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Title

Tolerance optimization with statistical approach in the CAT environment

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Abstract

Nowadays, product development in all its phases plays a fundamental role in the industrial chain. The need for a company to compete at high levels, the need to be quick in responding to market demands and therefore to be able to engineer the product quickly and with a high level of quality, has led to the need to get involved in new more advanced methods/ processes.

In recent years, we are moving away from the concept of 2D-based design and production and approaching the concept of Model Based Definition. By using this approach, increasingly complex systems turn out to be easier to deal with but above all cheaper in obtaining them. Thanks to the Model Based Definition it is possible to share data in a lean and simple way to the entire engineering and production chain of the product. The great advantage of this approach is precisely the uniqueness of the information. In this specific thesis work, this approach has been exploited in the context of tolerances with the aid of CAD / CAT software.

Tolerance analysis or dimensional variation analysis is a way to understand how sources of variation in part size and assembly constraints propagate between parts and assemblies and how that range affects the ability of a project to meet its requirements. It is critically important to note how tolerance directly affects the cost and performance of products.

Worst Case Analysis (WCA) and Statistical analysis (RSS) are the two principal methods in DVA. The thesis aims to show the advantages of using statistical dimensional analysis by creating and examining various case studies, using PTC CREO software for CAD modeling and CETOL 6σ for tolerance analysis. Moreover, it will be provided a comparison between manual and 3D analysis, focusing the attention to the information lost in the 1D case.

The results obtained allow us to highlight the need to use this approach from the early stages of the product design cycle.

Summary

1. Introduction

The quality of a product is based on 3 main requirements: functional, performance and technological point of view.

The object function and its construction are defined by a technical drawing, also called engineering drawing. As we are referring to a precise diagram or plan that express information about the object, we are talking about technical drawing. The drawings, defining the requirements for engineering products, are used by engineers as guides when constructing or repairing object. Moreover, them are used as technical manuals and as trouble-shooting tools for identifying the weak spots in a mechanical design. Mechanical drawings depend on precise mathematical equations to accurately represent the mechanism and its component parts.

The Mechanical Designer has the responsibility for the quality of the product, taking care of the regulations, from ISO to ASME, and redacting all the documents that could help to know all the product's requirements. Quality plays a relevant role in the product's lifecycle as tolerancing activities plays it too. Three-dimensional statistical tolerance analysis allows to ensure the appropriate quality, reducing the costs.

In order to reduce the number of costly amendments and allowing engineers to complete their jobs efficiently, a Model Based Definition, from now on MBD, provides them with a clear comprehension and method to use the design and development process for communication.

1.1 Drawing based design and its limits

Over the time 2D drawing has become more and more inadequate. 2D drawing is the base of the product definition but 3D drawing is taking over for some several reasons:

1. 3D visualization is more complete and simpler to understand because of the improving of technologies. Moreover, it has a logical and a common interpretation accessible to anyone;
2. 2D is the logical consequence of the 3D drawing, indeed 2D is created from 3D drawing.

It's important to concatenate be-dimensional with three-dimensional drawing to eliminate inaccuracies and errors. Both drawings get to a lot of documentations at our disposal.

Looking even further ahead we can introduce the Model Based Definition that helps us to reduce the need to generate 2D drawings and it also enables downstream applications to directly access this information for automating tasks such as tolerance stack-up analysis, CNC programming, and CMM[1] analysis using other PLM[2] products as well as third party applications. The real gaining is to have vast access to the right data at the right time with the right amount of detailed information without errors and the needing of update data.

The MBD is an approach to create a 3D model that is actually effective if all the definitions of a product are realized. The suppliers and all the manufacturers chain can use the model in direct product data in 3D models. So that, the MBD becomes the main source for all technical design activities. This allows them to create a single source of data for the entire extended team which avoids errors and saves valuable time.

MBD is the latest revolution in product development and manufacturing offering an integrated technology that provides all the manufacturing data in 3D CAD (Computer Aided Design) drawings without a single 2D engineering drawing being required at all.

Within the 3D CAD software, all the information that were traditionally added to 2D engineering drawings, can now be applied directly to the 3D model instead, including:

- manufacturing notes;
- design intent;
- standard dimensions and geometric dimensions and tolerancing (GD&T) data;
- bill of materials (BOM);
- configurations for engineering.

[1] A coordinate measuring machine (CMM) is a device that measures the geometry of physical objects by sensing discrete points on the surface of the object with a probe. Various types of probes are used in CMMs, including mechanical, optical, laser, and white light. Depending on the machine, the probe position may be manually controlled by an operator or it may be computer controlled. CMMs typically specify a probe's position in terms of its displacement from a reference position in a three-dimensional Cartesian coordinate system (i.e., with XYZ axes). In addition to moving the probe along the X, Y, and Z axes, many machines also allow the probe angle to be controlled to allow measurement of surfaces that would otherwise be unreachable.

[2] Product lifecycle management (PLM) is the process of managing a product's lifecycle from inception, through design and manufacturing, to sales, service, and eventually retirement. As a technology, PLM software helps organizations to develop new products and bring them to market.

Benefits of MBD:

- 2D drawings don't need to be created. Cost reduction is the logical consequence;
- minimization in scrap and rework. Before the model-based definition there were a lot of misinterpretation of info from the 2D drawing;
- now all the quality requirements can be added directly to the 3D model, which is used directly to manufacture the component, thus helping to maximize the product's quality.

1.2 Top 5 reasons to use MBD approach

To manufacture a product quickly and disseminate information easily, MBD can be leveraged. Thanks to MBD, it is possible to improve the handling and processing of the engineering change orders (ECO's) [1], after a product has been put into production and changes are made to it. When the model is updated with the appropriate changes, all the related data are automatically updated in the master 3D model. That eliminates the risk of wrong drawings being sent to suppliers or suppliers not updating the drawings for manufacture.

1) MBD further automates manufacturing with software-readable product and manufacturing information (PMI) [2]

The Computer Aided Manufacturing (CAM) software programs read CAD models to automate Numerical Control (NC) code generation.

Generally, in 2D drawings are defined tolerances and surface finishes. Because of that, CAM software cannot read drawings. Therefore, manufacturing engineers have to look back and forth between drawings and CAM programs to manually extract and re-enter these requirements. This step not only slows down the process, but also introduces data duplication, human interpretation and re-entry errors.

MBD provides us to define software-readable PMI directly in 3D models, rather than in 2D drawings. The 3D PMI can be read and modified by the CAM software. This avoids that human has to enter again data, which speeds up production and reduces errors.

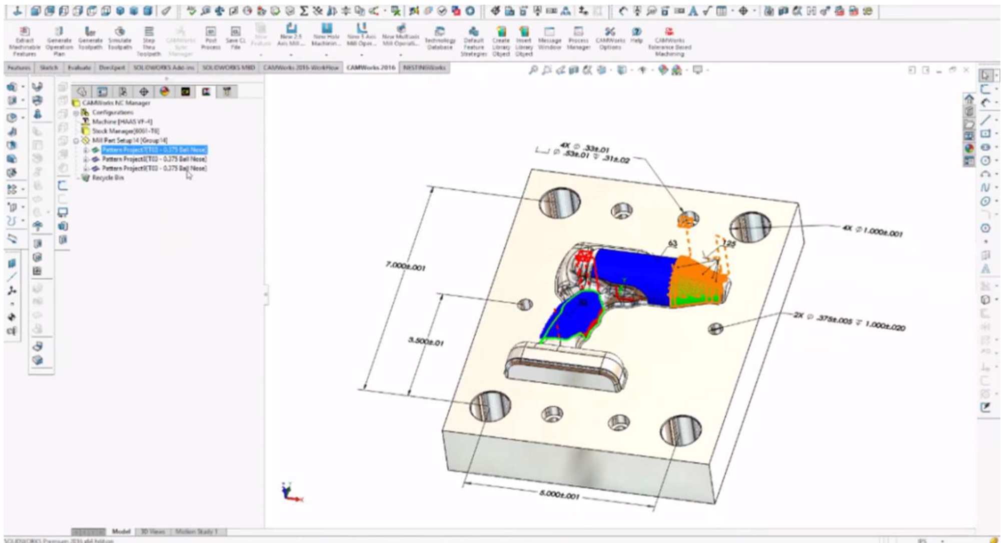


Figure 1. CAM-Works reuses defined 3D surface finishes to automate NC programming

MBD can automate many other procedures such as cost analysis, quoting, process planning, robot programming, tolerance stack-up analysis.

The National Institute of Standards and Technology (NIST) in the United States conducted the study: "*Testing the Digital Thread in Support of Model-Based Manufacturing and Inspection*" [3], to prove the benefits of MBD.

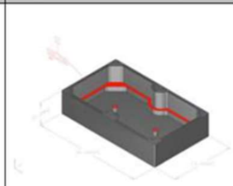
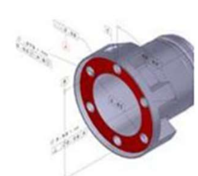
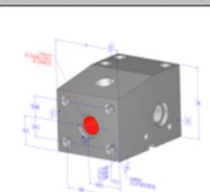
Test Case	1 (Full Annotation)		2 (Hybrid Annotation)		3 (Reduced Annotation)	
Model						
Approach	Drawing	MBD	Drawing	MBD	Drawing	MBD
Net hours	83.1	18.1	60.2	14	37.7	13.5

Figure 1.1 Time saved during three steps: annotation, machining and inspection

Chart

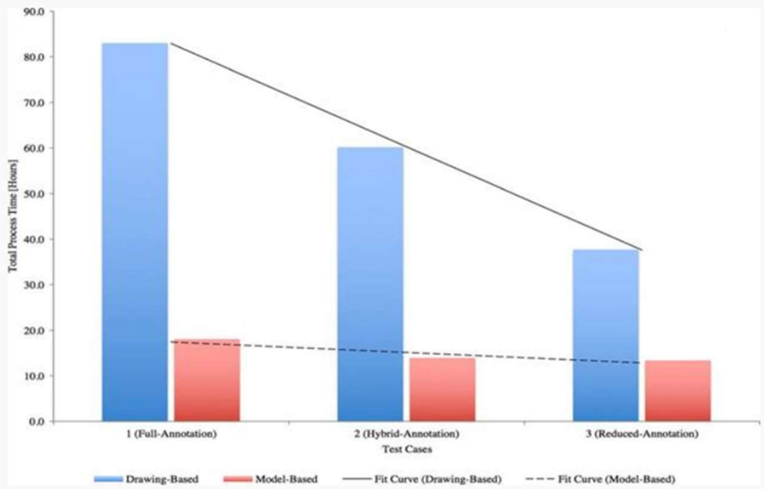


Figure 1.2 Time saved during three steps: annotation, machining and inspection

The research team compared drawing-based and MB method side by side in three passages: annotation, machining and inspection. The model-based approach saved over 60 percent of the net hours through different functional test models as shown in figure 1.1. The time savings primarily came from the automations powered by the software-readable PMI.

2) MBD increases technical communication efficiencies [2]

When an object has to be produced, we have to project 3D objects down to a 2D plane. Then, the normal process wants to re-create the 2D drawing into 3D again.

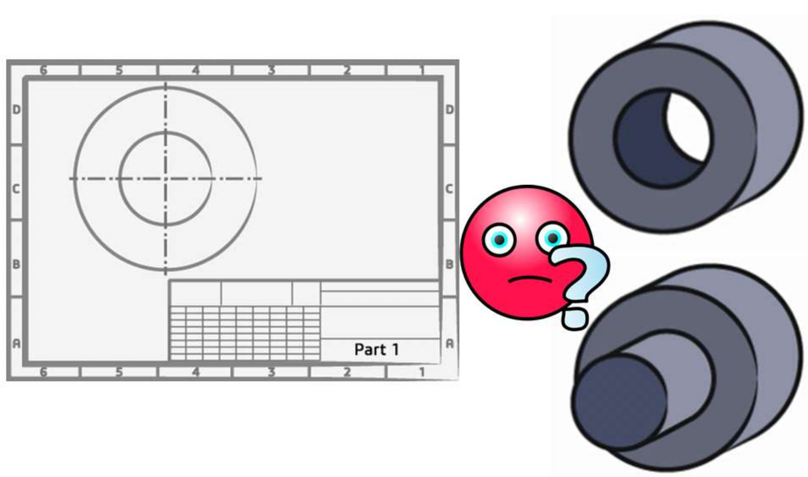


Figure 1.3 Ambiguity in a simple 2D drawing. Is it a cut or an extrusion?

Figure 1.3 is a representation of a detour, and it becomes excessive when you consider that most designs are built as 3D CAD models anyway. It is difficult for the drawer to interpret the drawing, as there is a discrepancy. That leads to the drawing being ambiguous and time being lost in understanding it. To interpret the drawing, we have to look for another view and multiple perspective. A simple drawing may be quick to figure out, but if we have to interpret a normal drawing, such as in Figure 1.4, there are a lot of things to consider. This can make communication even harder and less efficient.

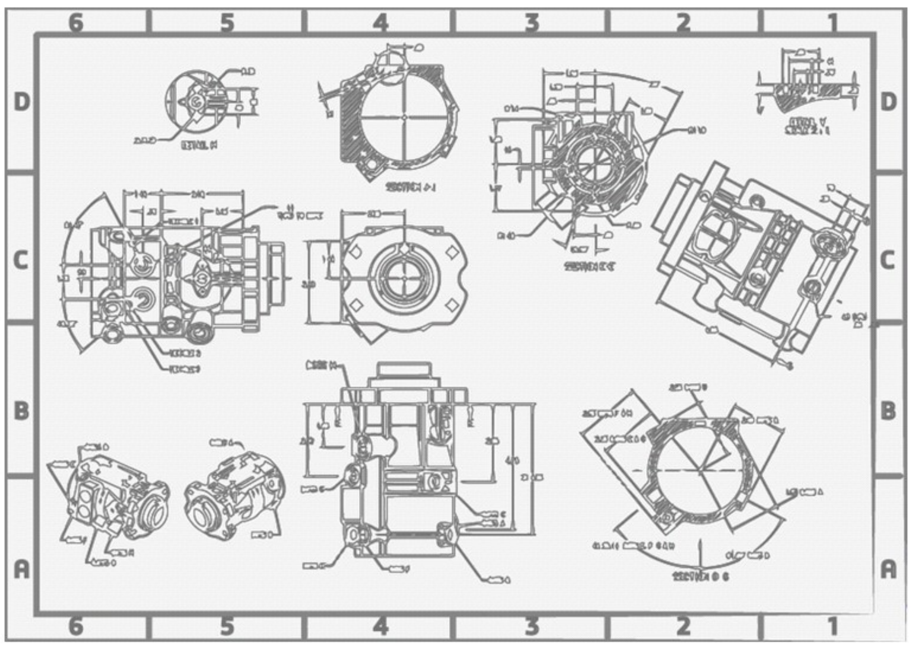


Figure 1.4 A normal drawing

For example, in the NIST study in Figure 1.1, Rockwell Collins sent to two suppliers three model to be done by two main steps. One supplier used the model-based approach as an experiment and the other one used the drawing-based method as a controlled comparison.

The vendor who used the MBD delivered parts in five weeks, while the one that used the drawing-based takes about eight months. The root cause was that the drawing-based supplier had to raise 12 questions related to interpreting the product definition from drawings, which led to work stoppages because the job had to be removed from the queue until clarifications were provided. On the contrary, the model-based supplier did not ask for any information during his activity.

In today's manufacturing industry, the links between the "actors" are becoming more and more weaving, leading to give more importance to them. For example, a Boeing 787 Dreamliner contains about 2.3 million parts according to Jeff Plant with Boeing commercial airplanes. These are just final parts.

Bob Deragisch with Parker Aerospace highlighted that one modification to a mere manifold created 1,700 changes to other related models and systems. The engineering change order (ECO) drawings would be 100 pages for this single change alone. If all the drawings of an airplane were printed, the package would be even bigger than the airplane, to which Deragisch declared "I can't do that anymore with drawings!"

If a picture is worth 10000, then a model is worth a 10 million words because it's in 3D and we can rotate and query it. The growing in complexity of today's manufacturing requires the use of MBD.

The Model Based Definition provides a 3D presentation rather than a 2D abstraction. It minimizes the ambiguity and time being lost in understanding it as well as miscommunication. In addition, dedicated MBD capabilities such as the cross-highlighting from a callout to its corresponding features provides an instant visual confirmation as shown in Figure 1.5.

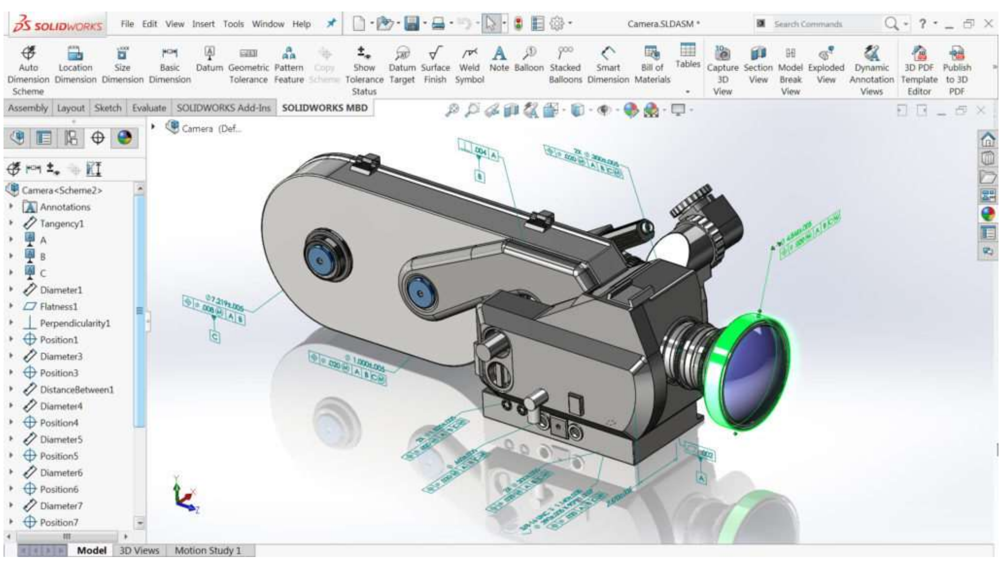


Figure 1.5 Cross-highlighting from a 3D callout to corresponding features

Thanks to MBD it is possible to save time avoiding 2D drawing. However, we need to create certain 3D callouts in models. Moreover, 3D callouts are faster to create than 2D thanks to the feature-based 3D PMI automation. Therefore, the real saving comes from the data consumption side. In fact, with MBD the data is created only once but it is used many times.

3) MBD improves product quality [2]

MBD can improve a lot the quality of a product. Although the NIST report quoted the net hours and the total delivery time in a side-by-side comparison between drawing based and MBD approaches, it is proved that time is not the only improvement. There were also major quality differences between drawing based and MDB approaches.

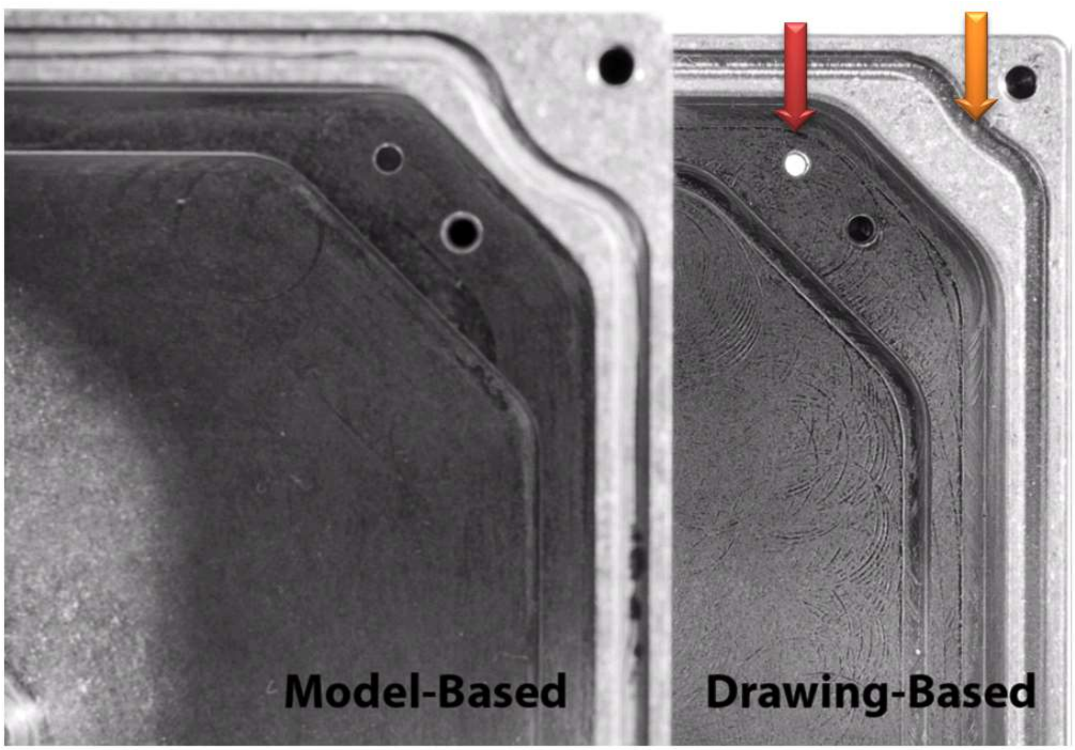


Figure 1.6 An unintended through-hole and a misshaped groove in the drawing-based part

The unintended hole overthrown the whole part because there was no convenience way to fill it up and make it sightless again. The root cause was that the drawing sent to the supplier missed a hole depth callout as shown in Figure 1.7.

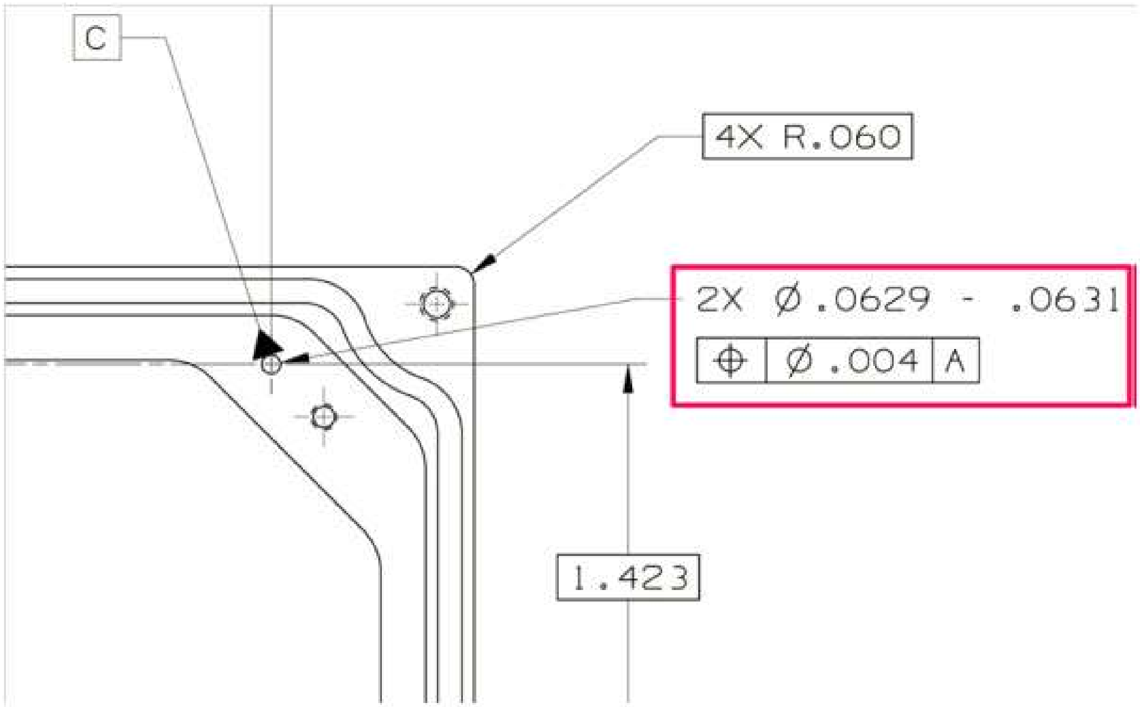


Figure 1.7 The hole depth callout was missing in the drawing

Without the depth, a hole defaults to a through-cut in drawings. Just looking to Figure 1.7 the machinist and even the inspector instinctively interpreted it as a through-cut. It didn't even occur to them that this could be a blind one because there was no way to tell visually. As a comparison, the model-based supplier caught this issue because it used the model as the authority in numerical code (NC) programming.

In Figure 1.6, notice the surrounding seal groove on the drawing-based part on the right-hand side didn't match the original design. This may not be a major issue but does demonstrate another quality discrepancy due to the drawing-based approach. This type of issue increases the cycle time and decreases manufacturer's margin as well as can compromise customer satisfaction.

Are these quality issues the result of mismatching between 3D models and 2D drawings? If the drawings had matched the models perfectly, these issues would have been prevented.

According to some manufacturers, up to 60 percent of be-dimensional drawings don't match three-dimensional designs. The link between models and drawings are busted, intentionally or unintentionally.

The solution is to create drawings perfectly concatenate to models i.e., put drawings and models together. Even better if we bypass drawings putting 3D PMI directly into models.

4) MBD establishes manufacturing competitive advantages

In the public sector, the Department of Defense (DoD) in the United States released the Military Standard 31000 revision A in 2013 to specifically define the requirements and best practices for its supply chain. The Model Based manufacturing is one of the four pillars in General Eletric (GE) factory initiative, along with automation powered by sensors and the Industrial Internet of Things, process prototyping and informatics.

The governing body of the Japan Industrial Standards (JIS[3]) is the Japan Electronics and Information Technology Association, or equivalent of ISO in Europe, or equivalent of ASME standards in the U.S. In 2014, JEITA members began to learn about MBD visiting software suppliers and manufacturers across Europe and the United States of America. Japan is trying to implement a new JIS standard for MBD. [2]

These driving forces from the top of the global supply chain are generating strong ripple effects in the manufacturing industry. Manufacturers have to catch up and plan ahead in order to be competitive on the market. For example, Figure 1.8 shows growing percentages of SOLIDWORKS customers using or planning to use MBD.

[3] Japanese Industrial Standards (JIS) (日本産業規格, Nihon Sangyō Kikaku, formerly 日本工業規格 Nihon Kōgyō Kikaku until June 30, 2019) are the standards used for industrial activities in Japan, coordinated by the Japanese Industrial Standards Committee (JISC) and published by the Japanese Standards Association (JSA). The JISC is composed of many nationwide committees and plays a vital role in standardizing activities across Japan. Standards are named in the format "JIS X 0208:1997", where X denotes area division, followed by four digits designating the area (five digits for ISO-corresponding standards), and four final digits designating the revision year.

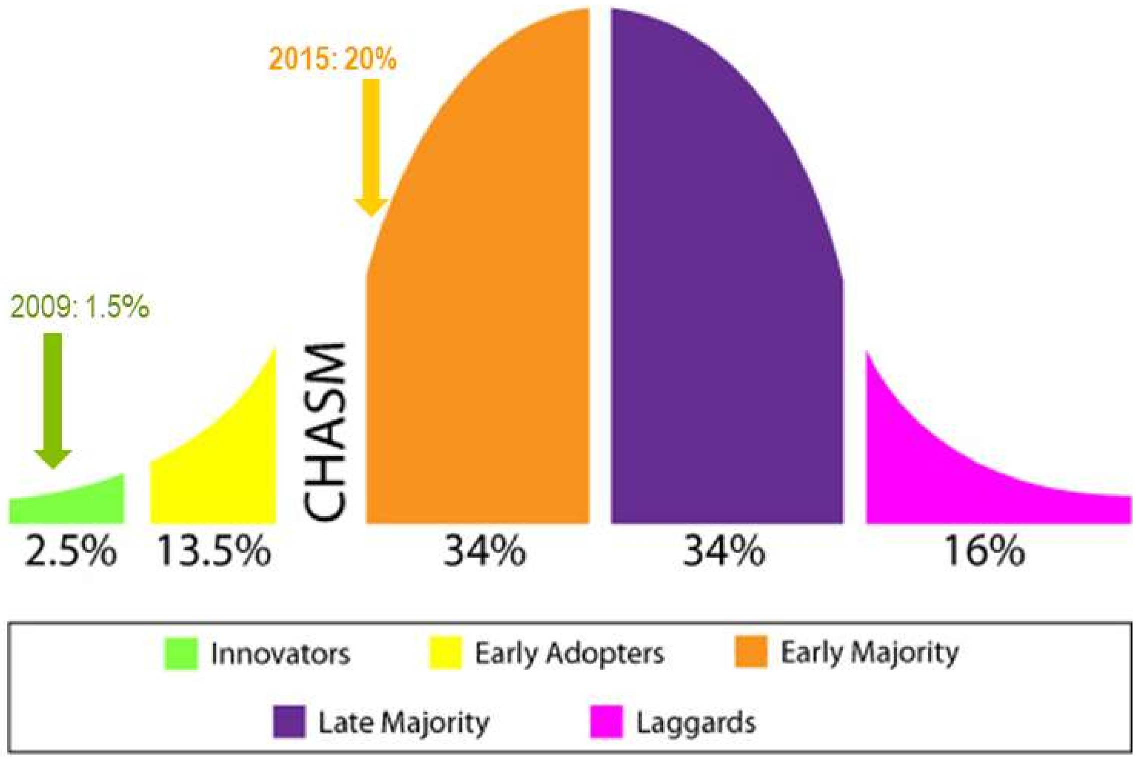


Figure 1.8 Growing percentages of SOLIDWORKS customers using or planning to use MBD (Survey sample sizes: 700 in 2009 and 524 in 2015)

5) MBD unleashes the power of emerging technologies [2]

Manufacturing is improving day by day thanks to emerging technologies. We are talking about 3D printing, big data analysis, sensors, artificial intelligence and connected machines. There have been many initiatives around the globe, such as Industrial Internet of Things in the United States, Industry 4.0 in Germany and Made in China 2025.

MBD facilitates the improving of these new technologies. For example, 3D printing a part is easier with a 3D CAD model. Otherwise, 3D printing is impossible to obtain with 2D drawings. The part has to be checked after printing using 2D drawings, referring to its dimensioning and tolerancing requirements. However, it is worthless to use 2D drawing only for examination purposes. It is better to insert PMI directly into the 3D models.

Generally, all the tolerances are defined and locked in 2D drawings. Engineers have to visually read and manually product a spreadsheet from drawings to calculate all the main tolerances needed. MBD can analyze digital tolerances through software.

Moreover, the downstream quality and cost data can be extrapolated and correlated back with the upstream tolerances to obtain better designs. The GE "Brilliant" Factory initiative in Figure 1.9 illustrated as-built quality and cost data and as-designed tolerances as the production feedback loop and the design feedback loop. The closed-loop analysis can reveal significant intuition to cut costs while improving quality. The cost and quality goals may sound at the antipodes, but the reality is most tolerances are too much conservative. We all end up with large tolerance just to be safe, but it is possible to loosen them to increase pass rates, but we do not know where to loosen without compromising the asset/quality.

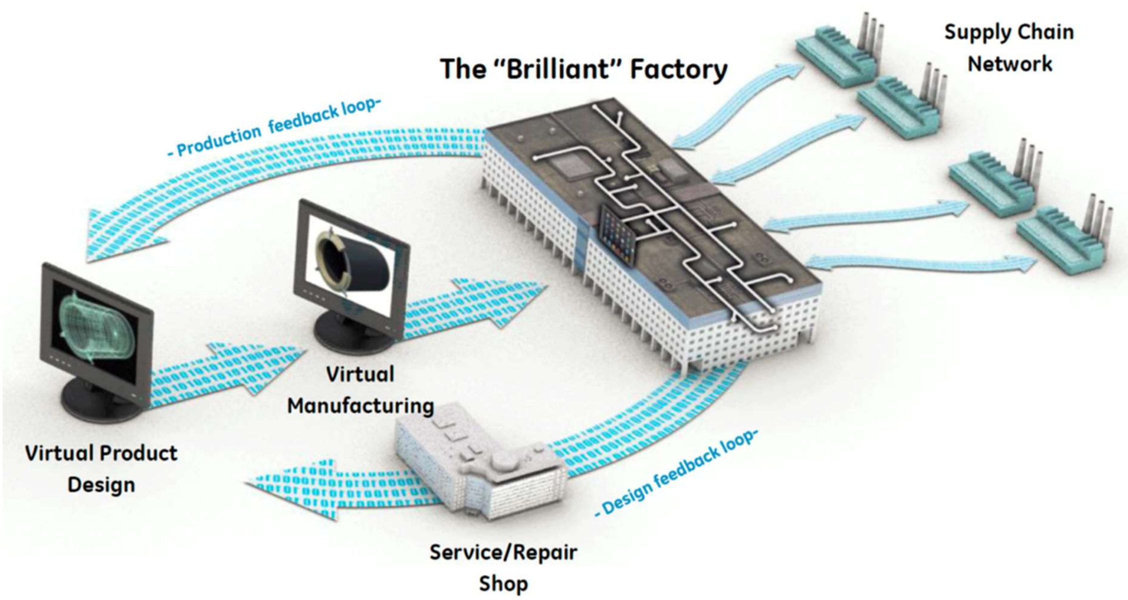


Figure 1.9 The GE "Brilliant" Factory initiative: the closed-loop tolerance

1.3 Product Data Quality (PDQ)

Product Data Quality (PDQ) is a term that defines the standard for establishing the quality level of the CAD models. International standards (e.g., SASIG, VDA, JAMA, MIL-STD-31000A) are beginning to describe PDQ recommendations for MBD entities. In addition, quality-criteria (QC) definition may also be based on the process-driven criteria (PDC), which is a smaller subset of the international standards. The model is analyzed for the quality requirements for finite element method analysis (FEM), metrology, manufacturing, using the process-driven criteria (PDC). MIL-STD-31000 Revision A, Appendix C [12] provides a set of recommended numerical thresholds for geometry-validation criteria.

Table 1 Automotive industry threshold values from MIL-STD-31000 Rev. A, Appx. C [11]. Table lenged: O—optional when required by design, A—value agreed by sender and receiver, U—user preference, NR—not required, and T—logical value true.

Criteria/use case	Native data Design	Analysis	Manufacturing	Derivative data ISO 10303	Visualization	Translation
Curve criteria						
Large curve or segment gap (G-CU-LG)	0.01 mm max.	0.01 mm max.	0.01 mm max.	0.01 mm max.	NR	0.01 mm max.
Non-tangent curves or segments (G-CU-NT)	2 max.	O	O	3 max.	O	O
Tiny curve or segment (G-CU-TI)	0.01 mm min.	0.01 mm min.	0.005 mm min.	0.005 mm min.	NR	0.005 mm min.
Self-intersecting curve (G-CU-IS)	0.01 mm	0.01 mm	0.01 mm	0.01 mm	NR	0.01 mm
Surface criteria						
Nontangent surfaces or patches (G-SU-NT)	2 max.	O	O	3 max.	NR	3 max.
Narrow surface or patch (G-SU-NA)	0.01 mm min.	A	0.005 mm min.	0.005 mm min.	NR	0.005 mm min.
Self-intersecting surface (G-SU-IS)	0.01 mm	0.01 mm	0.01 mm	0.01 mm	NR	0.01 mm
Edge loop criteria						
Self-intersecting loop (G-LO-IS)	0.01 mm	0.01 mm	0.01 mm	0.01 mm	NR	0.01 mm
Face criteria						
Large edge face gap (G-FA-EG)	0.01 mm max.	0.01 mm max.	0.01 mm max.	0.01 mm max.	NR	0.01 mm max.
Narrow face (G-FA-NA)	0.01 mm min.	0.01 mm min.	0.01 mm min.	0.01 mm min.	NR	0.01 mm min.
Embedded faces (G-FA-EM)	NA	A	0.01 mm min.	0.01 mm min.	NR	0.01 mm min.
Inconsistent face on surface (G-FA-IT)	T	T	T	T	T	T
Shell criteria						
Large face gap (G-SH-LG)	0.01 mm max.	0.01 mm max.	0.01 mm max.	0.01 mm max.	NR	0.01 mm max.
Over-used edge (G-SH-NM)	>2	A	>2	>2	>2	>2

Figure 1.10 Automotive Industry Threshold Values

2. Tolerance Analysis

Tolerance analysis [4] is a way of understanding how sources of variation in part dimensions and assembly constraints propagate across parts and assemblies, and how that total variation affects the capability of a design to achieve its design requirements within the process capabilities of manufacturing organizations and supply chains. During the tolerance analysis process are determined sources of the variation, stack-up[4]. The only way to understand more the source of dimensional variability identifying future issues before they become problems, reducing scraps, assembly fit issues and meeting the dimensions quality necessities, we have to analyze the effects of dimensional part and process variation (**tolerance analysis**).

