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# Reconstruction of non-prompt charmed baryon $\Lambda_c$ with Boosted Decision Trees technique

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# Abstract

ALICE (A Large Ion Collider Experiment) is the heavy-ion experiment at the CERN Large Hadron Collider. It is focused to study the physics of strongly interacting matter at extreme energy densities when a transition occurs and a phase of matter called Quark Gluon Plasma (QGP) forms. In these conditions, the ALICE experiment investigates how the QGP expands and cools down. Due to its short average lifetime, the QGP cannot be studied directly. It can only be proved when the QGP cools down and releases the particles that constitute the matter. In this respect, the study of the so-called " hard probes " (hadrons containing charm or beauty quarks) represents an outstanding tool, since such particles are created in the early stages and are expected to lose a significant fraction of their energy when crossing the deconfined medium. The aim of this thesis is to reconstruct charmed Lambda baryon  $\Lambda_c$  and divide non-prompt signal from prompt one, using Boosted Decision Trees technique. The analysis is performed using a multivariate analysis approach which allows to consider simultaneously multiple event properties, exploiting most of the available information through machine learning techniques.

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# Introduction

ALICE (A Large Ion Collider Experiment) is one of the four main experiments at the Large Hadron Collider (LHC) at CERN. It is optimized to study heavy-ion (Pb-Pb nuclei) collisions at a centre of mass energy of 5.5 TeV per nucleon pair. These collisions offer the best experimental conditions to produce the Quark Gluon Plasma (QGP), which is the fifth state of matter where quarks and gluons are free and no longer confined in hadrons. Such state of matter is predicted by the theory of strong interaction (Quantum Chromodynamics, QCD) at both extremely high temperature and densities. The QGP quickly cools down until the individual quarks and gluons recombine into a blast of ordinary matter that spreads away in all directions. The ALICE detectors are designed to detect the particles produced in these collisions. Therefore, one of the fundamental tools to investigate the properties of the QGP is the study of heavy quarks. Due to their large mass, these particles are created in the early stages after the collisions of Pb-Pb nuclei. The interaction of these heavy quarks with the medium can modify their hadronisation and this gives useful information on the evolution of the QGP.

In the third chapter of the thesis, it is analysed the charmed Lambda baryon  $\Lambda_c$  in its channel decay  $\Lambda_c \longrightarrow p + K_S^0$  and the non-prompt's contribution to the total signal. The reconstruction of this particle is challenging, because of its extremely short average lifetime, low ratio between signal and background and the limitation of the data sample collected by the ALICE detectors. This is based on the observation and analysis of multiple statistical outcome variables at the same time, and it allows to study different event properties. To achieve this, it is used a machine learning technique that guarantees to gather most of the information from the acquired data. In particular, for data analysis, it is used the Boosted Decision Trees technique. The last chapter illustrates the main functionality of the algorithm and how it is applied to the simulated data.

# Chapter 1

# **Physics of the ALICE experiment**

### **1.1 Introduction to the Standard Model**

The Standard Model (SM) of particle physics is the mathematical theory that describes the electromagnetic, strong and weak interactions, between leptons and quarks; the fundamental particles of Standard Model. The SM is based on the theoretical framework of the quantum field theory that merges the classical field theory, special relativity and quantum mechanics, but not general relativity's description of gravity.

The Standard Model [1] asserts that the matter in the Universe is made up of fundamental fermions<sup>\*</sup> interacting through fields, of which they are sources. The elementary fermions of the Standard Model, as shown in figure 1.1, are of two types:

- *leptons*, which interact only through the *electromagnetic interaction* (if they are charged) and the *weak interaction*;
- *quarks*, which interact through the *electromagnetic*, the *weak interaction* and also through the *strong interaction*.

The particles associated with the interaction fields are bosons, characterized by having a value of spin s = 1. It has been distinguished four types of interaction fields in nature:

- The *photons*  $\gamma$  are the massless quanta of the *electromagnetic interaction field* between electrically charged fermions;
- The charged  $W^+$  and  $W^-$  bosons and the neutral  $Z^0$  boson are the quanta of the *weak interaction field* between fermions. These bosons are characterized by having mass, meaning that the weak interaction is short ranged. For the uncertainty principle a particle of mass M can exist as part of an intermediate state for a time  $\hbar/Mc^2$ . In addition, in this time the particle can travel a distance no longer than  $\hbar c/Mc^2$ ;

<sup>\*</sup>**Fermion** is a particle that follows Fermi-Dirac statistics and has half odd integer spin in units of  $\hbar$ . **Boson** is a particle that follows Bose-Einstein statistics and has an integer value of spin.

• The gluons  $g_1, \ldots, g_8$  are the massless quanta of the strong interaction field and, like photons, might be expected to have infinite range. Instead, the interactions between quarks and gluons become asymptotically weaker as energy scale increases and the corresponding scale decreases (asymptotic freedom).

The fundamental interactions are described by the *gauge theory*. This is a type of field theory in which the Lagrangian does not change under local transformations from certain *Lie groups*. Quantum electrodynamics (QED) is an abelian gauge theory with the symmetry group U(1) and it has only one gauge field which is the photon (the gauge boson of QED). The Standard Model is a non-abelian gauge theory with the symmetry group  $U(1) \times SU(2) \times SU(3)$  and has a total of twelve gauge bosons described above.



Figure 1.1: Standard Model of elementary particles. Antileptons and antiquarks with same masses and spin, but reverse internal quantum numbers, have also reported.

It has been proved that a given symmetry law might not always be followed under certain conditions. This phenomenon is called *Spontaneous Symmetry Breaking* and it could be understood by introducing the *Higgs mechanism*. The interaction between the Higgs boson field, the thirteenth boson of Standard Model with spin s = 0, and bosons and fermions, explains the

generation mechanisms of property mass of bosons and fermions. Furthermore, this mechanism can explain the short range of the weak interaction, the renormalizability of the theory, and also the unification of electroweak interaction.

The Standard Model is the result of extensive experimental data and inspired theoretical endeavour, covering more than fifty years of research.

### **1.2 Quantum ChromoDynamics**

Quantum Chromodynamics (QCD) [2], the gauge field theory of strong interaction, is based on colour symmetry,  $SU(3)_c$ . QCD describes the strong interactions of quarks and gluons, which are the fundamental particles that constitute the hadrons. The two main types of hadron are:

- *mesons*, composed of one quark and one antiquark;
- baryons, composed of three quarks.

Quarks carry a colour index, and interact with the gluon fields which mediate the strong force. The  $SU(3)_c$  group is known to have eight infinitesimal generators, which are the Gell-Mann's matrices, whose physically represents the eight gluons. In QCD, we have three fields for each flavour of quark. These are put into *colour triplets*, which are the three types of colour: red, green, blue and three corresponding anticolors, as consequence of the color charge conservation in particle-antiparticle creation and annihilation. All three colors mixed together, or one color (anticolor) and its complementary, is *colorless* or *white* and has a net color charge of zero. The color charge differs from electric charge because the latter has only one kind of value, instead the color charge has three types. However, the color charge and electric charge are similar in that both have corresponding negative for each kind of value.

One of the main characteristics of QCD is the *colour confinement*, according to which, free quarks and free gluons are not observed and they must have a color charge zero. Gluons are electrically neutral and do not carry intrinsic quantum numbers (apart from three types of colour). The quantum numbers of hadrons are given by the quantum numbers of their constituent quarks and antiquarks. Most of the mass of the nucleon is not due to the Higgs boson but to gluons. Quarks interact weakly at high energies, allowing perturbative calculations. At low energies the interaction becomes strong, leading to the confinement of quarks and gluons within composite hadrons. Instead, in confinement regime, it is necessary to carry out a non-perturbative calculations because of high intensity of interactions.

#### **1.2.1** Lattice Quantum Chromodynamics

Lattice Quantum Chromodynamics (LQCD) is a non-perturbative theoretical approach to calculate quark-quark interactions. In conditions described above, it is necessary the LQCD that uses a discrete set of spacetime points (*lattice*), to reduce the analytically intractable path integrals of the continuum theory. The LQCD is one of the principal techniques to study the phase transition from ordinary matter to quark-gluon plasma.

### **1.3 Quark-Gluon Plasma**

In QCD, hadrons are colour-neutral bound states of basic pointlike coloured quarks and gluons. Hadronic matter can turn into Quark-Gluon Plasma (QGP) at high temperatures and/or densities [3]. Since the temperature is beyond the critic temperature  $T_c$  and critical density  $\mu_c$ , a transition phase occurs: the quarks and gluons are deconfined, freely to move into the *plasma* and these allow free colour charges. The existence of such phase and its properties, are important issues in QCD for the understanding of confinement and chiral symmetry restoration.

