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Theory of semileptonic $B \to D^{(*)}\ell\nu$ decays and sensitivity studies to new physics effects

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Science is built of facts the way a house is built of bricks;
but an accumulation of facts is no more science
than a pile of bricks is a house.
Henri Poincaré

Do not trust a theory until it has been supported by experiment;
and do not trust experimental results
until they have been supported by theory.
Sir Arthur Eddington

Sommario

In questa tesi, si vuole investigare l'effetto che la fisica oltre il modello standard ha sui decadimenti semileptonici $B \to D^{(*)}\ell\nu_\ell$, dove ℓ può essere un τ, un μ o un elettrone. Questo lavoro è stato spronato dalle cosiddette *anomalie della fisica del B in processi a corrente carica*, che consistono in un tasso del decadimento $B \to D^{(*)}\tau\nu_\tau$ più alto rispetto alle predizioni teoriche. Tra i vari tipi di fisica oltre lo standard model introdotti per spiegare questa ed altre anomalie del B, la soluzione con un contributo di tipo tensoriale appare la più convincente e studiata nella letteratura recente, principalmente per la sua generalità e semplicità. Altri tipi di contributi, come i leptoquarks e il modello a due Higgs, vengono considerati. Per studiare questi effetti, la tesi è stata organizzata in due parti: una teorica ed una sperimentale. Nella prima, dopo una introduzione sulle recenti osservazioni sperimentali che hanno evidenziato queste anomalie, vengono presentate le conoscenze teoriche necessarie per affrontare l'argomento. Sono quindi spiegati i fattori di forma, che sono cruciali per descrivere i decadimenti semileptonici, e viene considerata la loro importanza per fare predizioni affidabili. Si deriva in dettaglio l'ampiezza di decadimento sia per il caso $B \to D$ sia per il $B \to D^*$, mostrando come costruire osservabili sensibili al contributo di nuova fisica. Ignorando gli effetti del rivelatore nella ricostruzione dei decadimenti, si attua uno studio sulla sensibilità ai fattori di forma e alla fisica oltre lo standard model. Dopo aver introdotto i metodi Monte Carlo, si descrive l'approccio usato per effettuare lo studio di fattibilità sia per il caso di solo modello standard, sia per il caso di nuova fisica. Infine, vengono mostrati i risultati dello studio, e vengono presentate le conclusioni e prospettive future riguardo questa area di ricerca.

Abstract

In this thesis, we investigate the effects that physics beyond the Standard Model has on semileptonic decays $B \to D^{(*)}\ell\nu_\ell$, where ℓ can be a τ, a μ or an electron. This work has been motivated by the so called *B physics anomalies in charged current processes*, which consist in an anomalous higher rate of the decay $B \to D^{(*)}\tau\nu_\tau$, compared to the theoretical predictions. Among the various kind of beyond standard model physics introduced to explain this, and other anomalies in the B decays, the tensor-like contributions appear the most convincing and studied in the recent literature, mainly for its generality and simplicity. Other kind of contributions, like leptoquarks and two-Higgs-Doublet model are also considered. To study these effects, the thesis is organized into two main parts: a theoretical one and an experimental one. In the former part, after an introduction on recent experimental observations that have highlighted these anomalies, the theoretical knowledge required to deal with the topic is illustrated. The form factors, which are crucial to describe the semileptonic decays, are then explained, and their relevance for reliable predictions are considered. We derive in detail, the decay amplitude for both the $B \to D$ and $B \to D^*$ cases, showing how to build observables sensitive to new physics contributions. Ignoring the detector effects in the reconstruction of these decays, we perform studies on the sensitivity to the form factors and to physics beyond the standard model. After introducing the Monte Carlo methods, we describe the approach used to perform these feasibility studies for both the standard model case only and in the new physics case. Finally, the results of the studies are shown, and the conclusions and future perspectives relating to this area of research are presented.

Contents

Introduction

Since the born of quantum mechanics, physicists have tirelessly worked to understand the intimate structure of nature, carving one of the most fascinating theory human mind has ever given birth to.

Developed in the early 1970s, the Standard Model is currently the best description of the subatomic world. It has successfully explained almost all experimental results in particle physics and precisely predicted a wide variety of phenomena. The particles are the nature building blocks and they are splitted in two big families: quarks and leptons. The interactions among them are interpreted as an exchange of other particles called gauge bosons.

Every number that we can compare is of utmost importance, because this is the only thing we have to build a proper interpretation of facts. Due to the counterintuitive nature of quantum mechanics and its abstract mathematic formalism, it is very easy to confuse the real with the way we depict it. The strength of the theory is in its predictions capability only, while its intepretation is subject to change.

However, despite being one of the most successful theory of particle physics so far, it has issues that must be addressed, such as strong CP problem, neutrino oscillations, matter-antimatter asymmetry and the nature of dark matter and dark energy. For this reason, Beyond Standard Model Physics is essential to push forward our knowledge of the universe. Recently, two collaborations, LHCb at the LHC, Belle at the KEKB, have reported an excess of the decay probability in the processes $\bar{B} \to D\ \tau\ \bar{\nu_\tau}$ and $\bar{B} \to D^*\ \tau\ \bar{\nu_\tau}$ with respect to SM predictions. These results are in very good agreement with an earlier observation from the BABAR collaboration for the same decay channels. The decay rates measured by BABAR in 2012 exceeded the SM predictions by about 3.4σ. When combined with the new results from LHCb and Belle, the significance of the discrepancy rises to 3.1-3.7σ depending on the SM prediction considered, making it one of the largest departures from the SM seen up to now. The HFLAV collaboration reported in Ref.[1] a comprehensive summary of the most recent results, and the comparison with the SM predictions. The world average of the existing experimental results of the decay widths of $\bar{B} \to \bar{D}\tau\bar{\nu}_\tau$, normalized to the widths of the corresponding modes having a light lepton $l = e$, μ in the final state [1], are

$$
\begin{aligned}
R(D) &= \frac{\Gamma(\bar{B} \to \bar{D}\tau\bar{\nu}_\tau)}{\Gamma(\bar{B} \to \bar{D}l\bar{\nu}_l)} = 0.340 \pm 0.0027 \pm 0.013, \\
R(D^*) &= \frac{\Gamma(\bar{B} \to \bar{D}^*\tau\bar{\nu}_\tau)}{\Gamma(\bar{B} \to \bar{D}^*l\bar{\nu}_l)} = 0.295 \pm 0.011 \pm 0.008,
\end{aligned}
$$

where the first and second error are the statistic and systematic uncertainty, respectively. These results have to be compared with the SM predictions

$$
\begin{aligned}
R(D) &= \frac{\Gamma(\bar{B} \to \bar{D}\tau\bar{\nu}_\tau)}{\Gamma(\bar{B} \to \bar{D}l\bar{\nu}_l)} = 0.299 \pm 0.003, \\
R(D^*) &= \frac{\Gamma(\bar{B} \to \bar{D}^*\tau\bar{\nu}_\tau)}{\Gamma(\bar{B} \to \bar{D}^*l\bar{\nu}_l}) = 0.258 \pm 0.005,
\end{aligned}
$$

which are given by averaging different predictions from differents groups. This discrepancy is usually refererred in the literature as B-flavour anomaly in charged current processes [11].

The B meson decays have been studied at pp and e^+e^- colliding beam facilities. At e^+e^- a large production of B mesons has been collected by the B-Factories BABAR and Belle. At the B-Factories, the e^+ and e^- of the beams annihilate with an energy in the C.o.M. of 10.58 GeV and produce a particle, commonly refered to as Υ $(4S)$, which decays to B^+B^- or $B^0\overline{B}^0$ pairs. The maximum production rate at the B-Factories was achieved at the KEK accelerator.

In pp collisions at the typical energy of LHC, the B mesons are instead produced by the fragmentation of the b-quarks produced mainly by the gluon splitting process $gg* \to b\bar{b}$. The cross section for the $pp \to b\bar{b}$ is about five orders of magnitude larger than of the $e^+e^- \to \Upsilon(4S)$ at B-Factories. But the much larger backgrounds present in pp colliders, requires great care in the selection of the event.

The decays like $\overline{B} \to \overline{D}l\nu_l$ are called semileptonic because in the final state there are both leptons and hadrons. These decays involves only the b quark, while the light quark $\bar{d}$ (or $\bar{u}$ for the B^+ case) act simply as a spectator. The b goes into a c emitting a lepton (τ , μ or e) and the corresponding antineutrino. In the SM, the semileptonic decays are described by a single tree-level amplitude like the one shown in figure 1. Because there are no interactions among the leptons and the hadrons in the final state, usually the predictions about semileptonic decays have small theoretical uncertainties. Moreover, the QCD corrections are limited to the $B \to D$ current and there are theoretical tools that allow to compute systematically these corrections, for example the Lattice-QCD.

In the semileptonic decays the final state always contains at least one neutrino, making the full reconstruction of the kinematic challenging. Furthermore, in almost every theoretical approach to the problem, the neutrino is considered left-handed. [1].

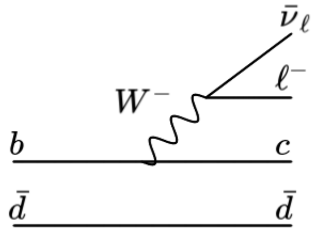

Figure 1: Feynman Diagram of the $\bar{B}$ to $\bar{D}l\nu_l$ decay.

The study of $\bar{B}$ meson physics is fundamental to investigate and understand the crucial puzzles of our universe. In order to have some hints on its importance, theoretically speaking, one might be able to explain the matter-antimatter asymmetry using CP violating B meson oscillations in conjunction with baryon number violating decays. The theory suggests a baryogenesis scenario beginning in the pre-nucleosynthesis early universe with the decays of a long lived scalar into b quarks and anti-quarks. These decays are assumed to take place late enough and at low enough temperature to allow hadronization. Most of the b quarks form $\bar{B}^0_s$ and $\bar{B}^0$ mesons. The neutral B mesons then oscillate and decay, sometimes to baryons or anti-baryons, resulting in the observed asymmetry. There are models exploring the connection of the leptoquark solution to the B-meson anomalies, involving an hypothetical new particle able to allow leptons and quarks to interact, carrying both lepton and baryon numbers, with a mechanism of neutrino mass generation and a viable dark matter candidate. Without any doubts B meson physics is a new and promising field of study, but a lot of work has yet to be done.

In this thesis we will analyse the sensitivity to new physics scenario and viable methods to distinguish all the different models. In the first chapter, we will describe the theoretical background needed, with an introduction to the standard model and flavour physics. The complete theory of the B decay is shown in the second chapter, with the focus on the form factors and the effect of their parametrisations, demonstrating how to incorporate new physics effects on sensible quantities, like the forward-backward asymmetry. In the third chapter, we will implement the new physics into the SL Decay software from [9], to conduct a feasibility study on the sensible quantities.

[1]One could consider a right-handed neutrino falling in a much more cumbersome calculations. But it seems that right-handed models are disfavoured from LHCb data

Chapter 1

Theoretical Background

1.1 Flavour and CKM Matrix

In 1964, two physicists, Murray Gell-Mann and George Zweig, independently proposed the quark model, explaining how the hadrons were not elementary particles but were in fact made of quarks and antiquarks. In this scheme, quarks are fermions, with spin 1/2 and electric charge $\pm 1/3$ or $\pm 2/3$, such that baryons are made of three quarks and mesons are made of quark-antiquark couples.
But there was a strong objetion to the quark model: it appears to violate the Pauli exclusion principle. Two particles with half integer spins cannot occupy the same state. The problem arised with the discovery of hadrons made by three identical quarks in the same state, like the Ω^- (sss). Greenberg proposed a way out of this dilemma suggesting that quarks come in three colors (red, blue, green), such that each of the quarks carried a different color making them no more identical and invalidating the Pauli principle.
Since all the particles in nature are colorless, meaning that either the total amount of each color is zero or all three colors are present in equal amounts, the only allowed combination of quarks are $q\bar{q}$, qqq, $\bar{q}\bar{q}\bar{q}$.
In 1965, all these concept were mathematically formalized within a non-abelian gauge group $SU(3)$, giving birth to the QCD (quantum chromodynamics), where the quarks could interact thanks to eight vector bosons: the gluons.
Parallel to the QCD, the electroweak theory was developing, unifying electromagnetism and weak interaction. Leptons and quarks can interact via an exchange of four bosons: $W_\pm$ and Z_0, that are massive, and the photon, that is massless. However, the presence of massive bosons, even if confirmed by experiments, was a nontrivial theoretical issue. The cornerstone of modern physics is the idea of gauge symmetry, i.e. mathematical transformations that do not change the physics, regulating redundant degrees of freedom. When a theory has a Lagrangian invariant under certain Lie groups of local transformations, is called gauge theory and the symmetries dictate the form of the interactions. Such that

one has a field theory where the fields can be redefined, but the physics is always the same. The issue with the electroweak model was that any mass term appearing in the Lagrangian would have ruined the gauge invariance property and was then forbidden. To solve this riddle, the idea of spontaneous symmetry breaking comes into play. To understand it qualitatively, the ferromagnet analogy is often used in literature. Ferromagnet can be modeled as spins on a grid. At high temperature the spins point in random directions, such that the ferromagnet is invariant under rotations. The expectation value of macroscopic quantities vanish. When the temperature is lowered below the critical temperature, the Curie point, the spins begin to align. The magnetization becomes non-zero and points in some specific direction. Thus the alignment of the spins breaks the invariance of the ferromagnet under global rotation.
When a continuous symmetry is spontaneusly broken, the Goldstone theorem imposes the appearance of massless bosons, one for each generator of the symmetry that is broken. These bosons were unavoidable but no one had never seen them; it seemed an impasse. Higgs recognized that in a local gauge symmetry, like the one of the original Yang-Mills theory, the Goldstone boson turns into the helicity-zero part of a gauge boson. It is usally said that the Goldstone boson is eaten by the gauge boson, giving it a mass. At the base of the Higgs mechanism is the assumption of a new field, the Higgs field, whose non-zero vacuum expectation value breaks the gauge symmetry of the Lagrangian.
The search for the Higgs boson became a major objective of experimental particle physics and, on 4 july 2012, it was finally observed at CERN.

To present knowledge, matter is made out of three kinds of particles: leptons, quarks and mediators. Adding everything up, including antimatter counterparts, the nature counts a large number of elementary particles: 12 leptons, 36 quarks, 12 mediators, plus the Higgs boson, to reach a minimum of 61 elementary particles, divided in three generations. In particular, there are six different types of quarks, known as flavours: u (up), d (down), s (strange), c (charme), b (bottom) and t (top).

Fermions			Bosons
I	II	III	
u	c	t	gluons G_μ
d	s	b	photons γ
e	μ	τ	$W_\pm$ & Z
ν_e	ν_μ	ν_τ	Higgs H

Table 1.1: Standard Model particles

Interacting with all the forces, they come with a set of quantum numbers, listed in table[1.2].

	I generation		II generation		III generation	
	u	d	c	s	t	b
Q	$+2/3$	$-1/3$	$+2/3$	$-1/3$	$+2/3$	$-1/3$
I	1/2	1/2	0	0	0	0
I_z	$+1/2$	$-1/2$	0	0	0	0
Mev/c^2	2.3	4.8	95	1275	173210	4180

Table 1.2: Quarks properties

The Standard Model, created from the work on the quark model and the electroweak theory, is formally a gauge theory, with gauge group $SU(3)_c \times SU(2)_W \times U(1)_Y$, where $SU(3)$ models the strong interactions of *colored* quarks and gluons, while $SU(2) \times U(1)$ is the Glashow-Weinberg-Salam model of the electroweak interactions. The last ingredient of the SM is the Brout-Englert-Higgs field H, a collection of complex scalars and has an expectation value taken to be $\frac{1}{\sqrt{2}}(0, \nu)$.

The Yukawa interaction between the Higgs and quarks causes not only the masses but even a phenomenon called *mixing*. Studying hadron decays, it is clear that there are transitions between different flavours of quarks. The interpretation of β decay, $n \to p + e^+ + \bar{\nu}_e$, is a good example of how the theory evolved, going from a pointlike interaction to an exchange of a W boson. This is an example of semileptonic decay, involving leptons and hadrons.
Leptons and quarks participate in weak interactions through charged currents constructed from pairs of left-handed fermion states with the same universal coupling. After spontaneous symmetry breaking, the most general quark mass term, expressed in the gauge eigenstates, takes the form [8]

$$\left(\bar{d}'_L, \bar{s}'_L, \bar{b}'_L\right) M^D \begin{pmatrix} \bar{d}'_R \\ \bar{s}'_R \\ \bar{b}'_R \end{pmatrix} + \left(\bar{u}'_L, \bar{c}'_L, \bar{t}'_L\right) M^U \begin{pmatrix} \bar{u}'_R \\ \bar{c}'_R \\ \bar{t}'_R \end{pmatrix}$$

The mass matrices M are complex 3×3 and their parameter depend on the Yukawa couplings and the vacuum value v of the Higgs field, and are always diagonalisable by a biunitary transformation. Exspressing the interaction currents in the new basis, the neutral currents (those mediated by the Z boson) do not give any flavour changing and it is said that the standard model is free of flavour changing neutral currents (FCNC). The charged currents (mediated by the $W_\pm$) are, on the other hand, written as

$$J^-_\mu = \left(\bar{d}_L, \bar{s}_L, \bar{b}_L\right) \gamma_\mu V^\dagger \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}$$

$$J^+_\mu = \Big(\bar{u}_L, \bar{c}_L, \bar{t}_L\Big)\gamma_\mu V \begin{pmatrix} \bar{d}_L \\ \bar{s}_L \\ \bar{b}_L \end{pmatrix}$$

This lead to the conclusion that weak interaction couples to the linear superposition of mass eigenstates, forcing the introduction of matrix V, called CKM.

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3×3 unitary matrix, which originates from this misalignment in flavour space of the up and down components of the $SU(2)_L$ quark doublet of the SM. In the quark mass eigenstate basis, the CKM matrix appears in the SM charged-current interaction Lagrangian

$$\mathcal{L}^{cc} = \frac{g}{2\sqrt{2}} \sum_{i,j} \bar{u}_i \gamma_\mu (1-\gamma_5)(V_{CKM})_{ij} d_j W^\mu + h.c.$$

where the quark fields are $u_i = (u, c, t)$ and $d_i = (d, s, b)$, while g is the weak coupling constant and W^μ is the field which creates the vector boson W^-. The CKM matrix can be written in terms of four real parameters and in the Wolfenstein parametrization is

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

which is an expansion in the small parameter $\lambda = sin(\theta_c)$, with θ_c being the Cabibbo angle. The expansion reflects the hierarchy of the magnitudes of the matrix elements. With four independent parameters, the unitary matrix cannot forced to be real-valued, and hence CP violation arises from the fact that the couplings for quarks and antiquarks can take different phases. The unitarity poses two important conditions:

$$\sum_j |V_{ij}|^2 = 1$$

meaning that the coupling of any *up* quark with any *down* quark is always the same, regardless of the generation. This relation was called *universality of weak interactions* by Nicola Cabibbo. And

$$\sum_k V_{ik} V^*_{jk} = 0$$

that is a bond on 3 complex number. The six vanishing combinations can be represented as triangles in a complex plane, creating the so called *unitary triangle*. Among these, the ones obtained by taking scalar products of neighboring rows or columns are nearly degenerate. The areas of all triangles are the same. Since their angles and sides are directly verifiable, a series of experiment aim to check the closure of the triangles, testing the SM and looking for CP violating effects at the same time.

