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Supersymmetry Breaking from 4D String Moduli

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Abstract

The low energy limit of String Theory can give rise to $N = 1$ supersymmetric effective field theories which represent a very promising framework for Beyond the Standard Model Physics. Moreover string compactifications naturally include mechanisms for spontaneous supersymmetry breaking in the hidden sector due to moduli stabilisation. Soft supersymmetry breaking terms are then generated due to the gravitational interaction between moduli and visible sector fields. This thesis focuses on Type IIB 4D models within the so-called LARGE Volume Scenario for moduli stabilisation. In particular it considers K3 fibred Calabi-Yau compactifications where the the Minimal Supersymmetric Standard Model (MSSM) is supported on D7 branes wrapped around 4-cycles in the geometric regime. String loop corrections to the Kähler potential play a crucial rôle for moduli stabilisation and supersymmetry breaking. Moreover, this class of string compactifications has been shown to be particularly suitable for realising cosmic inflation. After a discussion on the form of the Kähler metric for visible sector matter fields, the thesis determines the structure of resulting soft supersymmetry breaking terms for different brane set-ups. Finally, it analyses the main phenomenological bounds on hidden sector and Standard Model superpartner mass spectra together with the requirement of obtaining a correct Higgs mass. The final outcome is that this framework does not allow for both successful inflationary model building and a visible sector which is given exactly by the MSSM. Possible way-outs would require either a different inflationary mechanism or a minimal extension of the MSSM (like the NMSSM).

Abstract (in Italiano)

Il limite di bassa energia della Teoria delle Stringhe può dare luogo a teorie di campo effettive con supersimmetrie $N = 1$ che rappresentano uno scenario molto promettente per la Fisica oltre il Modello Standard. Inoltre, le compattificazioni delle stringhe includono naturalmente meccanismi per la rottura spontanea della supersimmetria nel settore nascosto a causa della stabilizzazione dei moduli. I termini che rompono la simmetria nel visibile (soft supersymmetry breaking terms) sono poi generati attraverso l'interazione gravitazionale tra i moduli e il settore osservabile. La tesi è incentrata sui modelli 4D per le stringhe Type IIB all'interno del cosiddetto LARGE Volume Scenario per la stabilizzazione dei moduli. In particolare, considera compattificazioni di Calabi-Yau di tipo K3 fibrato in cui il Minimal Supersymmetric Standard Model (MSSM) è supportato su brane D7 avvolte su 4-cicli nel regime geometrico. Le correzioni di string loop al potenziale di Kähler rivestono un ruolo cruciale per la stabilizzazione dei moduli e la rottura della supersimmetria. Inoltre, questa classe di compattificazioni di stringa si è dimostrata particolarmente appropriata per la realizzazione dell'inflazione cosmica. Dopo una discussione sulla forma della metrica di Kähler per i campi di materia del settore visibile, la tesi determina la struttura dei soft supersymmetry breaking terms risultanti per differenti sistemazioni delle brane. Infine, analizza i principali vincoli fenomenologici sugli spettri di massa del settore nascosto e dei superpartner del Modello Standard, insieme con il requisito dell'ottenimento di una massa dell'Higgs corretta. L'esito finale indica che questa modellizzazione non permette contemporaneamente la presenza di una buona descrizione dell'inflazione e di un settore visibile dato esattamente dall'MSSM. Delle possibili vie d'uscita necessiterebbero o di un modello inflazionario differente o di una estensione minimale dell'MSSM (come il NMSSM).

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Notation and Conventions

The reduced Planck system of units is employed, unless differently stated, i.e. the fundamental constants c, $\hbar$ and $8\pi G$ are set equal to 1:

$$c \equiv \hbar \equiv 8\pi G \equiv 1.$$

Therefore the reduced Planck mass M_P is equal to 1 too, while its value in physical units reads:

$$M_P = \sqrt{\frac{\hbar c}{8\pi G}} = 2.4 \cdot 10^{18} \mathrm{GeV}/c^2.$$

However, the use of physical units is accomplished by leaving $c = \hbar = 1$, as customary in the literature.

The Minkowski metric tensor, invariant under the Lorentz group, is defined with the $(+ - --)$ signature:

$$\eta_{\mu\nu} \equiv \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \equiv \eta^{\mu\nu},$$

while the $SL(2,\mathbb{C})$ invariant tensor is normalised as:

$$\epsilon_{\alpha\beta} \equiv \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix} \equiv -\epsilon^{\alpha\beta}.$$

Einstein summation convention is always understood where logically admissible.

The Kähler potential and the superpotential for string-derived supergravity are defined in reduced Planck mass units, i.e. they are considered dimensionless and physical mass dimensions are restored via multiplications by M_P.
For instance, the general string moduli Kähler potential and superpotential in the Einstein frame read, in physical units:

$$K = \left[-\ln\left(S + \bar{S}\right) - \ln\left(-i \int_X \Omega \wedge \bar{\Omega}\right) - 2\,\ln\left(\mathcal{V} + \frac{\xi'}{2}\left(\frac{S + \bar{S}}{2}\right)^{\frac{3}{2}}\right)\right] M_P^2,$$

$$W = \left[\int_X G_3 \wedge \Omega + \sum_i A_i e^{-a'_i(\tau_i + i\psi_i)}\right] M_P^3.$$

Introduction

Quantum Field Theory and General Relativity are undoubtedly the most striking successes of modern Physics. On their basis, the Standard Model of Particle Physics and Cosmology provides outstanding experimental predictions.

However, it is well known that Theoretical Physics still lacks in providing a unified description of the fundamental particles and interactions of Nature for high energy scenarios as well as a precise account of the details of the structure and evolution of Universe in its entirety.

String Theory is one of the most promising research areas in the way towards a more fundamental physical description. In its low energy limit - i.e. up to scales just some orders of magnitude below the Planck scale - it can provide natural extensions of the Standard Model able to account for both Particle Physics and Cosmology generalisations in a comprehensive scenario. The final purpose of String Phenomenology is then the accomplishment of a fully realistic description of Nature up to huge scales of energy.

Typically, Standard Model generalisations arising from String Theory rely on an extension of the symmetries of the model including supersymmetry, i.e. a symmetry which essentially involves the existence of a fermionic or bosonic partner with suitable couplings for any of the bosons and fermions of the theory, respectively.

Generally, these string-derived supersymmetric models involve two sectors of particles:

- the observable sector, represented by the Standard Model particles and their supersymmetric partners, plus possible further 'observable' particle doublets. Because of the breaking of supersymmetry - broken in the observable sector by the so-called "soft breaking terms" - all of the supersymmetric partners are new particles yet to be detected owing to their very large masses;

- the hidden sector, whose particles are responsible for the spontaneous breaking of supersymmetry. These particles are very heavy and are 'hidden' from observations in the sense that they are all coupled to the observable sector with gravitational-strength couplings.

In principle, such an enhanced particle spectrum can allow both to solve Particle Physics problematic issues and to individuate suitable candidates for cosmologically relevant particles such as dark matter candidates and the inflaton field.

Nevertheless, things are not easy because of the incredible vastness of phenomena that a reliable model must account for. Moreover, the lack of observational evidences from particle physics experiments - due to the large masses involved - and the uncertainty in the details of several cosmological processes render the task even harder.

This thesis falls within the scope of the individuation of a possibly reliable Standard Model supersymmetric extension by proposing the general analysis of a modelling arising from Type IIB String Compactification in the LARGE Volume Scenario. The hidden sector is represented by string moduli - i.e. low energy remnants of string theory - while the observable sector is identified with the Minimal Supersymmetric Standard Model particle content - i.e. the simplest Standard Model supersymmetric extension - assuming it to be supported on D7 branes wrapping some of the Calabi-Yau volume 4-cycles.

More specifically, the set-up takes place within the well known context of compactifications of Calabi-Yau manifolds with K3 Fibration Structure in the LARGE Volume Scenario, including string loop corrections. Notoriously, Kähler moduli emerge naturally as possible hidden sector fields and their mass spectrum is determined by studying their stabilisation by both perturbative and non-perturbative corrections. Then, the possible realisation of the Minimal Supersymmetric Standard Model on D7 branes wrapping large - i.e. volume-controlling - Calabi-Yau 4-cycles is introduced and the observable sector supersymmetry breaking terms are analysed. In particular, this fact essentially allows to determine the expected Standard Model supersymmetric partner mass spectrum in such a set-up and constitutes the original part of this thesis.

The whole analysis is performed in parallel with a modelling arising from string compactifications on $\mathbb{P}_{[1,1,1,6,9]}$, similar in many aspects as far the general spontaneous supersymmetry breaking is concerned, but which departs significantly when taking into account the realisation of the Minimal Supersymmetric Standard Model. Indeed, it must be supported on a tiny 4-cycle in this case. The model is a widely studied topic in the literature and served as a guide throughout the development of the thesis.

The outcomes of this work can be interesting as they help in pointing out some general conclusions about the model taken under exam. An analysis of possible inflation mechanisms based on the same general framework is present in the literature, so the bounds set on hidden sector and soft breaking terms outlined here can help in having a better viewpoint about the relationships between the Particle Physics and Cosmology implications of such a kind of scenarios.

To summarise and help the reading, the thesis structure is synthetically outlined below.

- Chapter 1 provides an essential introduction to supersymmetric extensions of the Standard Model, outlining their main features and the general context they can arise from.

- Chapter 2 summarises the main features of Type IIB String Compactifications in the LARGE Volume Scenario referring to the general characteristic of spontaneous supersymmetry breaking in the hidden sector by Kähler moduli.

- Chapter 3 describes the stabilisation of Kähler moduli in three specific models. Model I resumes the KKLT scenario in order to explain why different set-ups are needed, Model II outlines the framework of string compactifications on $\mathbb{P}_{[1,1,1,6,9]}$ and Model III delineates the scenario of compactifications with K3 Fibration Structure. Particular attention is dedicated to the determination of mass spectra.

- Chapter 4 contains an overview on the structure of soft breaking terms coming from moduli stabilisation and then describes the computation of such soft terms arising both from D7 branes on blow-up 4-cycles - in relation to Models II and III - and from D7 branes on large 4-cycles - in relation to Model III.

- Chapter 5 essentially analyses the mass spectra predicted by the previous models, with particular attention to the necessary conditions which must hold in such a way as to guarantee the realisation of a reliable Minimal Supersymmetric Standard Model from D7 branes on large cycles.

Chapter 1

Supersymmetric Standard Model

The Standard Model of particle physics is one of the principal achievements in modern science for its outstandingly successful predictions.
However, despite of its accuracy in currently accessible observations, it lacks in providing explanations to some crucial issues and therefore many extensions to a more fundamental theory exist in which it is regarded as an effective model valid at low energies.
This Chapter provides a brief list of Standard Model main problems and overviews synthetically the fundamental ideas which lie at the basis of its supersymmetric extensions, outlining the reasons for which such problems would be solved and placing it into the context of the general fundamental theory it is supposed to be embedded in, i.e. String Theory.
The most instructive way to proceed is the gradual description in this order of:

- the introduction of supersymmetries in quantum field theories, addressing some of the Standard Model problems;
- the extensions of global supersymmetries to local supersymmetries, i.e. supergravity, which complete the process of Standard Model generalisation;
- the arising of supersymmetric field theories as effective low energy models coming from the fundamental context of String Theory, with the main implications in supersymmetry breaking.

1.1 Standard Model Problems

The Standard Model turns out to come up against some primary issues whose fixing is necessary in order to approach a more fundamental description of Nature.
An essential summary of them is outlined in the following list.

- The impossibility to provide a satisfactory description of Quantum Gravity. Indeed, the gravitational interaction carried by the gravitational field $g_{\mu\nu} = g_{\mu\nu}(x)$ can be studied only at a classical level and is not included in the Standard Model because it corresponds to a non-renormalisable theory. Therefore a unified description of all of the fundamental interactions of Nature is not possible within the context of the Standard Model. Far below the reduced Planck scale $M_P \simeq 2.4 \cdot 10^{18}$ GeV, however, a classical description of gravity is theoretically consistent.

- The "Hierarchy Problem", i.e. the presence of two totally different energy scales, the electroweak scale $M_{EW} \sim 10^2$ GeV, fixed by the Higgs boson H scalar potential, and the reduced Planck scale M_P and, most importantly, the necessity to perform a fine-tuning order by order in perturbation theory to prevent the electroweak scale from taking values around the Planck scale because of quantum loop corrections. Indeed there is no symmetry protecting the Higgs mass and the electroweak symmetry breaking scalar potential has to be introduced by hand.

- The inability to give justifications for some of the Standard Model main structural features. For instance, the existence in the Standard Model of three families of leptons - i.e. electrons e, muons μ and tauons τ, with the corresponding neutrinos ν_e, ν_μ and ν_τ - and three families of quarks - i.e. up u and down d, charm c and strange s, top t and bottom b - and the presence of the general Standard Model general gauge group - $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$, with the corresponding gauge fields, gluons g, W and B bosons - are not based on any first principles.

- The cosmological constant problem, indeed experimental observations indicate a very small vacuum energy density for the Universe, thus requiring striking cancellations up to several orders of magnitude for the different contributions coming from the different components after quantum corrections.

- Cosmology unsolved issues. Standard Cosmology provides a very efficient description of Universe in its entirety and its evolution, but still the Standard Model fails in individuating a mechanism for baryon asymmetry as well as suitable particle candidates for inflaton and dark matter.

Supersymmetric field theories offer an interesting possibility to overcome these difficulties by leaving unaffected the Standard Model physics at low energies.
Indeed, within the more general framework of supergravity, supersymmetric models can be theorised as generalisations to higher energies of the Standard Model, where the Poincaré symmetry group is extended to include further transformations which establish a symmetry between the bosons and the fermions of the theory.

1.2 Supersymmetry

The study of Supersymmetric Quantum Field Theory is an extremely wide subject. In the following the essential theoretical features are briefly outlined and the Minimal

Supersymmetric Standard Model is introduced on the grounds of the basic introductions Ref. [1] and Ref. [2] as well as the classic Ref. [3]. The scope is placing into context this thesis and fixing the nomenclature employed throughout this work.

1.2.1 Supersymmetry Algebra

Supersymmetry transformations are the most general extension of the Poincaré group in Quantum Field Theory.
Indeed, a renowned result known as Haag-Łopuszański-Sohnius theorem essentially states that, under very general conditions, the most general S-matrix symmetry group in a quantum field theory is generated by Poincaré generators $M_{\mu\nu}$ and P_μ, internal symmetry group generators T_i and N couples of spinor generators $(Q^A)_\alpha$ and $(\bar{Q}^A)_{\dot{\alpha}} = ((Q^A)_\alpha)^\dagger$, $A = 1,2, ..., N$, which satisfy the general supersymmetry algebra. Spinor generators are Grassmann-odd valued operators which give rise to a graded algebra.

The general supersymmetry algebra of course does not modify the Poincaré and internal symmetry group algebra. The extension of the Poincaré algebra for the generators Q_α reads:

$$\Big[Q_\alpha, M^{\mu\nu}\Big] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta, \qquad \Big[Q_\alpha, P^\mu\Big] = 0, \tag{1.1}$$

and similarly for the conjugate $\bar{Q}_{\dot{\alpha}}$, and:

$$\begin{aligned} \Big\{Q_\alpha, Q_\beta\Big\} = 0, \qquad & \Big\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\Big\} = 0, \\ \Big\{Q_\alpha, \bar{Q}_{\dot{\beta}}\Big\} &= 2\,(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu, \end{aligned} \tag{1.2}$$

where $\sigma^\mu = (1_2, \sigma^i)$, $\bar{\sigma}^\mu = (1_2, -\sigma^i)$, σ^i being the Pauli matrices, and $\sigma^{\mu\nu} = i/4 \cdot (\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$.
It can be shown that chirality is preserved only in the presence of a single couple of spinor generators, so the discussion is directly referred to $N = 1$.

1.2.2 Supermultiplets

The single-particle states of a supersymmetric theory fall into irreducible representations of the supersymmetry algebra, called supermultiplets.

A first notable feature is that each supermultiplet contains both boson and fermion degrees of freedom in equal number, i.e. $n_B = n_F$, as can be inferred straightforwardly from the spin-statistic theorem and the supersymmetry algebra. Then, bosons and fermions of a supermultiplet are usually organised in couples of superpartners.

A second fundamental characteristic is the equality of masses among all of the particles of a supermultiplet. Indeed, the squared-mass operator $C_m = P^\mu P_\mu$ turns out to be a Casimir operator for the supersymmetry algebra as well as for the Poincaré algebra.

The simplest supermultiplets with the same number of bosonic and fermionic degrees of freedom turn out to be suitable for reliable generalisations of the Standard Model, as outlined below. In particular, they are:

- chiral (or matter) supermultiplets, which describe a single Weyl fermion field, with $n_F = 2$, and two real scalar fields, or one complex scalar field, with $n_B = 2$ (moreover, anti-chiral supermultiplets are similar but with reversed Weyl spinor chirality);
- vector (or gauge) supermultiplets, which describe a massless vector field, with $n_B = 2$, and a massless Weyl fermion field, with $n_F = 2$ (a massless spin-3/2 fermion gives a non-renormalisable theory);
- if supergravity is included, then the spin-2 massless graviton, with $n_B = 2$, and the massless spin-3/2 gravitino, with $n_F = 2$, form the gravity supermultiplet.

There are other possible supermultiplets, but they are always reducible to combinations of chiral and gauge supermultiplets if the theory is renormalisable.
The supersymmetry generators also commute with the generators of gauge transformations, therefore particles in the same supermultiplet are also in the same representation of the gauge group.

Solution of the Hierarchy Problem

The fundamental features of supersymmetric multiplets allow to realise qualitatively the main mechanism which lies at the basis of the supersymmetric solution to the Hierarchy Problem.

The Higgs mass receives quantum corrections from the virtual effects of every particle which couples, directly or indirectly, to the Higgs field. It can be observed that the problematic divergences which plague the Higgs mass differ in sign for bosons and fermions. In the Standard Model, all particles give of course different contributions.

Within a supersymmetric context, though, the presence of a superpartner for any of the Standard Model particles with suitable couplings and masses - precisely fixed by the supersymmetry invariance requirements - entails a remarkable systematic cancellation of the problematic divergences.
This mechanism provides a striking phenomenological solution of the Hierarchy Problem and constitutes one of the main reasons why supersymmetry is so interesting as a possible theory to extend the Standard Model.

1.2.3 Supersymmetry Lagrangian

Although unnecessary, the most elegant and efficient way to build Lagrangians invariant under the super-Poincaré symmetry group relies on the superfield formalism.

Basically, the fundamental feature is that chiral and vector supermultiplets can be described by general fields, known as superfields, which are defined over a superspace, i.e. the usual Minkowski space-time plus four Grassmann-odd valued extra-dimensions. In particular chiral and vector superfields can be associated respectively to the triplets $\Phi = (\varphi, \psi, F)$ and $V = (\lambda, V^\mu, D)$ which result from a Taylor expansion in the fermionic dimensions, where φ and ψ are the scalar and Weyl spinor chiral supermultiplet fields and V^μ and λ are the vector and Weyl spinor vector supermultiplet fields, while F and D are auxiliary fields which can always be integrated out.
In general, models with such supermultiplets are invariant under supergauge symmetries, i.e. generalised gauge symmetries within supersymmetric models.

Then, typically the definition of three functions determines a general supersymmetric Lagrangian:

- the Kähler potential K, a real function of chiral and antichiral superfields, i.e. a vector superfield, with the possible inclusion of vector superfields in such a way as to be supergauge invariant;
- the superpotential W, a holomorphic function of the chiral superfields, i.e. a chiral superfield itself, invariant under supergauge symmetries;
- the gauge kinetic function f_{ab}, i.e. a dimensionless holomorphic function of the chiral superfields, i.e. a chiral superfield itself, with the subscripts a, b running over the gauge group.

As a matter of fact, given a set of chiral and vector superfields Φ^i, $i = 1,2, ..., N$, and V_a, $a = 1,2, ..., n$, the general supersymmetric Lagrangian can be written as:

$$\mathcal{L}_{\text{susy}} = \Big(K(\Phi, \tilde{\Phi})\Big)_D + \Big(\Big(f_{ab}\,(\Phi)\, W_a W_b + W\,(\Phi)\Big)_F + \text{h.c.}\Big) \tag{1.3}$$

where the subscripts D and F indicate the auxiliary-field parts of the vector and chiral superfields involved and W_a is the generalised vector superfield strength tensor, conveniently setting $\tilde{\Phi}^{\bar{i}} \equiv (\Phi^\dagger e^{2gV_aT^a})^{\bar{i}}$ to preserve supergauge invariance, being T^a the generators of the gauge group.

In particular, the general Lagrangian $\mathcal{L}_{\text{susy}}$ turns out to describe the chiral and vector supermultiplets dynamics and also contain terms depending on the associated auxiliary fields. Integrating the latter out, scalar potentials are generated. A more detailed analysis allows to conclude that such scalar potentials are potential generators of supersymmetry breaking.

1.2.4 Minimal Supersymmetric Standard Model

In a supersymmetric extension of the Standard Model, each of the known particles must be in either a chiral or a vector supermultiplet.

In principle, some of the known particles could be supposed to be the superpartners of other known particles, but this possibility turns out to be forbidden by a multitude of wrong implications. Therefore the Standard Model superpartners are all really new particles undetected so far.

The simplest supersymmetry-invariant formulation of the Standard Model is known as Minimal Supersymmetric Standard Model - MSSM for short - and essentially consists in a formulation in which the particle content is represented only by the Standard Model particles and their superpartners. Actually, two Higgs fields are necessary due to phenomenological reasons.

MSSM Particle Content

The Minimal Supersymmetric Standard Model particle content can be organised into two great classes.

- Chiral supermultiplets.
  Standard Model spin-1/2 fermions, leptons and quarks, must reside in chiral supermultiplets because of chirality, therefore their superpartners are spin-0 scalars known as sleptons and squarks.
  Higgs fields must reside in chiral supermultiplets, too, and their superpartners are called higgsinos. The neutral scalar coincident with the physical Standard Model Higgs boson corresponds to a linear combination of the neutral components of these Higgs fields.

- Vector supermultiplets.
  Standard Model spin-1 gauge bosons must reside in vector supermultiplets and their superpartners are spin-1/2 fermions referred to as gauginos.

The standard symbols and gauge representations are summarised in Tables 1.1 and 1.2.

MSSM Lagrangian

Indicating with H_1, H_2, Q, L, $\bar{u}$, $\bar{d}$ and $\bar{e}$ the chiral superfields corresponding to the chiral supermultiplets in Table 1.1, grouping the doublets with obvious notation, and with g, W and B the usual vector superfields corresponding to vector multiplets in Table 1.2, multiplied by the gauge group generators with obvious notation, it is immediate to express the general Minimal Supersymmetric Standard Model Kähler potential, superpotential and gauge kinetic function.
The following indices must be intended as follows: $\alpha, \beta = 1{,}2$ represent $SU(2)_L$ weak isospin indices, $a = 1{,}2{,}3$ is a color index lowered and raised in the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations of $SU(3)_c$ and, if needed, $i, j = 1{,}2{,}3$ are family indices, in obvious notation.

Particle Names	spin-0	spin-1/2	$SU(3)_c \times SU(2)_L \times U(1)_Y$
squarks, quarks: $(u,d)=(u,d),(c,s),(t,b)$	$(\tilde{u}_L,\, \tilde{d}_L)$	$(u_L,\, d_L)$	$(\mathbf{3},\mathbf{2},+1/6)$
	$\tilde{u}^*_R$	$\bar{u}_R$	$(\bar{\mathbf{3}},\mathbf{1},-2/3)$
	$\tilde{d}^*_R$	$\bar{d}_R$	$(\bar{\mathbf{3}},\mathbf{1},+1/3)$
sleptons, leptons: $e=e,\mu,\tau$	$(\tilde{\nu}_e,\, \tilde{e}_L)$	$(\nu_e,\, e_L)$	$(\mathbf{1},\mathbf{2},-1/2)$
	$\tilde{e}^*_R$	$\bar{e}_R$	$(\bar{\mathbf{1}},\mathbf{1},+1)$
Higgses, higgsinos	$(H_1^+,\, H_1^0)$	$(\tilde{H}_1^+,\, \tilde{H}_1^0)$	$(\mathbf{1},\mathbf{2},+1/2)$
	$(H_2^0,\, H_2^+)$	$(\tilde{H}_2^0,\, \tilde{H}_2^+)$	$(\mathbf{1},\mathbf{2},-1/2)$

Table 1.1. Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars and the spin-1/2 fields are left-handed Weyl spinors.

Particle Names	spin-1/2	spin-1	$SU(3)_c \times SU(2)_L \times U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\mathbf{8},\mathbf{1},0)$
winos, W bosons	$\tilde{W}^\pm,\ \tilde{W}^0$	$W^\pm,\ W^0$	$(\mathbf{1},\mathbf{3},0)$
bino, B boson	$\tilde{B}^0$	B^0	$(\mathbf{1},\mathbf{1},0)$

Table 1.2. Vector supermultiplets in the Minimal Supersymmetric Standard Model. The spin-1/2 fields are left-handed Weyl spinors and the spin-1 fields are real vector fields.

Therefore, in the global supersymmetry framework, the Kähler potential K_{MSSM} leading to a renormalisable theory and canonically normalised fields reads:

$$\begin{aligned} K_{\mathrm{MSSM}} =\ & \bar{Q}^{a,\alpha}\big(e^{2c_Q g}\big)_a{}^b\big(e^{2g_Q W}\big)_\alpha{}^\beta\big(e^{2g'_Q B}\big)Q_{b,\beta} + \bar{u}^a\big(e^{2c_u g}\big)_a{}^b\big(e^{2g'_u B}\big)u_b \\ & + \bar{d}^a\big(e^{2c_d g}\big)_a{}^b\big(e^{2g'_d B}\big)d_b + \bar{L}^\alpha\big(e^{2g'_L W}\big)_\alpha{}^\beta\big(e^{2g_L B}\big)L_\beta + \bar{e}\big(e^{2g'_e B}\big)e \\ & + \bar{H}_1^\alpha\big(e^{2g'_{H_1} W}\big)_\alpha{}^\beta\big(e^{2g_{H_1} B}\big)H_{1,\beta} + \bar{H}_2^\alpha\big(e^{2g'_{H_2} W}\big)_\alpha{}^\beta\big(e^{2g_{H_2} B}\big)H_{2,\beta}, \end{aligned} \tag{1.4}$$

and similarly, suitably labelling gauge groups and gauge couplings, the gauge kinetic function $f_{AB} = f_A\delta_{AB}$ is diagonal, constant and such that:

$$\mathrm{Re} f_A^{\mathrm{MSSM}} = \frac{1}{4g_A^2}. \tag{1.5}$$

The Minimal Supersymmetric Standard Model superpotential W_{MSSM} reads:

$$\begin{aligned} W_{\mathrm{MSSM}} = \; & \bar{u}^{i,a} Y_{u,i}{}^{j} Q_{j,a,\alpha} H_{1,\beta} \epsilon^{\alpha\beta} - \bar{d}^{i,a} Y_{d,i}{}^{j} Q_{j,a,\alpha} H_{2,\beta} \epsilon^{\alpha\beta} \\ & - \bar{e}^{i} Y_{e,i}{}^{j} L_{j,\alpha} H_{2,\beta} \epsilon^{\alpha\beta} + \mu H_{1,\alpha} H_{2,\beta} \epsilon^{\alpha\beta}, \end{aligned} \tag{1.6}$$

where $Y_{u,i}{}^{j}$, $Y_{d,i}{}^{j}$ and $Y_{e,i}{}^{j}$ represent the well-known Yukawa couplings suitably organised in matrix notation, while μ is the supersymmetric version of the Higgs boson mass in the Standard Model.

A straightforward computation yields to the general - though extremely long and complex - Minimal Supersymmetric Standard Model Lagrangian $\mathcal{L}_{\mathrm{MSSM}}$, which can be naïvely intended as follows:

$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{SM\,susy}} + \mathcal{L}_{\mathrm{int}}^{\mathrm{SM-SM\,susy}}, \tag{1.7}$$

where $\mathcal{L}_{\mathrm{SM}}$ is the Standard Model Lagrangian, with kinetic terms and interactions, $\mathcal{L}_{\mathrm{SM\,susy}}$ is the Standard Model superpartners Lagrangian, with kinetic terms and interactions, and $\mathcal{L}_{\mathrm{int}}^{\mathrm{SM-SM\,susy}}$ is the Lagrangian interaction term between Standard Model particles and their superpartners. Of course the above expression is only qualitative because actually the Higgs sector in supersymmetric extensions is different from its Standard Model form as two Higgs fields are involved.

It is important to underline that in general the mass eigenstates of the Lagrangian mix the Standard Model superpartners to give new particles. So, supersymmetry phenomenology is expected to individuate the Standard Model particles and other new particles defined via suitable linear combinations of superpartners.

Soft Supersymmetry Breaking

Of course the Lagrangian $\mathcal{L}_{\mathrm{MSSM}}$ cannot be the complete Lagrangian describing Nature in vicinity of its ground state because no Standard Model superpartner has ever been observed so far. If supersymmetry were unbroken, Standard Model particles and their superpartners would have equal masses, so they both would be easily detectable.

The only way to account for this undisputed evidence is supposing that supersymmetry is broken in the ground state which Nature has selected.
Therefore, a mechanism which breaks supersymmetry must exist and a further non-supersymmetric term $\mathcal{L}_{\not{\mathrm{susy}}}$ which entails a globally non-supersymmetric Lagrangian must be included. An important clue as to the nature of supersymmetry breaking comes from the solution to the Hierarchy Problem.

Unbroken supersymmetry guarantees the systematic cancellation of the problematic divergent corrections to the Higgs mass. This fact relies on the exact correspondences which are created by supermultiplets between boson and fermion masses and couplings. Evidently, such relationships are generally not expected to be maintained also in the presence of supersymmetry breaking terms.

However, if the supersymmetry breaking Lagrangian contains only supersymmetry breaking terms like mass terms and coupling parameters with positive mass dimension, quadratic divergences are guaranteed to be absent by the same mechanism as in a supersymmetric model due to the right relationships between dimensionless couplings. A supersymmetry breaking Lagrangian of this kind is then called *soft supersymmetry breaking* Lagrangian.

Indicating generally the scalar fields and the gauginos of the model as ϕ^α and λ_a respectively, the soft supersymmetry breaking Lagrangian can be written as:

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -\frac{1}{2}\left(m^2\right)_{\alpha\bar{\beta}}\phi^\alpha\bar{\phi}^{\bar{\beta}} - \left(\frac{1}{2}M_a\lambda_a\lambda_a + \text{h.c.}\right) \\ & - \left(\frac{1}{2}b_{\alpha\beta}\phi^\alpha\phi^\beta + \text{h.c.}\right) - \left(\frac{1}{6}t_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma + \text{h.c.}\right),\end{aligned} \tag{1.8}$$

where $(m^2)_{\alpha\bar{\beta}}$ and M_a are scalar and gaugino masses, while $b_{\alpha\beta}$ and $t_{\alpha\beta\gamma}$ are bilinear and trilinear scalar couplings. Of course, soft masses take part to the effective superpartner masses, provided a correct Lagrangian diagonalisation, and in general they actually turn out to be the prevalent contribution since they must be larger.

Summing up, it is natural to conclude that the expected Lagrangian describing the minimal Standard Model supersymmetric extension reads:

$$\mathcal{L} = \mathcal{L}_{\text{MSSM}} + \mathcal{L}_{\text{soft}}. \tag{1.9}$$

Although in principle an explicit breaking of supersymmetry looks arbitrary, more general models which extend global supersymmetry to supergravity remarkably generate naturally a soft supersymmetry breaking Lagrangian contribution in vicinity of the ground state. This topic is outlined below and constitutes the fundamental idea at the basis of the rest of this thesis.

1.3 Supergravity

Supergravity Field Theory is a quantum field theory invariant under local supersymmetry transformations. It is an enormous topic and this Section, fully inspired by Ref. [4], is only intended to give the main ideas which motivate its introduction within the context of Standard Model extensions as low energy limits of String Theory.

1.3.1 Standard Model Supergravity Extension

Quantum Gravity and Supergravity

In short, it is well known that the natural way in which interactions are included within the Standard Model is the promotion of a symmetry of the action from global - i.e.

space-time independent - to local - i.e. space-time dependent - with the consequent introduction of new fields to save the action invariance properties. In particular:

- local internal symmetries entail the presence of spin-1 gauge fields;

- local space-time symmetries, i.e. general coordinate transformations, entail the presence of a spin-2 field, the graviton.

The requirement of invariance under general coordinate transformations corresponds to the inclusion of General Relativity into the Standard Model, including gravitational interactions in a natural way.
Unfortunately, this inclusion is highly problematic because such interactions are non-renormalisable. Therefore the basic idea is that a consistent theory of gravity must be finite order by order in perturbation theory.

Similarly to what happens with the systematic cancellations in global supersymmetry, the basic purpose could be the embedding of locality within supersymmetry transformations. Of course supersymmetry transformations are closely related to space-time transformations, as it emerges from the supersymmetry algebra (1.1) and (1.2), so local supersymmetry necessarily corresponds to generalised general coordinate transformations.
Evidently, in the presence of local supersymmetry, besides chiral and vector supermultiplets, the gravity supermultiplet is involved too. The physical fields belonging to the gravity supermultiplet are the graviton $g_{\mu\nu} = g_{\mu\nu}(x)$, a massless spin-2 field with two vector indices, and the gravitino $\Psi^{\alpha}_{\mu} = \Psi^{\alpha}_{\mu}(x)$, a massless spin-3/2 field with a spinor and a vector index.

Anyway, even though the general behaviour of gravitational divergences could be somehow improved, still supergravity does not remove all of the infinities from the model. This fact means that local supersymmetries cannot give a self-consistent solution to the introduction of gravitation within the Standard Model. Nevertheless, within the context of a more general theory such as String Theory, supergravity turns out to have a crucial rôle.

Supergravity from String Theory

Supergravity extensions of the Standard Model are very interesting from a phenomenological point of view because they naturally emerge as effective low energy descriptions of a more general model, String Theory, which is theorised as the correct description of Nature at very high energies including quantum gravity and the other interactions with matter.
Moreover, it turns out that several supergravity models coming from String Theory exist which, in vicinity of their ground state, result in an effective description with a globally Supersymmetric Standard Model Lagrangian plus soft supersymmetry breaking terms.

This property is modelled as the natural result of a spontaneous supersymmetry breaking, as introduced in the following Sections and studied in concrete models in the rest of the thesis.

To conclude, the general idea can be summarised as follows and is schematised in Figure 1.1:

- around the Planck scale M_P, String Theory is (presumably) the correct fundamental description of Nature;
- below a string scale M_S, of course smaller than M_P, supergravity emerges as the effective low energy limit of String Theory;
- below a supersymmetry breaking scale M_{susy}, the supersymmetric Standard Model with explicit breaking of supersymmetry emerges as a low energy limit of supergravity.

Of course, within a String Theory context like this one, the non-renormalisability is no longer a problem.

1.3.2 Supergravity Lagrangian

Lagrangians invariant under local supersymmetry transformations can be written via a generalisation of the superfield formalism. More precisely, they are completely specified by two functions of the involved superfields:

- the Kähler function G, which is a combination of the Kähler potential $K = (\Phi, \bar{\Phi})$ and the superpotential $W = W(\Phi)$:

  $$G = K(\Phi, \tilde{\bar{\Phi}}) + \ln \bar{W}(\bar{\Phi})W(\Phi),$$

  with K and W with the same properties as in global supersymmetry and the additive invariance under Kähler transformations $K' = K + F + \bar{F}$, $W' = e^{-F}W$, with $F = F(\Phi)$ an arbitrary function;
- the gauge kinetic function $f_{ab} = f_{ab}(\Phi)$, with the same properties as in global supersymmetry.

A general supergravity Lagrangian describes a model with scalar and spinor fields coming from chiral supermultiplets, spinor and vector fields coming from vector supermultiplets and the gravitino and the graviton fields arising from the gravity supermultiplet. Moreover auxiliary F-fields and D-fields associated to chiral and vector supermultiplets respectively emerge as in global supersymmetry theories.
Such a Lagrangian has in general non-canonical kinetic terms, which means that fields must be normalised, and both renormalisable and non-renormalisable interaction terms.

Such non-renormalisable terms are a consequence of the non renormalisability of supergravity. Evidently, all of the chiral and vector supermultiplet fields are coupled to the graviton via an overall factor $e = (-\text{det} g_{\mu\nu})^{1/2}$.
Very detailed expressions are derived and commented exhaustively in Ref. [3].

As a final remark about notation, while for general discussions it is preferable to deal with the Kähler function G, for practical calculations it is more convenient to deal directly with the Kähler potential K and the superpotential W, fixing a gauge F.

Scalar Fields Contribution to Supergravity Lagrangians

The scalar field part of the general supergravity Lagrangian is reported in order to exemplify the main characteristics o supergravity Lagrangians. Moreover, evidently it is such a contribution which plays the fundamental rôle in spontaneous supersymmetry breaking due to the scalar potential.

The general scalar kinetic terms and the associated scalar potential in a local supersymmetry invariant model reads, with indices i, $\bar{j}$ and a, b running over chiral superfields and gauge group respectively:

$$\begin{aligned} e^{-1}\,\mathcal{L}_{\text{scalar}} = \; & G_{i\bar{j}}\,\partial_\mu\,\phi^i\,\partial^\mu\,\bar{\phi}^{\bar{j}} - e^G\,\left(G_i G^{i\bar{j}} G_{\bar{j}} - 3\right) \\ & - \frac{1}{2}\,\text{Re}\Big[f_{ab}\left((f^{-1})^{ac}\bar{\phi}^{\bar{i}}(T^c)_{\bar{i}}^{\;j} G_j\right)\left((f^{-1})^{bd}\bar{\phi}^{\bar{k}}(T^d)_{\bar{k}}^{\;l} G_l\right)\Big]\,, \end{aligned} \tag{1.10}$$

where subscripts in G indicate derivations with respect to the scalar fields and upper indices indicate the inversion operation, with obvious notation, while $(T^a)_{\bar{i}}^{\;j}$ are the gauge group generators. Evidently, the overall factor e represents the ubiquitous gravitational coupling, while the factor $G_{i\bar{j}}$ multiplying the pure kinetic terms indicate generally non-canonically normalised fields. The scalar potential has a factor e^G and a generally non-constant gauge kinetic function, indicating the non-renormalisability of the theory. Nevertheless, in the vicinity of the ground state it can give rise to scalar masses.

It is fundamental to specify that the scalar potential comes from the integration of the chiral and vector supermultiplet auxiliary fields. Before being integrated out, they appear in the general Lagrangian through the contributions, respectively:

$$\mathcal{L}_F = -V_F = -G_{i\bar{j}} F^i \bar{F}^{\bar{j}} + 3\,e^G,$$

$$\mathcal{L}_D = -V_D = -\frac{1}{2}\text{Re}\big[f_{ab} D^a D^b\big]\,.$$

Then, substituting the F- and D-terms coming from the field equations, discarding some further irrelevant spinor-dependent contributions:

$$F^i = e^{G/2} G^{i\bar{j}} G_{\bar{j}}, \tag{1.11}$$

$$D^a = (f^{-1})^{ab}\bar{\phi}^{\bar{i}}(T^b)_{\bar{i}}^{\;j} G_j, \tag{1.12}$$

the total scalar potential $V = V_F + V_D$ appearing above is generated.

1.3.3 Spontaneous Supersymmetry Breaking

Of course the general condition for spontaneous supersymmetry breaking is that the variation under a supersymmetry transformation of the vacuum expectation value of a field χ_A is different from zero, i.e. $\langle \delta\chi_A \rangle \neq 0$.
Generally, it turns out that the only variations under local supersymmetry transformations with a parameter ϵ which can acquire non-vanishing expectation values without breaking Lorentz invariance are the variations of chiral spinor fields ψ^i and of gauginos λ^a. They can be written as:

$$\langle \delta\psi^i \rangle = \sqrt{2} \langle F^i \rangle \epsilon,$$
$$\langle \delta\lambda^a \rangle = 2 \langle D^a \rangle \epsilon.$$

This fundamental result indicates that the basic condition for spontaneous supersymmetry breaking to occur is that at least one of the auxiliary fields acquire a non-vanishing vacuum expectation value.
This is a dynamical problem which is governed by the minimisation of the scalar potential $V = V_F + V_D$:

$$\frac{\partial V}{\partial \phi^i}(\phi) \Big|_{\phi^i = \langle \phi^i \rangle} \equiv 0.$$

Evidently, depending on the form of the scalar potential, some scalar fields can take non-vanishing vacuum expectation values giving rise to the spontaneous breaking of supergravity.
As a remark, clearly the vacuum energy density $E_\Lambda = \langle V \rangle$ can take a positive, negative or vanishing value.

A fundamental consequence of the spontaneous breaking of supergravity is the so-called super-Higgs effect. Indeed, via the study of the general form of the supergravity Lagrangian, it can be shown that when certain scalar fields acquire non-vanishing expectation values determining the breaking of supersymmetry, the gravitino becomes massive. The mechanism very similar to the well known Higgs mechanism. Indeed, the two degrees of freedom of the goldstino, i.e. a suitable linear combination of fermion superpartners of the supersymmetry breaking scalar fields, are absorbed by the massless gravitino to give a massive spin-3/2 particle, i.e. a massive gravitino.
In case of a F-term supersymmetry breaking, the resulting gravitino mass $m_{3/2}$ in physical units turns out to be:

$$m_{3/2} = e^{G/2} M_P. \tag{1.13}$$

In a "gravity mediated" scenario like the one considered in the thesis, if supersymmetry is broken by a vacuum expectation value $\langle F \rangle$, then, in physical units, the supersymmetry breaking scale is estimated as:

$$M_{\text{susy}} \sim \sqrt{\langle F \rangle},$$

while soft masses are expected to be roughly around:

$$m_{\mathrm{soft}} \sim \frac{\langle F \rangle}{M_P},$$

by dimensional analysis.
With D-term supersymmetry breaking, results are similar.

Hidden Sector Supersymmetry Breaking

The most striking feature of spontaneous supersymmetry breaking is the natural generation of soft terms which can occur.

Models based on String Theory typically predict a supergravity effective Lagrangian description with a particle spectrum organised into two groups:

- the observable sector, which - suitably structuring the Higgs sector - consists of the Standard Model particles and their superpartners, with the typical gauge and Yukawa interactions, in the Minimal Supersymmetric Standard Model, plus possible further observable supermultiplets in more advanced extensions;
- the hidden sector, which consists of particles belonging to gauge-singlets under the observable gauge group, with neither gauge interactions nor Yukawa couplings with the observable sector and thus coupled to the latter only gravitationally and with very weak couplings.

Generally - discarding for the sake of simplicity gauge fields, which can be introduced straighforwardly - denoting as h_I and C_α the unnormalised hidden and observable sector fields respectively, with indices running accordingly, the Kähler potential and superpotential of such models are such as to be essentially expandable as [5]:

$$\mathcal{K} = K_H\left(h, \bar{h}\right) + \tilde{K}_{\alpha\bar{\beta}}\left(h, \bar{h}\right)\, C^\alpha \bar{C}^{\bar{\beta}} + \left[\frac{1}{2} Z_{\alpha\beta}\left(h, \bar{h}\right) C^\alpha C^\beta + \mathrm{h.c.}\right], \tag{1.14}$$

$$\mathcal{W} = W_H\left(h\right) + \frac{1}{2}\mu_{\alpha\beta}\left(h\right) C^\alpha C^\beta + \frac{1}{6} Y_{\alpha\beta\gamma}\left(h\right) C^\alpha C^\beta C^\gamma, \tag{1.15}$$

where, recalling the expressions for K_{MSSM} and W_{MSSM}, the terms concerning observable sector fields clearly give rise to possible supersymmetric Standard Model extensions, with canonical normalisation and bilinear and trilinear couplings depending on hidden sector fields, while K_H and W_H are typically non-canonical contributions which definitely extend the particle spectrum to the hidden sector.
Evidently, general observable sector terms are modelled as subleading contributions to hidden sector terms K_H and W_H.

Remarkably, this structure with both a hidden and an observable sector gives rise to a supergravity Lagrangian which undergoes a spontaneous supersymmetry breaking

in the hidden sector, thus making the gravitino massive, and generating in the meanwhile precisely the soft supersymmetry breaking terms in the visible sector as effective by-products.
Then, although the observable sector does not participate directly to the breaking of supersymmetry, it undergoes its consequences indirectly with the natural generation of soft terms. The way this striking mechanism takes place is a very active research area and is taken under consideration in some particular frameworks in the following Chapters.

Because of its very weak interactions with observable fields, the hidden sector is difficultly detectable at present. Nevertheless, it has fundamental effects on the structure of soft terms and in Cosmology. Indeed, it both determines the characteristic of supersymmetry breaking and has sensible effects in the evolution of Universe due to its gravitational interactions.

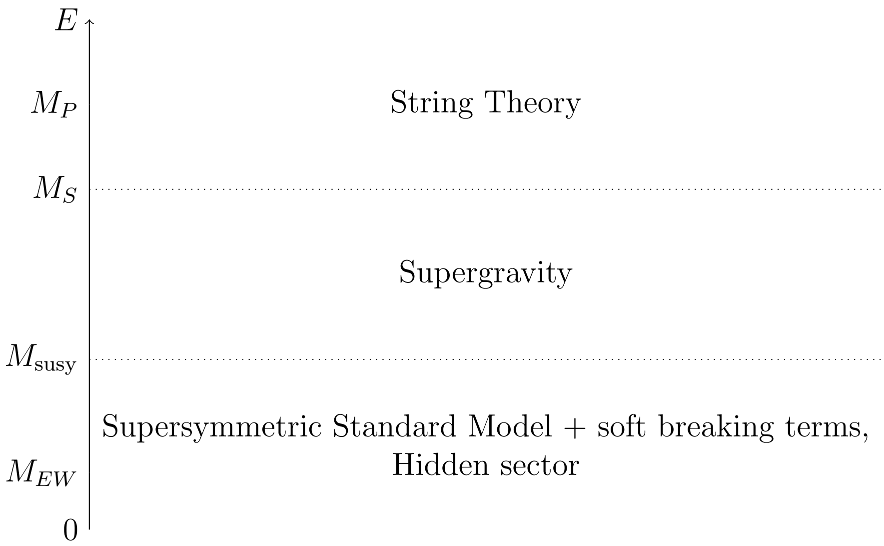

Figure 1.1. Schematic representation of the scenarios arising from the low energy limit of possible String Theory models.

Chapter 2

Moduli and String Compactifications

Several attempts to extend the Standard Model have been studied in which supersymmetries are present as general symmetries of Nature. Since supersymmetry has never been tested on experimental grounds, it must be a broken symmetry of the theory at least in the ground state.
String Theory provides mechanisms which can account account for this scenario in the low energy limit. In detail, the way supersymmetry gets explicitly and/or spontaneously broken at low energies can be studied according to the specific features that the underlying string theory eventually determines.
This Chapter is devoted to the introduction to some of the general characteristics of Type IIB String Theory, which at low energies give origin to an interesting model for the spontaneous breaking of supersymmetry in the hidden sector and the generation of soft breaking terms. For its developing, Ref. [6] and Ref. [7] have been the fundamental guide in order to have an account of the general framework of String Phenomenology.

2.1 Low Energy Limit of String Theory

Among string theories, the most important ones in the context of Standard Model extensions are Heterotic String Theories and Type IIB String Theory, that are perturbatively consistent and, roughly speaking, actually reproduce the Standard Model physics in their low energy limit [6] [7].
More specifically, these low energy limits give origin to supersymmetric theories in which supersymmetries turn out to be broken. Heterotic String Theories result in theories containing non-Abelian gauge group symmetries, while at a first glance Type IIB strings do not. Anyway, via the study of branes these symmetries can be found in the latter too. As they are also technically easier to study, the main features deriving from low energy Type IIB Strings are described below.

Type IIB Strings are characterised by a string scale M_S below which a ten dimensional $N = 2$ supergravity emerges as an effective theory. This is known as the Type IIB $D = 10$, $N = 2$ supergravity. The string scale M_S is assumed to be smaller than the reduced Planck scale $M_P = 2.4 \cdot 10^{18}\,\mathrm{GeV}$ but much larger than the electroweak scale $M_{EW} \simeq 10^2\,\mathrm{GeV}$. It is possible to equivalently consider the string length $l_S = 1/M_S$ as the physical scale of reference. Below the string scale, where only massless strings are present, the more conventional notion of quantum field is recovered [8] [9]. In this situation, it can be shown that the universal ten dimensional action $S_{D=10}$ in the Einstein frame has a pure gravitational contribution of the kind [8] [9]:

$$S_{D=10}^{\text{graviton}} = -\frac{1}{2} M_S^8 \int \mathrm{d}^{10}x \; g_{D=10}^{1/2} R_{D=10},$$

where $g_{D=10}$ and $R_{D=10}$ are the modulus of the determinant of the ten dimensional metric of the space, g_{MN}, $M, N = 0,1, ...9$, and the ten dimensional Ricci scalar respectively. The ten dimensional space X_{10} is assumed to be factorised in a product:

$$X_{10} = \mathbb{R}^{1,3} \times Y_6,$$

meaning that the line element can be written as:

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}\,x^\mu\,\mathrm{d}\,x^\nu + g_{mn}\,\mathrm{d}\,y^m\,\mathrm{d}\,y^n, \quad \mu,\nu = 0,1,2,3,\; m,n = 4,5,\cdots 9,$$

where Y_6 is a six dimensional manifold known as Calabi-Yau manifold depending on the string theory model and which is typically compactified at low energies.
The volume of the Calabi-Yau manifold is usually parametrised via a dimensionless volume in string units $\mathcal{V}$ by the definition:

$$\mathrm{Vol}\,(Y_6) = \int_{Y_6} \mathrm{d}^6y \; g_{D=6}^{1/2} \equiv \mathcal{V}\; l_S^6. \tag{2.1}$$

Then, the four dimensional gravitational contribution to the universal action $S_{D=4}$ can be expressed both as:

$$S_{D=4}^{\text{graviton}} = -\frac{1}{2} M_S^8 \int_{Y_6} \mathrm{d}^6y \int \mathrm{d}^4x \; g_{D=6}^{1/2} g_{D=4}^{1/2} R_{D=4},$$

with obvious notation, and of course as the well known Einstein-Hilbert action:

$$S_{D=4}^{\text{graviton}} = S_{EH} = -\frac{1}{2} M_P^2 \int \mathrm{d}^4x \; g_{D=4}^{1/2} R_{D=4},$$

in such a way as to naturally identify $M_S^8 \int_{Y_6} \mathrm{d}^6y \; g_{D=6}^{1/2} \sim M_P^2$. So it is immediate to conclude that the relation between the string and the Planck scale is:

$$M_S \sim \mathcal{V}^{-1/2}\, M_P. \tag{2.2}$$

Of course the expected scenario is such that $\mathcal{V} \gtrsim 1$ because the volume of the Calabi-Yau manifold is at least of the same order of magnitude as the reference string volume, but anyway a very large value $\mathcal{V} \gg 1$ is likely in a wide class of models, as investigated e.g. in Ref. [10] and Ref. [11]. Experiments eventually suggest an upper bound around $\mathcal{V} \sim 10^{30}$ since present day LHC tests exclude the possibility that M_S is smaller than roughly around 1 TeV as no string presence has ever been observed around that scale so far [12].

