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Learning and accepting quantum physics. Re-analysis of a teaching proposal

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Non è ricaduta
inerte nel passato
e neppure regressione
nel guscio delle cose già sapute
questo
ritorno della strada
spesso
su se medesima,
ma nuova
conoscenza, forse,
ed illuminazione
di un bene avuto e non ancora inteso.

(Mario Luzi)

Ai miei genitori

A tutti gli amici
da sempre o da oggi
compagni nel va e vieni del viaggio

Abstract

Nella tesi è analizzata nel dettaglio una proposta didattica sulla Fisica Quantistica elaborata dal gruppo di ricerca in Didattica della Fisica dell'Università di Bologna, in collaborazione con il gruppo di ricerca in Fisica Teorica e con ricercatori del CNR di Bologna. La proposta è stata sperimentata in diverse classi V di Liceo scientifico e dalle sperimentazioni sono emersi casi significativi di studenti che non sono riusciti ad *accettare* la teoria quantistica come descrizione convincente ad affidabile della realtà fisica (casi di *non accettazione*), nonostante sembrassero aver capito la maggior parte degli argomenti e essersi 'appropriati' del percorso per come gli era stato proposto.

Questo dato empirico ha posto due questioni, affrontate in dettaglio nella tesi: (1) qual è la natura di questa non accettazione? Rispecchia una presa di posizione epistemologica o è espressione di una mancanza di comprensione profonda? (2) Nel secondo caso, è possibile individuare precisi meccanismi cognitivi che possono ostacolare o facilitare l'accettazione della fisica quantistica?

L'analisi di interviste individuali degli studenti ha permesso di mettere in luce tre principali esigenze cognitive (*cognitive needs*) che sembrano essere coinvolte nell'accettazione e nell'apprendimento della fisica quantistica: le esigenze di *visualizzabilità, comparabilità* e di '*realtà*'.

I '*cognitive needs*' sono stati quindi utilizzati come strumenti di analisi delle diverse proposte didattiche in letteratura e del percorso di Bologna, al fine di metterne in luce le criticità. Sono state infine avanzate alcune proposte per un suo miglioramento.

Contents

Introduction

From its advent, Quantum Physics has deeply revolutionized scientific thinking in its formal, methodological and philosophical dimensions. Today it represents the basis upon which most of the modern theoretical frameworks are developed, and has been proved in its implications with the greatest accuracy ever. However, despite its fruitful successes, quantum physics still challenges scientists' conceptions about physics foundations and arouses a fascinating debate upon the meaning of some of its fundamentals.

With the Italian reform of secondary schools introduced by the Education Minister Mariastella Gelmini in 2010-2011, contents of quantum physics have become part of the official curriculum of all the "scientific *Licei*", because of their cultural significance and their essential role in the comprehension of recent technological developments and applications.

This dissertation is situated precisely in the context of the scientific research on teaching quantum physics in secondary schools and, in particular, it aims at contributing for the improvement of the teaching proposal developed by the research group in Physics Education at the University of Bologna, in collaboration with the research group in Theoretical Physics of the CNR of Bologna. The author of this Dissertation has followed the last stage of the development of the path, its implementations in three classes of scientific Liceo and taught in a further experimentation in a scientific high school in Castel san Pietro.

The main goal of this thesis is to build analytic tools to interpret one of the main evidences that emerged from the quantum path's implementations: the presence of significant cases of students who did not accept quantum physics as an adequate and personally reliable explanation of reality. The cases of *non-acceptance*, as we called them, concern also students who appeared to be confident with the formalism and also to have appropriated the path as it was proposed. Indeed, non-acceptance of quantum physics is not a new issue, both in the historical development of the theory and in didactical physics' research; however, it has never been specifically addressed in detail. The dissertation provides a rather detailed analysis of students' acceptance dynamics and, on the basis of the achieved results, it points out critical points to be strengthened for improving the teaching proposal.

The dissertation is articulated in four chapters, followed by some concluding remarks.

In chapter 1, after a brief sketch of the ministerial guidelines, an overview on the state of art about the different approaches for teaching quantum physics is presented. In this context, it is also collocated the proposal Bologna's group; its main didactical choices and its conceptual structure are pointed out, as well as the school contexts in which it was implemented.

In chapter 2 some cases of clear non-acceptance are reported and, after being contextualised in the research framework where they occurred, they are deeply analysed. From a methodological point of view, a hermeneutic qualitative approach has been chosen, and some specific cognitive requirements, which we called *cognitive needs*, were found and used to interpret students' resistance in accepting quantum physics. A comparative analysis allowed us to conjecture that they could be grouped in three different categories: the need of *visualisation*, *comparability*, and *reality*, in the specific nuances described in the chapter.

In Chapter 3 some critical points of the teaching proposal are presented, namely the *uncertainty principle* and the *superposition state*. The cognitive needs were used as analytical tools for interpreting students' reactions and, by triangulating the findings with literature, some specific suggestions for improving the path were developed. In particular, the issue of the *ontological shift* from classical to quantum physics has been recognised as a crucial cognitive mechanism that can foster or hinder the process of accepting quantum physics.

In Chapter 4 some proposals for improving the teaching path and satisfying the cognitive requirements are developed along the lines of what we found in chapters 2 and 3.

CHAPTER 1

Teaching quantum physics in upper secondary schools

1.1.1 Italians' ministerial instructions for Scientific Licei

A new Physics syllabus for the scientific Licei was introduced by the Education Minister Mariastella Gelimini in the year 2010-11, with the aim of updating scholastic curricula and making them closer and more relevant to present-day issues. The purpose of secondary education in Italian Licei is to:

> "*Provide students with the cultural and methodological tools needed to achieve a thorough understanding of reality; to follow the development of scientific and technological research; to identify interconnections between different forms of knowledge* [...]"
> (Article 2, sub-section 2, "Revisione dell'assetto ordinamentale, organizzativo e didattico dei Licei", 2008)

The choice is to emphasize the knowledge developed in the 20^{th} century. As far as the teaching of Physics in Scientific Licei and Scientific Licei – Option of Applied Science is concerned, the curriculum includes basic concepts of Quantum Physics (QP), since they are considered essential for the comprehension of recent technological developments and applications:

"*The establishment of the light quantum model can be introduced through the study of the thermal radiation and of Planck's hypothesis (perhaps adopting a merely qualitative approach). This model will be developed, on one hand, through the study of the photoelectric effect and its interpretation by Einstein and, on the other hand, by discussing theories and experimental results highlighting the presence of discreet energy levels in the atom. This conceptual itinerary can be concluded by mentioning the experimental evidence for the wave nature of matter, as postulated by de Broglie, as well as the uncertainty principle. Further emphasis on the experimental dimension of physics can be achieved through activities to be carried out in the school's didactic lab, in academic and research institutes, in connection with orientation projects for higher education*" (National Guidelines for Licei, Ministerial Decree n. 211, October 7th, 2010).

The National Guidelines outline a typical qualitative and pseudo-historical approach focused almost exclusively on the "old quantum theory". This approach is the most traditional one and can be found in most textbooks (see for instance (Amaldi, 2015) and (Halliday, Resnik & Walker, 2001)).

Some research on teaching QP has revealed that it is experienced by students and teachers as a sort of 'patchwork' of conceptually disconnected information, often kept together only by the chronological presentation of discoveries. One problematic consequence of this conceptual fragmentation is that students, in the attempt for filling the gaps between these 'chunks' of information, end up associating classical properties to quantum systems. This leads also to disappointing results, which often reveal a deep skepticism towards QP (Tarozzi, 2005).

Despite its numerous drawbacks, the historical approach can be useful – with the necessary precautions –to reflect on the epistemic nature of the subject and on its methodological laws, which may not be explicitly examined in other approaches. Furthermore, the ministerial guidelines leave room for teachers to introduce insights into specific topics pertaining to QP or to choose different approaches.

1.1.2 State of the art

As alternatives to the historical perspective, other paths have been designed based on a *logico-philosophical* and/or a *phenomenological* approach. In this section these approaches are briefly outlined so as to better contextualize the educational choices that led the group of Bologna to design their own path. Further elements of analysis of the approaches are provided in chapter 3.

- ***Logico-philosophical* approach**

The *logico-philosophical* approach originates from the present-day structure of quantum theory, namely from its ‘axiomatic’ structure (Haber-Schaim, 1975; Lawrence, 1996). Although the mathematical formalism cannot be fully developed at a secondary-school level, as Pospiech claims, QP main ideas can be understood by focusing on the concept of *spin*, which does not have any classical counterpart, and *Pauli's matrices*, as it is “*impossible to understand QP without mastering its mathematical structures*” (Pospiesch, 1999). By introducing the concept of spin from scratch, it is possible to move to the *superposition principle* and other elements of QP axiomatic structure, without making use of semi-classical representations which students risk sticking to.

The application of the formalism to various experimental configurations (such as Stern-Gerlach experiments) is aimed to help students to understand the connection between the theoretical and experimental dimensions, so that they can envision a less abstract image of QP. In her works, Pospiech demonstrates the effectiveness of this approach in highlighting the fundamental aspects of QP related to topics as the principles of superposition and indeterminacy, complementarity, entanglement, indistinguishability and measurement process (Pospiech, 2010), and also points out the feasibility of teaching this kind of formalism in secondary schools.

- ***Phenomenological* approach**

This latter approach presents the concepts of QP through a phenomenological analysis of experimental results, in order to build up the theoretical framework on the logical base of what is observed from the experiments themselves.

An exemplification of this approach is the one based on Ghirardi's introduction to QP (Ghirardi, 1997), whose argumentation is based upon experiments about the polarization of light carried out with Polaroid filters and bi-refringent crystals. The experimental outcomes guide the construction of hypothetical interpretative models, explicating each one's inner logic, up until introducing the superposition principle as the only reasonable logic expression for accounting them. Such an approach has been put into didactical practice by the research group in Udine (Michelini, Ragazzoni, Santi, & Stefanel, 2000; Stefanel, 2007). The results of the Udine's group experiments, reported in Stefanel (2007), indicate that – on a conceptual level – most students were able to understand the difference between quantum state and classical state, even if the consequences of the existence of non-compatible observables were not always clear. One further difficulty, linked to indeterminacy, is the fall of classical determinism. As far as formalism is concerned, Stefanel claims it to not hinder students' learning processes, but rather to help them.

A second, interesting, implementation of the phenomenological approach was proposed by the research group in Physics Education at the University of Pavia (Malgieri, 2015). The teaching proposal is based on Feynman's paths method, and it benefits from the support of interactive simulations created with the open-source software GeoGebra. These simulations provide activities of exploration and inquiry that would otherwise be carried out with difficulty in a laboratory. Beside its constructive part being based on Feynman reflections, it is chosen to follow a phenomenological process through different experiments for construction of photon model. Among these experiments are: classical experiments of light diffraction and interference; Young's experiment with single photons, electrons, neutrons and C_{60} molecules; Grangier experiment; Mach-Zehnder and Zhou-Wand-Mandel experiments; experiments with confined quantum particles. The proposal has been implemented in contexts of teacher education and in a class of secondary students (Malgieri, 2015). The results of this latter experimentation will be further considered in the analyses reported in chapters 2 and 3.

1.2 BOLOGNA'S GROUP TEACHING PROPOSAL

1.2.1 Main didactical choices

The Bologna's research group works within the context of the aforementioned experimentations of didactical paths for teaching QP, and has developed different proposals throughout the years.

The first path was described in (Tarozzi, 2005) and (Levrini & Fantini, 2013) and it was designed to create a rich and complex learning environment, in which students could navigate through different personal trajectories; it was divided in two parts, each characterized by a different approach (historical-philosophical in the first one, phenomenological and formal in the second). The leading thread was the concept of 'object' from the 'Old Quantum Physics' to its systematisation through the interpretation of Stern-Gerlach experiments with the Dirac notation for states and Pauli's matrices. The results of this experimentation show that students' difficulties had been turned into cultural challenges, producing a widespread involvement despite their personal attitude towards physics. The learning path, in fact, was built according to some guiding criteria, chosen to problematize knowledge and to foster its cultural value, according to an idea of science in which many points of view are legitimate and possible. The guiding criteria were:

- *Multi-perspectiveness*: the same physical contents (phenomenologies) are analyzed from different perspectives so as to encourage multiple connections among the content and conceptual routes;
- *Multi-dimensionality*: the different perspectives and multiple connections are analyzed and compared also for their philosophical-epistemological peculiarities, as well as for their relations with experiments and formalism;
- *Longitudinality*: the ''game'' of modelling quantum phenomena is systematically analysed and compared with the models already encountered by the students during the study of other physics topics (classical mechanics, special relativity and thermodynamics) (Levrini & Fantini, 2013),

A second proposal was built by a group of researchers of the Physics and Astronomy Department, in collaboration with the CNR-IMM in Bologna to be implemented in a lab for volunteer secondary students. The lab was part of the activities of the 'Piano Lauree Scientifiche' (PLS) and aimed to provide students the chance to understand the essential elements of quantum perspective, starting from the 'The most beautiful experiment in Physics' (MBE), that is the double slit experiment with single electrons, firstly realized in Bologna in 1974 (http://l-esperimento-piu-bello-dellafisica.bo.imm.cnr.it/) (Lulli, 2013), (Levrini, Lulli, Bertozzi, Ercolessi, Matteucci, Monzoni & Pecori, 2014; Stefanini, 2013). The main feature of the path was its multidimensionality, being the epistemological, formal, logical, experimental and applicative aspects of QP discussed and critically analysed.

The experimentation was analysed by Lucia Stefanini who, in her dissertation (2013), wrote:

> "*Students were able to accept the mathematics featured in the course, and they did not perceive it as being out of their league. Mathematical formalism was also regarded as a useful tool for interpreting and understanding the experiments that were carried out.*
> *Questionnaires were handed out to students, aimed at obtaining feedback on several aspects of the course. Results brought to light a great variety of interests, and this is proof that the course's multi-dimensionality was effective in stimulating curiosity and to encourage multiple approaches to scientific knowledge. Students have also grasped the language of formalism in the context of quantum applications, often using it when re-elaborating what they had learnt.*
> *This project has therefore obtained very positive feedback from both students and teachers, as well as showing great potential for possible use as a learning path in fifth-year scientific classes*" (Stefanini, 2013)

The two previous paths converged into a third one that was designed to be implemented in real classes of scientific *Liceo*. This third path is the one analysed also for the purposes of the present dissertation. A detailed description is reported in Lodovico (2016). In the next sections we simply sketch its macro structure and the contexts where it was implemented. The most delicate parts of the proposal (uncertainty principle and the introduction of the quantum interpretative apparatus) are described in some details in chapter 3, where we try to localize some specific difficulties met by the students.

1.2.2 Design of Bologna's teaching proposal

The proposal was designed by a working group of people who involved researchers in physics education, 4 physics and math teachers, post-doc students and undergraduate students (among which the author of this dissertation). The group met every three weeks from December 2014 to May 2015 in order to analyse the previous paths and adapt them to the new school contexts. The challenge was to account both for the National Indications and for the results in physics education.

The core idea developed by the group was to join up a destructive part belonging to the 'old quantum physics' (the *pars destruens*) with a constructive framework (*pars construens*) by using the MBE as an epistemological, experimental and conceptual *junction*. As suggested by Feynman, in fact, this experiment touches the very core of quantum physics, leading to face directly with some contradictions and interpretative limits of classical paradigms.

The *pars destruens* revolves around the four fundamental phenomena related to the "old quantum theory" and foreseen in the National Indications: black body, photoelectric effect, Compton effect and Bohr's atomic model. Even if the choice of dealing with these issues was somehow obliged by ministerial guidelines, the attempt was to strongly bet on this part, in order to foster the discrete-continuous debate. The latter was chosen as leading thread to connect in a sensible way the various phenomena and situate them into a "significance framework".

The *junction* part has the role of leading students towards the *pars construens* by presenting the first steps that led to the search for a new comprehensive theoretical framework that could account for all those phenomena that challenged and put in crisis the classical paradigms. The topics treated are the uncertainty relations, complementarity and the MBE, in the ways that we deeply describe in chapter 3 and in chapter 4. A special role was played by the contribution of Giorgio Lulli, senior researcher at IMM-CNR, and by his line for presenting the experimental and interpretative challenges opened with the MBE (Lulli, 2013).

As far as *pars construens* is concerned, the group chose to follow the path developed for the PLS context by Elisa Ercolessi and Vittorio Monzoni. It focused on Stern-Gerlach experiments, so as to build a constructive framework not linked to classical-like properties and to avoid any semi-classical misconception. According to Pospiech, the researchers decided to focus the construction of the genuine interpretative apparatus on something completely new, as the *spin* of Ag atom.

1.3 Experimentations and research questions

The latter teaching proposal has been implemented in four different schools within the 2014-2015 scholastic year:

- a class of the "A. Einstein" Liceo in Rimini (teacher: Paola Fantini)
- a class of the "A. Righi" Liceo in Bagno di Romagna (teacher: Laura Branchetti)
- a class of the "Archimede" Liceo in San Giovanni in Persiceto (teacher: Elisa Garagnani)
- a class of the "Malpighi Visitandine" high school in Castel san Pietro (teacher: Giovanni Ravaioli, author of the present dissertation)

The teachers were part of the working group who designed the materials of the learning path. A conference of Giorgio Lulli on the MBE was organised for all the classes just as introduction to the *pars construens*.

A broad and detailed analysis of the experimentation held in Rimini by Paola Fantini was carried out by Luca Lodovico and is reported in his master degree dissertation (Lodovico, 2016). The aim of the analysis was to build a comprehensive picture of what happened in class and to check the effectiveness of the learning path to foster processes of *appropriation,* as it is meant by Levrini and co-workers (2015). What was pointed out is an acceptable conceptual understanding and a general good level for students' involvement with the proposed path. Some issues in the classroom dynamics was pointed out as a

possible factor hindering appropriation. Nevertheless, what is mainly important here, was the discovery of cases of non-acceptance of QP, namely students who couldn't accept the theory as a reliable description of physical reality, and a more detailed study on this issue was claimed for. Since this type of cases was pointed out also in the implementation of Pavia proposal, we elevated this phenomenon to 'evidence', worth to be investigated.
This it the very goal of the present dissertation. More specifically, it aims at addressing to the following research questions:

RQ1: *How can the occurring of cases of non-acceptance be interpreted? What is their very nature? Do they simply mirror an epistemological stance, or do they reveal a cognitive lack in the understanding process?*

RQ2: *Is there a particular hidden cognitive mechanism preventing or fostering acceptance of quantum physics? If so, can it be pointed out?*

Chapters 2 and 3 aim to answer respectively these two questions. The answers will orient the revision and improvement of the teaching proposal, as we describe in Chapter 4.

