ALMA MATER STUDIORUM · UNIVERSITÀ DI BOLOGNA

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Theory and Phenomenology of a Massive Spin-2 Particle

Relatore:
Prof. Roberto Soldati

Correlatore:
Prof. Fabio Maltoni

Presentata da:
Riccardo Costa

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Sommario

Il presente lavoro di tesi si prefigge di fornire uno studio della teoria alla base della descrizione di una particella di spin 2 massiva, attraverso la teoria dei campi quantistici. Dopo una dettagliata derivazione del modello, il lavoro si concentra sullo studio di due possibili costruzioni che differiscono tra loro per una diversa struttura dell'accoppiamento tra la particella di spin 2 e il resto del Modello Standard (MS). In particolare la differenza consiste nel fatto che, nel primo caso la costante di accoppiamento è universale mentre nel secondo modello l'accoppiamento con ogni campo del MS è caratterizzato da un valore distinto della relativa costante di accoppiamento. Questo ultimo modello presenta, ad alte energie, un problema di unitarietà che la tesi si propone di investigare per la prima volta in dettaglio, soprattutto per quanto riguarda la fenomenologia al collider, nel tentativo di fornire una linea guida per la costruzione di un modello migliorato che permetta la non universalità degli accoppiamenti senza compromettere l'unitarietà della teoria.

Abstract

This thesis work aims to give a study on the theory underlying the description of a massive spin-2 particle through quantum field theory. After a detailed derivation of the model the work focuses on the study of two possible constructions, which differ from each other by the coupling between the spin-2 particle and the rest of the Standard Model (SM). In the first case the coupling constant is universal while in the second class of models the coupling between every SM field is characterized by a different value of the related coupling constant. This last model presents, at high energy, a unitarity problem which the thesis aims to investigate for the first time detail, in particular with respect to collider phenomenology, in the attempt to give a guideline for the building of a improved model which allows the non-universality of the couplings without compromising the unitarity of the theory.

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Introduction

For almost a century physicists have been fascinated by the possibility of providing a consistent quantum theory of spin-2 particles. Right after the Einstein's general relativity it was realized that the same equations it was based on, could be derived from a classical massless field theory for a symmetric tensor field which corresponds to a spin-2 particle for the Poincaré group representation. At the beginning the interest was totally focused in finding a proper quantum theory of gravitation and the massive formulation was studied as a generalization of general relativity to address the accelerated expansion of the universe thanks to long distances corrections, without invoking exotic concepts such as dark energy. However is now well established that massive gravity is experimentally ruled out because it does not works properly in reproducing the the correct results usual general relativity gives at small to medium ranges.
Very recently the interest for a massive spin-2 theory has enormously grown also in high energy particle physics due to the later disclaimed hints of a 750 GeV spin-2 resonance at the Large Hadron Collider at both CMS and ATLAS experiments. Even if, as said, this claim has been refuted with the Run 2, is more topical the ever the necessity of deepening the understanding of spin-2 theories and explore their phenomenology. There are a lot of different theoretical models which predict the existence of this type of particles, and many experimental searches on going.
The aforementioned motivations are the starting point for this work in which we will derive and study the model in one of its most general formulations. This thesis is organized as follows. In Chapter 1 we derive the action of the massive spin-2 particle starting from one of the most interesting model among those which give rise to such kind of particles. In the Randall-Sundrum model, a five dimensional theory originally developed to address the huge hierarchy between the gravitational energy scale (the Planck mass $\sim 10^{19}$ GeV) and the electro-weak scale (the vacuum expectation value of the Higgs field at $\sim 10^{2}$ GeV). Like every other five dimensional model it gives rise to an infinite set of massive spin-2 gravitons, similar to each others but with different masses. The main purpose of the thesis will be to analyze in detail the theory for one of these massive graviton excitations. In particular in Section 1.2 we will build the Randall-Sundrum model from scratch in its simplest formulation and then in Section 1.3 we will describe one of its most interesting generalizations which will be studied the rest of the thesis. In Chapter 2 we will drive our attention on the actual effective field theory for the massive spin-2 particle coupled with the Standard Model as previously derived. In fact we will solve the free theory and study the quantization in Section 2.1, then we will focus

on the interaction term of the theory and in especially on its mathematical structure, describing schematically all the kinds of different new interaction vertices the theory presents and finally in Section 2.3 we will develop a useful formalism which will allow us to describe in a easier way the phenomenology the high energy limit of the theory and will help us for the following considerations. Finally in Chapter 3 we will describe the actual model of interest, with a special attention on its most important feature and problem, consisting in a strong unitary violation. We will inspect directly a process that exhibits such behaviour both analytically and with numerical computations in Sections 3.1 and 3.2 and then we will propose a possible modification of the model to be taken under consideration for further studies in the attempt of restoring its unitarity.

1. The Randall-Sundrum Model

The existence of extra dimensions in our world is the basis of many modern theories and it has been extensively studied during the last century in different approaches and formulations. This idea was presented for the first time in the 1920's by Theodor Kaluza in the attempt of unifying electromagnetism and gravitation [1]. Kaluza's theory was a simple extension, in five space-time dimensions, of Einstein's general relativity in which both the electromagnetic and gravitational fields were components of the unique five dimensional metric tensor. Few years later, Oskar Klein gave a quantum interpretation of Kaluza's work [2] giving birth of what is called Kaluza-Klein theory.

More recently, many theories have been proposed in which five or even more dimensions are involved. The most remarkable and famous example of them all is probably string theories but, from the last years of the 90's up to nowadays, the interest in these kind of scenarios has increased mostly because the realization that extra dimensions theories can provide a incredibly natural solution to the hierarchy problem in Standard Model. Obviously, in order to not be in conflict with our observation of a four dimensional world, the theory has to provide an explanation to hide the extra dimensions to all the experimental results we got so far. The most plausible way to achieve such result is to assume the extra dimension to be finite and small enough to avoid every detection at the energy scales of our current experiments.

In this chapter, after a brief review of some basic and common features in Kaluza-Klein-like theories with compact extra dimensions, we will focus on the so-called Randall-Sundrum model, which will give the theoretical foundations of our effective spin-2 model. Actually, the effective theory we will take under consideration is a more general and universal construction. Its structure can be obtained in a general way from any theory which wants to describe a massive spin-2 particle. Despite the universality of this model, we will present it as derived from the Randall-Sundrum framework. Even though it is not the only model leading to such massive spin-2 theory, it helps to give a strong theoretical background and also to introduce different features of the model which can be naturally obtained form different formulations and generalizations of the Randall-Sundrum construction.

What follows is proposed to be a first analysis of the subject. There is no way to give a comprehensive and satisfactory overview on such a big topic as extra dimensions theories. For more detailed and complete works see for example [3, 4].

1.1 Kaluza-Klein States and Extra Dimension Size

Before getting started with the actual presentation of the topics, it is mandatory (although a little boring) to briefly set the conventions and notations for everything that follows. We will always use the mostly-minus convention for the Minkowski metric tensor, i.e. $\eta_{\mu\nu} = diag(1, -1, -1, -1)$ where the greek lowercase indicies are used as the usual four dimension Lorentz indicies $\mu = 0, 1, 2, 3$ while the latin uppercase ones are used as five dimensional indicies $M = 0, 1, 2, 3, 4$ so that the set of space-time coordinates will be denoted as $x^M = x^\mu, y$. Moreover we will use natural units ($c = \hbar = 1$) throughout the whole work.
First of all let us consider a five dimensional theory with the extra dimension compactified in a circle of radius r. This operation, commonly referred as toroidal compactification, can be easily accomplished by defining the fifth dimension coordinate up to a periodical equivalence:

$$y \sim y + 2\pi r.$$

In this space-time manifold, consider a massless 5D scalar field $\Phi(x^M)$ with the action

$$S = \int d^5x \; \partial_M \Phi \; \partial^M \Phi, \tag{1.1.1}$$

which leads to the field equation

$$\partial_M \partial^M \Phi = 0. \tag{1.1.2}$$

The periodical condition imposed on y suggest to perform a Fourier expansion of the field:

$$\Phi(x^M) = \sum_{n=-\infty}^{+\infty} \phi_n(x^\mu) \, e^{i \frac{n}{r} y}. \tag{1.1.3}$$

It is important to notice that the Fourier coefficients $\phi_n(x^\mu)$ can be interpreted as usual four dimensional fields whose equations of motion can be derived from (1.1.2) by substituting the decomposition we have just obtained:

$$\left(\Box + \frac{n^2}{r^2} \right) \phi_n(x^\mu) = 0. \tag{1.1.4}$$

These are an infinite number of Klein-Gordon equations for massive scalar particles of mass $m_n = \frac{n}{r}$. What is happening in this model can be understood by noting that, due to the compactification, the five dimensional field Φ has a quantized momentum in the fifth dimension:

$$p^4 = \frac{n}{r}, \tag{1.1.5}$$

who is responsible for what we can view as an "effective mass"for all these fields $\phi_n(x^\mu)$, the Kaluza-Klein states.

This simple construction can be applied to fields with arbitrary spin and, in the end, at the entire Standard Model, and the same results can be obtained. Every time we extend our field theory to a fifth compactified dimension, the phenomenology include an infinite number of KK states with increasing mass. This means that, in order to have theory consistent with experimental observations, the energy scale r^{-1} must be greater than the TeV scale at which we are currently working. This experimental bound impose a strong constraint on the radius:

$$r \lesssim 10^{-23}\, m.$$

A way to soften very much the constraint is to assume that only the gravity has access to the fifth dimension while the Standard Model fields remain confined on our four dimensional world. In this way the constrain on the radius comes only from gravitational experiments, whose set the upper bound to about a millimeter, and nevertheless keeps the fifth dimension undetectable.

1.2 Warped Extra Dimensions

The general setup, when dealing with extra dimensions, is to have a compactified dimension that connect together two or more *branes* through a *bulk*. A d-brane is some kind of slice of the whole five dimensional space-time that has d spatial dimensions and on which we want to localize some of the theory fields or we wish to study the model phenomenology. For example there is always a 3-brane that rappresent our four dimensional (including time) world.
As said before, with the freedom that extra dimension model building gives, the amount of different possible scenarios is huge. We will focus only on the Randall-Sondrum model and some of its extensions. It belongs to a category where the extra dimension is warped, meaning that, in opposition to the flat extra dimensions models, we take into account the backreaction on the space-time geometry due to the presence of fields and branes in the bulk. The main effect will be to have to deal with a curved space-time in five dimensions while making sure that at least the brane that represents our world remains flat in order to preserve Lorentz invariance on it. The usual way to achieve this result is to include a non-vanishing cosmological constant in the bulk, as we will see in the following.

1.2.1 The Randall-Sundrum Background

Now we can start building the model setup. Let us assume that the extra dimension is compactified in a circle of radius r in which the upper half is identified with the lower one, i.e. we are formally building an extra dimension as an $S^1/\mathbb{Z}^2$ orbifold as we can see in figure (1.1), and this is nothing but the correct geometrical definition of a segment with length $L = \pi r$. On both boundaries of the segment stands a four dimensional

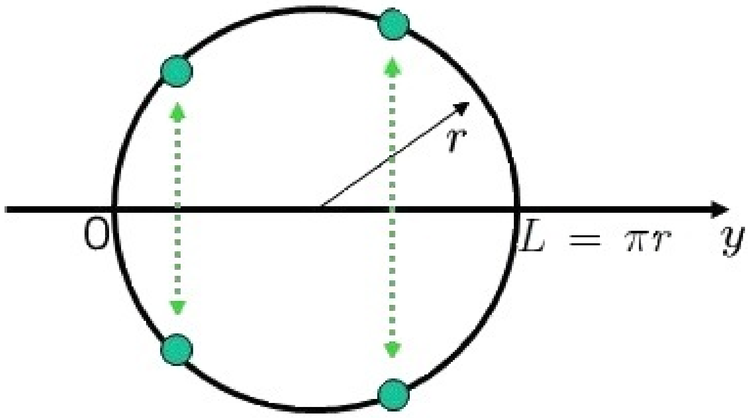


Figure 1.1: The $S^1/\mathbb{Z}^2$ orbifold.

Minkowski-like brane so that we have two flat branes, at distance L, enclosing a one dimensional bulk, along the fifth dimension.
The most general metric that fulfills all the properties needed is

$$ds^2 = e^{-A(y)}\, \eta_{\mu\nu}\, dx^\mu dx^\nu - dy^2. \tag{1.2.1}$$

The curvature of the extra dimension depends on the function $e^{-A(y)}$, called warp-factor, where $A(y)$ is an unknown function, to be determined solving the Einstein's equation.
While this could be the most intuitive form of the metric, we are free to apply a coordinate transformation on it. In particular we can go in a coordinate system where there is an overall pre-factor in front of a five dimensional metric tensor which also becomes flat, so that we are left with a conformally flat metric. This is by far the simplest coordinate system to find $A(y)$. Performing such kind of transformation we have to be careful that it does not depends on x^μ, otherwise we will be left with off-diagonal terms in the final metric. To accomplish this task the new coordinate z has to be connected to y by the differential relation

$$e^{-\frac{A(z)}{2}}\, dz = dy, \tag{1.2.2}$$

such that we are left with a metric becomes

$$ds = e^{-A(z)}\, \big(\eta_{\mu\nu}\, dx^\mu dx^\nu - dz^2\big). \tag{1.2.3}$$

It is now straightforward to notice that this is a conformally flat metric, namely it is connected by a conformal transformation, a simple overall rescaling, to the flat Minkowski metric in five dimensions:

$$g_{MN} = e^{-A(z)}\, \eta_{MN}. \tag{1.2.4}$$

This is exactly what we were looking for, because now we can use a very powerful and remarkable result from conformal theories. Exists in fact a relation that connects the Einstein tensors, calculated from two different metrics, connected by a conformal transformation in arbitrary numbers of dimensions. Denoting with G_{MN} and $\mathcal{G}_{MN}$ the Einstein tensors from the metrics g_{MN} and $\mathbf{g}_{MN}$ respectively, connected by a conformal transformation exactly as (1.2.4), the relation in five dimensions is:

$$G_{MN} = \mathcal{G}_{MN} + \frac{3}{2}\left[\frac{1}{2}\,\nabla_M A \nabla_N A + \nabla_M \nabla_N A - \mathbf{g}_{MN}\left(\nabla_R \nabla^R A - \frac{1}{2}\,\nabla_R A \nabla^R A\right)\right], \quad (1.2.5)$$

where ∇ is the covariant derivative with respect the metric $\mathbf{g}$. In our case of interest $\mathbf{g}_{MN} = \eta_{MN}$, hence the covariant derivative simplifies as the usual derivative ∂_M. With a bit of simple algebraic work we can evaluate all the non-vanishing components of the Einstein tensor, obtaining:

$$G_{\mu\nu} = -\frac{3}{2}\,\eta_{\mu\nu}\left(\frac{1}{2}\,A'^2 - A''\right), \quad (1.2.6a)$$

$$G_{44} = \frac{3}{2}\,A'^2. \quad (1.2.6b)$$

Now that we have worked out the left hand side of the Einstein equation we have to compute the right hand side of them, that is the energy-momentum tensor. To do it we have to recollect the Hilbert-Einstein action for gravity, extend it in five dimensions, and include a bulk cosmological constant Λ_5:

$$S = \int d^5x\,\sqrt{|g|}\,\left({M_5}^3\,R + \Lambda_5\right), \quad (1.2.7)$$

where M_5 is the five dimensional Planck scale, related to the Newton constant[1] by:

$$8\pi G_N = \frac{1}{2{M_5}^3}. \quad (1.2.8)$$

The energy-momentum tensor is defined as usual as

$$T^{MN} = \frac{2}{\sqrt{|g|}}\,\frac{\delta S_m}{\delta g_{MN}}, \quad (1.2.9)$$

with S_m the action part related to the matter content of the theory, in our case the term proportional to Λ_* in (1.2.7). Recalling that

$$\frac{\delta\sqrt{g}}{\delta g_{MN}} = \frac{1}{2}\,\sqrt{g}\,g_{MN}, \quad (1.2.10)$$

[1]Note that in five dimension the Newton constant has the dimension of inverse cubic power of a mass $[G_N] = M^{-3}$. More generically in n dimensions $[G_N] = M^{2-n}$.

it is easy to see that every term in the action like $C\sqrt{|g|}$, with C a costant, gives a contribution to the energy-momentum tensor equal to Cg_{MN}. Therefore we are now able to write down the Einstein equation:

$$G_{MN} = 8\pi G_5\, T_{MN} = \frac{\Lambda_5}{2{M_5}^3}\, g_{MN}. \tag{1.2.11}$$

To begin solving it let us consider the 44 component, that using (1.2.6b) reads

$$\frac{3}{2}A'^2 = -\frac{\Lambda_5}{2{M_5}^3}\, e^{-A}. \tag{1.2.12}$$

It is worth noting that a real solution for A exists only if $\Lambda_5 < 0$ and this means that whatever solution we will find we expect it to be an Anti-de Sitter space-time[2]. If we define a new function

$$f(z) \equiv e^{-\frac{A(z)}{2}}, \tag{1.2.13}$$

then we can recast (1.2.12) as

$$\frac{f'}{f^2} = -\frac{1}{2}\sqrt{-\frac{\Lambda_5}{3{M_5}^3}}. \tag{1.2.14}$$

The solution of this equation is

$$f(z) = \frac{1}{kz - c}, \tag{1.2.15}$$

and according to the definition of $f(z)$ we find

$$e^{-A(z)} = \frac{1}{(kz-c)^2}, \tag{1.2.16}$$

where c is an irrelevant constant of integration, because different values of it correspond to a rescaling of z, and can be set to $c = -1$ imposing $e^{-A(0)} = 1$, and we defined

$$k = \sqrt{-\frac{\Lambda_5}{12{M_5}^3}}. \tag{1.2.17}$$

As a last step we have to take care of making the solution symmetric under a $\mathbb{Z}^2$ transformation, i.e. $z \to -z$, since the solution has to live on the $S^1/\mathbb{Z}^2$ orbifold. Therefore the final solution for the Randall-Sundrum background metric can be written in the form

$$ds^2 = \frac{1}{\big(k|z|+1\big)^2}\,\big(\eta_{\mu\nu}\, dx^\mu dx^\nu - dz^2\big). \tag{1.2.18}$$

[2]The case where $\Lambda_5 > 0$, i.e. the de-Sitter solution, in which $A \in \mathbb{C}$ presents an oscillating warp-factor that does not concern the actual model.

It would seem that we have been able to solve the Einstein equation only using the 44 component, but this is not the case. We have to be sure that this solution satisfies also the remaining components, that from (1.2.6a) are given by

$$-\frac{3}{2}\,\eta_{\mu\nu}\left(\frac{1}{2}\,{A'}^2 - A''\right) = \frac{\Lambda}{2{M_5}^3}\,g_{\mu\nu}. \tag{1.2.19}$$

Due to the absolute value in the solution we found, the presence of the second derivative A'' leads to the appearance of delta function contributions where the branes are situated. This should not be a surprise because we expected the branes to affect the actual solution. As a matter of fact it is impossible to have a flat space-time induced on the branes with a presence of a non-vanishing cosmological constant in the bulk, without an energy contribution from the branes themselves to compensate it. However, given that the solution has already been completely determined, the energy density on the branes can only be constant.
To start we can see from (1.2.18) that

$$A = -\ln\left(\frac{1}{\big(k|z|+1\big)^2}\right), \tag{1.2.20}$$

and since we have to deal with the absolute value derivatives it is useful to remind that

$$\frac{d|x|}{dx} = \text{sign}(x) = \theta(x) - \theta(-x), \tag{1.2.21a}$$

$$\frac{d\theta(x)}{dx} = \delta(x). \tag{1.2.21b}$$

Here $\text{sign}(x)$ is the sign function and $\theta(x)$ is the Heaviside function, while $\delta(x)$ is the Dirac delta. With this in mind the derivatives of $A(z)$ can be easily found as

$$A' = \frac{2k\,\text{sign}(x)}{k|z|+1}, \tag{1.2.22a}$$

$$A'' = -\frac{2k^2}{(k|z|+1)^2} + \frac{4k}{k|z|+1}\,\big[\delta(z) - \delta(z-z^*)\big]. \tag{1.2.22b}$$

Few features of A'' deserve to be commented in some details. The first thing we notice is the presence of two delta functions, one in $z = 0$ and the other in $z = z^*$ where $z^* = k^{-1}$. This happens because, since we expect the solution to be $\mathbb{Z}^2$ invariant with respect both the extremes of the extra dimension which are the two branes, $A(z)$ has to have two cusps and both give rise to a delta function. It is moreover notable that the two delta's contributions have opposite sign, and this will lead to some important physical consequences. Now let us go back to equation (1.2.19), substituting the expressions for

$A(z)$ derivatives and performing some algebraic simplifications involving (1.2.17), we are left with:

$$\frac{3}{2}\,\eta_{\mu\nu}\left(\frac{4k^2}{\big(k|z|+1\big)^2}-\frac{4k}{k|z|+1}\,\big[\delta(z)-\delta(z-z^*)\big]\right)=\frac{6k^2}{\big(k|z|+1\big)^2}\,\eta_{\mu\nu},\tag{1.2.23}$$

and we can see how the first term in the right hand side of the equation matches precisely the left hand side, while the two delta contributions are not compensated, as we expected. It is quite clear now that we need to add energy contributions localized on the branes, in the action (1.2.7) to be able to solve the four dimensional part of Einstein equation. The right way to write such terms, keeping in mind that they have to be constant in order to maintain (1.2.18) as the solution, is given by

$$S_{brane}=\int d^5x\,\sqrt{|g|}\frac{V\,\delta(z-z_0)}{\sqrt{|g_{44}|}},\tag{1.2.24}$$

where z_0 is the brane fifth coordinate, V its energy density and the factor $\sqrt{g_{44}}$ in the denominator is there to ensure the right induced metric determinant in the brane. An action term in this form implies an energy-momentum contribution

$$T_{\mu\nu}=\frac{V\,\delta(z-z_0)}{\sqrt{|g_{44}|}}\,g_{\mu\nu}.\tag{1.2.25}$$

Putting everything together and dropping the terms not proportional to the delta functions, the four dimensional Einstein equation with two energy contributions localized in $z=0$ and $z=z^*$ now reads

$$\eta_{\mu\nu}\,\frac{6k}{k|z|+1}\,\big(\delta(z)-\delta(z-z^*)\big)=\frac{1}{2{M_5}^3}\left(\frac{V_0\,\delta(z)+V_*\,\delta(z-z^*)}{k|z|-1}\right)\eta_{\mu\nu},\tag{1.2.26}$$

and to satisfy this equality the branes energy density have to be opposite and with value

$$V_0=-V_*=12k{M_5}^3.\tag{1.2.27}$$

This means that the brane located in $z=z^*$ has to have a negative energy density to match the delta contribution in the right hand side of equation (1.2.23).
Therefore we found that the metric of equation (1.2.18) is actually the background spacetime metric for the Randall-Sundrum model with respect the action

$$S=\int d^5x\,\sqrt{|g|}\left({M_5}^3\,R+\Lambda_5-\frac{\sqrt{-12\Lambda{M_5}^3}\,\delta(z)}{\sqrt{|g_{44}|}}+\frac{\sqrt{-12\Lambda{M_5}^3}\,\delta(z-z^*)}{\sqrt{|g_{44}|}}\right).\tag{1.2.28}$$

Now that we have completely solved the Einstein equation we can go back to the original and more physical related coordinates. Since the relation between z and y is given by equation (1.2.2) now we are able to write it as

$$\frac{dz}{k|z|+1} = dy, \tag{1.2.29}$$

which entails, choosing $y = 0$ corresponding to $z = 0$

$$\log\big(k|z|+1\big) = k|y|. \tag{1.2.30}$$

Finally, the Randall-Sundrum metric becomes:

$$ds^2 = g_{MN}\, dx^M dx^N = e^{-2k|y|}\, \eta_{\mu\nu}\, dx^\mu dx^\nu - dy^2, \tag{1.2.31}$$

where y varies from 0 to L, as said before.

1.2.2 Hierarchy Solution

Now that the setup has been fully presented and defined, we can start investigating the main features of the model in its simplest formulation, i.e. with all the Standard Model fields confined on the second brane at $y = L$.
The first remarkable property is how this construction addresses the Hierarchy Problem between the Plank Scale and the Electro-Weak scale [3]. To study it let us consider the Higgs scalar field and its action evaluated in the four dimensional brane at $y = L$, given by

$$S_{Higgs} = \int d^4x \sqrt{|g|}\Big|_{y=L} \left[g^{\mu\nu}\Big|_{y=L} D_\mu H^\dagger D_\nu H - \lambda\big(H^\dagger H - v^2\big)^2 \right], \tag{1.2.32}$$

and by virtue of the solution we have found in equation (1.2.31), it can be recast as

$$S_{Higgs} = \int d^4x\; e^{-4kL} \left[e^{2kL}\; \eta^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda\big(H^\dagger H - v^2\big)^2 \right]. \tag{1.2.33}$$

It is easy to see that if we redefine a new field $\mathcal{H} = e^{-kL} H$ the action takes the form of

$$S_{Higgs} = \int d^4x \left[\eta^{\mu\nu} D_\mu \mathcal{H}^\dagger D_\nu \mathcal{H} - \lambda\big(\mathcal{H}^\dagger \mathcal{H} - e^{-2kL}\; v^2\big)^2 \right]. \tag{1.2.34}$$

What we are left with is actually the normal action for the Higgs field, in which the vacuum expectation value is exponentially suppressed and takes an effective value

$$v_{eff} = e^{-kL}\; v, \tag{1.2.35}$$

[3] This is the main reason why this model was initially conceived indeed.

and consequently, every mass parameter in the Standard Model is submitted to the same exponential suppression. By virtue of this relation we can allow the actual Higgs vacuum expectation value v to be of the same order of magnitude of M_5, and then warp down the effective one v_{eff} to the electro-weak scale, tuning the size of the extra dimension.
On the other hand, before claiming any important result, we have to work out the four dimensional Planck Scale also, to check if and how it is affected by the warping factor. To achieve this we have to consider the gravitational part of the action in equation (1.2.28), that is

$$S = {M_5}^3 \int d^5x \sqrt{|g|}\, R, \tag{1.2.36}$$

and try to extract from it the four dimensional component which has to be proportional to the Planck Mass squared as in the usual Hilbert-Einstein action. It is straightforward, but a bit tedious, to compute the Ricci tensor for the Randall-Sundrum background metric and find out that the only non-vanishing components are $R_{\mu\nu}$ and R_{44}, so that the Ricci scalar is simply $R = g^{\mu\nu}R_{\mu\nu} + g^{44}R_{44}$ (see Appendix (A.1) for the details). Then it is easy to see that the four dimensional part of the action can be written as

$$S_{4D} = {M_5}^3 \int d^4x \frac{\sqrt{|g|}}{\sqrt{|g_{44}|}} \int_0^L dy\, \sqrt{|g_{44}|}\, g^{\mu\nu} R_{\mu\nu}. \tag{1.2.37}$$

From this expression now we can read off the relation between M_5 and the Planck Mass M_{Pl}

$${M_{Pl}}^2 = {M_5}^3 \int_0^L dy\, e^{-k|y|} = \frac{{M_5}^3}{k}\Big(1 - e^{-kL}\Big). \tag{1.2.38}$$

As we can see M_{Pl} is barely influenced by the warping factor for moderately large extra dimension size.
Now it is clear that equation (1.2.35), together with equation (1.2.38), gives us a natural way to solve the Hierarchy Problem. We can set the five dimensional Planck Mass to obtain the right value for M_{Pl}, and then set the real vacuum expectation value to be of the same order of M_5. Then, since we want to have $v_{eff} \approx 10^{-16} M_{Pl}$ we can set the value of the extra dimension to be

$$kL \approx \ln 10^{-16} \approx 35, \tag{1.2.39}$$

that is a value large enough to satisfy the requirement imposed by equation (1.2.38).
In conclusion we have shown that we can have all the parameters of the theory to be determined by one unique scale M_5 and still generate the hierarchy between the gravitational and the electro-weak scales as we observe in nature through an exponential warping.

1.2.3 Graviton Modes

Now we want to understand how gravity works in the Randall-Sundrum model and, to do so, we need to find the Kaluza-Klein decomposition of the graviton, which corresponds to small fluctuations around the background metric. In general we would expect to have four different kinds of fluctuations from the unperturbed metric we found in equation (1.2.31), that we can parametrize as:

$$\begin{aligned} ds^2 = e^{-2k|y|} \Big[\eta_{\mu\nu}\Big(1+\psi\big(x^M\big)\Big) + h_{\mu\nu}\big(x^M\big) + \Big]\, dx^\mu dx^\nu \\ + 2e^{-2k|y|}\, A_\mu\big(x^M\big)\, dx^\mu dy - \Big(1+\phi\big(x^M\big)\Big)\, dy^2. \end{aligned} \tag{1.2.40}$$

However, it can be proven that the vector fluctuations A_μ and the scalar one ψ do not correspond to physical degrees of freedom and can be ignored without losing meaningful informations, while the other scalar fluctuation ϕ do have some important physical interpretation but we will come back to it only later and, for now, we will ignore it too. With all these considerations in mind it is better to go back again to the conformal frame for the metric of equation (1.2.18) and, as we said, keep only the tensor fluctuations $h_{\mu\nu}$ that represent the graviton

$$ds^2 = e^{-A(z)}\, \Big(\eta_{MN} + h_{MN}\big(x^R\big)\, dx^M dx^N\Big), \tag{1.2.41}$$

where, for sake of simplicity of the following calculations, we have parametrized again $e^{-2A(z)} = \big(k|z|-1\big)^{-2}$. What we have to do then is to compute the Einstein equations for the fluctuations, i.e. the linearized ones. In order to do so we will use again the relation (1.2.5), where this time the starting metric is the one enclosed in the squared brackets in the perturbed metric above. The first, and very convenient thing to do is decide to work with a particular choice of gauge in which the fluctuations do not have extra dimensions components and are transverse and traceless:

$$h_{M5} = 0, \tag{1.2.42a}$$

$$\partial_\mu\, h^{\mu\nu} = 0, \tag{1.2.42b}$$

$$\eta^{\mu\nu}\, h_{\mu\nu} = h^\mu_\mu = 0, \tag{1.2.42c}$$

and, in this gauge, we can evaluate the Einstein tensor $\mathcal{G}_{MN} = R_{MN} - \frac{1}{2}\,\eta_{MN}R$ for the starting metric, up to linear contributions (see Appendix (A.2) for the details of calculation for the Ricci tensor and the Ricci scalar), obtaining:

$$\mathcal{G}_{\mu\nu} = -\frac{1}{2}\,\partial_4\partial^4\, h_{\mu\nu}, \tag{1.2.43a}$$

$$\mathcal{G}_{44} = 0, \tag{1.2.43b}$$

$$\mathcal{G}_{\mu 4} = 0. \tag{1.2.43c}$$

In order to work out the Einstein tensor for our metric we have to be careful dealing with the covariant derivatives, because this time they do not always reduce to the usual derivatives. Considering that they act only on a scalar function, all first order derivatives reduce to the classical ones, however, when a double derivative is involved a term proportional to the Christoffel symbols has to appear. The two case of interest are then, for a general scalar function f:

$$\nabla_M \nabla_N f = \nabla_M \, \partial_N f = \partial_M \partial_N f - \Gamma^R_{MN} \, \partial_R f, \tag{1.2.44a}$$

$$\nabla_M \nabla^M \, f = \nabla_M \partial^M f = \partial_M \partial^M f + \Gamma^M_{MR} \, \partial^R f. \tag{1.2.44b}$$

With this in mind we can se that, in our case, equation (1.2.5) becomes

$$\begin{aligned} G_{MN} = \mathcal{G}_{MN} + \frac{3}{2}\left[\frac{1}{2}\,\partial_M A \partial_N A + \partial_M \partial_N A - \Gamma^R_{MN}\,\partial_R A\right] \\ -\frac{3}{2}\,\mathbf{g}_{MN}\,\left[\partial_R \partial^R A + \Gamma^R_{RS}\,\partial^S A - \frac{1}{2}\,\partial_R A \partial^R A\right]. \end{aligned} \tag{1.2.45}$$

We can simplify this expression a lot since A is a function of z only, using equations (A.2.4) and (1.2.42c) and plugging in the expressions (1.2.43), obtaining

$$G_{\mu\nu} = -\frac{1}{2}\,\partial_R \partial^R \, h_{\mu\nu} - \frac{3}{4}\, h'_{\mu\nu}\, A' + \frac{3}{4}\,(\eta_{\mu\nu} + h_{\mu\nu})\big(2A'' - A'^2\big), \tag{1.2.46a}$$

$$G_{\mu 4} = 0, \tag{1.2.46b}$$

$$G_{44} = \frac{3}{2}\, A'^2. \tag{1.2.46c}$$

Now that we have the Einstein tensor we have, on the other hand, to compute the Energy-Momentum tensor, which once again comes from the Randall-Sundrum action we wrote down in equation (1.2.28) and since $h_{\mu\nu}$ appears only in the $\mu\nu$ components of the Einstein tensor we are interested in the Energy-Momentum four dimensional part as well, that is

$$\begin{aligned} T_{\mu\nu} &= \Big[\Lambda_5 - \sqrt{-12\Lambda_5 {M_5}^3}\; e^{\frac{A(z)}{2}}\big(\delta(z) - \delta(z - z^*)\big)\Big]\, g_{\mu\nu} \\ &= \Big[\Lambda_5\, e^{-A(z)} - \sqrt{-12\Lambda_5 {M_5}^3}\; e^{-\frac{A(z)}{2}}\big(\delta(z) - \delta(z - z^*)\big)\Big]\big(\eta_{\mu\nu} + h_{\mu\nu}\big). \end{aligned} \tag{1.2.47}$$

Recollecting relations (1.2.12), (1.2.20) and (1.2.22) we are now able to write the right hand side of Einstein equations as

$$\begin{aligned} \frac{1}{{M_5}^3}\, T_{\mu\nu} &= \Big[-\frac{3}{2}\, A'^2 + \frac{3}{4}\,\big(A'^2 - 2A''\big)\Big]\big(\eta_{\mu\nu} + h_{\mu\nu}\big) \\ &= \frac{3}{4}\,\Big[2A'' - A'^2\Big]\big(\eta_{\mu\nu} + h_{\mu\nu}\big). \end{aligned} \tag{1.2.48}$$

