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The foundational case of the parabolic motion: design of an interdisciplinary activity for the IDENTITIES project

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Abstract

Questa tesi si inserisce nel campo di ricerca in Didattica della Fisica. In particolare, il lavoro si colloca all'interno del progetto Erasmus+ IDENTITIES, avviato nel settembre 2019, in collaborazione con le università di Montpellier, Creta, Parma e Barcellona. IDENTITIES ha lo scopo di sviluppare moduli didattici interdisciplinari (fisica – matematica – informatica), rivolti ai futuri insegnanti. I moduli riguardano sia temi curricolari sia temi STEM come contesto in cui sviluppare competenze interdisciplinari e progettare nuovi modelli di co-teaching.

I temi di IDENTITIES hanno ispirato e guidato l'attuazione di un corso rivolto a insegnanti di scuola secondaria di secondo grado, organizzato dal PLS di Fisica di Bologna assieme al PLS di Matematica e il POT di Bologna. Il corso, svoltosi tra novembre e dicembre 2019 ha rappresentato la principale fonte di materiale e di riflessioni per questo lavoro.

L'obiettivo di questa tesi è contribuire al progetto tramite la creazione di un'attività didattica, rivolta ai futuri insegnanti, sul tema della parabola e del moto parabolico. L'attività è stata progettata con lo scopo di guidare attraverso i principali passaggi che hanno caratterizzato, da un punto di vista epistemologico, l'evoluzione del pensiero fisico dalla teoria sul moto del proiettile di Tartaglia fino alla dimostrazione della traiettoria parabolica del proiettile di Galileo.

Nella tesi sono descritti il quadro teorico di base per il lavoro, la rielaborazione del materiale del corso PLS per costruire lenti per l'analisi dei libri di testo, l'analisi di un capitolo del libro di testo sulla cinematica bidimensionale e la conseguente progettazione dell'attività didattica. Nelle conclusioni sono discussi i principali risultati ottenuti, tra i quali la produzione delle griglie originali per l'analisi testuale, l'individuazione della simmetria e dell'indipendenza dei moti come attivatori epistemologici nel capitolo e la produzione dell'attività.

Introduction

The present thesis is framed in the research field of Didactic of Physics. In particular, this work is included in the Erasmus+ IDENTITIES project (www.identitiesproject.eu). The project started in September 2019, with the purpose of designing novel teaching approaches on interdisciplinarity in science and mathematics to innovate pre-service teacher education for contemporary challenges. The partnership includes, as well as the University of Bologna (coordinator), the universities of Barcelona, Crete, Montpellier and Parma. Operationally, the project aims to design and test teaching modules on both curricular topics (e.g. parabola and parabolic motion, cryptography, gravitation and geometry) and STEM advanced topics (e.g. artificial intelligence, nanotechnologies, quantum computing). The modules will be implemented in a platform for also blended or online tertiary teaching (in courses for preservice teachers, like the Curriculum in Physics Education and History of physics in the Master's Degree in Physics, in Bologna).

The themes of IDENTITIES inspired and guided the organization of a course for in service teachers, organised by the Physics PLS of Bologna in collaboration with the PLS of Mathematics and with the POT of Bologna. The course was carried out in the period of November- December 2019 in Bologna and represented the main source of material and insights for the present work. In fact, the main purpose of the course was to present interdisciplinary tools for the analysis and comprehension of the scientific texts. Moreover, during the first meeting, the mathematical object of parabola was historically analysed, both in its mathematical and physical development, with the help of excerpts from the original works of Apollonius, Kepler, Guidobaldo Del Monte and Galileo.

The main objective of this thesis is to contribute to the IDENTITIES project by creating activities, addressed to pre-service teachers, on the specific topic of parabola and parabolic motion. The activities have been designed to guide through the main epistemological breakthroughs that characterised the evolution of the physical thinking, from Tartaglia's theory of the projectile motion to the Galilean demonstration of the parabolic trajectory.

The thesis is articulated in four chapters plus the conclusions. In the first chapter the theoretical framework of the work is delineated. This section focuses on the characterization of the basic entities around which the discourse develops, namely the notions of discipline and interdisciplinarity, and the theme of mathematics and physics interplay. The second chapter contains a description of the authentic disciplinary and interdisciplinary elements that emerged from the historical analysis of the parabola, as it was described in the PLS course. Additionally, it proposes a selection (always from the PLS material) of operative tools, coming from the field of linguistic and philosophy, for the analysis of the scientific texts. This material is re-elaborated in the chapter, in order to build disciplinary and an interdisciplinary grids for the analysis of the textbooks. The third chapter mainly focuses on the disciplinary and interdisciplinary analysis of the chapter four of Walker's 5th edition of *Physics,* titled *Two-Dimensional Kinematics*. The analysis is carried out through the systematic application to the text of the two grids built in the previous chapter, and by comparing the key-moments in the disciplinary argumentation with the correspondent excerpts taken from the original works of Galileo and Guidobaldo. Finally, the fourth chapter is dedicated to the development of a teaching activity, based on the main results of the analyses of chapter three. This activity focus on the roles of symmetry, motions independence and demonstration as epistemological activators for the evolution of the theory of two-dimensional kinematics in the 16th and 17th century.

On the methodological plane, the research questions were progressively delineated during the work. Initially, we searched for a way of selecting and re-organizing specific elements from the PLS material in order to build an operative tool for the analysis of the text. Secondly, the average inadequacy of the modern textbooks in conveying the authentic disciplinary elements and the structural role of mathematics led to the research of a valid candidate among the physics textbooks, that could represent a basis for the developing of the activities. Lastly, the results of the Walker's analysis raises the issue of what way of designing the activity was more suitable in order to help preservice teachers to develop interdisciplinary skills from the specific topic of projectile motion.
As briefly said earlier, the PLS course was the main source of the material for this thesis. In addition to that, various types of sources were consulted during the work, spanning from the original memories, to upper secondary-school physics textbooks.

1. Research Framework

1.1 The concepts of discipline and interdisciplinarity

In this section we will dig deeper in the construct, definitions and characterizations that we need for the rest of the work, starting from the concept of *discipline*.

The term discipline has a medieval origin and was traditionally used referring to catalogue the expanding body of information gathered by scholarly community. In this acceptation, it refers to the particular *form of organisation of the knowledge* that guarantees an efficient transmission of information to the *disciple*s. This definition is quite concise; from our experience, it is evident that not every form of organization we choose could be called discipline. For this reason, we need to structure it a bit more.

We could establish, as Krishnan proposed (Krishnan, 2009), a series of criteria which indicates whether a subject is indeed a distinct discipline, as having:

1. A particular *object of study* (eventually shared with other disciplines);
2. A *body of accumulated specialist knowledge*, referred to their object of study;
3. *Theories and concepts* that can organise this knowledge;
4. A *specific terminology* and *technical language*;
5. *Specific research methods*;
6. Some *institutional manifestation*.

In the present discussion, we will neglect the sixth point, which is more concerned on the sociological aspect of the term.

These criteria allow us to see the concept emerging more clearly. While the object of study and the body of knowledge alone do not allow a proper distinction between disciplines, the theories that organise that knowledge and the specific methodologies applied in the research do the trick. In fact, following Krishnan historical dissertation on the concept, the term *paradigm* was coined by Kuhn to express this precise idea that disciplines are organised around certain ways of thinking or larger theoretical frameworks. Moreover, when the borders between two disciplines become blurry, in a more practical sense, the research method is often the only factor that allows to properly distinguish the two. Let us consider, for example, anthropology and sociology, where the object of study is almost identical, but the methodological attitude is different in the two cases (the first involving the ethnographic method, the other emphasizing the statistical analysis of data).

The list of criteria helped to clarify the concept of discipline. Now we would need a more operational and specific characterisation, one that makes us able to recognise the significant disciplinary structures in a text. In this regard, rather than trying to define what a discipline is, we focus on a “levels characterization” that prof. Paola Fantini suggested in the PLS course to point out the specificity of physics discourse when it was in its early stages of establishment. Fantini suggests the multi-level lens through which the disciplinary knowledge can be recognised in its organization. The approach led her to identify:

- An *ontological* level: referring to the basic entities that compose the object of study;
- An *epistemological* level: concerning how concepts are defined and laws are obtained;
- An *explicative* level: targeting how events and processes are explained;
- A methodological level: referring to the methods used to produce knowledge.

This lens will be presented and applied in the next chapter. Here we just wished to stress that in this thesis, instead of trying to answer the question what a discipline like physics is, we will try to analyse how a disciplinary discourse is characterised along those four levels. We indeed retain of crucial importance, to outline a disciplinary discourse, the concept of *characterizations*, rather than definitions. In fact, defining a discipline is a problematic task. Every proposal captures certain aspects of the concept, looking it through particular lenses, and ignores others. For instance, if you consider the same notion philosophically, sociologically, historically, as an anthropology or in an educational perspective, you will find completely different definitions, all equally agreeable. The following chart wants to visually prove this point:

Table 1.1 – The concept of discipline as intended in different fields.

Disciplinary Perspectives on Disciplines Matrix

	Philosophy	Anthropology	Sociology	History	Management	Education
Paradigm	Knowledge	Culture	Social Organisation	Time	Market	Personality Development
What Factors Encourage Disciplin-arity?	Language games/ discourses	Cultural identity and segmentation	Professionalization/ Power Structures	Leadership of talented founders of a discipline	Past success of disciplinary organisation	Curriculum and the need for structured or 'disciplined' learning
What Factors Encourage Inter- /Transdisciplinarity?	Universa-lisation of knowledge	New forms of community and identity	Social Change/Decline of Professions	Maturation of a discipline/ lack of leadership	Better adaptation to the market	Changes of knowledge structures/new approaches to teaching
On Balance	Disciplines are needed for validating claims to truth	Disciplines offer a stable identity and are similar to tribal structures	Disciplinary structures are difficult to overcome because of the self-interest of power groups	Historically the number of disciplines has constantly expanded rather than declined	Disciplines are an obsolete form of the organisation of science and universities	Educators are more in favour of disciplinary education because of a concern that students may only be confused by competing claims to truth and world views

This fact motivates our initial approach and moved us to establish some criteria that do not claim to be exhaustive, instead they give an idea of what we are referring to and about the direction we want to take. On the one hand they characterise the concept, on the other hand they provide some insights to actively develop a disciplinary analysis.

Until this point, we focused our attention on the concept of discipline. Logically, this notion had to be clarified before we could talk more extensively about interdisciplinarity. As said in the introduction, the term interdisciplinarity is polyhedral. Every possible interpretation hides in the background a specific epistemological view. The set of all these different perspectives produced a large *taxonomy* of interdisciplinarity. As example, we could refer to *methodological*, *theoretical* or *pseudo* interdisciplinarity meaning slightly different things. Having introduced this linguistical issue, we are ready to consider the following table, proposed by J.T. Klein (Integration and Implementation Sciences (I2S), 2014):

Table 1.1 – Degrees of cognitive integration.

DEGREE OF COGNITIVE INTEGRATION →

UNIDISCIPLINARY	MULTIDISCIPLINARY	INTERDISCIPLINARY	TRANSDISCIPLINARY
Focusing	Juxtaposing	Integrating	Transcending
Concentrating	Sequencing	Interacting	Transgressing
Analyzing	Coordinating	Linking	Transforming
Segmenting		Focusing	Overarching
		Blending	
		Hybridizing	
		Synthesizing	

Just as we did for the notion of discipline, we want to give a characterization of the term. Moving in this direction, we could focus on the key verbs that denotes important methodological patterns in the process of interdisciplinarity. Based on the above representation, J.T. Klein (Integration and Implementation Sciences (I2S), 2014) states that:

"*Interdisciplinarity* (ID) integrates information, data, methods, tools, concepts, and/or theories from two or more disciplines focused on a complex question, problem, topic, or theme. Scope and goals differ: from borrowing to large initiatives. The most common criterion is *Integration*."

Now, we can confront this proposition with the disciplinary criteria exposed earlier. The basic elements of a discipline, that is information, methods and tools, theories and concepts, persist and are integrated with the structural components of another discipline. They are not *juxtaposed*, as for the multidisciplinary approach, and they do not *transform* in something new, as in transdisciplinarity.

Considering the didactical dimension, while the disciplinary knowledge provides to the students significant forms of reasoning and contributes to the formation of argumentative and epistemic skills, a interdisciplinary approach targets the *context* in which ideas born and evolve, and in which the knowledge concerning a particular object of study is gathered in the first place. This kind of knowledge differs to the disciplinary one, it is not yet structured or stratified. It is collected by the mutual and constant interaction between disciplines, which investigate a specific problem with various lenses, igniting fruitful discussions and reciprocally stimulating innovations. Fully comprehend this vibrant and polyphonic context determines a more authentic and complete understanding of the disciplinary knowledge.

This section concludes the discussion about the notions of discipline and interdisciplinarity. In the following chapters, when these terms will pop up, they will be intended with the acceptation specified above. Now, having spent some time building up the basic concepts of discipline and interdisciplinarity, we now need to curb the field of research and discuss about the peculiar relation between mathematics and physics.

1.2 Mathematics and Physics as a relevant case of interdisciplinary approach:

Rather than considering mathematics and physics as separate and self-sustaining entities, with few mutual contaminations, following Tzanakis (Tzanakis, 2016), we prefer to say that "in teaching and learning mathematics or physics, neither *history* can be ignored, nor their *close interrelation* can be circumvented or bypassed."
It directly follows that neither maths nor physics has to be identified solely with their deductively-structured and logically coherent corpus of knowledge. They also include all the *processes* that lead to the formation of that knowledge. In other words, quoting Tzanakis:

"Mathematics and physics should be conceived (hence, taught and learnt) both as the result of *intellectual enterprises* and as the *procedures* leading to these results. Knowledge gained in their context has an evolutionary character; by its very nature, *historicity* is a deeply-rooted characteristic."

Therefore, considering all the procedures that lead to the structuration of knowledge in time means to recognise an *interdisciplinary quality* in the evolution of the disciplines. Historically, mathematics and physics developed simultaneously in a "close, continuous, uninterrupted, bidirectional, multifaceted and fruitful way" . New, unsolved physical problems often preceded and triggered the creation of appropriate mathematics. Vice versa, many concept, methods, theories originated in a purely mathematical context and were successively integrated in physics.
These considerations account for the existence of an articulated bond between mathematics and physics, that is closely linked to their genesis and evolution, and that could be further investigated with the help of the right interdisciplinary tools.

The last aspect we want to take care of, that will be useful during the analysis phase, is the question of the role of mathematics in physics. We could say that referring to mathematics as an *instrumental tool* for physics (emphasizing its technical role) is quite reductive. If analysed more precisely, mathematics has also a *communicative function*, acting as a language, and a *structural function*, providing a way of logical deductive reasoning (Uhden et al, 2011). Moreover, "at a deeper level mathematics penetrates into the construction of the physical concept itself and, precisely at this point, the distinction between conceptual and mathematical becomes artificial". This statement becomes clear if we consider velocity or acceleration. They are not just physical entities, but also intrinsically mathematical ones, being them rates of change.

So, mathematics not only evolves by relating to physics (as seen above), but also permeates it in a variety of different ways. During the analysis of the Walker we will directly deal with these several functions, confronting the textbook with original memories of great scientists in order to make a parallel with the valuable paradigms in the history of the discipline.

This part, concerning specifically the relation between mathematics and physics, concludes the theoretical framework that we will need for the rest of our work. Now, we can devote the next chapter to the in depth presentation of the object of study and explanation of the disciplinary analyses proposed in the PLS course.

2. The case of parabola

2.1 The choice of the topic, the PLS course and methodological issues

In the current chapter the disciplinary and interdisciplinary analysis of the topic of parabola proposed in the PLS (*Piano Lauree Scientifiche*) course is presented.
The course, organised by the Physics PLS of Bologna (http://www.pls.unibo.it/it/fisica/attivita/a.a.-2019-2020), in collaboration with the PLS of Mathematics, was addressed to upper secondary school teachers. In the course, the crucial theme of interdisciplinarity between mathematics and physics was explored and discussed, starting from the specific cases of the parabola and the parabolic motion. Different type of materials, from textbooks to original memories, concerning the topics were analysed by means of operative tools coming from linguistics, epistemology, mathematics, and physics education. The main goal of the analysis was to highlight the authentic interdisciplinarity between mathematics and physics, as it appeared in one of the decisive moments of the evolution of scientific thinking.
Overall, the course was articulated in three meetings and one conclusive workshop, carried out in the period of November- December 2019 and lasting three hours each. The disciplinary analyses of the topic, focusing mainly on the historical and epistemological aspects of the parabola, were held by professors L. Branchetti, A. Cattabriga and P. Fantini. The second and third meeting were dedicated respectively to the presentation of the linguistic tools for the analysis of the texts, exposed by V. Bagaglini and M. Viale, and the epistemological tools for the argumentative analysis of the texts, treated by S. Moruzzi. The final workshop was dedicated to a wider debate among the participants, on the possible contributions and implementations of the proposed analyses in the didactical dimension of the class.
The course was a pilot activity of the project Erasmus+ IDENTITIES (www.identitiesproject.eu).
The parabola embodies the perfect candidate for the type of research we want to develop, having a deeply rooted and interdisciplinary history spanning across the centuries. Moreover, the correspondent physics topic of projectile motion is a fairly simple one, compared to more complicated subjects like, for example, the blackbody on which the research group has carried out an interdisciplinary analysis (Branchetti, Cattabriga, Levrini, 2019). Lastly, the parabolic motion, the geometric concept of parabola and the topic of two-dimensional kinematics are foundational objects of study in high school, taught in the early stages of mathematics and physics education. It is therefore a great opportunity for the students to encounter a critical comparison of the two disciplinary approaches since the beginning and, through this, to recognise the foundations and the specificities of disciplinary thinking. In order to reach this purpose, materials to guide pre-service teachers to

develop an authentic, comprehensive view of this theme is needed, in order for them to effectively transmit it to their future pupils.