CHAPTER 2

The 'acceptance' problem in quantum physics education

2.1 FOCUSING ON 'ACCEPTANCE'

While analysing some student's interviews and discussions, we clashed into an important aspect that appeared to be a common problematic issue in most of experimentations about quantum physics (QP) education: the issue of *acceptance*, as we refer to it here. Generally speaking, we noticed that a significant number of students found it difficult to accept the theory as a personally reliable and adequate description of reality and of the accounted phenomenology.

The cases of students who did not accept QP, namely cases of *non-acceptance*, occurred even though they seemed to have understood and also appropriated the theory as it was proposed. The acceptance issue strongly characterizes the history of QP, but we are prone to think that it assumes a special meaning in QP's education, where it gets in relation with students' learning and appropriation processes. We indeed conjectured that it refers also to a cognitive dimension and not only to an epistemological or philosophical one. As will be shown throughout our analysis, the *acceptance* issue has already emerged in different studies about teaching QP; nevertheless, it has never been addressed in detail as a specific research problem in physics education research.

The aim of this chapter is to show the effective reality of the problem of *acceptance* and to shape it through the analysis of some selected interviews. This study follows up the work reported in the article of Levrini and Fantini (2013) that is a post-analysis of an experimentation based on the teaching proposal (2012 version) of Bologna's research group, conducted in two secondary-school classes. In that study all the students describe the formalism as intelligible and a necessary requirement, but for some of them it seems to be insufficient to completely comprehend the theory, or to accept it. As said by a student: "*The problem was not understanding but accepting the consequences of the theory*". Already there, it was clear that understanding the basic ideas and the formalism's rules is not enough to have the feeling of "getting it". A deeper and more sophisticated elaboration is needed to re-conceptualise – and accept - the strong detachment from classical conceptual categories, which are of course deeply rooted in students' ways of thinking about physical reality. As will be shown, the whole analysis points out personal and specific commitments that some students are not easily disposed to renounce in order to accept QP as a reliable explanation of reality. The strength of these requirements, which we will call *cognitive needs*, is strictly related to each one's inclinations, personality and cultural background.

From a methodological point of view, a hermeneutic qualitative approach has been chosen. It is focused on a selection of cases to analyse in depth so as to capture the very origin of the phenomenon we observed (the difficulty of accepting QP).

In order to make the analysis reliable, and to grasp problems that could be as much as possible context-independent, we considered materials from different teaching experiments. In particular, we refer here to the experimentations conducted by the research group of the University of Bologna in 2012 and in 2015 in Rimini (yet partially analysed in Levrini and Fantini (2013), Stefanini (2014) and Lodovico (2016)), in 2015 in Bagno di Romagna and in 2016 in Castel san Pietro (not yet analysed); some results are also cited from the experimentations conducted by the research group in physics education of University of Pavia in 2014 and 2015 (Malgieri, 2015).

The analysis started by focusing on three cases of evident non-acceptation that came out in three different contexts. Students' profiles were built so as to report their personal ways to describe their perplexities on QP. The profiles allowed to recognise, in particular, the words used by the students and the topics they found to be particularly puzzling. Thanks to the construction of the profiles, we could recognise the semantic fields to which students' words belonged and, hence, to formulate an hypothesis about the cognitive dimension and its articulation behind non-acceptance. This hypothesis has been hence developed.

This whole process is reported in this chapter according to the following outline.

This section (2.1) is completed with an analysis of the research literature, aimed to position the acceptance issue.

Section (2.2) reports the three emblematic cases of clear non-acceptance, two from Bologna's results and one from Pavia's, the analysis of which allowed us to point out the involvement of the *cognitive needs* in acceptance dynamics.

In section (2.3) a detailed description of the *cognitive needs* is developed, grouped in three main categories: need of *visualisation*, need of *comparability*, and need of *reality*.

The whole analysis led us to point out the critical points of the teaching proposal, besides giving also the criteria to zooming in and interpret them. This will be the aim of chapters 3 and 4.

As to not get lost with the names and the experimentations of the students analysed in the next sections, we report them here in table below, divided in cases of *non-acceptance* and *acceptance*, as will be explained in the next chapter.

Cases of non-acceptance	
Marco	Rimini
Federico	Rimini
Alice	Castel san Pietro
Cases of acceptance	
Andrea	Bagno di Romagna
Anna	Rimini
Cheng	Pavia
Silvia	Rimini[1]
Jessica	Rimini[1]
Simone	Rimini[1]
Luigi	Rimini[1]
Michele	Rimini[1]

Table 2.1: names and corresponding experimentations of the students analysed in this chapter

[1] This students are taken form the article by Levrini & Fantini (2013), and refer to an experimentation led in Rimini by Paola Fantini in 2005

2.1.1 How did we get at this stage? Re-analysis of literature

Before investigating acceptance's dynamics in students' interviews, we contextualize the *non-acceptance* issue, as it emerged in classrooms, in the whole international panorama. Here we briefly recall and re-analyse the main phases of the research in QP education, until mentioning the most innovative learning paths where *non-acceptance* occurred.

The development of international research in QP education, both at a university and pre-university level, went through a lot of drastic route's changes, but also through many enrichments. The long process that brought to the actual situation highlights three main typologies of problems, which have had to be faced off over the past 30 years:

- the diagnosis of the main conceptual difficulties which occurred in students dealing with QP;
- the design of new learning paths which could be incisive for gathering the essence of quantum theory;
- the problem of moving through the strenuous debate upon the foundations of QP and of deciding which interpretative approach had to be chosen.

C*riticalities of historical-like approaches*

A first phase of research was characterized by the analysis of students' conceptions in order to investigate the real effectiveness of the commonly used teaching approaches, that is the approaches that mainly focused on those transition phenomena which belong to the so-called 'Old Quantum Physics'. Until about two decades ago, all over the world, most of the university and secondary school textbooks appeared to follow an historical approach to guide students to enter QP. This approach started with the discussion of the black-body radiation, the photoelectric effect, the Compton effect and Bohr's atom, as well as Heisenberg's uncertainty and Bohr's complementarity principles.

Plenty of surveys allowed students' misconceptions to be brought out. A rich and interesting review is reported in Malgieri (2015). For our purposes, the main result concerns the teaching of QP at a secondary school level and, in particular, the implications of paths that deal *only* with the old QP. It is well-known that these approaches have the weakness of providing sets of disconnected information. The most problematic consequence of this conceptual fragmentation is that students tend to fill the gaps by associating classical properties to quantum systems, reaching a sort of scepticism towards quantum theory itself (as introduced in chapter 1 for the results reported by Tarozzi (2005)). The main critique towards these historical-like paths can be synthesized in the lack, next to a demolition process of the Physics of late '800, of a constructive apparatus based on quantum formalism and its interpretative categories (Levrini et al., 2015b).

Building up formal constructive frameworks

Over the years, and especially since the late 90's, the studies on the foundations and education of QP moved to the development of courses and materials aimed to introduce, also at the secondary school level, genuine quantum concepts within logical and consistent formal frameworks. As a reaction, the main trend in physics education was to design approaches, also for secondary schools, that aimed to completely detach teaching from the historical evolution of quantum theory. As introduced in Chapter 1, different proposals and approaches have been produced along this research line. The *logical-philosophical* approach (Lawrence, 1996) (Pospiech, 1999) is, for example, based upon the axiomatic structure of QP and starts from the introduction of spin and Pauli's matrices in order to lead the students closer and closer to the mathematical structures of QP. Other proposals follow a *phenomenological* approach, namely using key experiments to derive quantum description of the world; examples are the proposals based on the phenomenological analysis of light polarization or of phenomena of double refraction through calcite crystals (Ghirardi, 1997; Stefanel, 2008); other examples are the proposals based on the experiments of diffraction and double slit interference reanalysed through Feynman's approach of the sum over paths (Feynman, 1985; Taylor et al, 1998; Rinaudo, 2010).
All these efforts generally brought to a real and remarkable enhancement in students' comprehension of quantum foundations, in the sense that students appear to make strong progresses in solving problems and exercises concerning genuine and deep quantum concepts (Michelini et al., 2010).

Interpretation matters

It is in this cultural and didactical context that Baily and Finkelstein carried out one of the most interesting and original studies on QP's teaching (Baily & Finkelstein, 2010). They focused their attention on the effective relevance of teachers' choices to foster a proper detachment from classical categories, and in particular those choices concerning issues of *interpretative nature*. With *interpretative* the authors mean those issues that concern the philosophical stances in interpreting QP's formalism.

The study regards a statistical survey conducted in two university classes, dealing with courses on QP. The courses' structure was quite the same, but the respective teachers chose two different positions about dealing with interpretative issues: one opted for an 'agnostic' position, strongly and explicitly characterised by avoiding any kind of interpretative nuance; the other one chose to deal explicitly with them, often taking a 'realist' stance on the electron description (for example explicitly assuming that in the double slit interference experiment the electron passes through only slit, being it a tiny particle).

For addressing student responses the authors individuated three main possible positions: *realist* (the electron is a tiny particle, the probability density is so widespread for our ignorance), *quantum* (the electron interferes with itself, being describe by a probability wave), and *agnostic* (QP is only about predicting the outcomes of experiments.). The results (fig. 2.1) show that those students who dealt with the 'agnostic' teacher (PHYS3B in figure) tended to maintain more easily a 'realist' and still purely classical visualisation of the phenomenon and of the electron itself.

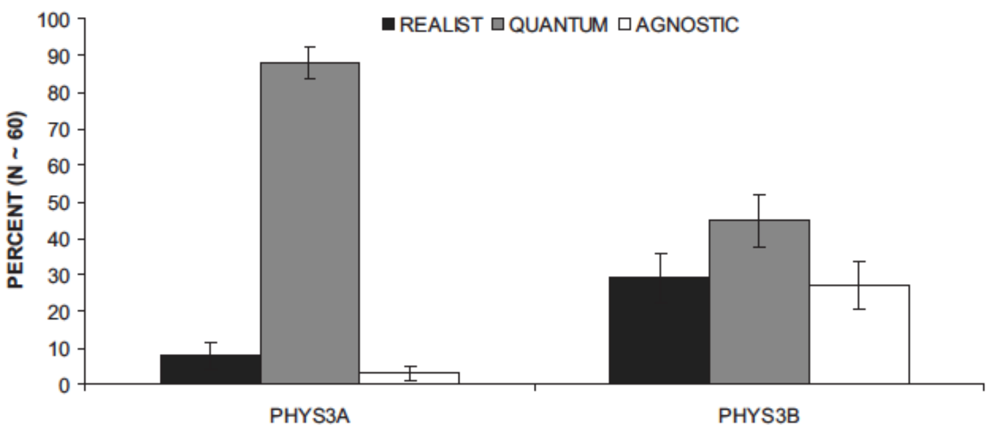

Figure 2.1: plot form (Baily & Finkelstein, 2010). An agnostic stance in teaching QP (PHYS3B) can produce a still classical way of thinking about quantum phenomena

Thus, ironically, despite what kind of philosophical stance is chosen, without any interpretative introduction to quantum phenomena and to its formalism, still stands the risk of getting back to a naïve realism, related to those misconceptions that were to be avoided with the introduction of a constructive quantum framework.

This study opens the very delicate point about what interpretative perspective can be chosen in teaching and why. If, on one hand, it is unrealistic and educationally idle to re-present the complicate debate that historically occurred for interpreting QP, what interpretative aspects are needed to recognise the new paradigm of QP and to overcome classical views? Of course, the study reported in this dissertation cannot answer this enormous question, but it is positioned exactly there: it aims to provide a contribution to analyse the interpretative issue from a cognitive perspective. The problem is particularly relevant because most of the teachers struggle with the interpretative issues as, despite being the main teaching trend for avoiding any interpretation, they feel uncomfortable in front of such a broad variety of stances.

In this perspective, a study conducted in 2013 by M. Shlosshauer and colleagues (Shlosshauer, Kwiat & Zeilinger, 2013) aimed at investigating the views of 33 participants (27 of which physicists, 5 philosophers and 3 mathematicians) of an international conference on QP ("Quantum Physics and the Nature of Reality", July 2011, International Academy Traunkirchen, Austria). The survey concerned critical topics, like randomness of individual events, measurement process, Bell's inequalities, quantum information, quantum computers, interpretations of the state and of quantum physics in general.

The results show a widespread spectrum of position on most of the questions, and in particular it's interesting what found for some specific issues, which we report below:

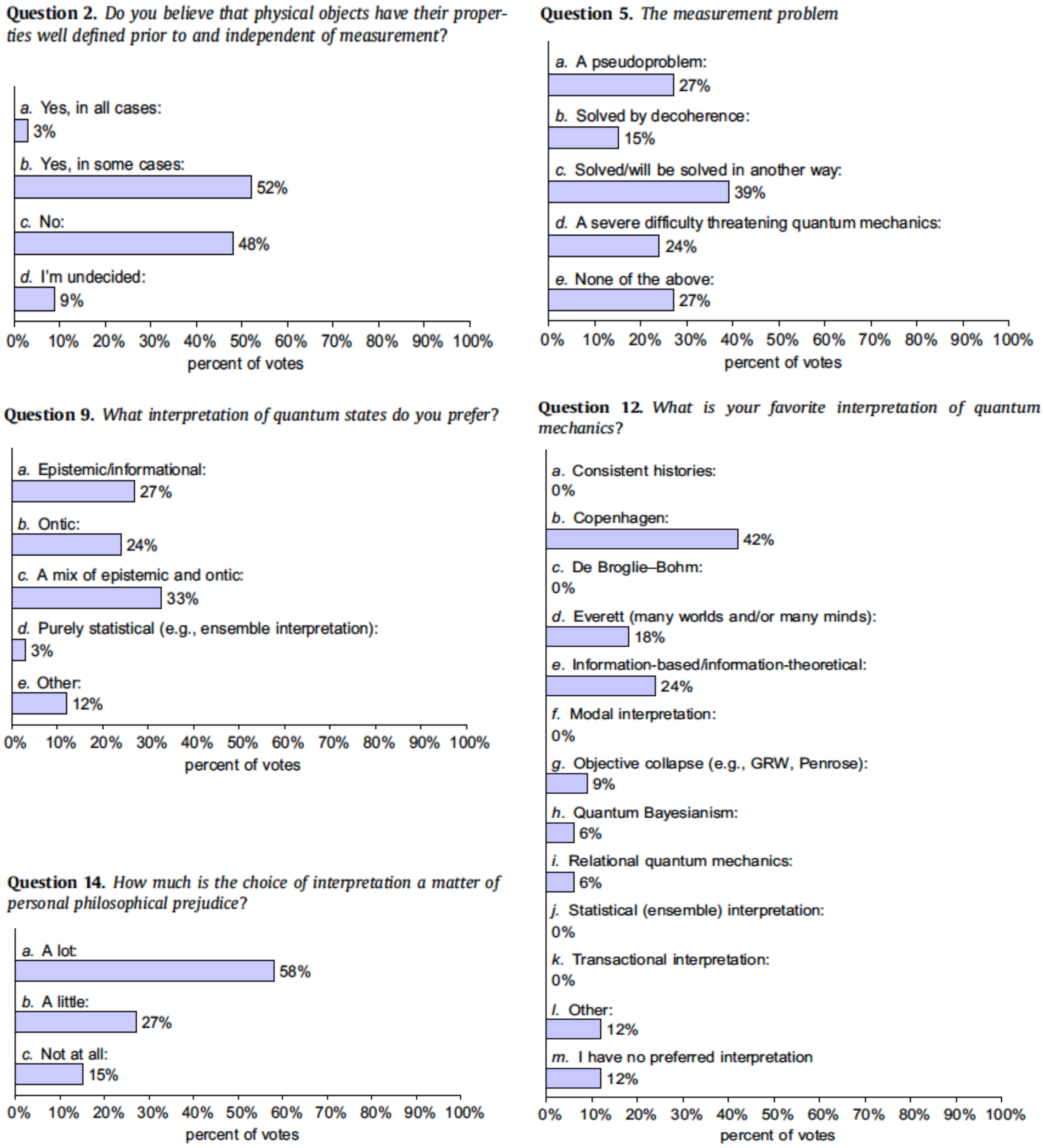

Figure 2.2: results from Shlosshauer and co-workers (Shlosshauer, Kwiat & Zeilinger, 2013)

In this context it is not surprising that a consistent part of teachers chooses to keep a forced agnostic stance, as there isn't an unanimously shared interpretation which might better fit with teaching/learning goals. However the cited article from Baily & Finkelstein (2010) reveals a new common feeling that a meta-reflection on the conceptual changes that quantum theory imposes is crucial to enter QP as a reliable way of conceiving and understanding phenomena.

This is maybe the reason why very recent proposals for teaching QP at the secondary school level, although still based on those fundamental constructive frameworks developed up until late ’90, arrive to deal, more or less explicitly, with interpretation issues. It is in the implementation of these approaches including an interpretative dimension that the acceptance issue emerges as the most relevant result.

We will present and discuss the proposals in the next chapter, after that we have unpacked the acceptance issue as it is complained by the students. This will allow us to present the proposals by focusing on the critical interpretative details that can foster or hinder acceptance.

The problem of non-acceptance

To sum up, the research in QP education produced important results to reveal the misconceptions induced by historical-like paths and, hence, fostered the design of paths that were focused, in a first moment, on the problem of how to build constructive genuine frameworks and, in a second moment, on how to introduce also epistemological and interpretative issues.

Despite encouraging successes, this is the very state of the art in which our own results upon *non-acceptance* arise. As explained, even though most of the students seem to have learnt the physics contents in its formal, experimental and epistemological dimensions, they complain that the quantum theory is not a reliable and adequate description of reality. In the following analysis of students’ excerpts we try to better understand these positions so as to understand if they are related to a specific epistemological view or if they are the expression of a surface and merely technical understanding.

In the history of physics relevant physicists did not accept QP according to their legitimate ‘realist’ stance. Thus, our analysis aims to unpack if students’ non-acceptance can be considered a philosophical option eventually embraceable or if it is related to a sort of ‘naïve realism’ due to the lack of awareness or to cognitive difficulties to grasp the new paradigm.

This latter possibility is a very interesting challenge for research since it points out that a generic insistence on interpretative issues, which is necessary and already in use in most of proposals, is not enough to remove the pillars of this classical-like attitude towards QP. A more reasoned and precise interpretative approach has to be developed.

2.2 CASES OF NON-ACCEPTANCE

In this paragraph three cases of clear non-acceptance of QP are brought out, two of which from Bologna's experimentations and one from Pavia's. In order to investigate acceptance dynamics, we focused on those students who generally well understood the formalism, and also have a fairly high degree of appropriation; this provide a focalisation on the personal aspects which overcome in learning process, leaving aside those cases in which non-acceptance is due to a lack of preparation. The first two cases presented (Marco and Cheng) show an explicit refusal of quantum description of reality, and still claim for a more 'realistic' theory to be found in future. In the third case (Alice), non-acceptance is not explicitly expressed in the requirement of a new theoretical framework, but it emerges as a difficulty in feeling confident with the description proposed.