Putting together the two sides of the Einstein equations it is easy to see that we are left with

$$\frac{1}{2}\,\partial_R\partial^R\,h_{\mu\nu} + \frac{3}{4}\,h'_{\mu\nu}\,A' = 0, \tag{1.2.49}$$

which is exactly the part involving the fluctuations and that now we are going to solve and study the solution in some details.
To start, we want to get rid of the first derivative term making a rescaling

$$h_{\mu\nu} \to e^{\alpha A}\,h_{\mu\nu}, \tag{1.2.50}$$

with α a constant that we will fix in order to simplify the expression we will obtain as much as possible. Substituting our rescaling into the equation we find

$$\frac{1}{2}\,\partial_R\partial^R\,h_{\mu\nu} + \Big(\frac{3}{4}-\alpha\Big)A'h'_{\mu\nu} + \bigg[\Big(\frac{3}{4}\,\alpha - \frac{\alpha^2}{2}\Big)A'^2 - \frac{1}{2}\,\alpha A''\bigg]h_{\mu\nu} = 0, \tag{1.2.51}$$

and, chosing $\alpha = \frac{3}{4}$ the coefficient of $h'_{\mu\nu}$ vanishes and the equation reduces to

$$\frac{1}{2}\,\partial_R\partial^R\,h_{\mu\nu} + \bigg(\frac{9}{32}\,A'^2 - \frac{3}{8}\,A''\bigg)h_{\mu\nu} = 0. \tag{1.2.52}$$

We are now ready to perform the Kaluza-Klein decomposition

$$h_{\mu\nu} = \sum_{n=0}^{+\infty} h^n_{\mu\nu}(x)\,\psi_n(z), \tag{1.2.53}$$

where the four dimensional part $h^n_{\mu\nu}$ satisfy the condition

$$\Big(\partial_\rho\partial^\rho + {m_n}^2\Big)h^n_{\mu\nu} = 0. \tag{1.2.54}$$

And inserting this decomposition in the equation we find a Schrödinger-like equation for $\psi_n(z)$ which reads

$$-\,\psi''_n(z) + \bigg(\frac{9}{16}\,A'^2 - \frac{3}{4}\,A''\bigg)\psi_n(z) = {m_n}^2\psi_n(z), \tag{1.2.55}$$

that looks like a Schrödinger-like equation equation with a potential we can now explicitly write using equations (1.2.22)

$$\begin{aligned} V(z) &= \frac{9}{16}\,A'^2 - \frac{3}{4}\,A'' \\ &= \frac{9}{16}\,\frac{4k^2}{\big(k|z|+1\big)^2} - \frac{3}{4}\left[-\,\frac{2k^2}{(k|z|+1)^2} + \frac{4k}{k|z|+1}\,\big[\delta(z)-\delta(z-z^*)\big]\right] \\ &= \frac{15}{4}\,\frac{k^2}{\big(k|z|+1\big)^2} - \frac{3k}{k|z|+1}\,\big[\delta(z)-\delta(z-z^*)\big], \end{aligned} \tag{1.2.56}$$

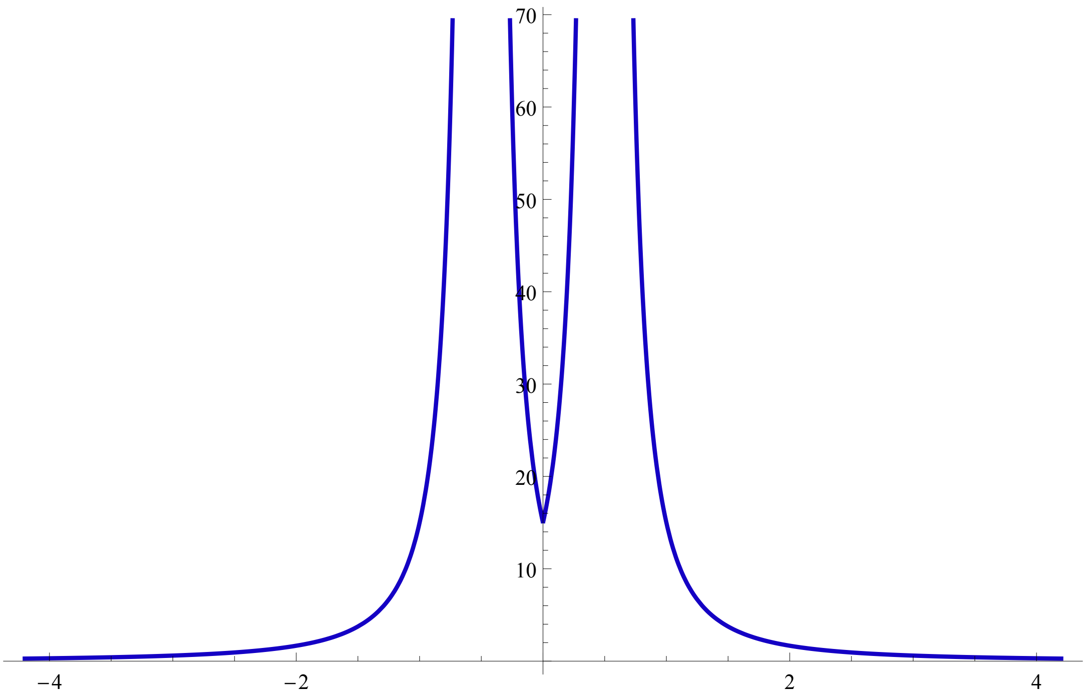


Figure 1.2: Plot of $V(z)$ for $k = 2$.

the shape of which we can see in figure (1.2). Moreover we need the boundary conditions on the two branes, and to obtain them we integrate the equation around the branes locations. For $z = 0$ we get

$$\begin{aligned} \int_{0^-}^{0^+} dz \left[-\psi_n''(z) + V(z)\psi(z) \right] &= \int_{0^-}^{0^+} dz\, {m_n}^2\, \psi_n(z) \\ -\psi_n'(0^+) + \psi_n'(0^-) - 3k\psi(0) &= 0, \end{aligned} \tag{1.2.57}$$

where the right hand side and the first term coming from the potential vanish because, by definition, the wave function has to be $\mathbb{Z}^2$ invariant, or in other words, it has to be an even function. If this is so, then its first derivative has to be an odd function, consequently we get the boundary condition at the first brane

$$\psi_n'(0) = -\frac{3k}{2}\, \psi_n(0). \tag{1.2.58}$$

In the same way we can obtain the condition on the brane at $z = z^*$. Reminding that the wave function has to be even also with respect this point we get

$$\psi_n'(z^*) = -\frac{3k}{2\,(k|z^*| + 1)}\, \psi_n(z^*). \tag{1.2.59}$$

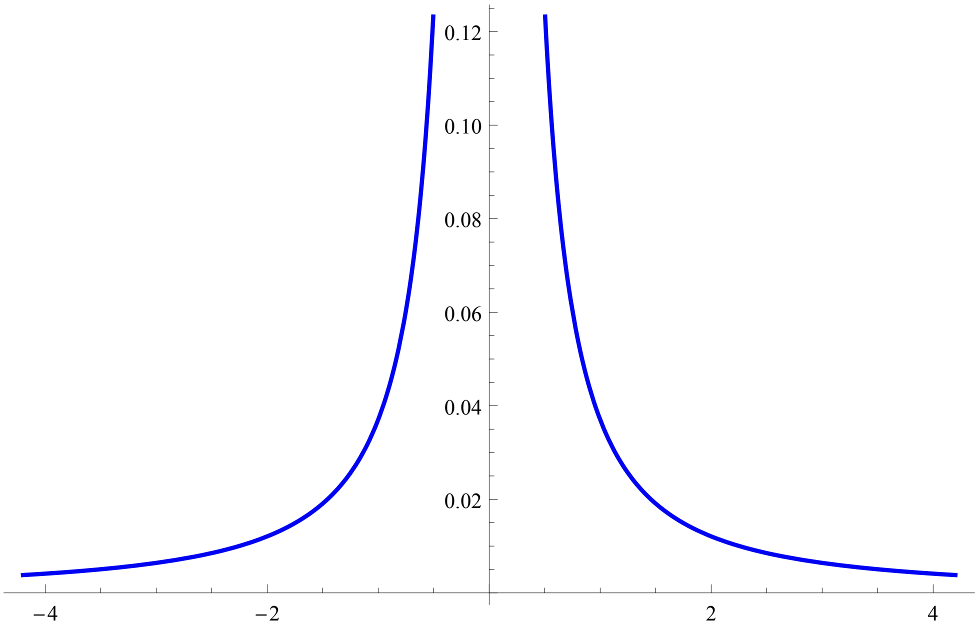


Figure 1.3: Plot of $\psi_0(z)$ for $k = 10$.

We are now ready to solve the equation starting with finding the zero mode, that is the solution for $n = 0$. This case is particularly simple because it is a massless mode since, as always happens in Kaluza-Klein's decompositions, $m_0 = 0$. The equation we have to solve then is

$$-\psi_0'' + \left(\frac{9}{16}\, A'^2 - \frac{3}{4}\, A''\right)\psi_0 = 0, \tag{1.2.60}$$

and it is pretty easy to check that the solution is given by

$$\psi_0(z) = e^{-\frac{3}{4}\,A(z)} = (k|z| + 1)^{-\frac{3}{2}}, \tag{1.2.61}$$

and that it satisfies the boundary conditions we found. This solution is remarkable most of all because we see (cf. figure (1.3)) that the zero mode wave function is peaked on the brane at $z = 0$ and then it is exponential suppressed. This ultimately explains how the hierarchy problem is solved in this model. Since the graviton zero mode, that is the predominant mediator of the gravitational interactions (as we will see in the next section), is strongly localized on the first brane, on the second one, our world brane, we feel only the exponential suppressed tail of it, and this is the explanation why we experience gravity as a incredibly weak force compared to the others.
Going back on the solution of the equation, now we have to solve it for all the massive Kaluza-Klein modes, that between the boundaries becomes

$$-\psi_n'' + \left(\frac{15}{4}\,\frac{k^2}{\left(k|z| + 1\right)^2} - {m_n}^2\right)\psi_n = 0. \tag{1.2.62}$$

This is a Bessel equation of order 2 and its solutions is a linear combination of first and second kind Bessel function:

$$\psi_n(z) = \sqrt{\left(|z| + k^{-1}\right)}\left[a_n\, J_2\Big(m_n\,\left(|z| + k^{-1}\right)\Big) + b_n\, Y_2\Big(m_n\,\left(|z| + k^{-1}\right)\Big)\right], \qquad (1.2.63)$$

with a_n and b_n the coefficients of the linear combination. Without going into the details, one can determine the coefficients and the expression for the masses of the Kaluza-Klein modes by imposing on the solution the two boudary conditions and requiring the normalization of the wave function. Hence one can find that the masses are given by

$$m_n = ke^{-kL} j_n, \qquad (1.2.64)$$

where j_n are the zeros of the first kind Bessel function of first order $J_1(j_n) = 0$. From this expression something not really trivial comes out. Despite the fact that naively one could expect the separation between the various modes would be of order of the Plank Scale since k is supposed to be of that order, once again the exponential warping appears and shrinks it to the TeV Scale. Moreover one can see that all these massive modes are strongly peaked on the second brane unlike ψ_0, therefore their coupling with the matter content of brane at $z = z^*$ would be enhanced compared to the gravitational zero mode. The combination of these two factors, the masses and the displacement between them at the TeV order and the profile of the wave function peaked around our world brane, implies that the Kaluza-Klein modes are theoretically observable as individual resonance at the colliders, making them a reasonable field of interest.

1.2.4 Newtonian Limit

Now what we want to do is to verify whether or not the interactions mediated by the graviton modes we found to reproduce correctly the Newton's gravitation. For this purpose we have to add matter content on the second brane and carry out the couplings between the graviton modes and this additional matter.
The action is then composed by a gravity part which is the action we have used since now and wrote down in equation (1.2.28) and that now we refer to as S_G, and a part S_M for the matter content and its interaction with gravity

$$S = S_G + S_M = S_G + \int d^5x\, \sqrt{|g|}\, \mathcal{L}_M\big(\Phi, g_{MN}\big), \qquad (1.2.65)$$

where Φ stands for the various fields that we want to introduce on the brane. Once again we want to work in the small fluctuation regime around the background metric $g_{MN} = e^{-A}\,\eta_{MN}$, so we parametrize it again as

$$\tilde{g}_{MN} = e^{-A}\big(\eta_{MN} + h_{MN}\big), \qquad (1.2.66)$$

imposing the same gauge choices we used before and expressed in equations (1.2.42). We then expand the matter lagrangian up to the first order

$$\mathcal{L}_M\big(\Phi, \tilde{g}_{MN}\big) = \mathcal{L}_M\big(\Phi, g_{MN}\big) + h_{\mu\nu}\, \frac{\delta \mathcal{L}_M}{\delta \tilde{g}_{\mu\nu}}\bigg|_{\tilde{g}_{\mu\nu}=g_{\mu\nu}} + \mathcal{O}\big(h^2\big). \tag{1.2.67}$$

Then, recollecting the definition of the Energy-Momentum tensor, which we can rewrite as

$$T^{\mu\nu} = 2\frac{\delta \mathcal{L}_M}{\delta \tilde{g}_{\mu\nu}}\bigg|_{\tilde{g}_{\mu\nu}=g_{\mu\nu}} - g^{\mu\nu}\mathcal{L}_M, \tag{1.2.68}$$

and expanding the squared root of the determinant to the first order as well

$$\sqrt{|\tilde{g}|} = \sqrt{|g|}\,\left(1 - \frac{1}{2}\, h_{\mu\nu}\, g^{\mu\nu} + \mathcal{O}\big(h^2\big)\right) = \sqrt{|g|}\,\left(1 - \frac{h}{2} + \mathcal{O}\big(h^2\big)\right), \tag{1.2.69}$$

and $h = h_{\mu\nu}\, g^{\mu\nu}$, we can write down the expansion of the S_M integrand

$$\begin{aligned}
\sqrt{|\tilde{g}|}\, \mathcal{L}_M\big(\Phi, \tilde{g}_{MN}\big) \simeq& \sqrt{|g|}\,\left(1 - \frac{h}{2}\right) \mathcal{L}_M\big(\Phi, \tilde{g}_{MN}\big) \\
\simeq& \sqrt{|g|}\,\left[\left(1 - \frac{h}{2}\right)\mathcal{L}_M\big(\Phi, g_{MN}\big) + h_{\mu\nu}\, \frac{\delta \mathcal{L}_M}{\delta \tilde{g}_{\mu\nu}}\bigg|_{\tilde{g}_{\mu\nu}=g_{\mu\nu}}\right] \\
\simeq& \sqrt{|g|}\,\left[\mathcal{L}_M\big(\Phi, g_{MN}\big) + \frac{1}{2}\, h_{\mu\nu}\, T^{\mu\nu}\right].
\end{aligned} \tag{1.2.70}$$

Here we have not used the trace gauge condition (1.2.42c) in order to more clearly recognise the expression for the Energy-Momentum tensor, nevertheless it is important to keep it in mind for the future.
Obviously the same expansion has to be performed on S_G too. The calculation is very similar to what we have done for S_M and so quite straightforward but a bit more time consuming. We are not going through it but if one would do it, he would get a term independent of $h_{\mu\nu}$ which it has to be set to 0 when imposing the vanishing of the effective cosmological constant; a linear term, that is the one leading to the equations of motion which then vanishes on shell; and finally a quadratic part that is exactly the Fierz-Pauli Lagrangian we would become very familiar with in the next chapter. Remembering the Kaluza-Klein decomposition of equation (1.2.53), after a rescaling by a factor $e^{\frac{3}{2}\,A}$ and imposing the canonical normalization of the lagrangian we finally obtain the expression for the coupling between matter and graviton modes

$$\mathcal{L}_{int} = \sum_n\, \frac{e^{\frac{3}{2}\,A}}{2\sqrt{M_5}^3}\, \psi_n(z)\, h^n_{\mu\nu}(x)\, T^{\mu\nu}, \tag{1.2.71}$$

from which we can read the expression for the coupling constants to the different modes

$$k_n = \frac{e^{\frac{3}{2}\,A}\,\psi_n(z)}{2\sqrt{{M_5}^3}}. \tag{1.2.72}$$

From this we can extrapolate, in the non-relativistic limit, the expression for the gravitational potential between two particles with unitary TeV mass living on the brane at $z = z^*$, generated by the exchange of the zero-mode as well as all the Kaluza-Klein tower of massive modes. The potential one can obtain is given by

$$V(r) = -\sum_{n=0}^{\infty} \frac{{a_n}^2}{4\pi}\,\frac{e^{-m_n r}}{r}, \tag{1.2.73}$$

and since we have the explicit expression for the wave function of the zero-mode in equation (1.2.61), we easily find its contribution to the potential that is

$$V_0(r) = -\frac{1}{16\pi {M_5}^3}\,\frac{1}{r} = -\frac{G_N}{r}, \tag{1.2.74}$$

where, for the second equality comes form the relation that connects the Newton's constant and the higher dimensional Planck Mass (equation (1.2.8)). We can see that the zero mode reproduces the Newtonian gravity only by itself, while the massive modes give merely small deviations because of the exponential suppression due to the factor $e^{-m_n r}$, these contributions in fact lead to corrections that are negligible up to distances of order of the fermi $r \lesssim 10^{-15}$ m.

1.2.5 Radius Stabilization

We are now going to discuss an important feature of the model but we will treat it without any claim to be comprehensive because, despite its general importance, it will not play any prominent role in following discussions.

Until now we have considered the size L of the extra dimension to be an arbitrary and fixed constant that we can set as we want to address the hierarchy problem. In other words, at this stage there is no dynamical mechanism taking care of fixing the value of the extra dimension radius. This means that there has to be an effective theory of a scalar field in the bulk, commonly called *radion*, corresponding to the fluctuations of the length around its main value. Moreover, we can naively understand that the radion has to be massless by realizing that, if there were not for the hierarchy argument, the Randall-Sundrum solution we found would work for arbitrary values of the radius. In the effective theory language it means the scalar field has no potential nor mass since there has not to be any prefered value for L fixed by the radion equation of motion. However, if this would be the case, we will be left with a new force in violation of the

equivalence principle, since a massless radion would affect the Newton's law. This is an obvious phenomenological constraint that forces us to find a mechanism to stabilize the radius, i.e. to give a mass to the radion.
The most common way to achieve the stabilization has been proposed by Goldberger and Wise. The main idea to dynamically obtain a non-trivial radius is to create a situation in which, adding terms to the Randall-Sundrum action, in addition to the kinetic term which drives the extra dimension size to very large values there would be some others trying to keep it as smaller as possible. This can be achieved by introducing a mass term for the scalar field which wants to have the smallest possible radius in order to minimize the potential and two branes energy contributions set in a particular way to have a non-trivial profile for the potential and then a non-trivial minimum. The way to build it is to introduce the branes potentials which have different minima values from each others so as to obtain a vacuum expectation value that varies along the extra dimension. This means that now we have to deal with an action that is composed by the usual Randall-Sundrum contribution S_G of equation (1.2.28) and in addiction we have the action for the radion that we denote as Φ. Using the coordinates $x^M = \{x^\mu, y\}$ we have

$$S = S_G + \int d^5x \, \sqrt{|g|} \left[\frac{1}{2} \, \partial_M \Phi \partial^M \Phi - V(\Phi) - V_1(\Phi)\delta(y) - V_2(\Phi)\delta(y - L) \right]. \quad (1.2.75)$$

Now one would have to simultaneously solve both Einstein equation and the radion field equation, to have the the effect of its presence on the background metric under control. Just to give the idea of the calculation one should face, by requiring and maintain the Lorentz invariance we have to do the same ansaz for the metric as we have always done

$$ds^2 = e^{-2A(y)} \, \eta_{\mu\nu} \, dx^\mu dx^\nu - dy^2, \quad (1.2.76)$$

and it is necessary to restrict the dependence of the scalar field on the extra dimension coordinate only

$$\Phi\big(x^M\big) = \Phi(y). \quad (1.2.77)$$

The equation of motion for the scalar field is the usual covariant Klein-Gordon equation

$$\frac{1}{\sqrt{|g|}} \, \partial_M \Big(\sqrt{|g|} \, g^{MN} \, \partial_N \Phi \Big) = \frac{\partial V_{tot}}{\partial y}, \quad (1.2.78)$$

while the Einstein equation comes out to be

$$4A'^2 - A'' = -\frac{2k^2}{3} \, V(\Phi_0) - \frac{k^2}{3} \Big[V_1(\Phi_0)\delta(y) - V_2(\Phi_0)\delta(y - L) \Big], \quad (1.2.79a)$$

$$A'^2 = \frac{k^2}{12} \, {\Phi_0'}^2 + \frac{k^2}{6} \, V(\Phi_0). \quad (1.2.79b)$$

The first equation comes the $\mu\nu$ components of the Einstein equation while the second from the 44 one and we denoted as Φ_0 the solution of the field equation. This system is quite hard to solve generally but for some special form of the potential $V(\Phi)$, deriving from some function usually referred to as *superpotential*, one can actualy solve the equations obtaining the expression of the radius length as function of the two different vacuum values of the two branes Φ_1 and Φ_2

$$L = \frac{k}{\lambda} \ln\left(\frac{\Phi_1}{\Phi_2}\right), \tag{1.2.80}$$

where λ is a constant parameter of the potential. What we have seen is that the radius is determined by the equation of motion and moreover one can obtain the right value for extra dimension length to address the hierarchy problem $kL \approx 35$ without any fine tuning of the initial parameters.

1.3 The Standard Model in the Bulk

The Randall-Sundrum model as presented so far, even though works perfectly fine in solving the hierarchy problem and in reproducing the right gravitation law in the Newtonian limit, appears to have some great issues mainly concerning flavour physics. This led to a lot of effort in finding some extension and improvement of the model. The most studied and valued of them is the framework where all the Standard Model gauge and matter fields are extended to live in the bulk while only the Higgs field remains confined on the world brane. This setup is very appealing because it offers a natural explanation of the flavour physics of the Standard Model, for example addressing the mass hierarchy of the fermions. Since a lot of work has been done in this direction it is impossible to cover in details this huge topic. We will concentrate only in reviewing the construction of the model and some remarkable results which will be helpful in the following. For many others details and aspects that will not be mentioned see [5–9] as well as many other good reviews that can be found.

1.3.1 The Bulk Field Actions

We will study the situation in which the back-reaction for the presence of all the additional fields on the bulk can be neglected in orderd to preserve the solution for the background we found previously. The first thing we have to do is to learn how to extend the Standard Model fields to the bulk.

Scalar Field

We start from the simplest case even if it is not the interesting one, the scalar massive field case denoted by Φ, of which action is given by

$$S_\Phi = \int d^5x\ \sqrt{|g|}\ \Big[\partial_M\Phi\partial^M\Phi + m_\Phi{}^2\Phi^2\Big], \tag{1.3.1}$$

And by the variation of it we can derive the equations of motion

$$\partial^2\Phi - e^{2ky}\partial_4\Big(e^{-4ky}\ \partial_4\Phi\Big) - e^{-2ky}\ m_\Phi{}^2\Phi^2 = 0, \tag{1.3.2}$$

where here $\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$. Then, performing the by now usual Kaluza-Klein decomposition and normalizing the field

$$\Phi\big(x^M\big) = \frac{1}{\sqrt{L}}\sum_n\ \Phi_n(x^\mu)\phi_n(y). \tag{1.3.3}$$

Assuming the mass m_Φ to be defined in units of the scale k, that means supposing $m_\Phi = ak$ with a dimensionless, we can find the general solution for the zero-mode extra dimension profile

$$\phi_0(y) = C_1e^{(2-\alpha)ky} + C_2e^{(2+\alpha)ky}, \tag{1.3.4}$$

with $\alpha = \sqrt{4+a}$ and C_1 and C_2 two arbitrary constants. However one can see that, given the boundary conditions given by the variation of the action as it is, both C_1 and C_2 vanish, implying that there is no zero-mode solutions. To obtain it we have to add boundary mass terms in the action which will me parametrized in units of k once again with dimensionless coefficient b

$$S_{bounduary} = -\int d^5x\ \sqrt{|g|}2bk\ \Big[\delta(y) - \delta(y-L)\Big]\Phi^2. \tag{1.3.5}$$

With this new term the boundary conditions becomes

$$\Big(\partial_4\phi_0(y) + bk\phi_0(y)\Big)\Big|_{y=0,L} = 0, \tag{1.3.6}$$

leading to some non-vanishing solution for $b = 2 \pm \alpha$. So, the profile of the zero mode wave function along the extra dimension is found to be

$$\phi_0(y) \propto e^{1\pm\sqrt{4+a}ky}, \tag{1.3.7}$$

and we can se that, with the freedom given by the free parameter a, we can choose the localization of the first mode. The general solution for the massive modes is given again in terms of Bessel functions and we can find an approximate expression for their mass in the limit of $kL \gg 1$ (which is quite good since we know that kL has to be ~ 35)

$$m_n \approx= \left(n + \frac{1}{2}\ \sqrt{4+a} - \frac{3}{4}\right)\pi ke^{-kL}, \tag{1.3.8}$$

and they can not be localized arbitrarily but rather forced to stay near the brane at $y = L$.

Fermion Field

Let us next consider the extension of a fermion field on the bulk. In five dimensions the fundamental spinor representation has four components, so fermions are described by Dirac spinors Ψ. The five dimensional action for it is

$$S_\Psi = \int d^5x \, \sqrt{|g|} \, \left[\bar{\Psi}\Gamma^M \nabla_M \Psi - m_\Psi \bar{\Psi}\Psi \right]. \tag{1.3.9}$$

Few points have to be defined and explained in this equation. Since we are building the action in curved five-dimension space-time geometry there are five gamma matrices

$$\Gamma^M = e^A_M \, \gamma_A, \tag{1.3.10}$$

where e^A_M is the *funfbein* defined by $g_{MN} = e^A_M \, e^B_N \, \eta_{AB}$ and $\gamma_A = (\gamma_\alpha, i\gamma_5)$ the gamma matrices in flat space. The curved covariant derivative ∇_M is composed by two terms $\nabla_M = D_M + \Omega_M$ of which the first is the usual gauge covariant derivative and the second is the part due to the curvature. This second part is expressed in terms of the so called *spin connection* ω^{AB}_M which is the extension to spinors bundles of the Christoffel's symbols, required to define the affine connection on vector bundles. It can indeed be defined from the affine connection since it is related to the Christoffel's symbols by

$$\omega^{AB}_M = e^A_N \, \Gamma^N_{RM} \, e^{RB} + e^A_N \, \partial_M e^{NB}. \tag{1.3.11}$$

So the curvature covariant derivative can be expressed as

$$\Omega_M = -\frac{i}{4} \, \omega^{AB}_M \, \sigma_{AB}, \tag{1.3.12}$$

where $\sigma_{AB} = \frac{i}{2} \left[\gamma_A, \gamma_B\right]$. Once the formal structure of the action is somehow defined there is one other precaution we should take. Under $\mathbb{Z}^2$ a fermion transforms up to a phase as

$$\Psi(-y) = \gamma_5 \, \Psi(y), \tag{1.3.13}$$

which means that the combination $\bar{\Psi}\Psi$ is $\mathbb{Z}^2$-odd. Since the bulk action must be invariant (i.e. even) we have to introduce a mass parameter which is odd as well. Parametrizing it in terms of k then m_Ψ has to necessarily be given by

$$m_\Psi = c \, k \, \mathrm{sign}(y), \tag{1.3.14}$$

with c a dimensionless parameter again. The corresponding equations of motion coming from the variation of the action S_Ψ as we build are then

$$e^{ky} \, \eta^{\mu\nu}\gamma_\mu \partial_\nu \hat{\Psi}_- - \partial_4 \hat{\Psi}_+ + m_\Psi \hat{\Psi}_+ = 0, \tag{1.3.15a}$$

$$e^{ky} \, \eta^{\mu\nu}\gamma_\mu \partial_\nu \hat{\Psi}_+ + \partial_4 \hat{\Psi}_- + m_\Psi \hat{\Psi}_- = 0, \tag{1.3.15b}$$

in which $\hat{\Psi} = e^{-2k}\ \Psi$ and $\Psi = \Psi_+ + \Psi_-$ with $\Psi_\pm = \pm\gamma_5\Psi_\pm$. To solve the equation we have to perform the Kaluza-Klein decomposition for both Ψ_+ and Ψ_-

$$\Psi_\pm(x^M) = \frac{1}{\sqrt{L}} \sum_n \Psi^n_\pm(x^\mu)\psi^n_\pm(y), \tag{1.3.16}$$

where the $\Psi^n_\pm(x^\mu)$ now satisfy the usual Dirac four dimensional equation. With this ansaz then the equations (1.3.15) can be solved for the zero-mode and the general solutions are given by

$$\psi^0_\pm(y) = D_\pm\ e^{\mp cky}, \tag{1.3.17}$$

with $D_\pm$ arbitrary constants. The $\mathbb{Z}^2$ symmetry implies that one component between ψ^0_+ and ψ^0_- has to be odd and then vanish (depending on the chosen representation for the gamma matrices). This is in fact how one can recover the chirality in four dimensions. For the remaining component, the boundary conditions obtained again from the action are

$$\Big(\partial_4\hat{\psi}^0_\pm(y) \pm ck\hat{\psi}^0_\pm(y)\Big)\Big|_{y=0,L} = 0. \tag{1.3.18}$$

We can easily see that this boundary condition is exactly the equation of motion and then there is always a zero-mode. It turns out that, likewise the scalar field the profile of the zero mode contains a dependence from the parameter c

$$\psi_0(y) \propto e^{\frac{1}{2}(1-2c)ky} \tag{1.3.19}$$

allowing us to localize it wherever we would like to. The non-zero modes solutions are once again given by a combination of Bessel functions and are always localized near the world brane.

Vector Field

Last but not least, we want to study how gauge vector bosons field can be implemented. We denote V_M the five dimensional gauge field and we will restrict the discussion for a abelian gauge boson since the extension to non-abelian it is straightforward. The action is then given by

$$S_A = \int d^5x\ \sqrt{|g|}\ \Big[-\frac{1}{4}\ g^{MR}g^{NS}F_{RS}F_{MN}\Big], \tag{1.3.20}$$

with F_{MN} the field strength tensor defined as usual as

$$F_{MN} = \partial_M V_N - \partial_N V_M, \tag{1.3.21}$$

but it is important to notice that the definition does not involve the covariant derivative coming from the curvature thanks to the antisymmetry structure that cancels any affine connection terms. We can now use the gauge freedom to set $V_4 = 0$ which is a consistent choice with the assumption we are free to make that V_4 is a $\mathbb{Z}^2$-odd function. In this way we can totally eliminate the fifth components from the theory on the world brane without affecting the gauge invariance of the four dimensional theory we want to preserve. With this gauge choice done we can rewrite the action, integrating by part, as

$$S_A = -\frac{1}{4}\int d^5x \; \eta^{\mu\rho}\eta^{\nu\sigma}F_{\rho\sigma}F_{\mu\nu} + 2\eta^{\mu\nu}V_\nu\partial_4\big(e^{-2ky}\partial_4 V_\mu\big). \tag{1.3.22}$$

As we have become familiar by now, we can Kaluza-Klein decompose the vector field

$$V_\mu\big(x^R\big) = \frac{1}{\sqrt{L}}\sum_n V_\mu^n(x^\rho)v_\mu^n(y), \tag{1.3.23}$$

and substituting it into the action and integrating over y we obtain

$$S_A = \sum_n \int d^4x \; -\frac{1}{4}\,F^{\,n}_{\mu\nu}F^{\mu\nu(n)} - \frac{1}{2}\,{m_n}^2\eta^{\mu\nu}V_\mu^n V_\nu^n, \tag{1.3.24}$$

while the fifth dimension profile has to obey the equation

$$\partial_4\Big(e^{-2ky}\;\partial_4 v_n(y)\Big) + {m_n}^2 v_n(y) = 0. \tag{1.3.25}$$

We have obtained then a four dimensional action for a vector boson of which the zero-mode is massless, recovering the usual gauge invariance. The general solution can be found as a combination of first order Bessel's function of first and second kind and the remarkable fact is that, in this case, the zero-mode is constant along the extra dimension

$$v_0(y) = \frac{1}{\sqrt{L}}, \tag{1.3.26}$$

while the others are localized in the same way as the scalar and the fermion ones.

1.3.2 The Standard Model Bulk Action

Now that we know how to build the action for the fields we can construct a bulk action for the Standard Model. Recalling that the warping factor to address the hierarchy problem affects only the Higgs boson, we have to be careful and must localize the Higgs field very near the world brane or, as many times assumed for simplicity, confine it at $y = L$.

The main consequence of extending the fermions to the bulk is, by virtue of the freedom we have to localize their zero-mode anywhere, to generate in a very natural way the hierarchy between the various Yukawa couplings. In fact, localizing every fermionic zero-mode field in a different position along the extra dimension, one can tune the strength

of their interaction with the Higgs field as required. So we will have a situation in which the electron field, being the lightest fermion, is the furthest away from the world brane, while the top field is the nearest.
With this in mind let us analyze for sake of briefness of the expressions (which will be quite long anyway) only the electro-weak sector since the extension to strong interaction is straightforward. The ingredients needed for write down the total action are the gauge fields of $SU(2)_L \times U(1)_Y$, we denote as W^a_M and B_M. We choose their fifth components W^a_4 and B_4 to be $\mathbb{Z}^2$-odd while the vector components W^a_μ and B_μ to be even, in order to ensure that the zero-modes correspond to the usual Standard Model gauge bosons. The Higgs field is, exactly as in the normal construction, a doublet we denote by Φ. And finally we need the fermion fields, we denote as L^i and Q^i respectively the lepton and the quark $SU(2)_L$ doublets and with e^i, u^i and d^i the singlets of lepton, up and down quarks while $i = 1, 2, 3$ is the generation index so that for example $e^i = \{e, \mu, \tau\}$ the three right handed leptons or $u^i = \{u, c, t\}$ the up, charm and top quarks. So the total action is composed by several pieces we are now going to study briefly:

- S_{RS}, the usual and already well known Randall-Sundrum background action of equation (1.2.28);
- S_H, the brane localized Higgs-sector action, which we have already studied in section (1.2.2);
- S_G, the gauge part of the theory composed by three different pieces, corresponding to the gauge bosons terms S_B, the gauge-fixing term S_{GF} and the Faddeev-Popov ghost part S_{FP};
- S_F, the fermionic part, divided in two pieces, the first involving kinetic terms S_f and the second concerning the Yukawa couplings S_Y.