The ten dimensions are compactified below another scale M_{KK}, known as Kaluza-Klein scale, which emerges naturally when lengths are smaller than the Calabi-Yau characteristic distances. It is possible to write roughly:

$$M_{KK} \sim \frac{1}{R} \sim \frac{1}{\mathrm{Vol}\,(Y_6)^{1/6}},$$

in such a way as to relate the Kaluza-Klein scale to the Planck scale as:

$$M_{KK} \sim \mathcal{V}^{-2/3}\, M_P. \tag{2.3}$$

Of course, M_{KK} is also expected to be smaller than M_S because of the condition $\mathcal{V} \gtrsim 1$.

A detailed account of Calabi-Yau compactifications in Type IIB String Theory can be found in Ref. [6], where the compactifications of Ref. [13] are taken under exam.
If the internal manifold is very simple (like a six-sphere), it can be shown that the corresponding theory from the $D = 10$, $N = 2$ supergravity in four dimensions is a $D = 4$, $N = 8$ supergravity model.
The $D = 10$, $N = 2$ supergravity bosonic spectrum can be readily summarised as follows [8] [9]:

- Neveu-Schwarz fields (NS), with the metric tensor g_{MN}, $M, N = 0,1,...,9$, the dilaton ϕ and the two-form B_2;
- Ramond-Ramond fields (RR), with the zero-form C_0, the two-form C_2 and the four-form C_4;

while in $D = 4$ supergravity they are reorganised in different kinds of fields, as described below.
Before proceeding further, it is important to notice that $N = 8$ supersymmetric theories are not chiral, unlike the Standard Model, i.e. unlike what is expected as a property of any reliable low energy model. Nevertheless, Calabi-Yau manifolds allow to break 1/4 of the supersymmetries that are present. Moreover, a particular projection known as orientifold involution allows to reduce the eventual $N = 2$ supergravity to the actual $N = 1$ supergravity which is theorised to be the correct supergravity model underlying the Minimal Supersymmetric Standard Model and thus the "effective" Standard Model too.
A qualitative idea of the general set-up is represented in Fig. 2.1.

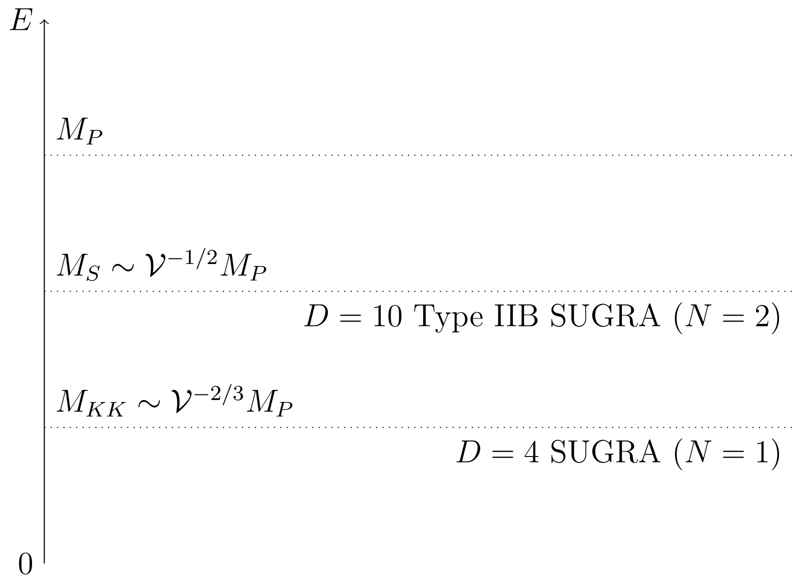

Figure 2.1. Typical energy scales for descriptions derived from String Theory: $(D = 4, N = 1)$ Supergravity Models emerge as the low energy limits of more general $(D = 10, N = 8)$ Supergravity Models, which in turn come from a general String Theory.

2.2 4D String Moduli

Following the previous Section, it can be shown that Type IIB String Theory below the scale M_{KK} results in a $D = 4$, $N = 1$ supergravity model involving a special kind of scalar fields, collectively indicated as 'moduli'.

In general, it is possible to define moduli as uncharged scalar fields which interact with ordinary matter only gravitationally. This implies that they are expected to be massive because, if massless, they would mediate unobserved long-range fifth-forces [14]. Moreover, it turns out that all the features of the corresponding effective field theory, like the particle mass spectrum, the gauge and the Yukawa couplings, depend on the vacuum expectation value of the moduli [4], since they generally represent the hidden sector, as explained below. In order to avoid a lack of predictability of the low energy four dimensional theory, it is therefore crucial to study *moduli stabilisation*, that is how the moduli develop a potential which gives them a vacuum expectation value and a non-zero mass.

The moduli of the supergravity effective theory are arranged in three general groups [8] [9]:

- the axio-dilaton modulus S, defined as:

$$S = e^{-\phi} + iC_0. \tag{2.4}$$

The field ϕ is commonly known as dilaton and its expectation value defines the string coupling constant g_S as $e^{-\langle\phi\rangle} = 1/g_S$;

- Kähler moduli T^i, also known as size moduli, which can be defined as, separating the real and imaginary components:

$$T^i = \tau^i + i\psi^i, \qquad i = 1,2,...,h^{1,1}_+, \tag{2.5}$$

where τ^i represent essentially the volumes of the 4-cycles $\Sigma_{4,i}$, i.e. particular compact sub-manifolds of the Calabi-Yau manifold depending on the metric tensor g_{MN}, in string length units:

$$\tau^i = \frac{\mathrm{Vol}\,(\Sigma_{4,i})}{l_S^4}, \tag{2.6}$$

while ψ^i are fields known as axions coming from the definition:

$$\psi^i = \int_{\Sigma_{4,i}} C_4, \tag{2.7}$$

$h^{1,1}_+$ being topological numbers (Hodge numbers) depending on the Calabi-Yau manifold and on the particular orientifold projection. These fields have a shift symmetry valid at perturbative level of the kind $\psi^i \mapsto \psi^i + \zeta^i$ known as 'Peccei-Quinn shift symmetry';

- complex structure moduli U^α, also known as shape moduli, which are fields coming from the metric tensor:

$$U^\alpha = U^\alpha[g], \qquad \alpha = 1,2,...,h^{2,1}_-, \tag{2.8}$$

$h^{2,1}_-$ being again other Hodge numbers.

In general, the fields S, τ^i and U^α need to be massive, as explained before, but actually axions ψ^i can turn out to be uncoupled directly to observable matter and therefore can be massless.
Actually one more kind of fields arises from the previous bosonic spectrum:

$$G^j = b^j[B_2] + iSc^j[C_2], \qquad j = 1,2,\cdots,h^{1,1}_-,$$

$h^{1,1}_-$ being again other Hodge numbers. Generally these ones can be ignored for choices of the orientifold projection which imply $h^{1,1}_- = 0$.

For the further discussion it is fundamental to specify that the dimensionless volume of the Calabi-Yau manifold $\mathcal{V}$ can be expressed as a function of Kähler moduli. In fact, it is possible to write it as [8] [9]:

$$\mathcal{V} = \frac{1}{6}k^{ijk}t_i t_j t_k, \qquad i,j,k = 1,2,...,h^{1,1}_+, \tag{2.9}$$

where t_i are the volumes of 2-cycles of the Calabi-Yau manifold, i.e. compact submanifolds as well as 4-cycles, and k^{ijk} are intersection numbers, i.e. roughly speaking the number of times in which the three 4-cycles τ_i, τ_j and τ_k intersect. These 4-cycles are defined as:

$$\tau^i = \frac{\partial \mathcal{V}}{\partial t_i} = \frac{1}{2} k^{ijk} t_j t_k, \qquad i = 1,2,...,h_+^{1,1}, \tag{2.10}$$

in such a way as to deduce the expression $\mathcal{V} = \mathcal{V}(\tau)$ after inverting (2.10) and inserting it in (2.9).

To conclude this Section, it is important to evidence that moduli turn out to be natural candidates for the hidden sector since:

- these fields have only gravitational interactions with the ones described in the Standard Model and their supersymmetric partners, coherently with the previous discussion and the analysis in this thesis;

- these fields spontaneously break supersymmetry, as will be described in more detail in the rest of this and the following Chapters.

Such items indicate the plausibility of supersymmetric extensions of the Standard Model coming along with moduli stabilisation and are taken into account in the thesis in order to study some elementary proposals of a supersymmetric extensio of the Standard Model.

2.3 Supergravity Effective Theory

In this Section, the spontaneous breaking of supersymmetry by moduli is introduced. Observable sector terms are interpreted as subleading contributions and therefore they can be ignored when considering the breaking of supersymmetry in the hidden sector.

It can be shown that the general supergravity model involving the axio-dilaton, Kähler moduli and complex structure moduli is described by the tree-level Kähler potential [13] [15]:

$$K_0 = -\ln\left(S + \bar{S}\right) - \ln\left(-i\int_{Y_6} \Omega \wedge \bar{\Omega}\right) - 2\,\ln\left[\mathcal{V}\left(T + \bar{T}\right)\right], \tag{2.11}$$

where $\Omega = \Omega(U)$ is a topological quantity whose details are irrelevant for the following description because of its independence from Kähler moduli, as will be clear soon, and by the tree-level superpotential:

$$W_0 = 0,$$

when G-fields are not present. Actually the latter is a very trivial model and it cannot be interesting because the scalar potential is manifestly vanishing. Anyway, if the possibility to deal with "background fluxes" $G_3 = F_3 - iSH_3$ is taken into account, with $F_3 = dC_2$ and $H_3 = dB_2$, then it results that the superpotential becomes [16]:

$$W_0 = \int_{Y_6} G_3(S) \wedge \Omega(U).$$

As a matter of fact, it is worthwhile to simply consider:

$$W_0 = W_0\left(S, U\right). \tag{2.12}$$

again because of the independence from Kähler moduli.

Now it is possible to write the scalar potential V_F^0 generated by the auxiliary fields and then to find its minimum.
Thanks to the well known formulae, the scalar potential generated by F-terms can be written in general as:

$$V_F = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3W\bar{W}\right), \tag{2.13}$$

in the usual notation, indicating collectively as Φ^I the moduli fields S, U^α and T^i.
Then, since the tree-level Kähler metric $K_{I\bar{J}}$ is block-diagonal and W_0 is independent from the Kähler moduli according to (2.11) and (2.12), the scalar potential V_F^0 can be conveniently arranged as:

$$V_F^0 = K_{0\,S\bar{S}} F^S \bar{F}^{\bar{S}} + K_{0\,\alpha\bar{\beta}} F^\alpha \bar{F}^{\bar{\beta}} + e^{K_0} W_0 \bar{W}_0 \left(K_0^{i\bar{j}} K_{0\,i} K_{0\,\bar{j}} - 3\right), \tag{2.14}$$

having explicited the auxiliary fields, which are generally defined as:

$$F^I = \frac{W}{|W|}\, e^{K/2} K^{I\bar{J}} D_{\bar{J}} \bar{W}, \tag{2.15}$$

and writing α and i instead of U^α and T^i for the sake of simplicity.
It is a well known result that for standard Kähler potentials of the kind $K_0 = -2\,\ln \mathcal{V}\left(T + \bar{T}\right)$ the relation $K_0^{i\bar{j}} K_{0\,i} K_{0\,\bar{j}} = 3$ holds true, thus leading to the cancellation of the third macro-addendum.

In fact, in a more general fashion, it is possible to consider $K_0(\tau) = -p \ln \mathcal{V}(\tau)$, where p is a real number and the volume $\mathcal{V}$ is a homogeneous function[1] of the real parts

[1]A function $f : \mathbb{R}^n \to \mathbb{R}$ is a homogeneous function of degree k, $k \in \mathbb{R}$ if it is verified that:

$$f(\alpha \underline{x}) = \alpha^k f(\underline{x}) \qquad \forall \underline{x} \in \mathbb{R}^n, \forall \alpha \in \mathbb{R}^+.$$

Moreover, according to Euler Theorem, if the function $f : \mathbb{R}^n \to \mathbb{R}\backslash\{0\}$ is continuosuly differentiable, then it is homogeneous of degree k if and only if it is verified that:

$$\underline{x} \cdot \frac{\partial f}{\partial \underline{x}}(\underline{x}) = k f(\underline{x}) \qquad \forall \underline{x} \in \mathbb{R}^n\backslash\{0\}.$$

of Kähler moduli of degree k, with k a real number too. Typical values are $p = 2$ and $k = 3/2$, as in the following models. Then, in the usual notation, it is immediate to show that:

$$\tau^i K_{0\,i} = -\frac{1}{2}pk, \tag{2.16}$$

and, after a derivation with respect to τ_j, to conclude that:

$$K_{0\,i} = -2K_{0\,i}{}^{j} K_{0\,j}. \tag{2.17}$$

Finally, a combination of these two results gives the important relation:

$$K_0^{ij} K_{0\,i} K_{0\,j} = pk, \tag{2.18}$$

Therefore, a special cancellation occurs for the part depending explicitly on the Kähler moduli and the scalar potential turns out to be simply given by:

$$V_F^0 = K_{0\,S\bar{S}} F^S \bar{F}^{\bar{S}} + K_{0\,\alpha\bar{\beta}} F^{\alpha} \bar{F}^{\bar{\beta}}. \tag{2.19}$$

This result is often referred to as 'No-Scale Structure' [17].

Kähler metric and its inverse are symmetric and positive definite, then the scalar potential V_F^0 has a vanishing minimum corresponding the solutions $\langle S \rangle$ and $\langle U^\alpha \rangle$ of the system of equations (it is possible to prove that it always admits a solution [6] [7]):

$$\begin{cases} D_S W_0 \left(\langle S \rangle, \langle U^\alpha \rangle\right) = & 0, \\ D_\alpha W_0 \left(\langle S \rangle, \langle U^\alpha \rangle\right) = & 0. \end{cases} \tag{2.20}$$

Indeed auxiliary fields associated to the fields S and U^α must be vanishing in the ground state:

$$\langle F^S \rangle = 0, \qquad \langle F^\alpha \rangle = 0. \tag{2.21}$$

Evidently, this discussion shows that Kähler moduli T^i are arbitrary because they undergo a completely flat potential due to the No-Scale Structure.

2.4 Moduli Stabilisation

The fact that the ground state of the very generic model which has just been described cannot fix the vacuum expectation values of Kähler moduli in any way implies that they should be massless particles, while this property is in clear contrast with experimental observations since their presence has never been observed so far.

Therefore it is necessary to conclude that in order to build a reliable model it is required to consider some perturbative and/or non-perturbative corrections to the Kähler potential (2.11) and the superpotential (2.12) depending on Kähler moduli in such a way as to make them have phenomenologically acceptable masses in the ground state of the theory. This topic goes specifically by the name of "moduli stabilisation" and constitutes a fundamental tool in order to reconcile theoretical predictions and experimental evidences.

Starting from the Kähler potential (2.11) and the superpotential (2.12), as a matter of fact it is possible to build more realistic models by adding corrections to them. Indeed, the general forms of the Kähler potential and of the superpotential are [6] [7]:

$$K = K_0 + \delta K^{\mathrm{p}}\left(T+\bar{T}\right) + \delta K^{\mathrm{np}}\left(T,\bar{T}\right), \tag{2.22}$$

$$W = W_0 + \delta W^{\mathrm{np}}(T), \tag{2.23}$$

where the labels 'p' and 'np' denote perturbative and non-perturbative corrections respectively (of course the superpotential is not corrected perturbatively). No explicit dependences on S and U^α in both perturbative and non-perturbative corrections have been included since the dilaton and the complex structure moduli are stabilised at tree-level, and so they can be considered just as constants in the corrections to K_0 and W_0 after a very simple reasoning.
Indeed, a straighforward computation shows that the scalar potential V_F can be conveniently written as:

$$V_F = V_F^0\left(S,\bar{S};U,\bar{U}\right) + V_F^{\text{Kähler}}\left(T,\bar{T}\right),$$

where:

$$V_F^{\text{Kähler}} = e^K\left(K^{i\bar{j}} D_i W D_{\bar{j}}\bar{W} - 3W\bar{W}\right). \tag{2.24}$$

Now, following the discussion of the previous Section, it is evident that $V_F^{\text{Kähler}}$ is not exactly vanishing because of the newly introduced corrections. Its value is lifted from zero by these corrections, so it is worthwhile to first minimise V_F^0 with respect to S and U^α:

$$\frac{\partial V_F}{\partial S} \sim \frac{\partial V_F^0}{\partial S} = 0, \qquad \frac{\partial V_F}{\partial U^\alpha} \sim \frac{\partial V_F^0}{\partial U^\alpha} = 0,$$

obtaining the same results as before, i.e.:

$$\langle F^S\rangle = 0, \qquad \langle F^\alpha\rangle = 0. \tag{2.25}$$

and then to minimise V_F, i.e. actually $V_F^{\text{Kähler}}$, with respect to the Kähler moduli, of course after evalutating it in $\langle S\rangle$ and $\langle U^\alpha\rangle$.

The process of Kähler moduli stabilisation and especially its interplay with phenomenology are comprehensively encoded by the diagonalisation and canonical normalisation of their Lagrangian, which are outlined below.
The addendum $\langle K_{S,U}\rangle$, corresponding to the vacuum expectation value of the Kähler potential terms depending on S and U^α, is not reported in the following. Indeed, it comes into play only in a negligible overall scaling factor $k = e^{\langle K_{S,U}\rangle/2}$ in the scalar potential, S and U^α taking vacuum expectation values roughly around unity.

Kähler Moduli Lagrangian

The effective functions to deal with are the Kähler potential and the superpotential depending only on the Kähler moduli T^i. Moreover, since the Kähler potential essentially depends only on the real parts of the Kähler moduli, it is better to express all the quantities as functions of the real and imaginary parts of $T^i = \tau^i + i\psi^i$.

The Kähler potential and the superpotential are then generally expressed as:

$$K = -2\,\ln\mathcal{V}\left(\tau\right) + \delta K^{\mathrm{p}}\left(\tau\right) + \delta K^{\mathrm{np}}\left(\tau,\psi\right), \tag{2.26}$$

$$W = W_0 + \delta W^{\mathrm{np}}\left(\tau,\psi\right), \tag{2.27}$$

and generate the scalar potential V_F:

$$V_F = e^K\left(K^{i\bar{j}} D_i W D_{\bar{j}}\bar{W} - 3W\bar{W}\right), \tag{2.28}$$

whose minimum determines the vacuum expectation values of the fields τ^i and ψ^i respectively and most importantly their masses. Now the notation is slightly different as all quantities are referred only to Kähler moduli in this context.

The general Lagrangian describing Kähler moduli dynamics is derived and commented below. The analysis is not referred to the most general potentials as possible, even though such a description would be easily done in quite the same way, but a particular class of scalar potentials is taken into account in such a way as to fix the notation for the rest of this work.
The scalar Lagrangian associated to a Kähler potential K and a superpotential W is generically expressed in terms of moduli fields as:

$$e^{-1}\mathcal{L}^{\text{Kähler}} = K_{i\bar{j}}\left(T,\bar{T}\right)\partial_\mu T^i\,\partial^\mu \bar{T}^{\bar{j}} - V_F\left(T,\bar{T}\right),$$

which means, in terms of the fields τ^i and ψ^i, ignoring tiny non-perturbative corrections to K:

$$e^{-1}\mathcal{L}^{\text{Kähler}} = K_{i\bar{j}}\left(\tau\right)\partial_\mu\,\tau^i\,\partial^\mu\,\tau^{\bar{j}} + K_{i\bar{j}}\left(\tau\right)\partial_\mu\,\psi^i\,\partial^\mu\,\psi^{\bar{j}} - V_F\left(\tau,\psi\right). \tag{2.29}$$

In order to determine a canonical Lagrangian which is appropriate for the description of the ground state of the theory, it is necessary to parametrise the fluctuations of the fields from their vacuum expectation values $\langle\tau^i\rangle$ and $\langle\psi^i\rangle$. It is customary to define:

$$\tau^i = \left\langle\tau^i\right\rangle + \delta\tau^i,$$
$$\psi^i = \left\langle\psi^i\right\rangle + \delta\psi^i,$$

to get:

$$e^{-1}\mathcal{L}^{\text{Kähler}} = K_{i\bar{j}}\left(\delta\tau\right)\partial_\mu(\delta\tau^i)\,\partial^\mu(\delta\tau^{\bar{j}}) + K_{i\bar{j}}\left(\delta\tau\right)\partial_\mu(\delta\psi^i)\,\partial^\mu(\delta\psi^{\bar{j}}) - V_F\left(\delta\tau,\delta\psi\right).$$

Then, an expansion in Taylor series up to the leading order around the ground state gives the unnormalised bosonic kinetic Lagrangian:

$$\begin{aligned}\mathcal{L}_{\rm kin}^{\rm Kähler} &= \langle K_{ij}\rangle\, \partial_\mu \left(\delta\tau^i\right) \partial^\mu \left(\delta\tau^j\right) + \langle K_{ij}\rangle\, \partial_\mu \left(\delta\psi^i\right) \partial^\mu \left(\delta\psi^j\right) - \langle V_F\rangle \\ &- \frac{1}{2}\left\langle \frac{\partial^2 V_F}{\partial\,\tau^i\,\partial\,\tau^j}\right\rangle \delta\tau^i\delta\tau^j - \frac{1}{2}\left\langle \frac{\partial^2 V_F}{\partial\,\psi^i\,\partial\,\psi^j}\right\rangle \delta\psi^i\delta\psi^j - \left\langle \frac{\partial^2 V_F}{\partial\,\tau^i\,\partial\,\psi^j}\right\rangle \delta\tau^i\delta\psi^j,\end{aligned}$$

with obvious notation, where the stationarity conditions of the scalar potential have been taken into account. Of course the vielbein determinant is set $e = 1$ because around the ground state the metric essentially becomes Minkowskian, i.e. $g_{\mu\nu} \equiv \eta_{\mu\nu}$.
This is a general result which can be specialised to the models which will be described below. If the scalar potential is such that the two following identities are verified:

$$\left\langle \frac{\partial^2 V_F}{\partial\,\psi^i\,\partial\,\psi^j}\right\rangle = \left\langle \frac{\partial^2 V_F}{\partial\,\psi^{i\ 2}}\right\rangle \delta_{ij}, \qquad \left\langle \frac{\partial^2 V_F}{\partial\,\tau^i\,\partial\,\psi^j}\right\rangle = 0,$$

then the Lagrangian can be written in the more simple form:

$$\begin{aligned}\mathcal{L}_{\rm kin}^{\rm Kähler} &= \langle K_{ij}\rangle\, \partial_\mu \left(\delta\tau^i\right) \partial^\mu \left(\delta\tau^j\right) + \langle K_{ij}\rangle\, \partial_\mu \left(\delta\psi^i\right) \partial^\mu \left(\delta\psi^j\right) - \langle V_F\rangle \\ &- \left(M^2\right)_{ij} \delta\tau^i\delta\tau^j - \left(M'^2\right)_i \left(\delta\psi^i\right)^2,\end{aligned} \tag{2.30}$$

where the unnormalised mass matrix is defined as:

$$\left(M^2\right)_{ij} \equiv \frac{1}{2}\left\langle \frac{\partial^2 V_F}{\partial\,\tau^i\,\partial\,\tau^j}\right\rangle, \tag{2.31}$$

and equivalently axions unnormalised masses are:

$$\left(M'^2\right)_i \equiv \frac{1}{2}\left\langle \frac{\partial^2 V_F}{\partial\,\psi^{i\ 2}}\right\rangle. \tag{2.32}$$

In the hypothesis of stabilised real components of Kähler moduli, the Kähler metric and the unnormalised mass matrix are both positive semi-definite (the former positive definite) and symmetric. These conditions are more than enough for a very simple diagonalisation of the Lagrangian.
Indeed, an immediate simultaneous diagonalisation of matrices $\langle K_{ij}\rangle$ and $(M^2)_{ij}$ can be performed. Focusing on the terms concerning fields $\delta\tau^i$, it is worthwhile to define the normalised mass matrix as:

$$\left(m^2\right)^i{}_j \equiv \left\langle K^{ik}\right\rangle \left(M^2\right)_{kj}. \tag{2.33}$$

If the matrix $\left(m^2\right)^i{}_j$ is diagonalisable, writing its eigenvalues and eigenvectors as m_l^2 and $u^i_{(l)}$ respectively in such a way that:

$$\left(m^2\right)^i{}_k\, u^k_{(l)} = m_l^2 u^i_{(l)}, \tag{2.34}$$

then it can be proved that matrices $\langle K_{ij}\rangle$ and $(M^2)_{ij}$ are simultaneously diagonalised as:

$$\left(P^T\right)_i^{\ m} \langle K_{mn}\rangle\, P^n_{\ j} = \delta_{ij}, \tag{2.35}$$

$$\left(P^T\right)_i^{\ m} \left(M^2\right)_{mn} P^n_{\ j} = m_i^2\ \delta_{ij}, \tag{2.36}$$

where P is the matrix whose columns are the eigenvectors of $(m^2)^i_{\ j}$:

$$P^k_{\ l} \equiv u^k_{(l)}. \tag{2.37}$$

In this way, it is possible to define canonically normalised τ-fields by setting:

$$\delta\tau^i \equiv \frac{1}{\sqrt{2}} P^i_{\ j}\, \varphi^j. \tag{2.38}$$

Moreover, thanks to the especially simple form of the axionic terms, it is immediate to diagonalise the Lagrangian and individuate the normalised axions masses m_i' of the canonically normalised axionic fields θ^i.

In conclusion, the bosonic Lagrangian (2.30) is finally expressed as:

$$\mathcal{L}_{\rm kin}^{\text{Kähler}} = -\langle V_F\rangle + \frac{1}{2}\,\partial_\mu\,\varphi^i\,\partial^\mu\,\varphi^i + \frac{1}{2}\,\partial_\mu\,\theta^i\,\partial^\mu\,\theta^i - \frac{1}{2}m_i^2\left(\varphi^i\right)^2 - \frac{1}{2}m_i'^2\left(\theta^i\right)^2. \tag{2.39}$$

This is the general kinetic term of the Lagrangian describing Kähler moduli dynamics and will be one of the landmarks of the analysis that follows as it is the fundamental tool in order to determine some of the basic features of hidden sector particles.

The analysis of moduli interactions is not undertaken in this work because moduli are studied with particular interest in the determination of soft terms in vicinity of the ground state of the theory.

However, such interactions can be studied by simply expanding all of the related terms in the general supergravity Lagrangian and play an important rôle especially in cosmological implications.

Chapter 3

Kähler Moduli Stabilisation

In this Chapter some simple models of moduli stabilisation are described and commented to help make it clear how some more advanced ones are progressively needed in order not to conflict with naturalness and phenomenological issues. Two of the models under analysis then lay the basis on which reliable Supersymmetric Standard Models can be estabilished, as shown in Chapter 4.
The full mathematical derivation of the following results is reported in detail in Appendix A, which constitutes a computational complement to the present Chapter.

3.1 Model I

A very simple possible model of moduli stabilisation goes by the name of KKLT scenario [18]. It involves one single Kähler modulus $T = \tau + i\psi$ and entails only non-perturbative corrections to the superpotential.

The volume $\mathcal{V}$ in the presence of a single Kähler modulus is expressed as:

$$\mathcal{V}(\tau) = \tau^{3/2},$$

while non-perturbative corrections to the superpotential are expressed as exponentially vanishing contributions.
More precisely, the Kähler potential is defined as:

$$K = -3 \ln \tau, \tag{3.1}$$

coming from $K = -2 \ln \tau^{3/2}$, and the superpotential is written as:

$$W = W_0 + Ae^{-a(\tau+i\psi)}, \tag{3.2}$$

where W_0 and A are complex numbers which is useful to express as:

$$W_0 = |W_0| e^{i\theta}, \qquad A = |A| e^{i\alpha},$$

while a is a positive real number. It is fundamental to notice that the basic assumption of a very large volume $\mathcal{V}$ is accomplished by the logical condition:

$$a\tau \gg 1, \tag{3.3}$$

which is a characteristic feature of every large volume model, as will be evident in the following.

3.1.1 Minimisation

According to (2.28), these very simple Kähler potential and superpotential give the as much simple scalar potential:

$$V_F = \frac{4a\,|A|}{3\tau}\left[a\,|A|\,e^{-2a\tau} + \frac{3\,|W_0|}{\tau}\cos\left(\alpha_s - \theta - a\psi\right)e^{-a\tau} + \frac{3\,|A|}{\tau}e^{-2a\tau}\right]. \tag{3.4}$$

The individuation of its minimum can proceed as follows. As regards the axion ψ, it is clear that its vacuum expectation value is simply:

$$\langle\psi\rangle = \frac{\alpha - \theta}{a} + (2n+1)\,\frac{\pi}{a}, \qquad n \in \mathbb{Z}. \tag{3.5}$$

Then it is possible to write the scalar potential in the axion vacuum expectation value, so that it now depends only on the variable τ:

$$V_F = \frac{4a\,|A|}{3\tau}\left[a\,|A|\,e^{-2a\tau} - \frac{3\,|W_0|}{\tau}e^{-a\tau} + \frac{3\,|A|}{\tau}e^{-2a\tau}\right], \tag{3.6}$$

and find its minimum straightforwardly.
The stationarity condition is of course:

$$\frac{\partial V_F}{\partial\tau} = \frac{4a\,|A|}{3\tau^3}\left[3\,(a\tau+2)\,|W_0|\,e^{-a\tau} - \left(2a^2\tau^2 + 7a\tau + 6\right)|A|\,e^{-2a\tau}\right] = 0,$$

and it can be solved by recalling the assumption (3.3) in a very rough way, which is enough for the purposes of this work. In fact, after discarding subleading contributions to the factors of the two macro-addenda above, the stationarity condition becomes:

$$\left\langle\frac{\partial V_F}{\partial\tau}\right\rangle \overset{a\langle\tau\rangle\gg 1}{\sim} \frac{4a^2\,|A|}{3\,\langle\tau\rangle^2}e^{-a\langle\tau\rangle}\left[3\,|W_0| - 2a\,\langle\tau\rangle\,|A|\,e^{-a\langle\tau\rangle}\right] = 0,$$

which implies a vacuum expectation value $\langle\tau\rangle$ such that:

$$a\,\langle\tau\rangle\,e^{-a\langle\tau\rangle} = \frac{3}{2}\frac{|W_0|}{|A|}. \tag{3.7}$$

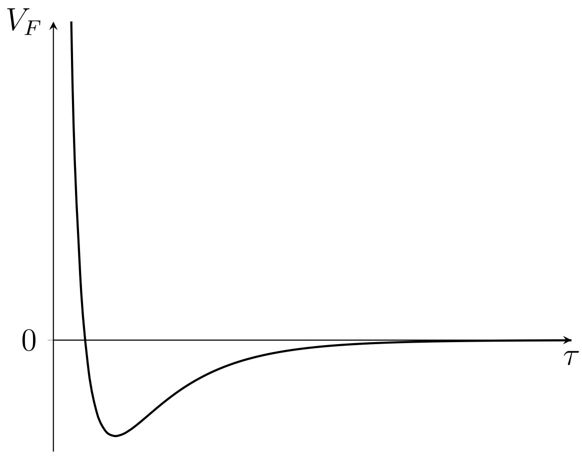

Figure 3.1. Plot of the scalar potential $V_F = V_F(\tau)$.

A rough estimate is enough because the leading order solution is sufficient to evidence the unnaturalness of this model. Indeed the conditions of large volume (3.3) and the solution (3.7) are reconcilable if and only if the modulus of the unperturbed superpotential $|W_0|$ is exponentially smaller than the modulus of the constant coefficient $|A|$ of non-perturbative corrections.

Anyway, it is very instructive to conclude the analysis of this model and to compute the cosmological constant, to consider the supersymmetry breaking and to evaluate the masses of the fields τ and ψ.
Indicating with the apex or subscript 'eff' the results of calculations which results from approximations by condition (3.3) of the same kind as the previous ones, it is easy to get the vacuum energy density, in physical units:

$$\left\langle V_F^{\text{eff}} \right\rangle = -\frac{3\,|W_0|^2}{\langle \mathcal{V} \rangle^2}\, M_P^4 \equiv E_\Lambda, \tag{3.8}$$

which indicates an anti-de Sitter universe, and a vanishing auxiliary field vacuum expectation value:

$$\left\langle F^T_{\text{eff}} \right\rangle = 0, \tag{3.9}$$

which means that in this model supersymmetry is not broken.

3.1.2 Moduli Masses

As concerns the computation of moduli masses, the starting point is the bosonic Lagrangian corresponding to the Kähler potential (3.1) and superpotential (3.2):

$$e^{-1}\mathcal{L}^{\text{Kähler}} = K_{T\bar{T}}\,\partial_\mu T\,\partial^\mu \bar{T} - V_F(T,\bar{T}).$$

Its especially simple form allows a straightforward derivation of its canonically normalised expression, which is performed explicitly for the sake of completeness.

Considering the real and complex components of the Kähler modulus T, it is necessary to define:

$$\tau = \langle \tau \rangle + \delta\tau, \qquad \psi = \langle \psi \rangle + \delta\psi,$$

to get:

$$e^{-1}\mathcal{L}^{\text{Kähler}} = \frac{3}{4\left(\langle \tau \rangle + \delta\tau\right)^2}\Big[\partial_\mu\left(\delta\tau\right)\partial^\mu\left(\delta\tau\right) + \partial_\mu\left(\delta\psi\right)\partial^\mu\left(\delta\psi\right)\Big] - V_F(\delta\tau, \delta\psi).$$

An expansion of the above expression around the ground state results in:

$$\begin{aligned}\mathcal{L}^{\text{Kähler}}_{\text{kin}} =& \frac{3}{4\langle \tau \rangle^2}\,\partial_\mu\left(\delta\tau\right)\partial^\mu\left(\delta\tau\right) + \frac{3}{4\langle \tau \rangle^2}\,\partial_\mu\left(\delta\psi\right)\partial^\mu\left(\delta\psi\right)\\ &- \frac{3a^2\left|W_0\right|^2}{\langle \tau \rangle^3}\left(\delta\tau\right)^2 - \frac{3a^2\left|W_0\right|^2}{\langle \tau \rangle^3}\left(\delta\psi\right)^2 - \left\langle V_F^{\text{eff}}\right\rangle,\end{aligned}$$

and, in physical units, defining canonically normalised fields as:

$$\frac{\phi}{M_P} = \left(\frac{3}{2\langle \tau \rangle^2}\right)^{1/2}\delta\tau, \qquad \frac{\theta_\phi}{M_P} = \left(\frac{3}{2\langle \tau \rangle^2}\right)^{1/2}\delta\psi,$$

the canonically normalised Lagrangian is determined:

$$\mathcal{L}^{\text{Kähler}}_{\text{kin}} = -E_\Lambda + \frac{1}{2}\,\partial_\mu\,\phi\,\partial^\mu\,\phi + \frac{1}{2}\,\partial_\mu\,\theta_\phi\,\partial^\mu\,\theta_\phi - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_{\theta_\phi}^2\theta_\phi^2. \tag{3.10}$$

where masses are given by:

$$m_\phi^2 \equiv m_{\theta_\phi}^2 \equiv \frac{4a^2\left|W_0\right|^2}{\langle \tau \rangle}\,M_P^2, \tag{3.11}$$

The equality between masses is necessary because of the vanishing of the auxiliary field, i.e. the unbroken supersymmetry.

It is customary to confront masses with physically notable quantities, such as the mass $m_{3/2} = \left\langle e^{K/2}\left|W\right|\right\rangle$ of the gravitino. In physical units, it is explicitly given by:

$$m_{3/2} = \frac{\left|W_0\right|}{\langle \tau \rangle^{3/2}}\left(1 - \frac{3}{2a\langle \tau \rangle}\right)M_P,$$

therefore at leading order Kähler moduli masses read:

$$m_\phi = m_{\theta_\phi} \simeq 2a\langle \tau \rangle\, m_{3/2}, \tag{3.12}$$

which means that moduli masses are larger than the gravitino one.

3.2 Systematics of Moduli Stabilisation

Some general considerations on moduli stabilisation are now in order as they will help in finding the best way to solve the unnaturalness problem. The problem of the presence of spontaneous supersymmetry breaking instead is not treated here but of course it must be a necessary feature of any reliable model.
More specifically, the task is here to determine qualitatively the magnitude of perturbative and non perturbative corrections to the scalar potential.

The scalar potential can be written after the expansion of covariant derivatives as:

$$V_F = e^K \left(K^{i\bar{j}} W_i \bar{W}_{\bar{j}} + K^{i\bar{j}} W K_i \bar{W}_{\bar{j}} + K^{i\bar{j}} \bar{W} W_i K_{\bar{j}}\right) + e^K \bar{W} W \left(K^{i\bar{j}} K_i K_{\bar{j}} - 3\right),$$

without any assumption on the number of Kähler moduli.

The first model which can be considered is the model with only non-perturbative corrections, which are assumed to affect just the superpotential. Indeed non-perturbative corrections are expected to be largely sub-dominant with respect to perturbative ones in the Kähler potential, which will be considered below. This fact means that, in the usual notation, the model is defined by:

$$\begin{aligned} K =& K_0, \\ W =& W_0 + \delta W^{\rm np}, \end{aligned}$$

assuming that the general form of the correction is of the kind $\delta W^{\rm np} \sim A e^{-a\tau}$, neglecting phases. Then, according to the above general expansion, the behaviour of the scalar potential $V_F = e^K \left(K^{i\bar{j}} W_i \bar{W}_{\bar{j}} + K^{i\bar{j}} W K_i \bar{W}_{\bar{j}} + K^{i\bar{j}} \bar{W} W_i K_{\bar{j}}\right)$, in a very rough estimate, is of the kind:

$$V_F^{\rm np} \sim e^K \left(\tau^2 \left(\delta W^{\rm np}\right)^2 + \tau W_0 \delta W^{\rm np}\right).$$

On the other hand, if only perturbative corrections are taken into account, then, the model arises from:

$$\begin{aligned} K =& K_0 + \delta K^{\rm p}, \\ W =& W_0. \end{aligned}$$

In this way, the scalar potential is $V_F = e^K W_0^2 \left(K^{i\bar{j}} K_i K_{\bar{j}} - 3\right)$, again neglecting phases. According to relations (2.16), (2.17) and (2.18), it is possible to write:

$$K^{i\bar{j}} K_i K_{\bar{j}} \sim K_0^{i\bar{j}} \left(K_{0\,i} K_{0\,j} + K_{0\,i} \delta K^{\rm p}_{\bar{j}} + K_{0\,\bar{j}} \delta K^{\rm p}_i\right) \sim kp + \tau \frac{\partial\, \delta K^{\rm p}}{\partial\, \tau},$$

so that, with the typical values $p = 2$ and $k = 3/2$, the scalar potential takes the form:

$$V_F^{\rm p} \sim e^K W_0^2 \delta K^{\rm p}.$$

Now it is possible to compare the orders of magnitude of the two scalar potentials by evaluating the ratio:

$$R = \frac{V_F^{\rm np}}{V_F^{\rm p}} \sim \frac{\tau^2 \left(\delta W^{\rm np}\right)^2 + \tau W_0 \delta W^{\rm np}}{W_0^2 \delta K^{\rm p}}.$$

An unnatural condition, which emerges for example in the model above, is the one where:

$$W_0 \sim \tau \delta W^{\rm np},$$

indeed this feature causes the ratio to be unnaturally large:

$$R \sim \frac{W_0^2}{W_0^2 \delta K^{\rm p}} \gg 1.$$

A much more natural situation is the one where non-perturbative corrections are very small, i.e.:

$$W_0 \gg \tau \delta W^{\rm np},$$

which allows for a more reliable hierarchy. As a matter of fact, the ratio becomes:

$$R \sim \frac{\tau \delta W^{\rm np}}{W_0 \delta K^{\rm p}} \sim \frac{A}{W_0} \frac{\tau e^{-a\tau}}{\delta K^{\rm p}} \sim \frac{A}{W_0} \frac{\tau e^{-a\tau}}{\tau^{-n}} \gtreqless 1,$$

where a general dependence $\delta K^{\rm p} \sim \tau^{-n}$ is assumed for the sake of simplicity.
In models involving one single Kähler modulus the ratio is close to zero. On the other hand, it is clear that in models involving two or more Kähler moduli the ratio can take values around unity. For instance, if some "big" fields $\tau_{\rm big}$ and "small" fields $\tau_{\rm small}$ are taken into account, then it is natural imagine to deal with ratios of the kind:

$$R \sim \frac{A_{\rm small}}{W_0}\, e^{-a_{\rm small}\tau_{\rm small}} \tau_{\rm small} \tau_{\rm big}^n \sim 1.$$

This is a very interesting class of models because it does not deal with any *ad hoc* assumption and therefore looks like a more reliable phenomenlogical description.

To conclude, the above discussion invites to look for a model with two Kähler moduli at least and with both perturbative and non-perturbative corrections. Indeed such a modelling should guarantee the absence of naturalness problems as well as the (hoped) presence of spontaneous supersymmetry breaking.
Such 'natural' models have been developed in Ref. [10] and go by the name of Large Volume Scenarios, where the volume $\mathcal{V}$ turns out to be exponentially large. Actually, more correctly, these ones should be called 'LARGE Volume Scenarios' to distinguish them from other similar scenarios where the volume is large altough not exponentially.
Two of the simplest examples of these scenarios are studied in detail below. They will be referred to as Models II and III.

3.3 Model II

The simplest model which deals with two Kähler moduli whose scalar potential is generated by both perturbative and non-perturbative corrections to the Kähler potential and superpotential respectively comes from string compactification on $\mathbb{P}_{[1,1,1,6,9]}$. By calling the fields as:

$$T_b = \tau_b + i\psi_b, \qquad T_s = \tau_s + i\psi_s,$$

the 'b' and 's' subscripts indicating a 'big' and a 'small' modulus respectively, the volume $\mathcal{V}$ is given by [19]:

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_b^{3/2} - \tau_s^{3/2}\right).$$

The geometry of this model can be imagined as a large volume, whose approximate value corresponds to the volume $\tau_b^{3/2}$ of a large 4-cycle $\Sigma_{4,b}$, where a small hole is present, which is represented by a small 4-cycle $\Sigma_{4,s}$ of volume $\tau_s^{3/2}$. This property means that the fundamental assumption is:

$$\tau_b^{3/2} \gg \tau_s^{3/2}.$$

It is worthwhile to clarify that the notation adopted indicates explicitly the moduli T^i (notice upper indices) with lower subscripts as T_b and T_s: this will be the convention from now on, except where explicitly declared as different.

The Kähler potential and the superpotential are expressed according to Ref. [10][1]:

$$K = -2\,\ln\left(\mathcal{V}\left(\tau_b, \tau_s\right) + \frac{\xi'}{2g_S^{3/2}}\right), \tag{3.13}$$

$$W = W_0 + A_s e^{-a_s'(\tau_s + i\psi_s)}, \tag{3.14}$$

where ξ' is a real constant, generally positive, which characterises the so called α' perturbative corrections to the Kähler potential, while, similarly to Model I, W_0 and A_s are complex numbers parametrised as:

$$W_0 = |W_0|\, e^{i\theta}, \qquad A_s = |A_s|\, e^{i\alpha_s},$$

and a_s' is a positive real number. The explicit correction to the superpotential does not involve an analog contribution $\delta W^{\mathrm{np}} = A_b e^{-a_b'(\tau_b + i\psi_b)}$ depending on τ_b and ψ_b because it can be assumed to be largely subdominant as τ_b is expected to be some orders of magnitude greater than τ_s. Nevertheless, it must be assumed that:

$$a_s'\tau_s \gg 1,$$

[1]This fundamental reference also contains a very detailed list of references which further motivate the forms of the Kähler potential and the superpotential.

since the volume of the blow-up cycle is greater than unity.
It is important to notice that by expliciting the perturbative correction to the Kähler potential in a Taylor expansion as:

$$\delta K^{\alpha'} = -\frac{\xi'}{g_S^{3/2}\mathcal{V}} + O\left(\frac{\xi'}{g_S^{3/2}\mathcal{V}}\right)^2,$$

it becomes clear that the form of the tree-level Kähler potential $K_0 = -2\ln\mathcal{V}(\tau_b, \tau_s)$ is the typical one allowing for No-Scale Structure.
It is fundamental to specify that the string coupling g_S must be small - i.e. $g_S \ll 1$ - as the effective action is derived under the perturbative approximation[2].

For the sake of symplictity, it is worthwhile to redefine the fields in such a way as to absorb the annoying $9\sqrt{2}$ factor by requiring the volume to be expressed as:

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}. \tag{3.15}$$

This redefinition can be improved by defining the constant parameters:

$$\xi \equiv 9\sqrt{2}\frac{\xi'}{2g_S^{3/2}}, \qquad a_s \equiv \frac{a_s'}{\left(9\sqrt{2}\right)^{2/3}},$$

in order to let expressions look more plain.

Summing up, the model under analysis is the one described by the Kähler potential and superpotential:

$$K = -2\ln\left(\tau_b^{3/2} - \tau_s^{3/2} + \xi\right), \tag{3.16}$$

$$W = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)}, \tag{3.17}$$

with the assumptions of a large exponential argument and of a large volume $\tau_b^{3/2}$ with respect to $\tau_s^{3/2}$ as well as to the constant ξ:

$$a_s\tau_s \gg 1, \tag{3.18}$$

$$\tau_b^{3/2} \gg \tau_s^{3/2} \sim \xi, \tag{3.19}$$

according to the previous discussion.

[2]In general, it must be at most of order 10^{-1}.

3.3.1 Minimisation

The scalar potential V_F can be computed easily and, after the straightforward minimisation with respect to the axion ψ_s, which takes the vacuum expectation value:

$$\langle \psi_s \rangle = \frac{\alpha_s - \theta}{a_s} + (2n+1)\,\frac{\pi}{a_s}, \quad n \in \mathbb{Z}, \tag{3.20}$$

while the axion ψ_b remains arbitrary as there is no dependence on it, under the assumptions (3.18) and (3.19) its leading order effective expression is [20]:

$$V_F^{\text{eff}} = \frac{8}{3}\frac{a_s^2 \tau_s^{1/2}}{\tau_b^{3/2}}\,|A_s|^2\, e^{-2a_s\tau_s} - 4\frac{a_s\tau_s}{\tau_b^3}\,|A_s|\,|W_0|\,e^{-a_s\tau_s} + \frac{3}{2}\frac{\xi}{\tau_b^{9/2}}\,|W_0|^2\,. \tag{3.21}$$

This potential determines the vacuum expectation values of the fields τ_b and τ_s:

$$\langle \tau_b \rangle^{3/2} = \frac{3}{4}\frac{\langle \tau_s\rangle^{\frac{1}{2}}}{a_s}\frac{|W_0|}{|A_s|}\; e^{a_s\langle\tau_s\rangle}\,\frac{1 - \dfrac{1}{a_s\langle\tau_s\rangle}}{1 - \dfrac{1}{4a_s\langle\tau_s\rangle}}, \tag{3.22}$$

$$\langle \tau_s \rangle^{3/2} = \xi\,\frac{\left(1 - \dfrac{1}{4a_s\langle\tau_s\rangle}\right)^2}{\left(1 - \dfrac{1}{a_s\langle\tau_s\rangle}\right)}. \tag{3.23}$$

These expressions of the vacuum expectation values are not explicit, however they are what is needed in order to go on and interpret further results.
First of all, a Taylor expansion in $1/a_s\langle\tau_s\rangle$ readily gives:

$$\langle \tau_b \rangle^{3/2} = \frac{3}{4}\frac{\langle \tau_s\rangle^{\frac{1}{2}}}{a_s}\frac{|W_0|}{|A_s|}\; e^{a_s\langle\tau_s\rangle}\left(1 + O\left(\frac{1}{a_s\langle\tau_s\rangle}\right)\right),$$

$$\langle \tau_s \rangle^{3/2} = \xi\left(1 + O\left(\frac{1}{a_s\langle\tau_s\rangle}\right)\right),$$

so it is evident that the 'small Kähler modulus' τ_s fixes its minimum close to the value of $\xi^{2/3}$, which means around unity, while the 'big Kähler modulus' τ_b is exponentially large if compared to the small one. It is important to notice that the expectation value of τ_b is exponentially sensible to the string coupling.

Moreover, in a very qualitative approximation, the following relations hold true:

$$\langle \tau_b \rangle^{3/2} \simeq \langle \mathcal{V} \rangle\,,$$

$$a_s\langle\tau_s\rangle \simeq \ln\langle \mathcal{V}\rangle\,.$$

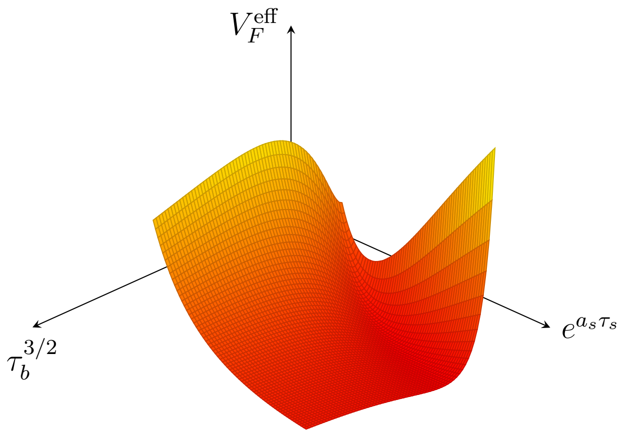

Figure 3.2. Detail of the of the scalar potential plot $V_F^{\text{eff}} = V_F^{\text{eff}}(\tau_b^{3/2}, e^{a_s\tau_s})$ in vicinity of the minimum: it is worthwhile to express it as a function of $\tau_b^{3/2}$ and $e^{a_s\tau_s}$ because in this way the two variables are roughly of the same order of magnitude in this zone. Moreover this dilatation makes it more evident the presence of a minimum.

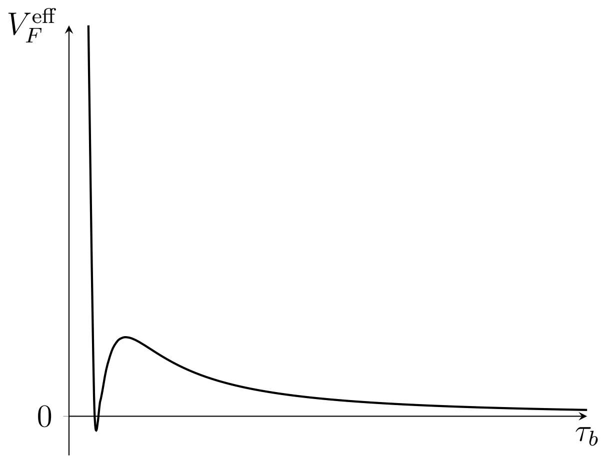

Figure 3.3. Plot of the scalar potential $V_F^{\text{eff}} = V_F^{\text{eff}}(\tau_b)$ in the minimum condition with respect to τ_s.