#### **1.3.1** Phase transition of QCD

The phase transition of QCD is therefore determined by two parameters: *temperature T* and *baryochemical potential*  $\mu$ . The baryochemical potential is defined as  $\mu = \partial E / \partial N_b$ , where  $N_b$  is the number of baryons of the system: it measures the baryon density.

The phase diagram of such a transition resembles the one of figure 1.2, which shows what happens to strongly interacting matter in the limit of high temperature and densities. It is expected at T = 0, in vacuum, that quarks dress themselves with gluons to form the constituent quarks that make up hadrons (*diquark matter - hadronic matter transition*). The last transition (*QGP - diquark matter*) occurs if the attractive interaction between quarks leads from the deconfined phase to the formation of coloured bosonic diquark pairs. At low temperature, these diquarks can condense to form a colour superconductor.



Figure 1.2: The phase diagram of QCD. [3]

The transition from hadronic matter to QGP can be explained by a simple model. For an ideal gas of massless pions<sup> $\dagger$ </sup>, the pressure is given by the Stefan-Boltzmann relation:

$$P_{\pi} = 3\frac{\pi^2}{90}T^4,\tag{1.1}$$

where factor 3 represents the three charge states of the pion. The correspondence of this pressure to QGP phase with two flavours and three colours is:

$$P_{qg} = \left\{ 2 \times 8 + \frac{7}{8} (3 \times 2 \times 2 \times 2) \right\} \frac{\pi^2}{90} T^4 - B = 37 \frac{\pi^2}{90} T^4 - B.$$
(1.2)

The first term in the curly brackets accounts for the two spins and eight colour degrees of freedom of the gluons, whereas the second addend for three colours, two flavours, two spin and two particle-antiparticle degrees of freedom of the quarks, with 7/8 to obtain the correct statistics. *B* is the bag pressure and, according to the hadron spectroscopy, its value is  $B^{1/4} \simeq 0.2$  GeV. It takes into account the difference between the physical vacuum and the ground state of quarks and gluons in a medium. The behaviour of temperature described by the two previous equations is shown in figure 1.3 (*left*).

This model leads to a splitting of the strongly interacting matter, with hadronic phase up to:

$$T_c = \left(\frac{45}{17\pi^{32}}\right)^{1/4} B^{1/4} \simeq 0.72 B^{1/4}$$
(1.3)

and QGP above this temperature. Thus, we obtain the deconfinement temperature:

$$T_c \simeq 150 MeV.$$



Figure 1.3: Pressure (left) and Energy (right) of two-phase ideal gas model. [3]

<sup>&</sup>lt;sup>†</sup>**Pion** is a meson, it is the lightest hadrons and can have three forms according to its charge and combination of quarks: neutral  $\pi^0$ , positive  $\pi^+$  or negative  $\pi^-$ .

The energy densities of the two phases are given by:

$$\varepsilon_{\pi} = \frac{\pi^2}{10} T^4 \tag{1.4}$$

and for the QGP phase:

$$\varepsilon_{qg} = 37 \frac{\pi^2}{30} T^4 + B.$$
 (1.5)

The dependence of energy density is shown in figure 1.3 (*right*). The transition occurs in relation to  $T_c$ , here the energy density increases abruptly by the latent heat of deconfinement.

### **1.4 Heavy-Ion Collisions**

To reproduce the conditions of quark-gluon matter on earth, the terrestrial laboratories are projected to make collisions of heavy nuclei (also called *heavy-ions*) at ultrarelativistic energies. Predictions of QCD establish that a phase transformation occurs between deconfined quark and confined hadrons. The relativistic nuclear collision analysis may help to completely understand the mechanisms of this QCD process.

#### **1.4.1 Relativistic Kinematics**

In nucleus-nucleus collisions, it is convenient to use kinematic variables: a particle is characterized by its 4-momentum,  $p_{\mu} = (E, \mathbf{p})$ . Nuclei approach each other with velocities compared to the speed of light; therefore, according to the Lorentz's contraction, beams of particles could be treated as thin disks of radius  $2R_A \simeq 2A^{1/2} fm$ , where A is the number of nucleons.

#### 1.4.2 Rapidity Variable

A greater centrality of collisions means a greater likelihood of creating suitable conditions for the formation of QGP. Along the beam direction, the coordinate (conventionally *z*-axis) is called *longitudinal*; the perpendicular coordinate to the longitudinal one is named *transverse* (x-y). The 4-momentum can be decomposed into the longitudinal  $p_z$  and transverse ones  $p_T$ , where the latter is a vector quantity which is invariant under Lorentz boost along the *z* direction.

Description of particle trajectories can be computed with the *dimensionless variable rapidity y*, defined by:

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z},$$
 (1.6)

where E and  $p_z$  are the energy and momentum of beam. The rapidity changes by an additive constant under longitudinal Lorentz boost.

For a free particle (for which  $E^2 = m^2 + p^2$ ), the 4-momentum has only three degrees of freedom and can be represented by  $(y, p_T)$ .

In these terms, we can express the 4-momentum as:

$$E = m_T \cosh y, \tag{1.7}$$

$$p_z = m_T \sinh y, \tag{1.8}$$

being  $m_T$  the *transverse mass* which is defined as:

$$m_T^2 = m^2 + p_T^2. (1.9)$$

The advantage of rapidity is that the shape of the rapidity distribution remains the same under a longitudinal Lorentz boost.

#### **1.4.3** Time evolution

One of the main goals of heavy-ion experiments is to study the evolution through time of a strongly interacting system.



Figure 1.4: Light-cone diagram showing the longitudinal evolution of an ultrarelativistic ion collision. Proper time  $\tau$  appear as hyperbolas,  $\tau = (t^2 - z^2)$ .

The system is created after the collisions and it is characterized of different properties during the passing of time. In cases the phase transition does not occur, the space-time evolution is studied with relativistic hydrodynamics. Time history of the central heavy nuclei collisions [4] is shown in the space-time diagram of figure 1.4. Below are briefly explained the phases after the collisions.

**Pre-equilibrium 0**  $< \tau < \tau_0$ : after the head-on collision of two beams of nucleons, many virtual quanta and/or coherent field configuration of the gluons are excited. It takes a proper time,  $\tau_{de}$ , for these quanta to be de-excited to real quarks and gluons. The de-excitation time would typically be a fraction of 1 fm/c, or it could be much less than 1 fm/c. The state of matter for  $0 < \tau < \tau_{de}$  is said to be in the *pre-equilibrium* stage.

**Thermalization:** particles after the collisions continue to mutually interact, giving rise to a region of high matter and energy density. This is formed at the thermal equilibrium from which the QGP can be produced in less than 0.3 fm/c. This stage is referred to as *thermaliza-tion*.

**Expansion and hadronization**  $\tau_0 < \tau < \tau_f$ : once the local thermal equilibrium is reached at  $\tau_0$ , the system expands due to the internal pressure. The thermalized system expands and the energy density decreases. In this period, the phase transition occurs: the QGP evolved to hadronic plasma.

**Freeze-out and post-equilibrium**  $\tau_f < \tau$ : *freeze-out* of the hadronic plasma happens at the time  $\tau_f$ . One can think of two kinds of freeze-outs:

- the *chemical freeze-out*, after which the number of each hadronic species is frozen, while the equilibrium in the phase-space is maintained;
- the *thermal freeze-out*, after which the kinetic equilibrium is no longer maintained.

The chemical freeze-out temperatures must be higher than the thermal freeze-out. After this transition, the distances between the hadrons become greater than the radius of strong interactions.

### **1.5** Experimental aspects

The QGP cannot be studied directly because of its short average lifetime. Therefore, theoretical models predict some properties of the final state of the collision. These can be studied as signatures of the production of the QGP. Experimental validations are based on the direct measurements and they are divided in:

• *soft probes* are non-perturbative regime of QCD characterized by the measurements of particles in the low transverse momentum or *soft* regime. It represents the majority of created particles;

• hard probes are perturbative regime of QCD, with high transverse momentum.

#### 1.5.1 Soft probes

**Strangeness Enhancement:** one of the first measured signatures of QGP was strangeness enhancement in heavy-ion collisions. It consists of increased production of strange hadrons after the collisions. Production of strange particles [5] is expected to be enhanced in relativistic nucleus-nucleus collisions relative to the scaled up *pp* data.