Gauge Action

Let us start by inspect the gauge bosons action given by

$$S_B = -\frac{1}{4} \int d^5x \, \sqrt{|g|} \, g^{MR} g^{NS} \Big[W^a_{MN} W^a_{RS} + B_{MN} B_{RS} \Big], \tag{1.3.27}$$

where W^a_{MN} is the non-abelian field strength tensor

$$W^a_{MN} = \partial_M W^a_N - \partial_N W^a_M + i g_5' \big[W^a_M, W^a_N \big], \tag{1.3.28}$$

and finally g_5' is the five dimensional gauge coupling of $SU(2)_L$, while we denote with g_5 the one corresponding to $U(1)_Y$. We can also perform the usual field redefinition to diagonalize the mass terms

$$W_M^{\pm} = \frac{1}{\sqrt{2}} \Big(W_M^1 \mp W_M^2 \Big), \tag{1.3.29a}$$

$$Z_M = \frac{1}{\sqrt{{g_5}^2 + {g_5'}^2}} \Big(g_5\, W_M^3 - g_5'\, BM \Big), \tag{1.3.29b}$$

$$A_M = \frac{1}{\sqrt{{g_5}^2 + {g_5'}^2}} \Big(g_5'\, W_M^3 + g_5\, BM \Big). \tag{1.3.29c}$$

The kinetic terms contain some mixed combinations between the actual gauge bosons and their extra dimension components $W_4^{\pm}$, Z_4 and A_4 and, from the Higgs kinetic terms arise contributions involving the gauge bosons and the Goldstone fields we call $\varphi^{\pm}$ and φ^3, from the usual decomposition of the Higgs complex doublet

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\sqrt{2}\varphi^+(x) \\ v + h(x) + i\varphi^3(x) \end{pmatrix}. \tag{1.3.30}$$

All these mixed terms can be removed introducing a gauge-fixing Lagrangian that reads

$$\begin{aligned} \mathcal{L}_{GF} = &- \frac{1}{2\xi} \Big(\partial_\mu A^\mu - \xi \partial_4 e^{-2ky} A_4 \Big)^2 \\ &- \frac{1}{2\xi} \Big[\partial_\mu Z^\mu - \xi \Big(\delta(y-L) M_Z \varphi^3 + \partial_4 e^{-2ky} Z_4 \Big) \Big]^2 \\ &- \frac{1}{\xi} \Big[\partial^\mu W_\mu^+ - \xi \Big(\delta(y-L) M_W \varphi^+ + \partial_4 e^{-2ky} W_4^+ \Big) \Big] \\ &\times \Big[\partial^\mu W_\mu^- - \xi \Big(\delta(y-L) M_W \varphi^- + \partial_4 e^{-2ky} W_4^- \Big) \Big], \end{aligned} \tag{1.3.31}$$

where the five dimensional mass parameters are

$$M_W = \frac{v g_5}{2}, \tag{1.3.32a}$$

$$M_Z = \frac{v}{2} \sqrt{{g_5}^2 + {g_5'}^2}. \tag{1.3.32b}$$

It is also important to say that there is no problem in squaring the delta functions because the derivatives of $W_4^{\pm}$ and Z_4 contain other delta contributions which cancel exactly the one explicitly written.

After having introduced the gauge fixing terms we can proceed in decomposing in Kaluza-Klein modes all the bosonic fields denoting the general n-th profile along the fifth dimension with χ_n^a with $a = A, W, Z$ and the masses of the modes as well as m_n^a. Substituting the decompositions in the action we can find the equations of motion for the profiles

$$- \partial_4\Big(e^{-2ky}\, \partial_4\chi_n^a(y)\Big) = (m_n^a)^2\chi_n^a(y) - \delta\big(y-L\big)M_a^2\ \chi_a^n(y), \tag{1.3.33}$$

with the boundary conditions as well. Solving the equation, inserting the solution in the action and integrating over y we find that the quadratic part of the action takes the usual and expected form

$$\begin{aligned} S_B^{(2)} + S_{GF}^{(2)} = &\sum_n \int d^4x \Bigg\{ - \frac{1}{4}\, F_{\mu\nu}^{\,n} F^{\mu\nu(n)} - \frac{1}{2\xi}\, \big(\partial^\mu A_\mu^n\big)^2 + \frac{1}{2}\, \big(m_n^A\big)^2 A_\mu^n A^{\mu(n)} \\ &- \frac{1}{4}\, Z_{\mu\nu}^{\,n} Z^{\mu\nu(n)} - \frac{1}{2\xi}\, \big(\partial^\mu Z_\mu^n\big)^2 + \frac{1}{2}\, \big(m_n^Z\big)^2 Z_\mu^n Z^{\mu(n)} \\ &- \frac{1}{4}\, W^{+}{}_{\mu\nu}^{(n)} W^{-\mu\nu(n)} - \frac{1}{\xi}\, \partial^\mu W^{+}{}_{\mu}^{(n)} \partial^\mu W^{-}{}_{\mu}^{(n)} + \big(m_n^W\big)^2 W^{+}{}_{\mu}^{(n)} W^{-\mu(n)} \\ &+ \frac{1}{2}\, \big(\partial_\mu\varphi_A^n\big)^2 - \frac{1}{2}\, \xi\big(m_n^A\big)^2\big(\varphi_A^n\big)^2 + \frac{1}{2}\, \big(\partial_\mu\varphi_Z^n\big)^2 - \frac{1}{2}\, \xi\big(m_n^Z\big)^2\big(\varphi_Z^n\big)^2 \\ &+ \partial_\mu\varphi^{+}{}_W^{n}\, \partial^\mu\varphi^{-}{}_W^{n} - \xi\varphi^{+}{}_W^{n}\varphi^{-}{}_W^{n} \Bigg\}. \end{aligned} \tag{1.3.34}$$

We see that, for each Kaluza-Klein mode, the action is almost identical to the Standard Model, the only difference is in the mass term for the photon form the $n \geq 1$ mode. It follows then that we need n Fadeevv-Popov ghost actions, one for every mode, which is totally analogous to the Standard Model with n different ghost fields, one for every component of the Kaluza-Klein tower, so that

$$S_{FP} = \sum_n S_{FP}^n, \tag{1.3.35}$$

where every S_{FP}^n is the Standard Model Faddeev-Popov action with the n-th ghost fields.

Fermionic Action

Let us now proceed to the fermion content of the theory. From the last section we learnt how to build the action for a Dirac spinor in five dimensions. In the following expression we write the kinetic term for the Standard Model S_f in which we already worked out the curvature contribution to the covariant derivative, leading to different exponentials in front of the various pieces. And also a mass term is needed but it has nothing to do with

the actual mass terms coming from the Yukawa part. It is only the piece concerning the five dimensional "mass" parameters needed for localize the fields in the desired point of the bulk.
With this in mind we can write

$$\begin{aligned} S_f = \sum_i \int d^5x \Bigg\{ e^{-3ky} \Big[\sum_{F=L,Q} \bar{F}^i i\Gamma^M D_M F^i + \sum_{q=e,u,d} \bar{q}^i i\Gamma^M D_M q^i \Big] \\ - e^{-4ky}\,\mathrm{sign}(y) \Big[\sum_{F=L,Q} \bar{Q}^i\, \mathbf{M}_Q\, Q^i + \sum_{q=e,u,d} \bar{q}^i\, \mathbf{M}_q\, q^i \Big] \\ - e^{-2ky} \Bigg[\sum_{F=L,Q} \bar{F}^i{}_L \partial_4 \Big(e^{-2ky} F^i_R \Big) - \bar{F}^i{}_R \partial_4 \Big(e^{-2ky} F^i_L \Big) \\ \sum_{q=e,u,d} \bar{q}^i{}_L \partial_4 \Big(e^{-2ky} q^i_R \Big) - \bar{q}^i{}_R \partial_4 \Big(e^{-2ky} q^i_L \Big) \Bigg] \Bigg\}. \end{aligned} \tag{1.3.36}$$

Here the various $\mathbf{M}$ are diagonal matrices containing the bulk masses and the chiral left and right components denoted with the L and R subscript are defined in the same way as in the previous section. These components are chosen with the right transformation under $\mathbb{Z}^2$ in order to recover the desired zero mode. In particular the left-handed components of the doublets L^i and Q^i are even, while the right-handed ones are odd. Conversely, for the singlets e^i, u^i and d^i the right-handed components are even and the left-handed are odd. This is the right way to assign the $\mathbb{Z}^2$ parities since we know that the zero mode of the even components are the one which correspond to the Standard Model fields while the odd ones vanish. Finally, we chose to write the action in a base of the flavour space in which the bulk masses terms are diagonal, this can be proven to be always possible without loss of generality. However, the action S_f itself would give rise to a massless Weyl fermion for every five dimensional field (plus their respective Kaluza-Klein towers) switching to the effective theory in four dimensions. This is why we need to introduce the Yukawa interactions that take care of removing the massless zero modes and replace them with the massive Standard Model fermions, which will be still accompanied by the massive Kaluza-Klein modes. Therefore we can write an action with the Yukawa mass terms in the form

$$S_Y = \sum_i \sum_{q=u,d,e} \int d^5x \; -\delta\big(y-L\big) e^{-3ky} \frac{v}{\sqrt{2}} \Big[\bar{q}^{i\,doub}_L\; \mathbf{Y}_q\; q^i_R + \bar{q}^i{}_R\; \mathbf{Y}^\dagger_q\; q^{i\,doub}_R \Big], \tag{1.3.37}$$

where with $q^{i\,doub}$ we denote the correspondent Weyl component coming from the $SU(2)_L$ doublets and the $\mathbf{Y}$ are the Yukawa five dimensional matrices connected with the usual four dimensional one by the relation

$$\mathbf{Y} = \frac{2}{k}\, \mathbf{Y}^{(4D)}. \tag{1.3.38}$$

Now for every field in the theory one have to perform the Kaluza-Klein decomposition and, inserting it into the action, find the equations of motion that are similar to the one we found in the previous section with the only difference regarding the additional Yukawa's term localized on the world brane. The solution it is also a straightforward generalization and can be written in terms of Bessel function but this time the zero mode, on the brane $y = L$ acquire a mass.

Now that the model is built we can ask ourselves what are the majors changes that the extension of the Standard Model in the bulk provides. The most important for the purposes of our discussion is that now, if one performs again the Newtonian limit of the theory (exactly as we done in section (1.2.4)) and finds the interaction between the graviton and the Standard Model particles, the coupling constants wouldn't be the same for every field. Since now all the Standard Models fields have their own and different bulk profile, the strength of the interaction with the gravitational field should change consequently the coupling. As an example, for a generic Standard Model fermion zero mode with profile $\psi(y)$, one can find that its coupling with the n-th graviton massive mode with profile $\chi_n(y)$ is given by

$$k \propto \int_0^L dy\ \psi(y)\chi_n(y), \tag{1.3.39}$$

up to some normalization. A similar result can be obtained for every coupling constant in the model, leading a theory in which the couplings with the graviton are not universal. This gives rise to some important features that are the main topic of this work.

2. The Spin-2 Effective Model

Now that we have an understanding on the origin of the effective model we want to study, we can proceed to analyze it, starting from the classical field theory for a free spin-2 particle, to the coupling with the Standard Model and finally perform the quantization and find the Feynman rules.
Of course the interest on a quantum field theory for a massive spin-2 particle is not something recent. Already in the late 1930's Markus Fierz and Wolfgang Pauli explored the possibility of having a field theory for a spin 2 particle [10], and since then the idea has been studied and applied in many different topics. The most investigated field is of course connected with gravity, in particular the Massive Gravity theory (see [11] for example), which most of all tried to address the acceleration of the universe without involving a cosmological constant since the huge discrepancy between the value needed to fit the experimental observation and the so much larger value that quantum field theory arguments suggest. And also the different attempt to obtain a quantum field theory for gravity have led to a lot of literature on the topic (one reference over all is the work from John Donoghue on gravity effective field theory [12]).
Despite all the different fields connected to this topic, the aim of this thesis work is, more humbly, to study the quantum field theory model for a spin-2 particle coupled with the Standard Model and in particular the non-unitary behaviour that this theory has under some circumstances, without pretending to give some new theory for massive gravity, but rather in the attempt to give the guideline for the construction of a simpler and consistent model that can be applied in many different applications, for example a simplified model for Dark Matter.
Therefore, in the following, we will study the quantum effective field theory for a spin-2 massive particle, thinking about it as the effective theory in the four dimensional world brane of one of the massive Kaluza-Klein mode of the graviton from the Randall-Sundrum model.

2.1 The Fierz-Pauli Action

As we already pictured in section (1.2.4), when one performs the expansion for small fluctuations from the background metric of the Randall-Sundrum action (1.2.28) and then inserts the Kaluza-Klein decomposition for the fluctuation field, one eventually finds that the action which describes the fluctuation field is the Fierz-Pauli action, given by

$$S = \int d^4x \Big\{ -\frac{1}{2}\,\partial_\rho h_{\mu\nu}\,\partial^\rho h^{\mu\nu} + \partial_\mu h_{\nu\rho}\,\partial^\nu h^{\mu\rho} - \partial_\mu h^{\mu\nu}\,\partial_\nu h + \frac{1}{2}\,\partial_\rho h\,\partial^\rho h - \frac{1}{2}\,m^2\big(h_{\mu\nu}\,h^{\mu\nu} - h^2\big)\Big\}, \tag{2.1.1}$$

where $h = h^\mu_\mu$. To start te discussion we can notice that this action contains all the possible contractions up to order of h^2 with at maximum two derivatives. The terms which do not depend on the mass are built in such a way to be gauge invariant under the usual symmetry

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{2.1.2}$$

even though the mass terms explicitly brakes gauge invariance. This is required because, allowing the kinetic term to have a more general structure, we would eventually end up propagating too many degrees of freedom than the expected, with the additional problem that the extra degrees of freedom are ghost-like[1]. Moreover we can notice that the mass term has already the right relative coefficient of -1 between the two different pieces it is composed by. It is important to remark this feature because there are no general principles that enforce this particular coefficient between them, nevertheless with every other value one would end again with some extra and ghost-like degrees of freedom propagating in the theory. This means that the theory we are describing is the full-fledged only correct description of a spin-2 particle without any other additional and unwanted degrees of freedom. Let us then count the actual number of degrees of freedom that we have. To do so we need to use the equations of motion which reads

$$G_{\mu\nu} + m^2\big(h_{\mu\nu} - h\,\eta_{\mu\nu}\big) = 0, \tag{2.1.3}$$

and here $G_{\mu\nu}$ is the usual Einstein tensor. If now we take the divergence with respect one of the two free indices of this equation we are left with

$$\partial_\mu h^{\mu\nu} - \partial^\nu h = 0, \tag{2.1.4}$$

since the divergence of the Einstein tensor vanishes by virtue of the Bianchi's identity. So, expanding the field in plane waves

$$h^{\mu\nu} = \tilde{h}^{\mu\nu} e^{ik\cdot x} + c.c. \tag{2.1.5}$$

and plug it into the equation we get

$$k_\mu \tilde{h}^{\mu\nu} - k^\nu \tilde{h}^\rho_\rho = 0. \tag{2.1.6}$$

[1] This is exactly the reason why, for a massive spin-1 particle, we keep the kinetic term as $F_{\mu\nu}F^{\mu\nu}$ even if the gauge invariance is broken by the mass term.

These are four equations and therefore four constraints on the symmetric polarization tensor $\tilde{h}^{\mu\nu}$ driving its 10 independent components to 6. Moreover, if we take another divergence of the equations of motion we get

$$\partial_\mu \partial_\nu h^{\mu\nu} - \partial_\rho \partial^\rho h = 0, \tag{2.1.7}$$

which is exactly the condition of having a traceless Einstein tensor since $G^\mu_\mu = \partial_\mu \partial_\nu h^{\mu\nu} - \partial_\rho \partial^\rho h$. It is important to notice that this happens only by virtue of the coefficient -1 we have in the mass term. By virtue of this relation, which tells us that the the Einstein tensor trace vanishes on the equations of motion, if we now take the trace of the equations we are left with

$$h = 0, \tag{2.1.8}$$

that implies, in plane waves, that the polarization tensor is traceless. So in the end we are left with two conditions on the polarization tensor which now read as

$$k_\mu \tilde{h}^{\mu\nu} = 0, \tag{2.1.9a}$$

$$\tilde{h}^\mu_\mu = 0, \tag{2.1.9b}$$

and there are now five equations which reduce the independent components of $\tilde{h}^{\mu\nu}$ to 5 that is the right number of degrees of freedom one should expect from a massive spin-2 particle. Inserting now these conditions in the equations of motion we can find that they reduce to

$$\left(\Box + m^2\right) h_{\mu\nu} = 0. \tag{2.1.10}$$

This is the usual Klein-Gordon equations which as the general solution

$$h_{\mu\nu}(x) = \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2\omega_\mathbf{k}}}\, \tilde{h}_{\mu\nu}(\mathbf{k}) e^{ik\cdot x} + \tilde{h}^*_{\mu\nu}(\mathbf{k}) e^{-ik\cdot x}, \tag{2.1.11}$$

with $\omega_\mathbf{k} = \sqrt{\mathbf{k}^2 + m^2}$ and the momenta in the equations are understood to be on shell $k^\mu = (\omega_\mathbf{k}, \mathbf{k})$. Now we expand the Fourier coefficients $\tilde{h}_{\mu\nu}(\mathbf{k})$ over some basis of tensor indexed with $\lambda = 1, 2, 3, 4, 5$

$$\tilde{h}_{\mu\nu} = \sum_\lambda a_{\mathbf{k},\lambda} \varepsilon_{\mu\nu}(\mathbf{k}, \lambda). \tag{2.1.12}$$

Imposing the conditions (2.1.9) and demanding the orthonormality of the basis

$$\varepsilon^{\mu\nu}(\mathbf{k}, \lambda) \varepsilon^*_{\mu\nu}(\mathbf{k}, \lambda') = \delta_{\lambda\lambda'}, \tag{2.1.13}$$

this basis forms a symmetric tensor traceless representation of the rotation group $SO(3)$, that is the little group for a massive particle of spin-2 and satisfy the completeness relation

$$\sum_\lambda \varepsilon^{\mu\nu}(\mathbf{k}, \lambda) \varepsilon^{*\rho\sigma}(\mathbf{k}, \lambda) = \frac{1}{2}\Big(P^{\mu\rho}P^{\nu\sigma} + P^{\mu\sigma}P^{\nu\rho}\Big) - \frac{1}{3}\, P^{\mu\nu}P^{\rho\sigma}, \tag{2.1.14}$$

where

$$P^{\mu\nu} = -\eta^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}. \tag{2.1.15}$$

Thus the general solution reads

$$h^{\mu\nu}(x) = \sum_{\mathbf{k},\lambda} a_{\mathbf{k},\lambda}\, u^{\mu\nu}_{\mathbf{k},\lambda}(x) + {a^*}_{\mathbf{k},\lambda}\, {u^*}^{\mu\nu}_{\mathbf{k},\lambda}(x), \tag{2.1.16}$$

where we defined the mode functions $u^{\mu\nu}_{\mathbf{k},\lambda}(x)$ as

$$u^{\mu\nu}_{\mathbf{k},\lambda}(x) = \frac{1}{\sqrt{(2\pi)^3 2\omega_{\mathbf{k}}}}\, \varepsilon^{\mu\nu}(\mathbf{k},\lambda)\, e^{ik\cdot x}. \tag{2.1.17}$$

The inner product on the space of solutions to the equations of motion we found is given by

$$(h,h') = \int d^3x\, h^{*\mu\nu} i \overleftrightarrow{\partial_0} h'_{\mu\nu}\Big|_{t=0}, \tag{2.1.18}$$

with respect to which the mode functions are orthonormal

$$(u_{\mathbf{k},\lambda}, u_{\mathbf{k}',\lambda'}) = \delta\big(\mathbf{k}-\mathbf{k}'\big)\delta_{\lambda\lambda'}, \tag{2.1.19a}$$

$$(u^*_{\mathbf{k},\lambda}, u^*_{\mathbf{k}',\lambda'}) = -\delta\big(\mathbf{k}-\mathbf{k}'\big)\delta_{\lambda\lambda'}, \tag{2.1.19b}$$

$$(u_{\mathbf{k},\lambda}, u^*_{\mathbf{k}',\lambda'}) = 0. \tag{2.1.19c}$$

In the quantum theory following the canonical quantization, the coefficients of the expansion ${a^*}_{\mathbf{k},\lambda}$ and $a_{\mathbf{k},\lambda}$ become creation and annihilation operators satisfying the usual commutation relations.
The last thing we have to do now is to find the propagator, inverting the kinetic term. To do so, integrating by part, we can rewrite the Fierz-Pauli action as

$$S = \int d^4x\, \frac{1}{2}\, h_{\mu\nu}\, \mathcal{O}^{\mu\nu\rho\sigma} h_{\rho\sigma}, \tag{2.1.20}$$

with

$$\mathcal{O}^{\mu\nu}_{\rho\sigma} = \Big[\eta^{(\mu}_{(\rho}\eta^{\nu)}_{\sigma)} - \eta^{\mu\nu}\eta_{\rho\sigma}\Big]\big(\Box + m^2\big) - 2\partial^{(\mu}\partial_{(\rho}\eta^{\nu)}_{\sigma)} + \partial^\mu\partial^\nu\eta_{\rho\sigma} + \partial_\rho\partial_\sigma\eta^{\mu\nu}, \tag{2.1.21}$$

where, with the bracket notation for the indicies $(\mu\nu)$, we intend the symmetric combination. The propagator is then obtained by solving in momentum space the equation

$$\mathcal{O}^{\mu\nu\alpha\beta}\mathcal{D}_{\alpha\beta,\rho\sigma} = \frac{i}{2}\left(\delta^\mu_\rho\delta^\nu_\sigma + \delta^\mu_\sigma\delta^\nu_\rho\right), \tag{2.1.22}$$

The right hand side of the equation is nothing but the identity operator on the space of symmetric tensor. The solution of this equation gives the expression of the propagator

$$\mathcal{D}^{\mu\nu\rho\sigma} = \frac{-i}{p^2 - m^2}\left[\frac{1}{2}\Big(P^{\mu\rho}P^{\nu\sigma} + P^{\mu\sigma}P^{\nu\rho}\Big) - \frac{1}{3}\, P^{\mu\nu}P^{\rho\sigma}\right]. \tag{2.1.23}$$

2.2 Effective Coupling with the Standard Model

Now that the free theory for a free massive spin-2 particle and its quantization are understood we can focus on the study of the interaction with the Standard Model. In equation (1.2.71) we already saw how the effective interaction Lagrangian looks like, so for a generic massive spin-2 particle it would take the form

$$\mathcal{L}_{int} = -\frac{k}{\Lambda}\, h_{\mu\nu}\, T^{\mu\nu}_{SM}, \tag{2.2.1}$$

for some coupling constant k, with Λ the relevant cut-off energy scale of the effective theory and $T^{\mu\nu}_{SM}$ the Standard Model energy-momentum tensor

$$T^{\mu\nu}_{SM} = 2\, \frac{\delta \mathcal{L}_{\mathcal{SM}}}{\delta g_{\mu\nu}}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}} - \eta^{\mu\nu}\, \mathcal{L}_{SM}. \tag{2.2.2}$$

Before proceeding with the calculations for the actual Standard Model it is useful and instructive to take a little while to compute a Feynman rule by hand for a toy model. In particular let us consider a real massive scalar field ϕ with the usual Lagrangian given by

$$\mathcal{L}_\phi = \frac{1}{2}\, \partial_\mu\phi\, \partial^\mu\phi - \frac{1}{2}\, {m_\phi}^2\phi^2. \tag{2.2.3}$$

From this expression we can easily find the energy-momentum tensor which reads

$$T^{\mu\nu}_\phi = \partial^\mu\phi\, \partial^\nu\phi - \frac{1}{2}\, \eta^{\mu\nu}\big(\partial_\rho\phi\, \partial^\rho\phi - {m_\phi}^2\phi^2\big). \tag{2.2.4}$$

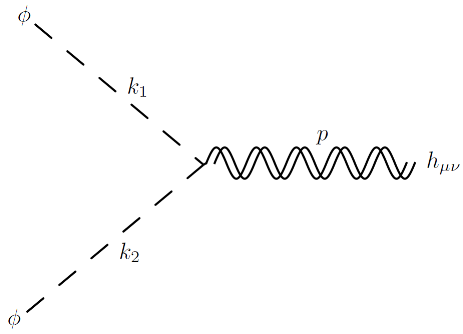
We know that the interaction term is obtained by the contraction between this tensor and the graviton field and we now should remember that the spin-2 tensor is actually traceless, which means that the only effective interaction contribution is given by the Lagrangian term

$$\mathcal{L}_{int} = -\frac{k}{\Lambda}\, h_{\mu\nu}\, \partial^\mu\phi\, \partial^\nu\phi, \tag{2.2.5}$$

that can be written also, for sake of clarity when extracting the Feynman rule, as

$$\mathcal{L}_{int} = -\frac{k}{2\Lambda}\, h_{\mu\nu}\, \big(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\big)\partial_\rho\phi\, \partial_\sigma\phi, \tag{2.2.6}$$

by virtue of the total symmetric structure of the contraction. The Feynman rule then follows immediately as reads



$$= -\frac{ik}{2\Lambda}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\right)k_1^\rho k_2^\sigma, \tag{2.2.7}$$

where all the momenta are meant to be incoming.
This is, in principle, what one should do for the whole Standard Model. However the calculation becomes extremely tedious and time consuming, insomuch that to resort on some software turns out to be the only viable option, most of all regarding to the Feynman rules evaluation. The first thing to do anyway is to find the expression for the energy-momentum tensor of every Standard Model field. If one actually does all the computations needed should eventually find the following results [13]:

$$\begin{aligned}
T_{\mu\nu}^{Higgs} &= \partial_\mu H \partial_\nu H + g_Z m_Z Z_\mu Z_\nu H + \frac{{g_Z}^2}{4}\, Z_\mu Z_\nu H^2 \\
&+ \left[g_W m_W W_\mu^+ W_\nu^- H + \frac{{g_w}^2}{4}\, W_\mu^+ W_\nu^- H^2 + (\mu \longleftrightarrow \nu) \right] \\
&- \eta_{\mu\nu}\left[\frac{1}{2}\, \partial_\rho H \partial^\rho H - \frac{1}{2}\, {m_H}^2 H^2 - \frac{g_W {m_H}^2}{4 m_W}\, H^3 - \frac{g_W {m_H}^2}{32 {m_W}^2} H^4 \right. \\
&- \sum_f \frac{g_W m_f}{2 m_W}\, \bar{\psi}_f \psi_f H + \frac{1}{2}\, g_Z m_Z Z_\rho Z^\rho H + \frac{1}{8}\, {g_Z}^2 Z_\rho Z^\rho H^2 \\
&\left. + g_W m_W W_\rho^+ W^{-\rho} H + \frac{1}{4}\, {g_W}^2 W_\rho^+ W^{-\rho} H^2 \right];
\end{aligned} \tag{2.2.8a}$$

$$T_{\mu\nu}^{Z-Boson} = -Z_\mu^\rho Z_{\nu\rho} + {m_Z}^2 Z^2 - \eta_{\mu\nu}\left[-\frac{1}{4}\, Z_{\rho\sigma} Z^{\rho\sigma} + \frac{1}{2}\, {m_Z}^2 Z^2 \right]; \tag{2.2.8b}$$

$$
\begin{aligned}
T_{\mu\nu}^{W-Bosons} &= -\Big[W^{+\rho}{}_{\mu}W^{-}_{\nu\rho} - {m_W}^2 W^{+}{}_{\mu}W^{-}_{\nu} + (\mu \longleftrightarrow \nu)\Big] \\
&- \eta_{\mu\nu}\Big[-\frac{1}{2}\,W^{+}{}_{\rho\sigma}W^{-\rho\sigma} + {m_W}^2 W^{+\rho}W^{-}_{\rho} + (\mu \longleftrightarrow \nu)\Big];
\end{aligned} \tag{2.2.8c}
$$

$$
\begin{aligned}
T_{\mu\nu}^{Photon} &= -F^{\rho}_{\mu}F_{\nu\rho} + \partial_\mu\partial^\rho A_\rho A_\nu + \partial_\nu\partial^\rho A_\rho A_\mu - \eta_{\mu\nu}\Big[-\frac{1}{4}\,F_{\rho\sigma}F^{\rho\sigma} \\
&+ \partial^\rho\partial^\sigma A_\rho A_\sigma - \frac{1}{2}\,\Big(\partial^\rho A_\rho\Big)^2\Big];
\end{aligned} \tag{2.2.8d}
$$

$$
\begin{aligned}
T_{\mu\nu}^{Gluons} &= -G^{a,\rho}_{\mu}G^{a}_{\nu\rho} + \partial_\mu\partial^\rho G^a_\rho G^a_\nu + \partial_\nu\partial^\rho G^a_\rho G^a_\mu - \eta_{\mu\nu}\Big[-\frac{1}{4}\,G^a_{\rho\sigma}G^{a,\rho\sigma} \\
&+ \partial^\rho\partial^\sigma G^a_\rho G^a_\sigma - \frac{1}{2}\,\Big(\partial^\rho G^a_\rho\Big)^2\Big];
\end{aligned} \tag{2.2.8e}
$$

$$
\begin{aligned}
T_{\mu\nu}^{Leptons} &= \Big[\sum_{L=l,\nu}\frac{1}{2}\,\bar{\psi}_L i\gamma_\mu D_\nu\psi_L - \frac{1}{4}\,\partial_\mu\big(\bar{\psi}_L i\gamma_\nu\psi_L\big) + (\mu \longleftrightarrow \nu)\Big] \\
&+ \Big[-\frac{g_W}{2\sqrt{2}}\,U_{ij}\bar{\psi}_{l_i}\gamma_\mu\big(1-\gamma_5\big)\psi_{\nu_j}W^{-}_{\nu} + h.c. + (\mu \longleftrightarrow \nu)\Big] \\
&- \eta_{\mu\nu}\Big[\sum_{L=l,\nu}\bar{\psi}_L\big(i\gamma^\rho D_\rho - m_L\big)\psi_L - \frac{1}{2}\,\partial^\rho\big(\bar{\psi}_L i\gamma_\rho\psi_L\big) \\
&- \Big(\frac{g_W}{2\sqrt{2}}U_{ij}\bar{\psi}_{l_i}\gamma^\rho\big(1-\gamma_5\big)\psi_{\nu_j}W^{-}_{\rho} + h.c.\Big)\Big];
\end{aligned} \tag{2.2.8f}
$$

$$
\begin{aligned}
T_{\mu\nu}^{Quarks} &= \Big[\sum_{Q=u,d}\frac{1}{2}\,\bar{\psi}_Q i\gamma_\mu D_\nu\psi_L + Q - \frac{1}{4}\,\partial_\mu\big(\bar{\psi}_Q i\gamma_\nu\psi_Q\big) + (\mu \longleftrightarrow \nu)\Big] \\
&+ \Big[-\frac{g_W}{2\sqrt{2}}\,V_{ij}\bar{\psi}_{u_i}\gamma_\mu\big(1-\gamma_5\big)\psi_{d_j}W^{-}_{\nu} + h.c. + (\mu \longleftrightarrow \nu)\Big] \\
&- \eta_{\mu\nu}\Big[\sum_{Q=u,d}\bar{\psi}_Q\big(i\gamma^\rho D_\rho - m_Q\big)\psi_Q - \frac{1}{2}\,\partial^\rho\big(\bar{\psi}_Q i\gamma_\rho\psi_Q\big) \\
&- \Big(\frac{g_W}{2\sqrt{2}}V_{ij}\bar{\psi}_{u_i}\gamma^\rho\big(1-\gamma_5\big)\psi_{d_j}W^{-}_{\rho} + h.c.\Big)\Big].
\end{aligned} \tag{2.2.8g}
$$

Here the covariant derivative is understood to be

$$
D_\mu = \partial_\mu + ig_s T^a G^a_\mu + ig_W \sin\theta_W Q_f A_\mu + ig_Z\Big[\frac{t^3}{2}\,\big(1-\gamma_5\big) - Q_f\sin^2\theta_W\Big]Z_\mu, \tag{2.2.9}
$$

notice that there are no terms involving the W-bosons in the derivative because they are explicitly written in the fermionic energy-momentum tensors. Moreover the field-strength tensors for the various gauge bosons are

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g_W \sin\theta_W \Big[W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^- \Big], \tag{2.2.10a}$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c, \tag{2.2.10b}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu + i g_W \cos\theta_W \Big[W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^- \Big], \tag{2.2.10c}$$

$$W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \mp i g_W \Big[\sin\theta_W W_\mu^\pm A_\nu + \cos\theta_W W_\mu^\pm Z_\nu + (\mu \longleftrightarrow \nu) \Big]. \tag{2.2.10d}$$

These expressions are obviously far too long and complex to be handled with pen and paper. This is why we employ the Mathematica [14] package FeynRules [15, 16]. It is a very useful tool which allows to implement a model and derives automatically the Feynman rules and also the analytical expressions for the decay width of the various particles of the model. Moreover a massive spin-2 model has already been realized and its phenomenology has been studied extensively [17] and is currently available. To list and give the analytical formulas of all the possible Feynman rule is impracticable due to their enormous quantity and complexity. In the following discussions we will have the opportunity to write down and inspect directly the structure of few of them, however in figure (2.1) the different kinds of vertices are summarized.This gives only an idea of how rich and interesting the phenomenology of this model is.
FeynRules itself can not clearly be enough to carry on the entire study of the model. It is not indeed the only software indispensable for this work. We also need a tool which allow us, starting from the model we implemented in FeynRules, to make predictions about the possible production of such particle and signatures for example at the Large Hadron Collider (LHC). All this can be achieved in MadGraph [18] which is a framework providing the possibility to perform numerical computations of matrix elements and cross-sections, event generation and simulation as well as a variety of different instruments needed for the analysis of the results. With such a tool one can for example study the different production channel for the spin-2 particle, computing the total cross-section for every one of them at LHC, that means with a proton-proton scattering process, and also the best signature for a possible detection studying the different decay channels (we refer again to [17] for all the detail of these investigations).
To our concerns we will see all this software equipment in action in the following chapter, in which we will study in details the unitary violating behavior the theory originating from allowing the Standard Model in the bulk has.