Flavour physics is then the study of quarks, their spectrum and transmutations among them, and so one of its first objectives is to precisely calculate the CKM parameters, since they are not predicted by the Standard Model.
While the transitions occur at quark level, it is possible only to observe transitions among hadrons, since quarks are bound in hadrons by strong interactions. Nevertheless, this does not mean they are not accessible from the experiments, one can explore the interior of a proton in much the same way as Rutherford probed the inside of an atom, with collisions, and this is what particle accelerators are for. For example, exclusive and inclusive semileptonic B decays have been used to measure the value of $|V_{cb}|$ that is in between the two determinations $|V_{cb}|_{(ex)} = (38.7 \pm 1.1) \times 10^{-3}$ and $|V_{cb}|_{(in)} = (41.5 \pm 0.7) \times 10^{-3}$.

1.2 S-Matrix

In a real scattering or decay process, the particles only interact briefly. The initial states cannot be exact momentum eigenstates, since such states are not spatially localized, and cannot be energy eigenstates to the extent that their profiles in position space change with time. Instead, the initial particles are given by wave packets which are somewhat localized in both position and momentum, describing the relative approach.
The probability of the interaction can be factorized into the product of the probability for the particles to meet, times the probability for the reaction to occur. The former depends on the wave packet and the latter is an intrinsic property of the interaction. Suppose, then, that the complete Hamiltonian, H, is written as $H = H_0 + H_{int}$, where the terms are respectively the free Hamiltonian and the interaction one. The Hilbert space, $\mathcal{H}$, for the full system is divided into two parts, $\mathcal{H} = \mathcal{B} \bigoplus \mathcal{S}$, in which $\mathcal{S}$ are the states of the theory with Hamiltonian H_0. For example, if the system consisted of electrons and protons interacting electromagnetically, then $\mathcal{S}$ might contain freely-moving electrons and protons, but $\mathcal{B}$ might contain bound hydrogen atoms.
Let $|\alpha\rangle$ be the eigenstates of H_0 such that $H_0 |\alpha\rangle = E_\alpha |\alpha\rangle$. The wave packets of these states is [8]

$$|\phi_g\rangle = \int d\alpha \; g(\alpha) \, |\alpha\rangle$$

where $g(\alpha)$ defines an appropriate normalizable packet. It is required now the existence of an *out* state and an *in* state such that

$$\lim_{t>>T} e^{-iHt} |\phi_g\rangle\rangle_{out} = \lim_{t>>T} e^{-iH_0 t} |\phi_g\rangle$$

$$\lim_{t<<-T} e^{-iHt} |\phi_g\rangle\rangle_{in} = \lim_{t<<-T} e^{-iH_0 t} |\phi_g\rangle$$

and it is possible to define in this way the idealized scattering eigenstates of the full Hamiltonian

$$\lim_{t>>T} e^{-iHt} |\alpha\rangle\rangle_{out} = \lim_{t>>T} e^{-iH_0 t} |\alpha\rangle$$

$$\lim_{t<<-T} e^{-iHt} |\alpha\rangle\rangle_{in} = \lim_{t<<-T} e^{-iH_0 t} |\alpha\rangle$$

Any scattering event can be found from the limiting amplitude for the ideal process where the initial and final state are approximately energy eigenstates. The matrix of all possible amplitudes is

$$S_{\beta\alpha} = {}_{out}\langle\langle\beta|\alpha\rangle\rangle_{in}$$

and it is called the S-Matrix. Defining the operator S

$$\langle\beta| S |\alpha\rangle = S_{\beta\alpha}$$

and the Møller operator Ω

$$\Omega(t) = e^{iHt} e^{-iH_0 t}$$

one has

$$|\alpha\rangle\rangle_{out} = \lim_{t>>T} \Omega(t)|\alpha\rangle$$

$$|\alpha\rangle\rangle_{in} = \lim_{t<<-T} \Omega(t)|\alpha\rangle$$

In this way the S matrix becomes

$$S = \lim_{t\to\infty} \lim_{t'\to-\infty} \Omega^*(t)\Omega(t')$$

Now, the operator $\Omega^*(t)\Omega(t')$ can be re-expressed in form of a solution to a differential equation

$$i\frac{d}{dt}\Big[\Omega^*(t)\Omega(t')\Big] = e^{iH_0 t}(H - H_0)e^{-iH(t-t')}e^{-iH_0 t'} = V(t)\Omega^*(t)\Omega(t')$$

with the initial condition $\Omega^*(t)\Omega(t') = 1$

$$\Omega^*(t)\Omega(t') = 1 - i\int_{t'}^{t} d\tau V(\tau)\Omega^*(\tau)\Omega(t')$$

that has an iterative solution

$$\Omega^*(t)\Omega(t') = \sum_{n=0}^{n=\infty}(-i)^n \int_{t'}^{t} d\tau_1 \int_{t'}^{\tau_1} d\tau_2 ... \int_{t'}^{\tau_{n-1}} d\tau_n V(\tau_1)V(\tau_2)...V(\tau_n)$$

So that, using the time-ordering operator T, the manifestly Lorentz invariant S-matrix becomes

$$S = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{\infty}^{\infty} d^4x_1...d^4x_n T\Big[H_I(x_1)...H_I(x_n)\Big]$$

where H_I is the Hamiltonian density such that

$$V(t) = \int d^3x H_I(x,t)$$

Using energy and momentum eigenstates one can employ the identity

$$\langle\beta|O(x)|\alpha\rangle = \langle\beta|e^{-iPx}O(0)e^{iPx}|\alpha\rangle = e^{i(p_\alpha - p_\beta)x}\langle\beta|O(0)|\alpha\rangle$$

to factor an overall energy-momentum conserving factor out of the matrix

$$S_{\beta\alpha} = \delta_{\beta\alpha} - i\mathcal{M}_{\beta\alpha}(2\pi)^4\delta(p_\beta - p_\alpha)$$

The $\mathcal{M}_{\beta\alpha}$ is called the matrix element for the transition from the state α to the state β.

1.3 LSZ reduction formula

The Lehmann-Symanzik-Zimmermann reduction formula is a method to calculate Amplitudes from the time-ordered correlation functions of a quantum field theory. It will be used during the form factors derivation, so a brief explanation of this formula is in order. The expansion of a free real scalar field (this demonstration can be generalized to particles with spin) in terms of annihilation and creation operator can be inverted to give [8]

$$\frac{a_k}{(2E_k)^{\frac{1}{2}}} = i \int d^3 x e^{ikx} \overleftrightarrow{\partial_0} \phi_{free}$$

$$\frac{a_k^\dagger}{(2E_k)^{\frac{1}{2}}} = i \int d^3 x e^{ikx} \overleftrightarrow{\partial_0} \phi_{free}$$

In the limit $t \to \infty$ the theory is expected to be free, since all the particles are infinitely far away and there will be no difference between a free and a bound state.

$$\phi(x) \to Z^{\frac{1}{2}} \phi_j(x)$$

with j being the initial state if $t \to +\infty$ and the final state if $t \to -\infty$ and Z is a c-number known as function renormalization.

$$\frac{a_k^{\dagger(in)} - a_k^{\dagger(out)}}{(2E_k)^{\frac{1}{2}}} = iZ^{\frac{1}{2}} \int d^x \partial_0 (e^{-ikx} \partial_0 \phi) = \int d^4 x \Big[e^{-ikx} \partial_0^2 \phi - \phi (\nabla^2 - m^2) e^{-ikx} \Big]$$

that is the covariant form. Therefore, writing the S Matrix in the Schrödinger picture $S = 1 + i\mathcal{T}$ and using the above relations, it is possible to remove a particle with momentum k from the initial state acting with the $a_k^\dagger$ operator, inserting the integral

$$iZ^{\frac{1}{2}} \int d^4 x e^{ikx} (\Box + m^2) \phi(x)$$

Iterating the procedure

$$\langle p_1 ... p_n | \, i\mathcal{T} \, | k_1 ... k_n \rangle = (iZ^{-\frac{1}{2}})^{n+m} \int \prod_{i=1}^{m} d^4 x_i \prod_{j=1}^{n} d^4 y_i e^{[i \sum_{j=1}^{n} p_j y_j - i \sum_{i=1}^{m} k_i x_i]} (\Box_{x_1} + m^2) ...$$

$$... (\Box_{y_n} + m^2) \langle 0 | \, T \big[\phi(x_1) ... \phi(y_n) \big] \, | 0 \rangle$$

Defining the N-point Green's function

$$G(x_1 ... x_n) = \langle 0 | \, T \big[\phi(x_1) ... \phi(x_n) \big] \, | 0 \rangle$$

and using the property

$$(\Box_{x_j} + m^2)G(x_1...x_n) = -\int \prod_{i=1}^{n} \frac{d^4k_i}{(2\pi)^4}(k_j^2 - m^2)e^{-i\sum_{i=1}^{n} x_i k_i}\tilde{G}(k_1...k_n)$$

the LSZ formula is

$$\prod_{i=1}^{m}\int d^4x_i e^{-ik_ix_i}\prod_{j=1}^{n} d^4y_j e^{-ik_jy_j}\langle 0|\, T\left[\phi(x_1)...\phi_(y_n)\right]|0\rangle$$
$$= (\prod_{i=1}^{m}\frac{i\sqrt{Z}}{k_i^2 - m^2})(\prod_{j=1}^{n})\langle p_1...p_n|\, i\mathcal{T}\,|k_1...k_n\rangle$$

that is a relation between the scattering amplitude and the vacuum expectation value of a time-ordered product of fields.

1.4 Form Factors

Form factors are a common and comfortable way to bypass the issue that is not possible to exactly solve pieces of the Amplitude needed to compute the decay width. The form factors of a general local operator are defined in terms of its matrix elements between single-particle states. Thus, if we write [10]

$$\langle B(p')|\, \mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(0)\, |A(p)\rangle = \bar{u}_B(p')\mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(p,p')u_A(p)$$

where $\bar{u}_A$ and u_B are the wave functions in momentum space of the particles A and B, $\mathcal{O}(p,p')$ is the most general tensor that can be constructed out of the momenta of the two particles and making use of the LSZ reduction formula

$$\langle B(p')|\, \mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(0)\, |A(p)\rangle = \mp\bar{u}_B(p')\int d^4x e^{-ip'x}\, \langle 0|\, T\mathcal{J}_B(x)\mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(0))\, |A(p)\rangle$$

$$\langle B(p')|\, \mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(0)\, |A(p)\rangle = \mp\bar{u}_B(p')\int d^4x e^{-iqx}\, \langle 0|\, T\big[\mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(x)\mathcal{J}_B(0)\big]\, |A(p)\rangle$$

Here $\mathcal{J}_B$ is the source of the particle B and $q = p - p'$. Now p' is considered at his mass-shell value. Working on the rest frame of particle A a list of kinematic relations comes up

$$q_0 = -\frac{1}{m_A}qp = -\frac{1}{2m_A}\Big(q^2 - m_A^2 + m_B^2\Big)$$

$$|q| = \sqrt{q^2 + q_0^2} = \frac{1}{2m_A}\Big|\Big(q^2 - m_A^2 + m_B^2\Big)\Big|\Big[1 + \frac{4m_A^2q^2}{\Big(q^2 - m_A^2 + m_B^2\Big)^2}\Big]^{\frac{1}{2}}$$

An important theoretical progress comes in help from the QCD sum rules, that allows to evaluate certain operators sandwiched between two hadron states by studying the correlation functions of the type [7]

$$\int d^4y d^4x e^{-iqy}e^{-iqx}\, \langle 0|\, T\big[\mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(x)\mathcal{J}_B(0)\big]\, |0\rangle \approx \langle 0|\, \mathcal{O}_1\, |0\rangle\, \frac{1}{m_A^2 - p_A^2}\, \langle B|\, J\, |A\rangle\, \frac{1}{m_B^2 - p_B^2}\, \langle B|\, \mathcal{O}_2\, |0\rangle$$

In the limit $q^2 \to \infty$ the dominant behavior of the integral depend on the region where

$$\langle 0|\, T[\mathcal{O}_{\alpha_1\alpha_2\ldots\alpha_j}(x)\mathcal{J}_B(0)]\, |A(p)\rangle = \mathcal{H}_{\alpha_1\alpha_2\ldots\alpha_j}(x,p)u_A(p)$$

In agreement with the postulate that the light-cone is the place where the variation is the most rapid (because the function is singular there), then for $q^2 \to \infty$ the behavior depends on $\mathcal{H}_{\alpha_1\alpha_2...\alpha_j}(x,p)u_A(p)$ near $x^2 = 0$. Using the light-cone expansion, i.e. the idea that near $(x-y)^2 = 0$ it is possible to write the product of any two operators as a sum of terms, consisting of a c-number function $\mathcal{C}^n_{\alpha_1\alpha_2...\alpha_j}(x-y)$ containing all the singularities and a bilocal regular operator $O^n(x,y)$, the product becomes

$$T[\mathcal{O}_{\alpha_1\alpha_2...\alpha_j}(x)\mathcal{J}_B(0)] = \sum_n \mathcal{C}^n_{\alpha_1\alpha_2...\alpha_j}(x-y)O^n(x,y)$$

thus with $x^2 \to 0$

$$\mathcal{H}_{\alpha_1...\alpha_j}(x,p) = \sum_n \mathcal{C}^{(n)}_{\alpha_1...\alpha_j}(x)\langle 0|\, O^{(n)}(x,0)\, |A(p)\rangle = \sum_n \mathcal{C}^{(n)}_{\alpha_1...\alpha_j}(x,p)g^{(n)}(x,p)u_A(p)$$

The last equation can be viewed as a representation of $\mathcal{H}_{\alpha_1...\alpha_j}(x,p)$ near the light-cone as a sum of its singularities multiplied for their respective residues $g^{(n)}$. The operators are actually restricted to the set of the needed ones for the theory: unit operator, scalar and pseudoscalar operators, vector and axial operators, symmetric and antisymmetric tensor operators.

In short, the method begins with the LSZ formula, writing the matrix element as an integral, then with the aid of the QCD sum rules one can make a connection between the matrix element and correlation functions and as a last step, it is possible to expand the correlation functions with an operator product expansion.

In the case of $B \to D/D^*$ decays, the correlation function of two quark currents taken between the vacuum and the on-shell B meson state is written as [15].

$$F(p,q) = i\int d^4x e^{ipx}\langle 0|\, T\bar{d}\Gamma_a c, \bar{c}\gamma_\mu(1-\gamma_5)b\, |B(p_B)\rangle$$

Where $\Gamma_a = m_c i\gamma_5$ for the D meson and $\Gamma_a = \gamma_\mu$ for D^* meson and the B meson momentum being on-shell, $p_B = (p+q)^2$. The correlation function is related to the form factors of interest via the hadronic dispersion relation, such that

$$F(p,q) = \frac{\langle 0|\, \bar{d}\gamma_a c\, |D^*(p)\rangle \langle D^*(p)|\, \bar{c}\gamma_\mu(1-\gamma_5)b\, |B(p+q)\rangle}{m^2_{D^*} + p^2} + ...$$

The right hand side of the equation contains decay constants (where the ellipses indicate excited states contributions) that are determined by the standard definitions of the form factors in the $B \to D$ case

$$\langle D(p)|\, c\gamma_\mu b\, \left|\bar{B}(p+q)\right\rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)q^\mu$$

And $B \to D^*$

$$\langle D^*|\bar{c}\gamma_\mu(1-\gamma_5)b|\bar{B}\rangle = -\frac{2V(q^2)}{m_B+m_D^*}i\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p_B^\alpha p_{D^*}^\beta - \Bigg\{(m_B+m_D^*)\Big[\epsilon_\mu^* - \frac{\epsilon^*\cdot q}{q^2}q_\mu\Big]A_1(q^2)$$

$$-\frac{\epsilon^*\cdot q}{(m_B+m_D^*)}\Big[(p_B+p_D^*)_\mu - \frac{m_B^2-m_{D^*}^2}{q^2}q_\mu\Big]A_2(q^2) + (\epsilon^*\cdot q)\frac{2m_{D^*}}{q^2}q_\mu A_0(q^2)\Bigg\}$$

Due to the fact that the D^* state is a vector state one has to take into account the polarisation vector ϵ^* and a set of new form factors.