Despite their inaccuracy, it is useful to always keep them in mind in order to readily interpret further results.

Results (3.22) and (3.23) allow to determine the vacuum energy density, in physical units:

$$\left\langle V_F^{\text{eff}} \right\rangle = -\frac{3}{4} \frac{\xi \left|W_0\right|^2}{a_s \left\langle \tau_s \right\rangle \left\langle \tau_b \right\rangle^{9/2}} M_P^4 \equiv E_\Lambda, \tag{3.24}$$

which denotes an anti-de Sitter Universe, and to observe that this model truly breaks supersymmetry as the vacuum expectation values of the auxiliary fields are, in physical units:

$$\begin{aligned}\left\langle F^b_{\rm eff}\right\rangle &= -\frac{2}{\langle\tau_b\rangle^{\frac{1}{2}}}\,|W_0|\left[1+\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_b\rangle^{\frac{3}{2}}}-\frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\left(1+\frac{3}{8a_s\langle\tau_s\rangle}-\frac{3}{16a_s^2\langle\tau_s\rangle^2}\right)\right]M_P^2,\\ \left\langle F^s_{\rm eff}\right\rangle &= -\frac{3}{2}\frac{\langle\tau_s\rangle}{a_s\langle\tau_b\rangle^{\frac{3}{2}}}\,|W_0|\left[1+O\left(\frac{1}{a_s\langle\tau_s\rangle}\right)\right]M_P^2,\end{aligned} \tag{3.25}$$

up to order I in $\langle\tau_s\rangle^{3/2}/\langle\tau_b\rangle^{3/2}$ and $\xi/\langle\tau_b\rangle^{3/2}$.

3.3.2 Moduli Masses

The canonically normalised bosonic Lagrangian of the model can be written as shown in Subsection 2.4.

The free unnormalised Lagrangian describing τ-field fluctuations around the ground state is of course given by:

$$\mathcal{L}^{\tau-\text{fields}}_{\text{kin}} = \langle K_{ij}\rangle\,\partial_\mu\left(\delta\tau^i\right)\partial^\mu\left(\delta\tau^j\right)-\left(M^2\right)_{ij}\delta\tau^i\delta\tau^j.$$

Under assumptions (3.18) and (3.19), the normalised mass matrix can be shown to be:

$$\left(m^2_{\rm eff}\right)^i{}_j = \frac{a_s\langle\tau_s\rangle^{\frac{1}{2}}\,\xi\,|W_0|^2}{\langle\tau_b\rangle^{\frac{9}{2}}}\begin{pmatrix} -9\langle\tau_s\rangle^{\frac{1}{2}}\left(1-\frac{7}{4}\sigma\right) & 6a_s\langle\tau_s\rangle^{\frac{1}{2}}\langle\tau_b\rangle\left(1-\frac{5}{4}\sigma+\sigma^2\right)\\ -6\langle\tau_b\rangle^{\frac{1}{2}}\left(1-\frac{5}{4}\sigma+\frac{1}{4}\sigma^2\right) & 4a_s\langle\tau_b\rangle^{\frac{3}{2}}\left(1-\frac{3}{4}\sigma+\frac{3}{8}\sigma^2+\frac{1}{8}\sigma^3\right)\end{pmatrix}, \tag{3.26}$$

up to order I in $\langle\tau_s\rangle^{3/2}/\langle\tau_b\rangle^{3/2}$ and $\xi/\langle\tau_b\rangle^{3/2}$, where $\sigma\equiv 1/a_s\langle\tau_s\rangle$.

As proven in Subsection 2.4, its eigenvalues correspond to the masses of canonically normalised τ-fields. Denoting these newly introduced fields as χ and ϕ, at leading order (see Section A.2 in the Appendix for accurate expressions) their masses are:

$$m^2_\chi \simeq \frac{81}{8}\frac{\xi\,|W_0|^2}{a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}, \tag{3.27}$$

$$m^2_\phi \simeq \frac{4\,a_s^2\langle\tau_s\rangle^{1/2}\,\xi\,|W_0|^2}{\langle\tau_b\rangle^3}. \tag{3.28}$$

It is important to notice that the field χ is by far much lighter than the field ϕ. In particular, an important reference for the order of magnitude of these fields is given by a comparison with the gravitino one, $m_{3/2}=\left\langle e^{K/2}\,|W|\right\rangle$, as usual. The latter is given at leading order by, in physical units:

$$m_{3/2}\simeq\frac{|W_0|}{\langle\tau_b\rangle^{\frac{3}{2}}}\,M_P,$$

and, again at leading order, it is easy to observe that (3.22) also gives:

$$\ln \frac{M_P}{m_{3/2}} \simeq a_s \langle \tau_s \rangle .$$

Then it is evident that the mass of the field χ is heavily reduced with respect to the field ϕ one because:

$$m_\chi \simeq \frac{9\sqrt{2}}{4} \left(\frac{\xi}{\langle \mathcal{V} \rangle} \right)^{\frac{1}{2}} \left(\ln \frac{M_P}{m_{3/2}} \right)^{-\frac{1}{2}} m_{3/2}, \tag{3.29}$$

$$m_\phi \simeq 2 \ln \left(\frac{M_P}{m_{3/2}} \right) m_{3/2}. \tag{3.30}$$

Indeed since the logarithm is not exponentially large unlike $\langle \mathcal{V} \rangle$, these relationships mean that only m_ϕ is essentially of the same order of magnitude as $m_{3/2}$, while m_χ is lighter by a factor roughly scaling as $\langle \mathcal{V} \rangle^{1/2}$.

On the other hand, the normalised eigenvectors $u^i_{(l)}$ of the matrix $\left(m^2_{\text{eff}}\right)^i{}_j$ turn out to be at leading order:

$$u^i_\chi \simeq \begin{pmatrix} \frac{2\sqrt{3}}{3} \langle \tau_b \rangle \\ \frac{\sqrt{3}}{a_s} \end{pmatrix},$$

$$u^i_\phi \simeq \begin{pmatrix} \sqrt{6} \langle \tau_s \rangle^{3/4} \langle \tau_b \rangle^{1/4} \\ \frac{2\sqrt{6}}{3} \langle \tau_s \rangle^{1/4} \langle \tau_b \rangle^{3/4} \end{pmatrix},$$

where of course their normalisation is such that $\left\langle K^{\text{eff}}_{ij} \right\rangle u^i_{(l)} u^j_{(k)} = \delta_{lk}$. Then, the change of basis which gives a canonically normalised diagonal Lagrangian, in physical units, is:

$$\begin{pmatrix} \delta\tau_b \\ \delta\tau_s \end{pmatrix} \simeq \frac{\sqrt{2}}{2} \begin{pmatrix} \frac{2\sqrt{3}}{3} \langle \tau_b \rangle & \sqrt{6} \langle \tau_s \rangle^{3/4} \langle \tau_b \rangle^{1/4} \\ \frac{\sqrt{3}}{a_s} & \frac{2\sqrt{6}}{3} \langle \tau_s \rangle^{1/4} \langle \tau_b \rangle^{3/4} \end{pmatrix} \begin{pmatrix} \chi/M_P \\ \phi/M_P \end{pmatrix}. \tag{3.31}$$

It is important to underline that this change of basis shows that the 'big' field τ_b fluctuation is mainly projected into the light field χ, while the 'small' field τ_s fluctuation is predominantly aligned to the heavy field ϕ. Indeed the transformations can be written very roughly as:

$$\begin{cases} \delta\tau_b \simeq \frac{\sqrt{6}}{3} \langle \tau_b \rangle \frac{\chi}{M_P}, \\ \delta\tau_s \simeq \frac{2\sqrt{3}}{3} \langle \tau_s \rangle^{1/4} \langle \tau_b \rangle^{3/4} \frac{\phi}{M_P}. \end{cases}$$

Now only axions are left. The scalar potential is such that conditions cited in Section 2.3 are verified. In particular the axionic Lagrangian is:

$$\mathcal{L}_{\text{kin}}^{\text{axion}} = \langle K_{ij} \rangle \, \partial_\mu \left(\delta\psi^i \right) \partial^\mu \left(\delta\psi^j \right) - \left(M'^2 \right)_i \left(\delta\psi^i \right)^2 ,$$

with the leading order expression for the only nonzero unnormalised axionic mass:

$$\left(M'^2 \right)_s \simeq \frac{3 a_s^2 \xi \left| W_0 \right|^2}{2 \left\langle \tau_b \right\rangle^{9/2}} .$$

The diagonalisation and canonical normalisation of the Lagrangian are immediate. Indeed it is sufficient to normalise the two fields by requiring canonical pure kinetic terms and eventually discarding in the expression of the kinetic Lagrangian an irrelevant interaction term. So the normalised fields θ_χ and θ_ϕ can be defined via the relations at leading order:

$$\begin{cases} \delta\psi_b & \simeq \dfrac{\sqrt{6}}{3} \left\langle \tau_b \right\rangle \dfrac{\theta_\chi}{M_P}, \\ \delta\psi_s & \simeq \dfrac{2\sqrt{3}}{3} \left\langle \tau_s \right\rangle^{1/4} \left\langle \tau_b \right\rangle^{3/4} \dfrac{\theta_\phi}{M_P}, \end{cases} \tag{3.32}$$

which actually coincide with the leading order transformations of the Kähler moduli real parts, thus justifying the notation.
The masses of canonically normalised axionic fields are then at leading order:

$$m_{\theta_\chi}^2 = 0, \tag{3.33}$$

$$m_{\theta_\phi}^2 \simeq \frac{4 \; a_s^2 \left\langle \tau_s \right\rangle^{1/2} \xi \left| W_0 \right|^2}{\left\langle \tau_b \right\rangle^3}, \tag{3.34}$$

i.e., in terms of the gravitino mass and in physical units:

$$m_{\theta_\chi} = 0, \tag{3.35}$$

$$m_{\theta_\phi} \simeq 2 \, \ln \left(\frac{M_P}{m_{3/2}} \right) m_{3/2}. \tag{3.36}$$

First of all, it is fundamental to notice that within this framework the fact that the field θ_χ is massless does not give rise to a phenomenological inconsistency because, since it appears neither in the Kähler potential nor in the superpotential, it does not have any kind of interactions with every other field. Secondly, it is important to underline that the fact that $m_{\theta_\phi}^2$ is equal to m_ϕ^2 is verified at leading order.

In the end, the kinetic Lagrangian describing moduli dynamics within the Supersymmetric Standard Model descending from the present construction is:

$$\begin{aligned} \mathcal{L}_{\text{kin}}^{\text{Kähler}} = & - E_\Lambda + \frac{1}{2} \, \partial_\mu \, \chi \, \partial^\mu \, \chi + \frac{1}{2} \, \partial_\mu \, \phi \, \partial^\mu \, \phi + \frac{1}{2} \, \partial_\mu \, \theta_\chi \, \partial^\mu \, \theta_\chi + \frac{1}{2} \, \partial_\mu \, \theta_\phi \, \partial^\mu \, \theta_\phi \\ & - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} m_{\theta_\phi}^2 \theta_\phi^2 . \end{aligned} \tag{3.37}$$

This model was studied in detail in a multitude of papers, e.g. in Ref. [21], which will be taken under exam for the computation of soft terms. This fact drives interest towards more sophisticated models, one of which is the next one that will be analysed in detail.

3.4 Model III: Leading Results

The present model comes from compactifications of Calabi-Yau manifolds with $K3$ Fibration Structure [22], which in one of the simplest realisations gives a volume $\mathcal{V}$:

$$\mathcal{V} = \lambda_1 t_1 t_2^2 + \lambda_2 t_3^3,$$

in terms of three 2-cycles t_1, t_2 and t_3, where it can be shown that t_3 is negative, with λ_1 and λ_2 model-dependent positive real numbers. Of course it is customary to express everyithing in terms of the corresponding 4-cycles τ_1, τ_2 and τ_3, which are related to 2-cycles by the relations:

$$\begin{cases} \tau_1 = \lambda_1 t_2^2, \\ \tau_2 = 2\lambda_1 t_1 t_2, \\ \tau_3 = 3\lambda_2 t_3^2, \end{cases} \tag{3.38}$$

and their inverse:

$$\begin{cases} t_1 = (4\lambda_1)^{-\frac{1}{2}}\, \tau_1^{-\frac{1}{2}} \tau_2, \\ t_2 = \lambda_1^{-\frac{1}{2}} \tau_1^{\frac{1}{2}}, \\ t_3 = -\,(3\lambda_2)^{-\frac{1}{2}}\, \tau_3^{\frac{1}{2}}. \end{cases} \tag{3.39}$$

Then the volume $\mathcal{V}$ can be written in the more familiar form:

$$\mathcal{V} = \alpha\left(\tau_1^{1/2}\tau_2 - \gamma\tau_3^{3/2}\right), \tag{3.40}$$

with:

$$\alpha \equiv \frac{1}{(4\lambda_1)^{1/2}}, \qquad \gamma \equiv \frac{2}{3}\left(\frac{\lambda_1}{3\lambda_2}\right)^{1/2}.$$

Of course, the geometry associated to this model in the large volume scenario is again such that a large compactification volume $\mathcal{V}$ contains a small 4-cycle of volume $\alpha\gamma\,\tau_3^{3/2}$, $\mathcal{V}$ being given essentially by a product of the fields τ_1 and τ_2, $\mathcal{V} \simeq \alpha\,\tau_1^{1/2}\tau_2$.
In order to readily compare this situation to the one of Model II, it is worthwhile to redefine the fields by absorbing the numerical constants α and γ and to replace the label '3' with the more familiar 's'.
Then, the present model is easily seen to be a sort of generalisation of the previous one. Indeed it deals with three Kähler moduli which can be conveniently labeled as:

$$T_1 = \tau_1 + i\psi_1, \qquad T_2 = \tau_2 + i\psi_2, \qquad T_s = \tau_s + i\psi_s,$$

with the volume $\mathcal{V}$ expressed as:

$$\mathcal{V} = \tau_1^{1/2}\tau_2 - \tau_s^{3/2}, \tag{3.41}$$

then making completely clear the analogy with Model II. Essentially the rôle of the field τ_b is here replaced by the couple of fields τ_1 and τ_2 as:

$$\tau_b^{3/2} = \tau_1^{1/2}\tau_2,$$

giving to the volume $\mathcal{V}$ a more sophisticated structure which can give rise to a variety of additional phenomenlogical features, as will be shown in the following. Of course, the rôle of the blow-up cycle is again assumed by the field τ_s.

In the end, under these premises, the Kähler potential and the superpotential are expressed as:

$$K = -2\,\ln\left(\tau_1^{1/2}\tau_2 - \tau_s^{3/2} + \xi\right), \tag{3.42}$$

$$W = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)}, \tag{3.43}$$

where ξ is a real constant that characterises perturbative corrections to the Kähler potential, while W_0 and A_s are complex constants commonly written as:

$$W_0 = |W_0|\, e^{i\theta}, \qquad A_s = |A_s|\, e^{i\alpha_s},$$

and a_s is a positive real number. It is important to notice that the tree-level Kähler potential $K_0 = -2\,\ln\mathcal{V}$ is the typical one allowing for No-Scale Structure.
Of course the constitutive assumptions underlying this construction can be explicited as:

$$a_s\tau_s \gg 1, \tag{3.44}$$

$$\tau_1^{1/2}\tau_2 \gg \tau_s^{3/2} \sim \xi, \tag{3.45}$$

according to the well-known properties of the model, i.e. a large exponent $a_s\tau_s$ and a large volume $\tau_1^{1/2}\tau_2$ compared to $\tau_s^{3/2}$ and ξ.

3.4.1 Minimisation

The scalar potential V_F can be computed explicitly. Again, the vacuum expectation value of the axion ψ_s is readily individuated in:

$$\langle\psi_s\rangle = \frac{\alpha_s - \theta}{a_s} + (2n+1)\,\frac{\pi}{a_s}, \quad n \in \mathbb{Z}, \tag{3.46}$$

while the axions ψ_1 and ψ_2 undergo a completely flat potential and thus remain unfixed. Then, under assumptions (3.44) and (3.45), the leading order effective expression of the scalar potential reads:

$$V_F^{\text{eff}} \equiv \frac{8}{3}\frac{a_s^2\tau_s^{1/2}}{\left(\tau_1^{1/2}\tau_2\right)}|A_s|^2 e^{-2a_s\tau_s} - 4\frac{a_s\tau_s}{\left(\tau_1^{1/2}\tau_2\right)^2}|A_s||W_0|e^{-a_s\tau_s} + \frac{3}{2}\frac{\xi}{\left(\tau_1^{1/2}\tau_2\right)^3}|W_0|^2. \quad (3.47)$$

This potential definitely makes it evident the close relationship which connects this model with the previous one. Indeed the very same approximations lead in both cases to the same scalar potential, provided that in the second case the rôle of the field τ_b is split among two fields τ_1 and τ_2. In the end, expression (3.47) suggests the presence of a flat direction given by a suitable combination of τ_1 and τ_2. This fact is manifestly confirmed by calculations.

An immediate consequence of the form of the scalar potential V_F^{eff} is that of course it gives the fields τ_1, τ_2 and τ_s vacuum expectation values such that:

$$\langle\tau_1\rangle^{1/2}\langle\tau_2\rangle = \frac{3}{4}\frac{\langle\tau_s\rangle^{\frac{1}{2}}}{a_s}\frac{|W_0|}{|A_s|}\,e^{a_s\langle\tau_s\rangle}\frac{1-\dfrac{1}{a_s\langle\tau_s\rangle}}{1-\dfrac{1}{4a_s\langle\tau_s\rangle}}, \quad (3.48)$$

$$\langle\tau_s\rangle^{3/2} = \xi\frac{\left(1-\dfrac{1}{4a_s\langle\tau_s\rangle}\right)^2}{\left(1-\dfrac{1}{a_s\langle\tau_s\rangle}\right)}, \quad (3.49)$$

coherently with solutions (3.22) and (3.23).
Evidently the very same considerations that are listed in Section 3.3.1 are similarly valid in the present situation provided the identification $\langle\tau_b\rangle^{3/2} = \langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$. In particular, the vacuum energy density turns out to be, in physical units:

$$\left\langle V_F^{\text{eff}}\right\rangle = -\frac{3}{4}\frac{\xi|W_0|^2}{a_s\langle\tau_s\rangle\langle\tau_1\rangle^{\frac{3}{2}}\langle\tau_2\rangle^3}M_P^4 \equiv E_\Lambda. \quad (3.50)$$

On the other hand, auxiliary fields expectation values are, in physical units:

$$\begin{aligned}
\left\langle F^1_{\text{eff}}\right\rangle &= -\,2\frac{\langle\tau_1\rangle^{\frac{1}{2}}}{\langle\tau_2\rangle}|W_0|\left[1-\frac{1}{2}\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\left(1-\frac{3}{4a_s\langle\tau_s\rangle}-\frac{27}{16a_s^2\langle\tau_s\rangle^2}\right)+\frac{1}{2}\frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\right]M_P^2,\\
\left\langle F^2_{\text{eff}}\right\rangle &= -\,\frac{2}{\langle\tau_1\rangle^{\frac{1}{2}}}|W_0|\left[1-\frac{1}{2}\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\left(1-\frac{3}{4a_s\langle\tau_s\rangle}-\frac{27}{16a_s^2\langle\tau_s\rangle^2}\right)+\frac{1}{2}\frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\right]M_P^2,\\
\left\langle F^s_{\text{eff}}\right\rangle &= -\,\frac{3}{2}\frac{\langle\tau_s\rangle}{a_s\langle\tau_1\rangle^{1/2}\langle\tau_1\rangle}|W_0|\left[1+O\left(\frac{1}{a_s\langle\tau_s\rangle}\right)\right]M_P^2,
\end{aligned} \quad (3.51)$$

up to order I in $\langle\tau_s\rangle^{3/2} / \langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ and $\xi / \langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ and thus indicate again the breaking of supersymmetry.

3.4.2 Moduli Masses

The guide to determine the kinetic Lagrangian describing the dynamics of Kähler moduli is of course the procedure outlined in Subsection 2.4.

The well-known free unnormalised Lagrangian describing τ-fields fluctuations about the ground state is:

$$\mathcal{L}_{\text{kin}}^{\tau-\text{fields}} = \langle K_{ij}\rangle \partial_\mu \left(\delta\tau^i\right) \partial^\mu \left(\delta\tau^j\right) - \left(M^2\right)_{ij} \delta\tau^i \delta\tau^j .$$

Under assumptions (3.44) and (3.45), the normalised mass matrix up to order I in $\langle\tau_s\rangle^{3/2} / \langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ and $\xi / \langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ can be written as:

$$\left(m_{\text{eff}}^2\right)^i{}_j = \begin{pmatrix} \left(m_{\text{eff}}^2\right)^{i'}{}_{j'} & \left(m_{\text{eff}}^2\right)^{i'}{}_s \\ \left(m_{\text{eff}}^2\right)^s{}_{j'} & \left(m_{\text{eff}}^2\right)^s{}_s \end{pmatrix}, \qquad i', j' = 1{,}2, \tag{3.52}$$

where primed indices run over apices and subscripts 1 and 2 only, with:

$$\left(m_{\text{eff}}^2\right)^{i'}{}_{j'} = \frac{a_s \langle\tau_s\rangle \xi |W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}} \langle\tau_2\rangle^3} \begin{pmatrix} -3\left(1 - \dfrac{7}{4}\sigma\right) & -6\,\dfrac{\langle\tau_1\rangle}{\langle\tau_2\rangle}\left(1 - \dfrac{7}{4}\sigma\right) \\ -3\,\dfrac{\langle\tau_2\rangle}{\langle\tau_1\rangle}\left(1 - \dfrac{7}{4}\sigma\right) & -6\left(1 - \dfrac{7}{4}\sigma\right) \end{pmatrix},$$

$$\left(m_{\text{eff}}^2\right)^{i'}{}_s = \frac{a_s \langle\tau_s\rangle \xi |W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}} \langle\tau_2\rangle^3} \begin{pmatrix} 6a_s \langle\tau_1\rangle \left(1 - \dfrac{5}{4}\sigma + \sigma^2\right) \\ 6a_s \langle\tau_2\rangle \left(1 - \dfrac{5}{4}\sigma + \sigma^2\right) \end{pmatrix},$$

$$\left(m_{\text{eff}}^2\right)^s{}_{j'} = \frac{a_s \langle\tau_s\rangle \xi |W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}} \langle\tau_2\rangle^3} \left(-2\,\frac{\langle\tau_2\rangle}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_s\rangle^{\frac{1}{2}}}\left(1 - \frac{5}{4}\sigma + \frac{1}{4}\sigma^2\right),\ -4\,\frac{\langle\tau_1\rangle^{\frac{1}{2}}}{\langle\tau_2\rangle^{\frac{1}{2}}}\left(1 - \frac{5}{4}\sigma + \frac{1}{4}\sigma^2\right)\right),$$

$$\left(m_{\text{eff}}^2\right)^s{}_s = \frac{a_s \langle\tau_s\rangle \xi |W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}} \langle\tau_2\rangle^3} \left(4\frac{a_s \langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}{\langle\tau_s\rangle^{\frac{1}{2}}}\left(1 - \frac{3}{4}\sigma + \frac{3}{8}\sigma^2 + \frac{1}{8}\sigma^3\right)\right).$$

This way of writing is coherent with Model III being a proper generalisation of Model II. Comparing the expression (3.52) to its analogue (3.26) it is evident that the row and the column b are splitted into rows and columns 1 and 2 in a natural way, indeed the 'components' of matrix (3.52) can be seen as direct 'extensions' of the elements of matrix (3.26) respectively:

$$\left(m_{\text{eff}}^2\right)^b{}_b \rightsquigarrow \left(m_{\text{eff}}^2\right)^{i'}{}_{j'}, \qquad \left(m_{\text{eff}}^2\right)^b{}_s \rightsquigarrow \left(m_{\text{eff}}^2\right)^{i'}{}_s, \qquad \left(m_{\text{eff}}^2\right)^s{}_b \rightsquigarrow \left(m_{\text{eff}}^2\right)^s{}_{j'}$$

Of course the element $\left(m^2_{\text{eff}}\right)^s{}_s$ is the same provided the substitution $\langle\tau_b\rangle^{3/2} = \langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$. This property implies the fact that rows 1 and 2 as well as columns 1 and 2 are linearly dependent, thus entailing the existence of a vanishing eigenvalue, i.e. a vanishing mass, which is the physical manifestation of a flat direction in the scalar potential.

Now, calling ζ, χ and ϕ the canonically normalised τ-fields, according to Subsection 2.4, their masses at leading order (see Section A.2.2 in the Appendix for detailed expressions) are:

$$m^2_\zeta = 0, \tag{3.53}$$

$$m^2_\chi \simeq \frac{81}{8}\frac{\xi\,|W_0|^2}{a_s\,\langle\tau_s\rangle\,\langle\tau_1\rangle^{3/2}\,\langle\tau_2\rangle^3}, \tag{3.54}$$

$$m^2_\phi \simeq \frac{4\;a_s^2\,\langle\tau_s\rangle^{1/2}\,\xi\,|W_0|^2}{\langle\tau_1\rangle\,\langle\tau_2\rangle^2}. \tag{3.55}$$

As expected, there are now three fields of which one, the newly introduced ζ, is massless, while the other two, χ and ϕ, obtain the same masses as in Model II. Therefore all of the considerations on m^2_χ and m^2_ϕ are valid for this situation too. In particular, since the gravitino mass $m_{3/2} = \left\langle e^{K/2}\,|W|\right\rangle$ is given at leading order by, in physical units:

$$m_{3/2} \simeq \frac{|W_0|}{\langle\tau_1\rangle^{\frac{1}{2}}\,\langle\tau_2\rangle}\,M_P,$$

and, thanks to (3.48), is such that:

$$\ln\frac{M_P}{m_{3/2}} \simeq a_s\,\langle\tau_s\rangle\,,$$

the orders of magnitude of the masses of canonically normalised fields are, in physical units:

$$m_\zeta = 0, \tag{3.56}$$

$$m_\chi \simeq \frac{9\sqrt{2}}{4}\left(\frac{\xi}{\langle\mathcal{V}\rangle}\right)^{\frac{1}{2}}\left(\ln\frac{M_P}{m_{3/2}}\right)^{-\frac{1}{2}} m_{3/2}, \tag{3.57}$$

$$m_\phi \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2}. \tag{3.58}$$

Following the well-known reasoning, the change of basis which allows to diagonalise the moduli Lagrangian is carried by the matrix whose columns are the properly normalised eigenvectors of $\left(m^2_{\text{eff}}\right)^i{}_j$. Then, at leading order and in physical units, the defining relation of fields ζ, χ and ϕ is accomplished by:

$$\begin{pmatrix} \delta\tau_1 \\ \delta\tau_2 \\ \delta\tau_s \end{pmatrix} \simeq \frac{\sqrt{2}}{2} \begin{pmatrix} -\frac{2\sqrt{6}}{3}\langle\tau_1\rangle & \frac{2\sqrt{3}}{3}\langle\tau_1\rangle & \sqrt{6}\,\frac{\langle\tau_1\rangle^{3/4}\langle\tau_s\rangle^{3/4}}{\langle\tau_2\rangle^{1/2}} \\ \frac{\sqrt{6}}{3}\langle\tau_2\rangle & \frac{2\sqrt{3}}{3}\langle\tau_2\rangle & \sqrt{6}\,\frac{\langle\tau_2\rangle^{1/2}\langle\tau_s\rangle^{3/4}}{\langle\tau_1\rangle^{1/4}} \\ 0 & \frac{\sqrt{3}}{a_s} & \frac{2\sqrt{6}}{3}\langle\tau_1\rangle^{1/4}\langle\tau_2\rangle^{1/2}\langle\tau_s\rangle^{1/4} \end{pmatrix} \begin{pmatrix} \zeta/M_P \\ \chi/M_P \\ \phi/M_P \end{pmatrix}. \tag{3.59}$$

This definition is enlightening as it shows how the original Kähler fields fluctuations $\delta\tau_1$, $\delta\tau_2$ and $\delta\tau_s$ combine to give the new fields. As a matter of fact, roughly speaking, $\delta\tau_1$ and $\delta\tau_2$ mix themselves and give the massless field ζ and the light field χ:

$$\begin{cases} \delta\tau_1 & \simeq -\frac{2\sqrt{3}}{3}\langle\tau_1\rangle\frac{\zeta}{M_P} + \frac{\sqrt{6}}{3}\langle\tau_2\rangle\frac{\chi}{M_P}, \\ \delta\tau_2 & \simeq +\frac{\sqrt{3}}{3}\langle\tau_1\rangle\frac{\zeta}{M_P} + \frac{\sqrt{6}}{3}\langle\tau_2\rangle\frac{\chi}{M_P}. \end{cases}$$

These features are coherent with $\delta\tau_b$ essentially projected into χ in Model II, indeed the present situation is analogous but for the presence of an additive degree of freedom represented by the massless field ζ. Finally, $\delta\tau_s$ is substantially aligned with the heavy field ϕ:

$$\delta\tau_s \simeq \frac{2\sqrt{3}}{3}\langle\tau_1\rangle^{1/4}\langle\tau_2\rangle^{1/2}\langle\tau_s\rangle^{1/4}\frac{\phi}{M_P}, \tag{3.60}$$

again in accordance with Model II.

As concerns axionic fields, the situation is once again similar to the previous one. Of course the scalar potential (3.47) satisfies conditions cited in Section 2.4, then the kinetic axionic Lagrangian reads:

$$\mathcal{L}_{\text{kin}}^{\text{axion}} = \langle K_{ij}\rangle\,\partial_\mu\left(\delta\psi^i\right)\partial^\mu\left(\delta\psi^j\right) - \left(M'^2\right)_i\left(\delta\psi^i\right)^2,$$

with one only unnormalised axionic mass, which is at leading order:

$$\left(M'^2\right)_s \simeq \frac{3a_s^2\xi\,|W_0|^2}{2\langle\tau_1\rangle^{3/2}\langle\tau_2\rangle^3}.$$

Then the redefinition of the axionic fields as in the leading order expressions for their corresponding real parts allows for a simple diagonalisation and canonical normalisation of the Lagrangian, up to some irrelevant couplings:

$$\begin{cases} \delta\psi_1 & \simeq -\dfrac{2\sqrt{3}}{3}\left\langle\tau_1\right\rangle\dfrac{\theta_\zeta}{M_P}+\dfrac{\sqrt{6}}{3}\left\langle\tau_2\right\rangle\dfrac{\theta_\chi}{M_P}, \\ \delta\psi_2 & \simeq +\dfrac{\sqrt{3}}{3}\left\langle\tau_1\right\rangle\dfrac{\theta_\zeta}{M_P}+\dfrac{\sqrt{6}}{3}\left\langle\tau_2\right\rangle\dfrac{\theta_\chi}{M_P}, \\ \delta\psi_s & \simeq +\dfrac{2\sqrt{3}}{3}\left\langle\tau_1\right\rangle^{1/4}\left\langle\tau_2\right\rangle^{1/2}\left\langle\tau_s\right\rangle^{1/4}\dfrac{\theta_\phi}{M_P}, \end{cases} \tag{3.61}$$

as can be argued by direct substitution. Of course the same features on the projection between the old and the new fields hold true as for the real parts of Kähler moduli. Canonically normalised axionic masses at leading order turn out to be:

$$m^2_{\theta_\zeta} = 0, \tag{3.62}$$

$$m^2_{\theta_\chi} = 0, \tag{3.63}$$

$$m^2_{\theta_\phi} \simeq \frac{4\, a_s^2 \left\langle\tau_s\right\rangle^{1/2} \xi \left|W_0\right|^2}{\left\langle\tau_1\right\rangle\left\langle\tau_2\right\rangle^2}, \tag{3.64}$$

which means essentially that the axionic partners of ζ and χ are massless while θ_ϕ has a mass of the same order of magnitude as ϕ. In particular, in physical units they can be written as:

$$m_{\theta_\zeta} = 0, \tag{3.65}$$

$$m_{\theta_\chi} = 0, \tag{3.66}$$

$$m_{\theta_\phi} \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2}. \tag{3.67}$$

Of course the fact that the axions θ_ζ and θ_χ are massless does not give rise to a problem since in this model they are non-interacting as they appear neither in the Kähler potential nor in the superpotential.

To conclude, the kinetic Lagrangian describing canonically normalised Kähler moduli descending from the Kähler potential (3.42) and superpotential (3.43) reads:

$$\begin{aligned} \mathcal{L}^{\text{Kähler}}_{\text{kin}} = &- E_\Lambda + \frac{1}{2}\partial_\mu\,\zeta\,\partial^\mu\,\zeta + \frac{1}{2}\partial_\mu\,\chi\,\partial^\mu\,\chi + \frac{1}{2}\partial_\mu\,\phi\,\partial^\mu\,\phi + \frac{1}{2}\partial_\mu\,\theta_\zeta\,\partial^\mu\,\theta_\zeta \\ &+ \frac{1}{2}\partial_\mu\,\theta_\chi\,\partial^\mu\,\theta_\chi + \frac{1}{2}\partial_\mu\,\theta_\phi\,\partial^\mu\,\theta_\phi - \frac{1}{2}m^2_\chi\chi^2 - \frac{1}{2}m^2_\phi\phi^2 - \frac{1}{2}m^2_{\theta_\phi}\theta^2_\phi. \end{aligned} \tag{3.68}$$

A simple argument is sufficient to conclude that such a Lagrangian cannot be suitable in order to modelise the hidden sector in the Supersymmetric Standard Model. Indeed, interacting massless fields like ζ are forbidden because then they should give rise to interactions for which there is no reason why they have never been observed so far.

The next step must then be the search for possible corrections neglected so far that lift the scalar potential V^{eff}_F in such a way as to make the field ζ massive.

3.5 Model III: Subleading Corrections

In order to study possible subleading corrections to the scalar potential V_F that make all Kähler moduli massive, a crucial observation is that there exists a change of variables that greatly simplifies the calculations.

In a generalisation of Model III, it is reasonable to expect that the Kähler potential K and the superpotential W receive furhter corrections as:

$$K_{\rm tot} = K + \delta K^{\rm p}, \tag{3.69}$$

$$W_{\rm tot} = W + \delta W^{\rm np}, \tag{3.70}$$

in such a way as to get a corrected total scalar potential of the kind:

$$V_F^{\rm tot} = V_F + \delta V_F, \tag{3.71}$$

where V_F is the original scalar potential and δV_F is a subleading correction, i.e. with:

$$\left|V_F^{\rm tot}\right| \sim |V_F| \gg |\delta V_F| \,.$$

The notation adpoted is chosen in order to keep it evident that K and W are not unperturbed potentials, but they are the contributions which have already been studied in Model III and that are part of the total potentials $K_{\rm tot}$ and $W_{\rm tot}$ as the main contributions to the total scalar potential.

In order to minimise $V_F^{\rm tot}$, the great difference between the orders of magnitude of its two contributions suggests that it is possible to first consider the leading contribution V_F only and then to study the contribution of δV_F as a residual correction to V_F which slightly lifts it. A crucial observation offers the possibility to make the calculations quite easy.

Indeed, after the usual approximation that leads to the effective form of the scalar potential $V_F^{\rm eff}$ and eventually considering further conditions concerning the newly introduced corrections to get the effective contribution $\delta V_F^{\rm eff}$, it is possible to define the effective total scalar potential:

$$V_F^{\rm tot,\,eff} = V_F^{\rm eff} + \delta V_F^{\rm eff}. \tag{3.72}$$

Then, defining the approximate volume:

$$\mathcal{V}' \equiv \tau_1^{1/2}\tau_2, \tag{3.73}$$

and changing the former triplet[3] of variables, (τ_1, τ_2, τ_s), to the new one, $(\tau_1, \mathcal{V}', \tau_s)$, it becomes clear that the scalar potential $V_F^{\rm eff}$ can be written as a function of only $\mathcal{V}'$ and τ_s:

$$V_F^{\rm eff} \equiv \frac{8}{3}\frac{a_s^2\tau_s^{1/2}}{\mathcal{V}'}\left|A_s\right|^2 e^{-2a_s\tau_s} - 4\frac{a_s\tau_s}{\mathcal{V}'^2}\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s} + \frac{3}{2}\frac{\xi}{\mathcal{V}'^3}\left|W_0\right|^2, \tag{3.74}$$

[3] Of course there is an axionic dependence, which here is left implicit.

in such a way that the vacuum expectation values for $\mathcal{V}'$, as a substitute of the product $\tau_1^{1/2}\tau_2$, and τ_s are at leading order:

$$\langle \mathcal{V}' \rangle \simeq \frac{3}{4}\frac{\langle \tau_s \rangle^{\frac{1}{2}}}{a_s}\frac{|W_0|}{|A_s|}\, e^{a_s\langle \tau_s \rangle}\frac{1-\dfrac{1}{a_s\langle \tau_s \rangle}}{1-\dfrac{1}{4a_s\langle \tau_s \rangle}}, \tag{3.75}$$

$$\langle \tau_s \rangle^{3/2} \simeq \xi \frac{\left(1-\dfrac{1}{4a_s\langle \tau_s \rangle}\right)^2}{\left(1-\dfrac{1}{a_s\langle \tau_s \rangle}\right)}. \tag{3.76}$$

It is worthwhile to specify that if δV_F^{eff} does not depend on τ_s, then the expression (3.75) is exact since it comes from the condition of vanishing derivative of the scalar potential with respect to the blow-up cycle.
The change of variables allows to take advantage of the solutions (3.75) and (3.76) in a very simple way. As a matter of fact, now it is possible to take into account the correction δV_F by considering the whole potential V_F^{tot} computed in the vacuum expectation values $\langle \mathcal{V}' \rangle$ and $\langle \tau_s \rangle$:

$$V_F^{\text{tot, eff}}(\tau_1, \langle \mathcal{V}' \rangle, \langle \tau_s \rangle) = \left\langle V_F^{\text{eff}} \right\rangle + \delta\, \mathcal{V}_F^{\text{eff}}(\tau_1, \langle \mathcal{V}' \rangle, \langle \tau_s \rangle), \tag{3.77}$$

and to simply minimise it with respect to τ_1. Once it is evaluated in the vacuum expectation value $\langle \tau_1 \rangle$, the contribution δV_F definitely allows the computation of Kähler moduli masses.

3.5.1 Non-perturbative Corrections

The most natural attempt to modelise a suitable hidden sector by keeping the general features depicted in Model III consists in taking into account more detailed non-perturbative corrections to the superpotential (3.43). Indeed the scalar potential which has been studied so far derives from:

$$W_{\text{tot}} = W_0 + \sum_{i=1,2,s} A_i\, e^{-a_i(\tau_i + i\psi_i)},$$

that is commonly approximated as:

$$W = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)},$$

because of the hierarchy between fields values around the ground state, which evidently entails the predominance of corrections depending on the blow-up cycle.

Nevertheless, the field ζ is substantially a combination of fluctuations around the ground state of fields τ_1 and τ_2, then it is presumable that non-perturbative corrections to the superpotential as:

$$\delta W^{\mathrm{np}} = \sum_{i'=1,2} A_{i'}\, e^{-a_{i'}(\tau_{i'}+i\psi_{i'})},$$

under the obvious conditions:

$$|A_{i'}|\, e^{-a_{i'}\tau_{i'}} \overset{\tau_{i'}\gg\tau_s}{\ll} |A_s|\, e^{-a_s\tau_s}, \qquad i' = 1{,}2,$$

lift the scalar potential in such a way as to make the field ζ massive.

For the sake of simplicity, the following calculations involve non-perturbative corrections depending only on τ_1, neglecting once again those depending on τ_2. This choice can be motivated somehow by assuming that the field τ_1 takes a vacuum expectation value smaller than that of τ_2, thus entailing, thanks to the exponential damping:

$$|A_2|\, e^{-a_2\tau_2} \overset{\tau_2>\tau_1}{\ll} |A_1|\, e^{-a_1\tau_1}.$$

Anyway, beyond this reasoning, the simpler model involving τ_1 will be enough to show how these kinds of superpotentials without any else modifications to Model III give unacceptable results. Stated differently, the following description can be seen as an instructive example to make it clear that non-perturbative corrections alone are not sufficient to properly improve and complete Model III.

To sum up, the Kähler potential and the superpotential under exam are:

$$K_{\mathrm{tot}} = -2\,\ln\left(\tau_1^{1/2}\tau_2 - \tau_s^{3/2} + \xi\right), \tag{3.78}$$

$$W_{\mathrm{tot}} = W_0 + A_s e^{-a_s(\tau_s+i\psi_s)} + A_1 e^{-a_1(\tau_1+i\psi_1)}, \tag{3.79}$$

with the same notation and conventions as before, where of course a_1 is a positive real number and A_1 is a complex constant commonly expressed as:

$$A_1 = |A_1|\, e^{i\alpha_1},$$

with the further obvious condition:

$$a_1\tau_1 \gg a_s\tau_s. \tag{3.80}$$

Minimisation

It is possible to compute the scalar potential explicitly and it is convenient to write it as:

$$V_F^{\mathrm{tot}} = V_F + \delta V_F,$$

where V_F is the scalar potential which appears in Model III, i.e. discarding the newly introduced corrections, while δV_F is the contribution coming from these ones and is such that:

$$\left|V_F^{\text{tot}}\right| \sim |V_F| \gg |\delta V_F| .$$

Under conditions (3.44), (3.45) and (3.80) it is possible to identify the effective contributions to the total scalar potential:

$$V_F^{\text{tot, eff}} = V_F^{\text{eff}} + \delta V_F^{\text{eff}},$$

where V_F^{eff} is the well known potential in (3.47), including the minimisation with respect to ψ_s, while:

$$\delta V_F^{\text{eff}} = \frac{4a_1}{\tau_2^2}\, |A_1|\, e^{-a_1\tau_1} \left[a_1\tau_1\, |A_1|\, e^{-a_1\tau_1} - \left(|W_0| + 2a_s\tau_s\, |A_s|\, e^{-a_s\tau_s}\right) \right], \tag{3.81}$$

after a typical minimisation with respect to the axion ψ_1 in:

$$\langle\psi_1\rangle = \frac{\alpha_1 - \theta}{a_1} + (2n+1)\,\frac{\pi}{a_1}, \quad n \in \mathbb{Z}, \tag{3.82}$$

Expression 3.81 requires particular attention: as a matter of fact, since τ_1 is larger than τ_s, the first term in brackets is expected to be negligible with respect to the other one unless the constant $|A_1|$ is exponentially large. This fact suggests that the vacuum expectation values coming from this model will probably require some unnatural features for some of its parameters. Otherwise the first addendum should be neglected, thus entailing the lack of a stabilisation for the modulus.

Anyway, following the discussion at the beginning of Subsection 3.5, it is convenient to find the minimum of $V_F^{\text{tot, eff}}$ by first considering the leading contribution V_F^{eff} only and then to consider the slight lifting by δV_F^{eff}.
Changing the variables from (τ_1, τ_2, τ_s) to $(\tau_1, \mathcal{V}', \tau_s)$, V_F^{eff} gives $\mathcal{V}'$ and τ_s the leading order vacuum expectation values (3.75) and (3.76) respectively. Then, setting the variables $\mathcal{V}'$ and τ_s in these configurations, the total scalar potential takes the effective form:

$$V_F^{\text{tot, eff}}(\tau_1, \langle\mathcal{V}'\rangle, \langle\tau_s\rangle) = \left\langle V_F^{\text{eff}}\right\rangle + \delta V_F^{\text{eff}}(\tau_1, \langle\mathcal{V}'\rangle), \tag{3.83}$$

where, discarding the term proportional to $e^{-a_1\tau_1}e^{-a_s\langle\tau_s\rangle}$:

$$\delta V_F^{\text{eff}} \simeq \frac{4a_1\tau_1\, |A_1|\, e^{-a_1\tau_1}}{\langle\mathcal{V}'\rangle^2} \left[a_1\tau_1\, |A_1|\, e^{-a_1\tau_1} - |W_0| \right]. \tag{3.84}$$

Then, it is immediate to conclude that the vacuum expectation value $\langle\tau_1\rangle$ of the field τ_1 is such that:

$$a_1\,\langle\tau_1\rangle\, e^{-a_1\langle\tau_1\rangle} = \frac{|W_0|}{2\,|A_1|}. \tag{3.85}$$

As expected, this result allows to conclude that the introduction of perturbative corrections only to the superpotential is not satisfactory. In fact, (3.85) shows clearly that the latter would require an exponentially large value of $|A_1|$, which would mean again a lack of naturalness in the model. Moreover it seems evident that the introduction of a further non-perturbative correction to the superpotential depending on τ_2 cannot solve this problem.

Then, the discussion of the present Section can only help in looking for the presence of different kinds of corrections. Fortunately they actually do exist, as explained in the following.

3.5.2 Perturbative Corrections

It is now well known that the Kähler potential K receives corrections from string loops besides α'-corrections, although unfortunately their exact form for a generic Calabi-Yau manifold compactification is unknown. Nevertheless, it it possible to argue that their general structure for Model III can be expressed as [23] [24]:

$$\delta K^{\mathrm{p}} = \delta K^{\mathrm{KK}} + \delta K^{\mathrm{W}}, \tag{3.86}$$

where δK^{KK} are called 'Kaluza-Klein string loop corrections':

$$\delta K^{\mathrm{KK}} = \frac{g_S\, C_1}{\tau_1} + \frac{g_S\, C_2}{\tau_2}, \tag{3.87}$$

at leading order, while δK^{W} are called 'winding string loop corrections':

$$\delta K^{\mathrm{W}} = \frac{C_W}{\tau_1 \tau_2}, \tag{3.88}$$

again at leading order, with C_1, C_2 and C_W real numbers depending on the complex structure moduli, i.e. real numerical constants within the context of low-energy Kähler moduli stabilisation, which can be generally taken to be around unity.

Expressions (3.87) and (3.88) require to pay attention. Indeed, in the hypoteshis where very roughly $\tau_1 \sim \tau_2$:

- first of all it is evident that, in the absence of particular conditions on the constants C_1, C_2 and C_W, Kaluza-Klein corrections are larger than winding ones in the large volume scenario:
$$\left|\delta K^{\mathrm{KK}}\right| \gg \left|\delta K^{\mathrm{W}}\right|;$$
- next, the most striking feature is actually the fact that Kaluza-Klein corrections are more significant than α'-ones, which at leading order can be written as:
$$\delta K^{\alpha'} = -\frac{\xi}{\tau_1^{1/2} \tau_2},$$

in such a way that:

$$\left|\delta K^{\mathrm{KK}}\right| \gg \left|\delta K^{\alpha'}\right|.$$

This relationship in turn seems to entail the inaccuracy of Model III because, before considering α'-corrections and the following problems, Kaluza-Klein should have been taken into account.

In the end, according to this reasoning, the most natural way to proceed is the one in which perturbative corrections are progressively introduced, if needed, in the order: δK^{KK}, then $\delta K^{\alpha'}$ and in the end δK^{W}.

Actually, a result referred to as 'Extended No-Scale Structure' [24] shows that Kaluza-Klein corrections to a tree-level Kähler potential lead to a vanishing leading order correction to the scalar potential. More precisely, in the context of Calabi-Yau manifold compactification leading to $N = 1, D = 1$ supergravity its general statement can be summarised as follows:

"Let the Kähler potential and superpotential of a system be:

$$K = K_0 + \delta K,$$
$$W = W_0,$$

where K_0 is the tree-level Kähler potential, δK is a loop correction and W_0 the well-known constant superpotential. If the correction δK to K_0 is a homogeneous function in the 2-cycle volumes of degree n, then the scalar potential at leading order is:

$$V_F \simeq -\frac{n\,(n+2)}{4}\frac{|W_0|^2}{\mathcal{V}^2}\delta K, \tag{3.89}$$

assuming that K_0 is such as to give rise to the usual No-Scale Structure."

In the case under exam, recalling the definitions (3.38) of the fields τ_1 and τ_2 as functions of the 2-cycles t_1 and t_2, Kaluza-Klein and winding corrections are homogeneous functions of the 2-cycles of orders $n^{\mathrm{KK}} = -2$ and $n^{\mathrm{W}} = -4$ respectively:

$$\delta K^{\mathrm{KK}} = \frac{g_S\,C_1}{\tau_1} + \frac{g_S\,C_2}{\tau_2} = \frac{C_1'}{t_2^2} + \frac{C_2'}{t_1 t_2}, \qquad n^{\mathrm{KK}} = -2,$$

$$\delta K^{\mathrm{W}} = \frac{C_W}{\tau_1\tau_2} = \frac{C_W'}{t_1 t_2^3}, \qquad n^{\mathrm{W}} = -4,$$

conveniently fixing the constants C_1', C_2' and C_W'.

Therefore, in a model with only Kaluza-Klein corrections, their first non-vanishing contribution is expected to be scaled somehow by the factors $g_S^2 C_1^2/\tau_1^2$ and $g_S^2 C_2^2/\tau_2^2$, i.e. turns out to be subleading with respect to the contribution coming from α' corrections, which is scaled by factors as $\xi/\tau_1^{1/2}\tau_2$, and roughly of the same order as the contribution descending from winding corrections, which is again weighted by $C_W/\tau_1\tau_2$.

This fact means that for the sake of symplicty it is reasonable to study at first α' corrections, while at a deeper level of accuracy both Kaluza-Klein and winding corrections must be taken into account.

In the end, the model which correctly generalises Model III is described by the Kähler potential and the superpotential:

$$K_{\text{tot}} = -2 \ln \left(\tau_1^{1/2} \tau_2 - \tau_s^{3/2} + \xi \right) + \frac{c_1}{\tau_1} + \frac{c_2}{\tau_2} + \frac{C_W}{\tau_1 \tau_2}, \tag{3.90}$$

$$W_{\text{tot}} = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)}, \tag{3.91}$$

with the same notation and assumptions as in Model III, where the effective parameters c_1 and c_2 have been defined as $c_1 = g_S\, C_1$ and $c_2 = g_S\, C_2$ for the sake of simplicity, plus the evident additional conditions:

$$\left| \frac{c_1}{\tau_1} \right| \sim \left| \frac{c_2}{\tau_2} \right| \ll 1, \qquad \left| \frac{C_W}{\tau_1 \tau_2} \right| \ll 1 \tag{3.92}$$

and more specifically the hierarchy in the large volume scenario:

$$\left| \frac{c_1}{\tau_1} \right| \sim \left| \frac{c_2}{\tau_2} \right| \gg \frac{\tau_s^{3/2}}{\tau_1^{1/2} \tau_2} \sim \frac{\xi}{\tau_1^{1/2} \tau_2} \gg \left| \frac{C_W}{\tau_1 \tau_2} \right| \sim \frac{c_1^2}{\tau_1^2} \sim \frac{c_2^2}{\tau_2^2}. \tag{3.93}$$

Actually, these several relationships are only indicative and disregard the (generally) slight modifications which would emerge by considering the effects of a small string coupling g_S around $g_S \sim 10^{-1}$, but will be assumed for a linear description without loss of generality because:

- if the compactification volume is huge, these relationships are generally true at the needed level of approximation;

- if the compactification volume is large but not enormous, some anisotropies between τ_1 and τ_2 can emerge and must be treated carefully. Nevertheless, in the cases of interest, the slight changes in the above hierarchies do not affect the relevant parts of the discussion, as can be observed by direct inspection following the general outline discussion in Chapter 5.

