virtualization, with:

• Portability: possibility to run uniformly and consistently across any platform or cloud being abstracted away from the host operating system;



Figure 2.2: Virtualization vs. Containerization. The structure of container is simpler and requires less overhead. [6]

- Speed and efficiency: the lightweight property of containers makes the start-up faster;
- Easy management: a container orchestration platform can automate the installation and handling of containers' workloads and services (e.g., rolling out new versions of applications, provide logging and debugging).

#### 2.3.1 Microservices and service composition

The microservice architecture is an approach to break a complex application into a collection of many specialized services, called *microservices*.

The microservice architecture is opposed to the monolithic application approach where components such as user interface (UI), business logic and data access functions are implemented in a single software application responsible for every step needed to complete a service.

The main benefit introduced by the in microservice architecture is the possibility to focus the development phase on a specific task, without having an impact on the whole service, making the development, test and deployment of applications faster.

Microservices works well if combined with containerization, since they can inherit all the benefits of virtualization with containers.

A complete service is obtained by the composition of multiple microservices, this objective is obtained through Application Program Interfaces (APIs) and REST interfaces (such as HTTP).



Figure 2.3: Monolithic vs microservices.[6]

In case of large compositions it is possible to adopt a *service mesh*, which is a facility dedicated to the service-to-service communication between services or microservices using proxies: when microservices needs to communicate the requests are routed between them through proxies that are integrated in their own infrastructure layer.

The service mesh captures every aspect of the inter-service communications, without it the communication needs to be defined with specific rules in the implementation of the microservice itself, making the management more difficult.

#### 2.3.2 Kubernetes

In the scope of container orchestration, *Kubernetes*, known also as k8s, is a popular open source framework that automates container-related functions[7]. It works with common containers engines, such as Docker, to schedule and automate the deployment, management and scaling of containerized applications.

The Kubernetes software is made up by multiple components, including:

• Working nodes: they are working machines on which containers are executed.

- Clusters: it is a set of working nodes;
- Control plane components: they make decisions about the cluster, detect and respond to cluster event;
- Node components: they are the components running on each working node, maintaining running pods (components of the application work-load) and enabling the Kubernetes running environment.

Kubernetes features are:

- Service discovery;
- Load balancing;
- Automated rollouts and rollbacks;
- Self healing;
- Secret and configuration management.

#### 2.4 6G

The Sixth Generation (6G) of mobile networks is currently at the conception phase, this new technology is expected to improve the 5G networks with [8, 9]:

- Higher throughput;
- Ultra low-latency;
- Advanced security;
- Environmental sustainability.

In the context of cloud-native, it is expected that 6G will offer new advanced applications thanks to network function virtualization and service orchestration. In particular, the virtualization with containers and the microservice architecture introduce new opportunities to the development and provisioning of applications on 6G networks.

In addition, it is expected that 6G will make use of artificial intelligence (AI) to improve the efficiency of the network, for instance by optimizing network resource usage, implementing new smart beamforming techniques and customize the user experience based on its preferences.

## Chapter 3

## Methodologies

In this chapter the methodologies applied in the experimental part of the thesis are introduced. Before dealing with model theory, some general concepts related to artificial intelligence and machine learning are covered.

### 3.1 Artificial intelligence and machine learning

The artificial intelligence (AI) refers to the ability to design techniques that enable machines to mimic human intelligence, meaning the ability to perceiving and inferring information.

Machine learning (ML) is a field of studies of artificial intelligence, it exploits statistical methods to give computers the ability to learn without being explicitly programmed.

ML algorithms are divided in different categories:

- Supervised learning;
- Unsupervised learning;
- Reinforcement learning.

Supervised machine learning is referred to algorithms that maps an input x on an *output label* y. It learns from being given "right answer" examples. Applications of this category include speech recognition, translation, advertising, self-driving cars etc...

The most common algorithms of this class are:

- Regression, aiming to predict a number among infinitely many possible outputs;
- Classification, aiming to predict a class/category among a small number of possible outputs (classes/categories are not real number, they belong to a finite set).

Unsupervised machine learning on the other hand works on data which is not associated to any output label, this kind of ML aims to find hidden structures in the data. Given some input data, the algorithm can group them in different clusters (clustering applications, like google news, DNA microarray types, marketing segmentation).

Examples of unsupervised ML algorithms are:

- Clustering, aiming to group similar data points together (e.g., Google News, DNA microarray types, market segmentation);
- Anomaly detection, aiming to find unusual data inputs/events;

#### **3.2** Neural networks and deep learning

Artificial neural networks (NN) are models to perform machine learning. The name "neural network" derives from the fact this architecture is loosely inspired on mammal's brain neural networks.

The architecture of NN is composed by a collection of processing nodes (or *units*) that are densely interconnected. Units are organized in layers, whose configuration provide a specific transformation of the input data. Data travel from the first layer, the *input layer*, to the last one, the *output layer*, passing through the intermediate layers. Data travelling in the NN can have different types (e.g., real numbers or categorical data) and can be *labelled* (i.e., meaning of data is known) or *unlabelled* (i.e., meaning of data is unknown). Units of a layer are interconnected with the ones of the subsequent layer by means of *edges*. Each edge is characterized by a weight, a coefficient that is multiplied to the data passing through it and increasing or decreasing the impact of that data on the final output. All units are characterized by an *activation function* and a *bias*.

#### 3.2. NEURAL NETWORKS AND DEEP LEARNING

The first and simplest model of NN representing a single unit is the *Perceptron*, demonstrated by Frank Rosenblatt in 1957. The aim of the Perceptron was to perform binary classification.



Figure 3.1: Artificial neuron.

Figure 3.1 shows a network with a single hidden layer and one unit. The output of the generic "j" unit is related to the inputs by the relationship:

$$o_j = \phi(\theta_j + \sum_{i=1}^n w_{ij} x_i)$$

Where:

- $x_i$  is the i-th input;
- $w_{ij}$  is the weight of the i-th input edge;
- $\theta_j$  is the bias;
- $\phi(\cdot)$  is the activation function.

Common choices for the activation function are:

- Logistic;
- Tanh;
- Sigmoid;

- Step;
- Linear;
- Rectified linear unit.

The NN composed by multiple intermediate units and layers are known as *Deep Neural Networks* (DNN) and the algorithm to train them is usually referred as to *Deep learning*. The layers placed between the input layer and the output layer are called *hidden layers* since their results can't be seen directly. Figure 3.2 represent a DNN.



Figure 3.2: DNN with 2 hidden layer of 4 units each, 3 units input and 1 unit output.

#### 3.2.1 Feedforward neural networks

*Feedforward neural network* (FNN) is an architecture of artificial neural network where connections between the nodes do not form a cycle, this kind of networks are the basis for most complex networks architecture . It is a general model to approximate non-linear mappings between two sets of variables [10]:

$$\hat{y} = f(\bar{x}) \tag{3.1}$$

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Where:

- $\bar{x}$  is a vector of inputs (or *features*);
- $\hat{y}$  is the prediction for y (the target).

This model can be extended to multivariate prediction by implementing multiple output neurons. FNN can have multiple hidden layers; the number of hidden layers and the approximation capability concern the *Universal approximation theorem* [11]: a neural network with suitable number of hidden layers and units is a "Universal approximator".

The training of a NN is the process of determining the values of the weights that minimize the error in the approximation. The training is performed on a training set (a set of examples, previously collected, on which the task is learned); feedforward NN are usually trained with the *Back-propagation algorithm*. The error of a neural network is provided by the *error function* (sometimes called cost function or loss), which is specific to the task to be approximated. Once the network is trained, the network is able to process new data.

Feedforward NN are suitable models to solve regression and classification problems. In the regression case the task is learned by minimizing the *mean-square error function* between the target and the prediction given features over the training set.

The mean-square error function is defined as:

$$E(\bar{w}) = \frac{1}{2m} \sum_{i=1}^{m} [f(\bar{x}^{(i)}; \bar{w}) - t^{(i)}]^2$$
(3.2)

Where:

- $\bar{x}^{(i)}$  is the i-th input vector in the training set;
- *m* is the cardinality of the training set;
- $t^{(i)}$  is the i-th target in the training set;
- $f(\bar{x}^{(i)}; \bar{w})$  is the prediction of the target in function of the i-th input and weights  $\bar{w}$

The larger is the training set, the less is the bias and variance of the model. In the limit case of infinite training set the mean-squared error function turns out to be:

$$E = \lim_{m \to \infty} \frac{1}{2m} \sum_{i=1}^{m} [f(\bar{x}^{(i)}; \bar{w}) - t^{(i)}]^2 = \iint [f(\bar{x}; \bar{w}) - t]^2 p(t, \bar{x}) dt d\bar{x} \qquad (3.3)$$

The error can be minimized in function of  $f(\bar{x}; \bar{w})$  by differentiation:

$$\frac{\partial E}{\partial f(\bar{x};\bar{w})} = 0 \tag{3.4}$$

By substituting 3.3 in 3.4 and remembering  $p(t, \bar{x}) = p(t|\bar{x})p(\bar{x})$  the approximating function that minimize the error (i.e., the optimal estimator) is:

$$f(\bar{x};\bar{w}) = \mathbb{E}[t|\bar{x}] \tag{3.5}$$

This means the network approximates the conditional average of the target (conditioned to the input vector).



Figure 3.3: The MSE estimator function on a set of training points.[12]

In the case of classification, the task is learned by minimizing the crossentropy error function and the approximated average gives the posterior probability of a class conditioned on the input features.

#### **3.2.2** Mixture density networks

Feedforward conventional neural networks for regression (prediction of a continuous variable) sometimes provide a limited description of the approximation of the target value, the conditional average of several correct values is not necessarily the correct prediction of a target but it only minimizes the sum-of-squared error function. To have a more complete description of the target, in particular for target with high variability, it is possible to model the *conditional probability distribution* of the target data, given the input vector.

In order to model the conditional probability of the target, a model combining a conventional FNN and a mixture density model can be adopted. This model is called *Mixture Density Network (MDN)* [12] and provides a general framework to approximate conditional density functions of targets by modelling the probability parameters as functions of the features.

In case the target follows a Normal distribution the parameters of its distribution would be  $(\mu, \sigma)$ , it is possible to write:

$$p(t|\bar{x}) = p(t|\mu(\bar{x}), \sigma(\bar{x})) \tag{3.6}$$

The MDN aim to model the conditional probability density as a linear combination (*mixture model*) of kernel functions:

$$p(t|\bar{x}) = \sum_{c=1}^{C} \alpha_c(\bar{x})\phi_c(t|\bar{x})$$
(3.7)

Where:

- $p(\bullet)$  is the probability density function (pdf) of the target variable;
- C is the number of kernel components in the mixture;
- $\alpha_c(\bar{x})$  is the mixing coefficient of the c-th kernel, it represents the prior probability of that kernel conditioned on  $\bar{x}$ ;
- $\phi_c(t|\bar{x})$  is the c-th kernel, it represents the conditional density function of the target t.