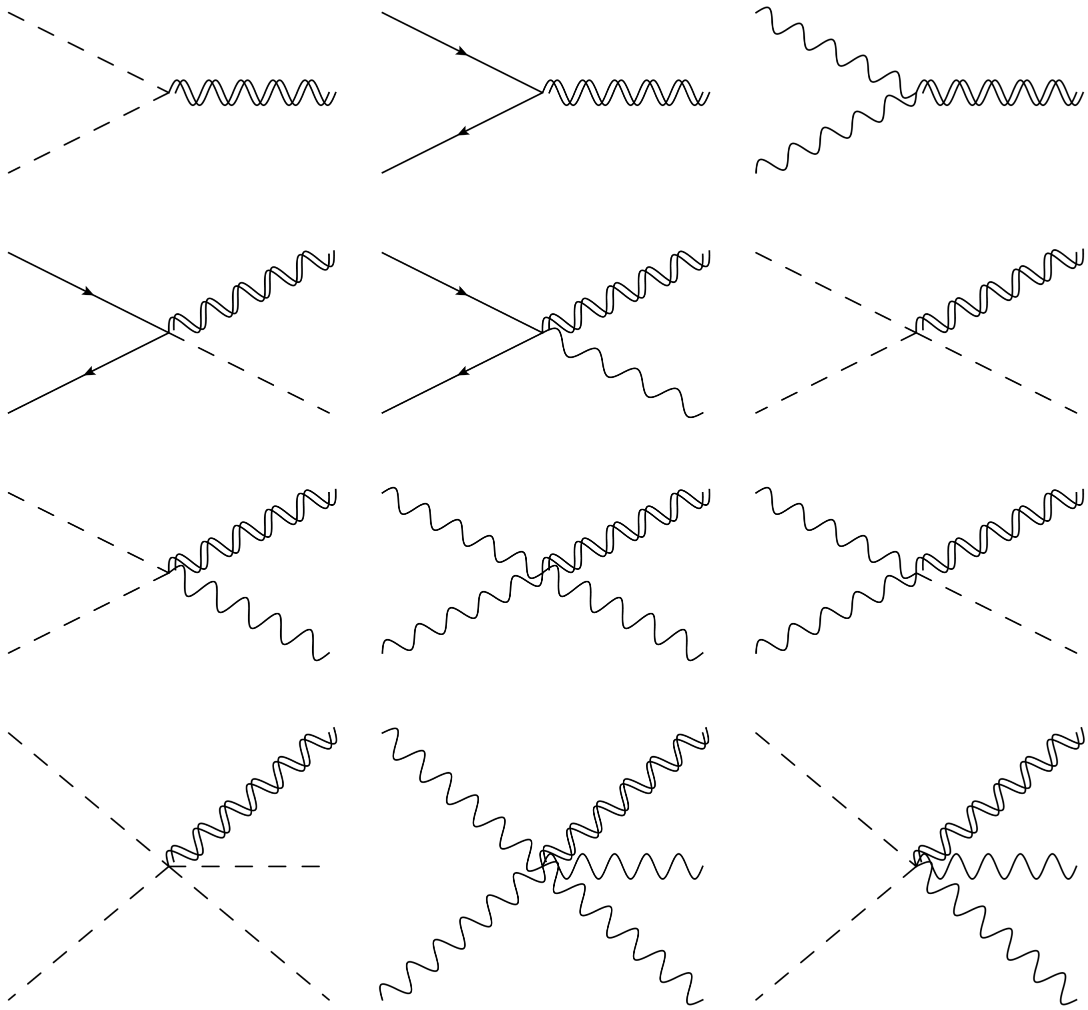


Figure 2.1: We present a schematic overview of all the different kinds of vertices in the theory involving the spin-2 particle. Here the dashed line stands for the Higgs boson, the solid line represents a generic fermionic particle and the curly line depicts any spin-1 boson, so either a photon, a gluon or a weak boson.

2.3 The Stückelberg Trick

In the previous sections we solved and described the theory for a massive spin-2 particle coupled through the energy-momentum tensor to the Standard Model. Now for further discussions we introduce a useful formalism, the Stückelberg trick, which makes the high energy limit particularly clear. By high energy limit here we mean the limit in which momenta are much larger than the mass of the particle $p \gg m$. For the massive spin-2, its five degrees of freedom are formally in the same representation little group and this means that each one of them can be cast in one other by suitable Lorentz transformations. Approaching the high energy regime however this reshuffle of the polarizations becomes increasingly tricky to achieve. This makes perfect sense since we know that in the massless limit (or equivalently the speed of light limit) the helicity is a Lorentz invariant quantity preventing them from changing under frame transformations. This is a clear suggestion that when $p \gg m$ is convenient to decompose the five massive polarizations into the little group for a massless particle representations. So the claim we are making is that in the high energy limit the five degrees of freedom can be thought as five helicity states of the particle, meaning that we can have $h = \pm 2, \pm 1, 0$, five different projections of the angular momentum of the particle onto its direction of motion.
The implementation of this trick at the Lagrangian level is quite simple. Even if we are starting with a massive theory without gauge invariance, because the mass term explicitly breaks it, when we want to approach the high energy limit we have somehow to restore it since this limit can be thought also as a massless limit. In this formulation so we have to end up with a tensor field $h_{\mu\nu}$ describing only the helicity $h = \pm 2$ which has to satisfy the usual and standard gauge invariance relation

$$\delta h_{\mu\nu} = \partial_\mu \, \xi_\nu + \partial_\nu \, \xi_\mu. \tag{2.3.1}$$

Moreover we should have a vector field V_μ, taking care of helicity $h = \pm 1$, along with the gauge invariance with respect the transformation

$$\delta V_\mu = \partial_\mu \Omega, \tag{2.3.2}$$

and for describing the longitudinal helicity $h = 0$ we need a scalar field ϕ. Now the trick is to replace in the Lagrangian, whenever we see an $h_{\mu\nu}$ with a new filed $H_{\mu\nu}$ which takes the form

$$H_{\mu\nu} \equiv h_{\mu\nu} + \partial_\mu V_\nu + \partial_\nu V_\mu + 2\partial_\mu \partial_\nu \phi. \tag{2.3.3}$$

This new field is now invariant with respect to two transformations

$$\begin{cases} h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\,\xi_\nu + \partial_\nu\,\xi_\mu, \\ V_\mu \to V_\mu - \xi_\mu, \end{cases} \tag{2.3.4a}$$

$$\begin{cases} V_\mu \to V_\mu + \partial_\mu \Omega, \\ \phi \to \phi - \Omega, \end{cases} \tag{2.3.4b}$$

where $\xi_\mu(x)$ and $\Omega(x)$ are generic.
Up to now what we have done is have introduced more fields supplemented with gauge invariances so that we can always choose a gauge in which the two extra field vanish, or in other words we can chose $\xi_\mu(x)$ and $\Omega(x)$ accordingly to set them to zero, since V_μ and ϕ are simply shifted by them. In this gauge, which we call *Unitary* gauge, it is trivial to see that we are left with $H_{\mu\nu} = h_{\mu\nu}$, our starting spin-2 field. This means that we have done nothing physically speaking and our rewriting in terms of these new fields is completely equivalent to the starting formulation, we only add a redundancy in our description of the model that can be eliminated with a suitable choice of the transformation parameters. However this parametrization of the theory is extremely useful, and to realize that, let us explicitly perform the trick. The first thing we have to do then is to replace $H_{\mu\nu}$ instead of $h_{\mu\nu}$ in the action. In doing so we have to realize that the kinetic part of the action is gauge invariant and the substitution we are going to make is nothing but a gauge transformation, namely

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\big(V_\nu + \partial_\nu\phi\big) + \partial_\nu\big(V_\mu + \partial_\mu\phi\big), \tag{2.3.5}$$

so when performing the trick we are not going to get additional terms from the kinetic term or, rephrasing the statement, $S_{kin}[H_{\mu\nu}] = S_{kin}[h_{\mu\nu}]$. The only two terms we have to take care are then the mass term mostly and the interaction one which is obviously simpler. For sake of simplicity let us substitute the part involving V_μ, so that we obtain

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{kin} - \frac{1}{2}\,m^2\big(h_{\mu\nu}\,h^{\mu\nu} - h^2\big) - \frac{1}{2}\,m^2 F_{\mu\nu}F^{\mu\nu} - 2m^2\big(\partial_\mu V_\nu\,h^{\mu\nu} - \partial_\mu V^\mu h\big) \\ + \frac{k}{\Lambda}\,h_{\mu\nu}T^{\mu\nu} + \frac{2k}{\Lambda}\,\partial_\mu V_\nu\,T^{\mu\nu}, \end{aligned} \tag{2.3.6}$$

where $F_{\mu\nu}$ is the usual field-strength tensor for V_μ. Now we can clearly see the need for introducing the scalar field also. After having rescaled the vector field $V_\mu \to m^{-1}V_\mu$ to normalize its kinetic term, if we go to the high-energy limit (formally the $m \to 0$ limit) we are left with a theory for a massless graviton and a massless vector filed which in total has 4 degrees of freedom instead of the required five. Introducing then the scalar field also, the Lagrangian takes the form

$$\begin{aligned}\mathcal{L} =&\mathcal{L}_{kin} - \frac{1}{2}\, m^2\big(h_{\mu\nu}\, h^{\mu\nu} - h^2\big) - \frac{1}{2}\, F_{\mu\nu}F^{\mu\nu} - 2m\big(\partial_\mu V_\nu\, h^{\mu\nu} - \partial_\mu V^\mu h\big) \\ &- 2\big(\partial_\mu\partial_\nu\phi\, h^{\mu\nu} - \Box\phi\, h\big) + \frac{k}{\Lambda}\, h_{\mu\nu}T^{\mu\nu} + \frac{2k}{m\Lambda}\, \partial_\mu V_\nu\, T^{\mu\nu} + \frac{2k}{m^2\Lambda}\, \partial_\mu\partial_\nu\phi\, T^{\mu\nu},\end{aligned} \tag{2.3.7}$$

where we have also rescaled the vector and the scalar field respectively as $V_\mu \to m^{-1}V_\mu$ and $\phi \to m^{-2}\phi$.
A quick but important remark has to be done here. One could in principle argue that the whole substitution of equation (2.3.3) would have led to more terms, for example one could have expected to see a term like $\big(\partial_\mu\partial_\nu\phi\big)^2$ coming from the first of the two parts of the mass term. The reason why we do not see them in the final result is due to the Fierz-Pauli tuning we have in the mass term, which allow, after an integration by part, to exactly cancel $\big(\partial_\mu\partial_\nu\phi\big)^2$ with the term $\Box\Box\phi$ coming from the second piece of the mass term. It is extremely important that this happens, otherwise we would have had a four-derivatives kinetic terms, which implies the double degrees of freedom, unavoidably leading to a ghost-like scalar field. On the other hand, now one could be worried about the apparent lack of kinetic term for the scalar. We do not see it because now we have a Lagrangian with a mixed quadratic term involving both the graviton and the scalar field which, at this stage, are not eigen-fields any more. The first step to diagonalize the Lagrangian is to consider a reshuffle between ϕ and $h_{\mu\nu}$, performing the following substitution

$$h_{\mu\nu} \to h_{\mu\nu} - \frac{2}{3}\, \eta_{\mu\nu}\phi. \tag{2.3.8}$$

In doing so we have to be careful because this is not a gauge transformation and then the kinetic term is not invariant. Under this change it actually becomes

$$\mathcal{L}_{kin} = \mathcal{L}_0 - \frac{4}{3}\partial_\mu\phi\partial^\mu h + \frac{4}{3}\partial_\mu\phi\partial_\nu h^{\mu\nu} + \partial_\mu\phi\partial^\mu\phi. \tag{2.3.9}$$

In this expression $\mathcal{L}_0$ stands for the kinetic term for the graviton (the new $h_{\mu\nu} field$) only and we now have the explicit form for the kinetic term of the scalar field together with some interactions. To do the same substitution in the rest of the Lagrangian is only a tedious but straightforward algebraic exercise and, with a bit of endurance one can go through this calculation and find

$$\begin{aligned}\mathcal{L} =&\mathcal{L}_0 - \frac{1}{2}\, m^2\big(h_{\mu\nu}\, h^{\mu\nu} - h^2\big) - \frac{1}{2}\, F_{\mu\nu}F^{\mu\nu} + 3\phi\big(\Box + 2m^2\big)\phi \\ &- 2m\big(\partial_\mu V_\nu\, h^{\mu\nu} - \partial_\mu V^\mu h\big) + 3m^2\, h\phi + 6m\, \partial_\mu V^\mu \phi \\ &+ \frac{k}{\Lambda}\, h_{\mu\nu}T^{\mu\nu} + \frac{2k}{m\Lambda}\, \partial_\mu V_\nu\, T^{\mu\nu} + \frac{2k}{m^2\Lambda}\, \partial_\mu\partial_\nu\phi\, T^{\mu\nu} - \frac{k}{\Lambda}\, \phi\, \eta_{\mu\nu}T^{\mu\nu}.\end{aligned} \tag{2.3.10}$$

We have to be aware that now the gauge symmetry for the graviton (2.3.4a) has been changed by this replacement we have done and now it reads

$$\begin{cases} h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \, \xi_\nu + \partial_\nu \, \xi_\mu + m \, \Omega \, \eta_{\mu\nu}, \\ V_\mu \to V_\mu - m\xi_\mu, \end{cases} \tag{2.3.11}$$

while the other one stays untouched.
To eventually obtain the diagonalized form we have to fix the gauge with a suitable choice which let us to get rid of the mixed terms. By imposing the conditions

$$\begin{cases} \partial^\nu h_{\mu\nu} - \frac{1}{2} \, \partial_\mu h + mV_\mu = 0, \\ \partial_\mu V^\mu + \frac{1}{2} \, mh + 3m\phi = 0, \end{cases} \tag{2.3.12}$$

or alternatively, introducing a gauge-fixing Lagrangian

$$\mathcal{L}_{GF} = -\left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \, \partial_\mu h + mV_\mu\right)^2 - \left(\partial_\mu V^\mu + \frac{1}{2} \, mh + 3m\phi\right)^2, \tag{2.3.13}$$

after an integration by part and a rescaling of the scalar field $\phi \to \frac{1}{\sqrt{6}} \, \phi$ and the vector field $V_\mu \to \frac{1}{\sqrt{2}} \, V_\mu$ to recover the right normalization of their kinetic term, we finally get the result

$$\begin{aligned} \mathcal{L} =& \frac{1}{2} \, h_{\mu\nu}\big(\Box - m^2\big)h^{\mu\nu} - \frac{1}{4} \, h\big(\Box - m^2\big)h + \frac{1}{2} \, V_\mu\big(\Box - m^2\big)V^\mu + \frac{1}{2} \, \phi\big(\Box - m^2\big)\phi \\ &+ \frac{k}{\Lambda}\left[h_{\mu\nu} + \frac{1}{\sqrt{2}m} \, \Big(\partial_\mu V_\nu + \partial_\nu V_\mu\Big) + \frac{\sqrt{2}}{\sqrt{3}m^2} \, \partial_\mu\partial_\nu\phi - \frac{1}{\sqrt{6}} \, \phi \, \eta_{\mu\nu}\right]T^{\mu\nu}. \end{aligned} \tag{2.3.14}$$

The propagators for these fields are now

$$\mathcal{D}^{Grav}_{\mu\nu\rho\sigma} = \frac{-i}{p^2 - m^2}\left[\frac{1}{2} \, \eta_{\mu\rho}\eta_{\nu\sigma} + \frac{1}{2} \, \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{1}{2} \, \eta_{\mu\nu}\eta_{\rho\sigma}\right], \tag{2.3.15a}$$

$$\mathcal{D}^{Vect}_{\mu\nu} = \frac{-i\eta_{\mu\nu}}{p^2 - m^2}, \tag{2.3.15b}$$

$$\mathcal{D}^{Scal} = \frac{-i}{p^2 - m^2}. \tag{2.3.15c}$$

3. Non-Universal Couplings Model

We are now ready to study the non-universal extension of the massive spin-2 model. The only difference in the action is that, as we already argued, the spin-2 field now couples to different Standard Model fields with a specific coupling constant, determined by the various bulk profile each distinct Standard Model field has. Formally the only modification we need to do is replace in the Lagrangian the interaction term of equation (2.2.1) with

$$\mathcal{L}_{int} = -\frac{1}{\Lambda}\, h_{\mu\nu} \sum_i k_i\, T_i^{\mu\nu}, \tag{3.0.1}$$

where the various $T_i^{\mu\nu}$ are the energy-momentum tensor of every Standard Model particle and of course $\sum_i T_i^{\mu\nu} = T_{SM}^{\mu\nu}$. This apparently harmless modification affects the model in a very strong way conversely. The problem is that the spin-2 particle is not coupled to a conserved source any more

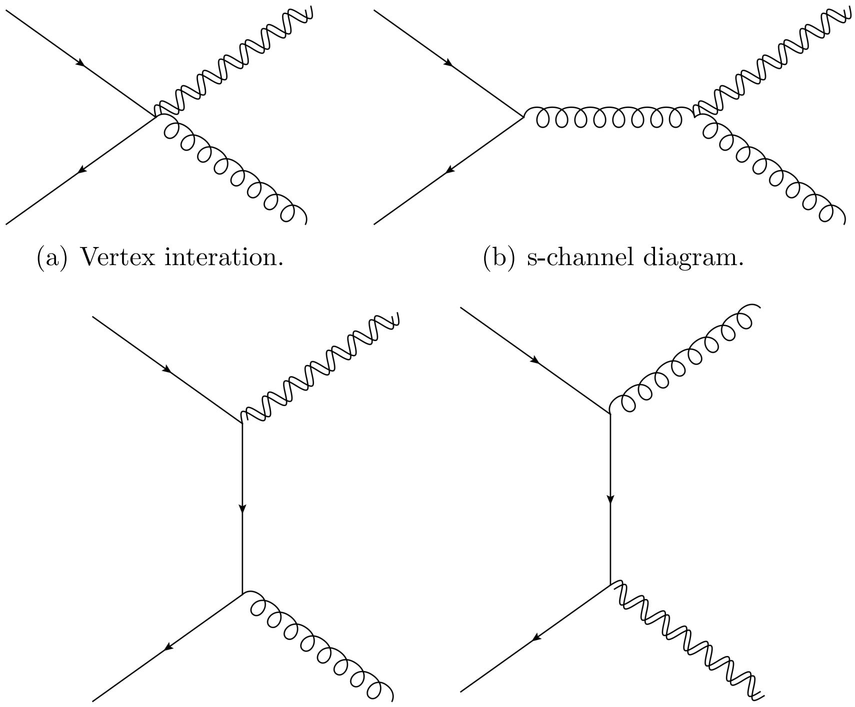
$$\partial_\mu J^{\mu\nu} = \partial_\mu \sum_i k_i T_i^{\mu\nu} \neq 0. \tag{3.0.2}$$

The conservation of the Standard Model energy-momentum tensor was preventing the theory to exhibit an anomalous and increasing energy dependence of some scattering amplitudes, and so a faster unitary loss than the expected from a dimension five effective field theory. In the universal case everything was fine because, every time the spin-2 propagator of equation (2.1.23) was hitting the source, the terms with a momentum dependence coming form the structure $m^{-2}\, p^\mu p^\nu$ were vanishing due to the conservation equation, which in Fourier's transform reads exactly $p^\mu T_{\mu\nu} = 0$. In the non-universal case these terms are no longer cancelled and they lead to a strong energy growth and a resultant non-unitary behaviour as we will explicitly show now.

3.1 Unitarity Violation

Before getting started it is useful to carry out a little change of notation. From now on we shall refer to the spin-2 field as $X_{\mu\nu}$ instead of $h_{\mu\nu}$ in order to avoid confusion between the graviton we will label as X and the Higgs boson usually denoted with h.

Let us now consider a production process for the graviton and compute the explicit expression for the amplitude squared and check its energy dependence. In particular we can focus on the study of the quark and anti-quark annihilation into a graviton and



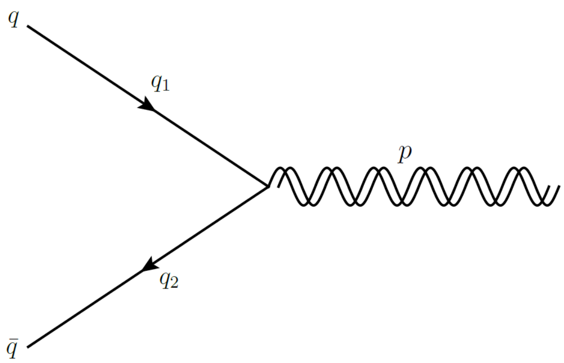
(a) Vertex interation. (b) s-channel diagram.

(c) t-channel diagram. (d) u-channel diagram.

Figure 3.1: The four diagrams involved in the $q\bar{q} \to gX$ process.

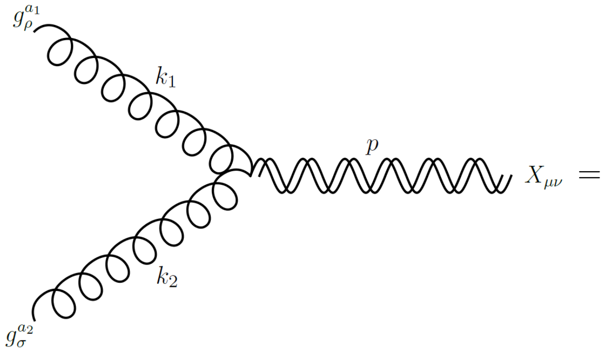
a gluon in the approximation of massless quarks. There are four different diagrams which contribute to the process we want to study and they are visualized in Figure (3.1). For sake of completeness and also to give an idea of the structure originating from the coupling through the Standard Model energy-momentum tensor we list now all the Feynamn rules needed to write down these diagrams and thus the total amplitude of the process.

The first rule we need is the three point vertex between the quark and anti-quark couple with the graviton. Assuming all the momenta to by incoming the rule reads



$$X_{\mu\nu} = \frac{ik_q}{4\Lambda}\Big[\big(q_2-q_1\big)^{\mu}\gamma^{\nu}+\big(q_2-q_1\big)^{\nu}\gamma^{\mu}\Big]. \quad (3.1.1)$$

The other rule involving the graviton is the three point vertex with two gluons which has the form



$$
\begin{aligned}
= \; & -\frac{ik_g}{\Lambda}\,\delta^{a_1a_2}\Big[\eta^{\rho\sigma}\Big(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu\Big) \\
& -\eta^{\mu\sigma}\Big(k_1^\nu k_2^\rho + k_1^\nu k_1^\rho\Big) - \eta^{\nu\rho}\Big(k_1^\sigma k_2^\mu + k_2^\sigma k_2^\mu\Big) \\
& -\eta^{\mu\rho}\Big(k_1^\sigma k_2^\nu + k_2^\sigma k_2^\nu\Big) - \eta^{\nu\sigma}\Big(k_1^\mu k_2^\rho + k_1^\mu k_1^\rho\Big) \\
& + k_1\cdot k_2\Big(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\Big)\Big].
\end{aligned}
\tag{3.1.2}
$$

In both the expressions we already got rid of the terms proportional to $\eta^{\mu\nu}$ since they vanish when the contraction with the graviton filed occurs.
We are able now to write down the four amplitudes corresponding to the four different diagrams of figure (3.1). We denote with q_1 and q_2 the momenta of the quark and the anti-quark respectively, with p_1 the spin-2 particle momentum and with p_2 the gluon one. The first amplitude, corresponding to the vertex interaction, comes again from the Feynman rules and it reads

$$
\mathcal{M}_v = -\frac{g_s k_q}{2\Lambda}\,T^a X_{\mu\nu}(p_1)\,\varepsilon_\rho^a(p_2)\,\bar{v}(q_2)\Big(\gamma^\mu\eta^{\nu\rho} + \gamma^\nu\eta^{\mu\rho}\Big)u(q_1),
\tag{3.1.3}
$$

where g_s is the strong interaction coupling constant, T^a the $SU(3)$ generator and $\varepsilon_\rho^a(p_2)$ the polarization vector of the gluon. The second amplitude is the one related to the s-channel diagram and, after few simplifications involving the gluon polarization vector transverse condition, take the form

$$
\begin{aligned}
\mathcal{M}_s = & -\frac{g_s k_g}{\Lambda(q_1+q_2)^2}\,T^a X_{\mu\nu}(p_1)\,\varepsilon_\rho^a(p_2)\,\Big[\eta^{\rho\sigma}\Big(p_2^\mu(q_1+q_2)^\nu + p_2^\nu(q_1+q_2)^\mu\Big) \\
& -\eta^{\mu\rho}\Big(p_2^\nu p_2^\sigma + p_2^\nu(q_1+q_2)^\sigma\Big) - \eta^{\nu\rho}\Big(p_2^\mu p_2^\sigma + p_2^\mu(q_1+q_2)^\sigma\Big) \\
& -\eta^{\mu\sigma}(q_1+q_2)^\nu(q_1+q_2)^\rho - \eta^{\nu\sigma}(q_1+q_2)^\mu(q_1+q_2)^\rho \\
& + p_2\cdot(q_1+q_2)\Big(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\Big)\Big]\,\bar{v}(q_2)\,\gamma^\sigma\,u(q_1).
\end{aligned}
\tag{3.1.4}
$$

The other two diagrams we need to write the amplitude are the t-channel and the u-channel, which are quite similar in the structure. The first one is given by

$$
\begin{aligned}
\mathcal{M}_t = -\frac{g_s k_q}{4\Lambda(q_1-p_1)^2}\,T^a X_{\mu\nu}(p_1)\,\varepsilon_\rho^a(p_2)\,\bar{v}(q_2)\gamma^\rho\big(\not{p}_1 - \not{q}_1\big)\Big[\gamma^\mu(p_2-q_2+q_1)^\nu \\
+ \gamma^\nu(p_2-q_2+q_1)^\mu\Big]u(q_1),
\end{aligned}
\tag{3.1.5}
$$

while the second is

$$\mathcal{M}_u = -\frac{g_s k_q}{4\Lambda(q_1-p_2)^2}\, T^a X_{\mu\nu}(p_1)\, \varepsilon^a_\rho(p_2)\, \bar{v}(q_2)\Big[\gamma^\mu(q_1-p_2-q_2)^\nu + \gamma^\nu(q_1-p_2-q_2)^\mu\Big]\big(\not{p}_2-\not{q}_1\big)\gamma^\rho u(q_1). \tag{3.1.6}$$

The total amplitude of the process is obviously the sum of these four contributions and we can notice that the colour component, identified by the generator T^a, is common to every amplitude and then it can be factorised out. This means that, even in there is a gluon in the final state, the amplitude is essentially QED-like or, in other words, when we perform the sum over its polarizations after the squaring, we can simply use the relation

$$\sum_\lambda \varepsilon^a_\rho(p_2,\lambda)\varepsilon^{*b}_{\ \sigma}(p_2,\lambda) = \eta_{\rho\sigma}\,\delta^{ab}. \tag{3.1.7}$$

The colour factor therefore comes out to be

$$C = Tr\big[T^aT^b\big]\delta^{ab} = \frac{1}{2}\,\delta^{ab}\delta^{ab} = 4. \tag{3.1.8}$$

To perform the computation by hand is almost impossible given the length and the difficulty of the expressions involved. However, as said before, we can rely on the package FeynCalc and perform the analytical calculation on Mathematica and in the Appendix (B.1) one can find the whole notebook used for this purpose. Here we provide only the final expression of the calculation, which is already a well known result [19, 20]

$$\begin{aligned}|\mathcal{M}|^2 = \frac{g_s^2}{27\Lambda^2M^4stu}\Big\{&3k_g^2M^4\Big[2M^4-2M^2(t+u)+t^2+u^2\Big]\Big[M^4-2M^2(t+u)+4tu\Big]\\ &+6k_gM^4s(k_q-k_g)\Big[M^6+M^2s(s+2u)-2su(s+u)\Big]\\ &+s(k_q-k_g)^2\Big[6M^{10}-6M^8(t+u)+3M^6(t^2+u^2)-12M^4tu(t+u)\\ &+2M^2tu(t^2+12tu+u^2)-2tu(t^3+t^2u+tu^2+u^3)\Big]\Big\},\end{aligned} \tag{3.1.9}$$

where M is the spin-2 particle mass. Unlike the results we can find in the Appendix, we have reorganized the terms in such a way to make as evident as possible the effect of the non-universal couplings scheme. We can clearly see that if we impose again the universality of the couplings, i.e. $k_q = k_g$, only the first line remains in the expression above, while the rest, being proportional to the difference between the two couplings, disappear. It is relevant to specify that this result has been also checked and confirmed

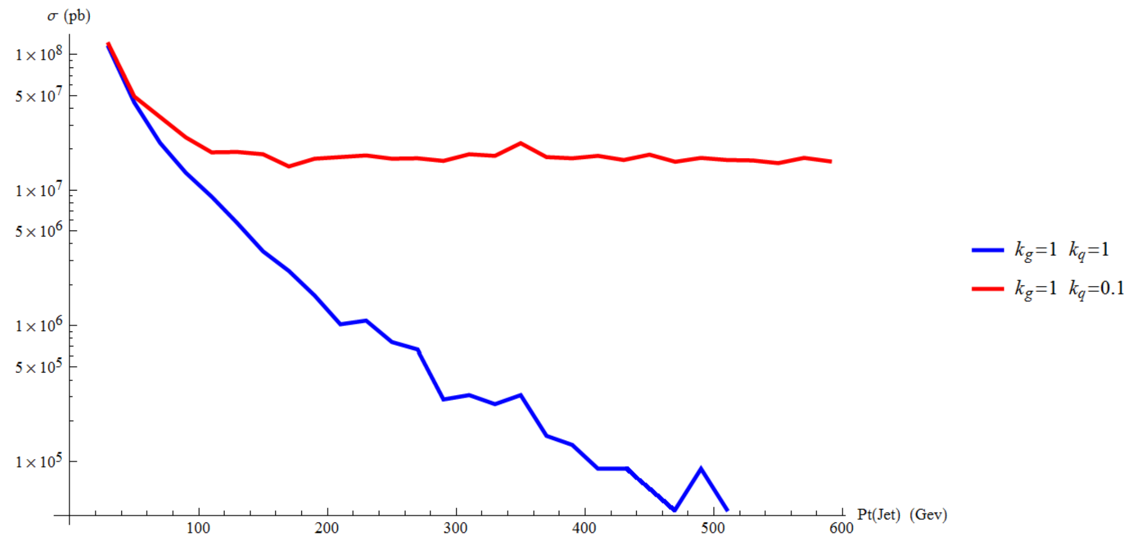


Figure 3.2: The transverse momentum distribution of the jet in the process $pp \to jX$ with energy $s = 13$ TeV. The mass of the spin-2 particle has been fixed to $M = 100$ GeV and the cut-off scale $\Lambda = 1$ TeV.

with the numerical value obtained with MadGraph for various phase space points.
From the optic theorem we know that, in order to be unitary, the matrix element squared as to be at most constant as function of the energy. In our case of interest however, since it is an effective theory with a dimension five operator, we should expect to have an amplitude squared with a linear dependence on the energy. This set a natural limit to the validity range of the theory and,consequently, to the value of the cut-off scale Λ. Now if we look at the expression for the amplitude squared in the limit of equal coupling we can easily see that it grows as fast as $\Lambda^{-2}s$, and it is the expected behaviour indeed. In the non-universal case however there are terms, on the contrary, which have a quite stronger energy dependence and they grow as fast as $\Lambda^{-2}M^{-4}s^3$. These terms, together with the milder but still badly behaved terms that goes as $\Lambda^{-2}M^{-2}s^2$ are the responsible of a much faster unitarity loss and then a significant lowering in the cut-off scale and are the one we want to focus our study on.
We can visualize this feature of the model with the help of MadGraph by simulating a proton-proton collision at LHC for the production of a spin-2 particle plus one jet, and look at the results. Note that, if we take such an initial state, not only the process we studied is present, but also the process $gg \to gX$ and all their crossing. However the situation is simpler that the expected. Of course the processes $q\bar{q} \to gX$ and its crossing are unitary-violating but the other with two gluons in the initial state is not. Naively we can understand this because it is obvious that this process involves only Feynman rules with the coupling constant for the gluons k_g. This means that, as far as this process in concerned, only the energy-momentum tensor of the gluon field is involved and, by

itself, it is conserved and it does not lead to any bad behaved term in energy. In figure (3.2) we plot the transverse momentum of the jet in the process mentioned above, in the universal and in the non-universal scenario and it is utterly explicit the non-unitary profile for the distribution when the couplings are different.

