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Distributed optimization methods for cooperative Beamforming in satellite communications

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Contents

Abstract

This thesis analyzes various Beamforming techniques and theories of space information networking (SIN), with the aim of merging and using them in two real applications casted as optimization problems and solved in a distributed fashion. We propose the distributed optimization algorithm known as Dual Subgradient Method to solve two different problems linked to satellites and Beamforming. The first one shows a cluster of satellites that performs collaborative Beamforming to reach an Earth user, while reducing interference in secondary directions. Whitin the second problem, we consider an application for hybrid satellite-terrestrial relay networks (HSTRNs), where multiple geostationary satellites transmit signals to multiple Earth terminals, with the help of multiple single-antenna relays. Moreover, we provide for both scenarios numerical simulations showing the effectiveness of the proposed solutions.

Introduction

In a world where the demand for wireless communication services is constantly increasing, devices and techniques that allow in increasing the transmission capability, while reducing interferences to and from other devices, are really needed.
Beamforming is a versatile and powerful approach to receive, transmit, or relay signals of interest in a spatially selective way in the presence of interferences and noise [1]. It is a spatial filtering technique that allows in concentrating signals along a desired direction, while lowering disturbance due to interference and noise in other directions. Beamforming is a field of interest for a lot of different applications, like radars, sonars, communications, biomedicine, radio astronomy, seismology and others. On the communication end, demand for high data transmission and the number of users are constantly increasing, and so techniques like Beamforming are used to increase the channel capacity, making it possible to communicate with multiple users on the same frequency, without having to serve each user using different time slots. Unluckily, performing Beamforming on extensive systems is computationally hard, and thus powerful computers are needed.
There are multiple ways in which we can obtain Beamforming, such as with reflector multi-beam antennas, lens multi-beam antennas and array multi-beam antennas. The first and second ones are widely used these days, because they are easy to implement. Recently, array multi-beam antennas are more attractive due to their high aperture efficiency and no leakage loss. Moreover, they can explore digital Beamforming techniques, which are very flexible for reconstructing beams for different situations. In a lot of situations, direct communication between transmitter and receiver is impossible because of obstacles or because they are too far apart. In this scenario comes in handy the use of relays, antennas that stay between the source and the

users and can receive the signal from the source, amplify it and transmit it forward to the users. Also in this scenario there exist some Beamforming techniques, capable of increasing the gain of the desired signal while diminishing the other ones.
In the last few decades, the usage of satellites for communication purpose has undergone an huge growth, with a considerable variety of applications, e.g. space-to-space, hybrid Earth-to-space and also Earth-space-Earth communications. Also the number of services that nowadays rely on satellites has been increasing, in particular to link places on Earth which would be unreachable via other ways. Downside to satellites are their higher maintenance issues, fixed orbit, limited complexity and power capability.
To overcome the high computing demand for Beamforming and the limited capacity of satellites, a distributed approach comes in handy. A distributed approach allows in dividing the computational load of Beamforming over each agent of the system, splitting the problem into easier to solve subproblems, such that each agent has a simpler problem than the original one. Also a well implemented distributed approach allows the system to be highly scalable, with each agent needing knowledge of a limited portion of the global framework.

Literature

A collection of notions about Beamforming and convex optimization-based Beamforming can be found in [1], where receive, transmit and relay network Beamforming are presented and analyzed. They also report convex formulations for optimization problems based on all three cases. Then in [2] it is proposed a novel Virtual Beamforming (VBF) concept: instead of a single array multi-antennas to perform Beamforming, multiple single antennas collaborate to perform Beamforming. Also some key technologies are investigated for VBF, as in [3], [4], [5], where different methods to manage difficulties and complications useful to implement VBF are analyzed.
About satellite communications and networks, in [6] there are considerations about in-orbit resource allocation for a SIN backbone based on optical intersatellite links, that can be useful research topic for satellite systems. Then in [7] is presented a scenario where collaborative Beamforming is performed

by a satellite cluster: a Mixed Integer LP optimization problem is proposed to find the optimal formation of satellite that have to collaborate to perform Beamforming. In [8], [9] [10] theories and methods to study and implement transmit Beamforming via analysis of the vector manifold of the signal are presented.
A distributed approach for terrestrial relay networks is proposed in [11], where multiple clusters of terrestrial single antennas interfere one with each others, in a scenario where source-user pairs communicate through a set of single antenna Amplify and Forward (AF) relays. In [12] a similar scenario where one single satellite needs to communicate to a terrestrial user via a set of single antenna relays AF, which have to perform Beamforming to reach the user while avoiding interferences to other terrestrial antennas.
In [13] are proposed different algorithms to solve optimization problem in a distributed fashion: the Distributed Dual Sub Gradient Algorithm presented in [13] will be used in this thesis to solve two different optimization problems related to Beamforming and satellite communications.

Contributions

In this thesis we study and develop two different distributed algorithms associated to optimization problems of two different scenarios of satellite Beamforming. We start from the problem presented in [7]: a huge cluster of single antenna satellites has to find the optimal formation of a sub-set of satellites to perform Beamforming to reach a terrestrial user. We reformulate it as follows: a smaller cluster of single antenna satellites that have to perform all together cooperative Beamforming to reach a terrestrial user. Then we develop, starting from the centralized optimization problem of [7] a distributed solution via Distributed Dual Subgradient Algorithm ([13]).
Then, starting from an optimization problem solved in a distributed way in [11] for a terrestrial relay network, we extend it to a scenario where the sources are satellite antennas instead of terrestrial single antennas, similar to scenario presented in [12]. Then we propose a distributed solution using a different method as that presented in [11].

Organization

In Chapter 1 different kinds of Beamforming techniques are analyzed, with a focus on transmitting and relays Beamforming, because these are the ones used in the aforementioned problems that we worked on.
Then in Chapter 2 we study the different usages of satellites for communication purposes, the capability they have and other characteristics, such that we can better understand how a distributed approach can be used and what benefit it has to offer.
In Chapter 3 we present the Distributed Dual Subgradient Algorithm and its convergence properties.
Chapter 4 presents in detail the first problem. Then a Distributed Dual Subgradient Algorithm is proposed and numerical simulation are performed to verify the correctness and validity of the approach.
Finally in Chapter 5 we proposed a distributed approach to solve an optimization problem related to a hybrid satellite terrestrial scenario where satellite sources and terrestrial antennas collaborate by performing Beamforming to reach terrestrial users. After the implementation of a Distributed Dual Subgradient Algorithm, we perform simulation and numerical analysis to validate our results.

Chapter 1

Beamforming Techniques

This chapter is dedicated to the study of different Beamforming techniques, with the focus on those based on multi-antenna arrays. In order we analyze Beamforming at receiver ends, then transmission Beamforming and finally how to perform Beamforming via an antenna relays network.

1.1 Receive Beamforming

Receive Beamforming is a spatial filtering technique that allows to select a desired direction and boost up signals coming from that direction, while at the same time weakening interference from all other directions. Usually this is done via a multi-antenna array, which is composed by multiple single receiving antennas precisely spaced and located that allow to spatial filtering all incoming radiation, to improve the gain of the desired signal and reduce or nullify gain of unwanted signals.
In Fig. 1.1 from [2] we can see an example of a receiving multi-antenna array. Here, a set of receivers are spatially distributed with a fixed distance between them. We assume that the origin of signals is sufficiently far away with respect to the distancing between each receiver such that we can consider that the front wave of the signal is a straight line. In this scenario, depending on the position of the origin, each receiver would perceive the signal at different subsequent time steps. Referring to the two dimensional examples of Fig. 1.1, we can see that the receiver on the left will be the first to perceive the signal, then the one on its right and so on till the last one of the array. The dash line represents the front wave of the signal: as it is

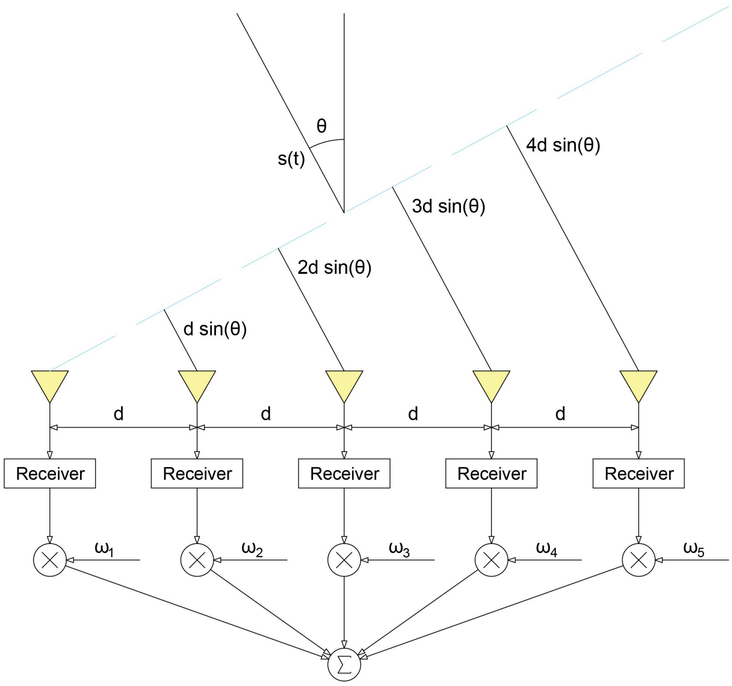
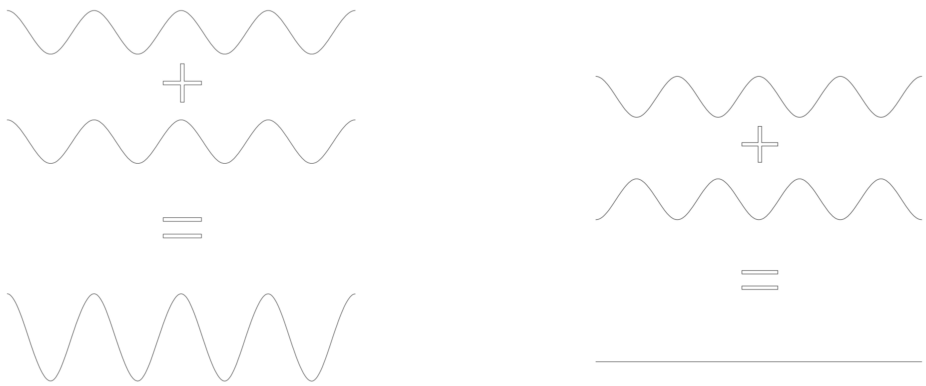


Figure 1.1: *Multi-antenna array receiver: the flat front wave of the signal reaches each receiver at a different time, depending on their position [2].*

a straight line and the receivers are separated by a fixed known distance, we may compute the time delay Δ between the arrival of signal in two subsequent receivers. As we can see in the figure, contribution of each node is then summed together, so if we add the necessary delay to each N node, we can obtain the summation of N times the signal making a constructive interference thanks to the delay, increasing its gain (Fig.1.2 a), while other signals arriving from other directions will in the best case nullified via a destructive interference (Fig.1.2 b), or in the worst case they will be just with nominal or reduced gain.

In the direction we want to receive the signal, by means of this spatial filtering technique, we define the main lobe, the set of direction that will benefit most from the constructive interference described above (see Fig. 1.3). Other than the main lobe, there will be a side lobe, a secondary direction in which the signal is weakened but not nullified. We have to notice also that between two consecutive lobes the gain is zero or almost zero: this direction is called nulls, because the signal is in perfect destructive



(a) Constructive Interference

(b) Destructive Interference

Figure 1.2: *Example of possible addition of the same signal with same phase (a) or in anti-phase (b).*

interference. A technique used in Beamforming, is to exploit this nulls to ignore certain well known disturbance by directing this nulls at it.
A single antenna is omni-directional, which means that it will receive signals from all directions in the same way. With just two antennas, it is possible to generate a very wide main lobe with few smaller side lobes. This means that it is possible to receive signals from the desired direction very well, but a lot of signals coming from directions near the desired one will be still considered with almost the same gain as the main one. By increasing the number of antennas the number of side lobe will increase, while the main lobe will become narrower and with also an increased gain with respect to other side lobes. This make possible a distinction of the main signal, which will have a greater gain, with all other disturbance signals coming to the side lobes and nulls.
Steering the beam is an operation that consists in changing the direction of the main lobe, directing it to the source, or to direct some interferences to the nulls.
Receive array multi-antenna Beamforming is very useful nowadays due to the high number of antennas and signal exchanged every second all over the world, which generate high noise in each communication. This technique permits to listen only in the desired direction, filtering all unwanted signals and also is versatile, because by changing the delay we can also change the listening direction, via steering the main lobe towards the new desired direction.

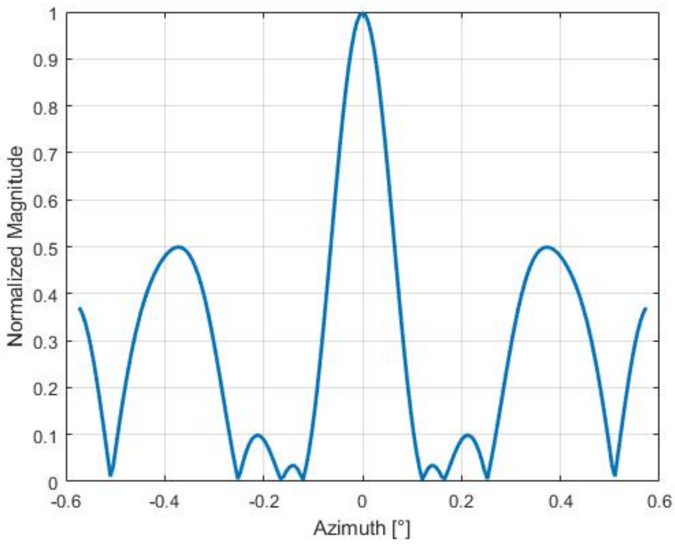


Figure 1.3: *Depending on the number of antennas, we can see how the main lobe will be even narrower and more stretched while the number of side lobes will increase.*

1.2 Transmission Beamforming

As for receive Beamforming, the concept of transmit Beamforming with a multi-antenna array is that we can combine multiple instance of the same signal to exploit constructive and destructive interference to send the signal only in the desired direction with increased gain. This works in the opposite way of the receive Beamforming: considering the two dimensional case, we have an array of single antennas, each of which can send the same signal independently from the others. By emitting the signal from each transmitter with a time delay correlated with the distance between two consecutive antennas and the direction in which we want to send the signal, it is possible to generate a unique beam in that direction (Fig.1.4).
As for the receiver case, depending on the number of antennas in the array, we can generate multiple side lobes and a narrower main lobe (Fig.1.5).
Transmitting multi-antenna Beamforming is very useful nowadays as it allows the same transmitter to use the same channel to communicate with different receivers, just changing the delay of each antenna makes it possible to steer the main lobe to the desired direction. The only information we need is the relative position of transmitter and receiver.