In the case of Normal kernel it is:

$$\phi_c(t|\bar{x}) = \frac{1}{\sqrt{2\pi}\sigma(\bar{x})} \exp\left\{\frac{|t - \mu_c(\bar{x})|^2}{2\sigma_c^2(\bar{x})}\right\}$$
(3.8)

Each kernel component is then characterized by a set of three parameters dependent on  $\bar{x}$ :  $(\alpha_c(\bar{x}), \mu_c(\bar{x}), \sigma_c(\bar{x}))$ .

The assumption of Normal kernel allows to approximate every probability density function with arbitrary accuracy [12]; this property is particularly convenient in case of multivariate target where the hypothesis of statistical independent component doesn't hold. However, the model can be generalized to different kernel choices (e.g., Exponential, Gamma or other distributions).

The mixture parameters, namely the mixing coefficients  $\alpha_c(\bar{x})$ , the means  $\mu_c(\bar{x})$  and the standard deviations  $\sigma_c(\bar{x})$  are assumed to be unknown continuous functions of  $\bar{x}$ .

In order to model the unknown functions:

- the first stage of the model is a *conventional FNN*, with input  $\bar{x}$ , weights  $\bar{w}$  and providing a parameters' vector  $\bar{y}(\bar{x}; \bar{w})$  as output (3.1);
- the output vector of the first stage is provided as input of a second stage represented by a *mixture model*.

In the mixture model the input parameters  $\bar{y}(\bar{x}; \bar{w})$  are processed by means of appropriate activation functions to model the mixture parameters (i.e., mixing coefficients, means and std deviations). The final output of the model turns out to be the conditional probability density (3.7). The combination of the two stages is referred to as a *Mixture Density Network* (MDN). The basic structure is represented in figure 3.4



Figure 3.4: Structure of a MDN

The adoption of this model requires the definition of some hyper-parameters (i.e., fixed parameters defining the specific implementation of the NN, they are not learned during training) such as the number of mixture components C, the kernel functions and the number of hidden layers and units.

The mixing coefficients  $\alpha_c(\bar{x})$  must satisfy the condition:

$$\sum_{c=1}^{C} \alpha_c(\bar{x}) = 1 \tag{3.9}$$

It can be achieved by choosing the *softmax* activation function for these outputs and the generic i-th mixing coefficient is:

$$\alpha_i = \frac{\exp(y_{\alpha_i})}{\sum_{c=1}^C \exp(y_{\alpha_c})} \tag{3.10}$$

where  $y_{\alpha_i}$  represents the FNN output related to the i-th mixing coefficient.

The variances  $\sigma_c(\bar{x})$  must satisfy the condition:

$$\sigma_c(\bar{x}) > 0 \tag{3.11}$$

It can be achieved by using *exponential* activation function:

$$\sigma_i = \exp(y_{\sigma_i}) \tag{3.12}$$

where  $y_{\sigma_i}$  represents the FNN output related to the i-th variance. The mean values  $\mu_c(\bar{x})$  are not constrained by any condition, they can be obtained by the FNN output with linear activation function:

$$\mu_i = y_{\mu_i} \tag{3.13}$$

The parameters  $\bar{w}$  of  $\bar{y}(\bar{x};\bar{w})$  are learned during training of the MDN through an examples set  $\{\bar{x}^{(q)}, t^{(q)}\}$  of cardinality m. This can be obtained by maximum likelihood estimation (MLE), indeed the objective of MLE is to find the set of parameters for which the observed data (the training set) have the highest joint probability. By assuming the training examples are drawn independently from the pdf given by 3.8, the likelihood function of the set can be written as:

$$\mathcal{L} = \prod_{q=1}^{m} p(t^{(q)}, \bar{x}^{(q)}) = \prod_{q=1}^{m} p(t^{(q)} | \bar{x}^{(q)}) p(\bar{x}^{(q)})$$
(3.14)

The maximum likelihood estimate is:

$$\widehat{\bar{w}} = \arg\max_{\bar{w}} \mathcal{L}(\bar{w}) \tag{3.15}$$

From the likelihood it is possible to derive the error function:

$$E = -\log \mathcal{L}(\bar{w}) = -\sum_{q=1}^{m} \log p(t|\bar{x}^{(q)})$$
(3.16)

The 3.16 is called negative log-likelihood (NLL), in its expression the  $p(\bar{x})$ factor is neglected since it does not depend on the NLL parameters. Its minimization is equivalent to the maximization of the likelihood.

By taking into account the mixture model 3.7, the NLL becomes:

$$E = \sum_{q=1}^{m} E^{(q)} = -\sum_{q=1}^{m} \log \sum_{c=1}^{C} \alpha_c(\bar{x}^{(i)}) \phi_c(t^{(i)} | \bar{x}^{(i)})$$
(3.17)

where  $E^{(q)} = -\log \sum_{c=1}^{C} \alpha_c(\bar{x}^{(q)}) \phi_c(t^{(q)} | \bar{x}^{(i)})$  is the error contribution of the q-th example.

Back-propagation procedure is used to minimize the error function, for this purpose the gradient of the error function with respect to the FNN output needs to be computed as reported in [10]. The software implementation of the back-propagation algorithm for this model can be inspired on regression FNN based on sum-of-squares error function. The modification of the error function is required to apply standard optimization procedures such as gradient descent.

The derivative of the error on the q-th example with respect to a generic weight w is:

$$\frac{\partial E^{(q)}}{\partial w} = \sum_{i} \frac{\partial E^{(q)}}{\partial y_i} \frac{\partial y_i}{\partial w} = \sum_{i} \delta_i^{(q)} \frac{\partial y_i}{\partial w}$$
(3.18)

where  $\delta_i^{(q)} = \frac{\partial E^{(q)}}{\partial y_i}$  is the derivative of the error with respect to the i-th FNN output that is back-propagated in the network. The partial derivative can be approximated by finite difference

$$\frac{E(w+\epsilon) - E(w)}{\epsilon} = \frac{\partial E}{\partial w} + \mathcal{O}(\epsilon)$$
(3.19)

20

At the end of the training, the MDN is able to approximate the conditional density function of the target data given the input feature, allowing to have a probabilistic description of the data generation process. The MDN results are useful to derive the specific statistics. By exploiting the 3.7 and linearity the conditional mean is given by

$$\mathbb{E}[t|\bar{x}] = \sum_{c=1}^{C} \alpha_c(\bar{x}) \int t \,\phi_c(t|\bar{x}) \,dt = \sum_{c=1}^{C} \alpha_c(\bar{x}) \,\mu_c(\bar{x}) \tag{3.20}$$

and the conditional variance

$$\operatorname{Var}(t|\bar{x}) = \mathbb{E}\left[(t - \mathbb{E}[t|\bar{x}])^2 \,|\, \bar{x}\right] \tag{3.21}$$

$$= \sum_{c=1}^{C} \alpha_c(\bar{x}) \left\{ \sigma_c^2(\bar{x}) + \left[ \mu_c(\bar{x}) - \sum_{j=1}^{C} \alpha_j(\bar{x}) \, \mu_j(\bar{x}) \right]^2 \right\}$$
(3.22)

### **3.3** Goodness of fit

The choice of a suitable model among several possibilities needs to be accounting its performance with a metric. The *goodness of fit* of a statistical model is a measure of how well a model fits data, typically this measure allows a data analyst to summarize the discrepancy between a sample of observed values and the predictions made by the model under evaluation. Examples of goodness of fit are Kolmogorov-Smirnov test (KS-test) and Bayesian information criterion (BIC).

#### 3.3.1 Kolmogorov-Smirnov test

Kolmogorov-Smirnov test is a non-parametric test used to compare a sample with a reference probability distribution (one-sample K-S test), or to compare two samples (two-sample K-S test)[13, 14]. The one-sample K-S test aims to quantify the distance between the empirical cumulative distribution function (eCDF) of a sample and the cumulative distribution function (CDF) of a reference distribution. For a random variable X with CDF F(x) and a sample  $(X_1, X_2, ..., X_n)$  with eCDF  $F_n(x)$  the test's hypothesis are:

$$H_0: F(x) = F_n(x), \ \forall x \tag{3.23}$$

 $H_1: F(x) \neq F_n(x), \text{ for some } x$  (3.24)

where the eCDF is defined as:

$$F_n(x) = \frac{\text{number of (elements in the sample \le x)}}{n}$$
(3.25)

The test's statistic (K-S distance) is defined as:

$$D_n = \sup_{x} |F_n(x) - F(x)|$$
 (3.26)

Under the null hypothesis, the K-S distance (3.26) is distributed according to the Kolmogorov distribution and converges to 0 in case  $n \to \infty$ . For a large sample (n > 50) goodness of fit test is made by setting a critical value  $K_{\alpha}$  on the Kolmogorov distribution and the null hypothesis is rejected at significance level  $\alpha$  if:

$$D_n > \frac{K_\alpha}{\sqrt{n}} \tag{3.27}$$

It is possible to perform the test with different variants of the null hypothesis, as implemented in many software libraries.

### 3.4 Overfitting, underfitting and regularization

The *overfitting* is a problem coming out in machine learning when a model becomes too specific to the training set, this leads to a model which seems very accurate at the end of the training phase (low value of the error function), but actually it shows a lack of generality being unable to fit new data. It is used to say the model is characterized by a *high variance* since slight changes on the training set alter the model very much.

The *underfitting*, as opposite to overfitting, is a problem coming out when the model is not able to fit well the training set, showing a high value of the cost function. It is used to say the model is characterized by a *high bias*.

In general underfitting happens when the model is too simple and it is not able to extract the framework inside the training data, reflecting the same issue on the new data. For instance, underfitting happens when a model has too few parameters. Overfitting happens when the model is too complex, for instance, it occurs when the model has too many parameters.

Techniques to overcome overfitting and underfitting includes:

- Collection of more training examples: it reduce the variance but sometimes it is not practicable because collect more data could be expensive.
- Feature selection: when the training set includes few examples n but a higher number of features m > n the overfitting may arise if all the features are accounted in the model. In order to avoid overfitting it is possible to select only the most relevant features (i.e., the ones showing higher correlation with the target). The main disadvantage of this technique is that possible important information may be lost.
- Regularization: in case a high number of features are included in the training set, the regularization technique aims to shrink the parameters related to the features that specialize too much the model, reducing their effect without completely eliminating it.

### 3.5 AI model's development environment

The experiments in the following parts of this thesis were developed by using different tools available in the Python language environment.

Python is a high-level object-oriented programming (OOP) language. The open-source, general-purpose and simplicity properties of this language are the strengths which have ensured its diffusion in many scientific fields, such as data science and machine learning. The Python's tools applied in the thesis belongs to different libraries such as:

- NumPy for scientific computing [15];
- Scipy for statistical functions [16];
- Matplotlib for graphical visualizations [17];
- Pandas for data manipulation [18];

• TensorFlow for machine learning [19].