In the case of $\bar{B} \to D$ it is possible to demonstrate that the form factors are real functions. Let T be the anti-unitary time reversal operator that, given two states $|a\rangle$ and $|b\rangle$, is defined by

$$\langle Ta|Tb\rangle = \langle a|b\rangle^*$$

Since the mesons are fully characterized by their momenta, the time reversed states describe particles with momenta $p'^\mu = (p^0, -\vec{p}) = \eta^{\mu\mu}p^\mu$ (with no sum over the index). The parametrization must hold also in the time-reversed frame

$$\eta^{\mu\mu}\Big(f_1(q^2)\eta^{\mu\mu}p^\mu + f_2(q^2)\eta^{\mu\mu}q^\mu\Big) = \Big(f_1(q^2)p^\mu + f_2(q^2)q^\mu\Big)^*$$

where on the left-hand side the overall factor $\eta^{\mu\mu}$ comes from the transformation of the current. Since $(\eta^{\mu\mu})^2 = 1$

$$f_1(q^2)p^\mu + f_2(q^2)q^\mu = \Big(f_1(q^2)p^\mu + f_2(q^2)q^\mu\Big)^*$$

1.5 Heavy Quark Symmetry

The strong interactions of systems containing heavy quarks are easier to understand than those of systems containing only light quarks. The effective coupling constant of QCD becomes weak in processes with large momentum transfer, corresponding to interactions at short distance scales. This is a unique feature of nonabelian gauge theories. When the mass of a quark Q is much larger than the scale of Λ_{QCD} then it is called an heavy quark. So, u, d and s are light quarks, whereas c, b and t are heavy. For them, the effective coupling constant is small, implying that on length scales comparable to the Compton wavelength, the strong interactions are perturbative and much like the electromagnetic ones. In fact, the quarkonium system $\bar{Q}Q$ is very much hydrogen-like. After the discovery of asymptotic freedom, their properties could be predicted before the observation of charmonium and bottonium states. For systems composed of an heavy quark and a light constituent, the heavy quark is surrounded by a complicated, strongly interacting cloud of light quarks, antiquarks and gluons. This cloud is known as *brown muck*, a term invented by Nathan Isgur to emphasize the fact that the properties of such systems cannot be calculated from first principles.
In the limit of $m_Q \to \infty$ in the hadron's rest frame, the heavy quark is at rest too. The wave function of the *brown muck* follows from QCD field equations with the boundary conditions of a static source at the location of the heavy quark. In this way, hadronic systems which differ only by the flavour or spin quantum number of the heavy quark have the same configuration.
Heavy Quark Symmetry is an approximate symmetry, and corrections of order $\frac{\Lambda_{QCD}}{m_Q}$ arise, but the condition $m_Q \gg \Lambda_{QCD}$ is sufficient to be very close to an actual symmetry. However, this is not a symmetry for the Lagrangian, but rather a symmetry of an effective theory. In the present context, only the transitions $b \to c + x$ qualify for this limit.
The velocity of the heavy quark is conserved with respect to soft processes and it is possible to remove the mass dependence from the momentum operator by a field redefinition. States are now characterized by their four-velocity v and v' and are related by an $SU(2)_f$ rotation at $vv' = 1$ (zero recoil of the D or D^*). The matrix element can be written with the Wigner-Eckart relation

$$\langle D(v')| \, J_\mu^{hadronic} \left|\bar{B}\right\rangle = \xi(vv')C(\Gamma)$$

where $C(\Gamma)$ is a Clebsch-Gordon coefficient (calculable for each $\Gamma = 1, \gamma_\mu, \gamma_5\gamma_\mu, ...$) and $\xi(vv')$ is a non-perturbative function called the Isgur-Wise function. So, the matrix element, in a heavy to heavy decay, is reduced, in HQ symmetry, to just one function. This approach can be used to calculate the V_{cb} element of the CKM matrix. The Isgur-Wise function is obtained from the analysis of a three-current-correlator, giving a pole-type function [27]

$$\xi(w) \approx \left(\frac{2}{w+1}\right)^{\beta(w)}$$

$$\beta(w) = 2 + \frac{0.6}{w}$$

For the semileptonic B decay, it is convenient to define a set of heavy-meson form factors $h_i(w)$, where it is used the variable w instead of the momentum transfer squared q^2, such that

$$w = v \cdot v' = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$

$$\langle D(v')|V_\mu|\bar{B}(v)\rangle = \sqrt{m_B m_D}\Big[h_+(w)(v+v') + h_-(w)(v-v')_\mu\Big]$$

$$\langle D^*(v')|V_\mu|\bar{B}(v)\rangle = i\sqrt{m_B m_{D^*}} h_V(w)\epsilon_{\mu\nu\alpha\beta}\epsilon^* {v'}^\alpha v^\beta$$

$$\langle D^*(v')|A_\mu|\bar{B}(v)\rangle = \sqrt{m_B m_{D^*}}\Big[h_{A_1}(w)(w+1)\epsilon^*_\mu - h_{A_2}(w)\epsilon^* \cdot v v_\mu - h_{A_3}(w)\epsilon^* \cdot v v'_\mu\Big]$$

where $V_\mu = \bar{c}\gamma_\mu b$ and $A_\mu = \bar{c}\gamma_\mu\gamma_5 b$ and ϵ_μ is the polarization vector of the D^*. These h_i form factors represent a linear combinations of the initial form factors but are more clearly related to the Isgur-Wise function, in fact in the infinite quark mass limit

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)$$

$$h_-(w) = h_{A_2}(w) = 0$$

Chapter 2

Theory of B decay

The B mesons are bound states of the quark $\bar{b}$, antiparticle of the b, in particular they are $B^+(u\ \bar{b})$, $B^0(d\ \bar{b})$, $B_s(s\ \bar{b})$ and $B_c(c\ \bar{b})$. While the binding is provided by the strong interaction, the decays can happen only through weak interactions. Since the b quark is the lighter element of the third generation, the decays of b-flavoured hadrons produce generation changing processes. Because of this, many interesting features such as loop and box diagrams, flavour oscillations, as well as large CP asymmetries, can be observed. Furthermore, the decays allow to extract the CKM parameters, $V_{cb}, V_{ud}, V_{ts}, V_{tb}$.
Semileptonic transitions are the simplest ones. The heavy b quark goes either into a c or a u, while the virtual W boson becomes a lepton pair. Experimentally, they have the advantage of large branching ratios and the characteristic signature of the energetic charged lepton. When it goes into a c the meson formed is called D meson. In particular: $D^+(c\ \bar{d})$, $D^-(\bar{c}\ d)$, $D^0(c\ \bar{u})$,$D_s^+(c\ \bar{s})$, $D_s^-(\bar{c}\ s)$, with their respective D^* as excited states. In this thesis, the focus is on the decay channels $\bar{B}^0 \to D^{*+}l^-\bar{\nu}$ and $\bar{B}^0 \to D^+l^-\bar{\nu}$, which have deviations in the decay ratios, with respect to the standard model predictions, that are motivating studies on possible new physics.
However, making those predictions in the Standard Model perspective is not an easy task, even if the $b \to c$ channel is less affected by QCD corrections cause leptons do not interact strongly and the theoretical calculations are believed to be more reliable.
First of all, one needs a suitable lagrangian for the problem, from which construct the decay amplitude that gives the decay rate (i.e. the probability per unit time that the particle will decay). But there are some issues; the lagrangian is known in the Standard Model, but to incorporate new physics effects one has to add new pieces, based on reasonable assumptions on what is missing in the Standard Model, like a new particle.

2.1 Effective Lagrangian

At low energy the most general approach is given by the effective lagrangian. The intuitive idea behind effective theories is that you can make calculations without knowing the exact theory. The most famous one is the Fermi theory of weak interactions, the first field theory in which the processes are described in terms of annihilation and creation of particles. Valid for energies below the W and Z bosons masses, it describes a point-like interaction weighted by the Fermi constant, borrowing the QED formalism. With the discovery of parity violation by madame Wu, in her famous experiment on the β-decay of the Cobalt-60, the theory evolved modifying the current to involve parity violation, taking the modern V-A form. Since $\bar{B}$ decay is a weak decay, this formalism will be used.
So, one has to establish the form of the local operators of the appropriate dimension, which might explain the observed anomalies and then searches for a suitable NP model. As used in many works, to account for NP one can simply add a tensor-like term in the SM Lagrangian. The effective lagrangian for the general $\bar{B}$ decay, assuming the SM neutrino, becomes [11]

$$\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{cb} \Big[(\bar{c_L}\gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \nu_L) + \epsilon_T^\ell (\bar{c}\sigma_{\mu\nu} b_L)(\bar{\ell}\sigma^{\mu\nu}\nu_L) \Big] + h.c.$$

where V_{cb} is the component of the CKM matrix for the quark mixing, ϵ_T^ℓ is a complex lepton-flavour dependent parameter weighting the operator $\sigma_{\mu\nu}$, the subscript Ψ_L means the left part i.e. $\Psi_L = \frac{(1-\gamma_5)}{2}\Psi$, G_F is the well known Fermi constants and $h.c.$ stands for hermitian conjugate. The Feynman diagram describing the decay is shown in FIG.1
Now, it is possible to write the amplitude $\mathcal{A}$ for the $\bar{B} \to D$ transition using the relation $\mathcal{L}_{eff} = -\mathcal{H}_{eff}$

$$\mathcal{A} = \left\langle D | \mathcal{H}_{eff} | \bar{B} \right\rangle$$

$$\mathcal{A} = \frac{4G_F}{\sqrt{2}} V_{cb} \left\langle D \right| \Big[(\bar{c}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu \nu_L) + \epsilon_T^\ell (\bar{c}\sigma_{\mu\nu} b_L)(\bar{\ell}\sigma^{\mu\nu}\nu_L) \Big] \left| \bar{B} \right\rangle$$

From the rules of quantum mechanics, it is known that the probability for this process is obtained by taking the squared modulus of the amplitude and summing over all the possible values of the momenta of the final state particles. The decay rate is therefore

$$d\Gamma = \frac{1}{2E_p} |\mathcal{A}|^2 \, d\Phi^{(n)}$$

where $d\Phi^{(n)}$ is the differential n-body phase space

$$d\Phi^{(n)} = (2\pi)^4 \delta^{(4)}(P_i - P_f) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

This approach is suitable for the μ decay, but cannot be used when quark states appear. Indeed, hadronic decays are not solvable cause one cannot exactly compute the hadronic current part of the amplitude. Furthermore, one does not know how to correctly write the quark bound state.
The trick is to parametrize all the ignorance on certain functions, the form factors as shown previously, and then study these functions to get fair approximation of the amplitude. Since the knowledge of those functions is quite poor, one knows the poles and the cut, and the general behavior, it is usually done a simplification to enhance implicitly or explicitly the known features of the form factors. These simplifications are the parametrisation.

Recalling the expression of the amplitudes of the $\bar{B}$ meson decays, decomposed in terms of the Lorentz invariant hadronic form factors [4]

$$\langle D(p')|\, c\gamma_\mu b \,|\bar{B}(p)\rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)q^\mu$$

and inserting the contribution of the tensor operator for the new physics part, obtaining a new form factor

$$\langle D(p')|\, c\sigma_{\mu\nu} b \,|\bar{B}(p)\rangle = -i\Big(p_\mu p'_\nu - p'_\mu p_\nu\Big)\frac{2f_T(q^2,\mu)}{m_B+m_D}$$

The differential rate can be written, in the Standard Model framework, as [4]

$$\frac{d\Gamma}{dq^2} = \frac{\eta_{new} G_F^2 \left|V_{cb}\right|^2 m_b \lambda^{\frac{1}{2}}}{192\pi^3}\Big(1-\frac{m_l^2}{q^2}\Big)\Big[c_+^l f_+^2(q^2) + c_0^l f_0^2(q^2)\Big]$$

where

$$r = \frac{m_D}{m_B}, \lambda = \Big(q^2 - m_B^2 - m_D^2\Big)^2 - 4m_B^2 m_D^2$$

$$c_+^l = \frac{\lambda}{m_B^4}\Big(1+\frac{m_l^2}{2q^2}\Big)$$

$$c_0^l = (1-r^2)^2\frac{3m_l^2}{2q^2}$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2}f_-(q^2)$$

In the limit of vanishing lepton mass the $f_0(0) = f_+(0)$ and its contribution becomes irrelevant. Indeed, it can always be neglected except for the decays into τ lepton. The factor $\eta_{new} = 1 + \frac{\alpha}{\pi}ln(\frac{M_z}{m_B}) \approx 1,0066$ takes into account short QED corrections. One can now parametrise the form factors to have a better access to their values. The main parametrisations are the BGL (Boyd-Grinstein-Lebed) and the CLN (Caprini-Lellouch-Neubert).

2.1.1 CLN parametrisation

The CLN parametrisation is the most used in literature and is based on the Heavy Quark Effective Theory (HQET). The form factors of the two-meson states contributing to the two-points function are related by heavy quark symmetry, i.e. the b quark is much more massive than is bounded light partner (like a $\bar{d}$) such that it behaves always in the same way, acting like a static colour source, regardless of the flavour of the partner. In the heavy quark limit, the form factors either vanish or are proportional to the universal Isgur-Wise function. This provides a simple formula valid within $\approx 2\%$.

$$f_+(z) = f_+(0)\Big[1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3)\Big]$$

$$\frac{f_0(z)}{f_+(z)} = \frac{4r^2}{(1+r)^2}\frac{1+w}{2}1.0036\Big[1 - 0.0068(w-1)^2 + 0.0017(w-1)^2 - 0.0013(w-1)^3\Big]$$

The ratio $\frac{f_0(z)}{f_+(z)}$ is fixed by the NLO (Next to Leading Order) HQET calculation. Any other form factor can be expressed in terms of $f_+(z)$ times a ratio computed at NLO in HQET.

2.1.2 BGL parametrisation

The BGL parametrisation relies on hadronic matrix elements and analycity. In the case of semileptonic B decays q^2 ranges from m_l^2 to $(m_B - m_D)^2$ but the form factors can be analytically extended in the q^2 complex plane. It is possible to define a new variable z such that

$$\frac{1+z}{1-z} = \sqrt{\frac{(m_B + m_D)^2 - q^2}{4m_B m_D}}$$

The change of variables $q^2 \to z$ maps the two sides of $q^2 > (m_B + m_D)^2$ to the unit circle $|z| = 1$, with the rest of the q^2 plane mapped to the interior of the unit circle. Calculations from perturbative QCD give the inequality on the contour C as the unit circle

$$\frac{1}{2\pi i}\int_C \frac{dz}{z}\Big|\phi_i(z)f_i(z)\Big|^2 \leq 1$$

where ϕ_i are weighting functions defined by [6]

$$\phi_i = m_B^{2-s}2^{2+p}\sqrt{kn_f}\Big[r(1+z)\Big]^{\frac{p+1}{2}}(1-z)^{\frac{s-3}{2}}\Big[(1-z)(1+r)+2\sqrt{r}(1+z)\Big]^{-s-p}$$

here $r = \frac{m_D}{m_B}$ and k, p, and s are parameters that depend on the form factors f_i (one can find in the literature a table with their values). One may form functions $P(z)$ that are products of terms of the form $\frac{(z-z_i)}{1-\bar{z}_i z}$ know as Blaschke factors

$$P_0 = P_1 = \prod_{j=5}^{8}\frac{z-z_j}{1-\bar{z}_j z}$$

$$P_2 = P_3 = \prod_{j=1}^{4}\frac{z-z_j}{1-\bar{z}_j z}$$

These are analytic on the unit disk $|z| \leq 1$ and serve to eliminate poles of f_i at each $z = z_j$

Taylor expanding $\phi_i P_i f_i$ about $z = 0$

$$f_i = \frac{1}{P_i(z)\phi_i(z)}\sum_{n=0}^{\infty} a_n z^n$$

with the condition

$$\sum_{n=0}^{\infty} |a_n|^2 \leq 1$$

The coefficients a_n are different for each form factor and must be determined by experiment. The accuracy of this parametrisation depends on when one decides to cut the series and in particular by truncating after N terms

$$max\Big|f_i(z) - f_i^N(z)\Big| = \frac{1}{P_i(z)\phi_i(z)}\sum_{n=N+1}^{\infty}|a_n|\, z^n \leq \frac{1}{P_i(z)\phi_i(z)}\frac{z_{max}^{N+1}}{\sqrt{1-z^2}}$$

2.2 Decay Width Calculation

2.2.1 $\bar{B}$ to D

It is worth now to explicitly calculate the decay width, from which it will be possible to have access to observable quantities. As already claimed, the amplitude is written

$$\mathcal{A} = \langle D|\mathcal{H}_{eff}|\bar{B}\rangle$$

$$\mathcal{A} = \langle D|\mathcal{H}_{SM} + \mathcal{H}_{NP}|\bar{B}\rangle$$

$$\mathcal{A} = \frac{4G_F}{\sqrt{2}} V_{cb} \langle D| \left[(\bar{c}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu \nu_L) + \epsilon_T^\ell (\bar{c}\sigma_{\mu\nu} b_L)(\bar{\ell}\sigma^{\mu\nu}\nu_L)\right] |\bar{B}\rangle$$

For the Standard Model part

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb} \langle D| (\bar{c}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu \nu_L) |\bar{B}\rangle$$

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb} \langle D| \bar{c}\gamma_\mu (1-\gamma_5) b \bar{\ell}\gamma^\mu (1-\gamma_5)\nu |\bar{B}\rangle$$

In the interaction picture, the evolution operator is $-i\int d^4x \mathcal{H}_{int}$ and thus the matrix element between an initial $|i\rangle$ and final $|f\rangle$ state is $-i\int d^4x \langle f| \mathcal{H}_{int} |i\rangle$. Using the space-time translation operator $\hat{P}^\mu$

$$\mathcal{H}_{int} = e^{i\hat{P}x}\mathcal{H}_{int}(0)e^{-i\hat{P}x}$$

$$-i\int d^4x \langle f| e^{i\hat{P}x}\mathcal{H}_{int}(0)e^{-i\hat{P}x} |i\rangle = -i\int d^4x e^{i(\hat{P}_f - \hat{P}_i)x} \langle f| \mathcal{H}_{int}(0) |i\rangle$$

$$= -i(2\pi)^4 \delta^{(4)}(P_f - P_i) \langle f| \mathcal{H}_{int}(0) |i\rangle$$

The factor $-i(2\pi)^4\delta^{(4)}(P_f - P_i)$ is reabsorbed in the definition of the matrix element such that it is defined only by the hamiltonian density evaluated at $x = 0$.