Although  $m_s \gg m_{u,d}$ , strange quarks and antiquarks can be created in QGP with temperature  $T > m_s$ . In addition, large gluon density in QGP leads to the efficient production of strangeness through gluon fusion  $gg \rightarrow s\bar{s}$ . Also, the threshold energy for strangeness production in the purely hadron-gas scenario is much higher than in QGP.

Abundance of strange quarks and antiquarks in QGP is expected to leave its imprint on the number of strange hadrons detected in the final state.



Figure 1.5: (a)(b) Enhancements in the rapidity range y = 0.5 as a function of the mean number of participants  $\langle N_{part} \rangle$ , showing ALICE (full symbols), RHIC and SPS (open symbols) data. Boxes on the dashed lines at unity indicate statistical and systematic uncertainties on the *pp* or p - Be reference. (c) Hyperon to pion ration as function of  $\langle N_{part} \rangle$ , for *AA* and *pp* collisions at LHC and RHIC. Lines mark thermal model predictions. [6]

In figure 1.5 (a) and (b), it is reported the enhancement of hyperons<sup>‡</sup> in Pb - Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV (full symblos). Comparing the ALICE measurements with those from the experiments NA57 at SPS (Pb - Pb collisions at  $\sqrt{s_{NN}} = 17.2$  GeV ) and STAR at RHIC

<sup>&</sup>lt;sup>‡</sup>any baryon containing at least one strange quarks, but not charm, bottom or top quark.

(Au - Au collisions at  $\sqrt{s_{NN}} = 200$  GeV), represented by open symbols, the enhancements are found to decrease with increasing centre of mass energy. These continue the trend established at lower energies.

**Elliptic Flow:** elliptic flow is one of the key observables in ultrarelativistic collisions of heavy nuclei, which confirms the collectivity and early thermalization in the created hot and dense matter. *Collective flow*, in particular anisotropic transverse flow, is the correlation between the position of matter and direction of flow, which is not necessary to be hydrodynamically evolving matter.



Figure 1.6: The  $p_T$ -differential  $v_2$  of  $\pi^{\pm}$ ,  $K^{\pm}$ ,  $p + \bar{p}$ ,  $\Lambda + \bar{\Lambda}$ ,  $K_S^0$  and the  $\phi$  – *meson* for various centrality classes. [7]

For a central collision (impact parameter  $b \rightarrow 0$ ) we expect an azimuthal isotropic space distribution. Instead, for a non-central collision (impact parameter  $b \neq 0$ ) of two spherical nuclei, the overlapping zone between the nuclei no longer remains circular in shape, rather it takes an almond shape. This initial spatial anisotropy of the overlapping zone is converted into momentum space anisotropy of particle distribution. This is possible via the action of azimuthally anisotropic pressure gradient, which gives rise to elliptic flow.

Elliptic flow coefficient  $v_2$  is quantified as the second Fourier coefficient of particle distribution in the  $p_T$  space:

$$\frac{dN(b)}{p_T dp_T dy d\phi} = \frac{dN(b)}{2\pi p_T dp_T dy} (b) \times [1 + 2\nu_1(p_T, b)\cos(\phi) + 2\nu_2(p_T, b)\cos(2\phi) + \dots],$$
(1.10)

where  $p_T$  is the transverse momentum and  $\phi$  the azimuthal angle. Elliptic flow coefficient  $v_2$  is the lowest non-vanishing anisotropic flow coefficient and it is the largest contribution to the asymmetry of non-central collisions.

In figure 1.6 it is shown the measurement of  $v_2$  in Pb-Pb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV in ALICE experiment. The flow coefficient is measured using the scalar product method, as shown in figure 1.6. The graphs are built in the rapidity range |y| < 0.5 as a function of transverse momentum  $p_T$ , at different centrality intervals between 0 - 70%, including ultracentral 0 - 1% collisions.

#### **1.5.2 Hard probes**

**Jet quenching:** a jet is a collimated beam of particles that have fragmented from the same parton<sup>§</sup>. In ultra-relativistic heavy-ion collisions, jets are used as a probe of the QGP. The partons that eventually fragment into jets are created in the very early stages of the collision.



Figure 1.7: Two quarks suffer a hard scattering: one goes out directly to the vacuum, radiates a few gluons, and hadronizes. The other goes through the dense plasma formed in the collision, suffers energy loss due to gluonstrahlung, and fragments outside into a quenched jet. [5]

<sup>&</sup>lt;sup>§</sup>**Partons** can be either quarks or gluons.

The partons, then transverse the medium and interact with the constituents of medium and lose energy by emission of gluons. Due to this interaction with QGP, the partons lose energy by radiating gluons, in a process called *jet quenching*.

In figure 1.7 is illustrated the attenuation of the hadrons, resulting from fragmentation of a parton due to energy loss in the dense plasma produced in the reaction. The energy lost by a particle in a medium,  $\delta E$ , provides fundamental information of its properties. In general,  $\delta E$  depends both on the particle characteristics (energy *E* and mass *m*), and on the plasma properties (temperature *T*, particle-medium interaction coupling  $\alpha$ , and thickness *L*). Direct information on the thermodynamical properties and transport properties of QGP is commonly obtained by a given observable  $\phi$ . This is measured in nucleus-nucleus (*AA* in QCD medium) collisions and in proton-proton (*pp* in QCD vacuum) collisions, as a function of: centre of mass energy  $\sqrt{s_{NN}}$ , transverse momentum  $p_T$ , rapidity *y*, reaction centrality (impact parameter *b*) and particle type (mass m). The standard method to quantify the effects of the medium on the yield of a hard probe in a *AA* reaction is given by the *nuclear modification factor*:

$$R_{AA}(p_T, y, b) = \frac{d^2 N_{AA}/dy dp_T}{\langle T_{AA}(b) \rangle \times d^2 \sigma_{pp}/dy dp_T},$$
(1.11)

where  $N_{AA}$  and  $\sigma_{pp}$  are charged particle yield in nucleus-nucleus (A - A) collisions and the cross section in pp collisions.  $\langle T_{AA} \rangle = \langle N_{coll} \rangle / \sigma_{inel}^{NN}$  is the nuclear overlap. The latter is related to the average number of binary nucleon-nucleon collisions  $N_{coll}$ . The numerator of this equation represents the number of particles after a collision AA per unit of transverse momentum  $p_T$  and rapidity y. Moreover,  $d^2\sigma_{pp}/dydp_T$  is the cross section of pp collision per interval of transverse momentum and rapidity.

Any observed *enhacements* or *suppressions* in the  $R_{AA}$  can be directly linked to the properties of strongly interacting matter. This factor measures the deviation of AA at b from an incoherent superposition of NN collisions. This normalisation is often referred as *binary collision scaling*. Instead, if we observe deviations from this value ( $R_{AA} \neq 1$ ), it indicates the presence of QGP. In particular, we have *suppression* when  $R_{AA} < 1$  and *amplification* for  $R_{AA} > 1$ .

In figure 1.8 are shown the results of ALICE measurements in Pb-Pb collision events at  $\sqrt{s_{NN}} = 2.76$  TeV. The figure illustrates that, for some centrality classes, when  $p_T > 10$  GeV/c, the nuclear modification factor of any particle species is small compared to large suppression ( $R_{AA} \ll 1$ ). Overall, it can be observed a suppression with minimum at  $p_T = 6 - 7$  GeV/c with respect to an overlap of binary *pp* collisions.



Figure 1.8: The nuclear modification factor  $R_{AA}$  as a function of  $p_T$  for different particle species. [8]

 $J/\psi$  production: the bound states of heavy quark and its antiquark is a flavourless neutral meson. These particles are called *quarkonia* and they are divided in *charmonium* ( $c\bar{c}$ ) and *bottonium* ( $b\bar{b}$ ).

Charmonium states are considered important signatures of the strongly interacting medium created in heavy-ion collisions. In the ALICE experiment, these probes can be investigated in the  $\mu^+\mu^-$  decay channel, in the rapidity region 2.5 < y < 4, with transverse momentum  $0 < p_T < 8$  GeV/c. Results on charmonia production in Pb-Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV are presented in figure 1.9. The nuclear modification factor depends on centrality and on the centrality and transverse momentum of the  $J/\psi$  particle. It is compared the ALICE experiment result with the PHENIX ones.

The medium dissociation probability of states should provide an estimate of the initial temperature reached in the collisions. The studies performed in the last twenty years have shown a reduction of the  $J/\psi$  production yield lower than expectations; this is due to the cold nuclear matter effects. These observations suggest the existence of an additional  $J/\psi$  production mechanism, which sets when higher  $\sqrt{s_{NN}}$  are reached. This mechanism can counteract the quarkonium suppression in the QGP. At high-energy density of the medium and large number of  $c\bar{c}$  pairs produced in central Pb-Pb collisions,  $J/\psi$  production mechanism should help to disentangle suppression and regeneration scenarios.