In the work the major contribution is given by TensorFlow. It is an open source framework developed by Google for building machine learning models, such as neural networks, decision trees, and regression tasks [19]. This tool offers many functionalities for the training and validation of models as well as instruments for the elaboration of results.

In TensorFlow deep networks are developed with the *Keras* API. The first step in the development requires the definition of the structure (i.e., number of layers, neurons and activation functions). Then it is necessary to make available data (i.e., import data in a "dataframe" structure) defining features and targets and performing features normalization to optimize the convergence of the model during training. When data is available, it is necessary to divide the dataset in three disjointed groups:

- Training set: the portion of the dataset used to train the model;
- Validation set: the portion of the dataset used to check the absence of overfitting during training;
- Test set: the portion of the dataset used after the training to check the generalization performance.

When results are available, they can be elaborated and summarized with charts and tables. The results of the thesis work are manipulated with the help of TensorFlow Probability, a library containing probabilistic tools that can be integrated in the models.

## Chapter 4

## Experiments

This chapter aim to show the application of methodologies described in the 3 on a real system. A description of the real system and its properties is provided to clarify the context of the work.

# 4.1 Cloud-native deployment scenario under analysis

#### 4.1.1 System description

The cloud-native deployment scenario under analysis in the experimental work of this thesis is represented by means of a *multi-layered queue system*. As a matter of fact, cloud service provisioning can be efficiently implemented using a multi-layered system approach.

The general implementation of a multi-layered queue system can be described by:

- requests: the system is fed with incoming requests sent by users which can be summarized by their average request rate  $\lambda[req/s]$ .
- replicas: they are multiple processing entities providing a microservice (i.e., an independent portion of processes related to a more complete service); as they are able to operate in parallel, a section with  $n_{rep}$  replicas can contemporary execute up to  $n_{rep}$  instances of microservices. The processing time related to a microservice performed by a replica is a random variable  $T_{ms}$  with unknown distribution.

- *queues*: each replica in the system has its own queue in which requests are queued in case they can't be immediately served.
- *load balancer*: it distributes the incoming requests to the replicas in case many replicas are present; the simplest way to do it is performing equal balancing.

In a cloud-native deployment a service is obtained through the chaining of several microservices, this composition is strictly related on the kind of service provided to the users.

With reference to a particular service, the structure described above can be stacked as many times as the number of microservice that make up the overall service, where each layer is responsible for a specific microservice. Figure 4.1 shows an example.



Figure 4.1: Example: a two-layered queue system with  $n_{rep}$  replicas for both layers. In the first layer, on average, the load balancer (LB) equally distributes requests among replicas.

The multi-layered queue system during its operations (e.g., load balancing, queuing, microservices processing) introduces a random response time to the users, this is referred as to TTR (time-to-resolution). The distribution of the time-to-resolution is unknown and it depends on the kind of service and the structure of the system.

The specific implementation of the system under analysis in the experiments' part is given by a two-layered queue system with different values of  $n_{rep} \in \{1, 2, 3, 4\}$ . The system is made up by using the *Kubernetes* platform and the replicas are deployed by means of *containers* distributed on a cluster.

The designated service for that system is an image processing application, in particular:

- the first layers is dedicated to an image pre-processing microservice (ms1) to simplify the operations of the second microservice;
- the second layer is dedicated to the actual image processing microservice (ms2).

The random processing times of a generic replica in the first and second layers are indicated as  $T_{ms1}$  and  $T_{ms2}$  respectively.

#### 4.1.2 System data collection

The described system has been tested on different conditions to collect data about time-to-resolutions and microservices processing times. The data collection is fundamental to characterize the system and aim to model its target quantities exploiting machine learning.

Data were collected performing multiple experiments on the system with:

- different number of replicas  $n_{rep} \in \{1, 2, 3, 4\}$
- for each number of replicas were produced n = 5000 incoming requests to the system with average request rate  $\lambda \in \{1, 2, ..., 30 * n_{rep}\}[req/s]$ in order to compare cases with different number of replicas but with the same average request rate per replica.

At the end of the experiment 300 data sets of 5000 samples were obtained for every target.

### 4.2 Motivations of the AI-based approach

The multi-layered queue system modelling and, in particular, the characterization of the response times can be done with two different approaches: adoption of theoretical-analytical queue modelling or data-based modelling [20].

The analytical approach aims to model the system exploiting queuing theory, which is a set of mathematical results that can be applied to the components of the system to obtain a characterization of its behavior. This approach is still an open field of research in telecommunications engineering; it is suitable to study simplified cases (e.g., stationary conditions and exponential service time), but can become hard to deal with in the study of more complex systems. The limitation of this approach justify the interest in data-based methods such as machine learning algorithms and the exploitation of simulations.

#### 4.3 Models

In this section the models for the characterization of the system described in the section 4.1.1 with the application of the methodologies described the Chapter 3 are presented.

#### 4.3.1 Model 1: characterization of microservice ms1 response time

The first proposed model aim to characterize by means of a mixture density network (MDN) the microservise ms1 response time, given the knowledge of the request rate to the system and the number of available replicas.

Mathematically the objective of the model is to approximate:

$$p(t_{ms1}|\lambda, n_{rep}) \tag{4.1}$$

Where:

- $p(\bullet)$  indicates the probability density function (pdf);
- $t_{ms1}[s]$  is the ms1 response time;
- $(\lambda, n_{rep})$  are respectively the request rate [req/s] and number of available replicas for ms1.
The approximation given by the model is useful to generalize the behaviour of the replicas in the first layer, perform analysis and exploit the pdf to make simulations.

The model is a DNN implemented with a Keras sequential model and TensorFlow probability, with the following characteristics:

- 2 input neurons: one input neuron for  $\lambda$  and one for  $n_{rep}$ ;
- 4 mixture components: the approximated pdf is the superposition of 4 Gamma's pdfs;
- 12 output neurons: the outputs represent the parameters of the components in the mixture model, every Gamma's pdf is characterized by 3 parameters (the weight in the superposition, the concentration and the rate)
- 2 hidden layer: each hidden layer has 7 neurons.

The hidden neurons are set with ReLU (Rectified Linear Unit) activation function. The activation functions for the output neurons depend on the parameter they represent.

The Gamma pdf is:

$$\phi(x,a,b) = \frac{x^{a-1}e^{-bx}b^a}{\Gamma(a)} \tag{4.2}$$

where a > 0 is the concentration and b > 0 is the rate.

During initial trials the Gamma and Normal pdfs were tried. The Gamma pdf was chosen because it provided the best fit among the two, this best fit can be explained by the fact that the Gamma distribution generalizes the Exponential distribution, often used to model times. In particular the Gamma distribution has support on the positive time axis, instead the Normal distribution has support on the whole real axis and so its tails considers also negative times that can't exists

At the end of the training process the ms1 response time is characterized in the following way:

$$p(t_{ms1}|\lambda, n_{rep}) = \sum_{c=1}^{4} \alpha_c(\lambda, n_{rep}) \phi(t_{ms1}, a_c(\lambda, n_{rep}), b_c(\lambda, n_{rep}))$$
(4.3)

```
InputLayer = Input(shape=(2,)) # 2 input neurons
1
2
   Norm_layer = Normalization (axis = -1)
3
   Layer_1 = Dense(7, activation="ReLU")(Norm_layer(InputLayer))
4
   Layer_2 = Dense(7, activation = "ReLU")(Layer_1)
5
   alpha = Dense(4, activation="softmax")(Layer_2)
6
   concentration = Dense(4, activation=lambda x: tf.nn.elu(x) + 1 + 1
\overline{7}
       e-15)(Layer_2) # The constant term 1e-15 avoids null values
       that leads to 'NaN' loss
   scale = Dense(4, activation=lambda x: tf.nn.elu(x) + 1 + 1e-15)(
8
       Laver_2)
   y_real = Input(shape=(1,))
9
   rate =1/scale
10
   lossF = gammanll_loss(y_real, alpha, concentration, rate)
11
   mdn_ms1_model = Model(inputs=[InputLayer, y_real], outputs=[alpha,
12
        concentration, rate])
   mdn_ms1_model.add_loss(lossF)
13
   # Learning rate scheduler with keras.optimizers.schedules
14
   lr_schedule =tf.keras.optimizers.schedules.PolynomialDecay(
15
        initial_learning_rate=i_lr, end_learning_rate=e_lr,
16
           decay_steps = 10000
   )
17
   adamOptimizer = optimizers.Adam(learning_rate=lr_schedule)
18
   mdn_ms1_model.summary()
19
   mdn_ms1_model.compile(optimizer=adamOptimizer)
20
```

Listing 4.1: MDN model implementation

The neurons related to the weights  $\alpha_c$  are set with "Softmax" activation functions, the neurons related to the concentrations and the rates are set both with "Exponential" activation functions. Before the training, the dataset was shuffled and split in the following way:

- 70% is dedicated to the training;
- 20% is dedicated to the validation;



• 10% is dedicated to testing.

Figure 4.2: Training history of model 1. The final error is NLL=-4.775 on the validation set

As reported in figure 4.2a the NLL converged after 500 epochs of training, with a batch size of 512 samples. The procedure required approx. 30 min on a laptop with Intel(R) Core(TM) i7-8565U CPU @ 1.80GHz and 8GB RAM (GPU acceleration not exploited).

The error function is the Negative Log-likelihood (NLL), in the software it is defined thanks to the eqs. 4.6 and 3.17, during the training its derivative with respect to the weights of the network is approximated by means of finite differences.

```
\# Cost for the multiple components MDN using GAMMA distribution
1
    def gammanll_loss(y, alpha, concentration, rate):
2
         "" Computes the mean negative log-likelihood loss of y given
3
            the mixture parameters.
        ,, ,, ,,
4
5
        gm = tfd. MixtureSameFamily(
6
             mixture_distribution=tfd.Categorical(probs=alpha),
7
             components_distribution=tfd.Gamma(
8
                 concentration=concentration,
9
                 rate=rate))
10
        # Evaluate log-probability of y
11
        \log_{\text{likelihood}} = \text{gm.}\log_{\text{prob}}(\text{tf.transpose}(y)+1e-15)
13
        return -tf.reduce_mean(log_likelihood, axis=-1)
14
```

Listing 4.2: Implementation of the Gamma error function

For a first evaluation of the generalization performance of this model, figure 4.3 shows two examples of output belonging to the test set.

To summarize the performance figure 4.4 shows the Kolmogorov-Smirnov (KS) test on the entire test set: 18 out of 30 distributions shows a p-value greater than 0.05 (possible to conclude that the MDN is able to fit these distributions).