Minimisation

The total scalar potential can be computed explicitly and it can be expressed in the familiar form:

$$V_F^{\text{tot}} = V_F + \delta V_F,$$

where V_F is the exact scalar potential of Model III, i.e. without string loop corrections, and δV_F is the contribution coming from these ones. This fact means that computations confirm the discussion based on the Extended No-Scale Structure as string loop corrections turn out to introduce only subleading corrections beacuse:

$$\left|V_F^{\text{tot}}\right| \sim |V_F| \gg |\delta V_F| .$$

Under conditions (3.44), (3.45), (3.92) and (3.93) it is then worthwhile to identify the effective contributions to the total scalar potential as:

$$V_F^{\text{tot, eff}} = V_F^{\text{eff}} + \delta V_F^{\text{eff}},$$

where of course V_F^{eff} is the well known potential of Model III expressed in (3.47), including the minimisation with respect to ψ_s, while:

$$\delta V_F^{\text{eff}} = \frac{1}{2}\frac{|W_0|^2}{\tau_1\tau_2^2}\left(2\frac{c_1^2}{\tau_1^2} - 4\frac{C_W}{\tau_1\tau_2} + \frac{c_2^2}{\tau_2^2}\right). \tag{3.94}$$

According to the arguments described in Subsection 3.5, in order to minimise $V_F^{\text{tot, eff}}$ it is convenient to first change the variables as $(\tau_1, \tau_2, \tau_s) \mapsto (\tau_1, \mathcal{V}' = \tau_1^{1/2}\tau_2, \tau_s)$ and to individuate the minimum of the leading contribution V_F^{eff} only, getting the leading order vacuum expectation values (3.75) and (3.76) for $\mathcal{V}'$ and τ_s respectively, and then to study the effects caused by the slight lifting of δV_F^{eff} as the total scalar potential takes the effective form:

$$V_F^{\text{tot, eff}}(\tau_1, \langle\mathcal{V}'\rangle, \langle\tau_s\rangle) = \left\langle V_F^{\text{eff}}\right\rangle + \delta\,\mathcal{V}_F^{\text{eff}}(\tau_1, \langle\mathcal{V}'\rangle), \tag{3.95}$$

where:

$$\delta V_F^{\text{eff}} = \frac{1}{2}\frac{|W_0|^2}{\langle\mathcal{V}'\rangle^2}\left(2\frac{c_1^2}{\tau_1^2} - 4\frac{C_W}{\tau_1^{1/2}\langle\mathcal{V}'\rangle} + \frac{c_2^2\tau_1}{\langle\mathcal{V}'\rangle^2}\right). \tag{3.96}$$

Its minimum is readily individuated in the point $\langle\tau_1\rangle$ such that:

$$\langle\tau_1\rangle^{3/2} = \frac{C_W}{c_2^2}\left[\frac{|C_W|}{C_W}\left(1 + 4\frac{c_1^2c_2^2}{C_W^2}\right)^{1/2} - 1\right]\langle\mathcal{V}'\rangle, \tag{3.97}$$

thus confirming the fact that $\langle\tau_1\rangle$, $\langle\tau_2\rangle$ and $\langle\mathcal{V}\rangle^{2/3}$ are roughly of the same order of magnitude:

$$\langle\tau_1\rangle \sim \langle\tau_2\rangle \sim \langle\mathcal{V}\rangle^{2/3}.$$

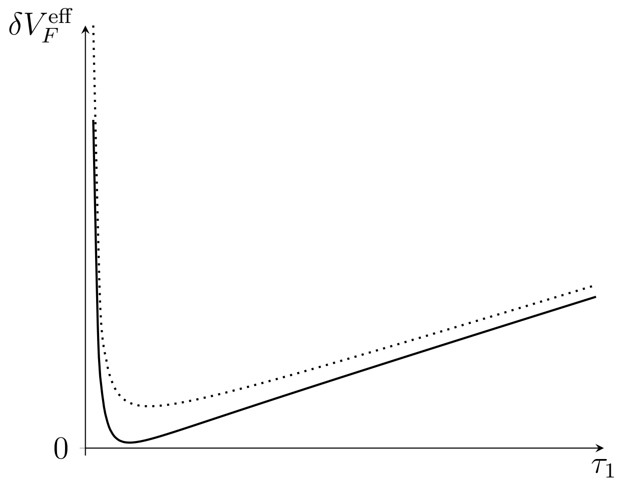

Figure 3.4. Plots of the correction to the scalar potential $\delta V_F^{\text{eff}} = \delta V_F^{\text{eff}}(\tau_1)$, solid and dotted lines indicating positive and negative C_W respectively.

Moduli Masses

According to Subsection 2.4, the kinetic term for Kähler moduli unnormalised Lagrangian is now:

$$\begin{aligned}\mathcal{L}_{\text{kin}}^{\text{Kähler}} =& \left\langle K_{ij}^{\text{tot}} \right\rangle \partial_\mu \left(\delta\tau^i\right) \partial^\mu \left(\delta\tau^j\right) + \left\langle K_{ij}^{\text{tot}} \right\rangle \partial_\mu \left(\delta\psi^i\right) \partial^\mu \left(\delta\psi^j\right) \\ &- \left\langle V_F^{\text{tot}} \right\rangle - \left(M_{\text{tot}}^2\right)_{ij} \delta\tau^i \delta\tau^j - \left(M'^2_{\text{tot}}\right)_i \left(\delta\psi^i\right)^2 ,\end{aligned} \tag{3.98}$$

with of course:

$$\left(M_{\text{tot}}^2\right)_{ij} \equiv \frac{1}{2} \left\langle \frac{\partial^2 V_F^{\text{tot}}}{\partial \tau^i \, \partial \tau^j} \right\rangle , \qquad \left(M'^2_{\text{tot}}\right)_i \equiv \frac{1}{2} \left\langle \frac{\partial^2 V_F^{\text{tot}}}{\partial \left(\psi^i\right)^2} \right\rangle ,$$

then, the normalised mass matrix can be defined as:

$$\left(m_{\text{tot}}^2\right)^i{}_j = \left\langle K_{\text{tot}}^{il} \right\rangle \left(M_{\text{tot}}^2\right)_{lj} . \tag{3.99}$$

Nevertheless, considering the effective form of the interesting quantities, the diagonalisation of the Lagrangian is immediate as the axionic dependence is the same as before and most importantly the normalised mass matrix receives corrections from the introduction of string loop corrections which affect in a sensible way only the field ζ, which is massless in Model III. Indeed it is possible to roughly write:

$$\left\langle K_{\text{tot}}^{ij} \right\rangle = \left\langle K^{ij} \right\rangle \left[1 + O\left(\frac{c_1}{\langle\tau_1\rangle}, \frac{c_2}{\langle\tau_2\rangle}\right)\right] ,$$

$$\left(M_{\text{tot}}^2\right)_{ij} = \left(M^2\right)_{ij} \left[1 + \frac{\langle\tau_1\rangle^{\frac{1}{2}} \langle\tau_2\rangle}{\xi} O\left(\frac{c_1^2}{\langle\tau_1\rangle^2}, \frac{c_2^2}{\langle\tau_2\rangle^2}, \frac{C_W}{\langle\tau_1\rangle \langle\tau_2\rangle}\right)\right] ,$$

in such a way that, very roughly:

$$\left(m^2_{\text{tot}}\right)^i_{\ j} = \left(m^2\right)^i_{\ j}\left[1 + \frac{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}{\xi}\, O\!\left(\frac{c_1^2}{\langle\tau_1\rangle^2}, \frac{c_2^2}{\langle\tau_2\rangle^2}, \frac{C_W}{\langle\tau_1\rangle\langle\tau_2\rangle}\right)\right].$$

This fact means that the leading order expressions (3.54) and (3.55) of m^2_χ and m^2_ϕ are not affected by string loop corrections. Moreover, the leading order expression of the change of basis (3.59) results to be the same too. Then, the only leading order expression which turns out to be modified is the vanishing mass m^2_ζ of the field ζ, while all of the rest of the diagonalisation is unaltered but at subleading order.

More specifically, the leading order correction $\delta\mathcal{L}^{\text{Kähler}}_{\text{kin}}$ to the normalised scalar Lagrangian (3.68) which makes it result into the Lagrangian (3.98) comes from the expansion of the new contribution $\delta V^{\text{eff}}_F(\tau_1)$ around the vacuum expectation value $\langle\tau_1\rangle$ after fixing $\mathcal{V}'$ and τ_s in their ground states:

$$\delta\mathcal{L}^{\text{Kähler}}_{\text{kin}} \simeq -\left\langle\delta V^{\text{eff}}_F\right\rangle - \frac{1}{2}\frac{\partial^2\,\delta V^{\text{eff}}_F}{\partial\langle\tau_1\rangle^2}\left(\delta\tau_1\right)^2.$$

The vacuum energy density is modified almost irrelevantly as:

$$E^{\text{tot}}_\Lambda = E_\Lambda + \left\langle\delta V^{\text{eff}}_F\right\rangle.$$

As regards masses, since it is possible to write the relative variation of the field $\mathcal{V}'$ as:

$$\frac{\delta\,\mathcal{V}'}{\mathcal{V}'} = \frac{1}{2}\frac{\delta\tau_1}{\tau_1} + \frac{\delta\tau_2}{\tau_2}$$

the following relationship holds true according to (3.59) in vicinity of the minimum of $\mathcal{V}'$ [25]:

$$\frac{\delta\,\mathcal{V}'}{\langle\mathcal{V}'\rangle} = \frac{\sqrt{6}}{2}\chi.$$

Of course the Lagrangian correction must be taken into account under the condition of fixed $\mathcal{V}'$, i.e. of fixed χ, which in turn entails the leading order equality:

$$\delta\tau_1 \simeq -\frac{2\sqrt{3}}{3}\langle\tau_1\rangle\,\zeta.$$

In the end, the mass of the field ζ turns out to be at leading order:

$$m^2_\zeta \simeq \frac{4}{3}\langle\tau_1\rangle^2\frac{\partial^2\,\delta V^{\text{eff}}_F}{\partial\langle\tau_1\rangle^2},$$

i.e.:

$$m^2_\zeta \simeq \frac{8\left|W_0\right|^2}{\langle\tau_1\rangle^{\frac{5}{3}}\langle\tau_2\rangle^{\frac{10}{3}}}\left(\frac{C_W^2}{16c_1}\right)^{\frac{2}{3}}\frac{C_W}{|C_W|}\left[\frac{C_W}{|C_W|}\left(1 + 4\frac{c_1^2c_2^2}{C_W^2}\right)^{1/2} + 1\right]^{\frac{1}{3}}\left(1 + 4\frac{c_1^2c_2^2}{C_W^2}\right)^{1/2}. \tag{3.100}$$

The mass m_ζ is smaller than m_χ and m_ϕ. Indeed, under the assumption in which c_1^2, c_2^2 and C_W are not different by several orders of magnitude, in terms of the gravitino mass in physical units it can be written as:

$$m_\zeta \simeq \left(\frac{\gamma}{\langle\mathcal{V}\rangle}\right)^{\frac{2}{3}} m_{3/2}, \tag{3.101}$$

γ being a factor around unity with resulting from (3.100). Then, m_ζ is reduced with respect to $m_{3/2}$ by a very large factor scaling as $\langle\mathcal{V}\rangle^{2/3}$.

Summing up, the canonically normalised kinetic term for the Kähler moduli Lagrangian in the properly corrected Model III describes four interacting massive particles, ζ, χ, ϕ and θ_ϕ, and two massless non-interacting particles, θ_ζ and θ_χ:

$$\begin{aligned}\mathcal{L}_{\text{kin}}^{\text{Kähler}} =& - E_\Lambda^{\text{tot}} + \frac{1}{2}\,\partial_\mu\,\zeta\,\partial^\mu\,\zeta + \frac{1}{2}\,\partial_\mu\,\chi\,\partial^\mu\,\chi + \frac{1}{2}\,\partial_\mu\,\phi\,\partial^\mu\,\phi\\ &+ \frac{1}{2}\,\partial_\mu\,\theta_\zeta\,\partial^\mu\,\theta_\zeta + \frac{1}{2}\,\partial_\mu\,\theta_\chi\,\partial^\mu\,\theta_\chi + \frac{1}{2}\,\partial_\mu\,\theta_\phi\,\partial^\mu\,\theta_\phi\\ &- \frac{1}{2}m_\zeta^2\zeta^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_{\theta_\phi}^2\theta_\phi^2,\end{aligned} \tag{3.102}$$

thus being suitable for a phenomenologically reliable introduction of soft terms.

Compactification Volume Anisotropy

The fact that Kaluza-Klein parameters are proportional to the string coupling can induce a slight anisotropy in the compactification volume, which has been ignored above, if $g_S \sim 10^{-1}$.

In fact, considering the general condition in which $c_1^2c_2^2 \ll C_W^2$, the vacuum expectation value of the field τ_1 reads, if C_W is positive:

$$\langle\tau_1\rangle^{3/2}\Big|_{C_W>0} \overset{c_1^2c_2^2\ll C_W^2}{\simeq} 2\,\frac{c_1^2}{C_W}\,\langle\mathcal{V}'\rangle\,, \tag{3.103}$$

or, if C_W is negative:

$$\langle\tau_1\rangle^{3/2}\Big|_{C_W<0} \overset{c_1^2c_2^2\ll C_W^2}{\simeq} 2\,\frac{|C_W|}{c_2^2}\,\langle\mathcal{V}'\rangle\,. \tag{3.104}$$

In the latter situations, the mass is simply, respectively:

$$m_\zeta^2\Big|_{C_W>0} \overset{c_1^2c_2^2\ll C_W^2}{\simeq} \frac{4\,|W_0|^2}{\langle\mathcal{V}'\rangle^{\frac{10}{3}}}\left(\frac{C_W^2}{4c_1}\right)^{\frac{2}{3}}, \tag{3.105}$$

and:

$$m_\zeta^2\Big|_{C_W<0} \overset{c_1^2c_2^2\ll C_W^2}{\simeq} \frac{4\,|W_0|^2}{\langle\mathcal{V}'\rangle^{\frac{10}{3}}}\left(\frac{c_2\,|C_W|}{4}\right)^{\frac{2}{3}}. \tag{3.106}$$

When the compactification volume is very large, e.g. a typical $\langle \mathcal{V}' \rangle \sim 10^{14}$, the anisotropy is evidently irrelevant as it does not affect sensibly the conditions (3.92) and (3.93) and the following analysis.
Anyway, for situations where the vacuum expectation value $\langle \mathcal{V}' \rangle$ is large but not huge, e.g. a possible $\langle \mathcal{V}' \rangle \sim 10^4$, the relative difference between $\langle \tau_1 \rangle$ and $\langle \tau_2 \rangle$ requires care in the analysis of perturbative corrections to the scalar potential. Nonetheless, no problems arise in the following modellings.

3.6 Energy Density Uplift

The modelling of a fully realistic scenario requires an approximately Minkowski minimum for a reliable description of present epoch Universe.
The vacuum energy density corresponding to Kähler moduli scalar potential is the leading contribution to the overall Universe vacuum energy density since the arising of Minimal Supersymmetric Standard Model fields is embedded as the manifestation of perturbations to hidden sector Kähler potential and superpotential, as outlined in Chapter 4.

The negative value set by perturbative and non-perturbative effects considered so far is in fact generally uplifted to almost Minkowskian values by several phenomena which affect the action.
They can emerge from various effects such as anti-D3 branes in warped throats [18], magnetised D7 branes [26][27][28], dilaton-dependent non-perturbative effects [29][30] or the effect of D-terms [31].
Anyway, in general, their presence can be accounted for by adding a further contribution V_{up} to the total F-term scalar potential of the kind [32]:

$$V_{\text{up}} = \frac{D}{\mathcal{V}^{\gamma}}, \tag{3.107}$$

where D is a positive term with suitable mass dimension and γ is a parameter taking a value depending on the details of the uplift mechanism, typically in the range $1 \leq \gamma \leq 3$.
In this way, the total scalar potential reads:

$$V = V_F^{\text{tot}} + V_{\text{up}}, \tag{3.108}$$

with the coefficient D taking a suitable value in such a way as to get an approximately Minkowski minimum.

As a final remark, it is fundamental to notice that the uplift potential generally must scale as $\mathcal{V}^{-3}$ in vicinity of the ground state. Then, at leading order it does not affect soft breaking terms, which generally scale with smaller powers of the inverse volume, as emerges for instance in Chapter 4.
The uplift mechanism is quite complicated and goes beyond the scope of this thesis, so it must be kept present that it exists but from now on it will not be described further.

Chapter 4

Soft Terms from Moduli Stabilisation

In a physical model with properly stabilised moduli it is possible to introduce fields belonging to the Standard Model as well as their supersymmetric partners in such a way as to describe a reliable Minimal Supersymmetric Standard Model in the low energy limit of the theory.
This modelling can be realised by adding suitable corrective terms to the moduli Kähler potential and superpotential, thus getting the description of particles and interactions in addition to modular ones. In other words, small fluctuations about the moduli vacuum configuration turn out to generate chiral matter as well as its interactions.
Notably, the introduction of phenomenologically suitable small perturbations to moduli Kähler potential and superpotential turn out to naturally entail the presence of soft terms.
The present Chapter is dedicated to the description of soft breaking term structure within moduli stabilisation and to the computation of soft terms arising - on the basis of Models II and III of Chapter 3 - with D7 branes wrapping Calabi-Yau 4-cycles.

4.1 Structure of Soft Terms

First of all, it is important to fix the notation for the scalar fields which are taken into account in what follows. Hidden sector fields, i.e. the axio-dilaton S, complex structure moduli $U^{\alpha'}$ and Kähler moduli T^i, are denoted collectively as Φ^I, while observable matter fields, i.e. Higgses, squarks and sleptons, are indicated as C^α. In other words, capital latin letters $I, J, ...$ run over moduli and lower case greek letters $\alpha, \beta, ...$ run over matter fields. Globally, italic capital latin letters $\mathcal{I}, \mathcal{J}, ...$ run over both of the two families of fields.
For the sake of simplicity, gauge fields Λ^a are neglected for the moment. However, lower case latin letters $a, b, ...$ are used to label gauge group indices.

In general, the Kähler potential $\mathcal{K}$ and superpotential $\mathcal{W}$ for a Supersymmetric Standard Model where the hidden sector is represented by moduli can be expressed as:

$$\mathcal{K} = K\left(\Phi, \bar{\Phi}\right) + \tilde{K}_{\alpha\bar{\beta}}\left(\Phi, \bar{\Phi}\right) C^{\alpha} \bar{C}^{\bar{\beta}} + \left[\frac{1}{2} Z_{\alpha\beta}\left(\Phi, \bar{\Phi}\right) C^{\alpha} C^{\beta} + \text{h.c.}\right], \tag{4.1}$$

$$\mathcal{W} = W\left(\Phi\right) + \frac{1}{2}\mu_{\alpha\beta}\left(S, U\right) C^{\alpha} C^{\beta} + \frac{1}{6} Y_{\alpha\beta\gamma}\left(S, U\right) C^{\alpha} C^{\beta} C^{\gamma}, \tag{4.2}$$

where K and W are the well known moduli Kähler potential and superpotential, while $\tilde{K}_{\alpha\bar{\beta}}$, $Z_{\alpha\beta}$, $\mu_{\alpha\beta}$ and $Y_{\alpha\beta\gamma}$ are the functions which parametrise observable field fluctuations. $\tilde{K}_{\alpha\bar{\beta}}$ is called Kähler matter metric, while $Y_{\alpha\beta\gamma}$ represent the unnormalised Yukawa couplings and $Z_{\alpha\beta}$ and $\mu_{\alpha\beta}$ are referred to as Z- and μ-parameters.
Typically, these functions come from the string theory which underlies the model under exam - as they depend on hidden sector fields - but are highly difficult or even impossible to compute explicitly. Nevertheless, some situations allow their determination through alternative approaches such as scaling arguments or geometrical intuitions. Of course, $\mu_{\alpha\beta}$ and $Y_{\alpha\beta\gamma}$ cannot depend on Kähler moduli T^i because of the shift symmetry.

4.1.1 General Soft Term Lagrangian

Kähler potentials and superpotentials of the form (4.1) and (4.2) determine of course the complete Lagrangian describing a Supersymmetric Standard Model. Evidently, its features depend on the fields involved and on the structure of the above defining functions. In particular, the structure of soft terms comes from the expansion of the total scalar potential:

$$\mathcal{V}_F = e^{\mathcal{K}}\left(\mathcal{K}^{\mathcal{I}\bar{\mathcal{J}}} \mathcal{D}_{\mathcal{I}} \mathcal{W} \mathcal{D}_{\bar{\mathcal{J}}} \bar{\mathcal{W}} - 3\mathcal{W}\bar{\mathcal{W}}\right). \tag{4.3}$$

Indeed, this one can be written in the moduli ground state neglecting non-renormalisable terms, thus considering the dynamics of matter fields only, as:

$$\mathcal{V}_F \simeq V_F + V_{\text{soft}}, \tag{4.4}$$

where V_F is the usual scalar potential concerning moduli, i.e. making them massive, while V_{soft} is the soft term scalar potential. For the sake of simplicity, in this Chapter from now on all of the quantities depending on moduli are to be intended as evaluated in moduli vacuum expectation values, even if this item is not explicited by the angled brackets employed so far.

More specifically, the soft term scalar potential can be written in the form:

$$V_{\text{soft}} = \left(m'^2\right)_{\alpha\bar{\beta}} C^{\alpha} \bar{C}^{\bar{\beta}} + \left[\left(\frac{1}{2} B'_{\alpha\beta} C^{\alpha} C^{\beta} + \frac{1}{6} A'_{\alpha\beta\gamma} C^{\alpha} C^{\beta} C^{\gamma}\right) + \text{h.c.}\right], \tag{4.5}$$

where the unnormalised soft parameters are [5]:

- the unnormalised scalar masses $(m'^2)_{\alpha\bar{\beta}}$:

$$\begin{aligned}\left(m'^2\right)_{\alpha\bar{\beta}} \equiv\; & \left(m_{3/2}^2 + V_F\right)\tilde{K}_{\alpha\bar{\beta}} \\ & - \left[\partial_{\bar{i}}\,\partial_j\,\tilde{K}_{\alpha\bar{\beta}} - \left(\partial_{\bar{i}}\,\tilde{K}_{\alpha\bar{\gamma}}\right)\tilde{K}^{\bar{\gamma}\delta}\left(\partial_j\,\tilde{K}_{\delta\bar{\beta}}\right)\right]\bar{F}^{\bar{i}}F^j;\end{aligned} \tag{4.6}$$

- the unnormalised trilinear couplings $A'_{\alpha\beta\gamma}$:

$$\begin{aligned}A'_{\alpha\beta\gamma} \equiv\; & \frac{\bar{W}}{|W|}\,e^{K/2}F^i\Big\{Y_{\alpha\beta\gamma}K_i + \partial_i\,Y_{\alpha\beta\gamma} \\ & - \left[Y_{\delta\beta\gamma}\tilde{K}^{\delta\bar{\rho}}\,\partial_i\,\tilde{K}_{\bar{\rho}\alpha} + (\alpha \leftrightarrow \beta) + (\gamma \leftrightarrow \beta)\right]\Big\};\end{aligned} \tag{4.7}$$

- the unnormalised bilinear couplings $B'_{\alpha\beta}$: cc

$$\begin{aligned}B'_{\alpha\beta} \equiv\; & \frac{\bar{W}}{|W|}\,e^{K/2}\Big\{F^i\Big[K_i\mu_{\alpha\beta} + \partial_i\,\mu_{\alpha\beta} \\ & - \left(\mu_{\delta\beta}\tilde{K}^{\delta\bar{\rho}}\,\partial_i\,\tilde{K}_{\bar{\rho}\alpha} + (\alpha \leftrightarrow \beta)\right)\Big] - m_{3/2}\mu_{\alpha\beta}\Big\} \\ & + \left(2m_{3/2}^2 + V_F\right)Z_{\alpha\beta} - m_{3/2}\bar{F}^{\bar{i}}\,\partial_{\bar{i}}\,Z_{\alpha\beta} \\ & + m_{3/2}F^i\Big\{\partial_i\,Z_{\alpha\beta} - \left[Z_{\delta\beta}\tilde{K}^{\delta\bar{\rho}}\,\partial_i\,\tilde{K}_{\bar{\rho}\alpha} + (\alpha \leftrightarrow \beta)\right]\Big\} \\ & - \bar{F}^{\bar{i}}F^j\Big\{\partial_{\bar{i}}\,\partial_j\,Z_{\alpha\beta} - \left[\tilde{K}^{\delta\bar{\rho}}\left(\partial_i\,\tilde{K}_{\bar{\rho}\alpha}\right)\left(\partial_{\bar{i}}\,Z_{\delta\beta}\right) + (\alpha \leftrightarrow \beta)\right]\Big\}.\end{aligned} \tag{4.8}$$

Actually, the soft scalar potential should contain one more term, i.e. the mass term should be given by $(m_0^2 + m'^2)_{\alpha\bar{\beta}}$, $(m_0^2)_{\alpha\bar{\beta}}$ being an additional parameter. Nevertheless, since $(m_0^2)_{\alpha\bar{\beta}}$ is vanishing whenever $\mu_{\alpha\beta}$ is zero, and this fact is always true in the situations under exam, it is not reported. The same could be said for some of the addenda of $B'_{\alpha\beta}$, but in this case they are all explicited in order to comprehensively analyse the calculation of the $B\tilde{\mu}$-term (defined below) in the following.

It is worthwhile to underline that the previous expressions come from more general ones in which all sums on Kähler moduli are generalised to sums on all kinds of moduli. Anyway, in the ground state Kähler moduli are the only ones with non-vanishing auxiliary fields.
This reasoning is accomplished because the exact form of the auxiliary fields is:

$$F^{\mathcal{I}} = \frac{\mathcal{W}}{|\mathcal{W}|}\,e^{\mathcal{K}/2}\mathcal{K}^{\mathcal{I}\bar{\mathcal{J}}}\mathcal{D}_{\bar{\mathcal{J}}}\bar{\mathcal{W}},$$

but the leading order expressions for modular auxiliary fields read:

$$F^I \simeq \frac{W}{|W|}\, e^{K/2} K^{I\bar{J}} D_{\bar{J}} \bar{W}, \tag{4.9}$$

being the additive terms associated to observable matter largely subdominant with respect to modular ones.

In order to make it easier to compare the results of the computations with other computations as well as with phenomenological issues and experimental bounds, soft terms are generally expressed in terms of the gravitino mass. Explicitly, this one is:

$$m_{3/2} = e^{\mathcal{K}/2} \left|\mathcal{W}\right|,$$

but, again, its leading order expression is:

$$m_{3/2} \simeq e^{K/2} \left|W\right|. \tag{4.10}$$

To conclude, the general soft term Lagrangian is reported for completeness.
Of course, the soft scalar potential (4.5) does not describe canonically normalised fields, which means that also its masses and couplings are not normalised.
As a matter of fact, according to expressions (4.1) and (4.2) the soft scalar Lagrangian $\mathcal{L}_{\text{soft}}^{\text{scalar}}$, which contains the pure kinetic terms for all of the fields appearing in the soft scalar potential and the soft scalar potential is not canonically normalised:

$$\mathcal{L}_{\text{soft}}^{\text{scalar}} = \tilde{K}_{\alpha\bar{\beta}}\, \partial_\mu\, C^\alpha\, \partial^\mu\, \bar{C}^{\bar{\beta}} - V_{\text{soft}}. \tag{4.11}$$

Evidently, the canonical normalisation takes place in the same way as does for Kähler moduli, i.e. suitably diagonalising the kinetic term:

$$\mathcal{L}_{\text{kin}}^{\text{soft scalar}} = \tilde{K}_{\alpha\bar{\beta}}\, \partial_\mu\, C^\alpha\, \partial^\mu\, \bar{C}^{\bar{\beta}} - \left(m'^2\right)_{\alpha\bar{\beta}} C^\alpha \bar{C}^{\bar{\beta}}, \tag{4.12}$$

by a simultaneous diagonalisation and normalisation of matrices $\tilde{K}_{\alpha\bar{\beta}}$ and $\left(m'^2\right)_{\alpha\bar{\beta}}$. Actually bilinear couplings would render this reasoning technically more complicated, but they are not considered because in general they are assumed to affect only Higgs fields, as explained below.

4.1.2 Soft Terms for Diagonal Matter Metric

In a multitude of situations it is reasonable to deal with a diagonal Kähler matter metric, as shown below, i.e. with a matter metric which can be expressed as:

$$\tilde{K}_{\alpha\bar{\beta}} = \tilde{K}_\alpha \delta_{\alpha\bar{\beta}}. \tag{4.13}$$

This fact allows to deal with much simpler soft terms as scalar fields are normalised straightforwardly. Indeed, in this case the pure kinetic term of the soft term Lagrangian reads:

$$\mathcal{L}^{\text{soft scalar}}_{\text{pure kin}} = \tilde{K}_\alpha \, \partial_\mu \, C^\alpha \, \partial^\mu \, \bar{C}^{\bar{\alpha}},$$

then canonically normalised observable scalar fields are evidently defined as:

$$\phi^\alpha \equiv (\tilde{K}_\alpha)^{1/2} C^\alpha. \tag{4.14}$$

Moreover, specialising the computation to the Minimal Supersymmetric Standard Model two further assumptions are in order.
First of all, the B-coupling is only relevant for the Higgses. Then, it is possible to set:

$$Z_{\alpha\beta} \equiv Z \left(\delta_{\alpha H_1} \delta_{\beta H_2} + \delta_{\alpha H_2} \delta_{\beta H_1} \right). \tag{4.15}$$

Secondly, the μ-term in the superpotential is expected to be vanishing [33]. However, it is instructive for the computation of the B-terms to consider:

$$\mu_{\alpha\beta} \equiv \mu \left(\delta_{\alpha H_1} \delta_{\beta H_2} + \delta_{\alpha H_2} \delta_{\beta H_1} \right), \tag{4.16}$$

before finally setting $\mu = 0$.

Under these assumptions, it is easy to show that the canonically normalised soft scalar potential can be written as:

$$V_{\text{soft}} = m_\alpha^2 \phi^\alpha \bar{\phi}^{\bar{\alpha}} + \left[\left(\frac{1}{6} A_{\alpha\beta\gamma} y_{\alpha\beta\gamma} \phi^\alpha \phi^\beta \phi^\gamma + B\tilde{\mu} H_1 H_2 \right) + \text{h.c.} \right], \tag{4.17}$$

where the soft parameters are identified as:

- the normalised scalar masses m_α^2:

$$m_\alpha^2 \equiv \left(m_{3/2}^2 + V_F \right) - \bar{F}^{\bar{i}} F^j \, \partial_{\bar{i}} \, \partial_j \left(\ln \tilde{K}_\alpha \right); \tag{4.18}$$

- the normalised trilinear parameters $A_{\alpha\beta\gamma}$:

$$A_{\alpha\beta\gamma} \equiv F^i \left[K_i + \partial_i \ln \left(Y_{\alpha\beta\gamma} \right) - \partial_i \ln \left(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma \right) \right]; \tag{4.19}$$

- the normalised $B\tilde{\mu}$-term:

$$\begin{aligned} B\tilde{\mu} \equiv \; & \left(\tilde{K}_{H_1} \tilde{K}_{H_2} \right)^{-\frac{1}{2}} \left\{ \frac{\bar{W}}{|W|} e^{K/2} \mu \Big[-m_{3/2} \right. \\ & + F^i \left(K_i + \partial_i \ln \mu - \partial_i \ln \left(\tilde{K}_{H_1} \tilde{K}_{H_2} \right) \right) \Big] \\ & + \left(2m_{3/2}^2 + V_F \right) Z - m_{3/2} \bar{F}^{\bar{i}} \, \partial_{\bar{i}} \, Z \\ & + m_{3/2} F^i \left[\partial_i \, Z - Z \, \partial_i \ln \left(\tilde{K}_{H_1} \tilde{K}_{H_2} \right) \right] \\ & \left. - \bar{F}^{\bar{i}} F^j \left[\partial_{\bar{i}} \, \partial_j \, Z - \left(\partial_{\bar{i}} \, Z \right) \partial_j \ln \left(\tilde{K}_{H_1} \tilde{K}_{H_2} \right) \right] \right\}; \end{aligned} \tag{4.20}$$

where the physical Yukawa couplings are defined as:

$$y_{\alpha\beta\gamma} \equiv \frac{\bar{W}}{|W|} \frac{e^{K/2}}{\left(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma\right)^{\frac{1}{2}}} Y_{\alpha\beta\gamma}, \tag{4.21}$$

while the rescaled μ-parameter reads:

$$\tilde{\mu} \equiv \left(\tilde{K}_{H_1} \tilde{K}_{H_2}\right)^{-1/2} \left[\frac{\bar{W}}{|W|} e^{K/2} \mu + m_{3/2} Z - \bar{F}^{\bar{i}}\, \partial_{\bar{i}}\, Z\right]. \tag{4.22}$$

The $\tilde{\mu}$-term turns out to correspond to Higgsino masses [2].

It is important to notice that the definition of B holds true provided the assumptions (4.15) and (4.16) and assuming a diagonal Higgs sector for the Kähler matter metric, without any further assumption on the rest of $\tilde{K}_{\alpha\bar{\beta}}$. In particular, under these conditions the relation is:

$$B'_{\alpha\beta} = \left(\tilde{K}_{H_1} \tilde{K}_{H_2}\right)^{1/2} B\tilde{\mu}\left(\delta_{\alpha H_1}\delta_{\beta H_2} + \delta_{\alpha H_2}\delta_{\beta H_1}\right).$$

Canonical Normalisation

In many situations with general Kähler matter metrics, unnormalised masses matrices turn out to assume especially simple expressions which allow a straighforward reconcilement to the situations with diagonal Kähler matter metrics. This is precisely what happens in the models considered below.
In particular, if the Kähler matter metric $\tilde{K}_{\alpha\bar{\beta}}$ of the model is such that:

- the unnormalised mass matrix can be written as:

$$m'^2_{\alpha\bar{\beta}} = m^2_\alpha \tilde{K}_{\alpha\bar{\beta}}; \tag{4.23}$$

- unnormalised trilinear couplings are such that:

$$A'_{\alpha\beta\gamma} = \frac{\bar{W}}{|W|} e^{K/2} Y_{\alpha\beta\gamma} A_{\alpha\beta\gamma}; \tag{4.24}$$

- the Kähler matter is diagonal in the MSSM-Higgs sector and conditions (4.15) and (4.16) hold true;

then the soft scalar Lagrangian can be written as:

$$\begin{aligned}\mathcal{L}^{\text{scalar}}_{\text{soft}} &= \tilde{K}_{\alpha'\bar{\beta}'}\, \partial_\mu\, C^{\alpha'}\, \partial^\mu\, \bar{C}^{\bar{\beta}'} + \tilde{K}_{H_i}\, \partial_\mu\, H'_i\, \partial^\mu\, \bar{H}'_i - m^2_{\alpha'} \tilde{K}_{\alpha'\bar{\beta}'} C^{\alpha'} \bar{C}^{\bar{\beta}'} - m^2_{H_i} \tilde{K}_{H_i} H'_i \bar{H}'_i \\ &- \left[\left(\frac{1}{6}\frac{\bar{W}}{|W|} e^{K/2} A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma + B\tilde{\mu}\left(\tilde{K}_{H_1}\tilde{K}_{H_2}\right)^{\frac{1}{2}} H'_1 H'_2\right) + \text{h.c.}\right],\end{aligned}$$

primed indices α', $\bar{\beta}'$ running over all of the scalar fields but the unnormalised Higgses H_1' and H_2'.
Since $\tilde{K}_{\alpha\bar{\beta}}$ is hermitian, it is possible to find a unitary matrix $(Q)^{\bar{\alpha}'}{}_{\bar{\beta}'}$ such that:

$$(Q^\dagger)_{\alpha'}{}^{\gamma'}\,\tilde{K}_{\gamma'\bar{\delta}'}\,(Q)^{\bar{\delta}'}{}_{\bar{\beta}'} = \tilde{K}_{\alpha'}\delta_{\alpha'\bar{\beta}'}, \tag{4.25}$$

$\tilde{K}_\alpha$ being Kähler matter metric eigenvalues. Then, it is possible to redefine the fields as:

$$\begin{cases} \bar{C}^{\bar{\alpha}'} & \equiv (Q)^{\bar{\alpha}'}{}_{\bar{\beta}'}\bar{C}'^{\bar{\beta}'}, \\ C^{\alpha'} & \equiv C'^{\beta'}(Q^\dagger)_{\beta'}{}^{\alpha'}, \end{cases} \tag{4.26}$$

getting the yet unnormalised Lagrangian:

$$\begin{aligned} \mathcal{L}^{\text{scalar}}_{\text{soft}} = {} & \tilde{K}_{\alpha'}\,\partial_\mu\,C'^{\alpha'}\,\partial^\mu\,\bar{C}'^{\bar{\alpha}'} + \tilde{K}_{H_i}\,\partial_\mu\,H_i'\,\partial^\mu\,\bar{H}_i' - m^2_{\alpha'}\tilde{K}_{\alpha'}C^{\alpha'}\bar{C}^{\bar{\alpha}'} - m^2_{H_i}\tilde{K}_{H_i}H_i'\bar{H}_i' \\ & - \left[\left(\frac{1}{6}\frac{\bar{W}}{|W|}e^{K/2}A_{\alpha\beta\gamma}Y'_{\alpha\beta\gamma}C'^{\alpha}C'^{\beta}C'^{\gamma} + B\tilde{\mu}\left(\tilde{K}_{H_1}\tilde{K}_{H_2}\right)^{\frac{1}{2}}H_1'H_2'\right) + \text{h.c.}\right], \end{aligned}$$

suitably redefining $Y'_{\alpha\beta\gamma}$ in such a way that:

$$Y_{\alpha\beta\gamma}A_{\alpha\beta\gamma}(Q^\dagger)_\delta{}^\alpha\,(Q^\dagger)_\epsilon{}^\beta\,(Q^\dagger)_\zeta{}^\gamma \equiv Y'_{\delta\epsilon\zeta}A_{\delta\epsilon\zeta}.$$

In the end, it is easy to define the canonically normalised fields as:

$$\phi^{\alpha'} \equiv (\tilde{K}_{\alpha'})^{1/2}C'^{\alpha'}, \tag{4.27}$$

$$H_i \equiv (\tilde{K}_{H_i})^{1/2}H_i', \tag{4.28}$$

and the canonically normalised Yukawa couplings as in (4.21), obtaining the correct canonically normalised Lagrangian:

$$\mathcal{L}^{\text{scalar}}_{\text{soft}} = \partial_\mu\,\phi^\alpha\,\partial^\mu\,\bar{\phi}^{\bar{\alpha}} - m^2_\alpha\phi^\alpha\bar{\phi}^{\bar{\alpha}} - \left[\left(\frac{1}{6}A_{\alpha\beta\gamma}y_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma + B\tilde{\mu}H_1H_2\right) + \text{h.c.}\right], \tag{4.29}$$

which is exactly the soft scalar Lagrangian for diagonal Kähler matter metrics.

However, although this reasoning allows the presence of very general Kähler matter metrics, there are strict experimental bounds connected to flavour-changing neutral currents which require very small off-diagonal elements, i.e. as a matter of fact tend to reduce the problem to diagonal matrices in any case.

Gaugino Masses

If gauge fields are taken into account, their properties are entirely encoded in the gauge kinetic function:

$$f_{ab} = f_{ab}\,(\Phi)\,, \tag{4.30}$$

for which the same considerations about modular dependence apply as before.
Assuming a diagonal gauge kinetic function for the sake of simplicity,

$$f_{ab} = f_a \delta_{ab},$$

it can be shown that, being Λ_a the unnormalised gauginos, canonically normalised gaugino fields are:

$$\lambda_a \equiv \left(\mathrm{Re}\left\{f_a\right\}\right)^{1/2} \Lambda_a, \tag{4.31}$$

and their normalised masses are:

$$M_a \equiv \frac{1}{2\mathrm{Re}\left\{f_a\right\}} F^i \, \partial_i \, f_a, \tag{4.32}$$

in such a way that gaugino mass terms in the soft Lagrangian is written as:

$$\mathcal{L}^{\mathrm{gaugino}}_{\mathrm{mass}} = -\frac{1}{2}\left(M_a \lambda_a \lambda_a + \mathrm{h.c.}\right). \tag{4.33}$$

4.2 MSSM on D7 Branes

In the context of moduli stabilisation in the Large Volume Scenario, one interesting feature is the possibility to study the arising of Supersymmetric Standard Models coming from D7 branes wrapping 4-cycles. Indeed, Standard Model fields and their supersymmetric partners can be modelled to come from the compactification of strings belonging to D7 branes that wrap one or several of the 4-cycles which form the volume $\mathcal{V}$ [8] [9].
Of course, the characteristics of the wrapping - i.e. in particular, dealing with D7 branes only, which of the 4-cycles are wrapped - determine the properties of the corresponding Supersymmetric Standard Model, such as the soft breaking terms.
The gauge kinetic function $f_a = f_a\left(\Phi\right)$ can be determined explicitly in Large Volume Scenarios. Instead, unfortunately, the exact expressions of functions such as $\tilde{K}_{\alpha\beta} = \tilde{K}_{\alpha\beta}\left(\Phi\right)$ and $Z_{\alpha\beta} = Z_{\alpha\beta}\left(\Phi\right)$ are unknown in most of the situations because of the extreme complexity of computations. In the following, two interesting scenarios related to Models II and III are described in which some physical intuitions allow to understand their behaviour.
Unless differently stated, the discussion is referred to the Minimal Supersymmetric Standard Model.

4.2.1 Gauge Kinetic Function

In the Large Volume Scenarios of interest, the effective expression of the gauge kinetic function for D7 branes wrapping the 4-cycles corresponding to the Kähler modulus T^i reads [21]:

$$f_i = \frac{T^i}{2\pi},$$

gauge group indices being associated to the cycle in whose branes the corresponding string modes exist. Specifically, the gauge couplings corresponding to such gauge kinetic functions are then:

$$g_i = \left(\frac{2\pi}{\tau^i}\right)^{1/2},$$

so gauge fields can only be supported on D7 branes wrapping 4-cycles which are not too large, otherwise this equality would imply extremely small couplings, unlike the Standard Model ones.

To conclude, gauge fields can generally arise from D7 branes wrapping blow-up 4-cycles, i.e.:

$$f_s = \frac{T_s}{2\pi}. \tag{4.34}$$

However, if the overall volume happens not to take huge values, then it is admissible to build models with gauge fields coming from D7 branes on large 4-cycles, i.e.:

$$f_1 = \frac{T_1}{2\pi}, \tag{4.35}$$

$$f_2 = \frac{T_2}{2\pi}. \tag{4.36}$$

4.2.2 Kähler Matter Metric for D7s on Blow-ups

The exact form of Kähler matter metrics arising from compactifications of D7 branes wrapping blow-up cycles is generally unkown. Nevertheless, some physical arguments allow to derive their leading order expressions, as shown in detail for instance in Ref. [33]. For the sake of completeness, here is reported the guideline of its discussion referred to Models II and III.

In scenarios in which the Standard Model is supported on a small cycle, associated to the field τ_s, within a very large bulk volume $\mathcal{V}$, on the one hand the leading order expression of the Kähler potential reads, according to Chapter 2:

$$K \simeq -2\,\ln\mathcal{V},$$

while on the other hand, assuming a diagonal Kähler matter metric for the sake of simplicity, the physically normalised Yukawa couplings are, according to 4.21:

$$y_{\alpha\beta\gamma} = \frac{\bar{W}}{|W|}\frac{e^{K/2}}{\left(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{\frac{1}{2}}}Y_{\alpha\beta\gamma}.$$

Since the Standard Model arises from branes wrapping the tiny volume of the blow-up cycle, by a locality principle all of the physical properties of the fields must be independent from the very large overall volume $\mathcal{V}$, while of course the unnormalised

Yukawa couplings $Y_{\alpha\beta\gamma}$ are also independent from Kähler moduli.
Therefore at leading order the diagonal Kähler matter metric is expected to be of the kind:

$$\tilde{K}_\alpha \simeq \frac{\tilde{k}_\alpha\left(\tau_s\right)}{\mathcal{V}^{2/3}},$$

where $\tilde{k}_\alpha$ is a unknown function of the axio-dilaton, the complex structure moduli and the small modulus. For the sake of brevity, only Kähler moduli dependence is reported. It can be proved that the function $\tilde{k}_\alpha$ can be expanded in a power series in τ_s as:

$$\tilde{k}_\alpha \simeq \tau_s^{1/3}\, k_\alpha,$$

for models in which all of the D7 branes wrap the same cycle.

Summing up, in a logical generalisation to non-diagonal situations, the effective Kähler matter metrics of this kind can be expressed for Model II as:

$$\tilde{K}_{\alpha\bar{\beta}} \equiv \frac{\tau_s^{1/3}}{\tau_b}\, k_{\alpha\bar{\beta}}, \tag{4.37}$$

and for Model III as:

$$\tilde{K}_{\alpha\bar{\beta}} \equiv \frac{\tau_s^{1/3}}{\tau_1^{1/3}\tau_2^{2/3}}\, k_{\alpha\bar{\beta}}. \tag{4.38}$$

The assumption of the very same functional dependence for all of the elements of Kähler matter metric is likely to involve the equality of soft parameters for all of the scalar fields. This property is confirmed in what follows.

4.2.3 Kähler Matter Metric for D7s on Large Cycles

In situations in which the large volume is structured as a combination of more than one cycle, e.g. in Model III, this structuring makes it reasonable to build the Standard Model on D7 branes wrapping the large volume cycle or part of it in quite an interesting way. Indeed a larger variety of fields can arise from compactifications of this kind, thus allowing for a possible diversification of soft breaking terms.
Following an idea of Ref. [34] and neglecting the blow-up cycle, the large volume (3.40) of Model III can be naïvely modelled as the torus shown in Figure 4.1.
The Standard Model can be realised on D7 branes wrapping the cycle τ_1 (case i), the cycle τ_2 (case ii) or both of them (case iii). Depending on which brane fields come from, their features turn out to be different as different is the Kähler matter metric they are referred to.
Of course it is reasonable to assume that there is no dependence on τ_s if the Standard Model is built on such kind of branes by a locality principle. Indeed, a small volume is expected not to affect the global behaviour of functions depending on the much larger structure it is included in.

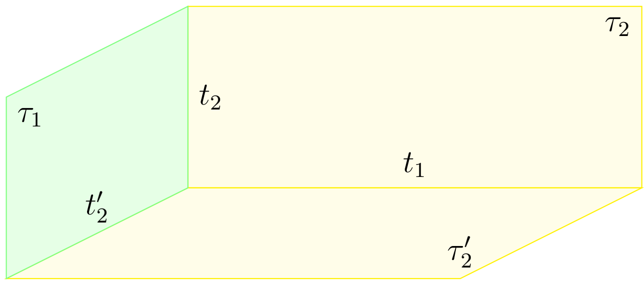

Figure 4.1. Volume structure graphical representation, $\mathcal{V} = \lambda_1 t_1 t_2^2 = \alpha \tau_1^{1/2} \tau_2$: t_1, t_2 and t'_2 are the sides of the parallelepiped $\mathcal{V}$, while τ_1 (in green), τ_2 and τ'_2 (both in yellow) are its faces areas. t_2 and t'_2 as well as τ_2 and τ'_2 are equivalent but are distinguished for future convenience.

Classification of D7 Fields on Large Cycles

In order to study Kähler matter metric dependences, it is necessary to individuate the general expressions which govern them in the general context of the wrapping by D7 branes. This Paragraph is devoted to this aim and is completely inspired by Ref. [35], from which the starting point general formulae are drawn.

Chiral fields in D7 branes are classifiable in three classes depending on their geometric origin (see Fig. 4.2 for a schematic representation):

- A-fields, which originate from general vector supermultiplets in higher dimensions. They come from massless modes of the gauge multiplets inside the D7 branes volume;
- Φ-fields, which originate from general scalar multiplet in higher dimensions. They correspond to massless modes coming from scalars in the transverse space;
- I-fields, which originate from the intersection of different branes. They correspond to massless fields coming from the exchange of open strings between intersecting D7 branes.

Considering three different sets of D7 branes D7^i, i = 1,2,3, each wrapping a 4-torus transverse to the i-th complex plane, it can be shown that the Kähler matter metrics dependences on Kähler moduli are, respectively:

$$\begin{aligned}
&A-\text{fields}: && \tilde{K}_{\left(\mathrm{D7}^i\mathrm{D7}^i\right)_j} = \frac{1}{2\tau_k}, && i \neq j \neq k, \\
&\Phi-\text{fields}: && \tilde{K}_{\left(\mathrm{D7}^i\mathrm{D7}^i\right)_i} = \frac{g_S}{2}, && \\
&I-\text{fields}: && \tilde{K}_{\left(\mathrm{D7}^i\mathrm{D7}^j\right)} = \left(\frac{g_S}{4\tau_k}\right)^{1/2}, && i \neq j \neq k,
\end{aligned} \tag{4.39}$$

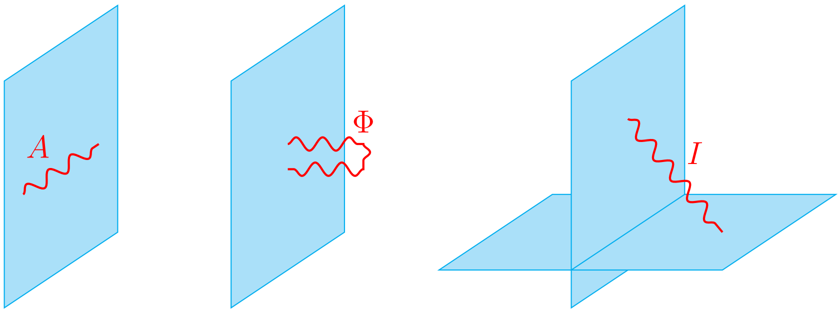

Figure 4.2. Representation of different matter fields from D7-branes wrapping: blue planes are D7 branes wrapping some cycle, while red lines are the possible string modes A, Φ and I.

where the subscripts $\mathrm{D7}^i$, $i = 1,2,3$, indicate the three types of D7 branes where the string extremities lie while the further subindices j, $j = 1,2,3$, indicate the complex plane the string is orthogonal to.

Kähler Matter Metric

Employing the results (4.39), it is possible to determine the expressions for Kähler matter metrics corresponding to the possible ways the Standard Model can be built in.

i. D7 branes wrapping τ_1.
   For D7 branes wrapping τ_1, the only possible fields are one kind of A-fields, called A', and one kind of Φ-fields, called Φ' (see Fig. 4.3).