Having introduced in a few words our object of study, we will now move on to the presentation of the PLS material, which is composed of an historically oriented analysis of the parabola from a mathematical and physical point of view. The materials refer to historical cases where parabola and parabolic motion were crucial topics to establish, on one hand, the disciplinary identity of mathematics and physics and, on the other, to explore and define the structural roles played by mathematics in physics.

In the following sections, we briefly present the historical cases with the details that are needed to show the disciplinary and interdisciplinary features that can characterise the discourse on the parabola. These features represent the foundation for the construction of analytic tools that will be applied in non-historical texts (like textbooks), in order to find out if and how the disciplinary and interdisciplinary discourses are articulated.

The construction of the analytic grids has been the first concrete goal of the present thesis. Their construction has been carried out in different phases. First of all, we reconstructed the authentic disciplinary and interdisciplinary features that emerged from the historical study of the parabola and of the parabolic motion in the PLS course. Secondly, always from the PLS's material, we chose the linguistic and argumentative tools that were more suitable for searching in the textbooks those distinctive elements found in the previous point. We then formulated two grids, one focused on the investigation of the disciplinary features in the text, the other addressing the interdisciplinary ones. In particular, the linguistic tools were included in the disciplinary grid, helping to clarify some relevant aspects in relation to the definitions of concepts and to the organization of the relevant information in the text; the argumentative tools were instead included in the interdisciplinary grid, since effective to promote the distinction of the role/s of mathematics in the text.

2.2 The construction of the disciplinary lens

2.2.1 Disciplinary levels for the analysis of an historical case

The physical analysis mainly focuses on the work of Galileo Galilei (1564-1642) and his correspondence with Guidobaldo del Monte (1545-1607), concerning the shape of the trajectory drawn by a projectile in air and the property of this particular motion. This collection of original memories is particularly valuable because it represents a crucial step in overcoming the Aristotelian duality of *natural* and *forced* motions, still rooted in the medieval and renaissance culture (Cerreta, 2019). As a matter of fact, until the sixteenth century, the motions were interpreted as combinations of *linear* and *circular* ones, as the circumference and the straight line were taught by Aristotle to be the only two irreducible figures. As an historical example of the predominance of this paradigm during the sixteenth century, we can consider the work of the Italian mathematician Niccolò Tartaglia (1499-1557). In his influent books *La nova scientia* (1537), Tartaglia refers to the trajectory drawn by the violent motion of an object, as being partially straight and partially curved, and the curved section being an arch of circumference (fig. 2.1).

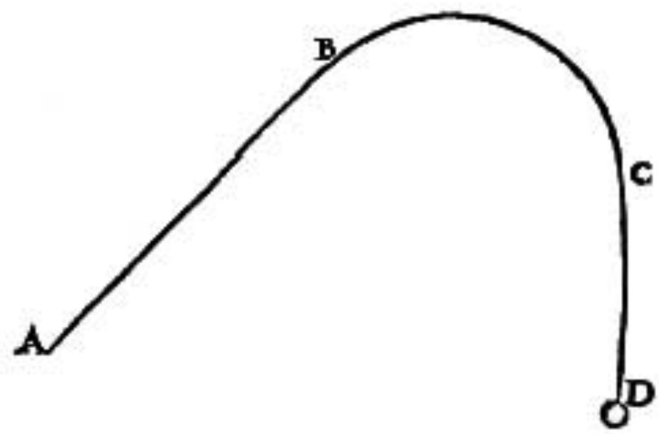
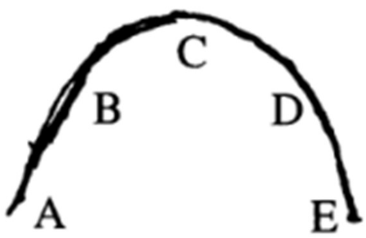


Fig.2.1 - Representation of the projectile motion in the second book of *La nova scientia.*

In the figure above, we can notice the characteristic asymmetry of the curve proposed. At that time, the parabola was not an appealing option mainly for two reasons. Firstly, the trajectory of the projectile was taught to be *intrinsically asymmetric* because the raise and falling of the object were determined by completely different causes, the violent motion on one hand and the natural motion toward the centre of the Earth on the other. Secondly, very few copies of Apollonius' *Conics,* a series of books in which the different conic sections were deeply studied, were circulating in Italy in that period. Unaware of Apollonius' work, Tartaglia could not find other curves that were able to suitably represent the trajectory. Thirteen year later, Gerolamo Cardano (1501-1576) in his *De subtilitate rerum* examines the same problem. Differently from Tartaglia, Cardano interprets the curved section of the trajectory as being similar to a *parabolic arc* rather than a circumference. In this forward step made by Cardano, historians have seen the acquisition of a high degree of familiarity with the opera of Apollonius, that was progressively rediscovered and studied. Following the considerations of Cerreta (2019), we can say that the conics, at the end of the sixteenth century, were able to provide new interpretative schemes for the theoretical understanding of the natural phenomena. This fact manifests its relevance especially in the works of Galileo.

Returning to our analysis, the excerpt by Guidobaldo (1592), regarding the experiment of throwing the inked ball, is a disciplinarily significant passage in the sense explained above. In few lines, Guidobaldo describes a characteristic symmetry in the trajectory drawn by an object, when it is thrown over the line of the horizon. Consequently, in a dialogue with Galileo, they search for the mathematical curve that could better represent this motion, designing an experiment to test their conjectures. Although a similarity with the parabola and hyperbole is evinced, they conclude considering the catenary as the most adequate candidate for the trajectory. The choice of considering a catenary arch hides an interesting theorical motivation; in fact, the dynamic composition of natural and violent motion in the trajectory of the object is optimally expressed, according to Galileo and Guidobaldo, by the balance between the weight of the chain and the tension applied in its extremities. In any case, at that historical moment, also mathematical literate scholars like Guidobaldo and Galileo were not able to distinguish between the two curves (Renn et al, 2000). Now, let us take a closer look to the excerpt. The colours refer to the analysis of the text that prof. Paola Fantini showed during the PLS course and that we will be commented below.

> "*If one throws a ball with a catapult or with artillery or by hand or by some other instrument above the horizontal line, it will take the same path in falling as in rising, and the shape is that which, when inverted under the horizon, a rope makes which is not pulled, both being composed of the natural and the forced, and it is a line which appearance is similar to a parabola and hyperbola. And this can be seen better with a chain than with a rope, since [in the case of] the rope abc, when ac are close to each other, the part b does not approach as it should because the rope remains hard in itself, while a chain or little chain does not behave in this way.*
>
> 
>
>
> *The experiment of this movement can be made by taking a ball coloured with ink, and throwing it over a plane of a table which is almost perpendicular to the horizontal. Although the ball bounces along, yet it makes points as it goes, from which one can clearly see that as it rises so it descends […] the violence that overcame [the path] from b to c, conserving itself, operates to that from c to d [the path] is equal to cb, and the violence which is gradually lessening when descending operates so that from d to e [the path] is equal to ba, since there is no reason from c toward de that shows that the violence is lost at all […].*"
> (Guidobaldo, 1592, in Damerow et al., 1992, p.151-152)

The text, following the analysis of Paola Fantini, shows that the discourse is articulated along three main levels, each one marked with a different colour. In dark orange, the first crucial passage for establishing physics as a discipline is described: from the description of an experience to the design of an experiment. Argument, originates and develops from the direct observation of the phenomenon, culminates with the ideation of the experiment. The profound epistemological change is highlighted in blue: these sentences regard considerations about the regularities and the symmetries of the motion. This represent the most counter-intuitive part of reasoning since it clashes with the current view of the motion of a thrown ball, and symmetry has to be explained in terms of combination of violent and natural motions. The sentences concerning the foundational question and its answer, "What shape can better represent the trajectory of the object?", are highlighted in green.

The three distinct planes concur to establish the methodological and epistemological foundations of physics as a discipline, and their combination outlines what can be called the *explicative level*. It can be noticed that the sequence of the colours varies in the order, and the register changes from the first to the second part. In fact, on the explicative level we are moving from the description of the phenomenon, to the interpretation of the observations, from the experience to the experiment. Within this explicative structure, Guidobaldo is delineating a new *method* to produce knowledge: he is telling us *which* relevant features of the phenomenon have to be visualised (the search for regularities in the motion) and *how* the hypotheses can be empirically tested, designing an experiment. Moreover, the excerpt is particularly interesting on the *epistemological level*. As already mentioned, it touches some crucial points in the evolution of the disciplinary knowledge. As a matter of fact, the motion is characterised as "*both being composed by the natural and the forced*", representing the first step in overcoming the medieval paradigm that keep the two distinct. The concept of symmetry is the epistemological knot that allowed this change in the point of view. This is the starting point for "mathematizing" the motion. In this excerpt, Guidobaldo arrives to retain that the curve is a catenary, because of its combination of matter and geometry that can make it more plausible than a "pure" conics. However, the door is open, and Galileo is ready to go on and to demonstrate that the combination of a "equable" and "accelerated" motion leads to a parabolic trajectory. Hence, in the short Guidobaldo's excerpt there are also the roots for the "mathematization" of the motion that is a completely new perspective, destined to become a structural pillar of physics in the following centuries.

This process of motion's mathematization goes on in Galileo's *Discourses and Mathematical Demonstration Relating to Two New Science* (1638) where the change of perspectives is represented by the transition from the catenary to the parabola in considering the projectile trajectory. At the end of the second day, Salviati presents a "*remarkable way*" of drawing the parabolic curve, originally studied by Apollonius. This method exploits the trace left by the motion of a spherical brass ball, thrown on an almost upright metallic mirror. Quoting the words of Salviati:

"*There are many ways of tracing these curves; I will mention merely the two which are the quickest of all. One of these is really remarkable; because by it I can trace thirty or forty parabolic curves with no less neatness and precision, and in a shorter time than another man can, by the aid of a compass, neatly draw four or six circles of different sizes upon paper. I take a perfectly round brass ball about the size of a walnut and project it along the surface of a metallic mirror held in a nearly upright position, so that the ball in its motion will press slightly upon the mirror and trace out a fine sharp parabolic line; this parabola will grow longer and narrower as the angle of elevation increases. The above experiment furnishes clear and tangible evidence that the path of a projectile is a parabola; a fact first observed by our friend and demonstrated by him in his book on motion which we shall take up at our next meeting. In the execution of this method, it is advisable to slightly heat and moisten the ball by rolling in the hand in order that its trace upon the mirror may be more distinct.*" (Galilei, 1638)

During the experiment with the brass ball, the dimension of the experiment, closely related to the physical aspects of the motion, intertwines with the mathematical considerations concerning the properties of the parabola. Salviati is referring on one hand to the physical object of the projectile trajectory, and on the other hand, to the mathematical object of parabola. More specifically, we could say that the parabola is now promoted to the rank of *ontology*, as it is one of the basic entities, a priori recognised as existent, that assumes a leading role in the description of the phenomenon. Now, when Salviati concludes the description of the experiment, he refers to the latter as furnishing "*clear and tangible evidence that the path of a projectile is a parabola*". This key statement implies that the physical object of the trajectory and the ontology of the parabola coincides. Reviewing both Guidobaldo and Galileo's work, we can say that this progressive process of identification has been possible only noticing the *symmetric quality* of the motion, and successively *mathematizing* it with the help of the right curve. In this sense, the symmetry acted as *epistemological* activator "An epistemological activator is an idea, a theme, or an activity, that has the potential either (a) to organise knowledge on a higher abstractive level, or (b) to set a new context where specific ideas can become leverage concepts. Because and by means of this potential, an epistemological activator also owns the power (c) to raise questions about the nature and the role of science itself." (Ravaioli, 2020). With this term, we want to address the profound epistemological change in the way of conceiving and describing the physical phenomenon of the projectile motion that the concept of symmetry could activate in Galileo's work.

In order to summarise our considerations, what we found during the analysis of the historical excerpts that ground the foundations of physics is the emergence of the different disciplinary dimensions that repeatedly intertwine in the text, more specifically:

- On an *explicative level*, the authors move from the initial description of the phenomenon to the actual interpretation of the observations.
- On a *methodological level*, the experiment is used to follow the conjectures on the motion and to support them. Moreover, mathematically the experiment clarifies some properties of the parabola.
- On an *epistemological level*, the paradigm that distinguishes natural and forced motion is at least partially surpassed; the mathematization and the consideration of the symmetry allowed the identification between trajectory and parabola.
- On an *ontological level*, the mathematical concept of parabola becomes an ontology in the physical context of the projectile motion (Fantini, 2019, from PLS presentation).

These considerations on the underlying structure of the text are very important because they enlighten some of the milestones in the evolution of the physical thinking, as the (implicit) definition of a method to produce knowledge, the progressive mathematization of the phenomena and the introduction of new paradigms that could substitute the previous ones. That is why in chapter three we will search for the echoes of these authentic disciplinary elements in the Walker textbook, confronting its meaningful passages with original memories taken from historical texts.

2.2.2 Linguistic tools for Disciplinary Identity (in Physics)

Let us start by making a premise: the focus of this work is to design criteria to analyse textbooks and original memories and point out possible hidden structures associated with their disciplinary features. For this reason, in this section, we report the aspects of a linguistic analysis that we selected as the most suitable for this task. We consciously omitted the linguistic tools that better match the didactical dimension of knowledge fruition (for example, the aspects that can be related to the comprehension and the engagement with the text).

Now we are ready to dig deeper into the linguistic aspects that will serve us in the analysis of the text and that look promising to become analytic lenses. They have been all stressed also during the PLS course by prof. Matteo Viale and Dr. Veronica Bagaglini.

First of all, the linguists strongly stress that a mathematics or physics textbook adopts a *mixed language*, combining elements of the natural language with specific terminologies, graphical representations, and particular lexical forms (Viale, 2019). It is also characterised by the communicative intent of *mono-referentiality*, for which the connections between the objects of study and their specific meaning is almost biunivocal. A consequence is that a scientific textbook is particularly *rigid* in its linguistic structural choices, in the sense that they are carried out to describe as exactly as possible phenomena, and use abstract constructs, and rigorous ways to illustrate experiments or report demonstrations. In most cases, the authors tend to leave little room, if any, to interpretations.

Having this concept in mind, Viale, in the PLS course, introduced one possible definition of *special language* (Cortellazzo, 2007). A special language is a functional variety of common language, that depends on a specific body of knowledge; it is used to satisfy the communicative needs of a group of people dedicated to that particular area of interest. This notion is fundamental if we want to approach a physics or mathematics textbook because we will inevitably deal with it. Linguistically, it is possible to denote the characteristics of a certain special language by conducting analyses structured on different planes. Among them, we will consider:

- *Lexicon*: the set of special terminology that appears. Scientific lexicon includes specific technicisms, redefinitions from the common language, eponyms, abbreviations that are all categories of this first level.
- *Syntax and Morphosyntax*. Scientific syntax and morphosyntax foresee the preference of some recurring forms or structures, as the length of the sentences, the extensive use of coordination or subordination, the degree of subordination.
- *Textuality*: Scientific textuality is characterised by the preference for particular genres. An analysis of the textual component regards the research of, for example, the order and the logical organization of the information; the type of cohesion of the text through the special use of connectives; the use of implicits or hypertextual references.

One important element that appears at the textual level is the role of the implicits. In the PLS course this level was addressed by Veronica Bagaglini, who stressed that implicits, in a text, can be either *implicatures or presuppositions* (Grice, 1975). Implicatures consist of the information that the speaker suggests or implies even though it is not literally expressed. In this sense, the implicature differs from the concept of *presupposition*. Differently from the first ones, the seconds stand for the implicit information on which the explicit message is built, therefore representing its foundation.

Both the notions of implicatures and presuppositions are valuable tools for our investigation because they address the problem of the critic assumptions that are not explicitly mentioned in the text. These assumptions are usually left to the reader to decode, relying on others section of the textbook or rather on the cultural background of the person.

With these last definitions we have all the material that we need. In order to briefly contextualise our case, we can say that approaching linguistically the Walker textbook will mean to study the special language of physics, in relation with the common language, the mathematical formulations and the figures that appear in the text.

Now, it is time to connect the dots and build the operative tool we need for our analysis.

On the basis of the re-analysis of the materials and reflections that were developed during the PLS course, I designed the following grid for the in depth examination of non-historical textbook:

DISCIPLINARY LENS

OVERALL STRUCTURE	How is the material organised in the chapter? Are the different elements effectively correlated? Is there a leading thread?
	Is it possible to recognise choices (special reasons, goals, criteria…) in the selection of the contents and in the order of the material? Could the contents be re-arranged in a different and more effective or relevant way? If so, how and why?
CORE CONCEPTS, ONTOLOGIES AND EPISTEMOLOGICAL KNOTS	Which are the conceptual knots in the text? Are these concepts defined or do they simply appear in the chapter? Does the reader need to deduce the definition?
	Among the conceptual knots previously pinpointed, what are the basic entities – the ontologies – on which the discourse is grounded?
	What are the epistemological knots in the text? What is their contribution in characterizing disciplinary knowledge?
	Are there any epistemological knots that act as epistemological activators in the text?
EXPLANATION AND METHODOLOGY	Does the text guide the reader to follow the reasoning and to grasp how the phenomena are analysed and interpreted? Does the explanation suggest methodological approaches typical of physics and of science?
LEXICON, SYNTAX, TEXTUALITY	What about the linguistic quality of the text? Is it comprehensible? In the key-moments of the text, are the linguistic aspects coherent enough to convey messages about the disciplinary identity? In other words, does scientific language emerge as a “special language”?
FIGURES	What types of figures are present in the text? What role/s do they play? Can a figure have more than one purpose?
	What kind of information do figures convey? How? Do they need any additional reasoning/information to be understood and interpreted?

