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Scuola di Scienze
Dipartimento di Fisica e Astronomia
Corso di Laurea Magistrale in Fisica

Superspace Formulation of Higher-Derivative Actions from D-branes

Relatore:
Prof. Michele Cicoli

Correlatore:
Dott. David Ciupke

Presentata da:
Sara Maraventano

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Abstract

Higher-derivative operators play a fundamental role in the development of effective field theories. This thesis deals with the study of such operators in the context of the effective low-energy theory of type IIB superstring theory. In particular, we revisit a past attempt to find the supersymmetric form of the higher-derivatives for the D7-brane modulus in an orbifold compactification at the level of a globally supersymmetric theory. These higher-derivatives descend from the Dirac-Born-Infeld action of the D7-brane. Starting from the component Lagrangian the task is to identify the supersymmetric higher-derivative operator which reproduces the component terms. To this end, we develop a new systematic approach to determine this supersymmetric form and test it for the present example.

Sommario

Gli operatori alto-derivativi giocano un ruolo fondamentale nello sviluppo delle teorie di campo effettive. La presente tesi è volta ad affrontare lo studio di tali operatori nel contesto della teoria effettiva a basse energie della teoria di superstringa di tipo IIB.

In particolare, questo lavoro rivisita un precedente tentativo di individuare la forma supersimmetrica delle derivate di ordine superiore per il modulo di D7-brana in una compattificazione di tipo orbifold al livello di una teoria supersimmetrica globale. Tali derivate di ordine superiore provengono dall'azione di Dirac-Born-Infeld. Partendo dalle componenti della Lagrangiana, il nostro proposito è identificare l'operatore supersimmetrico alto-derivativo che riproduca correttamente i termini delle componenti. A tal fine sviluppiamo un nuovo approccio sistematico volto a determinare la forma supersimmetrica della Lagrangiana e lo applichiamo al caso in esempio.

Indice

Capitolo 1

Introduction

Despite its incredible predictive power and its many achievements, the Standard Model is considered an incomplete theory today. One of the most annoying issues is the hierarchy problem (i.e. the fact that the Higgs particle receives large corrections from loop diagrams). It would be desirable to find a guiding principle as powerful and as predictive as the Lorentz invariance and the gauge symmetry principle are in the construction of the Standard Model. Such a lighthouse in the night could be *supersymmetry* (SUSY). Supersymmetry solves the hierarchy problem in a very elegant way. Indeed, supersymmetry transformations provide for the exchange of bosons and fermions into each other through the action of fermionic operators. The invariance under supersymmetry requires a superpartner for each existing particle: this superpartner is nothing more than a particle having the same mass and quantum numbers but with spin differing by one-half, this means that every boson has its fermionic superpartner and vice versa. What happens then is that the bosonic and fermionic loop corrections cancel each other exactly, giving rise to the *miraculous cancellation* and solving the hierarchy problem; this solution comes effortlessly and in a completely natural way. The simplest supersymmetric extension of the Standard Model is known as Minimal Supersymmetric Standard Model (MSSM): it includes the smallest number of particles and interactions necessary for an agreement with phenomenology.

The miracle of cancellation is the most fruitful reason to guess the existence of supersymmetry. There are however many other reasons to consider the study of supersymmetric theories interesting. For example, the fact that supersymmetry acts by inducing an exchange between bosonic and fermionic particles, could lead us to think that it is a mere internal symmetry, but it is not so. The algebra of the supersymmetry charges shows that the supersymmetric transformations are intrinsically and intimately related to the spacetime transformations: acting with two consecutive supersymmetric transformations one obtains the starting field but evaluated in a different spacetime point. Thanks to this surprising property of supersymmetric transformations, passing from a global supersymmetric theory to a local one, one obtains Einstein's General Relativity, in the same way, by making $U(1)$ local, we obtain electromagnetism. This is why local SUSY is called *supergravity.* The natural occurrence of supergravity is another reason which makes the study of supersymmetry really interesting.

As we have anticipated, a theory invariant under supersymmetry requires the existence of superpartners, which unfortunately are not observed in nature. This is not enough, however, to discourage research in this field. For instance, in the Standard Model it is necessary to resort to a spontaneous symmetry breaking, the Higgs mechanism, in order to make the gauge bosons $W^{\pm}$ and Z massive. One could therefore expect that something analogous happens with supersymmetry. This is the reason why many scientists are interested in the study of supersymmetry breaking phenomena. A way to break SUSY is *spontaneous symmetry breaking* (SSB): in this case the Lagrangian is still symmetric but the ground state no longer is, the effects of this breaking are found in the absence of symmetry in the spectrum of the states. Since SSB mechanisms in the MSSM are not (yet) known, one is urged to investigate different ways to break this symmetry, for instance, one can break supersymmetry by hand, by adding terms which are not invariant under supersymmetry. In these cases we talk about *explicit supersymmetry breaking.* But this breaking cannot take place in a completely random way, otherwise we would lose all the benefits given by the introduction of supersymmetry, for instance the miraculous cancellation of quadratic divergences. We are therefore willing to introduce some renormalizable terms which break supersymmetry *softly*, the so called *soft-breaking terms.* In this context, auxiliary fields play a fundamental role. They are fields introduced so that the number of bosonic and fermionic off-shell components (i.e. before the equations of motion are imposed) is the same: we will see that SSB occurs when the vacuum expectation value of these fields is non-vanishing. One of the reasons why the soft breaking terms are so interesting is that one of the most popular proposals considers the assumption of the existence of a *hidden sector* in which the symmetry is spontaneously broken: this breaking induces some repercussions on the observable sector (for example MSSM) through the arising of soft-breaking terms. This interaction between the hidden sector and the observable sector occurs through the so-called *gravity mediation.*

As we have anticipated, despite its numerous advantages, the introduction of supersymmetry makes things a bit more involved: the algebra of the generators must be extended in order to include the fermionic generators of supersymmetry, which mix in a non-trivial way internal and spacetime symmetries. The formal setup in which it is most simple to develop a supersymmetric theory is the superspace formalism: thanks to the properties of the Grassmann variables (which, as we will see in the first chapter, constitute the coordinates of the superspace, together with the ordinary space-time coordinates) the formalism of the superspace makes the formulation of supersymmetric theories very simple, or at least simpler than it would be using a classic approach à la Quantum Field Theory on an ordinary space-time .

Another reason why the study of supersymmetry is so exciting comes from string theory: a string theory involving the same number of bosons and fermions naturally gives rise to a supersymmetric theory. In particular, it is possible to understand supergravity as a low-energy effective string theory. Within this project we are interested in type IIB string theory, whose low-energy limit gives rise to the type IIB 10-dimensional $\mathcal{N} = 2$ supergravity, where one has four space-time dimensions and six extra dimensions. The absence of experimental evidences for the extra dimensions leads to the introduction of a geometric trick known as compactification: the extra dimensions are confined into very small sizes, wrapped on some suitable compact manifolds. This panorama includes objects inherited from string theory: the Dp-branes. These are $(p + 1)$-dimensional objects whose position in the internal manifold is parametrized by scalar fields known as position moduli. The brane world-volume is embedded in a higher dimensional spacetime known as *bulk*. Then, the geometric background of the low-energy effective theory of a string theory gives rise to the moduli, which play a fundamental role, for instance, in inflation theories where the bulk- or the brane-modulus corresponds to the inflaton [**?**, **?**, **?**, **?**].

Our work will focus on the study of the D7-brane dynamics on a torus orbifold compactification. Being an 8-dimensional object in 10 dimensions, its position in the internal manifold can be described by a single complex scalar field. We will study small fluctuations of the branes in the normal direction, with particular attention to the higher order corrections arising from the expansion of the Dirac-Born-Infeld Lagrangian. As we will see these corrections are of higher-derivative type.

The study of higher-derivative operators is interesting for many reasons: integrating out massive states generically generates higher-derivative operators in the low energy effective action, their discussion is therefore unavoidable. In particular, this holds for the effective supergravities of string theories, see e.g. [**?**].

There are also some interesting motivations to investigate higher-derivative operators in the setup of the D-branes. In particular, it is not clear (yet) how to determine the supersymmetric form of an effective action obtained from some string compactification including higher-derivative operators. Some attempts have been made, but the subject remains mostly unexplored [**?**, **?**, **?**, **?**]. Our study is aimed at conceptually understanding how a systematic matching can in principle be obtained. This will be clarified by the simple D7-brane example. The D7-brane moduli have the advantage that it is easy to include higher-derivative contributions to their action. Moreover, the multiplet structure of the moduli is usually easy to identify and higher-derivative terms for both the real and imaginary part of the chiral superfield corresponding for example to the D-brane position, are available. On the contrary the bulk moduli investigated in [**?**, **?**] combine into multiplets in a more complicated way and their higher-derivative terms are only partially known. Therefore, in order to develop this investigation, we start in Chapter 2 with an overview on global and local supersymmetry. In this chapter we show how the formalism of superspace is useful in the construction of a supersymmetric Lagrangian and we provide a list of the supersymmetric higher-derivative operators. Furthermore, we introduce the notion of supersymmetry breaking presenting first the F-terms breaking, which is an example of spontaneous supersymmetry breaking, and then the soft supersymmetry breaking arising in the observable sector. In Chapter 3 we provide a brief introduction on type IIB supergravity, compactification and orientifolding. Then we present the Dirac-Born-Infeld (DBI) and the Chern-Simons (CS) actions for the D-branes: we focus on the scalar potential induced by the background fluxes from the Dirac-Born-Infeld action for the D7-branes and we show that it is exactly the one obtained from the $\mathcal{N} = 1$ supergravity computation. In the end, we show how the DBI and CS reduction works in the case of a single D3-brane. In Chapter 4 we discuss the role of supersymmetric higher-derivative operators in the effective theory for the position moduli of type IIB D7-branes. We provide the list of the four derivative terms and integration-by-part identities necessary to operate the match between the higher-derivative component Lagrangian and the supersymmetric higher-derivative operators.

Capitolo 2

Supersymmetry and Supergravity

We begin this section with an overview on supersymmetry and supergravity following the outline of [?] and [?], defining the key concepts of superspace and superfield. Focusing our attention on the chiral superfields, we provide the supersymmetric Lagrangian of these special superfields. Next we introduce the general higher-derivative supersymmetric Lagrangian, supplying a list of the independent four-derivative operators. After an introduction on supergravity, we briefly analyze supersymmetry breaking, first in a global supersymmetry context and then extending the treatment to supergravity with a particular interest in the soft breaking of supersymmetry.

2.1 Superfields and Superspace

In this section we provide a summary of the main features of supersymmetry. At first we present the *graded algebra*, an unavoidable block in the construction of supersymmetric theories. Supersymmetry provides an extension of the ordinary concept of symmetry as it also accounts for fermionic variables. For this reason we need an algebra which includes fermionic operators. We proceed with the basics on *superspace* and, through the definition of the *Grassmann variables*, we introduce some extra coordinates which are added to the ordinary space-time ones in order to shape the supercoordinates on the superspace. *Superfields* are fields living on the superspace, in the following we present their main proprieties. Finally we treat *supersymmetry transformations* as a natural extension of the Poincaré symmetry in which the added fermionic operators are a representation of the spinorial generators found in the graded algebra. The introduction of the supersymmetric transformations allows us to define the *chiral covariant derivative* and, as a consequence, the *chiral superfields.*

2.1.1 Graded Algebra

The goal of this paragraph is to extend the well-know Poincaré algebra by adding fermionic operators. Let O_b be the operators of a Lie algebra, then:

$$O_a O_b - (-1)^{\gamma_a \gamma_b} O_b O_a = i C^d_{ab} O_d \tag{2.1}$$

where γ_a are the *gradings*:

$$\gamma_a = \begin{cases} 0 & \text{if } O_a \text{ is a bosonic generator} \\ 1 & \text{if } O_a \text{ is a fermionic generator} \end{cases} . \tag{2.2}$$

In supersymmetry both the usual Poincarè generators P^μ, $M^{\mu\nu}$ and the spinor generators Q^A_α, $\bar{Q}^A_{\dot{\alpha}}$, where $A = 1, ..., \mathcal{N}$ must be taken into account. We will only consider the cases in which $\mathcal{N} = 1$.

The commutation relations of the extended Poincaré algebra are found to be [?]:

$$\begin{aligned} &[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta, \qquad \sigma_{\mu\nu} = \frac{i}{4}(\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu) \\ &[Q_\alpha, P^\mu] = [\bar{Q}_{\dot{\alpha}}, P^\mu] = 0 \\ &\{Q_\alpha, Q_\beta\} = 0 \\ &\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu \end{aligned} \tag{2.3}$$

So we can see that two fermionic symmetry transformations have the effect of a translation. Consider a bosonic state $|B\rangle$ and a fermionic state $|F\rangle$, then the following relations hold

$$Q_\alpha |F\rangle = |B\rangle, \quad \bar{Q}_{\dot{\beta}} |B\rangle = |F\rangle \Rightarrow \quad Q\bar{Q}|B\rangle \mapsto |B(translated)\rangle. \tag{2.4}$$

2.1.2 Basics on Superspace and Superfields

As we have seen in the introduction, the most comfortable setup for the development of a supersymmetric theory is the superspace formalism. Superspace is a manifold which is provided with four fermionic coordinates in addition to the usual bosonic spacetime ones. A point in superspace is labelled by $(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$ where θ^α and $\bar{\theta}_{\dot{\alpha}}$ are two−component Grassmann number spinors with mass dimensions $[\theta] = -1/2$.

The main property of a single Grassmann variable is that θ squares to zero, $\theta^2 = 0$, which implies that when expanding any function of θ as a power series, only the first two terms are non-zero:

$$f(\theta) = \sum_{k=0}^{\infty} f_k \theta^k = f_0 + f_1 \theta. \tag{2.5}$$

One defines the Berezin integral:

$$\int d\theta \frac{df}{d\theta} := 0 \Rightarrow \int d\theta = 0 \tag{2.6}$$

$$\int d\theta\theta := 1 \Rightarrow \delta(\theta) = \theta \tag{2.7}$$

where the derivation is computed as usual. This implies that the integral over a function $f(\theta)$ corresponds to its derivative,

$$\int d\theta f(\theta) = f_1 = \frac{df}{d\theta} \tag{2.8}$$

So, defining the squares of Grassmann spinors as

$$\begin{aligned} &\theta\theta := \theta^\alpha\theta_\alpha, \qquad \bar{\theta}\bar{\theta} := \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} \\ \Longrightarrow\ &\theta^\alpha\theta^\beta = -\frac{1}{2}\epsilon^{\alpha\beta}\theta\theta, \qquad \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}\theta\theta \end{aligned} \tag{2.9}$$

the integration is now defined via:

$$\int d^2\theta := \frac{1}{2}\int d\theta^1 \int d\theta^2, \qquad \int d^2\theta\ \theta\theta = 1, \qquad \int d^2\theta \int d^2\bar{\theta}(\theta\theta)(\bar{\theta}\bar{\theta}) = 1 \tag{2.10}$$

and one can again identify integration and differentiation.

Due to these proprieties of the fermionic coordinates of the superspace, one can express a general scalar superfield $S(x, \theta, \bar{\theta})$ as an expansion in powers of θ and $\bar{\theta}$ carrying a finite numbers of non-zero terms. Its components are functions of x^μ:

$$\begin{aligned} S(x, \theta, \bar{\theta}) =&\phi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta M(x) + \bar{\theta}\bar{\theta}N(x) + (\theta\sigma^\mu\bar{\theta})V_\mu(x) \\ &+ (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\rho(x) + \theta\theta)(\bar{\theta}\bar{\theta})D(x). \end{aligned} \tag{2.11}$$

Note that the components of the superfield are 8 bosonic fields ($\phi(x)$, $M(x)$, $N(x)$, $D(x)$, $V_\mu(x)$) and 4 two-component fermionic fields (ψ, $\bar{\chi}$, $\bar{\lambda}$, ρ) and all of them are complex functions of x^μ. So the number of bosonic and fermionic degrees of freedom is exactly the same.