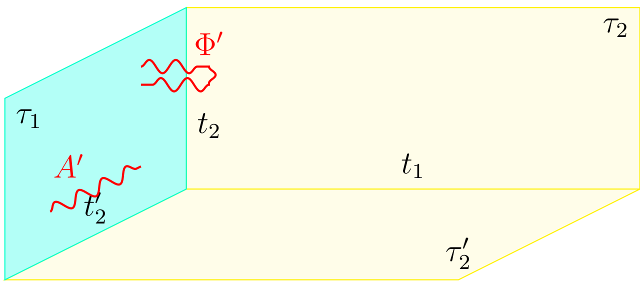

Figure 4.3. case i: graphical representation of D7 branes wrapping τ_1.

The corresponding Kähler matter metrics read:

$$\begin{aligned} \tilde{K}_{A'} &= \tilde{K}_{\left(\mathrm{D7}^1\mathrm{D7}^1\right)_2} = \frac{1}{2\tau_2}, \\ \tilde{K}_{\Phi'} &= \tilde{K}_{\left(\mathrm{D7}^1\mathrm{D7}^1\right)_1} = \frac{g_S}{2}. \end{aligned} \tag{4.40}$$

ii. D7 branes wrapping τ_2.
For D7 branes wrapping τ_2, the possible fields are two kinds of A-fields, A'' and A''', and one kind of Φ-fields, called Φ'' (see Fig. 4.4).

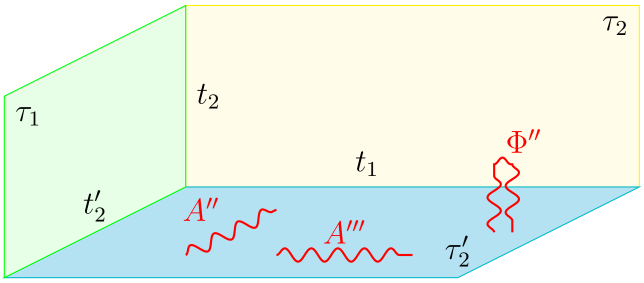

Figure 4.4. case ii: graphical representation of D7 branes wrapping τ_2.

The corresponding Kähler matter metrics are:

$$\begin{aligned}
\tilde{K}_{A''} &= \tilde{K}_{(\mathrm{D7^2D7^2})_{2'}} = \frac{1}{2\tau_1}, \\
\tilde{K}_{A'''} &= \tilde{K}_{(\mathrm{D7^2D7^2})_{1}} = \frac{1}{2\tau_2}, \qquad (4.41)\\
\tilde{K}_{\Phi''} &= \tilde{K}_{(\mathrm{D7^2D7^2})_{2}} = \frac{g_S}{2}.
\end{aligned}$$

iii. D7 branes wrapping both τ_1 and τ_2.
For D7 branes wrapping both τ_1 and τ_2, of course all of the previous fields are possible as well as I fields (see Fig. 4.5).

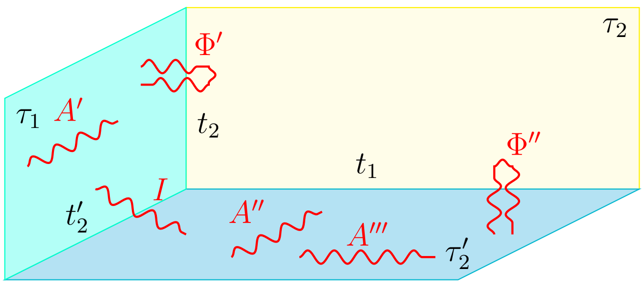

Figure 4.5. case iii: graphical representation of D7 branes wrapping both τ_1 and τ_2.

Then Kähler matter metrics are (4.40), (4.41) and:

$$\tilde{K}_I = \tilde{K}_{(\mathrm{D7^1D7^2})} = \left(\frac{g_S}{4\tau_2}\right)^{1/2}. \qquad (4.42)$$

Evidently, the last situation is the most general one as it includes the former two as special cases.

To sum up and adapt the notation to the previous one, it is possible to conclude that, in Model III, the most general possible prototype of Supersymmetric Standard Model can arise from observable fields coming from D7 branes wrapping both τ_1 and τ_2.
These observable fields can be associated to a block-diagonal Kähler matter metric:

$$\tilde{K}_{\alpha\bar{\beta}} = \begin{pmatrix} \tilde{K}_{A'\bar{B}'} & & & & & \\ & \tilde{K}_{A''\bar{B}''} & & & & \\ & & \tilde{K}_{A'''\bar{B}'''} & & & \\ & & & \tilde{K}_{\Phi'\bar{\psi}'} & & \\ & & & & \tilde{K}_{\Phi''\bar{\psi}''} & \\ & & & & & \tilde{K}_{I\bar{J}} \end{pmatrix}, \tag{4.43}$$

in which every block corresponds to one or more fields of the kinds described above, indices A', A'', A''', Φ', Φ'' and I running accordingly.
Specifically, according to expressions (4.40), (4.41) and (4.42), the blocks can be arranged in the form:

$$\begin{gathered} \tilde{K}_{A'\bar{B}'} = \frac{1}{\tau_2} k_{A'\bar{B}'}, \qquad \tilde{K}_{A''\bar{B}''} = \frac{1}{\tau_1} k_{A''\bar{B}''}, \qquad \tilde{K}_{A'''\bar{B}'''} = \frac{1}{\tau_2} k_{A'''\bar{B}'''}, \\ \tilde{K}_{\Phi'\bar{\psi}'} = k_{\Phi'\bar{\psi}'}, \qquad \tilde{K}_{\Phi''\bar{\psi}''} = k_{\Phi''\bar{\psi}''}, \\ \tilde{K}_{I\bar{J}} = \frac{1}{\tau_2^{1/2}} k_{I\bar{J}}, \end{gathered} \tag{4.44}$$

after absorbing numerical constants such as g_S in the constant matrix elements. The restriction to special cases such as D7 branes wrapping only τ_1 or τ_2 is straightforward.

4.2.4 Giudice-Masiero Term

The function $Z_{\alpha\beta}$ is generally unknown and very hard to compute. In the Minimal Supersymmetric Standard Model, assuming for the sake of simplicity to deal with a diagonal matter metric at least in the Higgs sector and setting $Z_{\alpha\beta} = Z\,(\delta_{\alpha H_1}\delta_{\beta H_2} + \delta_{\alpha H_2}\delta_{\beta H_1})$, Z can be assumed to scale as $(\tilde{K}_{H_1}\tilde{K}_{H_2})^{1/2}$ [34], i.e. in a general Model II:

$$Z = \frac{\tau_s^{\lambda_z}}{\tau_b^{p_z}} z, \tag{4.45}$$

and in a general Model III:

$$Z = \frac{\tau_s^{\lambda_z}}{\tau_1^{n_{1z}}\tau_2^{n_{2z}}}, \tag{4.46}$$

being λ_z, p_z, n_{1z} and n_{2z} the arithmetic means of the corresponding exponents for the fields H_1 and H_2.

4.3 Soft Terms in Model II

Model II offers the instructive possibility to easily compute soft terms for the Minimal Supersymmetric Standard Model arising from D7 branes wrapping the small cycle τ_s. Kähler matter metric is expressed as:

$$\tilde{K}_{\alpha\bar{\beta}} \equiv \frac{\tau_s^\lambda}{\tau_b^p}\, k_{\alpha\bar{\beta}}, \tag{4.47}$$

where the exponents λ and p are not explicited in order to evidence the special cancellations which occur when they (especially p) assume their correct values $\lambda = 1/3$ and $p = 1$.
The following computation is performed also in Ref. [21], which served as one of the main inspirations of the thesis.

4.3.1 F-terms and Gravitino Mass

Auxiliary fields and the gravitino mass can be computed easily according to the standard definitions (4.9) and (4.10), recalling the vacuum expectation values (3.22) and (3.23) and the assumptions (3.18) and (3.19).

In reduced Planck units, auxiliary fields read, up to leading order, which turns out to be sufficient for the present purpose:

$$F^b = -2\,\frac{|W_0|}{\tau_b^{1/2}}\left[1 + O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}}, \frac{\xi}{\tau_b^{3/2}}\right)\right], \tag{4.48}$$

$$F^s = -\frac{3}{2}\frac{|W_0|}{a_s\tau_b^{3/2}}\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right]. \tag{4.49}$$

while the gravitino mass, again up to leading order, can be expressed as:

$$m_{3/2} = \frac{|W_0|}{\tau_b^{3/2}}\left[1 + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_b^{\frac{3}{2}}}, \frac{\xi}{\tau_b^{\frac{3}{2}}}\right)\right]. \tag{4.50}$$

Moreover, one further identity holds true. In fact, it is possible to write at leading order:

$$a_s\tau_s = \ln\left(\frac{1}{m_{3/2}}\right)\left[1 - \frac{\ln\bar{m}}{\ln m_{3/2}}\right], \tag{4.51}$$

where $\bar{m}$ is a quantity with the dimensions of a mass:

$$\bar{m} = \frac{4}{3}a_s\tau_s\frac{|A_s|}{\tau_s^{3/2}}\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right].$$

This simple expression allows to conclude that the correction by the ratio of logarithms is subdominant with respect e.g. to the one by $O(1/a_s\tau_s)$. Indeed the mass $\bar{m}$ turns out to be around unity, then, evidently:

$$\ln \bar{m} \overset{\bar{m}\sim 1}{\sim} 0.$$

Combining the previous expressions, it is easy to express the auxiliary fields in terms of the gravitino mass. In fact, the following equalities hold true up to leading order:

$$F^b = -2\tau_b\, m_{3/2}\left[1 + O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}}, \frac{\xi}{\tau_b^{3/2}}\right)\right], \tag{4.52}$$

$$F^s = -\frac{3}{2}\,\tau_s\,\frac{m_{3/2}}{\ln\left(\frac{1}{m_{3/2}}\right)}\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right]. \tag{4.53}$$

These expressions are interesting because if in the soft terms the dependence on the field F^b turns out to be cancelled - and this cancellation is precisely accomplished by the correct value $p = 1$, as shown below - then their natural scale is set to $m_{3/2}$ divided by a factor $\ln(M_P/m_{3/2})$ in physical units.

4.3.2 Diagonal Kähler Matter Metric

For the sake of simplicity, the first case under exam is the one with a diagonal Kähler matter metric.
In order to have a more instructive computation in view of Model III, it can be taken of the form:

$$\tilde{K}_\alpha = \frac{\tau_s^{\lambda_\alpha}}{\tau_b^{p_\alpha}}k_\alpha, \tag{4.54}$$

with general exponents λ_α and p_α, and similarly the Z-term is taken as:

$$Z = \frac{\tau_s^{\lambda_z}}{\tau_b^{p_z}}z. \tag{4.55}$$

with a general exponent p_z.

Scalar masses (4.18), trilinear parameters (4.19) and the $B\tilde{\mu}$-term (4.20) (assuming $\mu = 0$) under assumptions (3.18) and (3.19) read:

$$\begin{aligned} m_\alpha^2 = &\left[1 - p_\alpha\left(1 + O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}}, \frac{\xi}{\tau_b^{3/2}}\right)\right)\right]m_{3/2}^2 \\ &+ V_F + \frac{9\lambda_\alpha}{16}\frac{m_{3/2}^2}{\ln^2\left(1/m_{3/2}\right)}\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right], \end{aligned} \tag{4.56}$$

$$\begin{aligned} A_{\alpha\beta\gamma} = 3\,m_{3/2}&\left[1+O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}},\frac{\xi}{\tau_b^{3/2}}\right)-\frac{p_\alpha+p_\beta+p_\gamma}{3}\left(1+O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}},\frac{\xi}{\tau_b^{3/2}}\right)\right)\right] \\ &+\frac{3}{4}\left(\lambda_\alpha+\lambda_\beta+\lambda_\gamma\right)\frac{m_{3/2}}{\ln\left(1/m_{3/2}\right)}\left[1+O\left(\frac{1}{a_s\tau_s}\right)\right], \end{aligned} \tag{4.57}$$

$$\begin{aligned} B\tilde{\mu} = &\frac{z}{\left(k_{H_1}k_{H_2}\right)^{\frac{1}{2}}}\frac{\tau_b^{\frac{p_{H_1}+p_{H_2}}{2}-p_z}}{\tau_s^{\frac{\lambda_{H_1}+\lambda_{H_2}}{2}-\lambda_z}}\left\{2m_{3/2}^2\left[1-\frac{1}{2}\left(p_{H_1}+p_{H_2}\right)\left(1+O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}},\frac{\xi}{\tau_b^{3/2}}\right)\right)\right.\right. \\ &\left.-\frac{1}{2}p_z\left(p_z+1-\left(p_{H_1}+p_{H_2}\right)\right)\left(1+O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}},\frac{\xi}{\tau_b^{3/2}}\right)\right)\right]+V_F \\ &+\frac{3}{4}\frac{m_{3/2}^2}{\ln\left(1/m_{3/2}\right)}\left(\left(\lambda_{H_1}+\lambda_{H_2}\right)\left(1-p_z\right)\right. \\ &\left.-2\lambda_z\left(\frac{p_{H_1}+p_{H_2}}{2}-p_z\right)\right)\left[1+O\left(\frac{1}{a_s\tau_s}\right)\right] \\ &\left.+\frac{9\lambda_z}{16}\left[\lambda_{H_1}+\lambda_{H_2}-\left(\lambda_z-1\right)\right]\frac{m_{3/2}^2}{\ln^2\left(1/m_{3/2}\right)}\left[1+O\left(\frac{1}{a_s\tau_s}\right)\right]\right\}. \end{aligned} \tag{4.58}$$

Moreover, gaugino masses are (absorbing an overall phase into gauginos):

$$M_a=\frac{3}{4}\frac{m_{3/2}}{\ln\left(\frac{1}{m_{3/2}}\right)}\left[1+O\left(\frac{1}{a_s\tau_s}\right)\right]. \tag{4.59}$$

In the end, physical Yukawa couplings and the $\tilde{\mu}$-term read:

$$y_{\alpha\beta\gamma}=e^{-i\theta}\left(\frac{\tau_b^{\left(p_\alpha+p_\beta+p_\gamma-3\right)}}{\tau_s^{\left(\lambda_\alpha+\lambda_\beta+\lambda_\gamma\right)}k_\alpha k_\beta k_\gamma}\right)^{\frac{1}{2}}Y_{\alpha\beta\gamma}\left[1+O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}},\frac{\xi}{\tau_b^{3/2}}\right)\right], \tag{4.60}$$

$$\begin{aligned} \tilde{\mu} =&\frac{z}{\left(k_{H_1}k_{H_2}\right)^{\frac{1}{2}}}\frac{\tau_b^{\frac{p_{H_1}+p_{H_2}}{2}-p_z}}{\tau_s^{\frac{\lambda_{H_1}+\lambda_{H_2}}{2}-\lambda_z}}\left\{m_{3/2}\left[1-p_z\left(1+O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}},\frac{\xi}{\tau_b^{3/2}}\right)\right)\right]\right. \\ &\left.+\frac{3\lambda_z}{4}\frac{m_{3/2}}{\ln\left(1/m_{3/2}\right)}\left(1+O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}},\frac{\xi}{\tau_b^{3/2}}\right)\right)\right\} \end{aligned} \tag{4.61}$$

Despite the seeming complexity of these expressions in a hypothetical general case, in the model under exam, assuming the known dependence for Z, the fact that $p_\alpha\equiv 1$ simplifies surprisingly the results because it provides a cancellation of the leading terms. The remaining contributions from the cancelled terms are weighted by factors scaling with the inverse volume and thus turn out to be subdominant with respect to the terms which are scaled by the logarithm of $M_P/m_{3/2}$ in physical units.

As a matter of fact, choosing the overall values $\lambda = 1/3$ and $p = 1$, up to the leading order the soft terms are, in physical units:

$$m_\alpha \simeq \frac{\sqrt{3}}{4} \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}, \tag{4.62}$$

$$M_a \simeq \frac{3}{4} \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}, \tag{4.63}$$

$$A_{\alpha\beta\gamma} \simeq \frac{3}{4} \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}, \tag{4.64}$$

$$B\tilde{\mu} \simeq \frac{z}{4\left(k_{H_1}k_{H_2}\right)^{\frac{1}{2}}} \left(\frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}\right)^2, \tag{4.65}$$

with:

$$y_{\alpha\beta\gamma} \simeq e^{-i\theta} \frac{Y_{\alpha\beta\gamma}}{\tau_s^{\frac{1}{2}}\left(k_\alpha k_\beta k_\gamma\right)^{\frac{1}{2}}}, \tag{4.66}$$

$$\tilde{\mu} \simeq \frac{z}{4\left(k_{H_1}k_{H_2}\right)^{\frac{1}{2}}} \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}. \tag{4.67}$$

As was pointed out before, for all of the soft masses, i.e. for Higgses, squarks, sleptons and gauginos, the reference value is:

$$\tilde{m}_{\rm soft} \simeq \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}, \tag{4.68}$$

which means that they turn out to be suppressed by a factor $\ln(M_P/m_{3/2})$ compared to gravitino mass $m_{3/2}$.

4.3.3 General Kähler Matter Metric

The previous results allow to discuss easily what happens for more general situations. Assuming for the moment the possibility to have a general Kähler matter metric of the kind (4.47), where actually the Higgs sector is still assumed to be diagonal with conditions (4.15) and (4.16), as a matter of fact, previous results turn out to be unaltered.

Indeed it is possible to write, defining $\lambda_s = \lambda$ and $\lambda_b = -p$:

$$\tilde{K}_{\alpha\bar{\beta}} = \Big(\prod_{l=s,b} \tau_l^{\lambda_l}\Big) k_{\alpha\bar{\beta}},$$

in such a way as to have:

$$\partial_i \tilde{K}_{\alpha\bar{\beta}} = \frac{\lambda_i}{2\tau_i}\tilde{K}_{\alpha\bar{\beta}}, \qquad \partial_j\,\partial_i\,\tilde{K}_{\alpha\bar{\beta}} = -\frac{\lambda_i}{4\tau_i\tau_j}\left(\delta_{ij} - \lambda_j\right)\tilde{K}_{\alpha\bar{\beta}},$$

and:

$$\partial_i \ln \tilde{K}_{\alpha\bar{\beta}} = \frac{\lambda_i}{2\tau_i}, \qquad \partial_j\,\partial_i \ln \tilde{K}_{\alpha\bar{\beta}} = -\frac{\lambda_i}{4\tau_i^2}\delta_{ij}.$$

Then, unnormalised scalar masses (4.6) and trilinear couplings (4.7) turn out to be expressible as:

$$m'^2_{\alpha\bar{\beta}} = \left[m^2_{3/2} + V_F - \bar{F}^{\bar{i}} F^j\,\partial_{\bar{i}}\,\partial_j \ln \tilde{K}_{\alpha\bar{\alpha}}\right] \tilde{K}_{\alpha\bar{\beta}},$$

$$A'_{\alpha\beta\gamma} = \frac{\bar{W}}{|W|} e^{K/2} Y_{\alpha\beta\gamma} F^i \left[K_i + \partial_i \ln\left(Y_{\alpha\beta\gamma}\right) - \partial_i \ln\left(\tilde{K}_{\alpha\bar{\alpha}}\tilde{K}_{\beta\bar{\beta}}\tilde{K}_{\gamma\bar{\gamma}}\right)\right],$$

by taking advantage of the possibility to write e.g. $\lambda_i/2\tau_i = \partial_i \ln \tilde{K}_{\alpha\bar{\alpha}}$.
In other words, these expressions can be eventually written as:

$$m'^2_{\alpha\bar{\beta}} = m^2_\alpha \tilde{K}_{\alpha\bar{\beta}}, \tag{4.69}$$

$$A'_{\alpha\beta\gamma} = \frac{\bar{W}}{|W|} e^{K/2} Y_{\alpha\beta\gamma} A_{\alpha\beta\gamma}, \tag{4.70}$$

thus making it clear that the general situation can always be brought back to the one with a diagonal Kähler matter metric without loss of generality by simply performing the canonical normalisation of the Lagrangian.

4.4 Soft Terms in Model III

Model III is suitable for the study of Supersymmetric Standard Models arising from D7 branes wrapping some of the cycles associated to the corresponding Calabi-Yau manifold. According to Subsections 4.2.2 and 4.2.3, it is worthwhile to analyse a general Kähler matter metric expressed as:

$$\tilde{K}_{\alpha\bar{\beta}} = \frac{\tau_s^{\lambda_{\alpha\bar{\beta}}}}{\tau_1^{n_{1\alpha\bar{\beta}}}\tau_2^{n_{2\alpha\bar{\beta}}}}\, k_{\alpha\bar{\beta}}. \tag{4.71}$$

Nevertheless, below it is shown that under very general assumptions, i.e. a block-diagonal matrix, the very same results can be inferred from a simpler diagonal Kähler matter metric.

4.4.1 F-terms and Gravitino Mass

Auxiliary fields vacuum expectation values as well as the gravitino mass can be computed following the standard definitions (4.9) and (4.10), recalling the vacuum expectation values (3.48) and (3.49) and the assumptions (3.44) and (3.45).

In reduced Planck units, auxiliary fields read, up to leading order corrections:

$$F^1_{\text{tot}} = -2\frac{\tau_1^{\frac{1}{2}}}{\tau_2}\,|W_0|\left[1 - \frac{1}{2}\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\!\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.72}$$

$$F^2_{\text{tot}} = -2\frac{1}{\tau_1^{\frac{1}{2}}}|W_0|\left[1 + \frac{1}{2}\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.73}$$

$$F^s_{\text{tot}} = -\frac{3}{2a_s\tau_1^{\frac{1}{2}}\tau_2}|W_0|\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right], \tag{4.74}$$

while the gravitino mass can be written, up to leading order corrections, as:

$$m_{3/2} = \frac{|W_0|}{\tau_1^{\frac{1}{2}}\tau_2}\left[1 + \frac{1}{2}\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right]. \tag{4.75}$$

Furthermore, the following identity holds true at leading order:

$$a_s\tau_s = \ln\left(\frac{1}{m_{3/2}}\right)\left[1 - \frac{\ln\bar{m}}{\ln m_{3/2}}\right], \tag{4.76}$$

where, again, $\bar{m}$ is the quantity with the dimensions of a mass:

$$\bar{m} = \frac{4}{3}a_s\tau_s\frac{|A_s|}{\tau_s^{3/2}}\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right],$$

with, evidently:

$$\ln\bar{m} \overset{\bar{m}\sim 1}{\sim} 0.$$

In conclusion, arranging the previous expressions allows to write the auxiliary fields eventually expliciting their dependence on the gravitino mass as:

$$F^1_{\text{tot}} = -2\tau_1 m_{3/2}\left[1 - \frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.77}$$

$$F^2_{\text{tot}} = -2\tau_2 m_{3/2}\left[1 - \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.78}$$

$$F^s_{\text{tot}} = -\frac{3}{2}\tau_s\frac{m_{3/2}}{\ln\left(1/m_{3/2}\right)}\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right]. \tag{4.79}$$

It is important to compare the structure of these expressions with their analogues (4.52) and (4.53) of Model II. By introducing string loop corrections, in the absence of an analogous mechanism to the Extended No-Scale Structure cancellation for auxiliary fields, these ones get Kaluza-Klein contributions which are in a delicate interplay with inverse volume and blow-up cycle corrections.

4.4.2 Diagonal Kähler Matter Metric

Now that auxiliary fields are determined and their structure is clear, the computation of soft terms is straightforward.
In particular for a diagonal Kähler matter metric of the kind:

$$\tilde{K}_\alpha = \frac{\tau_s^{\lambda_\alpha}}{\tau_1^{n_{1\alpha}}\tau_2^{n_{2\alpha}}}\, k_\alpha, \tag{4.80}$$

and a Z-term:

$$Z = \frac{\tau_s^{\lambda_z}}{\tau_1^{n_{1z}}\tau_2^{n_{2z}}}\, z, \tag{4.81}$$

expressions (4.18), (4.19) and (4.20) give the following scalar masses, trilinear parameters and $B\tilde{\mu}$-term:

$$\begin{aligned} m_\alpha^2 = {} & (1 - n_{1\alpha} - n_{2\alpha})\, m_{3/2}^2 + V_F \\ & + 2n_{1\alpha} m_{3/2}^2 \left[\frac{c_1}{\tau_1} + O\!\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] \\ & + n_{2\alpha} m_{3/2}^2 \left[\frac{c_2}{\tau_2} + O\!\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] \\ & + \frac{9\lambda_\alpha}{16} \frac{m_{3/2}^2}{\ln^2\left(1/m_{3/2}\right)} \left[1 + O\!\left(\frac{1}{a_s\tau_s}\right)\right], \end{aligned} \tag{4.82}$$

$$\begin{aligned} A_{\alpha\beta\gamma} = 3m_{3/2} \Bigg[& 1 - \frac{1}{3}\left(n_{1\alpha} + n_{1\beta} + n_{1\gamma}\right) \\ & - \frac{1}{3}\left(n_{2\alpha} + n_{2\beta} + n_{2\gamma}\right) + O\!\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\Bigg] \\ & + \left(n_{1\alpha} + n_{1\beta} + n_{1\gamma}\right) m_{3/2} \left[\frac{c_1}{\tau_1} + O\!\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] \\ & + \frac{1}{2}\left(n_{2\alpha} + n_{2\beta} + n_{2\gamma}\right) m_{3/2} \left[\frac{c_2}{\tau_2} + O\!\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] \\ & + \frac{3}{4}\left(\lambda_\alpha + \lambda_\beta + \lambda_\gamma\right) \frac{m_{3/2}}{\ln\left(1/m_{3/2}\right)} \left[1 + O\!\left(\frac{1}{a_s\tau_s}\right)\right], \end{aligned} \tag{4.83}$$

$$
\begin{aligned}
B\tilde{\mu} = \frac{Z}{\left(\tilde{K}_{H_1}\tilde{K}_{H_2}\right)^{\frac{1}{2}}} \Bigg\{ & 2m_{3/2}^2 \Bigg[1 - \frac{1}{2}\left(1 + n_{1z} + n_{2z}\right)\left(n_{1z} + n_{2z}\right) \\
& - \frac{1}{2}\left(1 - n_{1z} - n_{2z}\right)\Big(\left(n_{1H_1} + n_{1H_2}\right) + \left(n_{2H_1} + n_{2H_2}\right)\Big)\Bigg] + V_F \\
& + m_{3/2}^2 \Bigg[2n_{1z}\left(1 + n_{1z} + n_{2z} - \frac{1}{2}\left(n_{2H_1} + n_{2H_2}\right)\right) \\
& + \left(n_{1H_1} + n_{1H_2}\right)\left(1 - 2n_{1z} - n_{2z}\right)\Bigg]\left[\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] \\
& + m_{3/2}^2 \Bigg[n_{2z}\left(1 + n_{1z} + n_{2z} - \frac{1}{2}\left(n_{1H_1} + n_{1H_2}\right)\right) \\
& + \frac{1}{2}\left(n_{2H_1} + n_{2H_2}\right)\left(1 - 2n_{2z} - n_{1z}\right)\Bigg]\left[\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] \\
& + \frac{3}{4}\frac{m_{3/2}^2}{\ln\left(1/m_{3/2}\right)}\Bigg[\left(\lambda_{H_1} + \lambda_{H_2}\right)\left(1 - \left(n_{1z} + n_{2z}\right)\right) + 2\lambda_z\left(n_{1z} + n_{2z}\right) \\
& - \lambda_z\left(\left(n_{1H_1} + n_{1H_2}\right) + \left(n_{2H_1} + n_{2H_2}\right)\right)\Bigg]\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right] \\
& - \frac{9\lambda_z}{16}\frac{m_{3/2}^2}{\ln^2\left(1/m_{3/2}\right)}\left(\lambda_z - 1 - \left(\lambda_{H_1} + \lambda_{H_2}\right)\right)\left[1 + O\left(\frac{1}{a_s\tau_s}\right)\right]\Bigg\}.
\end{aligned}
\tag{4.84}
$$

The latter expressions are extremely long and articulated, but are structured in such a way as to evidence the hierarchies between the various contributions.
According to 4.32, gaugino masses take different values depending on which cycle gauge fields arise from, the apexes in brackets accounting for it in obvious notation:

$$
M_a = \begin{cases}
M_a^{(1)} & = m_{3/2}\left[1 - \dfrac{c_1}{\tau_1} + O\left(\dfrac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \dfrac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \\[2ex]
M_a^{(2)} & = m_{3/2}\left[1 - \dfrac{1}{2}\dfrac{c_2}{\tau_2} + O\left(\dfrac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \dfrac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \\[2ex]
M_a^{(s)} & = \dfrac{3}{4}\dfrac{m_{3/2}}{\ln\left(1/m_{3/2}\right)}\left[1 + O\left(\dfrac{1}{a_s\tau_s}\right)\right].
\end{cases}
\tag{4.85}
$$

Finally, physical Yukawa couplings (4.21) and the $\tilde{\mu}$-parameter (4.22) read:

$$\begin{aligned} y_{\alpha\beta\gamma} = e^{-i\theta}\, & \frac{\tau_1^{\frac{n_{1\alpha}+n_{1\beta}+n_{1\gamma}}{2}-\frac{1}{2}}\tau_2^{\frac{n_{2\alpha}+n_{2\beta}+n_{2\gamma}}{2}-1}}{\tau_s^{\frac{\lambda_\alpha+\lambda_\beta+\lambda_\gamma}{2}}\left(k_\alpha k_\beta k_\gamma\right)^{\frac{1}{2}}} \\ & \cdot\left[1+\frac{1}{2}\frac{c_1}{\tau_1}+\frac{1}{2}\frac{c_2}{\tau_2}+O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right]Y_{\alpha\beta\gamma}, \end{aligned} \tag{4.86}$$

$$\begin{aligned} \tilde{\mu} = & \frac{Z}{\left(\tilde{K}_{H_1}\tilde{K}_{H_2}\right)^{\frac{1}{2}}}\Bigg\{m_{3/2}\Bigg[1-n_{1z}-n_{2z} \\ & + n_{1z}\left(\frac{c_1}{\tau_1}+O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right)+\frac{n_{2z}}{2}\left(\frac{c_2}{\tau_2}+O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right)\Bigg] \\ & + \frac{3\lambda_z}{4}\frac{m_{3/2}}{\ln\left(1/m_{3/2}\right)}\left[1+O\left(\frac{1}{a_s\tau_s}\right)\right]\Bigg\}. \end{aligned} \tag{4.87}$$

D7s on Blow-ups

The Minimal Supersymmetric Standard Model can be supported on D7 branes wrapping the blow-up cycle, in the same way as for Model II. According to (4.38), the values for the exponents which determine the Kähler matter metric are:

$$n_{1\alpha}\equiv 1/3, \qquad n_{2\alpha}\equiv 2/3, \qquad \lambda_\alpha\equiv 1/3, \tag{4.88}$$

the same values holding for the Z-term too, as argued in Subsection 4.2.4.

The corresponding soft terms undergo the same kind of cancellations which characterises Model II. Indeed soft terms are up to leading order in physical units:

$$m_\alpha \simeq \frac{\sqrt{3}}{4}\frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}, \tag{4.89}$$

$$M_a \simeq \frac{3}{4}\frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}, \tag{4.90}$$

$$A_{\alpha\beta\gamma} \simeq \frac{3}{4}\frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}, \tag{4.91}$$

$$B\tilde{\mu} \simeq \frac{z}{4\left(k_{H_1}k_{H_2}\right)^{\frac{1}{2}}}\left(\frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}\right)^2, \tag{4.92}$$

with:

$$y_{\alpha\beta\gamma} \simeq e^{-i\theta}\frac{Y_{\alpha\beta\gamma}}{\tau_s^{\frac{1}{2}}\left(k_\alpha k_\beta k_\gamma\right)^{\frac{1}{2}}}, \tag{4.93}$$

$$\tilde{\mu} \simeq \frac{z}{4\left(k_{H_1} k_{H_2}\right)^{\frac{1}{2}}} \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)}. \tag{4.94}$$

Although these parameters receive generally non-vanishing Kaluza-Klein corrections, since they turn out to be subleading contributions, the soft terms essentially correspond to the ones of Model II. In particular, they are universally of order $\tilde{m}_{\text{soft}}$.

D7s on Large Cycles

If the Standard Model is estabilished on D7 branes wrapping the large cycles, soft terms show an interesting diversification which gives rise to a more complicated phenomenology. According to Subsection 4.2.3, the most general situation as possible in Model III shows the presence of different kinds of fields, each of which associated to special exponents of the Kähler matter metric:

$$A-\text{fields}: \quad \begin{cases} A': & n_{1A'} = 0, \quad n_{2A'} = 1, \quad \lambda_{A'} = 0, \\ A'': & n_{1A''} = 1, \quad n_{2A''} = 0, \quad \lambda_{A''} = 0, \\ A''': & n_{1A'''} = 0, \quad n_{2A'''} = 1, \quad \lambda_{A'''} = 0, \end{cases} \tag{4.95}$$

$$\Phi-\text{fields}: \quad \begin{cases} \Phi': & n_{1\Phi'} = 0, \quad n_{2\Phi'} = 0, \quad \lambda_{\Phi'} = 0, \\ \Phi'': & n_{1\Phi''} = 0, \quad n_{2\Phi''} = 0, \quad \lambda_{\Phi''} = 0, \end{cases} \tag{4.96}$$

$$I-\text{fields}: \qquad I: \quad n_{1I} = 0, \quad n_{2I} = 1/2, \quad \lambda_I = 0. \tag{4.97}$$

The Z-term is assumed to be scaled as the geometrical mean of the Kähler matter metric elements correponding to Higgs fields (see Subsection 4.2.4).
Evidently, as regards the computation of soft terms, A'''-fields can be treated as A'-fields as well as Φ'- and Φ''-fields can be studied globally as Φ-fields.
Then, the possible kinds of scalar fields are $N_0 = 4$, which means that the expected parameters are:

- 4 different scalar masses, one for each kind of field:

$$N_{\text{sc. masses}} = N_0 \overset{N_0=4}{=} 4;$$

- 20 different trilinear couplings, i.e. the number of possible matchings of 3 elements among the possible N_0 families of fields:

$$N_{\text{tril. couplings}} = \binom{N_0+3-1}{3} \overset{N_0=4}{=} 20;$$

- 10 possible B-terms, corresponding to the number of possible couples of 2 Higgses - in the Minimal Supersymmetric Standard Model - with both of the Higgses possibly belonging to any of the N_0 families:

$$N_{B-\text{terms}} = \binom{N_0 + 2 - 1}{2} \overset{N_0=4}{=} 10.$$

Under these premises, soft breaking terms can be finally determined and classified according to their order of magnitude.

Scalar masses can be sorted into two families as two different orders of magnitude can emerge. Indeed, A-fields turn out to have masses scaled by the factors $\sqrt{|c_1|/\tau_1}$ or $\sqrt{|c_2|/\tau_2}$ with respect to $m_{3/2}$, being:

$$m_{A'}^2 = m_{3/2}^2 \left[\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] + V_F, \tag{4.98}$$

$$m_{A''}^2 = 2\, m_{3/2}^2 \left[\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] + V_F, \tag{4.99}$$

while Φ- and I-fields are heavier as they have masses comparable to the gravitino mass:

$$m_\Phi^2 = m_{3/2}^2 + V_F, \tag{4.100}$$

$$m_I^2 = \frac{1}{2}\, m_{3/2}^2 \left[1 + \frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] + V_F. \tag{4.101}$$

It is worthwhile to notice that the sign of c_1 and c_2 is crucial. If negative, such parameters would severely complicate the expansions which have lead to the observable matter fields expressions. Therefore, if not otherwise claimed, they are to be considered positive.

Trilinear terms can be divided into two classes too, weak and strong parameters. As a matter of fact, interactions among only A-scalars turn out to be scaled as c_1/τ_1 and/or

c_2/τ_2 with respect to the scale $m_{3/2}$, i.e. couplings are weak:

$$A_{A'A'A'} = \frac{3}{2}\, m_{3/2} \left[\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.102}$$

$$A_{A'A'A''} = m_{3/2} \left[\frac{c_1}{\tau_1} + \frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.103}$$

$$A_{A'A''A''} = 2\, m_{3/2} \left[\frac{c_1}{\tau_1} + \frac{1}{4}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.104}$$

$$A_{A''A''A''} = 3\, m_{3/2} \left[\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.105}$$

instead, all of the parameters involving at least one among Φ- or I-fields are of the same order of magnitude as $m_{3/2}$. In particular, terms which involve two A-fields are:

$$A_{A'A'\Phi} = m_{3/2} \left[1 + \frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.106}$$

$$A_{A''A''\Phi} = m_{3/2} \left[1 + 2\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.107}$$

$$A_{A'A''\Phi} = m_{3/2} \left[1 + \frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.108}$$

$$A_{A'A'I} = \frac{1}{2}\, m_{3/2} \left[1 + \frac{5}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.109}$$

$$A_{A''A''I} = \frac{1}{2}\, m_{3/2} \left[1 + 4\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.110}$$

$$A_{A'A''I} = \frac{1}{2} m_{3/2} \left[1 + 2\frac{c_1}{\tau_1} + \frac{3}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.111}$$

while those involving one only A-field read:

$$A_{A'\Phi\Phi} = 2 m_{3/2} \left[1 + \frac{1}{4}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.112}$$

$$A_{A''\Phi\Phi} = 2 m_{3/2} \left[1 + \frac{1}{2}\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.113}$$

$$A_{A'\Phi I} = \frac{3}{2}\, m_{3/2} \left[1 + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.114}$$

$$A_{A''\Phi I} = \frac{3}{2}\, m_{3/2} \left[1 + \frac{2}{3}\frac{c_1}{\tau_1} + \frac{1}{6}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.115}$$

$$A_{A'II} = m_{3/2} \left[1 + \frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.116}$$

$$A_{A''II} = 2m_{3/2} \left[1 + \frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right]. \tag{4.117}$$

Finally, terms involving only Φ- and I-fields are:

$$A_{\Phi\Phi\Phi} = 3m_{3/2} \left[1 + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.118}$$

$$A_{\Phi\Phi I} = \frac{5}{2}\, m_{3/2} \left[1 + \frac{1}{10}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.119}$$

$$A_{\Phi II} = 2m_{3/2} \left[1 + \frac{1}{4}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right], \tag{4.120}$$

$$A_{III} = \frac{3}{2}\, m_{3/2} \left[1 + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right]. \tag{4.121}$$

The coupling $B\tilde{\mu}$ is characterised by an analogous structure. If the Higgses are both A-fields, it is scaled by the factors c_1/τ_1 and/or c_2/τ_2, indeed it can take the values, with obvious notation:

$$B\tilde{\mu}\big|_{\{A',A'\}} = \eta \cdot \left[V_F + m_{3/2}^2 \left(\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right) \right], \tag{4.122}$$

$$B\tilde{\mu}\big|_{\{A',A''\}} = \eta \cdot \left[V_F + m_{3/2}^2 \left(\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right) \right], \tag{4.123}$$

$$B\tilde{\mu}\big|_{\{A'',A''\}} = \eta \cdot \left[V_F + 2m_{3/2}^2 \left(\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2} \right) \right) \right], \tag{4.124}$$

otherwise, when one only A-field is present, it can be:

$$B\tilde{\mu}\big|_{\{A',\Phi\}} = \eta\cdot\left[V_F + \frac{3}{4}\,m_{3/2}^2\left(1+\frac{c_2}{\tau_2}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right)\right], \tag{4.125}$$

$$B\tilde{\mu}\big|_{\{A'',\Phi\}} = \eta\cdot\left[V_F + \frac{3}{4}\,m_{3/2}^2\left(1+2\,\frac{c_1}{\tau_1}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right)\right], \tag{4.126}$$

$$B\tilde{\mu}\big|_{\{A',I\}} = \eta\cdot\left[V_F + \frac{5}{16}\,m_{3/2}^2\left(1+3\,\frac{c_2}{\tau_2}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right)\right], \tag{4.127}$$

$$B\tilde{\mu}\big|_{\{A'',I\}} = \eta\cdot\left[V_F + \frac{5}{16}\,m_{3/2}^2\left(1+4\,\frac{c_1}{\tau_1}+\frac{c_2}{\tau_2}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right)\right], \tag{4.128}$$

and when only Φ- and I-fields are involved it can read:

$$B\tilde{\mu}\big|_{\{\Phi,\Phi\}} = \eta\cdot\left[V_F + 2\,m_{3/2}^2\right], \tag{4.129}$$

$$B\tilde{\mu}\big|_{\{\Phi,I\}} = \eta\cdot\left[V_F + \frac{21}{16}\,m_{3/2}^2\left(1+\frac{1}{3}\frac{c_2}{\tau_2}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right)\right], \tag{4.130}$$

$$B\tilde{\mu}\big|_{\{I,I\}} = \eta\cdot\left[V_F + \frac{3}{4}\,m_{3/2}^2\left(1+\frac{c_2}{\tau_2}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right)\right]. \tag{4.131}$$

Of course the overall coefficient η is defined as:

$$\eta = \frac{z}{(k_{H_1}k_{H_2})^{\frac{1}{2}}}.$$

Gaugino masses turn out to be around $m_{3/2}$, with subleading terms depending on the cycle gauge fields come from. Indeed they are:

$$M_a^{(1)} = m_{3/2}\left[1-\frac{c_1}{\tau_1}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right], \tag{4.132}$$

$$M_a^{(2)} = m_{3/2}\left[1-\frac{1}{2}\frac{c_2}{\tau_2}+O\Big(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2},\frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\Big)\right]. \tag{4.133}$$

Physical Yukawa coupings show a more complex hierarchy. Writing them as:

$$y_{\alpha\beta\gamma} = \tau_1^{\frac{n_{1\alpha}+n_{1\beta}+n_{1\gamma}}{2}-\frac{1}{2}}\tau_2^{\frac{n_{2\alpha}+n_{2\beta}+n_{2\gamma}}{2}-1}\tau_s^{-\frac{\lambda_\alpha+\lambda_\beta+\lambda_\gamma}{2}}y'_{\alpha\beta\gamma},$$

with:

$$y'_{\alpha\beta\gamma} \equiv \frac{e^{-i\theta}}{(k_\alpha k_\beta k_\gamma)^{\frac{1}{2}}} \left[1 + \frac{1}{2}\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right] Y_{\alpha\beta\gamma},$$

for A-fields only, they are roughly of the same order of magnitude as unnormalised couplings as they are:

$$y_{A'A'A'} = \tau_1^{-1/2}\tau_2^{1/2}\ y'_{A'A'A'}, \tag{4.134}$$

$$y_{A'A'A''} = \ y'_{A'A'A''}, \tag{4.135}$$

$$y_{A'A''A''} = \tau_1^{1/2}\tau_2^{-1/2}\ y'_{A'A''A''}, \tag{4.136}$$

$$y_{A''A''A''} = \tau_1\tau_2^{-1}\ y'_{A''A''A''}, \tag{4.137}$$

otherwise they turn out to be scaled. Couplings which involve two A-fields read:

$$y_{A'A'\Phi} = \tau_1^{-1/2}\ y'_{A'A'\Phi}, \tag{4.138}$$

$$y_{A''A''\Phi} = \tau_1^{1/2}\tau_2^{-1}\ y'_{A''A''\Phi}, \tag{4.139}$$

$$y_{A'A''\Phi} = \tau_2^{-1/2}\ y'_{A'A''\Phi}, \tag{4.140}$$

$$y_{A'A'I} = \tau_1^{-1/2}\tau_2^{1/4}\ y'_{A'A'I}, \tag{4.141}$$

$$y_{A''A''I} = \tau_1^{1/2}\tau_2^{-3/4}\ y'_{A''A''I}, \tag{4.142}$$

$$y_{A'A''I} = \tau_2^{-1/4}\ y'_{A'A''I}, \tag{4.143}$$

while those involving one only A-field are:

$$y_{A'\Phi\Phi} = \tau_1^{-1/2}\tau_2^{-1/2}\ y'_{A'\Phi\Phi}, \tag{4.144}$$

$$y_{A''\Phi\Phi} = \tau_2^{-1}\ y'_{A''\Phi\Phi}, \tag{4.145}$$

$$y_{A'\Phi I} = \tau_1^{-1/2}\tau_2^{-1/4}\ y'_{A'\Phi I}, \tag{4.146}$$

$$y_{A''\Phi I} = \tau_2^{-3/4}\ y'_{A''\Phi I}, \tag{4.147}$$

$$y_{A'II} = \tau_1^{-1/2}\ y'_{A'II}, \tag{4.148}$$

$$y_{A''II} = \tau_2^{-1/2}\ y'_{A''II}, \tag{4.149}$$

and those involving only Φ- and I-fields:

$$y_{\Phi\Phi\Phi} = \tau_1^{-1/2}\tau_2^{-1}\ y'_{\Phi\Phi\Phi}, \tag{4.150}$$

$$y_{\Phi\Phi I} = \tau_1^{-1/2}\tau_2^{-3/4}\ y'_{\Phi\Phi I}, \tag{4.151}$$

$$y_{\Phi II} = \tau_1^{-1/2}\tau_2^{-1/2}\ y'_{\Phi II}, \tag{4.152}$$

$$y_{III} = \tau_1^{-1/2}\tau_2^{-1/4}\ y'_{III}. \tag{4.153}$$

This particular structure affects sensibly the effective couplings $t_{\alpha\beta\gamma} = y_{\alpha\beta\gamma}A_{\alpha\beta\gamma}$ between triplets of scalars. The analysis of such couplings is left to Chapter 5 referred to the specific situations.

Finally, the parameter $\tilde{\mu}$ is again suppressed as c_1/τ_1 and/or c_2/τ_2 if only A-fields are involved, as:

$$\tilde{\mu}\big|_{\{A',A'\}} = \eta\cdot\frac{m_{3/2}}{2}\left[\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.154}$$

$$\tilde{\mu}\big|_{\{A',A''\}} = \eta\cdot\frac{m_{3/2}}{2}\left[\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.155}$$

$$\tilde{\mu}\big|_{\{A'',A''\}} = \eta\cdot m_{3/2}\left[\frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.156}$$

otherwise it is around $m_{3/2}$. Indeed for a single A-field it can be:

$$\tilde{\mu}\big|_{\{A',\Phi\}} = \eta\cdot\frac{m_{3/2}}{2}\left[1 + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.157}$$

$$\tilde{\mu}\big|_{\{A'',\Phi\}} = \eta\cdot\frac{m_{3/2}}{2}\left[1 + \frac{c_1}{\tau_1} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.158}$$

$$\tilde{\mu}\big|_{\{A',I\}} = \eta\cdot\frac{m_{3/2}}{4}\left[1 + \frac{3}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.159}$$

$$\tilde{\mu}\big|_{\{A'',I\}} = \eta\cdot\frac{m_{3/2}}{4}\left[1 + 2\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.160}$$

and for Φ- and I-fields it can read:

$$\tilde{\mu}\big|_{\{\Phi,\Phi\}} = \eta \cdot m_{3/2}, \tag{4.161}$$

$$\tilde{\mu}\big|_{\{\Phi,I\}} = \eta \cdot \frac{3}{4} m_{3/2}\left[1 + \frac{1}{6}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right], \tag{4.162}$$

$$\tilde{\mu}\big|_{\{I,I\}} = \eta \cdot \frac{m_{3/2}}{2}\left[1 + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}, \frac{\xi}{\tau_1^{\frac{1}{2}}\tau_2}\right)\right]. \tag{4.163}$$

These possibilities are coherent with what happens to the term $B\tilde{\mu}$.

In conclusion, this framework gives rise to soft terms of three possible orders of magnitude, restoring the angled brackets for vacuum expectation values and in physical units:

- soft masses of A-fields undergo leading order cancellations which set their values around the scale:
$$m'_{\text{soft}} \simeq \left(\frac{\kappa}{\langle\mathcal{V}\rangle}\right)^{1/3} m_{3/2}, \tag{4.164}$$
κ being a constant standing roughly for both $|c_1|^{3/2}$ and $|c_2|^{3/2}$;
- soft couplings involving only A-fields undergo significant cancellations which set their values around the scale:
$$m''_{\text{soft}} \simeq \left(\frac{\kappa}{\langle\mathcal{V}\rangle}\right)^{2/3} m_{3/2}; \tag{4.165}$$
- soft terms which concern Φ- and/or I-fields and gaugino masses at leading order are around the scale of the gravitino mass:
$$m'''_{\text{soft}} \simeq m_{3/2}. \tag{4.166}$$

4.4.3 General Kähler Matter Metric

The general model takes into account a general Kähler matter metric which is block-diagonal, where every block corresponds to a different group of scalar fields, i.e. to a special dependence on Kähler moduli. In particular:

- for D7 branes wrapping the blow-up cycle, the block is just one, i.e. the metric is a general non-diagonal matrix, where every element shares the same dependence as the others, as in Subsection 4.2.2;

- for D7 branes on large cycles, there is a block for every of the families of fields which are involved, as described in Subsection 4.2.3 by the relations (4.44).

It is assumed that the necessary conditions hold for a diagonal Higgs sector.

Under these premises, the Kähler matter metric can be conveniently written by expliciting the exponents of every block after choosing e.g. those coming from diagonal elements as the representative ones:

$$\tilde{K}_{\alpha\bar{\beta}} = \frac{\tau_s^{\lambda_{\alpha\bar{\alpha}}}}{\tau_1^{n_{1\alpha\bar{\alpha}}}\tau_2^{n_{2\alpha\bar{\alpha}}}}\, k_{\alpha\bar{\beta}}. \tag{4.167}$$

Setting $\lambda_{s_\alpha} \equiv \lambda_{\alpha\bar{\alpha}}$, $\lambda_{1_\alpha} \equiv -n_{1\alpha\bar{\alpha}}$ and $\lambda_{2_\alpha} \equiv -n_{2\alpha\bar{\alpha}}$, it is useful to express it as:

$$\tilde{K}_{\alpha\bar{\beta}} = \left(\prod_{l=1,2,s} \tau_l^{\lambda_{l_\alpha}}\right) k_{\alpha\bar{\beta}}.$$

Indeed this form allows to write the simple relations:

$$\partial_i \tilde{K}_{\alpha\bar{\beta}} = \frac{\lambda_{i_\alpha}}{2\tau_i}\tilde{K}_{\alpha\bar{\beta}}, \qquad \partial_j\,\partial_i\,\tilde{K}_{\alpha\bar{\beta}} = -\frac{\lambda_{i_\alpha}}{4\tau_i\tau_j}\left(\delta_{ij} - \lambda_{j_\alpha}\right)\tilde{K}_{\alpha\bar{\beta}},$$

and:

$$\left(\partial_i \ln \tilde{K}_{\alpha\bar{\beta}}\right)\tilde{K}_{\alpha\bar{\beta}} = \frac{\lambda_{i_\alpha}}{2\tau_i}\tilde{K}_{\alpha\bar{\beta}}, \qquad \left(\partial_j\,\partial_i \ln \tilde{K}_{\alpha\bar{\beta}}\right)\tilde{K}_{\alpha\bar{\beta}} = -\frac{\lambda_{i_\alpha}}{4\tau_i^2}\delta_{ij}\tilde{K}_{\alpha\bar{\beta}},$$

which turn out to be extremely important in bringing the problem back to the one with a diagonal Kähler matter metric.