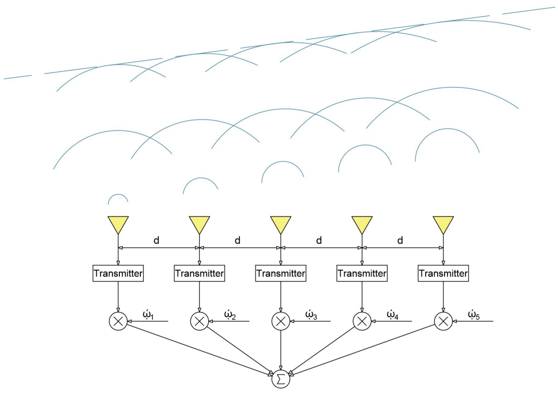
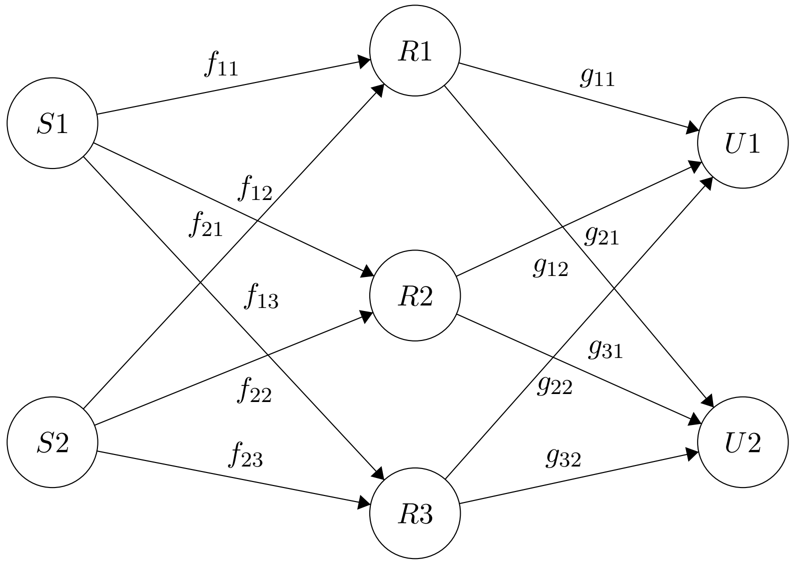


Figure 1.4: *Array antenna transmitter: the flat front wave of the signal is generated with multiple instance of the signal, generated with certain time delay.*

To improve this approach even further, one can simultaneously use Beamforming on both receiving and transmitting ends, to benefit from this approach doubly.

1.3 Relay Network Beamforming

Sometimes wireless communications cannot stand on just a source destination pair, because there could be obstacles between them or they could be too far apart. Also, sometimes a receiver and a transmitter can be made of just one single antenna, and not an array of it. Cooperative approaches for wireless communications have the potential for significant performance improvement, such as extended coverage of the network, throughput enhancement and energy savings. With a network made of single antenna source and user, we can add single antenna relays to create a virtual array multi-antenna (made of all relays) to perform Beamforming. In the graph below we can see an example of disposition of sources, relays and users.



The graph represents simultaneous communication between 2 sources and 2 destinations with the help of 3 relays. The signal transmitted from source 1 (S1) is intended for user 1 (U1), while the signal transmitted from source 2 (S2) is intended for user 2 (U2). Signals from S1 and S2 that reach U2 and U1, respectively, are considered interference. Also, the figure shows the channel gains between sources and relays, and between relays and users.
In detail, each relay receives signals from all directions indiscriminately, then it amplifies it and transmits it to all directions again. This because as previously said in this scenario we have just a single antenna, and so transmission and receiving occur without spatial filtering. Also, sources will send the signal to all relays that can receive it and destination will perceive all signals that can perceive. With a certain control over the value with which each relay multiplies the received signal, we can perform Beamforming, considering the totality of relays as an array multi-antenna. At the end of the day, each destination can perceive the desired signal better, limiting interference and noise via destructive interference effect. A difficulty that occurs in network Beamforming is that relays can hardly exchange information about their received signals, so that it will require a distributed approach to realize the Beamforming. In general, a network will require some knowledge of Channel State Information (CSI), to better adapt itself to system needs.

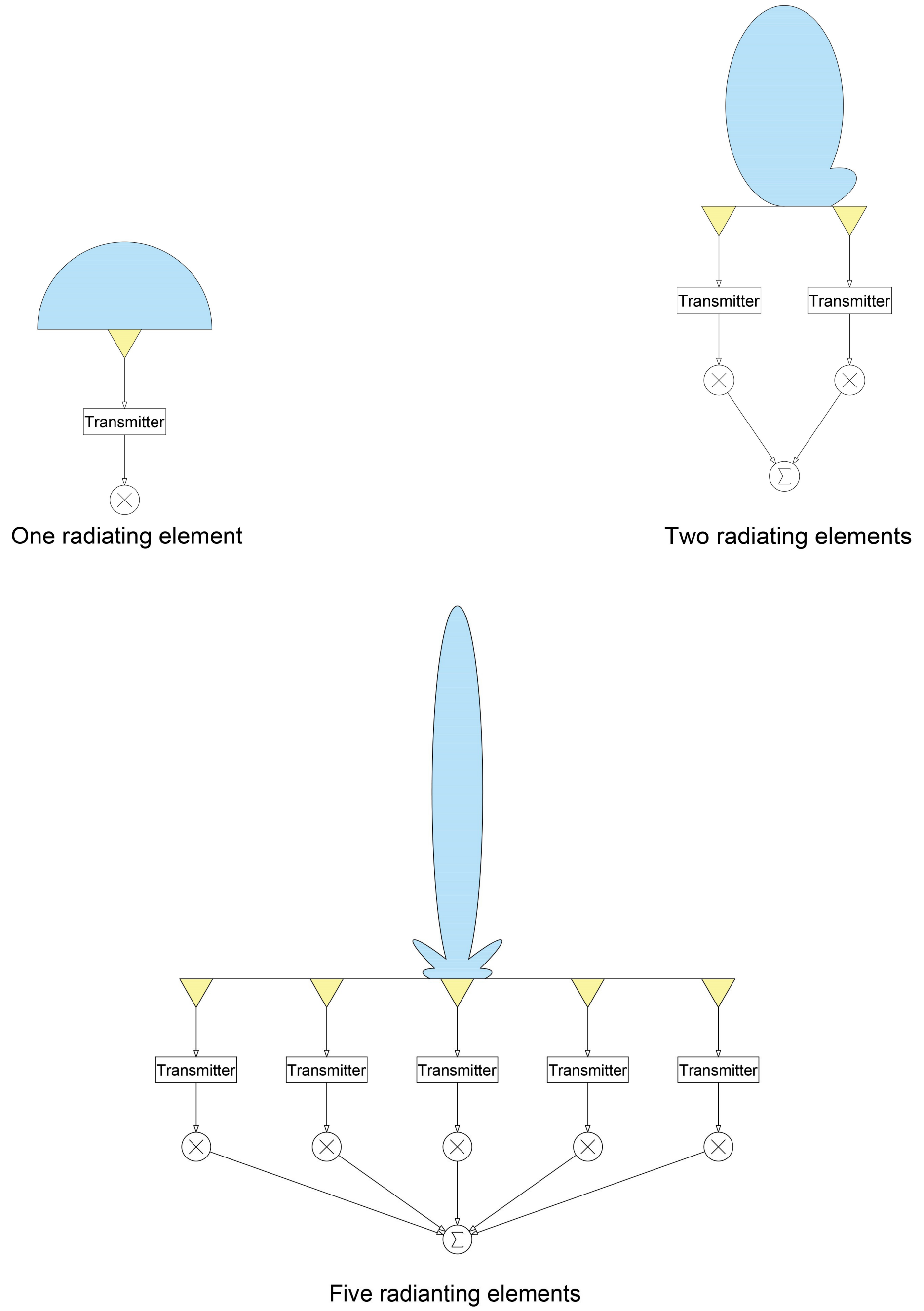


Figure 1.5: *An explanatory and simplified representation of lobe depending on number of antennas in a transmitting Beamforming scenario. This is very analogous to receiving Beamforming, with main and side lobe and nulls representing the direction in which the signal is transmitted respectively stronger, weaker and not transmitted at all.*

Chapter 2

Space Information Networking

In this chapter we want to introduce some basic theories of Space Information Networking (SIN). A SIN is a network able to achieve real-time acquisition, transmission and processing of space information. Generally speaking, a SIN is a network that uses different space platforms such as Geostationary Earth Orbit (GEO) satellites, Middle Earth Orbit (MEO) satellites, Low Earth Orbit (LEO) satellites, stratospheric balloons, manned and unmanned aircrafts and more. A SIN includes a satellite-terrestrial network and a deep-space network. Thanks to the satellite network around the Earth, global seamless communication becomes possible.

A SIN can comprehend different types of Distributed Satellite Systems (DSS), each one with different configurations and functions. Usually a DSS is composed by multiple satellites of the same type.

- Constellations: A constellation is a DSS where all satellites are distributed along the same orbit, usually equally spaced. Constellations main focus is on coverage, like for GPS service and internet connection;
- Clusters: Close formation of satellites that fly together in a geometrical formation, used for interferometry;
- Swarms: A huge number of small satellites with no specific topology that work together, usage of this DSS are an active research field;
- Trains: Few satellites that fly in a single line, close to each other, used for synergistic measurementes;

- Fractionated: Small groups of different satellites, with different functionalities, that work in a distributed fashion, with eventually a master satellite and some smaller slaves.

We will focus on Distributed Satellite Cluster (DSC), that is the formation we consider in one of the main topics of this thesis. The core research field about DSC can be divided in three aspects according to [2]: dynamic and optimum networking, high data rate transmission and multi-dimension information fusion.

- Dynamic and Optimum Networking: the mutual position of the satellites varies rapidly and required services are various. We need an highly dynamic graph theory, which is a novel topic for researchers.

- High data rate transmission: the topology of the network is variable in the time-space dimensions, hence this is a challenge to traditional information theory.

- Multi-dimensional information fusion studies how to efficiently and optimally acquire, process, share and apply multi-dimensional information.

For our purpose, we want to focus on a specific technique, Beamforming, to increase the data rate transmission, applied to DSC. There already exist single satellites that can perform Beamforming on their own, but it is also possible to perform Beamforming via a DSC of single antenna satellites that work together: this is called Virtual Beamforming, and we account for benefits and weaknesses in the following sections.

2.1 Virtual Beamforming

Virtual Beamforming (VBF) is a special case of array multi-antennas Beamforming. The concept is to use multiple antennas (i.e. multiple satellite) as a unique multi-beam antenna. We assume there are M users to be served and that the electromagnetic power from each antenna can cover all M users. If a traditional Spot Beam system (high gain antenna that emit a signal that covers a limited region on Earth) is considered, we must use different time slots (different frequencies) for each users. But if we also want to use a single carrier frequency, we cannot serve all the users simultaneously. The VBF can do this. The increasing capacity of VBF with respect to SB is at the expense of the bit error rate (BER) loss. When BER exceeds the threshold, extra power is used. The idea of VBF is taken from multi-user MIMO transmission theory, used in fourth and fifth generation mobile communications. The differences are for CSI (Channel State Information), antenna design and satellite's feature (such that the long distance and time delay between satellites and users). In [2] we can see the introduction of four key techniques for virtual multi-Beamforming: Channel Capacity theoretical Analysis, Opportunistic Beamforming, Multi-Beamforming theory for DSC and Resource Management for DSC. Each one is a suggested topic to explore in order to perform VBF at its best.

2.1.1 Channel Capacity Theoretical Analysis

The sum capacities of the AWGN (additive white Gaussian noise), Rayleigh and Rician channels have been deeply studied, but there is no theoretical analysis on the channel capacity of the multi-user virtual multi Beamforming system. Three main difficulties can be taken into account.

First, the VBF is a complex and hybrid system, which is a complicated multipoint-to-multipoint transmission system. This require knowledge from both classical information theory and network theory.

Then we should consider that the channel links between satellite to terrestrial and satellite to satellite have a long time delay between each node, and so it is hard to obtain the real-time CSI of the link. Moreover, satellites move at high speed, hence the Beamforming and the link to a terrestrial user can terminate due to the motion of the satellites.

Finally, serviced users are really heterogeneous, and so are the capacity and features of their antennas. This differentiation must be take into account.

All these problems must be considered while elaborating a mathematical model of the Virtual Multi-Beamforming.

2.1.2 Opportunistic Beamforming

Usually the channel model between satellite and a terrestrial user is viewed as an AWGN or Rician channel, hence with small fluctuations. According to [3], it is known that multi-user diversity depends on the rate and dynamic range of the channel fluctuations. Therefore in this scenario it seems hard to achieve large channel capacity due to small fluctuations of the channels. Prof. Tse in [3] showed that opportunistic Beamforming is an attractive technique to obtain multi-user diversity, even in this condition. To design valuable opportunistic Beamforming algorithms, we can draw inspiration from recently popular research, that is, the concept of the power domain introduced by non-orthogonal multiple access (NOMA) and so on. To implement opportunistic Beamforming we should consider developing a proper mathematical model for satellites, taking into account that a satellite has strict conditions that the design must accomplish, such as energy limitation and volume limitation. Also we must avoid Multiple-Access Interference (MAI), due to transmission to multiple users via the same frequency resource. To construct Multi-User MIMO transmission, satellites need to know the CSI of each user, but as previously said, it is hard to obtain such information in real-time and also we have low feedback and long time delays. In [4], a compound strategy that uses one bit feedback for multi-user diversity has been proposed.

Opportunistic Beamforming Using Dumb Antennas

Here we analyze article [3], to find useful information for virtual multi-Beamforming. Wireless channel suffers of fading due to constructive and destructive interference between multi-paths. To overcome this problem we need to have diversity, obtained over time, frequency and space. The basic idea is to improve performance by creating several independent signal paths between the transmitter and the receiver. These diversity modes pertain to a point-to-point link. Recent results point to another form of diversity, inherent in a wireless network with multiple users. This multi-user diversity is what we need for multi-Beamforming. This article suggests that to obtain this channel diversity, we can randomize the fading and the variation of each channel, such that the signal transmitted and received increase in variability, and thus improving the SNR (signal-to-noise-ratio).

The paper proposes a scheme that induces random fading when the environment has little scattering and/or the fading is slow. Multiple antennas at the base station are used to transmit the same signal from each antenna modulated by a gain whose phase and magnitude changes in time in a controlled but pseudorandom fashion. The gain in the different antennas varies independently. Channel variation is induced through the constructive and destructive addition of signal paths from the multiple transmit antennas to the receive antenna of each user. The overall SINR is tracked by each user and is fed back to the base station to form a basis for scheduling. The channel tracking is done via a single pilot signal which is repeated at the different transmit antennas, just like the data. Then transmit Beamforming can be performed by matching the powers and phases of the signal sent on the antennas to the channel gains in order to maximize the received SNR at the user.

If we consider an even more limited feedback of only the overall channel SNR, true Beamforming cannot be achieved. Hence, in a large system with many independently fading users, we can consider an Opportunistic Beamforming, considering that there will be likely a user whose instantaneous channel gains are close to matching the current power and phases allocated at the transmit antennas: the transmit power and phase are randomized and transmission is scheduled to the user which is close to being in the Beamforming configuration.

2.1.3 Multi-Beamforming Theory for DSC

As is known, channel capacity is related to the correlation among different antennas. The smaller is the correlation, the greater is the channel capacity. Therefore, if the antennas are located in different satellites, better channel capacity can be obtained due to the reduced correlation. It should be noted that for traditional multi-beam array antennas, the correlation between antennas is used for Beamforming, in opposition from MIMO.

2.1.4 Resource Management for DSC

For the DSC, there are different kind of resources, such as power, spectrum, time, space, antenna and orbit resources. Resource allocation is also one of the key technologies to satisfy end-to-end Quality of Service requirements at an acceptable cost. How to manage and allocate these resources is an interesting research topic. In [5] it is proposed an algorithm able to dynamically allocate bandwidth for SIN. In [6] there are consideration about in-orbit resource allocation for a SIN backbone based on optical intersatellite links.

Dynamic Allocate Bandwidth for SIN

In [5] we found an analysis on how to improve the overall QoS of the Space Information Network. In a multi user scenario we can divide the users in two classes: Quality of Service (QoS) and Best Effort (BE). The former is associated to delay sensitive applications while the latter is insensitive to delay. In this paper a method of dynamic deprivable bandwidth allocation (DDBA) is proposed for special delay-sensitive applications. With this method highest delay dependant application can deprive the bandwidth which has been allocated to the lower delay dependant application. All these consideration take into account also the Bit Error Rate, that in these high demanding applications needs to remain low.