Figure 4.3: Two test pdfs



Figure 4.4: KS test for ms1, the dashed line marks the 0.05 p-value



Listing 4.3: Implementation and plot of KS test

#### 4.3. MODELS

It is worth noting the distributions showing p-values less than 0.05 have maximum KS distance of 0.1 and KS test results are always pessimistic since it accounts for the maximum distance between data distributions and predictions, a more fair goodness of fit metric accounting for the could be used (e.g., a metric accounting for the average distance).

Once trained, the MDN is used to generalize the behaviour of the system and make comparisons. The figures 4.5a and 4.5b shows the comparisons of the mean and the 99-th percentile of ms1 response time in function of the average request rate per replica (defined as  $rps\_rep\_avg = \frac{\lambda}{n_{rep}}$ ), and  $n_{rep}$ .

A first evidence is the mean response time is a decreasing function of the average request rate per replica, and this applies for all the cases. This behaviour can be indicated as a *speed-up* of the processors depending on the request rate. The second evidence is the mean response time tends to be lower when multiple replicas are available with respect the case with only one replica. The same comparison on the 99-th percentile of the response time shows that the 99-th percentile is a decreasing function of the average request rate per replica as well, however the cases with multiple replicas shows higher 99-th percentiles than the case with only one replica, this trend is opposite with respect to what seen in the mean.





Figure 4.5: MS1 response time comparison for different cases of  $n_{rep}$ 



Figure 4.6: Comparison of some pdfs of ms1 response time

Focusing on the pdfs (figure 4.6) the comparison shows a bimodal tendency. In particular:

- For low values and high values of rps\_rep\_avg the bimodal tendency is weak, the response times are mainly distributed around the main mode.
- For intermediate values of rps\_rep\_avg the bimodal tendency is stronger, the ms1 processing tends to behave in two different ways (i.e., two main clusters of processing). Moreover, for cases with multiple replicas, the bimodal tendency start to arise for lower rps\_rep\_avg with respect the case

of one replica and is more focused on the cluster with lower processing times. and this focus is emphasized increasing rps\_rep\_avg.

A possible explanation for this phenomenon is an unequal and variable balancing of requests on replicas with a dynamic allocation of resources depending on the balancing: replicas fed with higher request rates belongs to the cluster with lower response times, on the other hand replicas fed with lower request rates belongs to the cluster of higher response times.

## 4.3.2 Model 2: characterization of microservice ms2 response time

This second model aim to characterize the microservice ms2 response time, similarly to what was done for ms1, to complete the characterization of the microservices of the system.

The objective is to approximate:

$$p(t_{ms2}|\lambda, n_{rep}) \tag{4.4}$$

The structure of the MDN and the division of the dataset for training, validation and testing is the same of model 1. For what concern the training, the MDN required 500 epochs, performed in 30 minutes with a batch size of 512 samples.



Figure 4.7: Training history of model 2. The final error is NLL=-4.337 on the validation set

The performance are summarized in figure 4.8. The KS test on the entire test set gives 29 out of 30 distributions showing a p-value greater than 0.05 (possible to conclude that the MDN is able to fit these distributions).



Figure 4.8: KS test for ms2, the dashed line marks the 0.05 p-value

The figure 4.9 shows the ms2 mean response time, the chart allows to do the same consideration on the speed-up made on ms1.



Figure 4.9: MS2 mean response time

Even for ms2 the bimodal tendency is present (figure 4.10) and the same considerations of ms1 can be done.



Figure 4.10: Comparison of some distributions of ms2 response time

### 4.3.3 Considerations about model 1 and model 2

The two models previously discussed are intended to characterize the response times of microservices ms1 and ms2. The properties of these response times have been investigate to:

- generalize the behaviour of the replicas of ms1 and ms2;
- perform analysis;
- obtain the probability density functions to make simulations.

The following conclusions may be drawn:

- The ms1 and ms2 average response times depends on the request parameters  $(\lambda, n_{rep})$ ;
- The MDN capture and generalize the dependency by means of probability density functions;
- In case the speed-up is significant, additional considerations on the tradeoff between speed-up and number of replicas can be done;
- The causes of the dependency needs to be investigated, for instance additional inputs can be identified and added in the model (e.g., the ones related to the resource allocation) to reduce the uncertainty (e.g., to separate the clusters of processing).

The pdfs produced by the model are suitable to get samples of the replicas response time that can be exploited in simulations of the overall system.

A way to perform simulations of cloud-native environments is the implementation of a *Digital twin* (DT), this software component make possible the real-time optimization and diagnostic of the system developed with *Kubernetes* (k8s).

The figure 4.11 shows a work in progress about the inference procedure of the time-to-resolution of the real K8S system by its digital twin *KubeTwin*, using the pdfs obtained through the model 1 and model 2.



Figure 4.11: K8S system performance and predicted performance by the DT

The ability of the DT to emulate the K8S are accurate up to  $\lambda = 20$  in terms of mean TTR and its 99-th percentile.

### 4.3.4 Model 3: characterization of time-to-resolution

This third model of MDN aims to characterize the time-to-resolution (TTR) experienced by users making requests on the system.

Mathematically, the model approximate:

$$p(ttr|\lambda, n_{rep}) \tag{4.5}$$

Where:

- $p(\bullet)$  indicates the probability density function (pdf);
- *ttr*[*s*] is the time-to-resolution;
- $(\lambda, n_{rep})$  are respectively the request rate [req/s] and the number of available replicas in each layer of the system.

The model is a DNN implemented with a Keras sequential model and Tensor-Flow probability, with the following characteristics:

- 2 input neurons: one input neuron for  $\lambda$  and one for  $n_{rep}$ ;
- 3 mixture components: the approximated pdf is the superposition of 3 Gamma's pdfs;
- 9 output neurons: the outputs represent the parameters of the components in the mixture model, every Gamma's pdf is characterized by 3 parameters (the weight in the superposition, the concentration and the rate)
- 2 hidden layer: each hidden layer has 6 neurons.

The architecture is similar to the previous models, the only difference is given by one less component in the mixture that has an impact on the number of hidden neurons too.

At the end of the training process the time-to-resolution is characterized in the following way:

$$p(ttr|\lambda, n_{rep}) = \sum_{c=1}^{3} \alpha_c(\lambda, n_{rep}) \phi(ttr, a_c(\lambda, n_{rep}), b_c(\lambda, n_{rep}))$$
(4.6)

```
InputLayer = Input(shape=(2,))
1
   Norm_layer = Normalization (axis = -1)
2
   Layer_1 = Dense(6, activation="ReLU") (Norm_layer(InputLayer))
3
   Layer_2 = Dense(6, activation = "ReLU")(Layer_1)
4
   alpha = Dense(3, activation="softmax")(Layer_2)
5
   concentration = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1
6
                       # The constant term 1e-15 avoids null values
       e-15)(Layer_2)
       that leads to 'NaN' loss
   rate = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1e-15)(
7
       Layer_2)
   y_{-real} = Input(shape=(1,))
8
   lossF = gammanll_loss(y_real, alpha, concentration, rate)
9
   mdn_ttr_model = Model(inputs=[InputLayer, y_real], outputs=[alpha,
10
        concentration, rate])
   mdn_ttr_model.add_loss(lossF)
11
   # Learning rate scheduler with keras.optimizers.schedules
13
   lr_schedule =tf.keras.optimizers.schedules.PolynomialDecay(
14
        initial_learning_rate=i_lr, decay_steps=100000,
           end_learning_rate=e_lr, power=1.0,
        cycle=False, name=None
16
   )
17
   adamOptimizer = optimizers.Adam(learning_rate=lr_schedule,
18
       clipvalue=1.0, clipnorm=1.) # clipnorm=1 avoid gradient
       explosion (reject all gradients with norm >1)
   mdn_ttr_model.summary()
19
   mdn_ttr_model.compile(optimizer=adamOptimizer)
20
   Norm_layer.adapt(features)
21
```

Listing 4.4: Model 3 implementation



Figure 4.12: Training history of model 2. The final error is NLL=-1.7162 on the validation set

As reported in figure 4.12 the NLL converged after 40 epochs of training, requiring approx. 38 min on the previously used laptop (GPU acceleration not exploited) with a batch size of 32 samples.

A first graphical evaluation of the generalization performance of this model is shown in figure 4.13 shows two examples of output belonging to the test set. The figure 4.13a shows a bad fit of the distribution.



Figure 4.13: Two test pdfs

#### 4.3. MODELS

To summarize the performance figure 4.14 shows the Kolmogorov-Smirnov (KS) test on the entire test set: 19 out of 30 distributions shows a p-value lower than 0.05 (possible to conclude that the MDN is not able to fit these distributions). The model is characterized by an high bias. In particular, the





maximum KS distance is 33% and it is related to the case  $\lambda = 26, n_{rep} = 1$ ). An analysis on the data highlighted the following issues:

- Non-stationarity
- Strong autocorrelation

These two properties arise for time-series related to requests parameters close to the condition  $rps\_rep\_avg = \frac{\lambda}{n_{rep}} = 25$ .

A possible explanation for this phenomenon can be that the system for this particular configuration of request parameters works in unstable conditions, meaning the service response time exceeds the inter-requests time and this bring to congestion with the queues in the system starting to grow rapidly.

To clarify, the non-stationa orty and high autocorrelation properties of an experiment with  $(\lambda = 26, n_{rep} = 1)$  are reported.

The figure 4.15 shows the time-series of the time to resolution with parameters ( $\lambda = 26, n_{rep} = 1$ ) and the moving average, both in function of the request identifier ("rid"), a progressive number identifying the requests during



Figure 4.15: Time-series  $ttr(\lambda = 26, n_{rep} = 1)$  and its moving-average computed on a window of 50 samples. The time is accounted by the request identifier.

experiments. From the figure it is possible to see the variation of the mean value of TTR during the experiment, that is an indicator of non-stationarity.

The upwards trends in the time-series are probably related to the instability condition in the system, vice versa the downward trends are related to the achievement of a stability condition due to the speed-up of the replica. The trends in the time-series turns out in a high autocorrelation (figure 4.16).



Figure 4.16: Autocorrelation function of time-series  $ttr(\lambda = 26, n_{rep} = 1)$ 

The considered MDN model can't fit these data, since it characterize the

TTR as a random variable and its training relies on the hypothesis of independent and identically distributed samples (i.i.d.) and this hypothesis doesn't hold due to non-stationarity and high autocorrelation.

A new model able to characterize  $\{TTR_i\}_{i\in\mathbb{N}}$  as a random process needs to be implemented.

### 4.3.5 Model 3-bis: characterization of time-to-resolution

The model 3 was extended to exploit the autocorrelation in the time-toresolution random process. In order to implement the new model, the input space is extended with additional features representing the previous realizations of the random process; as this architecture works with previous realization, it needs to have a temporal knowledge of the system.