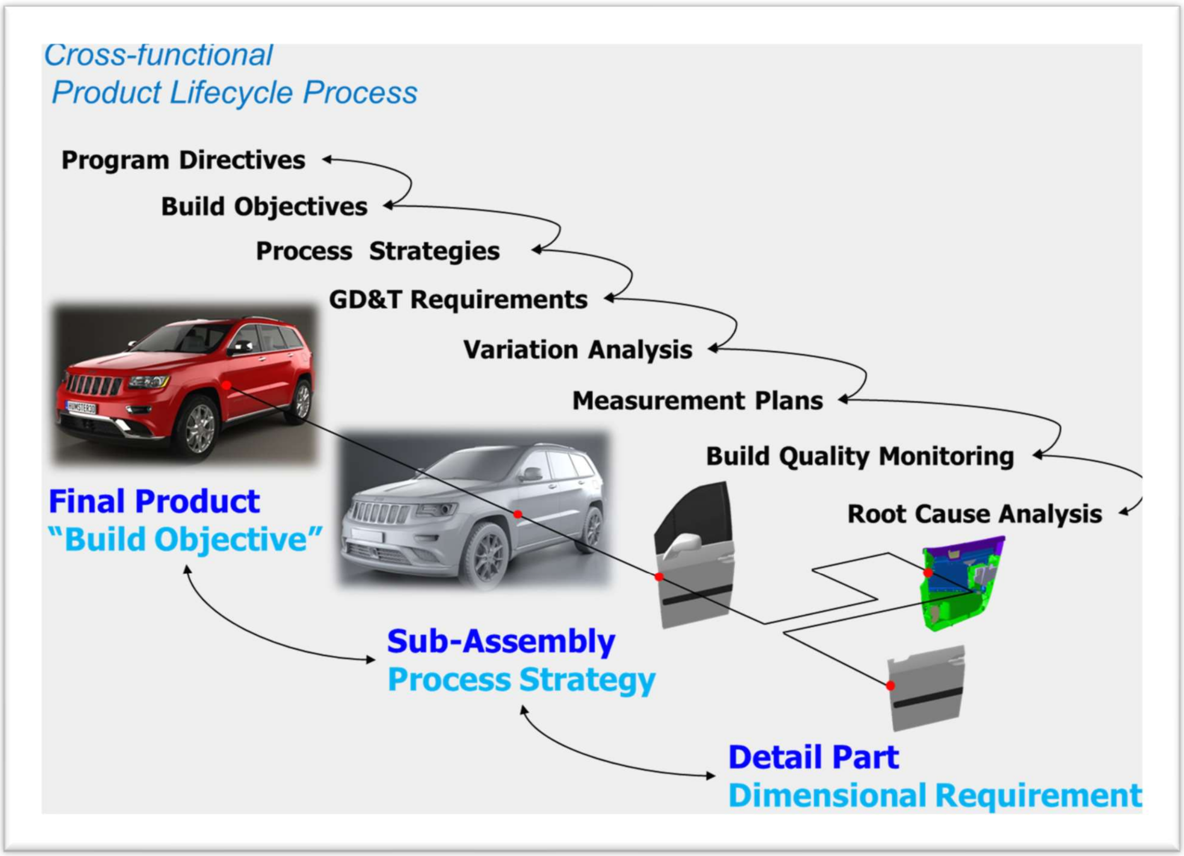


Figure 2.1 Tolerance analysis is an integral part of the engineering process

When manufactured, parts are not perfect aligned to the specifications previously designed. In fact, there may be a difference between what we designed and what we produced.

[4] The combined variation of all parts in a given assembly.

This variation is caused by material features and manufacturing processes. The nominal design is never achieved. In fact, parts are made larger or smaller. The range of variation acceptable in the design is defined as tolerance.

Tolerancing establish a dimensional tolerance that means to indicate the limits within which a certain size can vary. When defining a tolerance, we are designating a range where a given dimension, such as an edge, profile or hole size, could be assembled granting the functionality. Thanks to STD languages it is possible to better understand tolerancing methods.

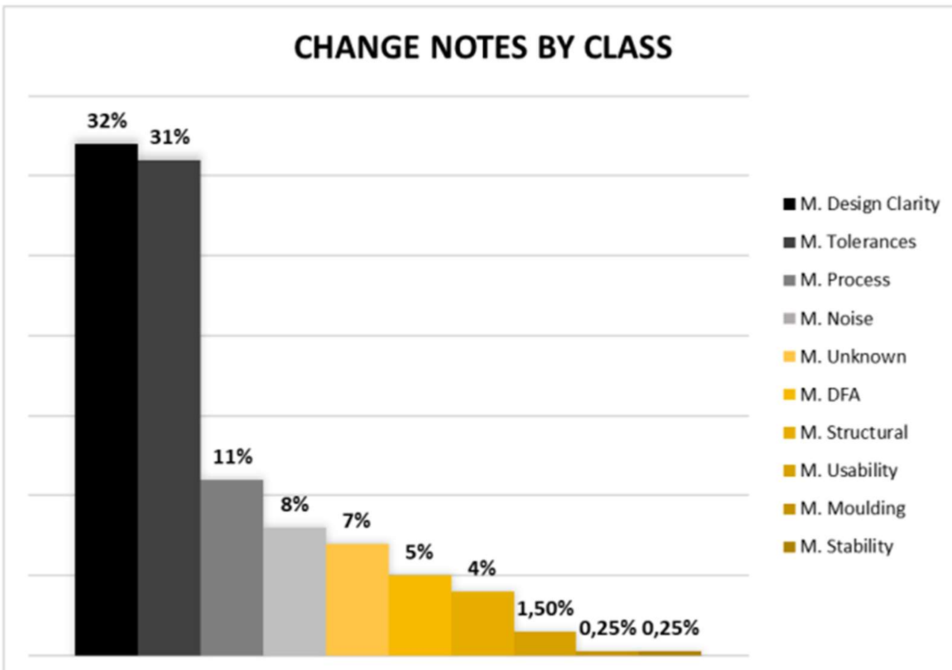


Figure 2.2 Mechanical Change Notes subcategorized into various issue-types. 63% of the change notes were related to Tolerance and so-called Design Clarity issues

M. Ebro [5] based his research on the # of Change Notes[5] collected during the production cycle. He states that changes in development of a new products are often late and that the 63% of late is caused by unclear concepts or tolerances. In order to manage the high costs caused by the late, it can be useful to use more robust concepts.

[5] The #of Change Notes was collected by making a simple query in the company's PDM system. This generated a report with 800 Change Notes, with a short description of what the problem was and what had been changed. A group consisting of the author, two quality managers and a technology manager categorized the change notes. First, they were categorized into software, hardware and mechanical issues and afterwards, the mechanical issues were subcategorized into structural failures, usability, tolerance issues etc. The results are shown in Figure 2.2.

These standard languages mentioned above consist in ISO and ASME Y14.5 2009 Standard for the design language of geometric dimensioning and tolerancing (GD&T). ISO and ASME standards set the uniqueness using od the GD&T to be adopted on engineering drawings and documents.

GD&T is an important part in defining part tolerances and quality. As stated by ASME, *"GD&T is an essential tool for communicating design intent — that parts from technical drawings have the desired form, fit, function and interchangeability. By providing uniformity in drawing specifications and interpretation, GD&T reduces guesswork throughout the manufacturing process — improving quality, lowering costs, and shortening deliveries."* The tolerance has a direct impact on the cost and performance of a product. The usage of a stamping die leads to decreasing costs instead of machining that has to be more precise. A component is expensive if it is difficult to manufacture and has very tight tolerances. Tolerances affect the performances of a product. For example, it will be a problem if a car door will not close well. In fact, if the tolerances are very large there will be a lot of road noise from a poor seal. Again, an engine will not work properly if the crankshaft has tighter or smaller tolerances instead of nominal tolerances.

In order to have lower prices and high-quality products, it is important to focus on tolerancing and tolerance analysis.

It is impossible to manufacture product without detours from the nominal shape. Deviations of size, form, orientation and location are the main problems that a workpiece could have.

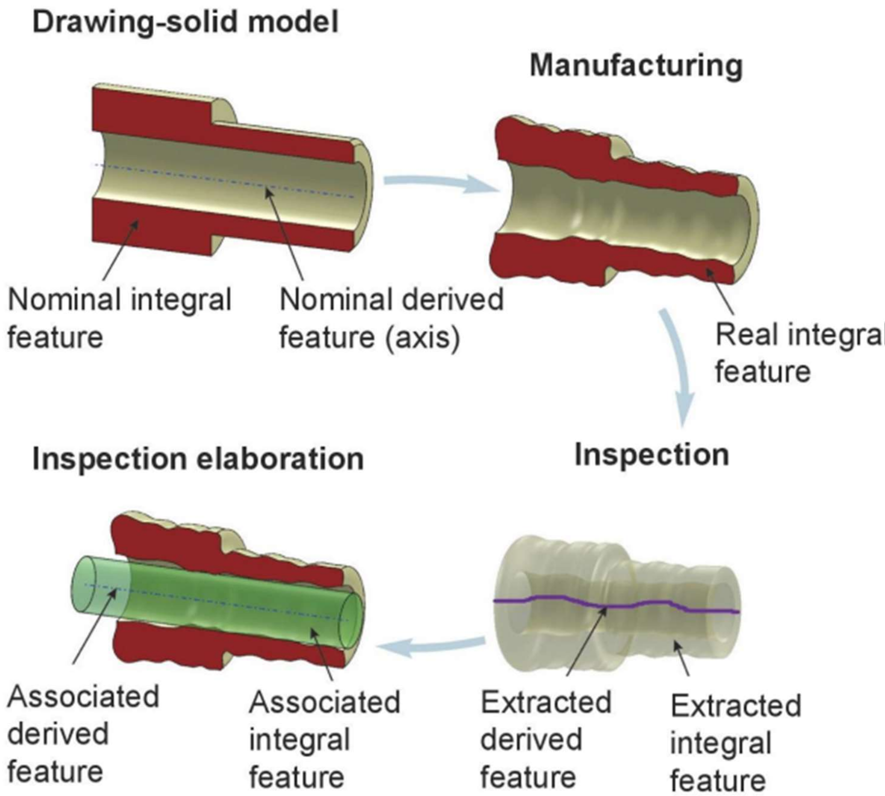


Figure 2.3 Real vs ideal part

Deviation needs to be monitored because a large deviation compromises the usability of the part. Trying to minimize it, manufacturing companies may not be competitive on the market, because of high costs of monitoring. Therefore, each property (size, form, orientation and location) must be tolerated. A good tolerancing lead to produce parts as precise as necessary and as economic as possible. Incompletely tolerated drawings result in:

1. questions for the production-planning engineer;
2. questions for the manufacturing engineer;
3. questions for the inspection engineer;
4. reworking;
5. defects, damages.

Two main standard organizations regulate dimensioning and tolerancing in order to have a common design language: in Europe, by ISO (International Organization for Standardization) and, in the U.S., by ASME (American Society of Mechanical Engineers). These international standards completely define representations and indications of dimensions and tolerances in the technical product documentations [6, 7, 8]. Dimensioning and tolerancing are two fundamental aspects of the technical documents' redaction, strictly connected each other. Dimensions specify the nominal form, size, orientation and location of part features. They can be classified as functional, manufacturing and inspection depending on the different approaches used, due to different aims or different requirements in technical drawing. Functional dimensions define the aim of the assembly and consequently of the single components involved, in strictly correlation with the notion of product interchangeability and repeatability of manufacturing processes. The standard UNI ISO 129-1 [6], integrated with the definitions and terms contained in UNI EN ISO 10209:2012 [9], defines principles for functional dimensioning. The designer should guarantee the correct functionality of the mechanism, without referring to a given determined scheme of dimensioning but considering each dimensioning case as a new case [10]. Manufacturing dimensions underline the dimensions involved in the specific manufacturing process, where reference systems are fundamental. In fact, the workpiece needs to be properly positioned for being correctly machining and for guarantee the machining repeatability of the others [11]. Inspection dimensions provides the information for evaluating the component conformity, according to its design intent. The dimensions and tolerances on the component have to establish a clear and measurable scheme for a certain and unambiguous evaluation [11]. Tolerances, instead, quantify the allowed errors along the entire production process of parts according to different aspects that have been classified in the following chart, Figure 2.4.

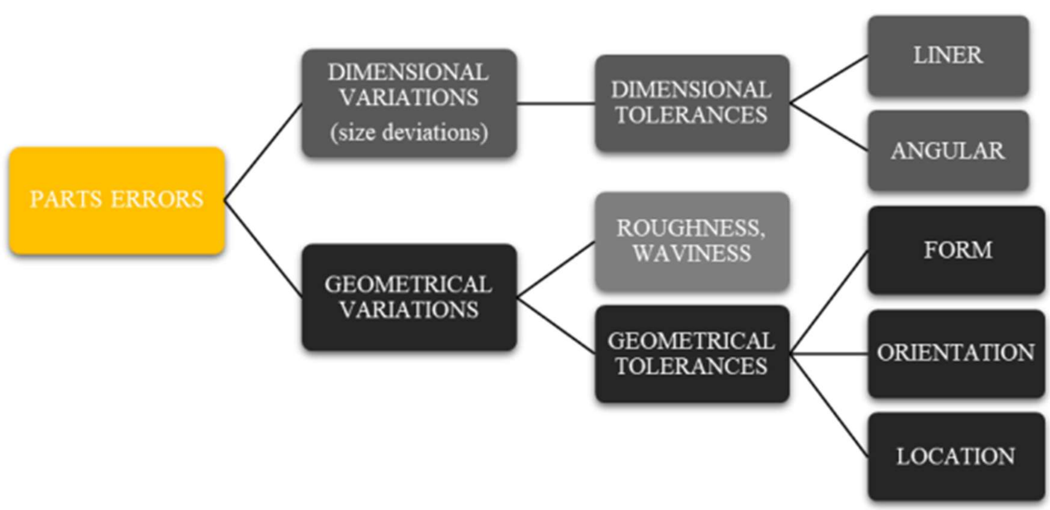


Figure 2.4 Part errors

The dimensional variation (or size deviation) is the difference between actual size and nominal size. We have: deviation from the nominal linear size or from the nominal angular size. The actual local linear sizes are evaluated by two-point measurements (ISO 8015, ISO 286 and ISO 14 660-2), while the actual local angular sizes are evaluated by angular measurements of averaged lines (ISO 8015, ISO 1947), see Figure 2.5. Typically, these tolerances are stated in the same units as the dimension, and the permitted variability range is expressed using the upper and lower limits (plus/minus).

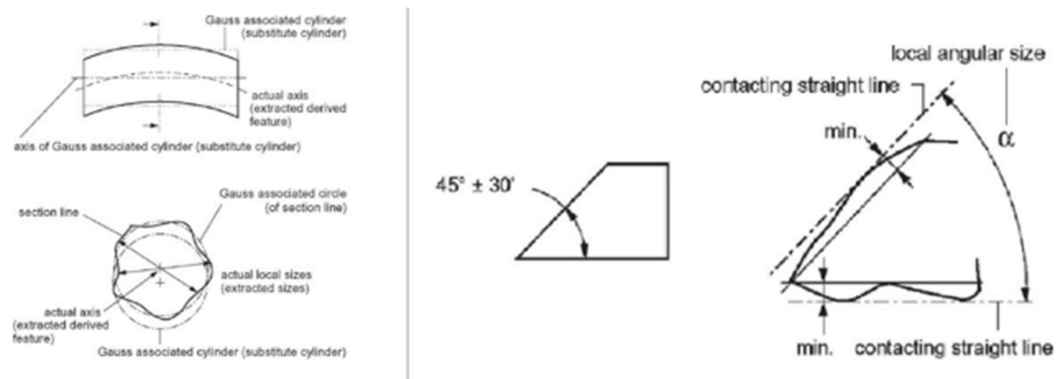
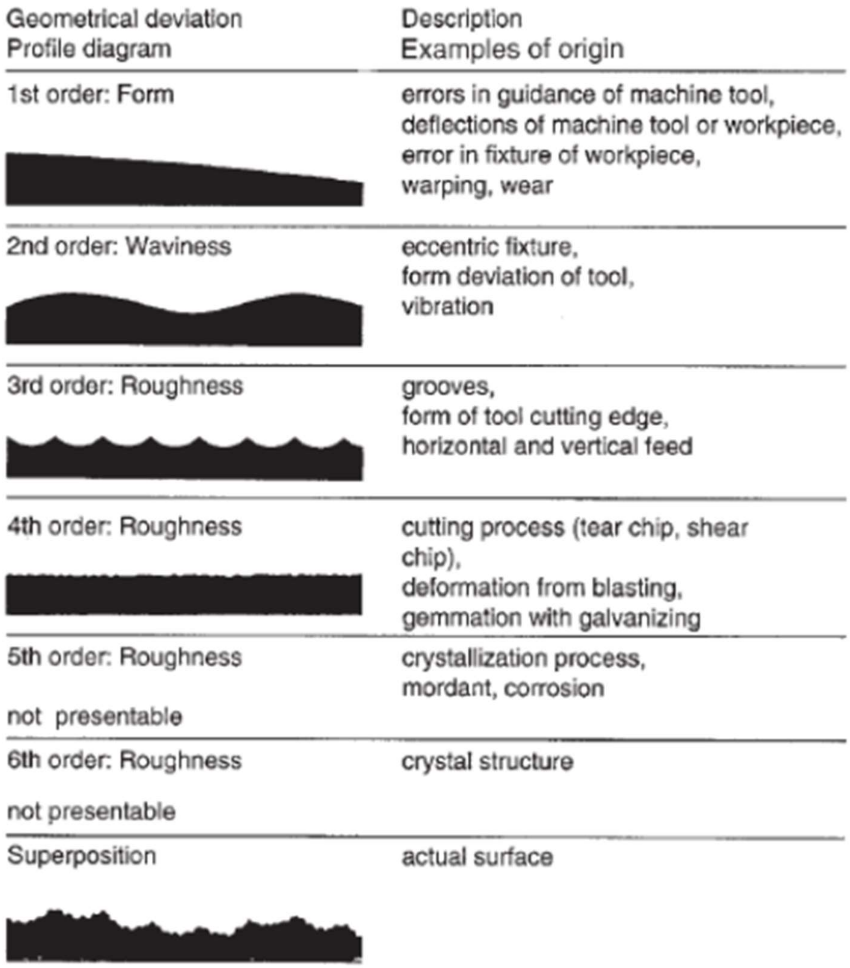


Figure 2.5 Dimensional tolerances measurements

With reference to geometrical variation, we can distinguish the micro-geometric errors (waviness and roughness) and the macro-geometric errors (form, orientation and position).

Waviness is a recurring defect of a product surface with spacings greater than the spacings of its asperity (DIN 4774). Generally, 1000:1 and 100:1 (VDI/VDE 2601) are the main ratios between spacing and depth of the waviness.

Waviness is originated by eccentric fixture during the manufacturing process or by form deviations of the cutter or by vibrations of the machine tool [12]. Roughness is periodic or non-periodic irregularities of a workpiece surface with small spacings due to the manufacturing process. The ratio between spacing and depth of the roughness is in general between 150:1 and 5:1 (VDI/VDE 2601). Roughness is originated by the direct effect of the cutting edges and cutting process. The combination of the two gives the real irregular surface (superposition), Figure 2.6.



Geometrical deviation Profile diagram	Description Examples of origin
1st order: Form	errors in guidance of machine tool, deflections of machine tool or workpiece, error in fixture of workpiece, warping, wear
2nd order: Waviness	eccentric fixture, form deviation of tool, vibration
3rd order: Roughness	grooves, form of tool cutting edge, horizontal and vertical feed
4th order: Roughness	cutting process (tear chip, shear chip), deformation from blasting, gemmation with galvanizing
5th order: Roughness not presentable	crystallization process, mordant, corrosion
6th order: Roughness not presentable	crystal structure
Superposition	actual surface

Figure 2.6 Superposition of surface deviations (DIN 4760)

The macro-geometric errors, instead, are classified in:

> **Form deviation.** It is the deviation of a feature (geometrical element, surface or line) from its nominal form, Figure 2.6. If it is not specified, the form deviation refers to the entire features. For example, form deviations are originated, by the looseness or error in tracks and bearings of the machine tool, deflections of the machine tool or the workpiece, error in the fixture of the workpiece, hardness deflection or wear.

> **Orientational deviation.** The deviation of a feature from its nominal form and orientation is called orientational deviation. Generally, is referred to one or more datum feature(s). The form deviation is included in the orientational one. If the workpiece is remounted on the machine tool and there is a positioning error, so we can talk about orientational deviation.

> **Locational deviation.** Referring to one or more datum features, it is the detour of a feature (surface, line, point) from its nominal place. The locational deviation includes both form and orientational deviations (of the surface, axis, or median face). They are originated similarly as size, form and orientational deviations [12].

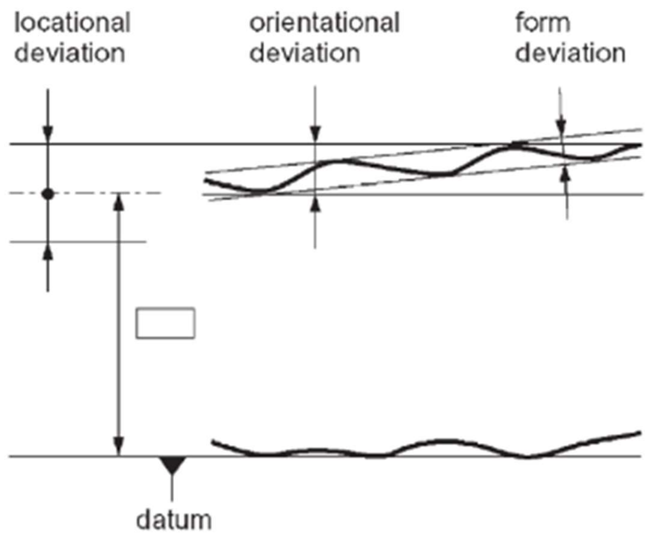


Figure 2.7 The hierarchical indication of form, orientational and locational deviation.

2.1 Traditional Methods of Tolerance Analysis

There are many methods of tolerance analysis. Even if they are so different, the result will be the same. As the complexity of the tolerance analysis increases, so does its accuracy and ability to account for more influencers in the process. Here below you have a synthesis from the simplest methods of tolerance analysis to the advanced ones 3D CAD based dimensional models.

1. ***Napkin Stacks*** – Stack-up Math on Paper -
   Pro – very fast. You are able to receive a fast response.
   Con – not so precise. Because of its simplicity, it should not be used for major programs or design changes. In fact, this method is good only to have a quick idea even if it could be approximated.
2. ***1D Stack Analysis*** – Excel Spreadsheet -

Pro – Like the Napkin Stack, this method can get answers quickly. Thanks to Excel macros, it is possible to enter the tolerances and the software calculates rapid stack-ups. It is one of the cheapest ways to calculate tolerances, like 1DCS (mono-dimensional Cad Software) and others based on Excel. As mentioned above, this method is good only to have a quick idea even if it could be approximated in fact, it is used for determining simple structures.

Con – low accuracy, lack of influencers, no root cause – It leaves out many influencers and gives no understanding of the root cause of build issues. This method should not be used if two parts do not fit together during assembly, because it does not give the precise answer.

3. ***2D Stack Analysis*** – Excel Spreadsheet or Software Tool

   Pro – This method provides the usage of Excel too. Merging all parts across a given plane gives a better understanding. Surface areas and simple structures are the perfect conditions to use it.

   Con – low precision, lack of influencers, no root cause – This method does not share 3D dimensional information determining good and bad parts in many workpieces. It is the best solution to examine a workpiece in 3D as all the pieces have effect on the geometry.

4. ***3D Stack Analysis*** – CAD Tool

   Pro – all influencers, root cause analysis, high degree of accuracy, process analysis– In order to determine the consequences on the characteristics of the pieces, the three-dimensional analysis merges all the pieces into an assembly. Thanks to this it is possible to carry out a detailed verification of the root cause. In addition to these benefits, many tolerance analysis software systems are used to model the assembly process. This gives a good comprehension of how your manufacturing and assembly process will affect and can be affected by variation.

   Con – Creating a tolerance analysis model needs experience. It is necessary to train the personnel to use 3D software since they are still complex processes to manage. This can be partially mitigated by creating simplified models, utilizing embedded (CAD) GD&T and Joints and Constraints or by reusing historical data and models.

Tolerance analysis can be used to reduce product cost while improving product quality.

A good tolerance analysis:

- Improve both visual and mechanical quality of your product.
- Define Gap and Flush conditions.

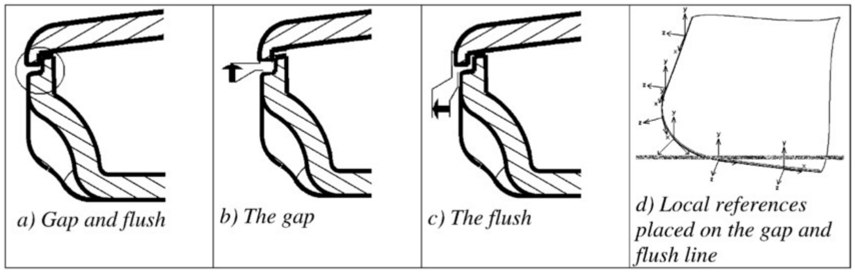


Figure 2.8 Gap and flush

- A good analysis allows us to obtain the best method and order obtained from process simulations (production and assembly).
- Decrease the total changes about compensating for it in design and reduce production errors using measurement data.
- Reduction of waste through GD&T.

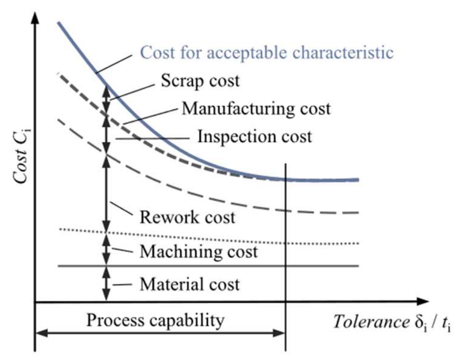


Figure 2.9 Tolerance cost function for a single process

2.2 Statistical Analysis Tools

Tolerances are statistical variables from manufacturing process to the assembly process. It is possible to reduce better the possibility of worst case by describing product variation as a statistical distribution.

Statistical simulation tools help us to understand how many scraps I could have. Moreover, we are able to know the probability that the product has to reach the requirements.

A dimensional tolerance analysis tool should be able to analyze the tolerances of the manufacturing process and the tolerances of the parts obtained. A useful tool in the reporting phase is certainly the QDM which checks that the quality expectations are verified by carrying out a statistical analysis. Thanks to the QDM, it is possible to compare the production results with the project results. This method is used in root cause analysis.

2.3 Geometric Dimensioning and Tolerancing (GD&T)

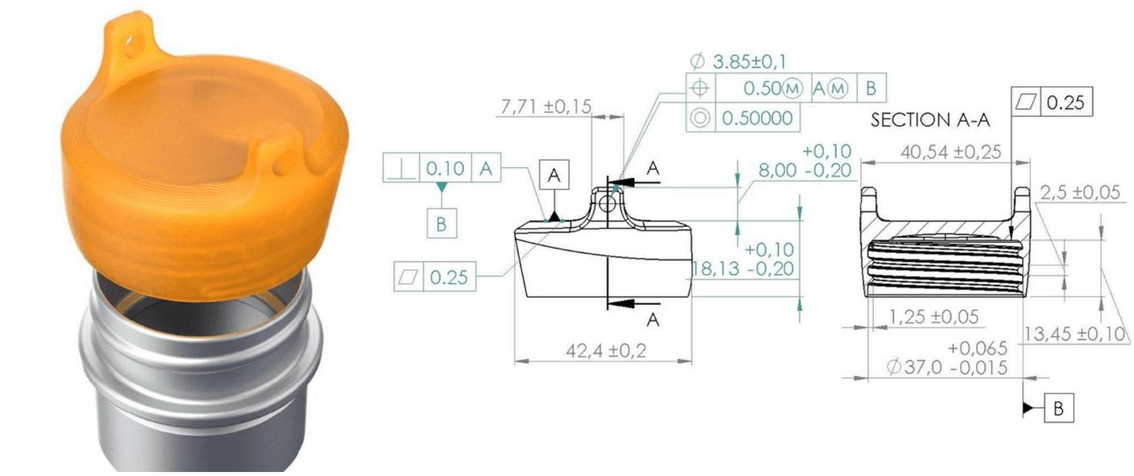


Figure 2.10 GD&T visualization example

Because of the changes in the manufacturing processes, manufactured products are different in size and dimensions from the original Computer Aided Design model. GD&T, short for Geometric Dimensioning and Tolerancing, is a unified language, used by engineers to manufacturers, which allows to share, communicate uniquely any variations of the product specifications.

GD&T standardizes the regulation allowed within the assembly also sharing this data with the manufacturers.

It is common to use GD&T to help engineers and manufacturers to control changes in manufacturing processes decreasing costs.

In the past, manufacturing features were defined by X-Y areas. The accepted area is a circle, therefore, a true tolerance specification to this feature can define the exact location of the hole. X-Y tolerancing leaves a zone in which inspection would have produced a false negative because while the hole is not within the X-Y square, it would fall within the circumscribed circle [13].

Stanley Parker, an engineer who was developing naval weapons during World War II, noticed this failure in 1940. Stanley Parker worked on a new system (a new operational process method) that was used as a new military STD.

Currently, the GD&T standard is defined by the American Society of Mechanical Engineers ASME Y14.5-2018 for the USA and ISO 1101-2017 for the rest of the world. It concerns mostly the overall geometry of the product, while other standards describe specific features such as surface roughness, texture, and screw threads.

GD&T is the main features to use workpieces with complex functionality or functional assemblies.

It is important that all components work together. We are talking about functional assemblies, multi-part products, or parts with complex functionality. Always granting the assemblability, it is important to specify the main features of the workpieces without interfering with the manufacturing process. Higher scrap rate and tooling variation lead to increasing of the costs, caused by tightening tolerances.

Thanks to GD&T, it is possible to describe the design intention instead of the outcoming geometry. That is, importance is given to its representation. For example, a feature standing at 90 degrees to a base surface can be tolerated on its perpendicularity to that surface. This will define two planes spaced apart, that the center plane of the feature must fall within. Or, when drilling a hole, it makes the most sense to tolerance it in terms of alignment to other features.

In fact, describing the geometry of the product by focusing attention on functionality and the production approach is easier than defining everything according to the linear dimensions. GD&T even allows statistical process control (SPC), reducing product reject rates, assembly failures, and the effort needed for quality control, saving organizations substantial resources. As a result, multiple departments are able to work together simplifying the communication because they have shared data together achieving the same results.

All the features of a part need to be shown by an engineering drawing, as well as the dimensions, the engineers must also indicate the tolerance with a minimum and maximum limit that can be drawn. For example, if we have a table that we would accept with a height between 10 mm and 45 mm, the tolerance would be 35 mm.

However, the tolerance for the table implies that we would accept a table that is 10 mm high on one side and 45 mm on the other or has a waved surface with 35 mm variation. The design intent of a flat surface is defined by both a tolerance that defines height and a flatness tolerance.

GD&T practices beyond simple max-min tolerancing for workpieces with complex shapes and changes.

GD&T is a set of symbols to convey such design intents.

Specifying the range of variation for all product characteristics is the basis of the concept of tolerance. The goal is to optimize the product taking into consideration the functionality and the process of obtaining the product itself.

The IT is an integral part of the metric system, for example, the 50H7 symbol indicates a hole with a diameter of 50 mm relative to a shaft. The tolerance values are tabulated, so you have to enter the table and search for the specific “hole”, with those characteristics.

Besides individual tolerances, engineers must consider system-level effects. If a part comes out with all the dimensions reaching the maximum allowed, we are referring to the Maximum Material Condition (MMC), while its counterpart is the Least Material Condition (LMC).

Tolerances also stack-up. If we create a chain link where each hole has a 0.1 mm plus tolerance and each shaft a 0.1 mm negative tolerance, that means we will still accept a 20 mm length difference at 100 links. If we have to install a repeated element (hole pattern for example), after the positioning of the pattern we have to specify interrelated distances instead of referring the holes to a plane or a fixed edge.

Designers, engineers and inspectors have to satisfy the standards. Everyone has to be informed about dimensions and tolerances, generally, digital micrometers and calipers, height gauges, surface plates, dial indicators, and a coordinate measuring machine (CMM) are used.

The Datum Reference Frame (DRF) is a theory space where the geometry exists, and where we can define and measure a part. The coordinate system at the origin of a space in 3D CAD is similar to the DRF. The measurement reference is given by the datum. Generally, it is a point, line or plane that exists in the DRF. We have to check that the datum features are referred the functionality of our part. Unless you are mating features of one part to those of others in an assembly, you can often use a single datum. Generally, the primary datum must have a certain location to derive other measurements from.

The workpiece without adding unnecessary complexity or limitations is a truly engineering drawing.

Generally, we can follow these guidelines:

- the lack of ambiguity is the most important thing, more than accuracy and completeness. Drawing dimensions and tolerances outside of the part's boundaries applying visible lines in true profiles can improve clarity. Employing a unidirectional reading direction brings the function of the part, group, and stagger dimension;

- to obtain minor costs it is important to design for the loosest feasible tolerance;
- usually, it is used a general tolerance defined at the bottom of the drawing for all dimensions of the part. If we use a specific tighter or looser tolerance it will replace the general tolerance;
- we have to tolerance functional features and their interrelations first, then move on to the rest of the part;
- generally, in the engineering drawing we have not to describe manufacturing processes as it will be done in the GD&T;
- if not specified dimensions and tolerances are valid at 20 °C / 101.3 kPa.

GD&T is feature-based, with each characteristic specified by various controls. GD&T has 5 symbols groups:

- ***Form controls*** specify the shape of features, including:
    - ***Straightness***. We have axis straightness and line element straightness.
    - ***Flatness*** in multiple dimensions.
    - A straightness curved into a circle is called C***ircularity*** or ***Roundness***.
    - The flatness bent into a barrel is called ***Cylindricity***. Difficult to inspect, it includes straightness, roundness, and taper.

- ***Profile controls*** describe the 3D tolerance zone around a surface:
    - ***Line Profile*** compares a 2D cross-section to an ideal form. Generally, two offset curves defined the tolerance range/zone.
    - ***Surface Profile*** is measured with a coordinate measuring machine because of its particularity. Two offset surfaces a Surface profile

- ***Orientation controls*** refers to angles:
    - ***Angularity***: When the reference line or plane is not at 90 then the angularity helps us to define the precision of an angle with respect to

the datum. As measure unit we prefer using millimeters instead of degrees.

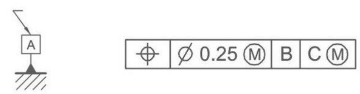
  - A flatness at 90 degrees to a datum is called ***Perpendicularity***.
  - ***Parallelism*** means straightness at a distance

- ***Location controls*** define feature locations using linear dimensions:
  - The most used control is the ***Position***. It defines the location of features relative to one another or to datums.
  - When comparing the location of a feature axis to the datum axis we are talking about ***Concentricity***.
  - When non-cylindrical parts are similar across a datum plane, we are referring to the concept of ***Symmetry***. CMM is used to control Symmetry.

- ***Runout controls***[6]:
  - When there is a need to refer for many different issues, we are defining the concept of ***Circular Runout***. The variation around the rotation axis is measured and evaluated by rotating the product on a spindle.
  - ***Total Runout*** controls straightness, profile, angularity and normally is measured on multiple points of a surface both describing the runout of a circular feature and of an entire surface.

[6] The amount by which a particular feature can vary with respect to the datums.

APPLICATION	TYPE	CHARACTERISTIC	SYMBOL
Individual Features	Form	Straightness	—
		Flatness	⏥
		Circularity	○
		Cylindricity	⌭
Individual or Related Features	Profile	Line Profile	⌒
		Surface Profile	⌓
Related Features	Orientation	Angularity	∠
		Perpendicularity	⊥
		Parallelism	//
	Location	Position	⌖
		Concentricity	◎
		Symmetry	⌯
	Runout	Circular Runout	↗ *
		Total Runout	⌰ *



Datum

Feature Control Frame

Figure 2.11 Both ANSI and ISO standards use these common symbols for tolerancing controls

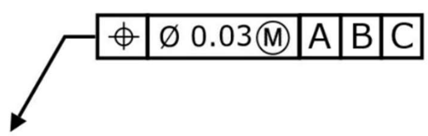


Figure 2.12 Feature Control Frame

The notation to add controls to the drawing is called Feature Control Frame. The geometric characteristic is defined on the left. The shape of the tolerance zone is indicated by the first symbol in the second compartment. The allowed tolerance is 0,03 mm. So, in the example above, we have a location control, a diameter as opposed to a linear dimension.

Then we have three separate boxes for each datum feature that the control refers to. Referring to datum A, B and C we will measure the location. Finally, on the right, we have an optional circled letter: the feature modifier.

Figure 2.13 Feature Control Frame specs

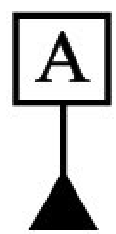
The following possibilities can occur:

- maximum Material Condition (MMC[7]) - M;
- least Material Condition (LMC[7]) - L;
- an unequal bilateral tolerance is defined by - U;
- we use "P" using a specific distance datum. We are talking about the Projected Tolerance Zone - P;
- (RFS[7]).

[7] The modifier RFS, MMC, or LMC applies to each geometric tolerance value applied on a feature of size.
Rule #2 of ASME states that **Regardless of feature size (RFS)** automatically applies, in a feature control frame, to individual tolerances of size features and to datum features of size. MMC and LMC must be specified when these conditions are required.
Maximum material condition (MMC) is the condition in which a feature of size contains the maximum amount of material within the stated limits of size, e.g. minimum hole diameter or maximum shaft diameter.
Least material condition (LMC), instead, is the condition in which a feature of size contains the least amount of material within the stated limits of size, e.g. maximum hole diameter or minimum shaft diameter [10].

