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**STUDY ON THE PRACTICAL  
REALIZATION OF A DEVICE ABLE TO  
GENERATE AN IN-SPACE  
3D LUMINOUS IMAGE**

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## **Abstract**

Gli ologrammi sono parte integrante della cultura pop a partire dagli anni 50, tanto che ad oggi sentirne parlare non desta più scalpore. Dal lato pratico, invece, solo negli ultimi anni sono state fatte ricerche approfondite con lo scopo di realizzarli. Fra i dispositivi attualmente in commercio, in pochi sono degni di nota e presentano numerose limitazioni, questo perché è molto difficile riuscire a progettare un sistema che permetta di illuminare dei punti specifici in uno spazio tridimensionale per lunghi periodi.

In questa tesi si illustrano i principi di funzionamento ed il progetto per un nuovo dispositivo, diverso da quelli fino ad ora realizzati, che sfrutti il decadimento spontaneo di atomi di rubidio eccitati tramite due fasci laser opportunamente incrociati. Nel punto di incrocio si produce luce visibile a 420 nm. Con un opportuno sistema di specchi che muovono velocemente il punto di intersezione tra i due fasci è possibile realizzare un vero ologramma tridimensionale visibile da quasi ogni angolazione.



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# Introduction

From the 50s, in almost every science fiction work, from books to films, there is no lack of some kind of device capable of creating what are called "holograms". Originally this term referred to the Dennis Gabor studies on the "holographic principle" and his method of reconstructing images from an interference pattern. Nowadays it's become a common word in the pop culture to indicate a full 3D representation of an object, created using light and projected in the tridimensional space. The first one to have described what we can assume to be an hologram was Isaac Asimov in his work "*The Encyclopedists*", where he describes how the protagonists discovered a prerecorded message in the form of a bright 3D moving figure in the air reproducing a speaking person. What sealed officially the word hologram and that type of representation was the famous film *Star Wars: A New Hope*. From that moment, whenever someone reads or see about holograms, no one is more surprised, because they know exactly what they are and how they look like.

Although the holograms have become very popular inside the pop culture, the researches conducted in this field aren't so promising. Only in the past twenty years various companies have invested time and money trying to replicate those 3D images, finding more and more problems and limitations realizing them. In the market there are some devices capable to create some sort of hologram, but they are often very small or are using optic illusions or are not even 3D, so only few of them are noteworthy.

The aim of this thesis is to define the state of the art on the topic and to propose some solutions on various difficulties to problems and obstacles that may be encountered during the study on this field.

In Chapter 1 the real holography and its popular imagination are described and compared, doing a historical prelude on the matter and analyzing the main features of the holograms. Then, a full description on what are the most famous and noteworthy devices able to create them is done, presenting their limitations and proposed solutions to overcome some problems.

Chapter 2 is based on describing the main features required to create a hologram. Both the problems encountered and some solutions to them are covered.

In Chapter 3 is described the project of a device, different from what it's actually on the market, able to generate a full 3D hologram. The main structure and principle of operation are covered, giving also more solutions on how to improve the different parts.





# Chapter 1

## Holography and holograms

At the beginning of the 20<sup>th</sup> century, one of the most studied in depth branches of science was the *microscopy*. The studies carried out by W. L. Bragg on the field of X-rays (for which he received the Nobel prize for Physics in 1915) led to the realization of the first X-ray microscopes (built by Bragg himself) in the late '40s. At the same time, with the help of Bragg's studies, the Hungarian-British physicist Dennis Gabor theorized a new method of using electronic waves to improve the already existent electronic microscopes<sup>[1]</sup>, coining the term *hologram* (from the Greek words *holos* "whole" and *graphē* "drawing"): the *holographic method*.

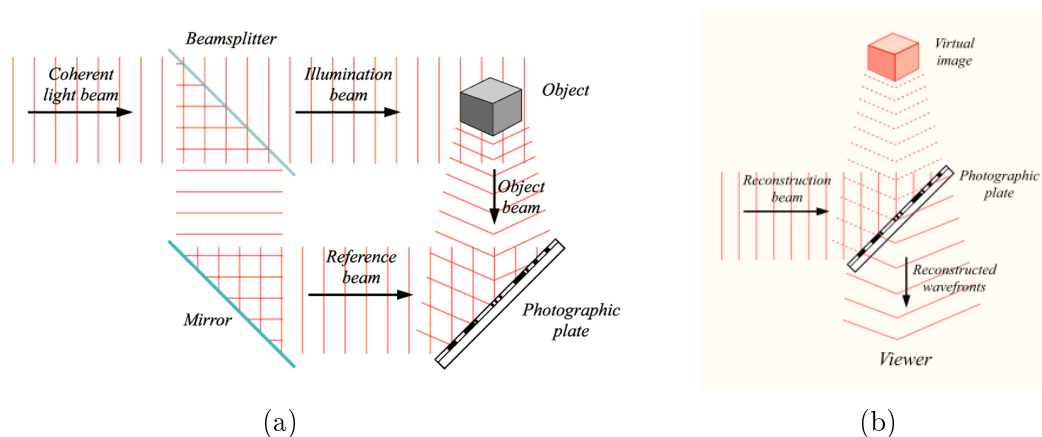
### 1.1 Holography in its pure meaning

The Gabor's goal was to reconstruct an image on a photographic film that would have all the optical means (such as parallax) using electron beams and to view the result with the visible light. However, the advancements into optical transfer function theory improved the quality of the images to a level that the holographic principle became irrelevant. It was only with the advent of the first optical laser 15 years later that holography became popular.

The holographic principle at the basis of Gabor's papers exploit the interference fringes pattern of light (shown in **Figure 1.1a**). To record an holographic image of an object, it is necessary to use a coherent light beam and some sort of beam splitter (like a *Fresnel biprism* or a *Lloyd's mirror*), in order to have 2 independent coherent light beams; the first one, called *Illumination beam*, must hit the body and be reflected, creating the *Object beam*, towards a photographic plate where is located a film; the second one, called *Reference beam*, should only be reflected towards the plate and cause interference with the object beam. The interference pattern thus formed is recorded by the film, containing all the information about the object wavefront. Now, to see the obtained holographic image, the film has to be developed like a photographic negative.

When the film is illuminated by the reference beam alone, now called *reconstruction beam*, a virtual image representing the object will be created, as shown in **Figure 1.1b**.

The holographic image obtained will preserve some optical features of the real object, such as the parallax, the binocular vision, ecc. (**Figure 1.2**). Using different laser wavelengths in the recording process will result in a different level of detail in the image, as using different reconstruction beams will produce aberrations: for instance, the reconstructed image is magnified if the laser used to reconstruct the hologram has a longer wavelength than the original laser<sup>[2]</sup>. That is a hologram.



**FIGURE 1.1:** Simplest apparatus of recording (a) and reconstruction (b) of a holographic image.



**FIGURE 1.2:** The famous hologram of a mouse. A shift in viewpoint causes the interception of a different part of the image wavefront, resulting in a different perceived image of the object, as it was 3D.

## 1.2 Holograms in popular imagination

Nowadays, when we think and speak about holograms outside the scientific sphere, we mean a total different thing and someone can ask why the word hologram has taken on such a different meaning. It all began in the '30s, with the invention and the distribution of the television. People were excited by the possibility to see a detailed moving person on a screen, and some were so thrilled that started to imagine new evolution of that device. One of many in particular has a revolutionary idea: an in-air 3D luminous projection that could be seen from any viewpoint. That was the idea of Isaac Asimov, a later became famous science fiction writer.

Among his many novels, he wrote, in 1942, "*The Encyclopedists*"<sup>[3]</sup>, an issue of *As-tounding Science-Fiction*, a science fiction magazine published in America since 1930. In that particular novel, Asimov described how the protagonists discovered a prerecorded message of Hari Seldon, one of the characters in the novel, described as a bright 3D moving figure in the air reproducing the person of Seldon. Asimov did not described it with the terms "hologram", as the word was not invented yet, but it definitely was the first "3D hologram" appearance.

What sealed officially the word hologram and that type of representation was the famous film *Star Wars: A New Hope*, released in 1977. At the beginning of the film, Luke Skywalker accidentally triggers a robot that reproduce a "holographic recording" of princess Leia asking for help to Obi-Wan Kenobi (**Figure 1.3**)

After that, the concept of holograms entered in everyday life thinking at the point that has become almost common and took for granted.



**FIGURE 1.3:** The hologram of princess Leia projected in front of Luke Skywalker, well defined and coloured.

One can say that this change in the meaning could be a misrepresentation of the original Gabor's theory, but Gabor himself said that the word "hologram" that he invented was not quite exact, because it didn't really represent the whole image, but only a part<sup>[1]</sup> (more than a photograph, but not the entire object). From now on we'll use the word "hologram" or "3D hologram" to indicate a luminous 3D representation of a generic static or moving object.