Mathematically, the model approximates:

$$p(ttr_i | \overline{ttr}_{i,h=5}, \lambda, n_{rep}) \tag{4.7}$$

Where:

- $p(\bullet)$  indicates the probability density function (pdf);
- $ttr_i[s]$  is the ttr experienced by the i-th request;
- $\overline{ttr}_{i,h=5} = (ttr_{i-1}, \ldots, ttr_{i-5})$  is the history vector, containing the five previously experienced ttrs;
- $(\lambda, n_{rep})$  are respectively the request rate [req/s] and number of available replicas for ms1.

The approximation given by the model is useful to make predictions about the ttr experienced by users given the previous five realizations. The model can be extended to larger history vector (in principle up to the maximum lags showing high autocorrelation).

The structure of the MDN requires the extension of the input layer and this have an impact on the other layers too.

The implementation is given by:

• 7 input neurons: two input neurons for  $\lambda$  and  $n_{rep}$  parameters, then other 5 neurons are for the history samples;

- 3 mixture components: the approximated pdf is the superposition of 3 Gamma's pdfs;
- 9 output neurons: the outputs represent the parameters of the components in the mixture model, every Gamma's pdf is characterized by 3 parameters (the weight in the superposition, the concentration and the rate);
- 2 hidden layer: each hidden layer has 8 neurons.

```
InputLaver = Input(shape=(7,))
   Norm_layer = Normalization (axis = -1)
2
   Layer_1 = Dense(8, activation="ReLU")(Norm_layer(InputLayer))
3
   Layer_2 = Dense(8, activation = "ReLU")(Layer_1)
4
5
   alpha = Dense(3, activation="softmax")(Layer_2)
6
   concentration = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1
7
                         \# The constant term 1e-15 avoids null values
      e-15)(Layer_2)
       that leads to 'NaN' loss
   rate = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1e-15)(
8
       Layer_2)
   y_real = Input(shape=(1,))
9
   lossF = gammanll_loss(y_real, alpha, concentration, rate)
   mdn_ttr_model = Model(inputs=[InputLayer, y_real], outputs=[alpha,
        concentration, rate])
   mdn_ttr_model.add_loss(lossF)
13
   # Learning rate scheduler with keras.optimizers.schedules
14
   lr_schedule =tf.keras.optimizers.schedules.PolynomialDecay(
        initial_learning_rate=i_lr, decay_steps=1e6, end_learning_rate
16
           =e_{lr}, power=1.0,
        cycle=False, name=None
17
18
   adamOptimizer = optimizers.Adam(learning_rate=lr_schedule,
19
       clipvalue=1.0, clipnorm=1.) # clipnorm=1 avoid gradient
       explosion (reject all gradients with norm >1)
   mdn_ttr_model.summary()
20
   mdn_ttr_model.compile(optimizer=adamOptimizer)
21
```

#### Listing 4.5: MDN model 3-bis implementation

The network training required 40 epochs, corresponding to a training time of 44 minutes with a batch of 32 samples.



Figure 4.17: Training history of model 3-bis. The final error is NLL=-1.9266 on the validation set

```
epochs = 40
1
   batch_size = 32
2
   val_split=0.22
3
   history_cache = mdn_ttr_model.fit ([features, ttr], #using an input
4
       to pass the real values to compute the NLL
                              verbose=1,
5
                              epochs=epochs,
6
                              validation_split=val_split ,
7
                              batch_size=batch_size)
8
   print('Final cost: {0:.4f}'.format(history_cache.history['loss'
9
      ][-1]))
```

Listing 4.6: MDN model code for training

At the end of the training the model, the eq. 4.7 allows to define two functions of prediction:

• The estimator for the time-to-resolution:

$$\overline{TTR_i} = \mathbb{E}[TTR_i \,|\, \overline{ttr}_{i,h=5}, \,\lambda, \,n_{rep}] \tag{4.8}$$

• The upper-bound, corresponding to the 95-th percentile.

```
def get_mean_prediction(a):
1
       a = np.array(a)
2
        alpha_pred, conc_pred, rate_pred = mdn_ttr_model.predict(list
3
           ((a, a)))
       gm = tfd.MixtureSameFamily(mixture_distribution=tfd.
4
           Categorical (probs=alpha_pred), components_distribution=tfd.
           Gamma(concentration=conc_pred ,rate=rate_pred))
        return gm.mean()
6
   def quantile_mixture (p, gm):
7
        (lambda x: gm.cdf(x))
8
        return inversefunc ((lambda x: gm. cdf(x) + 10e - 10*np. log(x)),
9
           y_values=p, image=[0,1]) # logarithm makes the CDF
           strictly monotonic
10
   def get_upper_bound(a):
11
       a = np.array(a)
12
        alpha_pred, conc_pred, rate_pred = mdn_ttr_model.predict(list
13
           ((a, a)))
       gm = tfd.MixtureSameFamily(mixture_distribution=tfd.
14
           Categorical (probs=alpha_pred), components_distribution=tfd.
           Gamma(concentration=conc_pred ,rate=rate_pred))
        return quantile_mixture(0.95, gm)
15
```

Listing 4.7: Estimator and upper bound functions definitions

#### 4.3. MODELS

Differently to what done in the previous models, now the output is given by the two quantities: the estimation and the upper bound. The performance of the model can't be evaluated in the same way of the previous case because now the datasets are treated as time-series rather than i.i.d. samples (not possible to perform KS tests and plot pdf vs. data distribution).

The performance are thus evaluated through the computation of the MSE (Mean Square Error) and MAE (Mean Absolute Error) on predictions.

As an example, figure 4.18 shows the application of the model on the timeseries of *ttr* for parameters ( $\lambda = 26, n_{rep} = 1$ )



Figure 4.18: Prediction of the with  $(\lambda = 26, n_{rep} = 1)$  and upper bound.



Figure 4.19: Performance (MAE and MSE) in the prediction of the with ( $\lambda = 26, n_{rep} = 1$ )

Finally, the figure 4.19 shows the performance of the model on the same example. The performance are given by  $MSE = 0.00395 s^2$  and MAE = 0.05023 s. The MAE corresponds to a mean absolute percentage error of 9%.

## 4.3.6 Considerations about model 3 and model 3-bis

The models 3 and 3-bis are intended to characterize the time-to-resolution (ttr) of the cloud-native system. The ttr can't be characterized like the response times of the microservices due to the issues described in the previous sections; as well the model 3-bis performance can't be evaluated with KS test. The conclusions after the implementation of these two models are:

- a degradation of the performance of the model is expected if the hypothesis of i.i.d. data doesn't hold;
- the MDN can exploit the autocorrelation:
  - the ttr can be predicted based on its history if it is available;
  - the approach accounts for the time and allow the modelling of ttr as a random process;
  - the history vector can be extended;
  - the performance of the predictions are measured in terms of MAE and MSE.

### 4.3.7 Summary of models

This section provide a recap of the presented models.

| Name        | Input                                      | Output  |
|-------------|--|---|
| Model 1     | $(\lambda, n_{rep})$                       | $p(t_{ms1} \lambda, n_{rep})$                         |
| Model 2     |  | $p(t_{ms2} \lambda, n_{rep})$                         |
| Model 3     |  | $p(ttr \lambda, n_{rep})$                             |
| Model 3-bis | $\overline{ttr}_{i,h=5}, \lambda, n_{rep}$ | $p(ttr_i   \overline{ttr}_{i,h=5}, \lambda, n_{rep})$ |

## Chapter 5

# Conclusion and future work

In conclusion, cloud-native has become an important paradigm for large-scale application deployment, enabling efficient management of infrastructures, microservices, and distributed applications. A cloud-native deployment can be described as a multi-layered queue system, where the management with platforms such as Kubernetes is critical to ensure service availability and system scalability.

However, theoretical queueing models have limitations, which can be overcome through the use of AI models. Indeed, the use of AI can be the way to analyze the performance of microservices more precisely and generalize their behavior, as well as to use the results to do simulations in a digital twin.

In the approach proposed in this thesis, it is shown that the use of AI, applied to the system with a MDN model, can lead to adequate performance in analyzing the microservices response times and in the use of results in a digital twin. In the specific case of the digital twin, accuracy up to a rate of requests in the system not exceeding 20 [req/s] is shown.

In addition, the same kind of model to characterize the microservice response time is applied to the time-to-resolution to have a description of the service time experienced by the users; this approach results inadequate since the i.i.d. assumption does not hold. The models is then adapted to the new case, extending the input space with collection of previous experienced timeto-resolution, allowing an estimate of time-to-response and an upper bound with good performance.

However, the future direction could be to improve the models by expanding the space of input variables (e.g., by adding the variables related to resource allocation of the replicas) and the adoption of other ML architectures, such as reinforcement learning models, or using ensemble learning techniques.

In conclusion, the use of AI in the cloud-native context offers many opportunities to improve the efficiency and scalability of distributed systems, and it is a field of research that could be widely explored in the future.