2.1.3 Supersymmetry Transformation

Let us define the differential operators acting on the superfields:

$$\mathcal{Q}_\alpha = -(i\frac{\partial}{\partial\theta^\alpha} + (\sigma^\mu)_{\alpha\dot\beta}\bar\theta^{\dot\beta}\frac{\partial}{\partial x^\mu})$$
$$\bar{\mathcal{Q}}_\alpha = i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \theta^\beta(\sigma^\mu)_{\beta\dot\alpha}\frac{\partial}{\partial x^\mu} \tag{2.12}$$

which satisfy the following commutation relations:

$$\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\alpha}\} = -i2(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu$$
$$\{\mathcal{Q}_{\dot\alpha}, \mathcal{Q}_\beta\} = 0 = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{Q}}_{\dot\beta}\} . \tag{2.13}$$

These operators are nothing more than a representation of the spinorial generators Q_α acting on functions of θ, $\bar\theta$ and x. Then the supersymmetry transformation of a general superfield S, parametrized by infinitesimal ϵ and $\bar\epsilon$, at the first order is given by:

$$\begin{aligned}\delta_\epsilon S &= i[S, \epsilon\mathcal{Q} + \bar\epsilon\bar{\mathcal{Q}}] = i(\epsilon\mathcal{Q} + \bar\epsilon\bar{\mathcal{Q}})S \\ &= S(x^\mu - i\sigma^\mu\bar\theta + i\theta\sigma^\mu\bar\epsilon, \theta + \epsilon, \bar\theta + \bar\epsilon) - S(x^\mu, \theta, \bar\theta),\end{aligned} \tag{2.14}$$

this shows how supersymmetry can be viewed as a translation in superspace. One can obtain the supersymmetry transformations of all the component fields of the superfields:

$$\begin{aligned}
\delta\phi &= \epsilon\psi + \bar\epsilon\bar\chi \\
\delta\psi &= 2\epsilon M + \sigma^\mu\bar\epsilon(i\partial_\mu\phi + V_\mu) \\
\delta\bar\chi &= 2\bar\epsilon N - \epsilon\sigma^\mu(i\partial_\mu\phi - V_\mu) \\
\delta M &= \bar\epsilon\bar\lambda - \frac{i}{2}\partial_\mu\psi\sigma^\mu\bar\epsilon \\
\delta N &= \epsilon\rho + \frac{i}{2}\epsilon\sigma^\mu\partial_\mu\bar\chi \\
\delta V_\mu &= \epsilon\sigma_\mu\bar\lambda + \rho\sigma_\mu\bar\epsilon + \frac{i}{2}(\partial^\nu\psi\sigma_\mu\bar\sigma_\nu - \bar\epsilon\bar\sigma_\nu\sigma_\mu\partial^\nu\bar\chi) \\
\delta\bar\lambda &= 2\bar\epsilon D + \frac{i}{2}(\bar\sigma^\nu\sigma^\mu\bar\epsilon)\partial_\mu V_\nu + i\bar\sigma^\mu\epsilon\partial_\mu M \\
\delta\rho &= 2\epsilon D - \frac{i}{2}(\sigma^\nu\bar\sigma^\mu\epsilon)\partial_\mu V_\nu + i\sigma^\mu\bar\epsilon\partial_\mu N \\
\delta D &= \frac{i}{2}\partial_\mu(\epsilon\sigma^\mu\bar\lambda - \rho\sigma^\mu\bar\epsilon).
\end{aligned} \tag{2.15}$$

Note that, if S is a superfield, $\partial_\mu S$ is a superfield too, but the same can't be said for $\partial_\alpha S$ because of the commutation relation $[\partial_\alpha, \epsilon Q + \bar{\epsilon}\bar{Q}] \neq 0$. One is thus induced to define the covariant derivative as:

$$
\begin{aligned}
D_\alpha &:= \partial_\alpha + i(\sigma^\mu)_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \\
\bar{D}_{\dot{\alpha}} &:= -\bar{\partial}_{\dot{\alpha}} - i\theta^\beta(\sigma^\mu)_{\beta\dot{\alpha}}\partial_\mu
\end{aligned}
\tag{2.16}
$$

This operator satisfies the commutation relations

$$
\begin{aligned}
&\{D_\alpha, \bar{D}_{\dot{\beta}}\} = 2i(\sigma^\mu)_{\alpha\dot{\beta}} \\
&\{D_\alpha, D_\beta\} = 0 = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} \\
&[D_\alpha, \epsilon Q + \bar{\epsilon}\bar{Q}] = 0
\end{aligned}
\tag{2.17}
$$

which implies that $D_\alpha S$ is a superfield.

At this point it is useful to note that the operators:

$$
\int d^2\theta D_\alpha \qquad \text{and} \qquad \int d^2\bar{\theta}\bar{D}_{\dot{\alpha}} \tag{2.18}
$$

each are a total derivative with respect to x^μ. It is also useful to note that the action of three consecutive D_α operators vanishes due to the commutation relation (**??**).

Chiral Superfields

A chiral superfield is defined as a superfield Φ such that $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$ (analogously we define a antichiral superfield as a superfiels that satisfies $D_\alpha\bar{\Phi} = 0$). This condition imposes some restrictions on the components of the superfields so we want to find the explicit component form of a chiral superfield. To this aim let us define

$$
y^\mu := x^\mu + i\theta\sigma^\mu\bar{\theta}. \qquad \Longrightarrow \qquad \Phi = \Phi(y, \theta, \bar{\theta}), \tag{2.19}
$$

In this basis one can quickly verify that [**?**]:

$$
\bar{D}_{\dot{\alpha}}\Phi = -\bar{\partial}_{\dot{\alpha}}\Phi = 0 \tag{2.20}
$$

so there is no $\bar{\theta}$-dependence and the expansion of the chiral field is quite simple:

$$
\Phi(y, \theta) = \phi(y^\mu) + \sqrt{2}\theta\psi(y^\mu) + \theta\theta F(y^\mu). \tag{2.21}
$$

Returning to the original basis one gets:

$$\begin{aligned}\Phi(x,\theta,\bar\theta) =&\phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(x)\\ &- \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar\theta - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial_\mu\partial^\mu\varphi(x)\end{aligned} \tag{2.22}$$

There are 4 bosonic complex components (ϕ, F) and 4 fermionic complex ones ψ_α. Under a supersymmetry transformation

$$\delta\Phi = i(\epsilon\mathcal{Q} + \bar\epsilon\bar{\mathcal{Q}})\Phi \tag{2.23}$$

the components are transformed as:

$$\begin{aligned}\delta\phi &= \sqrt{2}\epsilon\psi\\ \delta\psi &= i\sqrt{2}\sigma^\mu\bar\epsilon\partial_\mu\phi + \sqrt{2}\epsilon F.\\ \delta F &= i\sqrt{2}\bar\epsilon\bar\sigma^\mu\partial_\mu\psi\end{aligned} \tag{2.24}$$

It is also useful to remark that any holomorphic function $f(\Phi)$ of a chiral superfield Φ is chiral.

Vector Superfields

A superfield $V(x,\theta,\bar\theta)$ which satisfies the condition $V = \bar V$ is called *vector* superfield. The most general form of a vector superfield is given by:

$$\begin{aligned}V(x,\theta,\bar\theta) =&C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \frac{i}{2}\theta\theta(M(x)+iN(x)) - \frac{i}{2}\bar\theta\bar\theta(M(x)-iN(x))\\ &+ \theta\sigma^\mu\bar\theta A_\mu(x) + i\theta\theta\bar\theta(-i\bar\lambda(x) + \frac{i}{2}\bar\sigma_\mu\partial_\mu\chi(x))\\ &- i\bar\theta\bar\theta\theta(i\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)(D(x) - \frac{1}{2}\partial_\mu\partial^\mu C(x)).\end{aligned} \tag{2.25}$$

Where C, M, N, D, A_μ are 8 bosonic components and χ_α and λ_α are 4+4 fermionic ones. Under supersymmetry transformations the vector superfield's component variations are given by:

$$\begin{aligned}\delta A_\mu &= -i\epsilon(\bar\sigma^\mu\lambda + \partial_\mu\chi) + i\bar\epsilon(\sigma^\mu\bar\lambda + \partial_\mu\bar\chi)\\ \delta\lambda &= \epsilon D + \frac{i}{2}\sigma^\mu\bar\sigma^\nu(\partial_\mu A_\nu - \partial_\nu A_\mu)\\ \delta D &= \epsilon\sigma^\mu\partial_\mu\bar\lambda + \bar\epsilon\bar\sigma^\mu\partial_\mu\lambda\end{aligned} \tag{2.26}$$

The components of the chiral superfield S and of the vector superfield $V = i(S - \bar{S})$ are related by:

$$\begin{aligned}
&C = i(\phi - \bar{\phi}) \\
&\chi = \sqrt{2}\psi \\
&M + iN = 2F \\
&V_\mu = -\partial_\mu(\phi + \bar{\phi}) \\
&\lambda = D = 0.
\end{aligned} \tag{2.27}$$

2.2 Two-derivative Supersymmetric Lagrangian

In this section we want to build a Lagrangian of a chiral superfield which is invariant under supersymmetry transformations. We will now see that it is composed of D-type and F-type terms. This means that $(\theta\theta)$ and $(\theta\theta\bar{\theta}\bar{\theta})$ type terms will appear in the component Lagrangian. This fact allows to obtain the Lagrangian in the form of an integral on superspace. In order to build a supersymmetric Lagrangian $\mathcal{L}(\Phi)$ of a single chiral superfield let us consider the last lines of equations (**??**) and (**??**). The variations of the D and F terms are total derivatives:

$$\begin{aligned}
\delta D &= \frac{i}{2}\partial_\mu(\epsilon\sigma^\mu\bar{\lambda} - \rho\sigma^\mu\bar{\epsilon}) \\
\delta F &= i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi
\end{aligned} \tag{2.28}$$

This fact allows us to build a Lagrangian starting from D and F-terms in order to obtain a null (up to a total derivative) variation under a supersymmetry transformation $\delta\mathcal{L}$.

The most general supersymmetric Lagrangian for a collection of chiral superfields $\Phi_1, \ldots, \Phi_N$ can thus be written as:

$$\mathcal{L} = K(\Phi_i, \bar{\Phi}_{\bar{j}})|_D + W(\Phi_i)|_F + h.c. \tag{2.29}$$

where the function K is the *Kähler potential* which is a real function of Φ_i and $\bar{\Phi}_{\bar{j}}$ and W is a holomorphic function of Φ_i known as the *superpotential.* The notation $|_{D,F}$ means that we are considering D and F-terms of the corresponding superfields. This is an example of the power of the superspace formalism: it is possible to multiply an arbitrary number of chiral superfields and by extracting the F-term one obtains a superpotential and then a supersymmetric theory. Within the multitude of theories that could be obtained, we are only interested in renormalizable theories, i.e. with a Lagrangian having mass-dimension 4. Keeping in mind that the extraction of the F-type terms requires the use of the integral on superspace:

$$\int d^2\theta \tag{2.30}$$

which has mass-dimension 1, one has that the maximum number of chiral superfields of mass-dimension 1 which can be used for the construction of a renormalizable superpotential is three. In the same way, the extraction of the D-term increases the dimension by 2 through the integral on superspace:

$$\int d^4\theta. \tag{2.31}$$

Let us have a look at the dimensions of the objects appearing in the Lagrangian in order to obtain the explicit form of K and W in terms of the chiral fields Φ_i. We know that:

$$\begin{aligned} &[\mathcal{L}] = 4 \\ &[\Phi_i] = [\phi_i] = 1, \qquad [\psi_i] = \frac{3}{2} \end{aligned} \tag{2.32}$$

so we can quickly verify that $[\theta] = -\frac{1}{2}$. Now we have:

$$\begin{aligned} K|_D = (\theta\theta)(\bar{\theta}\bar{\theta})K_D \quad &\Rightarrow \quad [K_D] = 4,\ [K] = 2 \\ W|_F = (\theta\theta)W_F \quad &\Rightarrow \quad [W_F] = 4,\ [W] = 3 \end{aligned} \tag{2.33}$$

Then due to the form of the D and F-terms and the properties of the Grassmann variables, one can express the Lagrangian as an integral on superspace:

$$\mathcal{L} = \int d^4\theta\, K + \int d^2\theta\, W + h.c \tag{2.34}$$

In terms of the superfield components this is

$$\mathcal{L} = -K_{i\bar{j}}\partial_\mu\phi_i\partial^\mu\bar{\phi}_{\bar{j}} + K_{i\bar{j}}F^i\bar{F}^{\bar{j}} + \frac{\partial W}{\partial\phi_i}F_i + \frac{\bar{W}}{\partial\bar{\phi}_{\bar{i}}}\bar{F}^i \tag{2.35}$$

where we ignored the fermionic terms. Here $K_{i\bar{j}} = \dfrac{\partial^2 K}{\partial\phi_i\partial\bar{\phi}_{\bar{j}}} = (K^{i\bar{j}})^{-1}$ is the *Kähler metric.* A Kähler manifold is a special type of complex manifold such that the metric $K_{i\bar{j}}$ can be expressed as a second derivative of a scalar function K which is the Kähler potential. The metric and therefore the geometry of the Kähler manifold is invariant under the analytical transformations:

$$K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + F(\Phi) + \bar{F}(\bar{\Phi}) \tag{2.36}$$

which are called Kähler transformations.

We can see that F is a non-propagating auxiliary field which can be integrated out via the equations of motion:

$$\bar{F}^{\bar{i}} = -K^{j\bar{i}} \frac{\partial W}{\partial \phi_j}. \tag{2.37}$$

and substituting the result back into the Lagrangian one obtains

$$\mathcal{L} = -K_{i\bar{j}} \partial_\mu \phi_i \partial^\mu \bar{\phi}_{\bar{j}} - V_F(\phi) \tag{2.38}$$

where the scalar potential is

$$V_F(\phi_i, \bar{\phi}_{\bar{j}}) := K^{i\bar{j}} \frac{\partial W}{\partial \phi_i} \frac{\partial \bar{W}}{\partial \bar{\phi}_{\bar{j}}} \tag{2.39}$$

which appears in the Lagrangian with a minus sign. In the case of $N = 1$ we have $i = j = 1$.

Then, for a renormalizable theory, K and W take the following form:

$$K = \bar{\Phi}\Phi, \quad W = \alpha + \beta\Phi + \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3. \tag{2.40}$$

2.3 Four-derivative Supersymmetric Operators

In this section we discuss supersymmetric higher-derivative operators in superspace, neglecting the fermionic terms for simplicity. When higher-derivative operators are included in supersymmetric theories the Kähler potential becomes a real function of Φ, $\bar{\Phi}$ and their higher-derivatives. The superpotential instead becomes a function of chiral higher-derivative fields.

A general higher-derivative Lagrangian can be written as [**?**] :

$$\mathcal{L} = \int d^4\theta K(\Phi, \bar{\Phi}, D\Phi, \bar{D}\bar{\Phi}, D^2\Phi, ...) + \int d^2\theta W(\Phi, \bar{D}^2\Phi, ...) + h.c. \tag{2.41}$$

Higher-derivative operators of particular interest are the supersymmetric ones, in particular we are interested in real four-derivative operators: there are a lot of combinations which satisfy the reality requirement but they are not all independent, we can indeed relate one to the other by partial integration.

A set of independent supersymmetric four-derivative operators with each two chiral superfields Φ and two anti-chiral superfields $\bar{\Phi}$ is composed of the following three operators [?, ?, ?]:

$$
\begin{aligned}
\mathcal{O}_1 &= |\Phi|^2 D^2\Phi \bar{D}^2\bar{\Phi}|_D \\
&= 16|\phi|^2\Box\phi\Box\bar{\phi} + 20|F|^2\bar{\phi}\Box\phi + 20|F|^2\phi\Box\bar{\phi} + 16|F|^4 - 8|F|^2\partial_\mu\phi\partial^\mu\bar{\phi} \\
&+ 4|\phi|^2 F\Box\bar{F} + 4|\phi|^2\bar{F}\Box F - 8|\phi|^2\partial_\mu F\partial^\mu\bar{F} + 8\bar{\phi}F\partial_\mu\phi\partial^\mu\bar{F} \\
&- 8\bar{\phi}\bar{F}\partial_\mu\phi\partial^\mu F + 8\phi\bar{F}\partial_\mu\bar{\phi}\partial^\mu F - 8\phi F\partial_\mu\bar{\phi}\partial^\mu\bar{F} \\
\\
\mathcal{O}_2 &= \bar{\Phi}\bar{D}^2\bar{\Phi}D\Phi D\Phi|_D \\
&= 16\partial_\mu\phi\partial^\mu\phi\bar{\phi}\Box\bar{\phi} - 16|F|^2\bar{\phi}\Box\phi + 16|F|^2\partial_\mu\bar{\phi}\partial^\mu\phi - 16|F|^4 \\
&+ 16\bar{\phi}\bar{F}\partial_\mu\phi\partial^\mu F - 16\bar{\phi}F\partial_\mu\phi\partial^\mu\bar{F} \\
\\
\mathcal{O}_3 &= |\Phi|^2 D\bar{D}\bar{\Phi}\bar{D}D\Phi|_D \\
&= 8(\partial_\mu\phi\partial^\mu\bar{\phi})^2 + 8\phi\partial_\mu\bar{\phi}(\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\phi - \partial_\nu\phi\partial^\mu\partial^\nu\bar{\phi}) \\
&- 8|\phi|^2\partial_\mu\phi\partial^\mu\Box\bar{\phi} - 8|\phi|^2\partial_\mu F\partial^\mu\bar{F} - 8|F|^2\partial_\mu\phi\partial^\mu\bar{\phi} \\
&- 8\bar{\phi}F\partial_\mu\phi\partial^\mu\bar{F} - 8\phi\bar{F}\partial_\mu\bar{\phi}\partial^\mu F.
\end{aligned}
\tag{2.42}
$$

In principle, further operators with different numbers of $\Phi, \bar{\Phi}$ exist but we do not need them later. In appendix **??** we will demonstrate that these operators coincide exactly with those reported in [?] and [?].