2.4 Datum Feature

Datums are theoretically exact points, axes, lines, and planes or a combination thereof that are derived from datum features. A datum feature is the tangible surface or feature of size (comprised of multiple surfaces or revolved surfaces) that is indicated by the datum feature symbol. You can think of them as anchors for the entire part. They are the surface or feature where the other features are referenced from. It is usually an important functional feature that needs to be controlled during measurement as well.



Datums with GD&T symbols are used to help specify what geometrical control is needed on the part. This is not applied for the form tolerances (straightness, flatness, circularity and cylindricity).

It is important to know the difference between Datum Features and Datums. They are connected each other but they are so different.

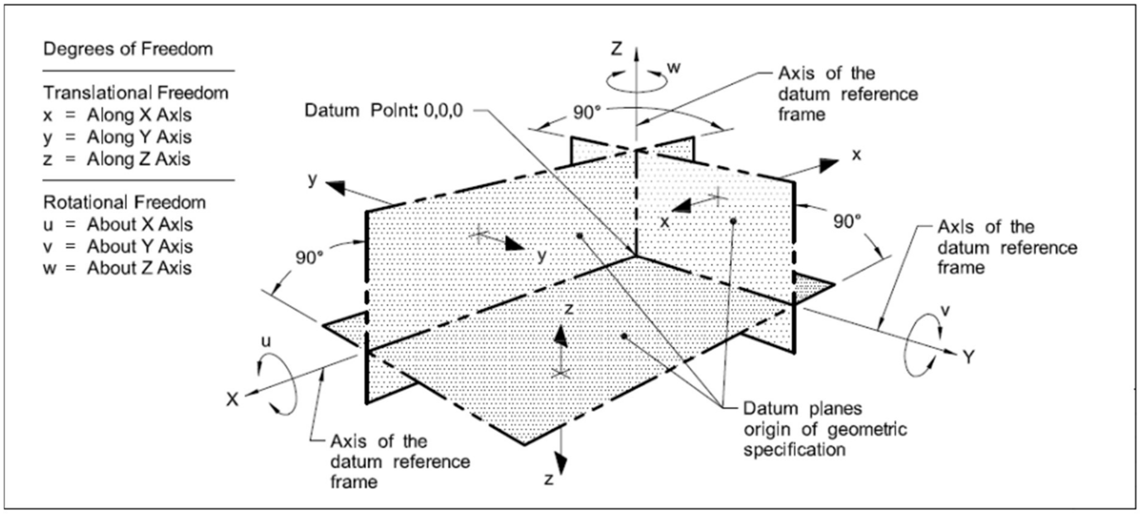


Figure 2.14 Datum Reference Frame

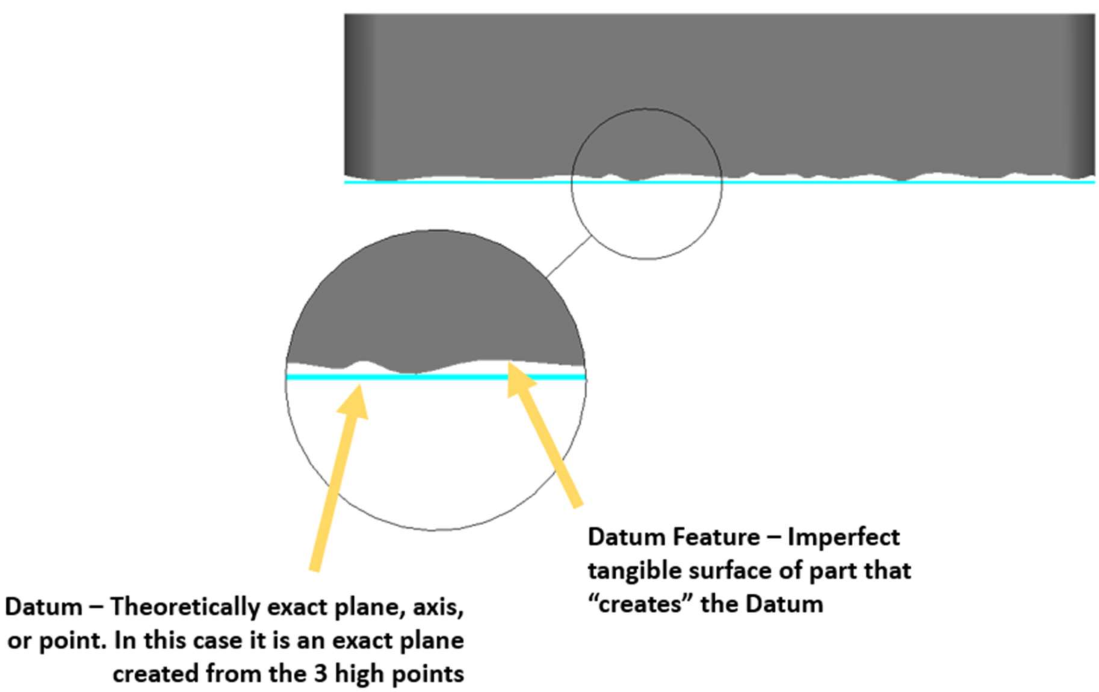


Figure 2.15 Datum vs Datum Feature

We are talking about ***Datums*** if it is theoretical and simulated by Measurement Equipment (Gauge pins, Granite slabs, angle plates, etc.)

While referring to tangible features we are talking about ***Datum Features.*** The measurement would be physically done.

A series of capital letters specify the datum features on a drawing. These letters are identified by a box and a black triangle. If a datum feature is a reference the letter will be applied in any feature control frame used. To lock all the necessary degrees of freedom (DOF), a feature control frame will reference as many datums features as necessary, creating a Datum Reference Frame.

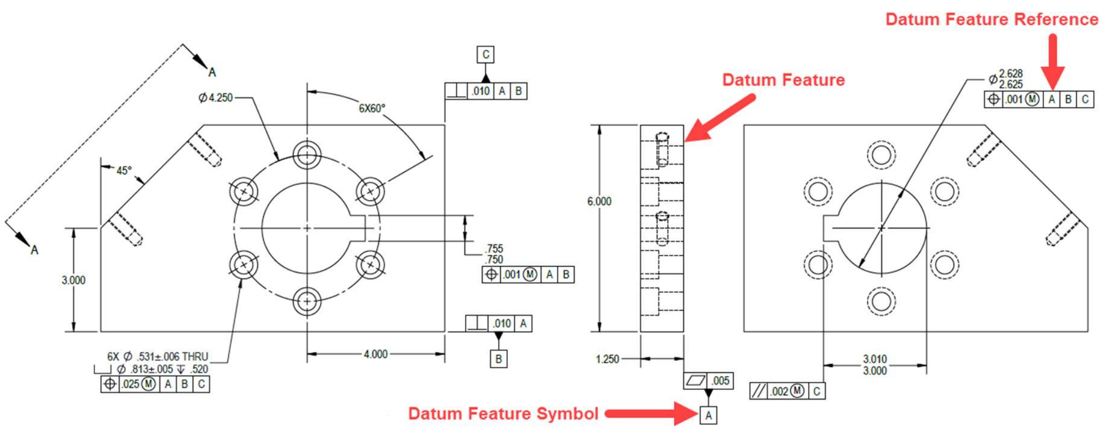


Figure 2.16 Datum vs Datum Feature view

Datum features[8] have to be defined correctly to ensure that the right type of feature is being controlled. Depending on how the symbol is applied, datum features can be: a surface or a feature of size.

[8] Points, axes, lines, and planes or a combination.

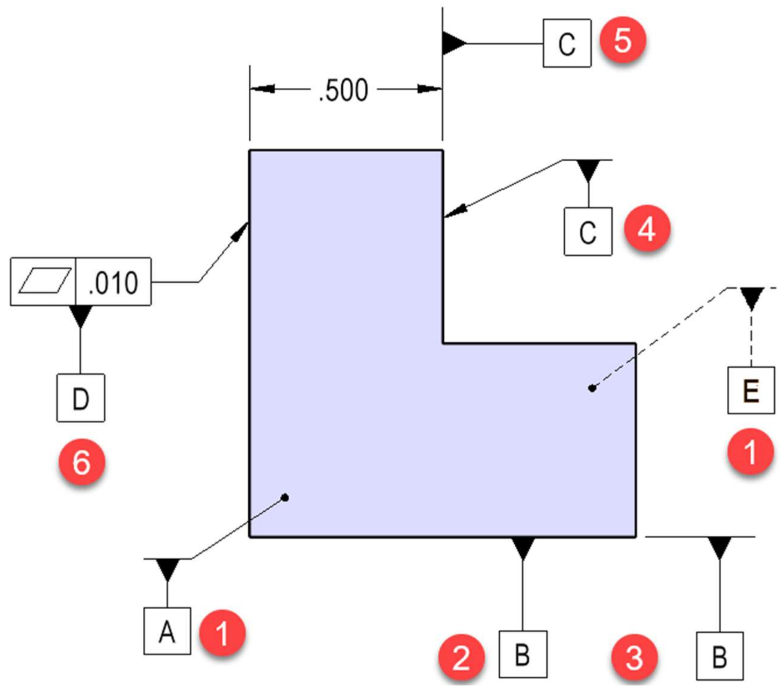


Figure 2.17 Datum Feature Symbol on a Surface

1. Point with a solid or dashed leader line with the datum feature symbol placed on it. A dashed leader line indicates the far side surface	2. Datum feature symbol placed directly on a planar surface	3. Symbol placed on an extension line off the datum feature surface
4. Leader arrow with the datum feature symbol pointing to a planar surface	5. Symbol placed on a size dimension but placed offset of the dimension arrows	6. Placed off a feature control frame that is applied to a surface. Datum feature is geometrically controlled

Figure 2.18 Datum Feature Symbol specs

When the symbol is indicated in the methods stated above this means that the datum feature is the surface of the associated with that symbol.

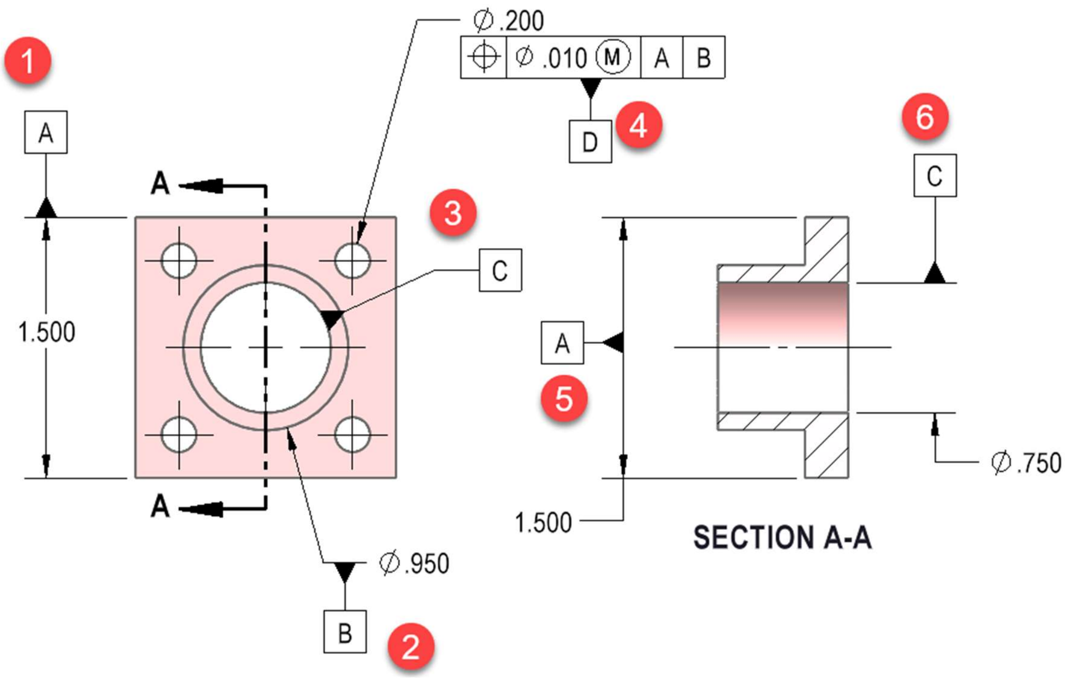


Figure 2.19 Datum Feature Symbol on a Feature of Size

1. Symbol is placed on a dimension line directly in line with the dimension arrows	2. Symbol placed on the leader arrow of a size dimension	3. Symbol placed on a non-planar feature of size
4. Placed off a feature control frame that is applied to a feature of size	5. Symbol placed on the arrows of a size dimension	6. Placed as one half in an open-ended size dimension (in line with the arrow)

Figure 2.20 Datum Feature Symbol on a Feature of Size Specs

Holes, cylinders and tabs are just some of the common functions of the Datum Features. Datum features are always imposed within a reference that is a plane or an axis. For example, datum feature C is referred to the axis of the hole and not to its surface.

Tolerancing Activity

The tolerancing activity is subdivided in three different steps: specification, allocation and analysis [14], Figure 2.21.

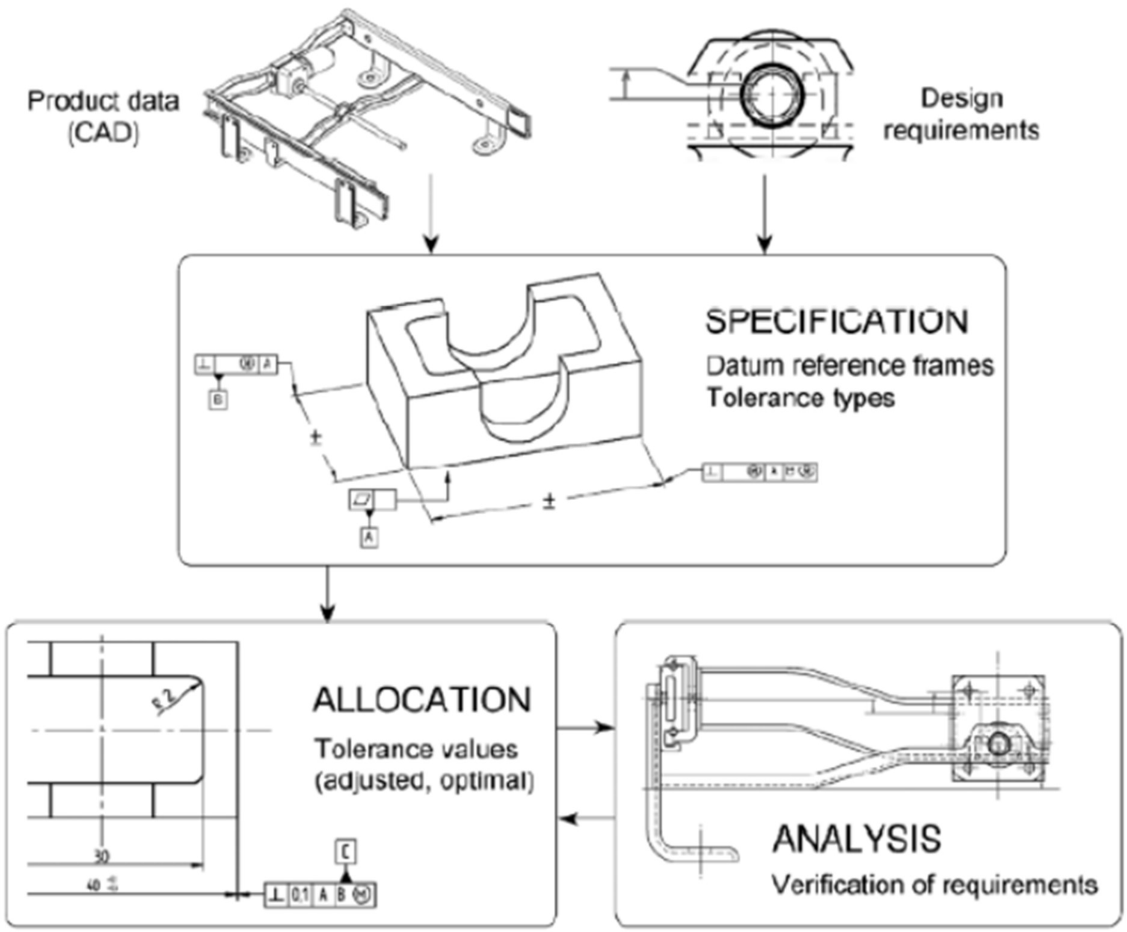


Figure 2.21 Tolerancing activity [14]

1. Specification: the tolerances specification is based on a tolerancing scheme able to reflect functional requirements.

2. Allocation: in the allocation phase, a numerical value is associated to each tolerance according to different approaches (costs, know-how, Taguchi, fuzzy logic, neural network, genetic algorithm etc.).

3. Verification of requirements: in this last phase we are concerned with verifying that the tolerance values are such that there are no problems with the component.

2.5 Sources of variation

The variation found on a finished product is influenced by different factors; however, three are the major sources of variation that must be addressed and included in every tolerance stack-up.

- Tolerances specified on the drawing.
- Variation encountered in the inspection process.
- Variation encountered in the assembly process.

At first, it is very common to include only the tolerances specified on the part and assembly drawings in the tolerance stack-up calculations. It is important that the inspection and assembly process are part of the possible variations applicable to the product. The inspection process may contribute variation where drawings are based on GD&T and datum features of size are referenced at MMC or LMC (MMB or LMB in ASME Y14.5-2009). Loading and application of forces also play a role in assembly-level variation, as parts are deformed as they are loaded (sometimes also the force of gravity may be important in terms of deformation) [14].

The stack-up includes assembly displacement, feature displacement, and specific tolerances. The other sources of variation are included here just for knowledge: manufacturing process limitations (process capability), tool wear, operator error and operator bias, variations in material, ambient conditions, difference in processing equipment, difference in process, poor maintenance, inspection process variation and shortcuts, assembly process variation.

2.6 Tolerance Stuck-Up

The expression *Tolerance Analysis* is a global term that includes:

1. the describing of the methods used to determine the meaning of individual tolerancing specifications;
2. Tolerance stack-up is the process of determining the possible range of variation between two or more features [14].

To describe the analysis of variation are used the terms tolerance analysis and tolerance stack-up even though some sources of variation do not directly derive from tolerances. So, tolerance analysis is the study of individual tolerances and their meanings, and it is the study of the cumulative variation between part features. Even if the features exist only virtually, or the parts have already been manufactured, the variation is analyzed by tolerance stack-ups.

The term "stack-up" is referred to the dimensions and tolerances added together, in order to have the total possible range. Dimensions and tolerances are stacked-up to form a chain of dimensions and tolerances. In fact, the tolerance stack-up is also called tolerance chain.

To verify a required clearance or to verify a required interference condition there are a lot of examples of a tolerance analysis. Figure 2.22 shows a common case of tolerance stack-up, in which each component has been correctly tolerated using the GD&T approach. To perform a tolerance analysis based on tolerance stacks it is necessary to define a stack coordinate system and the formulation of the stack path. See Figure 2.22 and Figure 2.23.

The stack equation yields to: $l = l0 + l1 + l2 + l3$ (1)

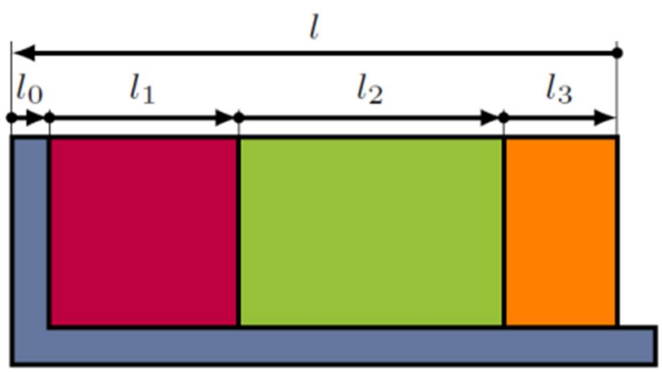


Figure 2.22 Tolerance stack-up, example

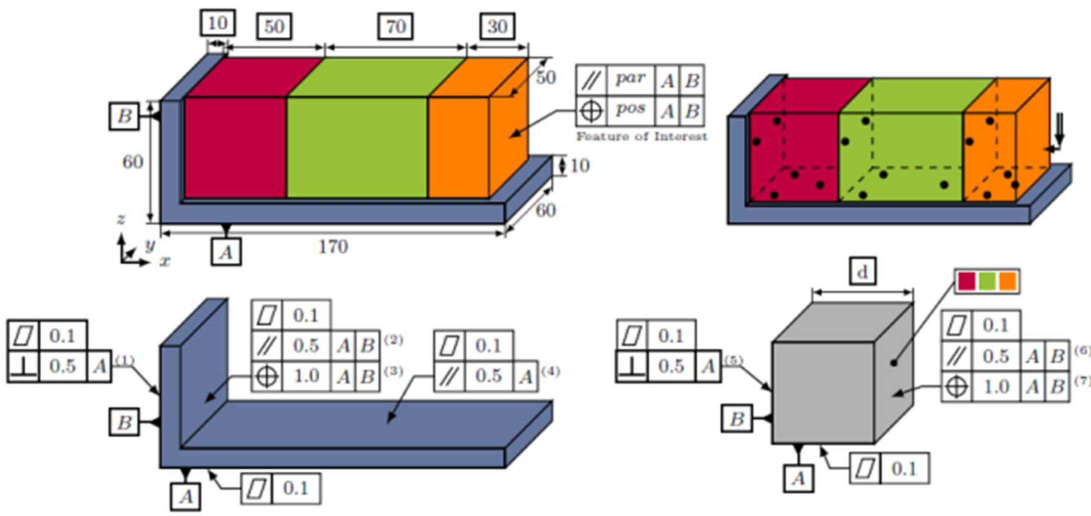


Figure 2.23 Tolerances specification of the single parts [15]

The information obtained by completing the tolerance stack-up can be used to determine if a change must be made to:

- the part and assembly geometry;
- to their dimensions and/or tolerances;
- to the dimensioning strategies used on the part and assembly drawings or annotated models;
- to the assembly process, or to the manufacturing process.

To reduce the assembly variation, the most effective way is to use a fixture to assembly parts. The fixturing features have to be used as datum features and to relate (tolerance) the features being controlled to them reducing the variation encountered at assembly. Therefore, in these conditions, the fixturing features become the principal locators for the mating parts. To compensate changes in variation, fixtures are usually manufactured to much tighter tolerances than the parts they assemble. Generally, fixture tolerances are in a percentage of 5%-10% of the part tolerances, so their costs impact can be limited.

Figure 2.24 What a tolerance stack-up allows the analyst to do [14]

In a tolerance stack-up dimensions and tolerances are defined by:

1. The geometry of parts and assemblies;
2. The schemes defining dimensions and tolerances on the drawings of the parts and assemblies;
3. The assembly process;
4. The direction of the dimensions and tolerances and the direction of the tolerance stack-up and.

Studying the geometry of the parts and assemblies is important to define which features affect the distance, how parts mate at assembly, which surfaces touch, which features locate the parts, etc.

To define which dimensions and tolerances must be included in the chain of dimensions and tolerances are also used the dimensioning and tolerancing schemes used on the part and assembly drawings. Thanks to the designer that tried to minimize the accumulation of the errors, the functional dimensioning and tolerancing allows to add fewer dimensions and tolerances to the chain. Drawings of parts that have been dimensioned and tolerated using plus/minus typically add more dimensions and tolerances to the chain of dimensions and tolerances, because the max-min system is imprecise, inaccurate and incapable of communicating the functional information. Unfortunately, this scheme of dimensioning and tolerancing is very common. The dimensioning and tolerancing scheme on the drawings can have a huge effect on a tolerance stack-up. In fact, a tolerance stack-up done to find the variation possible between two features on poorly dimensioned and tolerance drawings, and the same tolerance stack-up done on the same parts with revised drawings and functional dimensioning and tolerancing may be very different. A more compact tolerance stack-up with less ambiguity can be produced with the functionally dimensioned and tolerance drawings. The assembly process also plays a relevant role in which dimensions and tolerances are included in the chain. The assembly process can add or remove variation in several ways. In some cases (like large clearance holes locate one part to another), the assembly process may add more variation than the sum of the tolerances on the parts.

The direction of a linear tolerance stack-up is always along a line. Indeed, the methods described above are for linear, one-dimensional tolerance stack-ups. After that the direction is chosen, all the dimensions and tolerances that affect the distance being studied are included in the tolerance stack-up. Dimensions and tolerances on inclined surfaces respect the tolerance stack-up direction may need to be projected in that direction, using trigonometry.

The following two main methods can be used to develop the tolerance analysis:

1. <u>manually modelled:</u> strictly used for linear variation (mono-dimensional), done by hand, or using spreadsheet programs (Excel).
2. <u>computer modelled:</u> executed by computer simulation programs, that easily execute statistical (sometimes sampled) simulations. They are capable to perform 1D, 2D, and 3D analyses.

After that the tolerance analysis has been developed, it must be solved. Two are the main approaches used to demonstrate the tolerance analysis: arithmetic and statistical. The arithmetic tolerance analysis brings to the largest possible variation. Because of that, it is considered the "worst-case". While, if we have a long tolerance chain with different dimensions and tolerances involved, it is better to use a statistical tolerance analysis. With reference to the statistical analysis, the most common technique is the root-sum-square (RSS) method.

2.7.1 Assembly Response function

The functional requirements are translated into a set of product functional dimensions (y) which, in turn, are affected by the single dimensions and tolerances involved in the chain, x_i. These components are independent variables. The mathematical relationship between functional and components dimensions is called *Assembly Response Function*:

$$y = f(x_1, x_2, \ldots, x_n) \tag{2}$$

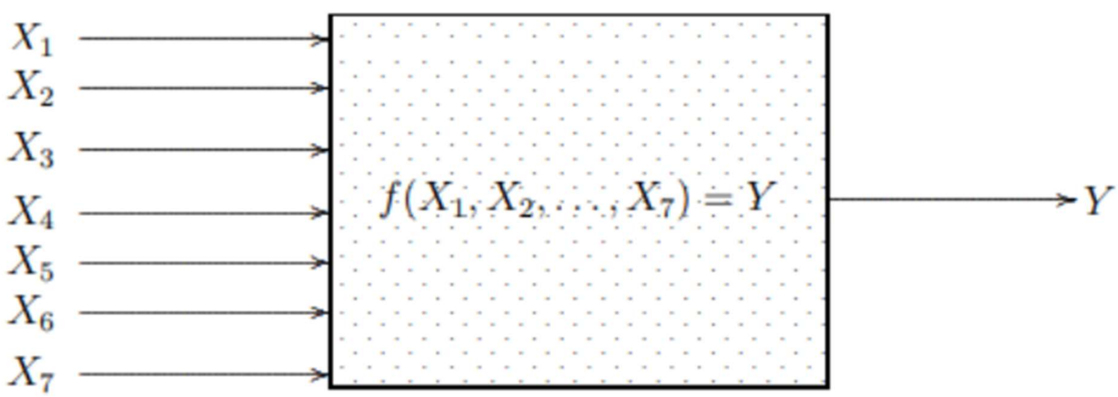


Figure 2.25 Assembly response function.

The resolution of the tolerance stack-up problem needs two methodological tools:

1. the *modelling method* is the mathematical way to describe the problem. It confirms the assembly response function (**y**), Figure 2.25, and it characterizes the association between each element involved in the chain;

2. the *solution method* is the way to solve the problem. It defines how to consider the contribution of each individual tolerance on the result. Usually, different resolution processes are used for the same modelling method.

2.7.2 Modelling methods

For tolerance analysis there are various models which can be divided into two categories:

- the model who defines the geometric functional requirements (part deviations) ***deviation accumulation methods***;
- the model who defines the subsets of multidimensional spaces (tolerance zones) ***tolerance accumulation approaches***.

For these two categories several models exist in the literature: vector loops (Gao et al., 1998), parametric tolerance analysis (Shah et al., 2007), simple tolerance stacks (Shen et al., 2005), solid offsets (Requicha, 1983), direct linearization method (Wittwer et al., 2004), Small Displacement Torsor (Bourdet et al., 1996; Li et al., 2015), Tolerance-Maps® (Ameta et al., 2011), deviation domains (Giordano et al., 2007), polytopes (Homri et al., 2015) [16] .

Vector Loop Model

The Vector Loop (VL) model uses a series of vectors, to represent the dimensions involved in the chain, for a given assembly. Each vector represents part dimensions or assembly functional dimension. The vectors are arranged in a loop at the same way of the real dimensions in the assembly. Using the model VL it can be simulated three types of variations: dimensional, geometric and kinematic [17].

In this model, the vector length (l_i) represents the nominal dimension value and the variation associated to the dimensional tolerances produce a change in the vector length.

The parts displacements resulting from changes in geometrical dimensions are described by the kinematic variations. Therefore, the kinematic joints are used to reproduce these kinematic variations. Moreover, in a kinematic joint each coupling condition is represented schematically. So, for each joint, the degrees of freedom allowed represent the assembly adjustments. Obviously, the Datum Reference Frame (DRF) must be defined.

The geometric variations (geometric tolerances) are modelled as additional degrees of freedom by displacement vectors and rotation matrices. The VL model is a simplification of the problem. In fact, in the VL model geometric tolerances are applied to the coupling points and act only in the directions permitted by the coupling. Normally, geometric tolerances act on the entire features.

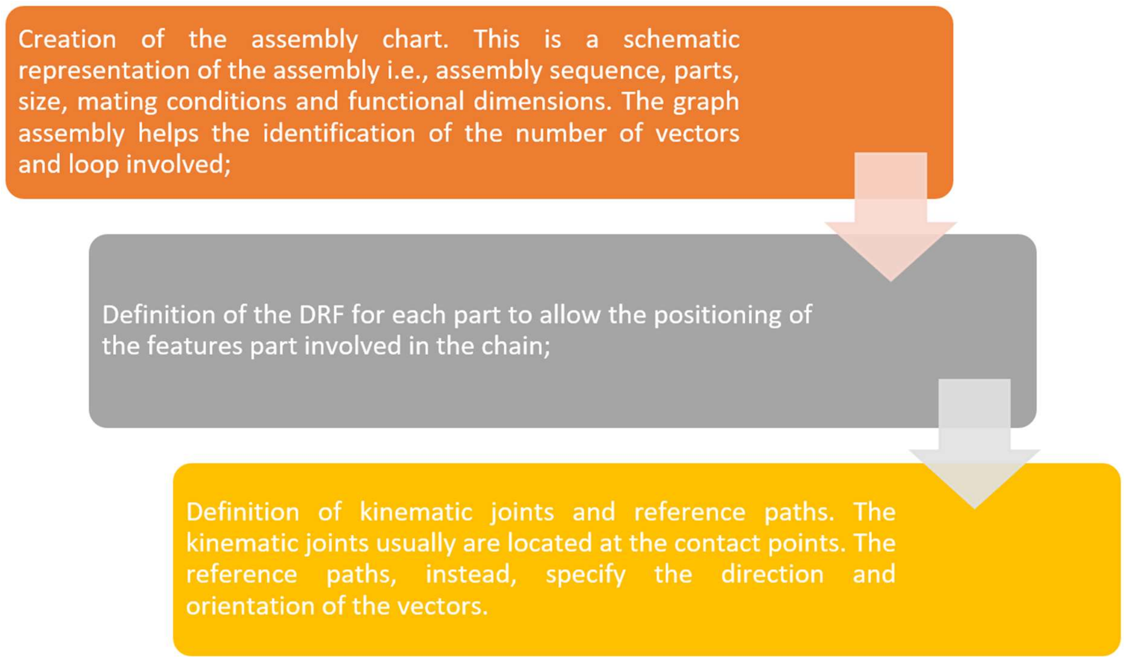


Figure 2.26 Steps of VL Model

The combination of the assembly graphs with the reference paths generates the Vector Loop model. As a result, the stack-up equations can be derived. Assembly constraints defined within the VL model can be mathematically represented as a concatenation of homogeneous transformation matrices for rigid bodies:

$$\boldsymbol{R_1} \cdot \boldsymbol{T_1} \cdot \ldots \cdot \boldsymbol{R_i} \cdot \boldsymbol{T_i} \cdot \ldots \cdot \boldsymbol{R_n} \cdot \boldsymbol{T_n} \cdot \boldsymbol{R_f} = \boldsymbol{H} \tag{3}$$

where $\boldsymbol{R_i}$ and $\boldsymbol{T_i}$ are the rotation and translation matrices for the vector, respectively: $\boldsymbol{R_f}$ is the final rotation matrix and $\boldsymbol{H}$ the final matrix.

An $\boldsymbol{x}$ vector of its relevant dimensions and an $\boldsymbol{\alpha}$ vector containing additional dimensions to consider geometric tolerances define each part. When parts are assembled, the resulting product is characterized by the assembly variables vector $\boldsymbol{u}$ and by the vector $\boldsymbol{g}$ of the measurable functional requirements [10]. For every stack, it is possible to write:

$$\boldsymbol{g} = \boldsymbol{H}(x, u, \alpha) \tag{4}$$

Variational Model

The Variational model manages the variability of an assembly, considering both the tolerances and the coupling conditions, due to a parametric mathematical model. Based on the first mathematical formulations [18, 19], numerous successive variants were made, transforming this model in a family of models.

The assembly is read directly from a CAD models, in which the nominal geometry of components is defined. Here, each feature involved in the tolerance stack-up is identified allowing the setup of the dimensional and geometric tolerances. Now, at a single feature is assigned a local reference system while a global reference system is given to each component. In nominal conditions a homogeneous transformation matrix (TN) is defined to identifies the location of the local reference systems relative to the global reference systems. But, in real conditions, the features are characterized by roto-translations from their nominal position, representative of the displacements associated with the dimensional and geometrical tolerances. These displacements are modelled through the differential homogeneous transformation matrix (DT). Easily, the features displacements can be referred to the global reference system instead the local reference one, by a matrix multiplication.

Using the parameters of the DT matrix it is possible to model different types and values variations. Therefore, the model is parametric. In this model it is possible to relate the location of a feature, affected by a variation, to other features of the part itself (also them tolerated) by a transformation. Moreover, the material modifiers (MMC, LMC, and RFS) can be defined through the parameters of the DT matrix. Differently from the VL model, the Variational model assumes the shape of the features as ideal, and it is not able to represent the geometrical form variation.

Finally, another group of differential homogeneous transformation matrices (DA) is defined in order to represent the shifts induced by the coupling conditions. These matrices are difficult to estimate, since they are influenced both by the tolerances of the parts in contact and the coupling conditions. Detailed studies, in the Literature, analyzed this problem.

Having all the transformation matrices, it is, now, possible to express all the feature variations respect the global reference system of the assembly. The functional requirements (FR) can be expressed using a function:

$$\boldsymbol{FR} = \boldsymbol{f}(\boldsymbol{p_1}, \boldsymbol{p_2}, \ldots, \boldsymbol{p_n}) \tag{5}$$

where $\boldsymbol{p_i}$ are the parameters of the model, and $\boldsymbol{f}$ is the feedback function of the assembly. This function (usually non-linear) is obtained by multiplying the matrices described above. The variational model can handle all kind of chain, as well as the VL model.

The function can be solved differently depending on several approaches founded in the literature [17, 21].

The approach consists in creating an assembly graph, which is a simplified diagram in which the assembly parts sequence, the features, the mating conditions, and the functional requirements are reported. Then a local DRF of each feature is identified together with the global DRF of each part and of the assembly. It is common to see the DRF of the assembly coincident with the DRF of the first part (hierarchical order).

DRFs define local parameters and the DT matrices, which allow to refer each feature of a part to the global DRF of the part itself. Then, using the assembly graph and the transformed features, the assembly conditions are extracted and so the assembly parameters of the DA matrix. All the features can be expressed in the same global DRF only knowing the assembly parameters.

Finally, the functional requirements are converted in terms of equations that the software is able to solve [10].

Matrix Model

All the possible variations coming from the different variability sources are mathematical representation of the boundaries of the space, these variations define the matrix model. For this reason, it uses only the worst-case approach. The model is based on TTRS criterion (Technologically and Topologically Related Surfaces) in which only basic dimensions and geometric tolerances are used according to the method of the minimum information. However, it is opposite to the actual drawing technical regulations ISO and ASME; therefore, both envelope and independent principle cannot be applied [10]. Since, the geometric features are considered ideal, this model does not allow to consider geometrical form errors, similar to the Variational model. The superimposition principle is applied if more tolerances are applied to the same part. Finally, the matrix model has good results when it is used with simple chains. On the contrary, the matrix is not able to manage interrelated chains that are complex structure.

Jacobian Model

The Jacobian model was born for the synthesis of tolerances. Starting from the assigned functional requirements it allows to get the tolerance values for the components involved in the chain. On the contrary, the reverse process (tolerance analysis), is more difficult to solve. The Jacobian model, as the Matrix model, is based on the TTRS criterion. So, it deals with the dimensional and the geometrical tolerances, but not with form tolerance. However, the tolerances of a generic drawing need to be converted in accordance with the previously defined criteria, before carrying out the tolerance analysis [20]. This model assumes that the parts are always in contact. Moreover, an important limitation is the impossibility to manage complex assemblies: only simple chain can be solved. Both the worst-case and statistical analyses can be performed [10].