Figure 1.9: the  $J/\psi R_{AA}$  is shown as a function of nucleons that participate to the collisions  $N_{part}$  in ALICE experiment and they are compared to the PHENIX result. [9]

#### **1.5.3 Direct Photons**

The term *direct photons* stands for the photons which emerge directly from a particle collision. In heavy-ion collision experiments, the detector captures all the emitted photons including those from the decay of final state hadrons.

More than 90% of the photons in spectrum are emitted from hadron decay. Photons whose are not direct decay product of hadrons represent an important tool for study the evolution of QGP. Different intervals of  $p_T$  correspond to photons emitted at different times. It is possible to subdivide this broad category of *direct photons* depending on their origin:

- prompt photons, which originated from initial hard scattering (hard probes);
- pre-equilibrium photons produced before the medium gets thermalized;
- *thermal photons* originated from quark-gluon plasma as well as from hadronic reactions in the hadronic phase;

#### • jet conversion photons created from a passage of jets through plasma.

In figure 1.10 are shown direct photons spectra measured in ALICE. It is evident that a greater number of photons are produced at low transverse momenta compared to high transverse momentum. The data are fitted with an exponential function  $\propto \exp(-p_T/T_{eff})$ . The extracted inverse slope parameter is  $T_{eff} = 297$  MeV in the range  $0.9 < p_T < 2.1$  GeV/c for the 0 - 20% class, and  $T_{eff} = 410$  MeV in the range  $1.1 < p_T < 2.1$  GeV/c for the 20 - 40% centrality class. These values indicate an initial temperature higher than the critical temperature at which the QGP is formed.



Figure 1.10: Direct photon spectra in Pb-Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV for 0 - 20%, 20 - 40% and 40 - 80% centrality classes. [10]

# Chapter 2

# **The ALICE experiment**

### 2.1 The Large Hadron Collider

The Large Hadron Collider (LHC) is located at CERN near Geneva. It is the world's largest and most powerful particle accelerator, with the first start-up of beams in 2008. *Large* due to its size, approximately 27 kilometres in circumference, *Hadron* because LHC accelerates protons or ions at energies up to the record energy 6.5 TeV per proton and *Collider* because these particles form two beams travelling in opposite directions. The two beams collide at four points, where the two rings of the machine intersect; these are in correspondence of the four particle detector: ALICE, ATLAS, CMS and LHCb, as it is shown in figure 2.1.



Figure 2.1: Schematic representation of LHC.

To avoid colliding with gas molecules inside the accelerator, the beams of particles must travel in the beam pipes in *ultralight vacuum*. The vacuum is equivalent to pressure to the order of  $10^{-10}$  to  $10^{-11}$  mbar. It is almost as rarefied as the pressure found on the surface of the Moon. LHC has three separate vacuum systems: one for the beam pipes, one for insulating the cryogenically cooled super-magnets and one for insulating the helium distribution line.

#### 2.1.1 Magnetic lattice

The trillions of accelerated charged particles (protons and heavy-ion) hurtle around the LHC at close the speed of light. They can circle the collider's tunnel 11.245 times per second. Before they reach the principal ring of LHC, the energy is boosted along the way in a series of interconnected linear and circular accelerators. Once particles reach the maximum speed that one accelerator can lead, they are shot into the next. Beams remain stable on the straight line trajectory due to more than 50 types of electromagnets (*dipole magnets*), which are needed to send particles along complex paths without them losing their speed.

One of the most complex parts of the LHC are dipole magnets. They generate powerful 8.3 Tesla magnetic field and they use a current of 11.080 amperes to produce field. In addition, a superconducting coil allows the high currents to flow without losing any energy because of electrical resistance. There are 1232 main dipoles, each 15 metres long and 35 heavy tonnes. In addition to just curving the beam, it is also necessary to focus it. Because protons are electrically charged a particle beam diverges if left on its own, due to Coulombian repulsion. *Quadrupoles* drive the beams and also keep the particles in a tight beam along the paths. These have four magnetic poles arranged symmetrically around the beam pipe to force the beam either vertically or horizontally. Schematic configuration of the quadrupoles is shown in figure 2.2.



Figure 2.2: Quadrupoles in LHC. They keep the beams tightly focused.

### **2.2** The ALICE experiment

ALICE [11] stands for *A Large Ion Collider Experiment*. The experiment is designed to study the physics of strongly interacting matter at extreme energy densities. This is possible due to the study of heavy-ion Pb-Pb nuclei collisions. In particular, ALICE is optimized for Pb-Pb collisions at a centre of mass energy up to 5.5 TeV per nucleon pair. Therefore, the ALICE Collaboration studies the QGP, the ways it expands and cools down. It tries to recreate laboratory conditions similar to those just a fraction of the first second after the Big Bang. It tries to show the events before quarks and gluons bind together to form hadron and heavier particles.

The experiment uses the 10.000 tonnes ALICE detector (26 m long, 16 m high and 16 m wide) shown in figure 2.3.



Figure 2.3: ALICE experiment.

#### 2.2.1 The ALICE detector

The ALICE detector is divided into 18 components and each component inspects a specific set of particle properties. These components are stacked in layers, and the particles go through the layers sequentially: outward from the collision point. At first, particles run into the tracking system, then into an electromagnetic (EM) and hadronic calorimeter and finally a muon system. The detectors are embedded in a magnetic field to bend the tracks of charged particles to measure momentum and charge determination. The aim is to identify all the particles that are coming out from the QGP system.



Figure 2.4: Schematic view of the ALICE detector system. [12]

ALICE uses a set of 18 different detectors that give information about the mass, the velocity and the electrical sign of particles. Schematic representation is shown in figure 2.4. The ALICE detectors can be divided in two main parts, according to the kinematic region that they cover.

At first, there is the *central barrel* that explore the central rapidity region, and it consists of:

- *tracking detectors* which are embedded in a magnetic field of 0.5 Tesla; this is produced by a huge magnetic solenoid bending the trajectories of the particles. From the curvature of the tracks, it is possible to derive the momentum of charged particles and their position by reconstructing the vertices of interaction. These are the ITS (*Inner Tracking System*), the TPC (*Time Projection Chamber*) and the TRD (*Transition Radiation Detector*);
- *Particle IDentification detectors* allow to know the identity of each particle, whether it is an electron, proton, kaon or a pion. This is done using the reconstructing trajectories of tracking detectors. These include the TOF (*Time-Of-Flight*) and the HMPID (*High Momentum Particle IDentification*);
- PHOS (*Photon Spectrometer*) is a high-resolution electromagnetic calorimeter installed in ALICE to provide data to test the thermal and dynamical properties of the initial phase of the collision between ions. In addition, EMCAL (*Electro-Magnet Calorimeter*) and DCAL (*Di-jet Calorimeter*) are installed in ALICE for similar purposes.

The second part is dedicated to the complementary rapidity region of the central barrel and it is dedicated to the muon and cosmic rays detection:

- *muon spectrometer*, formed by MCH (*Muon Chamber*) and MTR (*Muon Trigger*), studies the complete spectrum of heavy quarkonia via their decay in the  $\mu^+\mu^-$  channel;
- forward and trigger detectors which are the FMD (Forward Multiplicity Detector), the PMD (Photon Multiplicity Detector), AD (ALICE Diffractive), the ZDC (Zero Degree Calorimeter), V0 and T0;
- *cosmic ray trigger detector* ACORDE (*ALICE Cosmic Rays Detector*) detects ray showers by triggering the arrival of muons to the top of the ALICE magnet. The ALICE cavern is an ideal place for the detection of high energy atmospheric muons coming from cosmic ray showers.

#### 2.2.2 Inner Tracking System

The Inner Tracking System (ITS) [13] is the innermost tracking detector of the ALICE experiment.



Figure 2.5: Schematic view of the six ITS layers and their supporting cones.

Its purpose is to provide both primary and secondary vertices reconstruction and of improving the ALICE barrel tracking capabilities closer to the interaction point. Moreover, the ITS can recover particles which do not reach or are missed by the outermost layers, due to acceptance limitations and momentum cutoff. The ITS consists of six cylindrical layers of silicon detectors. Each layer has a hermetic structure and it is coaxial with the beam pipe. The ITS covers the pseudorapidity range  $|\eta| \le 0.9$  over the entire azimuthal angle, and its radius extends from 3.9 cm to 43 cm. The two innermost layers at 3.9 and 7.6 cm from the beamline are made of Silicon Pixel Detectors (SPD). The two central layers at 15 and 24 cm are made of Silicon Drift Detectors (SDD); the two outermost ones at 38 and 43 cm are of Silicon Strip Detectors (SSD). In figure 2.5 is shown a schematic view of the ITS layers and its supporting structure.