Fig.2.2 – Grid for the disciplinary analysis

Following a similar path, now we are going to shift our attention from physics to mathematics, from the linguistic analysis of the text to the argumentative one. In doing so, our goal is to build an interdisciplinary lens that will focus on the structural role of mathematics into the physics dissertation.

2.3 The construction of an interdisciplinary lens

2.3.1 Parabola in mathematics and the mathematics of parabola for physics

As stressed by Laura Branchetti in her intervention, in the PLS course, on parabola from a mathematical point of view, etymologically, the term "parabola" derives from the Ancient Greek word parabolé, descending of parabállō, which means "to set side by side; to parallel". This definition can be interpreted in a geometrical perspective, considering the parabola as the *conic section* formed by the intersection of a cone with a plane, *parallel* to a generatrix of that cone. This way of constructing the figure implicitly refers to one of the most influential work on the topic, *Conics* by Apollonius of Perga (262 a.C. – 190 a.C.). This production can be seen as the generalization of the results obtain by Euclid in the XI book of his *Elements*. The approach of the *Conics* has many innovative features, starting from the different construction of the cone (which now become a double cone, with a circular base) and the presentation of the parabola, hyperbole, and ellipse as different conic sections of the *same* cone, differently from Euclid.

For the present discussion, and following professors Branchetti and Cattabriga, we are interested in the general structure in which Apollonius' work is articulated and, particularly, in the use of demonstration as a mathematical practice with an intrinsic epistemic value for physics. For this reason, a brief contextualisation of the topic of demonstration, as intended and researched in the field of Didactics of Mathematics, is needed.

Formally, we cannot define a demonstration without explicitly considering the *context* in which it is formulated. This fact imposes to envision the concept of demonstration as included in a more general triplet of elements forming a theorem and consisting of a *statement*, its *demonstration*, the *theoretical framework* of reference (Mariotti, 2000). The theory is an essential aspect to consider; in fact, only with respect to a previously established set of axioms and definitions, a demonstration can be considered true or not. The fourth fundamental element that has to be added to the triplet is the *meta-theory*, i.e. the set of formal rules that allow to derive theorems from the starting group of axioms and definitions. The law of non-contradiction, the trichotomy law, the principle of bivalence are all possible elements of the metatheory (Mariotti, 2000). Differently from the theorem, the *conjecture* can be defined by the union of the *statement*, the *argumentation*, and the *system of references* needed.

This definition is important because it allows us to confront the two dimensions of demonstration and argumentation. Moreover, it logically follows that the set of demonstrations is strictly included in the wider set of argumentations. In fact, not every conjecture is a theorem, but every theorem could also be seen as a conjecture. These notions clarify the concepts of demonstrations and argumentations, but they are hard to implement in the analysis of the text. Hence, operatively, we can consider the *nature of the initial assumptions* as the main difference between demonstration and argumentation. In the first case, these assumptions are given and assumed as true; in the second, they are more negotiable and hypothetical. The aspect of *persuasion* is the other distinctive feature, mainly associated with the practice of argumentation. Demonstration, which starts from indisputable axioms and moves through conclusion with the help of the metatheory, has implicitly the assumption that the conclusion is true, without the need to convince the recipient. On the contrary, an effective argumentation requires persuasion as a structural component.

Now that we highlighted some key concepts regarding the demonstration, we can return to the work of Apollonius. Following the argumentative structure of Euclid's *Elements*, Apollonius starts his *Conics* with two sets of definitions, using them to demonstrate the numerous set of successive propositions. For example, in the first book, geometrical objects, like the cone or the conic section, are defined. After that, sixty propositions and various corollaries follows. These propositions concern the different curves that are generated by a plane secant to the cone (namely, the parabola, hyperbole, ellipse, and circumference), explaining their properties. The statements are progressively demonstrated, often with the support of figures. This example of rigorous and organic structuration of the dissertation is one of the valuable inheritances that comes from the ancient Greek geometry, crucial for the historical development and expansion of both mathematical *and* physical body of knowledge. In order to clarify this assertion, let us consider once again Galileo's *Discourses and Mathematical Demonstrations Relating to Two New Sciences*. The book is divided in four days, each one addressing a different areas of physics. It was written in a style that resembles the *Dialogue*s; three men, Salviati, Simplicio and Sagredo, debate the various questions that Galileo is seeking to answer. Looking at the title, the words "mathematical demonstrations" suggests what could be the configuration of the book. In fact, although Galileo privileges a dialogic style for his works, the organisation of the material is similar to the one used in the *Elements* or in the *Conics*. Once again, the process of argumentation moves from the definitions and axioms, to theorems and their mathematical demonstrations. As a relevant example, the third day of the *Discourses*, concerning the study of local motions, opens up with the fundamental definition of the uniform motion:

> *"By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal."*

From the above definition four axioms follows. They cover the mutual relations between the physical entities of time, space, and velocity for uniform motions in different cases. These axioms are used to demonstrate a sequence of theorems concerning the various properties of both uniform and accelerated motion. All this background architecture allows Galileo to adequately treat the case of a moving object, studying the composition of its uniform and accelerated motion. In particular, the first of the theorems that Salviati demonstrates is the following:

> "*Theorem 1 – Proposition 1*:
> *A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola*."

The interesting demonstration that Salviati adduces to prove the theorem will be discussed and analysed in the next chapter. For the moment, we conclude this part with some general considerations. As Apollonius does in his work, Galileo defines *all and only* those axioms that are needed for the following argumentation. In the *Discourses* framework, the Italian physicist mathematizes his space-time intuitions in order to geometrically demonstrate the theorems. These two operations create a solid correspondence between the mathematical objects and the phenomenological aspects underling them. In this context clearly emerges the structural role of mathematics, which provide a way of logical deductive reasoning for physics, moving organically from initial assumption to conclusions. On the contrary, in the *Discourses* mathematics does not assume an instrumental role, in a more pragmatic sense of "being a tool for computations".

2.3.2 Argumentative tools for interdisciplinarity

Argumentation, as already stressed, is deeply different from a mathematics demonstration, mainly because of the different "nature" of its assumptions. Following the reasoning of Professor Sebastiano Moruzzi developed in the PLS course, we can enter more precisely in the types of argumentations and, in particular, in the types of *arguments* that an argumentation produces. The argument coincides with the sequence of propositions, arranged in a precise order, and mutually connected by means of inferences. Through this system of inferences, an argument moves from initial hypotheses, assumptions, or preconditions to the conclusions. In stating this definition it is important to notice that the propositions of the argument can be also expressed through non-linguistic expressions, like diagrams or graphs. This fact imposes to include the graphical representations or the mathematical formulas in our analysis of the arguments. Additionally, while considering the argumentative structure of the text, we can distinguish:

- The *logical* structure of the argument, concerning how the inferences are organised in the text, moving from the premises to the conclusion;
- The *epistemological* structure of an argument, concerning the organization of the justification, more specifically the introduction of new justifications of the conclusion;
- The *dialectical* structure, i.e. how the argument is organised in relation to the knowledge and personal beliefs of the reader;
- The *rhetorical* structure, that is the set of strategies used in the argument to persuade the reader.

In relation to those, if the logical and epistemological structure of the argument do not present problems, it is said to be *argumentatively correct*. Complementarily, if the dialectical and rhetorical structure allow the argument to resonate with the reader, it is referred to as *argumentatively effective*. In the next chapter, we will focus more on the argumentative correctness of the texts, being the effectiveness more aligned with the didactical dialogue between the textbook and the reader. In particular, establishing if a text is argumentatively correct means to check its *logical* and *epistemological validity*. This is equals to verify *both* that the conclusions of the arguments are true, if the preconditions are also true, *and* that at least one new meaningful justification of the conclusions is acquired in the process. We can notice that, with the implementation of this validation method, defective arguments are cut off from the analysis. For example, despite circular arguments are logically valid, they do not produce new justifications of the conclusions. Therefore, we cannot say that they are argumentatively correct. To better visualise this point, let us consider the following example proposed by Moruzzi:

P1: Those animals are zebras.

P2: Zebras are not mules that were painted in stripes.

C: Those animals are not mules that were painted in stripes.

The argument, moving from the proposition P1 to the conclusion C, is logically valid. Anyway, if we examine the epistemological structure, we do not acquire a new justification of the conclusion. In other words, we do not learn anything new about the object of investigation. Arguments like the one just considered are called *complex circular arguments*. In a physical or mathematical context, the circularity of the argumentation becomes a crucial element to investigate. For instance, the conclusion of an argument could be implicitly and simultaneously taken as one of the starting assumptions, in this case structure of the implicatures collapse in the simple: “if P then P”. In the next chapter we will consider a similar case of complex circular argument in the textbook.

Analogously at what we did earlier in this chapter, by combining the reflections on the underlying mathematical structure of the *Discourses* with the notions concerning the analysis of the argumentation, we obtained the following operative tool:

INTERDISCIPLINARY LENS

ROLES OF MATHEMATICS	What type of mathematics is used in the text? What is its role (instrumental, communicative, structural, others)?
	Does mathematics support the understanding of physical properties and physical qualities or, instead, does the formalism tend to hinder the physical sense of the phenomenon?
	What connection is built between mathematical object and physical entity in the argumentation?
ARGUMENTATIVE STRUCTURE	In the key-moments of the text, what are the argumentative structures? What is the prevalent dimension of the arguments (logical, epistemological, dialectical, and/or rhetorical)? More specifically, is the argumentation valid from a logical, epistemological, dialectical, or rhetorical point of view? Does the reader gain new knowledge, following the argumentation? On the contrary, are some arguments in the text epistemically inadequate (for example circular arguments)?
	Are there interesting examples of arguments that can be used to reflect of the argumentative structures used in a physics textbook?
DEMONSTRATIVE STRUCTURE	Are there demonstrations? If so, are all the elements of a demonstration explicit? In particular, can a student recognise the initial assumptions, the framework theory, and the demonstration?
	Are the assumptions taken as fixed and indisputable or do they appear as negotiable and modifiable?
	Did the author previously define or demonstrate all the objects used in the demonstration? If not, what does this choice entail?

Fig.2.3 – Grid for the interdisciplinary analysis

2.4 Final comments: a disciplinary characterization from the analyses

Before applying the lenses to analyse a textbook, we want to briefly expose the relevant information that came to the surface by the historical analysis of the parabola, as presented in the PLS course:

1. Physics and mathematics progressively built *specific terminology*, around the concept of parabola, in order to address the new constructs that are introduced. In addition, this

terminology mostly differs in the two contexts; a physicist, for example, would more likely rely on words like trajectory, range, projectile rather than directrix, eccentricity or conic section. This fact can be exemplified by reading the proposition I.11 from Apollonius' *Conics*, where the parabola is characterised, and the demonstration of the Theorem 1- Proposition 1, from the fourth day of the *Discourses* by Galileo, concerning the shape of trajectory drawn by a projectile.

2. The *context* in which the research evolved is completely different. The same object of investigation was considered inside the physical topic of two-dimensional kinematic in the case of Galileo and Guidobaldo, into the geometric study on conic sections for Apollonius, in the optical study of reflection and refraction in the case of Kepler. In an epistemological perspective, the different bodies of knowledge, gathered in time, are complementary and represent an extension to the concept of parabola and to its applicability.
3. Mathematics and physics used different *methodologies* to produce new knowledge. The first, moving from definitions and axioms through the demonstration of theorems; the second, applying the scientific method, hence moving from the observation of the experience and the formulation of conjectures, to the designing of an experiment to text those conjectures. Moreover, physics used the same curves studied in the mathematical context of the conic sections, addressing to certain qualities of the phenomenon, therefore recognising the geometrical entities of parabola, hyperbole, and ellipse a priori as ontologically existent. Anyway, as we saw for Galileo's *Discourses*, the way of representing the collected knowledge in physics takes as a reference the demonstrative structure of the ancient Greek geometry, like the work of Euclid and Apollonius. In this framework, mathematics assumes a structural role for physics.
4. Here, we stressed that the mathematical notion of symmetry and parabola had been implemented in the study of the projectile motion in the work of Galileo and Guidobaldo. Furthermore, Galileo adopted a rigorous mathematical structure for his *Discourses*, make explicit the demonstrative intent of his book. This episode is an example of what Tzanakis (2016) means by referring at the mutual and fruitful interrelation of the two disciplines in their historical development. An important episode that moves from physics to mathematics in the history of parabola is the role of Kepler that here we did not consider. Kepler, in investigating the particular physical problem of reflection and refraction in his *Astronomiae Pars Optica* (1604), fostered the expansion of mathematical knowledge. In particular, the focal point of the parabola was defined, and the different conics were mutually deduced one from the others by a gradual shift of one mobile focal point along the symmetry axis.

It has to be specified that these points, representing some relevant features of physics and mathematics as disciplines, are contextualised to the present study on the parabola. They do not necessarily hold true in the totality of situations. The work of Kepler, briefly mentioned in the second and fourth point, was not shown in the present chapter.

3. Textbook Analysis

3.1 The text selection

In every meeting of the PLS course, after the introduction of the disciplinary and the interdisciplinary tools for the analysis of the texts, the teachers were divided in groups and they were asked to examine a series of textbooks of mathematics and physics with different lenses. More specifically, they were asked to analyse how the mathematical object of the parabola and the physical topic of the projectile motion were presented, from a disciplinary, linguistic and argumentative perspective. For the team work we had selected secondary school mathematics and physics textbooks, and their year of publication spanned over an arch of time of four decades.

The choice of the textbooks was very difficult since the discourse was, in all the textbooks, very simplified. Any reflection on the disciplinary and interdisciplinary dimensions risked to be only a ascertainment of weaknesses or lacks in the textbook. In order to avoid this, all the analytic activities were organised to suggest a positive and constructive attitude aimed to resolve textbooks problems with the design of further activities or with the integration of other materials.

Nevertheless, the distance between the textbooks and the type of discourse that the course was arguing to be effective to trigger a reflection on physics as discipline and on its integration with mathematics was definitively too huge to be filled in with those activities. The interpretation of such a gap is a core goal of this thesis and of this chapter, in particular.

The starting point of the analysis are the following important considerations that emerged from the various discussions concerning the analyses carried out by teachers:

1. The four disciplinary levels, that is the ontological, explicative, methodological and epistemological dimensions, were not easily recognisable and discernible in the texts;
2. In the physics textbooks, mathematics mainly appeared to assume an instrumental role, used mainly as an algebraic tool for computation;
3. The argumentative structure of the texts was very difficult to be pointed out, being the sentences more built to pass information that arguing;
4. From a linguistic point of view, the texts do not result coherent and the structure of the implicits did not appear precise enough to support reasoning.

In order to better comprehend these four points, let us consider one book as an explicative example. The text is articulated in two main sections, titled *The principle of composition of motions* and *Motions in two dimensions: the motion of the projectile*. The whole topic is entirely developed in about five pages: in less than two pages the parabolic motion is introduced and discussed, the remaining part is totally dedicated to guided examples and exercises. In this text, like in the others, the topic is addressed with peremptory sentences:

*"Let us consider the motion of a point particle in the plane. The motion of the point particle, once we fix a Cartesian system of coordinates, can be decomposed in two mutually **independent motions**, one along the x direction, the other along the y direction. The motion on the point particle is hence described by a position vector that varies in time, of components x(t) and y(t), which are respectively the laws of the time of the projections of the point on the two coordinate axes.*

*For the motion in the plane the principle of composition of motions is therefore valid: in two-dimensional motions, the **total displacement** is the vectorial sum of the displacements that take place in each one of the two dimensions:*

$$\vec{s}(t) = x(t)\hat{x} + y(t)\hat{y}$$ "

In this excerpt we read that, *once that a Cartesian frame of reference is fixed*, the motion of a point particle *can be decomposed* in two independent motions. The author does not specify what the term *independent motion* means and the proposition is not demonstrated, commented or exemplified in any way. It is given as a matter of fact. Then, the text proceeds assuming that, for a motion that can be described in a two-dimensional frame of reference, it is possible to apply the principle of composition of motions. These few lines are the only description we have of independent motions. After some examples and exercises, the discussion resumes in the second section:

*"The motion of a point particle of mass m, thrown with a certain initial speed v_0 and subjected only to the action of the force of gravity, is called **motion of the projectile**.*

Let us choose a frame of reference with the x axis parallel to the ground and the y axis perpendicular to the ground and directed upwards. The principle of composition of motions allows us to describe the motion of the projectile as the composition of a rectilinear uniform motion along the x axis and a uniformly accelerated motion with constant acceleration -g = -9,8 m/s^2 along the y axis.

Let us indicate with x_0 and y_0 the components of the initial position of the projectile and with v_{0x} and v_{0y} the components of its initial velocity; the laws of the time are: [...]

Let us determine the equation of the trajectory from the laws of time of the position, eliminating the time parameter; the Cartesian equation that is obtained is the one of a parabola: [...]"

The passage starts with a brief definition of the projectile motion. The situation has been already modelled to a certain extent (the author does not refer to a generic object, but to a point particle with a certain mass and initial speed); there is no trace of the initial assumptions or hypotheses that have been made to mathematize the phenomenon. The equations of motions are not deduced step by step but rather appear in the text as a consequence of having applied the principle of composition of motion. The section ends up with a brief mention that the quadratic relation between the x and y components, that the reader can obtain by doing a time substitution, is the analytic equation of a parabola. The algebraic passages are not shown.

We chose this example since it shows a typical way to present parabolic motion in physics textbooks: the presentation starts from the assumption of motions independence and, once a Cartesian frame is introduced, motions independence is translated into the mathematical rule of vector composition and decomposition. This mathematical apparatus is then applied to the projectile motion, where motions composition leads to find out parabolic equation for the trajectory. The style is what we described as peremptory, where assumptions are given as a matter of fact and mathematized.