# Appendix A

# Some more code

## A.1 Code for models 1 and 2

```
import numpy as np
1
   import pandas as pd
2
   import tensorflow as tf
3
   import scipy.stats as stats
4
   from tensorflow import keras
5
   from tensorflow_probability import distributions as tfd
6
   import matplotlib.pyplot as plt
7
   from sklearn.utils import shuffle
8
   import keras
9
   from keras import optimizers
10
   from keras.models import Sequential, Model
11
   from keras.layers import Dense, Activation, Layer, Input,
12
       Concatenate, Normalization, BatchNormalization
   import glob
13
14
   from scipy.stats import norm, kstest, gamma
15
   from sklearn.neighbors import KernelDensity
16
   from pynverse import inversefunc
17
18
   # Data import
19
20
   #Import the dataset
21
22
23
   # Data import of replica1
24
   path = ', '
25
```

```
filenames = glob.glob(path)
26
    i = 1
27
28
    ms1_list = []
29
30
    rep=1
31
32
    for f in filenames:
33
        index_rps=f.find('rps') + 3
34
        rps=int(float(f[index_rps:index_rps+2]))
35
        print(f)
36
        df = pd.read_csv(f, index_col=False, sep="\t", header = None,
37
            names=["rid","ms1"])
        df.insert(1, 'rps_rep_avg', rps/rep)
38
        df.insert(2, 'rep', rep)
39
        df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
40
            adding moving average
        ms1_list.append(df)
41
42
    filenames = glob.glob(path)
43
44
    rep=2
45
46
    for f in filenames:
47
        index_rps=f.find('rps') + 3
48
        rps=int(float(f[index_rps:index_rps+2]))
49
        print(f)
50
        df = pd.read_csv(f, index_col=False, sep='\t', header = None,
            names=['rid', 'ms1'])
        df.insert(1, 'rps_rep_avg', rps/rep)
        df.insert(2, 'rep', rep)
53
        df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
54
            adding moving average
        ms1_list.append(df)
55
56
    filenames = glob.glob(path)
57
58
    rep=3
59
60
    for f in filenames:
61
        index_rps=f.find('rps') + 3
62
        rps=int(float(f[index_rps:index_rps+2]))
63
        print(f)
64
        df = pd.read_csv(f, index_col=False, sep='\t', header = None,
65
            names=['rid', 'ms1'])
```

```
df.insert(1, 'rps_rep_avg', rps/rep)
66
        df.insert(2, 'rep', rep)
67
        df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
68
            adding moving average
        ms1_list.append(df)
69
70
    filenames = glob.glob(path)
71
    rep=4
73
74
75
    for f in filenames:
        index_rps=f.find('rps') + 3
76
        rps=int(float(f[index_rps:index_rps+3]))
77
78
        print(f)
        df = pd.read_csv(f, index_col=False, sep='\t', header = None,
79
            names=['rid', 'ms1'])
        df.insert(1, 'rps_rep_avg', rps/rep)
80
        df.insert(2, 'rep', rep)
81
        df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
82
            adding moving average
        ms1_list.append(df)
83
84
    data_train_val = pd.concat(ms1_list, axis=0, ignore_index=False)
85
86
87
88
    features = shuffle(data_train_val)
89
90
91
    rid = features.pop('rid')
92
    ms1 = features.pop('ms1')
93
    MA\_ms1 = features.pop('MA')
94
95
96
    def plot_loss(history, zoom=False):
97
      plt.plot(history.history['loss'], label='loss', )
98
      plt.plot(history.history['val_loss'], label='val_loss')
99
      plt.xlabel('Epoch')
      plt.ylabel('NLL - Negative Log-Likelihood')
102
      if zoom:
        plt.xlim(350, 500)
103
        plt.ylim(min(history.history['loss']), history.history['loss'
            ][350])
      plt.legend()
105
      plt.grid(True)
106
```

```
107
    def gammanll_loss(y, alpha, concentration, rate):
108
        "" Computes the mean negative log-likelihood loss of y given
109
            the mixture parameters.
        ,, ,, ,,
        gm = tfd. MixtureSameFamily(
             mixture_distribution=tfd.Categorical(probs=alpha),
113
             components_distribution=tfd.Gamma(
114
                 concentration=concentration,
                 rate=rate))
117
        \log_{-1} = gm. \log_{-1} (tf. transpose(y)+1e-15) \# Evaluate
118
             log-probability of y
119
        return -tf.reduce_mean(log_likelihood, axis=-1)
122
    InputLayer = Input(shape=(2,)) # 2 input neurons
123
124
    Norm_layer = Normalization (axis = -1)
125
126
    Layer_1 = Dense(7, activation="ReLU")(Norm_layer(InputLayer))
                                                                       #3
127
    Layer_2 = Dense(7, activation = "ReLU")(Layer_1)
128
    alpha = Dense(4, activation="softmax")(Layer_2)
129
    concentration = Dense(4, activation=lambda x: tf.nn.elu(x) + 1 + 1
130
                         # The constant term 1e-15 avoids null values
       e-15 (Layer_2)
        that leads to 'NaN' loss
    scale = Dense(4, activation=lambda x: tf.nn.elu(x) + 1 + 1e-15)(
        Layer_2)
    y_{-}real = Input(shape=(1,))
132
    rate =1/scale
133
    lossF = gammanll_loss(y_real, alpha, concentration, rate)
                                                                  #
134
        gammanll_loss(y_real, alpha, mu, sigma)
    mdn_ms1_model = Model(inputs=[InputLayer, y_real], outputs=[alpha,
135
         concentration, rate])
    mdn_ms1_model.add_loss(lossF)
136
137
    # Learning rate scheduler with keras.optimizers.schedules
138
    lr_schedule =tf.keras.optimizers.schedules.PolynomialDecay(
139
        initial_learning_rate=0.00007, end_learning_rate=0.00003,
140
            decay_steps = 10000
    ) #define the rate scheduler \rightarrow if we increase the number of
141
        layers we have to decrease the end learning rate (high
        complexity in the network)
```

```
142
    adamOptimizer = optimizers.Adam(learning_rate=lr_schedule)
143
        clipnorm=1 avoid gradient explosion (reject all gradients with
        norm >1)
    mdn_ms1_model.summary()
144
    mdn_ms1_model.compile(optimizer=adamOptimizer)
145
146
    epochs = 500
147
148
    history_cache = mdn_ms1_model. fit ([features, ms1], #notice we are
149
        using an input to pass the real values due to the inner
        workings of keras
                                 verbose=1, # write =1 if you wish to see
150
                                      the progress for each epoch
                                 epochs=epochs,
151
                                 validation_split = 0.22,
                                 batch_size=64)
153
    print('Final cost: {0:.4f}'.format(history_cache.history['loss'
154
        ][-1]))
    plt.rcParams["figure.figsize"] = [8,8]
156
    plot_loss(history_cache, zoom=True)
157
158
159
    #Test import
160
161
    filenames = glob.glob(path)
162
    i = 1
163
164
    ms1\_test = []
165
166
    rep=1
167
    for f in filenames:
168
         index_rps=f.find('rps') + 3
169
         rps=int(float(f[index_rps:index_rps+2]))
         print(f)
171
         df = pd.read_csv(f, index_col=False, sep="\t", header = None,
172
            names=["rid","ms1"])
         df.insert(1, 'rps_rep_avg', rps/rep)
173
         df.insert(2, 'rep', rep)
174
         df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
175
            adding moving average
         ms1_test.append(df)
177
    filenames = glob.glob(path)
178
```

```
179
180
    rep=2
181
    for f in filenames:
182
         index_rps=f.find('rps') + 3
183
         rps=int(float(f[index_rps:index_rps+2]))
184
         print(f)
185
         df = pd.read_csv(f, index_col=False, sep=' \ t', header = None,
186
            names=['rid', 'ms1'])
         df.insert(1, 'rps_rep_avg', rps/rep)
187
         df.insert(2, 'rep', rep)
188
         df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
189
            adding moving average
         ms1_test.append(df)
190
191
    filenames = glob.glob(path)
192
193
    rep=3
194
195
    for f in filenames:
196
         index_rps=f.find('rps') + 3
197
         rps=int(float(f[index_rps:index_rps+2]))
198
         print(f)
199
         df = pd.read_csv(f, index_col=False, sep='\t', header = None,
200
            names = ['rid', 'ms1'])
         df.insert(1, 'rps_rep_avg', rps/rep)
201
         df.insert(2, 'rep', rep)
202
         df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
203
            adding moving average
         ms1_test.append(df)
204
205
206
207
    filenames = glob.glob(path)
208
209
    rep=4
210
211
    for f in filenames:
212
         index_rps=f.find('rps') + 3
213
         rps=int(float(f[index_rps:index_rps+3]))
214
         print(f)
215
         df = pd.read_csv(f, index_col=False, sep=' \ t', header = None,
216
            names=['rid', 'ms1'])
         df.insert(1, 'rps_rep_avg', rps/rep)
217
         df.insert(2, 'rep', rep)
218
```
```
df.insert(3, 'MA', df.get('ms1').rolling(window=200).mean()) #
219
            adding moving average
        ms1_test.append(df)
220
221
    data_test = pd.concat(ms1_test, axis=0, ignore_index=False)
222
223
224
    def mixture_pdf(x,rps_rep_avg, rep):
225
226
        """ pdf function of mixture is the weighted superimposition of
227
             the pdfs """
228
         alpha_pred, conc_pred, rate_pred = mdn_ms1_model.predict(list
229
            ((np.array([rps_rep_avg, rep]), np.array([1,1])))) # The
            second array is dummy
         pdf = 0
230
         for i in range(0, alpha_pred.shape[1]):
231
             pdf = pdf + alpha_pred [0][i] * stats.gamma.pdf(x, a=
232
                conc_pred [0][i], scale=1/rate_pred [0][i])
        return pdf
233
234
    def mixture_cdf(x,rps_rep_avg, rep):
235
236
        """ cdf function of mixture is the weighted superimposition of
237
             the cdfs """
238
         alpha_pred, conc_pred, rate_pred = mdn_ms1_model.predict(list
239
            ((np.array([rps_rep_avg, rep]), np.array([1,1])))) # The
            second array is dummy
         cdf = 0
240
         for i in range(0, alpha_pred.shape[1]):
241
             cdf = cdf + alpha_pred [0][i] * stats.gamma.cdf(x, a=
242
                conc_pred [0][i], scale=1/rate_pred [0][i])
         return cdf
243
244
    \# Let's try to plot the predicted distribution for an arbitrary
245
        rps and rep and compare it with the distribution given in the
        test set
246
    rps = 27 \# The rps
247
    rep = 1 \# The rep
248
249
    plt.rcParams["figure.figsize"] = [9, 6]
250
251
```

```
df=data_test[(data_test['rps_rep_avg'] == rps/rep) & (data_test['
252
        \operatorname{rep}' = \operatorname{rep}
253
    ms1\_test = df.pop('ms1')
254
    ms1\_min = ms1\_test.min()
255
    ms1_max = ms1_test.max()
256
257
    bw = ms1\_test.std() * 0.182 \# Compute sub-optimal bandwidth for the
258
         histogram
    bins = np.arange(ms1_min, ms1_max, bw)
259
260
261
    # Plot PDF.
262
    x_p = np.linspace(0., 0.025, int(1e5), dtype=np.float32)
263
    X_r=x_p.reshape(-1, 1)
264
265
    plt.hist(ms1\_test.values, bins=bins, fc='g', alpha = 0.5, density=
266
        True, label = 'Data distribution')
    alpha_pred, conc_pred, rate_pred = mdn_ms1_model.predict(list((np.
267
        array ([rps/rep, rep]), np. array ([1,1])))) # The second array is
        dummy
    gm = tfd.MixtureSameFamily(mixture_distribution=tfd.Categorical(
268
        probs=alpha_pred), components_distribution=tfd.Gamma(
        concentration=conc_pred ,rate=rate_pred))
    \#plt.plot(x_p, mixture_pdf(x_p, rps/rep, rep), label = "Predicted"
269
       PDF", color = 'b')
    plt.plot(x_p, gm.prob(x_p), label = "Predicted PDF", color = 'b',
270
        linewidth=2)
    #for i in range(0, alpha_pred.shape[1]):
271
        #gm = tfd.MixtureSameFamily(mixture_distribution=tfd.
272
            Categorical (probs=[alpha_pred [0][i]]),
            components_distribution=tfd.Gamma(concentration=[conc_pred
            [0][i]], rate=[rate_pred[0][i]]))
        \#plt.plot(x_p, gm.prob(x_p), label = "Predicted PDF" + str(i)
273
            + "component" + "prob = " + str([alpha_pred[0][i]]))
    plt.title('MS1 pdf for (rps=' + str(rps) + ', rep=' + str(rep) +
274
        ')', fontsize = 18)
    plt.xlabel("ms1 [s]")
275
    plt.ylabel("PDF")
276
277
    plt.legend()
278
279
    # Perform KS-test
280
    plt.rcParams["figure.figsize"] = [20,7]
281
282
```

```
n_rep = 4
283
284
    rps_pl = []
285
    ks_distance = []
286
    ks_p = []
287
    fig, ax1 = plt.subplots()
288
289
    ax2 = ax1.twinx()
290
291
    for rep in range (1, n_rep + 1):
292
         for rps in range (1, n_rep * 30 + 1):
293
             df=data_test[(data_test['rps_rep_avg'] == rps/rep) & (
294
                 data_test['rep'] == rep)]
             if not df.empty:
295
                  ms1\_test = df.pop('ms1')
296
                  alpha\_pred, conc\_pred, rate\_pred = mdn\_ms1\_model.
297
                      predict (list ((np. array ([rps/rep, rep]), np. array
                      ([8,1])))
                  gm = tfd.MixtureSameFamily(mixture_distribution=tfd.
298
                      Categorical (probs=alpha_pred),
                      components_distribution=tfd.Gamma(concentration=
                      conc_pred ,rate=rate_pred))
                  testres=stats.kstest(ms1_test, cdf=mixture_cdf, args=(
299
                      rps/rep, rep), alternative='less')
                  ks_distance.append(testres.statistic)
300
                  ks_p.append(testres.pvalue)
301
                  rps_pl.append(rps)
302
         ax1.bar(rps_pl, ks_distance, label = "KS Distance for #rep=" +
303
              \operatorname{str}(\operatorname{rep}), \operatorname{alpha} = 0.5)
         ax2.scatter(rps_pl, ks_p, label = "KS p-value for #rep=" + str
304
             (rep), alpha = 0.7)
         rps_pl.clear()
305
         ks_distance.clear()
306
         ks_p.clear()
307
    ax2.plot(np.arange(1, 121), np.full(120, 0.05), 'k-')
308
    plt.xticks(np.arange(1, 120, 4.0))
309
    plt.grid(visible=True)
310
    plt.title('Test set')
311
    ax1.set_xlabel("RPS [req/s]")
312
    ax1.set_ylabel("KS Distance")
313
    ax2.set_ylabel("KS p-value")
314
    ax1.legend(loc='upper left')
315
    ax2.legend(loc='upper right')
316
    plt.show()
317
```