Torsor Model

The torsor is a classic three-dimensional tolerance analysis method. It uses three translational vectors and three rotational vectors to represent tolerance information in three-dimensional Euclidean space, capable to describe the motion of a rigid body (like a three-dimensional tolerance zone). Each real part surface is modelled by a substitution surface characterized by a set of screw parameters that model the deviations from the nominal, induced by the applied tolerances. Considering a generic point A on a given surface, in which $\boldsymbol{u_A}$, $\boldsymbol{v_A}$, $\boldsymbol{w_A}$ are the translation components and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ are the rotation angles (considered small), the corresponding torsor $\boldsymbol{T_A}$ is [20]:

$$\mathbf{T}_A = \begin{Bmatrix} \alpha & u_A \\ \beta & v_A \\ \gamma & w_A \end{Bmatrix}_{\mathbf{R}} \tag{6}$$

where $\boldsymbol{R}$ is the DRF used to evaluate the screw components.

To model the interactions between the parts of an assembly, the model considers three types of Small Displacement Torsor (SDT):

- a part SDT for each part of the assembly to model the displacement of the part;
- a deviation SDT for each surface of each part to model the geometrical deviations from nominal;
- a gap SDT between two surfaces linking two parts to model the mating relation.

The three Small Displacement Torsor contribute to describe the global action of the assembly. Drawing tolerances must be applied before running tolerance analysis as the torsor model operates under the hypothesis that TTRS and the positional tolerancing criteria are valid.

The Torsor model may be solved only by means of the WCA. It deals with the dimensional and the geometrical tolerances, but it does not support the form tolerances. Moreover, the model does not manage the Envelope or the Independence principle (applied to the dimensional tolerances) neither the MMC. It considers the interaction among the tolerance zones. Only linear stack-up function can be solved, but the joints may manage contact or clearance between the mating parts. The functional requirements of the assembly may be represented through features and points. Finally, the model is not able to distinguish the precedence among the datum [10, 20].

Skin model

Some of the methods mentioned above does not support the point cloud representation of the variant parts. Therefore, no geometrical form errors can be managed. It is a model for tolerance analysis based on the real representation of the workpieces (not ideal) and it allows the point clouds managing [16].

The models described above showed different advantages and disadvantages. Nevertheless, they present some common limitations which are:

- the impossibility to manage geometrical form deviations (except the VL method);
- the incomplete conformance to international standards ISO and ASME.

Differently, the Skin Model Shapes (SMS) uses a discrete geometry representation scheme (point clouds and surface meshes), to represent parts and assemblies. This allows to consider all the geometric tolerances includes the form deviations.

The tolerance analysis based on Skin Model Shapes, comprises:

- the generation of the discrete surface;
- the scaling of deviated workpiece due to specified tolerances,
- their processing using computational geometry algorithms for the relative positioning and assembly simulation;
- functional key characteristics (FKC) measurement from the simulated assemblies, Figure 2.27.

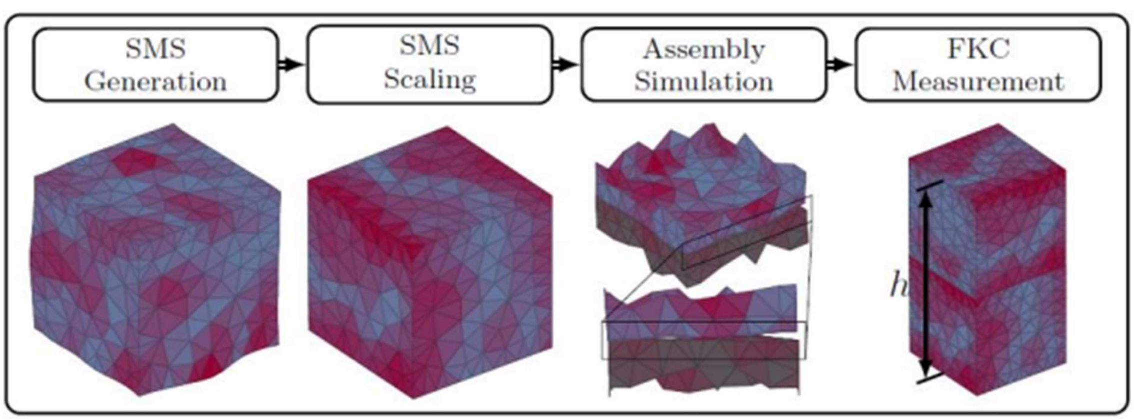


Figure 2.27 Tolerance analysis based on SMS model [15]

The Figure 2.28 shows an example of assembly with coarsened mesh and magnified form deviations. Usually, a refined mesh is applied on the mating surfaces, Figure 2.29.

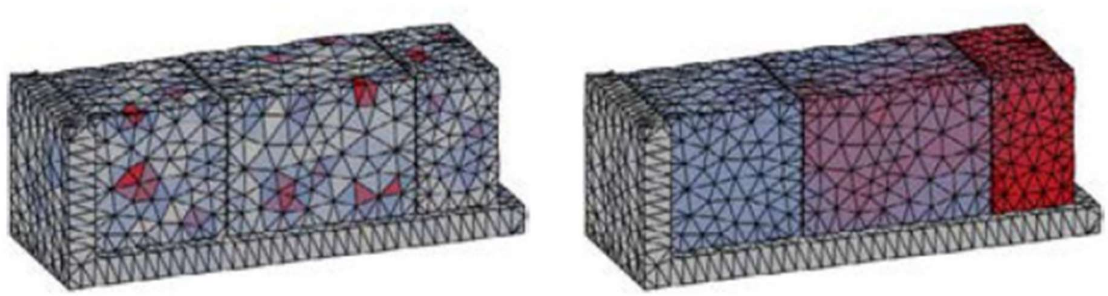


Figure 2.28 Coarsened Mesh and magnified Form Deviations: initial part deviations (left), accumulated deviations through the assembly (right) [15

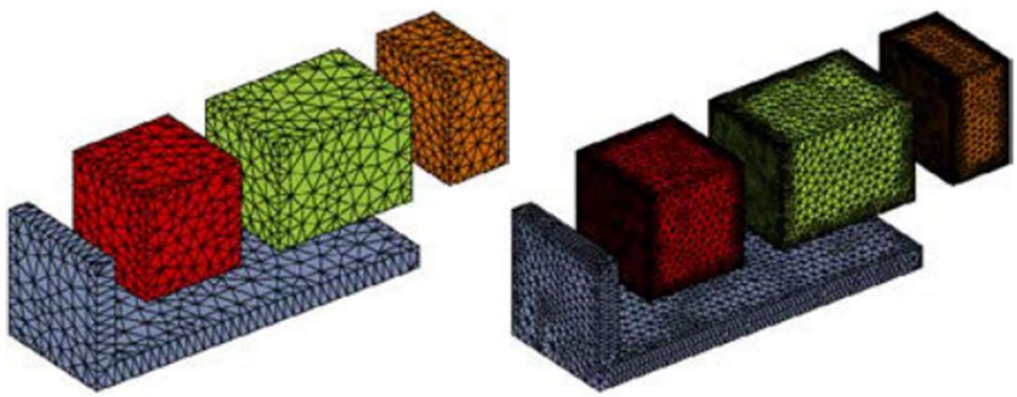


Figure 2.29 Surface meshes of the parts: coarse mesh for visualization (left), refined mesh on mating surfaces for computation (right) [15]

This new approach allows the consideration of form deviations in conformance to international standards. Thus, the tolerance analysis based on Skin Model Shapes allows a more realistic prediction of assembly characteristics in the development phase. However, some tolerance analysis problems still remain to be investigated, such as over-constrained assemblies or thermal expansion and part deformations [15].

Models Comparison

A comparison between the models described above, is made consulting the recent literature [10, 16, 17, 21, 20]. The main properties, required for tolerance analysis, are collected in the Table 1.

None of these models provide a complete representation of the tolerance analysis phenomena. Thus, also the CAT software shows the same constraints [20]. The main limitations may be here summarized:

- the application of the Envelope Rule and of the Independence Principle as prescribed by the ASME and ISO standards;
- the application of the geometric form tolerances (except for the Vector Loop model and the Skin model);
- impossibility of representing all the possible types of part couplings that may include clearance.

The Vector Loop and Variational models seem to be the most complete models, even though they do not completely respect the standards ISO and ASME; moreover, they do not allow to manage the interactions between the tolerance zones. The Variational model allows to consider the order of precedence of the datums and to apply the material modifiers [10].

The CAT software used in this thesis, Cetol 6**σ**® of Sigmetrix, is based on the Variational model (it used vector loops model in former versions) [16].

	Models				
	Vector loop	Variational	Matrix	Jacobian	Torsor
Tolerance kind	Dimensional/form/ geometrical (no form)	Dimensional/ geometrical (no form)	Dimensional/ geometrical (no form)	Dimensional/ geometrical (no form)	Dimensional/ geometrical (no form)
Envelope and independence	Envelope	Envelope	Envelope	Envelope	Envelope
Tolerance zones' interaction	No	No	Yes	Yes	Yes
Precedence among datum	No	Yes	No	No	No
Material modifier condition	No	No	No	No	No
Model parameters from tolerances	Yes	Yes	No	No	No
Tolerance stack-up function	Linear/network	Linear/network	Linear	Linear	Linear
Joint kind	Contact	Contact	Contact/clearance	Contact	Contact/clearance
Functional requirement schematisation	Feature/points	Feature/points	Feature	Feature/points	Feature/points
Analysis kind	Worst-case/statistical	Worst-case/statistical	Worst-case	Worst-case/approach	Worst-case

Table 1 - Taxonomy of models for tolerance analysis [20]

2.7.3 Solution methods

Before introducing the most common solution methods for tolerance stack-up calculations, a briefly introduction, on the two typical procedures in the tolerance design, is reported. The tolerance design can be categorized in tolerance analysis and tolerance allocation. The difference between the two is shown in the Figure 2.30. Tolerance analysis aims to the assembly variation calculation once the parts tolerances are known. Conversely, the tolerance allocation aims to spread or "allocate" the tolerance values to each dimension involved in the tolerance stack-up, starting from the design requirement and so from the functional assembly variation [22].

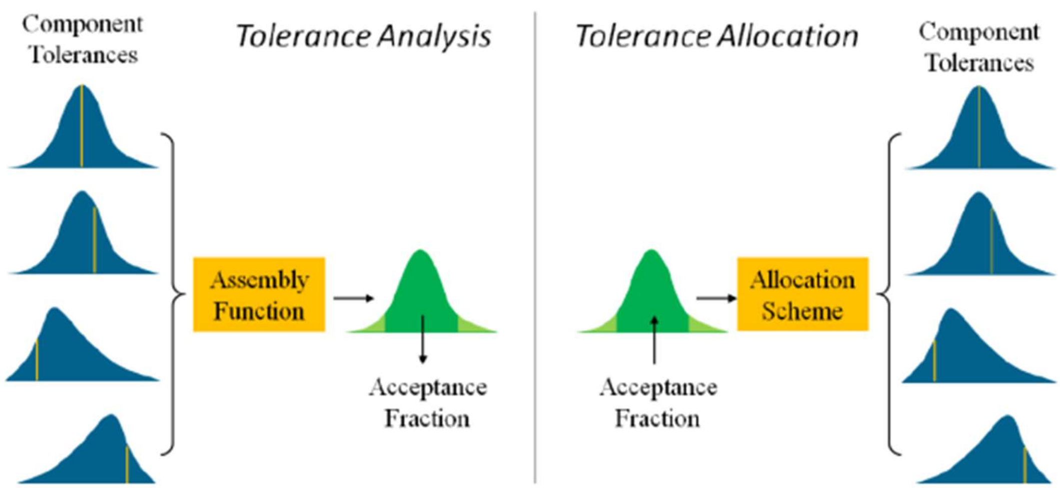


Figure 2.30 Tolerance Analysis vs Tolerance Allocation [22]

The literature describes different approaches for tolerance analysis, which can be grouped into three categories: deterministic, statistical and sampled, Figure 2.27. Moreover, in the Table 2, is reported a summary of the possible solution for tolerance analysis calculations (models and solution methods) [10].

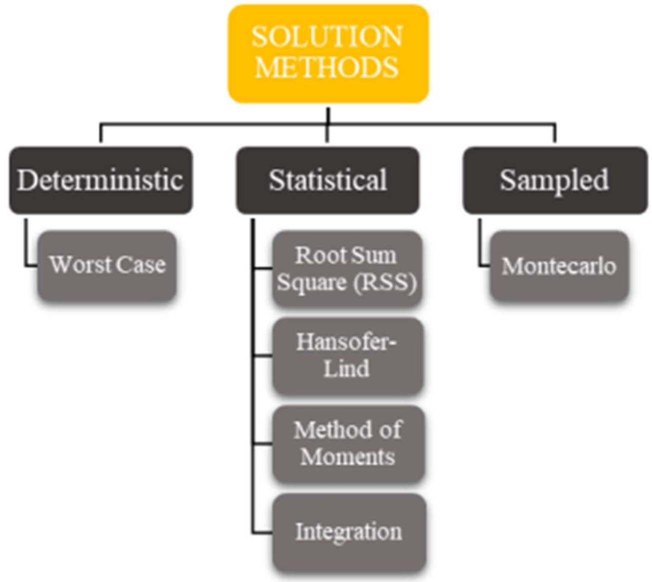


Figure 2.31 Tolerance stack-up, Solution methods.

Analysis	Vector Loop	Variational	Matrix	Jacobian	Torsor	Skin Model
Worst-Case	+	+	+	+	+	(+)
Statistical	+	+	-	+	-	(+)
Sampled	+	+	-	-	-	(+)

Table 2 - Models and Solution methods for tolerance analysis.

Deterministic approach

Worst Case Analysis

Worst-Case tolerance Analysis (WCA) determines the absolute maximum variation possible for a selected distance or gap. This method assumes that each dimension involved in the tolerance stack-up may have the same probability to occur within its tolerance range. Therefore, with an extremely pessimistic approach, it considers that all the chain dimensions are simultaneously in their worst-case conditions: summing up all the maximus and minimums values, the assembly variation extremes are achieved, Figure 2.32 [14, 10]. In the equation (7) T_{asm} is the total assembly variation, while the t_i is the $i-th$ tolerance in the chain.

WCA guarantees the 100% of the assembly and so it predicts the maximum and minimum variations, but at the same time it may lead to over-design. Because of the use of the WCA for long tolerance chain, we are in a very strict range of tolerance to respect our standards, but this brings to an increasing of manufacturing costs. Moreover, the probability to find these conditions is very low. This approach can be used in the design of safety and critical assembly systems or at least with short tolerance chains.

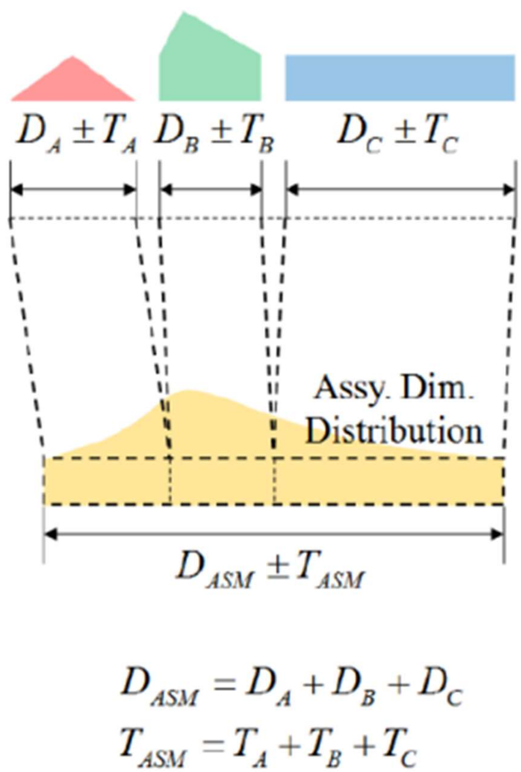


Figure 2.32 Worst Case Analysis (WCA)

$$T_{asm} = \sum_{i=1}^{n} t_i \tag{7}$$

In the Figure 2.33 is reported an easy tolerance stack-up example to better understand the worst-case approach together with the vector loop model, explained in its main phases. The loop method requires to identify all necessary components (and their tolerances) in the chain by systematically moving through the assembly from the start point to the end point of the functional assembly dimension [8]. Calculations are shown in Figure 2.34.

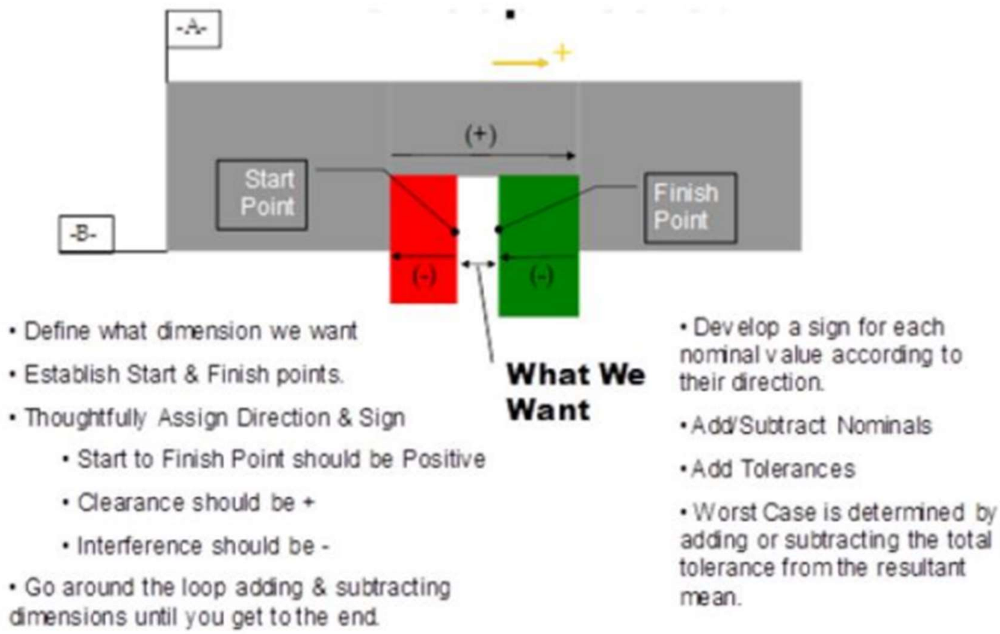


Figure 2.33 The Vector Loop method [8]

Figure 2.34 1-D Worst case analysis calculations [8]

Statistical approach

What is quality? Quality is referred to production yield and reliability. Through specific indices, which indicate the amount of parts outside specification, and so the probability of defects on the entire production batch. When engineers perform tolerance analysis, during the design phase, they essentially convert the design intent in to a statistical or probability-based design model [28].

Statistical tolerance analysis determines the probable or likely maximum variation possible for a selected functional dimension, Figure 2.35. This method is more realistic, because it assumes that it is highly improbable that all the dimensions involved in the chain will be at their worst-case limits simultaneously. So, a certain distribution is assigned at each dimension. In some cases, the normal distribution function well approximates the real variation of the processes, since it is quite common that a major part of the dimensions will be closer to their nominal value than either extreme. Usually, the statistical approach shows less variation than the worst-case analysis for the same tolerance stack-up. This is a benefit because it allows the design engineers to increase the tolerances and so to choose more convenient manufacturing processes or at least to get high quality parts and assemblies due to the tighter clearances [14].

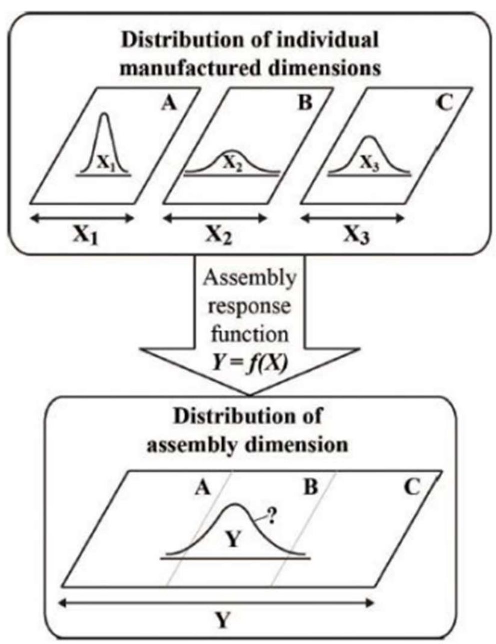


Figure 2.35 Statistical tolerance analysis approach [23]

In the literature, there are some general rules that guide in the choice of the better approach. A good indicator is, surely, the number of dimensions involved in the chain: higher is this number and better the statistical approach behaves. Moreover, there are other parameters such as the number of parts to be produces, the manufacturing process controls, the supplier quality, etc.

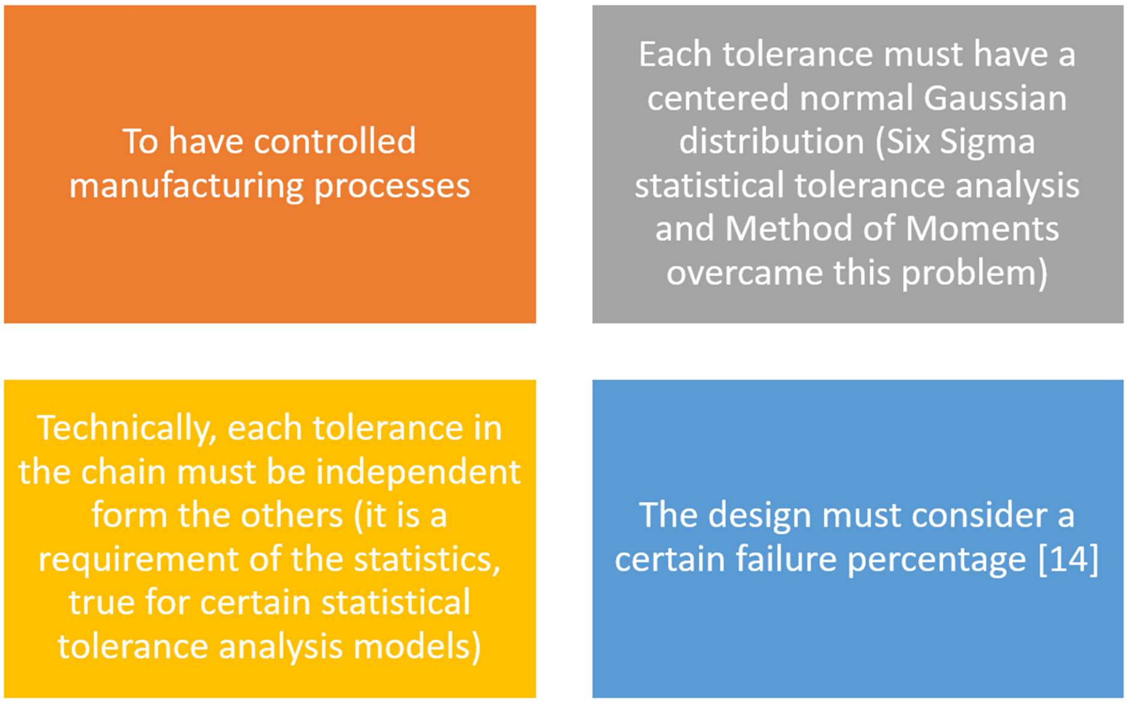


Figure 2.36 Conditions that should occur for safely adopt the Statistical tolerance analyses

There are different statistical methods for tolerance analysis. The most common are Root Sum Square (RSS) and Monte Carlo simulations.

Root Sum Square

The Root Sum Square is a tolerance analysis method, based on the following hypotheses:

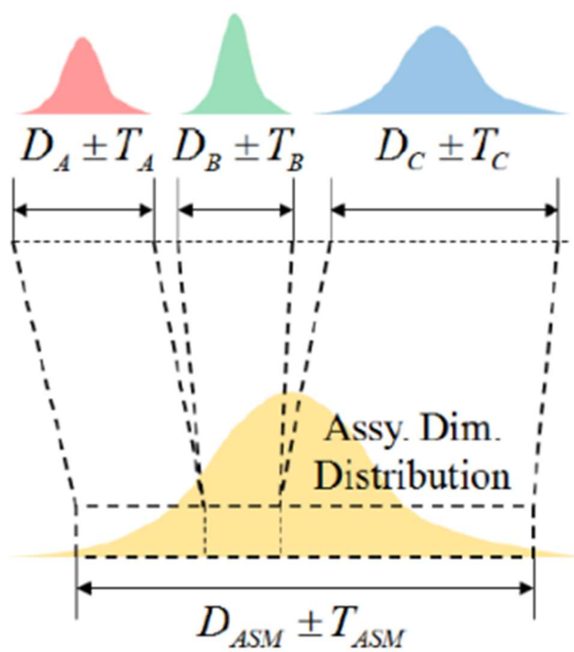
- the processes are statistically controlled, and each dimension has a normal Gaussian distribution, centered in the nominal value (quite strong limitation);
- all the dimensions of the loop are independent from each other;

- the variation of each tolerance is in between ±3σ, that means six times the standard deviation (σ) of the process.

The functional assembly deviation T_{asm} is calculated through the equation (8), where t_i are the single tolerances. Moreover, an example is shown in Figure 2.37.

$$T_{asm} = \sqrt{\sum_{i=1}^{n} t_i^2} \tag{8}$$

Generally, the calculations according to the RSS method are optimistic i.e., it predicts a lower number of defects compared to a real-assembly process [10]. This occur essentially, when the real distributions are quite different compared to the normal Gaussian ones.



$$D_{ASM} = D_A + D_B + D_C$$

$$T_{ASM} = \sqrt{T_A^2 + T_B^2 + T_C^2}$$

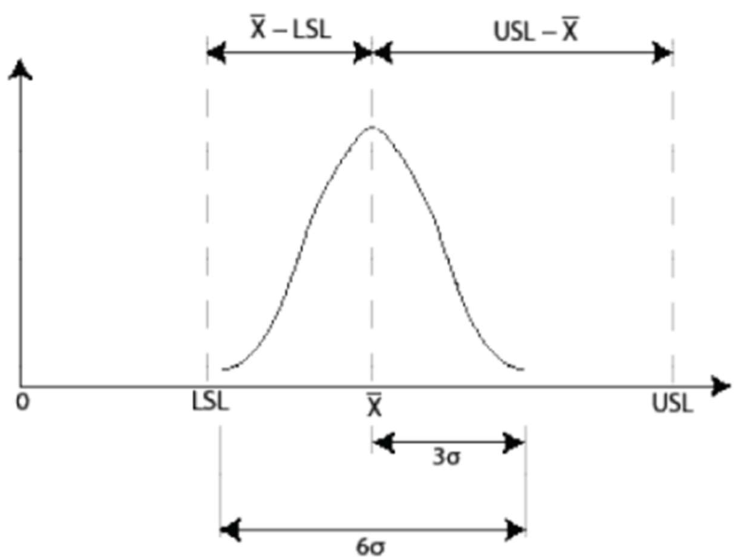
Figure 2.37 Root Sum Square approach [22]

Nevertheless, according to the central limit theorem of mathematical statistics, independent of the distribution kind of the single sizes, if more than four members contribute to the tolerance stack-up, the final functional variation approaches to a normal distribution [12]. Moreover, when there is a not normal dominant distribution in the chain, it will be necessary to have a lot of normal minor tolerances to compensate its effect.

To solve this problem, alternative methods were developed in the years. Some of them are based on correction factors (C_f), that simply move the midpoint of the distribution, to approach the real distribution.

Six Sigma

The Six Sigma method was developed from Motorola in the ’80s to achieve high-quality production processes. It is based on the observation that considering a variation range of $\pm 6\sigma$ even if the real distribution is likely different or shifted, there will still be gain before reaching a poor value of process capability. In the production world, the process quality is measured mainly by two statistical parameters: C_p and C_{pk}. They represent the process spread and process centering, respectively, Figure 2.38.



$$Cp = \frac{USL - LSL}{6\sigma}$$

$$Cpk_L = \frac{\bar{X} - LSL}{3\sigma}$$

$$Cpk_U = \frac{USL - \bar{X}}{3\sigma}$$

$$Cpk = \min\{Cpk_L, Cpk_U\}$$

Figure 2.38 Cp and Cpk meaning

The LSL (Lower Specification Limit) and USL (Upper Specification Limit) represent the functional limits requested from design engineers, and so they represent the statistical tolerance limits. The equations below, describe the new deviation $\boldsymbol{\sigma}_i$ and process capability C_{pi} to get a more realistic distribution. $\boldsymbol{\sigma}_{proc}$ is the manufacturing process standard deviation, while m_i is the mean displacement factor that consider the shifting of the process during the time [10].

$$\sigma_i = \frac{T_i}{3C_{pi}(1 - m_i)} \tag{9}$$

$$C_{pi} = \frac{USL - LSL}{6\sigma_{proc}} \tag{10}$$

In the Figure 2.39 has been represented a typical statistical production process trend (based on the $\pm 3\sigma$), in which the deviation in time, from the nominal value, occurs on the total manufacturing time.

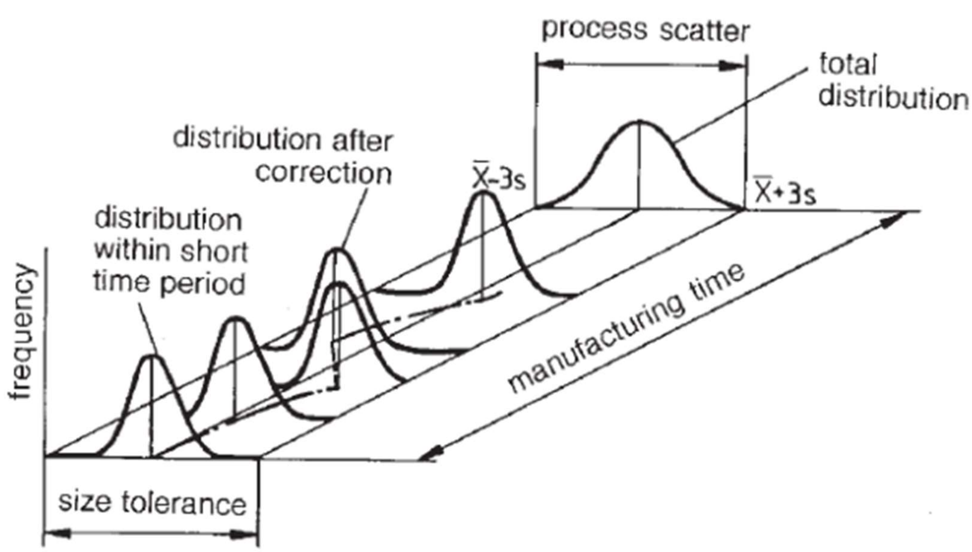


Figure 2.39 Shifting phenomenon during the production process

Second-order tolerance analysis (SOTA) and Method of System Moments (MSM)

The Second-order tolerance analysis (SOTA) is a method that consider real processes distributions can deviate from the centered normal distribution due to tool wear, form aging and other typical manufacturing phenomena. The variation from the centered normal Gaussian distribution is achieved through some “moments” that usually are four: indicating the mean shift, the spread, the asymmetry and the peak of the distribution, Figure 2.40.

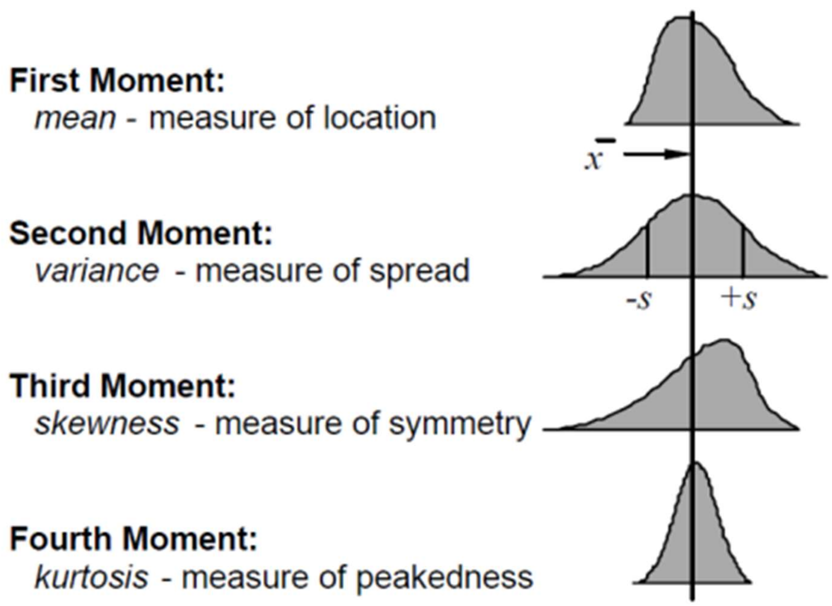


Figure 2.40 First four statistical moments

The second-order tolerance analysis method attempts to combine the advantages of the Linearized Method with the advantages of Monte Carlo simulation. The Method of System Moments (MSM) is used by the SOTA analysis, referring to implicit variables of a system of nonlinear equations [12]. The MSM expands the function of interest in Taylor series around its mean values. As example, the equation (11), truncated at the second-order term, is shown below [24]:

$$u_i = \bar{u}_i + \sum_{j=1}^{n} \frac{\partial u_i}{\partial x_j}(x_j - \bar{x}_j) + \frac{1}{2}\sum_{j=1}^{n} \frac{\partial^2 u_i}{\partial x_j^2}(x_j - \bar{x}_j)^2 \qquad (11)$$

$$+ \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \frac{\partial^2 u_i}{\partial x_j \partial x_k}(x_j - \bar{x}_j)(x_k - \bar{x}_k)$$

In this method only the first four terms are considered, the same described in the introduction. However, the math of this methods is completely reported in the cited paper [24]. Non-linear tolerance problems and at same time tolerance allocation, closed-loop constraints as well as non-normal input and output distributions can be solved by the SOTA method. It important to highlight that it does not present the Monte Carlo time limitation, in term of computation effort. Indeed, faster analyses can be achieved.

The SOTA method include a non-linear system solver, finite difference approximations for the first and second order partial derivatives, the Method of System Moments, and a Generalized Lambda Distribution (GLD) to empirically fit the calculated moments and approximate the distribution of the assembly dimensions [24].

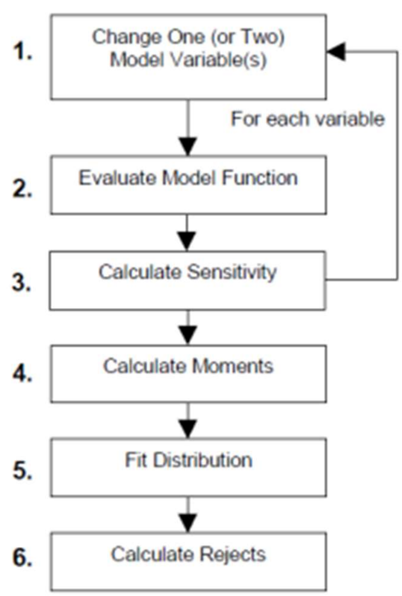


Figure 2.41 The SOTA method steps [24]

Finally, the research study (Glancy, 1999) shows a comparison between the Linearized, Monte Carlo and SOTA methods, highlighting the potentials of the latter. The SOTA is the solution method adopt by Cetol 6σ.

Features	Linearized Method	Monte Carlo Simulation	SOTA Method
Speed	√		√
Tolerance allocation	√		√
Closed-loop constraints	√		√
Nonlinear approximation		√	√
Non-normal input distributions		√	√
Non-normal output distributions		√	√

Table 3 - Methods comparison: Linearized, Monte Carlo, SOTA [24]

Sampled approach

Monte Carlo method

If we definitely want to use a non-linear-simple statistical analysis, we will adopt the ***Monte Carlo method***. On the basis of the distributions of the parts we will calculate the random values of the parts while for each set of values of the parts we will calculate the response function. Then, using the standard statistical formula we will calculate the moments of the sample of values of the function.

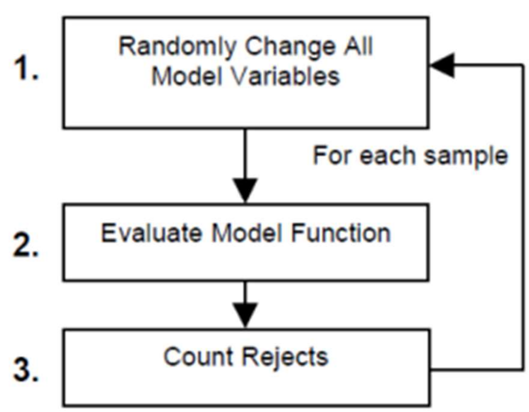


Figure 2.42 Monte Carlo method steps [24]

The Monte Carlo methods has different advantages due to its flexibility. It allows nonlinear tolerance analysis, tolerance allocation, any component distribution as input, Figure 2.43. On the other hand, Monte Carlo simulation is computationally expensive not allowing a rapid design iteration. Indeed, changing only one input parameter, the entire Monte Carlo simulation must be re-run [24]. Moreover, the Monte Carlo simulation does not produce a closed form solution because it changes with the number of simulations (samples) performed; the worst-case results are always different.