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb} \langle D| \left[\bar{c}\gamma_\mu (1-\gamma_5) b\right](0) \left[\bar{\ell}\gamma^\mu (1-\gamma_5)\nu\right](0) |\bar{B}\rangle$$

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb} \langle D| \left[\bar{c}\gamma_\mu (1-\gamma_5) b\right](0) |\bar{B}\rangle \langle \ell\nu| \left[\bar{\ell}\gamma^\mu (1-\gamma_5)\nu)\right](0) |0\rangle$$

The leptonic part is computed by [23]

$$\langle \ell\nu| \left[\bar{\ell}\gamma^\mu (1-\gamma_5)\nu\right](0) |0\rangle = \bar{u}(\ell)\gamma^\mu (1-\gamma_5) v(\nu_\ell)$$

where u and v are the spinor wave functions. The hadronic part has to be factorized. Since B and D are spin 0 particles, the meson state is described only by its four-momentum p^μ and since they have the same intrinsic parity (both pseudoscalars) only the vector current contributes to the matrix element. Therefore it is possible to write

$$\langle D|\, \bar{c}\gamma_\mu(1-\gamma_5)b \left|\bar{B}\right\rangle = \langle D|\, \bar{c}\gamma_\mu b \left|\bar{B}\right\rangle = f_+(q^2)p_\mu + f_-(q^2)q_\mu$$

The form factors depend uniquely by q^2 because is the only Lorentz invariant that can be constructed with $p^\mu = p_B^\mu - p_D^\mu$ and $q^\mu = p_B^\mu + p_D^\mu$. Hence

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[\bar{u}(\ell)\gamma_\mu(1-\gamma_5)v(\nu_\ell)\Big]\Big(f_+(q^2)p^\mu + f_-(q^2)q^\mu\Big)$$

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[\bar{u}(\ell)\gamma_\mu(1-\gamma_5)v(\nu_\ell)\Big]\Big(f_+(q^2)(p_B^\mu + p_D^\mu) + f_-(q^2)(p_B^\mu - p_D^\mu\Big)$$

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[\bar{u}(\ell)\gamma_\mu(1-\gamma_5)v(\nu_\ell)\Big]\Big(f_+(q^2)(2p_B^\mu - p_\ell^\mu - p_\nu^\mu) + f_-(q^2)(p_\ell^\mu - p_\nu^\mu\Big)$$

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[2p_B^\mu f_+\Big(\bar{u}\gamma_\mu(1-\gamma_5)v\Big) - \Big(f_+ - f_-\Big)\bar{u}(1+\gamma_5)\not{p}_\nu^\mu v - \Big(f_+ - f_-\Big)\bar{u}\not{p}_\ell^\mu(1-\gamma_5)v\Big]$$

Using the Dirac equation and assuming the SM neutrino ($m_\nu = 0$)

$$\mathcal{A}_{SM} = \frac{4G_F}{\sqrt{2}} V_{cb}\Big[2p_B^\mu f_+\Big(\bar{u}\gamma_\mu(1-\gamma_5)v\Big) + \Big(f_+ - f_-\Big)i\hbar m_\ell \bar{u}(1+\gamma_5)v\Big]$$

If there is no NP it is possible to compute $\mathcal{A}_{SM}$ in the limit $m_\ell = 0$ and in the B rest frame, summing over the polarization of the final lepton [23]

$$\Big(\bar{u}\gamma_\mu(1-\gamma_5)v\Big)^\dagger = \bar{v}\gamma_\mu(1+\gamma_5)u$$

$$u\bar{u} \to (\not{p}_\ell + m_l)$$

$$v\bar{v} \to \not{p}_{\nu_\ell}$$

$$|\mathcal{A}_{SM}|^2 = 32G_F^2|V_{cb}|^2 p^\mu p^\nu \mathbf{Tr}[\not{p}_{\nu_\ell}\gamma_\mu(1-\gamma^5)(\not{p}_\ell + m_\ell)(1+\gamma^5)\gamma_\nu]$$

$$|\mathcal{A}_{SM}|^2 \propto |V_{cb}|^2[2(pp_{\nu_\ell})(pp_\ell) - p^2(p_\ell p_{\nu_\ell})]$$

$$d\Phi^{(3)} = \frac{1}{32\pi^3} dE_\ell dE_D$$

$$\Gamma \approx G_F^2|V_{cb}|^2 m_B^2(1 - \frac{m_D^2}{m_B^2})$$

For the NP part instead

$$\mathcal{A}_{NP} = \frac{4G_F}{\sqrt{2}} V_{cb} \langle D | \, \epsilon_T^\ell (\bar{c}\sigma_{\mu\nu} b_L)(\bar{\ell}\sigma^{\mu\nu}\nu_L) \left| \bar{B} \right\rangle$$

and using the same procedure

$$\mathcal{A}_{NP} = \frac{4G_F}{\sqrt{2}} V_{cb}\epsilon_T^\ell \Big[-i \Big(p_\mu^B p_\nu^D - p_\mu^D p_\nu^B \Big) \frac{2 f_T(q^2,\mu)}{m_B + m_D} \Big(\bar{u}\sigma^{\mu\nu}(1-\gamma_5) v \Big) \Big]$$

the factor $p_\mu^B p_\nu^D - p_\mu^D p_\nu^B$ can be rearranged as

$$p_\mu^B p_\nu^D - p_\mu^D p_\nu^B = p_\mu^B \Big[p_\nu^B - (p_\nu^\ell + p_\nu^\nu) \Big] - \Big[p_\mu^B - (p_\mu^\ell + p_\mu^\nu) \Big] p_\nu^B$$

thus

$$\mathcal{A}_{NP} = \frac{4G_F}{\sqrt{2}} V_{cb}\epsilon_T^\ell \Big[i \frac{f_T}{m_B + m_D} \bar{u} \Big(p_B^\mu p_\ell^\nu - p_B^\nu p_\ell^\mu \Big) \sigma_{\mu\nu}(1-\gamma_5) v \Big]$$

now,

$$\mathcal{A} = \mathcal{A}_{SM} + \mathcal{A}_{NP}$$

$$|\mathcal{A}|^2 = \mathcal{A}\mathcal{A}^\dagger$$

$$|\mathcal{A}|^2 = |\mathcal{A}_{SM}|^2 + |\mathcal{A}_{NP}|^2 + \mathcal{A}_{SM}\mathcal{A}_{NP}^\dagger + \mathcal{A}_{NP}\mathcal{A}_{SM}^\dagger$$

assuming $\sigma_{\mu\nu} = \sigma_{\mu\nu}^\dagger = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ summing over the spins and calling $\mathcal{K}_\nu = m_B^2 p_\nu^D - p_B^\mu p_\mu^D p_\nu^B$

$$\mathcal{A}_{SM}\mathcal{A}_{NP}^\dagger = \frac{C}{m_B + m_D} f_+ f_T \epsilon_T^\ell \mathcal{K}_\nu \mathbf{Tr} \Big[\not{p}_{\nu_\ell} \gamma_\mu (1-\gamma^5)(\not{p}_\ell + m_l)\sigma^{\mu\nu}(1+\gamma^5) \Big]$$

but the in the trace only terms with even number of γ survive, then $\mathbf{Tr}[...] = 0$. Thus

$$|\mathcal{A}|^2 = |\mathcal{A}_{SM}|^2 + |\mathcal{A}_{NP}|^2$$

$$|\mathcal{A}_{NP}|^2 \propto \mathbf{Tr} \Big[\not{p}_\nu \sigma_{\mu\nu}(1-\gamma_5)(\not{p}_\ell + m_\ell)\sigma^{\mu\nu}(1+\gamma_5) \Big]$$

that is null for the same reason. So, this kind of NP does not affect the $\bar{B} \to \bar{D}$ channel. In truth, the trace has a dependence on the dimension d of the space-time

$$\mathbf{Tr}[...] \propto (4 - 5d + d^2) P_\mu^\ell P_\nu^\mu$$

such that is zero only for $d = 4$ and $d = 1$.
To see the footprints of NP is much better to investigate the $\bar{B} \to \bar{D}^*$, even because one has more variables (e.g. polarization of the $\bar{D}^*$) to play with.

2.2.2 $\bar{B}$ to D^*

The $\bar{B} \to D^*$ case is conceptually similar to the previous one but the hadronic part is parametrised with 4 form factors due to the polarization of the D^*. The $B \to D^* l \nu_l$ decay, with the subsequent D^* $D\pi$ decay, can be described by three angular variables and $q^2 = (p_B - p_{D^*})^2$, where p_B and p_{D^*} are the four-momenta of the B and D^* mesons, respectively. The three angular variables, indicated in Figure 2.1, are two helicity angles θ_l and θ_V, and the angle χ. The angle between the direction of the muon and the direction opposite to that of the B in the virtual W rest frame is called θ_l, while the angle between the direction of the D and the direction of the B in the D^* rest frame is called θ_V. Finally, χ is the angle between the two planes formed by the virtual W and D^* decay products in the B rest frame. The kinematics is represented in figure 2.1 .

The amplitude of the process is therefore [11]

$$\mathcal{A}(\bar{B} \to D^* \to D) = \mathcal{A}(\bar{B} \to D^*) \frac{i}{p_{D^*}^2 - m_{D^*}^2 + i m_{D^*} \Gamma(D^*)} \mathcal{A}(D^* \to D)$$

and the effective hamiltonian is the same as above. Being a transition between a pseudoscalar and a vector, the hadronic part is parametrised as follows

$$\langle D^* | \bar{c} \gamma_\mu (1 - \gamma_5) b | \bar{B} \rangle = - \frac{2V(q^2)}{m_B + m_D^*} i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_B^\alpha p_{D^*}^\beta - \Big\{ (m_B + m_D^*) \Big[\epsilon_\mu^* - \frac{\epsilon^* \cdot q}{q^2} q_\mu \Big] A_1(q^2)$$

$$- \frac{\epsilon^* \cdot q}{(m_B + m_D^*)} \Big[(p_B + p_D^*)_\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q_\mu \Big] A_2(q^2) + (\epsilon^* \cdot q) \frac{2 m_{D^*}}{q^2} q_\mu A_0(q^2) \Big\}$$

Where V and A are vector and axial for factors, ϵ is the Levi-Civita symbol and ϵ_μ^* is the polarization vector. Hence, the complete hadronic matrix element is written as a sum of

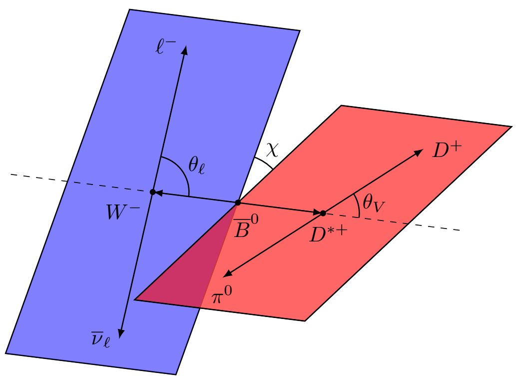

Figure 2.1: Schematic overview of the $B \to D^* l \nu_l$ decay, introducing the angles θ_V, θ_l and χ.

these terms (with the correct sign) with the condition

$$A_0(0) = \frac{m_B + m_{D^*}}{2m_{D^*}} A_1(0) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2(0)$$

$$\langle \bar{D}^* | \bar{c}\sigma_{\mu\nu}(1-\gamma_5) b | \bar{B} \rangle = -T_0(q^2) \frac{\epsilon^* \cdot q}{(m_B + m_D^*)^2} \epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{D^*}^\beta + T_1(q^2) \epsilon_{\mu\nu\alpha\beta} p_B^\alpha \epsilon^{*\beta}$$
$$+T_2(q^2)\epsilon_{\mu\nu\alpha\beta} p_{D^*}^\alpha \epsilon^{*\beta} + i \Big[T_3(q^2)(\epsilon_\mu^* p_{B\nu} - \epsilon_\nu^* p_{B\mu})$$
$$+T_4(q^2)(\epsilon_\mu^* p_{D^*\nu} - \epsilon_\nu^* p_{D^*\mu}) + T_5(q^2) \frac{\epsilon^* \cdot q}{(m_B + m_D^*)^2} (p_{B\mu} p_{D^*\nu} - p_{B\nu} p_{D^*\mu}) \Big]$$

where T are the form factors for the NP tensor part. The coupling constant ϵ^* has been determined using $R(D)$ and $R(D^*)$ from the Belle measurement. The benchmark point $Re(\epsilon) = 0.115$ has been chosen indeed to reproduce the amount of polarized D^* reckoned by Belle.

In the rest frame of the B meson with z axis along the trajectory of the D^*, a suitable basis for the lepton pair helicities is [11]

$$\epsilon_\mu(\pm) = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0)$$

$$\epsilon_\mu(0) = \frac{1}{\sqrt{q^2}}(|\mathbf{p}|, 0, 0, -q_0)$$

$$\epsilon_\mu(t) = \frac{1}{\sqrt{q^2}}(q_0, 0, 0, -|\mathbf{p}|)$$

where $q_0 = \frac{m_B^2 - m_{D^*}^2 + q^2}{2m_B}$ and $|\mathbf{p}| = \frac{\lambda^{\frac{1}{2}}(m_B^2, m_{D^*}^2, q^2)}{2m_B}$ with $\lambda^{\frac{1}{2}}(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$. They satisfy the normalization and completeness relations

$$\epsilon_\mu^*(m)\epsilon^\mu(m') = g_{mm'} \text{ , for } (m, m' = t, \pm, 0)$$

$$\sum_{m,m'} \epsilon_\mu(m)\epsilon_\nu^*(m') g_{mm'} = g_{\mu\nu}$$

Whereas for the D^* helicity basis

$$\epsilon_\alpha(\pm) = \mp \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$$

$$\epsilon_\alpha(0) = \frac{1}{m_{D^*}}(|\mathbf{p}|, 0, 0, E_{D^*})$$

with $E_D^* = \frac{m_B^2, m_{D^*}^2}{q^2}$ the energy of the D^* in the B rest frame.

$$\epsilon^*_\alpha(m)\epsilon^\alpha(m') = -\delta_{mm'}$$

$$\sum_{mm'} \epsilon_\alpha(m)\epsilon^\beta(m')\delta_{mm'} = -g_{\alpha\beta} + \frac{p_{D^*\alpha}}{p_{D^*\beta}}$$

It is possible to introduce now the helicity amplitudes, $H_\pm, H_0$ and H_t, describing the decay of a pseudo-scalar meson into the three helicity states of a vector meson and four helicity states of the lepton pair.

$$H_{mm}(q^2) = \epsilon(m)^\mu H_\mu(m) \ , \ \text{ for } m = 0, \pm$$

$$H_t(q^2) = \epsilon(m = t)^{\mu *} H_\mu(n = 0)$$

Where $H_\mu(m)$ is the corresponding hadronic matrix element, and m,n denote helicity projections of the D^* and lepton pair in B rest frame.
In the Standard Model, the helicity amplitudes can be written as

$$\begin{aligned} H_\pm(q^2) &= (m_B + m_{D^*})A_1(q^2) \mp \frac{2m_B}{m_B + m_{D^*}}|\mathbf{p}|V(q^2) \\ H_0(q^2) &= \frac{1}{2m_{D^*}\sqrt{q^2}}\left[(m_B^2 - m_{D^*}^2 - q^2)(m_B + m_{D^*})A_1(q^2) - \frac{4m_B^2|\mathbf{p}|^2}{m_B + m_{D^*}}A_2(q^2)\right] \\ H_t(q^2) &= \frac{2m_B|\mathbf{p}|}{\sqrt{q^2}}A_0(q^2) \end{aligned}$$

in this way the differential decay rate is

$$\frac{d^2\Gamma}{dq^2 d\cos\theta} = \frac{G_F^2|Vcb|^2|\mathbf{p}|q^2}{256\pi^3 m_B^2}\left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times$$
$$\Big\{(1-\cos\theta_l)^2|H_+|^2 + (1+\cos\theta_l)^2|H_-|^2 + 2\sin^2\theta_l|H_0|^2 +$$
$$\frac{m_\tau^2}{q^2}\left[\sin^2\theta_l(|H_+|^2 + |H_-|^2) + 2|H_t - H_0\cos\theta_l|^2\right]\Big\}$$

and integrating over $d\cos\theta_l$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|Vcb|^2|\mathbf{p}|q^2}{96\pi^3 m_B^2}\left(1 - \frac{m_l^2}{q^2}\right)^2\left[(|H_+|^2 + |H_-|^2 + |H_0|^2)\left(1 + \frac{m_l^2}{2q^2}\right) + \frac{3}{2}\frac{m_l^2}{q^2}|H_t|^2\right]$$

In the $l = \mu$ or e cases, $m_l \approx 0$ is a good approximation. From here, it is possible to write several quantities that should be sensible to NP, such as the ratio

$$R(q^2) = \frac{d\Gamma_\tau/dq^2}{d\Gamma_l/dq^2} = \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) + \frac{3m_\tau}{2q^2} \frac{|H_t|^2}{|H_+|^2 + |H_-|^2 + |H_0|^2}\right]$$

or the q^2-dependent forward-backward lepton asymmetry, that can be used to probe for the presence of right-handed $b \to c$ currents, since these contribute with opposite sign to $H_\pm$ relative to SM.

$$A_{FB}(q^2) = \frac{\left[\int_0^1 d\cos\theta_l \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d\cos\theta_l \frac{d^2\Gamma}{dq^2 d\cos\theta}\right]}{d\Gamma/dq^2}$$

It has been pointed out recently that the spin of the tau lepton can be inferred using the tau decay patterns. Therefore it is beneficial to compute the differential decay rates taking into account the helicity state ($\lambda_\tau = \pm\frac{1}{2}$), that are written using the completeness relations given above [25].

$$\begin{aligned}
\frac{d\Gamma_\tau}{dq^2}(\lambda_\tau = -\frac{1}{2}) &= \frac{G_F^2|Vcb|^2|\mathbf{p}|q^2}{96\pi^3 m_B^2}\left(1 - \frac{m_\tau^2}{q^2}\right)^2 (|H_+|^2 + |H_-|^2 + |H_0|^2) \\
\frac{d\Gamma_\tau}{dq^2}(\lambda_\tau = \frac{1}{2}) &= \frac{G_F^2|Vcb|^2|\mathbf{p}|q^2}{96\pi^3 m_B^2}\left(1 - \frac{m_l^2}{q^2}\right)^2 \frac{m_\tau^2}{2q^2}(|H_+|^2 + |H_-|^2 + |H_0|^2 + 3|H_t|^2)
\end{aligned}$$

It is possible to define a useful tau spin asymmetry

$$A_\lambda(q^2) = \frac{d\Gamma_\tau/dq^2(\lambda_\tau = -\frac{1}{2}) - d\Gamma_\tau/dq^2(\lambda_\tau = \frac{1}{2})}{d\Gamma_\tau/dq^2}$$

with the explicit form

$$A_\lambda(q^2) = 1 - \frac{6|H_t|^2 m_\tau^2}{(2q^2 + m_\tau^2)(|H_-|^2 + |H_0|^2 + |H_+|^2) + 3|H_t|^2 m_\tau^2}$$

and an angular asymmetry

$$\begin{aligned}
A_\theta(q^2) &= \frac{\left[\int_{-1}^0 d\cos\theta_l \frac{d^2\Gamma_\tau}{dq^2 d\cos\theta} - \int_0^1 d\cos\theta_l \frac{d^2\Gamma_\tau}{dq^2 d\cos\theta}\right]}{d\Gamma_\tau/dq^2} \\
&= \frac{3}{4}\frac{|H_+|^2 - |H_-|^2 + 2\frac{m_\tau^2}{q^2} Re(H_0 H_t)}{\left[(|H_-|^2 + |H_0|^2 + |H_+|^2)(1 + \frac{m_\tau^2}{2q^2}) + \frac{3m_\tau^2}{2q^2}|H_t|^2\right]}
\end{aligned}$$

Is it clear that the term H_t is of utmost importance, cause it distinguishes the τ case

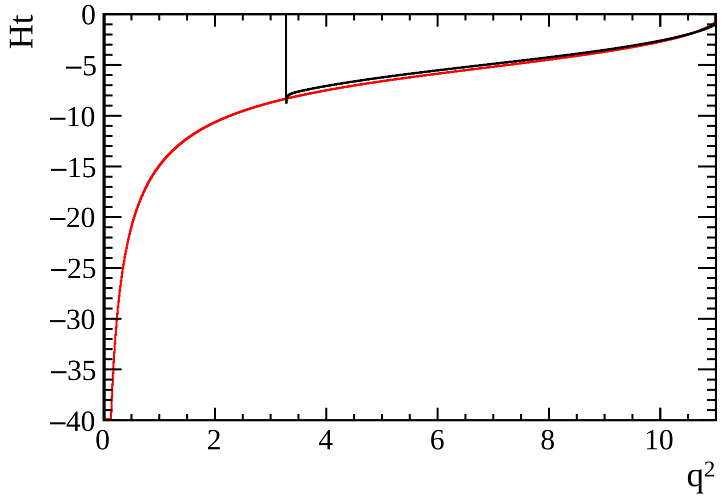

Figure 2.2: The red line is the H_t from [4], while the black line is the H_t calculated with the equation above. Due to the fact that $d\Gamma_\tau$ is used, $q^2_{min} = m^2_\tau$.

from the lepton case, and the $\lambda_\tau = \pm 1/2$ cases. Furthermore, it is directly connected to the form factors.
A way to directly evaluate H_t could be to use the decay widths, that ideally are know quantity from experiments.

$$\begin{cases} \sum\limits_{i=0,\pm} H_i^2 = \dfrac{1}{\mathcal{N}} \left(1 - \dfrac{m_\tau^2}{q^2}\right)^{-2} \dfrac{d\Gamma_\tau}{dq^2}(\lambda_\tau = -\dfrac{1}{2}) \\ \\ H_t^2 = \sum\limits_{i=0,\pm} H_i^2 \left[R(q^2) \left(1 - \dfrac{m_\tau^2}{q^2}\right)^{-2} - \left(1 + \dfrac{m_\tau^2}{2q^2}\right)\right] \dfrac{2q^2}{3m_\tau} \end{cases}$$

With $\mathcal{N} = \frac{G_F^2 |Vcb|^2 |\mathbf{p}| q^2}{96\pi^3 m_B^2}$. From a simulation of the decay widths, where the parameters of the CLN parametrization are taken from [4], in case of only SM contribution, it is possible to see that this analytic method is correct whitin uncertanities coming from the renormalization $\mathcal{N}$.
The main issue here is that NP contribution is not known so, if one can only see the value of the decay widths and parametrizes everything with the form factors, that are free functions, then it is possible to adjust the form factors to match the data, covering any NP contribution whatsoever. Thus, it is needed a way to check the value of H_t in case of only SM contribution. This is provided by the $\bar{B} \to D$ channel that has no NP.