### 1.3 3D Holographic devices: merits and limits

There are countless representations of 3D holograms in modern literature, from films to videogames (**Figure 1.4**), but only nowadays the studies carried out to make them possible are reaching an interesting point. The reasons behind that are likely to be found in the lack of adequate tools and technologies, as much as in the fact that it's an essentially recent research. In order to understand the progress made so far, I shall now present some techniques developed and used to recreate some sort of 3D holograms, all with their merits and limits, that were already used in the past and/or are still being improved.



**FIGURE 1.4:** Examples of holograms in the modern literature: (a) a hologram of a space station in the videogame *Mass Effect 3*, released by BioWare (2012); (b) a scene from the film *Avatar*, directed by James Cameron (2009).

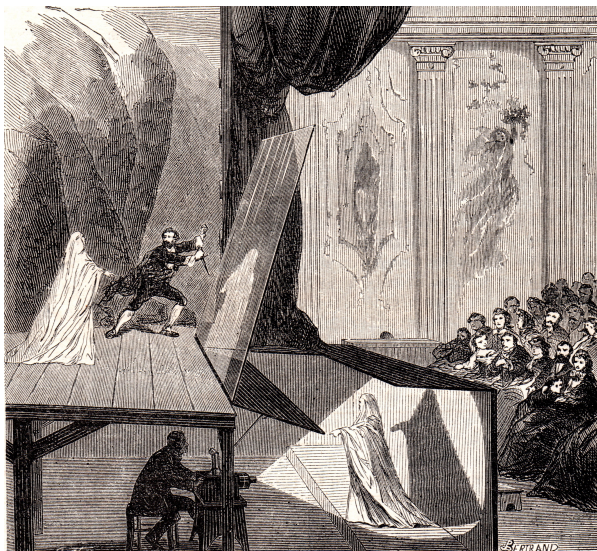
#### 1.3.1 Pepper's ghost

The first ever realization of a hologram was actually an illusion, called *Pepper's ghost*: it owes its name to its inventor John H. Pepper, who popularized the effect in a demonstration in 1862. The basic trick that he used involved a stage, a "substage" located under

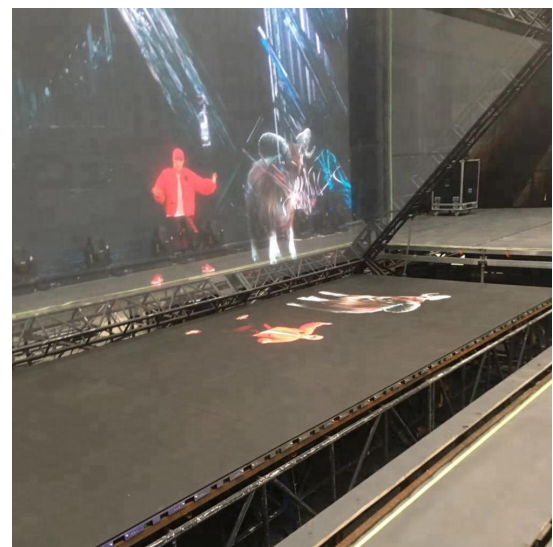
and a bit forward the main one, and a large glass plate placed on the stage, rotated around its horizontal axis (where the stage ends) at about  $45^\circ$  towards the public<sup>[4]</sup>. When the lights are bright in the substage and dimmed in the main stage, it starts to appear a reflection (a "ghost") of the scene played below, giving the optical illusion of a real person being on the stage. The entire setup is shown in **Figure 1.5a**.

This technique was then improved with new technologies and materials (**Figure 1.5b**). The most famous use of it was that by the company *Digital Domain*, in collaboration with AV concepts, in 2012, when they staged a concert in memory of the rapper Tupac, digitally reconstructing him and projecting on a large plexiglass sheet, making him sing a whole song (**Figure 1.6a**). In that case, there were not a substage or a scene played elsewhere, but was used a projector and various mirrors, in a completely dark location. After them, other companies used this technique for their performance: the BASE Hologram, for example, is one of the most present company in this sector and staged lots of performances in big theatres, with the help of their software and large glass-like plates (**Figure 1.6b**).

Of course, those are not a possible solutions for a "domestic type" of holograms, or for a small sized hologram in general, since there's the need for large systems, and whoever would be on the stage could not see the projected image. However, it remains a viable alternative for live lectures or conferences or performances.

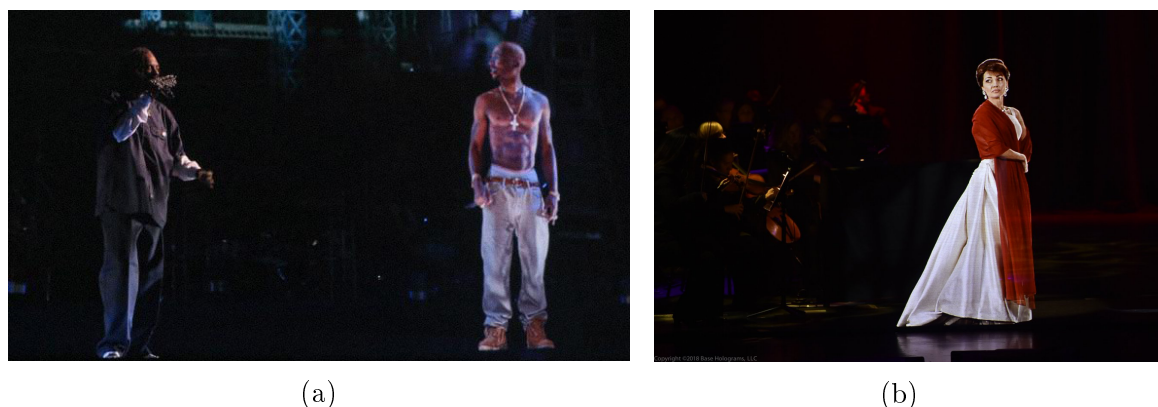


(a)



(b)

**FIGURE 1.5:** (a) Pepper's ghost scheme. It's clearly visible the substage and the plate; (b) a modern realization of the Pepper's ghost, using projectors.



**FIGURE 1.6:** (a) the hologram of Tupac (on the right), with the rapper Snoop Dog (on the left); (b) the hologram of the lyric singer Maria Callas, died in 1977, staged by Base Holograms.

### 1.3.2 Plasma holograms

In order to create a "real" hologram, it has to be generated a luminous figure in a 3D space, such as the air. Taking this essential definition as the main road, the Japanese company Burton Inc., along with Hamamatsu Photonics, developed a method to light up a small volume of air using plasma<sup>[5]</sup>. The principle behind that method is the ionization of molecules: using a focused high intensity laser light, it's possible to separate some electrons from their molecules, resulting in a gas full of ionized particles and free electrons. This new state of matter is called *plasma*<sup>[6]</sup>. Since the molecules and the electrons are in a high excitation level, the fluid dissipates energy emitting radiation in various forms, such as heat, light and sound, all in a short interval of time. In this way, a "bright point" is rapidly created and with the phenomenon of the persistence of vision, one can also rapidly move the laser in space creating lots of those points forming an image. The process and his results are shown in **Figure 1.7**. Such points in space are called *Voxel*, from the mix of the words "volume" and "pixel", in the meaning of a "single building block unit of a 3D image", as a pixel is the base unit of a 2D digital screen.

The idea of using plasma to create a voxel is carried out by another Japanese research group, the Digital Nature Group, that in 2015 created the project *Fairy Lights in Femtoseconds*<sup>[7]</sup>, which use a high intensity femtosecond laser controlled by a SLM (Spatial Light Modulator) and some lens to make a far smaller hologram (up to 1 cm<sup>3</sup>) creating between 4000 and 200000 voxels per second, and one can even "touch" it. In practical meanings, if you put a finger near the hologram, you'll experience a sting-like sensation caused by the high temperature of the plasma. Both the apparatus and the hologram are shown in **Figure 1.8**.

This method is the one that the most goes close to our definition of hologram, and offers some interesting viewpoints. First of all this doesn't need a particular chamber or gas to display the image and, second, the resulting image is very bright even in day time. On the contrary though a very high power laser is needed to create that voxels of plasma, and it's difficult to move at the necessary speed in order to have a smooth and detailed image. To do so, we have to downscale the entire figure, but at that point it's very difficult to see it. Also, plasma in this state is dangerous since it's not fully controlled by some sort of apparatus and relies only on the ambient conditions in which the hologram is created.