This example allows us to argue why we retain that textbook discourse not rigorously "disciplinary": the different levels (ontological, epistemological, explanatory, and methodological) that characterise an authentic disciplinary discourse do not appear. Moreover, mathematics is not used to structure an argument or a reasoning, but it is used as an algebraic tool for computation.

This is what emerged from the discussions and in the team-work of the PLS course. The discussions were very vivid and stimulating since the analysis acted effectively as "pars destruens": it allowed the teachers to point out the reasons of discomfort that they perceive when try to use the textbooks as basis for teaching. In particular, from the perspective of IDENTITES, it became evident why in the current textbooks neither authentic disciplinary elements, nor an explicit interplay of mathematics and physics on average appear.

After the course, we were left with the difficult task of finding a valid candidate to be analysed as basis to build activities for preservice teachers. The research of adequate textbooks is indeed a fundamental operation in the context of the IDENTITIES project.

The project is based on the idea that preservice teachers can be guided to develop disciplinary and interdisciplinarity tools also through the analysis of texts and through the recognition, in the texts, of "*epistemological"* and *"linguistic activators*", that are "epistemological and linguistic concepts or themes able to activate a meta-level of analysis from which the disciplines can be observed, compared and intertwined, moving, back and forth, from the details to the big picture." (www.identitiesproject.eu).

The previous statement is founded on the assumption that it is possible to conduct this kind of analysis of the textbooks. Nevertheless, as we already stressed, the first analysis of textbooks pointed out a serious problem because of the huge distance between school physics and the historical discourses that founded the discipline. Therefore, this thesis has been designed to check, validate or confute the assumption on which IDENTITIES goal ground: do suited physics textbooks exist on which it is possible to carry out a deep analysis using the disciplinary and interdisciplinary lenses created in the previous chapter and design, on the basis of that analysis, activities for teacher education?

In the period right after the end of PLS course we examined physics textbooks for university students, and we bumped into the 5th edition of *Physics* by James S. Walker (2017). In the fourth chapter of the text, dedicated to the two-dimensional kinematics, the motion of the projectile assumes a leading role and it is studied extensively. The independence of motions is introduced and debated under multiple perspectives; at a first glance the arguments appeared solid and structured and the text seemed coherent. Furthermore, at the end of the chapter, an entire section is dedicated to the symmetries of the motion; this fact also represented a possibility to make a parallel with the Galileo and Guidobaldo's excerpts examined in the PLS course. All these reasons encouraged us to opt for this textbook to examine it under the disciplinary and interdisciplinary lenses built in chapter 2.

We mainly referred to the original version, but we also compared it with the Italian translation: *Fondamenti di Fisica* (2020), edited by Giovanni Organtini and published by Pearson Italia.

3.2 The disciplinary analysis of the Walker textbook

Now we are ready to address our analysis of the Walker textbook. We focus on the fourth chapter of the textbook, titled *Two Dimensional Kinematics*. We will begin by presenting what emerges on a disciplinary level from the text. The results have been obtained by analysing the Walker through the disciplinary lens built in the previous chapter (see table 2.1).

3.2.1 Overall structure of the text

In Walker textbook, the chapter titled *Two-Dimensional Kinematics* is the forth chapter of the text and follows up the chapters dedicated to the one-dimensional kinematics and to the vectors in physics. In particular, some of the important results that were previously obtained are resumed and used. The fourth chapter consists of 21 pages, articulated in five paragraphs. The theoretical discussion of the topic is alternated by numerous guided examples.

The first paragraph is titled *Motion in Two Dimensions*; in the beginning, the motion of a "turtle" in a virtual two dimensions space is exploited in order to introduce the reader to the independence of motions. The motions independence is introduced by showing the analogy between two procedures: calculating the distance traveled by the turtle as $d = v_0 t$ along its straight line and, then, calculating the components of the distance along x and y by projecting the vector distance on the axis. The same result is obtained by, first, projecting the vector velocity on the two axes and, then, calculating the distances traveled along x and y. The agreements between the two results lead the author to conclude: “To summarise, we can think of the turtle’s actual motion as a combination of separate x and y motions.” (Walker, 2017, p.89).

This example is also used as a starting point to deduce the equations of motions in the general case, for an object moving in two dimension.

The second paragraph, titled *Projectile Motion: Basic Equations*, develops the physical model of the projectile motion as *an application* of the independence of motion. In the paragraph, the concept of projectile, as intended in physics, is defined and the starting hypotheses are established. In particular comments about the gravitational acceleration of the object, and the possibility to neglect the Earth rotation and the air resistance are reported. These assumptions are incorporated in the general equations of motion deduced in the second paragraph and the equations for the case of the projectile motion are obtained. The section concludes with a an “experiment” that the reader has to imagine. The experiment illustrates the independence of motions in a real-life situation.

The third paragraph is titled *Zero Launch angle* and discuss the particular case of a fully horizontal initial velocity of the projectile. In particular, the results obtained are used by the author to algebraically demonstrate that the trajectory of the object is parabolic. The last section is dedicated to obtaining the mathematical expression of the landing point. The fourth paragraph, titled *General Launch Angle*, is the logical continuation of the third one; the general equations of motion for the projectile are deduced.

The fifth and final paragraph is titled *Projectile Motion: Key Characteristics* and represents the most interesting part of the chapter. Here, the mathematical expressions for the *range* and *maximum range* are deduced. Most importantly, the author reserves a full section to the study of the symmetries in the projectile motion. More specifically, the properties of symmetries concerning the time of flight, the velocity vectors at a given height, and the range of the projectile are presented.

The map reported in fig. 3.1 provides a big picture of how the topic of two-dimensional kinematics is presented and developed in the chapter.

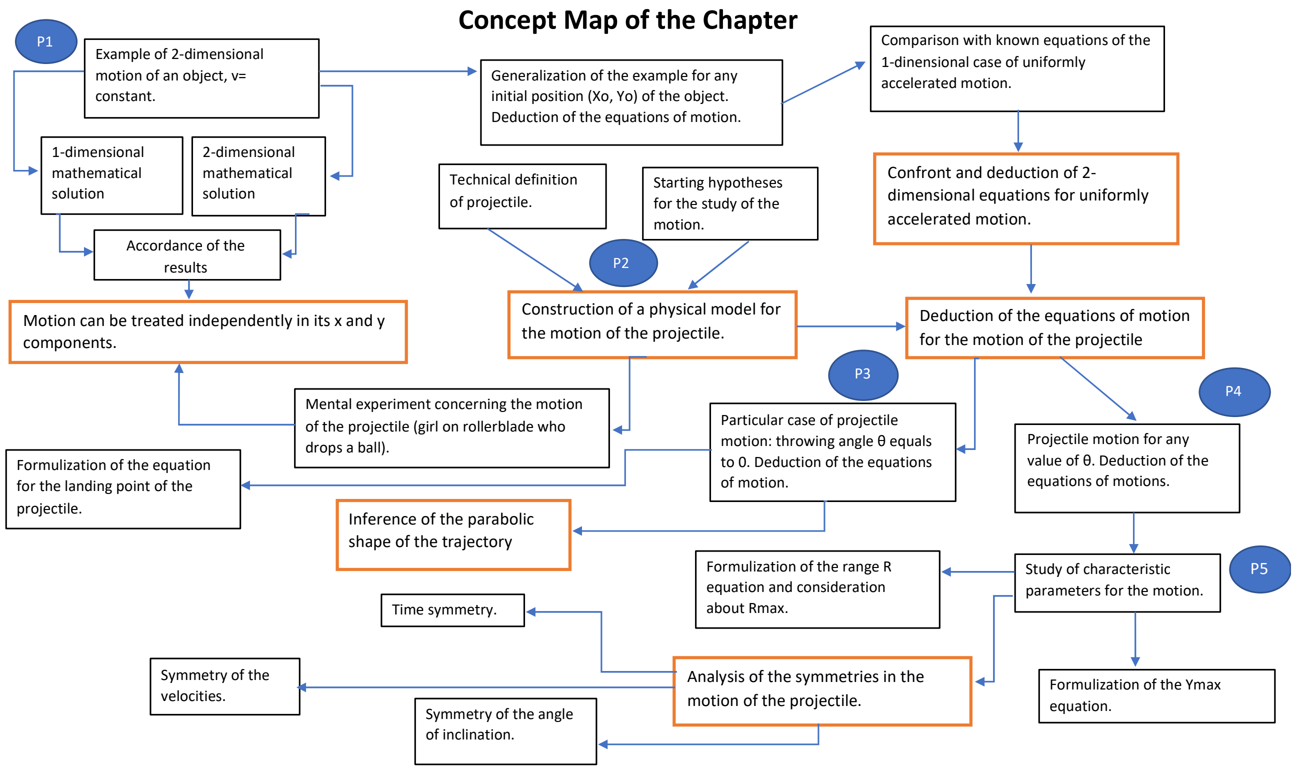


Fig. 3.1- Concept map summarizing the fourth chapter of the Walker textbook.

In figure 3.1, the blue circular frames indicate the relative paragraphs in the chapter; they were added as a further reference to the reader. More important, the orange boxes represent the central steps of the discussion. By looking at how these boxes are concatenated, we are able to effectively reconstruct the argument, as it was proposed by the author.

We can observe, first of all, that:

a) the projectile motion is defined only once the general concept of two-dimensional motion of an object is clarified and the independence of motions is exemplified;
b) the physical hypotheses and assumptions about the projectile motion are explicit and precede the deduction of the equations of motion;
c) the parabolic shape of the trajectory is discussed later on in the chapter, only when it can be mathematically proved.

Overall, if we follow the arrows in the figure 3.1, we move systematically from the top to the bottom of the map and from the beginning to the end of the chapter. The only anomaly is represented by the loop structure between the two orange boxes *Construction of a physical model for the motion of the* *projectile* and *Motion can be treated independently in its x and y components.* This loop represents a singularity in the chapter and needs to be further analysed; in particular, we will discuss in which sense the circularity of the structure vanishes during the interdisciplinary analysis (see Section 3.3). Lastly, we can say that the main idea of the chapter is that the horizontal and vertical motions are independent. This fact represents the leading thread of the discussion, as the author himself says in the introduction of the chapter. In fact, showing the independence of motions is the first step in deducing the general equations of motion for a point particle moving in two dimensions; it also constitutes one of the fundamental assumptions needed to create the mathematical model of the projectile motion.

It is possible to recognise that the material in the chapter is arranged in a conscious and explicit way. In this sense, it can be easy to add mastery materials, in case a teacher would like to address, in a deeper and wider way, some issues. As we will show in the next chapter, a more historically oriented activity can be designed to integrate the text. There the historical approach is used to stress the three breakthrough moments: the passage from the representation of motions in Tartaglia and Cardano to the symmetric trajectory built by Guidobaldo through the ink ball experiment and its contribution to assume the motions independence; the problematic interpretation of the curve of the trajectory and, in particular, the problem of distinguishing between a parabola and a catenary; Galileo's demonstration of the parabolic trajectory, thanks to the crucial step of introducing a third type of motion, the "neutral one", that allowed Galileo to overcome the Aristotelian distinction between forced and natural motions. As for the last point, following Guidobaldo and Galileo's works, the *Galilean* concept of *inertia* and the *law of free fall* could be coherently commented and stressed in their revolutionary scope.

Other possible integrative activities can regard the fundamental role of Apollonius' *Conics* in providing new interpretative schemes for the study of the physical phenomena or, as we have seen in Fantini's analysis, Guidobaldo and Galileo's excerpt (see Section 2.2.1) can be used as lens to recognise, in the textbook, the authentic elements that characterise physics as a discipline on the explicative, methodological, epistemological, and ontological level.

Furthermore, a historically informed presentation of the topic can certainly benefit from the implementation in the class of the original experiments, like the one designed in *Guidobaldo's Notebook* (Cerreta, 2019), and from the direct study the original memories.

3.2.2 Core concepts, ontologies, and epistemological knots

In the text, we pointed out the list of disciplinary knots that include "*ontological entities*" and "*epistemological activators*" that we represent in the following figure:

Disciplinary knots

Gravity, freefall, Cartesian frame of reference, uniform motion, accelerated motion, two-dimensional motion, trajectory, air resistance, launch angle, maximum height…

Ontological entities: projectile, parabolic trajectory

Epistemological activators: symmetry, independence of motions.

Fig. 3.2 - List of the conceptual knots and ontologies of the chapter.

It was not easy to find out the disciplinary knots, on which the discourse is based. The criterion we used is that of selecting only the "pillars" of disciplinary reasoning, that is those entities that belong to the discipline and that are necessary to support reasoning: without them disciplinary reasoning could not hold. Among them, there are disciplinary knots that are needed to create the context and others that represent the focal objects of the discourse. We called the latter *ontological entities* and, in this case, they are: *projectile* and *parabolic trajectory*.

Both of them are the result of a typical process of modelling and refer to entities that belong to a disciplinary discourse. The first one is defined as follows: "*a* ***projectile*** *is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity*" (Walker, 2017). The definition combines perceptual aspects ("*an object*"), actions that justify the etymology of the name (it *is "thrown, kicked, batted, or otherwise launched into motion"*), the only properties that matter and that situate it into an "inferential net", that is a system of links with other phenomena and their explanation (it "*follows a path determined solely by the influence of gravity").* The entity "projectile" was already introduced in the very first page of the chapter with these words: "*When you hear the word projectile, you probably think of an artillery shell or a home run into the upper deck. But the term projectile applies to any object moving under the influence of gravity alone. For example, a juggling ball undergoes projectile motion—and follows a parabolic path—as it moves from one hand to the other. In this chapter we explore the physical laws that govern projectile motion."* In the Italian version, the etymological meaning is added: *"the word projectile means "something that you can 'project", that is 'launch"*).

As well as the disciplinary knot represented by *projectile*, only a few are explicitly defined, like *free fall*, and *range*. The first term is recalled in one of the boxes in the introductory section (as the precise definition was given in the second chapter of the text): "*Objects in freefall moves under the influence of gravity alone"*; the second term is defined as "*the horizontal distance it [the projectile] travels before landing".* All the others are left to the common knowledge of the reader, like *trajectory*, *air resistance*, *gravity*, *maximum height*, *throwing angle*. A third category is made by those objects whose

definition is not explicit but has to be deduced from the discourse. This is true for the concepts of *two-dimensional motion* and *parabolic trajectory*.
As for *parabolic trajectory,* it represents the second *ontological entity*. It is one of the three big ideas, together with the definition of projectile and the big idea that horizontal and vertical motions are independent. The parabolic trajectory emerges from a mathematical reasoning that we will describe in the interdisciplinary analysis (see §3.3). We can anticipate that, even if it is mentioned in the list of the three big ideas, it does not have all the emphasis that it could deserve, given the historical revolution it introduced.
As for the third big idea, that is the independence of the horizontal and vertical motions, it does not represent an ontological entity but it is the leading thread and the core epistemological activator.
The concept of independent motions is articulated throughout the chapter. It is introduced by an exemplification of a 2-dimensional kinematics at the beginning of the first paragraph but it is never explicitly defined. In the context of this chapter, understanding what *independent motions* means should represent a priority, especially considering that the projectile motion in the second paragraph is mathematized exactly by *applying* the independence of motions.
Another concept could act as *epistemological activator*, but it is not only partially used as such in the text: the concept of *symmetry*, It, historically, led Guidobaldo and Galileo to question the Aristotelian view of motions and led them to speculate about the independence of the motions and about the shape of the projectile trajectory as a catenary. In the text, the intrinsically symmetric shape of the parabola represents the opportunity to discuss the "*many striking symmetries in projectile motion*", that is the mathematical properties of the projectile trajectory. The geometrical properties of the curve are highlighted as a way to search for interesting physical properties, exploring the relation between physics and mathematics. However, in the last paragraph, an interesting comment about the potential of symmetry is stressed. In particular, the author highlights that the research of symmetries is disciplinary characterised as follow: "*Symmetries such as these are just some of the many reasons why physicists find physics to be "beautiful" and "aesthetically pleasing." Discovering such patterns and symmetries in nature is really what physics is all about.*" (Walker, 2017).

3.2.3 Explanation and Methodology

Overall, the text guides the reader throughout an overall reasoning, following the concept of motions decompositions. In particular, the knowledge is progressively built exploiting various methodological approaches, typical of the physics research:

- In the first paragraph, the independence of motions along the x and y axes is shown with a *virtual example* of two-dimensional kinematics. As we already mentioned, in the problem, it is asked to find the distance travelled by a turtle, known the initial velocity, the time duration of the displacement and the angle between the velocity vector and the x axis. The solution is obtained firstly by decomposing the velocity vector after its calculation as the motion were in the one-dimension and, then, by combining the velocity components after their calculations along the x- and y- axis (in a two-dimensional formulation). The accordance of the two results is commented as the first formal argument supporting the possibility of decomposing the motion along the two dimensions. In this way, the independence of motions is not rigorously proved, but it starts acquiring some degree of plausibility.