3.2 Helicity Contributions

Until now we have only pointed out both analytically and numerically the most interesting although unpleasant feature of the model we are studying. We now need to go deeper in the understanding of this non-unitary behaviour and, in particular it would be very helpful to find out from where, in terms of the spin-2 particle proprieties, the strong energy growth comes from. For this reason we are now interested in find the analytical expressions for the various helicity contributions to the total matrix-element squared we considered in the previous section. Knowing them would be a strong hint about on what we should focus and what we should expect to modify in the model in order to address this unitarity problem.
There is no unique way to achieve such a goal but once again it would be better and easier to contrive a way to perform this calculation with the help of FeynCalc again. Moreover, since there in so actual clear decomposition of the polarization sum of equation (2.1.14) into helicity contributions, there is no evident way to carry out the computation either. However we can attain the desired result, exploiting the fact that the polarization tensors for the massive spin-2 can be obtained by taking specific combinations of products of the polarization vector for a massive vector field (see for example [21]). Denoting the transverse polarization vectors $\varepsilon^\mu_\pm$ and the longitudinal one ε^μ_0, we can build the five polarization tensors for the massive spin-2 particle using the Clebsch-Gordan coefficients to obtain the $X_\pm 1$ and X_0 from the transverse tensors

$$X^{\mu\nu}_{\pm 2} = \varepsilon^\mu_\pm \, \varepsilon^\nu_\pm, \tag{3.2.1a}$$

$$X^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}} \left(\varepsilon^\mu_\pm \, \varepsilon^\nu_0 + \varepsilon^\mu_0 \, \varepsilon^\nu_\pm \right), \tag{3.2.1b}$$

$$X^{\mu\nu}_{0} = \frac{1}{\sqrt{6}} \left(\varepsilon^\mu_+ \, \varepsilon^\nu_- + \varepsilon^\mu_- \, \varepsilon^\nu_+ + 2\, \varepsilon^\mu_0 \, \varepsilon^\nu_0 \right). \tag{3.2.1c}$$

The way to profit by this decomposition in a FeynCalc notebook is to choose a specific frame in which evaluate by hand the scalar products between all the four-vector (momenta and polarization vectors) involved in the calculation and instruct the software to substitute the result of such scalar product every time it appears in the expression. Again we stress the fact that this is not the only way to obtain the result but it is an easy and convenient way to make use of the software and of the decompositions relations

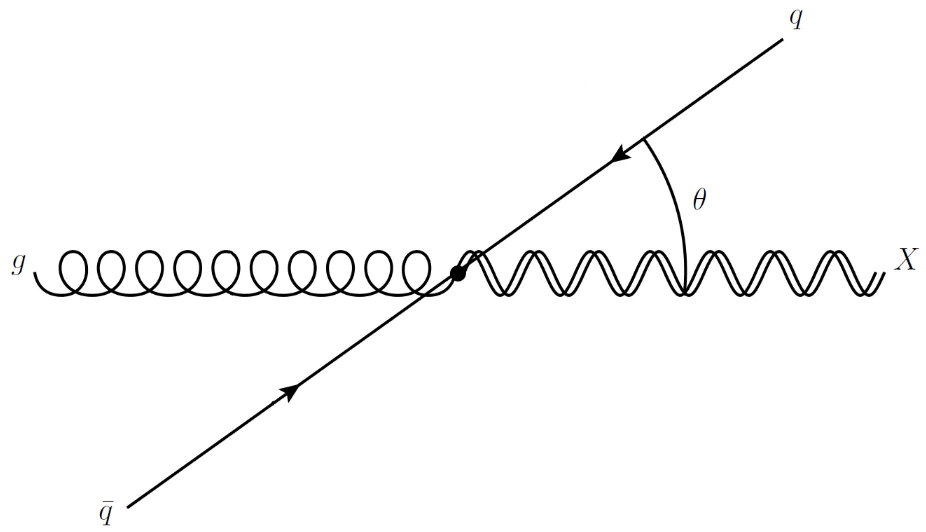


Figure 3.3: The process $q\bar{q} \to gX$ in the centre of momentum frame. The z-axis is meant to be parallel with the outgoing momentum of the spin-2 particle and the incoming quark is tilted with an angle θ along the y-axis.

we wrote down.
The best frame in which we can do the work is by far the centre of momentum frame with the z-axis aligned with the momenta of the outgoing particles and with θ the angle between the the spin-2 particle and the incoming quark (as represented in figure (3.3)). In this framework we can easily write down the explicit expression of every momenta involved in the proces componets by components

$$p_1^\mu = \big(E, 0, 0, p\big), \tag{3.2.2a}$$

$$p_2^\mu = \big(p, 0, 0, -p\big), \tag{3.2.2b}$$

$$q_1^\mu = \big(q, 0, q\ \sin\theta, q\ \cos\theta\big), \tag{3.2.2c}$$

$$q_2^\mu = \big(q, 0, -q\ \sin\theta, -q\ \cos\theta\big), \tag{3.2.2d}$$

where $E = \sqrt{p^2 + M^2}$ is the energy of the spin-2 particle and, form the conservation law follows

$$p = \frac{4q^2 - M^2}{4q}. \tag{3.2.3}$$

Moreover we also need the polarization vectors of a vector field, with the same momentum of the spin-2 particle, in order to build its polarization tensors. This is why we have chosen the z-axis in the same direction of p_1^μ. The expression for the polarization vector of a vector field along the z-axis are well-known and they read

$$\varepsilon_{+}^{\mu} = \frac{1}{\sqrt{2}}\Big(0, 1, -i, 0\Big), \tag{3.2.4a}$$

$$\varepsilon_{-}^{\mu} = \frac{1}{\sqrt{2}}\Big(0, -1, -i, 0\Big), \tag{3.2.4b}$$

$$\varepsilon_{0}^{\mu} = \frac{1}{M}\Big(p, 0, 0, E\Big), \tag{3.2.4c}$$

with the relation $\varepsilon^{*\mu}_{+} = -\varepsilon^{\mu}_{-}$.
Now the only thing we have to do is to evaluate the scalar products of the three polarization vectors with each others and with the momenta of the particles. Once we have done this we can then instruct MATHEMATICA to use the relations to simplify the results. In order to obtain the helicity contributions to the total amplitude squared, instead of replacing the whole polarization sum when performing the sum over the polarizations of the spin-2 particle as we have done in the previous calculation, now we substitute the explicit expression of the polarization tensors in term of $\varepsilon^{m}_{\pm}u$ and ε_0. If we want to calculate the contributions of the helicities $h = \pm 2$ then we perform the substitution

$$X^{\mu\nu}X^{*\rho\sigma} \to \varepsilon^{\mu}_{+}\varepsilon^{\nu}_{+}\varepsilon^{*\rho}_{+}\varepsilon^{*\sigma}_{+} + \varepsilon^{\mu}_{-}\varepsilon^{\nu}_{-}\varepsilon^{*\rho}_{-}\varepsilon^{*\sigma}_{-} = \varepsilon^{\mu}_{+}\varepsilon^{\nu}_{+}\varepsilon^{\rho}_{-}\varepsilon^{\sigma}_{-} + \varepsilon^{\mu}_{-}\varepsilon^{\nu}_{-}\varepsilon^{\rho}_{+}\varepsilon^{\sigma}_{+}, \tag{3.2.5}$$

while for the contributions form $h = \pm 1$ we have to replace

$$\begin{aligned} X^{\mu\nu}X^{*\rho\sigma} \to& \frac{1}{2}\left(\varepsilon^{\mu}_{+}\,\varepsilon^{\nu}_{0} + \varepsilon^{\mu}_{0}\,\varepsilon^{\nu}_{+}\right)\left(\varepsilon^{*\rho}_{+}\,\varepsilon^{*\sigma}_{0} + \varepsilon^{*\rho}_{0}\,\varepsilon^{*\sigma}_{+}\right) + \frac{1}{2}\left(\varepsilon^{\mu}_{-}\,\varepsilon^{\nu}_{0} + \varepsilon^{\mu}_{0}\,\varepsilon^{\nu}_{-}\right)\left(\varepsilon^{*\rho}_{-}\,\varepsilon^{*\sigma}_{0} + \varepsilon^{*\rho}_{0}\,\varepsilon^{*\sigma}_{-}\right) \\ =& -\frac{1}{2}\left(\varepsilon^{\mu}_{+}\,\varepsilon^{\nu}_{0} + \varepsilon^{\mu}_{0}\,\varepsilon^{\nu}_{+}\right)\left(\varepsilon^{\rho}_{-}\,\varepsilon^{\sigma}_{0} + \varepsilon^{\rho}_{0}\,\varepsilon^{\sigma}_{-}\right) - \frac{1}{2}\left(\varepsilon^{\mu}_{-}\,\varepsilon^{\nu}_{0} + \varepsilon^{\mu}_{0}\,\varepsilon^{\nu}_{-}\right)\left(\varepsilon^{\rho}_{+}\,\varepsilon^{\sigma}_{0} + \varepsilon^{\rho}_{0}\,\varepsilon^{\sigma}_{+}\right), \end{aligned} \tag{3.2.6}$$

and finally for the longitudinal contribution $h = 0$ we write

$$\begin{aligned} X^{\mu\nu}X^{*\rho\sigma} \to& \frac{1}{6}\left(\varepsilon^{\mu}_{+}\,\varepsilon^{\nu}_{-} + \varepsilon^{\mu}_{-}\,\varepsilon^{\nu}_{+} + 2\,\varepsilon^{\mu}_{0}\,\varepsilon^{\nu}_{0}\right)\left(\varepsilon^{*\rho}_{+}\,\varepsilon^{*\sigma}_{-} + \varepsilon^{*\rho}_{-}\,\varepsilon^{*\sigma}_{+} + 2\,\varepsilon^{*\rho}_{0}\,\varepsilon^{*\sigma}_{0}\right) \\ =& \frac{1}{6}\left(\varepsilon^{\mu}_{+}\,\varepsilon^{\nu}_{-} + \varepsilon^{\mu}_{-}\,\varepsilon^{\nu}_{+} + 2\,\varepsilon^{\mu}_{0}\,\varepsilon^{\nu}_{0}\right)\left(\varepsilon^{\rho}_{-}\,\varepsilon^{\sigma}_{+} + \varepsilon^{\rho}_{+}\,\varepsilon^{\sigma}_{-} + 2\,\varepsilon^{\rho}_{0}\,\varepsilon^{\sigma}_{0}\right). \end{aligned} \tag{3.2.7}$$

Even if it is not strictly necessary, for our procedure of computation, it can be interesting to look directly at the structure of the polarization tensors in the centre of momentum frame. We can write them in matrix form quite easily and they are given by

$$X^{\mu\nu}_{\pm 2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm i & 0 \\ 0 & \pm i & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3.2.8a}$$

$$X^{\mu\nu}_{\pm 1} = \frac{1}{2M}\begin{pmatrix} 0 & \mp p & -ip & 0 \\ \mp p & 0 & 0 & \mp E \\ -ip & 0 & 0 & -iE \\ 0 & \mp E & -iE & 0 \end{pmatrix}, \tag{3.2.8b}$$

$$X^{\mu\nu}_{0} = \frac{1}{\sqrt{6}M^2}\begin{pmatrix} 2p^2 & 0 & 0 & 2pE \\ 0 & -M^2 & 0 & 0 \\ 0 & 0 & -M^2 & 0 \\ 2pE & 0 & 0 & 2E^2 \end{pmatrix}. \tag{3.2.8c}$$

Further details on the calculation, such as the various expressions of the scalar products, can be found in the notebook written to perform this evaluation, reported in Appendix (B.2). Here we give and comment only the final results, which, as well as equation (3.1.9), have been verified with the numerical results we got with MadGraph.
The contribution coming from the transverse helicities of the spin-2 particle, i.e. $h = \pm 2$ we find

$$\begin{aligned} |\mathcal{M}_{\pm 2}|^2 = \frac{4{g_s}^2}{9\Lambda^2 s\,(M^2-s)^4}\,&\Big[M^4 - 2M^2(s+t) + s^2 + 2st + 2t^2\Big]\Big[{k_g}^2(M^2-s)^4 \\ &+ 2k_g k_q s(2M^2-s)(M^2-s)^2 + 2{k_q}^2 s^2(2M^4 - 2M^2 s + s^2)\Big]. \end{aligned} \tag{3.2.9}$$

It is quite easy to see that, even in the non-universal coupling configuration, this expression has the right energy behaviour for a amplitude squared involving one vertex coming from a dimension five operator, and it scales as $\Lambda^{-2}\ s$. This obviously means that the transverse helicities of the spin-2 particle are in no way the culprits for the non-unitary behaviour and for this reason, they do not require any intervention or modification. It should not be a real surprise because, as we learnt in section (2.3), the propagator of the transverse component of the spin-2 particle, as well as its polarization sum, do not have any momenta in its expression, but only metric tensors and then no strongly growing terms can arise from such a structure. The amplitude squared regarding the vector-like components of the helicity $h = \pm 1$ on the contrary reads

$$
\begin{aligned}
|\mathcal{M}_{\pm1}|^2 =& \frac{{g_s}^2}{9\Lambda^2 M^2 tu(M^2-s)^4}\Big\{8{k_g}^2t^2(M^2-s)^4(-M^2+s+t)^2 \\
&-4k_gk_qt(M^2-s)^2\Big[M^{10}-M^8(3s+5t)+M^6\left(3s^2+2st+8t^2\right) \\
&-M^4\left(s^3-15s^2t-8st^2+4t^3\right)-8M^2st\left(2s^2+3st+t^2\right)+4s^2t(s+t)^2\Big] \\
&+{k_q}^2\Big[M^{16}-2M^{14}(2s+3t)+M^{12}\left(7s^2+10st+14t^2\right)-4M^{10}\left(2s^3-3st^2+4t^3\right) \\
&+M^8\left(7s^4-12s^2t^2-48st^3+8t^4\right)-2M^6s\left(2s^4+5s^3t+50s^2t^2+16st^3-16t^4\right) \\
&+M^4s^2\left(s^4+6s^3t+126s^2t^2+160st^3+48t^4\right)-16M^2s^3t^2\left(3s^2+5st+2t^2\right) \\
&+8s^4t^2(s+t)^2\Big]\Big\}.
\end{aligned}
\tag{3.2.10}
$$

It is not the easiest expression to handle for sure but, with a careful inspection, we can notice that there are not term which grow faster than $\Lambda^{-2}M^{-2}s^2$. Again, this is not totally unexpected, since, performing the Stückelberg trick, we saw that the vector field in charge to mimic these helicities components, has a coupling with the energy-momentum tensor of the Standard Model involving one derivative, making the correspondent operator of dimension six. In any case the terms with this energy dependence are fully-fledged unitary-violating and need to be cured and compensated in order to restore the unitarity of the model. And last we can look into the contribution of the longitudinal helicity which, by process of elimination, should be the responsible of the worst behaving terms. For it, the squared amplitude is given by

$$
\begin{aligned}
|\mathcal{M}_0|^2 =& \frac{2{g_s}^2}{27\Lambda^2\left(M^3-Ms\right)^4}\, s\Big\{{k_g}^2\left(M^2-s\right)^4\left(M^4-2M^2(s+t)+s^2+2st+2t^2\right) \\
&+2k_gk_q\left(M^2-s\right)^2\Big[2M^8-2M^6(s+5t)+M^4\left(-3s^2+6st+10t^2\right) \\
&+2M^2s\left(2s^2+3st+2t^2\right)-s^2\left(s^2+2st+2t^2\right)\Big]+{k_q}^2\Big[13M^{12}-2M^{10}(9s+25t) \\
&+M^8\left(-3s^2+10st+50t^2\right)+4M^6s\left(s^2+13st+10t^2\right)+M^4s^2\left(9s^2-4st-12t^2\right) \\
&-2M^2s^3\left(3s^2+5st+4t^2\right)+s^4\left(s^2+2st+2t^2\right)\Big]\Big\}.
\end{aligned}
\tag{3.2.11}
$$

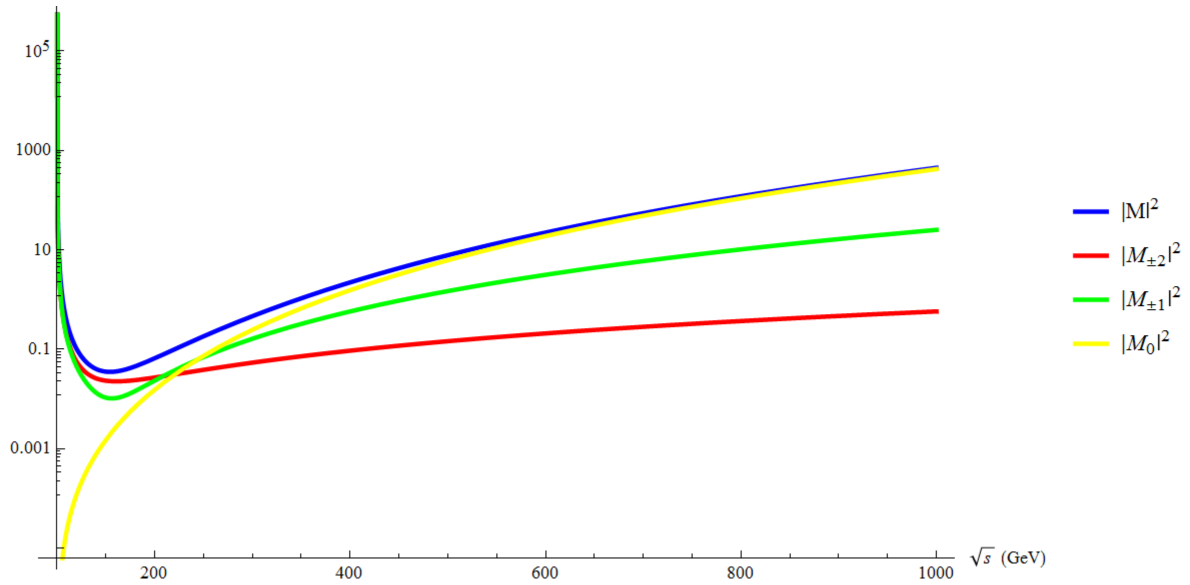


Figure 3.4: Plots of the total amplitude squared of the process $q\bar{q} \to gX$ compared with the different helicities contributions, with respect the total energy $\sqrt{s}$. The mass of the spin-2 particle has been chosen to be $M = 100$ GeV, the cut-off scale $\Lambda = 1$ TeV and the angle between the incoming quark and the graviton $\theta = \pi/2$. The coupling constants here are set to be $k_q = 1$ and $k_g = 0.1$ instead.

We can see as expected that the longitudinal contribution grows as fast as $\Lambda^{-2}M^{-4}s^3$, and this is also motivated by the fact that, in the Stückelberg formulation, the scalar field which is taking the part of the helicity $h = 0$ of the spin-2 particle, has a coupling term with a double derivative and so a dimension seven operator.
A quick check has to be done now in order to have an additional confirmation, other than the MadGraph numerical verification, that what we have done is right. The first thing we want to make us sure of, is that these three contributions sum up to the total amplitude squared of equation (3.1.9), since the various helicities are orthogonal to each other and no interference can happen between them. This exercise is very easy with Mathematica and much more time consuming by hand, but in the end one can prove that indeed

$$|\mathcal{M}|^2 = |\mathcal{M}_{\pm 2}|^2 + |\mathcal{M}_{\pm 1}|^2 + |\mathcal{M}_0|^2. \tag{3.2.12}$$

In figure (3.4) we can see the plots, in the non-universal couplings scheme, of the total squared amplitude compared with the various helicities contributions we previously found. In order to plot them one must use the expressions in the centre of momentum frame which are written in the Appendixes (B) and has to fix the angle θ between the spin-2 particle and the incoming quark (in the figure it has been chosen $\theta = \pi/2$). Looking at the plots it is manifest that almost immediately the contribution of the lon-

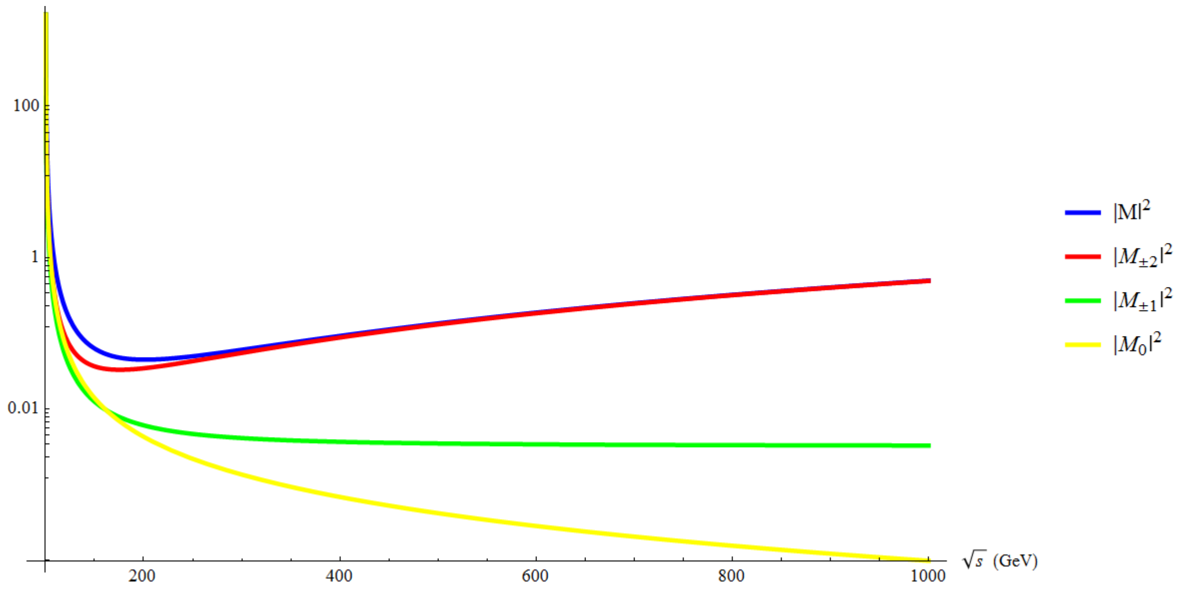


Figure 3.5: Plots of the total amplitude squared of the process $q\bar{q} \to gX$ compared with the different helicities contributions, with respect the total energy $\sqrt{s}$ in the universal-coupling case. The mass of the spin-2 particle is set to $M = 100$ GeV, the cut-off scale $\Lambda = 1$ TeV and the angle between the incoming quark and the graviton $\theta = \pi/4$.

gitudinal helicity is by far the leading one while the others grow to much slower and they soon become irrelevant. The vector-like and the transverse contribution are indeed, more or less, respectively two and four order of magnitude smaller than the dominant contribution already at not-so-high energy like $\sqrt{s} = 1$ TeV.
The last study we want to do now is to investigate the behaviour of the helicity amplitudes in the limit of equal couplings to see, in that regime, which one is dominating. The expressions are easily calculable and we write them in the centre of momentum frame, because is much easier to understand what happens in the universal-couplings framework, and they end up to be

$$|\mathcal{M}_{\pm2}^{Univ}|^2 = \frac{{g_s}^2 k^2 \big(\cos 2\theta + 3\big)\big(M^8 + s^4\big)}{9\Lambda^2 s (M^2 - s)^2}, \tag{3.2.13a}$$

$$|\mathcal{M}_{\pm1}^{Univ}|^2 = \frac{{g_s}^2 k^2 M^2 \big(\cos 2\theta + \cos 4\theta + 2\big) \csc^2\theta \big(M^4 + s^2\big)}{9\Lambda^2 (M^2 - s)^2}, \tag{3.2.13b}$$

$$|\mathcal{M}_{0}^{Univ}|^2 = \frac{4{g_s}^2 k^2 M^4 s \cos^2\theta}{3\Lambda^2 \left(M^2 - s\right)^2}. \tag{3.2.13c}$$

From these results, together with figure (3.5), we can realize that the situation is totally inverted. The contribution from the helicities $h = \pm 2$ is now the leading one and it gives rise to the $\Lambda^{-2}s$ terms. The amplitude squared of the vector-like components behave like a constant while the longitudinal part is actually a decreasing function of the energy, in strong opposition with the previous case.

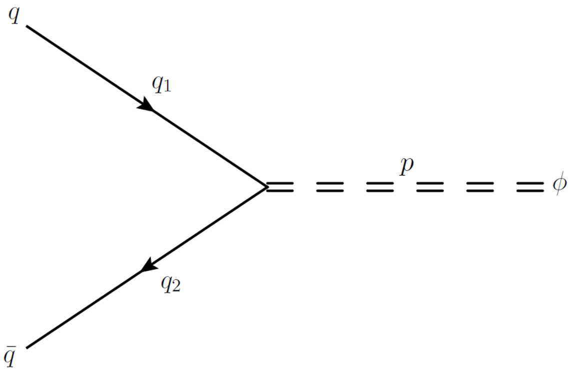
3.3 First Steps Towards Restoring Unitarity

Until now what we have done is to analyze in details the non-unitary behaviour of the model with non-universal couplings. Now what we can try to do is to address this problem the model has, with the intention to give a guideline about what could be a possible way to solve this issue and to write down a model which allows us to maintain the non-universal coupling scheme without compromising the unitarity.
The fundamental idea we want to pursue is to achieve the unitarity by adding to the theory new degrees of freedom, in the form of new fields, which, with some mechanism, have the aim to compensate the wrong-behaving energy term, leaving the final model with the expected behaviour of a dimension five effective filed theory. The most important observation we made is that the term which have a wrong dependence with respect the energy arise from the helicities $h = \pm 1, 0$ of the spin-2 particle. In our framework it means that we have to think about adding a scalar and a vector field to the theory in order to restore the unitarity by counterbalancing. Exactly for this reason we are interested to study in more details the vector and the scalar field which come from the Stückelberg trick we described in section (2.3). At this stage we want to think about them not as proper Stückelberg fields but, more generically, like new extra degrees of freedom to be used as we mentioned, utilizing the Stückelberg formulation as a suggestion about how these fields should couple with the Standard Model in order to obtain the desired outcome.
Let us start by recollecting the interaction Lagrangian we have derived and obtained in equation (2.3.14), which emerges form the trick

$$\mathcal{L} = \frac{k}{\Lambda}\left[\frac{1}{\sqrt{2}M}\,\Big(\partial_\mu V_\nu + \partial_\nu V_\mu\Big) + \frac{\sqrt{2}}{\sqrt{3}M^2}\,\partial_\mu\partial_\nu\phi - \frac{1}{\sqrt{6}}\,\phi\,\eta_{\mu\nu}\right]T^{\mu\nu}_{SM}. \tag{3.3.1}$$

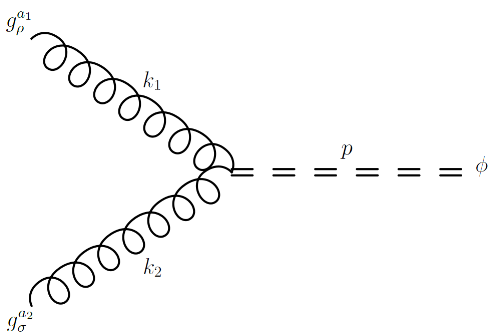
In order to perform the same kind of study we have done in the previous sections with the spin-2 particle we now need to implement this Lagrangian in FeynRules which will allow us to easily discover the Feynman rules concerning these fields and to produce the UFO (Universal FeynRules Output) file needed to perform simulation of this model in MadGraph. The implementation is actually a straightforward addition to the "DMspin2"model [17], where we simply introduced two new fields and we added the extra terms displayed in the previous equation. For the model file we refer to Appendix (C) in which we have written in red the modification we made. Note also that in the model we gave to the scalar and the vector field two different masses with respect the spin-2 mass to make even more explicit the fact that we are thinking about these fields

as totally unrelated from the graviton in their origin and in the following we will keep this generalization.
The intention is again to compute the total amplitude squared for the processes $q\bar{q} \to g\ V$ and $q\bar{q} \to g\ \phi$ and look at the analytic expressions, to see how these new fields are able to reproduce the contribution of the spin-2 particle we want to eliminate from the theory. It is quite intuitive to realize that the same topology of Feynman diagrams are involved as in the case of the precess $q\bar{q} \to g\ X$, in fact they are exactly the same of the diagrams in figure (3.1), but with the spin-2 particle replaced with the vector and the scalar field. To begin, it is useful to write down the Feynman rules which are the counterparts of the rules (3.1.1) and (3.1.2) for the vertex involving the massive graviton and the quark and anti-quark pair or the gluon pair respectively. Loading the model in FeynRules we find the following expression for the scalar vertex with the quark and anti-quark pair



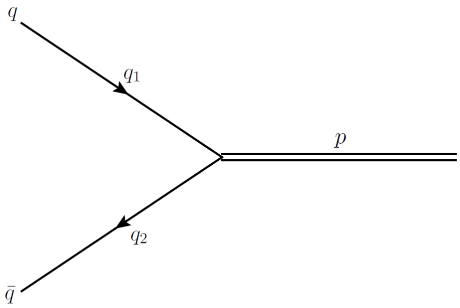
$$= \frac{ik_q}{\sqrt{6}\Lambda}\left[\frac{5}{2}\left(\not{q}_2 - \not{q}_1\right) + \frac{p\cdot(q_1-q_2)}{{M_\phi}^2}\,\not{p}\right], \qquad (3.3.2)$$

and for the vertex with two gluons



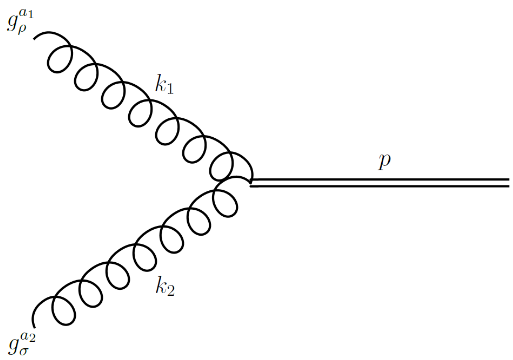
$$= \frac{\sqrt{2}ik_g}{\sqrt{3}\Lambda}\,\delta^{a_1a_2}\Bigg[\eta^{\rho\sigma}\left(\frac{2}{{M_\phi}^2}\,(k_1\cdot p)\,(k_2\cdot p) - k_1\cdot k_2\right) + \frac{2}{{M_\phi}^2}\Big((k_1\cdot k_2)\,p^\rho p^\sigma - (k_1\cdot p)\,(k_1+k_2)^\rho p^\sigma - (k_2\cdot p)\,(k_1+k_2)^\sigma p^\rho\Big) + 2\Big(k_1^\rho k_1^\sigma + k_2^\rho k_2^\sigma + k_1^\rho k_2^\sigma + k_2^\rho k_1^\sigma\Big)\Bigg]. \qquad (3.3.3)$$

The same procedure can be performed for the vector field also, leading to the result



$$V^\mu = \frac{k_q}{2\sqrt{2}\Lambda M_V}\left[p\cdot(q_2-q_1)\,\gamma^\mu+\left(q_2-q_1\right)^\mu\,\not{p}+2\left(\not{q}_1-\not{q}_2\right)\,p^\mu\right], \tag{3.3.4}$$

for the three point vertex with the quarks, and



$$\begin{aligned} V^\mu = \; & \frac{\sqrt{2}k_g}{\Lambda M_V}\,\delta^{a_1a_2}\Big[p^\rho\left(k_1^\sigma k_2^\mu+k_2^\sigma k_2^\mu-\eta^{\sigma\mu}k_1\cdot k_2\right)\\ & +p^\sigma\left(k_1^\rho k_1^\mu+k_2^\rho k_1^\mu-\eta^{\rho\mu}k_1\cdot k_2\right)\\ & +p^\mu\left(\eta^{\rho\sigma}k_1\cdot k_2-k_1^\rho k_1^\sigma-k_1^\rho k_2^\sigma-k_2^\rho k_1^\sigma-k_2^\rho k_2^\sigma\right)\\ & +k_1\cdot p\,\left(\eta^{\sigma\mu}k_1^\rho+\eta^{\sigma\mu}k_2^\rho-\eta^{\rho\sigma}k_2^\mu\right)\\ & +k_2\cdot p\,\left(\eta^{\rho\mu}k_1^\sigma+\eta^{\rho\mu}k_2^\sigma-\eta^{\rho\sigma}k_1^\mu\right)\Big], \end{aligned} \tag{3.3.5}$$

for the rule with the two gluons.