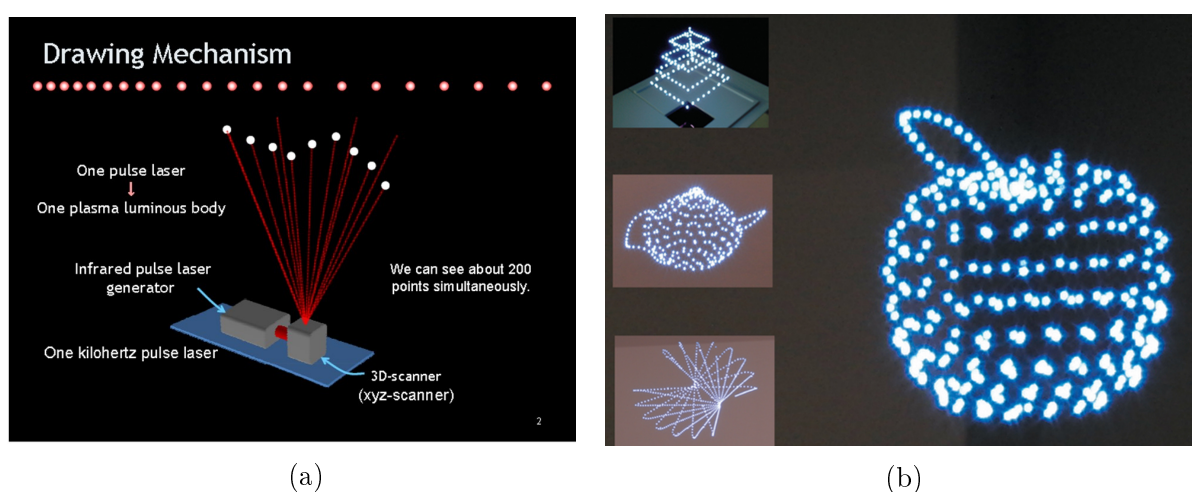


FIGURE 1.7: (a) the explained drawing mechanism provided by Burton Inc.; (b) various resulting holograms achieved with this technique.

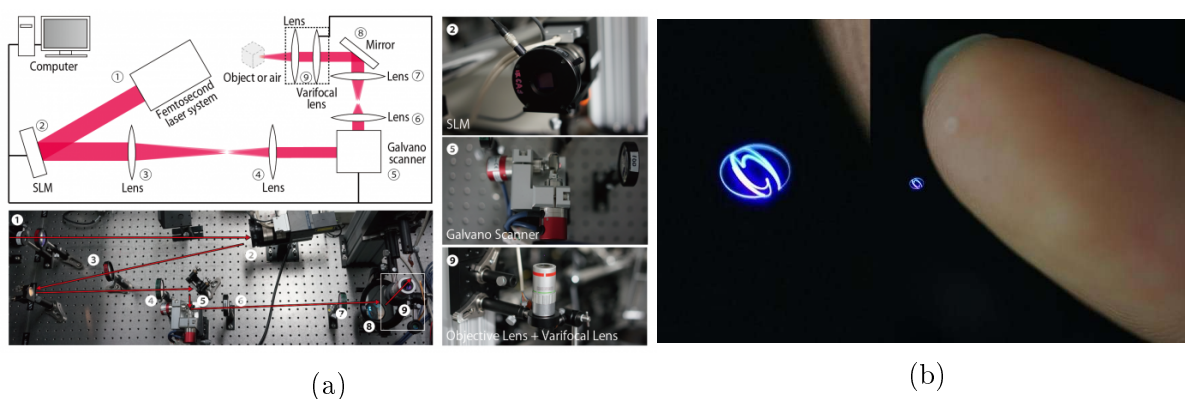


FIGURE 1.8: (a) the schematized apparatus used to create the *fairy lights*; (b) the resulting hologram scaled with a finger.

### 1.3.3 Volumetric displays

The last project we'll be discussing about regards a device made by the *Voxon Photonics*. The company was founded in 2009 by Gavin Smith and Will Tamblyn as a startup in a backyard shed. After many years, in 2013, they joined the forces with Ken Silverman, a well skilled programmer, and many other startups and in 2016 they first released the *Voxon VX1*, their first fully integrated commercial volumetric display<sup>[8]</sup>.

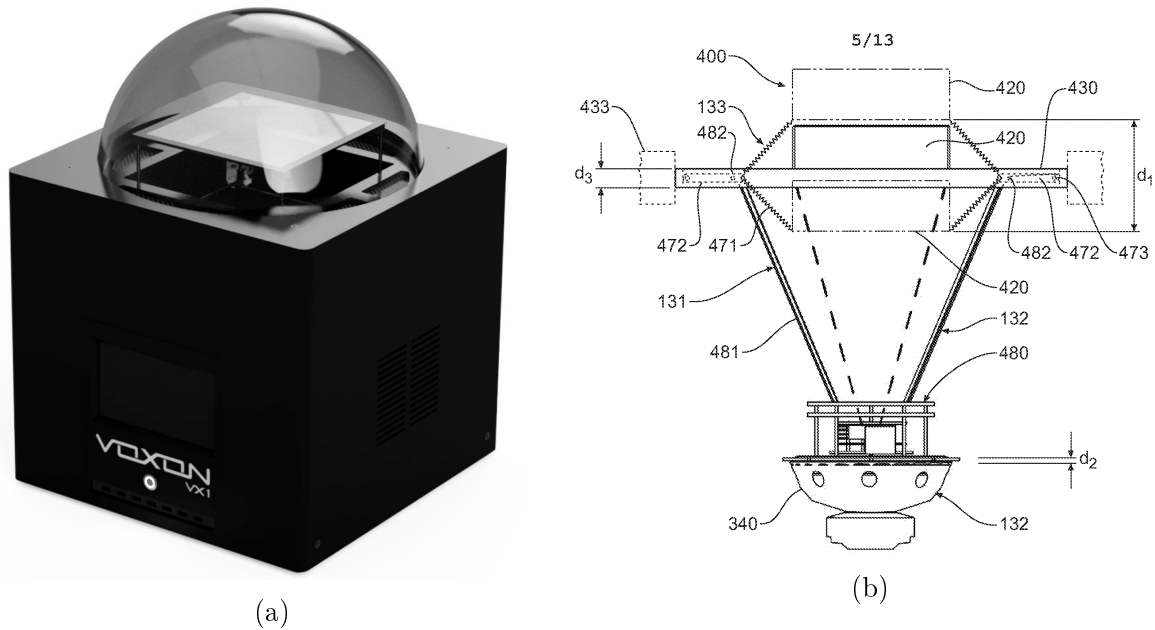
This volumetric display is capable to project a full 3D image in a  $18\text{ cm} \times 18\text{ cm} \times 8\text{ cm}$  volume, with an high level of detail, both coloured and dynamic, which means that it's possible even to play some simple games, as shown in **Figure 1.9**. This system was even used in the Mobile World Congress Americas 2018 event in Los Angeles to test the demos of some real time holographic video conferencing using the 5G technology<sup>[9]</sup>, showing its future possible application.

As described in both the manual and the patent<sup>[10][11]</sup>, as well as shown in **Figure 1.10**, the voxon VX1 is a  $39\text{ cm} \times 39\text{ cm} \times 42\text{ cm}$  device, with a hemispherical glass dome at the top containing a  $18\text{ cm} \times 18\text{ cm}$  screen. Below the screen, inside the body of the apparatus, there's a projector pointing upwards at the display. The screen is capable of oscillating along the vertical axis covering 4 cm in both directions (8 cm in total). To reproduce a 3D figure in input, a data processor slices up the image horizontally in 200 layers, each one consisting of  $1000 \times 1000$  pixel, for approximately a total of 200 million voxels, and project every slide on the screen, when it's in a different position, and so the 3D image is formed by persistence of vision. To do so, the screen movement and the projected images must be synchronized by the "synchronization system", which determines when a complete volume has been processed by the projector implying that the screen is about to start a new cycle.



**FIGURE 1.9:** (a) An holographic representation of a spine. The borders of the display are highlighted; (b) the game called *Space Out*, developed by the student Nathan Winckel specifically for the Voxon VX1.





**FIGURE 1.10:** (a) The *Voxon VX1*, consisting of the dome, with the screen, and the body; (b) An example of embodiment. It's easy to distinguish the screen (110), the projector (120) and the drive system, consisting of the actuator arrangement (131), a support structure (132) and the "resonating mounting arrangement" (133).

One of the most important factors in determining the quality of the 3D image is the frequency at which the screen has done a full cycle, called the *reciprocating frequency*. When the cycle starts at the lowest screen position, during a complete cycle the entire volume is covered two times. In one embodiment, a 20 Hz reciprocating frequency is capable of sending 200 frames/cycle (100 for a single traversal), for a total of 4000 frames/s, corresponding to 40 volumes/s.