Indeed it is easy to show that, taking advantage of this way of writing, unnormalised scalar masses (4.6) and unnormalised trilinear couplings (4.7) can be expressed as:

$$\left(m'^2\right)_{\alpha\bar{\beta}} = \left[\left(m_{3/2}^2 + V_F\right) - \bar{F}^{\bar{i}}F^j\,\partial_{\bar{i}}\,\partial_j \ln \tilde{K}_{\alpha\bar{\alpha}}\right]\tilde{K}_{\alpha\bar{\beta}},$$

$$A'_{\alpha\beta\gamma} = \frac{\bar{W}}{|W|}e^{K/2}Y_{\alpha\beta\gamma}F^i\left[K_i + \partial_i \ln Y_{\alpha\beta\gamma} - \partial_i \ln\left(\tilde{K}_{\alpha\bar{\beta}}\tilde{K}_{\beta\bar{\beta}}\tilde{K}_{\gamma\bar{\gamma}}\right)\right],$$

i.e., more clearly:

$$\left(m'^2\right)_{\alpha\bar{\beta}} = m_\alpha^2\tilde{K}_{\alpha\bar{\beta}}, \tag{4.168}$$

$$A'_{\alpha\beta\gamma} = \frac{\bar{W}}{|W|}e^{K/2}Y_{\alpha\beta\gamma}A_{\alpha\beta\gamma}, \tag{4.169}$$

where m_α^2 and $A_{\alpha\beta\gamma}$ are precisely the values computed before for a diagonal Kähler matter metric.

This fact proves that the model with a general Kähler matter metric can always be reduced to the one with a diagonal matrix without loss of generality.

Chapter 5

Hidden Sector and Soft Term Analysis

This Chapter is intended to provide an analysis of the main predictions of Models II and III with the Minimal Supersymmetric Standard Model supported on Calabi-Yau blow-up or large cycles. Particular attention is devoted to the latter construction since it is the most consistent part of the thesis.
The Chapter is structured as follows. At first, the main experimental and phenomenological bounds which constrain the values of hidden sector masses and soft terms are summarised and briefly commented. Then, an instructive account of the reliability of modellings with the Minimal Supersymmetric Standard Model supported on the blow-up cycles as in Model II and III is reported for the sake of completeness, although it is present in the literature dedicated to Model II. Finally, a critical analysis of the general predictions arising from the construction of Model III with the Minimal Supersymmetric Standard Model supported on large cycles is performed.

Note on the Expression of Masses and Couplings

This thesis is focused on the investigation on possible frameworks which account for reliable modellings of the hidden sector and the corresponding soft terms and does not claim the prediction of accurate parameters for detailed comparisons with real experimental data. It is purposed to determine just the orders of magnitude of masses and couplings involved.
Then, in the general hypothesis of no fine-tuned parameters W_0, a_s, ξ, C_1, C_2 and C_W, they are all considered to be roughly around unity. In this way, in order to render the physical analysis easier, it is legitimate to express all of the factors scaling with $\langle \mathcal{V} \rangle$ via a ratio between the gravitino mass $m_{3/2}$ and the reduced Planck mass M_P. Indeed, at leading order the relationship holds true $m_{3/2}/M_P \simeq |W_0| / \langle \mathcal{V} \rangle$. Moreover, physical units are employed.

5.1 Phenomenological Bounds

A reliable Supersymmetric Standard Model which arises from low energy supergravity with spontaneous supersymmetry breaking in a hidden sector must describe the evolution of the Universe in its entirety from Big Bang to present day observations. This ambitious task sets constraints to the modelling coming from both Particle Physics and Cosmology.

The main bounds on the hidden sector and soft breaking terms are summarised and briefly commented below.

- The Cosmological Moduli Problem (CMP) generally requires very heavy moduli with masses around at least the 10^2 TeV scale [36] [37], which can be defined as the reference smallest hidden sector mass scale:

  $$m_{\text{mod}}^{\text{min}} \sim 10^5 \,\text{GeV}. \tag{5.1}$$

  Indeed, it can be argued that string moduli expectation values after inflation are around the Planck scale and, if not sufficiently heavy, they could either decay after the Big Bang nucleosynthesis, thus spoiling its subtle but very successful predictions, or be still present in nowadays Universe, overclosing it. Some solutions to this problem have been proposed in the literature (for instance in Ref. [38]).

- The Hierarchy Problem is solved naturally in supersymmetric extensions of the Standard Model if soft masses are roughly around the 1 TeV scale [2] [39]:

  $$m_{\text{soft}} \sim 10^3 \,\text{GeV}.$$

  However, this is not a very restrictive bound, as shown e.g. in Ref. [40]. There, High-Scale Supersymmetric Models, i.e. in which all supersymmetric particles have masses around a common scale m_{soft} unrelated to the weak scale, are shown to be acceptable from this point of view as long as the soft mass scale is smaller than around 10^{11} GeV, which thus can be identified as the reference largest soft term scale:

  $$m_{\text{soft}}^{\text{max}} \sim 10^{11} \,\text{GeV}. \tag{5.2}$$

- The magnitude of Standard Model gauge couplings g_{SM} requires that, if τ_{SM} is the modulus on whose cycle the Minimal Supersymmetric Standard Model is supported, then its vacuum expectation value must be generally of order:

  $$\langle \tau_{\text{SM}} \rangle \sim 10^2. \tag{5.3}$$

A wide variety of further conditions emerge from Particle Physics and Cosmology implications, both model dependent and model independent. However, for a general overview on a reliable Supersymmetric Standard Model, the three bounds listed above are typically the most selective ones which String Phenomenology must tackle.

5.2 MSSM on Blow-up Cycle

Models II and III allow the possibility to build the Minimal Supersymmetric Standard Model on the blow-up cycle, controlled by the modulus τ_s. In the natural hypothesis of no fine-tuned parameters, a reasonable value of the overall volume vacuum expectation value, essentially controlled by τ_b and $\tau_1^{1/2}\tau_2$ respectively, is set by both the conditions of soft masses around $m_{\rm soft} \sim 1\,\text{TeV}$ and gauge couplings around $g_{\rm SM} \sim 10^{-1}$ to values roughly of order:

$$\langle \mathcal{V} \rangle \sim 10^{13}.$$

Then, the string scale M_S and the Kaluza-Klein scale M_{KK} can be estimated as:

$$M_S \sim 10^{12}\,\text{GeV}, \tag{5.4}$$

$$M_{KK} \sim 10^{9}\,\text{GeV}. \tag{5.5}$$

In more intuitive terms, such modellings require a gravitino mass roughly around:

$$m_{3/2} \sim 10^{5}\,\text{GeV}. \tag{5.6}$$

In both models the physical analysis of the mass spectrum and soft couplings is very similar, as emerges from Chapters 3 and 4 analysis. Then it is convenient to treat both of them together.

5.2.1 Hidden Sector

In Model II and in Model III, the hidden sector is represented by string moduli. Actually it is only the Kähler moduli to cause the spontaneous breaking of supersymmetry.

In both cases, Kähler moduli give origin to principally three groups of real scalar fields:

- the Kähler modulus controlling the blow-up cycle determines the presence of two heavy real scalar fields ϕ and θ_ϕ with masses around the $10^4\,\text{TeV}$ scale:

$$m_\phi \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^4\,\text{TeV}, \tag{5.7}$$

$$m_{\theta_\phi} \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^4\,\text{TeV}, \tag{5.8}$$

  according to (3.30) and (3.36) as well as (3.58) and (3.67);

- the Kähler moduli controlling the overall volume give rise with their real parts to light real scalar fields, ζ - actually present only in Model III - and χ, with masses around 1 MeV or even lighter:

$$m_\chi \sim \left(\frac{m_{3/2}/M_P}{\ln\left(M_P/m_{3/2}\right)}\right)^{1/2} m_{3/2} \sim 1\,\text{MeV}, \tag{5.9}$$

$$m_\zeta \sim \left(\frac{m_{3/2}}{M_P}\right)^{2/3} m_{3/2} \sim 0.1\,\text{MeV}, \tag{5.10}$$

according to (3.29) as well as (3.57) and (3.101), and to massless non-interacting scalar fields, θ_ζ - only in Model III - and θ_χ:

$$m_{\theta_\chi} = 0\,\text{GeV}, \tag{5.11}$$

$$m_{\theta_\zeta} = 0\,\text{GeV}, \tag{5.12}$$

according to (3.35) as well as (3.66) and (3.65).

The axio-dilaton and complex structure moduli fields, which have been ignored so far as a consequence of No-Scale Structure, in vicinity of the ground state can be canonically normalised and, in the large volume scenario, in a very rough estimate have masses around the scale:

$$m_S \sim m'_\alpha \sim g_S^2\, m_{3/2} \sim 1\,\text{TeV}, \tag{5.13}$$

as shown in Ref. [11].

5.2.2 Soft Terms

Soft breaking term masses turn out to be exatcly identical in both models at leading order.
Indeed, according to expressions (4.62) and (4.63) as well as (4.89) and (4.90), soft scalar masses and gaugino masses are universally given by, respectively:

$$m_\alpha \simeq \frac{\sqrt{3}}{4} \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)} \sim 1\,\text{TeV}, \tag{5.14}$$

$$M_a \simeq \frac{3}{4} \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)} \sim 1\,\text{TeV}. \tag{5.15}$$

Differences between the two models emerge only in subleading corrections.

All of the effective soft couplings, neglecting an overall phase and assuming the unnormalised Yukawa couplings to around unity, turn out to take values around the 1 TeV scale:

$$t_{\alpha\beta\gamma} = y_{\alpha\beta\gamma} A_{\alpha\beta\gamma} \sim \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)} Y_{\alpha\beta\gamma} \sim 1\,\text{TeV}\ Y_{\alpha\beta\gamma}, \tag{5.16}$$

$$B\tilde{\mu} \sim \frac{m_{3/2}^2}{\ln^2\left(M_P/m_{3/2}\right)} \sim (1\,\text{TeV})^2\,, \tag{5.17}$$

according to (4.64), (4.65), (4.66) and (4.91), (4.92), (4.93), while the $\tilde{\mu}$-parameter reads:

$$\tilde{\mu} \sim \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)} \sim 1\,\mathrm{TeV}, \tag{5.18}$$

according to (4.67) and (4.94).

5.2.3 Mass Spectrum Analysis

The reciprocal hierarchies between masses are exemplified in Fig. 5.1.

As regards the hidden sector, the previous analysis readily evidences the dangerous presence of very light interacting moduli, thus giving rise to the Cosmological Moduli Problem. More specifically, the situation can be summarised as follows.

- The axio-dilaton S and complex structure moduli $U^{\alpha'}$ are interacting moduli with masses smaller than 100 TeV, then in principle they could give rise to the Cosmological Moduli Problem. In spite of this, however, Ref. [20] and Ref. [11] show that the scalar potential which governs their dynamics provides a trapping mechanism thanks to which they 'prefer' to essentially sit at their minima, thus avoiding any cosmological problem.

- Heavy moduli ϕ and θ_ϕ have masses around the 10^4 TeV scale, i.e. far above 100 TeV, and are therefore free from the Cosmological Moduli Problem.

- Light moduli ζ and χ have very small masses, around or smaller than 1 MeV. This is a serious problem, indeed such fields would dramatically give origin to the Cosmological Moduli Problem. A possible solution to this inconsistency could be the dilution by a low-energy period of thermal inflation, as investigated e.g. in Ref. [38].

- Massless moduli θ_ζ and θ_χ are free from cosmological problems as they are non-interacting fields.

As concerns the soft breaking terms, instead, the models seem to be quite reliable. Indeed, as a matter of fact:

- sleptons, squarks, Higgses and gauginos have all masses around 1 TeV and thus could both explain why they have never been observed so far, of course except for a Higgs field, and at the same time save naturally the hierarchy problem solution without fine tuning;

- soft couplings and the $\tilde{\mu}$-term are all around 1 TeV too.

A peculiar characteristic of such models is the 'small' hiearchy between the gravitino mass $m_{3/2}$ and all the soft breaking terms by the factor $\ln M_P/m_{3/2}$, which allows the gravitino to be at least one order of magnitude or more heavier than Standard Model superpartners.

To conclude, these modellings are very attractive because of their general apparent naturalness, indeed they could possibily be in agreement with both Standard Cosmology and predict reliable Beyond the Standard Model Physics without the need of fine-tuned parameters.

The only but fatal exception is the unavoidable presence of the Cosmological Moduli Problem associated to the moduli controlling the overall compactification volume. This issue actually seems to require very particular mechanisms in order to avoid inconsistencies, thus appearently spoiling the models of their naturalness, unless the discovery of further motivations to describe early post-inflation Universe in such a way as to solve the Cosmological Moduli Problem.

Further discussion and more accurate explanations can be found in the guiding Ref. [20] and Ref. [21].

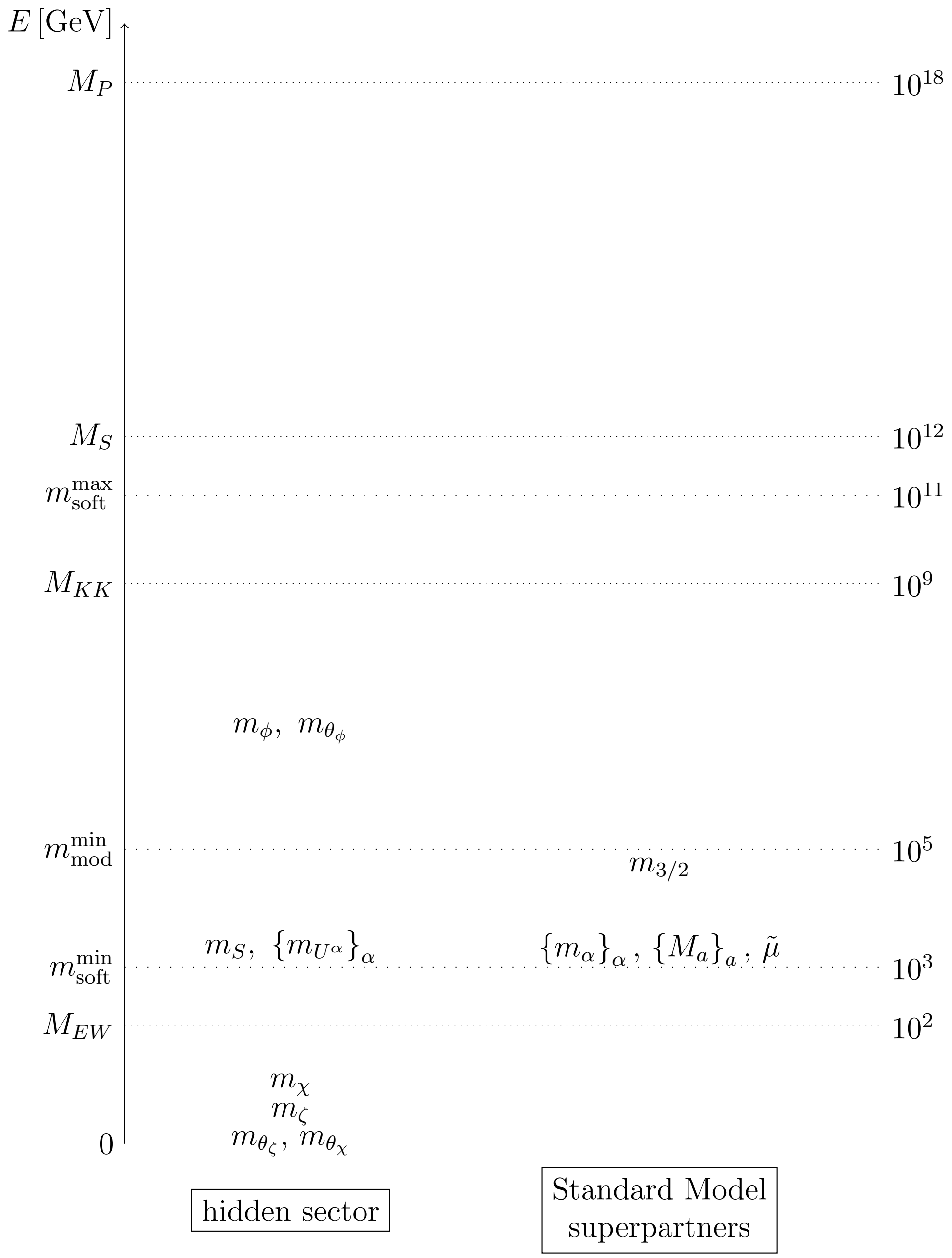

Figure 5.1. Typical mass scales for hidden sector fields and Standard Model supersymmetric partners in Models II and III, with the Minimal Supersymmetric Standard Model supported on the blow-up cycle, in realisations with $m_{3/2} \simeq 10^5$ GeV and no fine-tuned parameters. Curly brackets indicate a family of degenerate fields. The graph scale is logarithmic and only aims at providing a general idea of the reciprocal hierarchies.

5.3 Remarks about MSSM on Large Cycles

The study of Model III in Chapters 3 and 4 with the Minimal Supersymmetric Standard Model supported on large cycles evidences a fine structure in the particle mass spectrum and a large variety of soft term couplings.
The most general construction would be the one with the Minimal Supersymmetric Standard Model supported on both of the cycles associated to the fields τ_1 and τ_2. However, in this case a subtle unnaturalness feature can emerge.

As a matter of fact, the Kaluza-Klein parameters c_1 and c_2 are proportional to the string coupling g_S, while the winding parameter C_W is not, being actually $c_1 = g_S\, C_1$ and $c_2 = g_S\, C_2$. Then, in a likely situation with a string coupling g_S around $g_S \sim 10^{-1}$, according to the minimum conditions (3.103) and (3.104), a slight anisotropy in the compactification volume which has been ignored so far actually becomes relevant due to the necessary smallness of the wrapped cycle volumes, needed in such realisations.
More precisely, the two following situations do emerge assuming the natural values $|C_1| \sim |C_2| \sim |C_W| \sim 1$ with a small compactification volume.

- If C_W is positive, then the vacuum expectation value of the field τ_1 approximately reads:
$$\langle\tau_1\rangle^{3/2}\big|_{C_W>0} \simeq 2\, g_S^2\, \frac{C_1^2}{C_W}\, \langle\mathcal{V}'\rangle\, .$$
Then the Minimal Supersymmetric Standard Model can be supported naturally only on the cycle controlled by the Kähler modulus τ_1. Indeed, the vacuum expectation value of this field is set straightforwardly around $\langle\tau_1\rangle \sim 10^2$ by the Standard Model gauge coupling condition. This fact in turn entails the vacuum expectation value $\langle\tau_2\rangle \sim 10^4$, with a compactification volume set around the value $\langle\mathcal{V}\rangle \sim 10^5$. Evidently, the arising of the Standard Model fields from the cycle controlled by τ_2 is forbidden.
In such a delicate set-up, a direct inspection shows that the study of the scalar potential as outlined in Chapter 3 is still valid because the relevant conditions under which it fixes its minimum and determines the field ζ mass are satisfied, thus confirming the validity of the related analysis.

- If C_W is negative, then the vacuum expectation value of the field τ_1 approximately reads:
$$\langle\tau_1\rangle^{3/2}\big|_{C_W<0} \simeq 2\, \frac{|C_W|}{g_S^2 C_2^2}\, \langle\mathcal{V}'\rangle\, .$$
Then, reversing the previous situation, the Minimal Supersymmetric Standard Model can be supported naturally only on the cycle controlled by the field τ_2. Its vacuum expectation value is set of course around $\langle\tau_2\rangle \sim 10^2$ by the Standard Model gauge coupling condition. This estimate in turn entails the vacuum expectation value $\langle\tau_1\rangle \sim 10^4$, with a compactification volume set around the value

$\langle \mathcal{V} \rangle \sim 10^4$. Evidently, such a set-up forbids the arising of the Standard Model fields from the cycle controlled by τ_1.
Again, in such a modelling, a direct analysis shows the validity of the scalar potential analysis overviewed in Chapter 3.

The Minimal Supersymmetric Standard Model can be built on both of the cycles, instead, only provided a slight but precise tuning such that $g_S\,|C_1| \sim g_S\,|C_2| \sim |C_W|$. Indeed, according to (3.97), it is only in this situation that the condition:

$$\langle \tau_1 \rangle^{3/2} = \frac{C_W}{c_2^2}\left[\frac{|C_W|}{C_W}\left(1 + 4\frac{c_1^2 c_2^2}{C_W^2}\right)^{1/2} - 1\right]\langle \mathcal{V}' \rangle\,,$$

allows to consider $\langle \tau_1 \rangle \sim \langle \tau_2 \rangle \sim 10^2$. Once more, this case is treatable according to the standard approach.

5.4 MSSM on τ_1

Assuming natural parameters as discussed above, the Minimal Supersymmetric Standard Model is supported on the cycle associated to the Kähler modulus τ_1 if the vacuum expectation values of the fields τ_1 and τ_2 read respectively $\langle \tau_1 \rangle \sim 10^2$ and $\langle \tau_2 \rangle \sim 10^4$. This set-up is accomplished with a compactification volume $\langle \mathcal{V} \rangle \sim 10^5$, with positive winding string loop corrections.
In physical terms, this construction sets the string scale M_S and the Kaluza-Klein scale M_{KK} roughly around the very huge values:

$$M_S \sim 10^{16}\,\text{GeV}, \tag{5.19}$$

$$M_{KK} \sim 10^{15}\,\text{GeV}. \tag{5.20}$$

The gravitino mass is instead set at the very large scale:

$$m_{3/2} \sim 10^{13}\,\text{GeV}. \tag{5.21}$$

The corresponding hidden sector and Standard Model superpartners mass spectrum and soft couplings are outlined below.
In order to render the discussion easier to read, all of the quantities are expressed as functions of physically intuitively meaningful quantities such as $m_{3/2}$ and M_P. Due to the relevant anisotropy in the overall volume, sometimes the string coupling g_S is needed to be explicited and the volume-controlling fields vacuum expectation values are expressed as $\langle \tau_1 \rangle \sim (M_P/m_{3/2})^{2/5}$ and $\langle \tau_2 \rangle \sim (M_P/m_{3/2})^{4/5}$.

5.4.1 Hidden Sector

In Model III, the hidden sector is properly represented by Kähler moduli. Moreover, also the axio-dilaton and complex structure moduli are present.

Kähler moduli can be separated into massive interacting and massless non-interacting real scalar fields:

- interacting real scalars coming from Kähler moduli divide into very heavy fields, ϕ and θ_ϕ, with masses:

$$m_\phi \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{14}\,\text{GeV}, \tag{5.22}$$

$$m_{\theta_\phi} \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{14}\,\text{GeV}, \tag{5.23}$$

  according to (3.58) and (3.67), and lighter fields χ and ζ, with masses:

$$m_\chi \sim \left(\frac{m_{3/2}/M_P}{\ln\left(M_P/m_{3/2}\right)}\right)^{1/2} m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.24}$$

$$m_\zeta \sim \left(\frac{m_{3/2}}{M_P}\right)^{2/3} g_S^{-1/3} m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.25}$$

  according to (3.57) and (3.101);

- massless non-interacting scalar fields, θ_χ and θ_ζ:

$$m_{\theta_\chi} = 0\text{ GeV}, \tag{5.26}$$

$$m_{\theta_\zeta} = 0\text{ GeV}, \tag{5.27}$$

  according to (3.66) and (3.65).

The axio-dilaton and complex structure moduli fields, instead, as shown in Ref. [11] have masses roughly around the scale:

$$m_S \sim m'_\alpha \sim g_S^2\, m_{3/2} \sim 10^{11}\,\text{GeV}. \tag{5.28}$$

5.4.2 Soft Terms

In this modelling, Standard Model scalar superpartners can be only of the kind A' and Φ'. Moreover, only one kind of gaugino fields is present. These particles can be very heavy due to the the eventual absence of suppression factors with respect to the gravitino mass.

More specifically, soft breaking scalars and gauginos turn out to take values among two possible orders of magnitude. As a matter of fact, according to expressions (4.98), (4.100) and (4.132):

- A'-field masses are roughly estimated as:

$$m_{A'} \sim \frac{g_S^{1/2}}{\left(M_P/m_{3/2}\right)^{2/5}}\, m_{3/2} \sim 10^{10}\,\mathrm{GeV}; \tag{5.29}$$

- Φ'-field and gaugino masses are close to the gravitino mass:

$$m_\Phi \simeq m_{3/2} \sim 10^{13}\,\mathrm{GeV}, \tag{5.30}$$

$$M_a \simeq m_{3/2} \sim 10^{13}\,\mathrm{GeV}. \tag{5.31}$$

Soft couplings show a fine structure, where suppressions turn out to depend on the family of the fields involved.
Explicitly, they can be estimated from the detailed expressions in Chapter 4 as follows:

- all of the effective trilinear couplings $t_{\alpha\beta\gamma} = A_{\alpha\beta\gamma} y_{\alpha\beta\gamma}$ turn out to be remarkably suppressed with respect to the scale $m_{3/2}$. In order of intensity from the strongest to the weakest one, they organise as follows:

$$t_{A'A'\Phi} \sim \frac{m_{3/2}}{\left(M_P/m_{3/2}\right)^{\frac{1}{5}}} \sim 10^{12}\,\mathrm{GeV}\, Y_{A'A'\Phi}, \tag{5.32}$$

$$t_{A'\Phi\Phi} \sim \frac{m_{3/2}}{\left(M_P/m_{3/2}\right)^{\frac{3}{5}}} \sim 10^{10}\,\mathrm{GeV}\, Y_{A'\Phi\Phi}, \tag{5.33}$$

$$t_{A'A'A'} \sim \frac{g_S\, m_{3/2}}{\left(M_P/m_{3/2}\right)^{\frac{3}{5}}} \sim 10^{9}\,\mathrm{GeV}\, Y_{A'A'A'}, \tag{5.34}$$

$$t_{\Phi\Phi\Phi} \sim \frac{m_{3/2}}{\left(M_P/m_{3/2}\right)} \sim 10^{8}\,\mathrm{GeV}\, Y_{\Phi\Phi\Phi}; \tag{5.35}$$

- the Higgs bilinear coupling turns out to be of two possible orders of magnitude. If the Higgses are both A'-fields, it is suppressed with respect to $m_{3/2}^2$, otherwise it is of the same order of magnitude:

$$B\tilde{\mu}\big|_{\{A',A'\}} \sim \frac{g_S\, m_{3/2}^2}{\left(M_P/m_{3/2}\right)^{\frac{4}{5}}} \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.36}$$

$$B\tilde{\mu}\big|_{\{A',\Phi\}} \sim m_{3/2}^2 \sim \left(10^{13}\,\mathrm{GeV}\right)^2, \tag{5.37}$$

$$B\tilde{\mu}\big|_{\{\Phi,\Phi\}} \sim m_{3/2}^2 \sim \left(10^{13}\,\mathrm{GeV}\right)^2; \tag{5.38}$$

The behaviour of the $\tilde{\mu}$-term is similar to the one of the $B\tilde{\mu}$-coupling. If the Higgs fields are A'-fields, it is suppressed with respect to the gravitino mass, otherwise it is around $m_{3/2}$:

$$\tilde{\mu}\big|_{\{A',A'\}} \sim \frac{g_S\, m_{3/2}}{\left(M_P/m_{3/2}\right)^{\frac{4}{5}}} \sim 10^8\,\text{GeV}, \tag{5.39}$$

$$\tilde{\mu}\big|_{\{A',\Phi'\}} \sim m_{3/2} \sim 10^{13}\,\text{GeV}, \tag{5.40}$$

$$\tilde{\mu}\big|_{\{\Phi',\Phi'\}} \sim m_{3/2} \sim 10^{13}\,\text{GeV}. \tag{5.41}$$

5.4.3 Mass Spectrum Analysis

The reciprocal hierarchies between hidden sector and Standard Model superpartner masses are represented in Figure 5.2.

It is evident that Model III with the Minimal Supersymmetric Standard Model supported on large cycles with natural parameters is not reliable due to the large masses of some of the Standard Model superpartners.
More specifically, in this modelling:

- the hidden sector would be safe from the Cosmological Moduli Problem, being its interacting fields heavier than the critical minimum mass $m_{\text{mod}}^{\text{min}}$;
- superpartners whose masses are not suppressed with respect to the gravitino mass $m_{3/2}$, i.e. the scalars Φ'-fields and gauginos, are slightly heavier than the the maximum allowed scale $m_{\text{soft}}^{\text{max}}$, thus rendering this construction unacceptable. Instead, A'-fields would be passable.

To sum up, the situation described so far cannot be taken as a basic version of a final generalisation of the Standard Model, even though because of only a few orders of magnitude in some superpartner masses.
Then, it is easy to understand that a very small degree of fine tuning could allow to build an acceptable Minimal Supersymmetric Standard Model. This issue is argued in the following Sections.

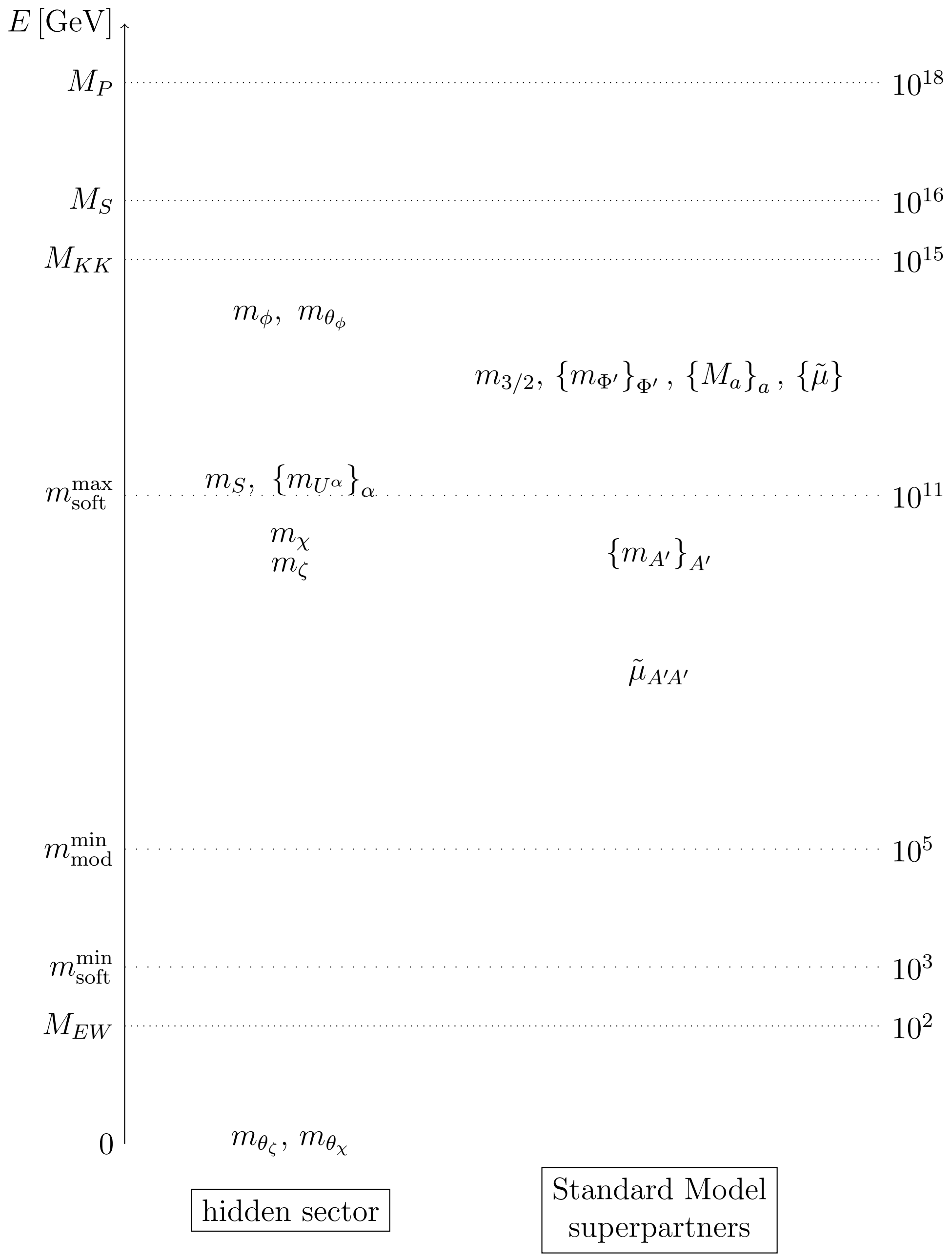

Figure 5.2. Typical mass scales for hidden sector fields and Standard Model supersymmetric partners in Model III with the MSSM supported on the τ_1 cycle and natural parameters. Curly brackets indicate a family of degenerate fields. The graph scale is logarithmic and only aims at providing a general idea of the reciprocal hierarchies.

5.5 Fine-Tuned MSSM on τ_1

An acceptable Minimal Supersymmetric Standard Model can be supported on the τ_1 cycle if the constant part of the superpotential $|W_0|$ is sufficiently small. Indeed, if it is set, following the conventions employed so far:

$$|W_0| \sim 10^{-n},$$

with $n \geq 2$, then the gravitino and consequently the heaviest Standard Model superpartners turn out to be as light as to give rise to a reliable model. Such a small value of $|W_0|$ is possible though not very likely in a realistic theory within the well known 'landscape' model [41].
In other words, assuming all of the other parameters to be naturally around unity, the only fine tuning of $|W_0|$ allows to accomplish for an interesting Standard Model extension to supersymmetry.

In the following, the model arising with a sample value $|W_0| \sim 10^{-3}$ is described and commented in order to analyse the reliability of such constructions.
The vacuum expectation values of the fields τ_1 and τ_2 must be taken respectively as $\langle\tau_1\rangle \sim 10^2$ and $\langle\tau_2\rangle \sim 10^4$, with a compactification volume $\langle\mathcal{V}\rangle \sim 10^5$.
In this framework, the string scale M_S and the Kaluza-Klein scale M_{KK} do not depend on $|W_0|$ and thus are fixed roughly around the huge values:

$$M_S \sim 10^{16}\,\text{GeV}, \tag{5.42}$$

$$M_{KK} \sim 10^{15}\,\text{GeV}. \tag{5.43}$$

The gravitino mass is instead suppressed down to the large but not huge scale:

$$m_{3/2} \sim 10^{10}\,\text{GeV}. \tag{5.44}$$

In such a modelling, all of the quantities are expressed as functions of physically meaningful quantities such as $m_{3/2}$ and M_P, with the inclusion of $|W_0|$. In particular, the compactification volume scales as $\langle\mathcal{V}\rangle \sim |W_0|\, M_P/m_{3/2}$, while the volume-controlling fields vacuum expectation values are expressed as $\langle\tau_1\rangle \sim (|W_0|\, M_P/m_{3/2})^{2/5}$ and $\langle\tau_2\rangle \sim (|W_0|\, M_P/m_{3/2})^{4/5}$.
The trace of the computation is exactly the same as in the previous Section, so it is reported more synthetically.

5.5.1 Hidden Sector

Kähler moduli turn out to be separated into massive interacting and massless non-interacting scalar real fields:

- interacting scalar fields are divided into very heavy fields, ϕ and θ_ϕ, with masses:

$$m_\phi \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{11}\,\text{GeV}, \tag{5.45}$$

$$m_{\theta_\phi} \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{11}\,\text{GeV}, \tag{5.46}$$

and lighter fields χ and ζ, with masses:

$$m_\chi \sim \left(\frac{m_{3/2}/\left(|W_0|\,M_P\right)}{\ln\left(M_P/m_{3/2}\right)}\right)^{1/2} m_{3/2} \sim 10^7\,\text{GeV}, \tag{5.47}$$

$$m_\zeta \sim \left(\frac{m_{3/2}}{|W_0|\,M_P}\right)^{2/3} g_S^{-1/3} m_{3/2} \sim 10^7\,\text{GeV}; \tag{5.48}$$

- massless non-interacting scalar fields, θ_χ and θ_ζ:

$$m_{\theta_\chi} = 0\ \text{GeV}, \tag{5.49}$$

$$m_{\theta_\zeta} = 0\ \text{GeV}. \tag{5.50}$$

The axio-dilaton and complex structure moduli fields, instead, have masses roughly around the scale:

$$m_S \sim m'_\alpha \sim g_S^2\, m_{3/2} \sim 10^8\,\text{GeV}. \tag{5.51}$$

5.5.2 Soft Terms

The Minimal Supersymmetric Standard Model soft breaking terms have masses among two possible orders of magnitude:

- A'-field masses are roughly estimated as:

$$m_{A'} \sim \frac{g_S^{1/2}}{\left(|W_0|\,M_P/m_{3/2}\right)^{2/5}}\, m_{3/2} \sim 10^7\,\text{GeV}; \tag{5.52}$$

- Φ'-field and gaugino masses are around the gravitino mass:

$$m_\Phi \simeq m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.53}$$

$$M_a \simeq m_{3/2} \sim 10^{10}\,\text{GeV}. \tag{5.54}$$

Of course the fine-tuning of the superpotential constant term does not alter the structure of soft breaking terms.

Soft coupling fine structure takes the following form:

- the effective trilinear couplings $t_{\alpha\beta\gamma} = A_{\alpha\beta\gamma} y_{\alpha\beta\gamma}$ in order of intensity from the strongest to the weakest one organise as:

$$t_{A'A'\Phi} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{\frac{1}{5}}} \sim 10^9\,\text{GeV}\, Y_{A'A'\Phi}, \tag{5.55}$$

$$t_{A'\Phi\Phi} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{\frac{3}{5}}} \sim 10^7\,\text{GeV}\, Y_{A'\Phi\Phi}, \tag{5.56}$$

$$t_{\Phi\Phi\Phi} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)} \sim 10^4\,\text{GeV}\, Y_{\Phi\Phi\Phi}, \tag{5.57}$$

$$t_{A'A'A'} \sim \frac{g_S\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{\frac{3}{5}}} \sim 10^3\,\text{GeV}\, Y_{A'A'A'}; \tag{5.58}$$

- the Higgs bilinear coupling can be:

$$B\tilde{\mu}\big|_{\{A',A'\}} \sim \frac{g_S\, m_{3/2}^2}{\left(|W_0|\, M_P/m_{3/2}\right)^{\frac{4}{5}}} \sim \left(10^7\,\text{GeV}\right)^2, \tag{5.59}$$

$$B\tilde{\mu}\big|_{\{A',\Phi\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\text{GeV}\right)^2, \tag{5.60}$$

$$B\tilde{\mu}\big|_{\{\Phi,\Phi\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\text{GeV}\right)^2. \tag{5.61}$$

The $\tilde{\mu}$-term can read:

$$\tilde{\mu}\big|_{\{A',A'\}} \sim \frac{g_S\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{\frac{4}{5}}} \sim 10^5\,\text{GeV}, \tag{5.62}$$

$$\tilde{\mu}\big|_{\{A',\Phi'\}} \sim m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.63}$$

$$\tilde{\mu}\big|_{\{\Phi',\Phi'\}} \sim m_{3/2} \sim 10^{10}\,\text{GeV}. \tag{5.64}$$

5.5.3 Mass Spectrum and Coupling Analysis

The hidden sector and Standard Model superpartner mass spectrum is schematised in Figure 5.3 with the sample value $|W_0| \sim 10^{-3}$.

Evidently, Model III with the Minimal Supersymmetric Standard Model supported on the τ_1 cycle with natural parameters but the passably tuned $|W_0|$ turns out to be generally reliable as it neither gives rise cosmological inconsitencies nor spoils the Hierarchy Problem supersymmetry solution.
In detail, the following items hold true:

- the hidden sector is safe from the Cosmological Moduli Problem, being its interacting fields heavier than the critical minimum mass $m_{\text{mod}}^{\text{min}}$. The smallest value which allows the lightest interacting moduli masses to be acceptable is $|W_0|_{\text{min}} \sim 10^{-5}$;

- superpartners whose masses are not suppressed with respect to the gravitino mass $m_{3/2}$, i.e. the scalar Φ'-fields and gauginos, are lighter than the the maximum Hierarchy Problem-free scale $m_{\rm soft}^{\rm max}$ as well as of course A'-fields. The biggest value which allows the heaviest interacting superpartners to be sufficiently light is $|W_0|_{\rm max} \sim 10^{-2}$.

Summing up, a fundamental conclusion is pointed out. A possibly natural set of conditions for the realisation of the minimal supersymmetric extension of the Standard Model supported on the τ_1 cycle in Model III framework is that:

- all perturbative and non-perturbative parameters in the supergravity modelling of the hidden sector are natural, i.e. around unity, in such a way that the vacuum expectation value of the field supporting the Minimal Supersymmetric Standard Model is compatible with Standard Model gauge couplings;

- the constant contribution to the superpotential, $|W_0|$, is tuned within the acceptable but unlikely range from $|W_0|_{\rm min} \sim 10^{-5}$ to $|W_0|_{\rm max} \sim 10^{-2}$.

Of course, a model satisfying such quite natural conditions is not necessarily reliable. Indeed, depending on the Inflation modelling and on the parameter choice for the Minimal Supersymmetric Standard Model parameters, many Cosmology and Particle Physics implications can arise and carry out further bounds.

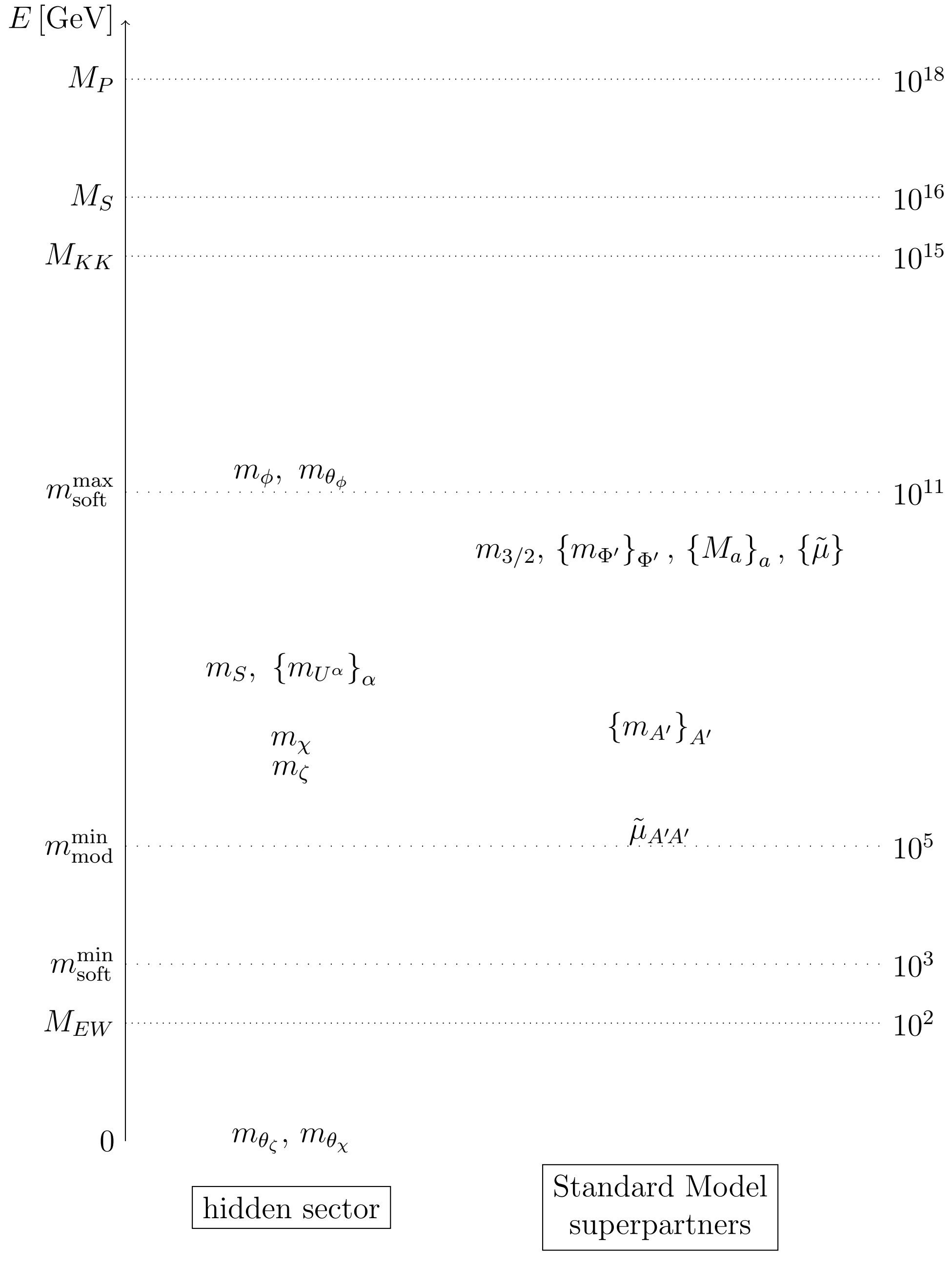

Figure 5.3. Typical mass scales for hidden sector fields and Standard Model supersymmetric partners in Model III with the MSSM supported on the τ_1 cycle and the fine-tuned parameter $|W_0| \sim 10^{-3}$. Curly brackets indicate a family of degenerate fields. The graph scale is logarithmic and only aims at providing a general idea of the reciprocal hierarchies.

5.6 Fine-Tuned MSSM on τ_2

Assuming natural parameters as discussed above, in principle the Minimal Supersymmetric Standard Model can be supported on the cycle associated to the Kähler modulus τ_2 when the vacuum expectation values of the fields τ_1 and τ_2 read respectively $\langle \tau_2 \rangle \sim 10^4$ and $\langle \tau_2 \rangle \sim 10^2$, with a compactification volume $\langle \mathcal{V} \rangle \sim 10^4$ and a negative winding string loop correction.
In physical terms, the string scale M_S and the Kaluza-Klein scale M_{KK} can be estimated around the enormous values:

$$M_S \sim 10^{16}\,\text{GeV}, \tag{5.65}$$

$$M_{KK} \sim 10^{15}\,\text{GeV}. \tag{5.66}$$

Unfortunately, the gravitino mass is set very close to the scale M_{KK}, being $m_{3/2} \sim 10^{14}\,\text{GeV}$ with some Standard Model superpartners with a similar mass. This situation is forbidden by internal consistency reasons, as pointed out in Ref. [42], which places the constraint $|W_0| \ll \langle \mathcal{V} \rangle^{1/3}$ in the Large Volume scenario framework, i.e., in other words, $m_{3/2} \ll M_{KK}$.

Then, actually it would be correct to say that the Minimal Supersymmetric Standard Model can be supported on the τ_2 cycle only if the constant term in the superpotential is sufficiently small. After the previous discussion, evidently the fine tuning of $|W_0|$ must be also such that the gravitino mass is below the largest allowed scale for soft terms.

In the following, the model corresponding to the sample value $|W_0| \sim 10^{-4}$ is outlined and commented. The gravitino mass is thus set at the large but acceptable scale:

$$m_{3/2} \sim 10^{10}\,\text{GeV}. \tag{5.67}$$

Again, the computation closely mirrors the previous ones and so only the main results are listed for the sake of brevity.

Hidden Sector

The fine structure of hidden sector mass spectrum is outlined below, with a sample value $|W_0| \sim 10^{-4}$.
The hidden sector in constituted by the axio-dilaton and complex structure moduli fields, with masses around the scale $m_S \sim m'_\alpha \sim g_S^2\, m_{3/2} \sim 10^8\,\text{GeV}$, massless non-interacting Kähler moduli and the potentially dangerous massive Kähler moduli.
The latter turn out to be divided into heavy fields, ϕ and θ_ϕ, with masses:

$$m_\phi \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{11}\,\text{GeV}, \tag{5.68}$$

$$m_{\theta_\phi} \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{11}\,\text{GeV}, \tag{5.69}$$

and light fields χ and ζ, with masses:

$$m_\chi \sim \left(\frac{m_{3/2}/\left(|W_0|\, M_P\right)}{\ln\left(M_P/m_{3/2}\right)}\right)^{1/2} m_{3/2} \sim 10^7\,\mathrm{GeV}, \tag{5.70}$$

$$m_\zeta \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{2/3} g_S^{1/3} m_{3/2} \sim 10^7\,\mathrm{GeV}. \tag{5.71}$$

Soft Terms

Soft breaking terms are A'' and A''' scalar fields, with masses:

$$m_{A''} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim 10^7\,\mathrm{GeV}, \tag{5.72}$$

$$m_{A'''} \sim \frac{g_S^{1/2}\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/4}} \sim 10^8\,\mathrm{GeV}, \tag{5.73}$$

and Φ'' scalar fields and gauginos with masses:

$$m_\Phi \simeq m_{3/2} \sim 10^{10}\,\mathrm{GeV}, \tag{5.74}$$

$$M_a \simeq m_{3/2} \sim 10^{10}\,\mathrm{GeV}. \tag{5.75}$$

Once again, a split between very heavy and heavy Standard Model superpartners is characteristic.