Satellite network topologies

As noted in [6], one major challenge faced by all satellites is the bottleneck in information relay from space to ground. To overcome the shortcomings of expansive bandwidth on radio frequency uplinks and downlinks, the concept of networked space-borne processing is introduced for data reduction and compression and to increase the value of space-based assets. The space-based information network considered in the article will serve mostly space users, such as sensors in space, space shuttles or communication satellites themselves. The design of this network will be different from a terrestrial and airborne network, because its requirements are different: different data types, different quality of service and location of space users, i.e. in which orbit they are in. To provide high-speed space-to-space communications between space-based assets and networked processing resources, the inter-satellite backbone is built using laser communications as the enabling technology. Laser communication systems operating at optical frequencies allow the use of small antennas due to the narrow beamwidths and thus reducing the power demand. Moreover, a single optical communication system can transmit up to terabits per second of information.
The design goal of this research is to find an optimized topology for the satellite constellation to serve all user demands with minimum cost and best QoS. Different topologies and different setups for the network are analyzed, with a different number of satellites per each scenario.

Chapter 3

Distributed Dual Subgradient Algorithm

In this Chapter we will report the Distributed Dual Subgradient Algorithm and the assumption needed to apply it as presented in [13]. We start by considering a constraint-coupled problem

$$\begin{aligned} \min_{\mathbf{x}_1,\ldots,\mathbf{x}_N} \quad & \sum_{i=1}^{N} f_i(\mathbf{x}_i) \\ \text{subj. to} \quad & \mathbf{x}_i \in X_i, \qquad\qquad i \in \{1,\ldots,N\} \\ & \sum_{i=1}^{N} \mathbf{h}_i(\mathbf{x}_i) = \mathbf{0}, \\ & \sum_{i=1}^{N} \mathbf{g}_i(\mathbf{x}_i) \leq \mathbf{0}, \end{aligned} \tag{3.1}$$

where $\mathbf{x}_i \in \mathbf{R}^{d_i}$, $(\mathbf{x}_1,\ldots,\mathbf{x}_N)$ is the global optimization vector stacking all the local variables, $X_i \subseteq \mathbb{R}^{d_i}$, $f_i : \mathbb{R}^{d_i} \to \mathbb{R}$, $\mathbf{h}_i : \mathbb{R}^{d_i} \to \mathbb{R}^M$ and $\mathbf{g}_i : \mathbb{R}^{d_i} \to \mathbb{R}^S$ are known by agent i only, for all $i \in \{1,\ldots,N\}$.
To derive the dual problem of (3.1), let us introduce two multiplier $\boldsymbol{\lambda} \in \mathbb{R}^M, \boldsymbol{\mu} \in \mathbb{R}^S$ associated to the coupling constraints $\sum_{i=1}^{N} \mathbf{h}_i(\mathbf{x}_i) = \mathbf{0}$ and $\sum_{i=1}^{N} \mathbf{g}_i(\mathbf{x}_i) \leq \mathbf{0}$ respectively. Thus, the Lagrangian is as follows

$$\mathcal{L}(\mathbf{x}_1,\ldots,\mathbf{x}_N,\boldsymbol{\lambda},\boldsymbol{\mu}) = \sum_{i=1}^{N} \Big(f_i(\mathbf{x}_i) + \boldsymbol{\lambda}^T \mathbf{h}_i(\mathbf{x}_i) + \boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i) \Big), \tag{3.2}$$

The dual problem of (3.1) is

$$\max_{\boldsymbol{\mu}\geq\mathbf{0}} q(\boldsymbol{\lambda},\boldsymbol{\mu}) = \max_{\boldsymbol{\mu}\geq\mathbf{0}} \sum_{i=1}^{N} q_i(\boldsymbol{\lambda},\boldsymbol{\mu}), \tag{3.3}$$

where the i-th term q_i of the dual function q is defined as

$$q_i(\boldsymbol{\lambda},\boldsymbol{\mu}) = \min_{\mathbf{x}_i\in X_i} f_i(\mathbf{x}_i) + \boldsymbol{\lambda}^T\mathbf{h}_i(\mathbf{x}_i) + \boldsymbol{\mu}^T\mathbf{g}_i(\mathbf{x}_i), \tag{3.4}$$

It is easy to see that (3.3) is a cost-soupled problem. We consider N agents in a network communicating as a connected, fixed and undirected graph, which aim to cooperatively solve a constraint-coupled problem 3.1 satisfying assumption A.1. The latter part of A.1 is Slater's constraint qualification and ensures that strong duality holds.
We recall that each agent i aims to compute only its portion $\mathbf{x}_i^*$ of the entire optimal solution $(\mathbf{x}1i^*,\dots,\mathbf{x}_N^*)$.
Then it is possible to apply a subgradiend method to the maximization of problem (3.3) that reads

$$\begin{aligned}
\boldsymbol{\lambda}_i^{t+1} &= \boldsymbol{\lambda}_i^t + \gamma^t\tilde{\nabla}q_i(\boldsymbol{\lambda}^t),\\
&= \boldsymbol{\lambda}^t + \gamma^t\sum_{i=1}^{N}\tilde{\nabla}q_i(\boldsymbol{\lambda}^t),\\
\boldsymbol{\mu}^{t+1} &= \mathcal{P}_{\boldsymbol{\mu}_i\geq\mathbf{0}}\Bigg(\boldsymbol{\mu}^t + \gamma^t\tilde{\nabla}q(\boldsymbol{\mu}^t)\Bigg),\\
&= \mathcal{P}_{\boldsymbol{\mu}_i\geq\mathbf{0}}\Bigg(\boldsymbol{\mu}^t + \gamma^t\sum_{i=1}^{N}\tilde{\nabla}q_i(\boldsymbol{\mu}^t)\Bigg),
\end{aligned} \tag{3.5}$$

A subgradient of q_i at $\boldsymbol{\lambda}^t$ and $\boldsymbol{\mu}^t$ can be computed by evaluating the dualized constraints $\mathbf{h}_i$, $\mathbf{g}_i$ at the minimizer of the Lagrangian, i.e.,

$$\mathbf{x}_i^t = \underset{\mathbf{x}_i\in X_i}{\text{argmin}} f_i(\mathbf{x}_i) + (\boldsymbol{\lambda}^t)^T\mathbf{h}_i(\mathbf{x}_i) + (\boldsymbol{\mu}^t)^T\mathbf{g}_i(\mathbf{x}_i), \tag{3.6}$$

In the following, we describe the distributed dual subgradient algorithm. Each node i maintains a local dual variable estimate $\boldsymbol{\lambda}_i^t$, $\boldsymbol{\mu}_i^t$ that is iteratively updated according to a distributed subgradient iteration described by (3.11), and a local primal variable $\mathbf{x}_i^t$, computed by minimizing the i-th term of the Lagrangian as in (3.10). Nodes initialize their local dual variables $\boldsymbol{\lambda}_i^t$, $\boldsymbol{\mu}_i^t$ to any vector in the positive orthant. The algorithm 1 formally summarizes the distributed dual subgradient algorithm for a constraint-coupled optimization problem (from the perspective of agent i). Being the following algorithm a distributed subgradient method, the usual convergence properties discussed in Chapter 2 of [13] apply.

Theorem. *Let Assumption A.1 hold. Let the communication graph be undirected and connected with weights a_{ij} satisfying Assumption A.2 and let the step-size γ^t satisfy Assumption A.3. Then, the sequence of dual variables $\{\boldsymbol{\lambda}_1^t, \dots, \boldsymbol{\lambda}_N^t\}_{t\geq 0}$, $\{\boldsymbol{\mu}_1^t, \dots, \boldsymbol{\mu}_N^t\}_{t\geq 0}$ generated by Algorithm 5 satisfies*

$$\begin{aligned} &\lim_{t\to\infty} \big|\big|\boldsymbol{\mu}_i^t - \boldsymbol{\mu}^*\big|\big| \quad i \in \{1, \dots, N\}, \\ &\lim_{t\to\infty} \big|\big|\boldsymbol{\lambda}_i^t - \boldsymbol{\lambda}^*\big|\big| \quad i \in \{1, \dots, N\}, \end{aligned} \tag{3.7}$$

where $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}^*$ are optimal solutions of problem (3.3)*, the dual of problem* (3.1)*. Moreover, let the sequence $\{\hat{\mathbf{x}}_i^t\}_{t\geq 0}$ be defined as $\hat{\mathbf{x}}_i^t = 1/t \sum_{\tau=0}^{t} \mathbf{x}_i^\tau$, for all t. Then, it holds*

$$\begin{aligned} &\lim_{t\to\infty} \sum_{i=1}^{N} f_i(\hat{\mathbf{x}}_i^t) = f^* \\ &\lim_{t\to\infty} \big|\big|\hat{\mathbf{x}}_i^t - \mathbf{x}^*\big|\big| = 0 \quad i \in \{1, \dots, N\}, \end{aligned} \tag{3.8}$$

where $\mathbf{x}^*$ and f^* denote an optimal solution and the optimal cost of problem (3.1), respectively.

Algorithm 1: Distributed Dual Subgradient Algorithm

Initialization: $\boldsymbol{\lambda}_i^0$, $\boldsymbol{\mu}_i^0 \geq 0$,

Evolution: for t=0,1,...

Gather $\boldsymbol{\lambda}_j^t$, $\boldsymbol{\mu}_j^t$, for $j \in \mathcal{N}_i$

Compute

$$\begin{aligned} \mathbf{v}_{\boldsymbol{\lambda},i}^{t+1} &= a_{ii}\boldsymbol{\lambda}_i^t + \sum_{j\in\mathcal{N}_i} a_{ij}\boldsymbol{\lambda}_j^t, \\ \mathbf{v}_{\boldsymbol{\mu},i}^{t+1} &= a_{ii}\boldsymbol{\mu}_i^t + \sum_{j\in\mathcal{N}_i} a_{ij}\boldsymbol{\mu}_j^t, \end{aligned} \tag{3.9}$$

$$\mathbf{x}_i^{t+1} \in \underset{\mathbf{x_i}\in X_i}{\operatorname{argmin}} f_i(\mathbf{x}_i) + \mathbf{v}_{\boldsymbol{\lambda},i}^{t+1\,T}\mathbf{h}_i(\mathbf{x}_i) + \mathbf{v}_{\boldsymbol{\mu},i}^{t+1\,T}\mathbf{g}_i(\mathbf{x}_i), \tag{3.10}$$

Update

$$\begin{aligned} \boldsymbol{\lambda}_i^{t+1} &= \boldsymbol{\lambda}_i^t + \gamma^t\mathbf{h}_i(\mathbf{x}_i^{t+1}), \\ \boldsymbol{\mu}_i^{t+1} &= \mathcal{P}_{\boldsymbol{\mu}_i\geq 0}\left(\boldsymbol{\mu}_i^t + \gamma^t\mathbf{g}_i(\mathbf{x}_i^{t+1})\right). \end{aligned} \tag{3.11}$$

Chapter 4

Beamforming for a Distributed Satellite Cluster

As already defined in Chapter 2, a satellite cluster is a set of satellites that fly close one to each other in a fixed formation on the same orbit. In [7] it is presented a collaborative approach to Beamforming, where a cluster of single antenna satellites are used to perform together Beamforming, working as a virtual multi-antenna array. This approach allows huge flexibility and fault resistance, together with an increased capability of channel transmission. We start by analyzing the optimization problem presented in the article and then, we will define a similar problem solvable in a distributed fashion.

4.1 Scenario

In the article [7] it is presented a scenario in which hundreds of mono antenna satellites fly in a fixed formation, evenly distributed inside a volume with side of 600 meters. The cluster is on a Geostationary Earth Orbit (GEO), and the objective is to perform Beamforming, using some satellites of the cluster to create a virtual multi-antenna array and then perform transmitting Beamforming. One of the main advantages of this setup is that due to Beamforming itself, it is possible to have multiple signals over the same frequencies overlapping, and thanks to the little interference we can have on unwanted directions, multiple beamformed signals can be active at the same time, even with sources close to each others. Another advantage of this method is that thanks to the quantity of satellites, if one of them stops

working, another one can take its place quickly, thus increasing the reliability of the system.
Starting from the scenario presented in the article, we will consider a slightly different one: instead of a huge number of satellites, with just a few actually utilized, we consider a smaller satellite cluster, where all the satellites work together to perform Beamforming. This will resemble a single array multi-antenna, and we can refers to it as a Virtual Large Aperture Array, as addressed previously. This VLAA made up by a Distributed Satellites Cluster, can perform Beamforming on its own, as a single satellite mounted with an array multi-antenna, but benefits from the distributed composition. First of all smaller satellites are cheaper than a single more complex one, and then, having more satellites than the minimum needed to perform Beamforming, a DSC increases the redundancy of the system and so the reliability: if some of the satellites of the cluster stop working, the distributed system is able to keep on its duty. Also, when talking about Space Information Networks, maintainability and updatability are two key factors: acting on such remote devices is really hard, and if you have to replace or repair a satellite, you have to interrupt its services for a while, but if you have an entire cluster of satellites, you can act on some of them, while keeping the others delivering their services.

4.2 Mathematical Model

A graphic representation of the model for a Distributed Satellite Cluster with N satellites is shown in Fig. 4.1, as presented in [7]. Information of the fixed location for the n^{th} satellite is $P_n(r_n, \theta_n, \phi_n)$, where $\theta_n \in [0, \pi], \phi_n \in [0, 2\pi]$ respectively represent the Elevation and Azimuth angles. The instantaneous actual location information is $P'_n(r'_n, \theta'_n, \phi'_n)$, because it is assumed that each satellite due to perturbation is random located around the fixed location inside a sphere of radius B. The desired direction of the beam that we want to form is denoted by $P_0(r, \theta_0, \phi_0)$.
We need to make some practical consideration about the mathematical assumption that arises in this context:

- All satellites in the cluster use the same type of antenna, and the array pattern function conforms to the pattern multiplication theorem;

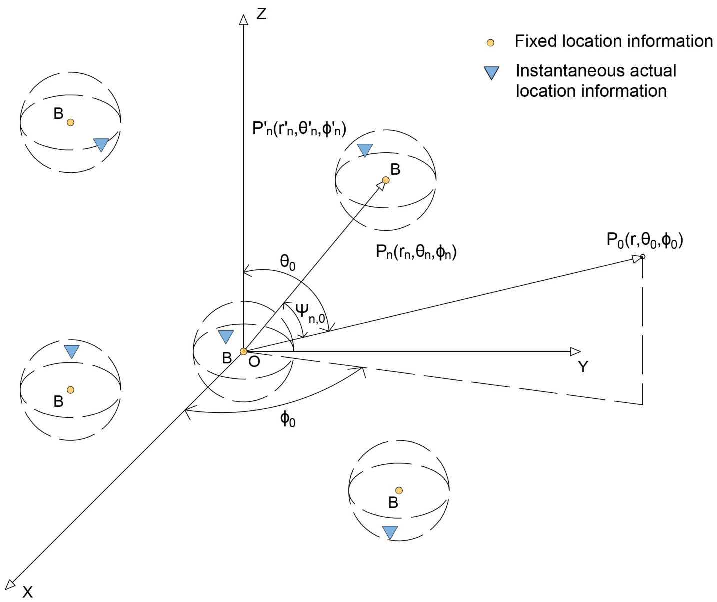


Figure 4.1: *Model of a distributed satellite cluster, with fixed position information that represents the expected position of the satellite, and instantaneous position information, that represents the actual position of the satellite.*

- The distance between the cluster to the ground is much larger than the maximum distance between the satellites in the cluster. Thus, the channel fading is almost the same for all the satellites in the DSC;
- The satellites are perfectly synchronized in carrier frequency, phase and time.

To evaluate the optimal beampattern we need to know the manifold vector for each satellite. In [7] they consider also the perturbation on the position of each satellite, but without loss of generality, and due to our interest in developing a distributed algorithm, we will consider each satellite fixed in its position. So we can compute the average manifold vector of the DSC, based on the random antenna array theory in [8], as follows:

$$\bar{A}_n(\theta, \phi) = e^{j*2\pi L_n(\sin\theta \sin\theta_n \cos(\phi-\phi_n)+\cos\theta \cos\theta_n)}, \quad (4.1)$$

with θ_n, ϕ_n, L_n angles and radius associated to position of satellite n in spherical coordinates.