As already shown, the results of this device are impressive, but it's limited in size since relies on an oscillating screen synchronized with a projector, the whole device needs the structure below it that contains all the systems described above and, most importantly, it has a big stream of data to process. Doing simple math, we know that we have  $N_V = 2 \cdot 10^8$  voxels (an HD screen  $1920 \times 1080$  has  $\approx 2 \cdot 10^6$  pixels, 2 orders of magnitude lower), so if we put 8 bits per RGB color (with a total of 24 bits per pixel), we'll use the **Equation 1.3.1** written below<sup>[12]</sup>:

$$N_{bit} = N_{px} \cdot N_{py} \cdot N_{pz} \cdot N_{bpc} \cdot N_{colors} , \quad (1.3.1)$$

that leaves us with  $4,8 \times 10^9$  bits.

The video data bandwidth required can be calculated with the **Equation 1.3.2** below

$$B_{ps} = N_{bit} \cdot R_{rps} , \quad (1.3.2)$$

where  $B_{ps}$  is the bandwidth and  $R_{rps}$  is the rate of volumes per second displayed. With our data of 40 volumes per second we'll have a bandwidth requirement of  $1920 \cdot 10^8$  bits/s, that is 24 GB/s, only to display that "little" volume of about  $2 \text{ dm}^3$ .

# Chapter 2

## Device design

For the purpose of making a hologram, we have to set some basic characteristics to achieve in our project. In this chapter we'll focus on the description of the supposed theoretical functioning along side its basic necessary components.

### 2.1 Lighting process: interaction of radiation with particles

One of the fundamental characteristics of a hologram is its being "made of light", so it is necessary to produce controlled light, in certain points in space, which spreads in every direction. The processes that can cause emission and absorption of electromagnetic radiation are well explained and studied in the *Quantum Electrodynamics Theory* (QED), described by Dirac while studying the transition rate between two different energy level of a quantum system<sup>[13]</sup>.

#### 2.1.1 Absorption and emission of radiation

In the presence of an electromagnetic field, the Hamiltonian of the quantum system formed by a particle and the radiation is modified in a way that causes transitions in the unperturbed system by changing the state of the particle and simultaneously decreasing or increasing the number of field quanta by one unit (causing the so called absorption and stimulated emission processes, respectively). Another way of seeing it is considering our particle as an electron bounded in atoms in specific energy levels so, when interacting with a electromagnetic radiation, there's a shift of electrons between the ground state and the excited levels. The simplest way to solve this problem is using the time dependent perturbation theory, where the energy eigenvalues of the Schrödinger's problem are a linear composition between the eigenvalues of the unperturbed system and the eigenvalues of the perturbation terms of the Hamiltonian. The aim of this thesis is

not to present an explanation of this method, findable in any text of quantum mechanics, but only to use its results to underline some properties. The diagram schematizing all the processes that we are going to discuss is shown in **Figure 2.1**.

To simplify the concept, let's consider the energy spectrum of an atom: in principle the spectrum will have a ground state and various excited levels of increasing energy, distinguished by an eigenfunction  $\tilde{n}$  and an energy value  $E_n$ . Let's call the ground state 1 and the immediately next upper level 2, with  $E_2 > E_1$ . If the system interacts with a quanta of radiation of energy  $\Delta E \approx E_2 - E_1$ , the photon is absorbed and the system makes a transition to the energetic level  $E_2$ . A notable effect is the attenuation of the intensity of light waves as they propagate through a medium, when the electromagnetic energy is mostly converted into thermal energy, increasing the temperature in the medium.

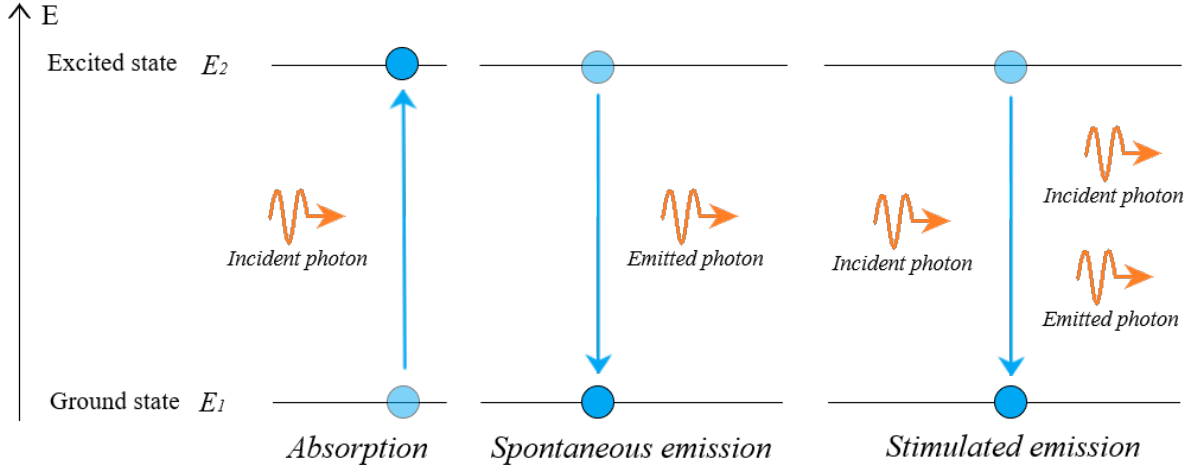
The approximation comes from the fact that the levels are actually continuous bands of amplitude calculable through the Heisenberg uncertainty principle, which states that the more precisely the energy of some particle is determined, the less precisely the time at which the measurement occurs can be predicted from initial conditions, and vice versa<sup>[14]</sup>. The formal inequality relating the standard deviation of energy  $\sigma_E$  and the standard deviation of time  $\sigma_t$  is shown in the **Equation 2.1.1** below:

$$\sigma_t \sigma_E \geq \frac{\hbar}{2}, \quad (2.1.1)$$

The general meaning of the "energy-time principle" is that a quantum state that exists for only a short time cannot have a definite energy. The reason is that the frequency of a state is inversely proportional to time and the frequency connects with the energy of the state, so to measure the energy with good precision, the state must be observed for many cycles.

The explanation behind the stimulated emission is due to Albert Einstein who showed, in 1916, studying the spectral distribution curve of temperature radiation similarity to Maxwell's velocity distribution curve, the following synthesized behaviour<sup>[15]</sup>. Let's assume that we have some atoms in an excited state  $E_2$  invested by a stream of photons of energy  $\Delta E = \hbar\omega$  and momentum  $\vec{p}$ . What happens is that the photons interaction induce the switching of electrons between the excited state to the ground level  $E_1$ , emitting another photon. The electromagnetic waves associated to those new photons share the same energy, wave length, phases and polarization state of the incident stream.

Along with these two processes there's another type of radiation emission from atoms, called the *spontaneous emission*. In 1892, Hendrik Antoon Lorentz presented his *electron theory* (that made him win the Nobel prize in 1902, together with Pieter Zeeman<sup>[16]</sup>), in which he explained how matter was also composed of electrons and that their oscillations, caused by thermal disturbance, induce light emission. With the advent of the new atomic orbital theory and Max Planck's theory of the quanta, they managed to give a more precise formulation to these phenomena. The spontaneous emission is the process in



**FIGURE 2.1:** The absorption, spontaneous emission and stimulated emission processes.

which a quantum mechanical system (such as an atom) transits from an excited energy state  $E_2$  to a lower energy state  $E_1$  and emits a photon with angular frequency  $\omega$  and an energy  $\hbar\omega = E_2 - E_1$ , but the phase of the photon in spontaneous emission is random as is the direction in which the photon propagates.