2.2.3 Comparison between BGL and CLN

Now that it is clear how to compute the amplitude for both $B \to D$ and $B \to D^*$ cases, it must be considered the fact the solution is not univocal. In fact, the parametrisation shown previously are enveloped into the calculation, such that the choice of BGL or CLN can change the results.

In Figure 2.3, it is clear how the different parametrisations can affect the distribution of the kinematic variables. The graphics are for the angles $cos\theta_V$ and $cos\theta_l$ (these are for the muon case but the effect is the same for the tau as well). A lot of information can be extract from the $2D$ plot of the $d\Gamma$, integrated over the other two angles. The $\frac{d\Gamma}{dq^2 dcos\theta_V}$ is symmetric around $cos\theta_V = 0$, showing a completely longitudinally polarized D^* at $q^2 = 0$, while the $\frac{d\Gamma}{dq^2 dcos\theta_l}$ is asymmetric due to parity violation.

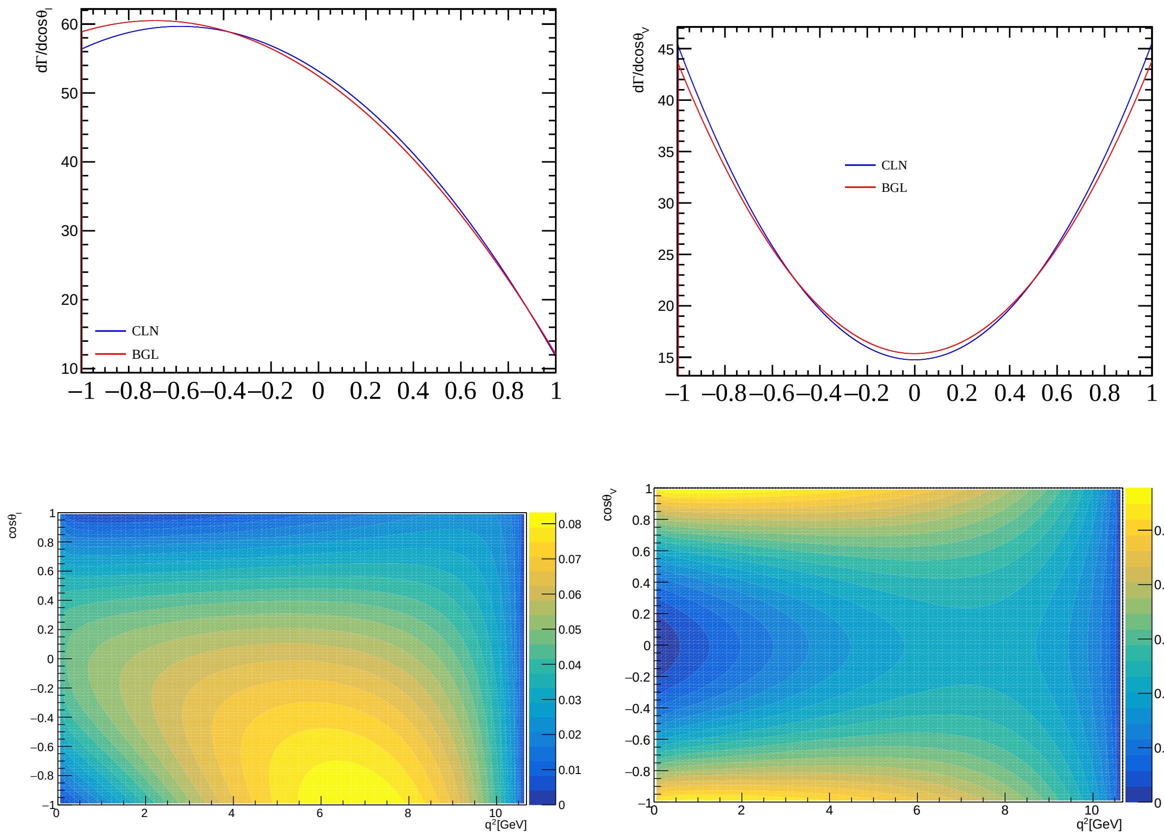

Figure 2.3: Distribution of $\cos\theta_l$ (top left) and $\cos\theta_v$ (top right) for both CLN and BGL parameterizations. The two-dimensional distribution of $\cos\theta_l$ (left) and $\cos\theta_v$ as a function of the q^2 are eported in the bottom.

In Figure 2.4 one can see the difference in the shape of the decay width (integrated over the other variables and normalized arbitrarlily) separately for $l = e$, $l = \mu$ and $l = \tau$ in the final state. Here the plots shown are for the $B \to D^*$ decay, which is the interesting one.

It is worth to remind that is always possible to switch among parametrisations using appropriate relations that depend only on the meson masses and the four-velocities [5].

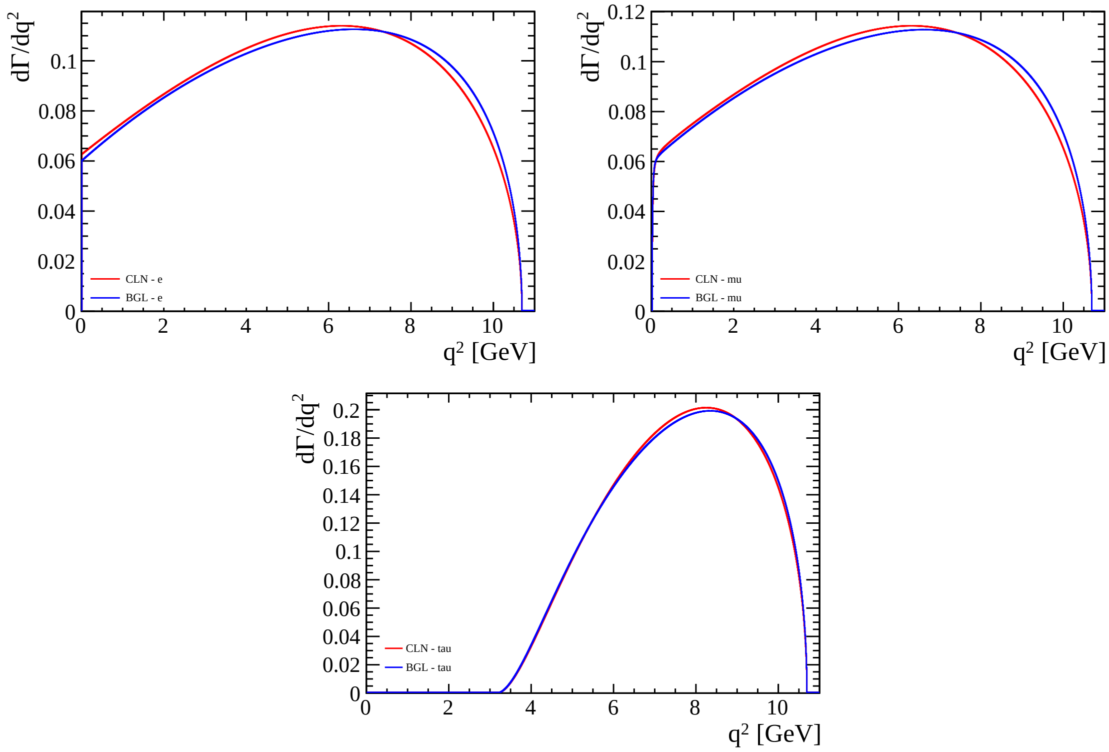

Figure 2.4: Distribution of $\frac{d\Gamma}{dq^2}$ for BGL and CLN, separately for electrons (top), muons(middle) and taus(bottom).

It can be easily seen how the lepton mass strongly affects these distributions, as, for example, the minimum q^2 is given by m_l^2.

2.3 Heavy Quark Expansion, Form Factor Ratios and New Physics

In the heavy quark effective theory, hadronix matrix elements of currents between two hadron states are expanded in inverse powers of the heavy quark masses. In the limit $m_Q \to \infty$, the effective Lagrangian for the strong interactions becomes [14]

$$\mathcal{L}_{HQET} = \bar{h}_Q iv \cdot D h_Q$$

where $D^\alpha = \partial^\alpha - ig_s t_a A^\alpha_a$ is the gauge-covariant derivative and $h_Q(v,x) = e^{im_Q v \cdot x} P_+(v) Q(x)$ is a redefined field which annihilates a heavy quark with velocity v^α. While $P_\pm = \frac{1}{2}(1 \pm \not{v})$ is an on-shell projection operatorn and $Q(x)$ is the conventional quark field in QCD. With finite mass, the lagrangian can be written in a power series of higher dimension operators.

$$\mathcal{L}_p = \frac{1}{2m_Q}\mathcal{L}_1 + \frac{1}{4m_Q^2}\mathcal{L}_2 + ...$$

The leading term is given by

$$\mathcal{L}_1 = \bar{h}_Q (iD)^2 h_Q + Z(m_Q/\mu)\bar{h}_Q s_{\alpha\beta} G^{\alpha\beta}$$

where $s_{\alpha\beta} = -\frac{1}{2}\sigma^{\alpha\beta}$ and $G^{\alpha\beta} = [iD^\alpha, iD^\beta] = ig_s t_\alpha G^{\alpha\beta}_a$ is the gluon field strength. Z is the renormalization factor for the chromo-magnetic moment operator, which is responsible, at order $1/m_Q$ of the expansion, for the mass splitting between the pseudoscalar and vector B mesons and it depends on the number of light quark flavours with mass below m_Q. Given the mass M of the mesons, the mass carried by the light degrees of freedom is $\bar{\Lambda} = M - m_Q$ and due to the field redefinition it governs the x-dependence of states.

$$|M(x)\rangle_{HQET} = e^{-i\bar{\Lambda} v \cdot x}|M(0)\rangle$$

The eigenstates of $\mathcal{L}_{HQET}$ differ from the states of the full theory. The mass shift in the meson rest frame is such that

$$\Delta m_M^2 = \frac{\langle M(v)|(-\mathcal{L}_1)|M(v)\rangle)}{\langle M(v)|h_Q^\dagger h_Q|M(v)\rangle}$$

To evaluate the matrix elements one can define $\mathcal{M}(v) = \sqrt{M}P_+(v)$ and the hadronic parameters λ_i, such that

$$\langle M(v)|\bar{h}_Q (iD)^2 h_Q|M(v)\rangle = -\lambda_1 tr[\bar{\mathcal{M}}\mathcal{M}] = 2M\lambda_1$$
$$\langle M(v)|\bar{h}_Q s_{\alpha\beta} G^{\alpha\beta} h_Q|M(v)\rangle = -\lambda_2 tr[i\sigma_{\alpha\beta}\bar{\mathcal{M}} s^{\alpha\beta}\mathcal{M}] = 2d_M M\lambda_2(\mu)$$

where $d_M = 3$ for a pseudoscalar and $d_M = -1$ for a vector meson.
For the meson form factors one is interested in a current like $\bar{Q}'\Gamma Q$, that has a short distance expansion

$$\bar{Q}'\Gamma Q = \sum_j C_j \bar{h}'\Gamma_j h + \frac{1}{2m_Q}\sum_j c_j \bar{h}'\Gamma_j^\alpha iD_\alpha h + ...$$

where c_j, C_j are perturbative coefficients. The matrix element of the leading term can be parametrized as

$$\langle M'|\bar{h}'\Gamma h|M\rangle = -\xi(w) tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}]$$

so that, at leading order, all matrix elements of currents between mesons are described by the Isgur-Wise function. At subleading order instead

$$\langle M'|\bar{h}'\Gamma^\alpha iD_\alpha h|M\rangle = -tr[\xi(v,v',\mu)_\alpha \bar{\mathcal{M}}'\Gamma^\alpha\mathcal{M}]$$

The most general decomposition of universal form factor ξ_α involves three real scalar functions.

$$\xi_\alpha(v,v',\mu) = \xi_+(w,\mu)(v+v')_\alpha + \xi_-(w,\mu)(v-v')_\alpha - \xi_3(w,\mu)\gamma_\alpha$$

Thanks to the equation of motion and the use of the projection operators, one has a relation between the scalar functions.

$$(w+1)\xi_+(w,\mu) - (w-1)\xi_-(w,\mu) + \xi_3(w,\mu) = 0$$

Insertions of operators of $\mathcal{L}_1$ into matrix elements of the leading order currents represent corrections to the wave functions and their effect is to change the structure of $\mathcal{M}(v)$.

$$\mathcal{M}(v) \to P_+(v)L_+^M(v,v') + P_-(v)L_-^M(v,v')$$

The general form of $L_\pm$ is

$$L_+^P = \sqrt{M}(-\gamma_5)L_1(w)$$
$$L_-^P = \sqrt{M}(-\gamma_5)L_4(w)$$

$$L_+^V = \sqrt{M}[\not\epsilon L_2(w) + \epsilon\cdot v' L_3(w)$$
$$L_-^V = \sqrt{M}[\not\epsilon L_5(w) + \epsilon\cdot v' L_6(w)$$

The L_i are corrections to the Isgur-Wise function such that the form factors are expressed as

$$h_+(w) = \xi(w) + \big(\frac{1}{2m_c} + \frac{1}{2m_b}\big)L_1(w)$$

$$h_1(w) = \xi(w) + \big(\frac{1}{2m_c} + \frac{1}{2m_b}\big)L_2(w)$$

$$h_{A_1}(w) = \xi(w) + \frac{1}{2m_b}\big[L_1(w) - \frac{w-1}{w+1}L_4(w)\big] + \frac{1}{2m_c}\big[L_2(w) - \frac{w-1}{w+1}L_5(w)\big]$$

with the zero recoil condition $h_+(1) = h_1(1) = 1$ from which it follows that $L_1(1) = L_2(1) = 0$.

Writing the decay widths as function of $w = v \cdot v'$ [3].

$$w = v \cdot v' = \frac{m_B^2 + m_{D^*(D)}^2 - q^2}{2m_B m_{D^*(D)}}$$

$$\frac{d\Gamma}{dw}(B \to D) = \frac{G_F^2|V_cb|^2\eta_{new}^2 m_B^5}{48\pi^3}(w^2-1)^{3/2}r_D^3(1+r_D)^2\mathcal{G}(w)^2$$

$$\frac{d\Gamma}{dw}(B \to D^*) = \frac{G_F^2|V_cb|^2\eta_{new}^2 m_B^5}{48\pi^3}(w^2-1)^{1/2}(w+1)^2 r_{D^*}^3(1-r_{D^*})^2$$

$$\times\left[1 + \frac{4w}{w+1}\frac{1-2wr_{D^*}+r_{D^*}^2}{(1-r_{D^*})^2}\right]\mathcal{F}(w)^2$$

Where $r_{D^*(D)} = m_{D^*(D)}/m_B$ and $\mathcal{F}(w)$ and $\mathcal{G}(w)$ are a combination of the form factors such that in the heavy quark limit, $\mathcal{F}(w) = \mathcal{G}(w) = \xi(w)$, the leading Isgur-Wise function.

$$\mathcal{G}(w) = h_+ - \frac{1-r_D}{1+r_D}h_-$$

$$\mathcal{F}(w)^2 = h_{A_1}^2\left[2(1-2wr_{D^*}+r_{D^*}^2)\left(1 + R_1^2\frac{w-1}{w+1}\right) + [(1-r_{D^*}) + (w+1)(1-R_2)]^2\right]$$

$$\times\left[(1-r_{D^*}^2) + \frac{4w}{w+1}1 - 2wr_{D^*} + r_{D^*}^2\right]$$

where $R_1(w) = \frac{h_V}{h_{A_1}}, R_2 = \frac{h_{A_3}+r_{D*}h_{A_2}}{h_{A_1}}$ are form factors ratios, and the $h(w)$ in the CLN parametrization are so defined:
for the $B \to D$

$$r = m_B/m_D$$

$$f_+(q^2) = \frac{1}{2\sqrt{r}}\left[(1+r)h_+(w) - (1-r)h_-(w)\right]$$

$$f_0(q^2) = \sqrt{r}\left[\frac{w+1}{1+r}h_+(w) - \frac{w-1}{1-r}h_-(w)\right]$$

while for the $B \to D^*$

$$V(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}}h_V(w)$$

$$A_1(q^2) = \sqrt{m_B m_{D^*}}\frac{w+1}{m_B + m_{D^*}}h_{A_1}(w)$$

$$A_2(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}}\left[h_{A_3}(w) + \frac{m_{D^*}}{m_B}h_{A_2}\right]$$

$$A_0 = \frac{1}{2\sqrt{m_B m_{D^*}}}\left[m_B(w+1)h_{A_1}(w) - (m_B - m_{D^*}w)h_{A_2}(w) - (m_B w - m_{D^*}h_{A_3}(w))\right]$$

Thus are related to the $H_{0,t,\pm}$.