Then, the average beampattern of a distributed cluster can be expressed as:

$$F(\theta, \phi) = \bar{\mathbf{A}}^{\mathbf{T}}\mathbf{W} = \frac{1}{N}\sum_{n=1}^{N} w_n \bar{A}_n(\theta, \phi) \tag{4.2}$$

where $\mathbf{W} = [w_1, w_2, \ldots, w_n]^T \in \mathbb{C}^N$ is the weighted vector for the manifold of the Virtual Large Aperture Array (VLAA), $w_n \in \mathbb{C}$ denotes the complex weighted value for the nth satellite. $\bar{\mathbf{A}} = [\bar{A}_1, \bar{A}_2, \ldots, \bar{A}_n]^T \in \mathbb{C}^N$ denotes the average manifold vector.

4.3 Optimization problem formulation

From Antenna Theory [9] the equivalent isotropic radiated power (EIRP) of a distributed satellite cluster can be expressed as:

$$EIRP_{CB} = G_{CB} P_{tot} = N^2 G_r P_s$$

where G_{CB} denotes the gain of the DSC array, P_{tot} denotes the total power of all transmitting satellites, N denotes the number of satellites to form a VLAA, G_r denotes the receive antenna gain, and P_s denotes the total power of a satellite.

Following the article, taking into account the bit-error-rate (BER) for BPSK and QPSK (binary and quadrature Phase Shift keying) and the free space propagation loss, to meet the requirement of the desired link's quality we need to solve

$$N_{th} = \arg_N \{EIRP_{CB} = N^2 P_s = EIRP_{CB,th}\} \tag{4.3}$$

where $EIRP_{CB,th}$ represents the equivalent isotropic radiation power threshold to meet the requirement of bit-error-rate performance.

As in the article, we will use typical value for satellite to Earth station communication, as presented in table 4.1.

4.3.1 Revised Scenario Problem Formulation

Starting from the optimization problem presented in [7], that is a Mixed Integer LP, we modify it to address the modified scenario we have taken

Parameters	Typical value
Altitude	36000Km (GEO)
Receive Antenna Gain	15dBi
Frequency	4GHz
Transmit Power	8W
Modulation	BPSK/QPSK
LDPC coding	coding gain = 6dB
Carrier information rate	36Mbit/s
Boltzmann constant	1.380×10^{-23}J/K
Threshold (Eb/N0)/dB	6
Receiver noise temperature Tn/K	30K
Transmitter gain	10dBi

Table 4.1: *Typical parameters for the satellite and ground station, according to [7].*

into account. In the new scenario we want all satellites to work together, and not just a subset of them. So, we remove that condition, and reformulate the problem in a more suitable manner for a distributed approach as follows

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{C}^N} \quad & \sum_{i=1}^{N} (|\mathbf{x}_i|)^2 \\ \text{subj.to} \quad & \sum_{i=1}^{N} \bar{A}_i(\theta_0,\phi_0)\mathbf{x}_i = 1, \\ & |\sum_{i=1}^{N} \bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i| \leq u, \quad s\in\{1,\dots,S\}, \\ & |\mathbf{x}_i| \leq 1, \quad i\in\{1,\dots,N\}, \end{aligned} \tag{4.4}$$

We had changed slightly the problem with respect to [7] to make the cost function composed by terms local to each agent. We recast it as the sum of the square of the modulus of complex weight $\mathbf{x}_i$, such that instead of minimizing the max power used by all satellites, we want to minimize the total power usage of the system: thus we obtain a cost function that is local to each agents, and also we grand convexity of the cost function. Noticing that the first constraint is a sum of complex number that adds up to a real value, due to consideration made in [10], we can split this constraint in, obtaining:

$$\begin{aligned} &\sum_{i=1}^{N} \mathcal{R}e\left(\bar{A}_i(\theta_0,\phi_0)\mathbf{x}_i\right) = 1, \\ &\sum_{i=1}^{N} \mathcal{I}m\left(\bar{A}_i(\theta_0,\phi_0)\mathbf{x}_i\right) = 0, \end{aligned} \tag{4.5}$$

with $\mathcal{R}e(\cdot)$ that returns the real part of a complex number and $\mathcal{I}m(\cdot)$ the imaginary part. Then we need to manage all inequality constraints on side directions, to make these constraints separable. Due to the presence of a norm, we need to rewrite these via a restriction of the feasible set, making this constraints tighter: the problem will became sub-optimal, but as we will see, the sub-optimal solution remains close to the optimal one.
First we need to evaluate the norm:

$$\sqrt{\left[\mathcal{R}e\left(\sum_{i=1}^{N} \bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right)\right]^2 + \left[\mathcal{I}m\left(\sum_{i=1}^{N} \bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right)\right]^2} \leq u, \tag{4.6}$$

then to remove the square root by elevating both left and right terms. Now we divide the constraints in two, restricting in fact the feasible set:

$$\left[\mathcal{R}e\left(\sum_{i=1}^{N} \bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right)\right]^2 \leq \frac{u^2}{2}, \quad \left[\mathcal{I}m\left(\sum_{i=1}^{N} \bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right)\right]^2 \leq \frac{u^2}{2}, \tag{4.7}$$

Now we can remove the square over the summation, obtaining

$$\begin{aligned} -\frac{u}{\sqrt{2}} \leq \mathcal{R}e\left(\sum_{i=1}^{N} \bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right) \leq \frac{u}{\sqrt{2}}, \\ -\frac{u}{\sqrt{2}} \leq \mathcal{I}m\left(\sum_{i=1}^{N} \bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right) \leq \frac{u}{\sqrt{2}}, \end{aligned} \tag{4.8}$$

that for practicality and looking forward to rewrite them with notation of [13], we rewrite them as follows:

$$\begin{aligned}
&\sum_{i=1}^{N} \mathcal{R}e\left(\bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right) - \frac{u}{\sqrt{2}} \leq 0,\\
&-\sum_{i=1}^{N} \mathcal{R}e\left(\bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right) - \frac{u}{\sqrt{2}} \leq 0,\\
&\sum_{i=1}^{N} \mathcal{I}m\left(\bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right) - \frac{u}{\sqrt{2}} \leq 0,\\
&-\sum_{i=1}^{N} \mathcal{I}m\left(\bar{A}_i(\theta_s,\phi_s)\mathbf{x}_i\right) - \frac{u}{\sqrt{2}} \leq 0,
\end{aligned} \tag{4.9}$$

Finally we can rewrite problem (4.4) as:

$$\begin{aligned}
\min_{\mathbf{x}_1,\ldots,\mathbf{x}_N} \quad & \sum_{i=1}^{N} f_i(\mathbf{x_i})\\
\text{subj.to} \quad & \mathbf{x}_i \in X_i, \qquad i \in \{1,\ldots,N\},\\
& \sum_{i=1}^{N} \mathbf{h}_i(\mathbf{x}_i) = 0,\\
& \sum_{i=1}^{N} \mathbf{g}_i(\mathbf{x}_i) \leq 0,
\end{aligned} \tag{4.10}$$

where

- $f_i : \mathbb{R} \to \mathbb{R},\ f_i(\mathbf{x}_i) = (|\mathbf{x}_i|)^2$,
- $\mathbf{h}_i : \mathbb{R} \to \mathbb{R}^2,\ \mathbf{h}_i(\mathbf{x}_i) = [h_{1,i}(\mathbf{x}_i), h_{2,i}(\mathbf{x}_i)]^T$,
- $\mathbf{g}_i : \mathbb{R} \to \mathbb{R}^{4S},\ \mathbf{g}_i(\mathbf{x}_i) = [\mathbf{g}_{1,i}(\mathbf{x}_i), \mathbf{g}_{2,i}(\mathbf{x}_i), \mathbf{g}_{3,i}(\mathbf{x}_i), \mathbf{g}_{4,i}(\mathbf{x}_i)]^T$,
- $h_{1,i} : \mathbb{R} \to \mathbb{R},\ h_{1,i}(\mathbf{x}_i) = \mathcal{R}e\left(\bar{A}_i(\theta_0,\phi_0)x_i\right) - \frac{1}{N}$,
- $h_{2,i} : \mathbb{R} \to \mathbb{R},\ h_{2,i}(\mathbf{x}_i) = \mathcal{I}m\left(\bar{A}_i(\theta_0,\phi_0)x_i\right)$,
- $\mathbf{g}_{1,i} : \mathbb{R} \to \mathbb{R}^S,\ \mathbf{g}_{1,i}(\mathbf{x}_i) = [g^1_{1,i}(\mathbf{x}_i), \ldots, g^S_{1,i}(\mathbf{x}_i)]^T$, with $g^s_{1,i}(\mathbf{x}_i) = \mathcal{R}e\left(\bar{A}_i(\theta_s,\phi_s)x_i\right) - \frac{u}{N\sqrt{2}}$, $s \in \{1,\ldots,S\}$,
- $\mathbf{g}_{2,i} : \mathbb{R} \to \mathbb{R}^S,\ \mathbf{g}_{2,i}(\mathbf{x}_i) = [g^1_{2,i}(\mathbf{x}_i), \ldots, g^S_{2,i}(\mathbf{x}_i)]^T$, with $g^s_{2,i}(\mathbf{x}_i) = -\mathcal{R}e\left(\bar{A}_i(\theta_s,\phi_s)x_i\right) - \frac{u}{N\sqrt{2}}$, $s \in \{1,\ldots,S\}$,

- $\mathbf{g}_{3,i} : \mathbb{R} \rightarrow \mathbb{R}^S$, $\mathbf{g}_{3,i}(\mathbf{x}_i) = [g^1_{3,i}(\mathbf{x}_i), \dots, g^S_{3,i}(\mathbf{x}_i)]^T$, with $g^s_{3,i}(\mathbf{x}_i) = \mathcal{I}m\left(\bar{A}_i(\theta_s, \phi_s)x_i\right) - \frac{u}{N\sqrt{2}}$, $s \in \{1, \dots, S\}$,
- $\mathbf{g}_{4,i} : \mathbb{R} \rightarrow \mathbb{R}^S$, $\mathbf{g}_{4,i}(\mathbf{x}_i) = [g^1_{4,i}(\mathbf{x}_i), \dots, g^S_{4,i}(\mathbf{x}_i)]^T$, with $g^s_{4,i}(\mathbf{x}_i) = -\mathcal{I}m\left(\bar{A}_i(\theta_s, \phi_s)x_i\right) - \frac{u}{N\sqrt{2}}$, $s \in \{1, \dots, S\}$,
- $X_i = \{\mathbf{x}_i \in \mathbb{C} : |\mathbf{x}_i| \leq 1\}$.

The problem is now formulated in a suitable way for distributed approach, and so, in the next sections, we will elaborate a distributed algorithm for solving this problem.
In our scenario, we consider a smaller cluster than that presented in [7], and also we will consider different configuration of satellites distributed in a grid disposition. This will lead us to use $N >= N_{th}$ satellites, with each one considered as an agent. Each satellite must be able to communicate others satellites of the cluster: we considered the worst case scenario, where each satellite can directly communicate only to satellites placed along orthogonal direction of the grid.
Due to this assumption, the network of N agents communicate according to a fixed, strongly connected and undirected graph $\mathcal{G}$ by construction, where $\mathcal{G} = (\{1, \dots, N\}, \mathcal{E})$, with $\mathcal{E} \subseteq \{1, \dots, N\} \times \{1, \dots, N\}$ the set of edges that links two nodes. That is, the edge (i, j) models the fact that node i and j exchange information. We denote by $\mathcal{N}_i$ the set of *neighbors* of node i in the fixed graph $\mathcal{G}$, i.e., $\mathcal{N}_i := \{j \in \{1, \dots, N\} | (i, j) \in \mathcal{E}\}$. So such as we have considered the cluster configuration, one satellite considers as its neighbours only those satellites in line of sight along direction parallel to the side of the cubic grid.

4.4 Development of a distributed Algorithm for Satellite Beamforming

Following [13] first we derive the Lagrangian with respect to the coupling constraints problem (4.10):

$$\mathcal{L}_1(\mathbf{x}_1, \dots, \mathbf{x}_N, \boldsymbol{\lambda}, \boldsymbol{\mu}) =$$
$$\sum_{i=1}^{N} f_i(\mathbf{x}_i) + \boldsymbol{\lambda}^T \left(\sum_{i=1}^{N} \mathbf{h}_i(\mathbf{x}_i) \right) + \boldsymbol{\mu}^T \left(\sum_{i=1}^{N} \mathbf{g}_i(\mathbf{x}_i) \right),$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ are Lagrangian multipliers associated with the coupling constraints of (4.10), with

- $\boldsymbol{\lambda} \in \mathbb{R}^2$,
- $\boldsymbol{\mu} \in \mathbb{R}^{4S}$.

and Eq. (4.4) can be recasted as

$$\mathcal{L}_1(\mathbf{x}_1, \ldots, \mathbf{x}_N, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{i=1}^{N} \left(f_i(\mathbf{x}_i) + \boldsymbol{\lambda}^T \mathbf{h}_i(\mathbf{x}_i) + \boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i) \right).$$

Now we can define the dual problem of (4.10) as

$$\begin{aligned} \max_{\boldsymbol{\lambda} \in \mathbb{R}^2, \boldsymbol{\mu} \in \mathbb{R}^{4S}} \quad & \sum_{i=1}^{N} q^i(\boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{subj.to} \quad & \mu_s \geq 0, \qquad s \in \{1, \ldots, 4S\}, \end{aligned} \tag{4.11}$$

where

$$q^i(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \min_{\mathbf{x}_i \in X_i} f_i(\mathbf{x}_i) + \boldsymbol{\lambda}^T \mathbf{h}_i(\mathbf{x}_i) + \boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i).$$

As it is, we have successfully divided the problem into N local problems, with respect to the decision variable $\mathbf{x}_i$, i.e. each $q^i(\boldsymbol{\lambda}, \boldsymbol{\mu})$ depends only on variable $\mathbf{x}_i$ and local knowledge of function g_i and h_i. Now the cost function is coupled only by the lagrangian multipliers $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$. To overcome this couplings, we will use a Distributed Dual Sub Gradient [13] that allows each agent i to maintain a local dual variable estimate of $\boldsymbol{\lambda}_i^t$ and $\boldsymbol{\mu}_i^t$ that is iteratively updated according to a distributed subgradient iteration described by (4.14), and a local primal variable $\mathbf{x}_i^t$, computed by solving $q_i(\boldsymbol{\lambda}_i^t, \boldsymbol{\mu}_i^t)$.
Each agent i also at each iteration compute a weighted average (4.12) of $\boldsymbol{\lambda}_j^t, \boldsymbol{\mu}_j^t$, $\forall j \in \mathcal{N}_i$, with $\mathcal{N}_i$ the set of neighbours of i as defined in Section 4.3.1. To grant the convergence of the algorithm to the correct value, we need to respect some conditions.
First of all, the graph $\mathcal{G}$ that represents the connection between all agents is fixed, connected and undirected for construction, as stated previously, and it is also strongly connected. Then we need to satisfy assumption on weights a_{ij} and step-size γ^t as stated in Chapter 3. Then we can formulate the Distributed Dual Subradient Algorithm for a satellite cluster that performs

collaborative Beamforming.

Algorithm 2: Distributed Dual Subgradient Algorithm for Satellite Beamforming

Initialization: $\boldsymbol{\lambda}_i^0$, $\boldsymbol{\mu}_i^0 \geq 0$,

Evolution: for t=0,1,...