The transition probabilities per unit time are given by<sup>[17]</sup>

$$\frac{dP_{\tilde{1}n \rightarrow \tilde{2}(n-1)}^{(1)}}{dt} = \frac{2\alpha|\omega|^3}{3c^2} |\langle \tilde{2} | \mathbf{r} | \tilde{1} \rangle|^2 \bar{n}, \quad (2.1.2a)$$

$$\frac{dP_{\tilde{1}n \rightarrow \tilde{2}(n+1)}^{(1)}}{dt} = \frac{2\alpha|\omega|^3}{3c^2} |\langle \tilde{2} | \mathbf{r} | \tilde{1} \rangle|^2 (\bar{n} + 1), \quad (2.1.2b)$$

for absorption and emission processes, respectively. Here  $\bar{n}$  is the average number of photons of a given frequency  $\omega$ . From both **Equation 2.1.2a** and **Equation 2.1.2b** we can extract some fundamental consequences:

- The probability of absorbing a photon is proportional to the intensity of the radiation field present before the transition, represented by the factor  $\bar{n}$ . However, the probability of emission consists of two terms: the first one also depends on the intensity of the radiation field (stimulated emission), while the second term, independent of the field intensity, allows the atom to decay from an excited state (spontaneous emission).
- The ratio  $(\bar{n} + 1)/\bar{n}$  is needed to preserve the correct thermal equilibrium of the radiation with a gas: in a gas at temperature  $T$ , the number of atoms in the states

$a, b$  is given by  $e^{-E_a/k_bT}$  and  $e^{-E_b/k_bT}$ , respectively. The condition for equilibrium is

$$P_{emission}e^{-E_a/k_bT} = P_{absorption}e^{-E_b/k_bT} \quad (2.1.3)$$

which yields

$$\bar{n} = \frac{1}{e^{\hbar\omega_{ab}/k_bT} - 1} \quad (2.1.4)$$

This deduction of Planck's law led Einstein in his explanation of the spontaneous and stimulated emission in quantum theory.

### 2.1.2 Selection rules

The transition probabilities are also proportional to the squared modulus of the matrix elements  $|\langle \tilde{2} | \mathbf{r} | \tilde{1} \rangle|^2$ , therefore if the value of the matrix elements is  $\sim 0$ , the transition almost doesn't happen and we say that is forbidden. With this method it's been possible to define a priori whenever one transition could happen or not, described by the *selection rules*.

For practical reasons we'll not cover the calculus behind the derivation of those rules, but we'll provide only the selection rules related to the electronic interactions in atoms: considering  $L$  as the angular momentum quantum number of an atom,  $S$  the spin quantum number,  $J = L + S$  the total angular momentum quantum number, and  $M_J$  the secondary total angular momentum quantum number, a transition between two levels it's possible if:

1.  $\Delta J = 0, \pm 1$ , but it's forbidden the transition between  $J_i = 0$  and  $J_f = 0$  ( $J = 0 \leftrightarrow 0$ );
2.  $\Delta M_J = 0, \pm 1$ ;
3.  $\Delta L = \pm 1$ ;
4. If  $\Delta S = 0 \Rightarrow \Delta L = 0, \pm 1$  ( $L = 0 \leftrightarrow 0$ )

## 2.2 Active material proposal: Rubidium

Defined the processes that controls the light emission, we need to find an *active material* to operate with, thus with the following characteristics: (i) it should be simple to excite, with relatively low energy cost; (ii) it must have an energy levels configuration to allow spontaneous emission of photons in the visible spectrum, thus propagating light in every direction in order to be seen from every point of view; (iii) it should be in a state with a

density value high enough to have a lot of excitable particles (and so a lot of luminous voxels).

The materials more reactive to the emission processes described above are the *hydrogen-like atoms/ion*, atomic nucleus bound to one electron and thus isoelectronic with hydrogen. Examples of hydrogen-like atoms/ions are  $\text{He}^+$ ,  $\text{Li}_{+2}$ . Because those are basically two-particle systems with an interaction depending only on the distance between the two particles, the electron and the atomic nucleus, their non-relativistic Schrödinger equation can be solved in analytic form, giving us detailed information about their energy spectrum. An approximation of hydrogen-like atoms are the neutral alkali metals whose electronic structure is composed by deep filled shells with a single electron in an  $nS$  state. The chemical properties are mainly due to this single electron that sees a nucleus whose charge is partially shielded by all the other electrons in the inner shells. Among all of the simplest possible choices, one of the alkali metals shows some interesting characteristics: the Rubidium.

### 2.2.1 Characteristics and general applications

Rubidium was discovered in 1861 by Robert Bunsen and Gustav Kirchhoff during some researches on the mineral lepidolite using flame spectroscopy. Because of the bright red lines in its emission spectrum, they chose a name derived from the Latin word *rubidus*, meaning "deep red"<sup>[18]</sup>. Rubidium is a very soft, silvery-white metal, it's the fourth alkali metal (so belonging to the first group with the electron configuration  $[\text{Kr}]5s^1$ ), with an atomic number  $Z$  of 37, and it's the first alkali metal to have higher density than water, with a standard atomic weight of  $85.467(8) \text{ u}$ <sup>[19]</sup>. On Earth, its isotopic composition is: 72% is a stable isotope  $^{85}\text{Rb}$ , and 28% is the metastable  $^{87}\text{Rb}$ , with a half-life of 49 billion years. Some of their physical properties are listed in the **Table 2.1** and **Table 2.2**; 24 additional isotopes have been synthesized in the past, but they all have a half-life less than 3 months and mostly are highly radioactive, so have few uses.

The studies conducted on Rubidium's energy levels composition led to various different applications: due to its moderate temperature required to obtain vapor pressures and its optical properties, Rubidium, particularly vaporized  $^{87}\text{Rb}$ , is one of the atomic species employed for laser cooling and Bose–Einstein condensation; the resonant element in atomic clocks utilizes the hyperfine structure of rubidium's energy levels, and rubidium is useful for high-precision timing. It is used as the main component of secondary frequency references (rubidium oscillators) in cell site transmitters and other electronic transmitting, networking, and test equipment.

### 2.2.2 Energy configuration and four-level scheme

We already said that the alkali metals are relevant to various quantum optics experiments, due to their optical electron in the outermost shell, and Rubidium is no exception. The

**TABLE 2.1:** Rubidium 85 Physical Properties.

|                            |                        |                         |
|----------------------------|------------------------|-------------------------|
| Nucleons Number            | $Z + N$                | 85                      |
| Relative Natural Abundance | $\eta(^{85}\text{Rb})$ | 72.17%                  |
| Nuclear Lifetime           | $\tau_n$               | (stable)                |
| Atomic Mass                | $m$                    | 84.911(8) u             |
| Density at 25 °C           | $\rho_m$               | 1.53 g/cm <sup>3</sup>  |
| Melting Point              | $T_M$                  | 39.30 °C                |
| Boiling Point              | $T_B$                  | 688 °C                  |
| Specific Heat Capacity     | $c_p$                  | 0.363 J/g·K             |
| Molar Heat Capacity        | $C_p$                  | 31.060 J/mol·K          |
| Vapor Pressure at 25 °C    | $P_v$                  | $5.2 \times 10^{-5}$ Pa |

**TABLE 2.2:** Rubidium 87 Physical Properties.

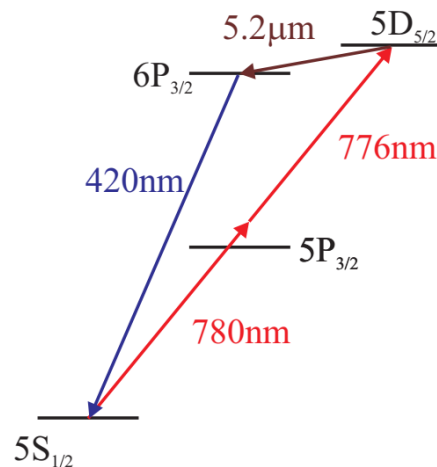
|                            |                        |                          |
|----------------------------|------------------------|--------------------------|
| Nucleons Number            | $Z + N$                | 85                       |
| Relative Natural Abundance | $\eta(^{87}\text{Rb})$ | 27.83%                   |
| Nuclear Lifetime           | $\tau_n$               | $4.88 \times 10^{10}$ yr |
| Atomic Mass                | $m$                    | 86.909(2) u              |
| Density at 25 °C           | $\rho_m$               | 1.56 g/cm <sup>3</sup>   |
| Melting Point              | $T_M$                  | 39.31 °C                 |
| Boiling Point              | $T_B$                  | 688 °C                   |
| Specific Heat Capacity     | $c_p$                  | 0.363 J/g·K              |
| Molar Heat Capacity        | $C_p$                  | 31.060 J/mol·K           |
| Vapor Pressure at 25 °C    | $P_v$                  | $4.0 \times 10^{-5}$ Pa  |



first thing to do is checking if it's possible to generate spontaneous emission of visible light, and so we have to find a suitable high energy state; the first transition possible is the  $5P_{1/2} \rightarrow 5S_{3/2}$ , but the corresponding wavelength (in vacuum) is  $\sim 780$  nm, thus in the infrared spectrum and not in the visible one, while the transition between the energy  $6^2P_{3/2} \rightarrow 5^2S_{1/2}$  is likely to be chosen: the associated wavelength is  $\sim 420$  nm, making theoretically possible to generate a pure blue light beam.