2.2.1 Marco: postulating 'well-defined properties'

Marco[2] (experimentation led in Rimini, 2015) is a student whose idiosyncratic idea of science is mainly founded in its utility and its possible applications: "*science has to be used in technical field* […] *to create, let's say, the great inventions*". Marco is considered by his classmates as a good and hardworking student, and the analysis reported in Lodovico (2016) shows that he appears to have generally appropriated the basic ideas of QP. Nevertheless, he consistently insists that he cannot accept it as a complete explanation of reality, and in particular as reliable description of quantum objects. For example when asked about the *superposition principle* (in the context of Stern-Gerlach experiments), he answers as follows:

Marco: *If I hypothetically can take a measure with a sufficiently sophisticated instrument, that object would have* [would reveal] *a well-defined property. The object itself does own a well-defined property, that's what I believe. […] As*

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[2] This students are taken form the article by Le

Einstein, mine postulate is that an object has to embody well-defined properties.

[E-1]

Or, for what concerns the *uncertainty principle*:

Interviewer: […] *What was the most useful way for you to comprehend the meaning of quantum uncertainty in its revolutionary holding?*

Marco: *So… if I have to be honest, none.*

Interviewer: *None of these* [he had listed them before]*?*

Marco: *All of these partly contributed, but none gave me a thorough explanation. Namely, what I was searching for as an explanation, I haven't found it in any of these.*

Marco: […] *[I was in] a great confusion, not mostly because of the mathematic part, [in fact] I could understand the concepts the teacher was talking about, […] they were logically comprehensible. The point is that I couldn't understand how couldn't a body have its own properties, well-defined properties…*

[E-2]

The two quotations above show that his need of classical-like properties plays for him the role of a real postulate, which can't be put aside. The strength of this requirement probably comes from his very personality and from his idiosyncratic idea of science, but in its nature it seems to be strictly founded on the categories of classical physics, which we are prone to say he has not yet overcome. This generally produces in Marco a form of scepticism towards QP, that affects his acceptance of the *uncertainty relation* and *superposition principle*, but also of the concept of '*quanton'* and *probability*, as he states:

Marco: *To me the word 'probability' is quite an 'escamotage'* [trick] *that we use to…to determine the phenomenon with certainty* […] *But, indeed, these are the errors induced by this way of representing this fundamental issue, namely the one of non-defined properties.*

[E-3]

Indeed, Marco's search for applications and technological developments serves him to partially postpone the problem of accepting QP in its implications. But in postponing the problem he always specifies his concern, for example, on uncertainty relation:

Marco: *…although I don't agree with it, I understood that Heisenberg's hypothesis* [of uncertainty] *in necessary in this moment.* […] *I notice that considering the quanton as non-defined particle, even though I don't agree, is in any event fruitful for the moment. Just like as your mother tries to convince you that black dogs are evil, and you know they're not* […], *but she gives you 50 euros every time you say: "yes, ok, ok".*

[E-4]

Hence, although Marco's idiosyncratic idea of science reveals a sort of empiricism, and the very reason that leads him to not accept QP is founded on epistemological requirements and considerations; he feels the necessity to "*find a more epistemologically accurate meaning*".

2.2.2 Cheng: "I would like to know more about reality"

Cheng is a Chinese student form the experimentation of the group of Pavia. His case is extensively investigated in Malgieri (2015). He seems to have well understood all the disciplinary contents of the course and, on the basis of the markers proposed in Levrini and colleagues (2015a), he seems to have also quite appropriated the theory and the proposed learning path, with the exception of its discourse to be *carrier of social relationships* (as reported in Malgieri's analysis, Cheng did not actively participate to any discussion conducted in classroom, maybe also because of his difficulties with the language).
As it appear in Malgieri's PhD Dissertation, Cheng correctly talks about the main historical developments of QP, describes the wave-particle duality from the point of view of some different scientists, and also explains in detail the most recent developments proposed in classroom (entanglement among the others). But, despite his confidence with all these issues, when interviewed he explicitly states that he cannot accept quantum theory as a 'final' explanation:

Interviewer: *So you are not convinced by the idea of a quantum objects, which is neither wave nor particle (...) You believe a better explanation exists.*

Cheng: *I think it exists, but hasn't been found yet.*

[…]

Cheng: *I would like to discover why it is that way.*

Interviewer: *Is it my impression or there is something that you do not accept.*

Cheng: *Exactly. I would like to know more about reality.*

Interviewer: *So you don't accept it. Sooner or later it will be discovered.*

Cheng: *Yes. Exactly.*

[E-5]

Cheng doesn't face any repulsion towards mathematics; on the contrary, he firmly believes in the explicating power of formalism: "*Images can help you understand, while the mathematical model simplifies everything. If we know how it works, it makes us remember everything at a glance*".

This confidence with mathematics leads him to consider QP understandable, as he demonstrates when speaking about 'Which Way' measurements: "*It is surprising because it does not follow the classical probability rule, but it's not incomprehensible, because it follows the quantum probability rule. So it is surprising, but only because it is computed in a different way"*. He shows also to have a precise idea of the relationship between physics and mathematics, as a description of intrinsic laws of Nature (which he demand to be the classical physics' ones):

Interviewer: *So you believe Newton's formula for gravitation exists somewhere, and we just discover it.*

Cheng: *It exists, in the sense that it's intrinsic. But it's not mathematics. We mathematize it.*

[E-6]

Hence, Cheng declares to understand QP and to be able to visualize, for example, Feynman's model; furthermore he seems peaceful to momentarily accept QP for its results in calculations. But at the same time, confronting his idea with those of the most important scientists who developed QP, he is sure that this is not the final answer, as he explains:

Cheng: *I believe objects to have a definite position and momentum. There is something that escapes our understanding. But it is not that uncertainty is due to measurement. It due to some other reason. Something which we still don't know.*

[E-7]

As it was for Marco, this need of more 'realistic' properties affects his acceptance of the uncertainty principle, and of the nature of *quanton*s. It is also interesting to notice that both Marco and Cheng consciously focused their attention on the formal apparatus and its relations with experimental devised, since they both consider QP useful and very effective for its technological applications. What they seem to keep faraway is the modelling game that the formalism seems to suggest to provide a new interpretation of the world.

2.2.3 Alice: "The ball is round, and the state?"

Alice is a student from 2016 Castel San Pietro's experimentation. Her personality shows to be always curious and ready to accept the challenge with every topic proposed in classroom. She likes to dialogue both with the teachers and her classmates, even if she is not sure to have the right answers to give. Alice suffers a slight linguistic fragility, which often leads her to not fully comprehend the texts, and which weakens some of her logic arguments; for instance, she does not feel comfortable with most of the metaphors proposed in the course, mainly because of her tendency to read them literally and to miss the appropriate connections. Despite this slight linguistic difficulty, Alice is considered to be quite a good student and physics is her favourite subject matter, so that her final dissertation was about gravitational waves and general relativity.

Alice showed a great interest towards QP course, and was the most participating student during the lessons.

Nevertheless, when interviewed, she expressed her difficulties in dealing with QP, some of which have still remained unresolved. In particular she felt bothered about the problem of *imaging* the quantum state:

Alice: *Quantum physics has been difficult to comprehend in respect to the other physics fields, because…it's a kind of physics that I cannot imagine, or* *contextualise* […]. *When we talk about an electron, I know that I cannot see it but, at least, I imagine it as it is drawn in the textbook. Quantum physics instead…namely, the quantum state is much more difficult to be imagined.*

Interviewer: […] *So, how had you imagined the state when we were talking about it in classroom?*

Alice: *…when you said that the* [a particular] *state comes to be defined only with a measurement…this shocked me a little, because that is not an 'intrinsic' characteristic, and so I really don't know how to visualize it…*

[E-8]

Alice's idea of comprehension is directly linked to the one of visualisation, as demonstrated from her example about the electron. When trying to visualise the quantum state, she searches for an intrinsic property that can characterize it and let her to use the imagination. We restate here for the word 'intrinsic' what still claimed for Marco's 'well-defined' properties, as they are indeed tacitly identified with properties held by a state or an object in a classical sense: properties that have a single, well-defined value to be discovered through measurement. In another extract, to get to the point, she enforces her argumentation through a metaphor:

Interviewer: *So, what is your concern with the quantum state?*

Alice: *I would like to understand better what it is. We didn't say: the state is this, or that…we only talked about some of its features…so to speak, the ball is round, and the state?*

[E-9]

Consistently, the role of measurements in determining the state seems to be an awkward point for her conception of science:

Alice: *I was used to think that all scientific subjects had to describe all the phenomena with certainty, but this issue of measurements changing the state…it makes a little bit perplexed".*

[E-10]

Even if not explicitly addressed by Alice herself, as it was instead for Marco and Cheng, we are prone to consider her case as a non-acceptance one. In fact, although she seems to have appropriated the teaching proposal, she does not feel comfortable with QP's description of the world, as clearly pointed out in the following:

Alice: *I'm used to think about the world and about reality through classical physics. Sure enough, even with relativity I had some difficulties in imaging its 'curvatures'…but for me quantum physics requires even a greater effort, because it's a too small world…it's too abstract. I haven't fully grasped it yet…*

[E-11]

2.3 COGNITIVE NEEDS

A comparative analysis of the three cases shows some main evidences: (1) all the students mention three main conceptual points against which their acceptance clashed, namely the concept of quantum object, the superposition state and the uncertainty relations; (2) the words used by the students to complain their difficulties can be grouped in three semantic fields, namely *visualization/imagination*, *to know more/better*, *reality/existing*. Some key expressions that mark problems of acceptance are "to know more about reality", "to give meaning to the formulas", or "compatibility with reality", and reveal the need to strengthen or establish an interpretative and epistemological connection between the new mathematical structure and the world. In some sense it seems that the modelling dimension, that is the hypotheses and the features of the new paradigm, is not completely grasped or accepted.

In front of these evidences we tried to recognise if behind non-acceptance there were basic cognitive requirements, which were not completely satisfied. As a result of the re-analysis, we pointed out the existence of *cognitive needs* that, we conjectured, could be grouped into three main categories: the needs of *visualisation*, *comparability*, and *'reality'*, in the particular nuances specified in the following sections.

We moreover conjectured that these needs do not belong only to those students who didn't accept the theory, but also to students that hardly work to accept it. These needs indeed seem to represent common cognitive elements; simply, in the context of QP, they clash with its deeply non-classical categories, giving rise to possible non-acceptance cases.

In order to elaborate the conjecture and to better characterise the cognitive needs, we considered excerpts taken from all students, making no difference between their personal appropriation or acceptance of QP, as instead was chosen for non-acceptance cases in section (2.2).

As anticipation, we can say that the analysis led us to see that, in all the teaching experiences we carried out, what seems to make the difference out is the *strength* of these cognitive requirements and, as a consequence, the 'degree' of acceptance strongly depended upon each student's cultural background, idiosyncratic idea of science and personality.

2.3.1 Need of visualisation

The cognitive *need of visualisation* seems to be one of the most important points students complain to be not satisfied in QP. From the interviews, this requirement essentially emerges as a sense of lack of mental images or metaphors to "see" processes or objects, or even to grasp concepts by intuition. Intuition of physical concepts is very seldom ascribed by students to mathematics and formalism and it is perceived as a vey complex and high level to reach. Marco (Rimini, 2015), for example, talking about the superposition principle, expresses his concerns about this issue as follows:

Marco: *In my opinion [these abstract concepts] are only simple mathematical tools, which however avoid what is the intuitive problem […]. We need to consider the mathematical side to take in account the measurements […]. It's the best way, because intuitively it* [facing the problem] *would be much more complex.*

[E-12]

One of the two classes under study in Levrini and Fantini (2013), where formalism was recognised from all the students as necessary to understand, generally recognized in the issue of *visualisation* of quantum phenomena a clear-cut point of detachment from classical physics. This generated a lively discussion in class, where different positions came to light. The case of Jessica is particularly interesting in this perspective.

Luca: *The picture of microscopic reality, in this case, is sufficiently supplied by the* *mathematical formalism. Therefore, in my opinion, to have a graphical representation is not important for scientific progress: What's the use of the graphical representation? It may help in explaining the object as it is to children. But mathematics already explains it. [...] In my opinion anyway, the picture of microscopic reality is already described well enough by mathematics. It is enough to have the tools for comprehending it and it seems to me that everyone can do so…*

[…]

Jessica: *But for me it [visualization] is necessary in order to understand…*

Luca: *Ah, but what if you can't do it…*

Jessica: *Because it is impossible to talk about something without trying to have a picture of what we are talking about, even unconsciously. It may help, in my opinion, also to give a meaning to formulas, because otherwise, even if we say that it is nonsense to represent the microscopic object, we make a picture anyway… I think so, although we decide not to draw it because we don't want to give a model that…[...] it helps me, it helps me to remember. [...] honestly I can explain the Compton effect by keeping in mind the drawing. [...] we know that to be untrue but…*

Pietro: *Ok, but it is just an icon, you could draw a little star to make a photon.*

Jessica: *Yes, exactly.*

[E-13]

When saying "*…to have a graphical representation is not important for scientific process*", Luca is accepting the impossibility to visualize quantum phenomena, founding his confidence in the possibility of scientific progress, and refusing any other need of description: "*…the picture of microscopic reality is already described well enough by mathematics*".

Jessica, instead, ascribes to her need of visualization a necessary role for understanding: the formalism has to be interpreted in terms of pictures, and being pictures implicitly connected with classical world, this allows for the use of ordinary language.

She restates many times that pictures does not have to be a 'true' representation of reality itself, as "*a little star*" isn't a fitting model of the photon; but visualisation, for Jessica, is an obliged way to travel through in order to face her necessity to "*give meaning to the* *formulas"*, to interpret the formalism in a more intuitive way, to talk about the model. As said before, this requirement for a *meaning*, seems to strongly influence students' acceptance of QP.

The authors of the article point out that this personal requirement someway recall the position interpreted by Schrödinger in the historical debate about formulations of Quantum Mechanics, as underlined in de Regt (1997):

> *"The association of visualizability with understanding rather than with realism may be elucidated by considering the German word Anschaulichkeit, which is the term Schrödinger used in his writings. This word does not only mean 'visualizability' but also 'intelligibility'"*

This connection between *visualisation* and *intelligibility* was for Schröedinger not only a useful way to comprehend the content of a theory, but concerned the very aim of science research itself: visualisation was considered not only a method, but quite a task, as can be seen from this extract (de Regt, 1997]:

> *"Physics does not consist only of atomic research, science does not consist only of physics, and life does not consist only of science. The aim of atomic research is to fit our empirical knowledge concerning it into our outer thinking. All of this other thinking, so far as it concerns the outer world, is active in space and time. If it cannot be fitted into space and time, then it fails in its whole aim and one does not know what purpose it really serves"*

This strong line of though isn't of course consciously taken from students as a philosophical stance, and indeed neither entirely in its methodological and epistemological implications, but still emerges from their words a strong connection between visualisation and understanding.

Another clear example of the need of visualisation is the case of Alice, as she express in the excerpts [E-8] and [E-11]. This need links her appropriation of quantum theory to the possibility to visualise the quantum state and she complains her difficulty in the lack of 'intrinsic' properties. In this case it is crucial that Alice focuses her attention on the "state", since this is a clear signal that Alice made a sort of ontological shift from the object to the state, but she needs to finalise and strengthen such a shift. This is a point deeply addressed in the next chapter.

The case of Federico (Rimini, 2015) is also interesting since he appropriated the basic concepts but he however felt that his acceptance process was not accomplished, since his imagination was stuck: he was not able to build a comprehensive picture ("image") that could sum up the logic connections between the concepts:

Federico:	*[…] Personally I'm still quite confused about uncertainty, I'm still quite confused…*
Interviewer:	*About what exactly? […] I would say that you have well understood it. So, what is not yet clear?*
Federico:	*The problem is this: […] it is difficult to conceptualize this modern concepts after have been exposed, let's say, to a whole classical path…*
Interviewer:	*Then you're saying: I cannot imagine them.*
Federico:	*Yes, I cannot imagine them, therefore sometimes I repeat* [the concepts] *only because I've heard them, I've studied them, but sometimes… I loose the logic connections between these concepts.*

[E-14]

Although Federico seems he well understood the formal implications of uncertainty relations, he still feels a kind of concern about it. Federico is searching for a deeper and more comprehensive conceptualization, which is strongly linked to what he call imagination and that is deeply different to represent the quantum object through a familiar picture. Visualization is hence a deep cognitive need that refers to requirement to have *a synthetic view able to guide and orient reasoning so as to guarantee its inner logic consistency*.

The same connection between intelligibility and visualisation stands out, even if more implicitly, in the words of Anna (Rimini, 2015). When asked, during the final interview, about her images of quantum objects, she answered as follows:

Anna: *In my head I've no ideas about the quanton […], I've not a clear image in mind. […] But I've made up the idea that this is quite a new stuff, and it seems almost unreachable, as it is not to be understood…*

[E-15]

Although Anna seems not to be prevented by her *need of visualisation* for accepting quantum theory, she clearly considers the image of 'quanton' at the same level of her understanding of the latter, of having an intelligible and comprehensive view to conceptualize it; as she cannot reach a clear image or idea of the 'quanton', it cannot be properly understood. It is also interesting that her personal justification for this impossibility, and the resulting acceptance of it, comes from her fascination for the cultural challenge she wants to accept for herself by removing all the 'prejudices' that, as she states, belong to classical physics' categories (her particular case of acceptance is analysed also in paragraph 3.2.2).

2.3.2 Need of comparability

The second need we pointed out behind the students' words concerns *comparability*, that is the cognitive need of bridging, both formally and imaginatively, quantum world to the classical one, so as to allow imagination to move from one to the other. In fact, the absence of a real demarcation line between classical and quantum domains often prevents this requirement to be fully satisfied in students' conceptions. As a consequence, students like Marco or Cheng still perceive the two domains completely detached from each other, and the quantum formalism comes to be a 'trick' to account for the experimental results without really interpreting the world.
Federico (Rimini, 2015), for example, when asked to compare his studies about QP to the others, answers as follows:

Federico: *[…] In the past two years* [Federico was exposed to the experimentation about relativity in the previous year] *my idea of physics has changed from the one where science had to determine everything, calculate, and tell us everything with certainty. Science has become an endless research of truths; truths that* *have to be proved wrong, or even made more true, by the following theory […].*

Interviewer: *Yes. In fact in your essay you were claiming that it's not clear yet how is possible the coexistence between classical and quantum worlds, with such great differences…*

Federico: *Yes, that's an issue I dealt with. […] What I can't explain it's how could they coexist, but just as how could relativity and classical mechanics coexist. […] This is closer to philosophy than to physics! Or maybe this is true physics, I don't know.*

[E-16]

In dealing with relativity and QP, as they were proposed in classroom, Federico's idea of science has been enriched and enlarged from those limits that were given to be fixed in classical domains. Science's development assumed the image of a dynamical process, where 'truths' are always to be questioned and deepened; this new horizon demanded for fixing possible relationships between different theories and domains.