This limit is strictly valid only at $w = 1$, but should hold if [24]

$$2\Lambda^2_{QCD}(w-1) << m^2_{b,c}$$

so it is possible to make an expansion of the form factor ratios with the leading $\xi(w)$ and the subleading $\eta(w)$ and $\chi(w)$ Isgur-Wise functions. In the Standard Model they are

$$R_1(w) = 1 + \alpha_s(C_{V_1} - C_{A_1}) - \frac{2}{w+1}(\epsilon_b L_4 + \epsilon_c L_5)$$

$$R_2(w) = 1 + \alpha_s(C_{A_3} + r_{D^*}C_{A_2}) - \frac{2}{w+1}(\epsilon_b L_4 + \epsilon_c L_5) + \epsilon_c\left[L_6(1 + r_{D^*} - L_3(1 - r_{D*})\right]$$

with the condition $R_{1,2}(w) = 1$ in the heavy quark limit. The coefficients C_i account for radiative corrections, while L_i for $\mathcal{O}(1/m_b)$ corrections. Their complete expression can be found in [2]. These ratios are usually fitted to the measured $\bar{B} \to \bar{D}^* l\bar{\nu}$ angular distributions.
Making the conformal mapping $z(w) = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$ and thanks to the unitarity constraints one has

$$\mathcal{G}(w)/\mathcal{G}(1) = 1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3 + ...$$

where ρ is the slope parameter. Keeping terms to $\mathcal{O}(\epsilon_{c,b}(w-1))$ one can approximate the subleading Isgur-Wise functions as

$$\chi_2(w) \approx \chi_2(1) + \chi_2(1)'(w-1)$$

$$\chi_3(w) \approx \chi_3(1)'(w-1)$$
$$\eta(w) \approx \eta(1) + \eta(1)'(w-1)$$

such that one can parametrize the $B \to D^*$ form factors in terms of six parameters. In this way, a simultaneus fit of $B \to D$ and $B \to D^*$ parameters can bring results without lattice calculations and other theory input but heavy quark expansion. As shown in [3] the global fit is in agreement with lattice predictions.

Making use of the ratios and the corrections to the Isgur-Wise function, it is possible to evaluate the effect of new physics. Taking the h_{A_1} in [11] and comparing it with [5]

$$h_{A_1}(z) = h_{A1}(1)[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3)]$$
$$\to \Big[C_1^5 + \epsilon_c\Big(L_2 - \frac{w-1}{w+1}L_5\Big) + \epsilon_b\Big(L_1 - \frac{w-1}{w+1}L_4\Big)\Big]\xi(w) = h_{A_1}(w)$$

with $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$. In this way it is possible to extract the Isgur-Wise function and evaluate the CLN parametrization functions.

$$h_V = \Big[C_1 + \epsilon_c(L_2 - L_5) + \epsilon_b(L_1 - L_4)\Big]\xi(w)$$
$$h_{A_2} = \Big[C_2^5 + \epsilon_c(L_3 - L_6)\Big]\xi(w)$$
$$h_{A_3} = \Big[C_1^5 + C_3^5 + \epsilon_c(L_2 - L_3 - L_5 - L_6) + \epsilon_b(L_1 - L_4)\Big]\xi(w)$$

The coefficients C_i account for radiative corrections, while the L_i for HQ expansion. These functions have been completely worked out in [3, 14]. There are analogous relation for the New Physics part as well

$$h_{T_1} = \Big[C_1 + \epsilon_c L_2 + \epsilon_b L_1\Big]$$
$$h_{T_2} = \Big[C_2 + \epsilon_c L_5 - \epsilon_b L_4\Big]$$
$$h_{T_3} = \Big[C_3 + \epsilon_c(L_6 - L_3)\Big]$$

Reminding that in the CLN parametrization the matrix elements are written as

$$\langle D^*(v',\epsilon)|\,\bar{c}\gamma_\mu b\,|\bar{B}(v)\rangle = \sqrt{m_B m_{D^*}}\, i h_v(w)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^{\alpha}v^{\beta}$$

$$\langle D^*(v',\epsilon)|\,\bar{c}\gamma_\mu\gamma_5 b\,|\bar{B}(v)\rangle = \sqrt{m_B m_{D^*}}\Big[h_{A_1}(w)(w+1)\epsilon^*_\mu - (h_{A_2}(w)v_\mu + h_{A_3}(w)v'_\mu)(\epsilon^*\cdot v)\Big]$$

$$\langle D^*(v',\epsilon)|\,\bar{c}\sigma_{\mu\nu} b\,|\bar{B}(v)\rangle = -\sqrt{m_B m_{D^*}}\epsilon_{\mu\nu\alpha\beta}\Big[h_{T_1}(w)\epsilon^{*\alpha}(v+v')^\beta + h_{T_2}(w)\epsilon^{*\alpha}(v-v')^\beta + h_{T_3}(w)v^\alpha v'^\beta(\epsilon^*\cdot v)\Big]$$

.

Such that the New Physics form factor are expressed as [11]

$$T_0(q^2) = -\frac{(m_B + m_{D^*})^2}{m_B m_{D^*}} \sqrt{\frac{m_{D^*}}{m_B}} h_{T_3}(w)$$

$$T_1(q^2) = \sqrt{\frac{m_{D^*}}{m_B}} (h_{T_1}(w) + h_{T_2}(w))$$

$$T_2(q^2) = \sqrt{\frac{m_{D^*}}{m_B}} (h_{T_1}(w) - h_{T_2}(w))$$

The form factors T_3, T_4, T_5 are related to T_0, T_1, T_2 by the identity $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$. So, $T_3 = T_1$, $T_4 = T_2$, $T_5 = -T_0$.
Thus, it is possible to evaluate the effect of the New Physics, for example for the forward-backward asymmetry.

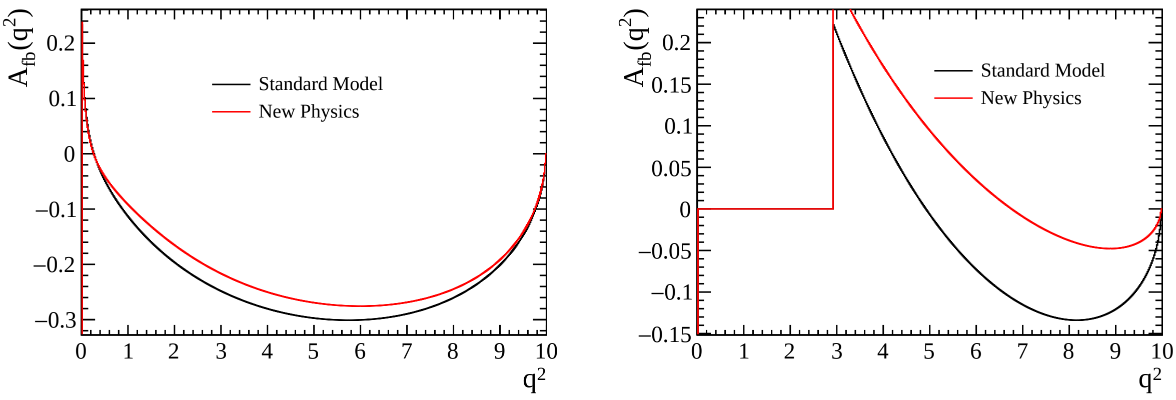

Figure 2.5: Forward-Backward asymmetry for muon and tau in case of SM and NP.

A more quantitave way to see the difference between Standard Model and New Physics is to calculate the definite integral of the A_{FB} in different q^2 range.

$$\bar{A}_{low} = \int_0^6 A_{FB} dq^2$$

$$\bar{A}_{high} = \int_6^{q^2_{max}} A_{FB} dq^2$$

	Standard Model	New Physics
$\bar{A}_{low}$	0.220164	0.44291
$\bar{A}_{high}$	-0.462698	-0.804904

2.4 A sketch on CP violation in B decay

Three discrete operations are potential symmetries of a field theory Lagrangian; Parity P, that sends $(t,x) \to (t,-x)$, Time reversal T, that sends $(t,x) \to (-t,x)$, and Charge conjugation C, that interchanges particles and antiparticles. The CP combination replaces a particle by its antiparticle and reverses momentum and helicity. The combination CPT is an exact symmetry in any local Lagrangian field theory.
To underand whether a given theory can accomodate CP violation, one needs to know the transformation properties of the fields under the various discrete symmetries. In particular for a Dirac spinor [28]

$$P\psi(t,x)P = \gamma^0\psi(t,-x)$$

$$T\psi(t,x)T = -\gamma^1\gamma^3\psi(-t,x)$$

$$C\psi(t,x)C = -i(\bar{\psi}(t,x)\gamma^0\gamma^2)^T$$

form this follow the bilinear transformation properties under CP

$$\bar{\psi}_i\psi_j \to \bar{\psi}_j\psi_i$$

$$i\bar{\psi}_i\gamma^5\psi_j \to -i\bar{\psi}_j\gamma^5\psi_i$$

$$\bar{\psi}_i\gamma^\mu\psi_j \to -(-1)^\mu\bar{\psi}_j\gamma^\mu\psi_i$$

$$\bar{\psi}_i\gamma^\mu\gamma^5\psi_j \to -(-1)^\mu\bar{\psi}_j\gamma^\mu\gamma^5\psi_i$$

with $(-1)^\mu = 1$ for $\mu = 0$ and $(-1)^\mu = -1$ for $\mu = 1,2,3$. Similarly, the CP transformation properties of scalar, pseudoscalar and vector boson fields, and derivative operator are rispectively

$$H \to H$$

$$A \to -A$$

$$W^{\pm\mu} \to -(-1)^\mu W^{\mp\mu}$$

$$\partial\mu \to (-1)^\mu\partial_\mu$$

The above rules imply that each of the combinations of the fields and derivatives in the Lagrangian transforms under CP to its hermitian conjugate. But, there are coefficients in front of these expressions, representing either couplings or masses, which do not transform under CP. If they are complex, then the coefficients in front of a CP-related Lagrangian terms are complex conjugate of each other. When the rates of physical processes are calculated, there can be CP-violating effects, namely rate differences between pairs of CP conjugate processes.

For any final state f, and for an amplitude A_f, the quantity $|\frac{\bar{A}_{\bar{f}}}{A_f}|$ is independent of phase conventions and physically meaningful (where $\bar{A}_f$ means the CP transformed). Such that, if the relation $|\frac{\bar{A}_{\bar{f}}}{A_f}| \neq 1$ is true, CP violation is present. So far, this effect has been observed only in K decays.
Beyond Standard Model theories usally introduce new fields and with them additional couplings and hence the possibility of additional CP violation phases.
Direct CP violation requires two contributions to the decay process which differ in both their strong and weak phases. Purely leptonic and semileptonic decays are dominated by a single diagram and thus are unlikely to exhibit any measurable direct CP violation. B decays can thus be grouped inot five classes.

1. Decays dominated by a single term: $b \rightarrow c\bar{c}s$. The Standard Model predicts zero direct CP violation because the second term is suppressed. Modes like $B^+ \rightarrow \psi K^+$ are examples of this class.

2. Decays with a small second term: $b \rightarrow c\bar{c}d$. The penguin-only contributions are suppressed and these modes will have small direct CP violation effects.

3. Decays with a suppressed tree contribution: $b \rightarrow u\bar{u}s$. Such as $B \rightarrow \rho K$. Small mixing angle and large interference effects.

4. Decays with no tree contribution: $b \rightarrow s\bar{s}d$. Penguin contributions with different charge quarks. Like $B \rightarrow KK$.

5. Radiative Decays: $b \rightarrow s\gamma$. The leading contribution comes from electromagnetic penguins. An example is $B \rightarrow K^*\gamma$.

CP violation is then a probe for new physics but requires a better knowledge of the decay amplitudes.

2.5 NP proposals

The most common approach to the problem of new physics in B decay is, as seen, to add a tensor operator to the lagrangian, but this is not the only one. In the literature, one can find different proposals to solve the puzzle; two honorable mentions are Leptoquarks and Two Higgs model.

2.5.1 Leptoquarks

New Physics explanations of the B-physics anomalies suggest a presence of one or more TeV scale mediators which couple to left-handed currents with predominantly third generation fermions; among the most prominent candidate are leptoquarks(LQs). Using effective theory approach, it was shown that among all possible single mediators only one particular vector LQ can generate suitable $V-A$ operators for the anomalies and satisfy both low and high energy constraints. An UV complete model is based on a $SU(5)$ Grand Unified Theory with two light scalar LQs. At the moment, the LHC is looking for LQs through two main leptoquark production mechanisms. Firstly, leptoquarks would be copiously produced in pairs via strong interactions followed by the prompt decay to leptons and jets. This is indeed a conventional assumption in most experimental searches. After creation, a leptoquark would split almost immediately into a quark and a lepton and could be identified by looking for their decay products. Quarks, since they cannot exist isolated, quickly create many quark-antiquark pairs and form a jet of particles that can be identified by the large energy deposition in the calorimeter.

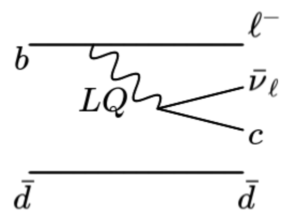

Figure 2.6: Feynman Diagram of the B to D decay with a LQ.

The second mechanism is the production of a single leptoquark in association with a lepton due to the direct quark-lepton coupling. This is an important complementary perspective which is not yet fully exploited by the experimental collaborations.

2.5.2 Two Higgs model

A well motivated class of models, compatible with the Higgs discovery, is given by extending the Standard Model Higgs sector by a second scalar $SU_L(2)$ doublet, the so-called Two Higgs Doublet Model. The minimal supersymmetric extension of the Standard Model is a prominent example of

BSM theories that features a Higgs sector with two Higgs doublets. This additional Higgs boson is usally charged and it generates new flavour-changing interactions. The coupling to fermion seems to grow with the mass, so now B decays with a τ lepton in the final state are studied to try to uncover these possible new effects.

Chapter 3

Feasibility Study

3.1 Introduction to LHCb

The European Organization for Nuclear Research, commonly known by the acronym CERN (Conseil européen pour la recherche nucléaire), is the largest laboratory in the world of particle physics, used by over 600 research institutes and universities from all over the world. Founded in 1954 by eleven european states, it is located on the border between Switzerland and France, on the western outskirts of the city of Geneva. The Large Hadron Collider (LHC) is at present the world's most powerful particle accelerator. Two beams of paricles are accelerated in the opposite directions and set to collide in four interaction points along the LHC ring. These points host the seven LHC experiments: ATLAS and LHCf, CMS and TOTEM, LHCb and MoEDAL, and ALICE.
Accelerators are mainly charaterised by their luminosity $\mathcal{L}$ and collision energy $\sqrt{s}$, defined as the energy available in the centre of mass [16]

$$\sqrt{s} = \left[(p_{b1} + p_{b2})^{\mu}(p_{b1} + p_{b2})_{\mu}\right]^{\frac{1}{2}}$$

LHC operated at $\sqrt{s} = 7$ and 8 TeV during Run I, and $\sqrt{s} = 13$ TeV since the start of Run II in 2015.
Luminosity is the measure of the number of occurrences per unit time for a process with a given cross section σ, such that if R is the rate of events featuring the given process, one has $R(t) = \sigma\mathcal{L}(t)$. The integrated luminosity $\mathcal{L} = \int dt\mathcal{L}(t)$ is the measure of the amount of pp collisions delivered by the LHC.
The beams of the collider are not continuous and assuming a Gaussian beam profile, the instantaneous luminosity is

$$\mathcal{L}(t) = \frac{f\gamma N_p^2 n_b}{4\pi\epsilon_n\beta^*}F$$

where n_b is the number of proton bunches, N_p is the number of protons per bunch,

f is the revolution frequency of the bunches, γ is the relativistic gamma factor, ϵ_n is the normalised transverse beam emittance, β^* is a quantity related to the transverse dimensions of the beam and F is a geometrical reduction factor due to the beams crossing angle.
Particle emitted from a collision are mainly emitted in the forward region, due to the asymmetry in the fraction of proton momentum. While collisions decrease the beam intensity, the distance between the beams, in the LHCb experiment, is reduced, keeping the instantaneous luminosity almost stable. The large centre of mass energy and the high rate of interactions allow the LHC to produce a huge sample of charm and beauty hadrons that the LHCb experiment has been designed to exploit.

The Large Hadron Collider beauty (LHCb) experiment was developed at the end of the 90's to investigate the slight differences between particles and antiparticles, which caused the universe to evolve into a matter only state, making the antimatter disappear just one second after the Big Bang. LHCb selects, during the proton-proton collision, couples of mesons or hadrons, containing the b quark, trying to reconstrunct their evolution. During the years, the experiment has broadened its goals, looking for more and more discrepancies between what is observed and what is known from the theory.

Inside the LHCb the particles created travel for a distance of few millimeters before decaying, so a crucial requirement of an experiment aimed at b-physics is to be able to faithfully reconstruct the trajectory of the particles and then to distinguish beween different decay modes. The tracking system is composed of a vertex detector, the VELO, and a system of forward tracking stations, the Tracker Turicensis (TT). The VELO enables the reconstruction of the trajectory of the decaying hadron. Combining the information from all the tracking systems it is possible to provide a measurement of the charge and momentum of the charged decay products, by tracing them through a magnetic field. Each track is then associated to a vertex. Alongside the tracking systems, there are: two Cherenkov radiation detectors (a radiation emitted when a charged particle moves in a medium faster than the speed of light in that medium) that can determine the mass of the particles; a calorimeter system, that is able to distinguish charged and neutral particles, providing the identification of electrons, photons and hadrons; and the muon system, that implements the identification of muons to a very high level of purity.