- In the second paragraph, the *physical model* of the projectile is created. The author starts from establishing the initial hypotheses, concerning air resistance, earth rotation and the acceleration of gravity, elucidates their validity range and translates them into mathematical form. After that, the equations of motion are deduced for this specific case. As already stresses, the concept of projectile is introduced, in the Walker, by highlighting the modeling process that leads to position it between the experiential level and the formal one.
- At the end of the second paragraph, the author guides the reader to imagine a situation and uses it to "demonstrate" the independence of motions. The situation is presented as it were reproducible in daily experience or, however, convincing because of its connections with daily experience. The experience is represented in the following figure:

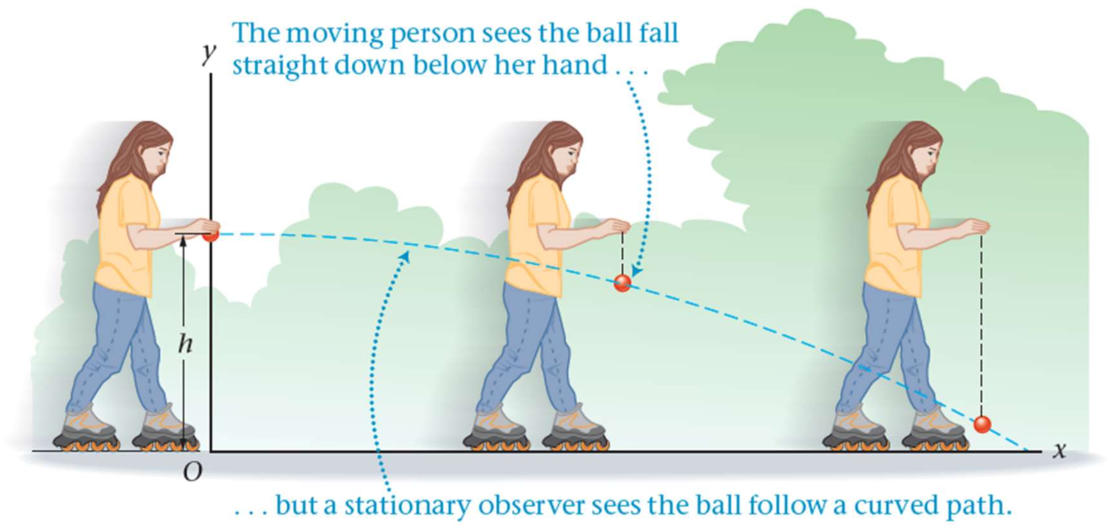


Fig 3.3 - Mental experiment on the independence of motion in the Walker textbook (p.93).

Figure 3.3 shows a girl, moving at uniform speed on the rollerblades. At a certain moment, she drops the ball to the ground. The book invites to image that the motion of the ball appears to the girl as directed straight down. On the contrary, a stationary observer will see a "curved path that combines horizontal and vertical motions" (p. 93).

Following author's argument, the reader is led to conclude that, since the horizontal motion of the girl had no effect on the vertical motion of the ball for the entire duration of the fall, then the horizontal and vertical motions are independent. This case is used to infer new justifications for the conclusion - the independence of the motions -, leveraging on imagination.

- In the fifth and last paragraph, the characteristic parameters of motion path, like the range or the maximum height, are deduced by means of *direct algebraic manipulations*. In this case, the reader is guided to acquire new knowledge on the projectile motion by exclusively operating in the mathematical realm.

It also needs to be said that the *physics experiment* as a fundamental method of producing knowledge and particularly of testing the conjecture cannot be covered on a textbook. Anyway, various simulations are included in the chapter. Furthermore, the phenomenological dimension of the experiment is switched on by the numerous real-life pictures in the chapter, whose main function is to convince the reader of the reliability of the conclusions obtained.

It is important to say that in the overall fourth chapter, the fundamental epistemological and ontological elements concerning the topic of two-dimensional kinematics are treated into an explicative structure typical of the physics discipline. In fact, differently from the textbook that we described in section 3.1, if we analyse the key moments in Walker's textbook, various disciplinary levels emerge. In order to prove this point, we propose the analysis of the second paragraph of the chapter, concerning the model of the projectile and the skater girl's example presented above. The analysis of the paragraph was carried out in the style of Fantini's analysis of the excerpt from Guidobaldo's notebook (see Section 2.2.1).

First of all, in order to contextualise the paragraph, we can say that at this point of the chapter we are already moving into a structured model of the 2-dimensional motion. In fact, in the previous pages we deduced the equations describing the general case of a uniformly accelerated object. We also have a clue (but not a proof) that motions in two dimensions can be treated independently, considering the horizontal and vertical component in the chosen frame of reference. The paragraph begins as follow:

We now <u>apply the independence</u> of horizontal and vertical motions to projectiles.

This is our starting point; the independence of motions is our initial assumption. The verb "to apply" leads to an ambiguous statement, meaning "to consider the motion of the projectile by assuming that its components can be dealt with independently of one other". This initial step is fundamental at the epistemological level because it specifies how the phenomenon is approached and what is its relationship with the concept of independence of motions.

Well, a projectile is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity. As you might expect, this covers a wide variety of physical systems.

The first step to take, on the ontological level, is specifying what we mean by "projectile". As already stressed, we obtain our definition invoking a perceptual dimension (the "object"), but also an experiential plane, introduced by the use of the verbs "to throw", "to kick", "to bat". Our characterisation circumscribes the process and contextualises it. In fact, we are not interested in what causes the motion. Additionally, we request that gravity is the only exerting force.

In studying the motion of the projectile we make the following assumptions:

- *Air resistance is ignored.*
- *The acceleration due to gravity is constant, downward, and has a magnitude equal to $g = 9.81\ m/s^2$.*
- *The Earth's rotation is ignored.*

Now, we can make some assumptions about the phenomenon. We are looking at the physical environment, imposing suitable constraints for the sake of the discussion. The intensity and direction of the gravity vector is the only reference to a mathematical quantity at this stage (and already discussed in previous chapters).

Air resistance can be significant if a projectile moves with relatively high speed or if it encounters a strong wind. In many everyday situations, however, like tossing a ball to a friend or dropping a book, air resistance is relatively insignificant. As for the acceleration due to gravity, $g = 9.81\ m/s^2$, this value varies slightly from place to place on the Earth's surface and decreases with increasing altitude. In addition, the rotation of the

Earth can be significant when we consider projectiles that cover great distances. Little error is made in ignoring the variation of g or the rotation of the Earth, however, in the examples of projectile motion considered in this chapter.

The previous paragraph represents a further step in the argumentation and contains the justification of the assumptions: the discussion of the validity domain of the phenomenological assumptions. The fact that the lexicon is still qualitative ("relatively high speed", "strong wind", "relatively insignificant", "this value varies slightly", "great distances", coloured in blue) and the situations considered are familiar ("encounter a strong wind", "tossing a ball to a friend", "dropping a book", coloured in orange) reveals that we are still in the everyday dimension. We cannot find limitations on the shape or density of the projectile (but precise objects are chosen: a ball, a book, not a feather; coloured in green); this information remains implicit and it has to be deduced.
The argument goes on as follows.

Let us incorporate the preceding assumptions into the equations of motion given in the previous section. Suppose as in FIGURE 4-2, that the x axis is horizontal and the y axis is vertical, with the positive direction upward. Noting that downward is the negative direction, it follows that

$a_y = -9.81\ m/s^2 = -g$

Gravity causes no acceleration in the x direction. Thus, the x component of acceleration is zero:

$a_x = 0$

We are ready to abstract and structure our observations. Even if it is not explicitly pointed out, the reader can infer that the situation depicted here is coherent with the model that was built in the previous section for an object moving in the 2-dimensional space, and therefore this model can be incorporated. In other words, our three initial hypotheses guarantee that acceleration is null on the horizontal component and constant on the vertical one. So, it is possible to establish a specific and simple frame of reference and to fix the proper values of the acceleration factors. As it shown by the sentence coloured in red, we shift from the "real", experiential space, to the geometrical one, quantified by our coordinates. Furthermore, our extensive object becomes a point particle.

The strangest choice of the book is the collocation of a box with the motion equations, just after the definition of "projectile" and before the example of the skater girl.
The words used to present the equations are:

With these acceleration components substituted into the fundamental constant acceleration equations of motion (Table 4-1) we find:

Table 3.1 - Equations of motion for the case of the projectile (p-93).

Projectile Motion ($a_x = 0, a_y = -g$)			
$x = x_0 + v_{0x}t$	$v_x = v_{0x}$	$v_x^2 = v_{0x}^2$	**4-6**
$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$	$v_y = v_{0y} - gt$	$v_y^2 = v_{0y}^2 - 2g\Delta y$	

The motion equations are obtained by manipulating the general formulas obtained in the previous section and represent the conclusion of this part related to the abstract world of our model, setting the rules that govern its motion.

After this, the skater girl example is introduced with these words:

A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion.

Here comes a thorny point in the argument, mainly due to the choice of the terminology. Why do we want to demonstrate the independence of motions by an experiment concerning the projectile? Was not it our starting point in the previous section? The risk of a circular argumentation emerges, unless we suppose that the demonstration hides an epistemic value that, however, has to be searched in a very careful way, being implicit.

First, while standing still, drop a rubber ball to the floor and catch it on the rebound. Notice that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second. Next, walk—or roller skate—with constant speed before dropping the ball, then observe its motion carefully.

We return to the dimension of experience; we drop a rubber ball to the floor while standing or walking. The lexicon is once again qualitative, but now the observative component, coloured in yellow, is highlighted ("notice", "observe"). We are moving closer to the modelised phenomenon of the previous section, by choosing a suitable situation, and witness what happens.

To you, its motion looks the same as before: It goes straight down, lands near your feet, bounces straight back up, and returns in about one second. This is illustrated in FIGURE 4-3. The fact that *you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion—the motions are independent.*

The result of the experiment is that the two considered motions "look the same", and hence horizontal and vertical ones are independent. This is the key consideration. We are not demonstrating, in mathematical sense, the independence of motions. We are looking at the physical world and searching for a correlation with our ideal model, showing that a relation exists. The difference between the verbs "to demonstrate" and "to show" is therefore important. In this sense, the argumentation overcome the circularity and obtain epistemic validity, acquiring new justifications of our conclusions.

To an observer who sees you walking by, the ball follows a curved path, as shown. The precise shape of this curved path—a parabola—is verified in the next section. Additional examples of this principle are shown in FIGURE 4-4.

These final propositions, that end the paragraph, foresee the next important results we are going to obtain, the exact shape of the curve path drawn by the object.

To sum up this first part of the textbook analysis, we can now argue why the independence of motion is the central theme of the discussion. Moreover, this theme is included in a rich and multi-level disciplinary structure, in fact:

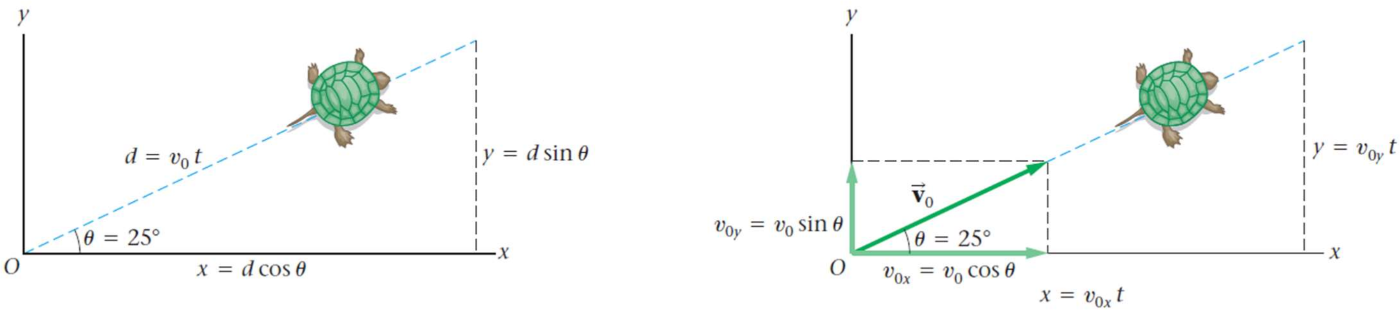
- On the explicative level, the independence of motions acts like the starting assumption in the development of the physical model of the projectile. Furthermore, the independence is also described in the plane of reasoning of the direct experience during the mental experiment.
- On the epistemological level, the modelling of the projectile is contextualised as being the application of the independence of motion.
- On the methodological level, the skater girl example allows the reader to find a correlation between the abstract model of the projectile and the phenomenological dimension, logically verifying that the horizontal and vertical motions do not influence each other in any way.

3.2.4 Lexicon, Syntax, Textuality

Linguistically, the text is comprehensible and easy to follow. Overall, simple and short sentences are preferred to longer and more articulated one. The information in well organised and the text results cohesive. Anyway, on the lexical plane, we can highlight an ambiguous use of the verb "to demonstrate" in the mental experiment described in the section 3.2.3. As we said earlier, the experiment does not demonstrate the independence of motions, instead it searches for a correspondence between the physical model of the projectile and the experiential domain. In this sense, the experiment could be intended as a "phenomenological demonstration" of the independence. This ambiguity is somewhat linked to the circular loop that we found in the conceptual map and it will be clarified with the in depth analysis of the arguments adduced by the author, done in section 3.3.

3.2.5 Figures

Looking at the figures from a disciplinary perspective, we found out that different images or graphs can have different purposes in the text. For instance, some of the figures *represent* the phenomenon



or the particular situation we are studying: They support the modeling and its mathematical formulation, see figure 3.4 as an example:

Fig. 3.4 - Example of a *representative* figure, supporting the virtual example of the motion of the turtle (p-89).

Figure 3.4 is used in the first paragraph to visually support the example of the turtle, previously mentioned in section 3.2.3.

Other figures that appear in the chapter are mainly used to *synthetize* results, features, or properties in a concise way. For instance, it is the case of tables or grids summarizing the main results obtained in a specific paragraph. However, the most relevant class of figures in a disciplinary analysis is the group composed by the *informative* ones (see fig.3.5). We use the term "informative", referring to those images that convey additional information respect to the written text.

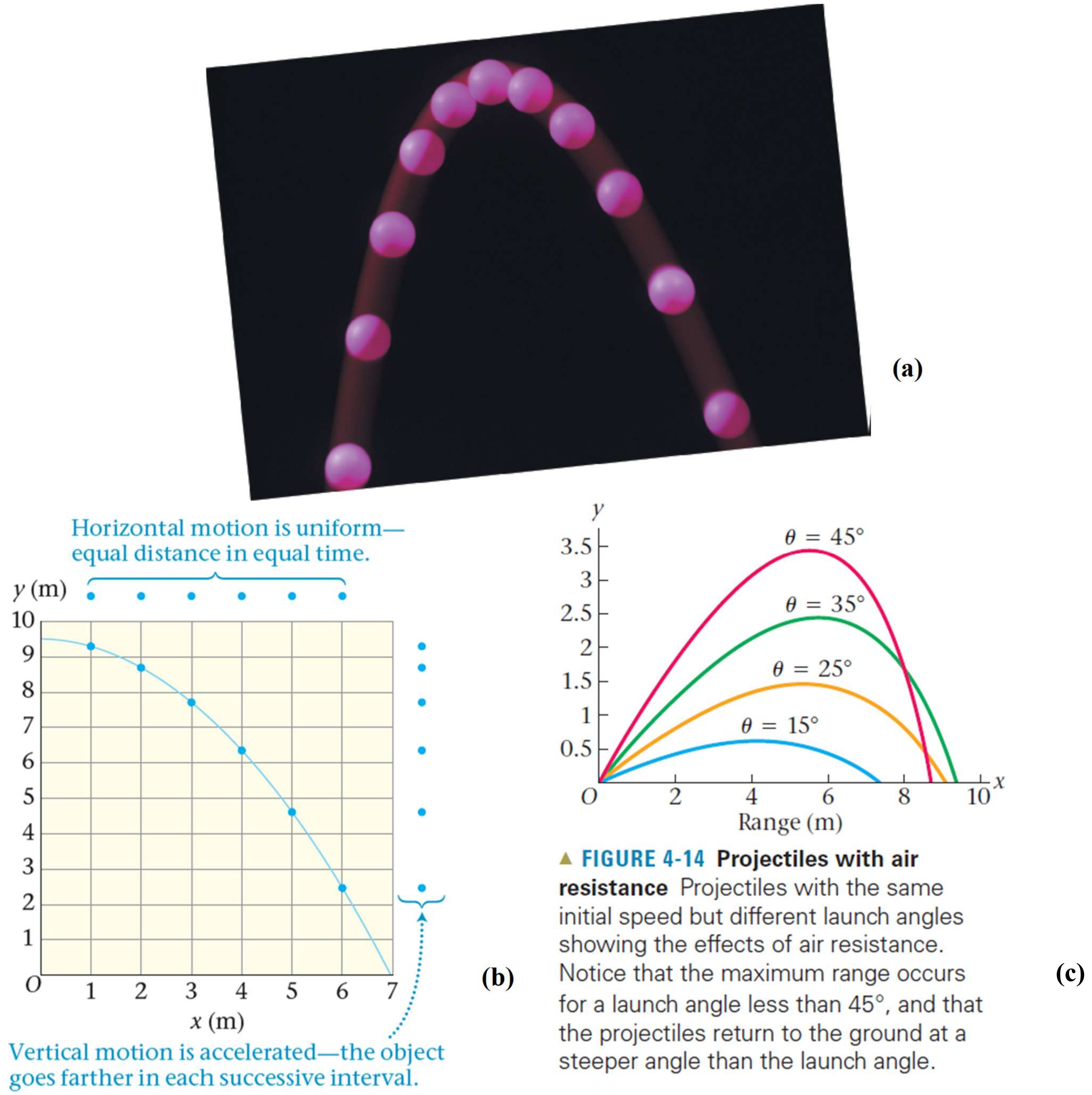


Fig.3.5 - Examples of *informative* figures in the chapter (p-88; 95; 104).

If we consider the informative figures, they can be said to condense and convey further pieces of knowledge. This could be done in different ways, like introducing information not yet explained in the chapter; expressing connections between ideas and/or data in different, meaningful ways; adding complementary facts that help understand the point made by the author, and so on. In some cases, the additional information is easy to decode; in others, some degree of mastery by the reader and/or explanatory descriptions are needed. Let us consider some examples. The image (a) in figure 3.5 is highly informative for a trained eye. It targets the fact that a projectile does not cover the same distances in consecutive time intervals. In additional to that, we perceive the symmetry of the trajectory drawn by the object in motion, created by the superposition of different time frames. This image is at the beginning of the chapter; therefore, all this information is not easily accessible to the average reader and needs further propositions to be understood. The chat (b) is shown in the paragraph relative to the projectile motion in the case of a null throwing angle. It combines the concepts of trajectory and independence of motions in an interesting new way, extracting from the graph both the uniform and the accelerated motion. Lastly, the graph (c) is shown while discussing on the maximum range of the projectile. It integrates external data to the argumentation. By representing various trajectories for different values of the throwing angle and by superimposing them, the reader can get a sense of how the effect of air resistance influences the motion.