Gather $\boldsymbol{\lambda}_j^t$, $\boldsymbol{\mu}_j^t$, for $j \in \mathcal{N}_i$

Compute

$$\begin{aligned} \mathbf{v}_{\boldsymbol{\lambda},i}^{t+1} &= a_{ii}\boldsymbol{\lambda}_i^t + \sum_{j \in \mathcal{N}_i} a_{ij}\boldsymbol{\lambda}_j^t, \\ \mathbf{v}_{\boldsymbol{\mu},i}^{t+1} &= a_{ii}\boldsymbol{\mu}_i^t + \sum_{j \in \mathcal{N}_i} a_{ij}\boldsymbol{\mu}_j^t, \end{aligned} \tag{4.12}$$

$$\mathbf{x}_i^{t+1} \in \underset{\mathbf{x_i} \in X_i}{\operatorname{argmin}} f_i(\mathbf{x}_i) + \mathbf{v}_{\boldsymbol{\lambda},i}^{t+1^T}\mathbf{h}_i(\mathbf{x}_i) + \mathbf{v}_{\boldsymbol{\mu},i}^{t+1^T}\mathbf{g}_i(\mathbf{x}_i), \tag{4.13}$$

Update

$$\begin{aligned} \boldsymbol{\lambda}_i^{t+1} &= \boldsymbol{\lambda}_i^t + \gamma^t \mathbf{h}_i(\mathbf{x}_i^{t+1}), \\ \boldsymbol{\mu}_i^{t+1} &= \mathcal{P}_{\boldsymbol{\mu}_i \geq 0}\left(\boldsymbol{\mu}_i^t + \gamma^t \mathbf{g}_i(\mathbf{x}_i^{t+1})\right). \end{aligned} \tag{4.14}$$

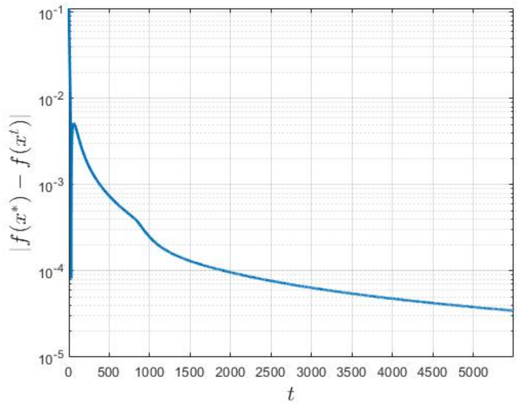
4.5 Numerical Simulations

In this section we propose some numerical examples and analysis in which we show the effectiveness of the proposed method.

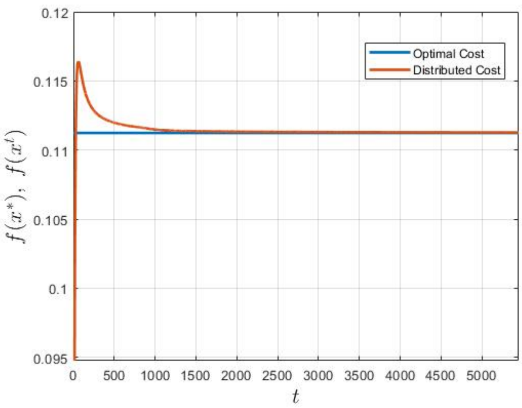
As a first experiment, we tested a small cluster of 9 satellites, disposed in square formation, with just 4 side directions in which lower interference is needed. The magnitude of the normalized manifold vector in those side directions must be lower than 0.6. The results shown in Fig. 4.2 are coherent with a dual subgradient algorithm: it can be seen that the proposed algorithm converges to the optimal cost with a sublinear rate, as expected for a subgradient method.

As we can see in Fig. 4.3, due to low number of side directions chosen, all inequality constraints are satisfied by the first iteration and never violated. The equality constraint already at iteration 1000 is violated in the order of 10^{-4}, that is a very good result.

Then we have tried to increment the number of satellites, reaching the number of 16 and maintaining a square formation. We also increase the number of side directions considered to 40. The algorithm still manages to converge



(a) Evolution of cost error in logarithmic scale



(b) Convergence of the cost to the optimal value

Figure 4.2: *Cost convergence for first example considered, with 9 agents.*

to optimal solution, as it is possible to see in Fig. 4.5, although the increasing of satellites, the algorithm quickly manages to converge to optimal solution, as around iteration 3000 the cost error is in the order of 10^{-4}.
The equality constraint violation decreases rapidly and inequality constraints are satisfied before iteration 250 (Fig. 4.6).
Then we have tried a third configuration, with 27 satellites organized in a cube formation, that has to satisfy 20 constraints on side directions. As before, it is good to notice that the cost converges quickly, as previously seen: after iteration 800 its error is lower than 10^{-4}.
At last, we want to show the evolution of the manifold at different iterations. In Fig. 4.8 we can see that at the beginning the cluster generates multiple lobes. Then, iteration after iteration, the system manages to start to perform

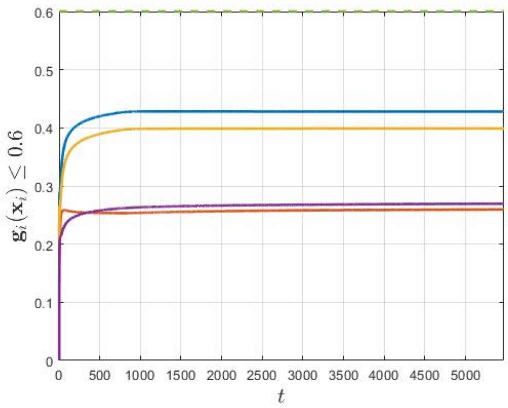


Figure 4.3: *Evolution of the constraints on side directions: satisfied if* ≤ 0.6 *(dashed line).*

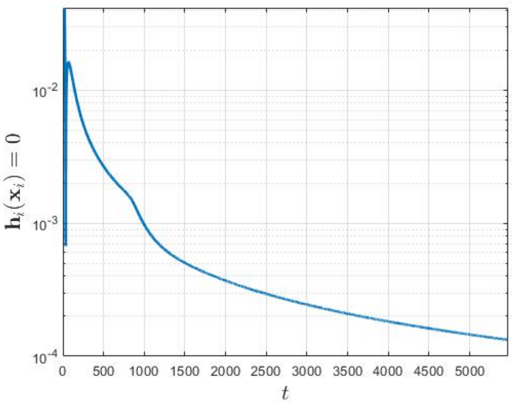


Figure 4.4: *Evolution of the equality constraint.*

Beamforming as desired. After some more iterations, the system reaches the optimal Beamforming, represented as a main lobe with gain one, and side lobes around it with gain lower than 0.6.

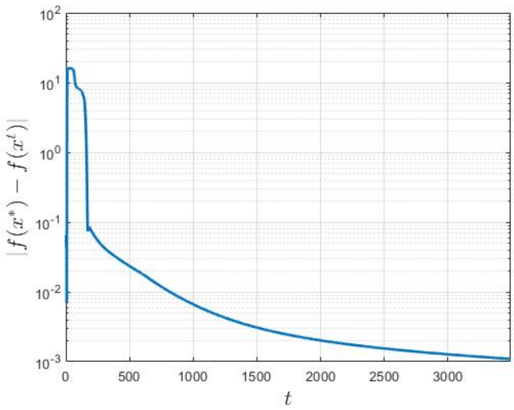
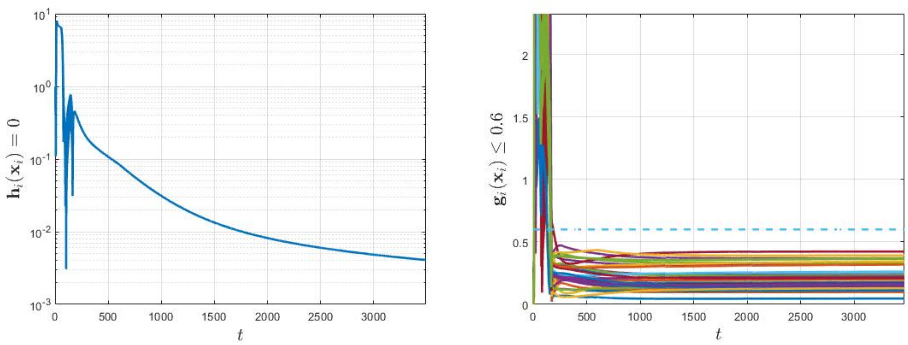


Figure 4.5: *Cost convergence for example with 16 agents and 40 side directions.*



(a) Equality constraint violation in logarithmic scale

(b) Side direction constraints

Figure 4.6: *Constraints for 16 agents experiment. Inequality constraints satisfied if* ≤ 0.6 *(dashed line).*

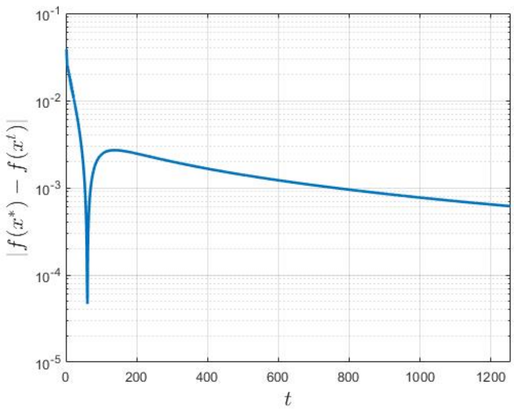
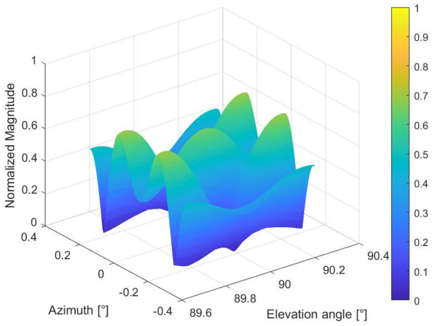
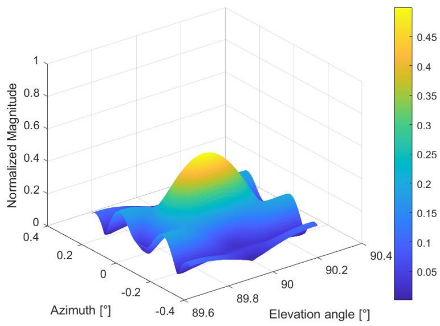


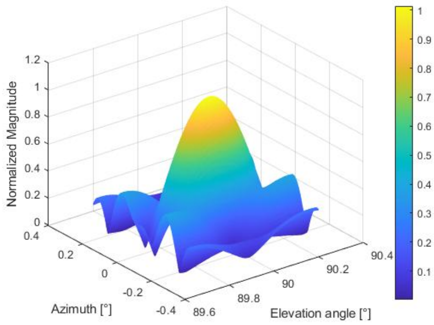
Figure 4.7: *Cost convergence for third example considered, with 27 agents.*



(a) Iteration 5



(b) Iteration 500



(c) Iteration 3000

Figure 4.8: *Evolution of manifold vector in desired direction for 16 satellites.*

Chapter 5

Distributed Cooperative Beamforming in Multi-Source Multi-destination Hybrid Earth-Space Clustered Systems

Satellite communication has been widely applied in various areas such as broadcasting, navigation and Internet coverage, due to the potential in providing wide coverage and achieving high data rate transmission. However, the direct link between the satellite and terrestrial user cannot be always achievable due to fixed satellite orbit and remoteness of Earth user. To overcome this problem, Hybrid Satellite Terrestrial relay networks (HSTRNs) are considered. As previously presented in Chapter 1, relay network Beamforming is an interesting research field: multiple signals are transmitted at once on the same frequency to multiple users via relays. While networks of Earth only antennas are already a case of study, hybrid Space-Earth networking is a new interesting research field. We want to implement a distributed optimization approach to a hybrid Space-Earth relay network, where sources are Geostationary Earth Orbit Satellite and on Earth there are multiple clusters of relays and users.

5.1 Scenario

We start by analyzing an Earth relay network, as presented in [11]. The scenario is the following: a multi-cluster network in which each cluster contains multiple single-antenna Earth source-destination pairs that communicate simultaneously over the same channel. There is also a set of cooperating Amplify and Forward (AF) relays, which perform Beamforming. In this scenario multiple clusters can interfere with each other, increasing the complexity of the problem.
The optimization aim is to minimize the total relay transmit power, knowing the channel second-order statistics, while meeting certain SINR (Signal-to-Interference-plus-noise ratio) constraints at the destinations.
As previously mentioned, in [11] they considered multiple clusters of antennas, each one composed by M source-user pairs and L relays. As assumption, they considered that none of source-user pairs can communicate directly and so they must interact via some of the relays in the cluster. The number of relays must be at least greater than that of source-user pairs. They consider each source, user and relay as a single antenna, capable only to transmit or receive a signal in a single channel. Relays are of the type Amplify and Forward (AF), such that the signal emitted is a weighted sum of all signals received. The signal propagation inside a single cluster works in two steps: first each source sends the signal to relays, then the relays amplify and send forward the signal to the users . The signal that arrives at each user is a combination of multiple signals emitted from various relays, so we need to accomplish certain quality requirements in terms of SINR in order to guarantee that each user is able to distinguish its desired signal from other interferences.
Then we replace all terrestrial sources with GEO Satellites that perform Beamforming on their own, such that their beam can reach multiple relays of the terrestrial network but not the users. To do so we will refer to [12] for satellite-terrestrial communication and channel analysis, while for the terrestrial part we will refer to [11]. In Fig. 5.1 we can see a simplified representation of a single cluster of our scenario.

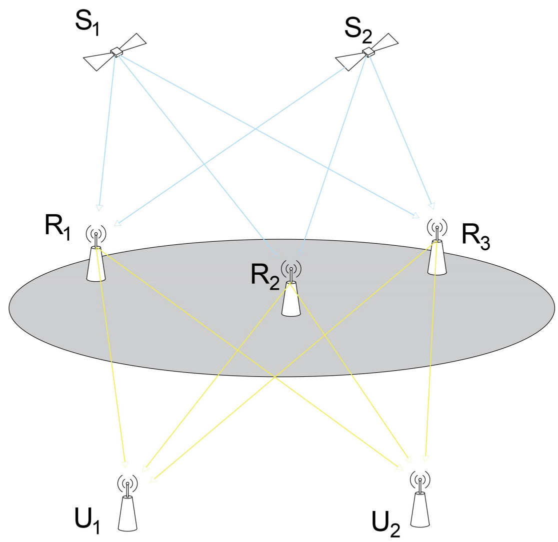


Figure 5.1: *Representation of a single cluster: 2 satellites communicate to terrestrial antenna relays inside the zone covered by satellites (grey area). Then relays communicate to terrestrial users.*

5.2 Network Connection and Interference

In the scenario presented in [11], as already mentioned, we had multiple single-antenna scattered on the ground in different patterns. These antennas are then divided into different clusters, and for each cluster we have some sources, some relays and some users. For our purpose, sources are satellites and not terrestrial mono antenna, but due to the huge distance between satellites and Earth's surface, the area covered by the beam may include multiple relays, also belonging to other clusters, and so the considerations for the connection of the network are not affected by this change. Depending on the position of each antenna of each cluster with respect to those of other clusters, we would have some clusters that exert negligible interference on other ones.

For explanation purpose, we can see in Fig. 5.2 an example of a possible real scenario with just two clusters: the area covered by satellites of both clusters overlap, and thus satellites of left cluster reach also relays of right cluster, and vice versa. Also relays of both clusters can communicate with users of both clusters, creating more interference. In a real scenario, the number of clusters can be larger and also the configuration and channel be-

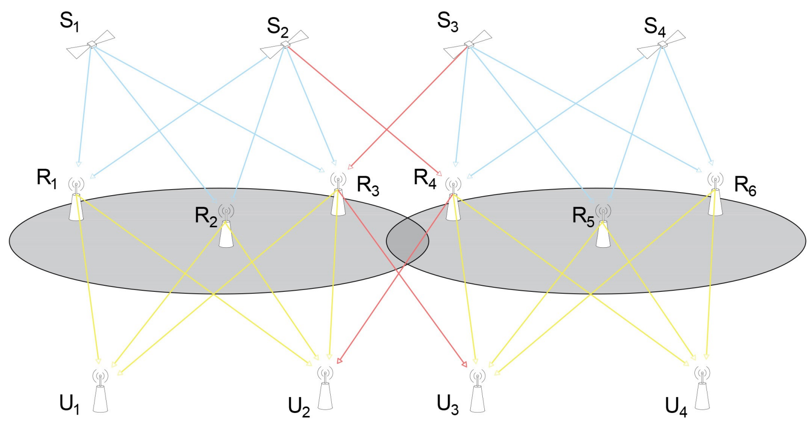


Figure 5.2: *Two cluster example.*

tween all single antennas may vary, i.d. if a cluster exerts non-negligible interference on another, it does not mean that all of its antennas have a non-negligible channel with antennas of the other clusters, but only that at least one antenna of the first cluster had a non-negligible channel with an antenna of the second one.