This very issue is one of the aspects that we attach to the *need of comparability*, and of course it emerges when students are asked to look beyond the wall of classical categories, being them the speed of light or the continuous nature of physical processes.

Federico is facing his need for a 'coexistence' of the different theories, probably making the implicit assumption that all of them are needed to explain the whole reality but he is certainly searching for a connection between them. It's interesting, from this perspective, what Federico states about everyday reality:

Federico: *The difficulty I encountered is, as I said before, that quantum concepts are so much distant from the Newtonian reality we experience every day.*

[E-17]

What is lacking to Federico is an explicit connection between the daily experience, which is to him undoubtedly assumed to be Newtonian, and the new quantum concepts (like *discreteness of the process* or *abstract spaces*, as himself will point out in other excerpts). Thus, the problem here goes beyond the need to establish a connection from the formalism

to its corresponding phenomenology: It seems to express the need to find a bridge between different theoretical "worlds".
On this way, it's also interesting what Silvia (Levrini & Fantini, 2013) points out during a discussion led in classroom:

Silvia:	*In relativity it was different […] there you have a demarcation line. If you apply our velocity in formulas, you re-find our formulas. [In relativity] the two things are compatible, here not. […] In relativity, in my opinion, there was a greater compatibility with reality.*

[E-18]

As Levrini and Fantini highlight in their analysis: "without such a demarcation line and hence a comparative criterion, the quantum formalism risks becoming nothing but a "*mechanism*", "*a mentality*" (Silvia) to jump into, lacking what she felt to be a way for making the worlds comparable.
Silvia (like Federico) was not compelling the impossibility of projecting classical images on the quantum world. She was instead manifesting the need of making the two 'worlds', however different, comparable, where comparability includes also the knowledge of where one fades in the other" (Levrini & Fantini, 2013).
Comparativeness seems to be, like visualization, another way to travel within the quantum world without feeling to get lost and to express in a not simplistic way the wish to reduce quantum world to the classical one. More than visualization, comparativeness seems to belong to the epistemological level regarding modelling and the features of the interpretative apparatus (the "world") that the new theory has built; apparatus that a contrastive approach with other "worlds" can highlight.

2.3.3 Need of 'reality'

This last *cognitive need* we saw behind students' words refers to the request for the quantum description to be more tied with 'reality', in the different nuances this word has for the students. This needs emerges quite systematically when talking about the quantum object, the superposition state and the uncertainty principle.

Andrea (Bagno di Romagna, 2015), for example, when asked about the nature of 'quanton', answers:

Andrea: *[This is] a word quite particular to describe it, but maybe it could be said to be mysterious, as up to now it's difficult to define what it really is; we don't know yet how to define it well, if particle, wave, or something which lies outside both natures.*
[...] 'quanton' is a totally new kind of thing, it's difficult to tell its properties... it's something that is not well definable.

[E-19]

In the attempt to find a definition of quantum objects Andrea implicitly makes the assumption of considering the words 'property' and 'definition' as strictly linked to classical quantities, associable to the object at any instant. In order to reach a more 'realistic' identification of the quanton, imagination searches for those classical-like properties on which students are used to rely and, thus, consider more 'real'.

Cheng in [E-5 and E-6] repeat the same need of more reality several times and in ways very similar to Marco (Rimini, 2015), as reported in the extract [E-2] about the uncertainty principle ("*The point is that I couldn't understand how couldn't a body have its own properties, well-defined properties...*"), and in [E-1] on the superposition state ("*the object itself does own a well-defined property, that's what I believe. [...] As Einstein, mine postulate is that an object has to embody well-defined properties*").

These positions seem somehow recall Einstein's position on the concept quantum state, although indeed with a more conscious philosophical stance (Einstein, 1953):

"*I am not ashamed to put the concept of «real state of a physical system» ["existing objectively, independently of any observation or measure, and that can in principle be described through the means of expression of physics"] at the very centre of my meditation*"

This requirement also appears in Alice's interview as a need of 'intrinsic' properties, as she says in [E-8] and [E-10]. What is interesting is the generality of this *cognitive need*, concerning all kind of properties associable with physical objects, including for example *spin* (as it is for Stern-Gerlach experiment), that is exquisitely quantum-like.

As can be seen also from the following quotation (Levrini & Fantini, 2013), the need of reality (more than the previous needs) emerges when very precise conceptual topics are addressed: the topics like uncertainty and the superposition state that mark the fall of classical determinism:

Simone: *The hardest point to understand has been giving up classical determinism […] Deterministic physics was an exact science, at least at a theoretical level. Quantum mechanics is upsetting since it requires facing the knowledge problem, it makes you ask if what we observe is really what it is.*

[E-20]

2.3.4 Cases of hard-won acceptance

By cases of 'hard-won' acceptance we mean those cases in which students' disposition seemed to change due to a post-reflection, both on the arguments and on themselves. We chose to report and analyse two particular cases, as they point out how some students, despite being dealing with their cognitive needs, found a personal justification for (partially) accepting QP.

- **Anna: "We have to remove our prejudices"**

Anna, attending at our last experimentation in Rimini, seems a case of significant understanding of almost all of the arguments proposed and she seems also to have accepted QP.

When talking about the *quanton*, she explicitly declares the impossibility of visualising it, but she can find a personally acceptable justification in her disposal to admit that QP is a new world where classical categories fail:

Anna: *I haven't any idea of the quanton in my head… […] Actually, I don't have a clear image in mind. […] However I developed the idea that it is something completely new, and it seems almost unreachable, not to be understood…* […]

…we entered a new world that we have to discover.

[E-21]

Also when speaking about superposition principle, she ascribes her difficulty to some prejudices which have to be faced off:

Anna: *... for Quantum Physics it* [the atom in Stern-Gerlach experiment] *is in a superposition state* […], *but indeed this is something very difficult to think about.* […] *Namely, it' quite shocking, because… we have such a lot prejudices, also Einstein told so…"*

[E-22]

During the interview she constantly returns on this point: "*we are in an entirely new world"*. Her conceptual needs seem to be overcame by a sort of personal challenge she feels to herself:

Anna: *We have to remove our prejudices.* […] *However, it is difficult, because in the past years we followed a path where we completely trusted science, "I don't even demonstrate it to you, because that's so", "Ah ok, well, so let's study it this way…"*

[…]

Interviewer: *Then, do you bring home something* [from this experience]?

Anna: *The 'quanton'! I bring home it* [laughing]*…meaning that… Also with Quantum Physics, and even before with Relativity, but certainly more in Quantum Physics, you bring home the fact that* [you find yourself saying]*: "Then it isn't as I though…". That is, you don't merely have to fix some concepts in your head… a part from demonstrations, which I've always wanted to do, but… some issues are not demonstrable, you have to deal with this.*

[E-23]

Anna's personality seems to be very shy, hence her approach may appear to be part of her attitude of accepting QP since she never tries to impose personal commitments.

Nevertheless from her statements, which are not incidental as they were repeated in many different ways throughout the interview, she seems to accept it mainly because of the cultural challenge she feels to herself. Anna doesn't talk explicitly of the new ontology and it is not clear what she ascribes to the quantum world as deeply new. Anyway, her fascination for this cultural challenge leads her to accept giving up classical world and to avoid any resistance to maintain it.

We are prone to think that this was facilitated by the teaching path itself, as it was constructed on the very idea of supporting students' involvement, by creating a rich, challenging and complex learning environment. In fact, as pointed out in Levrini & Fantini (2013), some forms of complexity can be productive for students' learning processes and unavoidable difficulties can be transformed in cultural challenges. This fact is strongly confirmed by the general results about students' involvement with the proposal (Stefanini, 2013), which revealed to be widely transversal to their personal scientific inclination: building up the materials for achieving a properly complex learning environment, seems to really facilitate not only the *appropriation* (Lodovico, 2016) of the teaching proposal, but also the *acceptance* of it.

- **Luigi and Michele: reorganizing relations between math and physical properties**

In the experimentation conducted in 2012 in Rimini and reported in Levrini and Fantini (2013), a lively discussion about the crisis of determinism was generated and developed around the following questions: should "real" be synonymous of "determined", "know in all detail", "know with certainty"? Why should a description based on uncertainty, on a "non-epistemic probability" be less realistic than a classical one? These are somehow the core questions that led Marco and Cheng (see section 2.2) to not accept Quantum Theory. The discussion was led by Luigi and Michele:

Luigi: *I think that realism is not lost. I mean that what we are talking about is something real and is not metaphysical, therefore realism is not excluded. We are talking about something that, so to say, is undeterminable because it has a non-epistemic probability. Realism is not excluded anyway; on the contrary it is defined in another way… let's say on the basis of its probability instead of its certainty.*

Mathematics (in this case) allows us to explain the superposition principle, the principle of uncertainty. And that's what I found somehow difficult to understand: how mathematics gave us an explanation of how nature is not something exact but is instead undeterminable.

Michele: *I think Luigi pointed out that mathematics has never been associated with the concept of realism, it has always been abstraction. Mathematics has always given us certainty, something certain and computable. So mathematics providing here a concept of probability and uncertainty can be a little disconcerting. But when has mathematics ever been associated with realism? It has always been abstraction, model.*

[E-24]

In the quotation above Luigi and Michele face the posed questions by re-thinking and reorganizing the relations between math and reality. For Luigi, "*Realism is not* lost", it is just "*defined in another way*".

The authors specify that Luigi probably uses the word 'realism' just for stressing the link to reality, without referring to its philosophical nuances. To him the main difficulty does not belong to mathematics itself, but in conceiving that mathematics gives us "*an explanation of how nature is not something exact but is instead undeterminable"* (Luigi). Michele then properly clarifies what is hidden in Luigi's discomfort: "*Mathematics has never been associated with the concept of realism*", namely, he points out the possible and undue association between math's certainty and "realism". As pointed out by the authors, "the detachment of maths from a strict and trivial link to reality led Luigi and Michele to find (even with a little discomfort) a new space of freedom for allowing maths to embody probability and uncertainty and to problematize the relationship between determinism (certainty) and realism".

This little episode confirms the strength of the cognitive needs like the need of reality and of comparability in students' learning processes, but at the same time points out that students can find personal justifications for accepting QP, as it is for Luigi and Michele who arrived to reconceptualise the relationship between maths and physics.

2.3.5 Remarks on the cognitive needs

The analysis pointed out specific challenges that students have to address to accept QP: the challenge to build a comprehensive view (*need of visualisation*), the challenge to recognise the features of the new world with respect to the others (*need of comparability*) and the challenge to reconceptualise how the mathematical formalism is related to the real word (*need of reality*). These challenges imply a wide and multidimensional re-arrangement of knowledge on different dimensions, that is imagination/perceptual, epistemological and ontological; in other words, what is implied is a process of positioning the new theory with respect to: (i) one's own way of imaging the world, (ii) other physical theories (other possible models) and (iii) reality.

To get it in simple characterising questions, the cognitive needs could be expressed as follows:

Need of visualisation: how can I imagine the quantum world?
Need of reality: how is its modelling related to reality?
Need of comparability: to what extent is it different from the classical one?

The students who appropriated the teaching proposal and are able to accept the theory are usually students very attracted by the intellectual challenges and by the conceptual changes. They are moreover fascinated by the power of knowledge in designing new worlds. Yet, the students who appropriated the contents but didn't accept completely the theory clashed against their resistance in changes and/or against deep knowledge problems; we are prone to say that the learning path can be improved in order to support also this type of students to satisfy their cognitive needs.

The analysis pointed out specific topics deserving an explicit revision: the definition of the *uncertainty relations*, the *superposition principle* and their relations with the definition of quantum object.

The questions addressed in the next chapter are: why did the teaching path fail to satisfy the cognitive needs we pointed out? How do other proposals address the issue? Are they more effective, as for the specific aim to satisfy the cognitive needs? How can we improve our proposal so as to support more students in addressing their own challenges?

And, more in general, what interpretative issues can be introduced and explicitly address for cognitive reasons?

CHAPTER 3

Re-thinking some critical points of Bologna's proposal

This chapter aims at individuating the most critical points of Bologna's path. In particular, we will focus on the topics which the analysis reported in Chapter 2 stressed as the most demanding for students: the *uncertainty relations* (3.1), the *superposition principle* (3.3) and their relation with the definition of quantum object.

The proposed *cognitive needs* will be operationally used as analytic tools for rereading students' reactions to the teaching proposal and individuating the specific details against which acceptance dynamics clashed.

For each topic, we briefly report how it was addressed in Bologna's path and students' specific reactions; then, we sketch the panorama of the research's literature to present the main didactical stances on the interpretative debate that has been occurring on these issues within the scientific community. On the basis of this triangulation between empirical results and literature, some specific suggestions to improve our path will be pointed out.

3.1 UNCERTAINTY RELATIONS

3.1.1 *Uncertainty relations* in Bologna's path (2015-16)

The approach to *uncertainty principle* chosen by the Bologna research group aimed to highlight some epistemological dimensions belonging to the quantum description of the world: the introduction of a non-epistemic probability, the fall of the principle of causality and of the concept of trajectories, the existence of 'conjugated' variables and the 'active' role of the measurement process. The slant given for introducing these fundamental themes was mainly historical-like, as it was based upon the Heisenberg's article (Heisenberg, 1927) "*On the intuitive content the quantum kinematics and mechanics*" and the debate with Bohr around the gamma rays microscope's thought experiment.

The specific order in which *uncertainty relations* were presented is approximately (as it has always to be appropriated and re-organized by teachers themselves) the following.

- They are introduced in the *junction* part, before the main formal concepts have been developed in the *pars construens*, and presented in their most famous formulation:

$$\Delta x \Delta p \geq \frac{1}{2}\hbar$$

The focus is on the epistemological role addressed to them by Heisenberg himself, who explicitly talks about the "intuitive content" of quantum theory, giving also a precise definition of intuition: "*We believe to intuitively comprehend a physics' theory when we're able to think qualitatively about its experimental consequences in all of the simplest cases, and when, at the same time, we recognize that the application of the theory does not imply any inner contradiction*".

- A critical analysis of Heisenberg'*s gamma ray microscope* (Heisenberg, 1927) is proposed to students, underlying the *operationalist* stance of his approach. This choice has been agreed with teachers of the research group, as most of the textbooks still make use of this argument as the most characterizing uncertainty principle, despite all the critics addressed to it up until 1927. As suggested in (Hadzidaki, 2006), Heisenberg's thought experiment, if critically analysed and contextualised in the historical debate with Bohr, can serve to "lead learners to an essential understanding of QM worldview". The choice is therefore to show not only its potentialities but also, and mainly, its limits, and in particular the two main weak points of Heisenberg's interpretation: his insistence on a disturb effect, bounded to an epistemic view of the uncertainty relation, and an underestimation of the *non-separability* of the object with the experimental set-up. The 'disturb interpretation' hides the implicit idea that the object does own precise values of position and velocity before the measurement and that the indeterminacy on velocity is produced by the act of localizing the electron through the gamma ray; in this sense the uncertainty would be experimental and not ontological. Bohr's critics to Heisenberg's interpretation is here touched upon (also through some video extracts taken from "Copenhagen" (Frayn, 2009)) in terms of complementarity, which he believed to be the real ground for explaining uncertainty relations.

- From complementarity, the existences of 'conjugated' variables is introduced, as to get closer to the modern interpretation of uncertainty principle, formally derived in 1929 by Robertson (Robertson, 1929) for non-commuting operators. This part aims at arguing that, as pointed out from Levy-Leblond and Balibar, "while for classical entities the physical properties take on unique and determined numerical values, for *quantons* [quantum objects] they are characterised by numerical spectra, extended sets of numerical values" (Levy-Leblond, Balibar, 1990), that for the case of pairs of non-commuting observables do exhibit correlations between their distribution amplitudes (as it is for example between the spatial e the momentum distributions).

In order to give an intuitive idea of this interpretation on uncertainty principle, a metaphor is proposed and analysed with students: the '*Chinese menu*', taken from a well-known text of Brian Green (Green, 2004):

> "*To understand it* [the uncertainty principle]*, think of the prix-fixe menus in certain Chinese restaurants. Dishes are arranged iin two columns, X and B, and if, for example, you order the first dish in column A, you are not allowed to order the first dish in column B; if you order the second dish in column A, you are not allowed to order the second dish in column B, and so forth. In this way, the restaurant has set up a dietary dualism, a culinary complementarity (one, in particular, that is designed to prevent you from piling up the most expensive dishes). On the prix-fixe menu you can have Peking Duck or Lobster Cantonese, but not both. Heisenberg's uncertainty*

principle is similar. It says, roughly speaking, that the physical features of the microscopic realm (particle positions, velocities, energies, angular momenta, and so on) can be divided into two lists, A and B. And as Heisenberg discovered, knowledge of the first feature from list A fundamentally compromises your ability to have knowledge about the iirst feature from list B; knowledge of the second feature from list A fundamentally compromises your ability to have knowledge of the second feature from list B; and so on. Moreover, like being allowed a dish containing some Peking Duck and some Lobster Cantonese, but only In proportions that add up to the same total price, the more precise your knowledge of a feature from one list, the less precise your knowledge can possibly be about the corresponding feature from the second list. The fundamental inability to determine simultaneously all features from both lists-to determine with certainty all of these features of the microscopic realm-is the uncertainty revealed by Heisenberg's principle".

- The following point of the path in which students meet uncertainty is in the Stern-Gerlach (SG) experiment performed with multiple magnets in series, in the particular disposition below (McIntyre, 2013):

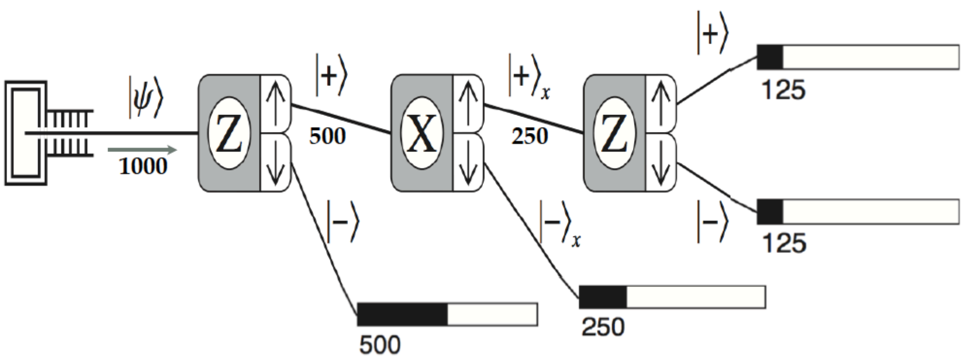

Figura 3.1: Stern-Gerlach magnets in series. The results are a consequence of uncertainty principle, as observables S_x and S_z do not commute.