We will also see how the fourth operator which appears in [?] is linearly dependent from the three operators (**??**) and is therefore not necessary in the constitution of a basis of supersymmetric operators. Making use of these operators one can eventually complete the component Lagrangian in order to obtain a manifestly supersymmetric Lagrangian.

2.4 Supergravity

Now we provide an overview on supergravity [?]. We give an interpretation of supergravity as a gauge theory and we summarize the formulation of supergravity on superspace. We analyze the supergravity Lagrangian $\mathcal{L}_{SG}$ coupled to matter and we show how, in the limit of flat space, one obtains the SUSY Lagrangian discussed in the previous section.

Making the parameter ϵ a function of spacetime coordinates $\epsilon(x)$ we extend supersymmetry to a local symmetry: the corresponding theory is called *supergravity*. We begin by explaining the theory of pure supergravity without matter-coupling. The gauge field of supergravity is the *gravitino* Ψ^μ_α which is coupled to the *supercurrent* $\mathcal{J}^\mu_\alpha$. This supercurrent gives rise to the supersymmetric charge

$$Q_\alpha = \int d^3x \mathcal{J}^0_\alpha. \tag{2.43}$$

The gravitino is part of the *supergravity multiplet* $(h_{\mu\nu}, \Psi^\mu)$ together with a graviton linear excitation $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \qquad \kappa = \sqrt{\frac{8\pi}{M_{pl}^2}} \tag{2.44}$$

Ψ^μ and $h_{\mu\nu}$ are governed by the Rarita Schwinger and the Einstein Hilbert actions respectively:

$$\begin{aligned} S_{RS}[\Psi] &= \frac{1}{2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma \\ S_{EH}[h] &= -\frac{1}{2}\int d^4x h^{\mu\nu}(R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R). \end{aligned} \tag{2.45}$$

In the Rarita Schwinger action we made use of the Dirac spinor notation whereas $R_{\mu\nu}$ and R, appearing in the Einstein Hilbert action, are the Ricci tensor and the Ricci scalar respectively. Promoting the parameter of the supersymmetry transformation to a function of the spacetime coordinates, the total action becomes:

$$\delta_\epsilon S_{total} = \delta_\epsilon(S_{RS}[\Psi] + S_{EH}[h]) = \int d^4x \mathcal{J}^\mu\partial_\mu\epsilon \tag{2.46}$$

and in close analogy with the electromagnetic case the interaction term

$$S_{int}[\Psi, h] = -\frac{\kappa}{2}\int d^4\mathcal{J}^\mu\Psi_\mu \tag{2.47}$$

can be used to restore the invariance.

The most convenient formulation of supergravity is the superspace formulation which makes use of an extension of the superfields to the local supersymmetry.

2.4.1 $\mathcal{N} = 1$ Supergravity Coupled to Matter

In this section we would like to couple supergravity to chiral superfields [?]. In this environment the definition of chiral field proposed in section (??) must adapt to curved spaces. For this reason we substitute the chiral derivative (??) with the covariant derivative $\mathcal{D}_\alpha$. Therefore the definition of chiral superfield on curved spaces is:

$$\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0 \tag{2.48}$$

which reduces to $\bar{D}_{\dot\alpha}\Phi = 0$ in the limit of flat space. Now the components of the chiral superfield are

$$(\phi, \chi_\alpha, F) \tag{2.49}$$

The theory in curved space is described again by K, W. The form in components of the Lagrangian is in fact given by:

$$\begin{aligned}
\mathcal{L} \propto & -\frac{1}{2}\mathcal{R} - K_{i\bar{j}}\partial_\mu\phi_i\partial^\mu\bar{\phi}_{\bar{j}} \\
& - iK_{i\bar{j}}\bar{\chi}_{\bar{j}}\bar{\sigma}^\mu\mathcal{D}_\mu\chi_i + \epsilon^{\mu\nu\rho\lambda}\bar{\Psi}_\mu\bar{\sigma}_\nu\tilde{\mathcal{D}}_\rho\Psi_\lambda \\
& - \frac{1}{2}\sqrt{2}K_{i\bar{j}}\partial_\mu\bar{\phi}_{\bar{j}}\chi_i\sigma^\mu\bar{\sigma}^\nu\Psi_\nu - \frac{1}{2}\sqrt{2}K_{i\bar{j}}\partial_\mu\phi_i\bar{\chi}_{\bar{j}}\bar{\sigma}^\mu\sigma^\nu\bar{\Psi}_\nu \\
& + \frac{1}{4}K_{i\bar{j}}[i\epsilon^{\mu\nu\rho\lambda}\Psi_\mu\sigma_\nu\bar{\Psi}_\rho + \Psi_\rho\sigma^\lambda\bar{\Psi}^\rho]\chi_i\sigma_\lambda\bar{\chi}_{\bar{j}} \\
& - \frac{1}{8}[K_{i\bar{j}}K_{k\bar{l}} - 2R_{i\bar{j}k\bar{l}}]\chi_i\chi_k\bar{\chi}_{\bar{j}}\bar{\chi}_{\bar{l}} \\
& - \exp(K/2)\{\bar{W}\Psi_\alpha\sigma^{\alpha\beta}\Psi_\beta + W\bar{\Psi}_\alpha\bar{\sigma}^{\alpha\beta}\bar{\Psi}_\beta \\
& + \frac{i}{2}\sqrt{2}(D_iW\chi_i\sigma^\alpha\bar{\Psi}_\alpha + \bar{D}_{\bar{i}}\bar{W}\bar{\chi}_{\bar{i}}\bar{\sigma}^\alpha\Psi_\alpha) \\
& + \frac{1}{2}\mathcal{D}_iD_jW\chi_i\chi_j + \frac{1}{2}\bar{\mathcal{D}}_{\bar{i}}\bar{D}_{\bar{j}}\bar{W}\bar{\chi}_{\bar{i}}\bar{\chi}_{\bar{j}} \\
& - \exp(K)[K^{i\bar{j}}(D_iW)(\bar{D}_{\bar{j}}\bar{W}) - 3\bar{W}W]\}
\end{aligned} \tag{2.50}$$

where the covariant derivatives are defined as:

$$\begin{aligned}
\mathcal{D}_\mu\chi_i &= \partial_\mu\chi_i + \chi_i\omega_\mu + \Gamma^i_{jk}\partial_\mu\phi_j\chi_k - \frac{1}{4}(K_j\partial_\mu\phi_j - K_{\bar{j}}\partial_\mu\bar{\phi}_{\bar{j}})\chi_i \\
\tilde{\mathcal{D}}_\mu\Psi_\nu &= \partial_\mu\Psi_\nu + \Psi_\nu\omega_\mu + \frac{1}{4}(K_j\partial_\mu\phi_j - K_{\bar{j}}\partial_\mu\bar{\phi}_{\bar{j}})\Psi_\nu \\
D_iW &= W_i + K_iW \\
\mathcal{D}_iD_iW &= W_{ij} + K_{ij}W + K_iD_jW + K_jD_iW - K_iK_jW - \Gamma^k_{ij}D_kW.
\end{aligned} \tag{2.51}$$

Where Γ^k_{ij} is the Christoffel symbol for the Kähler manifold, and ω_μ is the spin connection for spacetime. In the global limit the chiral covariant derivatives reduce to their counterpart in flat space, the curvature vanishes and one obtains the flat solution presented in the previous chapter.

Note that also the higher-derivative operators (**??**) have a generalization in curved superspace provided by [**?**]. It must be stressed that on curved superspace the list of higher-derivative operators is composed of four independent operators (not three as in flat space) and they can be traced back to the list (**??**) in the global limit. The presence of a fourth independent operator is attributable to the fact that in supergravity there exist high-derivative objects such as the Riemann tensor, which vanish in the limit of flat space. Note that in [**?**] are also listed two higher-derivative operators which do not have a rigid counterpart and which are intrinsically linked to the Riemann tensor.

2.5 Supersymmetry Breaking

In this section we start analyzing some generalities about SUSY breaking, including F-term breaking. In the following we introduce the concept of *soft* supersymmetry breaking and, after a brief introduction on supersymmetry breaking in supergravity, we have a look at the repercussions of supersymmetry breaking on the observable sector. What emerges is that the spontaneous breaking in the hidden sector induces soft-breaking terms in the observable one and the interaction between these two sectors occurs through the so-called gravity mediation.

2.5.1 Generalities on Supersymmetry Breaking

When the Lagrangian is invariant under a supersymmetry transformations but the vacuum state is not, we speak about *spontaneous supersymmetry breaking* (SSB). More generally, when the ground state $|vac\rangle$ satisfies

$$Q_\alpha |vac\rangle \neq 0 \tag{2.52}$$

one has a broken supersymmetry.

Two important results provided by [**?**] are that for any state $E \geq 0$ and that when supersymmetry is broken the energy is strictly positive.

Taking into account the variations of the components of a chiral superfield (**??**) under a supersymmetry transformation

$$\begin{aligned} \delta\phi &= \sqrt{2}\epsilon\psi \\ \delta\psi &= i\sqrt{2}\sigma^\mu\bar{\epsilon}\partial_\mu\phi + \sqrt{2}\epsilon F. \\ \delta F &= i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi \end{aligned} \tag{2.53}$$

we note that Lorentz invariance imposes:

$$\langle\psi\rangle = \langle\partial_\mu\phi\rangle = 0. \tag{2.54}$$

So the supersymmetry breaking condition (at least one of $\delta\phi, \delta\psi, \delta F \neq 0$) entails:

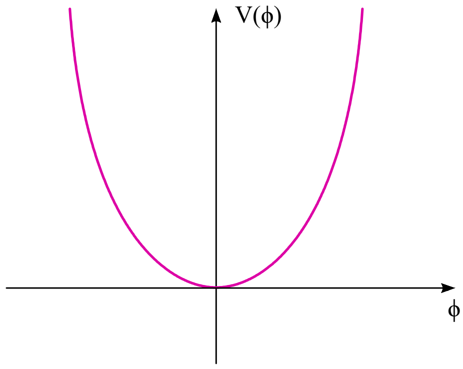
$$SU\!\!\!/SY \Longleftrightarrow \langle F\rangle \neq 0, \tag{2.55}$$

then we have

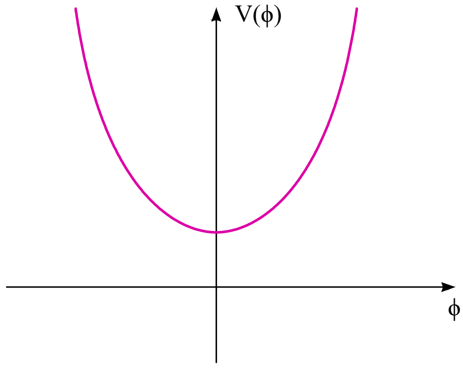
$$\delta\phi = \delta F = 0, \quad \delta\psi = \sqrt{2}\epsilon\langle F\rangle \neq 0. \tag{2.56}$$

Because of the form of the scalar potential one has:

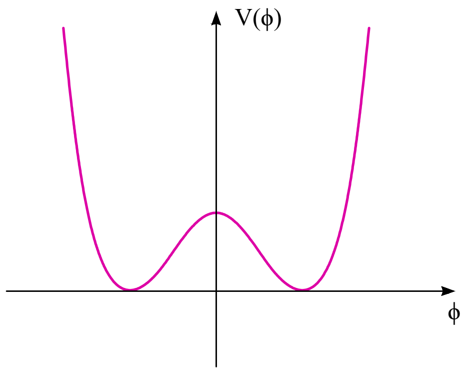
$$SU\!\!\!/SY \Longleftrightarrow \langle V_F\rangle > 0. \tag{2.57}$$



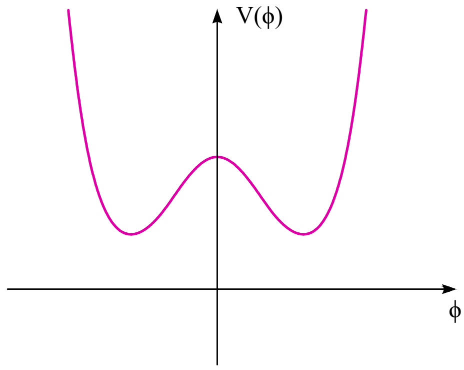
(a) *SUSY and gauge symmetry preserved.*



(b) *SUSY broken and gauge symmetry preserved.*



(c) *SUSY preserved and gauge symmetry broken*



(d) *SUSY and gauge symmetry broken*

Figura 2.1: In this figure we can see various symmetry breaking scenarios: as we discuss in the present section, whenever the minimum potential energy is zero, SUSY is preserved. Gauge symmetry breaking is instead associated to a non-zero field configuration [**?**].

2.5.2 Soft Supersymmetry Breaking

In the absence of a clear theoretical mechanism for spontaneous supersymmetry breaking, it is interesting to investigate the ways in which SUSY can be explicitly broken. To this end one introduces some particular terms called *soft* breaking terms which do not generate quadratic divergences, these terms are the non-supersymmetric ones. The general Lagrangian including soft terms is given by:

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft} \tag{2.58}$$

where $\mathcal{L}_{SUSY}$ refers to equation (**??**) and $\mathcal{L}_{soft}$ is given by [**?**]:

$$\mathcal{L}_{soft} = -m^2_{i\bar{j}}\phi^i\bar{\phi}^j - (\frac{1}{2}B_{ij}\phi^i\phi^j + A_{ijk}\phi^i\phi^j\phi^k + h.c.) \tag{2.59}$$

where the ϕ^i's are quantum scalar fields, m^2_{ij} and B_{ij} are mass matrices for the scalars, A_{ijk} is a trilinear coupling.

Supersymmetry Breaking in Supergravity

As we have seen in the previous section, the spontaneous supersymmetry breaking order parameters are the $\langle F \rangle$ v.e.v.. In the supergravity framework one needs to add another order parameter due to the presence of the gravitino in the supergravity multiplet. The gravitino gives rise to a fermionic component variation:

$$\delta\Psi_\mu \propto D_\mu\epsilon(x) + ie^{\frac{\kappa^2}{2}K}W\sigma_\mu\epsilon(x) \tag{2.60}$$

Note that in this context $F^i \propto e^{\frac{\kappa^2}{2}K}K^{i\bar{j}}\bar{D}_{\bar{j}}\bar{W}$. This equation restricts the vacuum spacetime-solution, this means that also the $\langle V_F \rangle = 0$ solution is possible after SUSY breaking in supergravity. A sufficient condition for supersymmetry breaking is given by

$$\langle F^i \rangle \neq 0 \ . \tag{2.61}$$

Nevertheless super-partners of ordinary particles have not been detected. This suggests the presence of a "hidden sector" in which supersymmetry is broken at energies higher than the electroweak scale since supersymmetry breaking in the "visible sector" would give rise to some super-partners which would be lighter than ordinary particles [**?**]. That is:

$$W = W_{\text{obs}} + W_{\text{hidden}} \tag{2.62}$$

in such a manner that

$$\langle \bar{F}^i \rangle = \langle e^{\frac{\kappa^2}{2}K}K^{\bar{i}j}D_jW_{\text{hidden}} \rangle \neq 0. \tag{2.63}$$

Gravity Mediation

At this point it is very interesting to identify the effects of supersymmetry breaking in the observable sector (which could, for instance, be given by the MSSM). Making use of the same notation of [**?**], we consider the observable charged matter fields Q^I and scalars T^i living in the hidden sector and assume $\langle Q^I \rangle = 0$. Then the superpotential takes the form:

$$\begin{aligned} W(T,Q) &= W_{obs}(T,Q) + W_{hidden}(T) \\ W_{obs} &= \frac{1}{2} m_{IJ}(T) Q^I Q^J + \frac{1}{3} Y_{IJK}(T) Q^I Q^J Q^K + ... \end{aligned} \tag{2.64}$$

In the hidden sector we take $\langle F^i \rangle \neq 0$ for some i, $\langle V \rangle = 0$ and fix the v.e.v. of T^i. We write the Kähler potential as a power series in Q^I:

$$K = \kappa^{-2} \hat{K}(T,\bar{T}) + Z_{IJ}(T,\bar{T}) \bar{Q}^I Q^J + (\frac{1}{2} H_{IJ}(T,\bar{T}) Q^I Q^J + c.c.) + \mathcal{O}(Q^3) \tag{2.65}$$

Taking the limit $M_{pl} \to \infty$ one finds [**?**] that the potential is given by:

$$\begin{aligned} V =& \frac{1}{4} g^2 (\bar{Q}^I Z_{IJ} T^a Q^J)^2 + \partial_I W Z^{IJ} \bar{\partial} \bar{J} \bar{W} \\ &+ m^2_{I\bar{J}} Q^I \bar{Q}^{\bar{J}} + (\frac{1}{3} A_{IJK} Q^I Q^J Q^K + \frac{1}{2} B_{IJ} Q^I Q^J + c.c.) \end{aligned} \tag{2.66}$$

where the soft supersymmetry breaking terms are encoded in the second line.