3.3 The interdisciplinary analysis of the Walker textbook

The previous paragraph concluded the section dedicates to the disciplinary analysis of the textbook. Now, in an analogous way, we want to conduct a complementary analysis of the chapter, focused on the interdisciplinary elements that are present in the chapter. Our starting point and principal tool for the investigation is the interdisciplinary lens exposed in the previous chapter (see table 2.2).

3.3.1 The roles of mathematics

When we proposed the example of the textbook in section 3.1, we showed that mathematics was not used as a structural component of the argumentation, but mainly as an algebraic computational tool. This is a common treat of many physics textbook and it has a dramatic effect on the reader's perception of the relations between mathematics and physics. For this reason, we selected a textbook in which different roles of mathematics could emerge as part of the physical argumentation, hence showing the potential to discuss with prospective teachers about the sense of authentic interplay between the two disciplines. It is interesting to notice that in every paragraph of Walker's fourth chapter, but for the fourth one (which repeats the reasoning of the third paragraph for a generic launch angle), mathematics assumes a different role in relation to the specific subtopic discussed. More specifically, the four roles are:

1. *Formal structure that establishes the rules of a virtual environment (simulation)*: math provides the rules to apply to simulate a motion in the virtual environment of the "turtle" that reminds the language of Logo and netLogo. From there, we can create different algorithms

and therefore infer unique features of the phenomenon. For example, in the first paragraph of the chapter, while considering the example of the turtle moving in two-dimensional space, we are able to deduce that the motion can be treated independently in its x and y components from the coherence of the results obtained in the one-dimensional and two-dimensional case (correspondent to different algorithms).

2. *Mathematization in a modelling process*: math provides the criteria to shape the mathematical model of the phenomenon, starting from physics hypotheses. This is the case of the second paragraph, in which we study the motion of the projectile in two-dimensional space and transform (although most of the passages are implicit) the physical intuition and hypotheses in mathematical equations. The equations of motion obtained are presented as analytic functions, following the formalism of Newtonian mechanics, as it is clear from the table 3.1, in section 3.2.3. Both the distance and the velocity are represented as integral products. This choice of the author implies a higher level of mathematical abstraction in the discussion that can be detrimental in preserving the relevant correspondences between mathematical object and physical quantities.

3. *Logical argumentation*: mathematics has a role in the argumentation process. In particular, there are cases in which mathematics allows the reasoning to be developed and conclusions to be inferred. We can mention the third paragraph, in which we are able to determine the shape of the projectile's trajectory, starting from the equations of motion for a horizontal throw through the application of strictly algebraic inferences.

4. *Epistemological activator*: last but not least, math can boost epistemological considerations about the phenomenon, stimulating a deeper comprehension of the latter. That is the case of the fifth paragraph, in which we study the symmetries of the motion. In this example, math allows to enter in a dimension where it overlaps and closely coexist with physics. More specifically, the symmetry in the parabola, seen as a geometrical object, acts as a trigger and leads on to consider a mechanical equivalent of this property for the trajectory of the projectile.

This richness is a great potential that can stimulate the design of activities aimed to explore more in depth all these roles and their specificities. In the next sections, instead, we will stress the main weaknesses of the text that our analysis have revealed.

3.3.2 Argumentative Structure

The historical texts that we have discussed in the previous chapter share the feature of having an explicit argumentative structure. Particularly, the inferences are logically organised in the text and new justifications of the conclusion are adduced in the argument. For instance, in the extract from Guidobaldo's *Notebook* (see section 2.2.1), the shape of the trajectory is logically inferred from the initial observation of the symmetry and the designed experiment of the ball coloured in ink contributes to epistemologically support the conclusion. Contrarily, in the Walker textbook, the argumentative structure tends to be more implicit or improperly addressed. The case of the skater girl analysed in section 3.2.3 is particularly critical, as the word "to demonstrate" is improperly used and the argumentative structure does not clearly emerge. In this passage, the author wants to provide "*a simple demonstration*" of the independence of horizontal and vertical motions in projectile motion, hence the motion of the projectile is here used to demonstrate the independence of motion. However, in the first few lines of the paragraph, the motion of the projectile is presented as an *application* of the independence of motions. Therefore, if we consider the logical structure of the argument adduced in the second paragraph, we are moving from the independence of motions to the independence itself in a circular way, as it shown it the upper-left block of the concept map (see fig.3.1). Anyway, as we said during the analysis of the paragraph, we are not demonstrating the independence in the rigorous sense of the word; this would mean to deduce it from a fixed set of pre-established axioms through the use of the metatheory's rules. In this case, we are looking at the physical world and searching for a correlation with our ideal model, showing that a relation actually exists. So, if we look at the epistemological structure of the argument, the projectile motion is used to acquire a new justification of the independence of motion on the phenomenological dimension. Only by considering this switch from the ideal to the phenomenological, the passage can be seen as argumentatively correct.

3.3.3 Demonstrative Structure

The fourth paragraph of the analysed chapter is dedicated to the motion of the projectile launched horizontally. In particular, at the end of the section, the author wants to *demonstrate* that the trajectory for this particular motion is parabolic. First of all, let us examine the relative passage of the text:

"Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2} g t^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = \frac{x}{v_0}$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \frac{1}{2} g \left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2v_0^2}\right) x^2$$

It follows that y has the form

$$y = a + bx^2$$

In this expression, a = h = constant and b = $-g/2v_0^2$ *= constant. This is the equation of a parabola that curves downward, a characteristic shape in projectile motion."*

As we can see in these few lines, the parabolic shape of the trajectory is deduced in a very typical way for physics textbooks, that is by means of algebraic steps. This use of mathematics is very effective to give all the elements for exercises but tends to move the attention away from the relevant physical quantity of the discourse. The comparison with the original demonstration by Galileo, for example, cuts the crucial steps that lead, first of all, physical properties to be turned into quantities and, then, to be formally related. In particular, the original demonstration includes the delicate passage that turns time into space, thanks to the definition of "equable motion" and that lead velocity to be a "quality" that can characterise different types of motions. In the "typical" textbooks demonstration time is a simple "real parameter" that can unify two laws thought a mere formal "variable substitution". In this sense, we can argue neither the epistemological status of mathematics, not that of physics is valued.

However, another relevant question is the following: is the above excerpt actually a demonstration? From the characterisation of demonstration we gave in the previous chapter, we know that an argument, in order to be considered the demonstration of some statement, must refer to a list of axioms (given as true) and be included a theory of reference (Mariotti, 2000). Furthermore, the inferences must be adduced using a metatheory. In the present case, all these elements are not explicit in the text and, if we search for a demonstration structure, they need to be retraced. The statement appears implicitly in the first two lines of the passage and can be reformulated as: "The shape of the curved path followed by a projectile launched horizontally is parabolic". Moving on, from the collocation of the demonstration in the chapter, we can infer that the considered assumptions for the demonstration are the one specified at the beginning of the second paragraph, when the motion of the project is modelled. Namely, they are the independence of motions, the absence of air resistance and effects due to Earth's rotation, and the constant value for the acceleration on the vertical direction. Furthermore, the theory of two-dimensional kinematics represents the frame that contextualise the statement. Finally, the set of algebraic rules used in the demonstration represent the metatheory, making the inferences logically valid.

From this brief dissertation it follows that the one proposed in the paragraph is actually a demonstration. However, its structure remains entirely implicit, to the point that also the verb "to demonstrate" does not appear in the text. This expositive choice is crucial in the sense that it prevents the reader to bring out the idea of demonstration as a fundamental *meta-object* (or mathematical structure) for the production of knowledge in physics. In fact, as we said in the previous chapter, the demonstration has always had a relevant interdisciplinary role for the evolution of the discipline as, for example, Galileo's works testimony.

Galileo's *Discourses and Mathematical Demonstrations Relating to Two New Sciences* (1638) clearly shows how mathematics is used to structure and direct the physical reasoning in a coherent and organic way. The demonstrative intent of the work emerges clearly as a distinctive feature. In this regard, we can analyse the passage of Salviati's demonstration of the parabolic trajectory, whose statement was briefly mentioned in the second chapter, in order to establish the structural differences that subsist between the historical text and the modern textbook. First of all we need to look at the general context in which the demonstration is presented. The third day of the *Discourses*, concerning the study of local motions, opens up with the definition of *uniform motion:*

"By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal."

This is the only definition that Galileo needs in order to deal with the topic of uniform motions. From the definition, four axioms are deduced:

> ***"Axiom I***
> *In the case of one and the same uniform motion, the distance traversed during a longer interval of time is greater than the distance traversed during a shorter interval of time.*
> ***Axiom II***
> *In the case of one and the same uniform motion, the time required to traverse a greater distance is longer than the time required for a less distance.*
> ***Axiom III***
> *In one and the same interval of time, the distance traversed at a greater speed is larger than the distance traversed at a less speed.*
> ***Axiom IV***
> *The speed required to traverse a longer distance is greater than that required to traverse a shorter distance during the same time-interval."*

The four axioms introduce clarify the relations between the three main physical entities of time, space and speed. Anyway, it is important to notice that the concept of "speed" does not explicitly appear in any of the axioms. In this sense, the speed can be interpreted as a *quality* of the motion, providing a way to relate the concepts of time and space. The initial definitions and these four axioms represents the assumptions of the third day and will be systematically used to demonstrate the following theorems. In particular, always in the third day the following theorem is demonstrated by Salviati:

> *"**Theorem I, Proposition I***
> *A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola."*

The demonstration adduced is the following:

"Let us imagine an elevated horizontal line or plane ab along which a body moves with uniform speed from a to b. Suppose this plane to end abruptly at b; then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular bn. Draw the line be along the plane ba to represent the flow, or measure, of time; divide this line into a number of segments, bc, cd, de, representing equal intervals of time; from the points b, c, d, e, let fall lines which are parallel to the perpendicular bn. On the first of these lay off any distance ci, on the second a distance four times as long, df; on the third, one nine times as long, eh; and so on, in proportion to the squares of cb, db, eb, or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from b to c with uniform speed, it also falls perpendicularly through the distance ci, and at the end of the time-interval bc finds itself at the point i. In like manner at the end of the time interval bd, which is the double of bc, the vertical fall will be four times the first distance ci; for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space eh traversed during the time be will be nine times ci; thus it is evident that the distances eh, df, ci will be to one another as the squares of the lines be, bd, bc. Now from the points i, f, h draw the straight lines io, fg, hl parallel to be; these lines hl, fg, io are equal to eb, db and cb, respectively; so also are the lines bo, bg, bl respectively equal to ci, df, and eh. The square of hl is to that of fg as the line lb is to bg; and the square of fg is to that of io as gb is to bo; therefore the points i, f, h, lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola."

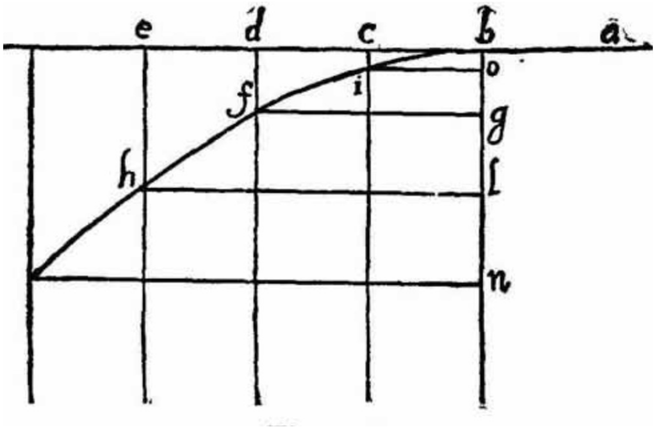


Fig. 3.6 Figure on the *Discourse* supporting the demonstration.

In the case of the *Discourses,* the initial axioms, the statement and the demonstration itself are all objects made explicit in the work. Furthermore Galileo introduces *all and only those elements* needed in order to demonstrate the various theorems. All these elements, unlike in Walker's book, contributes to bring out the demonstration explicitly as mathematical meta-object (Mariotti, 2000). Furthermore, Galileo's demonstration of the parabolic trajectory undoubtedly seems more intertwined than the one proposed in the Walker, but it brings out some relevant disciplinary and interdisciplinary elements. First of all, as the fig.3.6 shows, the motion of the projectile is *geometrically described*; the physical entities of time and space are mathematized through segments on the horizontal and vertical dimension. Secondarily, the time is explicitly *spatialized.* In fact, in the first line of the passage we read that the body is moving along the horizontal line ab. However, shortly after, the line *be* on the plane *ba* in drawn, representing *"the flow, or measure, of time".* This construction contains the intuition that time and space on the horizontal plane are proportional. In other words, given that the motion of the projectile if uniform on the horizontal axis, equal distances travelled by the object correspond to equal time intervals. So, by establishing a unit of distance, we are indirectly able to measure the time. This fact clarifies the physical meaning underlying the variable substitution $t = x/v_0$ in the Walker's demonstration. We can say that Galileo's *proportional* way of reasoning becomes *analytic* in the Walker and the relation between mathematical object and physical property, tends to vanish in the abstract formalism.

4. Activity for pre-service teachers

4.1 From the results of the analyses to the designing of the activity

The disciplinary and interdisciplinary analyses of the Walker textbook, topic of the previous chapter, brought out three fundamental elements that characterize the two-dimensional kinematics and projectile motion, namely the *symmetry* and *the independence of motion* as epistemological activators and the mathematical *demonstration* as an important meta-object for physics. The present chapter therefore dedicated to the design of an activity, addressed to preservice teachers, that can support and foster an active discussion regarding these topics.

The idea of the activity arises from the comparison between the historical development of the transition and the canonical textbooks' treatment of the projectile motion.

The main starting point for the design comes from the paper written by Renn, Damerow and Rieger (2000) entitled *Hunting the White Elephant: When and How did Galileo Discover the Law of Fall?*. Following the reasoning of Renn et al. on how the discovery of the symmetry of the parabolic trajectory has put in crisis the Aristotelian paradigm thus leading to the birth of modern physics, other materials have been considered and used. In particular, the original memories of Galileo and Guidobaldo, the problems proposed by the well-known *Force Concept Inventory* experiment (Hestenes, Wells, Swackhamer, 1992) and Walker's textbook.

Overall, the activity is mainly shaped around Renn's paper and it has been progressively delineated using a methodology of triangular construction and critical reflexion on the literature with the contribution of Professor Olivia Levrini and PhD student Sara Satanassi.

The activity has the main goal of highlighting the central aspects of symmetry, motions independence and the role of demonstration as *epistemological activators* in the historical development of the physical theory of kinematics during the 16th and 17th century. For this particular reason, we chose a way of modelling the activity that would not aim to reconstruct, step by step, the historical evolution of the concepts. Instead, we focused on the main breaking points that opened toward new ways of conceiving the projectile motion (and local motions), both respect to the symmetry, motions compositions and the methods used to produce the knowledge. In order to accomplish this task, we selected central passages in the work of Renn et al. and we integrated them with meaningful excerpts from Guidobaldo and Galileo's studies. More specifically, Tartaglia's theory of projectile motion is explored and confronted with Guidobaldo's experiment of the ink ball, analysed in chapter two. Later, the mathematical and physical reasoning that led to the understanding of the parabolic trajectory of the projectile are reconstructed, with the help of Renn's mathematical argument on the hyperbole and Galileo's demonstration from his *Discourse*s (see section 4.2 for further details). The chosen passages are also important from a didactic perspective because they resonate with important themes in

Didactic of Physics, such as the common-sense knowledge and the mental images of the students. Finally, the evolution of reasoning, from Tartaglia to Galileo is investigated on two complementary planes, the analysis of the *disciplinary arguments* and the analysis of the *figures* that support these arguments. In this way, the activity is supposed to foster preservice teachers to retrace not only the great ideas that produced fruitful epistemological changes in physics, but also the representation of those ideas.

4.2. Description of the macro-contents of the activity

The present activity, composed by four main tasks, is designed to be preferably carried out in groups, to foster the discussion and the comparison between members and to let emerge different possible views about the same topics.

The first task has the goal to retrace the path that lead, from the initial idea, to the representation of the projectile's trajectory. In particular, it is asked to compare two excerpts and the relative figures. The first excerpt, concerning Tartaglia's theory of projectile motion, is taken from Renn et al. (2000); the second, relative to Guidobaldo Del Monte's ink experiment, comes from his *Notebook*. The goal of this part is to determine, by the comparison of both the excerpts and figures, what are the main breaking points and innovation aspects of Guidobaldo's work respect to Tartaglia's theory. From the proposed questions, we expect some important aspects to emerge. First of all, Guidobaldo's observations highlight, from observations and then from the realization of an "experiment", the phenomenological *symmetry* of the trajectory, symmetry that has to be manifested also in the mathematical curve representing the motion. Secondly, from Guidobaldo's excerpt, the *natural* and *forced* motions appears as *composed* during the entire motion of the projectile. On the contrary, Tartaglia, following Cardano's idea that natural and forced motion cannot act simultaneously, strictly differentiates the *natural* from the *forced* in the object's trajectory. Finally, the third question address the fact that it was not possible, with the knowledge of the time, to rigorously *differentiate* between the catenary and the parabola. In fact, Galileo posed the problem of mathematically describing the catenary following the analogy that he maintained between it and the projectile's trajectory. Galileo thought that, as the projectile motion, in each instant, is subjected to two actions, one *natural* which pushes it downwards due to its weight and the other *forced* which has the direction of movement, therefore also in the catenary each ring is subjected to the same two actions. Galileo was able to demonstrate the parabolic nature in the case of the projectile motion and he tried to do the same in the case of hanging chain. Nevertheless, as Renn et al. stress, there were not yet the mathematical tools necessary to distinguish them (Renn et al., 2000). Galileo, as we will show below, arrived to "demonstrate" the parabolic shape, only after a redefinition of the two motions that have to be composed and, in particular, after the substitution of the forced motion along the direction of movement with the "neutral" (nor forced or natural) equable motion along the horizontal axis.