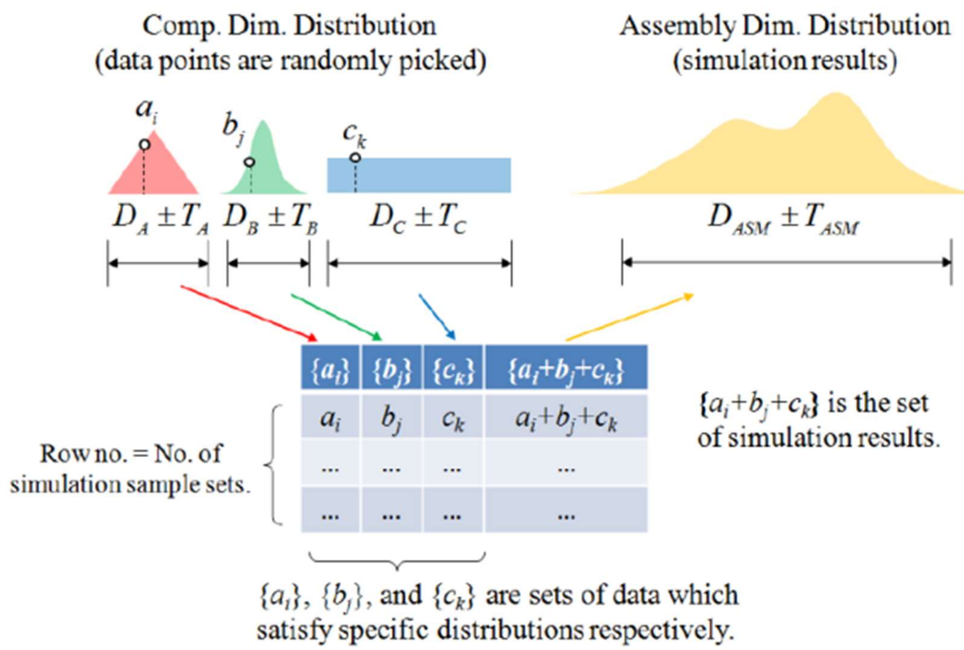


Figure 2.43 Monte Carlo method [22]

For its features the Monte Carlo method is widely adopted by different CAT tools.

Practical considerations

Concluding, the Linearized methods well behave when tolerances are in the order of 1/100 to 1/1000, compared to the nominal dimension. In research (Gao 1995) it is found that in these conditions the Linearized method accuracy corresponds to the Monte Carlo method one, with a sample size of 30.000 (quality level near ±3σ). In the case of highly not linear assemblies, the Linearized methods may not behave correctly. Due to the non-linearity of the assembly, even if the inputs are symmetrical, the resulting assembly distribution could be deformed and asymmetric. In any case, it is quite common and right to assume normal distributions as inputs, because in the design phase, rarely designer engineers know the real production data. However,

	Assembly model		Distributions		Efficiency
Analysis method	Linearized	Nonlinear	Normal	Non-normal	Relative CPU time
WC	X	X	NA	NA	1
RSS	X		X		1
Hasofer-Lind		X	X		6
Method of Moments		X	X	X	10
Integration		X	X	X	60
Monte Carlo	X	X	X	X	100

the central limit theorem holds. Often, even the manufacturers do not have much information about their manufacturing processes and tools. A summary table is here reported, to compare the methods seen before, Table 4.

Table 4 - Solution methods, comparison [10]

2.7 Computer-Aided Tolerancing: CAT Tools

At this point, it is quite clear how a correct tolerance analysis impacts on the quality of the final product, allowing a more conscious choice of the manufacturing processes to achieve the functional design targets.

From the end of 70's, computational instruments support technicians and engineers in their design activities, enable to increase the productivity, to improve the products quality and to better manage the technical documentations. Computer-Aided technology (CAx), is the general name given at these instruments in which it can be easily recognized: CAD (Computer Aided Design), CAE (Computer Aided Engineering), CAM (Computer Aided Manufacturing), etc.

Since tolerance analysis is part of the Concurrent Engineering (CE) approach, CAT (Computer Aided Tolerancing) tools have also been developed, based on the modelling methods previously described. Today different CAT software exists, with specific advantages and disadvantages, showing the same limitations of the modelling and solution methods they adopt. The most common CAT software are: CeTol **6σ**® (Sigmetrix), 3DCS® (DCS software solutions) and VSA® (Siemens/Tecnomatix) which use parametric approaches (CeTol 6σ® used vector loops in previous versions), MECAMaster®, which is based on the SDT (Small Displacement Torsor), and PolitoCAT®, which employs polytopes [16]. Some other CAD systems have an integrated module for tolerance analysis [29].

Different research and comparative reviews are present in the literature, but (Corrado, 2020) presents a useful comparative table about the three main software cited before: Cetol 6σ, 3DCS and VSA. The Table 5 has been updated in particular in the CAD compatibility sector. Conversely to the manual/spreadsheet calculations, these tools allow 3D, non-linear, statistical tolerance stack-ups calculations (someone uses the Monte Carlo approach). Moreover, they may directly import the Product Manufacturing Information (PMI) from the CAD model together with the assembly constraints. These features lead to a more accurate tolerance analysis performed in much less time.

When we use the contributors expressed as geometric parameters by defining the dimension of the functional assembly as an algebraic function i.e., using a parametric approach. Both dimensional and geometric tolerances can be included in the analysis. Then, the function is linearized or directly solved through a non-linear Monte Carlo simulation. In the post-process phase, commonly results are available: the contributors list, sensitivities, variance contributions (%), in both worst-case and statistical analysis. The CATS systems differ in how they interface with CAD software and which analysis they can provide [29].

Despite the wide capabilities of commercial CAT software, some important limits remain:

- not all the tolerances are supported (form, composite location, …), despite they comply with the ISO/ASME standards (Envelope and Independent principle critical);
- do not support more tolerances applied to the same surface;
- do not support assembly constraints refers to actual assembly operations;
- to consider dimensional and geometric tolerances, the CAT tools apply transformations to the perfect features within the zones (when form tolerances are neglected), which are simulated by putting limits on rotation and translation parts of the transformation matrix based on feature dimensions and tolerance values. When the form variations have

to be considered, the software use additional parameters to describe the features [16].

Requirements\CAT Software	CeTol 6σ®	VSA®	3DCS®
Tolerancing scheme			
Dimensional tolerances	Yes	Yes	Yes
Geometric product specification (GPS)	Yes	Yes	Yes
Automatic utilization of CAD model, once defined GPS data	No	No	No
Tolerance analysis			
Worst-case approach	Yes	Yes	Yes
Statistical approach	Yes	Yes	Yes
Sensitivity analysis	Yes	Yes	Yes
Uncertainty qualification methods			
Monte Carlo	No (SOTA)	Yes	Yes
Simplifying assumptions			
Rigid body	Yes	Yes	Partial
Limit on variation size	No	No	No
Further considerations			
Compatible CAD tools	PTC Creo, Catia V5-6, SolidWorks, NX	PTC Creo, Catia V5-6, SolidWorks, NX	Creo, CatiaV5-6, SolidWorks, NX, STEP, IGES
Distributed/parallel computing	No	No	No
Integration with external CAE modelling tools	No	No	No
Accommodation of assembly loads	No	No	Limited

Table 5 - CAT tools comparation

3. Processes and Appliances

The systematic approach used today from industries [25] is composed by sequential steps planned on various process stages with iterative loops. With particular reference to the automotive area, we can refer to the design process below.

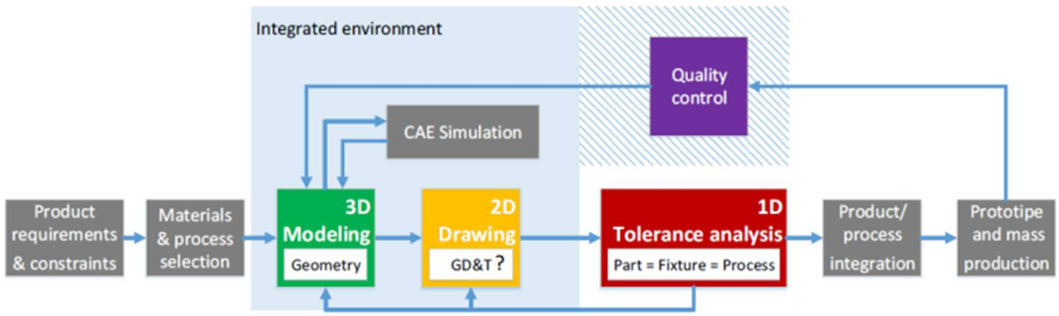


Figure 3.1 Traditional Design method [10]

In the beginning of the Traditional Design Method, we have two main phases:

1. we have to define the functional requirements as performance/structural and the project constraints as size/costs;
2. we have to choose the best materials and process technologies for the manufacturing processes.

After the main phases done, we can realize a 3D CAD Model usually with a parametric approach that guarantees future modifications simpler. This 3D Model has to pass different CAE (computer-aided engineering) simulations to check if it respects the requirements. If not, it starts an iterative loop till it is ok.

After the validation of the requirements of the 3D Model, it can be executed a 2D Drawing created from the 3D. It is important to create a 2D Drawing because of the containing of some important technical information: functional requirements, materials, manufacturing processes, manufacturing limitations and costs. The dimensional and geometrical tolerance's information are part of the 2D drawing. Generally, 1D tolerance stack-ups analyses (usually worst-case analyses) are performed to guarantee the assembly functional requirements. So, starting from the functional key characteristics (FKC) the tolerances are allocated down to the single parts involved in the chain, reducing the variations when it is necessary. Obviously, this means increasing the manufacturing accuracy i.e., increasing assembly costs. Sometimes this leads to

CAD modification in terms of nominal dimensions or parts and assembly design. Now the element is integrated in the process to achieve the actual product feasibility. The last step is the hands-on verification of the functional requirements, from prototypes to mass production. Different tests and check, act to verity the real performances of the assembly, are needful to agree the transition.

Even if we have a practical reference, it is important to verify the quality of the product for some reasons:

- it ensures the performance;
- it may help to have not deviations and problems.

However, we have to face some problems:

- The tolerance analyses are made in a separated environment from the modelling one. Using this approach, the two worlds (tolerance analysis and 3D modelling) never will be integrated, with consequent time consuming and lack of information. The CAE simulations often require a model modification due to the different software. Despite, inspection and quality controls use the 3d model data, also this phase is not fully integrated with the 3D environment. Concluding, only the 3D modelling and the 2D drawings are linked between them.
- The limits of the 1D tolerance analyses.
- The files management. Many files are produced due to the separation between the 3D and 2D file, with a consequent waste of time and lower quality control. Eventually, the updates, coming from the tolerance analyses, first require the 2D drawings modification and then the 3D models too.

3.1 The new approach

The new approach requires the changing in design mentality together with the use of different instruments to perform a more accurate tolerance analysis. CAT tools better integrate with the CAD software, providing a more realistic model of variation, especially for complex systems. Currently, these tools represent the most advanced solution for the 3D tolerance analysis simulations. It is an important step toward the adoption of the MBD approach, in which the availability of PMI would allow a unique integrated space with a rapid exchange of information, minimizing the interpretation errors.

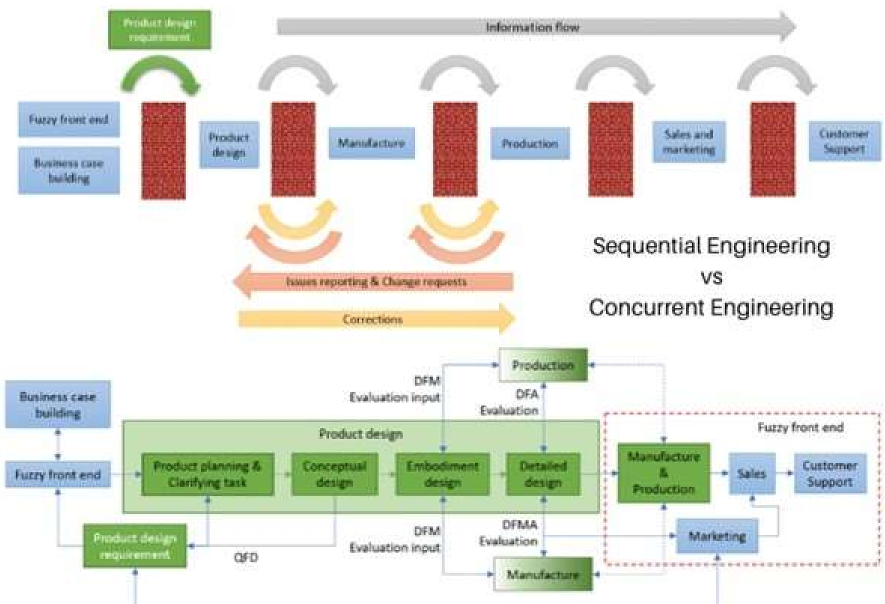


Figure 3.2 Sequential vs Concurrent Engineering

The purpose of tolerance analysis is to achieve a robust design recipe producing a thoughtful allocation of tolerances, limiting the costs. Dimensional Variation Analysis (DVA) methods are employed for correctly guarantee the assembly functional requirements, achieving a higher understanding of the tolerance values, driven by cost improvements or correction of reliability issues. The value of DVA and early, proactive, concurrent engineering[9] shows the

[9] Concurrent engineering (CE) is a work methodology emphasizing the parallelization of tasks (i.e. performing tasks concurrently), which is sometimes called simultaneous engineering or integrated product development (IPD) using an integrated product team approach. It refers to an approach used in product development in which functions of design engineering, manufacturing engineering, and other functions are integrated to reduce the time required to bring a new product to market.

importance of performing the appropriate DVA activities, early in the design cycle in order to achieve the future goal of optimum concurrent engineering leading to more rapid release of robust production designs [26], Figure 3.2.

As usual, each technique has its pros and cons:

- The 3D analysis are more accurate, non-linear stack-up calculations can be easily performed with less simplifications.
- The combined application of translational and rotational effects, due to the dimensional and geometrical tolerances, are internally managed and visible on CAD.
- Statistical tolerance analyses can be performed with different types of distributions.
- The PMI, if available, can be directly imported, from the CAD model, in the specific CAT analysis avoiding typing errors and saving time.
- Any modification in the CAT modeler space directly reflects in the results space, allowing a better understanding of the phenomena.
- Being conform to the international standards, ISO and ASME, the CAT tools also guide in respecting them in case of non-compliant indications on the CAD models/drawings. At the same time it avoids manufacturing problems for the nonunique design language adopted (GD&T).

- The CAT software requires specific skills, therefore training is essential for the correct use of the instrument.
- The modelling of a 3D non-linear stackup require much time compared to the simplified 1D linear tolerance stackup.
- CAT tools require the 3D CAD models to execute the analysis.

Figure 3.3 pros and cons CAD/CAT activities

It seems quite clear the big advantage of using 3D tolerance analysis in the design phase of complex assemblies as an engine. The better understanding of the tolerance system is unmatched. Moreover, in this way it is possible to define and view the effects of a specific assembly sequence and/or the influence of the fixturing systems, reaching a much more robust design method.

3.2 Purpose of the model

Optimizing the production process is only possible by taking into account the three key points of the process itself: costs, performance and assemblablity. Making changes to the tolerance ranges is possible considering the amount of data available to us. Generally, modifications could be improved when there are tolerance problems in the design or production phase or when we fail to reduce costs.

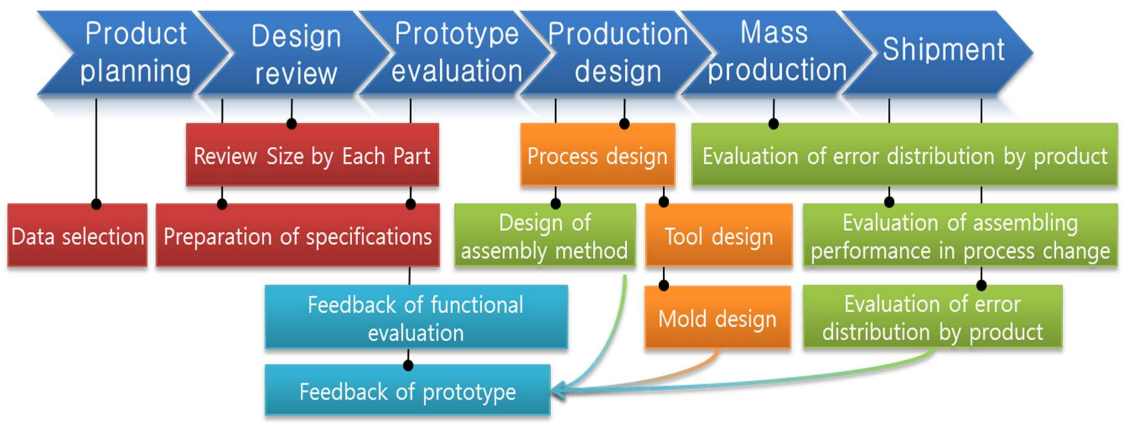


Figure 3.4 Product obtainment sequence

The main purposes of this thesis works are:

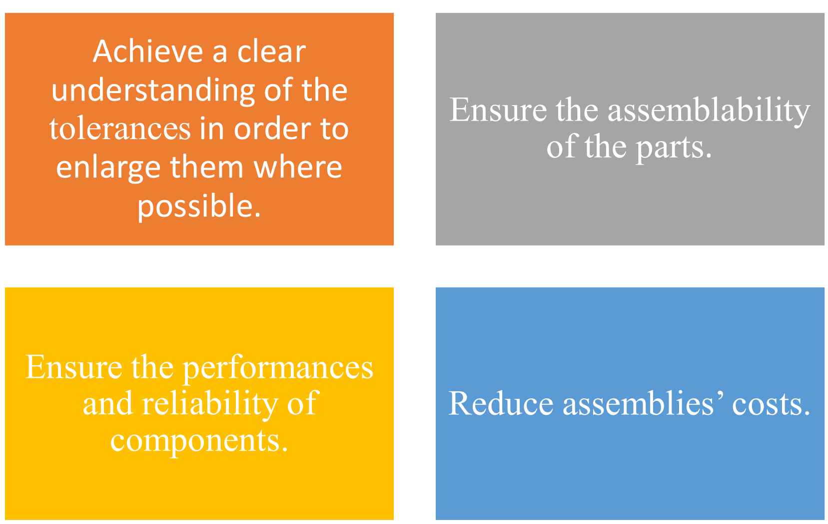


Figure 3.5 Main purposes of the model

The analyses were carried out using the actual 3D CAD models, and modelling the particular case on CAT tool that, in this case, works in parallel with the CAD software (PTC Creo Parametric). In the automotive industry the quality target for long term processes capability is set to $c_{pk} \geq 1{,}33$ i.e., ±4σ. So, the tolerance stack-ups approach using the Statistical analysis, belong to the VDA 6.3 automotive standard. Without real manufacturing data each distribution was considered Normal and Centered. In the Figure 3.6 are shown the requirements and the relation between σ and relative ppm.

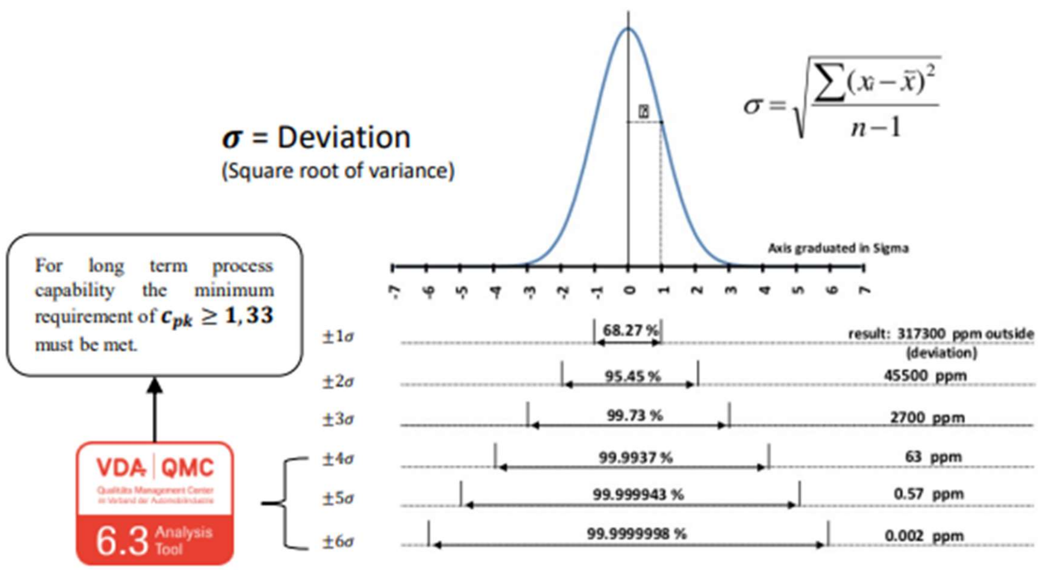


Figure 3.6 VDA 6.3 requirements: σ and relative ppm

3.3 The Proposed Methodology

The proposed methodology exploits the integrational capabilities of the Computer Aided Tolerancing (CAT) tool with the CAD software to share a common environment, Figure 3.7. The connection between the two allows, at least, the direct reading of the assembly parts. In turn, the features involved in the stack-up analysis may be easily imported together with the respective dimensional properties necessary for the particular analysis. In that way geometric modelling, geometrical product specification and tolerance stacks analysis occur in the same design environment. Indeed, all the information are stored into the same 3D CAD model improving the data management and limiting the scattering risks, approaching the MBD philosophy [10].

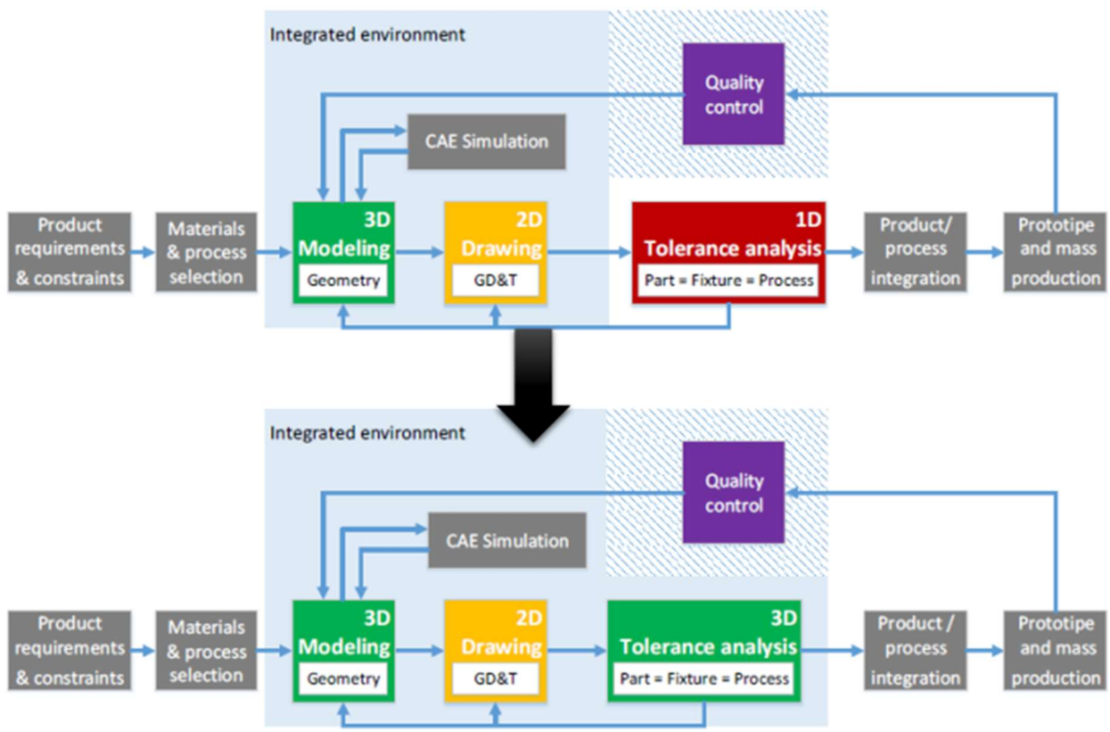


Figure 3.7 Proposed design method [10]

The CAT software are very rigid regarding the geometrical product specifications and sometimes they help to highlight unwanted GD&T errors and/or non compliances with ISO and ASME languages. In these cases, the usual procedure was to correct the tolerancing of the parts, at least to perform the CAT analysis. Often the different tolerancing scheme (for the same tolerances values) has shown different tolerance stack-up results, highlighting the importance of a correct dimensioning and tolerancing.

The adding value of this method is therefore the 3D tolerance analysis, which exploits the three-dimensional nature of the problem, avoiding the strong simplification made in the mono-dimensional analysis. This was possible only using a Computer-Aided tolerancing tool.

The CAT tool allows statistical tolerance analyses according to the Six-Sigma method or using the real statistical distributions when manufacturing and processes information are available.

The tolerance analyses are carried out under static conditions and thermal effect, thanks to the new Cetol **6σ**® V11.2 release. But, in some case, could be an adding value verify the functional requirements even in dynamic conditions. Some CAT software allow to do this, but the analysis of the internal FEM module can never be compared with a professional FEM tool.

Finally, a comparison with 1D analysis is useful to validate the tolerance stack-up simulation 3D made.

This thesis work does not aim to implement the 3D annotation (PMI), but as mentioned in the introduction, it could be the next step toward a Model-Based definition approach and finally toward Model-Based Enterprise[10] (MBE). Obviously, having the PMI in the 3D model may only be beneficial for tolerance analyses with CAT tool. Agree with (Bonazzi,2015), it shown a natural progressive process, Figure 3.8. Moreover, it will lead to a better time to market saving a lot of time in drawn up the 2D drawings.

[10] A Model-based Enterprise (MBE) is an organization that applies modeling and simulation technologies to integrate and manage its technical and business processes related to production and product lifecycle support. By using product and process models to define, execute, control, and manage all enterprise processes, and by applying science-based simulation and analysis tools to optimize processes at every step of the product life-cycle, it will be possible to substantially reduce the time and cost of product development and delivery.

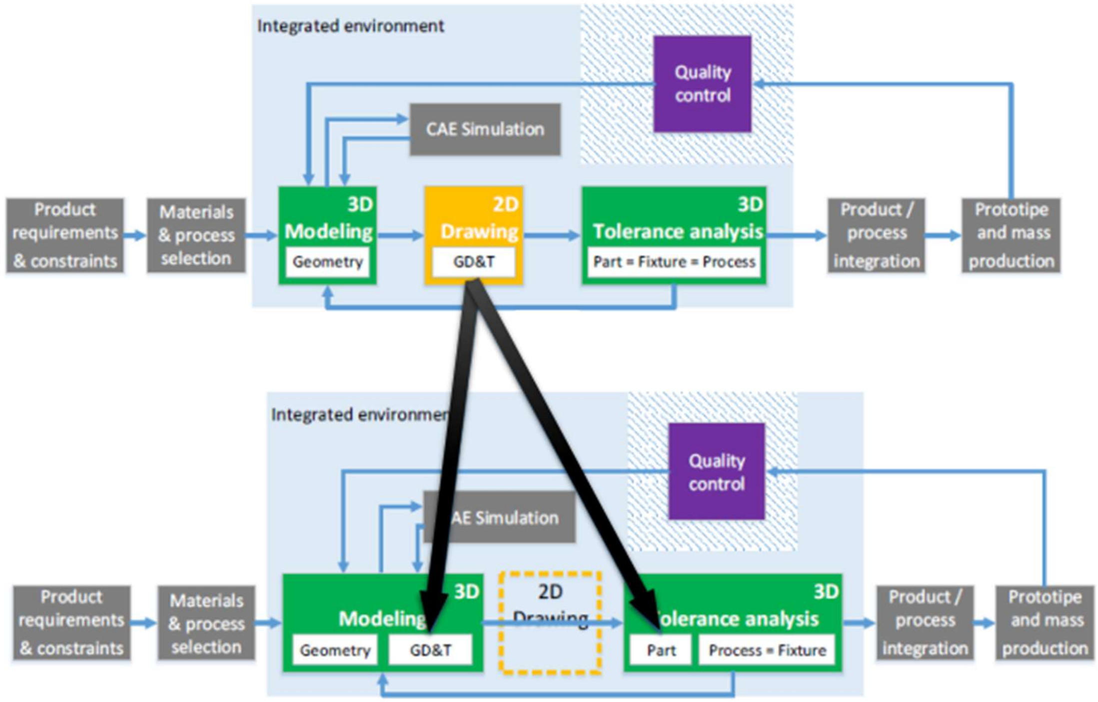


Figure 3.8 Future design method toward the MBD [10]

Concluding, the proposed tolerance analysis method consists of the following main steps:

- Functional requirements definition.
- Identification of the tolerance stack-up.
- Application of the correct GD&T approach.
- Integrated environment CAT/CAD.
- Tolerance stack-up CAT Modelling.
- 3D, linear or non-linear tolerance analysis.
- Statistical approach.
- Optimization of the results: reports.
- Deformed verification: FEM (if necessary).
- Validation with the real data (if necessary).

3.4 Computer Aided Tolerancing Phases

Performing the tolerance analysis via CAT tools it is necessary to approach the problem differently. A general procedure can be identified, independent upon the particular CAT software. The main phases are described below:

1. Functional analysis

The first phase is obviously the functional analysis of the specific assembly. It allows to understand the real engineering needs and to translate them in functional requirements (FKC). Usually, different parts are identified in the assembly, to understand which are the main features involved in the chain. So, dimensions and tolerances are identified too.

This phase, also, helps to understand the hierarchical order of the assembly: hence the assembly sequence. It is essential to ensure the correct functioning of the model method. Coupling features (pins, holes, surfaces, snaps, etc.) are therefore highlighted.

Finally, the feature necessary for set the functional measures (Linear, gap, flush, etc.) are identified.

2. Import components

As mentioned above, most of CAT tools work in parallel with the CAD software. The parts highlighted (3D models) are now imported in the CAT space, following the hierarchical order discovered before.

3. Tolerance specification and allocation

In this phase all the tolerances involved in the chain must be entered in the CAT model. The operation requires the selection of the part features (FOS or NOFOS) and the consequent tolerance characterization. Indeed, this phase is also called characterization. Usually, the operation starts with a 2D drawings analysis (or 3D models). If the Product Manufacturing Information are not available, the operator must manually enter the tolerances values. Here, the correctness of the DRFs and the tolerance method are checked.

4. Assembly modelling: constraints

Due to the functional analysis, the assembly sequence is known. The parts are linked between them with "joints" that describe the kinematic of the assembly. It is a critical phase that assigns the assembly behavior. Different connections can lead very different results.

5. Thermal expansion

During this phase it is necessary to define the thermal expansion effect creating a cloned state assembly and defining a reference temperature in the thermal tab. To ensure the operation's success it needs to be defined also the coefficient of thermal expansion in each part properties.

6. Functional key characteristics definition: Measures

This phase is the core of the analysis. Here the functional requirements are defined in terms of measures (linear, gap, angular, etc.) and acceptable limits. Then selecting the proper features, the measure can be set.

7. Simulation and analysis of results

After running the simulation, the results are analyzed and compared with the targets. Two situations may occur:

- In case of success, usually the post processing environment shows a list of the tolerances involved in the chain with their "weight" and "sensitivity". The weight indicates the variance contribution at the functional measure set, while the sensitivity indicates the slope of the response function. These indices indicate which tolerances should change in order to exploit the functional limits set in the measure, ensuring the quality targets (c_{pk}, σ, ppm[11], etc.).
- In case of a failure, there are mainly two options, depending on the size. A big inconsistency means that something in the model is wrong; the inappropriate use of the kinematic joints is usually the cause. A small inconsistency means that re-

[11] Ppm: number of defects indicated as "part per million"

allocation is necessary. This means changing the process or changing the functional limits.

Summarizing, the different input data of the CAT analysis can be grouped in these categories:

- Geometry input data: the analysis requires only the CAD models of the parts involved in the chain. All the other parts are useless; they make heavy the CAT model, only. In particular must be defined only the functional surfaces, in other words the surfaces necessary for the assembly. Sometimes characteristic feature (points and axes) must be imported for manage the contacts.
- Tolerance input data: most of tolerances (dimensional and geometrical) may be implemented. Currently the limit is represented from the form tolerances and the Envelope or Independence principle. Also, the composite location can be problematic. The values can be assigned via any statistical distributions.
- Assembly input data: they represent essentially the kinematic joint types. Though theme, the assembly response is defined.
- Measurement input data: They represent the functional requirements to investigate (linear, gap, angle, …). The choose of measure type may have a big impact on the correctness of the analysis. Often, points or directions are necessary to better set the measure.

3.5 Thermal Expansion

Atoms begin to vibrate when heat is added to the material. Because of the atoms' vibration, they move away each other causing the expansion of the material. (α) is the linear coefficient of thermal expansion. The length variation is described by (α). Length variation (Δl) to the total starting length (l_i) and change in temperature (ΔT).

$$\alpha = \frac{\Delta l}{l_i \Delta T} \tag{12}$$

If we know (α) it is possible to define (Δl). And the reverse is also true. If we cool down a component (remove energy) the object will contract due to the lowering of the temperature.

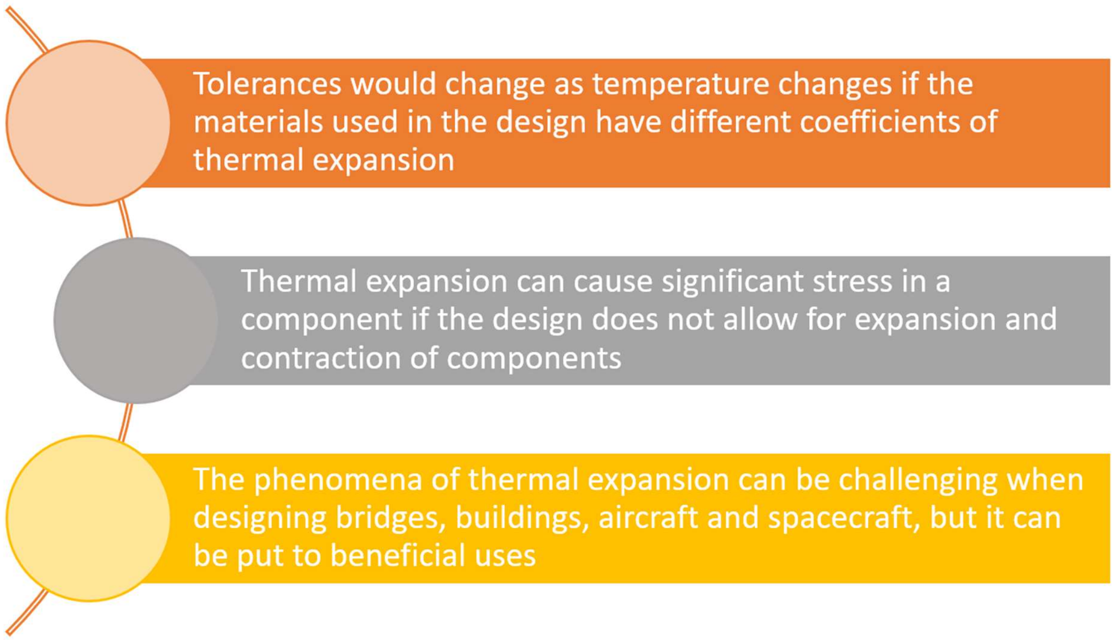


Figure 3.9 Thermal expansion (and contraction) must be taken into account when designing products with close tolerance fits.

Material	α (m/m/°K)	α (mm/m/°K)
Aluminum	23.8 x 10^{-6}	0.0238
Concrete	12.0 x 10^{-6}	0.011
Copper	17.6 x 10 -6	0.0176
Brass	18.5 x 10^{-6}	0.0185
Steel	12.0 x 10^{-6}	0.0115
Timber	40.0 x 10^{-6}	0.04
Quartz Glass	0.5 x 10^{-6}	0.0005
Polymeric Materials	40-200 x 10^{-6}	0.040-0.200
Acrylic	75.0 x 10^{-6}	0.075

Figure 3.10 Linear Coefficient of Thermal Expansion for a Few Common Materials

3.6 CETOL 6σ®

To develop the project, it has been adopted Cetol **6σ**® V11.2 (developed by Sigmetrix, LLC). That tool represents one of the most advanced solutions for the 3D statistical tolerance analysis and has been used together with PTC Creo Parametric 7.0.5. Cetol **6σ**® allows to solve non-linear problems faster than using Monte Carlo approach. Based on the reference manual, the main Cetol **6σ**® features are listed below.

3.6.1 About CETOL 6σ

Cetol 6σ is a full way to analyze and manage tolerances of parts and assemblies. Using that tool, the design engineers can identify the critical areas of the assembly and evaluate individual tolerance contributions to the overall assembly quality, see Figure 3.11. Based on product performance, Cetol 6σ makes the users able to model, analyze, and allocate tolerances while considering manufacturing process capabilities. Therefore, the tool allows a higher understanding of the assembly behavior leading to produce product in higher quality at a lower cost [27].