Capitolo 3

Compactification of IIB String Theory

3.1 Type IIB Supergravity

A popular superstring theory is type IIB string theory. At low-energies this theory can be described by ten-dimensional type IIB supergravity.

Type IIB supergravity is an $\mathcal{N}=2$ theory and is described by a single supermultiplet [?]:

$$G,\ B_2,\ C_2,\ \phi,\ C_0,\ C_4,\ \Psi^1_{M\alpha},\ \Psi^2_{M\alpha},\ \lambda^1_{\dot\alpha},\ \lambda^2_{\dot\alpha} \tag{3.1}$$

where one has, in the order, the graviton, two two-forms, the dilaton, a zero-form, a four-form, the gravitino and the dilatino.

Let us define the field-strengths of the supermultiplet components [?]:

$$H_3 = dB_2, \qquad F_3 = dC_2, \qquad F_5 = dC_4, \qquad F_1 = dC_0 \tag{3.2}$$

$$G_3 := F_3 - (C_0 + i\mathrm{e}^{-\phi})H_3\ , \qquad \tilde F_5 := F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3 \tag{3.3}$$

The type IIB low energy effective action in the 10-dimensional string-frame is given by [?]:

$$\begin{aligned} S^{(10)}_{IIB} = &-\int \mathrm{e}^{-2\phi}\left(\frac{1}{2}R * \mathbf{1} + 2d\phi \wedge *d\phi\right) \\ &-\frac{1}{4}\int\left(F_1 \wedge *F_1 + \frac{1}{3}G_3 \wedge *\bar G_3 + \frac{1}{240}\tilde F_5 \wedge *\tilde F_5\right) - \frac{i}{8}\int \mathrm{e}^{\phi} C_4 \wedge G_3 \wedge \bar G_3 \end{aligned} \tag{3.4}$$

where $*$ is the Hodge-$*$ operator and the self-duality condition on $\tilde F_5$ is imposed by hand.

In order to bring these new objects closer to something familiar, we note the analogy with the electromagnetic gauge field: within electromagnetism the aim of introducing

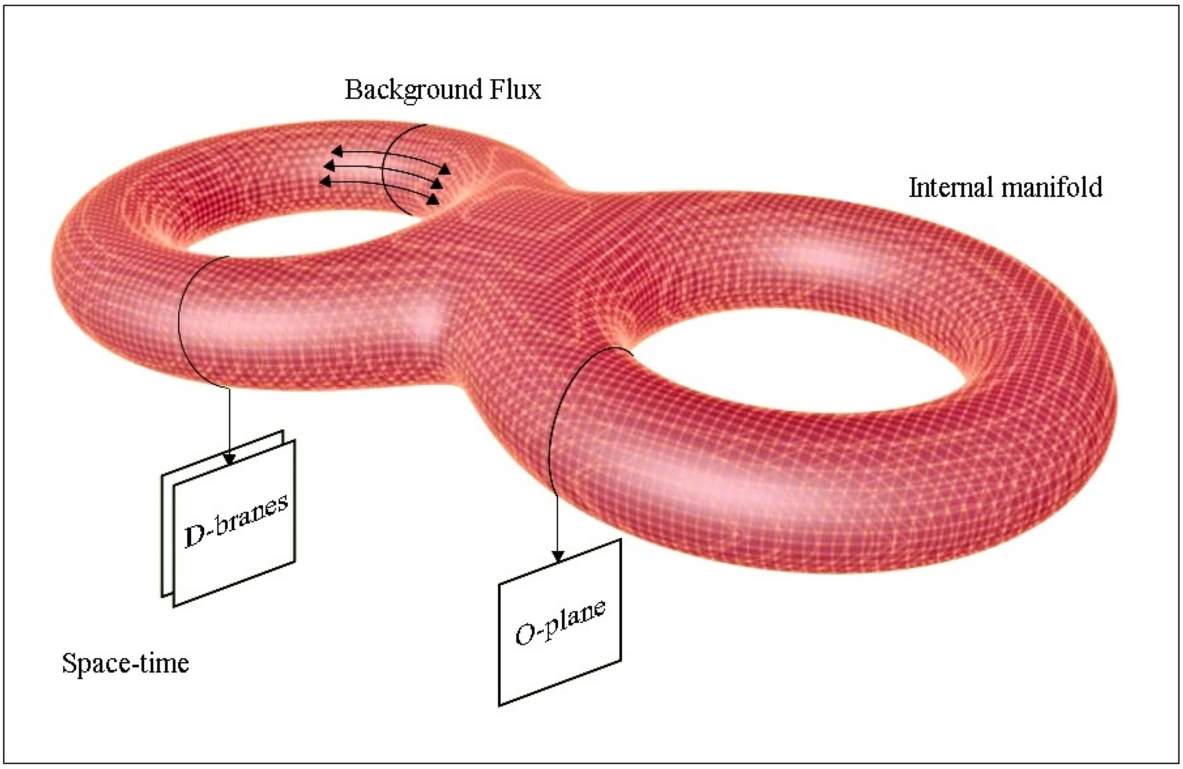


Figura 3.1: Brane-world scenario on four Minkowsky space-time and compact internal manifold: D-branes, orientifold planes and background fluxes.

the vector field A_μ is that of maintaining the invariance of the Lagrangian under local phase transformations. The gauge field is then introduced into the Lagrangian through the term:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{3.5}$$

which is invariant under the same phase transformation. Here we have defined the field-strength F as:

$$F = dA. \tag{3.6}$$

3.1.1 Compactifications of IIB Supergravity

The existence of extra dimensions which is necessary to describe type IIB supergravity is not experimentally evident. One possible explanation for this could be that such extra dimensions are wrapped up on themselves and confined in very small sizes (for six extra dimensions the typical radius should not be larger than 10^{-11}cm [**?**], but could be much smaller than that). This necessity leads to the notion of compactification, which allows to obtain a four-dimensional theory in which the extra dimensions are compactified on some suitable class of six-dimensional manifolds. An example of such manifolds is given by the Calabi-Yau manifolds or by the torus orbifolds. To compactify means the metric can be written as:

$$ds^2 = \tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu + R\, g_{mn}(y)dy^m dy^n \tag{3.7}$$

where y denotes a point in the six dimensions, $\tilde{g}_{\mu\nu}$, $\mu, \nu = 0, ..., 3$ is a Minkowski metric and g_{mn}, $m, n = 1, ..., 6$ is the compactified manifold metric. In the small radius limit $R \to 0$ the Minkowski term becomes more relevant and, as a consequence of compactification, we no longer see the extra dimensions. However, the fact that the extra dimensions are undetectable does not mean that there cannot be experimental evidence of their existence: an example is given by the moduli which arise as massless modes of the higher-dimensional fields on the compact manifold.

These theories give rise to $\mathcal{N} = 2$ supersymmetry in four-dimensions, but from a phenomenological point of view we need an $\mathcal{N} = 1$ supersymmetric theory. This problem is solved by what is known as *orientifolding*. Orientifolds are obtained starting from a particular compactification and introducing a parity transformation Ω at the string-theory level. Furthermore, we need a space-time isometry which includes an involution σ^* on the compact space. Calabi-Yau manifolds can only be subject to discrete isometries which act on the coordinates. The orientifolding process projects out some part of the spectrum and, furthermore, there exist fix-points with respect to σ^* which give rise to $O3/O7$-planes.

3.2 D-brane Action

Dp-branes are non-perturbative solutions of supergravity, they are important in order to give consistent solutions with $O3/O7$-planes. They are objects of particular interest for many reasons: the open strings end on the brane, the branes give also rise to non-abelian gauge group and to chiral matter. Moreover they can be thought of as a higher-dimensional generalization of point-particles charged under bulk U(1)s. Dp-brane are objects whose dynamics are governed by an action given by the sum of the Chern-Simons (CS) and Dirac-Born-Infeld (DBI) actions [?]. They are given by [?, ?]:

$$\begin{aligned} S^{\text{sf}}_{\text{DBI}} &= -\mu_p g_s^{-1} \int_{\mathcal{W}} d^{p+1}\xi \text{Tr} e^{-\phi} \sqrt{-det\left(\varphi^*(E_{\mu\nu}) + lF_{\mu\nu}\right) \det Q^n_m}, \\ S_{CS} &= \mu_p g_s \int_{\mathcal{W}} \text{Tr} \left(\varphi^*(e^{il\mathbf{i}_\phi \mathbf{i}_\phi} \sum_{q\,\textbf{even}} C^{(q)} e^B) e^{lF} \right). \end{aligned} \tag{3.8}$$

Where

$$Q^n_m = \delta^n_m + il[\phi^n, \phi^k]E_{km}, \quad n, m = 1, ..., 6 \tag{3.9}$$

and $l = 2\pi\alpha'$. For a single D-brane F is a $U(1)$ field strength which describes the $U(1)$ gauge theory of the endpoints of the open strings attached to the brane. The integration is performed over the $(p+1)$-dimensional world-volume $\mathcal{W}$, which is a submanifold of the entire ten-dimensional manifold M and is related to the latter by the map $\phi : \mathcal{W} \hookrightarrow M$. $\mu_p = (2\pi)^{-p}(\alpha')^{-\frac{1}{2}(p+1)}$ is the brane tension, $E_{\mu\nu}$ encodes the metric g and the B-field via:

$$E_{\mu\nu} = g_s^{1/2} g_{\mu\nu} + B_{\mu\nu}. \tag{3.10}$$

φ^* is the pull-back: it is necessary because $E_{\mu\nu}$ is defined in the entire ten-dimensional manifold M and we need to pull it back on the brane. To do this we have to take into account the fluctuations along the normal direction of the world-volume as in [9,10] and expand the pull-back as well. In the DBI action the field strength F also appears, which we will overlook both for simplicity and because it is part of a vector superfield, which is not our focus within this project.

3.2.1 Background Fluxes

Let us focus our attention on the case of D7-branes living in a 10-dimensional space treated with a compactification where the internal, compact manifold is an orbifold [?, ?]. In [?] we can see how type IIB 10-dimensional theory admits imaginary self-dual (ISD) 3-form fluxes as solutions of the equations of motion in warped Calabi-Yau backgrounds. In this section we consider the presence of such ISD 3-form fluxes G_3 acting as a background. In type IIB compactification these fluxes are of two different types [?] showing different tensor structures, one is a (0,3) tensor and one is a (2,1) tensor. We are particularly interested in the first type, which induces a supersymmetry-breaking soft term; the second one is involved in the induction of supersymmetric F-term masses to the chiral multiplets [?, ?, ?, ?, ?, ?].

In the following we discuss the scalar potential induced by the fluxes and we see how background fluxes lead to a vacuum in which supersymmetry is spontaneously broken.

3.2.2 Flux-induced Scalar Potential from DBI action

Let us now focus on torus orbifold compactifications in the presence of G_3 fluxes. The DBI action for D7-branes is given by:

$$S_{\text{DBI}} = -\mu_7 g_s^{-1} \int_{\mathcal{W}} d^8\xi \text{Tr} e^{-\phi} \sqrt{-det\left(\varphi^*(E_{\mu\nu}) + lF_{\mu\nu}\right) \det Q^n_m}. \tag{3.11}$$

As we can see in [?], neglecting the contribution coming from the field-strength F and from $\det Q^n_m$, which gives rise to the D-term scalar potential, computing the determinant one has

$$\det(\varphi^* E_{\mu\nu}) = -g_s^4 f(B)^2 \left[1 + 2Z\, l^2 D_\mu \phi D^\mu \phi\right] \tag{3.12}$$

where Z is a warp factor that depends on the internal coordinates and $f(B)$ is a function of the B-field given by:

$$f(B) = 1 + \frac{1}{2} Z^{-1} g_s^{-1} B^2. \tag{3.13}$$

Then the DBI action is written as:

$$S_{\text{DBI}} = -\mu_7 g_s^{-1} \int d^8\xi \text{Tr} \sqrt{f(B)^2 \left[1 + 2Z\, l^2 D_\mu \phi D^\mu \phi\right]}. \tag{3.14}$$

Now, recalling that the three-form fluxes G_3 are given by equation (**??**), one has:

$$H_3 = dB_2 = \frac{\text{Im}G_3}{\text{Im}\tau}\,, \tag{3.15}$$

where τ is the complex type IIB axio-dilaton $\tau = C_0 + i\text{e}^{-\phi}$. Integrating the previous expression one obtains the B-field induced on the brane by the presence of background fluxes G_3. The result in terms of the B-field components is given by [**?**]:

$$B_{12} = \frac{g_s\, l}{2i}(G^*_{(0,3)}\phi - G_{(2,1)}\bar{\phi})\,, \quad B_{\bar{1}\bar{2}} = -\frac{g_s\, l}{2i}(G_{(0,3)}\bar{\phi} - G^*_{(1,2)}\phi) \tag{3.16}$$

replacing this result in equation (**??**) one obtains:

$$f(\phi) = 1 + \frac{Z^{-1} g_s\, l^2}{4} |\mathcal{G}^* \phi - \mathcal{S}\bar{\phi}|^2, \tag{3.17}$$

where $\mathcal{G} \equiv G_{\bar{1}\bar{2}\bar{3}}$ and $\mathcal{S} \equiv \epsilon_{3jk} G_{3\bar{j}\bar{k}}$. Thus one obtains that the F-term contribution to the scalar potential is given by:

$$V(\phi) = \text{Tr}\left(\frac{Z^{-2} g_s}{2} |\mathcal{G}^* \phi - \mathcal{S}\bar{\phi}|^2\right). \tag{3.18}$$

So in the end we have seen that the presence of background fluxes induces a scalar potential term.

Shape of the scalar potential induced by fluxes: We are now interested in seeing what shape the potential takes after the orbifold projection. Following the footsteps of [**?**], we consider a Z_4 orbifold projection which splits the brane position into two non-vanishing components:

$$\phi = (H_u, H_d). \tag{3.19}$$

We can now express the scalar potential (**??**) in terms of these components. It takes the form:

$$V = \frac{Z^2 g_s}{2}[(|\mathcal{G}|^2 + |\mathcal{S}|^2)(|H_u|^2 + |H_d|^2) - 4\text{Re}(\mathcal{G}^* \mathcal{S}^* H_u H_d)]. \tag{3.20}$$

3.2.3 $\mathcal{N} = 1$ Supergravity Description

In this section we will see that the scalar potential (**??**) obtained by the three-form fluxes G_3 is identical to the one derived from a $\mathcal{N} = 1$ supergravity computation. The Kähler potential for type IIB supergravity with a stack of D7-branes is given by:

$$K = -log[(S + S^*)(U_3 + U_3^*) - \frac{\alpha'}{2}|H_u + H_d^*|^2] - 3log(T + T^*) \tag{3.21}$$

where $S = e^{-\phi} + i\,C_0$ is the type IIB complex dilaton (notice the slight different definition with respect to $\tau = C_0 + i\mathrm{e}^{-\phi}$), U_3 is the complex structure modulus of the $\mathbf{T}^2$ torus in the third complex direction and T is a diagonal Kähler modulus field.