In this context uncertainty occurs for S_z and S_x spin components (Ag atoms are used). This case is discussed to stress that Heisenberg's inequality does hold for all pairs of non-commuting observables, and not only for position and momentum (which represent only a particular case of canonically conjugated variables). The extension of the validity of uncertainty to operators that do not have a classical equivalent, as it is for spin, is used to someway overtake Bohr's complementarity and, at the same time, to strengthen the concept of *incompatible observables*. As to better understand the meaning of these results, a further metaphor is proposed, "*Erwin's socks*" (McIntyre, 2013):

> "*It was a dark and stormy night. Erwin huddled under his covers as he had done numerous times that summer. As the wind and rain lashed at the window, he feared having to retreat to the storm cellar once again. The residents of Erwin's apartment building sought shelter whenever there were threats of tornadoes in the area. While it was safe down there, Erwin feared the ridicule he would face once again from the other school boys. In the rush to the cellar, Erwin seemed to always end up with a random pair of socks, and the other boys teased him about it mercilessly.*
>
> *Not that Erwin hadn't tried hard to solve this problem. He had a very simple collection of socks – black or white, for either school or play; short or long, for either trousers or lederhosen. After the first few teasing episodes from the other boys, Erwin had sorted his socks into two separate drawers. He placed all the black socks in one drawer and all the white socks in another drawer. Erwin figured he could determine an individual sock's length in the dark of night simply by feeling it, but he had to have them pre-sorted into white and black because the apartment generally lost power before the call to the shelter.*
>
> *Unfortunately, Erwin found that this pre-sorting of the socks by colour was ineffective. Whenever he reached into the white sock drawer and chose two long socks, or two short socks, there was a 50% probability of any one sock being black or white.*
>
> *The results from the black sock drawer were the same. The socks seemed to have "forgotten" the colour that Erwin had determined previously. Erwin also tried sorting the socks into two drawers based upon their length, without regard to colour. When he chose black or white socks from these long and short drawers, the socks had also "forgotten" whether they were long or short. After these fruitless attempts to solve his*

problem through experiments, Erwin decided to save himself the fashion embarrassment, and he replaced his sock collection with a set of medium length brown socks. However, he continued to ponder the mysteries of the socks throughout his childhood.

After many years of daydreaming about the mystery socks, Erwin Schrödinger proposed his theory of "Quantum Socks" and become famous. And that is the beginning of the story of the quantum socks".

3.1.2 Empirical findings

- **Persisting 'disturb' interpretation**

Undoubtedly, the most significant finding is about the didactical use of *Heisenberg's microscope*. Even though the analysis of the though experiment has been widely investigated from its experimental and epistemological dimensions, and critically contextualised within (and beyond) the historical discussion with Bohr, it seems to persist in students the idea of uncertainty as being produced by a measurement disturb. For example, in Federico's interview (extended extract [E-14]):

Federico: *What I understood is that when you enlighten an electron to take a measurement, you give it some energy and so the measurement you take is no more the one you had to take…*[…] *About the microscope argument, even if, I repeat it, I still have to understand it mathematically at 100%, conceptually I understood that the problem of enlightening this electron is that it* [the act of enlightening] *changes its conditions, and so you take no more a measurement on the electron you had to measure.* [It's] *a changed electron, with more energy.*

Interviewer: *But then you're interpreting uncertainty as given by a disturb effect…*[…]

Federico: *The problem is that the concept of uncertainty is itself uncertain* […]*, I know that uncertainty is 'ontological' and I know that it is not experimental, like, let's say, the one I've just described you.* […] *Personally I'm still a bit confused about uncertainty.*

Interviewer: *About what, exactly?* […] *Because I would say that you well understood it. What is not yet clear?*

Federico: *The problem is this: […] it is difficult to conceptualize these modern concepts after have been exposed, let's say, to a whole classical path…*

Interviewer: *Then you're saying: I cannot imagine them.*

Federico: *Yes, I cannot imagine them, therefore sometimes I repeat* [the concepts] *only because I've heard them, I've studied them, but sometimes… I loose the logic connections between these concepts.*

The excerpt is only an example of what should be properly considered a tendency in our students' conceptions about uncertainty. Although Federico claims to be 'in theory' aware of the 'ontological' (in the sense of 'implicated from the foundations of the theory') nature of Heisenberg's relations, his imagination, and his explanation, is still trapped in a semi-classical image, close to the Heisenberg's one about the microscope. He seems to be searching for a reliable handhold to definitely give up the 'disturb' interpretation and consciously coordinate all the pieces of knowledge he has, but he can't find it in any of the proposed argumentations.

So, even if criticised, Heisenberg's reasoning seems to be so strong to remain in students' conceptions as still acceptable and, probably due to the absence of a strong cognitive ground upon which to build a new interpretation, the *non-epistemic* nature of uncertainty remains only a secondary epistemological shade. This approach failed in supporting several students to address their cognitive needs since, on one hand, no comprehensive view was suggested to replace the classical one (*need of visualisation*) and, on the other hand, a sharp detachment from classical views is stressed without providing enough constructive comparative criteria (*need of comparability*).

- **Ontology, rather than uncertainty**

Another difficulty that emerges also in Federico's words concerns the problem of *accepting* the real 'nature' of uncertainty: it not only fixes a limit to the precision with which couples of observables can be known, but someway restates the matrix nature of observables themselves, and of their spectra of values. In these terms, Federico is trying to *accept* this epistemological distinction, as he knows that it is the new interpretation, but he cannot find a cognitive support to take it as his personal explanation. This can be clearly noticed also in Marco's disappointment in extract [E-2], which we report here again:

Interviewer: […] *What was the most useful way for you to comprehend the meaning of quantum uncertainty in its revolutionary holding?*

Marco: *So… if I have to be honest, no one.*

Interviewer: *None of these* [he had listed them before]*?*

Marco: *All of these partly contributed, but none has given me a thorough explanation. Namely, what I was searching for as an explanation, I haven't found it in any of these.*

Marco: […] [I was in] *a great confusion, not mostly because of the mathematic part,* [in fact] *I could understand the concepts the teacher was talking about,* […] *they were logically comprehensible. The point is that I couldn't understand how couldn't a body have its own properties, well-defined properties…*

Marco's concern does not relate to uncertainty's experimental implications, but to its restatement of the 'superposed' description of quantum objects' properties, namely their ontology. This particular finding seems to be at the heart of our whole analysis around acceptance's dynamics, and will be highlighted also for what concerns the superposition state in section (3.2): there seems to be an 'ontological' dimension of learning (which is not the philosophical one about the stances upon the ontology of quantum physics, it's more likely a 'cognitive ontology') that strongly influences and shapes students' conceptions and mental images. This dimension, as proposed, is expressed in the *need of 'reality'* and it seems to have a crucial role in respect to the others for facing non-acceptance.

The examples show that, when facing uncertainty relations, the common sense of 'real' seems to get stuck either on the plane of *object's inner properties* (as from this latter extract) or on the plane of *measurement* (as also reported for example in [E-8] and [E-10] from Alice's interview): this latter, in fact, cannot be more considered in quantum physics (QP) as a passive process, just revealing objects' own properties, but it takes an active role in determining them, even though not classically disturbing the system.

3.1.3 Triangulating with literature: teaching the *uncertainty principle*

The *uncertainty principle* seems to have been broadly recognized as one of the most difficult issues for almost all the teaching proposals (as also extensively reported by Malgieri (2015)).

Consistently with the findings exposed in (3.1), two main misconceptions have been reported in literature concerning uncertainty (see for example Johnston, 1998; Müller, 2002; Ayene, 2011). The first is of considering the principle as expressing an experimental limitation, so that experiments cannot be performed more accurately than a certain limit); the second is of individuating the cause of uncertainty in the measurement apparatus itself, along the line of Heisenberg's interpretation of 'gamma rays microscope' thought experiment.

As already mentioned, the analysis of some students' profiles conducted in (Lodovico, 2016) about Bologna's 2015 experimentation in Rimini, showed that uncertainty was one tough topic that had to be re-thought, and the difficulty was mainly ascribed to the semi-classical fashion in which students were prone to interpret it, often associated with the 'disturb interpretation'. This problematic result led the group to seriously re-consider the choice of an historical approach.

However, the analysis of the literature shows that the issue is still deeper, since similar results have been obtained in other studies where the gamma rays experiment was not addressed.

For example, in the experimentation carried out by the research group of Pavia in 2015 (Malgieri, 2015, chapter 6) the *uncertainty principle* was the less scored topic in the final tests, despite having been improved from the previous experimentations. What is interesting in Pavia's findings is that "*a majority of students* […] *obtains a score of 2 or more* [between 0 and 4], *meaning they are able to identify the basic inadequacy in the presentation of the uncertainty principle as a perturbation due to measurement, although not always providing valid connections*". This is exactly what was underlined for Federico in (3.1.2), who is aware (in theory) that the disturb interpretation is no more acceptable, but cannot find a cognitive ground on which to build up the new interpretation, and logically connect it to the obsolete one. What is missing here, as well as in Bologna students, is the

confidence with *a new ontology* of objects and observables. There is the feeling that old categories do not work, but the new ones do not sound enough convincing and reliable.
Also the research group of Udine University, which follows a phenomenological approach to QP based upon the polarisation of light and the construction of the concepts of state and superposition principle, reports some difficulties in comprehending the consequences of the existence of incompatible variables, and in accepting the fall of determinism (Stefanel, 2007).
All these results show that a new approach has to be developed and the debate has to be moved explicitly to the *ontological* plane. This point is described in the next section, since it concerns also the cognitive problems of accepting quantum superposition principle. We only anticipate that by "ontology" we refer to a cognitive dimension and not to a philosophical or metaphysical one. Ontology, in our sense, refers to "small" and local issues and refers to how students conceptualize the models of objects and physical processes and ascribe a sense of existence or reality to elements of the physical description (Levrini & diSessa, 2008).
Dealing with the ontological issue would re-frame the current debate about teaching/learning uncertainty relations, that so far has been mainly focused on whether or not discussing Heisenberg thought experiment on gamma rays microscope, and to what contexts (experiments) seem more appropriate to introduce uncertainty.
Very briefly, for what concerns Heisenberg's thought experiment on gamma rays microscope, some authors take the clear stance of avoiding any type of reference to the uncertainty as produced by a disturbance (Fischler, 1992; Ireson, 2000); some others, instead, believe that an adequate introduction to the historical debate with Bohr could suffice for providing the critical instruments to accept the new interpretation (Hadzidaki, 2006; Velentzas, 2011); this was the line chosen also for the development of Bologna's proposal.
As for the issue of what context seems more appropriate, a recent proposal suggests to introduce uncertainty principle in association with the derivation of the ground state energy for a particle in a box (Dreyfus, 2015). In other textbooks, instead, uncertainty principle is introduced with the analysis of the single-slit diffraction experiment performed with single photons (Cutnell, 2007; Halliday 2010), as proposed by Muiño (2000) and by Rioux

(2005), and experimentally realized with electrons by Matteucci and co-workers (2010).

According to our cognitive needs, the analysis has pointed out the following weaknesses of our approach:

- the way we chose to introduce and address Heisenberg's thought experiment failed in providing to students a quantum-like reliable conception of uncertainty, for several reasons. On one hand, it resonated with a classical paradigm and the critics presented appeared to be mainly destructive; on the other hand, the path emphasised too much why the classical interpretation was not acceptable, without providing any effective *comprehensive picture* that students could *constructively compare* with the classical ones;
- an explicit ontological issue needs to be addressed so as to guide progressively the students to shift their attention from the objects, as the basic ontological unit, to the states. This is the most important result that also the following discussion on the superposition state points out.

3.2 SUPERPOSITION STATE

3.2.1 *Superposition state* in Bologna's path (2015-16)

In Bologna's proposal, *superposition state* was introduced and formally developed through the discussion of the "Most beautiful experiment" (MBE) and of Stern-Gerlach sets-up (SG). In the way it was introduced, the quantum state played the role, on one hand, of the mathematical ground for interpreting the results of some experiments and, on the other hand, also of a new descriptive imaginary of the object itself. The main passages (only the ones concerning the present discussion, see (1.2) for a general overview) have been proposed to students as follows:

- The MBE (presented by Giorgio Lulli (Lulli, 2013), researcher of the CNR-IMM of Bologna)[3] was the context where a reflection on the difference between quantum and

vrini & Fantini (2013), and refer to an experimentation led in Rimini by Paola Fantini in 2005
e teaching experiments (Lodovico, 2016).

classical interference was carried out and where it was qualitatively discussed the notion of indistinguishable alternatives and “which path?” (or “which way?”) configurations.

More specifically, the superposition principle was recalled for classical waves, so as to position it at the basis of the interference patterns produced with mechanical waves, or with intense beams of photons and electrons. The main massages were that “interference implies a superposition principle at its ground” and that, in the classical sense, interference implies the presence of two wave sources, as it is in the case of the double-slit set-up. The next step was to show that superposition does not necessarily implies waves in a classical sense. The crisis of a classical wave paradigm was stressed by comparing what happens when the intensity of the beam is progressively reduced until single object, and what the classical wave paradigm would have foreseen: instead of having interference fringes progressively less and less intense, dots appear.

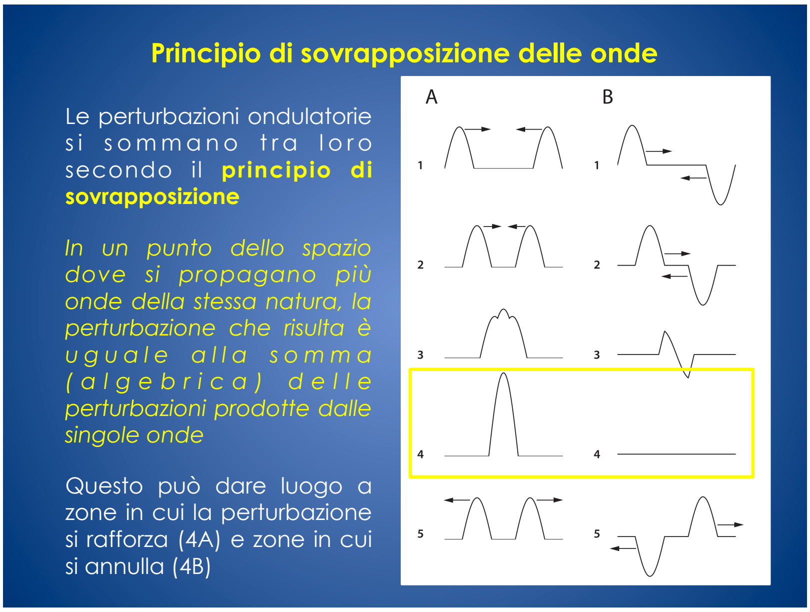

Figure 3.2

Figure 3.3

After the presentation of the results obtained with single electrons, the attention was focused on the oddity of the interference obtained with single particles, as for classical interference there would have been at least two perturbing sources. In order to capture such oddity the well-know Dirac's statement was quoted: "*Each photon* [or electron] *interferes with itself*" (Dirac, 1947) (see fig. 3.5).

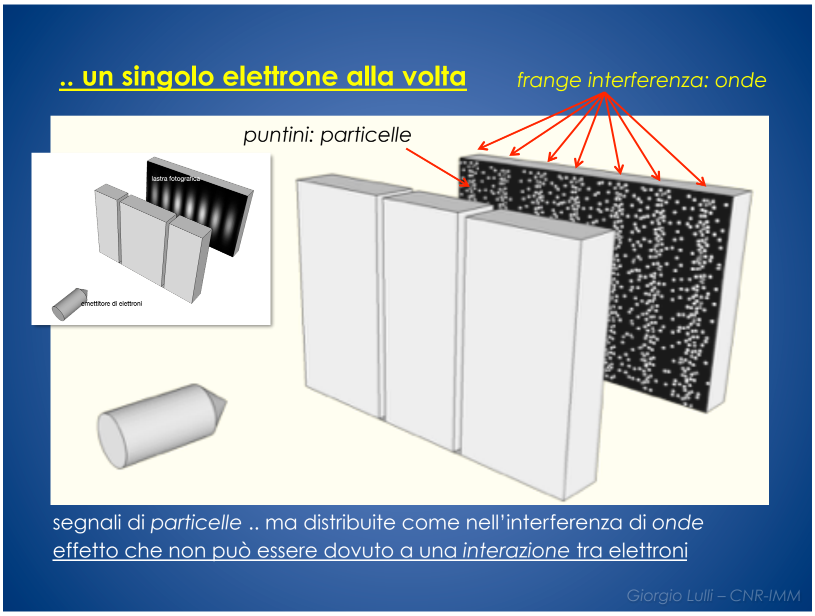

Figure 3.4

Figure 3.5

As the quantum object must have a peculiar characteristic which could explain the interference pattern, the "Which way?" (WW) apparatus was presented to better explore the experimental conditions for the interference to occur. In doing so, special emphasis was given to stress that these experimental configurations have been designed to not classically disturb the system, and as an example the experiment proposed in Scully (1991) was briefly described (see fig. 3.6).

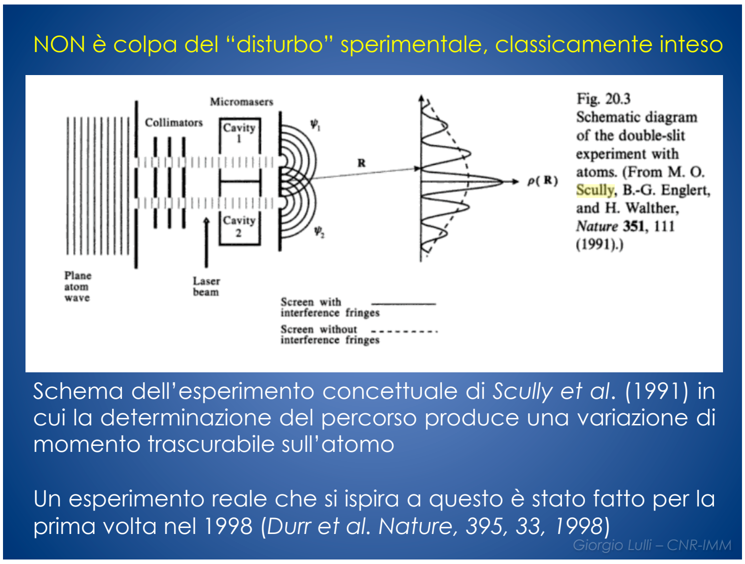

Figure 3.6

In order to explain the disappearance of the interference pattern due to the preparation of a Which Way set-up, Feynman words were used to point out that we are not talking about a classical object: "*regardless of the quantum system, any information – recorded or not – about the alternative taken by a quantum process capable of following more than one alternative, destroys the interference between alternatives*" (Feynman, 1965). The interference is hence stressed to occur not between classical waves but between *alternatives*, and the electron to form "*a world of potentialities, or possibilities*" (Heisenberg, 1962). In this sense the object can no more be described separately to the measurement apparatus, as the measurement process has a crucial role in determining the state of the object itself, despite doing so not in a classical fashion (for example practicing a force); this exquisitely quantum-like property has been stressed as a form of *entanglement* between the system and the object, even if this terminology didn't return until the final part of the course about the applications of QP.