Listing A.1: Code for ms1 characterization

## A.2 Code for model 3

```
import numpy as np
1
2
   import pandas as pd
   import tensorflow as tf
3
   import scipy.stats as stats
4
   #from tensorflow import keras. layers
5
   from tensorflow import keras
6
   from tensorflow_probability import distributions as tfd
7
   import matplotlib.pyplot as plt
8
   from sklearn.utils import shuffle
9
   from keras import optimizers
10
   from keras.models import Sequential, Model
11
   from keras.layers import Dense, Activation, Layer, Input,
12
       Concatenate, Normalization, BatchNormalization
13
   import glob
14
15
   from scipy.stats import norm, kstest, gamma
16
   from sklearn.neighbors import KernelDensity
17
   from pynverse import inversefunc
18
19
   # Train data import
20
21
   #Import the dataset
22
23
24
   datalist= []
25
26
   i = 1
27
   # Data import of replica1
28
29
   path=''
30
31
   filenames = glob.glob(path)
32
33
    for f in filenames:
34
        print(f)
35
        index_rps=f.find('rps') + 3
36
```

```
df = pd.read\_csv(f, index\_col=0)
37
        df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                        #
38
           Add in the firs column the corresponding rps
        print(int(float(f[index_rps:index_rps+3])))
39
                                  \# Add in the second column the
        df.insert (2, 'rep', 1)
40
           corresponding number of replicas
                                 # Append the dataframe containing the
        datalist.append(df)
41
           values for each rps in the datalist
        i\!=\!i\!+\!1
42
43
44
   # Data import of replica2
45
   filenames = glob.glob(path)
46
47
   i = 1
48
   for f in filenames:
49
        print(f)
50
        index_rps=f.find('rps') + 3
        df = pd.read_csv(f, index_col=0)
        df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                        #
           Add in the firs column the corresponding rps
        print(int(float(f[index_rps:index_rps+3])))
54
        df.insert (2, 'rep', 2)
                                  \# Add in the second column the
           corresponding number of replicas
        datalist.append(df)
                                 # Append the dataframe containing the
56
           values for each rps in the datalist
        i = i + 1
57
58
   # Data import of replica3
59
   filenames = glob.glob(path)
60
61
   i = 1
62
   for f in filenames:
63
        print(f)
64
        index_rps=f.find('rps') + 3
65
        df = pd.read_csv(f, index_col=0)
66
        df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                        #
67
           Add in the firs column the corresponding rps
        print(int(float(f[index_rps:index_rps+3])))
68
        df.insert(2, 'rep',3)
                                 \# Add in the second column the
69
           corresponding number of replicas
        datalist.append(df)
                                 # Append the dataframe containing the
70
           values for each rps in the datalist
        i = i + 1
71
72
```

```
# Data import of replica4
73
    filenames = glob.glob(path)
74
75
    i = 1
76
    for f in filenames:
77
        print(f)
78
         index_rps=f.find('rps') + 3
79
         df = pd.read_csv(f, index_col=0)
80
         df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                        #
81
            Add in the firs column the corresponding rps
         print(int(float(f[index_rps:index_rps+3])))
82
         df.insert(2, 'rep', 4)
                                  \# Add in the second column the
83
            corresponding number of replicas
                              # Append the dataframe containing the
         datalist.append(df)
84
            values for each rps in the datalist
         i = i + 1
85
86
87
    data_train_val= pd.concat(datalist, axis=0, ignore_index=True)
                                                                         #
88
        Concatenate the datalist in a dataframe for semplicity
89
    #data_train_val.head()
90
91
    # Make a copy
92
    train_val_copy = data_train_val.copy() \# make a copy
93
94
    features = shuffle(data_train_val)
95
96
97
    rid = features.pop('rid')
98
    ttr = features.pop('ttr')
99
100
101
    def plot_loss(history, zoom=False):
102
      plt.plot(history.history['loss'], label='loss', )
103
      plt.plot(history.history['val_loss'], label='val_loss')
104
      plt.xlabel('Epoch')
      plt.ylabel('NLL - Negative Log-Likelihood')
106
      if zoom:
107
         plt.xlim(35, 39)
108
         plt.ylim(min(history.history['loss']), history.history['loss'
109
            ][35])
      plt.legend()
      plt.grid(True)
111
112
```

70

```
113
    # GNLL loss for multiple components MDN using the Gaussian
114
        distributions
    def gnll_loss(y, alpha, mu, sigma):
115
         "" Computes the mean negative log-likelihood loss of y given
            the mixture parameters.
        ., ., .,
117
118
        gm = tfd. MixtureSameFamily(
119
             mixture_distribution=tfd.Categorical(probs=alpha),
120
             components_distribution=tfd.Normal(
                 loc=mu,
                 scale=sigma))
123
124
         \log_{\text{likelihood}} = \operatorname{gm.log_prob}(tf.transpose(y)) \# Evaluate \log_{\text{log}}
125
            probability of y
126
         return -tf.reduce_mean(log_likelihood, axis=-1)
127
128
129
130
131
    InputLayer = Input(shape=(2,))
    Norm_layer = Normalization (axis = -1)
    Layer_1 = Dense(6, activation = "ReLU") (Norm_layer(InputLayer))
135
    Layer_2 = Dense(6, activation = "ReLU")(Layer_1)
136
137
    alpha = Dense(3, activation="softmax")(Layer_2)
138
    concentration = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1
139
        e-15)(Layer_2)
                          # The constant term 1e-15 avoids null values
        that leads to 'NaN' loss
    rate = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1e-15)(
140
        Layer_2)
    y_real = Input(shape=(1,))
141
    lossF = gammanll_loss(y_real, alpha, concentration, rate)
142
    mdn_ttr_model = Model(inputs=[InputLayer, y_real], outputs=[alpha,
143
         concentration, rate])
    mdn_ttr_model.add_loss(lossF)
144
145
    # Learning rate scheduler with keras.optimizers.schedules
146
    lr_schedule =tf.keras.optimizers.schedules.PolynomialDecay(
147
         initial_learning_rate=i_lr, decay_steps=100000,
148
            end_learning_rate=e_lr, power=1.0,
         cycle=False, name=None
149
```

```
)
150
151
    adamOptimizer = optimizers.Adam(learning_rate=lr_schedule,
152
        clipvalue=1.0, clipnorm=1.) # clipnorm=1 avoid gradient
        explosion (reject all gradients with norm >1)
    mdn_ttr_model.summary()
153
    mdn_ttr_model.compile(optimizer=adamOptimizer)
154
    Norm_layer.adapt(features)
156
157
    epochs = 40 \# 20
158
159
    history_cache = mdn_ttr_model.fit ([features, ttr], #notice we are
160
        using an input to pass the real values due to the inner
        workings of keras
                                verbose=1, # write =1 if you wish to see
161
                                     the progress for each epoch
                                epochs=epochs,
162
                                validation_split = 0.22,
163
                                batch_size=32
164
    print ('Final cost: {0:.4f}'.format (history_cache.history ['loss'
165
       ][-1]))
```

Listing A.2: Code for ttr characterization

## A.3 Code for model 3-bis

```
import warnings
1
   warnings.filterwarnings('ignore')
2
3
   import numpy as np
4
   import pandas as pd
5
   import tensorflow as tf
6
   import scipy.stats as stats
7
   from tensorflow import keras
8
   from tensorflow.keras import layers
9
   from tensorflow_probability import distributions as tfd
10
   import matplotlib.pyplot as plt
   from sklearn.utils import shuffle
12
   from sklearn import metrics
13
   from keras import optimizers
14
   from keras.models import Sequential, Model
15
```