In the present case we consider a superpotential given by a constant term and a μ-term:

$$W = W_0(S, U) + \mu H_u H_d. \tag{3.22}$$

As we can also see in the component Lagrangian (**??**), the scalar potential is given by:

$$V = e^K[\sum_{T,S,U,H} K^{i\bar{j}}(D_i W)(\bar{D}_{\bar{j}}\bar{W}) - 3W\bar{W}], \tag{3.23}$$

We can now rearrange the terms as:

$$V = e^K[\underbrace{K^{T\bar{T}} D_T W \bar{D}_{\bar{T}}\bar{W} - 3W\bar{W}}_{=0} + \sum_{S,U,H} K^{\alpha\bar{\beta}} D_\alpha W \bar{D}_{\bar{\beta}}\bar{W}]. \tag{3.24}$$

We can see that the contribution from the first two terms cancels out, indeed one has:

$$D_T W = \partial_T W + W K_T \tag{3.25}$$

where $\partial_T W = 0$ because of the fact that W is T-independent. For the first two terms in equation (**??**), this implies:

$$\begin{aligned} &K^{T\bar{T}} W K_T \bar{W} K_{\bar{T}} - 3W\bar{W} \\ &= (K^{T\bar{T}} K_T K_{\bar{T}} - 3)W\bar{W} \\ &= \left(\frac{\cancel{(T+\bar{T})^2}}{\cancel{3}} \frac{\cancel{3}}{\cancel{(T+\bar{T})}} \frac{3}{\cancel{(T+\bar{T})}} - 3\right) W\bar{W} = 0 \end{aligned} \tag{3.26}$$

So that

$$V = e^K[\sum_{S,U,H} K^{\alpha\bar{\beta}} D_\alpha W \bar{D}_{\bar{\beta}}\bar{W}] \geq 0. \tag{3.27}$$

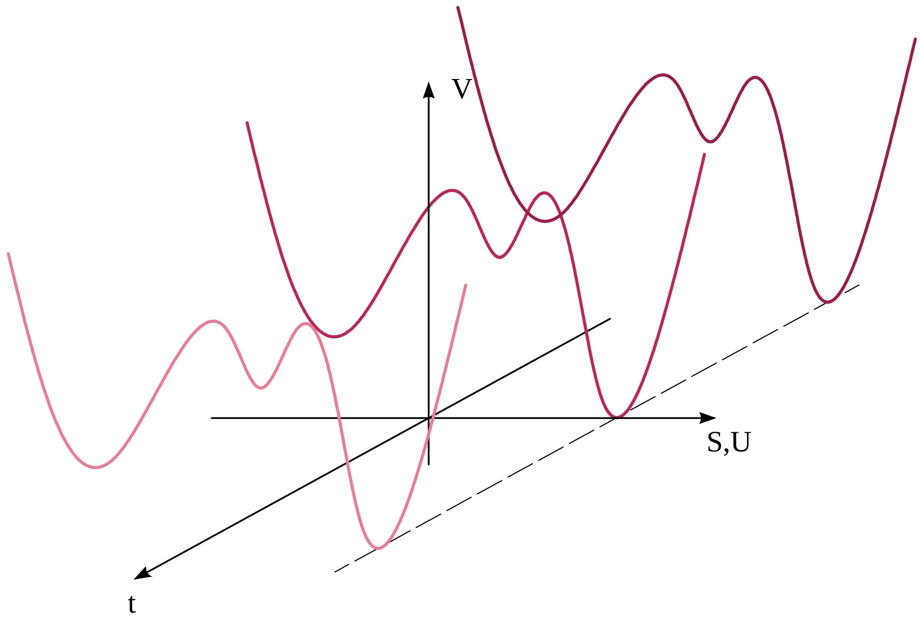


Figura 3.2: No-scale structure in the flat t-direction.

As we can see, now the scalar potential is always non-negative, and so a minimum of the potential occurs when:

$$\begin{aligned} &D_\alpha W = 0 \\ \Rightarrow &D_U W = D_S W = 0, \end{aligned} \tag{3.28}$$

from equation (**??**) one obtains:

$$\begin{aligned} &F^\alpha = e^{K/2} K^{\alpha\bar{\beta}} \underbrace{\bar{D}_{\bar{\beta}} \bar{W}}_{=0} = 0 \\ \Rightarrow &F^S = F^U = 0, \end{aligned} \tag{3.29}$$

meanwhile for F^T we have:

$$\begin{aligned} F^T &= e^{K/2} K^{T\bar{T}} \bar{D}_{\bar{T}} \bar{W} \\ &= e^{K/2} K^{T\bar{T}} K_{\bar{T}} \bar{W} \\ &= \frac{W_0}{(T+\bar{T})^{1/2}} = \frac{W_0}{\sqrt{t}} \xrightarrow[t\to\infty]{} 0 \end{aligned} \tag{3.30}$$

where $t = T + \bar{T}$ is the volume of a 4-cycle. So we have that F^T vanishes in the decompactification limit and, for finite t and $W_0 \neq 0$, one has that SUSY is broken because of the presence of the non-vanishing F^T-term:

$$F^T \neq 0 \Rightarrow S\not{U}SY \qquad \text{unless} \qquad W_0 = 0. \tag{3.31}$$

Thus, because of the no-scale structure of the Kähler potential, the scalar potential takes the simplified form:

$$V = e^K \left(\sum_{S,U,H} K^{\alpha\bar{\beta}} (D_\alpha W)(\bar{D}_{\bar{\beta}} \bar{W}) \right) \tag{3.32}$$

So one obtains that the scalar potential is given by:

$$V = (|M|^2 + |\hat{\mu}|^2)(|H_u|^2 + |H_d|^2) - 2M\hat{\mu}H_u H_d + h.c. \tag{3.33}$$

where:

$$\hat{\mu} = \frac{W_0 \mu s}{t^{3/2}\sqrt{s}}, \qquad M = -\frac{\bar{W}_0}{t^{3/2}\sqrt{s}}$$
$$s = (S + S^*), \quad t = (T + T^*). \tag{3.34}$$

Through the redefinitions:

$$\mathcal{G}^* = \left(\frac{g_s}{2}\right)^{-1/2} \frac{W_0^*}{t^{3/2}\sqrt{s}}, \qquad \mathcal{S}^* = -\left(\frac{g_s}{2}\right)^{-1/2} \frac{W_0 + \mu s}{t^{3/2}\sqrt{s}} \tag{3.35}$$

we have that the scalar potential (??) is exactly the one induced by the presence of ISD three-form fluxes (??). By comparing the expression (??) with the equation (??) we can see that the soft-part of the scalar potential is given by the two terms:

$$\frac{1}{2} B_{IJ} Q^I Q^J \equiv -2M\hat{\mu}H_u H_d \tag{3.36}$$

and

$$m^2_{I\bar{J}} Q^I \bar{Q}^{\bar{J}} \equiv |M|^2 (|H_u|^2 + |H_d|^2). \tag{3.37}$$

3.3 Single D3-Brane Effective Action at Two-Derivative Level

In this section we compute the low energy effective action of a single D3-brane by reduction of the Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions. We are going to see how the fluctuation of the brane position gives rise to a chiral superfield ϕ in the effective action. This superfield is charged under a $U(1)$-gauge group. In the case of the D3-brane we have a no-scale structure also for the field ϕ, which implies that there is no mass generation for this mode for $D_S W = D_U W = 0$.

Dirac-Born-Infeld Action: Let us give the bosonic contributions for the DBI action of a single D3-brane [?], in the string frame this is written as given by equation (??) which in the present case reduces to:

$$S^{\text{sf}}_{\text{DBI}} = -\mu_3 \int_{\mathcal{W}} d^4\xi e^{-\phi}\sqrt{-det(\varphi^* E_{\mu\nu})} \tag{3.38}$$

As we have seen in the previous section, when computing the pull-back of the metric we have to take into account the normal fluctuations. This approach allows us to treat the DBI action (??) as an expansion for small fluctuations of the brane. As is shown in [6] one can start by taking into account the expansion of the square root of the determinant:

$$\sqrt{\det(\mathbf{1}+A)} = 1 + \frac{1}{2}\text{Tr}A - \frac{1}{4}\text{Tr}A^2 + \frac{1}{8}(\text{Tr}A)^2 + ... \tag{3.39}$$

For the moment we limit our discussion to the first order of this expansion. As we show in appendix A, evaluating the pull-back one obtains:

$$\varphi^*(g)_{\mu\nu} = e^{2A(y_0)}\tilde{g}_{\mu\nu} + e^{-2A(y_0)}\, l g_{mn} D_\mu\phi^m D_\nu\phi^n + e^{2A(y_0)}\, l^2 \tilde{g}_{\mu\tau} R_n{}^\tau{}_{\nu m}\phi^n\phi^m \tag{3.40}$$

where y_0 is the position of the brane and D_μ is the covariant derivative which in general also contains the connection of the normal bundle [10],in our case however this connection vanishes due to the ansatz of the metric [6]. At the end one finds the DBI which, in the Einstein frame, is given by [6, 11]:

$$S^E_{DBI} = -\mu_3 \int_{\mathcal{W}} d^4\xi\sqrt{-g_4}\left(\frac{36e^{4A}}{\mathcal{K}^2_\omega}(1 + l^2 R_n{}^\tau{}_{\tau m}\phi^n\phi^m) + \frac{3l^2}{\mathcal{K}_\omega} g_{mn} D_\mu\phi^m D^\mu\phi^n\right) \tag{3.41}$$

where, as can be seen in [12], the relation between $\tilde{g}_{\mu\nu}$ is given by:

$$g_{\mu\nu} = \frac{\mathcal{K}_\omega}{6}\tilde{g}_{\mu\nu}, \qquad \mathcal{K}_\omega = 6\int_Y d^6\sqrt{\text{det}g_{mn}}e^{-4A}. \tag{3.42}$$

Chern-Simons Action: In the abelian case with the strength field F set to zero, the Chern-Simons action provided in [?] becomes:

$$S_{CS} = \mu \int_{\mathcal{W}} \varphi^*\left(\sum_{q\text{ even}} C^{(q)}e^B\right) = \mu \int_{\mathcal{W}} \varphi^*\left(C_4 + C_2\wedge B_2 + \frac{1}{2}B_2\wedge B_2\right) \tag{3.43}$$

in which we have to consider the four-form C_4 which is given by its background value contribution plus excitations:

$$\begin{aligned} C_4 &= C_{\mu\nu\rho\sigma}dx^\mu\wedge dx^\nu\wedge dx^\rho\wedge dx^\sigma \\ &\quad + D^\alpha_{(2)}(x)\wedge\omega^\alpha + \rho_\alpha\tilde{\omega}^\alpha \end{aligned} \tag{3.44}$$

where $C_{\mu\nu\rho\sigma}$ is related to the background value via:

$$\begin{aligned}&\frac{1}{4!}C_{\mu\nu\rho\sigma}dx^\mu\wedge dx^\nu\wedge dx^\rho\wedge dx^\sigma\\&=C_{0123}dx^0\wedge dx^1\wedge dx^2\wedge dx^3\\&=\alpha(y)dx^0\wedge dx^1\wedge dx^2\wedge dx^3.\end{aligned}\tag{3.45}$$

Computing the pull-back by making use of the formula (**??**), we must keep in mind that the only non-vanishing components of the expansion are those including $C_{\mu\nu\rho\sigma}$, $C_{\mu\nu mn}$ and C_{mnpq} type terms. This yields the following contribution to the CS action in the Einstein frame:

$$\begin{aligned}S^E=\mu_3\int_{\mathcal{W}}d^4\xi\sqrt{-g_4}\left(\frac{36e^{4A}}{\mathcal{K}_\omega^2}(1+l^2R_n{}^\tau{}_{\tau m}\phi^n\phi^m)\right)\\+\frac{\mu_3 l^2}{4}\int_{\mathcal{W}}(\phi^iD_\mu\bar{\phi}^{\bar{j}}-\bar{\phi}^{\bar{j}}D_\mu\phi^i)(\omega_\alpha)_{i\bar{j}}dx^\mu\wedge dD^\alpha_{(2)}\end{aligned}\tag{3.46}$$

where the last term is obtained by partial integration and using a complex basis. One notices that the first line gives the same contribution as the DBI action (**??**) but with opposite sign, so their sum gives a null contribution. We also have to include small deviations for the background value α, as the following expansion shows[6]:

$$\alpha=e^{4A}-h(y)\tag{3.47}$$

The contribution of the pull-back is given by:

$$\varphi^*(h)=h(y_0)+l\,\nabla_n\,h|_{y_0}\phi^n+\frac{1}{2}l^2\,\nabla_n\nabla_m h|_{y_0}\phi^n\phi^m=\frac{1}{2}l^2h_{nm}\phi^n\phi^m\tag{3.48}$$

where

$$h(y_0)=0,\qquad \nabla_nh|_{y_0}=0,\qquad \nabla_n\nabla_m\,h|_{y_0}\equiv h_{nm}.\tag{3.49}$$

Now we can write the complete Chern-Simons action:

$$\begin{aligned}S^E=\mu_3\int_{\mathcal{W}}d^4\xi\sqrt{-g_4}\left(\frac{36e^{4A}}{\mathcal{K}_\omega^2}(1+l^2R_n{}^\tau{}_{\tau m}\phi^n\phi^m)-\frac{18l^2}{\mathcal{K}_\omega^2}(h_{i\bar{j}\phi^i\bar{\phi}^{\bar{j}}}+h_{ij}\phi^i\phi^j+h.c.)\right)\\+\frac{\mu_3 l^2}{4}\int_{\mathcal{W}}(\phi^iD_\mu\bar{\phi}^{\bar{j}}-\bar{\phi}^{\bar{j}}D_\mu\phi^i)(\omega_\alpha)_{i\bar{j}}dx^\mu\wedge dD^\alpha_{(2)}.\end{aligned}\tag{3.50}$$

Adding the two contributions of DBI action (**??**) and CS action (**??**) we finally get

$$\begin{aligned}S^E_{bos}=\mu_3l^2\int_{\mathcal{W}}d^4\xi\sqrt{-g_4}\left(\frac{6i}{\mathcal{K}_\omega^2}v^\alpha(\omega_\alpha)_{i\bar{j}}D_\mu\phi^iD_\mu\bar{\phi}^{\bar{j}}-\frac{18l^2}{\mathcal{K}_\omega^2}(h_{i\bar{j}}\phi^i\bar{\phi}^{\bar{j}}+h_{ij}\phi^i\phi^j+h.c.)\right)\\+\frac{\mu_3 l^2}{4}\int_{\mathcal{W}}(\phi^iD_\mu\bar{\phi}^{\bar{j}}-\bar{\phi}^{\bar{j}}D_\mu\phi^i)(\omega_\alpha)_{i\bar{j}}dx^\mu\wedge dD^\alpha_{(2)}.\end{aligned}\tag{3.51}$$

Capitolo 4

Higher-derivative Terms in Supersymmetric Effective Theories

In this chapter we are going to discuss the role of supersymmetric higher-derivative operators in effective theories described in the previous chapter. In particular we are interested in four-derivative operators involving the same number of ϕ and $\bar{\phi}$ with mass dimension 8 (which we can heuristically indicate as $\partial^4\phi^4$) as we saw in section (**??**), ignoring those terms in which the higher-derivative orders are provided by higher-derivative objects as the Riemann tensor. The first step is to understand where higher-derivative operators come from: they typically arise from effective actions considering higher orders of the expansions. The question is, are they actually supersymmetric? In this regard we can consider two cases: in the first we have $F^i = 0$, so we have no scalar potential ($V_F = 0$), in the second case one has instead $F^i \neq 0$, which implies a non-vanishing scalar potential ($V_F \neq 0$). In the first case we have that the SUSY is preserved and the four-derivative terms arising from higher orders of the expansions can be recast into some suitable combination of superymmetric higher-derivative operators in which one can fully represent all terms supersymmetrically. As we have seen in the previous chapter in the second case, on the contrary, the SUSY is broken due to the presence of fluxes: soft terms arise and one can only partially represent all terms supersymmetrically. We will address this topic in more detail later.

In the next section we show a practical example of this method applied at the D7-Brane effective action.

4.1 Higher-derivative Operators for a D7-Brane

In this section we consider the issue of higher-derivative operators for the effective action for the position moduli of the Type IIB D7-branes. As proposed in [**?**] we consider the

Lagrangian at the two-derivative level in the form:

$$\mathcal{L} = -\,[1 + aV(\phi)]\partial_\mu\phi\partial^\mu\bar{\phi} - V(\phi) \tag{4.1}$$

where the factor a is given by:

$$a = \frac{Z}{\mu_z V_4 g_s} \tag{4.2}$$

where Z denotes the warp factor $e^{2A(y_0)}$, V_4 is the volume wrapped by the brane. We have only used one complex scalar field because D7-branes are 8-dimensional objects living in 10-dimensions. This Lagrangian arises from the reduction of the DBI action. In those terms, using the same ansatz for the metric (**??**) which we have used in the D3-brane example, as shown in appendix (**??**), the root expansion (**??**) gives:

$$\begin{aligned}&\sqrt{-det(g_s^{1/2}Z^{-1/2}\eta_{\mu\nu} + g_s^{1/2}Z^{1/2}\,l^2\partial\varphi_m\partial^m\varphi^m)} = \\ &g_s^2 Z^{-2}\{1 + 2Z\,l^2\partial_\mu\phi_i\partial^\mu\bar{\phi}^i \\ &-\frac{1}{2}Z^2\,l^4[(\partial_\mu\phi^i\partial^\mu\phi^j)(\partial_\nu\bar{\phi}_i\partial^\nu\bar{\phi}_j) + (\partial_\mu\phi^i\partial^\mu\bar{\phi}^j)(\partial_\nu\phi_j\partial^\nu\bar{\phi}_i) \\ &-(\partial_\mu\phi_i\partial^\mu\bar{\phi}^i)(\partial_\nu\phi_j\partial^\nu\bar{\phi}^j)]\}\end{aligned} \tag{4.3}$$

In case of a single complex scalar field, the previous expansion reduces to:

$$\begin{aligned}&\sqrt{-det(g_s^{1/2}Z^{-1/2}\eta_{\mu\nu} + g_s^{1/2}Z^{1/2}\,l^2\partial\varphi_1\partial^m\varphi_2)} = \\ &g_s^2 Z^{-2}\{1 + 2Z\,l^2\partial_\mu\phi\partial^\mu\bar{\phi} - \frac{1}{2}Z^2\,l^4[(\partial_\mu\phi\partial^\mu\phi)(\partial_\nu\bar{\phi}\partial^\nu\bar{\phi})]\}\end{aligned} \tag{4.4}$$

Then, as we can see in [**?**],the Lagrangian (**??**) at the four-derivative level becomes:

$$\mathcal{L} = -[1 + aV(\phi)]\partial_\mu\phi\partial^\mu\bar{\phi} + \frac{a}{2}|\partial_\mu\phi\partial^\mu\phi|^2 - V(\phi). \tag{4.5}$$

At this point our goal is to provide a supersymmetric operator which yields the same four-derivative contribution $|\partial_\mu\phi\partial^\mu\phi|^2$ appearing in the Lagrangian (**??**). We show the explicit calculation for the coefficient $a/2$ of the four-derivative term in the appendix (**??**). At this point our purpose is to find a suitable higher-derivative supersymmetric operator that reproduces the four-derivative term appearing in the previous Lagrangian. The higher-order terms which will appear in this operator together with the four-derivative term will constitute the supersymmetric high-derivative Lagrangian $\mathcal{L}^{HD}_{SUSY}$. This Lagrangian, together with the two-derivavive supersymmetric Lagrangian $\mathcal{L}_{SUSY}$ is the supersymmetric rappresentaion of the Lagrangian (**??**) but, as we will see later, this representation cannot be complete.