- The superposition state was next formalised in the context of SG experiments. After the introductive part about the physical properties of SG magnets, were it was pointed out that a SG magnet works exactly as a spin *analyser* (as the atoms' beam splits in two separated spots), a phenomenological argumentation was proposed in order to gather some logical conclusions. Some different experimental configurations with subsequent SG magnets were proposed, and the argumentation was developed as follows.

Performing the experiment with only one SG apparatus (either directed along the x-axis or the z-axis) and with single atoms[4], three annotations were made:

- each atom arrives either in the upper or in the lower spot;
- each atom arrives only in one spot;
- if repeated several times, half atoms will get the upper spot and half in lower. So they have 50% of probability to be revealed with spin-up or spin-down.

[4] The following images are taken from the presentation developed by E. Ercolessi

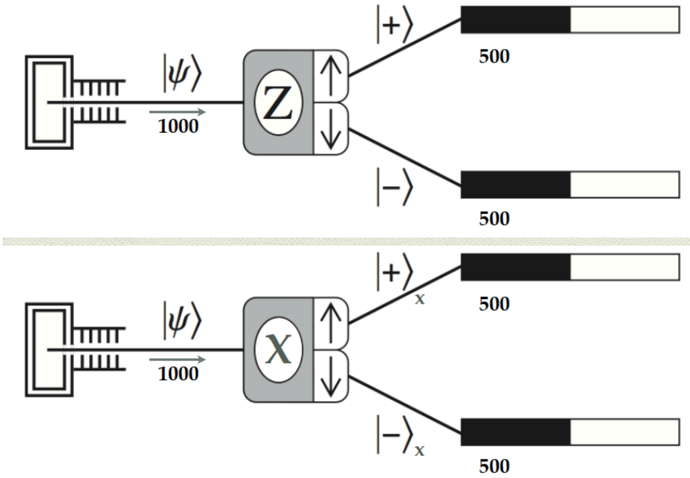

Figure 3.7

Here the concept of state was introduced for describing atoms' spin at the moment of measuring: |+⟩ for spin-up state, and |−⟩ for spin-down. An important annotation was that, given this situation, one could be prone to think that the atoms own a precise value of spin (spin-up or spin-down), and that the apparatus just reveal it as it was before passing through the magnet. Afterward, some more complex configurations with SG magnets were proposed and analysed as performed in several experiments, getting statistical results reported in fig. 3.8.

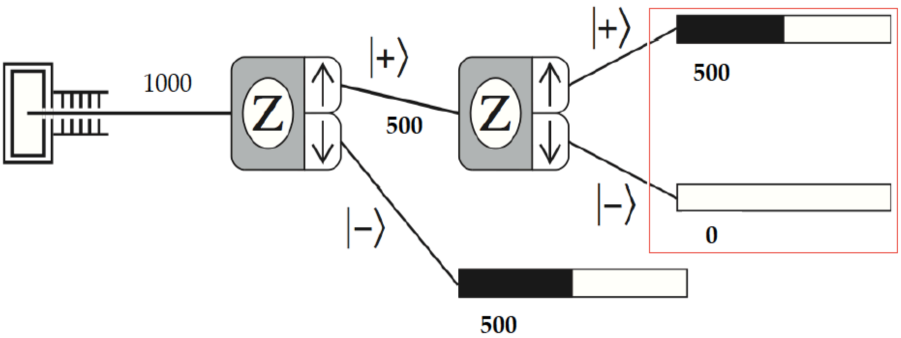

Figure 3.8

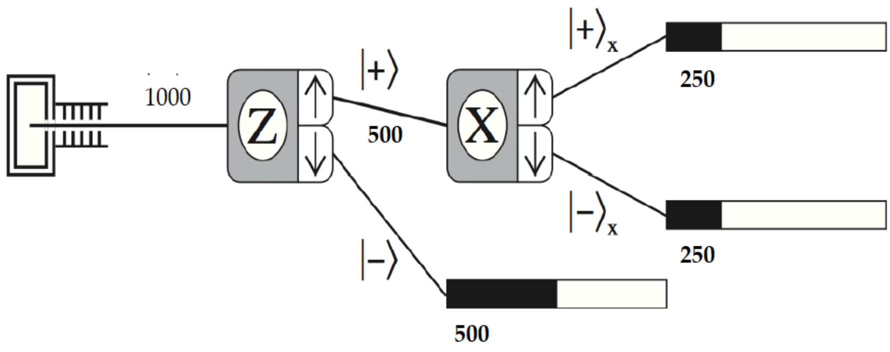

Figure 3.9

After the configuration reported in figure 3.3, a final configuration (fig. 3.9) was given and it often happens that students read it along a classical "path logic" and foresee a result of 50% and 50%, different from what experimentally observed.

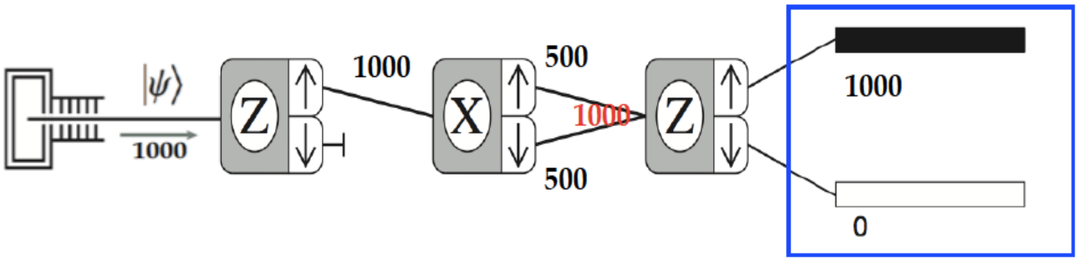

Figure 3.10

The surprising experimental outcomes impose to re-think about our implicit conception of property, and a reflection on this point was proposed in giving the question: *according to classical physics*[5], *what value of spin does the atom own between the two Z magnets*?

- **Both $|+\rangle$ and $|-\rangle$**: this is not an acceptable option, as all the experiments confirm that atoms do not split up;
- **Neither $|+\rangle$ nor $|-\rangle$**: non acceptable, as all the experiments confirm tha atoms do not vanish;

[5] Indeed, spin is not a classical property, but here the intention was to stress the implicit classical-like logic we are used to think with

- **Either $|+\rangle$ or $|-\rangle$**: again not acceptable, as the experiment in fig. 3.9 gives different outcomes.

This logical 'empasse' is resolved by pointing out that we were making the implicit assumption that revealing an atom with spin-up or spin-down meant that before the atom did own that particular value of spin. Dismissing this assumption, the *superposition state* was introduced, as describing a linear combination of the classically admitted alternatives, whose coefficients are the corresponding probability amplitudes; it was strongly underlined that this description does not belong to anyone of the previous logical option for the atom's spin.

Some notions about vectors and linear algebra were then resumed. The superposition state is expressed as the state vector $| +\rangle_z$:

$$|+\rangle_z = cos\theta|+\rangle_x + sin\theta|-\rangle_x$$

where 2θ is the angle between the directions of the magnetic field of the two magnets (for the case presented, X and Z). In this particular case, being $\theta = 45°$, we obtain:

$$|+\rangle_z = \frac{1}{\sqrt{2}}|+\rangle_x + \frac{1}{\sqrt{2}}|-\rangle_x$$
$$|-\rangle_z = \frac{1}{\sqrt{2}}|+\rangle_x - \frac{1}{\sqrt{2}}|-\rangle_x$$

Assuming this new definition of the state, together with the rules for calculating probability amplitudes (multiply in sequence - add for the final amplitude - square to get probability), the outcomes were justified and predicted. Finally, for allowing the description of spin states directed in any space direction, the more general complex linear combination was introduced:

$$|\theta, \phi\rangle_z = cos\theta|+\rangle_z + e^{i\phi}sin\theta|-\rangle_z$$

- To extend the validity of quantum states' formulation, so as to considered it in its generality and not only as an *ad-hoc* method, another experimental set-up was presented: the *Mach-Zender interferometer* (MZ). The apparatus was composed of a low-energy photon source, two beam splitters (semi-reflective surfaces, going through which the photon has a 50% probability of being reflected or transmitted), two mirrors, and two single-photon detectors, arranged as in the figure below.

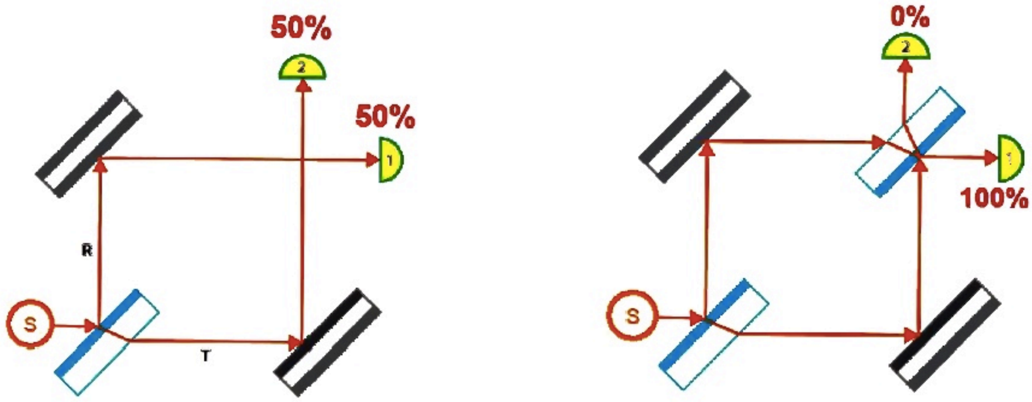

Figure 3.11

In the first configuration only one beam splitter is adopted and the results (again obtained statistically) do fit classical expectations (50% e 50%); therefore it can be perfectly explained on the basis of classical optics. But when another beam splitter is interposed, the outcome is again not predictable with classical probabilities, as they are explained only through the introduction of the superposition quantum state, with probability amplitudes. In this sense the beam splitter is completely equivalent to a SG magnet, despite being the two systems and the observables physically different.

- It was finally mentioned to students that the superposition state could be used also to reinterpret the *double-slit interference* with single electrons, in terms of all electron's possible paths. In principle there is a different path for every point of the revealing screen, and, on varying of the position x, a *wave function* $\psi(x)$ univocally individuate the state (and thus the probability amplitude) of the 'quanton'.

3.2.2 Empirical findings

This section is focused on the critical issues emerged by students' responses to the teaching proposal, with respect to the superposition state. In particular, two critical questions about were individuated as mostly preventing acceptance:

1. *How can an object have non-well defined properties (before being measured)?*
2. *Where do I find that formula* [related to the superposition state] *in reality?* That is, how can an apparent mathematical trick describe familiar physical phenomena?

After the analyses of the two questions, one minor problem are also highlighted in respect to the superposition state.

- **How cannot an object have well defined properties (before being measured)?**

It would be redundant to report again all the extracts found on this point (see (2.3.3), in particular the case of Marco); we just underline that in students' words the expression 'well-defined properties' often and tacitly implies 'classical properties'. This stands clear when noticing that students often consider a 'quanton' to be in a precise state only when measured, and so only when it lies on one of the possible alternatives, on a projected vector. As an example, we report only an extract of Alice's interview (Castel San Pietro, 2016):

Interviewer: [Regarding the superposition principle] *what were you imagining when we were talking about it?*

Alice: *When you said that the* [a particular] *state comes to be defined only through measurement…this shocked me a little, because that is not an 'intrinsic' characteristic, and so I really don't know how to visualize it…*

Interviewer: *So what would you say to be the quantum strangeness?*

Alice: *That the state is assumed only when measured, and not before. So one cannot really say what is it…*

Alice is rephrasing what the teacher told her by introducing a key-word: "intrinsic". With 'intrinsic' Alice seems to refer to the very classical ontology where objects are thought to have their own properties, which the act of measuring has to unveil. The superposition is not yet considered as a "real" property that characterizes the whole system (object and apparatus). In particular it seems that Alice didn't move to conceptualize the role of states as the main ontological elements that can define quantons: she didn't make any move to think that in QP objects should not be conceptualized as something *in* a state, but as *the states themselves*. In QP, indeed, completeness shifts from being a property of the object (cognitively perceived as the basic unit that keeps together the whole information that we can collect from the world) to being a property of the state. This very point, namely the *ontological shift from object to state*, will be deepened in section (4.2).

- **Where do I find that formula in reality?**

As highlighted in Stefanini (2013) and Lodovico (2016), the introduction of the superposition state seems to have been generally well greeted by students as a logically coherent answer for 'getting the counts fitted in'.

This point came out through questions posed by the students themselves, like the one we reported as title of this section (yet partially discussed in (2.3)). This question raised during a lesson (Castel san Pietro, 2016) led by the author of this dissertation, after had deeply analysed the state of the two classical alternatives superposition for describing SG outcomes. The question had a large consensus in the class when it was posed and, in our opinion, the it resonated mainly with both the cognitive needs of *visualization* and *comparability*. The formal expression of the superposition state *per se* is not effective for several students either to outline an intuitive picture of the new interpretation, or even to provide contrastive criteria to compare this interpretation against the classical idea that one project on more familiar phenomena. In particular it fails to provide an acceptable explanation of the double slit experiment, that offers to the students a more familiar picture of reality than the Stern and Gerlach one.

As we discussed in chapter 2, Marco (Rimini, 2015) for example complained that the mathematical definition of *quantum state* was not enough for supporting imagination and intuition [E-12]. Also in the excerpt [E-3], Marco claims for the lack of contact between the formalism and the 'reality' of phenomena, as he states that to him "*the word 'probability' is quite an 'escamotage'* [trick] *that we use to*".

- **A minor problem: Why that *minus sign*?**

A minor problem but still related to ontological issues concern the apparent conventionality of the sign when it is introduced in the expression for the components of spin in a given direction in SG experiments, which are as follows:

$$|+\rangle_z = \frac{1}{\sqrt{2}}|+\rangle_x + \frac{1}{\sqrt{2}}|-\rangle_x$$
$$|-\rangle_z = \frac{1}{\sqrt{2}}|+\rangle_x - \frac{1}{\sqrt{2}}|-\rangle_x$$

The question concerns in particular the reason for choosing the minus sign for the second superposition. Indeed, the choice of the sign is perfectly arbitrary, as what matters is only the relative phase between the states; but this question seems to hide a more demanding issue than the merely mathematical one, mainly because when answered this formally, they generally appear to be disappointed. Actually, in our opinion, what students are searching for is the physical meaning for that expression to be different from the first one, as they have not yet gathered the abstractive power of quantum formalism, which remains only as a limitation for understanding.

3.2.3 Triangulating with literature: the *ontological shift*

In the following sub-sections, we zoom in on the issue of the *ontological shift*. It will be contextualised both from a physics' foundations' point of view, and from a didactical research's one (indeed entangled to each other).

The first sub-section aims to move the analysis more explicitly from a philosophical to a cognitive plane. There are many reasons for this conceptual and methodological choice: (1) acceptance seems mainly influenced by cognitive needs, more referred to a 'cognitive ontology'; (2) the debate upon the interpretations of QP sounds too complex and idle for the teachers.

The second sub-section present the main results of an analysis of the teaching approaches to QP, that has been carried out with two aims: (1) to better and better characterize the "cognitive" ontological shift we are interested in, by distinguishing sources of cognitive difficulties from elements that can instead characterise legitimate epistemological stances; (1) to point out where and how the teaching path can be improved.

3.2.3.1 From philosophical interpretations to 'cognitive ontology'

The analysis of students' reactions stresses the evidence we already pointed out in the analysis of uncertainty relations: the crucial role of the ontological plane in learning QP. Also here the cognitive need of reality seems to highlight the lack of an accomplished 'ontological shift' from a classical view of a physical object to the quantum one, based on the very notion of quantum state.

This result is not new in physics education research. For example, Mannila & Koponen (2001) pointed out that "for students the main difficulty lies in the *conceptual shift* needed in order to form a new ontology", as they "are used to direct their attention to properties of entities (particles, bodies, etc.), create images and draw pictures, where illustrations concentrate on the behaviour of entities. A similar approach is very difficult in QP where the properties of basic entities are difficult to approach and one should really concentrate on properties of phenomena".

Moreover, is it well known that these difficulties do present despite students' confidence with the formalism; many of them, in fact, emerge also in students who strongly succeed in addressing the mathematical basis required by the courses, as in our own results, where the formalism often came to be considered as a mere "mathematical tool".

This is what pointed out also in the analysis of Greca & Freire (2003) concerning a course of engineering students, where "it seems evident that […] quantum concepts are fragmentary or mere mathematical expressions. […] The traditional introductory courses approaches do not favour students learning the quantum way of perceiving phenomena; and latter courses, more technical ones, also do not seem to succeed in this goal either".

Still, the results of the Pavia's research group confirm that QP's formalism is not enough to facilitate acceptance, as the "students can build satisfying mental models […], accepting them at least to the extent of considering them reliable sources for explanations", but "whether they accept them as a definitive description of reality is […] a more definitive issue to settle" (Malgieri, 2015, chapter 3).

From an historical-epistemological perspective, the most debated point is weather conceiving the quantum description as gathering the object in its *reality* or only as a theoretical framework for expressing the rules of our *observation* of the object; to use the words of J. Bell (Bell, 1987), the clash is on what the theory has to be about, *observables* or '*beables*'. These issues have been debates throughout decades, starting from the very beginning of QP's development. As it is well known, personalities of the calibre of Heisenberg, Born, Bohr, Schröedinger and of course Einstein were deeply engaged in resolving these ontological frictions, often guided by their personal epistemological views upon physics and science. The debate is still open and somehow vivid. The great number of interpretational theoretic frameworks developed through the years for answering these issues witnesses the actuality of QP's interpretive problem. A complete review of these frameworks is definitely out of the purposes of this dissertation, but we want to mention again the recent study conducted in 2013 by M. Shlosshauer and colleagues (Shlosshauer, Kwiat & Zeilinger, 2013). It shows very clearly the lack of consensus on the foundational themes of QP, and in particular about the philosophical interpretation of quantum objects' ontology.