For this reason, to better represent a real case, we will consider a randomized network: in each cluster, the channels between sources and relays are randomized, and also the distance between relays and users (the channel depends from the distance); the connection between two clusters is also randomized, and between two cluster that exert non-negligible interference one on the other, we have randomized which source exert non-negligible interference over which relays of the other cluster, and the same for relays and users. So, each cluster is considered an agent, and each agent that exert non-negligible interference on another one, we assume that can directly communicate to it. Also, if an agent can communicate with another one, the opposite is also true. Due to this assumption, the network of N agents communicate according to a fixed and undirected graph $\mathcal{G}$ by construction, where $\mathcal{G} = (\{1, \ldots, N\}, \mathcal{E})$, with $\mathcal{E} \subseteq \{1, \ldots, N\} \times \{1, \ldots, N\}$ the set of edges that links two nodes. That is, the edge (i, j) models the fact that node i and j exchange information. We denote by $\mathcal{N}_i$ the set of *neighbors* of node i in the fixed graph $\mathcal{G}$, i.e., $\mathcal{N}_i := \{j \in \{1, \ldots, N\} | (i, j) \in \mathcal{E}\}$. For communication purpose, we have assumed that two cluster that exert a non-negligible interference one on the other are considered neighbours and so by a distributed system view, they are two neighbour agents that share an undirected connection. Also, we assume the system consider as strongly connected, which is a plausible assumption also for a real environment, and is very important for the following considerations concerning distributed approaches.

5.3 Mathematical Model

As previously mentioned we are interested in a single channel communication over multiple antennas. We start by analyzing the problem as proposed in [11], and then we will formulate the problem for our scenario and express it using [13] notation. Consider now a network of an index set $\mathcal{N} = \{1, ..., N\}$ of clusters $C_n, \forall_n \in \mathcal{N}$, each one containing a set $\mathcal{M}_n = \{1, ..., M_n\}$ of pairs of source-user with as source we have a satellite and as user a single antenna receiver, and a set $\mathcal{L}_n = \{1, ..., L_n\}$ of dedicated single antenna relays. We denote the m-th user (destination) of the n-th cluster as $U_{nm}, \forall_n \in \mathcal{N}, \forall_m \in \mathcal{M}_n$, the respective source as $S_{nm}, \forall_n \in \mathcal{N}, \forall_m \in \mathcal{M}_n$, and the relays as $R_{nl}, \forall_n \in \mathcal{N}, \forall_l \in \mathcal{L}_n$. Note that for simplicity of notation and without loss of generality, from now on we will consider that all clusters have the same number of source-user pairs M and relays L.
As previously mentioned, communication happens in two subsequent stages: first from sources to relays, then from relays to users. In this multi-cluster scenario, other than interference intra-cluster produced by other sources of the same cluster, some sources can interfere also with relay antennas of other cluster and also relays of a cluster can interfere with users of other clusters. The received signal vector $\mathbf{x}_n$ at each relay of cluster C_n hence is a superposition of signals originating from each source of each cluster of the network:

$$\mathbf{x}_n = \sum_{j \in \mathcal{N}} \sqrt{P_0} \mathbf{F}_{jn} \mathbf{s}_j + \mathbf{v}_n,$$

where P_0 is the common transmit power of all sources, $\mathbf{s}_j = [s_{j1}, \cdots, s_{jM}]^T \in \mathbb{C}^M$ is the complex vector of, normalized to unit power, information symbol transmitted by sources of cluster j, $\mathbf{x}_n = [x_{n1}, \cdots, x_{nL}]^T \in \mathbb{C}^L$ and $\mathbf{v}_n = [v_{n1}, \cdots, v_{nL}]^T \in \mathbb{C}^L$ are respectively the complex vector of received signal and noise at each relay of cluster n, and the matrix

$$\mathbf{F}_{jn} = [\mathbf{f}_{j1,n} \cdots \mathbf{f}_{jM,n}] = \begin{bmatrix} f_{j1,n1} & \cdots & f_{jM,n1} \\ \vdots & \ddots & \vdots \\ f_{j1,nL} & \cdots & f_{jM,nL} \end{bmatrix} \in \mathbb{C}^{L \times M}$$

is defined as the channel state matrix containing the channels from all sources of cluster C_j to all relays of cluster C_n. Also, v_{nl} is the zero mean and unit variance noise at relay R_{nl} and $f_{jm,nl}$ denotes the channel gain between source S_{jm} and relay R_{nl}.

The channel $f_{jm,nl}$ between satellite source and each relays is defined as follows

$$f_{jm,nl} = \sqrt{\mathbf{G}_{FSL}\mathbf{G}_S}\tilde{f}_{SR},$$

where $\mathbf{G}_{FSL}$ represent the Free Space Loss of the channel and could be considered constant (distance from satellite to terrestrial antenna can be approximated as constant), $\mathbf{G}_{S,jm,nl}$ denotes the satellite beam gain on relay l of cluster n from satellite m of cluster j, and $\tilde{f}_{SR}$ is the random variable of channel fading, and so $\tilde{f}_{SR}$ follows the Shadowing-Rician distribution. Then:

- $\mathbf{G}_{FSL} = \left(\dfrac{\lambda_s}{4\pi d_s}\right)^2,$
- $\mathbf{G}_S = \mathbf{G}_{S,max}\left(\dfrac{J_1(u)}{2u} + 36\dfrac{J_3(u)}{u^3}\right)^2,$

where $u = 2.07123\frac{sin\theta}{sin(\theta_{3dB})}$, with θ the angle of the receiver's position with the beam boresight, and θ_{3dB} is the beam 3-dB angle. $J_1(u)$ and $J_3(u)$ are the first-kind Bessel function of order 1 and 3. $\mathbf{G}_{S,max}$ denotes the maximum antenna gain of satellite. λ_s represent the carrier wavelength of the signal transmitted by each satellites, while d_s the distance between the satellite and terrestrial relays. During the second communication stage each relay re-transmits the received signal in an AF fashion, that is a linear transformation of their respective received signals $\mathbf{x}_n$, i.e., $\mathbf{t}_n = \mathbf{W}_n\mathbf{x}_n$, where $\mathbf{t}_n \in \mathbb{C}^L$ denotes the forwarded signal vector and $\mathbf{W}_n \in \mathbb{C}^{L\times L}$ is the corresponding Beamforming matrix of cluster C_n. Because each relay has a single antenna, then the Beamforming matrix is diagonal, i.e., $\mathbf{W}_n = diag\{w_{n1}, \cdots, w_{nL}\} \in \mathbb{C}^{L\times L}$, where w_{nl} denotes the complex weight with which relay R_{nl} multiplies its received signal.

The received signal vector $\mathbf{y}_n \in \mathbb{C}^M$ for all users of each cluster C_n is now a superposition of signals from all relays of all clusters, and is expressed as

$$\mathbf{y}_n = \sum_{j\in\mathcal{N}} \mathbf{G}_{jn}\mathbf{t}_j + \mathbf{z}_n = \sum_{j\in\mathcal{N}} \left(\sqrt{P_0}\mathbf{G}_{jn}\mathbf{W}_j(\sum_{i\in\mathcal{N}} \mathbf{F}_{ij}\mathbf{s}_i) + \mathbf{G}_{jn}\mathbf{W}_j\mathbf{v}_j \right) + \mathbf{z}_n,$$

where $\mathbf{z_n} = [z_{n1}, \cdots, z_{nM}]^T \in \mathbf{C}^M$ denotes the vector of independent identically distributed (i.i.d.) random noise components $z_{nm} \sim \mathcal{CN}(0, 1)$ at user U_{nm}. The matrix $\mathbf{G}_j n \in \mathbb{C}^{M\times L}$ is the channel state matrix containing the channels from all relays of C_j to all the users of C_n, and is defined in a similar fashion as $\mathbf{F}_{jn}$, where $g_{jl,nm}$ defined the channel gain from relay R_{jl} of C_j to user U_{nm} of C_n, that is defined as:

$$g_{jl,nm} = \alpha_{jl,nm} c_{jl,nm} e^{j(\frac{2\pi}{\lambda_t})d_{jl,nm}},$$

where $\alpha_{jl,nm}$ represent the multi path fading that in [11] is assumed as Rayleigh fading, such that the gains $\alpha_{jl,nm}$ are independent and identically distributed circularly symmetric complex Gaussian random variables with zero mean and unit variance. λ_t denotes the wavelength of carrier waves of terrestrial communication between relays and terrestrial users, and we assume $\lambda_t = c/f = (3 \times 10^8)/(2.4 \times 10^9) = 0.125\,m$ which is a reasonable choice for wireless transmission utilizing ultra high frequency carrier waves (2.4 GHz). The Euclidian distance between relay R_{jl} and user U_{nm} is denoted by $d_{jl,nm}$ and $c_{jl,nm} = d_{jl,nm}^{-\mu/2}$, where $\mu = 3.4$ is the path loss exponent and represent the power fall-off rate. For simplicity, as in [11], we did not consider large-scale shadowing effects in terrestrial communication.

Each element y_{nm} of $\mathbf{y}_n$ represents the total received signal at each user U_{nm}. It is composed by a desired part, that is the signal of the source S_{nm} plus multiple interference and noises. First there is the Intra-Cluster Interference made by sources of the same cluster other than S_{nm} that are transmitted by relays of the same cluster. Then the Inter/Intra-Cluster interference, due to sources of other clusters that transmit the signal to relays of C_n. Next the Inter-Cluster Interference made by signals transmitted to user U_{nm} from relays of other clusters: note that in this section are considered all signals transmitted by all sources to relays of every other cluster other than C_n. At last there is the noise, composed both by noise z_{nm} of U_{nm} and noise transmitted and amplified by each relay.

By following the considerations in [11] we can obtain a formulation for SINR for each user m of cluster n as

$$
\begin{aligned}
\mathrm{SINR}_{nm} = \; & \mathbb{E}\Big(\underbrace{P_0 \left|\mathbf{g}_{n,nm}^T \mathbf{W}_n \mathbf{f}_{nm,n} s_{nm}\right|^2}_{\text{Desired}} \Big) \Big/ \\
& \mathbb{E}\Big(\underbrace{P_0 \sum_{i \in \mathcal{M}_n}^{i \neq m} \left|\mathbf{g}_{n,nm}^T \mathbf{W}_n \mathbf{f}_{ni,n} s_{ni}\right|^2}_{\text{Intra-cluster interference}} \\
& \quad + \underbrace{\sum_{i \in \mathcal{N}} \left|\mathbf{g}_{j,nm}^T \mathbf{W}_j \mathbf{v}_j\right|^2}_{\text{Noise}} \\
& \quad + \underbrace{P_0 \sum_{j \in \mathcal{N}}^{j \neq n} \sum_{k \in \mathcal{M}_j} \left|\mathbf{g}_{n,nm}^T \mathbf{W}_n \mathbf{f}_{jk,n} \mathbf{s}_{jk}\right|^2}_{\text{Inter/Intra-cluster interference}} \\
& \quad + \underbrace{P_0 \sum_{j \in \mathcal{N}}^{j \neq n} \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{M}_i} \left|\mathbf{g}_{j,nm}^T \mathbf{W}_j \mathbf{f}_{ik,j} \mathbf{s}_{ik}\right|^2}_{\text{Inter-cluster interference}} \\
& \quad + \underbrace{|z_{nm}|^2}_{\text{Noise}} \Big).
\end{aligned}
$$

that represents the ratio between the desired signal and the sum of all noises and unwanted interferences.

5.4 Optimization problem formulation

Now we have to formulate the optimization problem. The multi-cluster Beamforming problem entails finding $\mathbf{W}_n$ that solves the optimization problem presented in [11]

$$
\begin{aligned}
\min_{\{\mathbf{W}_n, \forall n \in \mathcal{N}\}} \quad & \sum_{n \in \mathcal{N}} P_T^n(\mathbf{W}_n) \\
\text{subj. to} \quad & \mathrm{SINR}_{nm}(\mathbf{W}_n) \geq \gamma_{nm}, \forall n \in \mathcal{N}, m \in \mathcal{M},
\end{aligned}
\tag{5.1}
$$

where P_T^n is the total power transmitted by relays of cluster n, and γ_{nm} is the threshold of user m of cluster n that SINR must stay above to grant that the desired signal is well perceived with respect to other interference and noises.

By following the notation of [11], we can rewrite the problem in an equivalent matrix form to obtain a more compact notation for each SINR constraint as

$$\begin{aligned} \min_{\{\mathbf{w}_n, \forall n \in \mathcal{N}\}} \quad & \sum_{n \in \mathcal{N}} \mathbf{w}_n^{\mathcal{H}} R_T^n \mathbf{w}_n \\ \textit{subj.to} \quad & \mathbf{w}_n^{\mathcal{H}} \mathbf{Q}^{nnm} \mathbf{w}_n + \sum_{j \in \mathcal{N}}^{j \neq n} \mathbf{w}_j^{\mathcal{H}} \mathbf{Q}^{jnm} \mathbf{w}_j \geq 1, \quad \forall n \in \mathcal{N},\, m \in \mathcal{M} \end{aligned} \tag{5.2}$$

where $\mathbf{w}_n^{\mathcal{H}} R_T^n \mathbf{w}_n = P_T^n(\mathbf{W}_n)$ and the matrices $\mathbf{Q}^{jnm}$ are introduced as in [11] to represent inter and intra cluster interferences. As it is, the optimization problem (5.2) belongs in the class of nonconvex Quadratically Constrained Quadratic Programming (QCQP) problems, which are NP-hard to solve. For this reason we will reformulate the problem as in [11], by defining $\mathbf{X}_n = \mathbf{w}_n \mathbf{w}_n^{\mathcal{H}}$, $\forall n \in \mathcal{N}$ and using the fact that $\mathbf{w}_j^{\mathcal{H}} \mathbf{Q}^{jnm} \mathbf{w}_j = \mathrm{Tr}(\mathbf{X}_j \mathbf{Q}^{jnm})$ we can express it in the equivalent form [14]

$$\begin{aligned} \min_{\{\mathbf{X}_n, \forall\, n \in \mathcal{N}\}} \quad & \sum_{n \in \mathcal{N}} \mathrm{Tr}(\mathbf{X}_n \mathbf{R}_T^n) \\ \text{subj.to} \quad & \mathrm{Tr}(\mathbf{X}_n \mathbf{Q}^{nnm}) + \sum_{j \in \mathcal{N}}^{j \neq n} \mathrm{Tr}(\mathbf{X}_j \mathbf{Q}^{jnm}) \geq 1, && \forall n \in \mathcal{N}, m \in \mathcal{M}, \\ & \mathbf{X}_n \in \mathbb{S}_+^L, && \forall n \in \mathcal{N}, \\ & \mathrm{rank}(\mathbf{X}_n) = 1, && \forall n \in \mathcal{N}, \end{aligned} \tag{5.3}$$

remembering that the problem is formulated for a set N clusters with $\mathcal{N}$ denoting $\mathcal{N} = \{1, \ldots, N\}$, each one with a set of source-user pairs $\mathcal{M} = \{1, \ldots, M\}$ and a set of relays $\mathcal{L} = \{1, \ldots, L\}$. We have that $\mathbf{X}_n \in \mathbb{R}^{L \times L}$ is the matrix representing complex power of each relay of cluster n, $\mathbf{R}_T^n \in \mathbb{R}^{L \times L}$ is a sort of transmission power at relays and $\mathbf{Q}^{jnm} \in \mathbb{R}^{L \times L}$ represents the influence of cluster j over user m of cluster n. Immediately we rewrite the problem in a more familiar notation for a distributed approach, as in [13].