4.2 Supersymmetric Match of Higher-Derivative Actions

As we have seen in section **??**, at the two-derivative level the presence of fluxes induces soft SUSY breaking and one can only partially write the Lagrangian supersymmetrically:

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft} \tag{4.6}$$

These soft terms occur when background fluxes lead to a vacuum in which supersymmetry is spontaneously broken, therefore leading to soft terms for the D7-brane modulus in the effective theory which is obtained after the bulk-moduli are made massive and gravity is neglected thus allowing to describe the theory through a global supersymmetric model.

When one looks at the higher-derivative terms something similar happens. Once again we cannot fully represent all terms of the Lagrangian supersymmetrically. This time, however, higher-derivative and higher-order terms concur both in the supersymmetric part and in the broken SUSY part.

$$\mathcal{L} = \underbrace{\mathcal{L}_{SUSY} + \mathcal{L}^{HD}_{SUSY}}_{SUSY\checkmark} + \underbrace{\mathcal{L}_{soft} + \mathcal{L}^{HO}_{\not{SUSY}}}_{\not{SUSY}} \tag{4.7}$$

where the $\mathcal{L}^{HO}_{\not{SUSY}}$ terms are *soft*-type terms which are of mass-dimensions > 4.

In the present case, from the reduction of the DBI action we got a component Lagrangian given by the sum of the scalar potential, the four-derivative and the two-derivative contributions:

$$\mathcal{L} = \mathcal{L}^{(4-der)}_{comp} + \mathcal{L}^{(2-der)}_{comp} + V_{comp} \tag{4.8}$$

where:

$$\begin{aligned} &\bullet \quad \mathcal{L}^{(4-der)}_{comp} \subset \mathcal{L}^{HD}_{SUSY} \\ &\bullet \quad \mathcal{L}^{(2-der)}_{comp} \subset \mathcal{L}_{SUSY} + \mathcal{L}^{HD}_{SUSY} + \mathcal{L}^{HO}_{\not{SUSY}} \\ &\bullet \quad V_{comp} \subset \mathcal{L}_{SUSY} + \mathcal{L}_{soft} + \mathcal{L}^{HD}_{SUSY} + \mathcal{L}^{HO}_{\not{SUSY}}. \end{aligned} \tag{4.9}$$

As we can see, the non-supersymmetric Lagrangians $\mathcal{L}_{soft}$ and $\mathcal{L}^{HO}_{\not{SUSY}}$ also contribute to the potential terms and $\mathcal{L}^{HO}_{\not{SUSY}}$ contributes to the two-derivative terms of the component Lagrangian (where the superscript "*HO*" stands for "higher order"), in addition to the two-derivative and the higher-derivative supersymmetric Lagrangians. This is the reason why we can only partially represent all terms supersymmetrically via the Lagrangian (**??**). In the next sections we will follow this philosophy in order to investigate the terms induced by the comparison between the higher-derivative terms of the component Lagrangian with the higher-derivative SUSY Lagrangian.

4.2.1 List of Four-Derivative Operators

When one obtains $(\partial^4\phi^4)$-type terms by expanding an effective action, it would be useful to be able to compare them with the ones appearing in the supersymmetric higher-derivative operators (**??**) or, eventually, in some other basis. The arbitrariness in the choice of the basis is due to the fact that the number of independent higher-derivative operators is higher than the number of the independent supersymmetric ones. This, as we see in the following, generates a difficulty in comparing the higher-derivative terms with their supersymmetric completion. In order to remedy this comparison problem, we draft a list of all possible $(\partial^4\phi^4)$-type operators limiting our discussion to a single complex scalar field ϕ. The complete list is given by:

$$\begin{aligned} A_1 &= \phi\phi\,\bar{\phi}\Box^2\bar{\phi} \\ A_2 &= \bar{\phi}\bar{\phi}\,\phi\Box^2\phi \\ A_3 &= \phi\phi\Box\bar{\phi}\Box\bar{\phi} \\ A_4 &= \bar{\phi}\bar{\phi}\Box\phi\Box\phi \\ A_5 &= \phi\bar{\phi}\Box\phi\Box\bar{\phi} \end{aligned} \tag{4.10}$$

$$\begin{aligned} A_6 &= \phi\phi\partial_\mu\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\bar{\phi} \\ A_7 &= \bar{\phi}\bar{\phi}\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi \\ A_8 &= \phi\bar{\phi}\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\bar{\phi} \end{aligned} \tag{4.11}$$

$$\begin{aligned} A_9 &= \phi\phi\partial_\mu\bar{\phi}\Box\partial^\mu\bar{\phi} \\ A_{10} &= \bar{\phi}\bar{\phi}\partial_\mu\phi\Box\partial^\mu\phi \\ A_{11} &= \phi\bar{\phi}\partial_\mu\phi\Box\partial^\mu\bar{\phi} \\ A_{12} &= \phi\bar{\phi}\partial_\mu\bar{\phi}\Box\partial^\mu\phi \end{aligned} \tag{4.12}$$

$$\begin{aligned} A_{13} &= \bar{\phi}\partial_\mu\phi\partial^\mu\phi\Box\bar{\phi} \\ A_{14} &= \phi\partial_\mu\bar{\phi}\partial^\mu\bar{\phi}\Box\phi \\ A_{15} &= \bar{\phi}\partial_\mu\phi\partial^\mu\bar{\phi}\Box\phi \\ A_{16} &= \phi\partial_\mu\phi\partial^\mu\bar{\phi}\Box\bar{\phi} \end{aligned} \tag{4.13}$$

$$\begin{aligned} A_{17} &= \bar{\phi}\partial_\mu\phi\partial_\nu\phi\partial^\mu\partial^\nu\bar{\phi} \\ A_{18} &= \phi\partial_\mu\bar{\phi}\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\phi \\ A_{19} &= \bar{\phi}\partial_\mu\phi\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\phi \end{aligned} \tag{4.14}$$

$$\begin{aligned} A_{20} &= \phi\partial_\mu\phi\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\bar{\phi} \\ A_{21} &= \partial_\mu\phi\partial^\mu\phi\partial_\nu\bar{\phi}\partial^\nu\bar{\phi} \\ A_{22} &= \partial_\mu\phi\partial_\nu\phi\partial^\mu\bar{\phi}\partial^\nu\bar{\phi}. \end{aligned} \tag{4.15}$$

Then we have 22 operators but they are not all independent: one can relate them by partial integration. The integration-by-part identities and complex conjugate identities are given by:

$$\begin{aligned}
A_{21} &= -(A_{14} + 2A_{20}) \\
A_{22} &= -(A_{18} + A_{16} + A_{20}) \\
A_1 &= -(2A_{11} + A_9) \\
A_{11} &= -(A_5 + A_{13} + A_{16}) \\
A_{11} &= -(A_8 + A_{17} + A_{20}) \\
A_{10} &= -(A_7 + 2A_{19}) \\
A_3 &= -(A_9 + 2A_{16}) \\
A_2 &= \bar{A}_1 \\
A_4 &= \bar{A}_3 \\
A_7 &= \bar{A}_6 \\
A_{10} &= \bar{A}_9 \\
A_{12} &= \bar{A}_{11} \\
A_{13} &= \bar{A}_{14} \\
A_{17} &= \bar{A}_{18} \\
A_{15} &= \bar{A}_{16} \\
A_{19} &= \bar{A}_{20}
\end{aligned} \tag{4.16}$$

To make sure that we have considered all integration-by-part-identities and that we have not forgotten any of them, one can perform the following consistency check: the number of integration-by-part identities in which each operator must be involved is equal to the number of the non-equivalent integration-by-parts that can be performed on it, keeping in mind that for each real operator, this number doubles. For instance, the first operator

$$A_1 = \phi\phi\,\bar{\phi}\Box^2\bar{\phi} \tag{4.17}$$

can be integrated by parts in a single way, taking into account one of the partial derivatives appearing in $\Box^2$, so that there is only one integration-by-part identity which involves it:

$$A_1 = -(2A_{11} + A_9). \tag{4.18}$$

On the contrary, for example, the operator

$$A_{16} = \phi\partial_\mu\phi\partial^\mu\bar{\phi}\Box \tag{4.19}$$

can be integrated by parts in three different ways, so that we find the three identities:

$$\begin{aligned} A_{22} &= -(A_{18} + A_{16} + A_{20}) \\ A_{11} &= -(A_5 + A_{13} + A_{16}) \\ A_3 &= -(A_9 + 2A_{16}) \end{aligned} \tag{4.20}$$

and so on. Also note that the real operators appear in the previously listed identity and in its complex conjugate. For instance the operator

$$A_{22} = \partial_\mu\phi\partial_\nu\phi\partial^\mu\bar{\phi}\partial^\nu\bar{\phi} \tag{4.21}$$

is involved in the identity

$$A_{22} = -(A_{18} + A_{16} + A_{20}) \tag{4.22}$$

and in its conjugate complex:

$$A_{22} = -(A_{17} + A_{15} + A_{19}) \tag{4.23}$$

obtained by making use of the complex conjugate identities, satisfies the requirement of being involved in two identities due to the its possibility of being integrated by parts in two different ways. Also note that, taking into account the complex conjugate of equations (**??**) we do not obtain more identities for A_{16}, which is not real. In the end we have a set of six independent four-derivative operators. For example, operators (**??**) can be written as:

$$\begin{aligned} &\mathcal{O}_1^{4-der}(A_5) = 16A_5 \\ &\mathcal{O}_3^{4-der}(A_{13}) = 16A_{13} \\ &\mathcal{O}_3^{4-der}(A_{22}, A_{18}, A_{20}, A_{11}) = 8(A_{22} + A_{18} - A_{20} - A_{11}). \end{aligned} \tag{4.24}$$

We note that in the previous expressions we used more than three operators from the list, this means that they can be further simplified by choosing a basis of three independent operators: we can choose, for instance, the operator basis (A_5, A_{13}, A_{20}) given by:

$$\begin{aligned} A_5 &= \phi\bar{\phi}\Box\phi\Box\bar{\phi} \\ A_{13} &= \bar{\phi}\partial_\mu\phi\partial^\mu\phi\Box\bar{\phi} \\ A_{20} &= \phi\partial_\mu\phi\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\bar{\phi}. \end{aligned} \tag{4.25}$$

Now the four-derivative parts of the operators (**??**) are written as:

$$\begin{aligned}\mathcal{O}_1^{4-der} &= 16A_5 \\ \mathcal{O}_2^{4-der} &= 16A_{13} \\ \mathcal{O}_3^{4-der} &= 8(A_5 + A_{13} - 2A_{20})\end{aligned} \tag{4.26}$$

As we have seen in section **??**, the supersymmetric four-derivative operators are only three: this means that, choosing a basis of three higher-derivative operators from the set of six independent ones and expressing the supersymmetric operators (**??**) in such a basis, the three remaining higher-derivative operators cannot be expressed in some supersymmetric way (but this feature is basis-dependent).

Therefore assuming that we have a non-supersymmetric Lagrangian including higher-derivative terms, we can now identify (making use of the identities (**??**)) the exact linear combination of the supersymmetric higher-derivative operators (**??**) which gives rise to the four-derivative term appearing in the Lagrangian.

4.2.2 Operator Matching for the D7-brane Model

We note that the four-derivative term in the Lagrangian (**??**) corresponds exactly to the four-derivative operator A_{21} in the list (**??**). Thanks to the identities (**??**), in particular making use of the first one and its complex conjugated:

$$\begin{aligned}A_{21} &= -(A_{14} + 2A_{20}) \\ A_{21} &= -(A_{13} + 2A_{19})\end{aligned} \tag{4.27}$$

one can quickly find the relation:

$$A_{21} = -\frac{1}{32}(\mathcal{O}_1^{4-der} + 2\mathcal{O}_2^{4-der} - 2\mathcal{O}_3^{4-der} + h.c.). \tag{4.28}$$

Including all the terms of the operators $\mathcal{O}_i$ in the last linear combination and making use of the integration-by-part method for those terms including the F and $\bar{F}$ summarized in appendix **??**, one obtains

$$\begin{aligned}A_{21} = |\partial_\mu\phi\partial^\mu\phi|^2 &\subset -\frac{1}{32}(\mathcal{O}_1 + 2\mathcal{O}_2 - 2\mathcal{O}_3 + h.c.) \\ &\equiv \frac{1}{16}D\Phi D\Phi\bar{D}\bar{\Phi}\bar{D}\bar{\Phi} = |\partial_\mu\phi\partial^\mu\phi|^2 - 2|F|^2\partial_\mu\phi\partial^\mu\bar{\phi} + |F|^4.\end{aligned} \tag{4.29}$$

This operator has appeared in several papers including [**?**, **?**, **?**, **?**, **?**]. As pointed out in [**?**] the operator $D\Phi D\Phi\bar{D}\bar{\Phi}\bar{D}\bar{\Phi}$ has some interesting properties: in this operator no kinetic terms for F appear, this means that F remains an auxiliary field; moreover it is Kähler-invariant and the bosonic part (**??**) of the operator $D\Phi D\Phi\bar{D}\bar{\Phi}\bar{D}\bar{\Phi}$ only contains D-type terms and all the lower components in θ and $\bar{\theta}$ vanish.

The above operator represents the unique supersymmetric completion of the four-derivative terms. As we have seen any other operator would not give the proper four-derivative contribution and would, thus, be incorrect.