The incredibly various panorama of interpretations, together with the actual widespread disagreement upon this kind of themes, impedes teachers to gather a precise stance on them, and creates a general concern about dealing with these issues. Furthermore, our own empirical results do not concern (so much) philosophical issues and conceptions, but instead, as yet pointed out, the cognitive need to "form a new ontology" (Mannila & Koponen, 2001) for accepting QP and relating it with the others fields of physics knowledge. A similar discovery was carried out in the teaching/learning of special relativity where most of the documented difficulties in understating the relativistic effects were traced to the lack of an ontological shift from "looking on terms of objects and phenomena" to "looking in terms of events" (Levrini & diSessa, 2008). In the cited paper the authors used the term "ontological" for a description that they felt is apt for the naïve idea that "something" exists and has somehow intrinsic properties. In special relativity, deep understanding and acceptance occurred when students not only recognised that objects did not have a fixed length and phenomena an inner duration, but also when they were guided to shift their sense of reality to something else. In that case that 'something else' was the ‘event’ that became the new ontological entity to which reality could be ascribed. In QP that entity is the quantum state. To foster such a shift means to re-think about the discourse and the language since the success passes through very precise and “small” details. Like in the case of relativity, the “object” ontology is so strong that also apparently insignificant aspects can be crucial.

Therefore, despite being great part of the debate upon teaching QP focused on interpretative issues, we mainly claim for the need to shift the ground on a cognitive plane, as to further move from an ‘object-ontology’ to a ‘state-ontology’. How do the other proposals address such an issue? This question is addressed in the next section.

3.2.3.2 Main didactical choices on the ontological shift

An analysis of the research literature explicitly carried out to answer the previous question led us to identify three main didactical approaches about the ‘ontological shift’:

1. a clear-cut *refusal of any reflection* about the ontology, being a controversial point:
2. a progressive *cleaning-up process* from the classical to quantum ontology, up to build up a new language to be applied to deal with quantum objects;
3. a *‘reasoned’ jump* to the quantum ontology through the introduction of a new interpretative scheme, for describing a wide range of phenomena.

In what follows these three main educational choices are outlined and briefly analysed.

1. *Refusal of any ontology*

This particular stance on the ontological problem, the most ‘orthodox’ one, finds its supporters in those who generally agree with the Copenhagen interpretation of QP. A famous statement by D. Mermin captures the mood: “*If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be: shut up and calculate*!”.

This perspective is emphatically sponsored by Feynman himself, as expressed in the following statements:

"*Physicists [...] understood that the essential point is not whether you like a theory or not, but if it provide forecasts in accordance with the experiments. The philosophical richness, ease, the reasonableness of a theory are all things that do not interest*" (Feynman, 1985).

"*What I am going to tell you about is what we teach our physics students in the third or fourth year of graduate school and you think I'm going to explain it to you so that you can understand it? No, you're not going to be able to understand it. Why then am I going to bother you with all this? Why are you going to sit here all this time, when you won't be able to understand what I am going to say? It is my task to convince you not to turn away because you don't understand it. You see, my physics students don't understand it either. That is because I don't understand it. Nobody does*" (Feynman, 1985)

Basically, the teaching proposals which explicitly take this stance on the ontological shift are the ones which faithfully follow the Feynman's model for explaining QP, based upon the *Path's integrals' theory* (Feynman, 1965). An example is the teaching proposal reported in Dobson (2000) for British schools, that completely entails Feynman's reflections and builds up the *phasor*'s methodology explicitly avoiding any interpretative question:

"*The gamma photons arrive randomly. How do they get from A to B?*
We have no way of answering this question. This fact is strongly emphasized to students. All we can observe is what happens at A and what happens at B. We don't try to talk about wave–particle duality—this confuses the issue. When students studied waves they saw 'interference' effects, which are fairly easily explained on a wave model—but how can photons get into this act? Or, how can a 'wave' pack itself into a space small enough to trigger a GM tube? We don't know. We don't really care" (Dobson, 2000)

Here the refuse of any interpretation is subtle since it is implicitly promoting an ontological shift from the object to the experiments results: to happenings in A and B. The approach is suggesting to avoid any attempt to model what happens between A and B and just trust formalism prediction power. This ontological shift can work for some students and Marco and Cheng explicitly adopted this view and they were successful in appropriating the physics contents. Marco, more than Cheng, however seems to complain that that ontology is "for him".

More in general, the approach that refuses any interpretative nuance not always succeed in promoting this ontological shift from object to experiments. Besides our own results, reported in (2.4), the findings of Baily & Finkelstein (2010) seem to show that generally students tend to remain still attached to that naïve realism that teachers wanted implicitly to avoid.

2. *Cleaning-up process from the classical ontology*

Another approach to the 'ontological shift' is the one that entails an accompanied process of refinement of the ontological descriptions of the object, so as to reach a language to be used for describing also other quantum entities.

Along this line are the proposals of the research group of Pavia University (Malgieri, Onorato, De Ambrosis, 2014), which is based on Feynman's approach, but it however deals with the interpretative issues, and the one of Udine University (Michelini, Santi, Stefanel, 2010) grounded on the proposal of Ghirardi (Ghirardi, 1997) about polarisation of light. In what follows some important points of each are briefly presented.

▪ Pavia's proposal bets on the strong points of Feynman's '*Sum over paths*' approach, which are, in their opinion:

- an *algorithm* that allows to face QP's mathematical difficulties in a way feasible by secondary school's students;
- an adequate *language* for expressing the most arduous and deep concepts;
- the *visualisation* of processes through a graphical representation of the mathematical model.

The authors recognize the cognitive limits of considering this a mere method and choose to follow further guidelines in order to "*facilitate meta-conceptual awareness* […] *and* *explicitly highlight* [to students] *those ontological categories that must be created anew"* (Malgieri, 2015, par. 2.3). The idea is to accompany the ontological shift of quantum objects through a step-by-step refinement process; this is made, mostly, through the analysis of modern quantum optics experiments so as to outline step by step the photons' quantum ontology (its properties and its features with respect to a classic object). The figure below is a conceptual map for the photon phenomenology, with the related misconceptions that can come out and that the following experiment aims to address:

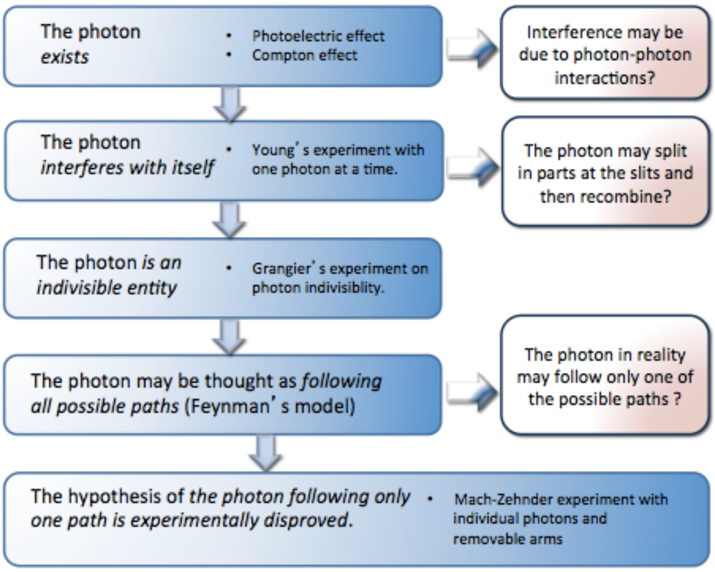

Figure 3.12

In the conclusions of Malgieri's PhD dissertation (Malgieri, 2015, chapter 7) an interesting remark concerns the visualisation of quantum processes through graphical simulations: "*simulations giving to students the wrong impression, even with subtle hints, that quantum objects retained classical features, such as trajectories, could be a decisive factor in leading them to produce inconsistent conceptions*".

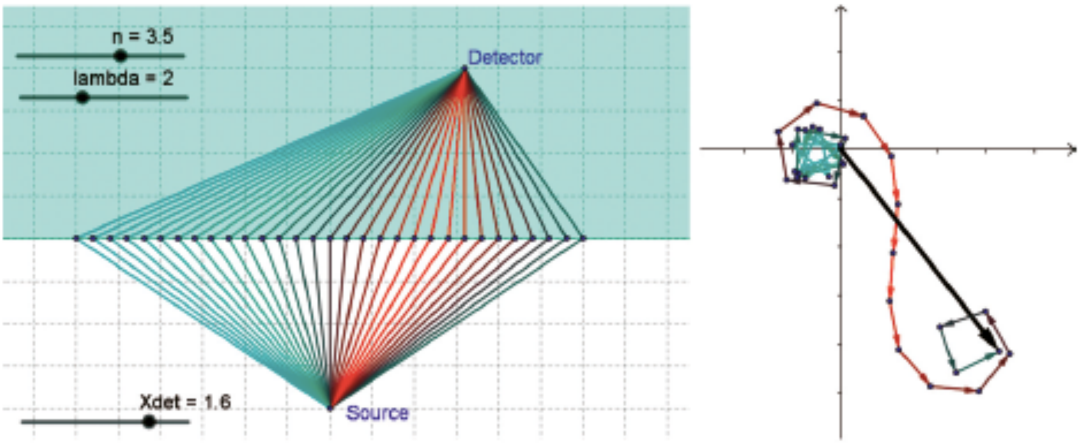

Figure 3.13

In this direction the attempt was to follow a “source-to-detector’ philosophy, so as to “*focus students' attention on the emission and detection events and the paths between them, rather than on the quantum object itself, which was never directly represented*”.

Therefore, what could seem a handhold for visualising quantum description, so to be considered a point of strength of Feynman’s approach, has been revealed in some cases to be leading to misconceptions about the object itself.

Without any presumption of drawing clear-cut conclusions, this approach is well-known to be very at risk to resonate with a classical ontology because of its language and its very insistence on the object’s possible paths, which somehow keeps fixed the attention on the object itself, and not on system as a whole; the graphic representation seems to be only emphasizing this implicit focusing. The researchers of Pavia were very conscious of that and this is the main reason that pushed them to carry out explicitly a cleaning-up process from the classical ontology. As we will describe in the next chapters about uncertainty principle and superposition state, this progressively “destructive” logic can clash against other needs and a linguistic shift toward the “state ontology” is not so easy.

- Ghirardi’s approach to QP choose to deal with the polarisation of light, as “*polarisation phenomena, and in particular the way in which polarisation states can combine with each other, present tight analogies with the way in which quantum states do combine in general, and thus allow to show the key principles of the formalism in a simple and direct fashion*” (Ghirardi, 1997).

Some experimental set-ups with *polaroids* and *birefringent crystals* are presented, equivalently to what done in Bologna's proposal for SG configurations, building up a logical argumentation about the results obtained with multiple birefringent crystals in series, with one photon at time: the photon cannot logically be considered to cross neither only one of the path in the crystal, neither both, and indeed neither none. The superposition state has to be introduced to account for the outcomes (Ghirardi, 1997, chapter 4).

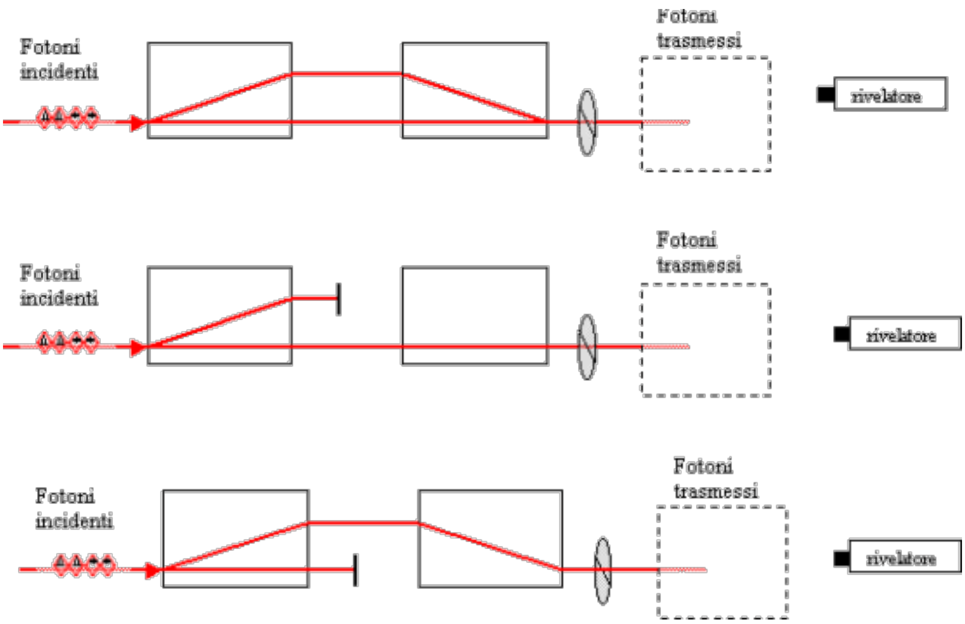

Figure 3.14

Even in this case the ontological shift is accompanied through a cleaning-up process guided by phenomenological evidences, concerning in this case the polarisation of the photon.

Keeping an eye on the ontological shift, we underline a particular point in Ghirardi's reasoning about the superposition state. As an introduction, he firstly highlights that "*the quantum mechanics' superposition principle is of an essentially different nature from the ones of any classical theory*".

However, in order to explore this difference, the language seems to be often forced to handle a still classical vocabulary, and the comparisons, also, find often their counterpart in classical examples. This is very evident in the following extract about the electro-magnetic fields in Mach-Zender interferometer (Ghirardi, 1997, p.83):

> "*Obviously, the fields propagating along the axes play the role of the 'wave function' of the photon, and their squares identify the probability for activating the detector on the x-axis or the one on the y-axis, respectively*".

Or equivalently:

> "*Reminding that the square of the* [electro-magnetic] *field rules the probability for particle to be detected along one path…*".

We are prone to say that such an use of similarities between polarisation and quantum superposition states, instead of characterizing the peculiarities of the latter, could even weaken their differences, making the two ontological planes to almost coincide.

3. 'Reasoned' jump to the quantum ontology

A third approach chosen for facing off the ontological shift is to 'jump' directly to the quantum formal and philosophical description of the phenomena, as to not get trapped in semi-classical views, and only after compare this picture with the classical paradigms.
An example of this stance is the didactical proposal of Pospiech, yet introduced in chapter 1, who choose a logical-philosophical approach for teaching quantum physics (Pospiech, 2010). In some of her works, the author points out that "most difficulties in understanding quantum theory arise from trying to develop quantum theory starting from classical concepts and then explaining the differences", "but it is just these classical concepts borne from daily experiences that have to be thrown away" (Pospiech, 2000).
In order to reach a appropriate understanding of quantum physics concepts, the proposal is to develop a formal framework starting from *spin*, which is indeed not classical and allows for getting inside simple formal tools as Pauli's matrices, which have been demonstrated to

be well greeted by students. Next, some core concepts of quantum physics can be consequently introduced, as uncertainty principle and entanglement, which form a logical structure to be compared with classical paradigms.

Such an approach could be read as a sort of accomplishment to the ontological shift, but quite opposite from the phenomenological one, as it starts giving a synthetic quantum picture (the jump) to be next compared with the classical ones from a formal, epistemological and philosophical point of view (‘reasoned’).

CHAPTER 4

Proposals for improvements

In this chapter we suggest three lines of improvement of Bologna's teaching path, in the light of the results achieved on students' non-acceptance reactions.
As we showed, the most delicate points seems to concern the uncertainty relations and the superposition state; these parts of the path are here re-designed in order to address explicitly the exposed cognitive needs.
In particular, in section (4.1.1) we suggest a deeply different approach to address the uncertainty principle, whilst in section (4.1.2) we zoom in on the specific step where the ontological shift appears, namely in the introduction of the *pars construens*. In section (4.1.3) we outline a possible conclusion of the path, where the formal framework, developed through Stern-Gerlach (SG) and Mach-Zender (MZ) apparatuses, is applied to the familiar phenomenology of the double slits experiment.
A draft version of the improvements has been informally tested in different contexts and the reactions of the audience were strongly supportive. The first two improvements were implemented by Levrini in two contexts of in service teacher education (meetings organised by schools in Fano and Piacenza) and in the course of Physics Education at the Master Degree in Physics, University of Bologna. The third improvement was implemented by the author of this Dissertation in the experimentation led in Castel San Pietro. More structured and designed tests will be probably carried out in the next year by the research group in Physics Education.

4.1 REVIEW OF UNCERTAINTY'S INTRODUCTION

Uncertainty relations deserve a deeply revised introduction so as to resonate with the cognitive needs, and to avoid the persistence of classical views on the quantum description of phenomena. In particular, the analysis led us to realize that we had to revise the whole approach and the general logic.

As underlined, in the previous implementations, we chose to follow an historical approach, that started from Heisenberg's article and the gamma rays thought experiment, as to next pass through Bohr's and Levy-Leblond's critics. Such an approach followed a progressively destructive logic, which aimed to clean up an obsolete image of uncertainty. From the empirical findings we can affirm that the constructive pieces we offered step by step didn't result in a comprehensive picture with a sufficient strength: It failed to stress the novelties of the new paradigm and to recompose the fragments we had sown along the path.

We are now suggesting a presentation based on a completely new logic, starting from the analysis of uncertainty relations *per se*, so as to provide the essence of the physical content in a synthetic way. Only after that, we contrast such a core view with the previous classical models. In building the core view, the presentation will pave the way to the ontological shift, that will be more explicitly addressed as the first stage of the *pars construens*.

This approach was also elevated to the role of a general methodology that we applied also to revise the other topics: (1) firstly individuating the ontological elements needed to accomplish the ontological shift (which is in this context the 'spectrum of values' or 'extended set of numbers' for describing the observables, and that will become the *state* in the *pars construens*), and next (2) using this new ontology for giving a comprehensive picture (visualisation) to be compared with other familiar or even only different models or contexts (comparability).

4.1.1 The core content: building a comprehensive picture

We suggest to introduce the uncertainty relations in their formal expression and to discuss how this apparent simple formula contains deeply revolutionary contents:

$$\Delta \boldsymbol{x} \Delta \boldsymbol{p} \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

where $\hbar = \frac{h}{2\pi}$ = 1,054x10^{-34}Js

As first step, the physical content of the relation is investigated through the analysis of a single-slit diffraction experiment executed with single photons [Muiño, 2000; Matteucci, 2010].