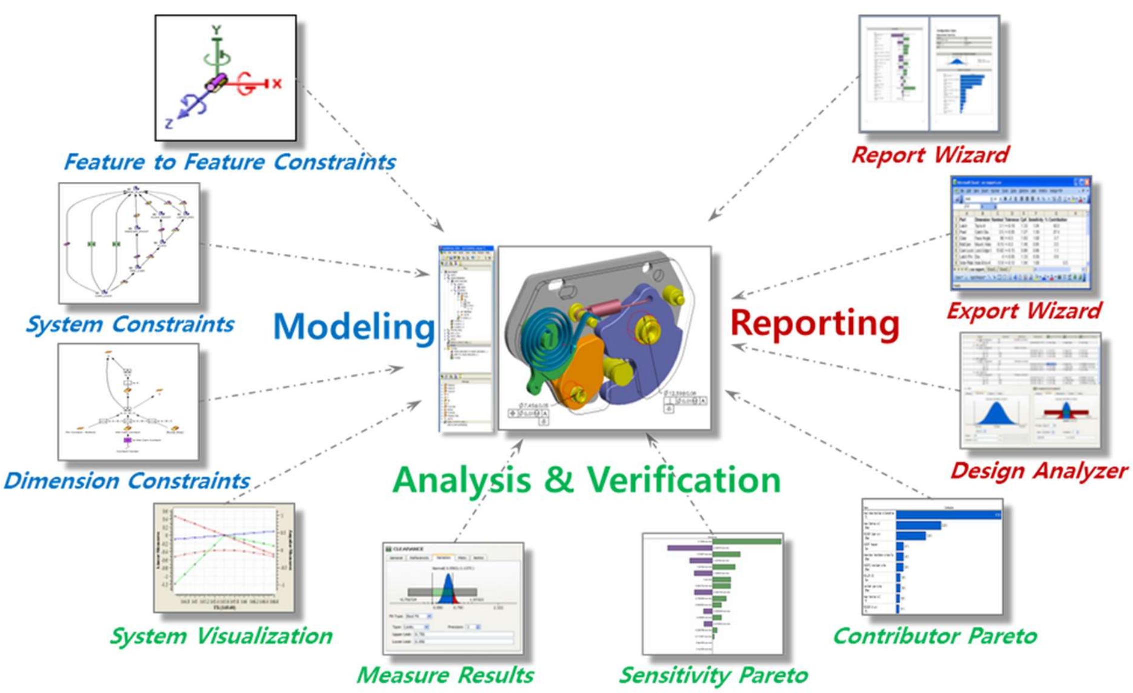


Figure 3.11 Cetol **6σ**

Cetol 6σ presents two primary components: the Cetol 6σ Modeler and the Cetol 6σ Analyzer, see Figure 3.12.

3.6.2 CETOL 6σ Modeler

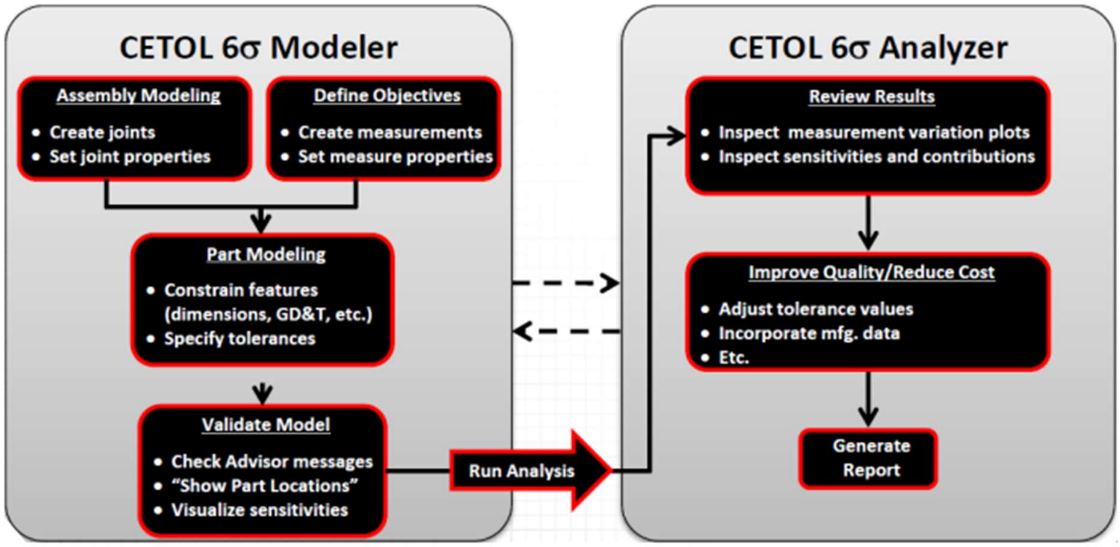


Figure 3.12 Cetol 6σ: phases

Advanced CAD Integration

Cetol 6σ is highly integrated with the CAD model. In fact, the model is built directly by the CAD geometry. It is not necessary to export CAD models to an external file. However, in case of assembly parts that present an assembly feature (boring, milling, etc.) the file must be exported in a common cad file (Step, Iges, …) to recognize the assembly feature as a part of the same assembly. The Cetol model includes that assembly structure and hierarchy as defined the CAD assembly.

All the data of the Cetol model are written into the CAD part and assembly files. After that the model data have been defined for a specific component (part or assembly), that data are reused in case of the part or assembly is used elsewhere in the assembly or in other assemblies. The Cetol 6σ model can also be updated to recognize model reworks from the original nominal CAD geometry. The CAD geometry is the base of the Cetol model because of the Cetol model is built directly on CAD model. Critical measurements and assembly constraints can be defined between the CAD surfaces and edges, rather than just between discrete points. Thus, clearance measurements between surfaces will find the actual minimum distance between the selected surfaces, rather than just between discrete points on the surfaces [27].

Kinematic Assembly Modeler

Assembly constraints are defined in Cetol 6σ using kinematic joints. A joint can be created by the user selecting two interfacing features on components within the assembly. The created joint can be characterized for example with float or bias at fasteners, Figure 3.13.

Thanks to that, the Cetol 6σ, through an easy-to-use interface, can characterize the assembly behavior of complex assemblies. For contact joints and tangent fastener joints, the selected features are defined to be in contact. The shape of the contact zone could be a point, a line, an arc, or a plane depending on the features selected and the relative orientation of those features. For centered fasteners, the contact is idealized at the center (point, axis, or plane) of the features.

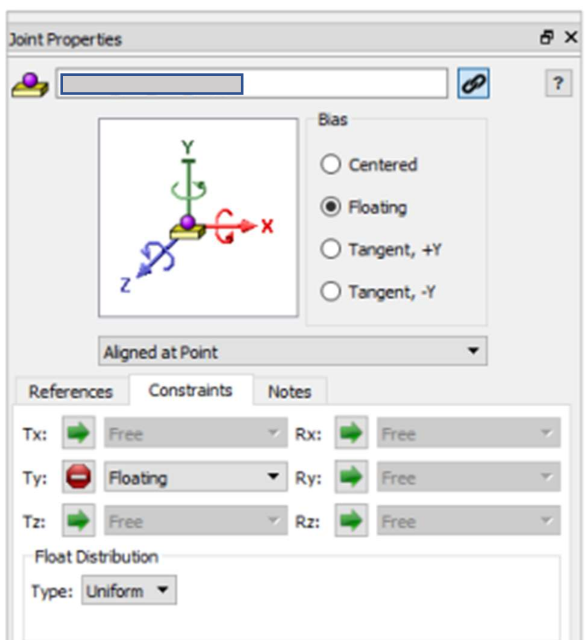


Figure 3.13 Joint properties window

The Cetol 6σ Modeler includes an iterative solver to solve for assembly closure in order to determine the location and orientation of each component in the assembly. This type of solver is essential for analyzing closed-loop assemblies such as mechanisms (valvetrain, suspensions, etc.).

A tolerance model with the DOFs set correctly is called “exactly constrained”, Figure 3.14. An exactly constrained assembly has all degrees-of-variance (not degrees-of-freedom) of each part constrained exactly once. An incorrect DOF settings may lead to:

- under-constrained parts;
- over-constraining joints;
- “not closed” condition.

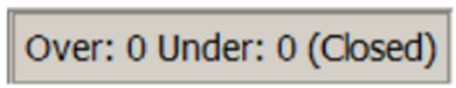


Figure 3.14 "Closed" condition

Usually, it is mandatory to obtain an “exactly constrained” condition to run the analysis because these situations can cause mathematical problems when analyzing tolerance models.

However, Cetol 6σ, even if the tolerance model isn’t “exactly constrained”, allows the run of the analysis.

Part Variation Modeler

Thanks to the Cetol, the user can precisely define the dimension and tolerance scheme for each component. Based on the International Standard ASME Y14.5:2009. Assigning the statistical variation of the location and orientation of the critical features of the model it is performed the statistical variation analysis. The variation for each variable can be defined with different distribution models (normal, lambda, custom) that can be calculated automatically based on the tolerance limits and an assumed quality metric for the component. If available, measured statistical manufacturing data can be used for the analysis.

Thermal expansion

The assembly state properties have an “Include Thermal Expansion” option. When that option is selected, the solved nominal, worst case results, and statistical results for each measurement in the assembly state include the effects of thermal expansion. All dimensions and tolerances are assumed to be specified at standard temperature (20°C/68°F). It is necessary to specify the coefficient of linear thermal expansion for each part and assembly. When the "Include Thermal Expansion" option is selected for an assembly state, all linear dimensional values are scaled according to the following formula:

$$Scale = 1 + [CTE * (T_{state} - T_{ref})]$$

- Scale: Scale factor due to thermal expansion.
- CTE: Coefficient of linear thermal expansion.
- T_{ref}: Reference temperature (20°C/68°F).
- T_{state}: Specified temperature for the assembly state.

All components included in the assembly state are assumed to be at steady state temperature as specified on the assembly state thermal properties. For multi-level assemblies, the temperature specified in the assembly state overrides the temperature value specified in lower-level assembly states.

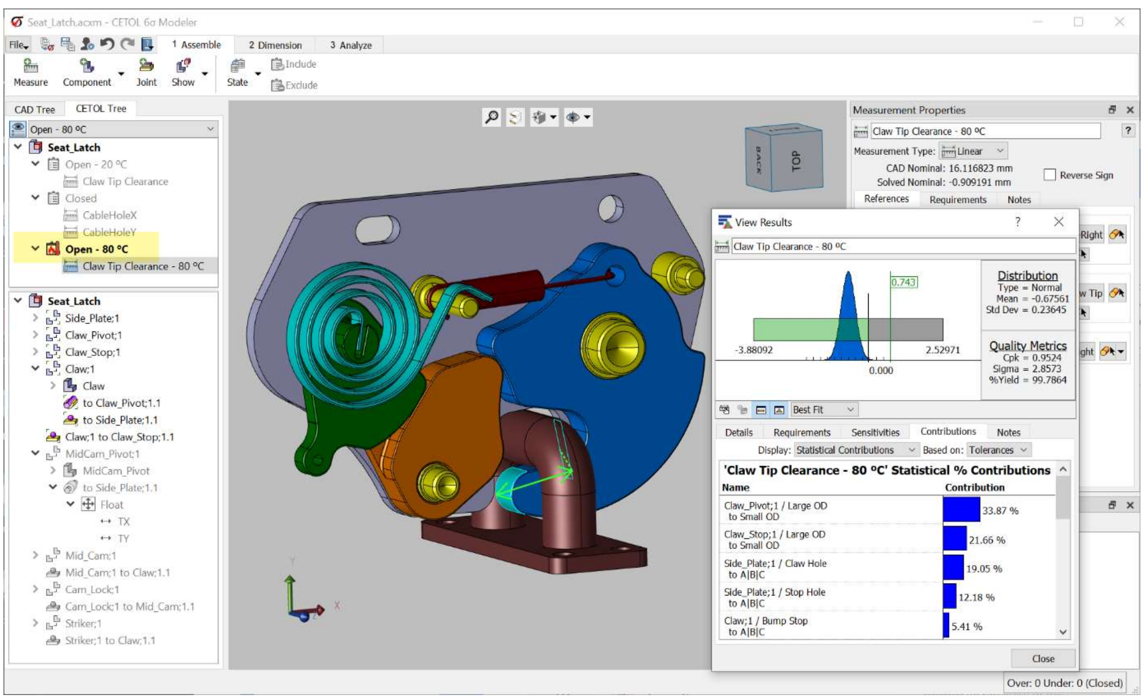


Figure 3.15 Thermal Expansions view

3.6.3 CETOL 6σ Analyzer

The Cetol 6σ Analyzer is the user interface designed for the post processing-phase in which the analysis results are visualized. The functional measures and a list of the tolerances involved in the chain are reported. The "sensitivities", "variance contribution" and "shift contribution" guide the post processing phase, Figure 3.16.

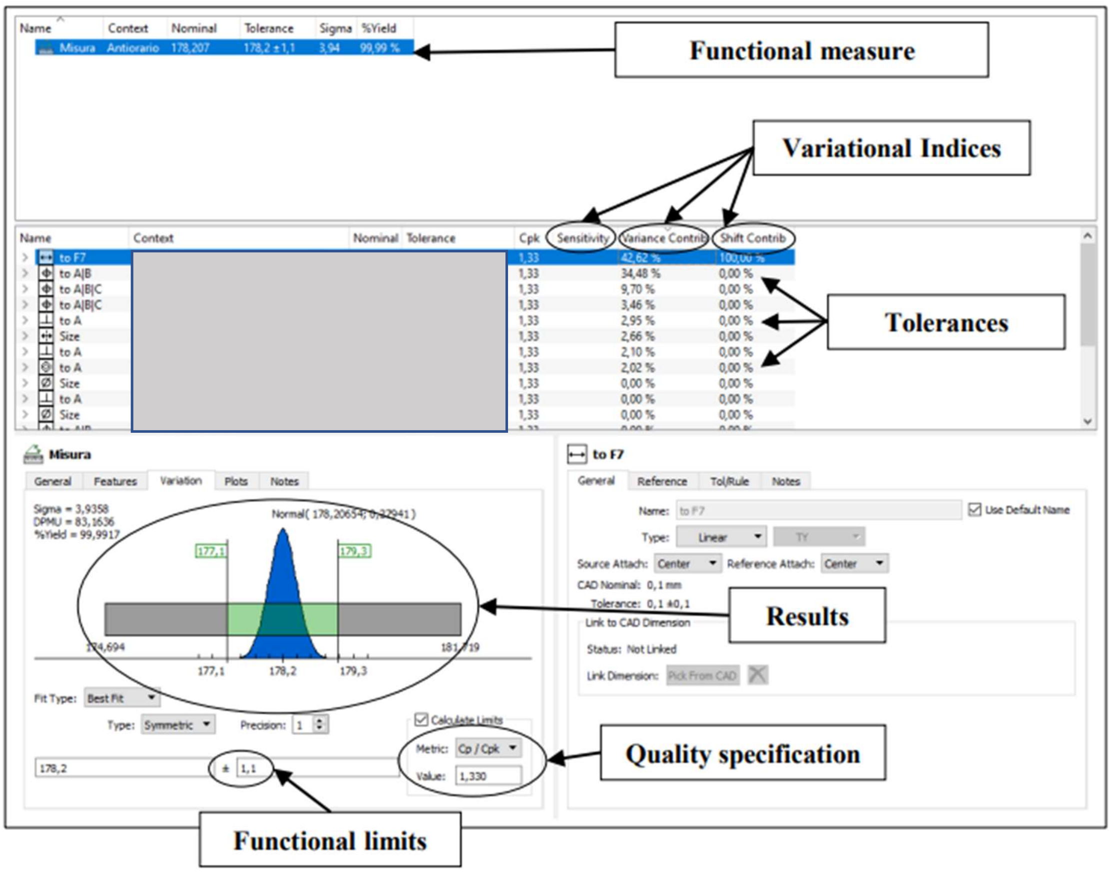


Figure 3.16 Analyzer environment: results

3.6.4 CETOL 6*σ* Derivative-based Analysis

CETOL uses a derivative-based analysis to calculate partial derivatives of each measurement to each dimension. These partial derivatives are commonly referred to as "sensitivities". As said, there are several vantages of derivative-based analysis compared to a Monte Carlo simulation.

First, in a derivative-based analysis, CETOL 6σ calculates the sensitivity of each measurement to each variable and the contribution of each dimension to the variation of each measurement. The sensitivities and contributions are very useful for understanding assembly behavior. They help in identifying potential manufacturing costs savings: dimensions with low sensitivity and contribution might represent an opportunity to enlarge the tolerance values therefore to reduce the costs.

Second, once the sensitivities have been calculated, each design and manufacturing variations, set in the model, can be evaluated instantly, the results of the analysis update immediately. In a Monte Carlo simulation, any changes, require the run of a new simulation. Thus, derivative-based analysis is generally a better approach, especially during the design phase of a project. Cetol 6σ can use both statistical and worst-case analyses.

Statistical Analysis

As said, statistical analysis can be easily carried out in Cetol. The 1-D RSS (Root Sum Squares) analysis is commonly performed manually or via spreadsheet. This equation (13) assumes that all processes have the same distribution type:

$$\sigma_{ASM} = \sqrt{\left(\sum_{i=1}^{n} \sigma_i{}^2\right)} \tag{13}$$

where σi is the standard deviation of the "i-th" dimension. For 2-D and 3-D analysis, the equation (14) is slightly more complex:

$$\sigma_{ASM} = \sqrt{\left(\sum_{i=1}^{n} \frac{\partial \mathrm{U}}{\partial \mathrm{x}_i} \sigma_i\right)^2} \tag{14}$$

where $\frac{\partial U}{\partial xi}$ is the first partial derivative of the functional requirement to the "i-th" dimension and σi is the standard deviation of the "i-th" dimension manufacturing process. However, the partial derivative terms (commonly referred to as sensitivities) are often difficult to calculate manually or with a simple spreadsheet, in particular for three-dimensional cases. This combined with the simplification of the RSS methods (independent variables, normal distributions, $\pm 3\sigma$, liner analysis) makes it not always a winning method. Cetol 6σ derivative-based analysis uses the Second-Order Tolerance Analysis method (SOTA) that have not the simplifying assumptions of the RSS method. The SOTA method include a nonlinear system solver, finite difference approximations for the first and second order partial derivatives, the Method of System Moments (MSM), and a Generalized Lambda Distribution (GLD) empirical fit.

Worst-Case Analysis

This solution method is also capable of calculating worst-case analyses. An underlying assumption of the solver is that all features are at one of the tolerance zone boundaries in the worst case. This is a good assumption in most real-world assemblies. However, it is possible that for some highly non-linear systems, the worst-case condition could actually occur when one or more features are not at the tolerance zone boundaries.

3.6.5 Crete a CETOL 6σ Model

CAD model Files

In this case the Cetol worked in parallel with PTC Creo Parametric 4.0. The Creo assembly and part files are required to create a Cetol model. Cetol 6σ queries the Creo model for geometric information about the parts. Since Cetol 6σ is dependent on the CAD model files, changes to the CAD models may affect the Cetol model file.

Assembly and Part Drawings

Drawings are not required to set up or analyze a tolerance study, but they can useful when a 3D annotation is missing. Drawings with tolerance information (datum feature definitions, tolerance values, G&T callouts) are particularly used.

Scope of Study

"Problem scope" refers to the type of measurement being performed and the portion of the assembly relevant to the study: functional key characteristic.

Identify Measurements

The first step in a tolerance analysis is to define exactly the objective of study: measurement. A measurement can be the interested length or angle on the CAD assembly model. A length measurement can be a gap or interference between two features, the overall height of a stack-up, a contact distance between two mating parts, or any other linear measurement. An angle measurement is any orientation measurement of one feature relative to another. Four different measurement types are supported:

> **Linear** – a linear measurement between two features

> **Gap** – a special case of a linear measurement between two features.

> **Flush** – a special case of a linear measurement between two features.

> **Angular** – an angular measurement between two features.

For a given assembly geometry imported in Cetol, different cases of study can be analyzed. This option allows to create different model version based on same assembly, changing for example joints, tolerances, measure types, etc. They are called "state" and different measures can be made in for each of them.

For linear measurements, Cetol finds the endpoints of the measurement based on the minimum distance between the measurement features, by default. But, for some types of features (e.g., cylinder, sphere, etc.) there is the opportunity to choose different options location (e.g., Near or Far).

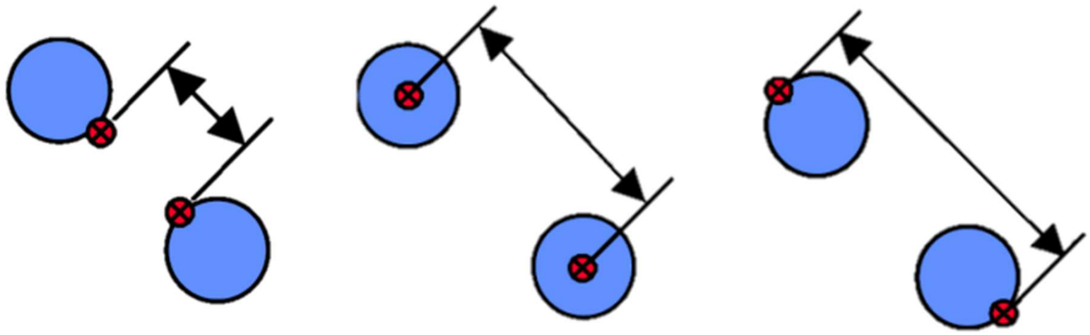


Figure 3.17 Different measure set-up

Also, specific direction can be selected to have the right measure, Figure 3.17.

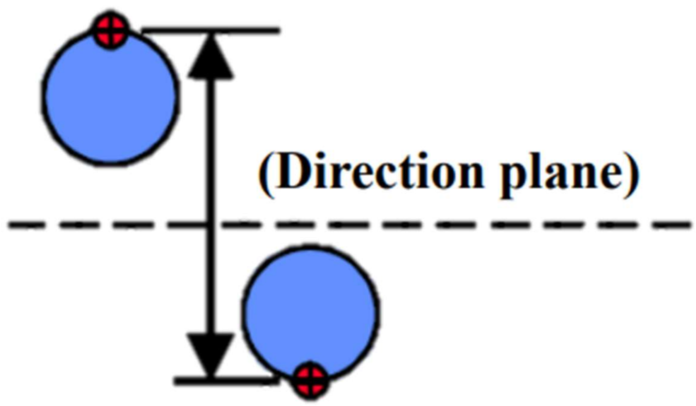


Figure 3.18 Measure direction, set-up

Simplify the CAD Assembly (Optional)

A tolerance stack-up seldom requires the inclusion of every part and feature in an assembly. Usually, only a small subset of the parts is relevant to the measurement. The nonrelevant parts in an assembly tend to add complexity to the CETOL modelling process. It is often helpful to simplify the CAD assembly model down to the set of relevant parts, although this is not required.

The graph tool

As shown in Figure 3.19, the Graph view shows the Cetol model graph, which is a schematic representation of the parts and features included in the model, the joints representing the assembly constraints, and the measurements that represent the fit and performance requirements of the assembly. It follows the hierarchical modelling order.

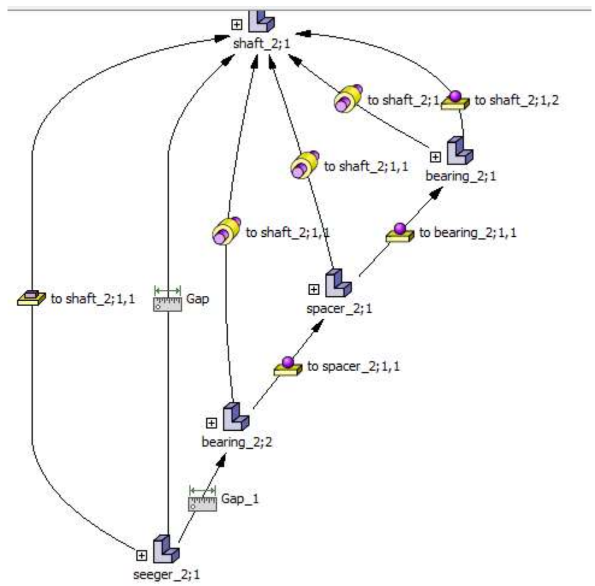


Figure 3.19 The graph view

Defining parts variation

In the tolerance definition phase, the appropriate variation rule can be chosen. The variation rule defines the relationships of the tolerance, the statistical distributions of the related variables, and the quality metric. Via the Variation Rule Editor, a specific distribution, metric, and quality parameters (C_p, C_{pk}, MMC, etc.) can be set, Figure 3.20.

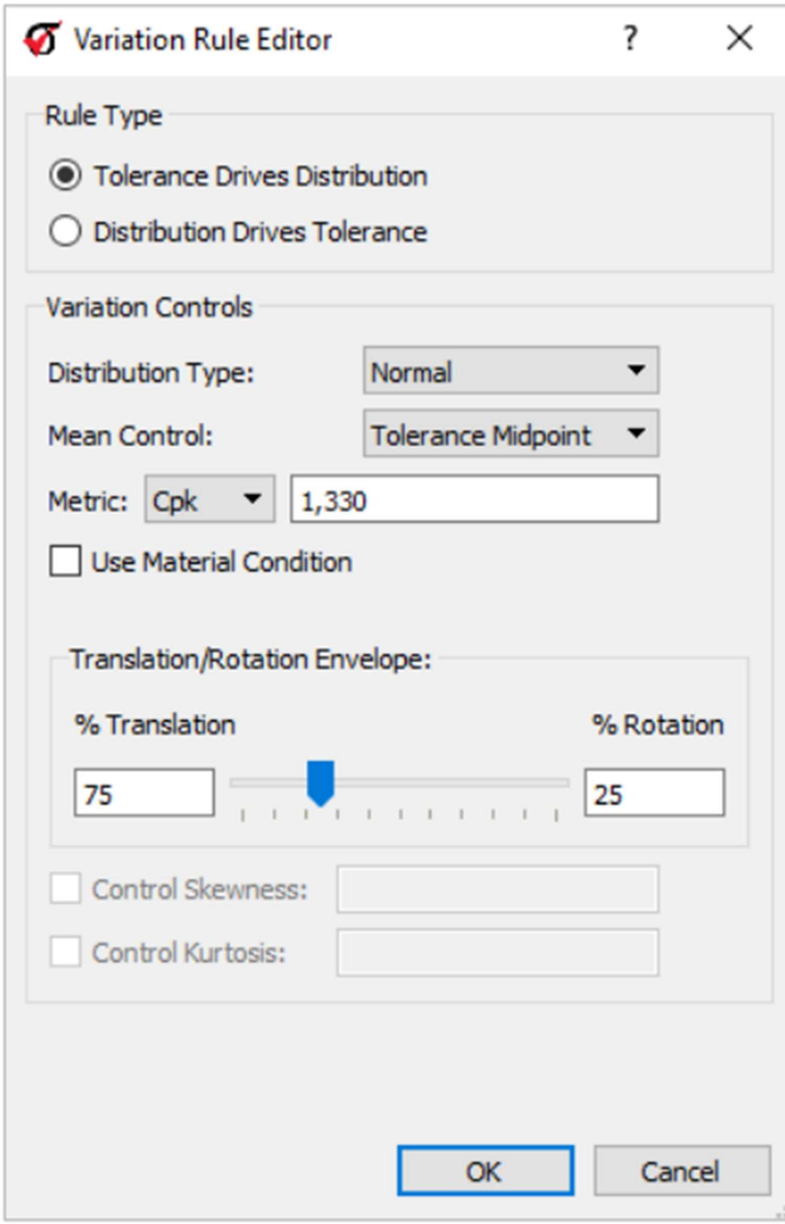


Figure 3.20 Variation Rule Editor

Summarizing, the modelling phases in Cetol can be grouped in 6 steps:

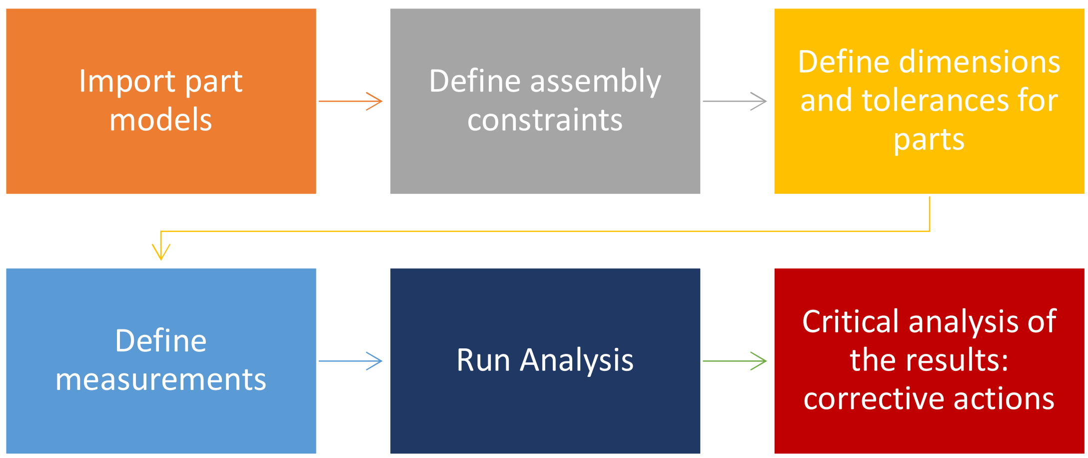
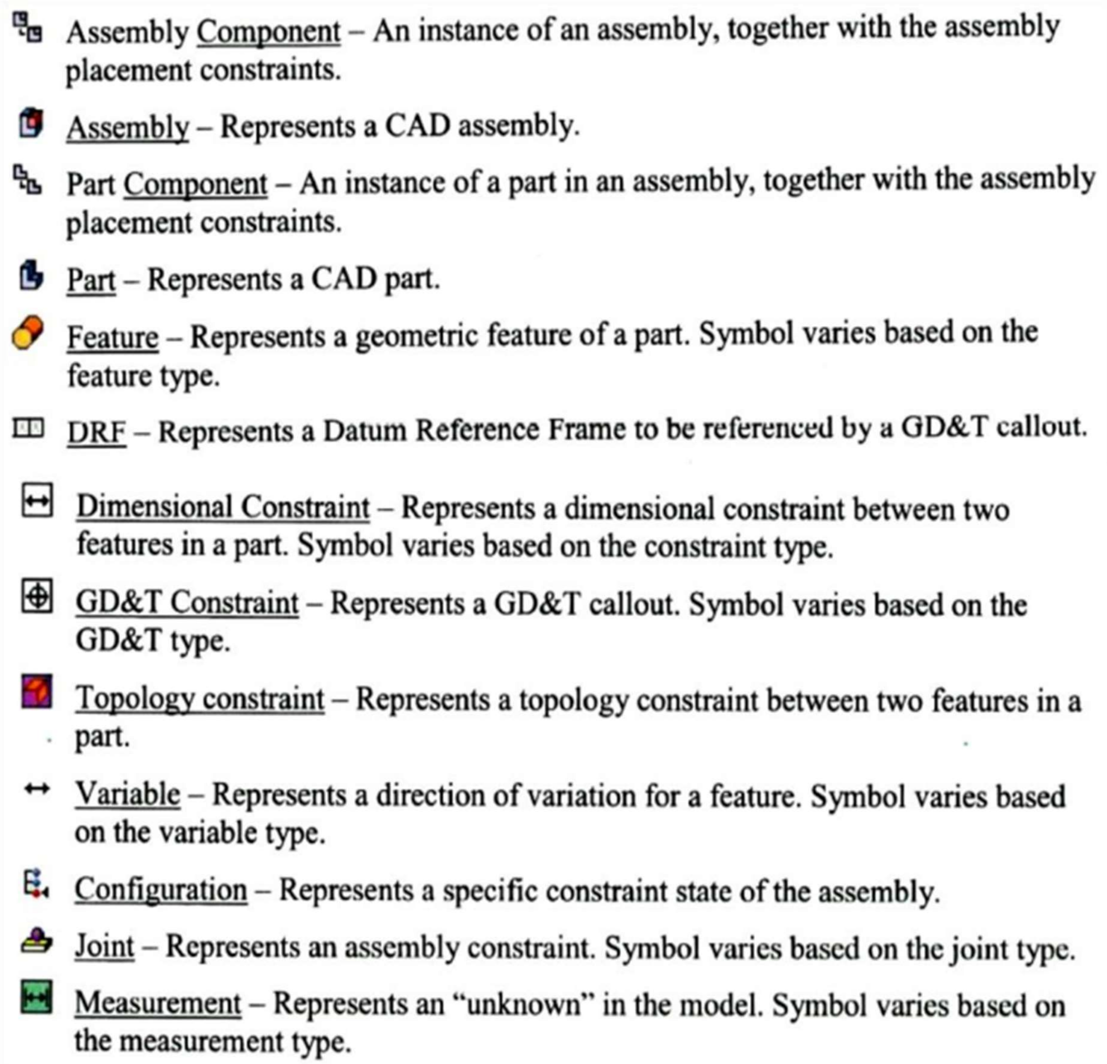


Figure 3.21 modelling phases

Sometimes can be useful to model each part involved in the stack-up analysis before being imported in the assembly model of Cetol. In this way each part contains all the information (features, DRF, tolerances, distributions) needed for the analysis. The only operation to do is the creation of the joints and the definition of “states” and “measures”. The order to proceed is quite indifferent.



Assembly Component – An instance of an assembly, together with the assembly placement constraints.

Assembly – Represents a CAD assembly.

Part Component – An instance of a part in an assembly, together with the assembly placement constraints.

Part – Represents a CAD part.

Feature – Represents a geometric feature of a part. Symbol varies based on the feature type.

DRF – Represents a Datum Reference Frame to be referenced by a GD&T callout.

Dimensional Constraint – Represents a dimensional constraint between two features in a part. Symbol varies based on the constraint type.

GD&T Constraint – Represents a GD&T callout. Symbol varies based on the GD&T type.

Topology constraint – Represents a topology constraint between two features in a part.

Variable – Represents a direction of variation for a feature. Symbol varies based on the variable type.

Configuration – Represents a specific constraint state of the assembly.

Joint – Represents an assembly constraint. Symbol varies based on the joint type.

Measurement – Represents an "unknown" in the model. Symbol varies based on the measurement type.

Figure 3.22 Cetol 6σ principal interface icons

3.7 From theory to practice

In this thesis one case study has been proposed: a shaft composed by 5 components. The goal is to have always the mountability of the Seeger on to the shaft.

The analysis helped me to familiarize with the software and understand its potential. The results obtained were instead of great help in defining the great differences that exist between the possible approaches to be adopted. In this thesis we will not use manual Excel calculations or sheets, however the main differences between these kinds of approaches and a statistical one will be defined.

This thesis aims to focus in particular on the great potential of the statistical approach to dimensional tolerances with the aid of two software in particular: PTC Creo CAD and Cetol 6$\boldsymbol{\sigma}$ CAT.