The second task guides the groups to reconstruct the physical and mathematical reasoning that led Guidobaldo and Galileo to establish the parabolic shape of the trajectory. The main goal of the task is to highlight the main steps that Galileo accomplished in overcoming Tartaglia's view of the projectile motion. For this part, we initially proposed an excerpt from Renn et al., concerning the confutation of the hyperbolic option. The confutation, given by Guidobaldo in his *protocol* (as Renn calls the ink experiment) consisted in the observation that the constantly decreasing ratio between violent and natural motion in descent is incompatible with the asymptotic behaviour of the hyperbola. From this asymptotic behaviour derives that this ratio should approach a non-zero constant (Renn et al., 2000).

> *"Moreover, the dynamical explanation given in the [Guidobaldo's] protocol implies, on closer inspection, that the constantly decreasing ratio between violent and natural motion in descent assumed in the protocol is incompatible with the asymptotic behavior of the hyperbola. It follows, in fact, from this asymptotic behavior that this ratio should approach a constant different from zero. But once the parabola has been chosen, its geometrical properties, well-known since ancient times, suggest certain assumptions about the forces and how they act together."*

After that, an analysis of the demonstration of the parabolic trajectory adduced by Galileo's in the fourth day of in his *Discourses* is asked.

"***Theorem I, Proposition I***

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola. [...]
Let us imagine an elevated horizontal line or plane ab along which a body moves with uniform speed from a to b. Suppose this plane to end abruptly at b; then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular bn. Draw the line be along the plane ba to represent the flow, or measure, of time; divide this line into a number of segments, bc, cd, de, representing equal intervals of time; from the points b, c, d, e, let fall lines which are parallel to the perpendicular bn. On the first of these lay off any distance ci, on the second a distance four times as long, df; on the third, one nine times as long, eh; and so on, in proportion to the squares of cb, db, eb, or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from b to c with uniform speed, it also falls perpendicularly through the distance ci, and at the end of the time-interval bc finds itself at the point i. In like manner at the end of the time interval bd, which is the double of bc, the vertical fall will be four times the first distance ci; for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space eh traversed during the time be will be nine times ci; thus it is evident that the distances eh, df, ci will be to one another as the squares of the lines be, bd, bc. Now from the points i, f, h draw the straight lines io, fg, hl parallel to be; these lines hl, fg, io are equal to eb, db and cb, respectively; so also are the lines bo, bg, bl respectively equal to ci, df, and eh. The square of hl is to that of fg as the line lb is to bg; and the square of fg is to that of io as gb is to bo; therefore the points i, f, h, lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola."

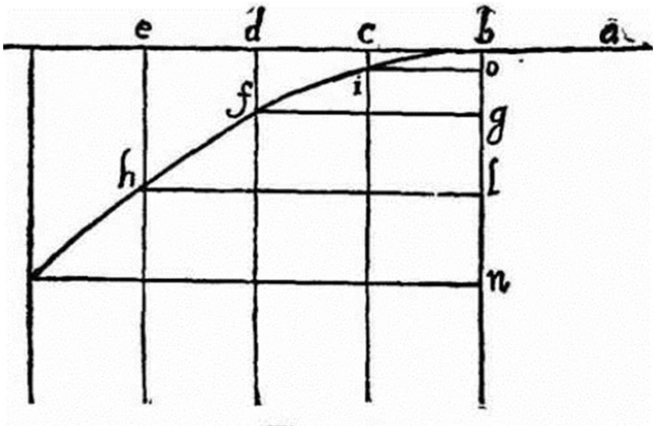


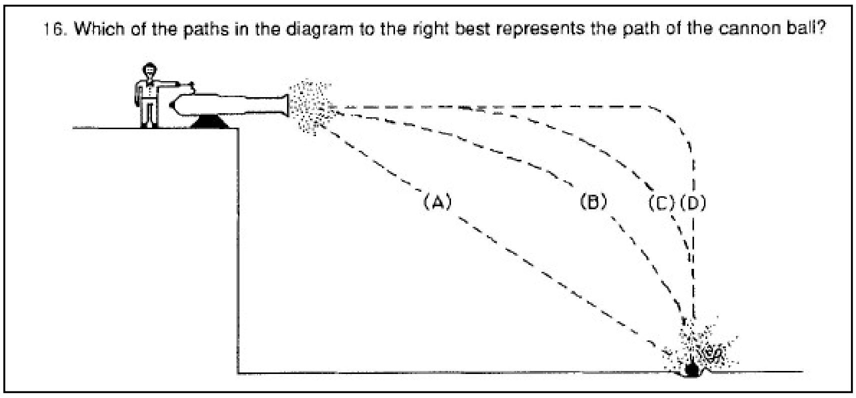
Fig.4.3 – Figure supporting Galileo's demonstration of the parabolic trajectory.

We expect some important aspects to be underlined, as the fundamental *role of the demonstration* in determining the curve, the necessity of the *uniform motion* as a "neutral one" (hence neither natural nor forced) and the mathematical shape of the curve as descending from the physical assumption regarding the motion.

The third task of the activity is a focus on the epistemological role of the symmetry. It is asked to resume the main epistemological aspects that emerged in the previous parts and use them to reflect on the role of symmetry in the evolution of kinematics. An excerpt from the fourth chapter of the Walker textbook, characterizing the symmetry in physics, is proposed as a starting point for a wider discussion on the possible roles that the symmetry has in physics:

> *"Symmetries such as these are just some of the many reasons why physicists find physics to be "beautiful" and "aesthetically pleasing." Discovering such patterns and symmetries in nature is really what physics is all about. A physicist does not consider the beauty of projectile motion to be diminished by analysing it in detail. Just the opposite—detailed analysis reveals deeper, more subtle, and sometimes unexpected levels of beauty."*

The last task, according to a common approach within IDENTITES, concerns an educational plan. It goal is to frame the historical and epistemological reflection on the disciplines and on interdisciplinarity within an educational problem. For this purpose, the questions 16 and 22 of the *Force Concept Inventory* (Hestenes, Wells, Swackhamer, 1992) have been selected and reported in this part of the activity with the percentage of given answers for each possibilities. The questions concern the expected trajectory of projectile motion and the forces that act on the projectile during its flight. Figure 4.1 shows the questions extrapolated from the questionnaire.



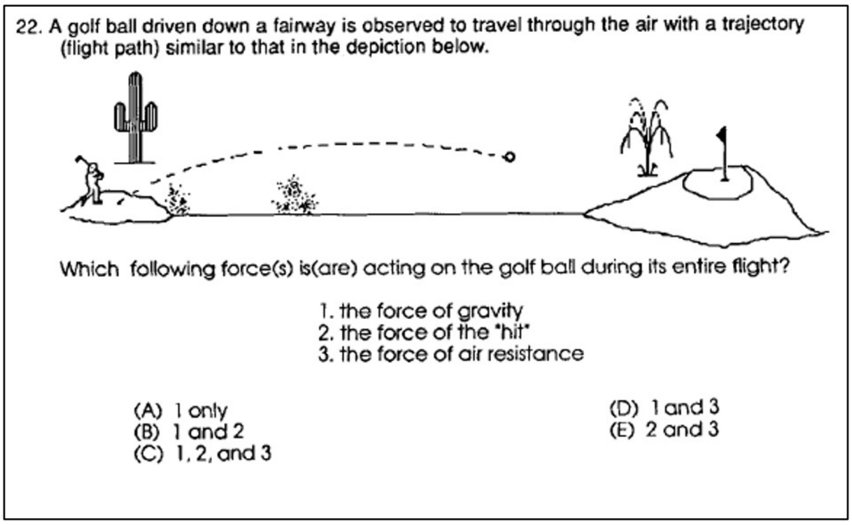
Answers percentual (pre-test; post-test):

A – less than 1%; less than 1%

[B] – 40.6%; 77.5%

C – 52.1%; 20.4%

D – 6.4%; 1.1%



22. A golf ball driven down a fairway is observed to travel through the air with a trajectory (flight path) similar to that in the depiction below.

Which following force(s) is(are) acting on the golf ball during its entire flight?

1. the force of gravity
2. the force of the "hit"
3. the force of air resistance

(A) 1 only
(B) 1 and 2
(C) 1, 2, and 3
(D) 1 and 3
(E) 2 and 3

Answers percentual (pre-test; post-test):

A – 1.9%; 11.9%

B –4.6%; 7.7%

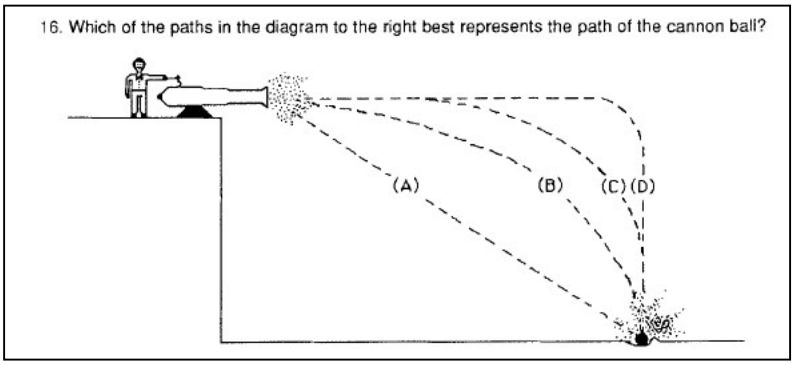
C –81.3%; 33.6%

[D] – 9.5%; 45.3%

Fig. 4.1 – Questions 16 and 22 from the *Force Concept Inventory* experiment.

These items are frequently used in the courses of Physics Education to show students' difficulties with projectile motion and, more deeply, with the concepts of force and motion. In this activity, they are used to invite preservice teachers to think about students' processes of recruiting knowledge and activating resources when they imagine a projectile trajectory. The answers to the question show, indeed, the counter-intuitive character of parabolic motion. For example, in question 16, as shown in figure 4.1 (a), the incorrect option C, chosen by the 52.1% of the students in the pre-test, reveals the idea of a trajectory that is originally straight, then progressively curves and finally returns straight. This idea could for instance be originated from the thought of separated effects acting on the projectile: the initial force impress to the object by the cannon and the successive action of gravity that curves it toward the ground. This reasoning shows to what extent the idea of motion independence can be counter-intuitive and really represented an epistemological innovation. In Tartaglia's conception, the shape of the trajectory was the result of a view that foresaw motion as generated by the separate contributions of *forced* and *natural* motion of the object (see part b of figure 4.2). In students' ideas is was the result of naïve knowledge that leads force to be seen as a mover (Hestenes et al., 1992).

(a)



(b)

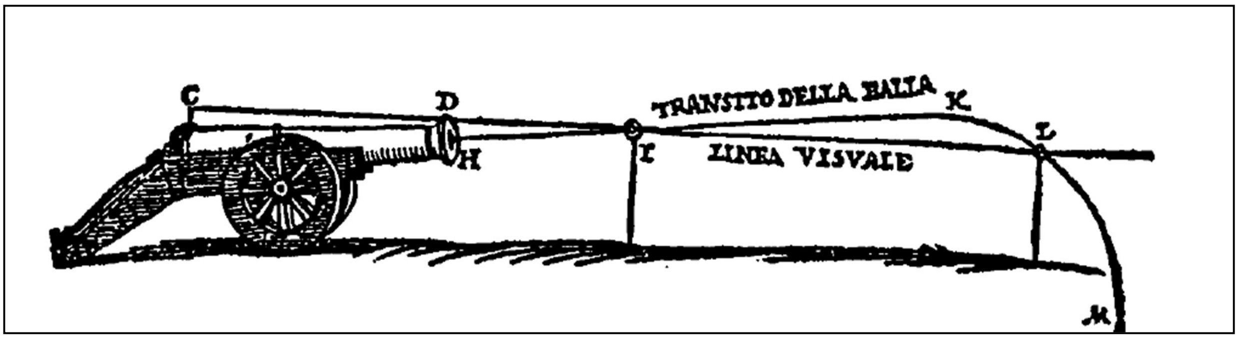


Fig.4.2 - Question 16 from the *Force Concept Inventory* (a) and the trajectory of a projectile shot by a cannon (b), as represented by Tartaglia in his *Nova Scientia*.

The same reasoning can be extended to question 22 of the *Force Concept Inventory*. Here, the interesting part is that the 83.3% of the students that answered the question (always in the pre-test) thought that, additionally to gravity and air resistance, also the force of the initial "hit" impressed to the projectile could act during the *entire* flight. This answer reflects a common belief of the students in considering the initial force impressed to the projectile as something that manifest itself during all the displacement of the object. This idea, again, shows the extent to which the shape of trajectory and the idea of force are related, like in the medieval discussion when motion representation revealed a view of forced motions. In that case, forced motions were thought as the result of "*acting through an impetus impressed by the mover into the moving body*" (Renn et al., 2000).

4.3 Pilot-test

The activity has been tested with the help of two couples of students. Three of them are at the end of their university path and they are attending the course of *Advanced Professional and Research Skills in Physical Sciences*, within the master program in Physics at the University of Bologna and the other is a PhD student in Data Science and Computation at the University of Bologna. The two couples had different level of knowledge on the main themes targeted by the activity. The first couple had already some personal insights on the role of mathematics in physics and on the parabola as an interdisciplinary topic. The other group did not share the same starting knowledge on the specific topic, but represented a typical target group for the activity.

The test was a very preliminary pilot test aimed to receive feedback comments about the effectiveness of the questions to stimulate discussion. More specifically, the test has been carried out to check:

a) the clarity and efficiency of questions formulation, of their number and their order;
b) the topics of discussion from which they were stimulated and the type of engagement (did the students meet moments of silence and discomfort? who spoke and how (in the sense, did the activity trigger a vivid exchange among the students or did only one regularly speak?)

The test was carried on in the form of interviews and the discussions have been audio-recorded. The coding scheme and the analysis have been carried out through debriefs among several researchers and the results triangulated.

For the analysis, three people listened the audio-recording and answered the questions reported in fig. 4.4:

Clarity and efficiency of questions of the questions.

Do they hesitate to understand the questions?

Do they have to share the meaning or reread them?

Are all the questions needed?

Is the order effective or does they force reasoning to go back?

Quality and effectiveness of the discussion

What topics did they discuss?

What was/were the more intense and interesting moment/s of the discussion?

Where the students able to recognise the key-points of reasoning?

a) The incorporation of Aristotelian idea in Tartaglia's motion representation;
b) The distinction between Tartaglia's representation of movement and the "phenomenological drawing of Guidobaldo";
c) The role of symmetry to move toward the concept of motion independence and the displacement of the debate on the shape of the symmetrical trajectory;
d) The crucial passage, to arrive at the demonstration of a parabolic trajectory, consisting of move from the combination of "a forced motion toward the direction of movement plus the natural movement which pushes the projectile downwards" to the combination of "a natural motion toward the horizontal plus the accelerated movement which pushes the projectile downwards".

e) The difficulties to recruit the idea of motion independence to interpret motions' representation in spontaneous knowledge;

f) The difficulties to give up an idea of "forced motion".

Engagement

What type of engagement did the students show?

Did they meet moments of silence and discomfort?

Who spoke and how (in the sense, did the activity trigger a vivid exchange among the students or did only one regularly speak?)

Fig. 4.4 - Grid of analysis.

4.4 Results and activity revision

The pilot study proved to be very fruitful and the results of the analysis show that the problems that students encountered in the activity were mainly due to the efficacy of the questions, and not to the contents selection that have been found interesting and very relevant, because of their role in founding the discipline. This consideration led us to re-elaborate the order of the various proposed tasks and to specify, reformulate and expand the questions, with the purpose of better guide the preservice teachers during the future implementation. After a rearrangement of the questions, our deep change has been carried out in task 4 that was, at the beginning, this first task but, instead of situating the activity within an educational framework, activated reflections that were not developed in the following tasks. For this reason, the task related to the didactical dimension was moved to the end with a different goals: to think about how the main epistemological and conceptual points emerged in the discussion could be implemented into the dimension of a class. More specifically, the new activity asks the pre-service teachers to build a conceptual map highlighting the main conceptual and epistemological knots that emerged from the precedents tasks and underline the weak connections that the question from the *Force Concept Inventory* enlighten. The map is thought also to be used in order to structure the design of innovative teaching modules on the topic of projectile motion.

The re-elaborated activity, in its final formulation, is presented below. The estimate time for the execution is approximately two hours for the first three tasks, and two hours for the last one.

Activity: Parabolic motion as foundational case to establish physics as discipline

Introduction:

The purpose of the activity is to stimulate a reflection about the conceptual and epistemological breaks and aspects, introduced by Guidobaldo and Galileo in their development of the theory of projectile motion, that led to the crisis of the Aristotelian paradigm and to the birth of the modern physics as we know it today.

In the following, we report some excerpts from:

a) the article *Hunting the White Elephant: When and How did Galileo Discover the Law of Fall?* (Renn, Damerow, Rieger, 2000);
b) Guidobaldo Del Monte's *Notebook* (ca. 1587-1592);
c) *Discourses and Mathematical Demonstrations Relating to Two New Sciences* (Galilei, 1638).

In particular, the passages from the works of Guidobaldo and Galileo have been selected because they ratified the birth of the *scientific methods* and the *mathematization* as foundational elements in science.