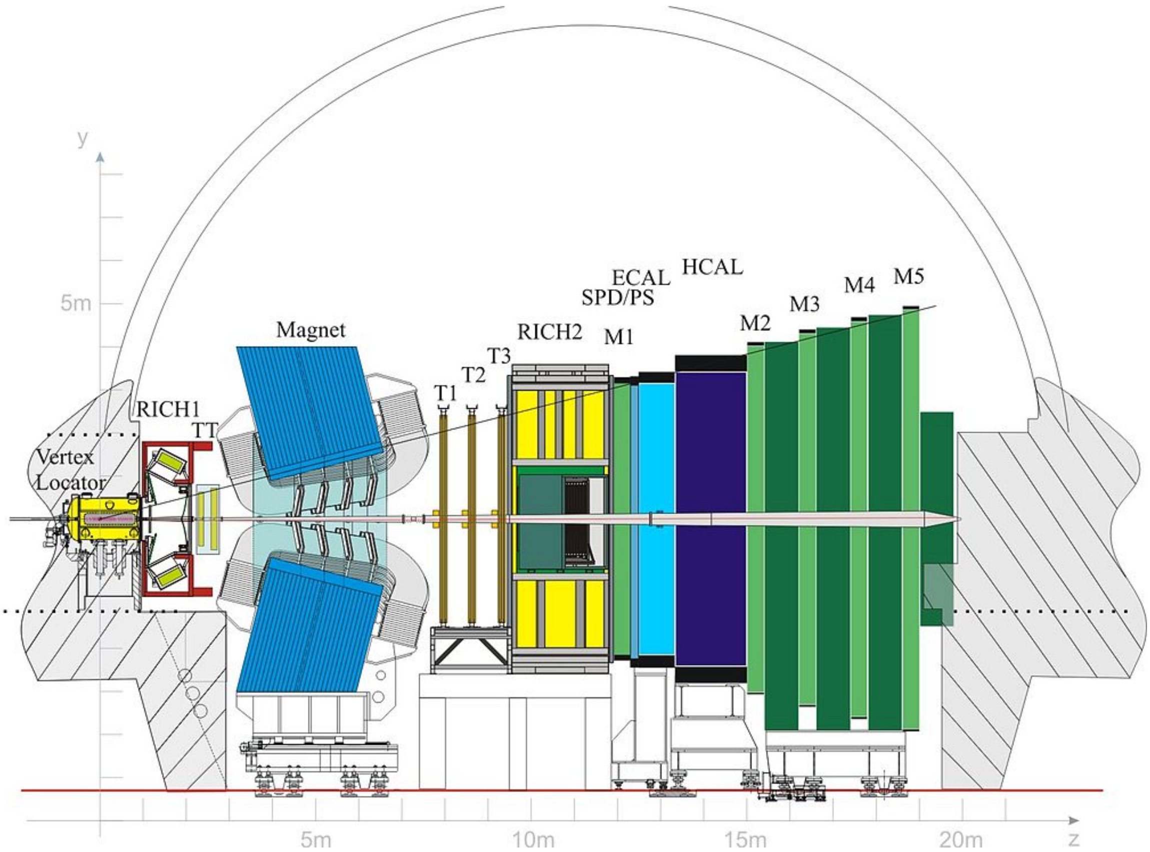

Figure 3.1: Schematic view of the LHCb detector.

Particle identifications is then completely performed combining information from all the systems.

The amount of raw data recorded at LHCb is too high to be stored, so a data acquisition system is used to select only the interesting events. In particular, a series of hardware and software based algorithms, called *trigger*, is employed to reduce the data generation rate, from an initial 40 MHz, down to a few kHz. Such that each physics analysis uses a small subset of data chosen on the base of topological and kinematic features of the seeked events.

3.2 Monte Carlo Method

Monte Carlo method is a broad variety of computational methods, based on random quantities, to obtain numerical results. Its most used application is for numerical integrals, and in physics, a good method to compute intergals, especially multidimensional ones, is crucial. Indeed, the efficiency of Monte Carlo method increases with respect to the other methods (i.e. trapezoid, Simpson and so on) when the dimension of the problem grows, albeit it is less accurate in simpler cases.
The technique to calculate the integral is called *hit or miss* and is based on the geometrical interpretation of an integral, that can be an area, a volume or an hypervolume. For semplicity, one can consider a function $g(x)$ with boundaries [29]

$$0 \leq g(x) \leq c, \qquad a \leq x \leq b$$

and the problem is to calculate the integral

$$I = \int_a^b g(x)dx$$

Let Ω be the rectangle defined as

$$\Omega = \{(x, y) : a \leq x \leq b,\ 0 \leq y \leq c\}$$

Let (X, Y) be a random vector uniformly distributed over the rectangle Ω with probability density function (p.d.f)

$$f_{XY}(x, y) = \begin{cases} \frac{1}{c(b-a)}, & \text{if } (x, y) \in \Omega \\ 0, & \text{otherwise} \end{cases} \tag{3.1}$$

Denoting $S = \{(x, y) : y \leq g(x)\}$

$$\text{area under } g(x) = \text{area } S = \int_a^b g(x)dx$$

the probability P that the random vector (X, Y) falls within the area under the curve $g(x)$ is

$$P = \frac{\text{area } S}{\text{area } \Omega} = \frac{\int_a^b g(x)dx}{c(b-a)} = \frac{I}{c(b-a)}$$

Generating N independent random vectors $(X_1, Y_1), (X_2, Y_2), ..., (X_N, Y_N)$, one can estimates P by

$$\hat{P} = \frac{N_H}{N}$$

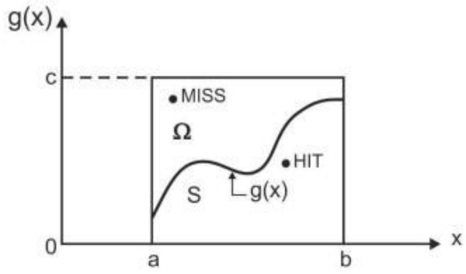

Figure 3.2: Graphical representation of the Hit or Miss method.

where N_H is the number of *hits*, i.e the number of occasions on which $g(X_i) \geq Y_i,\ i = 1, 2...N$, such that $N - N_H$ is the number of *misses*. So, the integral can be estimated by

$$I \approx \theta_1 = c(b-a)\frac{N_H}{N}$$

Since each of the N trials is a Bernoulli trial with probability P of a hit, then

$$E(\theta_1) = c(b-a)E(\frac{N_H}{N}) = c(b-a)\frac{E(N_H)}{N} = Pc(b-a) = I$$

that means θ_1 is an unbiased estimator of I. The variance of $\hat{P}$ is

$$\text{var}P = \text{var}(\frac{N_H}{N}) = \frac{1}{N^2}\text{var}(N_H) = \frac{1}{N}P(1-P) = \frac{1}{N}\frac{I}{[c(b-a)]^2}[c(b-a)-I]$$

Thus

$$\text{var}\theta_1 = [c(b-a)]^2\text{var}\hat{P} = \frac{1}{N}[c(b-a)-I]$$

and the standard deviation

$$\sigma_{\theta_1} = [\text{var}\theta_1]^{\frac{1}{2}} = N^{\frac{1}{2}}\{I[c(b-a)-I]\}^{\frac{1}{2}}$$

Now, the accuracy of the method is, as seen, dependent on the number of events N. One could freely set a lower limit to the accuracy and find out the minimum number of events needed asking, for example, that the probability

$$\mathcal{P}[|\theta_1 - I| < \epsilon] \geq \alpha$$

from Chebyshev's inequality,

$$\mathcal{P}[|\theta_1 - I| < \epsilon] \geq 1 - \frac{\text{var}}{\epsilon^2}$$

$$\alpha \leq 1 - \frac{\text{var}}{\epsilon^2}$$

Substituting

$$\alpha \leq 1 - \frac{P(1-P)[c(b-a)]^2}{N\epsilon^2}$$

and solving for N

$$N \geq \frac{(1-P)P[c(b-a)]^2}{(1-\alpha)\epsilon^2}$$

When N is large enough, one can apply the central limit theorem, such that the random variable

$$\hat{\theta_1} = \frac{\theta_1 - I}{\sigma_{\theta_1}}$$

is distributed according to the standard normal distribution

$$\mathcal{P}(\theta_1 \leq x) \approx \phi(x) = \frac{1}{\sqrt{2}\pi}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,dt$$

3.3 Monte Carlo Simulation

Computer simulations became, in the last decades, an excellent tool to investigate the behaviour of complex systems in a broad variety of field of studies, ranging from particle physics to climatology and engineering. They allow to compare different mathematical models, checking their reliability making predictions and studying various scenarios. As simulations gained popularity, the issue of their trustworthiness for generating new knowledge has grown. The relevant question is always whether or not the results of a particular computer simulation are accurate enough for their intended purpose. Besides numerical approssimations, a lot of subtle problems could come up; the results could be biased, the assumptions behind the simulation could be not adeguate or the simulation could just not be similar enough to the real experiment. For these reasons, the interplay between experiment, simulation, and theory is fundamental during the process of building a computer simulation.

As a last thought in this overview of Monte Carlo simulation, every Monte Carlo requires a good way to produce random numbers. A sequence of pure random numbers, namely numbers without any connections among them, that follow the same probability distribution, is indeed impossible to build artificially. The issue is that sooner or later the sequence will repeat itself. There's a long story of different methods developed to make the sequence longer as possible, reaching now a maximum period of $2^{19937} - 1$ numbers with the Twister algorithm.

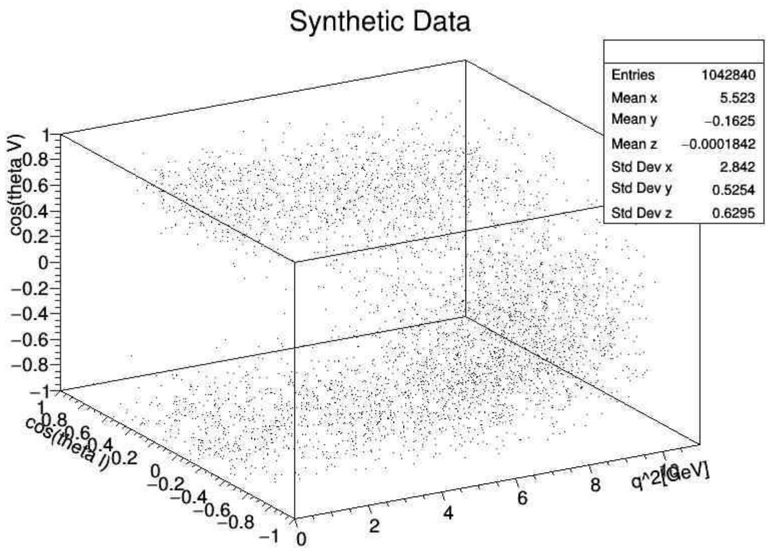

Figure 3.3: Simulation of 1 milion of B decays into τ and μ.

One of the most used and famous algorithm in the literature is the Monte Carlo Generator, that use randomness to calculate the properties of a mathematical model. In this work, a sample of millions of events has been built starting from the differential decay amplitude of the B decay. Knowing the amplitude, one can use it as a p.d.f. doing an *hit or miss* method to accept or reject the event. So, generating a random vector, if it is in the volume under the amplitude is accepted, otherwise is rejected. In this way, from random vectors, one ends up with a distribution of vectors that follow the decay amplitude. The distributions for the muon and the tau case are then summed up, with the proper weight, to have the final sample. The raw data generated appear like in figure 3.3. This method of producing data suffer of low effiency due to the shape of the amplitude, indeed the probability of a random vector to be accepted is $\approx 27\%$ for muon and $\approx 7\%$ for tau.
As shown in the previous sections, the amplitude is dependent on the form factors and their parametrization, so the data have been generated both for BGL and CLN parametrization.

3.3.1 Fit and Pseudoexperiments

A large class of problems in physics, when talking about making a fit, can be reduced to the problem of finding a minimum. Probably the most common application of minimization is χ^2 fitting, where the function to be minimized is the sum of squares of deviations, between measured values and predictions of a model containing variable parameters. Calling Y_n and σ_n the measured values and errors, and $T_n(x)$ the predicted values by the model, depending on a certain collection of parameters x, the function to minimize is

$$\chi^2(x) = \sum_{n=0}^{N} \frac{\Big(Y_n - T_n(x)\Big)^2}{\sigma_n}$$

The minimum gives the best set of parameters x with random errors σ. In this case the n runs over the binning, such that the χ^2 is a measure of the difference between events predicted and measured to be in a volume $\Delta q^2 \Delta cos(\theta_L) \Delta cos(\theta_V)$. While $T_n(x)$ is simply counted, one has

$$Y_n = \mathcal{A}\Big[c_\mu \frac{\int d\Gamma^\mu/d\Omega}{\Gamma^\mu_{tot}} + c_\tau \frac{\int d\Gamma^\tau/d\Omega}{\Gamma^\tau_{tot}}\Big]$$

where $\mathcal{A}$ is the appropriate normalization, c_μ and c_τ are coefficients such that Y_n is still a p.d.f. and $\mathrm{d}\Omega = \mathrm{d}q^2 \mathrm{d}cos(\theta_l) \mathrm{d}cos(\theta_V)$. The coefficients are linked to the fraction of B decaying into $D^*\tau\nu$, in fact integrating Y_n over all the phace space, one obtains the relation

$$c_\mu + c_\tau = 1$$

$$c_\mu = 1 - c_\tau$$

The parameter R_{D^*} can be written in terms of the numbers of muon and tau

$$R = \frac{BR(D^*\tau\nu)}{BR(D^*\mu\nu} = \frac{\Gamma_\tau}{\Gamma_\mu} = \frac{N_\tau}{N_\mu} = \frac{c_\tau}{1-c_\tau}$$

The number of tau that decay into muons is not observable. One can only see a surplus of muons.

$$N_{(\tau\to\mu)} = N_\tau \cdot BR(\tau \to \mu)$$

with $BR(\tau \to \mu) \approx 0.17$.
Working backward from this point, it is possible to calculate the coefficients for the Standard Model

$$c_\tau = \left[1 + \frac{1}{R \cdot 0.17}\right]^{-1} = 0.041$$

This method is supposed to find a minimum without considering the fact that the minimum could either not exist or be multiple. If the function has more than one local minimum, one is not guaranteed to find the global minimum or even the closest minimum to the starting point. In fact, it is usually assumed, when using these algorithms, that the function is unimodal (has one minimum) in the region of interest. For this FIT, a scan of the different parameters has been done to ensure, with reasonable certainty, that there are no multiple minima.

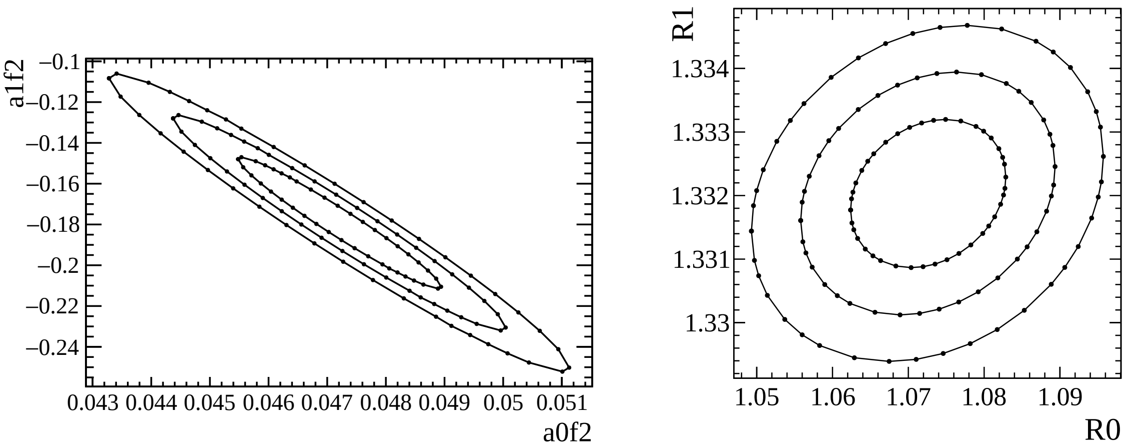

Figure 3.4: Contours plot for BGL and CLN parameters.

To compute the fit, minuit from ROOT has been used, which employs a gradient method to find the minimum. A gradient method is a technique which uses gradient, and maybe higher derivatives, to create a path towards the minimum. A first derivative can be estimated by

$$\frac{\partial F}{\partial x}\Big|_{x=0} \approx \frac{F(x_0+d)-F(x_0)}{d}$$

with d small. The error will be

$$\delta \approx \frac{d}{2}\frac{\partial^2 F}{\partial x^2}\Big|_{x=0}$$

at lowest order in Taylor's expansion. As soon as the function's first derivatives are known, it is natural to follow the direction of $-\nabla F(x)$ in seeking a minimum, since this is the direction in which the function is decreasing the fastest. So one starts from an arbitrary point x_0, calculate the gradient $P_k = -\nabla F(x)$ and iterate

$$x_{k+1} = x_k + \alpha_k P_k$$

where α_k is a real positive number that represent the step of the descent.

3.3.2 Fit Results

The fit has been done basically in two steps: the former has been to reproduce several times the pseudoexperiment to find how the parameter values and their uncertainities change, depending on the number of events, in the case of both CLN and BGL parametrisation. The values in [4] has been used as reference point to check the validity of the fit. The latter has been to introduce the contribution of new physics to see if, at least in ideal condition, it is possible to measure and clearly distinguish it from the standard model.

To reproduce the decay width and to make the analysis, it has been used a new C++ class, called SL Decay [9], created to implement all the various form factors in both CLN and BGL parametrisation. It can be used as a Monte Carlo generator of events and also the functions implemented can be employed as fitting functions to extract important parameters. The class has a method by which one can set the value of the coefficients of the form factors. In this way it has been possible to make the fit to test the accuracy of the fit itself and the correlation between the parameters.

As shown even by the contour plots, the parameters result to be correlated (or anticorrelated), but while for CLN is mainly due to statistical fluctuations, for BGL the first and second term of the expansion appear to be strongly correlated. This is due to the fact that in a linear fit the data points impose constraints on the form of the line, such that to incorporate all the points the parameters a_0 and a_1, in the expansion $a_0 + a_1 z + ...$, must be anticorrelated. Adding other terms in the fit would reduce the correlation between a_0 and a_1 .

	a_1^f	a_2^f	$a_1^{f_1}$	$a_0^{f_2}$	$a_1^{f_2}$	a_0^g	a_1^g
a_1^f	1.000	-0.954	0.644	-0.071	0.217	0.009	-0.031
a_2^f		1.000	-0.584	0.144	-0.310	-0.011	0.028
$a_1^{f_1}$			1.000	0.038	0.062	-0.152	0.027
$a_0^{f_2}$				1.000	-0.907	0.450	-0.449
$a_1^{f_2}$					1.000	-0.292	0.300
a_0^g						1.000	-0.971
a_1^g							1.000

Table 3.1: BGL Parameter Correlation Coefficients for 1 million events

	R_0	R_1	R_2
R_0	1.000	0.222	0.151
R_1		1.000	-0.430
R_2			1.000

Table 3.2: CLN Parameter Correlation Coefficients for 100K events.

	R_0	R_1	R_2
R_0	1.000	0.214	0.196
R_1		1.000	-0.420
R_2			1.000

Table 3.3: CLN Parameter Correlation Coefficients for 1M events.

The results are in agreement with [4] and the uncertanities decrease with the number of muons, as expected.
There are two important things to notice. The former is that in order to compute the fit, one has to build a sample of events, the synthetic data discussed earlier, that are collected into an histogram. The statistical uncertainity is affected by the choice of the binning. In figure 3.5 different cases of a reasonable binning for a real experiment are represented, in regards to the CLN parameter R_0.