#### 2.2.3 Time Projection Chamber

The Time Projection Chamber (TPC) [14] is a powerful detector for the three-dimensional tracking of charged particles and identification of ultra-high multiplicity events. The ALICE TPC has large volume (about 83  $m^3$ ) filled with gas as detection medium and it is the main particle tracking device. Only a conservative and redundant tracking device can guarantee reliable performance up to 800 charged particles per unit of rapidity.



Figure 2.6: Schematic view of the ALICE TPC. [15]

The identification of charged particles with low momenta occurs when they cross the gas of the TPC; they ionize the gas atoms along their path, liberating electrons that drift towards the end plates at the end of the detector. The average energy loss by charged particles through collisions with the atomic electrons of the medium is expressed by the relativistic Bethe-Bloch formula dE/dx.

The ALICE TPC has also the task of reconstructing the primary vertices of charged particles beginning from its outermost regions to its core. This is due to different density of charged particles tracks found across the layers: the density is lower in the outermost layers and increases when reaching the innermost ones. The TPC is cylindrical in shape with an active gas volume that ranges from about 85 cm to 250 cm, in its radial direction, and it has a length of 500 cm along the beam direction as is shown in figure 2.6.

#### 2.2.4 Transition Radiation Detector

Transition Radiation Detector (TRD) is the main electron and positrons detector in ALICE. It provides this identification using the emission of transition radiation. It is released X-rays when the particles cross many layers of thin materials.



Figure 2.7: Schematic representation of the TRD detector.

The TRD detector [16], shown in figure 2.7, consists of 522 readout chambers arranged in 6 layers surrounding the TPC. They are at a radial distance  $r (2.9 \le r \le 3.7 \text{ m})$  from the beam axis, with a maximum length of 7 m along the beam axis Each chamber is filled with *XeCO*<sub>2</sub> mixture (85:15). The gas volume is subdivided by cathode wire grid into a 3 cm drift region and 0.7 cm amplification region equipped with anode wires.



Figure 2.8: (*left*) Schematic cross-section of a TRD chamber including radiator. (*right*) Average pulse height as a function of drift time for pions and electrons with and without transition radiation.

Figure 2.8 (*left*) shows the cross-section of TRD chamber crossed by a pion and an electron. In addition, in figure 2.7 (*right*) is also shown the average pulse height, as a function of drift time for pions and electrons with a momentum of 2 GeV/c when passing through the TRD chamber. Comparing the shape of the electrical signal induced by an electron and a pion allows to separate the two particle species by the presence of a long-time peak due to transition radiation photon emitted by the electron in the radiator.

#### 2.2.5 Time of flight

Charged particles in the intermediate momentum range are identified in ALICE by the Time Of Flight (TOF) detector. Time measured with TOF [19] in conjunction with the momentum and with the track length measured by tracking detectors, is used to compute the particle mass. A time resolution of 50 ps will provide  $3\sigma$  separation up to 2.5 GeV/c for  $\pi/K$  and for K/p separation up to 4 GeV/c.

TOF principal goal is to study:

- The QCD thermodynamics via measurement of π, K and p transverse momentum distributions and particle ratios on an event-by-events basis;
- QGP formation via open charm and  $\phi$  meson production.



Figure 2.9: Schematic view of one of the 18 TOF supermodules positioned along the cylinder.

The TOF detector, shown in figure 2.9, is located at a distance of 3.7 m from collision axis. It covers a central region of pseudorapidity  $-0.9 < \eta < 0.9$  and covers in toto the azimuth. The TOF has a modular structure with 18 sectors in  $\phi$ , in addition, it has 5 modules along the axial direction. The modules contain a total of 1638 detector elements (*MRPC strips*), covering an area of 141  $m^2$  with 157.248 channels (*pads*).

#### 2.2.6 High-Momentum Particle Identification Detector

The High Momentum Particle IDentification (HMPID) is a detector used to identify charged hadrons with high transverse momentum  $p_T$ . The HMPID detector has been designed to extend the useful range for the identification of  $\pi/K$  up to 3 GeV/c and of K/p up to 5 GeV/c.

The ALICE HMPID [17], shown in figure 2.10, is formed by Ring Imaging Cherenkov (RICH) counters and consists of seven modules mounted in an independent support cradle. HMPID is fixed to the space frame at a distance of 4.9 m from the centre of collisions, with a surface of 12  $m^2$ . Cherenkov photons are emitted when a fast charged particle crosses the 15 mm thick layer of liquid  $C_6F_{14}$  (*perfluorohexane*) and the photons are detected by a photon

counter which exploits multi-wire proportional chamber (MWPC) with pad cathode covered with a thin layer of CsI. The Cherenkov photons refract out of the liquid radiator and reach the CsI-coated cathode. MWPC is located at a suitable distance that allows the reduction of the Cherenkov angle resolution because of means of geometrical aberration. The electrons released by ionizing particles in the proximity gap are filled with  $CH_4$  are prevented from entering the MWPC sensitive volume by a positive polarization of the collection of electrodes close to the radiator.



Figure 2.10: (*left*) Schematic representation of HMPID detector; (*right*) schematic representation of MWPC with cathode pads.

#### 2.2.7 Calorimeters

Calorimeters measure the energy of particles and determine whether they have electromagnetic or hadronic interactions. Particle identification in a calorimeter happens when deposition of particle energies occur, except from muons and neutrinos. This is possible because of production of electromagnetic and hadronic showers when the particles go through the calorimeters.

The electromagnetic calorimeter system of ALICE [18] consists of two detectors:

- PHOS is a high-precision photon spectrometer; its main goal of studying thermal properties of the hot strongly interacting matter created in heavy-ion collisions. To that end, PHOS measures direct photons radiation at low transverse momenta  $p_T$ , from hundreds of MeV to a hundred GeV;
- EMCal is a wide-aperture electromagnetic calorimeter used to explore parton energy loss in the QCD matter. This is achieved by measuring jet quenching as well as prompt photons and electrons at high  $p_T$  up to 250 GeV/c.

#### 2.2.8 Muon spectrometer

The ALICE muon spectrometer [19] studies the complete spectrum of heavy quarkonia  $(J/\psi, \psi', \Upsilon, \Upsilon')$  via their decay in the  $\mu^+\mu^-$  channel.

The spectrometer consists of a passive front absorber to absorb hadrons and photons from the interaction vertex, a high-granularity tracking system of 10 detection planes, a large dipole magnet and 4 planes of trigger chambers. Muons may be identified being the only charged particles able to pass, almost undisturbed, through the absorber.

It provides an essential tool to study the early and hot stages of heavy-ion collisions. In particular, the muon spectrometer is expected to be sensitive to QGP formation. The spectrometer covers the pseudorapidity interval  $2.5 \le \eta \le 4$ , and the resonances can be detected down to zero  $p_T$ . Muons may be identified using the just described technique because they are the only charged particles able to pass, almost undisturbed, through any material. It happens so because muons with momenta below a few hundred GeV/c do not radiate energy and do not produce electromagnetic showers.

# Chapter 3

# **Reconstruction of non-prompt charmed** baryon $\Lambda_c$ with Boosted Decision Trees technique

### **3.1 Introduction**

The study of charmed-baryon production at the LHC is one of the fundamental tools to verify the theoretical prediction of the state of strongly interacting matter both at high temperatures and densities. These conditions are realised in the early stages of heavy-ion collisions and they create the QGP, as seen in section 1.4.

Due to their high mass, charm quarks are created in the early stages of the collisions in partonic hard scattering. In this environment, the interaction of charm quarks with the medium constituents can modify the hadronisation of quarks: a significant fraction of low and intermediate momentum charm quarks can hadronise via recombination with other quarks from the medium, while at higher momenta fragmentation processes are dominant. The interplay between these two processes can be investigated with the study of the relative abundance of the different heavy-hadron species produced. In particular, the role of the hadronisation by coalescence can manifest in a baryon-to-meson enhancement for charmed hadrons. In this regard, the study of the  $\Lambda_c$  plays a key role.

The properties of charmed Lambda baryon<sup>\*</sup>  $\Lambda_c$  are illustrated in table 3.1.

Symbol	Quark content	Rest mass $(MeV/c^2)$	Mean lifetime (s)
$\Lambda_c^+$	udc	$2286.46 \pm 0.14$	$(2.00\pm0.06)\times10^{-13}$

Ta	ble	e 3	3.1	1:	Main	pro	perties	of	the	charmed	Lam	bda	bary	yon
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<sup>\*</sup>Lambda baryons are hadrons that contains one up quark, one down quark, and a third quark from a higher flavour generation.