On the other hand, soft couplings turn out to be organised as follows:

- the effective trilinear couplings $t_{\alpha\beta\gamma} = A_{\alpha\beta\gamma} y_{\alpha\beta\gamma}$ in order of intensity from the strongest to the weakest one organise as:

$$t_{A''A''\Phi''} \sim m_{3/2} \sim 10^{10}\,\mathrm{GeV}\, Y_{A''A''\Phi''}, \tag{5.76}$$

$$t_{A''A'''\Phi''} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/4}} \sim 10^9\,\mathrm{GeV}\, Y_{A''A'''\Phi''}, \tag{5.77}$$

$$t_{A''A''A'''} \sim \frac{g_S\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/4}} \sim 10^8\,\mathrm{GeV}\, Y_{A''A''A'''}, \tag{5.78}$$

$$t_{A''A''A''} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim 10^7\,\mathrm{GeV}\, Y_{A''A''A''}, \tag{5.79}$$

$$t_{A'''A'''\Phi''} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim 10^7\,\mathrm{GeV}\, Y_{A'''A'''\Phi''}, \tag{5.80}$$

$$t_{A''\Phi''\Phi''} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim 10^7\,\mathrm{GeV}\, Y_{A''\Phi''\Phi''}, \tag{5.81}$$

$$t_{A'''\Phi''\Phi''} \sim \frac{m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{3/4}} \sim 10^7\,\mathrm{GeV}\, Y_{A'''\Phi''\Phi''}, \tag{5.82}$$

$$t_{A''A'''A'''} \sim \frac{g_S\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim 10^6\,\mathrm{GeV}\, Y_{A''A'''A'''}, \tag{5.83}$$

$$t_{A'''A'''A'''} \sim \frac{g_S\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{3/4}} \sim 10^6\,\mathrm{GeV}\, Y_{A''A'''A'''}, \tag{5.84}$$

$$t_{\Phi''\Phi''\Phi''} \sim \frac{m_{3/2}}{|W_0|\, M_P/m_{3/2}} \sim 10^5\,\mathrm{GeV}\, Y_{\Phi''\Phi''\Phi''}, \tag{5.85}$$

$$\tag{5.86}$$

- the Higgs bilinear coupling can be:

$$B\tilde{\mu}\big|_{\{A'',\Phi''\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.87}$$

$$B\tilde{\mu}\big|_{\{A''',\Phi''\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.88}$$

$$B\tilde{\mu}\big|_{\{\Phi'',\Phi''\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.89}$$

$$B\tilde{\mu}\big|_{\{A'',A'''\}} \sim \frac{g_S\, m_{3/2}^2}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim \left(10^8\,\mathrm{GeV}\right)^2, \tag{5.90}$$

$$B\tilde{\mu}\big|_{\{A''',A'''\}} \sim \frac{g_S\, m_{3/2}^2}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim \left(10^8\,\mathrm{GeV}\right)^2, \tag{5.91}$$

$$B\tilde{\mu}\big|_{\{A'',A''\}} \sim \frac{m_{3/2}^2}{|W_0|\, M_P/m_{3/2}} \sim \left(10^8\,\mathrm{GeV}\right)^2. \tag{5.92}$$

The $\tilde{\mu}$-term can read:

$$\tilde{\mu}\big|_{\{A'',\Phi''\}} \sim m_{3/2} \sim 10^{10}\,\mathrm{GeV}, \tag{5.93}$$

$$\tilde{\mu}\big|_{\{A''',\Phi''\}} \sim m_{3/2} \sim 10^{10}\,\mathrm{GeV}, \tag{5.94}$$

$$\tilde{\mu}\big|_{\{\Phi'',\Phi''\}} \sim m_{3/2} \sim 10^{10}\,\mathrm{GeV}, \tag{5.95}$$

$$\tilde{\mu}\big|_{\{A'',A'''\}} \sim \frac{g_S\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim 10^7\,\mathrm{GeV}, \tag{5.96}$$

$$\tilde{\mu}\big|_{\{A''',A'''\}} \sim \frac{g_S\, m_{3/2}}{\left(|W_0|\, M_P/m_{3/2}\right)^{1/2}} \sim 10^7\,\mathrm{GeV}, \tag{5.97}$$

$$\tilde{\mu}\big|_{\{A'',A''\}} \sim \frac{m_{3/2}}{|W_0|\, M_P/m_{3/2}} \sim 10^5\,\mathrm{GeV}. \tag{5.98}$$

Mass Spectrum Analysis

The hidden sector and Standard Model superpartner mass hierarchies are schematised in Fig. 5.4 with the sample value $|W_0| \sim 10^{-4}$.

As for the previous construction, Model III with the Minimal Supersymmetric Standard Model supported on the τ_2 cycle with natural parameters but the passably tuned $|W_0|$ is acceptable because:

- the hidden sector is safe from the Cosmological Moduli Problem as long as $|W_0|$ is larger than $|W_0|_{\min} \sim 10^{-6}$;
- the heaviest superpartners are lighter than the maximum admitted scale as long as $|W_0|$ is smaller than $|W_0|_{\max} \sim 10^{-3}$.

Then, a possibly natural set of conditions for the realisation of a supersymmetric extension of the Standard Model supported on the τ_2 cycle in Model III framework reads:

- all perturbative and non-perturbative parameters in the supergravity modelling of the hidden sector are natural and compatible with Standard Model gauge couplings;
- the constant contribution to the superpotential, $|W_0|$, is tuned within the acceptable but unlikely range from $|W_0|_{\min} \sim 10^{-6}$ to $|W_0|_{\max} \sim 10^{-3}$.

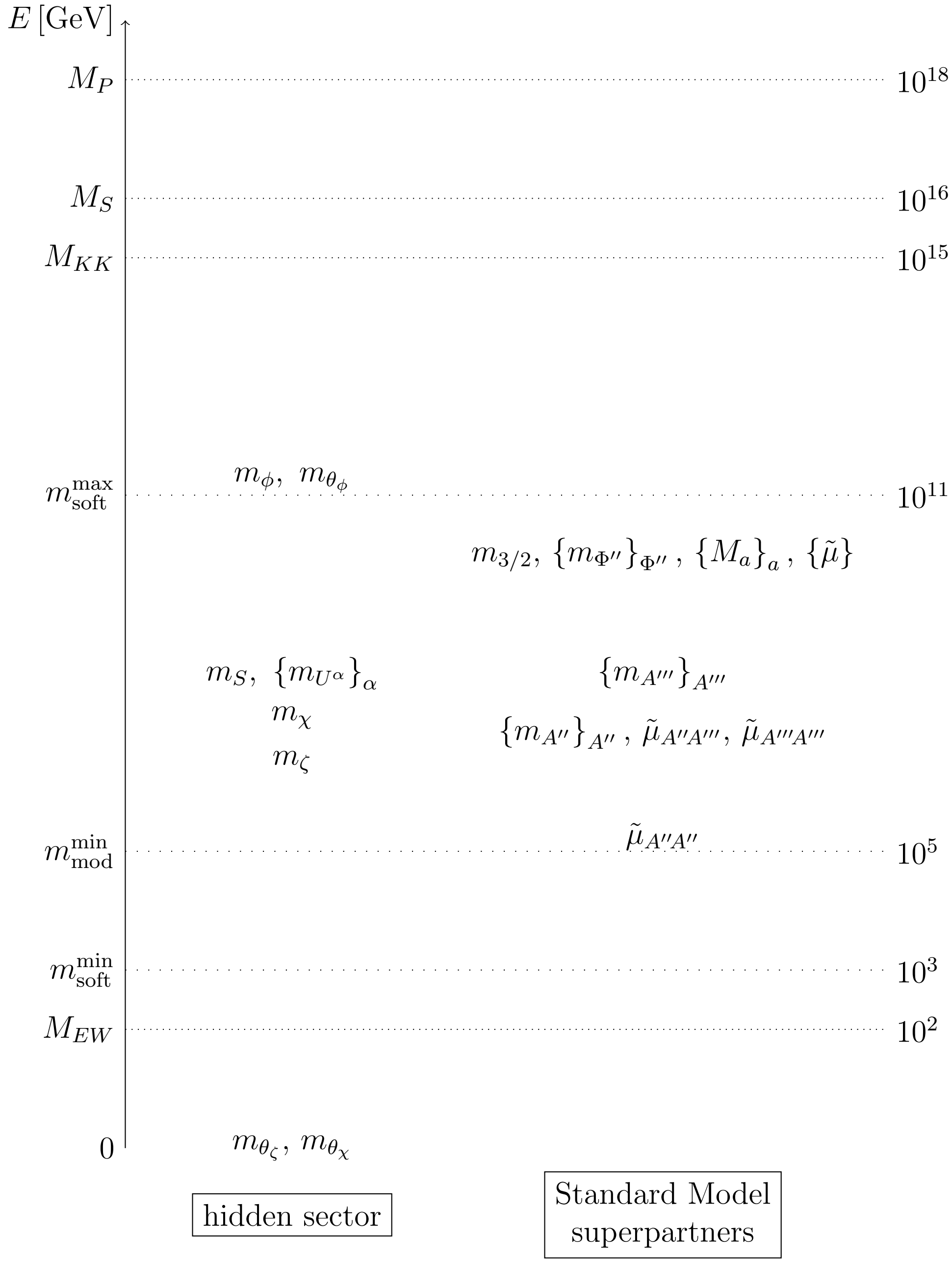

Figure 5.4. Typical mass scales for hidden sector fields and Standard Model supersymmetric partners in Model III with the MSSM supported on the τ_2 cycle and the fine-tuned parameter $|W_0| \sim 10^{-4}$. Curly brackets indicate a family of degenerate fields. The graph scale is logarithmic and only aims at providing a general idea of the reciprocal hierarchies.

5.7 Fine-Tuned MSSM on both τ_1 and τ_2

The last possibility to extend the Standard Model to a supersymmetric version within Model III is to consider fields coming from both of the large cycles. Independently of the sign of winding string loop corrections, such a set-up requires fine-tuned string loop corrections in such a way that, approximately, $g_S\,|C_1| \sim g_S\,|C_2| \sim |C_W|$, as shown before. Under these conditions, the necessity of Standard Model gauge couplings around $g_{\rm SM} \sim 10^{-1}$, sets inevitably the Calabi-Yau compactification volume vacuum expectation value to a quite small value:

$$\langle \mathcal{V} \rangle \sim 10^3,$$

with the desidered $\langle \tau_1 \rangle \sim \langle \tau_2 \rangle \sim 10^2$.

The string scale M_S and Kaluza-Klein scale M_{KK} are estimated respectively around the extremely large values:

$$M_S \sim 10^{17}\,\text{GeV}, \tag{5.99}$$

$$M_{KK} \sim 10^{16}\,\text{GeV}. \tag{5.100}$$

Similarly to the model arising on the only τ_2 cycle, the gravitino mass $m_{3/2}$ is incredibly large, i.e. $m_{3/2} \sim 10^{15}\,$GeV, since there are some Standard Model superpartners with a mass around the same order of magnitude. Moreover they would be too close to the Kaluza-Klein scale and thus must be avoided.
Then, a fine tuning of the constant part of the superpotential is necessary. In the following, a modelling with the value $|W_0| \sim 10^{-5}$, i.e. with a gravitino mass of order:

$$m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.101}$$

is briefly overviewed.

Hidden Sector

As usual, the critical massive and interacting Kähler moduli can be grouped into heavy fields, ϕ and θ_ϕ, with masses:

$$m_\phi \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{11}\,\text{GeV}, \tag{5.102}$$

$$m_{\theta_\phi} \simeq 2\,\ln\left(\frac{M_P}{m_{3/2}}\right) m_{3/2} \sim 10^{11}\,\text{GeV}, \tag{5.103}$$

and light fields, χ and ζ, with masses:

$$m_\chi \sim \left(\frac{m_{3/2}/\left(|W_0|\,M_P\right)}{\ln\left(M_P/m_{3/2}\right)}\right)^{1/2} m_{3/2} \sim 10^{7}\,\text{GeV}, \tag{5.104}$$

$$m_\zeta \sim \left(\frac{m_{3/2}}{|W_0|\,M_P}\right)^{2/3} m_{3/2} \sim 10^{7}\,\text{GeV}. \tag{5.105}$$

Soft Terms

As usual, soft breaking terms can turn out to have masses either suppressed with respect to the gravino mass or of the same order of magnitude as $m_{3/2}$. A'-, A''- and A'''- scalars have suppressed masses all of order:

$$m_A \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{1/3} m_{3/2} \sim 10^8\,\text{GeV}, \tag{5.106}$$

while Φ'-, Φ''- and I-fields around $m_{3/2}$:

$$m_\Phi \simeq m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.107}$$

$$m_I \sim \frac{\sqrt{2}}{2} m_{3/2} \sim 10^{10}\,\text{GeV}. \tag{5.108}$$

Gaugino masses are around 10^{10} GeV too:

$$M_a \simeq m_{3/2} \sim 10^{10}\,\text{GeV}. \tag{5.109}$$

Of course soft couplings show a fine structure. In spite of this, however, the small value of the ratio $m_{3/2}/M_P$ tends to smooth the hierachies.
Considerably, suppressions turn out to depend only on the family of the fields involved and not on the specific fields, i.e. the factors are the same for all of the three kinds of A-fields and the two kinds of Φ-fields.
In detail:

- remarkably, all of the effective trilinear couplings $t_{\alpha\beta\gamma} = A_{\alpha\beta\gamma} y_{\alpha\beta\gamma}$ turn out to be suppressed with respect to the scale $m_{3/2}$. In order of intensity from the strongest to the weakest, they organise as follows:

$$t_{AAI} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{1/6} m_{3/2} \sim 10^9\,\text{GeV}\, Y_{AAI}, \tag{5.110}$$

$$t_{AII} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{1/3} m_{3/2} \sim 10^9\,\text{GeV}\, Y_{AII}, \tag{5.111}$$

$$t_{AA\Phi} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{1/3} m_{3/2} \sim 10^9\,\text{GeV}\, Y_{AA\Phi}, \tag{5.112}$$

$$t_{A\Phi I} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{1/2} m_{3/2} \sim 10^8\,\text{GeV}\, Y_{A\Phi I}, \tag{5.113}$$

$$t_{III} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{1/2} m_{3/2} \sim 10^8\,\text{GeV}\, Y_{AII}, \tag{5.114}$$

$$t_{AAA} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{2/3} m_{3/2} \sim 10^7\,\mathrm{GeV}\, Y_{AAA}, \tag{5.115}$$

$$t_{A\Phi\Phi} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{2/3} m_{3/2} \sim 10^7\,\mathrm{GeV}\, Y_{A\Phi\Phi}, \tag{5.116}$$

$$t_{\Phi II} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{2/3} m_{3/2} \sim 10^7\,\mathrm{GeV}\, Y_{\Phi II}, \tag{5.117}$$

$$t_{\Phi\Phi I} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{5/6} m_{3/2} \sim 10^7\,\mathrm{GeV}\, Y_{\Phi\Phi I}, \tag{5.118}$$

$$t_{\Phi\Phi\Phi} \sim \left(\frac{m_{3/2}}{M_P}\right) m_{3/2} \sim 10^6\,\mathrm{GeV}\, Y_{\Phi\Phi\Phi}; \tag{5.119}$$

- the bilinear coupling between the Higgs fields turns out to be of two possible orders of magnitude. If the Higgses are both A-fields, it is suppressed with respect to $m_{3/2}^2$ as:

$$B\tilde{\mu}\big|_{\{A,A\}} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{2/3} m_{3/2}^2 \sim \left(10^8\,\mathrm{GeV}\right)^2, \tag{5.120}$$

otherwise, it is around $m_{3/2}^2$:

$$B\tilde{\mu}\big|_{\{A,\Phi\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.121}$$

$$B\tilde{\mu}\big|_{\{A,I\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.122}$$

$$B\tilde{\mu}\big|_{\{\Phi,\Phi\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.123}$$

$$B\tilde{\mu}\big|_{\{\Phi,I\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2, \tag{5.124}$$

$$B\tilde{\mu}\big|_{\{I,I\}} \sim m_{3/2}^2 \sim \left(10^{10}\,\mathrm{GeV}\right)^2. \tag{5.125}$$

The $\tilde{\mu}$-term has a similar behaviour. If the Higgses are A-fields, it is suppressed with respect to the gravitino mass:

$$\tilde{\mu}\big|_{\{A,A\}} \sim \left(\frac{m_{3/2}}{|W_0|\, M_P}\right)^{2/3} m_{3/2} \sim 10^8\,\mathrm{GeV}, \tag{5.126}$$

otherwise it is around the gravitino mass $m_{3/2}$:

$$\tilde{\mu}\big|_{\{A,\Phi\}} \sim m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.127}$$

$$\tilde{\mu}\big|_{\{A,I\}} \sim m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.128}$$

$$\tilde{\mu}\big|_{\{\Phi,\Phi\}} \sim m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.129}$$

$$\tilde{\mu}\big|_{\{\Phi,I\}} \sim m_{3/2} \sim 10^{10}\,\text{GeV}, \tag{5.130}$$

$$\tilde{\mu}\big|_{\{I,I\}} \sim m_{3/2} \sim 10^{10}\,\text{GeV}. \tag{5.131}$$

Mass Spectrum Analysis

The hidden sector and Standard Model superpartners mass spectra are represented in Fig. 5.5 with the sample value $|W_0| \sim 10^{-5}$.

Model III with the Minimal Supersymmetric Standard Model supported on both of the large cycles with slightly fine-tuned string loop corrections and fine-tuned $|W_0|$ is somehow reliable because:

- the hidden sector is safe from the Cosmological Moduli Problem as long as $|W_0|$ is larger than $|W_0|_{\text{min}} \sim 10^{-8}$;
- the heaviest superpartners are lighter than the maximum admitted scale as long as $|W_0|$ is smaller than $|W_0|_{\text{max}} \sim 10^{-4}$.

Then, a possibly natural set of conditions for the realisation of a supersymmetric extension of the Standard Model supported on both of the large cycles in Model III framework reads:

- all perturbative and non-perturbative parameters in the supergravity modelling of the hidden sector but the string loop ones are natural and compatible with Standard Model gauge couplings;
- string loop correction parameters are tuned in such a way that $g_S\,|C_1| \sim g_S\,|C_2| \sim |C_W| \sim 1$;
- the constant contribution to the superpotential, $|W_0|$, is tuned within the acceptable but unlikely range from $|W_0|_{\text{min}} \sim 10^{-8}$ to $|W_0|_{\text{max}} \sim 10^{-4}$.

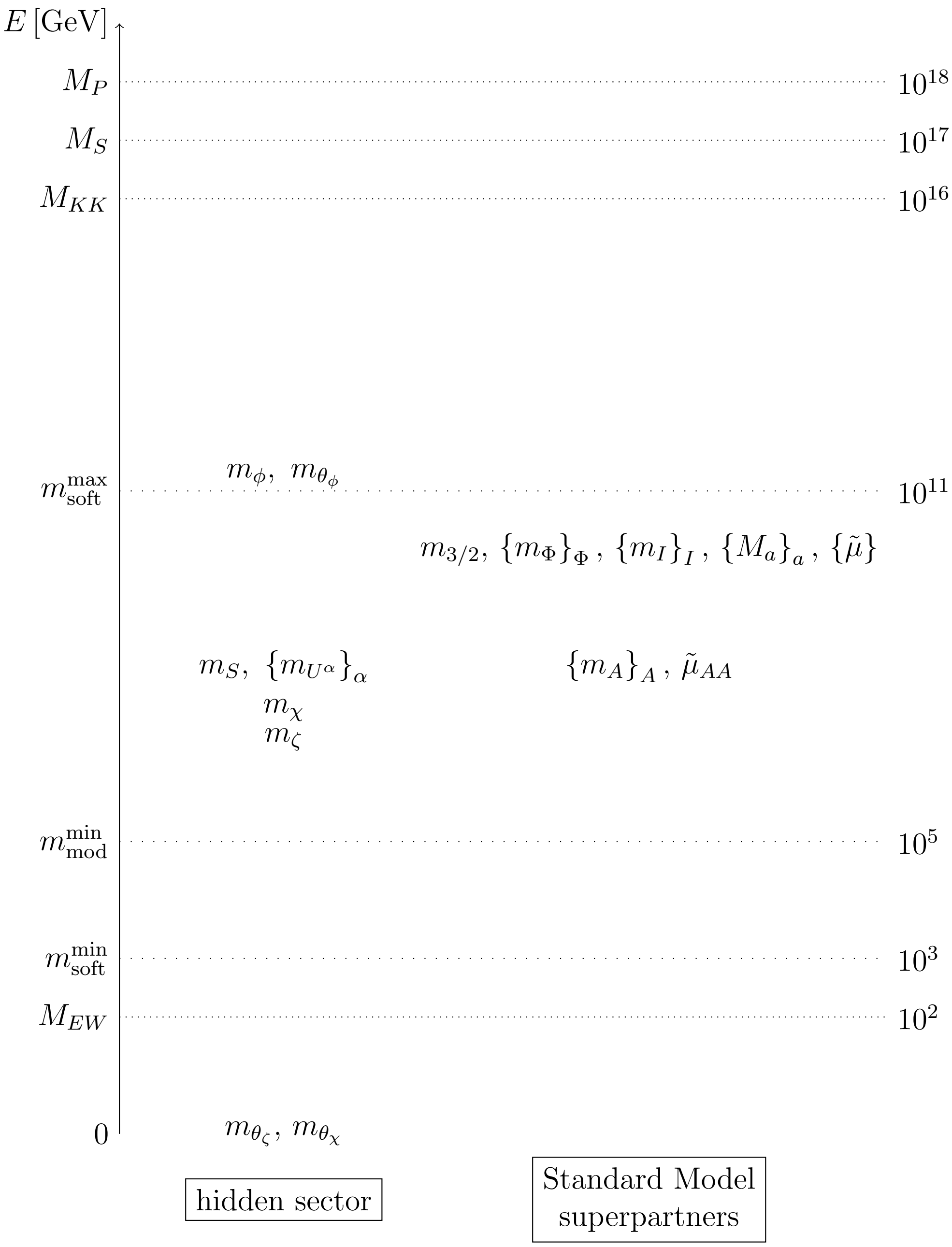

Figure 5.5. Typical mass scales for hidden sector fields and Standard Model supersymmetric partners in Model III with the MSSM supported on both of the large cycles. The basic assumptions are fine-tuned string loop parameters $g_S\,|C_1| \sim g_S\,|C_2| \sim |C_W| \sim 1$ and a fine-tuned constant superpotential parameter $|W_0| \sim 10^{-5}$. Curly brackets indicate a family of degenerate fields. The graph scale is logarithmic and only aims at providing a general idea of the reciprocal hierarchies.

5.8 Supersymmetry Breaking and Inflation

The modelling of a fully realistic scenario for the description of Nature is the ultimate aim of String Phenomenology. This accomplishment definitely relies on the realisation of a Model with consistent predictions in both the Theoretical Particle Physics and Cosmology fields, with special care about the mechanism of spontaneous supersymmetry breaking and inflation respectively.

Although the scenarios developed in this thesis related to Model III can eventually give rise to a set up able to describe current Particle Physics observations in a coherent way, as Figures 5.3, 5.4 and 5.5 account for, no mechanism of cosmological inflation has been considered.
Nevertheless, Ref. [22] and Ref. [43] show that the framework of Model III can be developed to realise a coherent inflationary scenario where essentially the lightest interacting Kähler modulus takes over the rôle of the inflaton field.
This possibility is very attractive but unfortunately it is incompatible with the Minimal Supersymmetric Standard Model scenario analysed in this Chapter. Indeed such a description of inflation requires the values:

$$|W_0|_{\mathrm{I}} \sim 1, \qquad \langle \mathcal{V} \rangle_{\mathrm{I}} \sim 10^4,$$

thus ruling out the fine-tuned modellings with the Minimal Supersymmetric Standard Model supported on branes associated to large cycles. In turn, the other only possibly suitable scenario without fine-tuned parameters, depicted in Figure 5.2, is ruled out by the presence of soft breaking terms which are much heavier than the largest consistent scale.

These considerations show how the deep interplay between spontaneous supersymmetry breaking and cosmological implications can set severe constraints on model building. Indeed, they show that within Model III it is impossible to describe a scenario which coherently accounts for the Minimal Supersymmetric Standard Model phenomenology and cosmological inflation with the right Higgs mass.

In the final analysis, two possible conclusions towards a fully realistic scenario within Model III are highlighted:

- the right particle content modelling is the one associated to the fine-tuned Minimal Supersymmetric Standard Model on large cycles, but the inflaton field arises from a different mechanism yet to be discovered;

- inflation is correctly described with the assumption of the lightest Kähler modulus as the inflaton field, with a particle content arising naturally only on the cycle controlled by τ_1 generalised to include further supermultiplets besides the Standard Model particle-superpartner ones. A concrete but very simple proposal of such a relisation consists in dealing with the Next-to Minimal Supersymmetric Standard Model - NMSSM for short - in which a further chiral supermultiplet is included.

Indeed, such a modelling involves less severe constraints on soft breaking scalar term masses, as evidenced in Ref. [44].

Of course, these conclusions are just simple proposals, but more complicated frameworks than Model III could be needed in order to realise a realistic description of Nature.

Conclusions

Type IIB String Theory represents a plausible candidate to extend the Standard Model. In principle it can provide a coherent and unified description of Nature far above the electroweak scale and account for both Particle Physics and Cosmology issues.

The subject of Kähler moduli stabilisation plays a crucial rôle within this context as Kähler moduli are serious hidden sector candidates. The thesis gives a summary of some interesting general models and evidences the most delicate points in LARGE Volume Scenario set-ups.

As a matter of fact, many kinds of phenomena must be taken into account in order to properly describe the mechanisms of moduli stabilisation. Therefore the study of all of the possible effects is a primary issue.

Moreover, the lightest interacting Kähler moduli are typically in a very delicate position. Indeed their masses turn out to be highly suppressed with respect to the gravitino mass, so models often have to face the Cosmological Moduli Problem. In the most optimistic cases, this problem at least sets precise constraints on model building. On the other hand, other moduli are generally problem-free.

Finally, the characteristics of moduli stabilisation affect soft supersymmetry breaking terms in a critical way. This fact is evidenced in this work and it emerges from the description of the general features of extended Standard Model realisations on both small and large cycles.

The accomplishment of the Minimal Supersymmetric Standard Model on D7 branes wrapping large cycles actually constitutes the main object of the thesis. Besides, also D7 branes on tiny cycles are considered for pedagogical reasons. Such a topic is discussed in the second part of the thesis.

As a matter of fact, the deep interplay between the hidden sector and soft supersymmetry breaking terms becomes even more manifest in the attempts of modelling a realistic scenario.

On the one hand, it is well known that Large Volume Scenarios with branes on blow-up cycles generally origin models which are interesting from the particle physics point of view - being the mass spectrum slightly above the electroweak scale - but undergoing the unavoidable presence of the Cosmological Moduli Problem - being the lightest Kähler moduli too light.

On the other hand, Large Volume Scenario set-ups with branes on volume-controlling cycles offer the possibility of different kinds of Standard Model supersymmetric extensions. These ones are of particular interest with respect to the individuation of a fully realistic model, at the day of writing the thesis, as some inflation scenarios in a generally compatible framework have been proposed in the literature.

Unfortunately, the computations carried out in this thesis prove that a coherent description of both soft breaking terms and the inflation mechanism introduced before is not realised. However, two clear possible indications are pointed out in the search for a realistic model. Either the inflation description is to be modified, or such an inflation scenario is to be embedded within a further enhanced supersymmetric extension of the Standard Model.

Appendix A

Moduli Stabilisation: Calculations

This Appendix contains the explicit calculations which lead to the individuation of the vacuum expectation values of Kähler moduli in the models that have been taken into account and to the determination of moduli masses. It must be intended as a mathematical complement to Chapter 3, to which is left every phenomenological consideration. Moreover, the computation auxiliary fields is reported as a complement to Chapter 4.

A.1 Model II

Model II deals with two Kähler moduli $T_b = \tau_b + i\psi_b$ and $T_s = \tau_s + i\psi_s$ whose Kähler potential and superpotential are:

$$K = -2\ \ln\left(\tau_b^{3/2} - \tau_s^{3/2} + \xi\right), \tag{A.1}$$

$$W = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)}, \tag{A.2}$$

under the assumptions:

$$(*) : \begin{cases} a_s\tau_s \gg 1, \\ \tau_b^{3/2} \gg \tau_s^{3/2} \sim \xi, \end{cases} \tag{A.3}$$

according to the previous discussion.

A.1.1 Minimisation

Now a straightforward computation gives the exact Kähler metric:

$$K_{i\bar{j}} = \frac{3}{8(\tau_b^{3/2} - \tau_s^{3/2} + \xi)^2} \begin{pmatrix} \dfrac{2\tau_b^{3/2} + \tau_s^{3/2} - \xi}{\tau_b^{1/2}} & -3\tau_b^{1/2}\tau_s^{1/2} \\ -3\tau_b^{1/2}\tau_s^{1/2} & \dfrac{\tau_b^{3/2} + 2\tau_s^{3/2} + \xi}{\tau_s^{1/2}} \end{pmatrix}, \tag{A.4}$$

with the first and second rows and columns referred to τ_b and τ_s respectively, and its inverse:

$$K^{i\bar{j}} = \frac{8}{3}\frac{\tau_b^{3/2} - \tau_s^{3/2} + \xi}{2(\tau_b^{3/2} - \tau_s^{3/2}) - \xi}\begin{pmatrix} \tau_b^{1/2}(\tau_b^{3/2} + 2\tau_s^{3/2} + \xi) & 3\tau_b\tau_s \\ 3\tau_b\tau_s & \tau_s^{1/2}(2\tau_b^{3/2} + \tau_s^{3/2} - \xi) \end{pmatrix}. \tag{A.5}$$

The exact scalar potential is then:

$$\begin{aligned} V_F =& \frac{1}{2(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}}) - \xi}\frac{8}{3\,(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} + \xi)^2}\Bigg\{\frac{9}{8}\xi\,|W_0|^2 \\ &+ \cos(\alpha_s - \theta - a_s\psi_s)\left[3a_s\tau_s(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} + \xi) + \frac{9}{4}\xi\right]|A_s|\,|W_0|\,e^{-a_s\tau_s} \\ &+ \left[(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} + \xi)\left(2\tau_b^{\frac{3}{2}}\tau_s^{\frac{1}{2}}a_s^2 + 3a_s\tau_s - a_s^2\tau_s^{\frac{1}{2}}(\tau_s^{\frac{3}{2}} - \xi)\right) + \frac{9}{8}\xi\right]|A_s|^2\,e^{-2a_s\tau_s}\Bigg\}. \end{aligned} \tag{A.6}$$

Recalling that $\tau_b^{3/2} \gg \tau_s^{3/2} \sim \xi$, it is very easy to minimise the potential with respect to the axion ψ_s, which takes a vacuum expectation value such that $\cos(\alpha_s - \theta - a\langle\psi\rangle) = -1$:

$$\langle\psi_s\rangle = \frac{\alpha_s - \theta}{a_s} + (2n+1)\frac{\pi}{a_s}, \quad n \in \mathbb{Z}. \tag{A.7}$$

Unfortunately, even though it now depends on two variables only, the scalar potential is still way too complex to be minimised exactly.
Anyway, under assumptions (A.3) it is possible to greatly simplify its expression. Indeed conditions (A.3) indicate the validity of a Taylor expansion of the scalar potential up to order I in the parameters $\tau_s^{3/2}/\tau_b^{3/2}$ and $\xi/\tau_b^{3/2}$. Moreover, it is coherent with the same conditions to discard all of the terms proportional to expressions like $(\tau_s^{3/2}/\tau_b^{3/2})\ e^{-a_s\tau_s}$ and $(\xi/\tau_b^{3/2})\ e^{-a_s\tau_s}$. Finally the condition $a_s\tau_s \gg 1$ allows further considerations useful in order to individuate the leading contribution to the factors of $|A_s|^2 e^{-2a_s\tau_s}$ and $|A_s|\,|W_0|\,e^{-a_s\tau_s}$.
Taking these considerations into account, the scalar potential can be approximated as:

$$V_F \overset{(*)}{\sim} V_F^{\text{eff}},$$

in such a way as to allow the definition of the 'effective' scalar potential V_F^{eff}:

$$V_F^{\text{eff}} \equiv \frac{8}{3}\frac{a_s^2\tau_s^{1/2}}{\tau_b^{3/2}}|A_s|^2\,e^{-2a_s\tau_s} - 4\frac{a_s\tau_s}{\tau_b^3}|A_s|\,|W_0|\,e^{-a_s\tau_s} + \frac{3}{2}\frac{\xi}{\tau_b^{9/2}}|W_0|^2. \tag{A.8}$$

Its derivatives are then immediately computed to be:

$$\begin{aligned}\frac{\partial V_F^{\text{eff}}}{\partial \tau_b} &= -4\,\frac{a_s^2\tau_s^{1/2}}{\tau_b^{5/2}}\,|A_s|^2\,e^{-2a_s\tau_s} + 12\frac{a_s\tau_s}{\tau_b^4}\,|A_s|\,|W_0|\,e^{-a_s\tau_s} - \frac{27}{4}\frac{\xi}{\tau_b^{11/2}}\,|W_0|^2\,,\\ \frac{\partial V_F^{\text{eff}}}{\partial \tau_s} &= \frac{4}{3}\frac{a_s^2\tau_s^{-1/2}}{\tau_b^{3/2}}(1-4a_s\tau_s)\,|A_s|^2\,e^{-2a_s\tau_s} - 4\frac{a_s}{\tau_b^3}(1-a_s\tau_s)\,|A_s|\,|W_0|\,e^{-a_s\tau_s},\end{aligned} \tag{A.9}$$

so that the stationarity conditions:

$$\begin{cases}\dfrac{\partial V_F^{\text{eff}}}{\partial \tau_b}\left(\langle\tau_b\rangle\,,\langle\tau_s\rangle\right)=0,\\ \dfrac{\partial V_F^{\text{eff}}}{\partial \tau_s}\left(\langle\tau_b\rangle\,,\langle\tau_s\rangle\right)=0,\end{cases} \tag{A.10}$$

are solved quite easily. As a matter of fact, the second equation of (A.10) readily gives:

$$\langle\tau_b\rangle^{3/2} = \frac{3}{4}\frac{\langle\tau_s\rangle^{\frac{1}{2}}}{a_s}\frac{|W_0|}{|A_s|}\;e^{a_s\langle\tau_s\rangle}\,\frac{1-\dfrac{1}{a_s\langle\tau_s\rangle}}{1-\dfrac{1}{4a_s\langle\tau_s\rangle}}, \tag{A.11}$$

and once that this value is substituted into the first one of (A.10), it gives:

$$\langle\tau_s\rangle^{3/2} = \xi\frac{\left(1-\dfrac{1}{4a_s\langle\tau_s\rangle}\right)^2}{\left(1-\dfrac{1}{a_s\langle\tau_s\rangle}\right)}. \tag{A.12}$$

It is fundamental to notice that these results are coherent with the approximations which allowed to the define V_F^{eff}. Indeed around the ground state it is evident that the terms which have been discarded behave as:

$$\frac{\tau_s^{3/2}}{\tau_b^{3/2}}\;e^{-a_s\tau_s} \overset{(*)}{\sim} O\left(\frac{\tau_s^{3/2}}{\tau_b^{3/2}}\right)^2, \qquad \frac{\xi}{\tau_b^{3/2}}\;e^{-a_s\tau_s} \overset{(*)}{\sim} O\left(\frac{\xi}{\tau_b^{3/2}}\right)^2.$$

This observation will be crucial in what follows.

F-terms

Auxiliary fields can be computed straightforwardly. Their exact expressions read:

$$F^b = -\frac{4\,\tau_b}{2\tau_b^{\frac{3}{2}}-2\tau_s^{\frac{3}{2}}-\xi}\left[|W_0| + (1+2a_s\tau_s)\,|A_s|\,e^{-a_s\tau_s}e^{-i(\alpha_s-\theta-a_s\psi_s)}\right], \tag{A.13}$$

$$F^s = -\frac{1}{3\left(2\tau_b^{\frac{3}{2}} - 2\tau_s^{\frac{3}{2}} - \xi\right)}\Bigg\{12\tau_s\left|W_0\right| \\ + \left[8a_s\tau_s^{\frac{1}{2}}\left(2\tau_b^{\frac{3}{2}} + \tau_s^{\frac{3}{2}} - \xi\right) + 12\tau_s\right]\left|A_s\right| e^{-a_s\tau_s} e^{-i(\alpha_s - \theta - a_s\psi_s)}\Bigg\}, \tag{A.14}$$

Then, under conditions (A.3), the effective auxiliary fields vacuum expectation values can be written as:

$$\left\langle F^b_{\text{eff}}\right\rangle = -2\,\frac{|W_0|}{\langle\tau_b\rangle^{\frac{1}{2}}}\left[1 + \frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_b\rangle^{\frac{3}{2}}} - \frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\left(1 + \frac{3}{8a_s\langle\tau_s\rangle} - \frac{3}{16a_s^2\langle\tau_s\rangle^2}\right)\right], \tag{A.15}$$

$$\left\langle F^s_{\text{eff}}\right\rangle = -\frac{3}{2}\,\frac{|W_0|}{a_s\langle\tau_b\rangle^{3/2}}\left[1 + \sum_{n=1}^{\infty}\frac{1}{(4a_s\langle\tau_s\rangle)^n}\right]. \tag{A.16}$$

A.1.2 Moduli Masses

Mass Matrix

It is possible to go on by writing the Hessian matrix of V_F^{eff} and to determine its vacuum expectation value. The second derivatives read:

$$\begin{aligned}
\frac{\partial^2 V_F^{\text{eff}}}{\partial\tau_b^2} &= 10\frac{a_s^2\tau_s^{1/2}}{\tau_b^{7/2}}\left|A_s\right|^2 e^{-2a_s\tau_s} - 48\frac{a_s\tau_s}{\tau_b^5}\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s} + \frac{297}{8}\frac{\xi}{\tau_b^{13/2}}\left|W_0\right|^2, \\
\frac{\partial^2 V_F^{\text{eff}}}{\partial\tau_b\,\partial\tau_s} &= -2\frac{a_s^2}{\tau_b^{5/2}\tau_s^{1/2}}\left(1 - 4a_s\tau_s\right)\left|A_s\right|^2 e^{-2a_s\tau_s} + 12\frac{a_s}{\tau_b^4}\left(1 - a_s\tau_s\right)\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s}, \\
\frac{\partial^2 V_F^{\text{eff}}}{\partial\tau_s^2} &= \frac{4a_s^2}{\tau_b^3}\left|A_s\right| e^{-a_s\tau_s}\left[-\frac{\tau_b^{3/2}}{6\tau_s^{3/2}}\left(1 + 8a_s\tau_s - 16a_s^2\tau_s^2\right)\left|A_s\right| e^{-a_s\tau_s} + \left(2 - a_s\tau_s\right)\left|W_0\right|\right].
\end{aligned}$$

By noticing from (A.11) and (A.12) that the following equalities hold true:

$$\begin{aligned}
a_s^2\langle\tau_s\rangle^{1/2}\langle\tau_b\rangle^3\left|A_s\right|^2 e^{-2a_s\langle\tau_s\rangle} &= \frac{9}{16}\xi\left|W_0\right|^2\left(1 - \frac{1}{a_s\langle\tau_s\rangle}\right), \\
a_s\langle\tau_s\rangle\langle\tau_b\rangle^{3/2}\left|A_s\right|\left|W_0\right| e^{-a_s\langle\tau_s\rangle} &= \frac{3}{4}\xi\left|W_0\right|^2\left(1 - \frac{1}{4a_s\langle\tau_s\rangle}\right),
\end{aligned}$$

a straightforward computation allows then to write the vacuum expectation value of the Hessian matrix or, equivalently, the unnormalised mass matrix $(M^2_{\text{eff}})_{ij} = \left\langle V_{ij}^{\text{eff}}\right\rangle/2$:

$$\left(M^2_{\text{eff}}\right)_{ij} = \frac{3}{2}\frac{\xi\left|W_0\right|^2}{\langle\tau_b\rangle^{13/2}}\begin{pmatrix} \frac{9}{4}\left(1 + \frac{1}{2}\sigma\right) & -\frac{3}{2}a_s\langle\tau_b\rangle\left(1 - \frac{5}{4}\sigma + \frac{1}{4}\sigma^2\right) \\ -\frac{3}{2}a_s\langle\tau_b\rangle\left(1 - \frac{5}{4}\sigma + \frac{1}{4}\sigma^2\right) & a_s^2\langle\tau_b\rangle^2\left(1 - \frac{3}{4}\sigma + \frac{3}{8}\sigma^2 + \frac{1}{8}\sigma^3\right)\end{pmatrix}, \tag{A.17}$$

where the parameter σ is defined as $\sigma = 1/a_s \langle\tau_s\rangle$.
In order to compute the normalised mass matrix and its eigenvalues and eigenvectors, some approximations are needed. According to (A.3) and thus following the same approximations which lead to the effective scalar potential, the simplest though non-trivial condition which can be assumed is that terms of order II or greater in $\langle\tau_s\rangle^{3/2}/\langle\tau_b\rangle^{3/2}$ and $\xi/\langle\tau_b\rangle^{3/2}$ are negligible.
The inverse Kähler metric vacuum expectation value $\langle K^{ij}\rangle$ can then be expanded up to order I in $\langle\tau_s\rangle^{3/2}/\langle\tau_b\rangle^{3/2}$ and $\xi/\langle\tau_b\rangle^{3/2}$ as:

$$\left\langle K^{ij}\right\rangle \overset{(*)}{\simeq} \left\langle K^{ij}_{\text{eff}}\right\rangle ,$$

where:

$$\left\langle K^{ij}_{\text{eff}}\right\rangle \equiv \begin{pmatrix} \frac{4}{3}\langle\tau_b\rangle^2\left(1+2\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_b\rangle^{\frac{3}{2}}}+2\frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\right) & 4\langle\tau_s\rangle\langle\tau_b\rangle\left(1+\frac{3}{2}\frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\right) \\ 4\langle\tau_s\rangle\langle\tau_b\rangle\left(1+\frac{3}{2}\frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\right) & \frac{8}{3}\langle\tau_s\rangle^{1/2}\langle\tau_b\rangle^{3/2}\left(1+\frac{1}{2}\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_b\rangle^{\frac{3}{2}}}+\frac{1}{2}\frac{\xi}{\langle\tau_b\rangle^{\frac{3}{2}}}\right) \end{pmatrix}. \tag{A.18}$$

Then, since $(M^2_{\text{eff}})_{ij}$ is by itself proportional to a factor $\xi/\langle\tau_b\rangle^{3/2}$ the computation is straightforward, so the normalised mass matrix $(m^2)^i{}_j$, after the very same approximations as above and following the same notation,

$$\left(m^2\right)^i{}_j = \left\langle K^{ik}\right\rangle\left(M^2\right)_{kj} \overset{(*)}{\simeq} \left\langle K^{ik}_{\text{eff}}\right\rangle\left(M^2_{\text{eff}}\right)_{kj} \overset{(*)}{\simeq} \left(m^2_{\text{eff}}\right)^i{}_j ,$$

takes the effective form:

$$\left(m^2_{\text{eff}}\right)^i{}_j \equiv \frac{a_s\langle\tau_s\rangle^{\frac{1}{2}}\xi\,|W_0|^2}{\langle\tau_b\rangle^{\frac{9}{2}}}\begin{pmatrix} -9\langle\tau_s\rangle^{\frac{1}{2}}\left(1-\frac{7}{4}\sigma\right) & 6a_s\langle\tau_s\rangle^{\frac{1}{2}}\langle\tau_b\rangle\left(1-\frac{5}{4}\sigma+\sigma^2\right) \\ -6\langle\tau_b\rangle^{\frac{1}{2}}\left(1-\frac{5}{4}\sigma+\frac{1}{4}\sigma^2\right) & 4a_s\langle\tau_b\rangle^{\frac{3}{2}}\left(1-\frac{3}{4}\sigma+\frac{3}{8}\sigma^2+\frac{1}{8}\sigma^3\right) \end{pmatrix}, \tag{A.19}$$

which must be diagonalised in order to determine the diagonalised kinetic bosonic Lagrangian.

Canonically Normalised Fields and Masses

The eigenvalues $m^2_{(l)}$ and eigenvectors $u^i_{(l)}$ of $(m^2_{\text{eff}})^i{}_j$ can be computed directly.
Under the same approximations as before, i.e. up to order I in $\langle\tau_s\rangle^{3/2}/\langle\tau_b\rangle^{3/2}$ and $\xi/\langle\tau_b\rangle^{3/2}$, on the one hand normalised masses turn out to be:

$$m^2_\chi \overset{(*)}{\simeq} \frac{81}{8}\frac{\xi\,|W_0|^2}{a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}, \tag{A.20}$$

$$m_\phi^2 \overset{(*)}{\simeq} \frac{4a_s^2 \langle\tau_s\rangle^{1/2} \xi \left|W_0\right|^2}{\langle\tau_b\rangle^3} \left[1 - \frac{3}{4a_s\langle\tau_s\rangle} + \frac{3}{8a_s^2\langle\tau_s\rangle^2} + \frac{1}{8a_s^3\langle\tau_s\rangle^3}\right], \tag{A.21}$$

while on the other hand unnormalised eigenvectors $v^i_{(l)}$ can be written as:

$$v^i_\chi \overset{(*)}{\simeq} \begin{pmatrix} \frac{2}{3}a_s\langle\tau_b\rangle \left[1 + \frac{1}{2a_s\langle\tau_s\rangle} + \frac{3}{4a_s^2\langle\tau_s\rangle^2} - \frac{9}{4a_s\langle\tau_s\rangle}\frac{\langle\tau_s\rangle^{3/2}}{\langle\tau_b\rangle^{3/2}}\left(1 - \frac{5}{4a_s\langle\tau_s\rangle}\right)\right] \\ 1 \end{pmatrix}, \tag{A.22}$$

$$v^i_\phi \overset{(*)}{\simeq} \begin{pmatrix} \frac{3}{2}\frac{\langle\tau_s\rangle^{\frac{1}{2}}}{\langle\tau_b\rangle^{\frac{1}{2}}}\left[1 + \frac{5}{4a_s\langle\tau_s\rangle} + \frac{3}{16a_s^2\langle\tau_s\rangle^2}\right] \\ 1 \end{pmatrix}, \tag{A.23}$$

and can be normalised as $u^i_{(l)} \equiv N_{(l)} v^i_{(l)}$ in such a way as to have:

$$\left\langle K^{\mathrm{eff}}_{ij}\right\rangle u^i_{(l)} u^j_{(k)} = \delta_{lk}.$$

The leading order of these expressions is enough for further discussions. Indeed it was shown in a general way that the bosonic Lagrangian can always be diagonalised by following the procedure which has just been described. The eigenvectors matrix turns out to be:

$$P^i_{\ j} \simeq \begin{pmatrix} \frac{2\sqrt{3}}{3}\langle\tau_b\rangle & \sqrt{6}\langle\tau_s\rangle^{3/4}\langle\tau_b\rangle^{1/4} \\ \frac{\sqrt{3}}{a_s} & \frac{2\sqrt{6}}{3}\langle\tau_s\rangle^{1/4}\langle\tau_b\rangle^{3/4} \end{pmatrix}, \tag{A.24}$$

and allows to define the canonically normalised τ-fields via the relation:

$$\begin{pmatrix} \delta\tau_b \\ \delta\tau_s \end{pmatrix} \simeq \frac{\sqrt{2}}{2} \begin{pmatrix} \frac{2\sqrt{3}}{3}\langle\tau_b\rangle & \frac{\sqrt{3}}{a_s} \\ \sqrt{6}\langle\tau_s\rangle^{3/4}\langle\tau_b\rangle^{1/4} & \frac{2\sqrt{6}}{3}\langle\tau_s\rangle^{1/4}\langle\tau_b\rangle^{3/4} \end{pmatrix} \begin{pmatrix} \chi \\ \phi \end{pmatrix}, \tag{A.25}$$

where of course $\delta\tau_b$ and $\delta\tau_s$ are the fields fluctuations from the ground state.
As regards the axionic fields, the effective scalar potential (A.6) is exactly of the kind analysed in Section 2.4, which means that[1]:

$$\left\langle \frac{\partial^2 V_F}{\partial\psi^i\,\partial\psi^j}\right\rangle = \left\langle\frac{\partial^2 V_F}{\partial{\psi^i}^{\,2}}\right\rangle\delta_{ij}, \qquad \left\langle\frac{\partial^2 V_F}{\partial\tau^i\,\partial\psi^j}\right\rangle = 0.$$

[1]The notation might be misleading: here V_F^{eff} is intended as still dependent on ψ_s, as it turns out to be when directly approximating (A.6) before minimizing with respect to the axions.

In particular, the effective axionic unnormalised masses are, after the usual approximation:

$$\left(M'^2_{\text{eff}}\right)_i = \frac{1}{2}\left\langle \frac{\partial^2 V_F^{\text{eff}}}{\partial \psi^{i\ 2}} \right\rangle = \begin{pmatrix} 0 \\ \frac{3a_s^2 \xi \left|W_0\right|^2}{4\left\langle \tau_b \right\rangle^{\frac{9}{2}}}\left(1 - \frac{1}{4a_s \left\langle \tau_s \right\rangle}\right) \end{pmatrix}.$$

The canonical normalisation of the Lagrangian is immediate: it is sufficient to normalise the two fields by requiring canonical pure kinetic terms, eventually discarding an irrelevant mixing term. More precisely, the normalised fields θ_χ and θ_ϕ can be defined via the relations:

$$\delta\psi_b \simeq \frac{\sqrt{6}}{3}\left\langle \tau_b \right\rangle \theta_\chi, \tag{A.26}$$

$$\delta\psi_s \simeq \frac{2\sqrt{3}}{3}\left\langle \tau_s \right\rangle^{1/4}\left\langle \tau_b \right\rangle^{3/4}\theta_\phi, \tag{A.27}$$

and the masses of canonically normalised axionic fields are then at leading order:

$$m^2_{\theta_\chi} = 0, \tag{A.28}$$

$$m^2_{\theta_\phi} \simeq \frac{4\ a_s^2 \left\langle \tau_s \right\rangle^{1/2} \xi \left|W_0\right|^2}{\left\langle \tau_b \right\rangle^3}. \tag{A.29}$$

A.2 Model III: Leading Results

Model III describes three Kähler moduli $T_1 = \tau_1 + i\psi_1$, $T_2 = \tau_2 + i\psi_2$ and $T_s = \tau_s + i\psi_s$ with the Kähler potential and the superpotential respectively expressed as:

$$K = -2\ \ln\left(\tau_1^{1/2}\tau_2 - \tau_s^{3/2} + \xi\right), \tag{A.30}$$

$$W = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)}, \tag{A.31}$$

under the assumptions:

$$(\star): \begin{cases} a_s\tau_s \gg 1, \\ \tau_1^{1/2}\tau_2 \gg \tau_s^{3/2} \sim \xi, \end{cases} \tag{A.32}$$

according to the well-known features.