Recent experiments<sup>[20][21][22]</sup> have reported the generation of coherent 420 nm blue light by pumping the Rb  $5S \rightarrow 5P \rightarrow 5D$  transition. In their experiment, there were considered a four-state scheme of the Rubidium shown in **Figure 2.2**: pumping the electron 2 times to the  $5^2D_{5/2}$  level using coherent infrared light, one can make the population inversion required; due to the selection rules, the  $5^2D_{5/2} \rightarrow 5^2S_{1/2}$  transition is prohibited, thus the system decays with a 35% probability to the intermediate state  $6^2P_{3/2}$  and then it decays spontaneously to the ground state directly with a 31% probability., emitting the desired photon<sup>[23]</sup>. In one example, the optimal conditions found include a Rb vapor pressure of  $10^{-3}$  mbar and coherent input excitation beams at available powers of 25 mW at 780 nm and 17 mW at 776 nm, producing about 1.1 mW of coherent blue light close to two-photon resonance of the input lasers.

**FIGURE 2.2:** The four-level scheme considered in our description. The spectroscopic notation of the energy levels and the optical transitions, with the corresponding wavelengths, are shown.



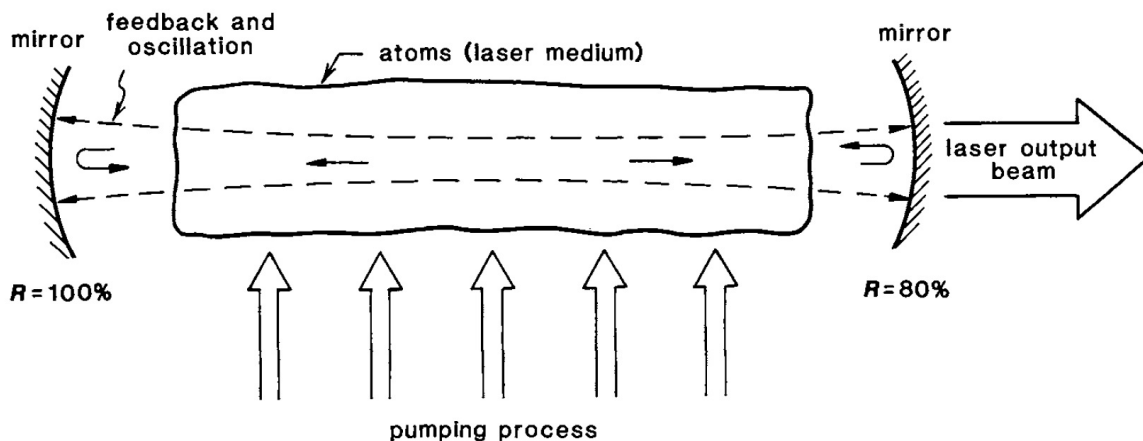
## 2.3 Radiation sources: lasers

Having defined in the previous section the characteristics of the material suitable for our purpose, we have to decide how to excite the atoms in order to make them do the transitions, and the most obvious choice falls on lasers as they are capable of producing coherent electromagnetic waves in the infrared spectrum.

The word LASER stands for Light Amplification by Stimulated Emission of Radiation<sup>[24]</sup> and was coined in the late '50s by Gould R. Gordon and built by Theodore H. Maiman. Actually, the forerunner of the laser was the the MASER, Microwave Amplification by Stimulated Emission of Radiation, built by Charles H. Townes, James P. Gordon, e Herbert J. Zeiger in 1953. This system is capable to produce coherent electromagnetic waves at microwave, radio and infrared frequencies. Five years later Gordon hypothesized that the same principle could be used to produce waves in the optical region, therefore, in the first instance, it was called "optical maser". Nowadays, the word laser refers to all the possible frequencies that the device can emit, with a specification in front of it if necessary (e.g. X-rays laser), though the word maser still remains in use indicating only the microwaves emission. The functioning principle is the same for all the lasers and masers and we'll provide a brief description.

### 2.3.1 Laser functioning principle

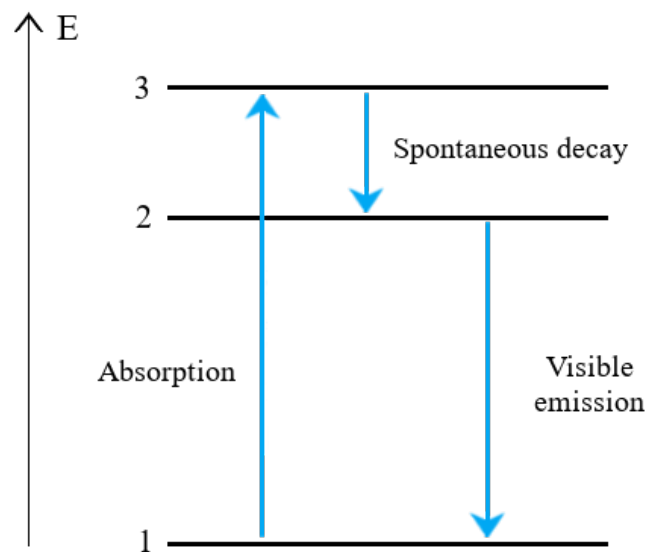
There exists various types of lasers, along with different structures, but all of them shares the same functioning principle. The essential elements of a laser device, as shown in **Figure 2.3**, are: (i) a laser medium consisting of a collection of atoms, molecules, ions, or in some instances a semiconducting crystal; (ii) a pumping process to excite the material (molecules, etc.) into higher quantum-mechanical energy levels, such as a flashing pump; and (iii) suitable mirrors, one of them only partially reflective, that allow a beam of radiation to either pass once through the laser medium or bounce back and forth repeatedly, resulting in an amplification of the beam.



**FIGURE 2.3:** Functioning structure of a typical laser, with common reflective coefficients  $R$ .

To produce a laser beam, we need to consider at least 3 energy levels ( $E_1 < E_2 < E_3$ ) of our medium, between which the transitions are possible (**Figure 2.4**). When the

laser is turned on, the pumping process excites ground state atoms to go on the level 3 and at the same time it stimulates the decay back to the ground state. A fraction of atoms at the level 3 naturally decay to level 2 by means, in general, of non-radiative transactions. Under normal circumstances level 2 is a metastable state and after some time the pumping process has brought most of the ground state atoms to the excited level 2 state, realizing the condition of population inversion. Now the amplification process takes place: the electrons on the overpopulated level 2 spontaneously decay in the level 1, producing many photons in various directions. Though most of them dies on the camera walls, it is enough for one to hit the mirrors to be reflected between them, passing over and over through the laser medium, triggering the stimulated emission of other photons, producing a coherent electromagnetic beam, that triggers other emissions and so on, resulting in the beam amplification, while the pumping process guarantees the electron flux on the upper levels. This process may also add some small phase shift, a certain amount of distortion, and a small amount of amplifier noise. However, with some optical elements it's simple to reduce those aberration, obtaining a coherent monochromatic output beam passing through the semi-reflective mirror<sup>[25]</sup>.



**FIGURE 2.4:** Schematic representation of the absorption and emission processes between the medium's 3 levels.

### 2.3.2 Point light source in rubidium

Summing up what we discussed above, let's define the method to create a voxel of light in rubidium:

1. To generate the desired light in a point occupied by rubidium, we'll need to use 2 lasers, one for the first transition  $5^2S_{1/2} \rightarrow 5^2P_{3/2}$  and the second one for the latter  $5^2P_{3/2} \rightarrow 5^2D_{5/2}$ ;
2. We then rely on the spontaneous decay  $5^2D_{5/2} \rightarrow 6^2P_{3/2}$  creating the population inversion on the level of emission of the optics photons;
3. The intersection of both lasers creates a voxel of light with the dimensions of the section of the smallest laser beam.

## 2.4 Persistence of vision: tilt-mirrors

Now that we covered the problem of the light production, another one comes along: the light produced by the stimulation of a laser it's visible as long as lasers are kept in position. So apparently, if we want to create a straight line we need to use a large number of lasers and keeping them in line. Obviously that's not an option, so we can work around the problem using the same method previously described in **chapter 1** used in the plasma holograms : the persistence of vision.

Persistence of vision traditionally refers to the optical illusion that occurs when visual perception of an object does not cease for some time after the rays of light proceeding from it have ceased to enter the eye. This phenomena is due to the finite rate at which any recording device can perceive light and, for the human eye, this rate is called *flicker fusion threshold*, that is defined as the frequency at which an intermittent light stimulus appears to be completely steady to the average human observer. There are other factors that can modify the perception of a continuous movement such as the brightness of the subject, the angle at which the image is seen, the brain processing, and even the intervention of other senses, but considering only the structure of a human eye, the critical flicker fusion (CFF) threshold value is been measured around 30~35 Hz<sup>[26]</sup>.