Task 1: From the idea to the representation of the projectile motion

a. The following passage is intended as a preliminary historical contextualization. It provides some ideas on how projectile motion was conceived within the medieval and renaissance paradigm that was used to interpret terrestrial motions. Please read it carefully and answer the questions below.

> In medieval and renaissance time, the Aristotelian duality of natural and forced motion still represented the main paradigm to interpret the local motions. According to Aristotle's theory, there are only two types of possible motions, namely the *violent* (or forced motion) and the *natural* one. Every spontaneous motion of the object (whether smoke in rising or massive bodies in falling) is a natural motion; on the contrary, every motion produced by a different cause is a violent one. Furthermore, in Aristotle's work *De cielo*, the philosopher states that the local motions are always *straight* or *circular*, or mixed of the two, because these two motions are the only simple ones.

- What distinguishes violent from natural motions?
- What are the possible shapes of motions?
- Can you give some examples of natural and violent motions?

b. The Aristotelian paradigm of violent and natural motion, introduced above, is still recognisable in the works of many influent scholars of the 15th century. For instance, consider the theory of projectile motion as descripted by Niccolò Tartaglia (1499-1557), in the following excerpt:

> "*According to Tartaglia's theory, the trajectory of a projectile consists of three parts. It begins with a straight part that is followed by a section of a circle and then ending in a straight vertical line (see figure [on the right]). This form of the trajectory also corresponds to Tartaglia's adaptation of the Aristotelian dynamics to projectile motion in the case of artillery [...].*
>
> *[...] The first part of the trajectory was conceived by Tartaglia as reflecting the initially dominant role of the violent motion, whereas the last straight part is in accord with the eventual dominance of the projectile's weight over the violent motion and the tendency to reach the center of the earth. [...]*
>
> *He claimed instead the curved part to be exclusively due to violent motion as is the first straight part of the trajectory."* (Renn, Damerow, Rieger & Giuliani 2000)

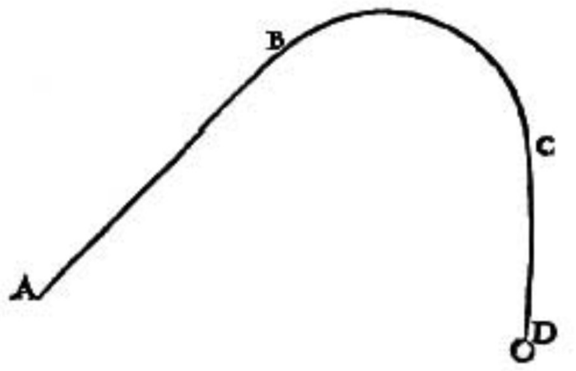


Fig.1.1 Tartaglia's representation of the projectile motion in his *Nova Scientia* (1537).

- Try, in your own words, to describe figure 1.1, using the Aristotelian paradigm of violent and natural motions.
- What kind of knowledge (intuitive, phenomenological, experience-based, philosophical, formal, aesthetical …) figure 1.1 incorporate? In what sense does the figure represent Aristotelian distinction between violent and forced motion? What about the shapes of the motions?
- In your opinion, is it a realistic representation of the projectile trajectory? Why?

c. Now, consider the following except from the *Notebook* of Guidobaldo Del Monte (1545-1607), in which the methodological fundaments of modern Science begin to be posed.

"If one throws a ball with a catapult or with artillery or by hand or by some other instrument above the horizontal line, it will take the same path in falling as in rising, and the shape is that which, when inverted under the horizon, a rope makes which is not pulled, both being composed of the natural and the forced, and it is a line which appearance is similar to a parabola and hyperbola. And this can be seen better with a chain than with a rope, since [in the case of] the rope abc, when ac are close to each other, the part b does not approach as it should because the rope remains hard in itself, while a chain or little chain does not behave in this way. The experiment of this movement can be made by taking a ball coloured with ink, and throwing it over a plane of a table which is almost perpendicular to the horizontal [(*)]. Although the ball bounces along, yet it makes points as it goes, from which one can clearly see that as it rises so it descends, and it is reasonable this way, since the violence it has acquired in its ascent operates so that in falling it overcomes, in the same way, the natural movement in coming down so that the violence that overcame [the path] from b to c, conserving itself, operates so that from c to d [the path] is equal to cb, and the violence which is gradually lessening when descending operates so that from d to e [the path] is equal to ba, since there is no reason from c towards de that shows that the violence is lost at all, which, although it lessens continually towards e, yet there remains a sufficient amount of it, which is the cause that the weight never travels in a straight line towards e." (Guidobaldo, 1592, in Damerow et al., 1992, p.151-152)

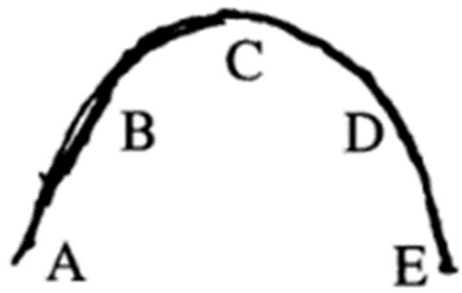


Fig.1.2 Guidobaldo's representation of the projectile motion in his *Notebook*.

Fig.1.3 (*) Reproduction of Guidobaldo's experiment (Cerreta, 2019).

- What methodological aspects emerge from Guidobaldo's excerpt? Which of them are still relevant in modern science from a methodological perspective?
- What kind of knowledge does fig 1.2 incorporate? What elements methodologically and epistemologically distinguish figures 1.1 and 1.2?
- How is the curve explained in terms of the Aristotelian paradigm of motions by Guidobaldo? In this sense, what are most "artificial" and obscure points in the argumentation?
- Given the geometrical shape of the trajectory, and the impossibility to explain it in terms of combination of straight lines and circles, how can the mathematical shape of the curve be deduced, in your opinion?

Task 2. Interpretation of the trajectory's curve

a. Consider the following excerpt from Renn et al. (2000), concerning Guidobaldo's experiment. This passage provides a mathematical explanation of why the hyperbole was not a valid candidate for the mathematical representation of the shape of the trajectory.

"Moreover, the dynamical explanation given in the [Guidobaldo's] protocol implies, on closer inspection, that the constantly decreasing ratio between violent and natural motion in descent assumed in the protocol is incompatible with the asymptotic behavior of the hyperbola. It follows, in fact, from this asymptotic behavior that this ratio should approach a constant different from zero. But once the parabola has been chosen, its geometrical properties, well-known since ancient times, suggest certain assumptions about the forces and how they act together."

- Starting from Guidobaldo's excerpt proposed in the previous section and following the passage above, try to reconstruct the reasoning that leads to exclude the hyperbole as a candidate for the trajectory.

b. Now, focus on the following extract from Galileo's *Discourses and mathematical demonstrations relating to two new sciences* (1638) in which the parabolic trajectory is demonstrated:

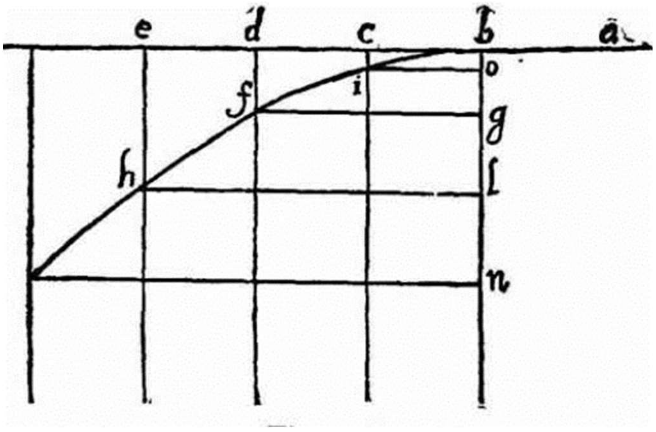


Fig.2.1 Figure supporting Galileo's demonstration.

“***Theorem I, Proposition I***

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola.

[...]

Let us imagine an elevated horizontal line or plane ab along which a body moves with uniform speed from a to b. Suppose this plane to end abruptly at b; then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular bn. Draw the line be along the plane ba to represent the flow, or measure, of time; divide this line into a number of segments, bc, cd, de, representing equal intervals of time; from the points b, c, d, e, let fall lines which are parallel to the perpendicular bn. On the first of these lay off any distance ci, on the second a distance four times as long, df; on the third, one nine times as long, eh; and so on, in proportion to the squares of cb, db, eb, or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from b to c with uniform speed, it also falls perpendicularly through the distance ci, and at the end of the time-interval bc finds itself at the point i. In like manner at the end of the time interval bd, which is the double of bc, the vertical fall will be four times the first distance ci; for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space eh traversed during the time be will be nine times ci; thus it is evident that the distances eh, df, ci will be to one another as the squares of the lines be, bd, bc. Now from the points i, f, h draw the straight lines io, fg, hl parallel to be; these lines hl, fg, io are equal to eb, db and cb, respectively; so also are the lines bo, bg, bl respectively equal to ci, df, and eh. The square of hl is to that of fg as the line lb is to bg; and the square of fg is to that of io as gb is to bo; therefore the points i, f, h, lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola.”

- What methodological aspects emerge from Galileo’s demonstration? Which of them are still relevant in modern science from a methodological perspective?
- What kind of knowledge does figure 2.1 represent?
- What are the main physical assumptions, not present in Guidobaldo’s argument, on which the demonstration is articulated?
- What physical innovations, with respect to Aristotelian paradigm of motions, are introduced in this demonstration?
- Which role does the uniform motion have in Galileo’s reasoning?
- What kinds of mathematics is used in the demonstration? What types of mathematical reasoning emerge?

Task 3: The epistemological role of symmetry

a. With the group, retrace the main epistemological steps that you highlighted in the previous activities; then answer these questions:
   - In your opinion, what role of the symmetry emerges in the evolution of the physical thinking around the topic of projectile motion?

- Is your answer to the first questions in parabolic motion changed? Now how would you describe the most revolutionary aspects incorporated in the recognition of the parabolic trajectory of a projectile?

b. Please, consider the following extract from the 5th edition of Walker's textbook *Physics* (2017):

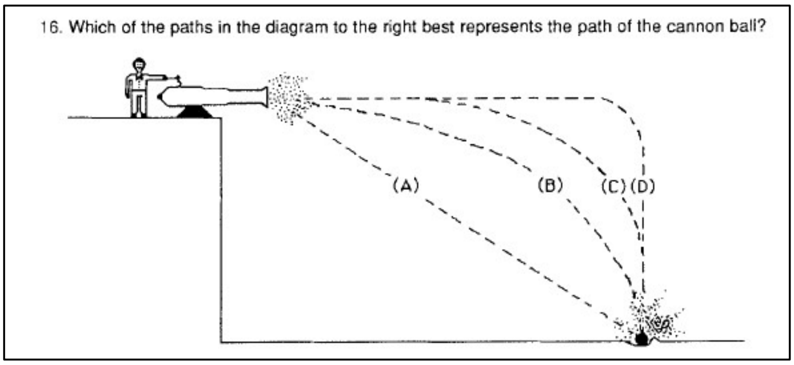
> *"Symmetries such as these are just some of the many reasons why physicists find physics to be "beautiful" and "aesthetically pleasing." Discovering such patterns and symmetries in nature is really what physics is all about. A physicist does not consider the beauty of projectile motion to be diminished by analyzing it in detail. Just the opposite—detailed analysis reveals deeper, more subtle, and sometimes unexpected levels of beauty."*

What do you think about this consideration about symmetry? Do you agree? Confront these few lines with idea of symmetry that emerged from the previous point.

Task 4: From the common sense to the designing of teaching modules

The following items are part of the *Force Concept Inventory*, a very well-known test on the concept of force, addressed to university students. All the possible choices resonate with images and conceptions that students possess. The correct choices are in square brackets, and the percentage of the answers given by the students for each possible solution are shown on the right side of the relative figures. The data refer to the experiment carried out by D. Hestenes, M. Wells and G. Swackhamer in 1992.

- Consider the correct answer in bold. What is the kind of knowledge activated by the student in order to choose this option?
- Now, consider the options marked in Italics. What is the kind of knowledge activated by the students in order to choose these other options?



Answers percentual (pre-test; post-test):

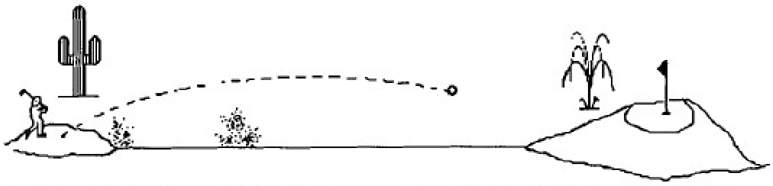
A – less than 1%; less than 1%

[B] – 40.6%; 77.5%

C – 52.1%; 20.4%

D – 6.4%; 1.1%

22. A golf ball driven down a fairway is observed to travel through the air with a trajectory (flight path) similar to that in the depiction below.



Which following force(s) is(are) acting on the golf ball during its entire flight?

1. the force of gravity
2. the force of the "hit"
3. the force of air resistance

(A) 1 only
(B) 1 and 2
(C) 1, 2, and 3
(D) 1 and 3
(E) 2 and 3

<u>Answers percentual (pre-test; post-test):</u>

A – 1.9%; 11.9%

B –4.6%; 7.7%

C –81.3%; 33.6%

[D] – 9.5%; 45.3%

E – 2.6%; 1.5%

- Now, keeping into consideration students difficulties in choosing the right answers to the previous questions and the whole historical path that led parabolic motion to be a foundational case for physics as discipline, how would you design your teaching on the topic of projectile motion? Please, build a conceptual map with the main conceptual and epistemological knots and mark in blue which ones can be particularly difficult for the students (epistemological obstacles a la Bachelard).

4.5 Final comments

In this chapter we presented the activity we designed on the three epistemological activators that emerged from the analysis of Walker textbook. Also the grids we designed for the analysis and reported in chapter two can be used to realize activities of text analysis. Chapter three showed their effectiveness for the analysis of Walker's textbook, but we imagine they can be used to analyse further texts.

This trial, as well as the next implementation of the activity on the epistemological activators, will be carried out next semester within the course of "*Insegnamento della Fisica: aspetti teorici e sperimentali*", led by O. Levrini, and in January-February 2021 within an IDENTITIES winter-school.

Conclusions

In the present thesis, we started from the selection and re-elaboration of the material presented in the PLS course with the main goal of building a set of analytic grids for the analysis of physics textbooks. In particular, we created a disciplinary and an interdisciplinary grid consisting of guiding questions for examining a text on multiple levels. The two grids were then applied to analyse, in depth, Walker's fourth chapter, leading us to highlight some important features of the text. From results obtained in the analysis, we were finally able to design a didactic activity targeted to preservice teachers, on the topic of the epistemological development of the theory of projectile motion during the 16th and 17th century.

In the following, a summary of the main results is presented.
Firstly, the re-elaboration of PLS materials led us to the production of *original grids* for the analysis of physics textbooks. These grids, exploring the disciplinary, argumentative, and linguistic aspects of the text, were proven to be powerful lenses since they allowed us to identify key-elements of the chapter that remained implicit at a first glance. For this reason, the grids seem to have the potential to guide the analysis of other physics textbooks.

Secondly, from the application of the analytic grids to Walker's fourth chapter, we obtained a set of important results. Initially, we highlighted the relevance of using articulated definitions to introduce the *ontological entities* on which the disciplinary discourse is based. For instance, in chapter four of Walker's textbook, the projectile was the only physical object rigorously defined. The ontological entities differed from the other concepts of the discussion, whose definitions had to be inferred from the context. Moreover, we recognised two objects, the motions independence and the symmetry, as two *epistemological activators* of the chapter. These two showed that the analysis of the textbook was compatible with the historical analysis of the parabola presented in the PLS course, in which the same elements emerged as fundamental for the evolution of the physical thinking (particularly from the work of Guidobaldo and Galileo).
On the level of the explanation-argumentation, by the means of the grids, it was possible to *reinterpret* an apparently *circular-argument*, the "*phenomenological demonstration*" of motions independence of the second paragraph, considering the epistemological aspects of the research of a physical correspondence between the theoretical model and the real life phenomenon. The combined study of the explicative and argumentative structure of the text allowed us to also enlighten a new praxis of the modern textbooks, i.e. the fact that the *mathematical structure of the discourse* does not explicitly emerge. In particular, as we noticed in the analysis of the parabolic trajectory's demonstration treated in the textbook, the explicit demonstrative structure disappeared. On the contrary, the historical

analysis of Galileo's *Discourses* showed the importance of the *demonstration* as a mathematical *meta-object* in physics. In addition, considering the equations of motions as main example, we highlighted how mathematical concepts are constructed without paying any attention to their underlying physical meaning. In fact, the Walker's textbook systematically employs a more abstract formalism of the analytic functions, preferred over a more intuitive one, focused on the expression of proportions (as, for example, Galileo did in his works).

On a more general perspective, the analysis of the Walker's textbook allowed us to build an argument supporting one of the main assumptions of the IDENTITIES project, namely the idea that preservice teachers can be guided to develop disciplinary and interdisciplinarity tools both through the analysis of texts and through the recognition, in the texts, of "*epistemological*" and "*linguistic activators*". In this sense, with the analysis of Walker's fourth chapter, we showed that, by choosing the adequate textbooks, this IDENTITIES assumption is indeed well-founded.

Starting from the individuation of the motions independence, the symmetry and the meta-object of demonstration as *epistemological activators* a teaching activity was designed and tested in the course of *Advanced Professional and Research Skills in Physical Sciences*. From the preliminary results of the pilot study the activity was reformulated and it will officially implemented for a more structured test in the course of "Insegnamento della fisica: aspetti teorici e sperimentali", in October-November 2020 (a course led by prof. Levrini within the Degree course in Physics in Bologna(and in the winter school of IDENTITES that will be organized in January-February 2021.

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