A.2.1 Minimisation

Refering the first, second and third columns and rows to τ_1, τ_2 and τ_s respectively, a straightforward computation gives the exact Kähler metric:

$$K_{i\bar{j}} = \frac{1}{(\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}} + \xi)^2} \begin{pmatrix} \frac{\tau_2\left(2\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}} + \xi\right)}{8\tau_1^{\frac{3}{2}}} & \frac{\tau_s^{\frac{3}{2}} - \xi}{4\tau_1^{\frac{1}{2}}} & -\frac{3\tau_2\tau_s^{\frac{1}{2}}}{8\tau_1^{\frac{1}{2}}} \\ \frac{\tau_s^{\frac{3}{2}} - \xi}{4\tau_1^{\frac{1}{2}}} & \frac{\tau_1}{2} & -\frac{3}{4}\tau_1^{\frac{1}{2}}\tau_s^{\frac{1}{2}} \\ -\frac{3\tau_2\tau_s^{\frac{1}{2}}}{8\tau_1^{\frac{1}{2}}} & -\frac{3}{4}\tau_1^{\frac{1}{2}}\tau_s^{\frac{1}{2}} & \frac{3\left(\tau_1^{\frac{1}{2}}\tau_2 + 2\tau_s^{\frac{3}{2}} + \xi\right)}{8\tau_s^{\frac{1}{2}}} \end{pmatrix}, \tag{A.33}$$

and its inverse:

$$K^{i\bar{j}} = \begin{pmatrix} K^{i'\bar{j}'} & K^{i'\bar{s}} \\ K^{s\bar{j}'} & K^{s\bar{s}} \end{pmatrix}, \tag{A.34}$$

primed indices standing only for rows and columns 1 and 2, with:

$$K^{i'\bar{j}'} = \frac{\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}} + \xi}{2(\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}}) - \xi} \begin{pmatrix} 8\,\tau_1^2 & 4\,\tau_1^{\frac{1}{2}}\left(2\tau_s^{\frac{3}{2}} + \xi\right) \\ 4\,\tau_1^{\frac{1}{2}}\left(2\tau_s^{\frac{3}{2}} + \xi\right) & \frac{2\,\tau_2\left[2\left(\tau_1^{\frac{1}{2}}\tau_2 + \tau_s^{\frac{3}{2}}\right) + \xi\right]}{\tau_1^{\frac{1}{2}}} \end{pmatrix},$$

$$K^{i'\bar{s}} = \frac{\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}} + \xi}{2(\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}}) - \xi} \begin{pmatrix} 8\tau_1\tau_s \\ 8\tau_2\tau_s \end{pmatrix},$$

$$K^{s\bar{s}} = \frac{\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}} + \xi}{2(\tau_1^{\frac{1}{2}}\tau_2 - \tau_s^{\frac{3}{2}}) - \xi} \cdot \frac{8}{3}\,\tau_s^{\frac{1}{2}}\,\left(2\tau_1^{\frac{1}{2}}\tau_2 + \tau_s^{\frac{3}{2}} - \xi\right),$$

being $K^{i'\bar{s}} = K^{s\bar{i}'}$ as the metric is symmetric.
Therefore, the exact scalar potential can be computed and results:

$$V_F = k\left(v_F^{(1)} + v_F^{(2)} + v_F^{(3)}\right), \tag{A.35}$$

where the following elements have been defined for the sake of clarity:

$$k \equiv \frac{1}{3\left(\tau_1^{\frac{1}{2}}\tau_2 + \tau_s^{\frac{3}{2}} - \xi\right)^2\left(2\tau_1^{\frac{1}{2}}\tau_2 - 2\tau_s^{\frac{3}{2}} - \xi\right)},$$

$$v_F^{(1)} \equiv -\Big[8a_s^2\xi^2\tau_s^{\frac{1}{2}} + 8a_s\tau_s^{\frac{1}{2}}\left(\tau_s^{\frac{3}{2}} - \tau_1^{\frac{1}{2}}\tau_2\right)\left(3\tau_s^{\frac{1}{2}} + a_s\left(2\tau_1^{\frac{1}{2}}\tau_2 + \tau_s^{\frac{3}{2}}\right)\right)$$
$$- \xi\left(9 + 24a_s\tau_s + 8a_s^2\left(\tau_1^{\frac{1}{2}}\tau_2\tau_s^{\frac{1}{2}} + 2\tau_s^2\right)\right)\Big] |A_s|^2 e^{-2a_s\tau_s},$$

$$v_F^{(2)} \equiv 6\cos\left(\alpha_s - \theta - a_s\psi_s\right)\left[3\xi + 4a_s\tau_s\xi + 4a_s\tau_1^{\frac{1}{2}}\tau_2\tau_s - 4a_s\tau_s^{\frac{5}{2}}\right] |A_s| \, |W_0| \, e^{-a_s\tau_s},$$

$$v_F^{(3)} \equiv 9\xi \, |W_0|^2 \,.$$

As usual, the minimisation of the scalar potential takes place if the axion ψ_s takes a vacuum expectation value such that:

$$\langle\psi_s\rangle = \frac{\alpha_s - \theta}{a_s} + (2n+1)\,\frac{\pi}{a_s}, \quad n \in \mathbb{Z}. \tag{A.36}$$

Again, the exact scalar potential is still way too complicated to be treated directly. Under conditions (A.32) it is reasonable to perform a Taylor expansion of the scalar potential up to order I in the parameters $\tau_s^{3/2}/\tau_1^{1/2}\tau_2$ and $\xi/\tau_1^{1/2}\tau_2$. Moreover, it is coherent with the same conditions to neglect all of the terms proportional to expressions like $(\tau_s^{3/2}/\tau_1^{1/2}\tau_2)\; e^{-a_s\tau_s}$ and $(\xi/\tau_1^{1/2}\tau_2)\; e^{-a_s\tau_s}$. The outcome of these approximations is:

$$V_F \overset{(\star)}{\simeq} V_F^{\text{eff}}$$

where the effective scalar potential is defined as:

$$V_F^{\text{eff}} \equiv \frac{8}{3}\frac{a_s^2\tau_s^{1/2}}{\tau_1^{1/2}\tau_2} |A_s|^2 e^{-2a_s\tau_s} - 4\frac{a_s\tau_s}{\left(\tau_1^{1/2}\tau_2\right)^2} |A_s| \, |W_0| \, e^{-a_s\tau_s} + \frac{3}{2}\frac{\xi}{\left(\tau_1^{1/2}\tau_2\right)^3} |W_0|^2 \,. \tag{A.37}$$

Subsequent calculations then confirm the presence of a flat directions according to the fact that the scalar potential (A.37) is exactly equal to (A.8) but for the expression of one variable as a product of two, i.e. $\tau_b^{3/2} = \tau_1^{1/2}\tau_2$.

The scalar potential derivatives are:

$$\begin{aligned}
\frac{\partial V_F^{\text{eff}}}{\partial \tau_1} &= -\frac{4}{3}\frac{a_s^2\tau_s^{1/2}}{\tau_1^{3/2}\tau_2} |A_s|^2 e^{-2a_s\tau_s} + 4\frac{a_s\tau_s}{\tau_1^2\tau_2^2} |A_s| \, |W_0| \, e^{-a_s\tau_s} - \frac{9}{4}\frac{\xi}{\tau_1^{5/2}\tau_2^3} |W_0|^2 \,, \\
\frac{\partial V_F^{\text{eff}}}{\partial \tau_2} &= -\frac{8}{3}\frac{a_s^2\tau_s^{1/2}}{\tau_1^{1/2}\tau_2^2} |A_s|^2 e^{-2a_s\tau_s} + 8\frac{a_s\tau_s}{\tau_1\tau_2^3} |A_s| \, |W_0| \, e^{-a_s\tau_s} - \frac{9}{2}\frac{\xi}{\tau_1^{3/2}\tau_2^4} |W_0|^2 \,, \\
\frac{\partial V_F^{\text{eff}}}{\partial \tau_s} &= \frac{4}{3}\frac{a_s^2\tau_s^{-1/2}}{\tau_1^{1/2}\tau_2}(1 - 4a_s\tau_s) |A_s|^2 e^{-2a_s\tau_s} - 4\frac{a_s}{\tau_1\tau_2^2}(1 - a_s\tau_s) |A_s| \, |W_0| \, e^{-a_s\tau_s}.
\end{aligned} \tag{A.38}$$

It is then immediate to notice that the stationarity conditions:

$$\begin{cases} \dfrac{\partial V_F^{\text{eff}}}{\partial \tau_1} \left(\langle \tau_1 \rangle , \langle \tau_2 \rangle , \langle \tau_s \rangle \right) = 0, \\ \dfrac{\partial V_F^{\text{eff}}}{\partial \tau_2} \left(\langle \tau_1 \rangle , \langle \tau_2 \rangle , \langle \tau_s \rangle \right) = 0, \\ \dfrac{\partial V_F^{\text{eff}}}{\partial \tau_s} \left(\langle \tau_1 \rangle , \langle \tau_2 \rangle , \langle \tau_s \rangle \right) = 0, \end{cases} \tag{A.39}$$

are directly reducible to their analogue (A.10) in Model II. Indeed it is possible to observe that the following identities hold true:

$$\frac{\partial V_F^{\text{eff}}}{\partial \tau_b} \left(\tau_b = \tau_1^{\frac{1}{2}} \tau_2, \tau_s \right) \equiv \frac{1}{3} \left(\frac{\tau_1}{\tau_2} \right)^{\frac{2}{3}} \frac{\partial V_F^{\text{eff}}}{\partial \tau_1} \left(\tau_1, \tau_2, \tau_s \right) \equiv \frac{2}{3} \left(\frac{\tau_2}{\tau_1} \right)^{\frac{1}{3}} \frac{\partial V_F^{\text{eff}}}{\partial \tau_2} \left(\tau_1, \tau_2, \tau_s \right),$$

$$\frac{\partial V_F^{\text{eff}}}{\partial \tau_s} \left(\tau_b = \tau_1^{\frac{1}{2}} \tau_2, \tau_s \right) \equiv \frac{\partial V_F^{\text{eff}}}{\partial \tau_s} \left(\tau_1, \tau_2, \tau_s \right),$$

where the left-hand sides of both expressions refer to the derivatives (A.9) of the scalar potential of Model II. This fact means that the system of equations (A.39) admits the solutions:

$$\langle \tau_1 \rangle^{1/2} \langle \tau_2 \rangle = \frac{3}{4} \frac{\langle \tau_s \rangle^{\frac{1}{2}}}{a_s} \frac{|W_0|}{|A_s|} \, e^{a_s \langle \tau_s \rangle} \frac{1 - \dfrac{1}{a_s \langle \tau_s \rangle}}{1 - \dfrac{1}{4 a_s \langle \tau_s \rangle}}, \tag{A.40}$$

$$\langle \tau_s \rangle^{3/2} = \xi \frac{\left(1 - \dfrac{1}{4 a_s \langle \tau_s \rangle} \right)^2}{\left(1 - \dfrac{1}{a_s \langle \tau_s \rangle} \right)}, \tag{A.41}$$

as expected.

F-terms

The exact expressions of the auxiliary fields can be easily shown to be:

$$F^1 = - \frac{4 \tau_1}{2 \tau_1^{\frac{1}{2}} \tau_2 - 2 \tau_s^{\frac{3}{2}} - \xi} \left[|W_0| + (1 + 2 a_s \tau_s) |A_s| \, e^{-a_s \tau_s} e^{-i(\alpha_s - \theta - a_s \psi_s)} \right], \tag{A.42}$$

$$F^2 = - \frac{4 \tau_2}{2 \tau_1^{\frac{1}{2}} \tau_2 - 2 \tau_s^{\frac{3}{2}} - \xi} \left[|W_0| + (1 + 2 a_s \tau_s) |A_s| \, e^{-a_s \tau_s} e^{-i(\alpha_s - \theta - a_s \psi_s)} \right], \tag{A.43}$$

$$F^s = -\frac{1}{3\left(2\tau_1^{\frac{1}{2}}\tau_2 - 2\tau_s^{\frac{3}{2}} - \xi\right)}\Bigg\{12\tau_s\left|W_0\right| + \left[8a_s\tau_s^{\frac{1}{2}}\left(2\tau_1^{\frac{1}{2}}\tau_2 + \tau_s^{\frac{3}{2}} - \xi\right) + 12\tau_s\right]\left|A_s\right| e^{-a_s\tau_s} e^{-i(\alpha_s-\theta-a_s\psi_s)}\Bigg\}. \tag{A.44}$$

Following the conditions (A.32), it is then possible to determine their effective vacuum expectation values:

$$\left\langle F^1_{\rm eff}\right\rangle = -\,2\,\frac{\left\langle\tau_1\right\rangle^{\frac{1}{2}}}{\left\langle\tau_2\right\rangle}\,\left|W_0\right|\Bigg[1+\frac{1}{2}\frac{\xi}{\left\langle\tau_1\right\rangle^{\frac{1}{2}}\left\langle\tau_2\right\rangle} - \frac{1}{2}\frac{\left\langle\tau_s\right\rangle^{\frac{3}{2}}}{\left\langle\tau_1\right\rangle^{\frac{1}{2}}\left\langle\tau_2\right\rangle}\left(1-\frac{3}{4a_s\left\langle\tau_s\right\rangle}-\frac{27}{16a_s^2\left\langle\tau_s\right\rangle^2}\right)\Bigg], \tag{A.45}$$

$$\left\langle F^2_{\rm eff}\right\rangle = -\,2\,\frac{1}{\left\langle\tau_1\right\rangle^{\frac{1}{2}}}\,\left|W_0\right|\Bigg[1+\frac{1}{2}\frac{\xi}{\left\langle\tau_1\right\rangle^{\frac{1}{2}}\left\langle\tau_2\right\rangle} - \frac{1}{2}\frac{\left\langle\tau_s\right\rangle^{\frac{3}{2}}}{\left\langle\tau_1\right\rangle^{\frac{1}{2}}\left\langle\tau_2\right\rangle}\left(1-\frac{3}{4a_s\left\langle\tau_s\right\rangle}-\frac{27}{16a_s^2\left\langle\tau_s\right\rangle^2}\right)\Bigg], \tag{A.46}$$

$$\left\langle F^s_{\rm eff}\right\rangle = -\frac{3}{2a_s\left\langle\tau_1\right\rangle^{\frac{1}{2}}\left\langle\tau_2\right\rangle}\,\left|W_0\right|\left[1+\sum_{n=1}^{\infty}\frac{1}{\left(4a_s\left\langle\tau_s\right\rangle\right)^n}\right]. \tag{A.47}$$

A.2.2 Moduli Masses

Mass Matrix

The second derivatives of $V_F^{\rm eff}$ read, in a close resemblance to their analogue in Model II:

$$\frac{\partial^2 V_F^{\rm eff}}{\partial\tau_1^2} = 2\frac{a_s^2\tau_s^{\frac{1}{2}}}{\tau_1^{\frac{5}{2}}\tau_2}\left|A_s\right|^2 e^{-2a_s\tau_s} - 8\frac{a_s\tau_s}{\tau_1^3\tau_2^2}\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s} + \frac{45}{8}\frac{\xi}{\tau_1^{\frac{7}{2}}\tau_2^3}\left|W_0\right|^2,$$

$$\frac{\partial^2 V_F^{\rm eff}}{\partial\tau_1\,\partial\tau_2} = \frac{4}{3}\frac{a_s^2\tau_s^{\frac{1}{2}}}{\tau_1^{\frac{3}{2}}\tau_2^2}\left|A_s\right|^2 e^{-2a_s\tau_s} - 8\frac{a_s\tau_s}{\tau_1^2\tau_2^3}\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s} + \frac{27}{4}\frac{\xi}{\tau_1^{\frac{5}{2}}\tau_2^4}\left|W_0\right|^2,$$

$$\frac{\partial^2 V_F^{\rm eff}}{\partial\tau_1\,\partial\tau_s} = -\frac{2}{3}\frac{a_s^2}{\tau_1^{\frac{3}{2}}\tau_2\tau_s^{\frac{1}{2}}}\left(1-4a_s\tau_s\right)\left|A_s\right|^2 e^{-2a_s\tau_s} + 4\frac{a_s}{\tau_1^2\tau_2^2}\left(1-a_s\tau_s\right)\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s},$$

$$\frac{\partial^2 V_F^{\rm eff}}{\partial\tau_2^2} = \frac{16}{3}\frac{a_s^2\tau_s^{\frac{1}{2}}}{\tau_1^{\frac{1}{2}}\tau_2^3}\left|A_s\right|^2 e^{-2a_s\tau_s} - 24\frac{a_s\tau_s}{\tau_1\tau_2^4}\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s} + 18\frac{\xi}{\tau_1^{\frac{3}{2}}\tau_2^5}\left|W_0\right|^2,$$

$$\frac{\partial^2 V_F^{\rm eff}}{\partial\tau_2\,\partial\tau_s} = -\frac{4}{3}\frac{a_s^2}{\tau_1^{\frac{1}{2}}\tau_2^2\tau_s^{\frac{1}{2}}}\left(1-4a_s\tau_s\right)\left|A_s\right|^2 e^{-2a_s\tau_s} + 8\frac{a_s}{\tau_1\tau_2^3}\left(1-a_s\tau_s\right)\left|A_s\right|\left|W_0\right| e^{-a_s\tau_s},$$

$$\frac{\partial^2 V_F^{\rm eff}}{\partial \tau_s^2} = \frac{4a_s^2}{\tau_1 \tau_2^2} |A_s| e^{-a_s\tau_s} \left[-\frac{\tau_1^{\frac{1}{2}}\tau_2}{6\tau_s^{3/2}} \left(1 + 8a_s\tau_s - 16a_s^2\tau_s^2\right) |A_s| e^{-a_s\tau_s} + (2 - a_s\tau_s) |W_0| \right].$$

Indeed, observing as before thanks to solutions (A.40) and (A.41) that:

$$a_s^2 \langle\tau_s\rangle^{1/2} \langle\tau_1\rangle \langle\tau_1\rangle^2 |A_s|^2 e^{-2a_s\langle\tau_s\rangle} = \frac{9}{16}\xi |W_0|^2 \left(1 - \frac{1}{a_s\langle\tau_s\rangle}\right),$$
$$a_s \langle\tau_s\rangle \langle\tau_1\rangle^{\frac{1}{2}} \langle\tau_2\rangle |A_s| |W_0| e^{-a_s\langle\tau_s\rangle} = \frac{3}{4}\xi |W_0|^2 \left(1 - \frac{1}{4a_s\langle\tau_s\rangle}\right),$$

the unnormalised mass matrix $(M^2_{\rm eff})_{ij} = \left\langle V^{\rm eff}_{ij}\right\rangle/2$ can be written for the sake of simplicity as:

$$\left(M^2_{\rm eff}\right)_{ij} = \begin{pmatrix} (M^2_{\rm eff})_{i'j'} & (M^2_{\rm eff})_{i's} \\ (M^2_{\rm eff})_{sj'} & (M^2_{\rm eff})_{ss} \end{pmatrix}, \tag{A.48}$$

with:

$$\left(M^2_{\rm eff}\right)_{i'j'} = \frac{3}{2}\frac{\xi |W_0|^2}{\langle\tau_1\rangle^{\frac{7}{2}}\langle\tau_2\rangle^3} \begin{pmatrix} \frac{1}{4}\left(1+\frac{1}{2}\sigma\right) & \frac{1}{2}\frac{\langle\tau_1\rangle}{\langle\tau_2\rangle}\left(1+\frac{1}{2}\sigma\right) \\ \frac{1}{2}\frac{\langle\tau_1\rangle}{\langle\tau_2\rangle}\left(1+\frac{1}{2}\sigma\right) & \frac{\langle\tau_1\rangle^2}{\langle\tau_2\rangle^2}\left(1+\frac{1}{2}\sigma\right) \end{pmatrix},$$

$$\left(M^2_{\rm eff}\right)_{i's} = \frac{3}{2}\frac{\xi |W_0|^2}{\langle\tau_1\rangle^{\frac{7}{2}}\langle\tau_2\rangle^3} \begin{pmatrix} -\frac{1}{2}a_s\langle\tau_1\rangle\left(1 - \frac{5}{4}\sigma + \frac{1}{4}\sigma^2\right) \\ -\frac{a_s\langle\tau_1\rangle^2}{\langle\tau_2\rangle}\left(1 - \frac{5}{4}\sigma + \sigma^2\right) \end{pmatrix},$$

$$\left(M^2_{\rm eff}\right)_{ss} = \frac{3}{2}\frac{\xi |W_0|^2}{\langle\tau_1\rangle^{\frac{7}{2}}\langle\tau_2\rangle^3} \cdot a_s\langle\tau_1\rangle^2 \left(1 - \frac{3}{4}\sigma + \frac{3}{8}\sigma^2 + \frac{1}{8}\sigma^3\right),$$

being $(M^2_{\rm eff})_{sj'} = (M^2_{\rm eff})_{j's}$ as the matrix is symmetric.
It is evident that this matrix has a vanishing eigenvalue. Indeed its first two rows are linearly dependent as well as it two first columns (and are proportional to their analogue in the 'b' rows and columns in Model II):

$$\left(M^2_{\rm eff}\right)_{2j} = 2\frac{\langle\tau_1\rangle}{\langle\tau_2\rangle}\left(M^2_{\rm eff}\right)_{1j}, \qquad \left(M^2_{\rm eff}\right)_{i2} = 2\frac{\langle\tau_1\rangle}{\langle\tau_2\rangle}\left(M^2_{\rm eff}\right)_{i1}.$$

The inverse Kähler metric up to order I in in $\langle\tau_s\rangle^{3/2}/\langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ and $\xi/\langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ takes the effective form:

$$\left\langle K^{ij}\right\rangle \overset{(\star)}{\simeq} \left\langle K^{ij}_{\rm eff}\right\rangle = \begin{pmatrix} \left\langle K^{i'j'}_{\rm eff}\right\rangle & \left\langle K^{i's}_{\rm eff}\right\rangle \\ \left\langle K^{sj'}_{\rm eff}\right\rangle & \left\langle K^{ss}_{\rm eff}\right\rangle \end{pmatrix}, \tag{A.49}$$

with, being $\left\langle K_{\text{eff}}^{sj'}\right\rangle = \left\langle K_{\text{eff}}^{j's}\right\rangle$ as the matrix is symmetric:

$$\left\langle K_{\text{eff}}^{i'j'}\right\rangle = \begin{pmatrix} 4\langle\tau_1\rangle^2\left(1+\frac{1}{2}\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\right) & 2\langle\tau_1\rangle^2\left(2\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}+\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\right) \\ 2\langle\tau_1\rangle^2\left(2\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}+\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\right) & 2\langle\tau_2\rangle^2\left(1+\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}+\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\right) \end{pmatrix},$$

$$\left\langle K_{\text{eff}}^{i's}\right\rangle = \begin{pmatrix} 4\langle\tau_1\rangle\langle\tau_s\rangle\left(1+\frac{1}{2}\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\right) \\ 4\langle\tau_2\rangle\langle\tau_s\rangle\left(1+\frac{1}{2}\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\right) \end{pmatrix},$$

$$\left\langle K_{\text{eff}}^{ss}\right\rangle = \frac{8}{3}\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle\langle\tau_s\rangle^{\frac{1}{2}}\left(1+\frac{1}{2}\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\right).$$

Then, after the same approximations as above, the normalised mass matrix $(m^2)^i{}_j$ can be expressed as:

$$\left(m^2\right)^i{}_j = \left\langle K^{ik}\right\rangle\left(M^2\right)_{kj} \overset{(\star)}{\simeq} \left\langle K_{\text{eff}}^{ik}\right\rangle\left(M_{\text{eff}}^2\right)_{kj} \overset{(\star)}{\simeq} \left(m_{\text{eff}}^2\right)^i{}_j\,,$$

where, again:

$$\left(m_{\text{eff}}^2\right)^i{}_j = \begin{pmatrix} \left(m_{\text{eff}}^2\right)^{i'}{}_{j'} & \left(m_{\text{eff}}^2\right)^{i'}{}_s \\ \left(m_{\text{eff}}^2\right)^s{}_{j'} & \left(m_{\text{eff}}^2\right)^s{}_s \end{pmatrix}, \tag{A.50}$$

with:

$$\left(m_{\text{eff}}^2\right)^{i'}{}_{j'} = \frac{a_s\langle\tau_s\rangle\xi|W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}}\langle\tau_2\rangle^3}\begin{pmatrix} -3\left(1-\frac{7}{4}\sigma\right) & -6\,\frac{\langle\tau_1\rangle}{\langle\tau_2\rangle}\left(1-\frac{7}{4}\sigma\right) \\ -3\,\frac{\langle\tau_2\rangle}{\langle\tau_1\rangle}\left(1-\frac{7}{4}\sigma\right) & -6\left(1-\frac{7}{4}\sigma\right) \end{pmatrix},$$

$$\left(m_{\text{eff}}^2\right)^{i'}{}_s = \frac{a_s\langle\tau_s\rangle\xi|W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}}\langle\tau_2\rangle^3}\begin{pmatrix} 6a_s\langle\tau_1\rangle\left(1-\frac{5}{4}\sigma+\sigma^2\right) \\ 6a_s\langle\tau_2\rangle\left(1-\frac{5}{4}\sigma+\sigma^2\right) \end{pmatrix},$$

$$\left(m_{\text{eff}}^2\right)^s{}_{j'} = \frac{a_s\langle\tau_s\rangle\xi|W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}}\langle\tau_2\rangle^3}\left(-2\frac{\langle\tau_2\rangle}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_s\rangle^{\frac{1}{2}}}\left(1-\frac{5}{4}\sigma+\frac{1}{4}\sigma^2\right),\ -4\frac{\langle\tau_1\rangle^{\frac{1}{2}}}{\langle\tau_2\rangle^{\frac{1}{2}}}\left(1-\frac{5}{4}\sigma+\frac{1}{4}\sigma^2\right)\right),$$

$$\left(m_{\text{eff}}^2\right)^s{}_s = \frac{a_s\langle\tau_s\rangle\xi|W_0|^2}{\langle\tau_1\rangle^{\frac{3}{2}}\langle\tau_2\rangle^3}\left(4\frac{a_s\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}{\langle\tau_s\rangle^{\frac{1}{2}}}\left(1-\frac{3}{4}\sigma+\frac{3}{8}\sigma^2+\frac{1}{8}\sigma^3\right)\right).$$

Of course the first two rows and columns are again linearly dependent.

Canonically Normalised Fields and Masses

The eigenvalues $m^2_{(l)}$ and eigenvectors $u^i_{(l)}$ of $\left(m^2_{\text{eff}}\right)^i{}_j$ can be computed directly. Under the well-known conditions (A.32), up to order I in $\langle\tau_s\rangle^{3/2}/\langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ and $\xi/\langle\tau_1\rangle^{1/2}\langle\tau_2\rangle$ normalised masses turn out to be:

$$m^2_\zeta = 0, \tag{A.51}$$

$$m^2_\chi \overset{(\star)}{\simeq} \frac{81}{8}\frac{\xi\left|W_0\right|^2}{a_s\langle\tau_s\rangle\langle\tau_1\rangle^{3/2}\langle\tau_2\rangle^3}, \tag{A.52}$$

$$m^2_\phi \overset{(\star)}{\simeq} \frac{4a_s^2\langle\tau_s\rangle^{1/2}\xi\left|W_0\right|^2}{\langle\tau_1\rangle\langle\tau_2\rangle^2}\left[1-\frac{3}{4a_s\langle\tau_s\rangle}+\frac{3}{8a_s^2\langle\tau_s\rangle^2}+\frac{1}{8a_s^3\langle\tau_s\rangle^3}\right]. \tag{A.53}$$

Unnormalised eigenvectors $v^i_{(l)}$ are then:

$$v^i_\zeta = \begin{pmatrix} -2\dfrac{\langle\tau_1\rangle}{\langle\tau_2\rangle} \\ 1 \\ 0 \end{pmatrix}, \tag{A.54}$$

$$v^i_\chi \overset{(\star)}{\simeq} \begin{pmatrix} \dfrac{2}{3}a_s\langle\tau_1\rangle\left[1-\dfrac{11}{12a_s\langle\tau_s\rangle}+\dfrac{5}{12a_s^2\langle\tau_s\rangle^2}+\dfrac{9}{4a_s\langle\tau_s\rangle}\dfrac{\langle\tau_s\rangle^{3/2}}{\langle\tau_b\rangle^{3/2}}\right] \\ \dfrac{2}{3}a_s\langle\tau_2\rangle\left[1-\dfrac{11}{12a_s\langle\tau_s\rangle}+\dfrac{5}{12a_s^2\langle\tau_s\rangle^2}+\dfrac{9}{4a_s\langle\tau_s\rangle}\dfrac{\langle\tau_s\rangle^{3/2}}{\langle\tau_b\rangle^{3/2}}\right] \\ 1 \end{pmatrix}, \tag{A.55}$$

$$v^i_\phi \overset{(\star)}{\simeq} \begin{pmatrix} \dfrac{3}{2}\dfrac{\langle\tau_1\rangle^{\frac12}\langle\tau_s\rangle^{\frac12}}{\langle\tau_2\rangle}\left[1-\dfrac{1}{2a_s\langle\tau_s\rangle}+\dfrac{1}{4a_s^2\langle\tau_s\rangle^2}+\dfrac{1}{4a_s^3\langle\tau_s\rangle^3}+O\left(\dfrac{1}{a_s\langle\tau_s\rangle}\right)^4\right] \\ \dfrac{3}{2}\dfrac{\langle\tau_s\rangle^{\frac12}}{\langle\tau_1\rangle^{\frac12}}\left[1-\dfrac{1}{2a_s\langle\tau_s\rangle}+\dfrac{1}{4a_s^2\langle\tau_s\rangle^2}+\dfrac{1}{4a_s^3\langle\tau_s\rangle^3}+O\left(\dfrac{1}{a_s\langle\tau_s\rangle}\right)^4\right] \\ 1 \end{pmatrix}. \tag{A.56}$$

They must be normalised as $u^i_{(l)} \equiv N_{(l)}v^i_{(l)}$ in such a way as to have:

$$\left\langle K^{\text{eff}}_{ij}\right\rangle u^i_{(l)}u^j_{(k)} = \delta_{lk}.$$

Again, the leading order is sufficient in order to determine the change of basis which diagonalises the scalar Lagrangian. It is easy to show that properly normalised eigenvectors at leading order are:

$$u^i_\zeta \simeq \begin{pmatrix} -\dfrac{2\sqrt{6}}{3}\left\langle\tau_1\right\rangle \\[2ex] \dfrac{\sqrt{6}}{3}\left\langle\tau_2\right\rangle \\[2ex] 0 \end{pmatrix}, \tag{A.57}$$

$$u^i_\chi \simeq \begin{pmatrix} \dfrac{2\sqrt{3}}{3}\left\langle\tau_1\right\rangle \\[2ex] \dfrac{2\sqrt{3}}{3}\left\langle\tau_2\right\rangle \\[2ex] \dfrac{\sqrt{3}}{a_s} \end{pmatrix}, \tag{A.58}$$

$$u^i_\phi \simeq \begin{pmatrix} \sqrt{6}\,\dfrac{\left\langle\tau_1\right\rangle^{3/4}\left\langle\tau_s\right\rangle^{3/4}}{\left\langle\tau_2\right\rangle^{1/2}} \\[2ex] \sqrt{6}\,\dfrac{\left\langle\tau_2\right\rangle^{1/2}\left\langle\tau_s\right\rangle^{3/4}}{\left\langle\tau_1\right\rangle^{1/4}} \\[2ex] \dfrac{2\sqrt{6}}{3}\left\langle\tau_1\right\rangle^{1/4}\left\langle\tau_2\right\rangle^{1/2}\left\langle\tau_s\right\rangle^{1/4} \end{pmatrix}. \tag{A.59}$$

Finally, as concerns axions, computations are very similar to their corresponding in Model II. The presence of one more field is in fact irrelevant as this field is obviously massless.

A.3 Model III: Non-perturbative Corrections

Considering non-perturbative corrections to Model III, an exemplary model comes from the Kähler potential and the superpotential:

$$K_{\text{tot}} = -2\,\ln\left(\tau_1^{1/2}\tau_2 - \tau_s^{3/2} + \xi\right), \tag{A.60}$$

$$W_{\text{tot}} = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)} + A_1 e^{-a_1(\tau_1 + i\psi_1)}, \tag{A.61}$$

under the assumptions:

$$(\triangle): \begin{cases} a_1\tau_1 \gg a_s\tau_s \gg 1, \\ \tau_1^{1/2}\tau_2 \gg \tau_s^{3/2} \sim \xi. \end{cases} \tag{A.62}$$

A.3.1 Minimisation

The exact scalar potential can be conveniently written as the sum of two contributions:

$$V_F^{\text{tot}} = V_F + \delta V_F, \tag{A.63}$$

where V_F is equal to the exact scalar potential (A.35) of Model III, while δV_F is:

$$\delta V_F = k\left(\delta v_F^{(1)} + \delta v_F^{(2)} + \delta v_F^{(3)}\right), \tag{A.64}$$

with k as given in Section A.2 and:

$$\begin{aligned}
\delta v_F^{(1)} &= 3\left[8a_1\tau_1\left(a_1\tau_1+1\right)\left(\tau_1^{\frac{1}{2}}\tau_2-\tau_s^{\frac{3}{2}}\right)+\xi\left(8a_1^2\tau_1^2+8a_1\tau_1+3\right)\right]\left|A_1\right|^2 e^{-2a_1\tau_1},\\
\delta v_F^{(2)} &= 6\cos\left(\alpha_1-\alpha_s-a_1\psi_1+a_s\psi_s\right)\Bigg[4\left(\tau_1^{\frac{1}{2}}\tau_2-\tau_s^{\frac{3}{2}}\right)\left(a_s\tau_s+a_1\tau_1+2a_1\tau_1a_s\tau_s\right)\\
&\quad+\xi\left(8a_1\tau_1a_s\tau_s+4a_1\tau_1+4a_s\tau_s+3\right)\Bigg]\left|A_1\right|\left|A_s\right| e^{-a_1\tau_1}e^{-a_s\tau_s},\\
\delta v_F^{(3)} &= 6\cos\left(\alpha_1-\theta-a_1\psi_1\right)\left[4a_1\tau_1^{\frac{3}{2}}\tau_2-4a_1\tau_1\tau_s^{\frac{3}{2}}+4a_1\tau_1\xi+3\xi\right]\left|A_1\right|\left|W_0\right| e^{-a_1\tau_1}.
\end{aligned}$$

Taking into account the conditions (A.62) it is possible to identify the effective contributions to the total scalar potential:

$$\begin{aligned}
V_F &\overset{(\triangle)}{\sim} V_F^{\text{eff}},\\
\delta V_F &\overset{(\triangle)}{\sim} \delta V_F^{\text{eff}},
\end{aligned}$$

where of course V_F^{eff} corresponds to the potential in (A.37) under the same approximations, including the minimisation with respect to ψ_s, while:

$$\begin{aligned}
\delta V_F^{\text{eff}} =& \frac{4a_1\tau_1\left|A_1\right|e^{-a_1\tau_1}}{\tau_1\tau_2^2}\Bigg[a_1\tau_1\left|A_1\right|e^{-a_1\tau_1}\\
&+\cos\left(\alpha_1-\theta-a_1\psi_1\right)\left(\left|W_0\right|+2a_s\tau_s\left|A_s\right|e^{-a_s\tau_s}\right)\Bigg],
\end{aligned}\tag{A.65}$$

discarding all but the leading terms in all of the three factors of $|A_1|$, $|W_0|$ and $|A_s|$ because of the extremely small overall scaling by $e^{-a_1\tau_1}$.

According to Subsection 3.5, it is reasonable to first minimise V_F^{eff} after the change of the variables (τ_1, τ_2, τ_s) to $(\tau_1, \mathcal{V}', \tau_s)$ with:

$$\tau_2 = \tau_1^{-1/2}\,\mathcal{V}',$$

getting the leading order vacuum expectation values (3.75) and (3.76) for $\mathcal{V}'$ and τ_s, and then to consider δV_F as a residual correction evaluated in this ground state. The usual reasoning on phases enables to conclude that the vacuum expectation value of the axion ψ_1 is such that:

$$\langle \psi_1 \rangle = \frac{\alpha_1 - \theta}{a_1} + (2n+1)\,\frac{\pi}{a_1}, \quad n \in \mathbb{Z}, \tag{A.66}$$

in such a way as to have actually:

$$\delta V_F^{\text{eff}} \overset{(\triangle)}{\sim} \frac{4 a_1 \tau_1 \, |A_1| \, e^{-a_1 \tau_1}}{\langle \mathcal{V}' \rangle^2} \Bigg[a_1 \tau_1 \, |A_1| \, e^{-a_1 \tau_1} - |W_0| \Bigg].$$

Then, the latter is readily minimised in the point $\langle \tau_1 \rangle$ such that:

$$a_1 \, \langle \tau_1 \rangle \, e^{-a_1 \langle \tau_1 \rangle} = \frac{|W_0|}{2\,|A_1|}. \tag{A.67}$$

A.4 Model III: Perturbative Corrections

The introduction of further perturbative corrections in Model III involves the Kähler potential and the superpotential:

$$K_{\text{tot}} = -2 \, \ln \left(\tau_1^{1/2} \tau_2 - \tau_s^{3/2} + \xi \right) + \frac{c_1}{\tau_1} + \frac{c_2}{\tau_2} + \frac{C_W}{\tau_1 \tau_2}, \tag{A.68}$$

$$W_{\text{tot}} = W_0 + A_s e^{-a_s(\tau_s + i\psi_s)}. \tag{A.69}$$

under the assumptions:

$$(\bullet) : \begin{cases} a_s \tau_s \gg 1, \\ \tau_1^{1/2} \tau_2 \gg \tau_s^{3/2} \sim \xi, \\ \left| \dfrac{c_1}{\tau_1} \right| \sim \left| \dfrac{c_2}{\tau_2} \right| \ll 1, \\ \left| \dfrac{C_W}{\tau_1 \tau_2} \right| \ll 1, \end{cases} \tag{A.70}$$

with the useful hierarchy between orders of magnitude in the large volume scenario:

$$\left| \frac{c_1}{\tau_1} \right| \sim \left| \frac{c_2}{\tau_2} \right| \gg \frac{\tau_s^{3/2}}{\tau_1^{1/2} \tau_2} \sim \frac{\xi}{\tau_1^{1/2} \tau_2} \gg \left| \frac{C_W}{\tau_1 \tau_2} \right| \sim \frac{c_1^2}{\tau_1^2} \sim \frac{c_2^2}{\tau_2^2}. \tag{A.71}$$

A.4.1 Minimisation

The exact scalar potential can be computed exactly and it results in an overwhelmingly long expression, which is not reported here explictily for the sake of brevity. It is worthwhile to arrange it as the sum of four contributions, respectively proportional to $|A_s|^2 e^{-2a_s\tau_s}$, $|A_s|\,|W_0|\,e^{-a_s\tau_s}$ and the last two to $|W_0|^2$, multiplied by an overall factor:

$$V_F^{\rm tot} = d\left(v_F^{(1)} + v_F^{(2)} + v_F^{(3)} + v_F^{(4)}\right), \tag{A.72}$$

where, expliciting the leading order corrections for the sake of clarity in following explanations:

$$d^{-1} = 12\;\tau_1^{11/2}\tau_2^8\left[1+\frac{c_1}{\tau_1}+O\left(\frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2},\frac{\xi}{\tau_1^{1/2}\tau_2}\right)\right],$$

$$v_F^{(1)} = 32\;a_s^2\tau_s^{1/2}\tau_1^5\tau_2^7\,|A_s|^2\,e^{-2a_s\tau_s}\left[1+2\frac{c_1}{\tau_1}+\frac{c_2}{\tau_2}+O\left(\frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2},\frac{\xi}{\tau_1^{1/2}\tau_2}\right)\right],$$

$$v_F^{(2)} = -48\;a_s\tau_s\tau_1^{9/2}\tau_2^6\,|A_s|\,|W_0|\,e^{-a_s\tau_s}\left[1+\frac{c_1}{\tau_1}+O\left(\frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2},\frac{\xi}{\tau_1^{1/2}\tau_2}\right)\right],$$

$$v_F^{(3)} = 18\xi\tau_1^4\tau_2^5\,|W_0|^2\left[1-\frac{c_2}{\tau_2}+O\left(\frac{c_1^2}{\tau_1^2},\frac{c_2^2}{\tau_2^2},\frac{C_W}{\tau_1\tau_2}\right)\right],$$

$$v_F^{(4)} = 6\;\tau_1^{\frac{9}{2}}\tau_2^6\,|W_0|^2\left[2\frac{c_1^2}{\tau_1^2}+\frac{c_2^2}{\tau_2^2}-4\frac{C_W}{\tau_1\tau_2}+\;O\left(\frac{c_1^3}{\tau_1^3},\frac{c_2^3}{\tau_2^3},\frac{c_1^2c_2}{\tau_1^2\tau_2},\frac{c_1c_2^2}{\tau_1\tau_2^2},\frac{c_1C_W}{\tau_1^2\tau_2},\frac{c_2C_W}{\tau_1\tau_2^2}\right)\right],$$

subleading terms being expressed according to the large volume scenario hierarchy as in (A.71).

The effective form for the scalar potential can be inferred by taking into account conditions (A.70) and following a generalisation of the usual approximations which lead e.g. to the effective scalar potential in Models II and III.

It is manifest that the leading order expression of the scalar potential corresponds to the well known one (A.37) which is found in the absence of string loop corrections, thus confirming the discussion about the Extended No-Scale Structure. Therefore, a Taylor expansion of expression (A.72) can be done assuming the well known behaviour:

$$e^{-a_s\tau_s}\overset{(\bullet)}{\sim}O\left(\frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2}\right)\overset{(\bullet)}{\sim}O\left(\frac{\xi}{\tau_1^{1/2}\tau_2}\right).$$

In this way, it becomes evident that the scalar potential can be approximated at a first attempt as:

$$V_F^{\text{tot}} \overset{(\bullet)}{\simeq} \frac{8}{3}\frac{a_s^2\tau_s^{1/2}}{\tau_1^{1/2}\tau_2}\left|A_s\right|^2 e^{-2a_s\tau_s}\left[1+\frac{c_1}{\tau_1}+\frac{c_2}{\tau_2}\right]-4\frac{a_s\tau_s}{\tau_1\tau_2^2}\left|A_s\right|\left|W_0\right|e^{-a_s\tau_s}$$
$$+\frac{3}{2}\frac{\xi}{\tau_1^{3/2}\tau_2^3}\left|W_0\right|^2\left[1-\frac{c_1}{\tau_1}-\frac{c_2}{\tau_2}+\frac{1}{3}\frac{\tau_1^{1/2}\tau_2}{\xi}\left(2\frac{c_1^2}{\tau_1^2}+\frac{c_2^2}{\tau_2^2}-4\frac{C_W}{\tau_1\tau_2}\right)\right],$$

where corrections to the addenda are of two possible orders of magnitude, as evidently:

$$\frac{\tau_1^{1/2}\tau_2}{\xi}\frac{c_1^2}{\tau_1^2}\sim\frac{\tau_1^{1/2}\tau_2}{\xi}\frac{c_2^2}{\tau_2^2}\sim\frac{\tau_1^{1/2}\tau_2}{\xi}\left|\frac{C_W}{\tau_1\tau_2}\right|\gg\left|\frac{c_1}{\tau_1}\right|\sim\left|\frac{c_2}{\tau_2}\right|.$$

This fact means that the leading correction to the scalar potential introduced by string loop corrections is scaled by factors which do not depend on ξ and which are proportional to c_1^2/τ_1^2, c_2^2/τ_2^2 and $C_W/\tau_1\tau_2$.

In the end, discarding the subleading corrections, it is possible to write the effective total scalar potential as:

$$V_F^{\text{tot}} \overset{(\bullet)}{\simeq} V_F^{\text{tot, eff}} = V_F^{\text{eff}} + \delta V_F^{\text{eff}}, \tag{A.73}$$

where V_F^{eff} is the well known scalar potential (A.37), while the correction is:

$$\delta V_F^{\text{eff}} = \frac{1}{2}\frac{\left|W_0\right|^2}{\tau_1\tau_2^2}\left(2\frac{c_1^2}{\tau_1^2}-4\frac{C_W}{\tau_1\tau_2}+\frac{c_2^2}{\tau_2^2}\right). \tag{A.74}$$

Following Subsection 3.5, it is convenient to first minimise V_F^{eff} after the change of variables from (τ_1,τ_2,τ_s) to $(\tau_1,\mathcal{V}',\tau_s)$, getting the leading order vacuum expectation values (3.75) and (3.76) for $\mathcal{V}'$ and τ_s, and then to consider the scalar potential lifting δV_F in this ground state, in such a way as to have :

$$\delta V_F^{\text{eff}} = \frac{1}{2}\frac{\left|W_0\right|^2}{\left\langle\mathcal{V}'\right\rangle^2}\left(2\frac{c_1^2}{\tau_1^2}-4\frac{C_W}{\tau_1^{1/2}\left\langle\mathcal{V}'\right\rangle}+\frac{c_2^2\tau_1}{\left\langle\mathcal{V}'\right\rangle^2}\right). \tag{A.75}$$

Its minimum is found straighforwardly in the point $\langle\tau_1\rangle$ such that:

$$\langle\tau_1\rangle^{3/2} = \frac{C_W}{c_2^2}\left[\frac{|C_W|}{C_W}\left(1+4\frac{c_1^2c_2^2}{C_W^2}\right)^{1/2}-1\right]\left\langle\mathcal{V}'\right\rangle. \tag{A.76}$$

F-terms

Auxiliary fields can be determined and, in the absence of a mechanism analogous to the Extended No-Scale cancellation, turn out to get leading corrections from Kaluza-Klein

terms.
Again, their explicit expressions are extremely long, so they are reported as:

$$F^1_{\rm tot} = \frac{f^1}{k}, \tag{A.77}$$

$$F^2_{\rm tot} = \frac{f^2}{k}, \tag{A.78}$$

$$F^s_{\rm tot} = \frac{f^s}{k}, \tag{A.79}$$

where the leading terms are:

$$k = 2\tau_1^{7/2}\tau_2^5\left[1 + 2\frac{c_1}{\tau_1} + \frac{c_2}{\tau_2} - \frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2} - \frac{1}{2}\frac{\xi}{\tau_1^{1/2}\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\right],$$

$$\begin{aligned} f^1 = &- 4\tau_1^4\tau_2^4\Bigg\{|W_0|\left[1 + \frac{3}{2}\frac{c_1}{\tau_1} + \frac{3}{2}\frac{c_2}{\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\right] \\ &+ 2a_s\tau_s\,|A_s|\,e^{-a_s\tau_s}e^{-i(\alpha_s-\theta-a_s\psi_s)}\left[1 + \frac{1}{2a_s\tau_s} + O\left(\frac{c_1}{\tau_1}, \frac{c_2}{\tau_2}\right)\right]\Bigg\}, \end{aligned}$$

$$\begin{aligned} f^2 = &- 4\tau_1^3\tau_2^5\Bigg\{|W_0|\left[1 + \frac{5}{2}\frac{c_1}{\tau_1} + \frac{c_2}{\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\right] \\ &+ 2a_s\tau_s\,|A_s|\,e^{-a_s\tau_s}e^{-i(\alpha_s-\theta-a_s\psi_s)}\left[1 + \frac{1}{2a_s\tau_s} + O\left(\frac{c_1}{\tau_1}, \frac{c_2}{\tau_2}\right)\right]\Bigg\}, \end{aligned}$$

$$\begin{aligned} f^s = &- 4\tau_s\tau_1^3\tau_2^4\Bigg\{|W_0|\left[1 + \frac{3}{2}\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\right] \\ &+ \frac{4}{3}a_s\tau_s\frac{\tau_1^{\frac{1}{2}}\tau_2}{\tau_s^{\frac{3}{2}}}\,|A_s|\,e^{-a_s\tau_s}e^{-i(\alpha_s-\theta-a_s\psi_s)}\Bigg[1 + \frac{5}{2}\frac{c_1}{\tau_1} + \frac{3}{2}\frac{c_2}{\tau_2} \\ &+ \frac{1}{2}\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}\left(1 + \frac{3}{2a_s\tau_s}\right) - \frac{1}{2}\frac{\xi}{\tau_1^{1/2}\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\Bigg]\Bigg\}. \end{aligned}$$

Then, more clearly, the auxiliary fields can be expressed as:

$$\begin{aligned} F^1_{\rm tot} = &- 2\frac{\tau_1^{\frac{1}{2}}}{\tau_2}\Bigg\{|W_0|\left[1 - \frac{1}{2}\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} + \frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2} + \frac{1}{2}\frac{\xi}{\tau_1^{1/2}\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\right] \\ &+ 2a_s\tau_s\,|A_s|\,e^{-a_s\tau_s}e^{-i(\alpha_s-\theta-a_s\psi_s)}\left[1 + \frac{1}{2a_s\tau_s} + O\left(\frac{c_1}{\tau_1}, \frac{c_2}{\tau_2}\right)\right]\Bigg\}, \end{aligned}$$

$$
\begin{aligned}
F^2_{\rm tot} = & -2\frac{1}{\tau_1^{\frac{1}{2}}}\Bigg\{ |W_0| \left[1 + \frac{1}{2}\frac{c_1}{\tau_1} + \frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2} + \frac{1}{2}\frac{\xi}{\tau_1^{1/2}\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\right] \\
& + 2a_s\tau_s\, |A_s|\, e^{-a_s\tau_s} e^{-i(\alpha_s-\theta-a_s\psi_s)} \left[1 + \frac{1}{2a_s\tau_s} + O\left(\frac{c_1}{\tau_1}, \frac{c_2}{\tau_2}\right)\right]\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
F^s_{\rm tot} = & -2\frac{\tau_s}{\tau_1^{\frac{1}{2}}\tau_2}\Bigg\{ |W_0| \Bigg[1 - \frac{1}{2}\frac{c_1}{\tau_1} - \frac{1}{2}\frac{c_2}{\tau_2} \\
& + \frac{\tau_s^{3/2}}{\tau_1^{1/2}\tau_2} + \frac{1}{2}\frac{\xi}{\tau_1^{1/2}\tau_2} + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\Bigg] \\
& + \frac{4}{3}a_s\tau_s\frac{\tau_1^{1/2}\tau_2}{\tau_s^{3/2}}\, |A_s|\, e^{-a_s\tau_s} \Bigg[1 + \frac{1}{2}\frac{c_1}{\tau_1} + \frac{1}{2}\frac{c_2}{\tau_2} \\
& + \frac{3}{2}\frac{\tau_s^{\frac{3}{2}}}{\tau_1^{\frac{1}{2}}\tau_2}\left(1 + \frac{1}{2a_s\tau_s}\right) + O\left(\frac{c_1^2}{\tau_1^2}, \frac{c_2^2}{\tau_2^2}, \frac{C_W}{\tau_1\tau_2}\right)\Bigg] e^{-i(\alpha_s-\theta-a_s\psi_s)}\Bigg\}.
\end{aligned}
$$

Finally, the effective vacuum expectation values of the auxiliary fields are:

$$
\begin{aligned}
\left\langle F^1_{\rm tot,\,eff}\right\rangle = & -2\frac{\langle\tau_1\rangle^{\frac{1}{2}}}{\langle\tau_2\rangle}\, |W_0| \Bigg[1 - \frac{1}{2}\frac{c_1}{\langle\tau_1\rangle} + \frac{1}{2}\frac{c_2}{\langle\tau_2\rangle} + \frac{1}{2}\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle} \\
& - \frac{1}{2}\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\left(1 - \frac{3}{4a_s\langle\tau_s\rangle} - \frac{27}{16a_s^2\langle\tau_s\rangle^2}\right)\Bigg],
\end{aligned}
\tag{A.80}
$$

$$
\begin{aligned}
\left\langle F^2_{\rm tot,\,eff}\right\rangle = & -2\frac{1}{\langle\tau_1\rangle^{\frac{1}{2}}}\, |W_0| \Bigg[1 + \frac{1}{2}\frac{c_1}{\langle\tau_1\rangle} + \frac{1}{2}\frac{\xi}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle} \\
& - \frac{1}{2}\frac{\langle\tau_s\rangle^{\frac{3}{2}}}{\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\left(1 - \frac{3}{4a_s\langle\tau_s\rangle} - \frac{27}{16a_s^2\langle\tau_s\rangle^2}\right)\Bigg],
\end{aligned}
\tag{A.81}
$$

$$
\left\langle F^s_{\rm tot,\,eff}\right\rangle = -\frac{3}{2a_s\langle\tau_1\rangle^{\frac{1}{2}}\langle\tau_2\rangle}\, |W_0| \left[1 + \sum_{n=1}^{\infty}\frac{1}{(4a_s\langle\tau_s\rangle)^n}\right]. \tag{A.82}
$$

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