We do not design 1D product anymore so why limiting ourselves to 1D dimensional tolerance analysis? Perhaps it is because defining all the mathematical equations takes too much time. The statistical approach adopted by Cetol makes it faster and easier. In fact, it allows you to solve even the simplest tolerance analysis problems in less time it would take you using a spreadsheet. Meanwhile you are looking for all data to insert in the spreadsheet, Cetol has concluded the analysis with both ***WCA*** and ***Statistical Results.***

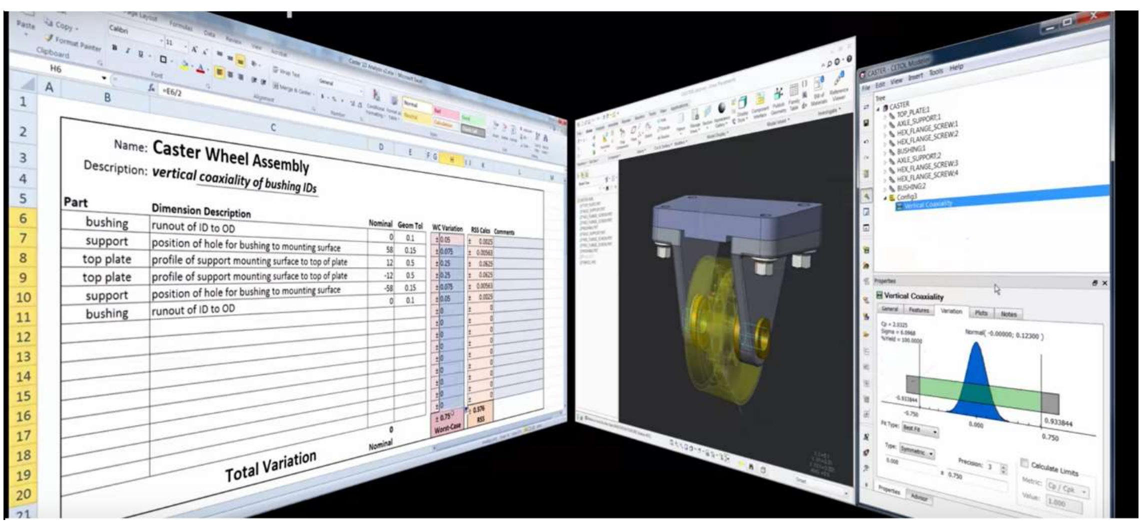


Figure 3.23 Spreadsheet Vs Excel: Analysis the vertical alignment of the axes in the two bushings

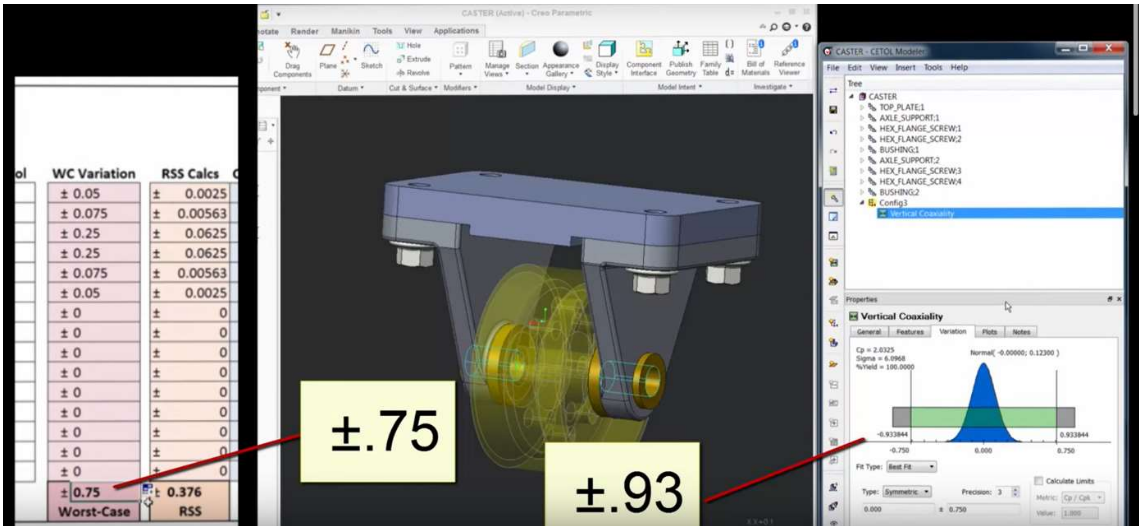


Figure 3.24 Spreadsheet Vs Excel Results: Analysis the vertical alignment of the axes in the two bushings

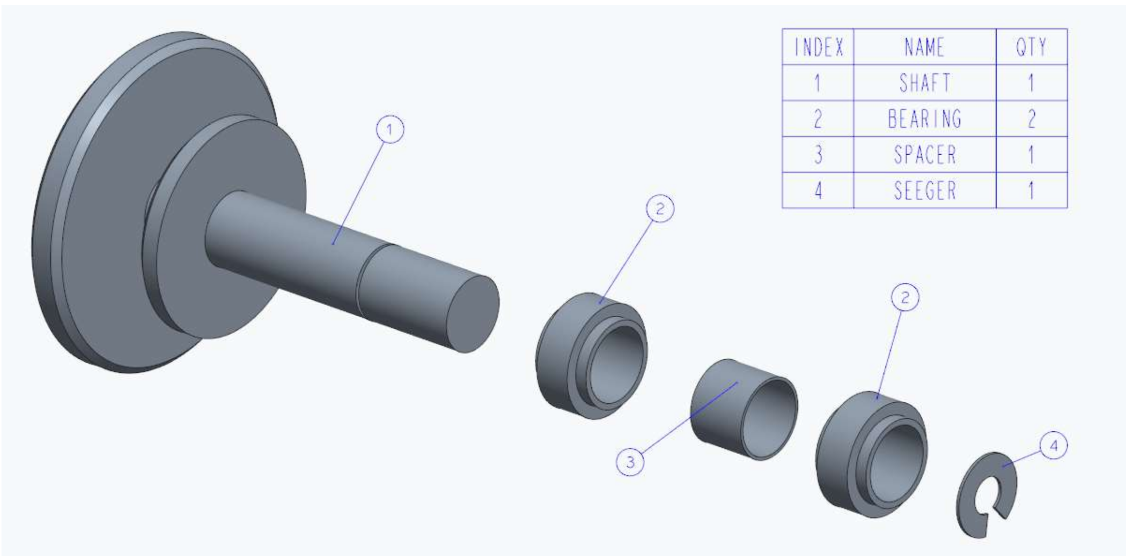
Here above an example of comparison between spreadsheet and Cetol Figure 3.23 and Figure 3.24.

Main goals of the statistical tolerancing:

Figure 3.25 Main goals of the statistical tolerancing

3.7.1 Case studies

First of all, I created a 3D example model using the PTC Creo software, able to help me to understand the subject matter in order to become familiar with the two platforms used. The assembly created is a simplified assembly consisting of 5 components: a Shaft, two Bearings, a Bushing and a Seeger. Thanks to the new release of Cetol Software, it is possible to improve the results obtained by inserting the operating temperatures of the parts and the thermal expansion coefficient. We want to check the mountability of the Seeger, in order to ensure the functionality of the parts both in static and hot conditions.



INDEX	NAME	QTY
1	SHAFT	1
2	BEARING	2
3	SPACER	1
4	SEEGER	1

Figure 3.26 Exploded view and BOM

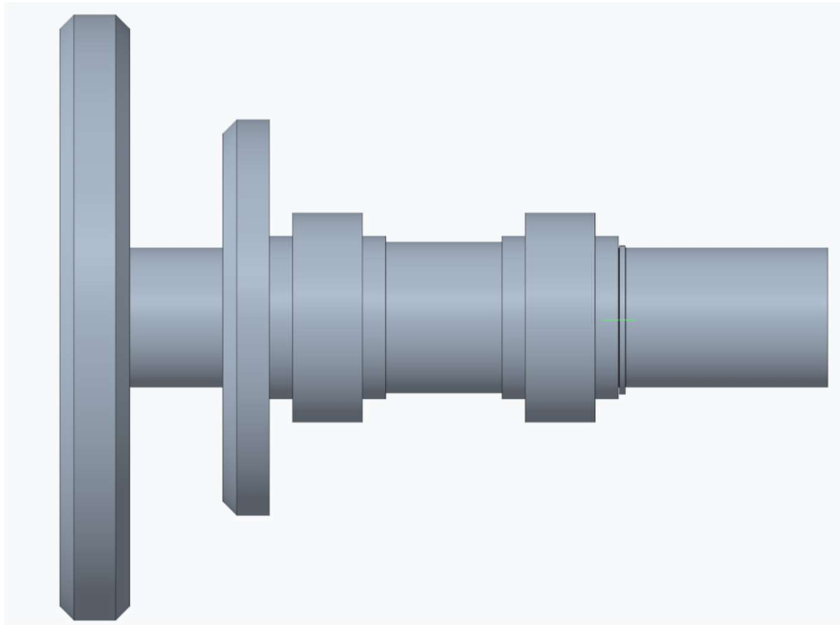


Figure 3.27 Shaft.asm

The analysis followed these steps:

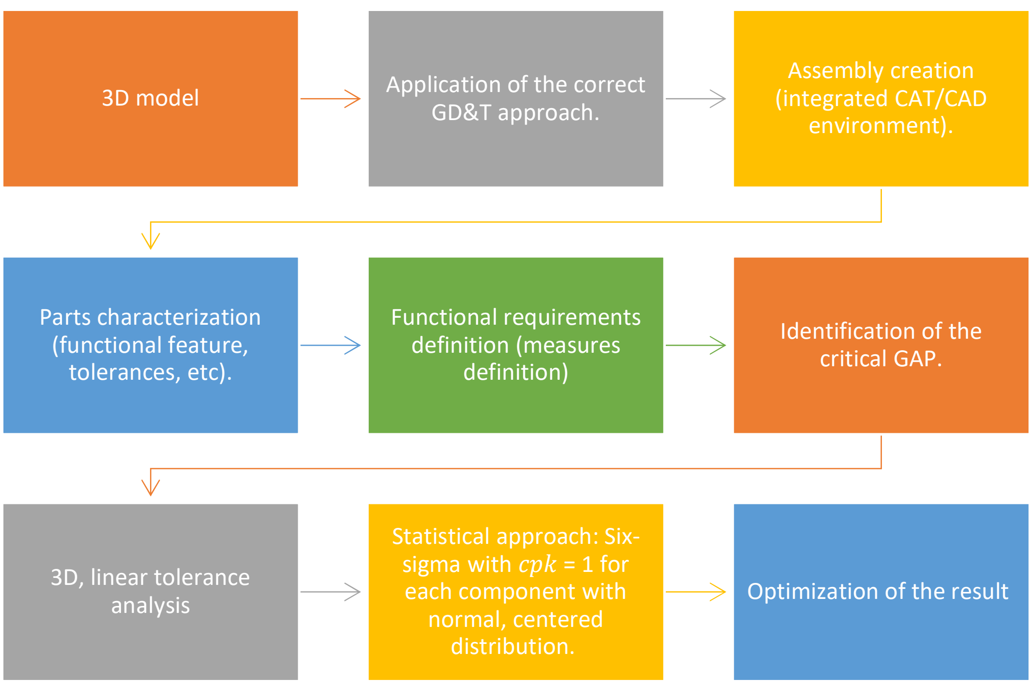


Figure 3.28 followed steps

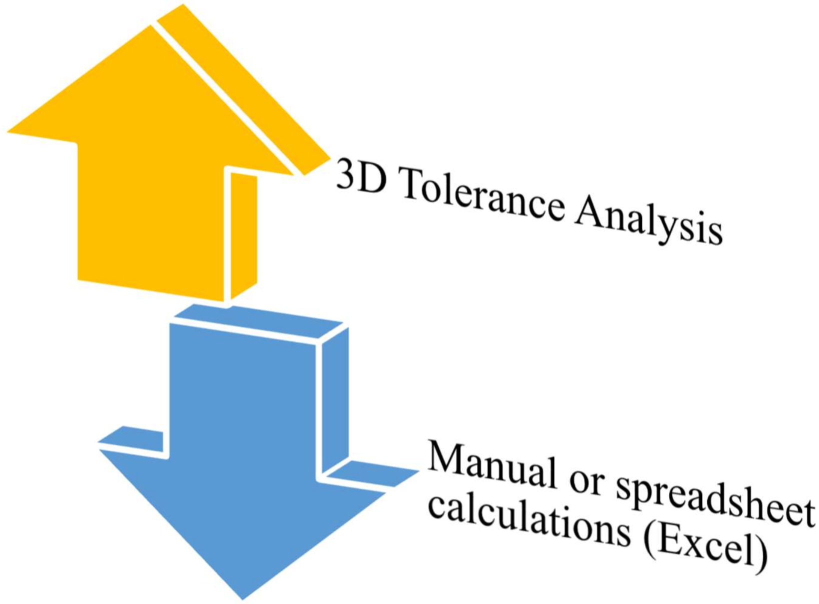


Figure 3.29 3D tolerance analysis is a way better than manual or spreadsheet calculations

Normally the tolerance analysis starts with the 2D drawings analysis of the components involved in the tolerance chain. However, as it is composed by simple components and it is developed only axially, it was simple to directly create a 3D CAD been characterize on Cetol. It is common in this case to choose a similar DRF for all the parts of the assembly, to better guarantee the mountability.

The assembly sequence plays a fundamental role, as the model could be more precise and truthful. Each part in space has six DOFs and the constraints must reflect its kinematics. Obviously, given the type of components making up the assembly, the joints were defined starting from the first bearing inserted in the shaft. The CAT model needs the definition of a correct DRF. As said before, the CAT tools, based on the "vector loop" or "variational" model, create a local coordinate system for each part and for each feature. The only way to link the global coordinate system with the local one is to create a DRF. The DRF is composed in that case by the shaft cylinder axis (A), by the shaft plane for the first bearing to be inserted (B); we complete the triad with a specific geometric tolerance the perpendicularity between (A and B). To improve accuracy, I have insert dimensional references, see Figure 3.30. This procedure has been made for all the 5 parts of the assembly.

Cetol does not recognize the entire geometry part, but it knows only the features and their relative information given via modelling. When this information is missing, there are two options:

1. coherent features with coherent values are assumed, verifying their impact in the analysis results.
2. invariant features (or invariant value) are added in the model, writing the assumption on the report.

Basically, it is necessary to insert some joints to concatenate the components as our goals is to return a fully constrained system.

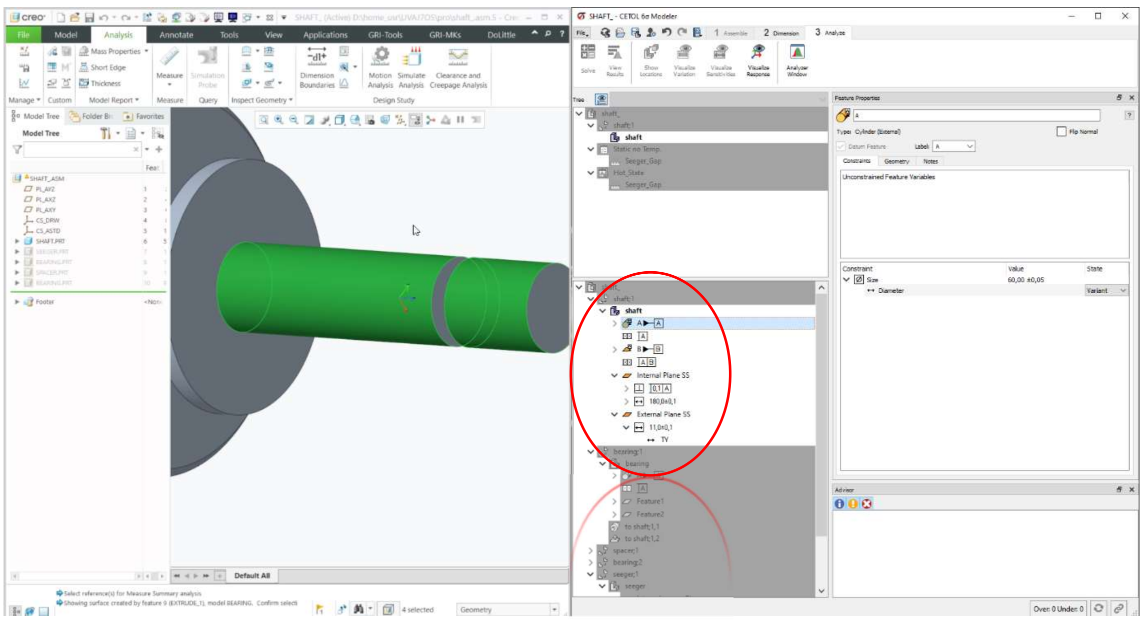


Figure 3.30 Datum Reference Frame and Datum Features

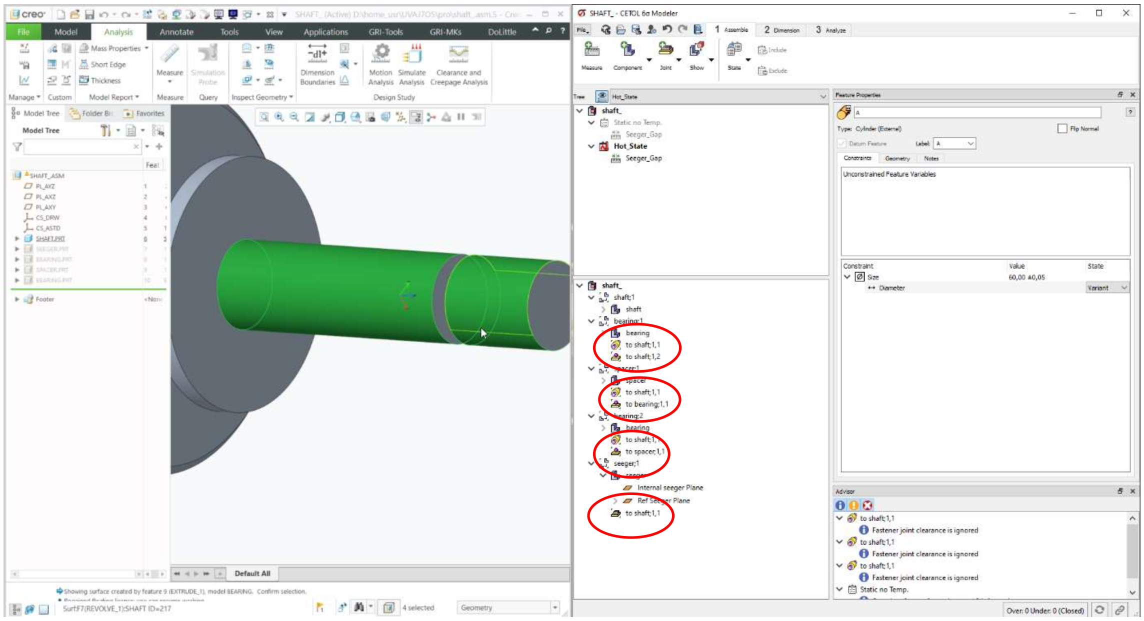


Figure 3.31 Joints

Cetol allows to create different case studies, called "***State***", into the same assembly model simply turn-on or turn-off to switch from one to another.

In each "***State***" the user can choose the measure we want to check, the components and joints. For example, it can import some parts active only for a given state or create different joints that simulate different assembly conditions. In our case the two states have been called "***Static***" and "***Hot***". Selecting one of them, the related parts, joints and properties set became active.

Note that Measurements can be evaluated at different temperature by specifying:

1. Assembly state at desired temperature containing the desired measurement/s;
2. Coefficient of thermal expansion of each part.

In my case I have created two linear measurements, each per "state". They differ for the start/end features. The first measure, "**Gap**", considers a Linear Gap between the internal plane of the Seeger seat of the shaft from to the Seeger's plane, the second measure," **Gap_1**", considers a Linear Gap from the bearing's plane to the Seeger's plane. During the final analysis it was clear that the measure "**Gap**" was not representative of the tolerance stack-up, but it was for sure a good exercise to go in for. In fact, we will focus only to the "**Gap_1".**

Regarding the thermal tool, the whole is based on assuming a steady state temperature across all components; linear expansion/contraction. Temperature could change per model but not per components, which is the reason why we have to create more "***States***". The "***Static***" and the "***Hot"*** state are representative of the two hypothetical operating conditions of the shaft. The "***Hot***" state considers an exercise Temperature of the shaft of about 200°C – steel shaft and aluminum material for all the parts, so that the thermal expansion coefficient will be:

$$\alpha = 12 \times 10^{-6}/°C$$ [12]

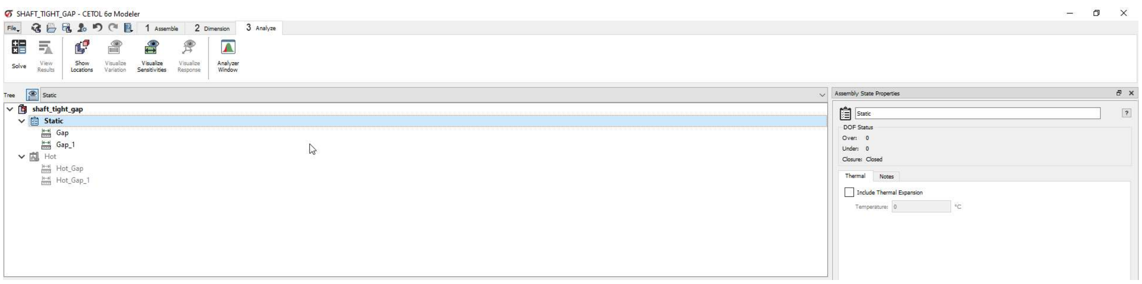


Figure 3.32 "Static" and "Hot" state

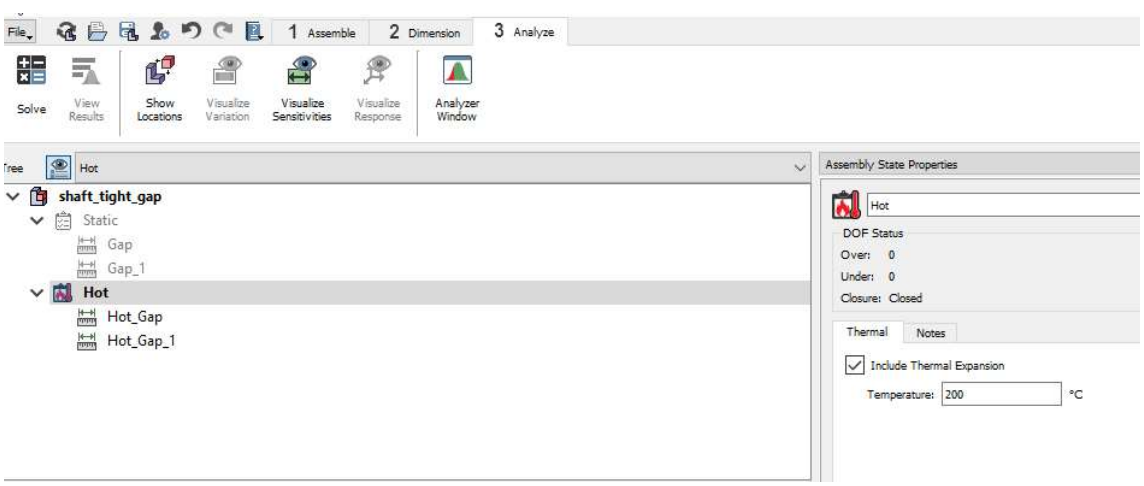


Figure 3.33 Hypothetic functional temperature

[12] Alpha values calculated considering a reference temperature of 20°C

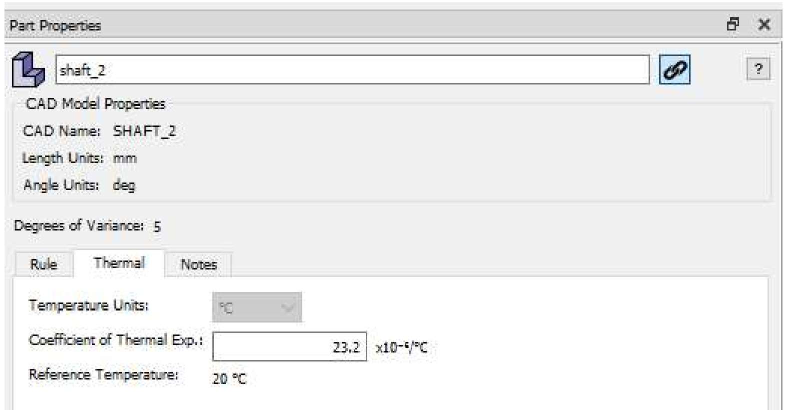


Figure 3.34 Thermal expansion coefficient

After the assembly definition all the useful information regarding the single component become integral part of the model. This information includes functional features involved in the chain, functional features used to set the measures, datum reference frame (DRF), dimensional and geometric tolerances, joints properties, etc. This phase is usually called component characterization.

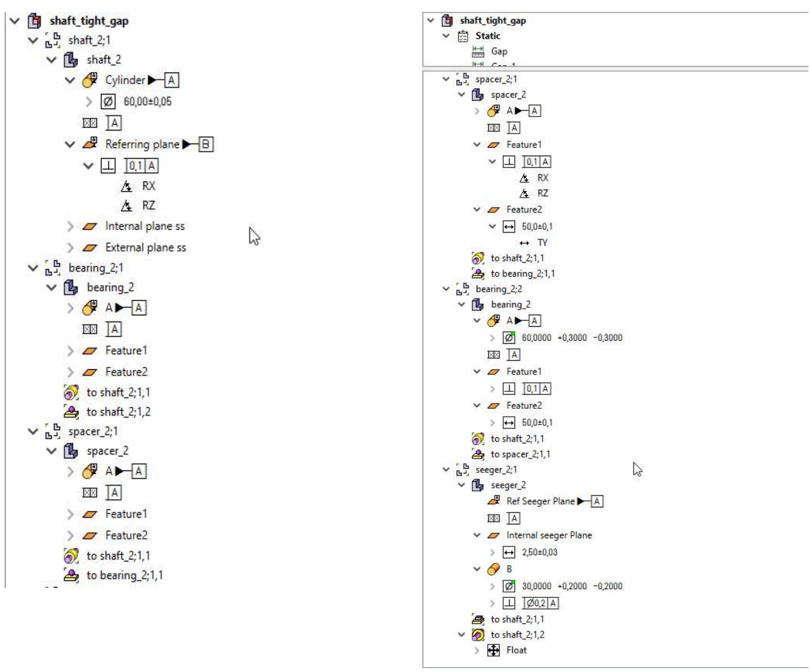


Figure 3.35 Tree View (DRF-Joints-Features-Dimensions-Geometric Tolerances-etc.)

If the PMI is not at our disposal, we will use the information present in the 2D model. Studying and analyzing this model it is possible to characterize the assembly. In the specific case, no particular information was used. Otherwise, it is necessary a direct importation of PMI for a faster and safer characterization phase. All the missing and/or non-compliant information to the ISO and ASME standards have been added manually.

"The more uncertain you are about the accuracy of the data entered into tolerance stack-up, the less certain you can be about the output, which is true of any mathematical exercise [14]."

The CAT file contains all the information useful to understand the characterization process of each component. Each indications constraint a certain number of DOVs for a given feature. When all the DOV have been constraint, each added specification becomes "redundant". In these cases, the operating way has been to constraint the feature DOVs with the tighter geometric tolerance or to specify a certain tolerances hierarchical order.

Now we have to set all the information regarded the simulation. In a 3D tolerance analysis, the following information must be chosen.

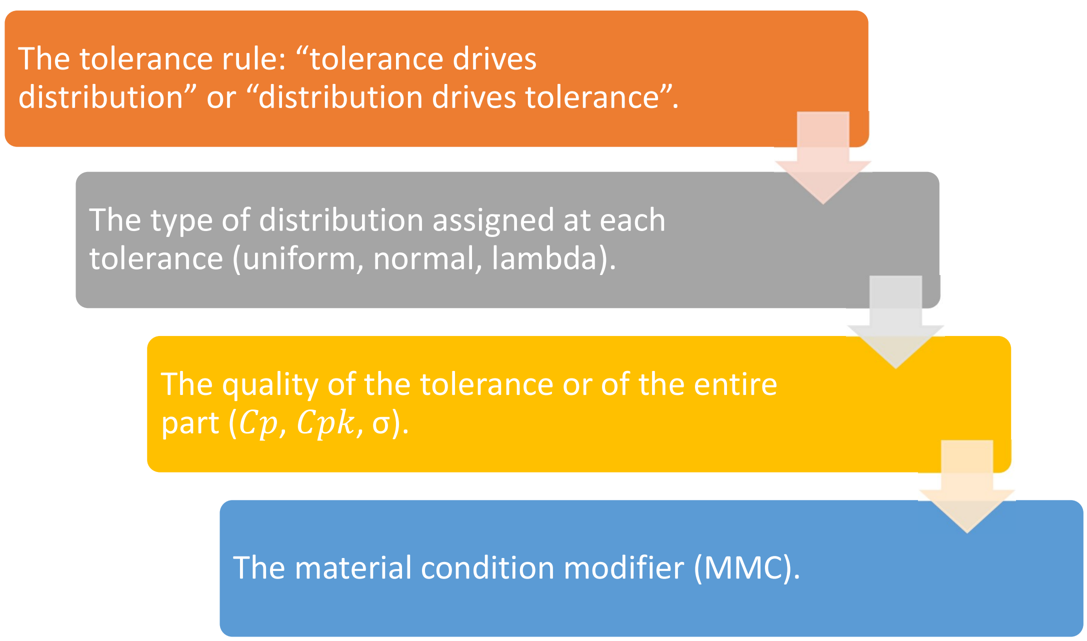


Figure 3.36 chosen information (see Figure 3.20)

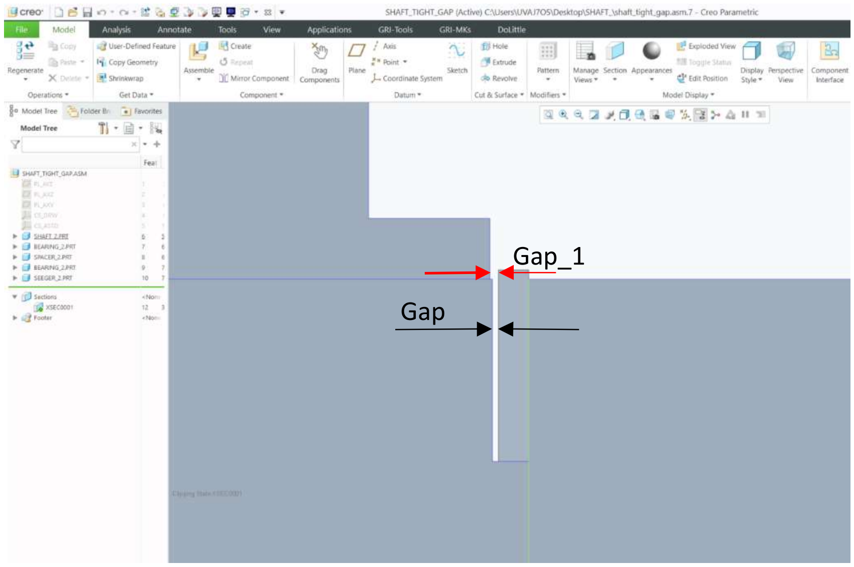


Figure 3.37 Setting measure section view

The set-up of the measures represents the most critical phase during the CAT modelling. As said, it is possible to assign different types of measurements, depending on the functional requirements but also on the mastery of the CAT tool. Indeed, often the “*gap*” measure must be substitute with a more general “linear” because the latter gives a better control of the stack-up direction and allows the positioning of the measure ends on particular point or lines. In that case it is wanted to have a gap that does not bring to the compenetrating of the parts. Moreover, we want to grant the mountability of the Seeger.

Set measures:

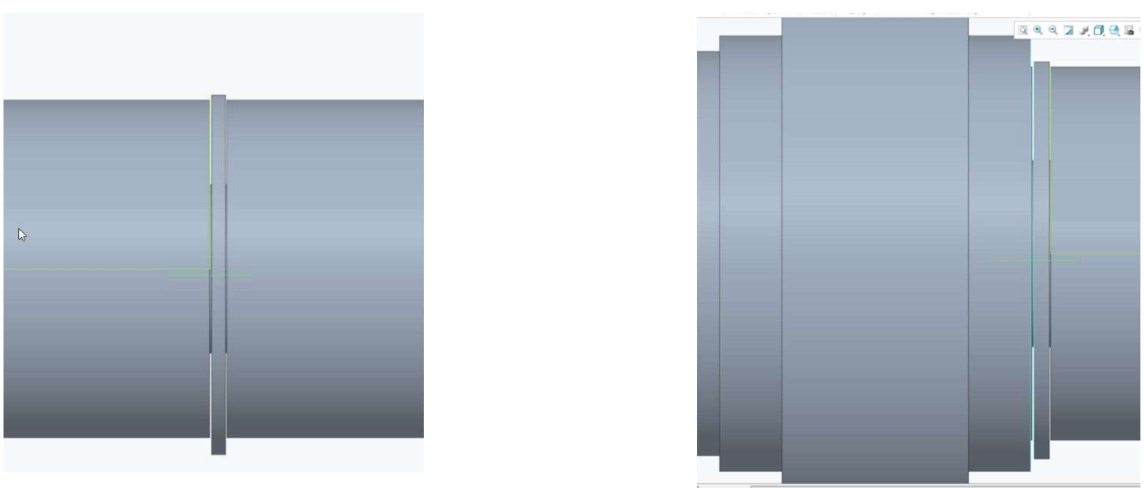


Figure 3.38 Gap from the internal plane of the Seeger seat of the shaft from to the Seeger’s plane (left); Gap_1 from the bearing’s plane to the Seeger’s plane (right)

After that we can finally set the solution method: 1st order analysis rather than 2nd order analysis in a proper control window and the simulation starts.

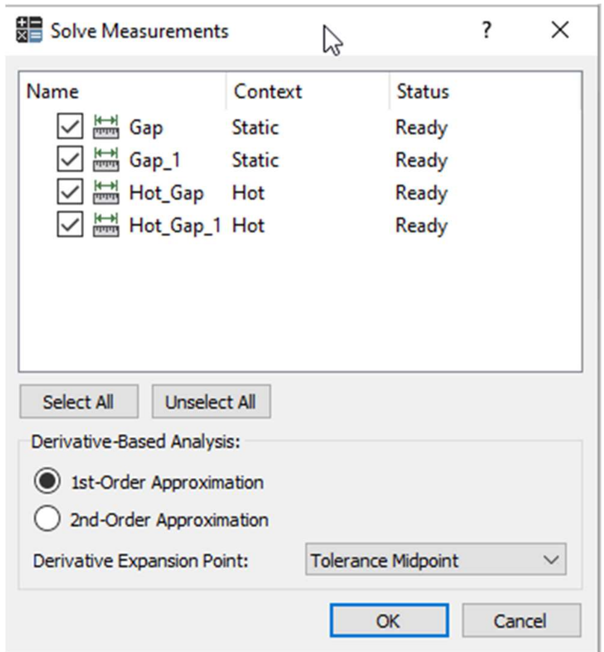


Figure 3.39 Solution method

3.7.2 Results

As said before we want to focus the attention on the Gap_1 and Hot_Gap_1 measures. In general, we must have always the possibility to assembly the Seeger. The bearing washer plane must remain at the same distance from the Seeger, granting the functionality. Generally, it is important to identify the seat position instead of modifying the Seeger, that normally is STD.

The measures Gap_1 and Hot_Gap_1 have a functional requirement *gap* $\geq 0{,}100mm$ to ensure the mountability of the Seeger and to avoid the contact between the Seeger and bearing's washer during the operating conditions. Being the nominal gap = 0,7 mm, the functional requirement was expressed defining the limits:

lower limit=0,1 mm and *upper limit*=0,950 mm

It is more common to define a symmetry tolerance gap[13]. The $c_{pk} = 1$ was applied to all the parts and tolerances included in the model therefore at the functional limits too.

A comparison with a mono-dimensional spreadsheet calculation was also made for validating the results with the CAT simulation, but also for highlighting the limits of the 1D analysis.

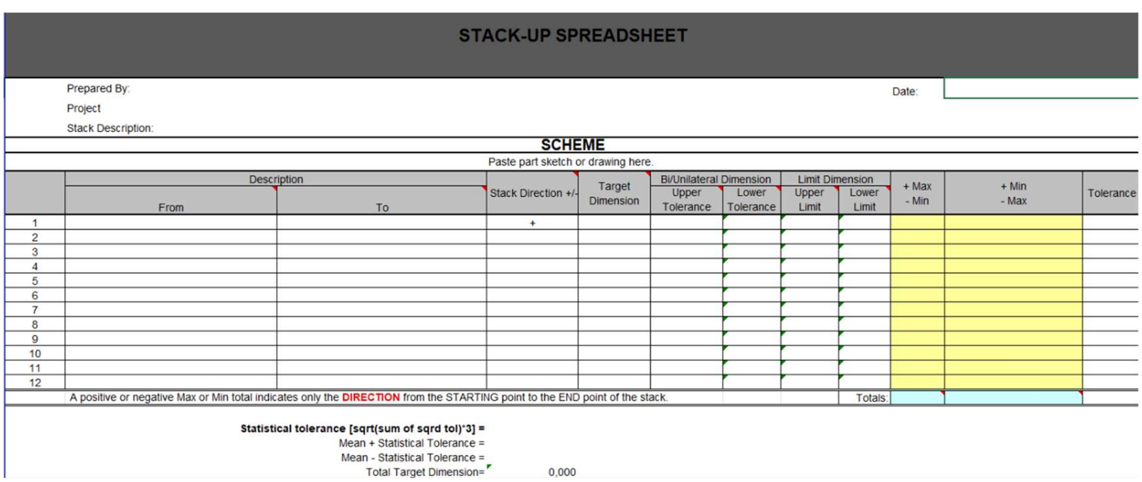


Figure 3.40 Spreadsheet view

[13] Symmetry tolerance gap: considering a functional requirement $gap \geq$ 1,000 mm, a nominal *gap*=1,500 mm, adopting the symmetric tolerance → $1\text{ mm} \leq \boldsymbol{gap} \leq 2\text{ mm}$.