So it's possible to reproduce our straight line moving only one radiation source at the CFF rate, creating the illusion of a steady image. The best way to do so is not to use directly the laser itself, because they can be very heavy and/or big for us to move at those rate, but add an optical element called *tip-tilt mirror*. A tip-tilt mirror is simply a mirror with a mount capable of tilting in different angular ranges. They are used especially by astronomers in the construction of large telescopes providing a way to do minimal corrections in real time. Those mirrors can be really light and compact thus they represent a perfect solution for our problem.

# Chapter 3

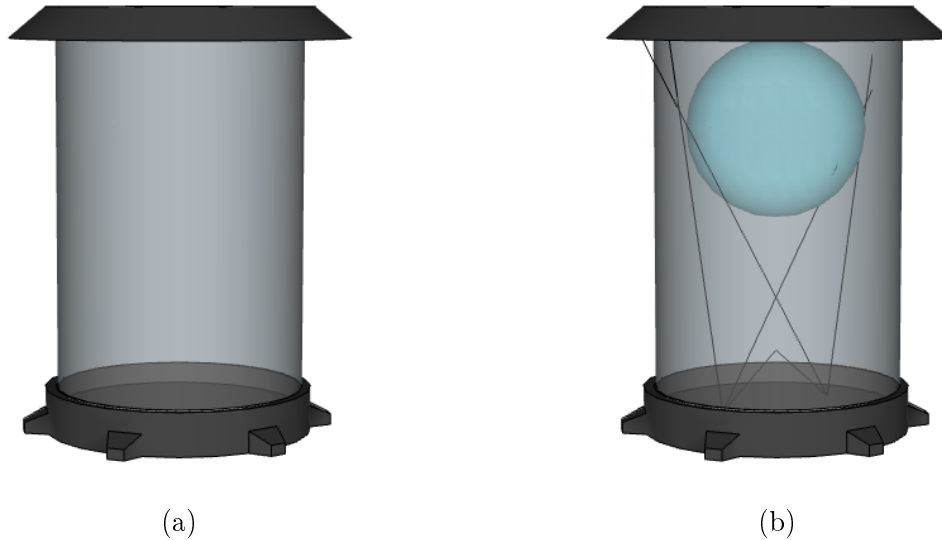
## Example of realization

Defined all the required parameters to illuminate a voxel, along with some proposed solutions to overcome the greatest difficulties, we'll be able to design a device capable, in theory, of realizing a full 3D hologram. In the following description we assume to work with a gas solution of  $^{85}\text{Rb}$  and  $^{87}\text{Rb}$  in natural abundance proportion as our active material, working between the energy levels discussed in **chapter 2**.

### 3.1 Main structure and container material

The main structure consists of a cylindrical transparent container, where the rubidium gas will be stored, enclosed by two plates that act as a cover and, more importantly, as support of all the structure. At the bottom of the container are stored 2 different lasers pointing towards 2 small tilt mirrors, kept separated from the rubidium gas by a transparent material covered with a magnification coating (discussed in the following sections). In the example render shown in **Figure 3.1**, the ratio between the height and the radius of the transparent container is  $\sim 2.8$ . Using these proportions, the blue holographic example sphere inside has a diameter  $\approx$  half of the height, and the maximum angular range of the mirrors is limited to  $\sim 41^\circ$ . In this way, the infrared rays of the lasers can all be restrained inside the container (with a small percentage covered by the upper plate). For example, with a height of 56 cm and 20 cm radius we can create a sphere of radius of 14 cm.

We'll leave aside the discussion on the covers material to focus on other more important matters. Assuming, as we said, to work with rubidium gas, the container must be able to withstand high temperatures and allow viewing of the hologram inside. The first most simple solution falls back on the glass-ceramics. This material has the fabrication advantage of glass, as well as special properties of ceramics. It's mostly produced in two steps: first, a glass is formed by a glass-manufacturing process, it's cooled down and it's then reheated in a second step. In this heat treatment the glass partly crystallizes. In



**FIGURE 3.1:** 3D render of the model design. The blue sphere represents a hypothetical hologram, while the black lines are guidelines of the possible range of the lasers beam (currently at  $\sim 41^\circ$ ).

most cases nucleation agents are added to the base composition of the glass-ceramic to aid and control the crystallization process.

A wide variety of glass-ceramic systems exist, based on the main composition of the nucleation agents added during the production. The most common is the LAS system, mainly referred to a mix of lithium, silicon, and aluminium oxides with additional components. Glass-ceramic from the LAS system has some interesting thermo-mechanical properties: it's a mechanically strong material and can sustain repeated and quick temperature changes up to  $800\text{--}1000\text{ }^\circ\text{C}$ <sup>[27]</sup>. The dominant crystalline phase of the LAS glass-ceramics has a strong negative coefficient of thermal expansion (CTE), keatite-solid solution as still a negative CTE. These negative CTEs of the crystalline phase contrasts with the positive CTE of the residual glass, balancing such that the thermal expansion coefficient becomes very close to zero. It can be transparent, translucent or opaque and even coloured by colouring agents.

Originally developed for use in the mirrors and mirror mounts of astronomical telescopes, LAS glass-ceramics have become known and entered the domestic market through its use in glass-ceramic cooktops, as well as cookware or as high-performance reflectors for digital projectors. As shown in the catalog of one of the most well-known brands of glass-ceramics, Eurokera (which produces glass for both stove and cooktops) it's even possible to apply special infra-red coatings<sup>[28]</sup> or anti-reflective coatings<sup>[29]</sup>, to reduce the heat dispersion and the sharpness of the image behind. The cost range varies on the

type, characteristics, shape, ecc., but a plain plate with standard thickness of 4mm it can cost about 500 €/m<sup>2</sup>.

## 3.2 2 laser method

In terms of the excitement procedure, we propose the use of two pulsed diode lasers synchronized with mirrors mounted on two different tip-tilt supports, placed at the base of the structure. The lasers will create an intersection where the rubidium atoms will be pumped, creating our voxel. They must work at 780 nm and 776 nm, with small beam diameter to produce a relatively small voxel (we'll discuss more about it on the next section) but with high intensity to be able to excite a large number of atoms. There are plenty of models from which to choose, all varying in cost and output power. We'll list 2 different possibilities:

- the QL78D6SA is a small low cost (5.50 €) laser diode, producing a beam 2 mm wide and 5 mW of power (technical sheet: <https://datasheetspdf.com/pdf-file/906086/QSI/QL78D6S-A/1>).
- the RLT7870MG is a more large and high cost (77.18 €) laser diode, producing an adjustable beam between 775÷798 nm, 4 mm wide and 70 mW of power (technical sheet: [http://www.roithner-laser.com/datasheets/ld\\_div/RLT7870MG.pdf](http://www.roithner-laser.com/datasheets/ld_div/RLT7870MG.pdf)).

The price list is available on <http://www.roithner-laser.com/pricelist.pdf>.

Leaving aside the mirrors' reflective material, what really matters is the support responsible of their movement. It must be small, capable of rapid angular shifts, thus with high response rate. One of the models found is the H-811.S2 6-Axis Motion Hexapod (**Figure 3.2**, an hexapod base occupying about 1 cm<sup>3</sup> capable of moving it's top in a range of  $\pm 21^\circ$  at the maximum speed of 625 mrad/s ( $\sim 35^\circ/s$ ) in every direction independently (technical sheet: [https://www.pionline.it/fileadmin/user\\_upload/physik\\_instrumente/files/datasheets/H-811.S2-Datasheet.pdf](https://www.pionline.it/fileadmin/user_upload/physik_instrumente/files/datasheets/H-811.S2-Datasheet.pdf)). If the speed of the mounts would not be enough to cover that angle, there are some ways to adjust our system. First of all, we can use divergent lenses to cover a wider angle, moving less the mirrors. The other solution would be using a series of fixed mirrors after the movable ones. In this way we increase the optical path, so small variations of the incident angle produce a larger difference in the optical path, again covering a wider angle.

### 3.2.1 Mean voxel volume

We discussed how to stimulate atoms in producing light using two synchronized laser pulse, thus only the atoms that are invested with both the laser beams have a chance to produce light. Therefore the two beams have to make an intersection to create a volume of emitting particles: that volume will be our voxel.