It would be desirable to understand the complete form of the action as a sum of supersymmetric and soft-like terms. However, a full answer cannot be given here. We can at best present a result for all supersymmetric terms. Therefore, the contribution to the 2-derivative part of the Lagrangian and the scalar potential as induced by (**??**) are incomplete. To give the full answer regarding the supersymmetric terms we also have to consider possible corrections to the Kähler potential and superpotential as well as to the correct chiral variables Φ. Now, since we cannot say anything about the non-supersymmetric contributions to the Lagrangian, it may in principle be that several cancellations between supersymmetric and non-supersymmetric terms arise so that many possible terms do not appear in the final component Lagrangian in eq. (**??**). We are not going to consider this possibility here, but merely focus on the terms induced by the operator $D\Phi D\Phi\bar{D}\bar{\Phi}\bar{D}\bar{\Phi}$ as well as the terms we know must be present in eq. (**??**). Concerning this, let us have a look at the supersymmetric higher-derivative Lagrangian:

$$
\begin{aligned}
\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{SUSY}^{HD} &= \int d^4\theta K(\Phi,\bar{\Phi}) + \int d^2\theta W(\Phi) + \frac{a}{2}\sum_{i=1,2,3}(\lambda_i\mathcal{O}_i + \rho_i\bar{\mathcal{O}}_i) \\
&= \int d^4\theta K(\Phi,\bar{\Phi}) + \int d^2\theta W(\Phi) - \frac{a}{2}\left[\frac{1}{32}(\mathcal{O}_1 + 2\mathcal{O}_2 - 2\mathcal{O}_3 + h.c.)\right] \\
&= -K_{\phi\bar{\phi}}|\partial\phi|^2 - K^{\phi\bar{\phi}}|F|^2 + \frac{a}{2}\left(|\partial\phi|^4 - 2|F|^2|\partial\phi|^2 + |F|^4\right)
\end{aligned}
\tag{4.30}
$$

where the coefficient $a/2$ has been fixed by looking at the four-derivative term of the Lagrangian (**??**). As we can see, in the previous Lagrangian an $|F|^4$-type term arises, which would give rise to a correction to V which seems not present in the component Lagrangian (**??**). In the above Lagrangian the chiral superfield Φ could be corrected as:

$$
\begin{aligned}
\Phi = \Phi_0 + \delta\Phi = \Phi_0 + \sum_{i=1}^{\inf} l^{i/2}\Phi_i, \quad l = 2\pi\alpha', \quad l_s = 2\pi\sqrt{\alpha'} \\
m_s = \frac{1}{l_s} = \frac{1}{2\pi\sqrt{\alpha'}} = \frac{1}{\sqrt{2\pi\, l}}, \quad l = \frac{1}{2\pi m_s^2},
\end{aligned}
\tag{4.31}
$$

where Φ_i has mass-dimension $i+1$. The corresponding variation of the scalar component ϕ is of the type:

$$
\phi = \phi_0 + \phi_1 + ... \tag{4.32}
$$

where ϕ_0 is the scalar component of the chiral superfield involved in the two-derivative theory and ϕ_1 is a holomorphic function of ϕ_0 (because of the absence of terms of the form: $(\partial\phi)^2$ and $(\partial\bar{\phi})^2$). Since this function is holomorphic it would merely correspond to a different choice of coordinates and not actually to a correction of the geometry and, therefore, without loss of generality we set $\phi_1 = 0$. One can expect a correction to the Kähler potential $K_0 = \Phi_0\bar{\Phi}_0$ of the two-derivative theory as:

$$K = K_0 + \delta K, \tag{4.33}$$

so that

$$K_{\phi\bar{\phi}} = (K_0)_{\phi\bar{\phi}} + (\delta K)_{\phi\bar{\phi}} \tag{4.34}$$

and

$$\begin{aligned} K^{\phi\bar{\phi}} = (K_{\phi\bar{\phi}})^{-1} &= [(K_0)_{\phi\bar{\phi}} + (\delta K)_{\phi\bar{\phi}}]^{-1} \\ &\simeq (K_0)^{\phi\bar{\phi}}(1 - (K_0)^{\phi\bar{\phi}}(\delta K)_{\phi\bar{\phi}}) + O(l^2) \\ &= 1 - \delta K_{\phi\bar{\phi}} \end{aligned} \tag{4.35}$$

Making use of the equation of motion (**??**) for the leading order auxiliary field

$$\begin{aligned} F &= -K^{\phi\bar{\phi}}\frac{\partial\bar{W}}{\partial\bar{\phi}} = -\bar{W}_{\bar{\phi}} \\ \bar{F} &= -K^{\phi\bar{\phi}}\frac{\partial\bar{W}}{\partial\bar{\phi}} = -W_{\phi}, \end{aligned} \tag{4.36}$$

and of the scalar potential definition (**??**)

$$V_F(\phi) := K^{\phi\bar{\phi}}\left|\frac{\partial W}{\partial\phi}\right|^2 = K^{\phi\bar{\phi}}\,|W_{\phi}|^2 \tag{4.37}$$

one obtains:

$$V_F = K^{\phi\bar{\phi}}|F|^2. \tag{4.38}$$

Thus, taking into account equations (**??**) and (**??**) the Lagrangian (**??**) can be written as:

$$\begin{aligned} \mathcal{L} &= -K_{\phi\bar{\phi}}|\partial\phi|^2 - K^{\phi\bar{\phi}}|W_{\phi}|^2 + \frac{a}{2}\left(|\partial\phi|^4 - 2|F|^2|\partial\phi|^2 + |F|^4\right) \\ &= -(1 + \delta K_{\phi\bar{\phi}})|\partial\phi|^2 - (1 - \overbrace{\delta K_{\phi\bar{\phi}})|W_{\phi}|^2}^{\delta V} - a|\partial\phi|^2|W_{\phi}|^2 + \frac{a}{2}(|\partial\phi|^4 + |F|^4) \\ &= -(1 + aV_F(\phi) + \delta K_{\phi\bar{\phi}})|\partial\phi|^2 - (V_F(\phi) + \delta V) + \frac{a}{2}(|\partial\phi|^4 + |F|^4) \end{aligned} \tag{4.39}$$

where δV is the correction to the scalar potential induced by the correction to the Kähler potential δK. Comparing this to eq. (**??**) we find that $\delta K = 0$, since the two-derivative correction coming from the operator $D\Phi D\Phi\bar{D}\bar{\Phi}\bar{D}\bar{\Phi}$ already gives the correct (supersymmetric) dimension-8 two-derivative correction in eq. (??). In principle, one may wonder whether δV can be chosen in such a way that it cancels the contribution from the $|F|^4$-term. To this end, the correction to the Kähler potential has to be:

$$\delta K = -\frac{a}{2} W\bar{W} + c.c. \tag{4.40}$$

which gives

$$\delta K_{\phi\bar{\phi}} = -\frac{a}{2}|W_\phi|^2 \tag{4.41}$$

in such a manner that

$$K = K_0 + \delta K = \Phi_0\bar{\Phi}_0 - \frac{a}{2}|W|^2 \tag{4.42}$$

so that the correction to the scalar potential is:

$$\delta V = -\delta K_{\phi\bar{\phi}}|W_\phi|^2 = \frac{a}{2}|W_\phi|^4 = \frac{a}{2}|F|^4. \tag{4.43}$$

Now, substituting these results back into the Lagrangian (??), one has:

$$-[1 + \frac{a}{2}V_F(\phi)]\partial_\mu\phi\partial^\mu\bar{\phi} + \frac{a}{2}|\partial_\mu\phi\partial^\mu\phi|^2 - V_F(\phi). \tag{4.44}$$

Again we find that the respective two-derivative terms would not match with the ones in eq. (??) implying that $\delta K = 0$. This means that there is no full match between the component form Lagrangian and the supersymmetric Lagrangian which is not surprising since we did not determine the non-supersymmetric contributions in eq. (??).

Capitolo 5

Conclusions

Higher-derivative operators are a fundamental part of the study of effective field theories: they arise naturally by integrating out massive states in the low energy effective action. In this work we were interested in clarifying how to determine the supersymmetric form of an effective action obtained from string compactification with the inclusion of higher-derivative operators. During this analysis we realized that we have to make an essential assumption in order to say something about the partial or full supersymmetric representation of the effective action: we assume that no cancellations between supersymmetric and non-supersymmetric terms arise. Under this hypothesis we started the analysis of the effective Lagrangian (**??**) for the scalar fields position moduli of type IIB D7-branes in torus orbifolds provided by [**?**]. We have chosen the simple D7-brane example in order to understand how a systematic matching can in principle be obtained. Looking at the four-derivative term in the Lagrangian (**??**), we have identified the only higher-derivative supersymmetric operator which reproduces the same four-derivative term:

$$\frac{1}{16}\int \mathrm{d}^4\theta D\Phi D\Phi \bar{D}\bar{\Phi}\bar{D}\bar{\Phi} = |\partial_\mu\phi\partial^\mu\phi|^2 - 2|F|^2\partial_\mu\phi\partial^\mu\bar{\phi} + |F|^4. \tag{5.1}$$

In order to identify this operator and to determine its uniqueness, we performed a systematic analysis of the higher-derivative supersymmetric operators each with two chiral superfields Φ and two anti-chiral superfields $\bar{\Phi}$. Starting from the four-derivative terms and making use of integration-by-part identities, we have identified the combination of supersymmetric higher-derivative operators (supersymmetric by construction as well) which gives rise to the four-derivative term appearing in the Lagrangian (**??**): this combination coincides exactly with the well-known operator (**??**).

The presence of the $|F|^4$-type term in operator (**??**) leads us to conclude that a full supersymmetric representation of the Lagrangian (**??**) is not possible, especially since the $|F|^4$-type term cannot be reabsorbed via corrections to the Kähler potential without inducing corrections also to the $(|\partial\phi|^2 V(\phi))$-type term. This is why we can at best present a result for all supersymmetric terms, provided by (**??**).

There is however another possibility we must take into account: The Lagrangian (**??**) which was obtained assuming several approximations (such as neglecting the CS-terms and the effect of bulk-terms on the brane-reduction and vice versa) which might turn out to be unjustified if one wants to fully match a Lagrangian including all dimension-8 operators. For instance, as we can see in [**?**], the DBI and the CS give the same contribution to the scalar potential so, in principle we cannot neglect the contribution from the CS action. Furthermore, one might think that the bulk terms must also be included. Another possibility we must consider is that out of the probe-limit the back-reaction of the supergravity background is no longer negligible [**?**].

A desirable goal to reach in the future would be to study the previous corrections in order to obtain a more complete description of the effective action including higher-derivative terms.

Appendice A

Consistency Checks for Supersymmetric Higher-Derivative Operators

In this appendix we will see how the supersymmetric higher-derivative operators provided by equation (??) are consistent with those provided by [?] and [?]. In listing these operators we have chosen the same basis of [?], who lists four supersymmetric higher-derivative operators. In addition, we will see that the fourth operator in [?] can be recast in terms of the other three. Furthermore, as one can see, the $\mathcal{O}_3$ operator in (??) written in its supersymmetric form $\mathcal{O}_3 = |\Phi|^2 D\bar{D}\bar{\Phi}\bar{D}D\Phi$ is manifestly self-adjoint but we cannot say the same for its D-terms. This is the reason which motivates us to express this operator in a basis where self-adjointness is manifest. In this regard we make use of the method shown in section (??). We begin by expressing the operators given in (??) using the four-derivative operators listed in (??) as is done in (??):

$$\begin{aligned} \mathcal{O}_1^{4-der} &= 16|\phi|^2 \Box\phi\Box\bar{\phi} \\ &\equiv \mathcal{O}_1^{4-der}(A_5) = 16A_5 \end{aligned} \tag{A.1}$$

$$\begin{aligned} \mathcal{O}_2^{4-der} &= 16\partial_\mu\phi\partial^\mu\phi\bar{\phi}\Box\bar{\phi} - 16|F|^2\bar{\phi}\Box\phi \\ &\equiv \mathcal{O}_2^{4-der}(A_{13}) = 16A_{13} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \mathcal{O}_3^{4-der} &= 8(\partial_\mu\phi\partial^\mu\bar{\phi})^2 + 8\phi\partial_\mu\bar{\phi}(\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\phi - \partial_\nu\phi\partial^\mu\partial^\nu\bar{\phi}) \\ &\equiv \mathcal{O}_3^{4-der}(A_{22}, A_{18}, A_{20}, A_{11}) = 8(A_{22} + A_{18} - A_{20} - A_{11}) \end{aligned} \tag{A.3}$$

In the following we will use the notation $\mathcal{O}_A$ to indicate the operators in [?] and the notation $\mathcal{O}_{(i|j)}$ for those in [?]. The operators of interest in terms of the four-derivative operators listed in (??) are the ones given by [?]:

$$\begin{aligned} \mathcal{O_A}^{4-der} &= 16A_5 \\ \mathcal{O_B}^{4-der} &= 16A_{13} \\ \mathcal{O_C}^{4-der} &= 8(A_5 + A_{14} + A_{15} + A_{21}) \end{aligned} \tag{A.4}$$

The first ones coincide explicitly while the last one has to be recast through integration by parts, for instance, into $\mathcal{O}^{4-der}_{(2|3)}$:

$$\begin{aligned} &\mathcal{O_C}^{4-der} = 8(\phi\bar{\phi}\Box\phi\Box\bar{\phi} + \phi\partial_\mu\bar{\phi}\partial^\mu\bar{\phi}\Box\phi + \phi\partial_\mu\phi\bar{\partial}^\mu\phi\Box\bar{\phi} + \partial_\mu\phi\partial^\mu\phi\partial_\nu\bar{\phi})\partial^\nu\bar{\phi} \\ &\equiv \mathcal{O_C}^{4-der}(A_5, A_{14}, A_{15}, A_{21}) = 8(A_5 + A_{14} + A_{15} + A_{21}) \end{aligned} \tag{A.5}$$

$$\begin{aligned} &\mathcal{O}^{4-der}_{(2|3)} = -4(\phi\partial_\mu\phi\partial^\mu\bar{\phi}\Box\bar{\phi} + 2\phi\partial_\mu\bar{\phi}\partial_\nu\phi\partial^\mu\partial^\nu\bar{\phi} + \phi\bar{\phi}\partial_\mu\phi\partial^\mu\Box\bar{\phi} + h.c.) \\ &\equiv \mathcal{O}^{4-der}_{(2|3)}(A_{16}, A_{20}, A_{11}) = -4(A_{16} + 2A_{20} + A_{11} + h.c.). \end{aligned} \tag{A.6}$$

We can quickly derive (??) from (??) making use of the first and the fourth identities of the usual list:

$$\begin{aligned} A_{21} + A_{14} &= -2A_{20} \\ A_5 + A_{13} &= -(A_{16} + A_{11}) \, , \end{aligned} \tag{A.7}$$

$$\begin{aligned} \mathcal{O_C}^{4-der} &= 8(A_5 + A_{13} + A_{14} + A_{21}) \\ &= 4[-(A_{16} + A_{11}) + A_{14} + A_{21} + h.c.] \\ &= -4(A_{16} + 2A_{20} + A_{11} + h.c.) = \mathcal{O}^{4-der}_{(2|3)}. \end{aligned} \tag{A.8}$$

In the same way, we can bring the operator of the present paper:

$$\begin{aligned} &\mathcal{O}^{4-der}_3 = 8(\partial\mu\phi\partial^\mu\bar{\phi})^2 + 8\phi\partial_\mu\bar{\phi}(\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\phi - \partial_\nu\phi\partial^\mu\partial^\nu\bar{\phi}) \\ &\equiv \mathcal{O}^{4-der}_3(A_{22}, A_{18}, A_{20}, A_{11}) = 8(A_{22} + A_{18} - A_{20} - A_{11}) \end{aligned} \tag{A.9}$$

to $\mathcal{O}^{4-der}_{(2|3)}$ using the second equation from the list (??):

$$A_{21} + A_{18} = -(A_{16} + A_{20}) \, , \tag{A.10}$$

which allow to write:

$$\begin{aligned} O^{4-der}_3 &= 8(A_{22} + A_{18} - A_{20} - A_{11}) \\ &= 4[-(A_{16} + A_{20}) - A_{20} - A_{11} + h.c.] \\ &= -4(A_{16} + 2A_{20} + A_{11} + h.c.) = \mathcal{O}^{4-der}_{(2|3)}. \end{aligned} \tag{A.11}$$

So we have shown that, despite the hidden self-adjointness of the operator $\mathcal{O}_3$ in (**??**), it coincides exactly with those provided by [**?**] and [**?**]. The fact that self-adjointness is hidden is because the operator terms A_{18} and A_{20} are not self-adjoint themselves but their difference cancels out the inconvenient contributions. Moreover we have seen how the method of tracing back the integration-by-part identities to simple linear systems can greatly simplify the problem of comparing higher-derivative operators.