Using the same notation we introduced in section (3.1) for the process $q\bar{q}\to gX$ we can now write down the amplitudes corresponding to the four diagrams involved in the process for the scalar field

$$\mathcal{M}_v^\phi = \frac{g_s k_q}{\Lambda}\,T^a\varepsilon_\rho^a(p_2)\bar{v}(q_2)\left(\frac{5}{\sqrt{6}}\,\gamma^\rho-\frac{\sqrt{2}}{\sqrt{3}{M_\phi}^2}\,p_1^\rho\,\not{p}_1\right)u(q_1), \tag{3.3.6a}$$

$$\begin{aligned} \mathcal{M}_s^\phi = & -\frac{\sqrt{2}g_s k_g}{\sqrt{3}\Lambda(q_1+q_2)^2}\,T^a\varepsilon_\rho^a(p_2)\Big[\eta^{\rho\sigma}p_2\cdot(q_1+q_2)+2(q_1+q_2)^\rho(q_1+q_2)^\sigma-(q_1+q_2)^\rho p_2^\sigma\\ & -\frac{2}{{M_\phi}^2}\,\Big(\eta^{\rho\sigma}(p_1\cdot p_2)\,p_1\cdot(q_1+q_2)+p_1^\rho\,p_1^\sigma\,p_2\cdot(q_1+q_2)+p_1^\rho\,(q_1+q_2)^\sigma\,p_1\cdot(q_1+q_2)\\ & -p_1^\rho\,p_2^\sigma\,p_1\cdot(q_1+q_2)+(q_1+q_2)^\rho\,p_1^\sigma\,p_1\cdot p_2\Big)\Big]\bar{v}(q_2)\,\gamma^\sigma\,u(q_1), \end{aligned} \tag{3.3.6b}$$

$$\mathcal{M}_t^\phi = \frac{g_s k_q}{\sqrt{6}\Lambda(q_1-p_1)^2}\, T^a \varepsilon^a_\rho(p_2)\, \bar{v}(q_2)\gamma^\rho(\not{p}_2-\not{q}_2)\Big[\frac{\not{p}_1}{{M_\phi}^2}\, p_1\cdot(p_2-q_2+q_1) - \frac{5}{2}\,(\not{p}_2-\not{q}_2+\not{q}_1)\Big]u(q_1), \tag{3.3.6c}$$

$$\mathcal{M}_u^\phi = \frac{g_s k_q}{\sqrt{6}\Lambda(q_1-p_2)^2}\, T^a \varepsilon^a_\rho(p_2)\, \bar{v}(q_2)\Big[\frac{\not{p}_1}{{M_\phi}^2}\, p_1\cdot(p_2-q_1+q_2) - \frac{5}{2}\,(\not{p}_2-\not{q}_1+\not{q}_2)\Big] \cdot(\not{p}_2-\not{q}_1)\gamma^\sigma u(q_1), \tag{3.3.6d}$$

where we made use of the transverse relation for the gluon polarization vector in order to simplify the expressions as much as possible.
Of course we can do the same for the vector field also, denoting $\varepsilon_\mu(p_1)$ the polarization vector of V^μ, we obtain the following amplitudes

$$\mathcal{M}_v^V = \frac{ig_s k_q}{\sqrt{2}\Lambda M_V}\, T^a\, \varepsilon_\mu(p_1)\, \varepsilon^a_\rho(p2)\, \bar{v}(q_2)\Big[\eta^{\mu\rho}\not{p}_1 + \gamma^\mu p_1^\rho\Big]u(q_1) \tag{3.3.7a}$$

$$\begin{aligned}\mathcal{M}_s^V = &\frac{\sqrt{2}ig_s k_g}{\Lambda M_V(q_1+q_2)^2}\, T^a\, \varepsilon_\mu(p_1)\, \varepsilon^a_\rho(p2)\Big[\eta^{\mu\sigma}\Big((q_1+q_2)^\rho\,(p_1\cdot p_2) - p_1^\rho\, p_2\cdot(q_1+q_2)\Big)\\ &+\eta^{\mu\rho}\Big(p_2^\sigma\, p_1\cdot(q_1+q_2) - p_1^\sigma\, p_2\cdot(q_1+q_2) - (q_1+q_2)^\sigma\, p_1\cdot(q_1+q_2)\Big)\\ &-\eta^{\rho\sigma}\Big((q_1+q_2)^\mu\, p_1\cdot p_2 + p_2^\mu\, p_1\cdot(q_1+q_2)\Big) + p_1^\sigma\, p_2^\mu\,(q_1+q_2)^\rho\\ &+p_1^\rho\Big(p_2^\sigma\,(q_1+q_2)^\mu - (q_1+q_2)^\sigma\,(q_1+q_2)^\mu\Big)\Big]\bar{v}(q_2)\,\gamma^\sigma\, u(q_1),\end{aligned} \tag{3.3.7b}$$

$$\mathcal{M}_t^V = -\frac{ig_s k_q}{2\sqrt{2}\Lambda M_V(q_1-p_1)^2}\, T^a\, \varepsilon_\mu(p_1)\, \varepsilon^a_\rho(p2)\, \bar{v}(q_2)\gamma^\rho(\not{p}_2-\not{q}_2)\Big[\gamma^\mu\, p_1\cdot(p_2-q_2+q_1) + \not{p}_1\,(p_2-q_2+q_1)^\mu\Big]u(q_1), \tag{3.3.7c}$$

$$\mathcal{M}_u^V = -\frac{ig_s k_q}{2\sqrt{2}\Lambda M_V(q_1-p_2)^2}\, T^a\, \varepsilon_\mu(p_1)\, \varepsilon^a_\rho(p2)\, \bar{v}(q_2)\Big[\gamma^\mu\, p_1\cdot(p_2-q_1+q_2) + \not{p}_1\,(p_2-q_1+q_2)^\mu\Big](\not{p}_2-\not{q}_1)\gamma^\rho u(q_1). \tag{3.3.7d}$$

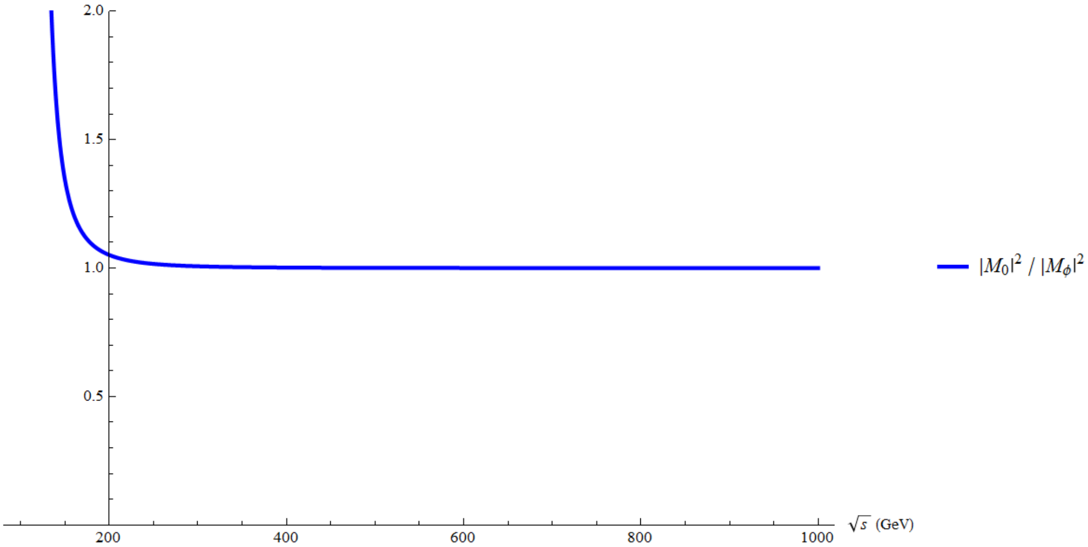


Figure 3.6: The ratio of the longitudinal helicity contribution of the spin-2 particle over the amplitude squared of the scalar field. The masses have been set to equal values $M = M_\phi = 100$ GeV and the scattering angle $\theta = \pi/4$.

The evaluation has been performed again using FeynCalc and goes on in the same way as in the previous calculation[1] and the final result for the scalar field is

$$|\mathcal{M}^\phi|^2 = \frac{2{g_s}^2\ s(k_g - k_q)^2\,(t^2 + u^2)}{27\Lambda^2 {M_\phi}^4}, \tag{3.3.8}$$

which can be recast in the centre of momentum frame as

$$|\mathcal{M}^\phi|^2 = \frac{{g_s}^2\ s(k_g - k_q)^2(\cos 2\theta + 3)\big({M_\phi}^2 - s\big)^2}{54\Lambda^2 {M_\phi}^4}. \tag{3.3.9}$$

At first glance this expression does not match at all with the longitudinal contribution of the spin-2 particle of equation (3.2.11), but we can immediately recognize that, at least, it has the correct energy behaviour, i.e. it grows like $\Lambda^{-2}{M_\phi}^{-4}s^3$ and moreover it vanishes identically in the universal couplings regime, which is also the asymptotic behaviour of $|\mathcal{M}_0|^2$ in the same limit, as checked in equation (3.2.13c). The difference between the expressions should not surprise us, in fact we learnt that the Stückelberg trick works in the limit of high energy, which means that the two amplitudes squared have to be similar only in that regime. We can prove this statement by commenting figure (3.6), in which is displayed the ratio between the longitudinal contribution of the

[1]It was considered appropriate to not give the Mathematica notebooks for these computations because they are almost identical to the one in the Appendix (B.1) and (B.2), one only needs to replace the sum over the tensor polarizations with the polarization sum for V^μ or with nothing for ϕ.

graviton $|\mathcal{M}_0|^2$ over the scalar amplitude squared $|\mathcal{M}^\phi|^2$ with respect the energy of the process. In the plot we can see that, after a initial range of energies around the values of the masses, in which the spin-2 amplitude squared is dominating, the ratio quickly stabilize at the value of 1, meaning that the two quantities are numerically identical. Moreover, the fact that at low energies the ratio diverges is a very good indication that the scalar field is totally negligible with respect the spin-2 particle, and so, adding it in the model would not modify the low energy behaviour of the theory which we know to be fine already.
Let us now write down the amplitude squared for the vector field also and discuss the result. Carrying out the whole calculation the expression ends up to be

$$|\mathcal{M}^V|^2 = \frac{2{g_s}^2(k_g - k_q)^2\big[{M_V}^2(t^2 + 4tu + u^2) - t^3 - t^2u - tu^2 - u^3\big]}{9\Lambda^2 {M_V}^4}, \tag{3.3.10}$$

or, in the centre of momentum frame,

$$|\mathcal{M}^V|^2 = -\frac{{g_s}^2(k_g - k_q)^2\left({M_V}^2 - s\right)^2\left[\cos 2\theta\left(2{M_V}^2 - s\right) - 2{M_V}^2 - 3s\right]}{18\Lambda^2 {M_V}^4}. \tag{3.3.11}$$

As for the scalar case, the expression does not look like the contribution of the helicities $h = \pm 1$ of the spin-2 particle, and, even worst, this amplitude squared grows with the energy exactly like the scalar one, that is as fast as $\Lambda^{-2}{M_V}^{-4}s^3$. Once again however we should have expected this kind of behaviour. Ultimately we are dealing with a massive vector field and therefore it has to have a longitudinal component too, and it is this which causes the matrix element squared to grow that fast. To appreciate this fact, let us then evaluate the contribution of the vector transverse polarizations as well as of the longitudinal one separately. The strategy to achieve the desired result follows exactly the construction we made for the spin-2 case in section (3.2) and so, when performing the polarization sum we actually make the replacement

$$\varepsilon^\mu\,\varepsilon^{*\nu} \to \varepsilon^\mu_+\varepsilon^{*\,\nu}_+ + \varepsilon^\mu_-\varepsilon^{*\,\nu}_- = -\varepsilon^\mu_+\varepsilon^\nu_- - \varepsilon^\mu_-\varepsilon^\nu_+, \tag{3.3.12}$$

for the transverse contribution while the substitution for the longitudinal one is

$$\varepsilon^\mu\,\varepsilon^{*\nu} \to \varepsilon^\mu_0\varepsilon^\nu_0. \tag{3.3.13}$$

The expressions, checked as usual with the numerical value obtained with the simulations in MadGraph, are given by

$$|\mathcal{M}^V_{\pm1}|^2 = \frac{8{g_s}^2 tu(k_g - k_q)^2}{9\Lambda^2 {M_V}^2}, \tag{3.3.14a}$$

$$|\mathcal{M}^V_0|^2 = \frac{2{g_s}^2 s(k_g - k_q)^2\,(t^2 + u^2)}{9\Lambda^2 {M_V}^4}, \tag{3.3.14b}$$

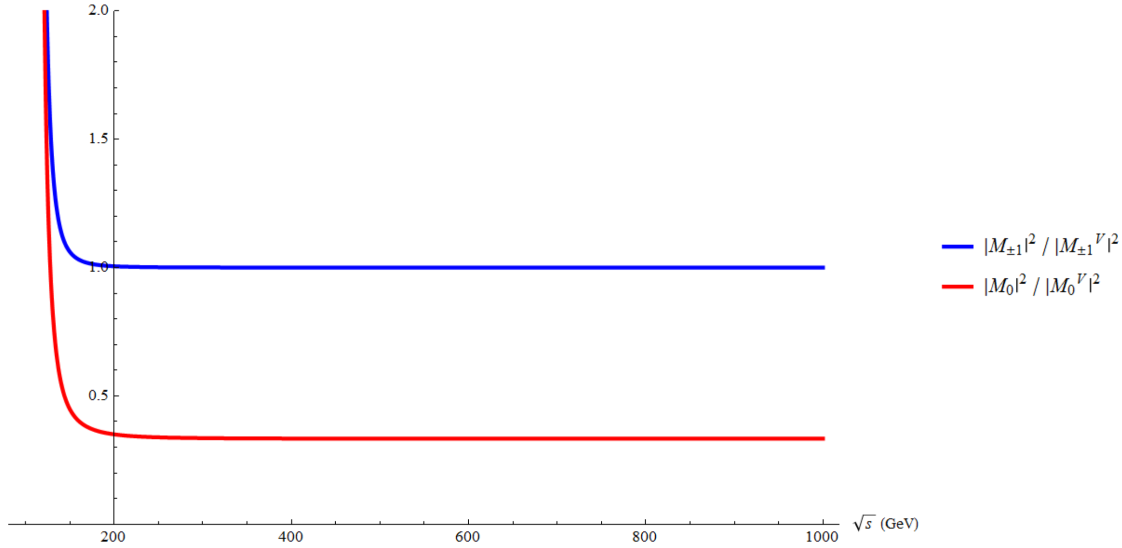


Figure 3.7: Ratio of the longitudinal helicity contribution of the spin-2 particle over the helicity $h = 0$ to amplitude squared of the vector field and the ratio between the respective contributions of the helicities $h = \pm 1$. The masses have been set to equal values $M = M_V = 100$ GeV and the scattering angle $\theta = \pi/4$.

or, equivalently in the centre of momentum frame

$$|\mathcal{M}^V_{\pm 1}|^2 = \frac{8{g_s}^2 tu(k_g - k_q)^2}{9\Lambda^2 {M_V}^2}, \tag{3.3.15a}$$

$$|\mathcal{M}^V_0|^2 = \frac{{g_s}^2 s(k_g - k_q)^2(\cos 2\theta + 3)\left({M_V}^2 - s\right)^2}{18\Lambda^2 {M_V}^4}. \tag{3.3.15b}$$

Inspecting the results we can realize that the different contributions behave in the expected way with respect the energy and, in particular, the transverse component grow as $\Lambda^{-2} M_V^{-2} s^2$ as required. Like the scalar the algebraic structure of the expressions is not similar to the spin-2 counterparts, but the hope is that they quickly become numerically equal in the high energy regime an analogy with the scalar field. To quickly check this out we plot the ratios of the $h = \pm 1$ and $h = 0$ separately as functions of the energy. As we can see in figure (3.7), the transverse contributions ratio approaches the unit value very soon, the ratio between the longitudinal parts instead stabilizes at $1/3$, while in both cases the low energy regime is dominated by the spin-2, which is a good feature as argued in the previous analysis. Now a problem arises for the longitudinal amplitudes since the vector contribution is three times bigger than the spin-2 particle, in fact looking at the expressions (3.3.8) and (3.3.14b) we realize that the proportion is exactly $|\mathcal{M}^V_0|^2 = 3\ |\mathcal{M}^\phi|^2$. This implies that we can not simply add these two fields as they are, because there would not be compensation between the longitudinal components. However, there could be at least two possible solutions to this issue, which allow the contributions to perfectly match. We could have naively expected to subtract both the vector and the scalar field to the starting spin-2 Lagrangian, or in other words, add the two fields but with inverted signs of the coupling with the Standard Model energy-momentum tensor in order to cancel the unitary-violating helicities. The first solution keeps this idea of flipping the signs of both the vector and the scalar field but needs also some kind of mechanism which enforces the transverse condition to V^μ even if it remains a massive vector field. In fact we can not make it a strictly massless field because the coupling with the Standard Model would diverge. This would leave in the theory only the degrees of freedom needed to ensure, at least at the amplitude squared level, the cancellation of all the terms with the strong energy dependence. Another and probably more natural solution would be to make use of the very precise proportionality between the longitudinal components of the three fields. The right combination suggested from the analysis seems to be to actually subtract the vector field and add, without flipping the sign, the scalar field with a further $\sqrt{2}$ enhancement for the coupling term, which would lead to double the value of its matrix element squared. In doing so we would end up with a perfect balance for the longitudinal contributions of the three involved fields without the necessity to impose more auxiliary conditions. This second way seems to be to more appealing and requires the minimal modification of the starting spin-2 theory and it would have been described by the Lagrangian

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{2}\,\partial_\rho X_{\mu\nu}\,\partial^\rho X^{\mu\nu} + \partial_\mu X_{\nu\rho}\,\partial^\nu X^{\mu\rho} - \partial_\mu X^{\mu\nu}\,\partial_\nu X + \frac{1}{2}\,\partial_\rho X\,\partial^\rho X \\
&- \frac{1}{2}\,M^2\big(X_{\mu\nu}\,X^{\mu\nu} - X^2\big) + \frac{1}{2}\,V_\mu\big(\Box - {M_V}^2\big)V^\mu + \frac{1}{2}\,\phi\big(\Box - {M_\phi}^2\big)\phi \\
&+ \frac{1}{\Lambda}\left[X_{\mu\nu} + \frac{2}{\sqrt{3}{M_\phi}^2}\,\partial_\mu\partial_\nu\phi - \frac{1}{\sqrt{3}}\,\phi\,\eta_{\mu\nu} - \frac{1}{\sqrt{2}M_V}\,\Big(\partial_\mu V_\nu + \partial_\nu V_\mu\Big)\right]\sum_i\,k_i T_i^{\mu\nu}.
\end{aligned}
\tag{3.3.16}
$$

Further and important comments will follow in the next section.

3.4 Outlooks

Clearly we are not making the statement of having solved the unitary problem of the model, because a lot more work has to be done on this topic before obtain some solid result. We are only trying to give a guideline for the construction of a model to be studied in more depths and, in the following, we want describe some further analysis which are currently under consideration, together with a possible application of this model.
As pointed out before, we have made a comparison between the different contributions

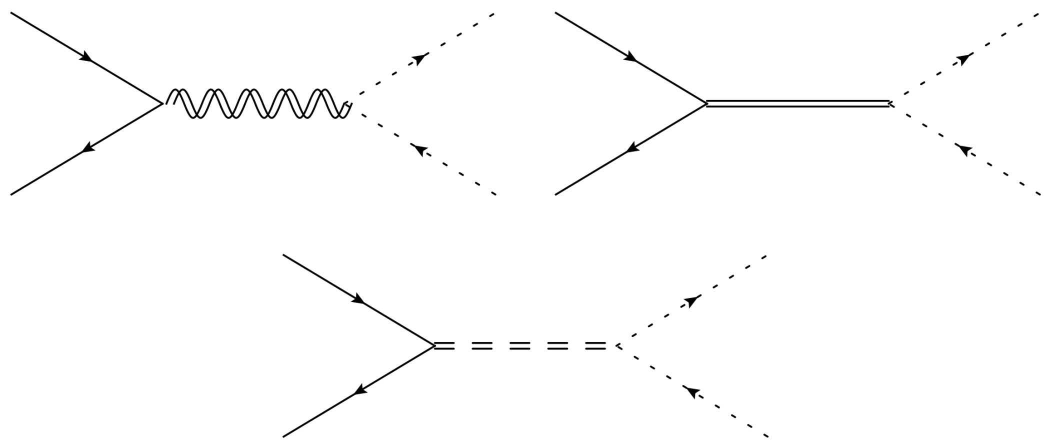


Figure 3.8: Three diagrams involved in the fermionic dark matter production process, where the dotted line stands for the dark matter particle.

of the spin-2 particle and the two new fields, at the amplitude squared level only. This is evidently not enough because, up to now, we treated these three fields as totally unrelated to each other. Even though it is a required step in the development of the model and it has lead to some useful insight, it is obviously that, if we want to really understand the effect of the new vector and scalar fields to the theory, we have to go one step further. Namely we should now take in consideration the situations in which these fields are allowed to interfere in one unique process and check whether or not the final result has a consistent energy dependence or is still unitary-violating. The interference can only happen if we allow the spin-2 (and consequently the other fields) to decay in some common final state. This framework is extremely interesting in the context of dark matter particle physics, where the coupling between the dark matter candidate and the Standard Model is achieved with a mediator which couples with both and always decays. We can then rephrase our model as a s-channel simplified dark matter model, adding for example a massive fermionic dark matter candidate to the theory, which couples with the three mediators through its energy-momentum tensor, with the form totally similar to the tensors for the Standard Model fermionic content we saw in equations (2.2.8f)and (2.2.8g), as prescribed by the Lagrangian of equation (3.3.16). This is exactly what it has been done. If one looks the last lines of the FeynRules model in Appendix (C) in fact will notice the construction of the interaction Lagrangian with the dark sector[2]. The first thing to realize is that, if we want to really test the unitarity of the model, we have to consider more complex process than the $2 \to 2$ ones. As already mentioned, when there is a vertex with a current simple enough to be conserved despite the non-universality of the couplings, i.e. when only the energy-momentum for a single field is involved, the

[2]Obviously the model has been modified, changing the coefficients and the signs of vector and scalar fields couplings according to equation (3.3.16).

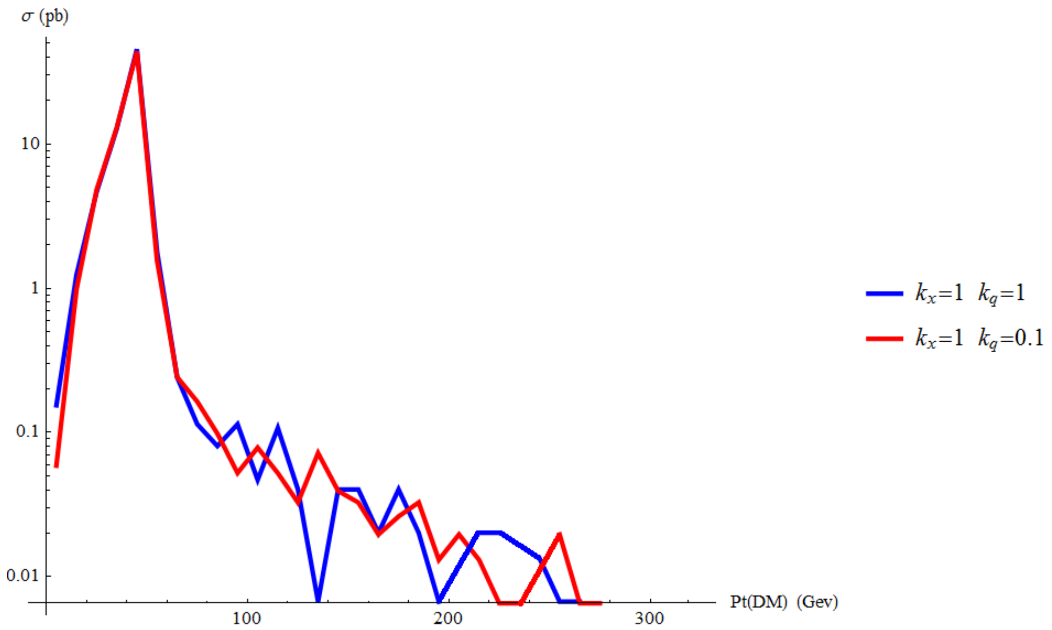


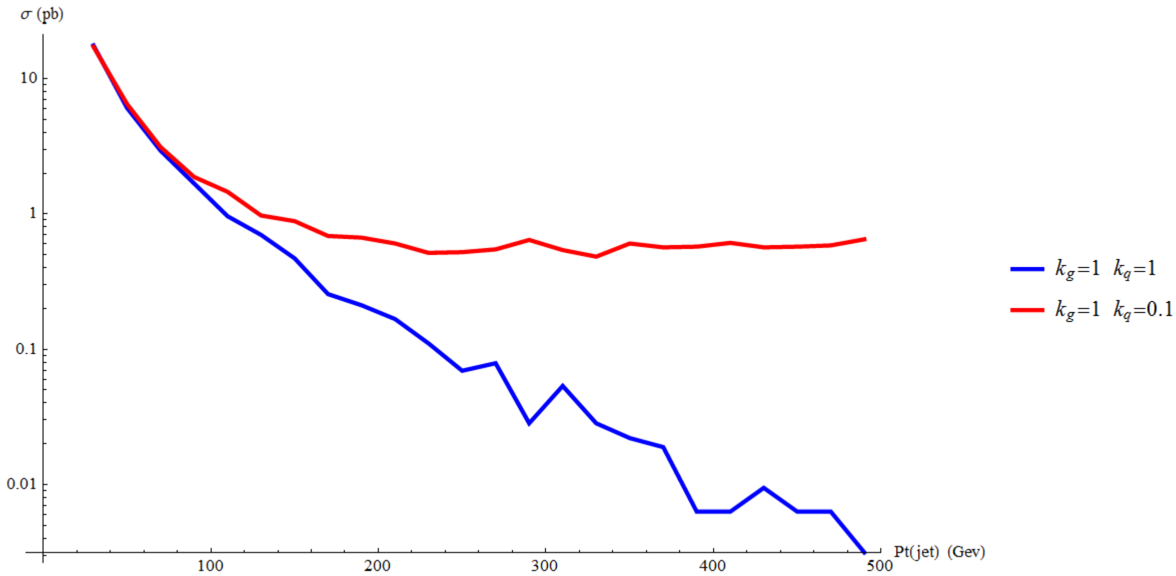
Figure 3.9: Transverse momentum distribution of the dark matter particle for the proton-proton collision production process at $\sqrt{s} = 13$ TeV.

amplitude squared turns out to have the right energy behaviour is is supposed to have. We can quickly check this statement by performing a simple calculation of the total amplitude squared for a dark matter pair production process with a quark anti-quark pair as initial state. The three diagrams involved are given in figure (3.8) and, with the help of the by now familiar computational routine, denoting m_x the dark matter particle mass and with k_x the respective coupling constant, we find the expression for the total amplitude squared

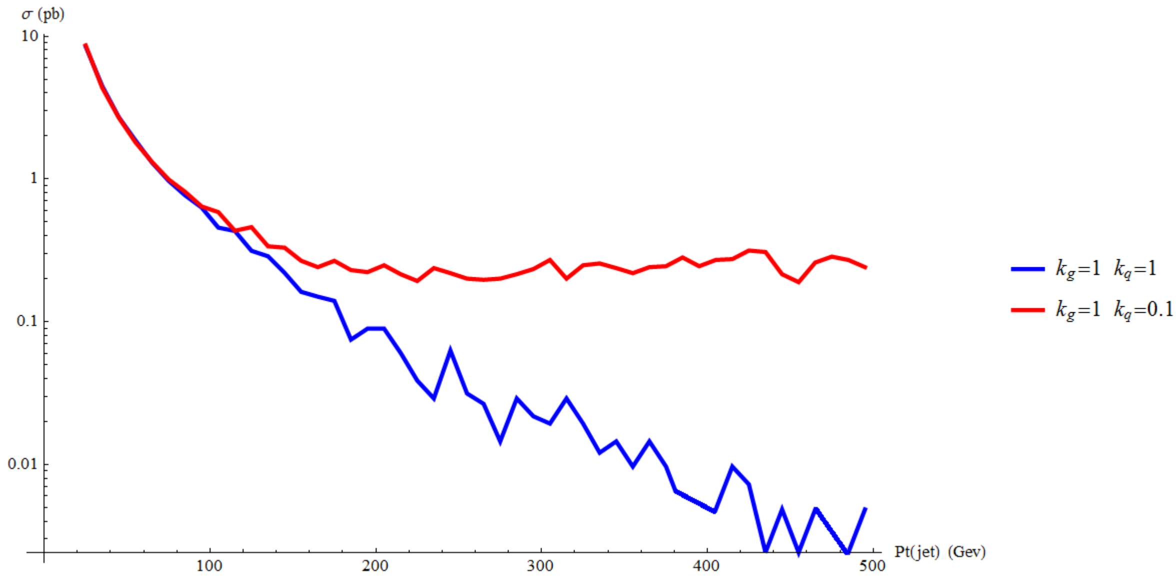
$$|\mathcal{M}|^2 = -\frac{{k_q}^2{k_x}^2}{96\Lambda^4\left(M^2-s\right)^2}\Big[8{m_x}^8 - 8{m_x}^6(t+u) - 10{m_x}^4(t-u)^2 + 4{m_x}^2\big(t^3+t^2u + tu^2+u^3\big) - t^4 + 6t^3u - 18t^2u^2 + 6tu^3 - u^4\Big]. \tag{3.4.1}$$

The expression grows as fast as $\Lambda^{-4}s^2$, that is the expected behaviour for an amplitude squared for a effective theory with two insertion of a dimension five operator, so no unitary problem arises beside the natural one. Moreover, if we evaluate the contributions of the vector and the scalar field separately we eventually find that they both vanish identically and this is another suggestion that they do what they are supposed to. Furthermore, the same results are confirmed numerically with MadGraph as we can see in figure (3.9) the transverse momentum distributions in the universal and in the

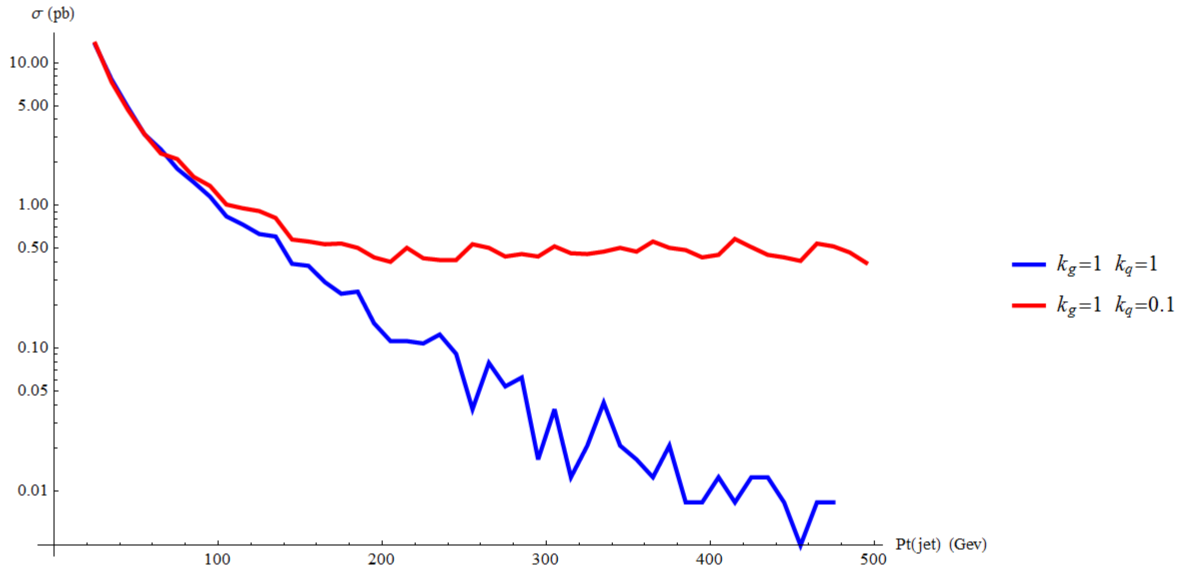
non-universal case are almost identical and very fast decreasing.
The real testing ground then is the slightly more complicated situation of a $2 \to 3$ process, namely the by this time well-known $q\bar{q} \to gX(V, \phi)$, where now the spin-2 particle (vector, scalar) decays in dark matter pair or in any other Standard Model final state. As figure (3.9) allows to visualize, this process once again exhibit the unitary-violating behaviour in every possible different decay channel for the spin-2 particle, of which only some examples are given, but the same behaviour can be found in each allowed case. In particular we present here the transverse momentum distributions for the decay respectively in: fermionic dark matter pair (a), in electron and positron pair (b), in two photons (c) and finally in two Z-bosons (d). This scenario is currently under analysis in order to produce, first of all the analytical results for the amplitudes squared of the processes simulated with the spin-2 particle only, and then to test the whole model with the three mediators to check if the cancellation really happens or more effort as to be expended to improve the model suggested in this work.



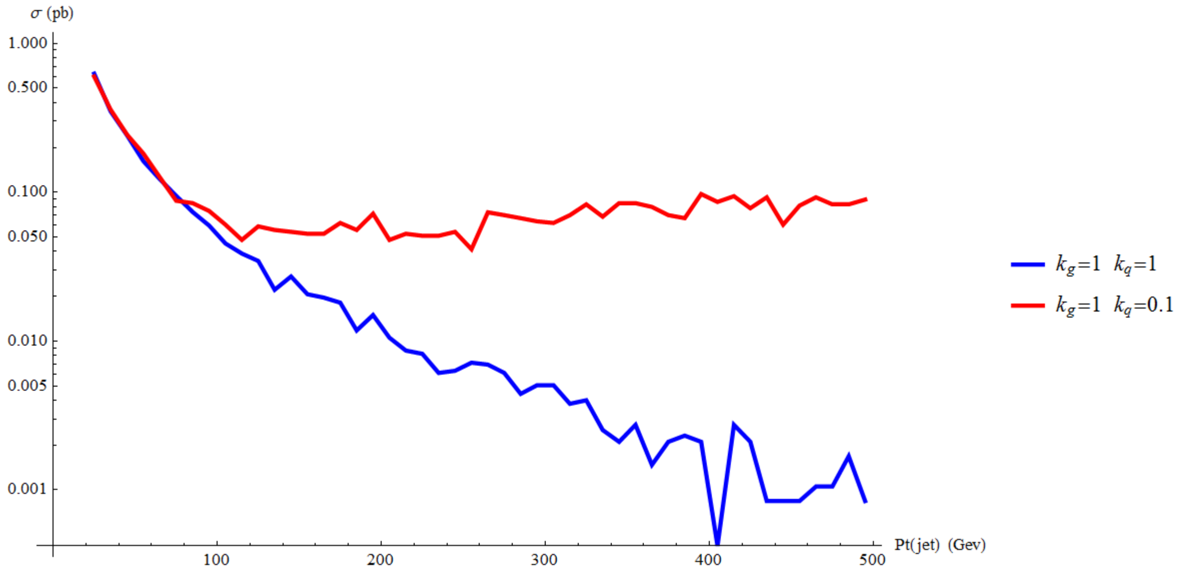
(a) $p\,p > j\ f_x\ f_x$



(b) $p\,p > j\ e^+\ e^-$



(c) $p\,p > j\,\gamma\,\gamma$



(d) $p\,p > j\,Z\,Z$

Figure 3.9: Transverse momentum distribution in $2 \rightarrow 3$ for different decay channels.

Conclusion

Before undertake the final conclusions, we can take a moment to recap the key point of this thesis work and do some final comments to the results we have obtained.
In the first chapter we fully work out the Randall-Sundrum model, which we decided to use as a theoretical foundation and origin for our massive spin-2 particle theory. In fact we have been able to show how this model, in the limit of small perturbation around the background metric, is able to give origin to the Fierz-Pauli action. In particular we pointed out that the Randall-Sundrum construction leads to the only possible formulation of a ghost-free massive spin-2 field, thanks to the Fierz-Pauli tuning for the mass term, which naturally arises without any external constraint, and ensures the right number of degrees of freedom propagating in the theory. Furthermore we demonstrate that is makes sense to study the theory for a massive spin-2 particle thinking about it as the first Kaluza-Klein excitation of the Randall-Sundrum graviton field without have to worry about the presence of an undetected massless zero mode because, due to the totally different five dimensional profile of the wave function, the couplings between the massive modes and the Standard Model are strongly enhanced, leaving the possibility to discover them before the massless mode. Finally we studied the differences between two possible framework constructions. The first, we used as first and simpler example to develop the model, where all the Standard Model field content is confined to live on the 3-brane corresponding to our visible four dimensional world and the second where we allowed everything, except the Higgs filed, to propagate through the five dimensional bulk. Beside the much more effort needed for the model building of the second instance, we focused the attention on the different structure of the couplings between the graviton modes and the various Standard Model fields, which correspond to the zero modes of every five dimensional bulk fermions and gauge bosons. In particular we saw that, even if the coupling with matter is obtained through the contraction between the graviton field and the energy-momentum tensor in both models, in the second the coupling constants values are influenced by the bulk profile of the Standard Model particles also, and because of that, they are different from a field to another, since there is the freedom to build the model in such a way to have the zero modes of the fermionic fields localized wherever we prefer to, along the extra dimension.
Motivated from this analysis, in the second chapter we proceeded in the detailed study of the massive spin-2 theory described by the Fierz-Pauli action, interacting with the Standard Model through its energy-momentum tensor as instructed. We have been able to solve the free theory, finding the mode decomposition, the structure of the tensor

polarizations and the form of the propagator. Then we moved on, investigating the interaction term in detail and giving an overview of the extremely vast and variegated structure of the Feynman rules. Lastly we described the Stückelberg trick, a useful construction which allows us to easily describe the theory in the high energy limit. It consist in the decomposition of the various helicities of the spin-2 particle into different degrees of freedom, respectively a vector and a scalar field, which have the purpose of mimic respectively to the $h = \pm 1$ and $h = 0$.