$$
\begin{aligned}
\min_{\mathbf{x}_1,\cdots,\mathbf{x}_N} \quad & \sum_{i\in\mathcal{N}} \mathrm{Tr}(\mathbf{x}_i\mathbf{R}_T^i) \\
\text{subj.to} \quad & \mathrm{Tr}(\mathbf{x}_i\mathbf{Q}^{iim}) + \sum_{j\in\mathcal{N}}^{j\neq i} \mathrm{Tr}(\mathbf{x}_j\mathbf{Q}^{jim}) \geq 1, \quad \forall i \in \mathcal{N}, m \in \mathcal{M}, \\
& \mathbf{x}_i \in \mathbb{S}_+^L, \qquad \forall i \in \mathcal{N}, \\
& \mathrm{rank}(\mathbf{x}_i) = 1, \qquad \forall i \in \mathcal{N},
\end{aligned} \tag{5.4}
$$

where $\mathbf{x}_i \in \mathbb{S}_+^L$ impose the convex constraint that matrix $\mathbf{x}_i$ belongs to the cone of symmetric, positive semidefinite matrices of dimension L, i.e. $\mathbf{x}_i$ is said to be positive semi-definite if $a^{\mathcal{H}}\mathbf{x}_i a \geq 0$ for all $a \in \mathbb{C}^L$. It's interesting to note that since $\mathbf{Q}^{jim}$ is Hermitian and $\mathbf{x}_j$ is symmetric, it follows that $\mathrm{Tr}(\mathbf{x}_j\mathbf{Q}^{jim}) = \mathrm{Tr}(\mathbf{x}_j\,\mathcal{R}e\left(\mathbf{Q}^{jim}\right))$, which means that the inequality constraint in (5.4) is well defined (with $\mathcal{R}e(\cdot)$ that returns the real part of a complex number). We can then compact the notation of the constraints, looking forward to express the problem in a way suitable for distributed optimization

$$
\begin{aligned}
\min_{\mathbf{x}_1,\cdots,\mathbf{x}_N} \quad & \sum_{i\in\mathcal{N}} \mathrm{Tr}(\mathbf{x}_i\mathbf{R}_T^i) \\
\text{subj.to} \quad & \sum_{i\in\mathcal{N}} \mathrm{Tr}(\mathbf{x}_i\mathbf{Q}^{ijm}) \geq 1, \quad \forall j \in \mathcal{N}, m \in \mathcal{M}, \\
& \mathbf{x}_i \in \mathbb{S}_+^L, \qquad \forall i \in \mathcal{N}, \\
& \mathrm{rank}(\mathbf{x}_i) = 1, \qquad \forall i \in \mathcal{N},
\end{aligned} \tag{5.5}
$$

Since the rank constraint is non convex, then, as done also in [11], we consider a relaxed version of problem (5.5) by removing this constraint. If the result of the relaxed problem has rank equal to 1, then that solution is optimal. Otherwise the solution found will be an approximation representing a lower bound for the original problem. Starting from the approximate solution it is possible to retrieve in terms of cost a feasible one via randomization techniques [15]. The problem we will address is then the relaxed one:

$$\begin{aligned} \min_{\mathbf{x}_1,\cdots,\mathbf{x}_N} \quad & \sum_{i\in\mathcal{N}} \mathrm{Tr}(\mathbf{x}_i \mathbf{R}_T^i) \\ \text{subj.to} \quad & \sum_{i\in\mathcal{N}} \mathrm{Tr}(\mathbf{x}_i \mathbf{Q}^{ijm}) \geq 1, \quad \forall j \in \mathcal{N}, m \in \mathcal{M}, \\ & \mathbf{x}_i \in \mathbb{S}_+^L, \qquad\qquad \forall i \in \mathcal{N}. \end{aligned} \tag{5.6}$$

The above formulation assumes knowledge of the second order statistics of channel state information (autocorrelation and autocovariance functions of the channel state): in a practical setting, this can be obtained based on past observations. Also, the inequality constraints in (5.6)) must be active at the optimal solution, because if this condition is not satisfied, we would be able to decrease the magnitudes of $\mathbf{x}_i$ further, thus invalidating the optimality assumption.
Now we can recast the problem with notation of [13], as follows

$$\begin{aligned} \min_{\mathbf{x}_1,\cdots,\mathbf{x}_N} \quad & \sum_{i=1}^{N} f_i(\mathbf{x}_i) \\ \text{subj.to} \quad & \sum_{i=1}^{N} \mathbf{g}_i(\mathbf{x}_i) \leq 0, \\ & \mathbf{x}_i \in X_i, \qquad \forall i \in \{1,\dots,N\}, \end{aligned} \tag{5.7}$$

with

- $f_i(\mathbf{x_i}) = \mathrm{Tr}(\mathbf{x}_i \mathbf{R}_T^i)$, $\forall i \in \{1,\dots,N\}$,
- $\mathbf{g}_i(\mathbf{x}_i) = [g_i^{11}(\mathbf{x}_i),\dots,g_i^{NM}(\mathbf{x}_i)]^T$, $\forall i \in \{1,\dots,N\}$,
- $g_i^{jm}(\mathbf{x}_i) = \frac{1}{|\mathcal{N}_j|+1} - \mathrm{Tr}(\mathbf{x}_i \mathbf{Q}^{ijm})$, if i exerts non-negligible interference on user m of cluster j,
- $g_i^{jm}(\mathbf{x}_i) = 0$ if i exerts negligible interference on user m of cluster j,
- $X_i = \{\mathbf{x}_i \in \mathbb{C}^{L\times L}, \mathbf{x}_i \in \mathbb{S}_+^L\}$,

with $g_i^{jm}(\mathbf{x}_i)$ that represents the influence of agent i on user m of cluster j, and $|\mathcal{N}_j|$ that is the cardinality of the set of neighbours of agent j.

5.5 Implementation of a Distributed Dual Subgradient Algorithm with Augmented Lagrangian

The problem, as we have recast it, fits perfectly with [13] notation, and thus we can straight forwardly try to develop a Dual Subgradient Algorithm. Starting from formulation (5.7) we evaluate its lagrangian introducing the set of multiplier $\boldsymbol{\mu} \in \mathbb{R}^{NM}$, $\boldsymbol{\mu} \geq 0$, associated to the coupled constraints:

$$\mathcal{L}_1(\mathbf{x}_1, \ldots, \mathbf{x}_N, \boldsymbol{\mu}) = \sum_{i=1}^{N} (f_i(\mathbf{x}_i)) + \sum_{i=1}^{N} \left(\boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i)\right),$$

and it is easily reformulated as

$$\mathcal{L}_1(\mathbf{x}_1, \ldots, \mathbf{x}_N, \boldsymbol{\mu}) = \sum_{i=1}^{N} \left(f_i(\mathbf{x}_i) + \boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i)\right),$$

to highlight the separability of the terms. Now we can define the dual problem, where all constraints are decoupled:

$$\max_{\boldsymbol{\mu} \geq 0} q(\boldsymbol{\mu}) = \max_{\boldsymbol{\mu} \geq 0} \sum_{i=1}^{N} q_i(\boldsymbol{\mu}), \tag{5.8}$$

The problem is now a cost-coupled problem [13], with

$$q_i(\boldsymbol{\mu}) = \min_{\mathbf{x}_i \in X_i} f_i(\mathbf{x}_i) + \boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i), \tag{5.9}$$

To implement a Distributed Dual Subgradient Algorithm, we need to respect all condition and assumption presented in Chapter 3:

- For all $i \in \{1, \ldots, N\}$, f_i is convex: $\alpha \in (0,1)$, $\mathrm{Tr}[(\alpha \mathbf{x}_1 + (1 - \alpha)\mathbf{x}_2)R_T^i] \leq \alpha \mathrm{Tr}[\mathbf{x}_1 R_T^i] + (1-\alpha)\mathrm{Tr}[\mathbf{x}_2 R_T^i]$ and it's satisfied by equality;
- For all $i \in \{1, \ldots, N\}, j \in \{1, \ldots, N\}, m \in \{1, \ldots, M\}$, g_i^{jm} convex: $\alpha \in (0,1)$, $\frac{1}{N} - \mathrm{Tr}[(\alpha \mathbf{x}_1 + (1-\alpha)\mathbf{x}_2)Q^{ijm}] \leq \alpha \left(\frac{1}{N} - \mathrm{Tr}[\mathbf{x}_1 Q^{ijm}]\right) + (1-\alpha)\left(\frac{1}{N} - \mathrm{Tr}[\mathbf{x}_2 Q^{ijm}]\right)$ and it's satisfied by equality;
- There exist $\overline{\mathbf{x}}_1 \in X_1, \ldots, \overline{\mathbf{x}}_N \in X_N$ such that $\sum_{i \in \mathcal{N}} \mathbf{g}_i(\overline{\mathbf{x}}_i) < 0$;
- For all $i \in \{1, \ldots, N\}$, $\mathbf{x}_i \in X_i$, is a convex, closed set;

The condition on the constraints $\mathbf{x}_i \in X_i$, $i \in \{1, \ldots, N\}$ needs that the set is non-empty, convex and compact, but the last one is not true, so, as it is,

the problem can not be solved via duality.
We want to apply to this problem the Distributed Dual Subgradient Algorithm [13], so we need to solve at each agent the almost unconstrained problem (5.9). As it is, the second order sufficient condition of optimality for unconstrained minimization A.4 is not satisfied, because the problem is linear with respect $\mathbf{x}_i$, and so, due to the lack of constraints, the problem is unbounded. To overcome this problem, we can use instead of the Lagrangian, the Augmented Lagrangian [16], such that the cost function of (5.9) becames strictly convex, and so a global minimum exists. The Augmented Lagrangians usually applied to equality constraints, and is possible to adapt it also to inequality ones, but as previously noted, the optimal solution of the primal problem requires that all inequality constraints are satisfied by equality. For this reason, we can add the penalty term of the constraints as it is, considering it as an equality constraint.
By adding the penalty term, the Augmented Lagrangian is then

$$\mathcal{AL}_1(\mathbf{x}_1, \dots, \mathbf{x}_N, \boldsymbol{\mu}) = \sum_{i=1}^{N} (f_i(\mathbf{x}_i)) + \sum_{i=1}^{N} \left(\boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i)\right) + \underbrace{\frac{\rho}{2} \left|\left| \sum_{i=1}^{N} \mathbf{g}_i(\mathbf{x}_i) \right|\right|}_{\textbf{Penalty Term}}, \tag{5.10}$$

where $\rho \geq 0$ should be properly setted for regularization purposes. Now the dual problem satisfies the Second order sufficient condition of optimality A.4.
To rewrite the dual problem as (5.8), we need a way to separate the variables in the penalty term, because as it is, it is not possible to divide the cost equation by the N agents. To do so, we choose to duplicate the penalty term for each user, and consider, with respect to agent i, all $\mathbf{g}_j(\mathbf{x}_j)$, $\forall j \in \{1, \dots, N\}$, $j \neq i$, as given parameters. Later we will discuss how each agent obtain this information.
The problem is then as (5.8), with

$$q_i(\boldsymbol{\mu}) = \min_{\mathbf{x}_i \in X_i} f_i(\mathbf{x}_i) + \boldsymbol{\mu}^T \mathbf{g}_i(\mathbf{x}_i) + \frac{\rho}{2} ||\mathbf{g}_i(\mathbf{x}_i) + \mathbf{G}_i||^2, \tag{5.11}$$

where $\mathbf{G}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathbf{g}_j(\mathbf{x}_j)$. With $\mathbf{G}_i$ assumed known for each agents, we now need to remove the coupling between each $q_i(\boldsymbol{\mu})$, represented by $\boldsymbol{\mu}$. This is done via the Distributed Dual Subgradient method, that allow each agent

i to maintain a local dual variable estimate $\boldsymbol{\mu}_i^t$ that is iteratively updated according to a distributed subgradient iteration described by (5.15), and a local primal variable $\mathbf{x}_i^t$, computed by solving $q_i(\boldsymbol{\mu}_i^t)$.

Each agent i also at each iteration compute a weighted average (5.13) of $\boldsymbol{\mu}_j^t$, $\forall j \in \mathcal{N}_i$, with $\mathcal{N}_i$ the set of neighbours of i. To grant the convergence of the algorithm to the correct value, we need to respect some conditions.

First of all, the graph $\mathcal{G}$ that represents the connection between all agents is fixed, connected and undirected for consideration made previously in Section 5.2. The adjacency matrix of $\mathcal{G}$ is symmetric and $(Adj+I)^N$ has all elements greater than 0, i.e. the graph is strongly connected.

Then we needs to satisfy assumption on weight a_{ij} and γ^t

Now we are almost ready to state the Distributed Dual Subgradient Algorithm, from the perspective of each agent i. While all other parameters and variables are now distributed over all agents, we have temporarily assumed $\mathbf{G}_i$ known for each agent i. We need now to define a way to overcome this coupling, because apparently $\mathbf{G}_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathbf{g}_j(\mathbf{x}_j)$ requires knowledge from all agents in the system, but actually each agent can communicate only to its neighbours.

Firstly, each agent will evaluate its own value of $\mathbf{g}_i(\mathbf{x}_i)$ at iteration t, $\bar{\mathbf{g}}_i^t = \mathbf{g}_i(\mathbf{x}_i^t)$, and communicate it to its neighbours. At the same time it will gather all $\bar{\mathbf{g}}_j^t$, from all j neighbours.

To proceed, we need to better analyze and highlight some peculiarities of the system. We need to remember the meaning of the constraints: each constraint (i, m) represents the SINR of all cluster on user m of cluster i. We can note that for our assumptions, we assumed that neighbour clusters of agent i are also the only clusters that exert non-negligible interference on users of cluster i, and so the complete knowledge of the (i, m) constraint is reachable from i.

Each constraint (j, m) has a sparse structure, it doesn't actually need all g_i^{jm}, $\forall i \in \mathcal{N}_j$, but only those who exert non-negligible interference on user m. For this reason, in the penalty term of agent i, those elements of $\mathbf{g}_i(\mathbf{x}_i$ associated to constraints on non neighbour agents are zero, and so all those rows of the vector $\mathbf{g}_i(\mathbf{x}_i) + \mathbf{G}_i$ are just constant value: for optimization purpose all those elements are irrelevant, because they just add up to the cost value, without interacting with the state $\mathbf{x}_i$. For this reason, agent i doesn't actually need them.

Then each agent i use all $\bar{\mathbf{g}}_j^t$ it has collected to evaluate a dedicated value $\mathbf{G}_j^i = [G_j^{i,11}, \dots, G_j^{i,NM}]^T$, $\forall j \in \mathcal{N}_i$, that is a vector with the same dimension of $\mathbf{g}_i$ where all elements are zeros, except for those correlated to SINR constraints on users of cluster i. To make everything clear, $\mathbf{G}_j^i$ is the term that agent i prepare for agent j. Those elements are defined as

$$G_j^{i,im} = \sum_{\substack{k \in \mathcal{N}_i^+ \\ k \neq j}} \bar{g}_k^{im}, \forall m \in \{1, \dots, M\}, j \in \mathcal{N}_i^+, \tag{5.12}$$

where $\mathcal{N}_i^+$ represents the set of neighbours of i plus agent i itself. That is, with $\mathbf{G}_j^i$ we represent all the contribution to constraints on agent i by all other agents except j. After the evaluation of $\mathbf{G}_j^i$, each agent will exchange this value to its neighbours and compute its personal term of the Augmented Lagrangian with $\mathbf{G}_i = \sum_{j \in \mathcal{N}_i^+} \mathbf{G}_i^j$.

Now we can formally state the algorithm.

Algorithm 3: Distributed Dual Subgradient (see [13])

Initialization: $\boldsymbol{\mu}_i^0 \geq 0$, $\mathbf{G}_j^{i,0} = \mathbf{0}_{NM}$, $\forall j \in \mathcal{N}_i^+$

Evolution: for t=0,1,...