Moreover, using the same method we can see that the combination:

$$\begin{aligned}&\frac{1}{4}(O_1^{4-der} + O_2^{4-der} - 2O_3^{4-der})\\&= \frac{1}{4}(16A_5 + 16A_{13} - 16(A_5 + A_{13} - 2A_{20}))\\&= 8A_{20}\end{aligned} \tag{A.12}$$

gives rise to the operator:

$$\begin{aligned}O_4^{4-der} &= \Phi^2 D\bar{D}\bar{\Phi}D\bar{D}\bar{\Phi}|_D\\&= -4(|\partial_\mu\phi\partial^\mu\phi|^2 + \phi\Box\phi\partial_\mu\bar{\phi}\partial^\mu\bar{\phi} + \phi^2\partial_\mu\partial_\nu\bar{\phi} + \phi^2\partial^\mu\partial^\nu\bar{\phi} + 2\phi\partial_\mu\phi\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\bar{\phi} + \phi^2\partial_\mu\bar{\phi}\partial^\mu\Box\bar{\phi})\\&= -4(A_{21} + A_{14} + A_6 + 2A_{20} + A_9) =\\&= -4(-2A_{20} + A_{20} + A_6 + A_9)\\&= 8A_{20}\end{aligned} \tag{A.13}$$

listed in [**?**]. Then, in the end, one has:

$$O_4^{4-der} = \frac{1}{4}(O_1^{4-der} + O_2^{4-der} - 2O_3^{4-der}). \tag{A.14}$$

Appendice B

Pull-back expansion along the normal direction

As we have seen in section (??), when expanding the pull-back we have to take into account the fluctuations in the normal direction of the world-volume [9,10]. As we can see in [6], these displacements in the normal direction are captured as section ξ of the world-volume normal bundle $\mathcal{NW}$ and we can now parametrize the normal direction ξ with a parameter t. This allows to naturally define the map:

$$\begin{aligned} \hat{\varphi} :& \mathcal{W} \times \mathcal{I} \hookrightarrow M \\ (x,t) &\longmapsto \hat{\varphi}, \end{aligned} \tag{B.1}$$

Where $\mathcal{I}$ is a tubular neighborhood of $\mathcal{W}$. This map is defined in such a way that

$$\hat{\varphi}(x,0) = \varphi(x) \quad \text{and} \quad \frac{d}{dt}\hat{\varphi}(x,0) = \xi|_y. \tag{B.2}$$

We can finally expand the pull-back of a general tensor T along the normal direction ξ parametrized by t:

$$\begin{aligned} (\hat{\varphi}^*T)|_{\hat{\varphi}(x,t)} &= \hat{\varphi}^*(e^{\nabla_{t\xi}}T)|_{\hat{\varphi}(x,0)} \\ &= \hat{\varphi}^*(T)|_{\hat{\varphi}(x,0)} + t\hat{\varphi}^*(\nabla_{t\xi}T)|_{\hat{\varphi}(x,t)} + \frac{1}{2}t^2\hat{\varphi}^*(\nabla_{t\xi}\nabla_{t\xi}T)|_{\hat{\varphi}(x,t)} + \ldots \quad . \end{aligned} \tag{B.3}$$

Considering the vanishing Lie-Bracket $[\partial_\mu, \partial_t]$, from the expression of the torsion:

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y] = 0 \tag{B.4}$$

one obtains

$$\nabla_\xi \partial_\mu = \nabla_{\partial_\mu}\xi. \tag{B.5}$$

Furthermore, from the expression of the Riemann tensor

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \tag{B.6}$$

and taking into account the fact that $\nabla_\xi \xi = 0$ one gets:

$$R(\xi, \partial_\mu)\xi = \nabla_\xi \nabla_{\partial_\mu} \xi = \nabla_\xi \nabla_\xi \partial_\mu . \tag{B.7}$$

Then the equation (**??**) for the metric gives:

$$\begin{aligned}(\hat{\varphi}^* g(\partial_\mu, \partial_\nu)|_{\hat{\varphi}^*(x,t)} =& \hat{\varphi}^* g(\partial_\mu, \partial_\nu)|_{\hat{\varphi}^*(x,0)} \\ &+ t \left[\hat{\varphi}^* g(\nabla_{\partial_\mu}\xi, \partial_\nu)|_{\hat{\varphi}^*(x,0)} + \hat{\varphi}^* g(\partial_\mu, \nabla_{\partial_\nu}\xi)|_{\hat{\varphi}^*(x,0)} \right] \\ &+ t^2 \left[\hat{\varphi}^* g(\nabla_{\partial_\mu}\xi, \nabla_{\partial_\nu}\xi)|_{\hat{\varphi}^*(x,0)} + \hat{\varphi}^* g(R(\xi,\partial_\mu)\xi, \partial_\nu)|_{\hat{\varphi}^*(x,0)} \right] .\end{aligned} \tag{B.8}$$

Setting $t = 1$ and introducing a dimensional factor l one has:

$$\begin{aligned}\varphi^*(g)_{\mu\nu} =& g_{\mu\nu} + l g_{\mu n} D_\nu \phi^n + l g_{m\nu} D_\mu \phi^n \\ &+ l^2 g_{mn} D_\mu \phi^n D_\nu \phi^m + l^2 g_{\mu\tau} R_n{}^\tau{}_{\nu m} \phi^n \phi^m + ...\end{aligned} \tag{B.9}$$

where the position modulus ϕ is involved via $\xi = \phi^n \partial_n$. In our case, the mixed terms $g_{\mu n}$ are all vanishing due to the ansatz of the metric. In close analogy, one obtains the expansion of a q-form pull-back as:

$$\begin{aligned}(\hat{\varphi}^* C^{(p)})|_{\hat{\varphi}^*(x,t)} = & [\frac{1}{p!} C^{(p)}_{\nu_1 \ldots \nu_p} + \frac{l}{p!} \phi^n \partial_n (C^{(p)}_{\nu_1 \ldots \nu_p}) - \frac{l}{(p-1)!} D_{\nu_1} \phi^n C^{(p)}_{n \nu_2 \ldots \nu_p} \\ &+ \frac{l^2}{2p!} \phi^n \partial_n (\phi^m \partial_m (C^{(p)}_{\nu_1 \ldots \nu_p})) - \frac{l^2}{(p-1)!} D_{\nu_1} \phi^n \phi^m \partial_m (C^{(p)}_{n\nu_2 \ldots \nu_p}) \\ &+ \frac{l^2}{2(p-2)!} D_{\nu_1} \phi^n D_{\nu_2} \phi^m C^{(p)}_{nm\nu_3 \ldots \nu_p} \\ &+ \frac{p-2}{2p!} l^2 R_n{}^\tau{}_{\nu_1 m} \phi^n \phi^m C^{(p)}_{\tau \nu_2 \ldots \nu_p}] dx^{\nu_1} \wedge \ldots \wedge dx^{\nu_p}\end{aligned} \tag{B.10}$$

Appendice C

Four-Derivative Term from the Square Root Expansion

In this appendix we summarize the computation of the four-derivative terms arising from the expansion of the square root (**??**). In order to have a lighter notation, let us ignore all coefficients for the moment. Moreover, as we are interested in the $(\partial\phi)^4$-type terms, let us also neglect the terms involving the Riemann tensor. Therefore, in order to obtain the four-derivative terms, the terms of interest in the following expansion:

$$\sqrt{\det(\mathbf{1}+A)} = 1 + \frac{1}{2}\text{Tr}A - \frac{1}{4}\text{Tr}A^2 + \frac{1}{8}(\text{Tr}A)^2 + ... \tag{C.1}$$

are the last two. First we consider the term $-\frac{1}{4}\text{Tr}A^2$, then we have:

$$\begin{aligned}\text{Tr}A^2 \supset & \text{Tr}(g_{mn}\partial_\mu\varphi^m\partial_\nu\varphi^n)^2 \\ =& \text{Tr}(\partial_\mu\varphi^m\partial_\nu\varphi_m)^2 \\ =& (\partial_\mu\varphi^m\partial_\nu\varphi_m)(\partial^\mu\varphi_p\partial^\nu\varphi^p)\end{aligned} \tag{C.2}$$

where φ^m are real fields. In order to rearrange them in a complex basis, let us split the index m in two set of consecutive indices as:

$$m = \underbrace{1,...,\frac{n}{2}}_{A}, \underbrace{\frac{n}{2}+1,...,n}_{\alpha} = (A,\alpha) \tag{C.3}$$

where we use uppercase Latin characters to indicate the first set of $n/2$ indices and Greek characters to indicate the second set of indices, where n is the number of real fields in the theory, in such a manner that the implicit sum $\partial\varphi^m\partial\varphi_m$ can be written as

$$\partial\varphi^m\partial\varphi_m = \partial\varphi^A\partial\varphi_A + \partial\varphi^\alpha\partial\varphi_\alpha \tag{C.4}$$

We can now rearrange the real fields in a complex basis defined via:

$$\phi^a = \varphi^A + i\varphi^\alpha, \qquad \bar{\phi}^a = \varphi^A - i\varphi^\alpha \tag{C.5}$$

so that

$$\varphi^A = \frac{1}{2}(\phi^a + \bar{\phi}^a), \qquad \varphi^\alpha = -\frac{i}{2}(\phi^a - \bar{\phi}^a). \tag{C.6}$$

Now we can expand the four-derivative term (**??**) in this complex basis as follows:

$$\begin{aligned}
&(\partial_\mu \varphi^m \partial_\nu \varphi_m)(\partial^\mu \varphi_p \partial^\nu \varphi^p) \\
&= (\partial_\mu \varphi^A \partial_\nu \varphi_A + \partial_\mu \varphi^\alpha \partial_\nu \varphi_\alpha)(\partial^\mu \varphi_B \partial^\nu \varphi^B + \partial^\mu \varphi_\beta \partial^\nu \varphi^\beta) \\
&= \frac{1}{4}[\partial_\mu(\phi^a + \bar{\phi}^a)\partial_\nu(\phi_a + \bar{\phi}_a) - \partial_\mu(\phi^a - \bar{\phi}^a)\partial_\nu(\phi_a - \bar{\phi}_a)] \\
&\times \frac{1}{4}[\partial^\mu(\phi^b + \bar{\phi}^b)\partial^\nu(\phi_b + \bar{\phi}_b) - \partial^\mu(\phi^b - \bar{\phi}^b)\partial^\nu(\phi_b - \bar{\phi}_b)] \\
&= \frac{1}{16}[\partial_\mu \cancel{\phi^a \partial_\nu} \phi_a + \partial_\mu \phi^a \partial_\nu \bar{\phi}_a + \partial_\mu \bar{\phi}^a \partial_\nu \phi_a + \partial_\mu \cancel{\bar{\phi}^a \partial_\nu} \bar{\phi}^a \\
&- (\partial_\mu \cancel{\phi^a \partial_\nu} \phi_a - \partial_\mu \phi^a \partial_\nu \bar{\phi}_a - \partial_\mu \bar{\phi}^a \partial_\nu \phi_a + \partial_\mu \cancel{\bar{\phi}^a \partial_\nu} \bar{\phi}^a)] \times (a \leftrightarrow b), (\partial_\mu \leftrightarrow \partial^\mu\ ) \\
&= \frac{1}{4}(\partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \phi^b \partial^\nu \bar{\phi}_b + \partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \bar{\phi}^b \partial^\nu \phi_b \\
&+ \partial_\mu \bar{\phi}^a \partial_\nu \phi_a \partial^\mu \phi^b \partial^\nu \bar{\phi}_b + \partial_\mu \bar{\phi}^a \partial_\nu \phi_a \partial^\mu \bar{\phi}^b \partial^\nu \phi_b) \\
&= \frac{1}{2}(\partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \phi^b \partial^\nu \bar{\phi}_b + \partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \bar{\phi}^b \partial^\nu \phi_b).
\end{aligned} \tag{C.7}$$

Therefore we have:

$$-\frac{1}{4}\mathrm{Tr}A^2 \supset -\frac{1}{8}(\partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \phi^b \partial^\nu \bar{\phi}_b + \partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \bar{\phi}^b \partial^\nu \phi_b). \tag{C.8}$$

Regarding the third term of the expansion (**??**) we have:

$$\begin{aligned}
(\mathrm{Tr}A)^2 =& (\partial_\mu \varphi^m \partial^\mu \varphi_m)^2 \\
=& \partial_\mu \varphi^m \partial^\mu \varphi_m \partial_\nu \varphi^p \partial^\nu \varphi_p
\end{aligned} \tag{C.9}$$

Note that in this case, unlike the first, the sum on the indices which take into account the real fields takes place under partial derivatives carrying the same space-time index.

Now considering the same change of basis, one obtains:

$$
\begin{aligned}
&(\partial_\mu \varphi^m \partial^\mu \varphi_m)^2 \\
&= (\partial_\mu \varphi^A \partial^\mu \varphi_A + \partial_\mu \varphi^\alpha \partial^\mu \varphi_\alpha)^2 \\
&= [\frac{1}{4}(\partial_\mu \phi^a + \partial_\mu \bar{\phi}^a)(\partial^\mu \phi_a + \partial^\mu \bar{\phi}_a) \\
&- \frac{1}{4}(\partial_\mu \phi^a - \partial_\mu \bar{\phi}^a)(\partial^\mu \phi_a - \partial^\mu \bar{\phi}_a)]^2 \\
&= [\frac{1}{4}(\partial_\mu \cancel{\phi^a \partial^\mu} \phi_a + \partial_\mu \cancel{\bar{\phi}^a \partial^\mu} \bar{\phi}_a + 2\partial_\mu \phi^a \partial^\mu \bar{\phi}_a \\
&- (\partial_\mu \cancel{\phi^a \partial^\mu} \phi_a + \partial_\mu \cancel{\bar{\phi}^a \partial^\mu} \bar{\phi}_a - 2\partial_\mu \phi^a \partial^\mu \bar{\phi}_a)]^2 \\
&= (\partial_\mu \phi^a \partial^\mu \bar{\phi}_a)^2 \\
&= \partial_\mu \phi^a \partial^\mu \bar{\phi}_a \partial_\nu \phi^b \partial^\nu \bar{\phi}_b.
\end{aligned}
\tag{C.10}
$$

So in the end we get:

$$\frac{1}{8}(\mathrm{Tr}A)^2 \supset \frac{1}{8}(\partial_\mu \phi^a \partial^\mu \bar{\phi}_a \partial_\nu \phi^b \partial^\nu \bar{\phi}_b), \tag{C.11}$$

In such a way that the total four-derivative contribution is:

$$-\frac{1}{8}(\partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \phi^b \partial^\nu \bar{\phi}_b + \partial_\mu \phi^a \partial_\nu \bar{\phi}_a \partial^\mu \bar{\phi}^b \partial^\nu \phi_b - \partial_\mu \phi^a \partial^\mu \bar{\phi}_a \partial_\nu \phi^b \partial^\nu \bar{\phi}_b). \tag{C.12}$$

Appendice D

Two-derivative terms including F and $\bar{F}$

Supersymmetric higher-derivative operators also contain terms including the auxiliary field F. In order to obtain a supersymmetric match the explicit knowledge of these kinds of terms and their integration-by-part identities could be useful. In the following we draft a list in close analogy with the one provided for the four-derivative terms.

$$\begin{aligned}
B_1 &= F\bar{F}\partial_\mu\phi\partial^\mu\bar{\phi} \\
B_2 &= F\bar{F}\phi\Box\bar{\phi} \\
B_3 &= F\bar{F}\bar{\phi}\Box\phi \\
B_4 &= F\phi\partial_\mu\bar{F}\partial^\mu\bar{\phi} \\
B_5 &= \bar{F}\bar{\phi}\partial_\mu F\partial^\mu\phi \\
B_6 &= F\bar{\phi}\partial_\mu\bar{F}\partial^\mu\phi \\
B_7 &= \bar{F}\phi\partial_\mu F\partial^\mu\bar{\phi} \\
B_8 &= \bar{F}\bar{\phi}\phi\Box F \\
B_9 &= F\phi\bar{\phi}\Box\bar{F} \\
B_{10} &= \phi\bar{\phi}\partial_\mu F\partial^\mu\bar{F}
\end{aligned} \tag{D.1}$$

provided with the following integration-by-parts identities:

$$\begin{aligned}
B_3 =& \bar{B}_2 \\
B_5 =& \bar{B}_4 \\
B_7 =& \bar{B}_6 \\
B_9 =& \bar{B}_8 \\
B_1 =& -(B_2 + B_7 + B_4) \\
B_4 =& -(B_9 + B_6 + B_{10}).
\end{aligned} \tag{D.2}$$

Appendice E

Computation of the four-derivative term coefficient

Observing the Lagrangian (??) we note that the multiplicative factor concerning the four-derivative term is different from that reported in [?] (eq. (3.1)):

$$\mathcal{L} = -[1 + aV(\phi)]\partial_\mu\phi\partial^\mu\bar{\phi} + |\partial_\mu\phi\partial^\mu\phi|^2 - V(\phi) \tag{E.1}$$

We report the explicit calculation which led us to the introduction of this factor for completeness. We start from eq. (2.7) in [?]:

$$\mathcal{L} = \frac{\mu_p V_{p-3} g_s}{Z}(1 + aV(\phi)) \left[1 + Z\, l^2|\partial\phi|^2 - \frac{1}{2}Z^2\, l^4|\partial_\mu\phi\partial^\mu\phi|^2\right], \tag{E.2}$$

comparing the potential term with what appears in the Lagrangian (??), one obtains:

$$-\frac{\mu_p V_{p-3} g_s}{Z} aV(\phi) = -V(\phi) \Longleftrightarrow a = \frac{Z}{\mu_p V_{p-3} g_s}. \tag{E.3}$$

Now imposing the match between the two-derivative terms, one finds that:

$$-\mu_p V_{p-3} z^{-1} g_s (1 + aV(\phi)) Z\, l^2 |\partial\phi|^2 = -(1 + aV(\phi))|\tilde{\phi}|^2 \tag{E.4}$$

where $\tilde{\phi}$ is defined as

$$\tilde{\phi} := \sqrt{\mu_p V_{p-3} g_s\, l^2}\,\phi. \tag{E.5}$$

Then, looking at the four-derivative term we finally get the four-derivative term coefficient:

$$\frac{\mu_p V_{p-3} g_s}{Z} aV(\phi)\frac{1}{2} z^2\, l^4 |\partial\phi|^4 = (\mu_p V_{p-3} g_s)^{-1}\frac{Z}{2}|\partial\tilde{\phi}|^4 = \frac{\mathbf{a}}{\mathbf{2}}|\partial\tilde{\phi}|^4. \tag{E.6}$$

The identification of this coefficient is important because the purpose of section (??) is precisely to find a correspondence between the higher-derivative operator which supplies the term $|\partial\phi|^4$ and the Lagrangian (??). As we have seen in section (??), the evidence of the matching failure between the Lagrangian and its supersymmetric completion is actually given by the fact that the two-derivative term $V(\phi)|\partial\phi|^2$ has a different proportionality to the coefficient a compared to the $|\partial\phi|^4$ term.

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