In the last chapter we finally took under consideration the model with non-universal couplings, focusing on the main feature it has, that is the unitary-violating behaviour. We explicitly saw it, with the concrete example of the spin-2 production process $q\bar{q} \to gX$, that, instead of grow with the energy as $\Lambda^{-2}s$, as expected from a dimension five effective filed theory, the amplitude squared has terms which grow as fast as $\Lambda^{-2}M^{-4}s^3$, clearly violating the unitarity and hugely lowering the cut-off scale for the effective theory validity. Then we tried to understand the origin of such behaviour and, evaluating separately the various helicities contributions to the total amplitude squared of the process we found out that the terms with the strong energy dependence are coming mainly from the longitudinal component and also from the $h = \pm 1$ helicities. Finally, we argued that a possible solution to restore the unitarity, without waive the non-universal couplings framework, could be to introduce new degrees of freedom in the model which cancel the contributions from the unitary-violating helicities. We studied in more details the vector and the scalar field arising from the Stückelberg trick, thinking them as more general fields suggested from the Stückelberg formulation, and we saw that, with some minor changes, there is an hope to use these fields to achieve the desired result since they give promising results at least at the amplitude squared level. Ultimately we proposed a possible Lagrangian, implementing the above mentioned fields for further studies in the direction of the unitarization of the massive spin-2 model we discussed.

For the last comment it is necessary to focus on the outlooks of this work. Even though the preliminary results obtained are promising indications that the model is worth studying in details, there is no substantial proof of its proper functioning. As mentioned, some aspects are under investigation already with the hope of being able to provide a consistent and unitary effective model for a spin-2 particle in the generic framework of non-universal couplings, which would be of extreme interest for both theoretical and experimental point of view.

A. Ricci Tensor

A.1 Ricci Tensor of Randall-Sundrum Background

We want to evaluate the Ricci Tensor for the Randall-Sundrum metric as in equation (1.2.31):

$$ds^2 = e^{-2k|y|}\,\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2 = g_{MN}(y)\,dx^M dx^N. \tag{A.1.1}$$

The first thing to do is to work out the Christoffel symbols

$$\Gamma^R_{MN} = \frac{1}{2}\,g^{RS}\,\big(\partial_M\,g_{NS} + \partial_N\,g_{MS} - \partial_S\,g_{MN}\big). \tag{A.1.2}$$

Since the only non-constant components of g_{MN} are the $\mu\nu$ ones and they are funcion of the extra dimension only we have

$$\partial_R\,g_{MN} = \partial_4\,g_{\mu\nu}, \tag{A.1.3}$$

while every other kind of derivatives vanish. This implies that there are only two types of relevant Christoffel symbols, that are

$$\begin{aligned}\Gamma^4_{\mu\nu} =& \frac{1}{2}\,g^{44}\,\big(\partial_\mu\,g_{\nu 4} + \partial_\nu\,g_{\mu 4} - \partial_4\,g_{\mu\nu}\big)\\ =& -\frac{1}{2}\,gx^{44}\,\partial_4\,g_{\mu\nu}\\ =& -k\,\mathrm{sign}(y)\,e^{-2k|y|}\,\eta_{\mu\nu},\end{aligned} \tag{A.1.4a}$$

and

$$\begin{aligned}\Gamma^\rho_{\mu 4} =& \frac{1}{2}\,g^{\rho\sigma}\,\big(\partial_\mu\,g_{4\sigma} + \partial_4\,g_{\mu\sigma} - \partial_\sigma\,g_{\mu 4}\big)\\ =& \frac{1}{2}\,g^{\rho\sigma}\,\partial_4\,g_{\mu\sigma})\\ =& -k\,\mathrm{sign}(y)\,\delta^\rho_\mu.\end{aligned} \tag{A.1.4b}$$

From these results we can work out the Ricci Tensor components, given by

$$R_{MN} = \partial_R\,\Gamma^R_{MN} - \partial_N\,\Gamma^R_{MR} + \Gamma^R_{RS}\,\Gamma^S_{MN} - \Gamma^R_{NS}\,\Gamma^S_{MR}, \tag{A.1.5}$$

obtaining

$$\begin{aligned} R_{\mu\nu} &= \partial_4 \, \Gamma^4_{\mu\nu} + \Gamma^\rho_{\rho 4} \, \Gamma^4_{\mu\nu} - \Gamma^4_{\nu\rho} \, \Gamma^\rho_{\mu 4} - \Gamma^\rho_{\nu 4} \, \Gamma^4_{\mu\rho} \\ &= -\, 2k \; \delta(y) e^{-2k|y|} \; \eta_{\mu\nu} + 2k^2 \; \eta_{\mu\nu} - k^2 \; e^{-2k|y|} \; \eta_{\mu\nu} \\ &= \Big[2k^2 e^{2k|y|} - k^2 - 2k \; \delta(y) \Big] g_{\mu\nu}, \end{aligned} \tag{A.1.6a}$$

$$\begin{aligned} R_{44} &= -\, \partial_4 \; \Gamma^\rho_{4\rho} - \Gamma^\rho_{4\sigma} \; \Gamma^\sigma_{4\rho} \\ &= -\, 2k \; \delta(y) - k^2 \; \delta^\rho_\sigma \; \delta^\sigma_\rho \\ &= -\, 4k^2 - 2k \; \delta(y), \end{aligned} \tag{A.1.6b}$$

while it is easy to see that the other components vanish

$$R_{\mu 4} = 0. \tag{A.1.6c}$$

A.2 Ricci Scalar of Perturbed Metric

We want to compute the Ricci scalar for the metric

$$ds^2 = \eta_{MN} + h_{MN}\big(x^R\big) \; dx^M dx^N, \tag{A.2.1}$$

that, with the gauge choice we made in equations (1.2.42) becomes:

$$ds^2 = \Big(\eta_{\mu\nu} + h_{\mu\nu}\big(x^R\big) \Big) \; dx^\mu dx^\nu - dz^2. \tag{A.2.2}$$

At the same time, by the fact that we want to do this calculation at the linear order, the Christoffel symbols reduce to

$$\Gamma^R_{MN} = \frac{1}{2} \; \big(\partial_M \; h^R_N + \partial_N \; h^R_M - \partial^R \; h_{MN} \big), \tag{A.2.3}$$

where the indices are rised and lowered with the flat metric η_{MN}. In the chosen gauge only two types of Christoffel symbols are non-vanishing:

$$\Gamma^\rho_{\mu\nu} \frac{1}{2} \left(\partial_\mu \; h^\rho_\nu + \partial_\nu \; h^\rho_\mu - \partial^\rho \; h_{\mu\nu} \right), \tag{A.2.4a}$$

$$\Gamma^\rho_{\mu 4} = \frac{1}{2} \; \partial_4 \; h^\rho_\mu, \tag{A.2.4b}$$

$$\Gamma^4_{\mu\nu} = -\frac{1}{2} \; \partial^4 \; h_{\mu\nu}. \tag{A.2.4c}$$

The next step is to evaluate the Ricci tensor, obtained again from equation (A.1.5), reminding once again the gauge conditions (in particular the tracelessness (1.2.42c)) and disregarding all non-linar terms

$$R_{\mu\nu} = \partial_R \, \Gamma^R_{\mu\nu} + \mathcal{O}(h^2) = -\frac{1}{2} \, \partial_R \partial^R \, h_{\mu\nu} + \mathcal{O}(h^2), \tag{A.2.5a}$$

$$R_{\mu 4} = 0, \tag{A.2.5b}$$

$$R_{44} = 0 + \mathcal{O}(h^2). \tag{A.2.5c}$$

Finally we can calculate the Ricci scalar contracting the Ricci tensor with the flat metric to keep the evaluation at the linear order

$$R = R_{MN} \, \eta^{MN} = R_{\mu\nu} \, \eta^{\mu\nu} + R_{44} \, \eta^{44} = 0 \tag{A.2.6}$$

for the traceless condition again.

B. Feyncalc Notebooks

B.1 Computation of $q\bar{q} \to gX$

Amplitudes of the Four Feynman Diagrams Involved:

vertex contribution

```
amp1 = 1 / Λ TensorFunction[X, μ, ν] * PolarizationVector[p2, ρ] * -gs * kq
  SpinorVBar[q2].(1 / 2 MT[μ, ρ] GA[ν] + 1 / 2 MT[ρ, ν] GA[μ]).SpinorU[q1]
```

$$-\frac{\text{gs kq}\,\bar{\varepsilon}^{\rho}(\text{p2})\,X(\mu,\nu)\,\bar{v}(\text{q2}).\left(\frac{1}{2}\bar{\gamma}^{\nu}\bar{g}^{\mu\rho}+\frac{1}{2}\bar{\gamma}^{\mu}\bar{g}^{\nu\rho}\right).(\text{u}(\text{q1}))}{\Lambda}$$

The s-channel diagram

```
amp2 = 1 / Λ TensorFunction[X, μ, ν] * PolarizationVector[p2, ρ] * gs *
  kg / SP[q1 + q2, q1 + q2] * (SP[(q1 + q2), p2] * (MT[μ, σ] MT[ν, ρ] + MT[μ, ρ] MT[ν, σ]) +
    MT[ρ, σ] * (FV[(q1 + q2), μ] FV[p2, ν] + FV[p2, μ] FV[(q1 + q2), ν]) -
    MT[μ, ρ] * (FV[p2, σ] FV[p2, ν] + FV[(q1 + q2), σ] FV[p2, ν]) -
    MT[ν, ρ] * (FV[p2, σ] FV[p2, μ] + FV[(q1 + q2), σ] FV[p2, μ]) -
    MT[μ, σ] * (FV[(q1 + q2), ν] FV[(q1 + q2), ρ]) - MT[ν, σ] *
     (FV[(q1 + q2), μ] FV[(q1 + q2), ρ])) * SpinorVBar[q2].GA[σ].SpinorU[q1]
```

$$\frac{1}{\Lambda(\overline{\text{q1}}+\overline{\text{q2}})^2}\,\text{gs kg}\,\bar{\varepsilon}^{\rho}(\text{p2})\,X(\mu,\nu)\,\bar{v}(\text{q2}).\bar{\gamma}^{\sigma}.(\text{u}(\text{q1}))$$
$$\Big(\left(\bar{g}^{\mu\sigma}\bar{g}^{\nu\rho}+\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma}\right)\left(\overline{\text{p2}}\cdot(\overline{\text{q1}}+\overline{\text{q2}})\right)+\bar{g}^{\mu\rho}\left(-\left(\overline{\text{p2}}^{\nu}\,\overline{\text{p2}}^{\sigma}+\overline{\text{p2}}^{\nu}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\sigma}\right)\right)-\bar{g}^{\nu\rho}\left(\overline{\text{p2}}^{\mu}\,\overline{\text{p2}}^{\sigma}+\overline{\text{p2}}^{\mu}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\sigma}\right)+$$
$$\bar{g}^{\rho\sigma}\left(\overline{\text{p2}}^{\nu}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\mu}+\overline{\text{p2}}^{\mu}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\nu}\right)-\bar{g}^{\mu\sigma}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\nu}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\rho}-\bar{g}^{\nu\sigma}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\mu}\left(\overline{\text{q1}}+\overline{\text{q2}}\right)^{\rho}\Big)$$

The t-channel diagram

```
amp3 = 1 / Λ TensorFunction[X, μ, ν] *
  PolarizationVector[p2, ρ] * -gs * kq / (SP[q1 - p1, q1 - p1])
  SpinorVBar[q2].(GA[ρ].(GS[(p1 - q1)]).((+1 / 4 * FV[(q1 - p1) + q1, ν] GA[μ] +
        1 / 4 * FV[(q1 - p1) + q1, μ] GA[ν]))).SpinorU[q1]
```

$$-\frac{\text{gs kq}\,\bar{\varepsilon}^{\rho}(\text{p2})\,X(\mu,\nu)\,\bar{v}(\text{q2}).\bar{\gamma}^{\rho}.\left(\bar{\gamma}\cdot\left(\overline{\text{p1}}-\overline{\text{q1}}\right)\right).\left(\frac{1}{4}\bar{\gamma}^{\mu}\left(2\,\overline{\text{q1}}-\overline{\text{p1}}\right)^{\nu}+\frac{1}{4}\bar{\gamma}^{\nu}\left(2\,\overline{\text{q1}}-\overline{\text{p1}}\right)^{\mu}\right).(\text{u}(\text{q1}))}{\Lambda(\overline{\text{q1}}-\overline{\text{p1}})^2}$$

The u-channel diagram

```
amp4 = 1 / Λ TensorFunction[X, μ, ν] * PolarizationVector[p2, ρ] *
  -gs * kq / (SP[q1 - p2, q1 - p2]) SpinorVBar[q2].
   ((+1 / 4 * FV[(q1 - p2) - q2, ν] GA[μ] + 1 / 4 * FV[(q1 - p2) - q2, μ] GA[ν]).
     (GS[(p2 - q1)]).GA[ρ]).SpinorU[q1]
```

$$-\frac{\text{gs kq}\,\bar{\varepsilon}^{\rho}(\text{p2})\,X(\mu,\nu)\,\bar{v}(\text{q2}).\left(\frac{1}{4}\bar{\gamma}^{\mu}\left(-\overline{\text{p2}}+\overline{\text{q1}}-\overline{\text{q2}}\right)^{\nu}+\frac{1}{4}\bar{\gamma}^{\nu}\left(-\overline{\text{p2}}+\overline{\text{q1}}-\overline{\text{q2}}\right)^{\mu}\right).\left(\bar{\gamma}\cdot\left(\overline{\text{p2}}-\overline{\text{q1}}\right)\right).\bar{\gamma}^{\rho}.(\text{u}(\text{q1}))}{\Lambda(\overline{\text{q1}}-\overline{\text{p2}})^2}$$

Computation of the Total Amplitude Squared

The total amplitude is:

```
amplitude = Simplify[amp1 + amp2 + amp3 + amp4]
```

$$\frac{1}{\Lambda} \mathrm{gs}\, \bar{\varepsilon}^{\rho}(\mathrm{p2})\, X(\mu,\nu) \Bigg(\frac{1}{(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^2} \mathrm{kg}\, \bar{v}(\mathrm{q2}).\bar{\gamma}^{\sigma}.(u(\mathrm{q1}))\Big((\bar{g}^{\mu\sigma}\bar{g}^{\nu\rho}+\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma})(\overline{\mathrm{p2}}\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})) - \overline{\mathrm{p2}}^{\nu}\bar{g}^{\mu\rho}\big(\overline{\mathrm{p2}}^{\sigma}+(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\sigma}\big) - \overline{\mathrm{p2}}^{\mu}\bar{g}^{\nu\rho}\big(\overline{\mathrm{p2}}^{\sigma}+(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\sigma}\big) + \bar{g}^{\rho\sigma}\big(\overline{\mathrm{p2}}^{\nu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\mu}+\overline{\mathrm{p2}}^{\mu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\nu}\big) - \bar{g}^{\mu\sigma}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\nu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\rho} - \bar{g}^{\nu\sigma}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\mu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\rho}\Big) - \mathrm{kq}\, \bar{v}(\mathrm{q2}).\left(\frac{1}{2}(\bar{\gamma}^{\nu}\bar{g}^{\mu\rho}+\bar{\gamma}^{\mu}\bar{g}^{\nu\rho})\right).(u(\mathrm{q1})) - \frac{\mathrm{kq}\, \bar{v}(\mathrm{q2}).\bar{\gamma}^{\rho}.(\bar{\gamma}\cdot(\overline{\mathrm{p1}}-\overline{\mathrm{q1}})).\left(\frac{1}{4}\big(\bar{\gamma}^{\mu}(2\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^{\nu}+\bar{\gamma}^{\nu}(2\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^{\mu}\big)\right).(u(\mathrm{q1}))}{(\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^2} - \frac{\mathrm{kq}\, \bar{v}(\mathrm{q2}).\left(\frac{1}{4}\big(\bar{\gamma}^{\mu}(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}})^{\nu}+\bar{\gamma}^{\nu}(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}})^{\mu}\big)\right).(\bar{\gamma}\cdot(\overline{\mathrm{p2}}-\overline{\mathrm{q1}})).\bar{\gamma}^{\rho}.(u(\mathrm{q1}))}{(\overline{\mathrm{q1}}-\overline{\mathrm{p2}})^2} \Bigg)$$

Squaring the amplitude we obtain a colour factor c=4 (we have $\tau_{ij}^{a}\, \tau_{ji}^{b} = \delta^{ab}/2$ from the trace of the generators and than we have another δ^{ab} from the gluon polarization sum, so, for every diagram, we obtain c= $\delta^{ab}\delta^{ab}/2=4$) so that the amplitude squared is:

```
amplsqrd = 4 * amplitude * ComplexConjugate[amplitude /. {μ → α, ν → β, ρ → γ, σ → δ}]
```

$$\frac{1}{\Lambda^2} 4\, \mathrm{gs}^2\, \bar{\varepsilon}^{*\gamma}(\mathrm{p2})\, \bar{\varepsilon}^{\rho}(\mathrm{p2})\, X(\alpha,\beta)\, X(\mu,\nu) \Bigg(\frac{1}{(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^2} \mathrm{kg}\, (\varphi(\overline{\mathrm{q1}})).\bar{\gamma}^{\delta}.(\varphi(\overline{\mathrm{q2}}))\Big((\bar{g}^{\alpha\delta}\bar{g}^{\beta\gamma}+\bar{g}^{\alpha\gamma}\bar{g}^{\beta\delta})(\overline{\mathrm{p2}}\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})) + \bar{g}^{\gamma\delta}\big(\overline{\mathrm{p2}}^{\beta}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\alpha}+\overline{\mathrm{p2}}^{\alpha}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\beta}\big) - \overline{\mathrm{p2}}^{\alpha}\bar{g}^{\beta\gamma}\big(\overline{\mathrm{p2}}^{\delta}+(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\delta}\big) - \overline{\mathrm{p2}}^{\beta}\bar{g}^{\alpha\gamma}\big(\overline{\mathrm{p2}}^{\delta}+(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\delta}\big) - \bar{g}^{\beta\delta}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\alpha}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\gamma} - \bar{g}^{\alpha\delta}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\beta}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\gamma}\Big) - \frac{1}{2}\mathrm{kq}\, (\varphi(\overline{\mathrm{q1}})).(\bar{\gamma}^{\beta}\bar{g}^{\alpha\gamma}+\bar{\gamma}^{\alpha}\bar{g}^{\beta\gamma}).(\varphi(\overline{\mathrm{q2}})) - \frac{\mathrm{kq}\, (\varphi(\overline{\mathrm{q1}})).\big(\bar{\gamma}^{\beta}(2\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^{\alpha}+\bar{\gamma}^{\alpha}(2\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^{\beta}\big).(\bar{\gamma}\cdot(\overline{\mathrm{p1}}-\overline{\mathrm{q1}})).\bar{\gamma}^{\gamma}.(\varphi(\overline{\mathrm{q2}}))}{4(\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^2} - \frac{\mathrm{kq}\, (\varphi(\overline{\mathrm{q1}})).\bar{\gamma}^{\gamma}.(\bar{\gamma}\cdot(\overline{\mathrm{p2}}-\overline{\mathrm{q1}})).\big(\bar{\gamma}^{\beta}(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}})^{\alpha}+\bar{\gamma}^{\alpha}(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}})^{\beta}\big).(\varphi(\overline{\mathrm{q2}}))}{4(\overline{\mathrm{q1}}-\overline{\mathrm{p2}})^2} \Bigg) \Bigg(\frac{1}{(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^2} \mathrm{kg}\, \bar{v}(\mathrm{q2}).\bar{\gamma}^{\sigma}.(u(\mathrm{q1}))\Big((\bar{g}^{\mu\sigma}\bar{g}^{\nu\rho}+\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma})(\overline{\mathrm{p2}}\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})) - \overline{\mathrm{p2}}^{\nu}\bar{g}^{\mu\rho}\big(\overline{\mathrm{p2}}^{\sigma}+(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\sigma}\big) - \overline{\mathrm{p2}}^{\mu}\bar{g}^{\nu\rho}\big(\overline{\mathrm{p2}}^{\sigma}+(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\sigma}\big) + \bar{g}^{\rho\sigma}\big(\overline{\mathrm{p2}}^{\nu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\mu}+\overline{\mathrm{p2}}^{\mu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\nu}\big) - \bar{g}^{\mu\sigma}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\nu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\rho} - \bar{g}^{\nu\sigma}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\mu}(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^{\rho}\Big) - \mathrm{kq}\, \bar{v}(\mathrm{q2}).\left(\frac{1}{2}(\bar{\gamma}^{\nu}\bar{g}^{\mu\rho}+\bar{\gamma}^{\mu}\bar{g}^{\nu\rho})\right).(u(\mathrm{q1})) - \frac{\mathrm{kq}\, \bar{v}(\mathrm{q2}).\bar{\gamma}^{\rho}.(\bar{\gamma}\cdot(\overline{\mathrm{p1}}-\overline{\mathrm{q1}})).\left(\frac{1}{4}\big(\bar{\gamma}^{\mu}(2\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^{\nu}+\bar{\gamma}^{\nu}(2\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^{\mu}\big)\right).(u(\mathrm{q1}))}{(\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^2} - \frac{\mathrm{kq}\, \bar{v}(\mathrm{q2}).\left(\frac{1}{4}\big(\bar{\gamma}^{\mu}(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}})^{\nu}+\bar{\gamma}^{\nu}(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}})^{\mu}\big)\right).(\bar{\gamma}\cdot(\overline{\mathrm{p2}}-\overline{\mathrm{q1}})).\bar{\gamma}^{\rho}.(u(\mathrm{q1}))}{(\overline{\mathrm{q1}}-\overline{\mathrm{p2}})^2} \Bigg)$$

Now we can set the Mandelstam variables,

```
SetMandelstam[s, t, u, q1, q2, -p1, -p2, 0, 0, M, 0];
```

Then we can sum over graviton and gluon polarizations:

```
step1 = amplsqrd /.
   PolarizationVector[p2, ρ] * ComplexConjugate[PolarizationVector[p2, γ]] →
    PolarizationSum[ρ, γ] /. TensorFunction[X, μ, ν] TensorFunction[X, α, β] →
   1 / 2 (PolarizationSum[μ, α, p1] PolarizationSum[ν, β, p1] +
       PolarizationSum[μ, β, p1] PolarizationSum[ν, α, p1]) -
    1 / 3 PolarizationSum[μ, ν, p1] PolarizationSum[β, α, p1]
```

$$-\frac{1}{\Lambda^2} 4\,\mathrm{gs}^2\, \bar{g}^{\gamma\rho}$$

$$\left(\frac{1}{2}\left(\left(\frac{\overline{\mathrm{p1}}^{\alpha}\,\overline{\mathrm{p1}}^{\nu}}{M^2}-\bar{g}^{\alpha\nu}\right)\left(\frac{\overline{\mathrm{p1}}^{\beta}\,\overline{\mathrm{p1}}^{\mu}}{M^2}-\bar{g}^{\beta\mu}\right)+\left(\frac{\overline{\mathrm{p1}}^{\alpha}\,\overline{\mathrm{p1}}^{\mu}}{M^2}-\bar{g}^{\alpha\mu}\right)\left(\frac{\overline{\mathrm{p1}}^{\beta}\,\overline{\mathrm{p1}}^{\nu}}{M^2}-\bar{g}^{\beta\nu}\right)\right)-\frac{1}{3}\left(\frac{\overline{\mathrm{p1}}^{\alpha}\,\overline{\mathrm{p1}}^{\beta}}{M^2}-\bar{g}^{\alpha\beta}\right)\left(\frac{\overline{\mathrm{p1}}^{\mu}\,\overline{\mathrm{p1}}^{\nu}}{M^2}-\bar{g}^{\mu\nu}\right)\right)$$

$$\Bigg(\frac{1}{(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^2}\mathrm{kg}\,(\varphi(\overline{\mathrm{q1}})).\bar{\gamma}^{\delta}.(\varphi(\overline{\mathrm{q2}}))\Big(\left(\bar{g}^{\alpha\delta}\bar{g}^{\beta\gamma}+\bar{g}^{\alpha\gamma}\bar{g}^{\beta\delta}\right)\left(\overline{\mathrm{p2}}\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})\right)+\bar{g}^{\gamma\delta}\left(\overline{\mathrm{p2}}^{\beta}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\alpha}+\overline{\mathrm{p2}}^{\alpha}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\beta}\right)-$$

$$\overline{\mathrm{p2}}^{\alpha}\bar{g}^{\beta\gamma}\left(\overline{\mathrm{p2}}^{\delta}+\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\delta}\right)-\overline{\mathrm{p2}}^{\beta}\bar{g}^{\alpha\gamma}\left(\overline{\mathrm{p2}}^{\delta}+\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\delta}\right)-\bar{g}^{\beta\delta}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\alpha}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\gamma}-$$

$$\bar{g}^{\alpha\delta}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\beta}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\gamma}\Big)-\frac{1}{2}\,\mathrm{kq}\,(\varphi(\overline{\mathrm{q1}})).\left(\bar{\gamma}^{\beta}\bar{g}^{\alpha\gamma}+\bar{\gamma}^{\alpha}\bar{g}^{\beta\gamma}\right).(\varphi(\overline{\mathrm{q2}}))-$$

$$\frac{\mathrm{kq}\,(\varphi(\overline{\mathrm{q1}})).\left(\bar{\gamma}^{\beta}\left(2\,\overline{\mathrm{q1}}-\overline{\mathrm{p1}}\right)^{\alpha}+\bar{\gamma}^{\alpha}\left(2\,\overline{\mathrm{q1}}-\overline{\mathrm{p1}}\right)^{\beta}\right).\left(\bar{\gamma}\cdot\left(\overline{\mathrm{p1}}-\overline{\mathrm{q1}}\right)\right).\bar{\gamma}^{\gamma}.(\varphi(\overline{\mathrm{q2}}))}{4(\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^2}-$$

$$\frac{\mathrm{kq}\,(\varphi(\overline{\mathrm{q1}})).\bar{\gamma}^{\gamma}.\left(\bar{\gamma}\cdot\left(\overline{\mathrm{p2}}-\overline{\mathrm{q1}}\right)\right).\left(\bar{\gamma}^{\beta}\left(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}}\right)^{\alpha}+\bar{\gamma}^{\alpha}\left(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}}\right)^{\beta}\right).(\varphi(\overline{\mathrm{q2}}))}{4(\overline{\mathrm{q1}}-\overline{\mathrm{p2}})^2}\Bigg)$$

$$\Bigg(\frac{1}{(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})^2}\mathrm{kg}\,\bar{v}(\mathrm{q2}).\bar{\gamma}^{\sigma}.(\mathrm{u}(\mathrm{q1}))\Big(\left(\bar{g}^{\mu\sigma}\bar{g}^{\nu\rho}+\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma}\right)\left(\overline{\mathrm{p2}}\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})\right)-\overline{\mathrm{p2}}^{\nu}\bar{g}^{\mu\rho}\left(\overline{\mathrm{p2}}^{\sigma}+\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\sigma}\right)-$$

$$\overline{\mathrm{p2}}^{\mu}\bar{g}^{\nu\rho}\left(\overline{\mathrm{p2}}^{\sigma}+\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\sigma}\right)+\bar{g}^{\rho\sigma}\left(\overline{\mathrm{p2}}^{\nu}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\mu}+\overline{\mathrm{p2}}^{\mu}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\nu}\right)-\bar{g}^{\mu\sigma}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\nu}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\rho}-$$

$$\bar{g}^{\nu\sigma}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\mu}\left(\overline{\mathrm{q1}}+\overline{\mathrm{q2}}\right)^{\rho}\Big)-\mathrm{kq}\,\bar{v}(\mathrm{q2}).\left(\frac{1}{2}\left(\bar{\gamma}^{\nu}\bar{g}^{\mu\rho}+\bar{\gamma}^{\mu}\bar{g}^{\nu\rho}\right)\right).(\mathrm{u}(\mathrm{q1}))-$$

$$\frac{\mathrm{kq}\,\bar{v}(\mathrm{q2}).\bar{\gamma}^{\rho}.\left(\bar{\gamma}\cdot\left(\overline{\mathrm{p1}}-\overline{\mathrm{q1}}\right)\right).\left(\frac{1}{4}\left(\bar{\gamma}^{\mu}\left(2\,\overline{\mathrm{q1}}-\overline{\mathrm{p1}}\right)^{\nu}+\bar{\gamma}^{\nu}\left(2\,\overline{\mathrm{q1}}-\overline{\mathrm{p1}}\right)^{\mu}\right)\right).(\mathrm{u}(\mathrm{q1}))}{(\overline{\mathrm{q1}}-\overline{\mathrm{p1}})^2}-$$

$$\frac{\mathrm{kq}\,\bar{v}(\mathrm{q2}).\left(\frac{1}{4}\left(\bar{\gamma}^{\mu}\left(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}}\right)^{\nu}+\bar{\gamma}^{\nu}\left(-\overline{\mathrm{p2}}+\overline{\mathrm{q1}}-\overline{\mathrm{q2}}\right)^{\mu}\right)\right).\left(\bar{\gamma}\cdot\left(\overline{\mathrm{p2}}-\overline{\mathrm{q1}}\right)\right).\bar{\gamma}^{\rho}.(\mathrm{u}(\mathrm{q1}))}{(\overline{\mathrm{q1}}-\overline{\mathrm{p2}})^2}\Bigg)$$

And take the average over the incoming quarks spin and colour that will lead to a term (1/2*1/2*1/3*1/3 = 1/36)

i) Trace over spin indices:

```
step2 = 1 / 36 * FermionSpinSum[Contract[step1]];
```

ii) Evaluation of the traces:

```
step3 = step2 /. DiracTrace → Tr;
```

And after few semplifications we obtain the result:

```
totamplsqrd = Contract[step3];
```

```
res = Simplify[TrickMandelstam[DiracTrick[Expand[totamplsqrd]], {s, t, u, M^2}] ]
```

$$\frac{1}{27\,\Lambda^2\,M^4\,s\,t\,u}\,\mathrm{gs}^2\,\Big(2\,\mathrm{kg}^2\,t\,u\left(M^4\left(7\,t^2+12\,t\,u+7\,u^2\right)-2\,M^2\left(t^3+7\,t^2\,u+7\,t\,u^2+u^3\right)+(t+u)^2\left(t^2+u^2\right)\right)-4\,\mathrm{kg}\,\mathrm{kq}\,s\,t\,u\left(-9\,M^6+4\,M^4\,(s+3\,u)-2\,M^2\left(s^2+7\,s\,u+6\,u^2\right)+s\left(s^2+2\,s\,u+2\,u^2\right)\right)+\mathrm{kq}^2\,s\left(6\,M^{10}-6\,M^8\,(t+u)+3\,M^6\left(t^2+u^2\right)-12\,M^4\,t\,u\,(t+u)+2\,M^2\,t\,u\left(t^2+12\,t\,u+u^2\right)-2\,t\,u\left(t^3+t^2\,u+t\,u^2+u^3\right)\right)\Big)$$

And we can obtain the result in the center of mass frame also, substituting t as a function of θ, the angle between q and X:

```
rescm = Simplify[ res /. u → (M^2 - s - t) /. t → (M^2 - s) (1 - Cos[θ]) / (2)]
```

$$\frac{1}{27\,\Lambda^2\,M^4\,s\,(\cos(\theta)-1)\,(\cos(\theta)+1)\left(M^2-s\right)^2}\,\mathrm{gs}^2\,\Big(\cos^4(\theta)\left(M^2-s\right)^4\left(\mathrm{kg}^2\left(6\,M^4-6\,M^2\,s+s^2\right)+2\,\mathrm{kg}\,\mathrm{kq}\,s\left(6\,M^2-s\right)+\mathrm{kq}^2\,s\left(s-6\,M^2\right)\right)+6\,M^2\,s\cos^2(\theta)\left(M^2-s\right)^2\left(2\,\mathrm{kg}^2\left(M^2-s\right)^2+2\,\mathrm{kg}\,\mathrm{kq}\left(M^4+3\,M^2\,s-2\,s^2\right)+\mathrm{kq}^2\left(-\left(M^4+2\,M^2\,s-2\,s^2\right)\right)\right)-\mathrm{kg}^2\left(6\,M^4+6\,M^2\,s+s^2\right)\left(M^2-s\right)^4-2\,\mathrm{kg}\,\mathrm{kq}\,s\left(12\,M^6+5\,M^4\,s-4\,M^2\,s^2-s^3\right)\left(M^2-s\right)^2-\mathrm{kq}^2\,s^2\left(25\,M^8+2\,M^6\,s-6\,M^4\,s^2+2\,M^2\,s^3+s^4\right)\Big)$$