This heavy particle was reconstructed in ALICE through two hadronic decay channels and a semileptonic one. In this chapter, it is presented the analysis of the hadronic decay  $\Lambda_c^+ \longrightarrow p K_S^0$ . The study is focused on the measurement of the production cross-section of *non-prompt*  $\Lambda_c$ , i.e.  $\Lambda_c$  coming from beauty-hadrons decays (addressed also as *feed-down*  $\Lambda_c$ in the following, while  $\Lambda_c$  directly coming from charm quark hadronization are referred as *prompt*  $\Lambda_c$ ). Figure 3.1 illustrates the two decay topologies analysed in this chapter. Due to the short lifetime of the  $\Lambda_c$  ( $c\tau = 60\mu m$ ), the low ratio between signal and background and the limited available statistics collected by the ALICE detector, the reconstruction of  $\Lambda_c$  decay was particularly challenging.



Figure 3.1: Decay of (on the left) feed-down  $\Lambda_c$  and (on the right) prompt  $\Lambda_c$ .

Recent results [20] suggested an enhanced production of  $\Lambda_c$  baryons at the LHC energies with respect to theoretical predictions and experimental results from previous experiments at lower energies. Models assuming a large feed-down from higher-mass states has been proposed to describe the data. In all the analysis published so far, the correction for the non-prompt component in the inclusive  $\Lambda_c$  measurements was performed based on theoretical calculations. Is therefore interesting to investigate the feasibility of *data-driven* correction approaches for the feed-down correction.

In this chapter it is considered a Machine Learning (ML) technique that uses multivariate analysis (MVA) for classification. For this type of analysis, it is used the TMVA package, distributed within the ROOT analysis framework.

# **3.2** The Toolkit for Multivariate Data Analysis

The Toolkit for Multivariate Data Analysis (TMVA) [21] is an integrated analysis framework of ROOT [22] that is an object-oriented program and library developed by CERN. The TMVA provides a broad variety of multivariate classification algorithms. All the multivariate techniques in TMVA belong to *supervised learning*. To reach this goal, the algorithms make use of training events, for which the desired output is known. The mapping function can contain various degrees of approximations and it may be a single global function or a set of local models. The machine learning technique chosen is the Boosted Decision Trees.

# **3.3** Boosted Decision Trees (BDT)

A *decision tree*, is a binary tree structured classifier composed by a consecutive set of questions: the *nodes*. A question divides the classification in two samples, called *branches*. Each question depends on the formerly given answers. The final verdict, called *leaf*, is reached after a given maximum number of nodes. Figure 3.2 illustrates the schematic view of a decision tree. The discriminating variable  $x_i$  is used to bifurcate the data from the *root node*. The spit variables used, at each node, provide the best discrimination between signal and background. Thus allowing multiple use of the same variable at several nodes whereas, others are useless. The ending decisions (*leaf nodes*) label the outputs according to *S* or *B* (signal and background respectively); the labelling depends on the events that end up in the different leaf nodes.



Figure 3.2: Schematic view of a decision tree.

A single decision tree is generally unstable because it is prone to statistical fluctuation in the training sample. Besides, a single decision tree can easily be *overtrained*, i.e. it has a heavy dependence from the characteristics of inputs. To enhance the stability of this method, it is used the *boosting* algorithm. It is a procedure that combines many *weak* classifiers to achieve a final powerful classifier. Boosting can be applied to any classification method.

To this end, different trees are produced; their combination, constitutes a *random forest*. These trees arise from the same training sample and they are built by giving more *weight* to events that are not correctly identified, as signal or background, by the previous tree. The final output is determined by combined decision of the trees. For the data analysis it has been used the *Gradient Boost* method, only available for decision trees.

#### **3.3.1 Gradient Boost**

Like other boosting methods, gradient boosting combines *weak learners* into a single strong learner in an iterative fashion. Gradient boosting technique finds the prediction function F(x) and this is assumed to the weighted sum of parametrised base functions  $f(x; a_m)$ . Each base function in this expansion corresponds to a decision tree:

$$F(x;P) = \sum_{m=0}^{M} \beta_m f(x;a_m); \qquad P \in \{\beta_m; a_m\}_0^M.$$
(3.1)

The boosting procedure is now employed to adjust the parameters P such that the deviation between the model response F(x) and the true value y obtained from the training sample is minimised. The deviation from this value is measured by the *loss-function* L(F, y). It is possible to demonstrate that the loss function entirely determines the boosting procedure.

The gradient boost implemented in the TMVA uses the binomial log-likehood loss:

$$L(F, y) = \ln(1 + e^{-2F(x)y}), \tag{3.2}$$

for classification. As the boosting algorithm corresponding to this loss function cannot be obtained straightforwardly, one has to restore to a steepest descent approach to approach the minimisation. This is done by computing the current gradient loss function and the growing of a regression tree whose leaf values are adjusted to match the mean value of the gradient in each region defined by the tree structure. By iteration of this procedure, it yields the set of decision trees which minimises the loss function.

### **3.4 BDT Training**

The result of running the Boosted Decision Tree algorithm and machine learning in general, can be expressed as a function y(x). This function [23] takes a new digit image x as input and that generates an output vector y, encoded in the same way as the target vectors. The shape of the function y(x) is determined during the *training* phase, also known as the *learning phase*, which is based on the training statistical data. Once the model is trained it can then determine the identity of the new digit images, which include a test set. The main task of these types of algorithms is to verify the ability to categorise correctly new events that differ from those used for training, this is known as *generalisation*. In our case, the algorithm uses the *multiclass method* that allows to recognise prompt  $\Lambda_c$ , feed-down  $\Lambda_c$  and the background signal.

#### 3.4.1 Multiclass

Decision tree learning naturally permit a binary classification. Instead, the *multiclass classification* problem supports the categorisation of more than two classes of signal. These can be

considered as an extension algorithm of the first binary classifier. Multiclass makes the assumption that from each sets of input, it is possible to assign one and only one label (it is not possible, for instance, that one sample can be at the same time prompt and non-prompt).

For the *training* phase, it is used an adaptation from the TMVA macro tutorials: *TMVA-Multiclass.C*; while for the *application phase* it used an adaptation from *TMVAMulticlassApplication.C*.

#### **3.4.2** Monte Carlo simulations

In order to obtain a ML algorithm able to make predictions, it is necessary to build the data sets on which the model training is performed and its performance is evaluated; these are called *training* and *test* sets. The prompt  $\Lambda_c$ , feed-down  $\Lambda_c$  and background candidates are constructed from Monte Carlo (MC) simulation of Pb-Pb collisions measured by the ALICE experiment. The MC consists of a simulation of PYTHIA6 [24] pp events containing charmed hadrons, embedded into an underlying Pb-Pb collision generated with HIJING [25] to obtain a better description of the multiplicity distribution observed in the data; the generated particles were then transported through the ALICE detector by using GEANT3 [26]. The presence of at least one  $\Lambda_c$  decaying via the hadronic decay channel under consideration in each simulated event was required in order to maximise the number of candidates.

### **3.5** Input variables

The gradient boost algorithm used for the data analysis used eleven input variables:

- 1. **massK0S:** it represents the rest mass of the neutral particle  $K_S^0$ , reconstructed from the tracks of its particle daughters after the  $\Lambda_c$  decay. The particle  $K_S^0$  can decay in  $\pi^+$  and  $\pi^-$ . Therefore, from the measurement of their momentum, it is possible to compute the momentum of the mother particle. It is assumed that for pions the mass is about 138  $MeV/c^2$  and the energy of the particle has the form of  $E_{\pi^\pm} = \sqrt{p_{\pi^\pm}^2 + m_{\pi^\pm}^2}$ . Finally, it is possible to derive the rest mass of  $K_S^0$  from  $m_K = \sqrt{(E_{\pi^+} + E_{\pi^-})^2 p_K^2}$ .
- 2. **tImpParBach:** proton impact parameter is defined as the minimum distance from the reconstructed track of proton and the position of the primary vertex.
- 3. **tImpParV0:**  $K_S^0$  impact parameter.
- 4. **ctK0S**:  $c\tau$  of  $K_S^0$  is calculated from the distance of the primary vertex and from the vertex at which the two pions are created. Then, it is multiplied for the  $K_S^0$  mass and divided by its momentum. This particle has  $c\tau \simeq 2.68$  cm. It is another way to prove that the reconstructed particle is effectively a  $K_S^0$ .