72

```
from keras.layers import Dense, Activation, Layer, Input,
16
       Concatenate, Normalization
   import keras
17
   import glob
18
19
   from scipy.stats import norm, kstest, gamma
20
   from pynverse import inversefunc
21
   plt.rcParams.update({ 'font.size ': 18})
23
24
   # Data import
25
26
   #Import the dataset
27
28
   path=''
29
   datalist= []
30
31
   i = 1
32
   h=5
33
   # Data import of replica1
34
35
    filenames = glob.glob(path)
36
    for f in filenames:
37
        print(f)
38
        index_rps=f.find('rps') + 3
39
        df = pd.read_csv(f, index_col=None)
40
        df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                        #
41
           Add in the firs column the corresponding rps
        print(int(float(f[index_rps:index_rps+3])))
42
        df.insert(2, 'rep', 1) # Add in the second column the
43
           corresponding number of replicas
        df1 = df.copy()
44
        for j in range(1,h+1):
45
            df. insert (3 + j, 'ttr_i - ' + str(j), dfl.get('ttr').shift(j)
46
                )) #Add in the columns from 4 to 8 the TTRs
                experienced in the previous h requests
        df = df.drop(index=range(0,h))
47
        datalist.append(df)
                                 # Append the dataframe containing the
48
            values for each rps in the datalist
        i = i + 1
49
50
51
   # Data import of replica2
   filenames = glob.glob(path)
53
54
```

```
i = 1
55
   for f in filenames:
56
        print(f)
57
        index_rps=f.find('rps') + 3
58
        df = pd.read_csv(f, index_col=None)
59
        df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                       #
60
           Add in the firs column the corresponding rps
        print(int(float(f[index_rps:index_rps+3])))
61
        df.insert(2, 'rep',2)
                                 \# Add in the second column the
62
           corresponding number of replicas
        df1 = df.copy()
63
        for j in range(1,h+1):
64
            df.insert(3 + j, 'ttr_i - ' + str(j), dfl.get('ttr').shift(j)
65
                )) #Add in the columns from 4 to 8 the TTRs
                experienced in the previous h requests
        df = df.drop(index=range(0,h))
66
        datalist.append(df)
                                # Append the dataframe containing the
67
           values for each rps in the datalist
        i = i + 1
69
   # Data import of replica3
70
   filenames = glob.glob(path)
71
72
   i = 1
73
   for f in filenames:
74
        print(f)
75
        index_rps=f.find('rps') + 3
76
        df = pd.read_csv(f, index_col=None)
        df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                       #
78
           Add in the firs column the corresponding rps
        print(int(float(f[index_rps:index_rps+3])))
79
        df.insert(2, 'rep',3)
                               # Add in the second column the
80
           corresponding number of replicas
        df1 = df.copy()
81
        for j in range(1,h+1):
82
            df. insert (3 + j, 'ttr_i - ' + str(j), dfl.get('ttr').shift(j)
83
                )) #Add in the columns from 4 to 8 the TTRs
               experienced in the previous h requests
        df = df.drop(index=range(0,h))
84
        datalist.append(df)
                                 # Append the dataframe containing the
85
           values for each rps in the datalist
        i = i + 1
86
87
   # Data import of replica4
88
   filenames = glob.glob(path)
89
```

74

```
90
    i = 1
91
    for f in filenames:
92
         print(f)
93
         index_rps=f.find('rps') + 3
94
         df = pd.read_csv(f, index_col=None)
95
         df.insert(1, 'rps', int(float(f[index_rps:index_rps+3])))
                                                                        #
96
            Add in the firs column the corresponding rps
         print(int(float(f[index_rps:index_rps+3])))
97
         df.insert(2, 'rep', 4)
                                 \# Add in the second column the
98
            corresponding number of replicas
         df1 = df.copy()
99
         for j in range(1,h+1):
100
             df.insert(3 + j, 'ttr_i-' + str(j) ,dfl.get('ttr').shift(j
101
                )) #Add in the columns from 4 to 8 the TTRs
                experienced in the previous h requests
         df = df.drop(index=range(0,h))
                                  # Append the dataframe containing the
         datalist.append(df)
            values for each rps in the datalist
         i = i + 1
105
106
    data_train_val= pd.concat(datalist, axis=0, ignore_index=True)
                                                                        #
107
        Concatenate the datalist in a dataframe for semplicity
108
    #data_train_val.head()
109
    # Make some copies
111
    train_val_copy = data_train_val.copy() # make a copy
    features = shuffle(data_train_val)
114
115
    rid = features.pop('rid')
116
    ttr = features.pop('ttr')
117
    st = features.pop('st')
118
119
120
    \# Cost for the multiple components MDN using GAMMA distribution
121
    def gammanll_loss(y, alpha, concentration, rate):
122
         "" Computes the mean negative log-likelihood loss of y given
123
            the mixture parameters.
        ,, ,, ,,
        gm = tfd. MixtureSameFamily(
126
             mixture_distribution=tfd.Categorical(probs=alpha),
127
```

```
components_distribution=tfd.Gamma(
128
                                          concentration=concentration,
129
                                         rate=rate))
130
131
                     \log_{-1} = gm. \log_{-1} (tf. transpose(y)+1e-15) \# Evaluate
                                log-probability \ of \ y \ np. \ clip ( \ tf. \ transpose(y) \ , \ 1e-8, \ 100.)
                     return -tf.reduce_mean(log_likelihood, axis=-1)
           def plot_loss(history, zoom=False):
136
                plt.plot(history.history['loss'], label='loss')
137
                plt.plot(history.history['val_loss'], label='val_loss')
138
                plt.xlabel('Epoch')
139
                plt.ylabel ('NLL - Negative Log-Likelihood')
140
                if zoom:
141
                     plt.ylim(min(history.history['loss']), min(history.history['
142
                             loss']) + 100)
                plt.legend()
143
                plt.grid(True)
144
145
146
          any_nan = np.any(np.isnan(ttr))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(features.get('rps'))|np.any(np.isnan(featu
147
                   ))) | np.any(np.isnan(features.get('rep')))
           for j in range (1,h+1):
148
                     any_nan = any_nan | np.any(np.isnan(features.get('ttr_i-' +
149
                             str(j)))
           print('Presence of NaN: ' + str(any_nan))
           InputLayer = Input(shape=(7,)) # 7 input neurons
155
156
157
          Norm_layer = Normalization (axis = -1)
158
159
           Layer_1 = Dense(8, activation="ReLU")(Norm_layer(InputLayer))
                                                                                                                                                                              #2
           Layer_2 = Dense(8, activation = "ReLU")(Layer_1)
                                                                                                                                               #2)
161
162
           alpha = Dense(3, activation="softmax")(Layer_2)
163
           concentration = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1
164
                   e - 15) (Layer_2)
                                                                \# The constant term 1e-15 avoids null values
                   that leads to 'NaN' loss
           rate = Dense(3, activation=lambda x: tf.nn.elu(x) + 1 + 1e-15)(
165
                   Layer_2)
```

```
y_real = Input(shape=(1,))
166
    lossF = gammanll_loss(y_real, alpha, concentration, rate)
                                                                  #
167
        gammanll_loss (y_real, alpha, mu, sigma)
    mdn_ttr_model = Model(inputs=[InputLayer, y_real], outputs=[alpha,
168
         concentration, rate])
    mdn_ttr_model.add_loss(lossF)
169
    # Learning rate scheduler with keras.optimizers.schedules
    lr_schedule =tf.keras.optimizers.schedules.PolynomialDecay(
172
        initial_learning_rate=0.0003, decay_steps=20,
173
            end_{learning_rate} = 0.00003, power=1.0,
        cycle=False, name=None
174
    ) #define the rate scheduler \rightarrow if we increase the number of
175
        layers we have to decrease the end learning rate (high
        complexity in the network)
    adamOptimizer = optimizers.Adam(learning_rate=lr_schedule,
177
        clipvalue=1.0, clipnorm=1.) # clipnorm=1 avoid gradient
        explosion (reject all gradients with norm >1)
    mdn_ttr_model.summary()
178
    mdn_ttr_model.compile(optimizer=adamOptimizer)
179
180
    Norm_layer.adapt(features) # Perform normalization of input
181
        features to improve the convergence of the MDN
182
    epochs = 40 \ \# epochs = 30
183
    batch_size = 32
184
    history_cache = mdn_ttr_model.fit ([features, ttr], #using an input
185
         to pass the real values to compute the NLL
                                verbose = 1,
186
                                epochs=epochs,
187
                                validation_split=0.22,
188
                                batch_size=batch_size)
189
    print('Final cost: {0:.4f}'.format(history_cache.history['loss'
190
       [-1])
191
    def quantile_mixture (p, gm):
192
        return inversefunc ((lambda x: gm.cdf(x)+10e-7*np.log(x)),
193
            y_values=p, image=[0,1]) # logarithm makes the CDF
            strictly monotonic
194
    def get_mean_prediction(a):
195
        a = np.array(a)
196
        alpha_pred, conc_pred, rate_pred = mdn_ttr_model.predict(list
197
            ((a, a)))
```

```
gm = tfd.MixtureSameFamily(mixture_distribution=tfd.
198
            Categorical(probs=alpha_pred), components_distribution=tfd.
            Gamma(concentration=conc_pred ,rate=rate_pred))
        return gm.mean()
199
200
    def get_upper_bound(a):
201
        a = np.array(a)
202
         alpha_pred, conc_pred, rate_pred = mdn_ttr_model.predict(list
203
            ((a, a)))
        gm = tfd.MixtureSameFamily(mixture_distribution=tfd.
204
            Categorical (probs=alpha_pred), components_distribution=tfd.
            Gamma(concentration=conc_pred ,rate=rate_pred))
         return quantile_mixture(0.95, gm)
205
206
207
    plt.rcParams ["figure.figsize"] = [16, 6]
208
    rps = 26
209
    rep=1
210
211
    filename=''
212
213
    df_plt = pd.read_csv(filename, index_col=None)
214
    st_test =df_plt.pop('st')
215
    ttr_test = df_plt.pop('ttr')
216
217
    plt.plot(np.arange(600,800), ttr_test[600:800], label = 'Ground
218
        truth ')
    plt.plot(np.arange(600,800), ttr_pred_mean[600:800], label = 'TTR
219
        estimation , h=5')
    plt.plot(np.arange(600,800), ttr_upper_bounds[600:800], label = '
220
        95-percentile, h=5', marker='_', alpha=0.7)
221
    plt.xlabel("req. id")
222
    plt.ylabel('TTR [s]')
223
    plt.title('TTR for (rps = + str(rps) + , rep = + str(rep) + ')')
224
    plt.legend()
225
226
227
    plt.figure(figsize = (7,7))
228
    plt.scatter(ttr_test[5:800], ttr_pred_mean[0:795], c='crimson',
229
        label="Test data")
230
231
    p1 = max(max(ttr_pred_mean[0:795]), max(ttr_test[5:800]))
232
    p2 = \min(\min(ttr_pred_mean[0:795]), \min(ttr_test[5:800]))
233
```

```
plt.plot([p1, p2], [p1, p2], 'b-')
234
                                    plt.xlabel('True Values', fontsize=18)
plt.ylabel('Predictions', fontsize=18)
235
236
                                    plt.axis('equal')
237
                                    t = ("MAE=" + str(round(mae_mean, 5)) + " s" " \ MSE=" + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" " \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " s" \ MSE= " + str(round(mae_mean, 5)) + " \ MSE= " + str(round(mae_mea
238
                                    \begin{array}{c} mse\_mean, 5)) + " s^2") \\ plt.text(0.02, 1.1, s=t) \end{array}
239
                                     plt.legend()
240
                                     plt.title("Predictor: mean given history")
241
                                     plt.show()
242
```

Listing A.3: Code for the with history characterization

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