B.2 Helicities Contributions

We go a little faster here, avoiding to show many outputs since they are identical to the notebook in Appendix (B.1) on which we refer for further details.

```
amp1 = 1 / Λ TensorFunction[X, μ, ν] * PolarizationVector[p2, ρ] * -gs * kq
   SpinorVBar[q2].(1 / 2 MT[μ, ρ] GA[ν] + 1 / 2 MT[ρ, ν] GA[μ]).SpinorU[q1];
```

```
amp2 = 1 / Λ TensorFunction[X, μ, ν] * PolarizationVector[p2, ρ] * gs *
    kg / SP[q1 + q2, q1 + q2] * (SP[(q1 + q2), p2] * (MT[μ, σ] MT[ν, ρ] + MT[μ, ρ] MT[ν, σ]) +
      MT[ρ, σ] * (FV[(q1 + q2), μ] FV[p2, ν] + FV[p2, μ] FV[(q1 + q2), ν]) -
      MT[μ, ρ] * (FV[p2, σ] FV[p2, ν] + FV[(q1 + q2), σ] FV[p2, ν]) -
      MT[ν, ρ] * (FV[p2, σ] FV[p2, μ] + FV[(q1 + q2), σ] FV[p2, μ]) -
      MT[μ, σ] * (FV[(q1 + q2), ν] FV[(q1 + q2), ρ]) -
      MT[ν, σ] * (FV[(q1 + q2), μ] FV[(q1 + q2), ρ])) * SpinorVBar[q2].GA[σ].SpinorU[q1];
```

```
amp3 = 1 / Λ TensorFunction[X, μ, ν] *
   PolarizationVector[p2, ρ] * -gs * kq / (SP[q1 - p1, q1 - p1])
   SpinorVBar[q2].(GA[ρ].(GS[(p1 - q1)]).((+1 / 4 * FV[(q1 - p1) + q1, ν] GA[μ] +
         1 / 4 * FV[(q1 - p1) + q1, μ] GA[ν]))).SpinorU[q1];
```

```
amp4 = 1 / Λ TensorFunction[X, μ, ν] *
   PolarizationVector[p2, ρ] * -gs * kq / (SP[q1 - p2, q1 - p2])
   SpinorVBar[q2].((+1 / 4 * FV[(q1 - p2) - q2, ν] GA[μ] + 1 / 4 * FV[(q1 - p2) - q2, μ] GA[ν]).
      (GS[(p2 - q1)]).GA[ρ]).SpinorU[q1];
```

The total amplitude is:

```
amplitude = Simplify[amp1 + amp2 + amp3 + amp4];
```

Squaring the amplitude we obtain a colour factor c=4 as we already know:

```
amplsqrd = 4 * amplitude * ComplexConjugate[amplitude /. {μ → α, ν → β, ρ → γ, σ → δ}];
```

Now we explicilty write down the scalar products between all the four-vectors involved in the calculation, polarization vectors included (respectively “ep” and “em” denote the two transverse and “el” the longitudinal):

```
SP[q1, q1] = 0;
SP[q2, q2] = 0;
SP[p1, p1] = M^2;
SP[p2, p2] = 0;
SP[q1, p1] = (s + M^2) / 4 - (s - M^2) * ((2 * t) / (s - M^2) + 1) / 4;
SP[q2, p2] = (s - M^2) / 4 * (1 - ((2 * t) / (s - M^2) + 1));
SP[q1, p2] = (s - M^2) / 4 * (1 + ((2 * t) / (s - M^2) + 1));
SP[q2, p1] = (s + M^2) / 4 + (s - M^2) * ((2 * t) / (s - M^2) + 1) / 4;
SP[q1, q2] = s / 2;
SP[p1, p2] = (s - M^2) / 2;
SP[ep, p1] = 0;
SP[ep, p2] = 0;
SP[ep, q1] = (Sqrt[s] * (2 / (s - M^2) * Sqrt[t * M^2 - t * s - t^2])) / (2 * Sqrt[2]);
SP[ep, q2] = - (Sqrt[s] * (2 / (s - M^2) * Sqrt[t * M^2 - t * s - t^2])) / (2 * Sqrt[2]);
SP[em, p1] = 0;
SP[em, p2] = 0;
SP[em, q1] = - (Sqrt[s] * (2 / (s - M^2) * Sqrt[t * M^2 - t * s - t^2])) / (2 * Sqrt[2]);
```

```
SP[em, q2] = (Sqrt[s] * (2 / (s - M^2) * Sqrt[t * M^2 - t * s - t^2])) / (2 * Sqrt[2]);
SP[el, p1] = 0;
SP[el, p2] = (s - M^2) / (2 * M);
SP[el, q1] = (s - M^2) / (4 * M) - ((s + M^2) * ((2 * t) / (s - M^2) + 1)) / (4 * M);
SP[el, q2] = (s - M^2) / (4 * M) + ((s + M^2) * ((2 * t) / (s - M^2) + 1)) / (4 * M);
SP[ep, ep] = 0;
SP[em, em] = 0;
SP[el, el] = -1;
SP[ep, em] = 1;
SP[ep, el] = 0;
SP[em, el] = 0;
```

Contribution for h=±2

We can perform the sum over the gluon polarization and the substitution we learnt in equation (3.2.5)

```
step1 = amplsqrd /.
   PolarizationVector[p2, ρ] * ComplexConjugate[PolarizationVector[p2, γ]] →
    PolarizationSum[ρ, γ] /. TensorFunction[X, μ, ν] TensorFunction[X, α, β] →
   FV[ep, μ] FV[ep, ν] FV[em, α] FV[em, β] + FV[em, μ] FV[em, ν] FV[ep, α] FV[ep, β]
```

$$-\frac{1}{\Lambda^2} 4\, gs^2\, \bar{g}^{\gamma\rho} \left(\overline{em}^{\mu}\, \overline{em}^{\nu}\, \overline{ep}^{\alpha}\, \overline{ep}^{\beta} + \overline{em}^{\alpha}\, \overline{em}^{\beta}\, \overline{ep}^{\mu}\, \overline{ep}^{\nu}\right)$$

$$\Bigg(\frac{1}{(\overline{q1}+\overline{q2})^2}\, kg\, (\varphi(\overline{q1})).\bar{\gamma}^{\delta}.(\varphi(\overline{q2}))\Big((\bar{g}^{\alpha\delta}\bar{g}^{\beta\gamma}+\bar{g}^{\alpha\gamma}\bar{g}^{\beta\delta})(\overline{p2}\cdot(\overline{q1}+\overline{q2}))+$$
$$\bar{g}^{\gamma\delta}\left(\overline{p2}^{\beta}(\overline{q1}+\overline{q2})^{\alpha}+\overline{p2}^{\alpha}(\overline{q1}+\overline{q2})^{\beta}\right)-\overline{p2}^{\alpha}\bar{g}^{\beta\gamma}\left(\overline{p2}^{\delta}+(\overline{q1}+\overline{q2})^{\delta}\right)-\overline{p2}^{\beta}\bar{g}^{\alpha\gamma}\left(\overline{p2}^{\delta}+(\overline{q1}+\overline{q2})^{\delta}\right)-$$
$$\bar{g}^{\beta\delta}(\overline{q1}+\overline{q2})^{\alpha}(\overline{q1}+\overline{q2})^{\gamma}-\bar{g}^{\alpha\delta}(\overline{q1}+\overline{q2})^{\beta}(\overline{q1}+\overline{q2})^{\gamma}\Big)-\frac{1}{2}kq\,(\varphi(\overline{q1})).(\bar{\gamma}^{\beta}\bar{g}^{\alpha\gamma}+\bar{\gamma}^{\alpha}\bar{g}^{\beta\gamma}).(\varphi(\overline{q2}))-$$
$$\frac{kq\,(\varphi(\overline{q1})).\left(\bar{\gamma}^{\beta}(2\,\overline{q1}-\overline{p1})^{\alpha}+\bar{\gamma}^{\alpha}(2\,\overline{q1}-\overline{p1})^{\beta}\right).(\bar{\gamma}\cdot(\overline{p1}-\overline{q1})).\bar{\gamma}^{\gamma}.(\varphi(\overline{q2}))}{4(\overline{q1}-\overline{p1})^2}-$$
$$\frac{1}{4(\overline{q1}-\overline{p2})^2}kq\,(\varphi(\overline{q1})).\bar{\gamma}^{\gamma}.(\bar{\gamma}\cdot(\overline{p2}-\overline{q1})).\left(\bar{\gamma}^{\beta}(-\overline{p2}+\overline{q1}-\overline{q2})^{\alpha}+\bar{\gamma}^{\alpha}(-\overline{p2}+\overline{q1}-\overline{q2})^{\beta}\right).(\varphi(\overline{q2}))\Bigg)$$

$$\Bigg(\frac{1}{(\overline{q1}+\overline{q2})^2}\, kg\, \bar{v}(q2).\bar{\gamma}^{\sigma}.(u(q1))\Big((\bar{g}^{\mu\sigma}\bar{g}^{\nu\rho}+\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma})(\overline{p2}\cdot(\overline{q1}+\overline{q2}))-\overline{p2}^{\nu}\bar{g}^{\mu\rho}\left(\overline{p2}^{\sigma}+(\overline{q1}+\overline{q2})^{\sigma}\right)-$$
$$\overline{p2}^{\mu}\bar{g}^{\nu\rho}\left(\overline{p2}^{\sigma}+(\overline{q1}+\overline{q2})^{\sigma}\right)+\bar{g}^{\rho\sigma}\left(\overline{p2}^{\nu}(\overline{q1}+\overline{q2})^{\mu}+\overline{p2}^{\mu}(\overline{q1}+\overline{q2})^{\nu}\right)-\bar{g}^{\mu\sigma}(\overline{q1}+\overline{q2})^{\nu}(\overline{q1}+\overline{q2})^{\rho}-$$
$$\bar{g}^{\nu\sigma}(\overline{q1}+\overline{q2})^{\mu}(\overline{q1}+\overline{q2})^{\rho}\Big)-kq\,\bar{v}(q2).\left(\frac{1}{2}(\bar{\gamma}^{\nu}\bar{g}^{\mu\rho}+\bar{\gamma}^{\mu}\bar{g}^{\nu\rho})\right).(u(q1))-$$
$$\frac{kq\,\bar{v}(q2).\bar{\gamma}^{\rho}.(\bar{\gamma}\cdot(\overline{p1}-\overline{q1})).\left(\frac{1}{4}\left(\bar{\gamma}^{\mu}(2\,\overline{q1}-\overline{p1})^{\nu}+\bar{\gamma}^{\nu}(2\,\overline{q1}-\overline{p1})^{\mu}\right)\right).(u(q1))}{(\overline{q1}-\overline{p1})^2}-$$
$$\frac{1}{(\overline{q1}-\overline{p2})^2}kq\,\bar{v}(q2).\left(\frac{1}{4}\left(\bar{\gamma}^{\mu}(-\overline{p2}+\overline{q1}-\overline{q2})^{\nu}+\bar{\gamma}^{\nu}(-\overline{p2}+\overline{q1}-\overline{q2})^{\mu}\right)\right).(\bar{\gamma}\cdot(\overline{p2}-\overline{q1})).\bar{\gamma}^{\rho}.(u(q1))\Bigg)$$

And then we take the average over the spin of the incoming quarks

```
step2 = 1 / 36 * FermionSpinSum[Contract[step1]];

step3 = step2 /. DiracTrace → Tr;

totamplsqrd = Contract[step3];
```

The result is

```
res2 = Simplify[DiracTrick[Expand[totamplsqrd]]]
```

$$\frac{1}{9\,\Lambda^2\,s\,(M^2-s)^4}4\,\text{gs}^2\left(M^4-2\,M^2\,(s+t)+s^2+2\,s\,t+2\,t^2\right)$$
$$\left(\text{kg}^2\left(M^2-s\right)^4+2\,\text{kg}\,\text{kq}\,s\left(2\,M^2-s\right)\left(M^2-s\right)^2+2\,\text{kq}^2\,s^2\left(2\,M^4-2\,M^2\,s+s^2\right)\right)$$

Which can be written also in the centre of momentum frame

```
rescm2 = Simplify[ res2 /. u → (M^2 - s - t) /. t → (M^2 - s) (1 - Cos[θ]) / (2)]
```

$$\frac{\text{gs}^2\,(\cos(2\,\theta)+3)\left(\text{kg}^2\left(M^2-s\right)^4+2\,\text{kg}\,\text{kq}\,s\left(2\,M^2-s\right)\left(M^2-s\right)^2+2\,\text{kq}^2\,s^2\left(2\,M^4-2\,M^2\,s+s^2\right)\right)}{9\,\Lambda^2\,s\left(M^2-s\right)^2}$$

Contribution of h=±1

Now we do the replacement with the instruction given by equation (3.2.6) and we suppress the output here and in the next section for sake of brevity

```
step1 = amplsqrd /.
   PolarizationVector[p2, ρ] * ComplexConjugate[PolarizationVector[p2, γ]] →
    PolarizationSum[ρ, γ] /. TensorFunction[X, μ, ν] TensorFunction[X, α, β] →
   -1 / 2 (FV[ep, μ] FV[el, ν] + FV[el, μ] FV[ep, ν])
     (FV[em, α] FV[el, β] + FV[el, α] FV[em, β]) -
    1 / 2 (FV[em, μ] FV[el, ν] + FV[el, μ] FV[em, ν])
     (FV[ep, α] FV[el, β] + FV[el, α] FV[ep, β])

step2 = 1 / 36 * FermionSpinSum[Contract[step1]];

step3 = step2 /. DiracTrace → Tr;

totamplsqrd = Contract[step3];
```

In this case we finally obtain

```
res1 = Simplify[DiracTrick[Expand[totamplsqrd]]]
```

$$\frac{1}{9\,\Lambda^2\,M^2\,t\left(M^2-s\right)^4\left(M^2-s-t\right)}$$
$$\text{gs}^2\Big(8\,\text{kg}^2\,t^2\left(M^2-s\right)^4\left(-M^2+s+t\right)^2-4\,\text{kg}\,\text{kq}\,t\left(M^2-s\right)^2\left(M^{10}-M^8\,(3\,s+5\,t)+M^6\left(3\,s^2+2\,s\,t+8\,t^2\right)-\right.$$
$$\left.M^4\left(s^3-15\,s^2\,t-8\,s\,t^2+4\,t^3\right)-8\,M^2\,s\,t\left(2\,s^2+3\,s\,t+t^2\right)+4\,s^2\,t\,(s+t)^2\right)+$$
$$\text{kq}^2\left(M^{16}-2\,M^{14}\,(2\,s+3\,t)+M^{12}\left(7\,s^2+10\,s\,t+14\,t^2\right)-4\,M^{10}\left(2\,s^3-3\,s\,t^2+4\,t^3\right)+\right.$$
$$M^8\left(7\,s^4-12\,s^2\,t^2-48\,s\,t^3+8\,t^4\right)-2\,M^6\,s\left(2\,s^4+5\,s^3\,t+50\,s^2\,t^2+16\,s\,t^3-16\,t^4\right)+$$
$$\left.M^4\,s^2\left(s^4+6\,s^3\,t+126\,s^2\,t^2+160\,s\,t^3+48\,t^4\right)-16\,M^2\,s^3\,t^2\left(3\,s^2+5\,s\,t+2\,t^2\right)+8\,s^4\,t^2\,(s+t)^2\right)\Big)$$

That in the centre of momenum frame reads

```
rescm1 = Simplify[ res1 /. u → (M^2 - s - t) /. t → (M^2 - s) (1 - Cos[θ]) / (2)]
```

$$\frac{1}{9\,\Lambda^2 M^2 \left(M^2-s\right)^2} 2\,\mathrm{gs}^2 \csc^2(\theta) \Big(\cos^4(\theta)\left(\mathrm{kg}^2\left(M^2-s\right)^4+2\,\mathrm{kg}\,\mathrm{kq}\left(M^4+2M^2 s-s^2\right)\left(M^2-s\right)^2+\mathrm{kq}^2\left(M^8+4M^6 s+6M^4 s^2-4M^2 s^3+s^4\right)\right)- \cos^2(\theta)\Big(2\,\mathrm{kg}^2\left(M^2-s\right)^4+2\,\mathrm{kg}\,\mathrm{kq}\left(M^4+4M^2 s-2s^2\right)\left(M^2-s\right)^2+ \mathrm{kq}^2\left(-M^8+4M^6 s+9M^4 s^2-8M^2 s^3+2s^4\right)\Big)+ \mathrm{kg}^2\left(M^2-s\right)^4+2\,\mathrm{kg}\,\mathrm{kq}\,s\left(2M^2-s\right)\left(M^2-s\right)^2+\mathrm{kq}^2 s^2\left(5M^4-4M^2 s+s^2\right)\Big)$$

Contribution of h=0

Finally we use equation (3.2.7) and substitute

```
step1 = amplsqrd /.
   PolarizationVector[p2, ρ] * ComplexConjugate[PolarizationVector[p2, γ]] →
    PolarizationSum[ρ, γ] /. TensorFunction[X, μ, ν] TensorFunction[X, α, β] →
   1 / 6 (FV[ep, μ] FV[em, ν] + FV[em, μ] FV[ep, ν] + 2 FV[el, μ] FV[el, ν])
    (FV[ep, α] FV[em, β] + FV[em, α] FV[ep, β] + 2 FV[el, α] FV[el, β])

step2 = 1 / 36 * FermionSpinSum[Contract[step1]];

step3 = step2 /. DiracTrace → Tr;

totamplsqrd = Contract[step3];
```

And find

```
res0 = Simplify[DiracTrick[Expand[totamplsqrd]]]
```

$$\frac{1}{27\,\Lambda^2\left(M^3-M\,s\right)^4} 2\,\mathrm{gs}^2 s\Big(\mathrm{kg}^2\left(M^2-s\right)^4\left(M^4-2M^2(s+t)+s^2+2st+2t^2\right)+2\,\mathrm{kg}\,\mathrm{kq}\left(M^2-s\right)^2 \left(2M^8-2M^6(s+5t)+M^4\left(-3s^2+6st+10t^2\right)+2M^2 s\left(2s^2+3st+2t^2\right)-s^2\left(s^2+2st+2t^2\right)\right)+ \mathrm{kq}^2\big(13M^{12}-2M^{10}(9s+25t)+M^8\left(-3s^2+10st+50t^2\right)+4M^6 s\left(s^2+13st+10t^2\right)+ M^4 s^2\left(9s^2-4st-12t^2\right)-2M^2 s^3\left(3s^2+5st+4t^2\right)+s^4\left(s^2+2st+2t^2\right)\big)\Big)$$

with the correspondent expression in the center of momentum frame

```
rescm0 = Simplify[ res0 /. u → (M^2 - s - t) /. t → (M^2 - s) (1 - Cos[θ]) / (2)]
```

$$\frac{\mathrm{gs}^2 s\left((\mathrm{kg}-\mathrm{kq})^2\left(M^2-s\right)^4+\cos^2(\theta)\left(\mathrm{kg}\left(M^2-s\right)^2+\mathrm{kq}\left(5M^4+2M^2 s-s^2\right)\right)^2\right)}{27\,\Lambda^2 M^4\left(M^2-s\right)^2}$$

C. FeynRules Model

```
(***** Setting for interaction order ******)

M$InteractionOrderLimit = {
 {DMT, 2}
};

M$InteractionOrderHierarchy = {
 {QCD, 1}, {DMT, 2}, {QED, 2}
};

(* ************************* *)
(* *****     Fields     ***** *)
(* ************************* *)
M$ClassesDescription = {

S[12] == { ClassName -> YS,
     SelfConjugate -> True,
     Mass -> {MYS, 1000.},
     Width -> {WYS, 1.},
     TeX -> S},

V[13] == { ClassName -> YV,
     SelfConjugate -> True,
     Mass -> {MYV, 1000.},
     Width -> {WYV, 1.},
     TeX -> V},

F[7] == { ClassName -> Xd,
          SelfConjugate -> False,
       Mass -> {MXd, 10.},
          Width -> 0,
          PDG -> 5000521,
          TeX -> Subscript[X,d],
          FullName -> "Dirac DM" },

T[1] == { ClassName -> Y2,
       SelfConjugate -> True,
       Symmetric -> True,
       Mass -> {MY2, 1000.},
          Width -> {WY2, 1.},
          PDG -> 5000002,
          TeX -> Subscript[Y,2],
          FullName -> "Spin-2 mediator" }

};
```

```
(* ************************** *)
(* *****   Parameters   ***** *)
(* ************************** *)

M$Parameters = {

Lambda == { ParameterType -> External,
             BlockName -> DMINPUTS,
          TeX -> \[CapitalLambda],
          Description -> "cut-off scale",
          Value -> 1000.0},

gTg == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,g],
      Description -> "g-Y2  coupling",
      Value -> 1. },

 gTw == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,W],
      Description -> "W-Y2  coupling",
      Value -> 1. },

 gTb == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,B],
      Description -> "B-Y2  coupling",
      Value -> 1. },

 gTq == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,q],
      Description -> "q-Y2  coupling",
      Value -> 1. },

 gTq3 == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,q3],
      Description -> "t-Y2  coupling",
      Value -> 1. },
```

```
gTl == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,l],
      Description -> "l-Y2  coupling",
      Value -> 1. },

 gTh == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,h],
      Description -> "h-Y2  coupling",
      Value -> 1. },

gTx == {
      ParameterType -> External,
      InteractionOrder -> {DMT, 1},
      BlockName -> DMINPUTS,
      TeX -> Subscript[gT,x],
      Description -> "Xd-Y2  coupling",
      Value -> 1. }
};

(*** Defining the cov derivatives ***)

covdelE[field_, mu_] :=
  Module[{j, a},  del[field, mu]
                 + I ee/cw 2 B[mu]/2 ProjP.field + I ee/cw B[mu]/2
ProjM.field + I ee/sw/2 ProjM.field Wi[mu,3]];

covdelN[field_, mu_] :=
  Module[{j, a}, del[field, mu] + I ee/cw B[mu]/2 ProjM.field - I ee/sw/2
ProjM.field Wi[mu,3]];

(*** Defining the energy-momentum tensor T[mu,nu] ***)

(* Fermions *)

TFf[mu_, nu_, ff_] :=   QLbar[ss, ii, ff, cc].Ga[mu, ss, ss1].DC[QL[ss1,
ii, ff, cc], nu] -    DC[QLbar[ss, ii, ff, cc], mu].Ga[nu, ss,
ss1].QL[ss1, ii, ff, cc] +
                                  uRbar[ss, ff, cc].Ga[mu, ss,
ss1].DC[uR[ss1, ff, cc], nu] - DC[uRbar[ss, ff, cc], mu].Ga[nu, ss,
ss1].uR[ss1, ff, cc] +
                                 dRbar[ss, ff, cc].Ga[mu, ss,
ss1].DC[dR[ss1, ff, cc], nu] - DC[dRbar[ss, ff, cc], mu].Ga[nu, ss,
ss1].dR[ss1, ff, cc];

TFhb[mu_, nu_] :=   QLbar[ss, 2, 3, cc].Ga[mu, ss, ss1].DC[QL[ss1, 2, 3,
cc], nu] -
   DC[QLbar[ss, 2, 3, cc], mu].Ga[nu, ss, ss1].QL[ss1, 2, 3, cc] +
   dRbar[ss, 3, cc].Ga[mu, ss, ss1].DC[dR[ss1, 3, cc], nu] -
   DC[dRbar[ss, 3, cc], mu].Ga[nu, ss, ss1].dR[ss1, 3, cc];
```

```
TFht[mu_, nu_] :=   QLbar[ss, 1, 3, cc].Ga[mu, ss, ss1].DC[QL[ss1, 1, 3,
cc], nu] -
   DC[QLbar[ss, 1, 3, cc], mu].Ga[nu, ss, ss1].QL[ss1, 1, 3, cc] +
   uRbar[ss, 3, cc].Ga[mu, ss, ss1].DC[uR[ss1, 3, cc], nu] -
   DC[uRbar[ss, 3, cc], mu].Ga[nu, ss, ss1].uR[ss1, 3, cc];

TFlq[mu_, nu_] := -ME[mu, nu] I/2 TFf[al, al, 1] + I/4 ( TFf[mu, nu, 1] +
TFf[nu, mu, 1]) - ME[mu, nu] I/2 TFf[al, al, 2] + I/4 ( TFf[mu, nu, 2] +
TFf[nu, mu, 2])-ME[mu, nu] I/2 TFhb[al, al] + I/4 ( TFhb[mu, nu] +
TFhb[nu, mu]);

TFt[mu_, nu_] := -ME[mu, nu] I/2 TFht[al, al] + I/4 ( TFht[mu, nu] +
TFht[nu, mu]);

feynmangaugerules =  If[Not[FeynmanGauge], {G0 | GP | GPbar -> 0}, {}];
yuk =  ExpandIndices[-yd[ff2, 3] CKM[3, ff2] QLbar[sp, ii, 3, cc].dR[sp,
3, cc] Phi[ii] - yu[3, 3] QLbar[sp, ii, 3, cc].uR[sp, 3, cc] Phibar[jj]
Eps[ii, jj], FlavorExpand -> SU2D];
yuk = yuk /. {CKM[a_, b_] Conjugate[CKM[a_, c_]] -> IndexDelta[b, c],
CKM[b_, a_] Conjugate[CKM[c_, a_]] -> IndexDelta[b, c]};

TYt[mu_, nu_] := -ME[mu, nu] (yuk + HC[yuk] /. feynmangaugerules)

TFlep[mu_, nu_] :=   LLbar[ss, ii, ff].Ga[mu, ss, ss1].DC[LL[ss1, ii,
ff], nu] -    DC[LLbar[ss, ii, ff], mu].Ga[nu, ss, ss1].LL[ss1, ii, ff] +
                                   lRbar[ss, ff].Ga[mu, ss,
ss1].DC[lR[ss1, ff], nu] - DC[lRbar[ss, ff], mu].Ga[nu, ss, ss1].lR[ss1,
ff] ;

TFl[mu_, nu_] := -ME[mu, nu] I/2 TFlep[al, al] + I/4 ( TFlep[mu, nu] +
TFlep[nu, mu]);

(* Higgs *)

Tscalar[mu_, nu_] := (2 DC[Phibar[ii], mu] DC[Phi[ii], nu]) - ME[mu, nu]
(DC[Phibar[ii], rho] DC[Phi[ii], rho] + muH^2 Phibar[ii] Phi[ii] - lam
Phibar[ii] Phi[ii] Phibar[jj] Phi[jj]);

(* Gauge bosons *)

TGg[mu_,nu_] := -ME[mu,nu] (-1/4 FS[G,rho,sig,a] FS[G,rho,sig,a]) -
FS[G,mu,rho,a] FS[G,nu,rho,a];
(*new lag for the weak sector before EWSB*)
(*Careful to check the gauge fixing term coefficient*)
TGB[mu_,nu_] := -ME[mu,nu](-1/4 FS[B,rho,sig]FS[B,rho,sig])-
FS[B,mu,rho]FS[B,nu,rho];
TGW[mu_,nu_] := -ME[mu,nu](-1/4 FS[Wi,rho,sig,ii]FS[Wi,rho,sig,ii])-
FS[Wi,mu,rho,ii]FS[Wi,nu,rho,ii];

(* Gauge fixing term is here because Madgraph takes the Feynman gauge for
massless gauge boson propagators *)
(* and unitary gauge for massive gauge boson propagators. *)
```

```
TGFg[mu_,nu_]:= -ME[mu,nu].( del[del[G[sig, a1], sig], rho].G[rho, a1]
+ 1/2 del[G[rho, a1], rho].del[G[sig, a1], sig] ) + del[del[G[rho, a1],
rho], mu].G[nu, a1] + del[del[G[rho, a1], rho], nu].G[mu, a1];

TGFa[mu_,nu_]:= -ME[mu,nu].( del[del[A[sig], sig], rho].A[rho] +1/2
del[A[rho], rho].del[A[sig], sig] ) + del[del[A[rho], rho], mu].A[nu] +
del[del[A[rho], rho], nu].A[mu];

(** Ghost **)

(*TGhost[mu_,nu_] := ( -ME[mu,nu].(DC[ghGbar,rho] DC[ghG,rho])
+DC[ghGbar,mu] DC[ghG,nu] + DC[ghGbar,nu] DC[ghG,mu] ); *)

LQCDGhs = -ghGbar[ii].del[DC[ghG[ii], mu], mu];

TGhost[mu_,nu_] := -ME[mu,nu](ExpandIndices[ LQCDGhs , FlavorExpand-
>SU2W]) + ( del[ghGbar[a], mu].(del[ghG[a], nu] - gs f[a,b,c] G[nu,c]
ghG[b] ) +   del[ghGbar[a], nu].(del[ghG[a], mu] - gs f[a,b,c] G[mu,c]
ghG[b] ));

(*** Writing the lagrangian ***)

L2f := -1/Lambda (gTq  TFlq[mu, nu] + gTq3 (TFt[mu, nu] + TYt[mu, nu]) +
gTl (TFl[mu, nu] + TYl[mu, nu])) (Y2[mu, nu] + (Sqrt[2])/(Sqrt[3]*MYS^2)
del[del[YS, nu], mu] - 1/Sqrt[6] ME[mu,nu] YS + 1/(Sqrt[2]*MYV)
(del[YV[nu],mu] + del[YV[mu],nu]));
L2v := -1/Lambda ExpandIndices[ ( gTg (TGg[mu,nu]+TGFg[mu,nu]) + gTw
TGW[mu,nu] + gTb TGB[mu,nu] + (gTb cw^2 + gTw sw^2) TGFa[mu,nu]) (Y2[mu,
nu] + (Sqrt[2])/(Sqrt[3]*MYS^2) del[del[YS, nu], mu] - 1/Sqrt[6]
ME[mu,nu] YS + 1/(Sqrt[2]*MYV) (del[YV[nu],mu] +
del[YV[mu],nu])),FlavorExpand->True];
L2gh := -1/Lambda (gTg TGhost[mu,nu]              ) (Y2[mu, nu] +
(2)/(Sqrt[3]*MYS^2) del[del[YS, nu], mu] - 1/Sqrt[6] ME[mu,nu] YS +
1/(Sqrt[2]*MYV) (del[YV[nu],mu] + del[YV[mu],nu]));
L2H  := -1/Lambda ExpandIndices[(gTh Tscalar[mu,nu]              )(Y2[mu,
nu] + (Sqrt[2])/(Sqrt[3]*MYS^2) del[del[YS, nu], mu] - 1/Sqrt[6]
ME[mu,nu] YS + 1/(Sqrt[2]*MYV) (del[YV[nu],mu] +
del[YV[mu],nu])),FlavorExpand->True] /. feynmangaugerules;

LY2YSYVSM := L2f + L2v + L2gh + L2H;

(**** DM sector ***)

TFqX[mu_,nu_] := (-ME[mu,nu] ( I Xdbar.(Ga[rho].del[Xd, rho]) -1/2 del[I
Xdbar.Ga[rho].Xd, rho]) +(I/2  Xdbar.Ga[mu].del[Xd, nu] - 1/4 I
del[Xdbar.Ga[nu].Xd, mu] + I/2  Xdbar.Ga[nu].del[Xd, mu] - 1/4 I
del[Xdbar.Ga[mu].Xd, nu] ));

TYqX[mu_,nu_] := -ME[mu,nu] ( - MXd Xdbar.Xd );

L2fX := -1/Lambda ( gTx (TFqX[mu,nu] + TYqX[mu,nu]) ) (Y2[mu, nu] +
(Sqrt[2])/(Sqrt[3]*MYS^2) del[del[YS, nu], mu] - 1/Sqrt[6] ME[mu,nu] YS +
1/(Sqrt[2]*MYV) (del[YV[nu],mu] + del[YV[mu],nu]));

L2DM := L2fX + LY2YSYVSM;
```

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Bibliography

[1] Theodor Kaluza. On the Problem of Unity in Physics. *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)*, 1921:966–972, 1921.

[2] Oskar Klein. Quantum Theory and Five-Dimensional Theory of Relativity. (In German and English). *Z. Phys.*, 37:895–906, 1926. [Surveys High Energ. Phys.5,241(1986)].

[3] Csaba Csaki. TASI lectures on extra dimensions and branes. In *From fields to strings: Circumnavigating theoretical physics. Ian Kogan memorial collection (3 volume set)*, pages 605–698, 2004. [,967(2004)].

[4] Maxime Gabella. The Randall-Sundrum Model. 2006.

[5] Tony Gherghetta. Les Houches lectures on warped models and holography. In *Particle physics beyond the standard model. Proceedings, Summer School on Theoretical Physics, 84th Session, Les Houches, France, August 1-26, 2005*, pages 263–311, 2006.

[6] S. Casagrande, F. Goertz, U. Haisch, M. Neubert, and T. Pfoh. Flavor Physics in the Randall-Sundrum Model: I. Theoretical Setup and Electroweak Precision Tests. *JHEP*, 10:094, 2008.

[7] Gustavo Burdman. Constraints on the bulk standard model in the Randall-Sundrum scenario. *Phys. Rev.*, D66:076003, 2002.

[8] J. L. Hewett, F. J. Petriello, and T. G. Rizzo. Precision measurements and fermion geography in the Randall-Sundrum model revisited. *JHEP*, 09:030, 2002.

[9] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo. Bulk gauge fields in the Randall-Sundrum model. *Phys. Lett.*, B473:43–49, 2000.

[10] M. Fierz and W. Pauli. On relativistic wave equations for particles of arbitrary spin in an electromagnetic field. *Proc. Roy. Soc. Lond.*, A173:211–232, 1939.

[11] Kurt Hinterbichler. Theoretical Aspects of Massive Gravity. *Rev. Mod. Phys.*, 84:671–710, 2012.

[12] John F. Donoghue. General relativity as an effective field theory: The leading quantum corrections. *Phys. Rev.*, D50:3874–3888, 1994.

[13] Kaoru Hagiwara, Junichi Kanzaki, Qiang Li, and Kentarou Mawatari. HELAS and MadGraph/MadEvent with spin-2 particles. *Eur. Phys. J.*, C56:435–447, 2008.

[14] Wolfram Research, Inc. Mathematica 9.0.

[15] Adam Alloul, Neil D. Christensen, Céline Degrande, Claude Duhr, and Benjamin Fuks. FeynRules 2.0 - A complete toolbox for tree-level phenomenology. *Comput. Phys. Commun.*, 185:2250–2300, 2014.

[16] Neil D. Christensen and Claude Duhr. FeynRules - Feynman rules made easy. *Comput. Phys. Commun.*, 180:1614–1641, 2009.

[17] Goutam Das, Celine Degrande, Valentin Hirschi, Fabio Maltoni, and Hua-Sheng Shao. NLO predictions for the production of a (750 GeV) spin-two particle at the LHC. 2016.

[18] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro. The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations. *JHEP*, 07:079, 2014.

[19] Priscilla De Aquino. *Beyond Standard Model Phenomenology at the LHC*. PhD thesis, Université Catholique de Louvain, 2012.

[20] P. Artoisenet et al. A framework for Higgs characterisation. *JHEP*, 11:043, 2013.

[21] Duane Dicus and Scott Willenbrock. Angular momentum content of a virtual graviton. *Phys. Lett.*, B609:372–376, 2005.