- 5. **cosPAK0S:** it represents the cosine of the pointing angle, i.e. the angle between the reconstructed direction of the  $K_S^0$  (based on the momenta of the two pions), and the straight line that links the primary vertex and the decay vertex of the  $K_S^0$ . It is expected that the value is close to unity.
- 6. **signd0:** the impact parameter of the proton defined with the sign. The sign is defined as positive if the sum of the x-coordinate of the secondary vertex multiplied by the momentum of the  $\Lambda_c$  candidate along the x axis and the y coordinate of the secondary vertex multiplied by the momentum of the  $\Lambda_c$  candidate along the y axis are greater than 0. The case with signd0>0 is much more compatible with a proton created by the decay of a  $\Lambda_c$  compared to the case with signd0<0, in which the proton appears to come from a point "*before*" the primary vertex.
- 7. **DecayLengthK0SXY:** it represents the length of the  $K_S^0$  decay, i.e. the distance between the decay vertex of  $K_S^0$ , from which the two pions start, and the primary vertex.
- 8. NormDecayLengthK0SXY: it is the length of decay of the meson  $K_S^0$ , i.e. the distance between the vertex decay and the primary vertex and in the transverse place XY, divided by the error of measure.
- 9. **distanceLcToPrimVtx:** it is the distance between the secondary vertex, i.e. decay vertex of the  $\Lambda_C$ , and the primary vertex.
- 10. **combinedProtonProb:** it quantifies the probability that particle assigned as a proton is effectively a proton. This is possible because of the combination of data recorded by the TOF detector and TPC detector. The TOF uses the difference between the measured flight time from the primary vertex to the detector TOF detector and the expected one in case the particle is a proton, divided by its time resolution. The smaller this difference, the greater the probability that the particle is actually a proton. This difference is usually expressed in units of measurement of the resolution of TOF. The same principle is followed by TPC detector, but in this case, the specific ionization dE/dx for evaluation of different hypothesis it is used.
- 11. **alphaArmLc:** it is the longitudinal variable that defines the asymmetry of the decay. If the  $p_{L,p}$  is defined as the parallel momentum of the proton to the momentum of  $\Lambda_c$  and  $p_{L,K_s^0}$  as the component of meson  $K_s^0$  parallel momentum to the momentum  $\Lambda_c$ , then:

$$\alpha = \frac{p_{L,p} - p_{L,K_s^0}}{p_{L,p} + p_{L,K_s^0}}.$$
(3.3)

The training of BDT was repeated independently for two intervals of transverse momentum  $p_T$ : [2,3] and [3,12] GeV/c. The distribution of the variables is illustrated in figures 3.3

and 3.4. In these figures are displayed the superimposition of the two signals (prompt  $\Lambda_c$  and feed-down  $\Lambda_c$ ) and the background.



Figure 3.3: Distribution of input variables for the two signals and background in the  $p_T$  interval [2,3] GeV/c.



Figure 3.4: Distribution of input variables for the two signals and background in the  $p_T$  interval [3,12] GeV/c.

It is useful to quantify the correlations between the input variables. The chosen variables are considered a good set if there is a low correlation between them. A correlation between variables exists when there is a relationship that links them. This relationship is quantified by a coefficient that equals 0 if the variables are not correlated. Instead, when the coefficient equals +1 or -1 the variables are totally correlated. The sign of this value states that the two variables are directly proportional (if positive) or indirectly proportional (if negative). The correlation matrices for the two signals and background, labelled respectively:

- *signal*: it represents feed-down  $\Lambda_c$ ;
- *bg0*: it represents prompt  $\Lambda_c$ ;
- *bg1*: it represents the background signal;

in the intervals [2,3] and [3,12] GeV/c are shown in figures 3.5 and 3.6. It is evident that for every matrix there is a high correlation between variables DecayLengthK0SXY and cosPAK0S.



Figure 3.5: Linear correlation coefficients for prompt  $\Lambda_c$ , background and feed-down  $\Lambda_c$ , created from the decay of  $\Lambda_b$  in the  $p_T$  interval [2,3] GeV/c.



Figure 3.6: Linear correlation coefficients for prompt  $\Lambda_c$ , background and feed-down  $\Lambda_c$ , created from the decay of  $\Lambda_b$  in the  $p_T$  interval [3,12] GeV/c.

### **3.6 BDT Configuration**

The TMVA was configured to use samples for signals and background coming from MC simulations. The good relationship between the performance results of the training and testing events can assure the success of the training process. The number of prompt  $\Lambda_c$  and feed-down  $\Lambda_c$  candidates are reported: overall in table 3.2, and for each interval of transverse momentum in table 3.3.

The configuration of the BDT is summarized below in table 3.4.

	feed-down $\Lambda_c$	prompt $\Lambda_c$	Background	Total Events
Number of events	34728	34728	2721306	2790762

Table	3 2. Sum of the	training and	testing events	
Table	5.2. Sum of the	e training and	testing events.	

$\Lambda_c p_T \operatorname{bin} (GeV/c)$	feed-down $\Lambda_c$	prompt $\Lambda_c$	Background
2-3	4734	4663	777280
3-12	14800	9851	685414

Table 3.3: Number of training and testing events for the two intervals of transverse momentum.

Option	Values	Description	
NTrees	1000	Number of trees in the forest	
MaxDepth	2	Maximum depth of the decision tree allowed	
MinNodeSize	2.5%	Minimum percentage of training events	
		required in a leaf node	
nCuts	20	Number of points in variable range used	
		in finding optimal cut in node splitting	
BoostType	Grad	Boosting type for the trees forest	
Shrinkage	0.10	Learning rate for GradBoost algorithm	
BaggedSampleFraction	0.50	Relative size of bagged event sample	
		to original size of the data sample	

Table 3.4: Configuration options used for BDT.

# 3.7 Training and performance evaluation

The BDT algorithm combines the discriminating power of several input variables to provide a classification tree, which can generate a score when run over data that allows separating signal candidates from background. After the training, the model is applied to the test and training sets to visualise the distributions of the BDT output scores obtained and to check their agreement between the two sets. In particular, in case of classification with three classes of candidates, three scores are provided by the ML algorithm, which represent the probability of belonging to each of the considered classes (non-prompt  $\Lambda_c$ , prompt  $\Lambda_c$ , and combinatorial background). By construction, for each candidate, the probabilities of belonging to the three classes sum to one.

Figures 3.7 and 3.8 show the distributions of the signal probability for candidates in the three categories, in the  $2 < p_T < 3$  GeV/c and  $3 < p_T < 12$  GeV/c intervals. From these figures one can see that, by applying the model over the two data sets, the same output is obtained. This means that the specific choice of a certain sample to train the algorithm does not introduce any bias in the output, i.e. the input data are statistically large enough to guarantee the lack of bias in the final results.



Figure 3.7: Overlapped response functions for signals and background in the  $p_T$  inverval [2,3] GeV/c. This is obtained from the training and testing BDT.



Figure 3.8: Overlapped response functions for signals and background in the  $p_T$  inverval [3,12] GeV/c. This is obtained from the training and testing BDT.

For each probability, the distribution corresponding to the respective true class is close to unity, while the other two distributions are shifted towards zero. Moreover, it is possible to observe that the distributions of combinatorial background candidates are typically more separated from those of prompt and non-prompt  $\Lambda_c$ , indicating that the separation between prompt and non-prompt  $\Lambda_c$ , as expected.

#### 3.7.1 Ranking input variables

Ranking the input variables means sorting them with criteria of importance and weight that they have within the training process. A ranking of the BDT input variables is derived by counting how often the variables are used to split decision tree nodes, and by weighting each split occurrence by the separation gain-squared it has achieved and by the number of events in the node.

Rank	Variable	Variable Importance
1	combinedProtonProb	3.059e-01
2	signd0	1.728e-01
3	alphaArmLc	9.757e-02
4	NormDecayLengthK0SXY	6.930e-02
5	tImpParBach	6.840e-02
6	distanceLcToPrimVtx	6.051e-02
7	massK0S	5.916e-02
8	tImpParV0	4.939e-02
9	DecayLengthK0SXY	4.731e-02
10	CtK0S	3.682e-02
11	cosPAK0S	3.280e-02

Tables 3.5 and 3.6 illustrate the BDT variables ranking for each  $p_T$  interval.