According to the Fisher, tolerance stack-ups are performed referring to these hypotheses:

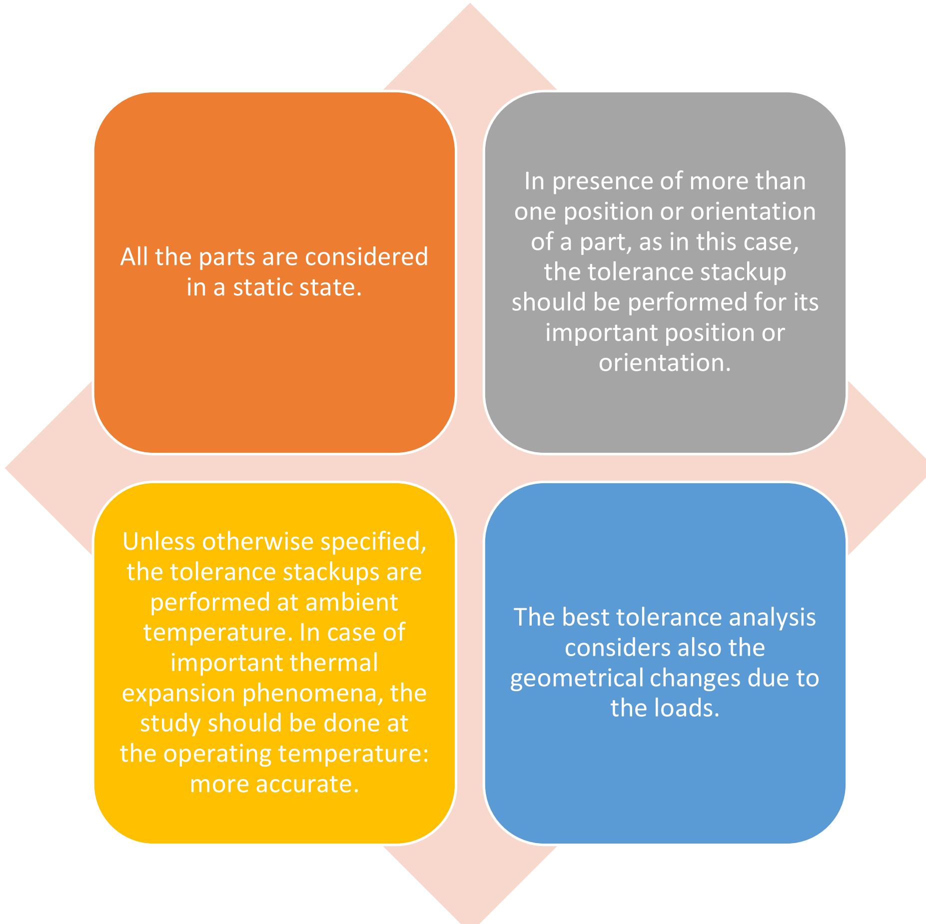


Figure 3.41 Hypothesis of the tolerance stack-up

Static Gap_1

The Cetol analyzer highlighted the main differences between the WCA and RSS. The green bar represents the WCA results of the same iteration. This means that the increase would not meet the functional requirements. The calculation according to the WCA provides that the lower limit for the WCA has a value of 0,166 mm, which respect our goal of $gap \geq 0{,}100$ mm. On the other side we can see that the WCA does not respect our functionality condition. In fact, our acceptable gap is $0{,}100 \text{ mm} \leq gap \leq 0{,}950$ mm, meanwhile WCA provides us a valor of 1,23 mm. That is actually over 0,300 mm out of tolerance. Too much to guarantee the mountability.

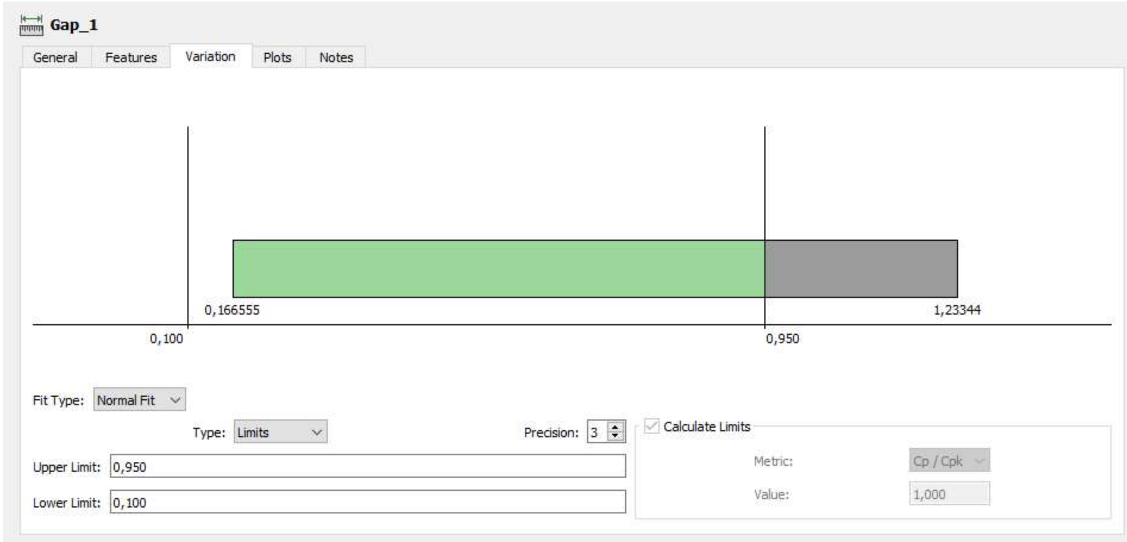


Figure 3.42 WCA results

Designing with a WC approach guarantees mostly 100% functionality of the products, and it is used when potential failures are absolutely not allowed. Its main drawback derives from the use of tolerances which in most cases are too precautionary for the real needs of the product.

This happens because it is assumed that the various dimensions of the chain are all simultaneously produced at the lower limit or at the upper specification limit, when instead the process objective is to approach the nominal value of each individual dimension. It is all a question of probability: what is the possibility, in the real world, that all the quotas in the chain are produced simultaneously at their lower or upper specification limits?

The answer is that this probability is very low and tends to decrease as the length of the tolerance chain increases. This results in high production costs to obtain the required accuracies or high number of rejects (high costs), not so much for a real need but due to an inadequate calculation method in providing a statistically reliable forecast.

Made this premise, on the other hand, we note instead that this limit is overcome by approaching it in a statistical way. We assign a statistical distribution to each variable in order to have a greater probability associated with the nominal value than the end values of the tolerance field:

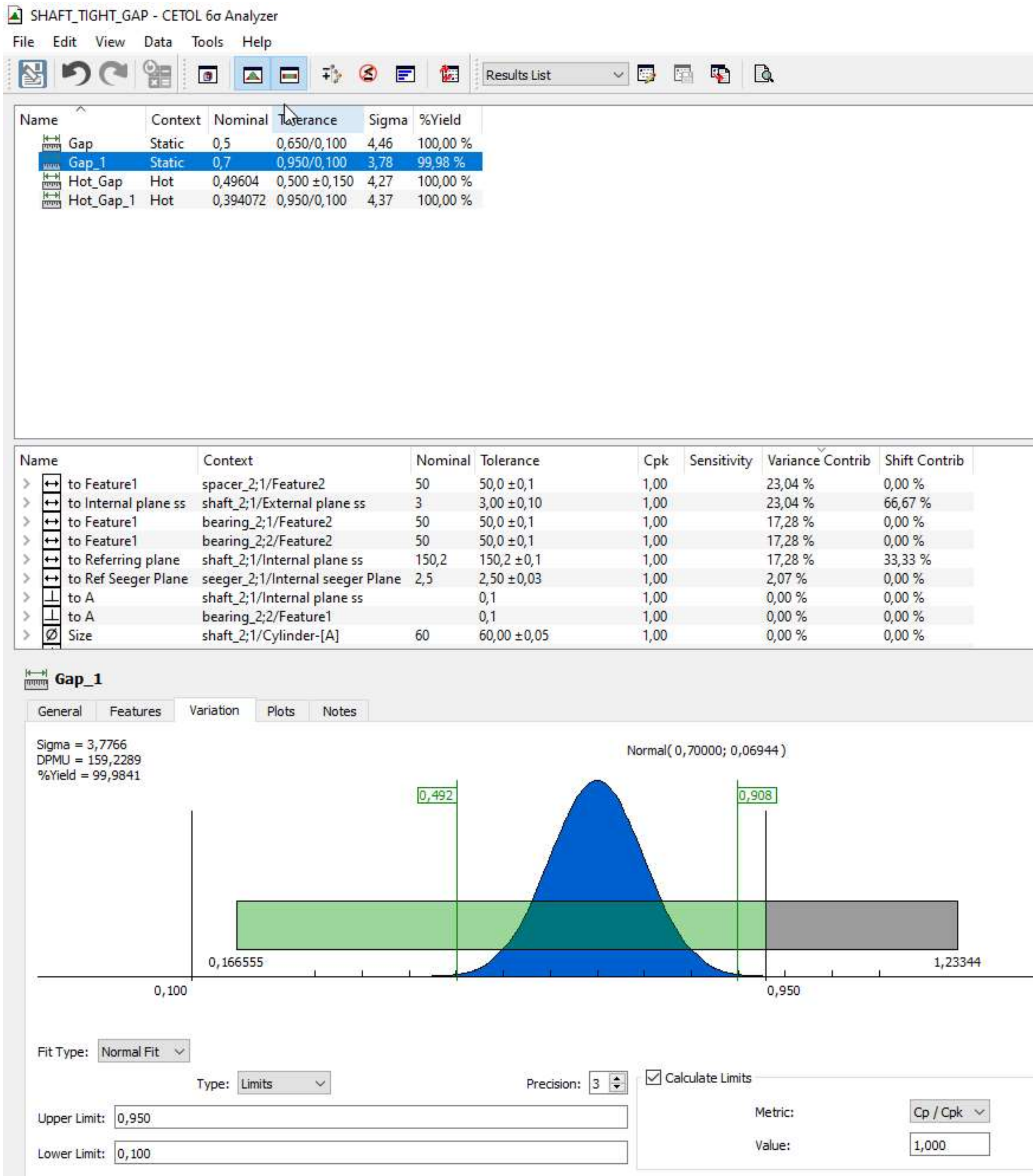


Figure 3.43 Gap_1_Results

The statistical distribution approach has led to a better solution in order to have mountability respecting the tolerances. As we can see in figure 3.43 something changed. With a statistical approach we are always in a functionality and mountability condition, we are even further from the 0,100 mm gap assumed necessary for the assembling. Moreover, on the other side, considering that the WCA is not exploitable we have a 0,908mm gap, which in any case is closer to the nominal value. Assuming the quality metric c_{pk} = 1 and $\sigma=\pm 3$ we have about 99,98% of yield[14].

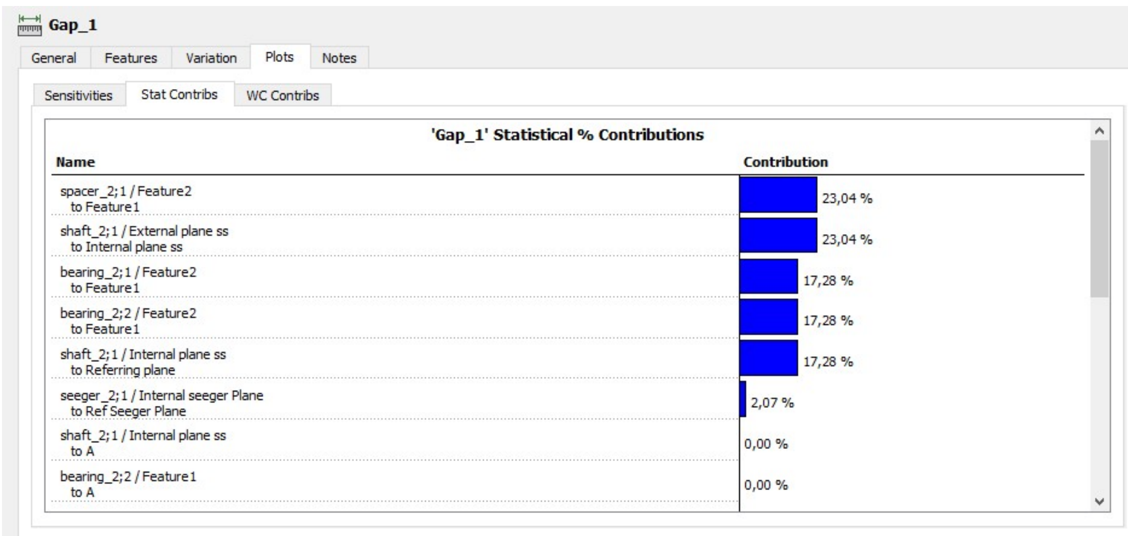


Figure 3.44 Variance contribution[15]

In that case the Seeger has no problem to be mounted on the shaft without interfering with the bearing washer. Defective units per million of units (DPMU) = 159,22. Thanks to this value we can obtain a statistical number of scraps.

Here below, Figure 3.44, the sensitivities results. CETOL 6σ solves for measurement results analysis using a derivative-based analysis, in which it calculates the sensitivity of the measurement to each variable in the model.

Sensitivities can be used to identify critical-to-quality dimensions. For a given measurement, variables with the highest sensitivity values (magnitude) are the most critical variables for that measurement. Conversely, sensitivities can also be used to identify non-

[14] Yield: the percent contribution to the mean shift (with respect to nominal) of the measurement.
[15] Variance contribution: the percent contribution to the statistical variance of the measurement.

critical dimensions. Variables with low or zero sensitivity values are not critical to the measurement. These variables are possible sources of cost savings. If all of the critical measurements in an assembly have a low sensitivity to a particular variable, it may be possible to use a low-cost process to manufacture the feature associated with that variable.

A positive sensitivity means that increasing the value of the variable increases the value of the measurement. A negative sensitivity means that increasing the value of the variable decreases the value of the measurement. The major tolerance contributors are the first two

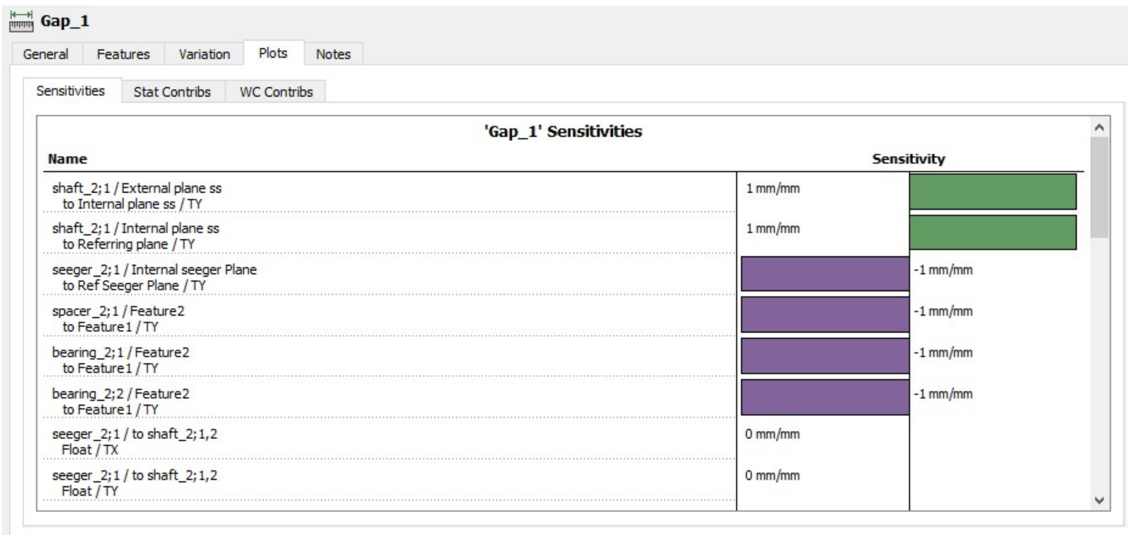


Figure 3.45 Sensitivity view

In that case it has been proposed also a comparison between the 1D analysis Vs the 3D analysis. For a relatively simple mechanical design with components all stacked in a single direction, a 1D stack-up analysis can work well. However, the most important limitation of 1D analysis is that all the geometric dimensions such as perpendicularity, parallelism, or concentricity are very difficult to represent. So, 2D or 3D stack-up analysis are the best solution when the design is really sensitive to geometric variations.

The 1D analysis has been made in WCA and RSS. Its results are similar to the one we have with the 3D because of the simple geometry. However, using the Excel was labored due to small issues occurred during the calculus, while the CAT tool has carried out the analysis in just some seconds. The actual gap with 1D approach is 0,15 mm $\leq$ *gap* $\leq$ 0,85 mm that is actually translated more to the left considering the Gaussian distribution on the 3D graph.

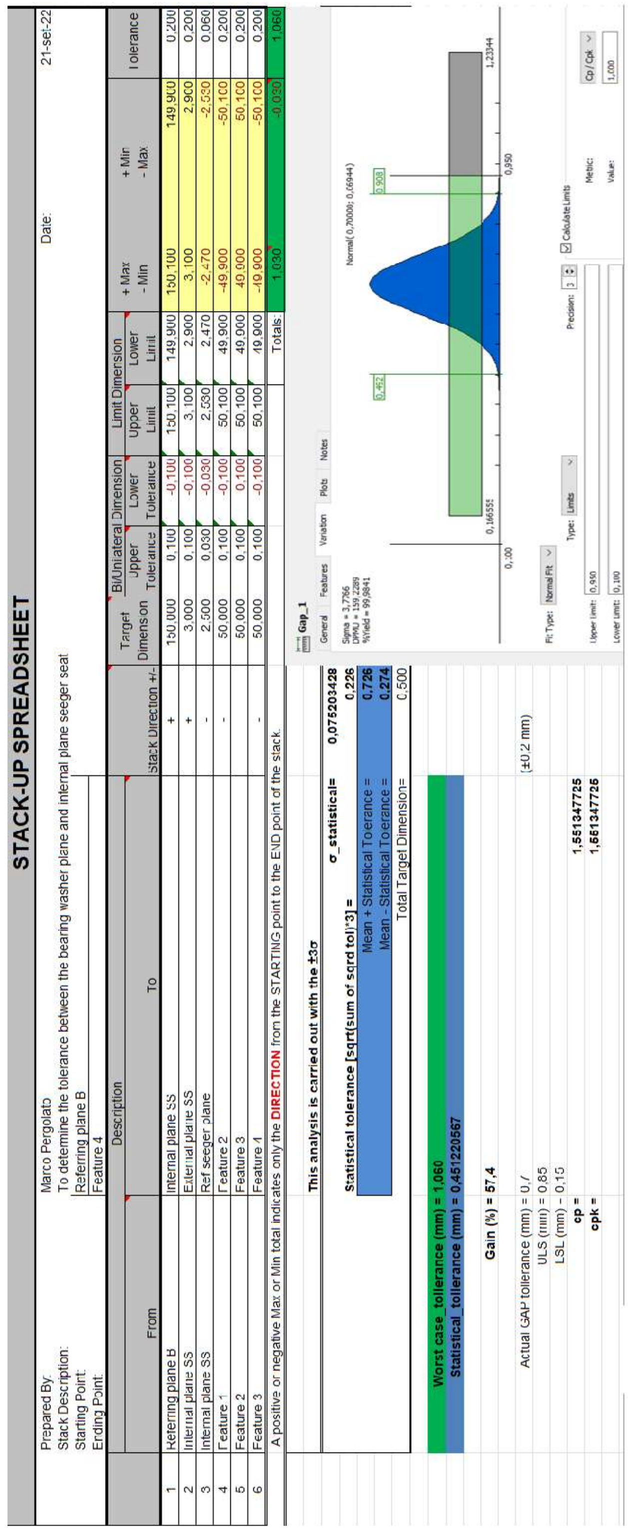


Figure 3.46 Excel vs CAT (1-D WCA and RSS)

Hot Gap_1

In that specific case, we want to ensure that, considering an operating temperature of 200°C, it is possible to mount the Seeger without compenetrating during the running.

In WC analysis we can notice that we have an opposite condition as compared to the static one. Unfortunately, we have compenetrating of the parts, in fact the lower limit in WCA is -0,141, that is actually not acceptable. Meanwhile the upper limit is ok, even if it is shifted towards the upper limit, not so far from the imposed gap boundary.

Figure 3.47 Hot_Gap_1_Results

Introducing the thermal operating conditions, with WCA it is not possible to have the correct mounting of the Seeger in its seat, so the functionality of the assembly is compromised. On the contrary a Statistical approach led again to a correct solution of the problem:

$$0{,}185 \le gap \le 0{,}603\text{mm}$$

It is actually a very good result considering the static one, in fact it shows also a 100% Yield with DPMU=12,281. There are no problems with SA and considering the nominal gap it is possible to continue reducing. Indeed, I could proceed using a lower nominal gap or using an asymmetric tolerance that is actually the same thing. To note that this kind of analysis is not possible to be done by an Excel or spreadsheet calculations as the thermal information regarding the working temperature and the material specifications (thermal expansion coefficient) could not be insert there.

The last example model wants to emphasize the potential of the statistical approach with the thermal. The ***Critical Gap*** is an important configuration to be done for investigating what would happen if we analyze only a ***Static State.***

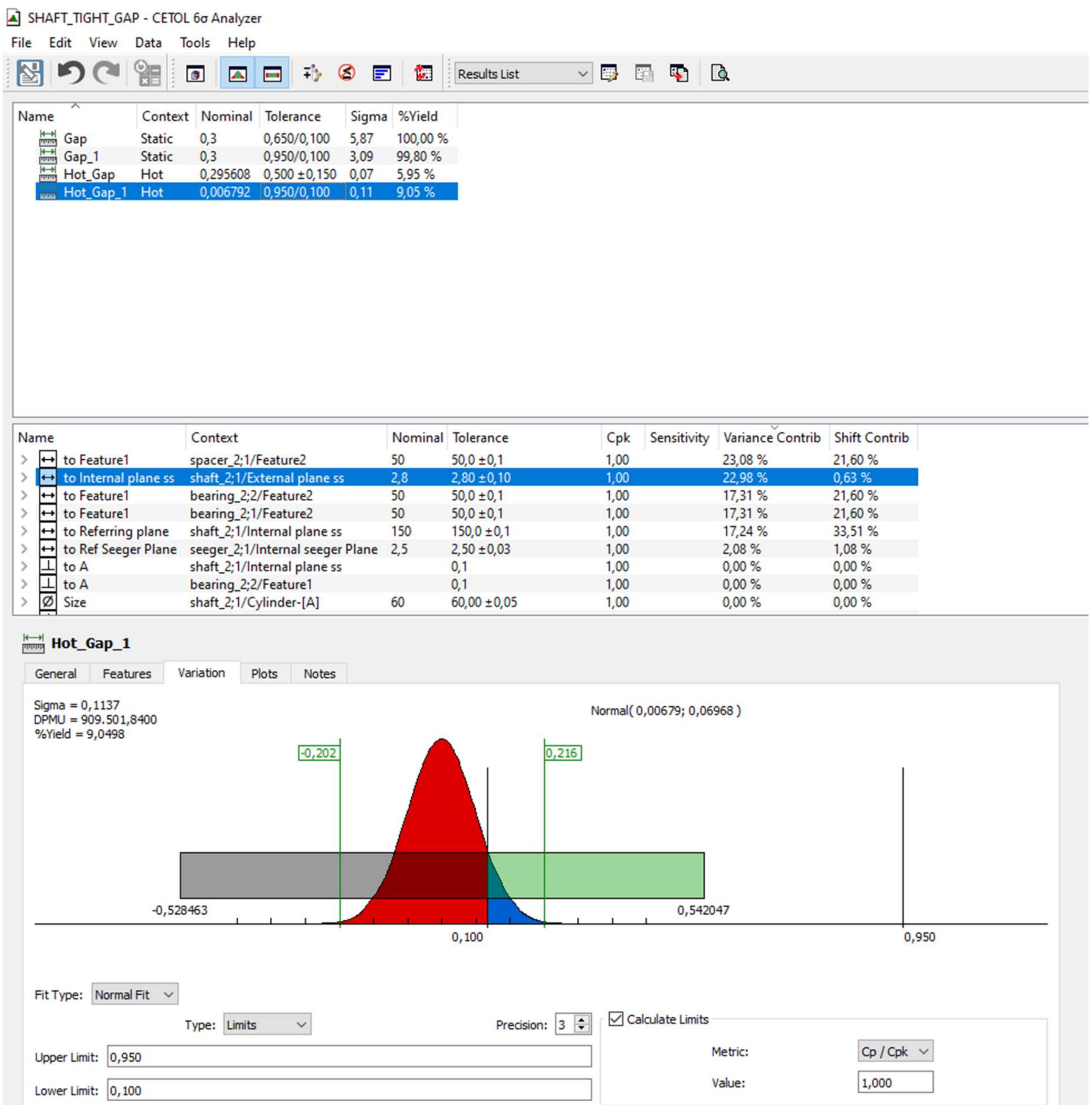


Figure 3.48 Critical Gap Hot results

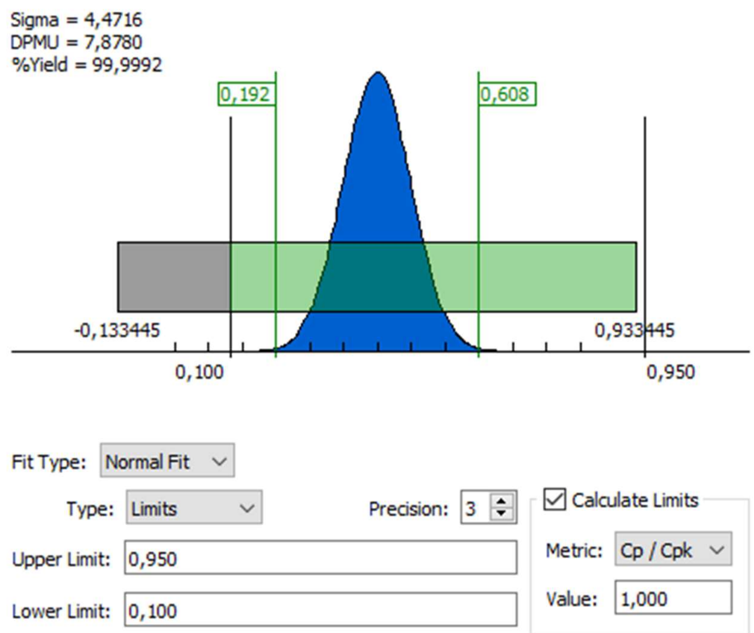


Figure 3.49 Critical Gap Static results

It could seem useless to have carried out this analysis with the thermal given the simplicity of the model, however the critical gap wants to highlight a fundamental aspect. If we had carried out the analysis exclusively in the static case, we would not have had any kind of problem, this is clear, however if a more realistic application the Seeger has to fit in the shaft in hot condition, we could have serious problems. In fact, we can see how in the hot case (which cannot be studied with the simple excel sheet) we have the interpenetration of the parts, which results in a non-assembling of the Seeger in the shaft.

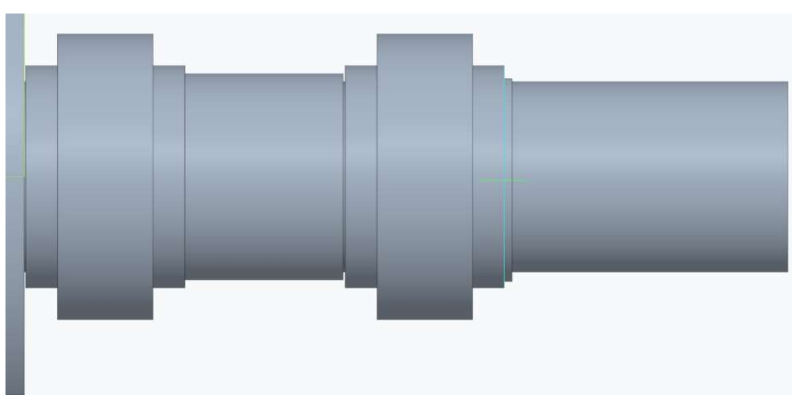


Figure 3.50 Compenetrating (figurative only)

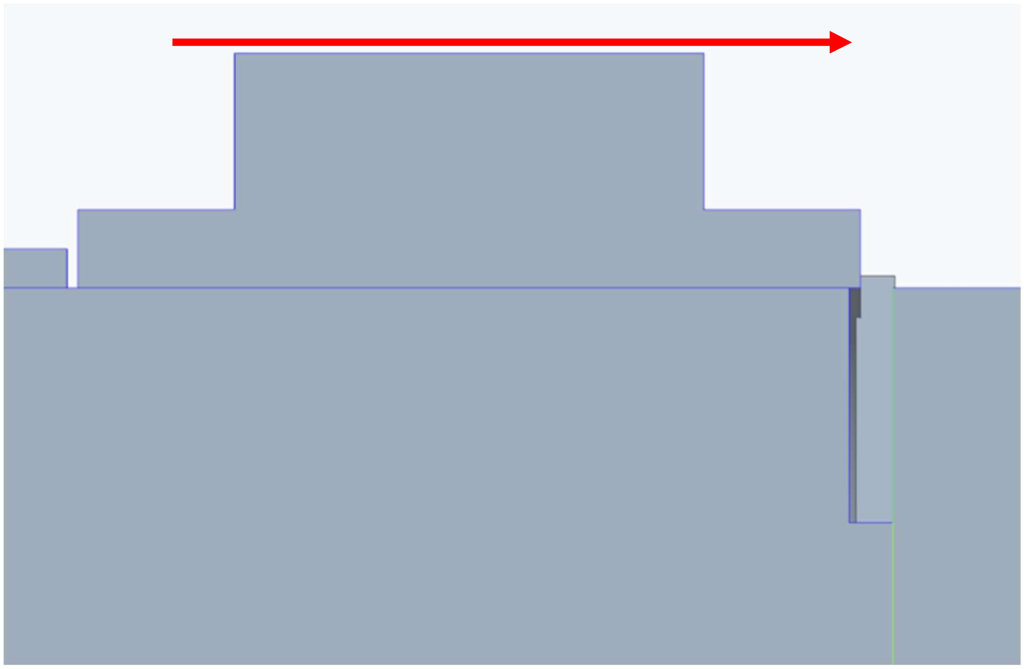


Figure 3.51 Compenetrating section view (figurative only)

4 Conclusions

It is not easy to choose the best approach to carry out a tolerance analysis. It depends on what we are trying to analyze from its complexity and the set of components that make up our system. For linear assembly relations and normal inputs, it is convenient to use the Root Sum Square. While for other scenarios, on average more complex, we move on to second order tolerance analysis or Monte Carlo method. Surely the CAT tools are adopted for simple and linear analysis, longer the chain better the analysis, even if the simplicity of the study, nowadays we prefer adopting CAT tools instead of spreadsheet.

What emerged from this study is certainly the great potential of the 3D statistical approach with the aid of CAD/CAT software that guarantees 3 main results:

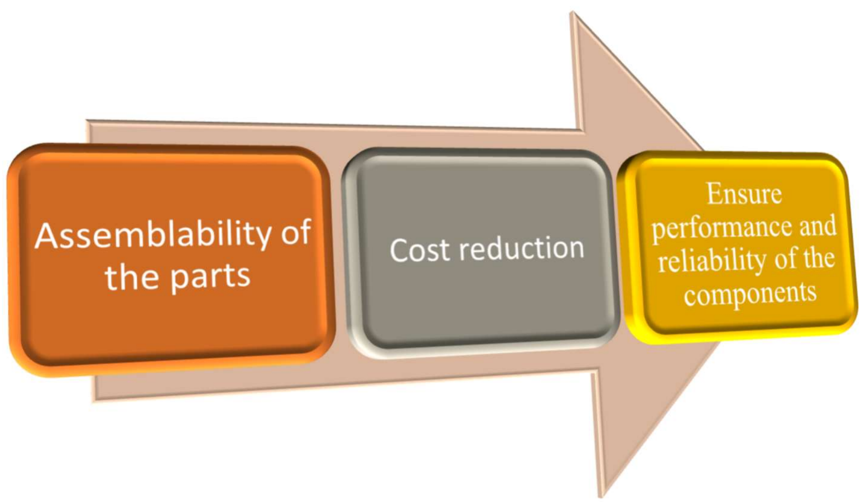


Figure 4 Obtained results.

However, this thesis wants to differentiate itself from previous studies also for having introduced thermal analysis which makes the overall model much closer to reality. In fact, excellent results can be obtained by introducing information that in a 1D analysis would make the system complex and slow in its execution, but which neglecting this information would still not give true results, only the thermal specs but even the geometric tolerances that are integral part of the model.

The three examples mentioned above are not real case studies, but they clarify that in a world where the faster you made a product the more you save, where if a product has a better quality the more you achieve the assemblability, the CAT tools represent a valid instrument for the three-dimensional statistical tolerance analysis in which each operation is expressed through parameters, i.e. numbers and mathematical formulas. These parameters (dimensional and geometrical) give rise to constraints, or relations (which are, in fact, "binding") between the parts: to put it more simply, they connect the parts together and set restrictions.

The differences with the classic approach (consisting in manual or via spreadsheet calculations) are concrete.

From now on, the use of a model based approach, and therefore of statistical 3D analysis, will certainly be increasingly used, because of the obtaining in cost reduction considering that it is possible to save time and to get quickly to the mountability of the assembly, even using a large tolerance that in the first phases of the product design were set and therefore a less effective cost due to the non-use of specific technologies. The best use we can do of this analysis is for economies of scale, where the cost plays a fundamental role and where oversizing is not allowed.

In the future surely, we could have a new integrated environment, where all the necessary information will be unified together, simplifying a lot more the process of developing and producing product.

Appendices

Appendix A

Abbreviations

CMM = coordinate measuring machine

GD&T= geometrical dimensioning and tolerancing

GPS = geometrical product specification

LMB = least material boundary

LMC = least material condition

LMR = least material requirement

LMS = least material size

LMVC = least material virtual condition

LMVS = least material virtual size

MCC = minimum circumscribed circle

MMB = maximum material boundary

MMC =maximum material condition

MMR = maximum material requirement

MMS = maximum material size

MMVC = maximum material virtual condition

MMVS = maximum material virtual size

RFS = regardless of feature size

SDT = small displacements torsor

RSS = roots sum square

SOTA = second-order tolerance analysis

TED = theoretically exact dimension

TP = true position, theoretically exact position or location

Appendix B

Standards

ISO 286-1	ISO System of limits and fits: bases of tolerances, deviations and fits
ISO 286-2	ISO System of limits and fits: tables of standard tolerance classes and limit deviations for holes and shafts
ISO 1101	Geometrical tolerancing
ISO 1660	Dimensioning and tolerancing of profiles
ISO 2768-2	General geometrical tolerances
ISO 2692	Maximum material requirement, least material requirement, reciprocity requirement
ISO 3040	Dimensioning and tolerancing of cones
ISO 4291	Methods for the assessment of departures from roundness: measurement of variations in radius
ISO 4292	Methods for the assessment of departures from roundness: measurement by two- and three-point methods
ISO 5458	Geometrical tolerancing: positional tolerancing
ISO 5459	Datums and datum systems for geometrical tolerancing
ISO TR 5460	Geometrical tolerancing: tolerancing of form, orientation, location and run-out; verification principles and methods; guidelines
ISO 6318	Measurement of roundness: terms, definitions and parameters of roundness
ISO 7083	Symbols for geometrical tolerancing: proportions and dimensions
ISO 8015	Fundamental tolerancing principle
ISO 10 360-1	Coordinate metrology Part 1 definitions and applications of the fundamental geometric principles
ISO 10 578	Projected tolerance zone
ISO 10 579	Dimensioning and tolerancing: non-rigid parts
ISO 12 181	Measurement of roundness deviations

ISO 12 180	Measurement of cylindricity deviations
ISO 12 780	Measurement of straightness deviations
ISO 12 781	Measurement of flatness deviations
ISO 13 715	Edges of undefined shape
ISO 14 253-1	GPS, decision rules for proving conformance
ISO 14 253-2	GPS, guide to the estimation of uncertainty (in preparation)
ISO TR 14 638	GPS Masterplan
ISO 14 660-1	GPS, geometrical features; general terms and definitions
ISO 14 660-2	Extracted median line, median surface, local size
ISO 15 530-1	CMM, determining measurement uncertainty, overview
ISO 15 530-2	CMM, determining measurement uncertainty, use of multiple measurement strategies
ISO 15 530-3	CMM, determining measurement uncertainty, use of calibrated workpieces
ISO 15 530-4	CMM, determining measurement uncertainty, use of computer simulation
ISO 15 530-5	CMM, determining measurement uncertainty, use of expert judgement
ISO TR 16 570	Linear and angular dimensioning and tolerancing: +/- limit specifications – step dimensions, distances, angular sizes and radii
ISO TS 17 450-1	Model for GPS, features, characteristics, operation, specification, verification
ISO TS 17 450-2	Operators and uncertainties
ASME Y14.5	Dimensioning and tolerancing
ANSI B89.3.1–1972	Measurement of out-of-roundness
DIN 4760	Form deviation, waviness, surface roughness; system of order, terms and definitions
DIN 6784	Edges of workpieces; terms, drawing indications
DIN 7167	Relationship between dimensional tolerances and form and parallelism tolerances; envelope requirement without indication

DIN 7184	Geometrical tolerances; definitions and drawing indications (superseded by DIN ISO 1101)
DIN 7186 T.1	Statistical tolerancing; definitions, applications, drawing indications
DIN 8570 T.3	General geometrical tolerances for welded parts
DIN 40680 T.2	General form tolerances for ceramic parts in electrical application
DIN 32 880-1	Coordinate measuring technique; geometrical basics and terms
VDI/VDE 2601 T.1	Requirements on the surface structure to cover function capability of surfaces manufactured by cutting; list of parameters
TGL 39 092	Methods of measuring geometrical deviations: general principles
TGL 39 093	Methods of measuring deviations from straightness
TGL 39 094	Methods of measuring deviations from flatness
TGL 39 095	Methods of measuring deviations from parallelism
TGL 39 096	Methods of measuring deviations from roundness
TGL 39 097	Methods of measuring deviations from cylindricity
TGL 39 098	Methods of measuring deviations of the longitudinal section profile
TGL 43 041	Methods of measuring straightness deviations of axes
TGL 43 042	Methods of measuring deviations from coaxiality
TGL 43 043	Methods of measuring the radial run-out deviations
TGL 43 044	Methods of measuring the axial run-out deviations
TGL 43 045	Methods of measuring the run-out deviations in a given direction
TGL 43 529	Methods of measuring the radial total run-out deviations

TGL 43 530	Methods of measuring the axial total run-out deviations
ST RGW 301-76	Geometrical tolerances: fundamental terms
ST RGW 368-76	Geometrical tolerances: drawing indications

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