**FIGURE 3.2:** A photo of the H-811.S2 6-Axis Motion Hexapod model.



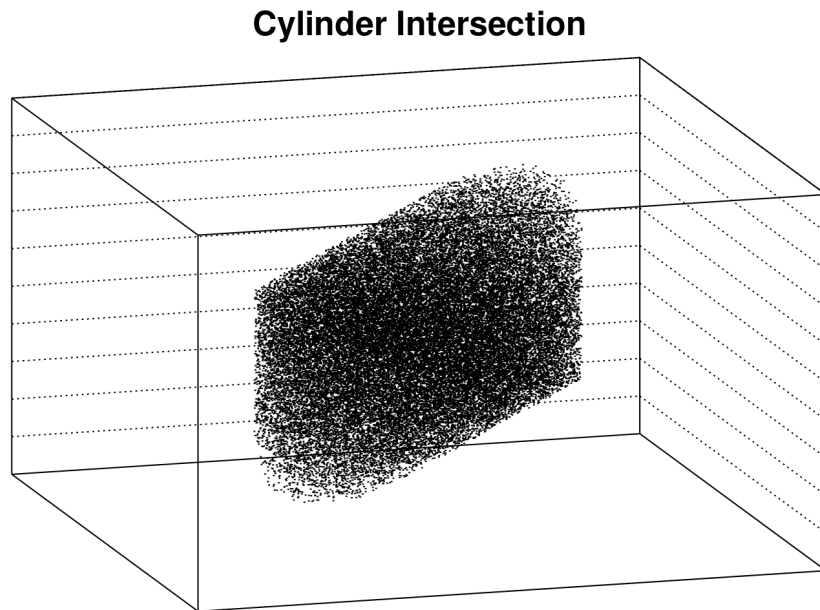
In our design, the lasers are at the bottom of the structure and are reflected in the cylinder above, so they intersect at various different angles. We need an analytical expression to find the volume of all the possible voxels in function of the intersection angle of the two beams. Considering the beams as perfectly cylindrical and with the same radius, proceeding by integrating by layers, we end up with the following expression for the volume:

$$V = \frac{8}{3}r^3(\tan(\theta/2) + \cot(\theta/2)) = \frac{8}{3}r^3 \left( \frac{2}{\sin\theta} \right), \quad (3.2.1)$$

where  $\theta$  is the angle between the beams direction and  $r$  is the beams radius. In our model proportions, the intersection angle range is  $22^\circ \div 63^\circ$ . Considering two beams of 1.5 mm radius, we end up with a volume range of  $48 \text{ mm}^3 \div 20 \text{ mm}^3$ , with a mean volume  $\bar{V} \approx 34 \text{ mm}^3$ . Having that one can easily calculate the number of atoms inside every voxels based on the density of the rubidium gas.

Considering our example sphere of 14 cm radius, the volume that would be occupied by our voxels in order to create its surface is about 3.4 million  $\text{mm}^3$ , which means that we need 100 thousand voxels. Another way of seeing it is considering a straight line: a 28 cm line (the diameter of our example sphere) will be formed by about 100 voxels (considering lasers of 3 mm diameter). Thus it could seem that the resolution of our object would be very poor. To resolve this problem there are different solutions. First of all we have to remember that we need to reproduce those voxels with a frequency of 30 Hz. Thus we can project our N voxels and then projecting N-1 voxels between them, both with a 15 Hz rate. In this way the resolution of the image is doubled, at the cost of some flickering. Another solution is to use a reversed beam expander: the lasers beam are made pass through two lenses, one convergent and the other divergent with a





**FIGURE 3.3:** Volume simulation created using the Monte Carlo method, realized using the data acquisition software ROOT.

collimator. In this way, the diameter of the laser beam can be decreased, allowing us to project more voxels, increasing the resolution.

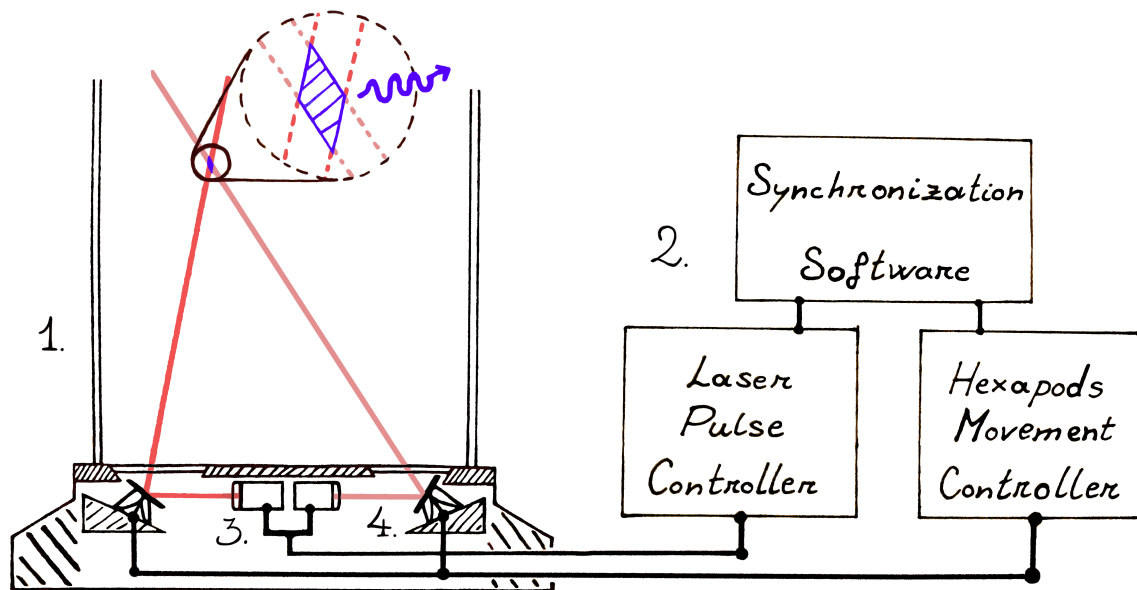
To have a more practical view of our voxels, we used the Monte Carlo method to randomly plot thousands of points and verifying if they were inside an intersection of 2 cylinders intersecting at an angle of  $45^\circ$  (it was more simple to set up and we need only a qualitative view of the distribution). The final plot is shown in **Figure 3.3**. We can clearly see how the atoms are located inside it, concentrating more towards the center and creating an elliptic-like distribution.

### 3.3 Complete design

In conclusion we present the complete device, schematized in **Figure 3.4**, and summarize its operation:

1. A rubidium gas solution is put at vapor pressure in the ceramic-glass container.
2. An external synchronization software is started. It analyzes the image to reproduce and sends signals to both the laser and the hexapods controller, synchronizing the pulses and the mirrors movements.

3. The infrared lasers and the hexapods are activated. The reflected beams form an intersection in the container full of rubidium, creating a voxel. The atoms in the volume are excited and make the transition discussed, emitting a perfect blue light of 420 nm in every direction.



**FIGURE 3.4:** The complete design of the proposed device. (1) Ceramic glass container; (2) synchronization software and controllers; (3) infrared laser diodes; (4) mirrors mounted on the hexapods.

# Conclusions

The fascination that 3D holograms instilled within pop culture has pushed, and continues to push, researchers and companies to experiment and make new attempts in designing devices able to make them. The work carried out was based on the theoretical design of a new device capable of generating real 3D holograms visible from almost every angle, and is proposed as a guideline for an alternative to the models already existent.

The first part of the work focused on identifying the main features that the device must achieve. For this purpose, an analysis was made on how to generate light beams by exploiting the possible two photon transitions between the excited states of matter, identifying the main variables involved, and how to reproduce them. The study then moved on to define what the voxels of light would be and how to replicate them by creating a 3D hologram. The device thus hypothesized is composed of a ceramic glass cylinder to which an anti-infrared coating is applied to reduce heat loss, enclosed between 2 circular covers. Inside the container there is a solution of vapor pressure rubidium gas, which will be used to produce bright voxels. At the base of the device there is another layer of ceramic glass that covers the lighting system: two adjustable laser diodes, working at 780 nm and 776 nm, that are directed towards two different mirrors, mounted on mobile tip-tilt supports. The mirrors and the lasers are drove by two different systems, both controlled by an external synchronizing software. The reflected rays pass through the glass layer and their intersection forms a light voxel. By quickly moving the mirrors in synchrony, the different voxels are lit in sequence. Taking advantage of the phenomenon of the residual image, if the voxels are reproduced at a frequency of about 30 Hz there will be the illusion of staring at a bright three-dimensional figure, that can change with time.

The possible applications of this type of device can be manifold. Being able to create a hologram visible from many angles and without theoretical size limits, it will be suitable to be used in the medical field for a real-time representation of internal organs or clusters of cancer cells. Even at the didactic level it would have its merits, being able to represent any three-dimensional model given as input, interacting in real time. Think, for example, of a hologram placed in the center of a classroom where the different shapes of the spherical harmonics are represented in sequence, or a problem of the three bodies rotating together.

The first design of this device, described in this thesis, is meant to be a starting point for further progress in the field of creating real 3D holograms. In the next future it'll be possible to find new materials or mixture of substances to operate with, improving this base idea with more ergonomics features and expanding its field of use.

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