Gather $\boldsymbol{\mu}_j^t$, $\mathbf{G}_i^{j,t}$ from neighbors $j \in \mathcal{N}_i$

Evaluate

$$\mathbf{G}_i^t = \sum_{j \in \mathcal{N}_i^+} \mathbf{G}_i^{j,t},$$

Compute

$$\mathbf{v}_i^{t+1} = \sum_{j \in \mathcal{N}_i} a_{ij} \boldsymbol{\mu}_j^t, \tag{5.13}$$

$$\mathbf{x}_i^{t+1} \in \underset{\mathbf{x}_i \in \mathbb{S}_+^L}{\text{argmin}}\ f_i(\mathbf{x}_i) + \mathbf{v}_i^{t+1^T} \mathbf{g}_i(\mathbf{x}_i) + \frac{\rho}{2} ||\mathbf{g}_i(\mathbf{x}_i) + \mathbf{G}_i^t||^2, \tag{5.14}$$

Update

$$\boldsymbol{\mu}_i^{t+1} = \mathcal{P}_{\boldsymbol{\mu}_i \geq 0} \left(\mathbf{v}_i^{t+1} + \gamma^t \mathbf{g}_i(\mathbf{x}_i^{t+1}) \right), \tag{5.15}$$

Evaluate $\bar{\mathbf{g}}_i = \mathbf{g}_i(\mathbf{x}_i^{t+1})$

Gather $\bar{\mathbf{g}}_j$ from neighbors $j \in \mathcal{N}_i$

Evaluate $\mathbf{G}_j^{i,t+1}, j \in \mathcal{N}_i^+$

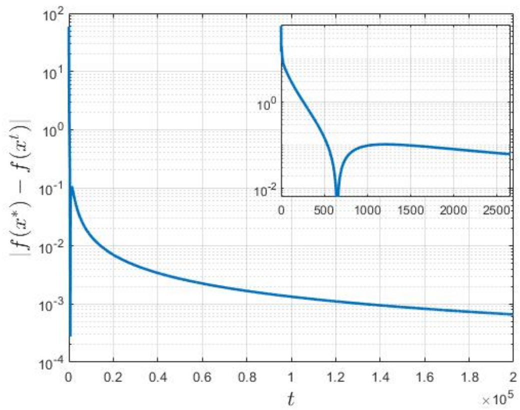
$$\mathbf{G}_j^{i,im,t+1} = \sum_{\substack{k \in \mathcal{N}_i^+ \\ k \neq j}} \bar{g}_k^{im}, \quad \forall m \in \{1, \ldots, M\},$$

$$\mathbf{G}_j^{k,km,t+1} = 0, \qquad \forall k \in \{1, \ldots, N\}, k \neq i, \forall m \in \{1, \ldots, M\},$$

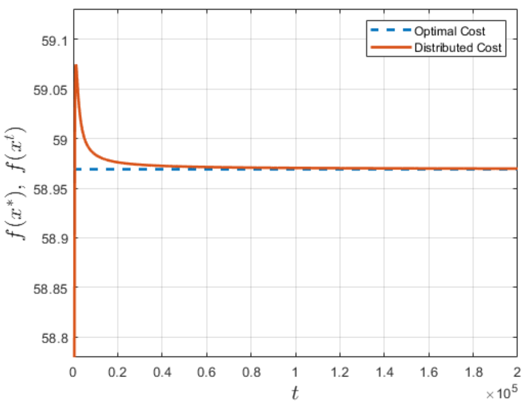
5.6 Numerical Simulations

In this section we propose some numerical examples and analysis in which we show the effectiveness of the proposed method. As stated in Section 5.2, the connection between each cluster and the interference inter-cluster will be randomized, with the constraint that the graph that represents the system connection must be strongly connected.

Firstly we consider a set of $N = 5$ cluster, with $M = 2$ source-users pairs and $L = 5$ relays each. For this first example, we have set that if a cluster exerts non-negligible interference over another cluster, all antennas of both can interfere with the other one. This represents a scenario with compact clusters spatially close to each others. In Fig. 5.3 we can see how the algorithm works, with the cost error that keeps converging to the optimal value. The results are coherent with a dual sub-gradient algorithm: it can be seen that the proposed algorithm converges to the optimal cost with a sublinear rate as expected for a subgradient method. Notice that the cost error is not monotone since the subgradient algorithm is not a descent method.



(a) Evolution of cost error in logarithmic scale



(b) Convergence of the cost to the optimal value

Figure 5.3: *Cost convergence for first example considered:* $N = 5$, $M = 2$, $L = 5$, *evaluated over* 2×10^5 *iterations.*

As we can see in Fig. 5.4, each constraint is satisfied before the 2000-th iteration. Then the system keep trying to make them all converge to 0: as stated before, the optimal solution requires that the constraints are satisfied by equality. In Fig. 5.5 we show the total violation of the inequality constraints, in logarithmic scale. In this example the total number of constraint is 10, with 2 constraints associated with each agent.

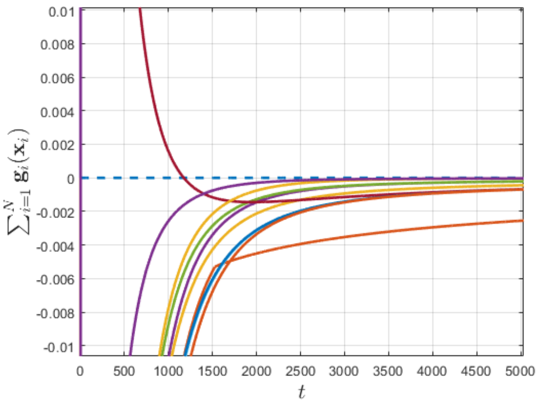


Figure 5.4: *Evolution of the constraints: satisfied for* < 0.

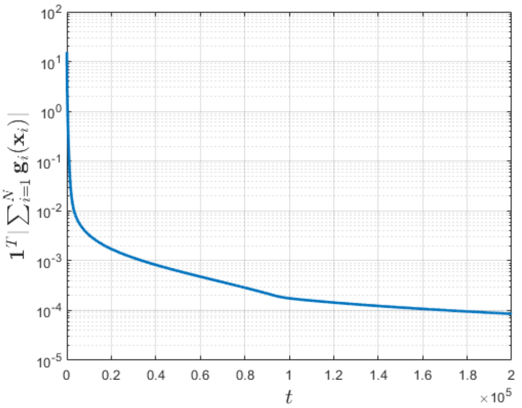


Figure 5.5: *Evolution of the total constraints violation for optimal solution.*

It is interesting to show that although the cost error seems slow to converge, due to the number of decision variables, the error on each state is really low, as we can notice in Fig. 5.6, where the sum of all state errors is in the order of 10^{-5} already before the 2000-th iteration, as we can see in the zoom box of the same figure.

We have preformed a Montecarlo simulation consisting of 50 trials. In Fig. 5.7 it is possible to notice that the behavior of the algorithm in all experi-

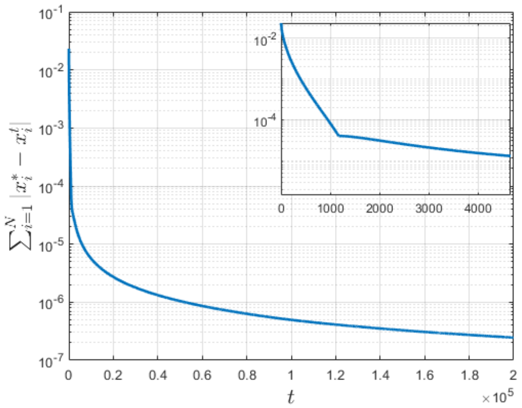


Figure 5.6: *Evolution of total state error in logarithmic scale.*

ments is coherent to the first experiment presented. In the figure we have represented the mean as a bold blue line, while with a shaded area the 1-standard deviation. Then to verify the scalability of the system and also

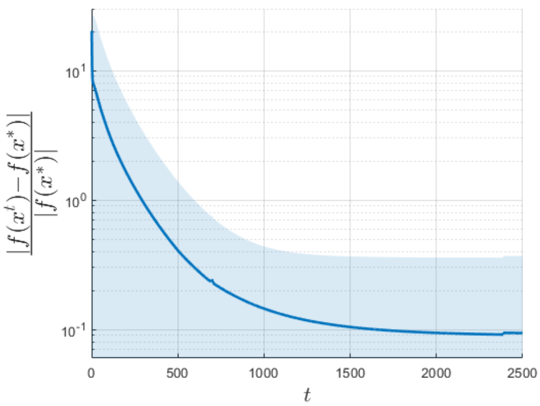
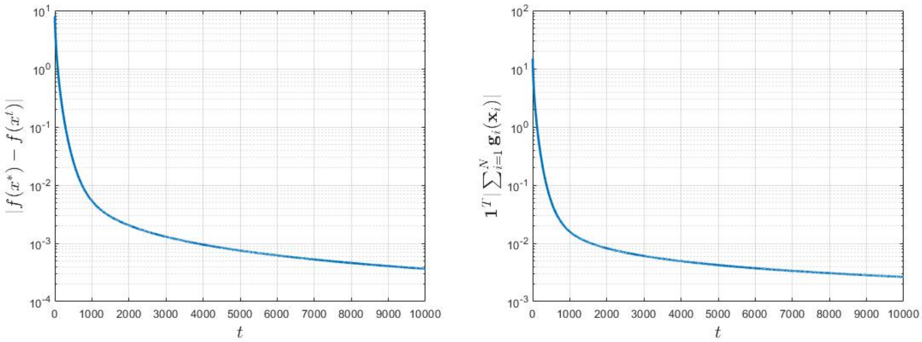


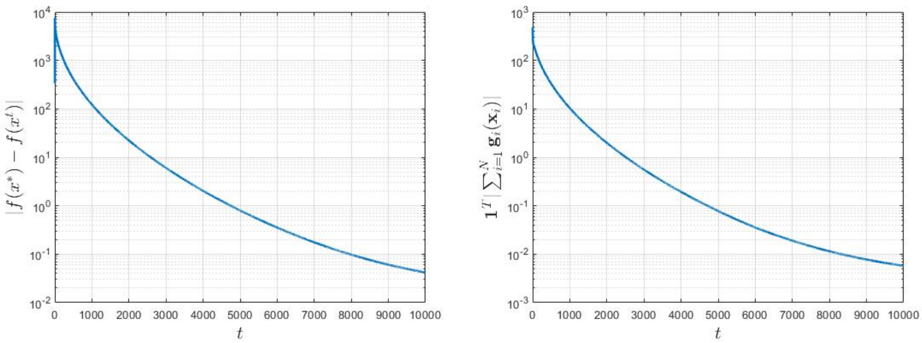
Figure 5.7: *Mean of the relative cost errors and 1-standard deviation band obtained with Monte Carlo simulations consisting of 50 trials, with randomized parameters and connections for the same scenario as the first example.*

the adaptability to different combinations of source-user pairs and relays, we have tested a configuration with just 3 agents, with an increased number of source-user pairs, with $M = 4$ and 6 relays per cluster, and then an experiment with 7 cluster, with $M = 2$ and 4 relays. The algorithm works as expected: with less agents the algorithm manage to converge faster to the optimal solution; for the case with more agents, it needs more iteration due to the higher number of constraints and interference inter-cluster.



(a) Cost error in logarithmic scale **(b)** Total constraints violation

Figure 5.8: *Result of experiment with* $N = 3$, $M = 4$, $L = 6$.



(a) Cost error in logarithmic scale **(b)** Total constraints violation

Figure 5.9: *Result of experiment with* $N = 7$, $M = 2$, $L = 4$.

Conclusions

This thesis proposes a distributed optimization approach to address two beamforming problems after an introductory overview of SIN and beamforming techniques.
Nowday, due to the constantly increasing demand for high data rate communications and high Quality of Service requested, methods that allow in improving both are very needed. We have analyzed different existing Beamforming techniques, classifying them in three main techniques: receive Beamforming, transmission Beamforming and relay network Beamforming. The former is very useful for users in a very noisy environment, where multiple signals with the same frequency carrier overlap and interfere one with the others. Via receive Beamforming the user is able to spatially filtering all received signals and to select ones coming from the desired direction. Transmission Beamforming, on the other hand, allows to spatially filtering an emitted signal, increasing the gain towards the desired direction and lowering the gain towards other directions: this is very useful in those situations where multiple signals use the same time slot and need to reach different users, increasing also the data rate transmission. Relay networks Beamforming is needed to convey the right signal to each user far from the source, with the capability to reduce interference to other transmissions and other users.
Then we have presented a brief analysis of space communication and satellite usages, with focus on Beamforming techniques used in space to Earth communication. We have studied different kinds of satellite configuration, with focus on satellite clusters. Then we have analyzed which knowledge is required to perform collaborative Beamforming with a satellite cluster, and which fields of research are interesting to solve this kind of problem.
After all studies and analysis on the topic, we had focused on a specific

scenario of collaborative Beamforming for satellite cluster, where multiple single antenna small satellites had to perform a Beamforming task to reach a terrestrial user, while limiting interference in other directions. The main advantage of this approach with respect to a centralized one, is that the system is autonomously able to adjust to changes and can sustain the fault of one of its agents. To implement a distributed approach we had proposed a Distributed Dual Subgradient Algorithm that lightens the computational heavy task of each agents and also makes the system scalable.
Then, we had analyzed a hybrid scenario, where satellites and terrestrial antennas collaborate to perform Beamforming, allowing multiple Geo Stationary satellites to reach terrestrial users via a set of single antenna Amplify and Forward relays. The Distributed Dual Subgradient Algorithm we had proposed for this Hybrid satellite terrestrial relay networks (HSTRNs) represents an easy to implement and reliable solution to the original problem. As shown, the algorithm manages to reach optimality, and it is also scalable with respect both number of clusters and number of elements in each cluster.

Appendix A

Assumption and Optimality conditions

A.1 Assumption for Distributed Dual Subgradient Algorithm

Assumption 22 of [13]: For all $i \in \{1, \dots, N\}$: each function f_i is convex, each constraint X_i is a non-empty, compact and convex set; each function $\mathbf{h}_i$, $\mathbf{g}_i$ is a component-wise convex function. Moreover, there exist $\bar{\mathbf{x}}_1 \in X_1, \dots, \bar{\mathbf{x}}_N \in X_N$ such that $\sum_{i=1}^{N} \mathbf{g}_i(\bar{\mathbf{x}}_i) < 0$.

A.2 Assumption on weights

Assumption 5 of [13]: Let the weights $a_{ij},\ i,\ j \in \{1, \dots, N\}$ be non-negative entries of $A \in \mathbb{R}^{N \times N}$ that match the graph $\mathcal{G}$, i.e., $w_{ij} \neq 0$ for all $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Moreover, they must satisfy:

- $\sum_{j=1}^{N} a_{ij} = 1,$ for all $i \in \{1, \dots, N\}$;
- $\sum_{i=1}^{N} a_{ij} = 1,$ for all $j \in \{1, \dots, N\}$;
- for all $i \in \{1, \dots, N\}, a_{ii} > 0$.

A.3 Assumption on step-size

Assumption 6 of [13]: Step-size sequence $\{\gamma^t\}_{t \geq 0}$, with $\gamma^t \geq 0$, must satisfy these conditions:

- $\sum_{t=0}^{\infty} \gamma^t = \infty$,
- $\sum_{t=0}^{\infty} \left(\gamma^t\right)^2 < \infty$.

A.4 Second Order Sufficient condition of optimality (uncostrained)

Let $f : \mathbb{R}^N \longrightarrow \mathbb{R}$ be twice continuously differentiable ($\mathcal{C}^2$) in $B(x^*, \epsilon)$ for some $\epsilon > 0$. Suppose that $x^* \in \mathcal{R}^n$ satisfies

$$\nabla f(x^*) = 0 \text{ and } \nabla^2 f(x^*) > 0 \,\text{(positive definite)}.$$

Then x^* is a strict (unconstrained) local minimum of f.

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