Table 3.5: Ranking of input variables in the  $p_T$  interval [2,3] GeV/c

Rank	Variable	Variable Importance
1	combinedProtonProb	2.622e-01
2	signd0	1.361e-01
3	CtK0S	1.097e-01
4	alphaArmLc	1.003e-01
5	DecayLengthK0SXY	9.570e-02
6	NormDecayLengthK0SXY	5.763e-02
7	massK0S	5.616e-02
8	tImpParBach	5.590e-02
9	tImpParV0	5.318e-02
10	distanceLcToPrimVtx	3.751e-02
11	cosPAK0S	3.562e-02

Table 3.6: Ranking of input variables in the  $p_T$  interval [3,12] GeV/c

# 3.8 Running the BDT

To evaluate the efficiency of the BDT method, the weights found during the *training phase* are applied to a validation sample of simulated events and the results are compared to the MC truth. This is done in order to simulate the procedures that can be used with the real experimental data for splitting prompt and non-prompt signals.

After running the *application phase*, the resulting data are selecting by applying ten different probability cuts on the signal probability output score. The same cuts, reported in table 3.7, are applied for two transverse momentum intervals. These cuts are used for both transverse momentum intervals.

0.01	0.02	0.03	0.04	0.05	0.07	0.10	0.20	0.30	0.40

Table 3.7: Values of cuts applied to the function response.

The selection of the data is performed by cutting the outputs of the BDT classifier, considering only candidates that lie above each probability cut. The invariant mass distribution of the selected  $\Lambda_c$ , including prompt and non-prompt candidates and combinatorial background, is fitted with a Gaussian function that models the invariant mass peak and a polynomial of the second order for the background. The cuts on BDT response allow to reduce the fake candidates below the  $\Lambda_c$  mass peaks, but it also implies necessarily a rejection of a certain fraction of interesting candidates.

In figures from 3.9 to 3.12 are shown the invariant mass distributions of  $\Lambda_c$  candidates overlapped with fit functions. The values of the signal-background ratio are computed considering a  $3\sigma$  range with respect to the mean of the Gaussian fit.

For every cuts and fits there is a correspondent equation of the type:

$$S = e_1 S_1 + e_2 S_2, \tag{3.4}$$

where the value of S is the total number of  $\Lambda_c$  signals (*prompt* and *non-prompt*) given by the fit results;  $S_1$  and  $S_2$  represent the number of non-prompt and prompt  $\Lambda_c$ ;  $e_1$  and  $e_2$  represent efficiencies of non-prompt and prompt computed from training outputs. Therefore, we have a system of ten equations (an equation for each cut) for each  $p_T$  interval in two unknowns,  $S_1$  and  $S_2$  respectively. The values of these parameters are given in tables 3.8 and 3.9.

To solve this problem, it is used the tool MINUIT [27]. It is an ensemble of libraries written to find the minimum value of multi-parameter functions, using numeric methods and analysing the shape of the function around the minimum. The two plots, 3.13 and 3.14, show the trend of the prompt and non-prompt signals throughout the ten cuts, their sums and signal given by the fit. The results of this calculation are given in table 3.10. These values are compared with the number of prompt and non-prompt  $\Lambda_c$  candidates in the MC validation sample. For a proper comparison, the reference number were extracted from a Gaussian fits of the invariant mass distributions. To be noted that these number are not the same as those reported in table 3.3, because the invariant mass distributions are not exactly Gaussian in shape. It can be seen that these values, given in table 3.11, have an acceptable agreement with MINUIT test. The compatibility of the values in these two tables means a good efficacy of the BDT in this analysis. Figure 3.15 shows graphically the accord of MINUIT results and the signals given by MC, for  $\Lambda_c$  non-prompt. It can be noted that only for the second  $p_T$  interval there is not a perfect agreement within the statistical uncertainties between results, but they are compatible.



Figure 3.9: Invariant mass distribution of  $\Lambda_c$  candidates for  $p_T$  interval [2,3] GeV/c for cuts (from top to bottom): 0.01, 0.02, 0.03, 0.04, 0.05.



Figure 3.10: Invariant mass distribution of  $\Lambda_c$  candidates for  $p_T$  interval [3,12] GeV/c for cuts (from top to bottom): 0.07, 0.1, 0.2, 0.3, 0.4.



Figure 3.11: Invariant mass distribution of  $\Lambda_c$  candidates for  $p_T$  interval [3,12] GeV/c for cuts (from top to bottom): 0.01, 0.02, 0.03, 0.04, 0.05.



Figure 3.12: Invariant mass distribution of  $\Lambda_c$  candidates for  $p_T$  interval [3,12] GeV/c for cuts (from top to bottom): 0.07, 0.1, 0.2, 0.3, 0.4.

	Signals	Signal	efficiencies $(e_1)$	efficiencies $(e_2)$
Cuts	( <i>S</i> )	errors	non-prompt	prompt
0.01	8535	623	1.0000	1.0000
0.02	7615	320	0.9295	0.7760
0.03	6699	172	0.8585	0.5874
0.04	6001	223	0.8152	0.4963
0.05	5256	189	0.7783	0.4500
0.07	4699	117	0.7096	0.3600
0.10	4104	102	0.6368	0.2819
0.20	2508	70	0.4129	0.1329
0.30	1409	50	0.2450	0.0568
0.40	755	35	0.1468	0.0172

Table 3.8: Values used to solve equation (3.4) for  $p_T$  interval [2,3] GeV/c.

	Signals	Signal	efficiencies $(e_1)$	efficiencies $(e_2)$
Cuts	<i>(S)</i>	errors	non-prompt	prompt
0.01	21073	605	0.9682	0.8980
0.02	20395	276	0.9361	0.8265
0.03	19581	243	0.9149	0.7671
0.04	18879	225	0.8943	0.7068
0.05	17420	200	0.8747	0.6464
0.07	16261	184	0.8436	0.5576
0.10	14822	168	0.8024	0.4576
0.20	11427	133	0.6682	0.2694
0.30	8839	112	0.5378	0.1659
0.40	6642	94	0.4203	0.0990

Table 3.9: Values used to solve equation (3.4) for  $p_T$  interval [3,12] GeV/c.



Figure 3.13: Trend of prompt (green line), non-prompt (blue line) signals, their sum (red dashed line) and signal computed from the fit (red solid line), through the probability cuts in  $p_T$  interval [2,3] GeV/c.



Figure 3.14: Trend of prompt (*green line*), non-prompt (*blue line*) signals, their sum (*red* dashed line) and signal computed from the fit (*red solid line*), through the probability cuts in  $p_T$  interval [3,12] GeV/c.

$\Lambda_c p_T \operatorname{bin} (GeV/c)$	feed-down $\Lambda_{c}(S_{1})$	prompt $\Lambda_c$ (S <sub>2</sub> )
2 - 3	4540.520±265.447	4397.740±411.805
3 - 12	$13161.700 \pm 268.364$	9618.230±274.846

Table 3.10: Values computed from MINUIT test.

$\Lambda_c p_T \operatorname{bin} (GeV/c)$	feed-down $\Lambda_c(S_1)$	prompt $\Lambda_c$ ( $S_2$ )
2-3	4427	4354
3-12	13783	9136

Table 3.11: Values computed from MC fit test. The compatibility with values of the table 3.10 means a good efficacy of the BDT in this analysis.



Figure 3.15: Comparison of *non-prompt* signal results between MINUIT and MC in both  $p_T$  intervals. The y-axis is expressed in logarithmic scale.

# Conclusions

Due to its short lifetime, the measurement of the  $\Lambda_c$  baryon is challenging: the reconstruction of this particle was performed by combining different decay channel, and different analysis approaches, using standard probability cuts, MVA techniques and a dedicated procedure to classify the different classes of signal, like Boosted Decision Trees used in this analysis.

Machine learning techniques are progressively spreading in high-energy physics, where small signals are submerged in large data sets. It has become essential to extract the majority of information available from a set of data.

The analysis performed in this thesis made use of the ROOT-integrated TMVA package, which allows to manage a large spectrum of multivariate classification algorithms. Among these methods, the Boosted Decision Trees algorithm was chosen, with the multi class classification so far available only with the Gradient Boost. This analysis was aiming at separating non-prompt  $\Lambda_c$  coming from beauty-hadron decays and prompt  $\Lambda_c$  in a data-driven fashion. The algorithm was trained with a MC simulation including prompt and non-prompt  $\Lambda_c$  candidates and a large combinatorial background. This training was used to determine a mapping function which was then applied to simulated data, in which the type of candidates was known. The extracted yields obtained after cutting on the output signal probability were minimized with MINUIT to get the individual contributions, and then compared to the MC truth. The results presented show that this approach is very promising; the numbers of non-prompt and prompt signals coming from the calculations are compatible with the signals in the MC simulation within the statistical uncertainties. This proves that the method can be safely applied to real data.

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