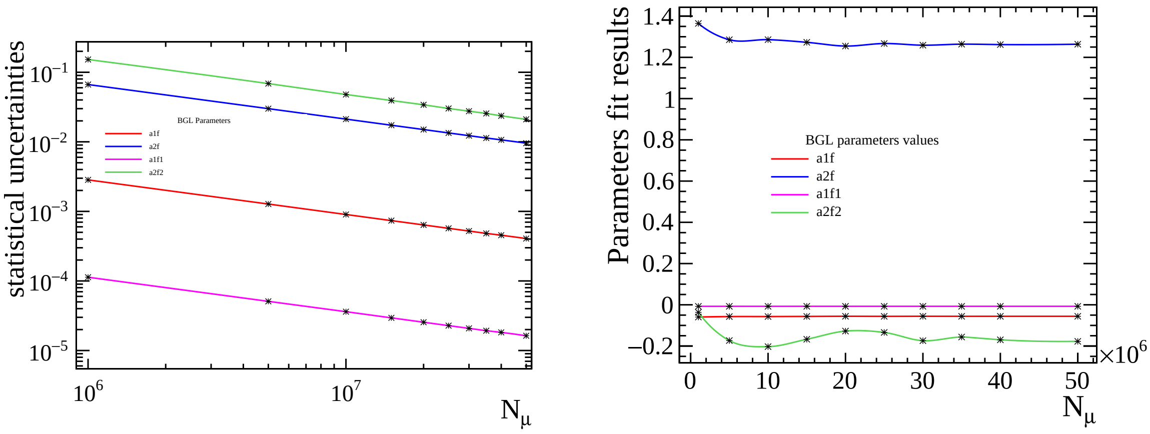

Figure 3.5: BGL fit results

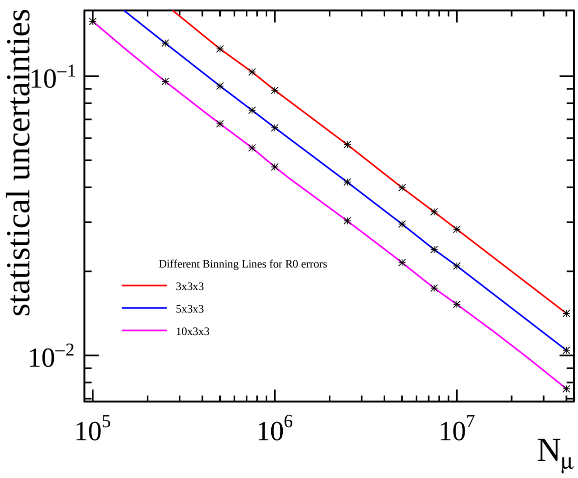

Figure 3.6: Dependence of R_0 from the binning of the fit

The latter is that all the values are biased due to the presence of an integral solved numerically, in particular with a simple trapezoid method. This means that the integral of a certain function $f(x)$, between two points a and b, becomes

$$\int_a^b f(x)dx \approx (b-a)\frac{f(b)+f(a)}{2}$$

and over all the domain

$$\sum_{n=0}^{m}\int_{a_n}^{a_{n+1}} f(x)dx \approx \sum_{n=0}^{m}\delta\frac{f(a_n)+f(a_{n+1})}{2}$$

such that $\delta = \frac{x_{max} - x_{min}}{m}$.

The smaller δ, the greater the precision of the integral. Nevertheless, if m is too high, the integral will be very time consuming (considering that in the program use the integral is done multiple times), so one has to find a balance between time efficiency and precision. In figure 3.7 is shown the bias from the integral on the R_0 values. The dotted line is the reference value from [4].

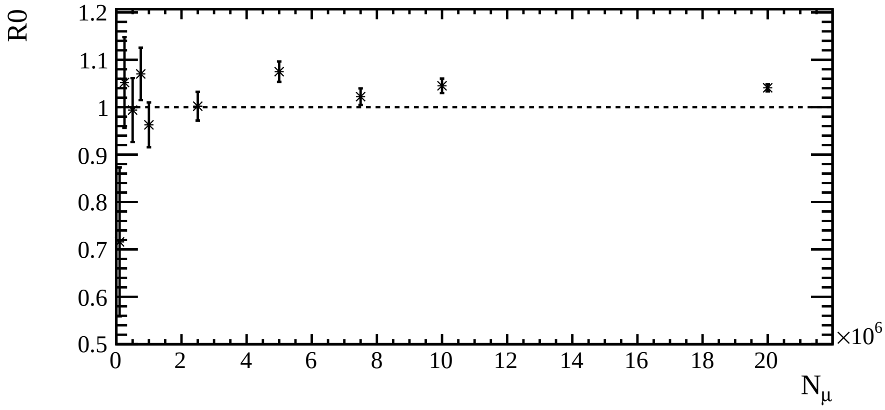

Figure 3.7: Trend of R_0 value with repect to the statistic.

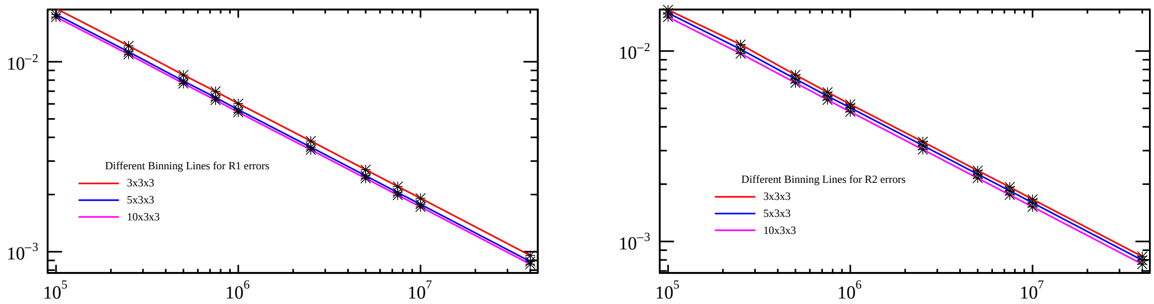

Figure 3.8: Dependence of R_1 and R_2 from the binning of the fit

It is believed that in the limit of $N_\mu \to \infty$ or with an infinte amount of steps for the integral, the fit is exactly convergent giving the reference value of $R_0 = 1$.

3.3.3 Asymmetry and new physics test

Previously, it has been shown how to evaluate the new physics effect on the forward-backward asymmetry. The SL Decay class however, does not implement new physics. A new macro has been built to realize simulations, using the SL Decay class as a backbone, with a numerical representation of the effect of the tensor-like contribution to the decay, to actually make a feasibility study. The new code, that can be intended as a subroutine to the SL Decay class, uses the analytical structure of [11], where the standard model form factors can be easily replaced by the new physics ones, that are seen as a sum of the standard model part plus a real and an imaginary contribution.
To complete the study, using the data from the various fit, a linear error has been introduced on the asymmetry, propagated from the uncertainities on the form factor parameters.

Considering A_{FB} as a function of the parameters

$$A_{FB}(q^2) = A_{FB}(q^2; R_0, R_1, R_2, \rho^2)$$

it is possible to find a deviation σ such that

$$A_{FB} \to A_{FB} \pm \sigma$$

.
Defining $\nabla A_{FB} = (\frac{\partial A_{FB}}{\partial R_0}, \frac{\partial A_{FB}}{\partial R_1}, \frac{\partial A_{FB}}{\partial R_2}, \frac{\partial A_{FB}}{\partial \rho^2})$ and calling Σ the covariance matrix

$$\sigma^2 = \nabla A_{FB} \Sigma \nabla A_{FB}^T$$

This quantity has been calculated several times at different number of events to see its evolution with the statistic. The avarage value of A_{FB} is barely affected, but the σ becomes smaller and smaller. The muon case has little error bars due to the dumping from the mass value. The tau case instead, has more evident error bars but still the separation between standard model and new physics remains clear.

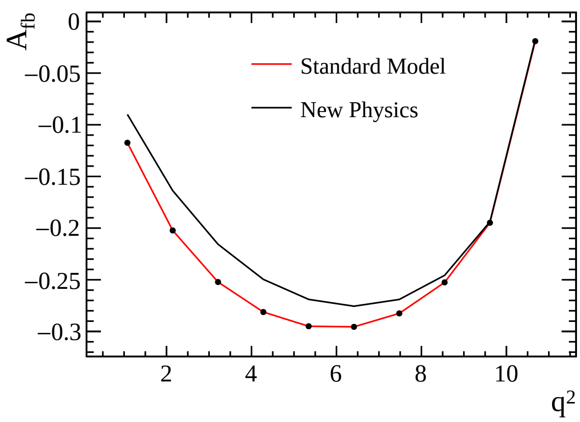

Figure 3.9: Effect of the statistic on A_{FB} and new physics for muon case at 100K events.

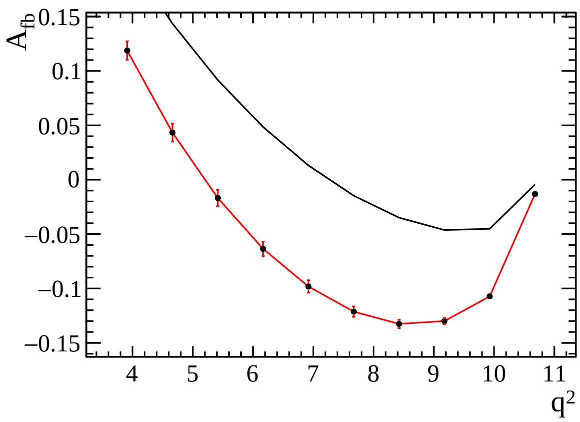

Figure 3.11: Effect of the statistic on A_{FB} and new physics comparison for tau case at 500K events.

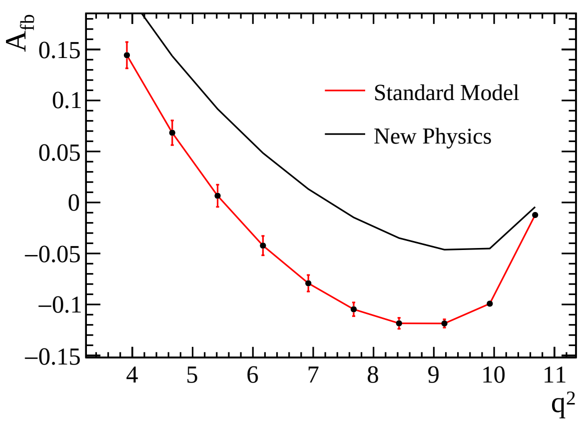

Figure 3.10: Effect of the statistic on A_{FB} and new physics comparison for tau case at 100K events.

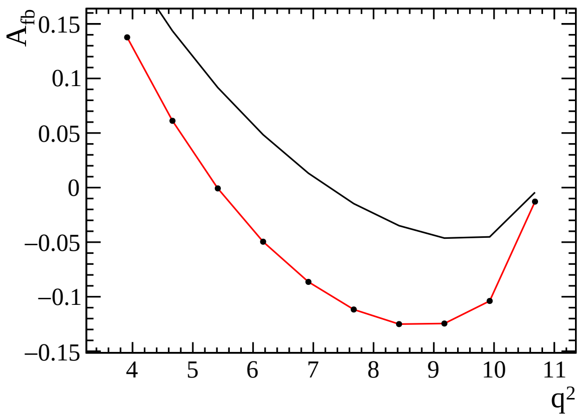

Figure 3.12: Effect of the statistic on A_{FB} and new physics comparison for tau case at 1M events.

3.3.4 Resolution effect

There is an experimental issue that has not been considered yet: when a B meson decays producing a τ lepton, is followed by the decay of the lepton as $\tau \to l\nu\nu$. So, there can be confusion between different decays, and one needs to be able to separate out the things. This is done exploiting the differences in the kinematics of the two processes, since the leptons have an high mass difference, and the τ produces extra neutrinos. It has been understood [19] that it is possible to construct several variables which usefully separate the cases: the momentum of the μ in the B rest frame; the missing mass $m^2 = (p_B - p_{x_c} - p_l)^2$, where the p are respectively the momenta of the B, the hadronic system and the lepton; and the momentum transfer q^2.
The B rest frame is approximately reconstructed, assuming that the proper velocity of the visible part, along the Z-axis, of the semileptonic decays is equal to the proper velocity of the B. The remaining components are determined from the primary vertex of the experiment, giving

$$|p_B| = \frac{m_B}{m_Y}(p_Y)_z\sqrt{1 + tan^2\alpha}$$

where α is the angle between the vertex and the Z-axis.

The resolutions on the variables of interest are then distributions, more or less symmetric, peaked on the avarage value. In Figure 3.13 the resolution for $B \to D^*\tau\nu_\tau$ and $B \to D^*\tau\nu_\mu$. The effect of this resolution is to mix the predicted values. To implement the resolution in the fit, one has to multiply the vector of theoretical values with a matrix, called migration matrix, to have the predicted rates in the reconstructed quantities, that can be directly comparable with the real experiment.

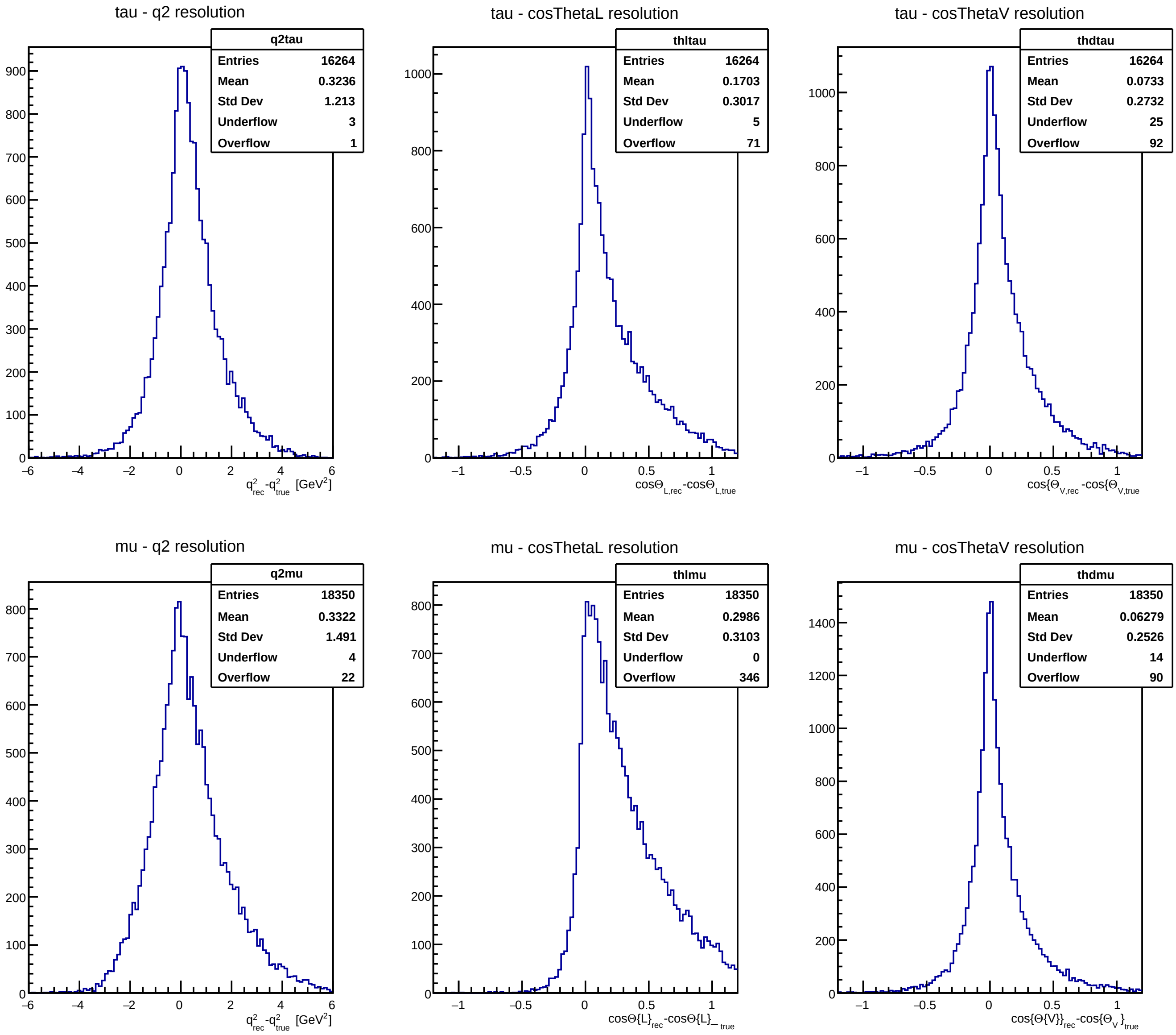

Figure 3.13: Resolution of the kinematic observables.

Chapter 4

Conclusions and Outlook

In this thesis, we tried to provide a theoretical explanation to the anomaly observed on the B decay. As said, we gave a possible solution inserting a tensor like contribution to the lagrangian, in an effective theory fashion. Although, this leaves unsolved the problem that we have no clue on where that operator comes from.

In the first chapter, we summarised the theoretical background needed to understand the topic. Beginning with an introduction on the Standard Model and flavour physics, to arrive at the problem of computing hadronic decays. To solve it, we introduced the form factors with their parametrisation, and we have seen how they can actually have a small influence on the results but, with enough data from experiments, it is possible to analytically derive them, in a completely independent way. In the second chapter, we have deeply analysed the two decay modes, $B \to D$ and $B \to D^*$, showing how it is possible to derive quantities sensible to new physics, such as the forward-backward asymmetry, even computing the difference in value with the standard model. In the third chapter, our feasibility study is reproduced, after a brief introduction to the Monte Carlo methods. Using the SL Decay class as an event generator, we managed to build pseudoexperiments that followed the correct distributions. In particular, a distribution given only by the contribution of the standard model, which served to check the correctness of the fit, and one with our hypothesis of new physics. The fit resulted to be in agreement with all the values from [4], paper used as a guide, demostrating its reliability. We took then the forward-backward asymmetry studied in the previous chapter and, with the pseudoexperiment, we were able to compute it for both $B \to D^*\mu\nu$ and $B \to D^*\tau\nu$, and with different number of events, to show how the statistics can affect the measurement. To deviate slightly from the perfect ideal case, we introduced an uncertainty on the asymmetry, propagated from the fitted parameters. The uncertainties appear as error bars that become smaller and smaller with the increasing number of events. As a last comment, we introduced the effect of the resolution, to come closer and closer to a real scenario.

We are now able to understand whether a possible real experiment is capable of distinguishing the contribution of new physics. The forward-backward asymmetry turned out to be a powerful observable to disentangle the Standard Model prediction from the new physics scenario we have considered. We believe therefore that the new physics has recognizable features that can be exploited to solve the B anomalies.

This has been a very preliminary study on how to explain the anomaly in the B decay and the focus has been only on one variable, even if we think the asymmetry is one of the most sensitive, but to understand completely the problem, it is mandatory to control all the possible ways in which deviations from the standard model can be observed. There are in fact several other quantities that can be studied.
As a final thought, we believe that, even if the measurement of the observables presented in this thesis could be challenging, in particular the decomposition of the amplitude in τ helicity, the incoming analyses at LHCb with the run 3 upgrade, expected in the next years, will be able to shed light on the topic and encourage new research.

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