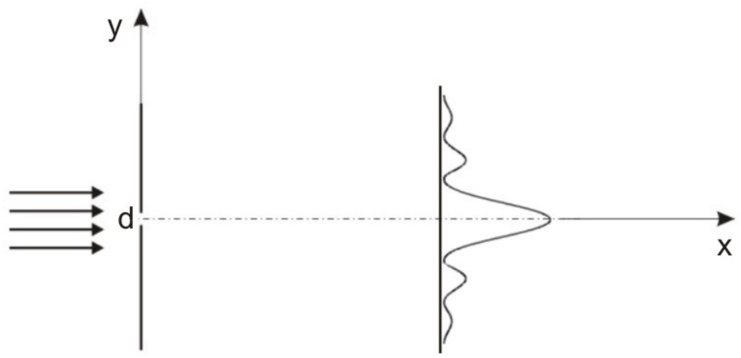

Figure 4.1: a single-slit apparatus scheme

Making a photon beam pass through the slit generates an interference pattern on the screen. It can be made clear to students that, thanks to the de Broglie's relation, this phenomenon occurs for any particle or atom.

The experiment is analysed by applying the knowledge that the students already have about light diffraction so as to stress that crucial step represented by the "single object configuration" and, hence, by the application of the De Broglie's relation. After that, the phenomenon is analysed in terms of *information*, considering the width of the slit as a measurement of the photon's position along *y* direction, and the spread of the diffraction pattern as indication of the uncertainty on the momentum component along y. Making smaller the slit means having a more precise information of position, but at the same time the diffraction pattern spreads more along y, which means having a less precise measurement of photon's momentum along y. The uncertainty obtained on the momentum can be theoretical estimated by applying the knowledge of optics ,together with the De

Broglie realtion, and compared with very recent experimental results [Matteucci, 2010] that are consistent with:

$$\Delta p_y = \frac{\hbar}{2} \Delta y$$

This case is hence re-discussed to outline the core contents of the uncertainty relations, which can be synthetized in the following sentences (the “core picture”):

- In quantum physics there exist pairs of variables (called also conjugated) linked by uncertainty relations. The more the knowledge of one variable is precise, the more the uncertainty of the other increases. One pair of conjugated variable are position and momentum but many other exist.
- There is a crucial difference between performing the diffraction experiment with a light ray, a collimated beam or with single photons. This difference, in particular, between the beam and the single object is crucial also because it allows us to introduce the so-called *quantum non-epistemic probability*, as the interpretation of deviation Δ is of a statistical mould for the first case (thus in principle reducible), and of a different nature, namely *ontological* to the description of the quantum object itself, for the latter. The “uncertainty” that emerges from Heisenberg relations cannot be reduced by increasing or refining our knowledge: it is an intrinsic feature of the relation between conjugated variables.

At this point the metaphor of the Chinese menu can be discussed with the students to stress the main core. The metaphor is puzzling since it is not well developed. Instead of being a limit, it is a typical case where students’ creativity can emerge and can be stimulated.

4.1.2 Comparing the picture with the classical descriptions

In order to resonate with the comparability need, the apparently simply formula is hence contrasted against the classical view. Three main points can be stressed for their deeply revolutionary potential:

1. Uncertainty (that in Italian has the more appropriate name of "indeterminazione") does NOT have an experimental nature, but it is inner at the existence of conjugated variables;
2. In quantum physics modelling, the properties of the object are assumed to have a spectrum of values and "NOT a well-determined value that measurement has to discover;
3. The concept of space-time trajectories and classical determinism falls down.

In more detail, the second aspect is probably the most delicate one, being the first moment that the issues of ontological shift can emerge. Reading from Lévy-Leblond and Balibar can help to stress that, "while for classical entities the physical properties take an unique and determined numerical values, for *quantons* they are characterised by numerical spectra, extended sets of numerical values" [Lévy-Leblond, Balibar, 1990]. When pairs of conjugated observables are taken into account, correlations between their distribution amplitudes emerge. This happens between the spatial e the momentum distributions, but it will be observed also with the components of the spin along x and z directions in the SG apparatus. This is the key concept that will also allow the metaphor of Erwin's socks to be grasped.

Also the third point is very delicate and not easy to be really appropriated. It this direction it seems to effective the reading of an excerpt from the original memory of Heinsenberg:

> "*But in the rigorous formulation of the law of causality - "If we know the present precisely, we can calculate the future" - it is not the conclusion that is faulty, but the premise . We simply cannot know the present in principle in all its parameters. Therefore all perception is a selection from a totality of possibilities and a limitation of what is possible in the future. Since the statistical nature of quantum theory is so closely to the certainty in all observations or perceptions, one could be tempted to conclude that behind the observed, statistical world a "real" world is hidden, in which the law of causality is applicable. We want to state explicitly that we believe such speculations to be both fruitless and pointless. The only task of physics is to describe the relation between observation. The true situation could rather be described better by the following: Because all experiments are subject*

> *to the laws of quantum mechanics and hence to equation (I), it follows that quantum mechanics once and for all establishes the invalidity of the law of causality*" (Heisenberg, 1927)

If teachers wish to comment the gamma rays microscope, since this is how the textbook addresses uncertainty, the suggestion is to do it after this presentation as a sort of exercise aimed to recognise its "classical" nature. When this strategy was used in the context of teacher education made the teachers very happy since they experiences that exercise as a confirmation that they "got the point".

4.2 FOSTERING THE ONTOLOGICAL SHIFT

As pointed out in Brookes & Etkina (2007), "*physicists' language encodes different varieties of analogical models through the use of grammar and conceptual metaphors*" and it seems (it was the hypothesis pursued in the article) that "*students categorize concepts into ontological categories based on the grammatical structure of physicists' language*".

What follows is a re-analysis of the language we used to introduce the interpretative apparatus.

Our path foresees a fundamental step between the discussion of the uncertainty relations and the interpretative apparatus: Bohr's complementarity and the overcoming of the wave-particle duality through Lévy-Leblond's metaphor of platypus. This is a part that we would maintain since it always reveals very effective to stress the need to search for a new ontology:

> "*That the true nature of quantum objects has long been misunderstood is proved by their still all too common description in terms of an alleged "wave-particle duality". It must be remarked first of all that this formulation is at best ambiguous. For it may be understood as meaning either that a quantum object is at once a wave and a particle, or that it is sometimes a wave and sometimes a particle. Neither one of these interpretations in fact make sense. "Wave" and "particle" are not things but concepts, and incompatible ones; as such, they*

definitely cannot characterise the same entity. While it is true that quantum objects may in some cases look like waves, and in other cases like particles, it is truer still that in most situations, particularly the ones explored by the elaborate modern experiments, they resemble neither one nor the other. The situation here is reminiscent of that encountered by the first explorers of Australia, when they discovered strange animals dwelling in brooks. Viewed from the forefront, they exhibited a duckbill and webbed feet, while, seen from behind, they showed a furry body and tail. They were then dubbed "duckmoles". It was later discovered that this "duck-mole duality" was of limited validity, and that the zoological specificity of these beasts deserved a proper naming, which was chosen as "platypus". Bunge's proposal to call them "quantons", building on the common terminology (electrons, photons, nucleons, etc.) and extending it to a common categorisation, is most to the point, and it is to be hoped that this terminology gradually gains ground" (Lévy-Leblond, 2003)

So the questions become: what are the properties of this new entity that we call *quanton*? Why does the quantum object deserve, like platypus, a new name?
The answer to these questions should go, in our opinion, along the direction of recognising the "states" as the basic ontological entities that define the quantum object. In other words, the aim is to arrive to use consistently linguistic expressions that move from the idea that "objects are *in* a state" to the idea that "objects *are* the states".
On the basis of the previous remarks, we zoom in on the precise moment where in our path a new ontology is introduced. As described in (3.2.1), this happens during the presentation of the double slit apparatus used to build criteria to interpret the single object interference. The discussion is articulated in two main conceptual passages:

1. The search for *something* that can explain the occurrence of the interference pattern;
2. The analysis of Which Way configurations to *better* investigate the interference's conditions.

In that context the Dirac's statement "*Each photon* [or electron] *interferes with itself*" is used to show the weirdness of the problem and, then, the words *states*, *alternatives*,

possibilities are introduced to provide a linguistic support for the fact that interference needs a superposition principle, but this superposition cannot occur between classical waves (that idea would clash against the experimental results at low intensity).
This is exactly the point where the ontological shift was linguistically introduced in our teaching experience, but, we observed, it failed to satisfy the needs of reality and of visualization.
What we are now suggesting is to make the shift more explicit, through a detailed and consistent linguistic analysis of the expressions used to describe the phenomenological situations.

In particular, we suggest to present Dirac's sentence as a *linguistic trick* that has two fundamental features: (1) it does real work to provide an apparently effective "picture" of the quantum object; (2) it cannot be read literally (literally it makes no sense) but is should be seen as a linguistic act that provides, through a synthetic idea, a cognitive prop.
This second aspect is not trivial and was stressed by Dirac himself when he wrote:

> *One may extend the meaning of the word 'picture' [mental model] to include any way of looking at the fundamental laws what makes their self-consistency obvious*. (Dirac, 1947)

In the words of Dirac, that linguistic act doesn't provide a real picture but suggests a "way of looking at the fundamental laws" that can orient to keep the consistency of reasoning.
Besides these pros, Dirac's statement has serious cons: it can hinder any ontological shift since it outline a "picture" and a narrative strongly object-focused.
More specifically, the statement pushes a reader to look at the interference apparatus by following the space-time story of the photon (or electron), like "there is a photon (an electron), it goes through the slits, interferes with itself and the interference pattern is produced".

The same type of analysis can be carried out on the other linguist act that we introduced: "what produces an interference patter is the superposition of possible states; 'the *interference of alternatives*' is the real characteristic of the quantum world" (Englert, 1999).

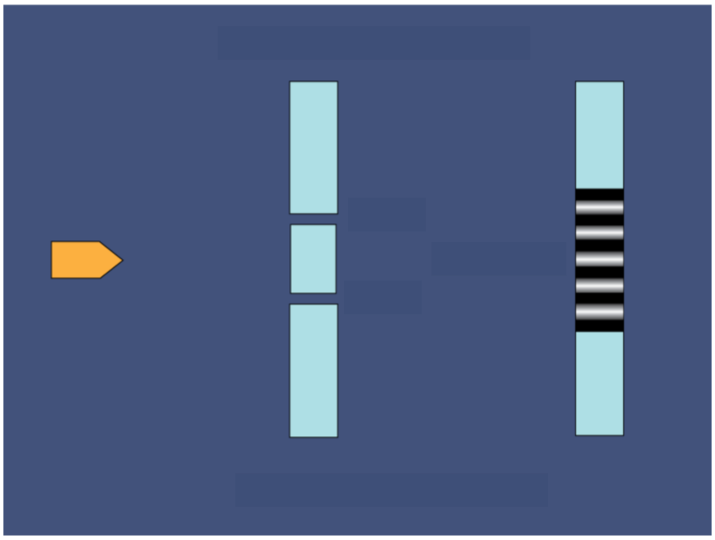

Fig. 4.1 – Double slit experiment

Unlike the statement of Dirac, this one deserves a sort of explanation since it is suggesting a real *gestalt* change. According to this statement, a reader that looks at the figure 4.1 is invited to focus her/his attention to the whole configuration and to recognise the *possible alternatives*; interference is said to come from their superposition. In this case, the narrative is not a space-time story of an object travelling through an apparatus, but a systemic a-temporal story built on the recognition of symmetries and indistinguishable paths (states).

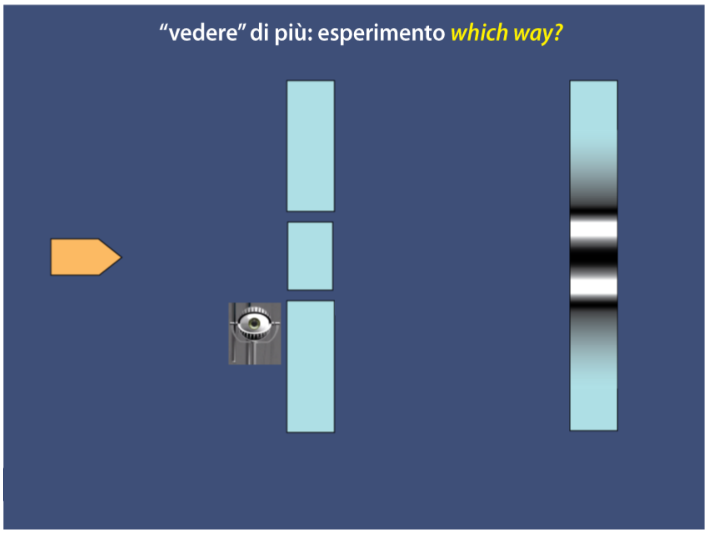

Fig. 4.2 – Double slit experiment (which way configuration)

In order to strengthen this gestalt change, figure 4.1 can be confronted with the Which Way configuration (Fig. 4.2), where information about the path can be gathered through a *non-disturbing measurement*. Here interference disappears but nothing happened to the object. What changed was the whole configuration. The only possibility for getting information about which path destroys interference; actually, it destroys the superposition of the two alternatives. As a result, the ontology of the superposition state has to be searched in the physical presence of indistinguishable alternatives and not in ordinary wave properties of the object. At this point the sentence of Feynman should sound less tricky: "*regardless of the quantum system, any information – recorded or not – about the alternative taken by a quantum process capable of following more than one alternative, destroys the interference between alternatives*" (Feynman, 1965).

In every context it was discussed, questions about the non-disturbing measurement were posed, like: "The whole argument seems reasonable but it is strongly dependent on that statement regarding the non-disturbing measurement. What does it mean?" (Matteo, Physics Education course, University of Bologna). In these cases, the Scully experiment (1991) already reported in fig. 3.6 has been discussed. Nevertheless an educational transposition of more recent experiments would be very useful.

As final remark, the expression "*interference of alternatives"* is a *trick* like Dirac's statement, a mere linguistic act that sounds strange if it is read literally. It is simply another cognitive prop that is supposed to help for checking the consistency of reasoning and, in this sense, to sustain a 'visualization need', just as Dirac's picture did. Nevertheless, its potential, unlike the Dirac's statement, lies in: (1) the narrative that it suggests, focused on a systemic view and a *state-ontology*, (2) its generalizability to every quantum system and configuration, including the SG ones, and (3) its closeness to the mathematical structure, that will be developed in the following stage of the proposal.

4.3 QUANTUM STATE AS A COMPREHENSIVE PICTURE TO INTERPRET FAMILIAR PHENOMENA

The issue of the *ontological shift* can also prevent, in our analysis, their attempt for building a comprehensive picture to interpret the experimental findings intentionally exposed to introduce the formalism (as it was for Stern-Gerlach experiments) and to compare it with more familiar phenomenologies (the double slit experiment, for example). We are indeed conjecturing that it is the lack of a proper ontology that ends up with considering the formalism as a mere mathematical tool for fitting the counts, but not adequate for *visualising* (getting it) the present phenomenon and for *comparing* it with other experimental situations, even belonging to the classical domain.

These considerations led us to develop a specific proposal that, as an attempt, follows the exposed requirements (and that could in future take the form of a *group activity*): a reflection on the maths-physics relationship based on the formal equivalence of TSI (two-slits interferometer), SG (Stern-Gerlach), and MZ (Mach-Zender) experiments.
The proposal is guessed to strengthen the ontological shift, since it is focused on an ontology of states and systems (and not of objects and its properties) and it is supposed to respond to the cognitive needs, by showing, once recognized the ontological unit (the state), how the mathematical structure built on the SG apparatus, is effective to interpret more familiar – and apparently "more realistic" – phenomenologies, like the double slits interference pattern. The operational and explicit strategy is to reframe *the SG apparatus as an interferometer*. This passage is the crucial one since it explicitly requires to overcome the idea that behind the interference there is a superposition of waves. In this sense, leading students to look at the SG apparatus as an interferometer implies guiding them to recognise that, also in this case, they have to focus their attention on the states and not on the objects.

In what follows, some remarks for outlining a possible activity are presented. In particular, the formal equivalence between the quantum description of physically different systems (as it is for all the two-ways interferometers) is pointed out. This allows to re-interpret the SG apparatus as an interferometer, so as to enforce a cognitive ontology based upon the states. Next, some observations about the generality of superposition principle are proposed, so as to distance its domain from the only wave formalism, and re-collocate it in familiar contexts, as the vector spaces in classical mechanics. Finally, a possible scheme for the activity is sketched out.

4.3.1 Remarks on the formal equivalence between different physical systems

The mathematical structure of the state vector $|S\rangle$,

$$|S\rangle = c_1|A_1\rangle + c_2|A_2\rangle$$

where $|A_1\rangle$ and $|A_2\rangle$ are the vector states for each alternative of the system and the coefficients c_1 and c_2 are the corresponding probability amplitudes, is indeed common to all two-state systems, or, to say, to all two-way interferometers, such as TSI (where the amplitudes correspond to two slits), SG (A_1 and A_2 corresponding to spin-up and spin-down), MZ (A_1 and A_2 corresponding to path-1 and path-2) or others, like 'bi-prism' interferometers, or Ramsey-Bordé interferometers for two-level atoms [Berman, 1997] [Miffre et al., 2008]. For symmetrical systems, the state comes to be expressed, equivalently as:

$$|S\rangle = \frac{1}{\sqrt{2}}(|A_1\rangle + |A_2\rangle)$$

Therefore, being the mathematic structure completely equivalent, the only difference stands in the physical interpretation of the state and the amplitudes; despite describing a photon or an electron, and despite being the observables the polarisation degrees or the position, the formalism is exactly the same. This is the great abstractive power of quantum formalism, gathering which, in our opinion, students could reinforce their conceptions about the quantum description of a system as a whole (object + apparatus).

More generally, as introduced to students after SG experiments, the most general expression for the state is described in the Bloch sphere, in dependence of two angles, as

$$|S(\theta,\phi)\rangle = \cos\theta|A_1\rangle + e^{i\phi}\sin\theta|A_2\rangle$$

that for symmetric interferometers comes to be in dependence of the only *interferometric phase difference* ϕ[6],

$$|S(\phi)\rangle = \frac{1}{\sqrt{2}}(|A_1\rangle + |A_2\rangle e^{i\phi})$$

The phase difference ϕ assumes different physical meanings depending on the experimental set-up: for the TSI, ϕ is determined by the site where the electron hits the screen, for MZ by the difference of the optical path's lengths, and for SG set-up by the spatial orientation of the second magnet (actually, a generic orientation in the space need also the angle θ to be described, that for TSI and MZ is determined by the position of electrons' source beside the slits, and by the possible variable reflectivity of the beam splitter, respectively).

For the state S, the interference pattern emerges clearly in the probability P of finding the superposition state $S(\phi)$[7],

$$P(S(\phi)) = |\langle S(\phi)|S\rangle|^2 = \frac{1}{2}(1 + \cos\phi)$$

where the interference fringes are clearly described by the dependence to $\cos(\phi)$. Thus, rereading the results of the basic SG and MZ set-ups (which were only of 100% or 0%), we can say that these are the maxima and the minima revealed both with a phase difference $\phi = 0°$ and $\phi = 90°$.

Therefore, changing the phase difference (i.e. moving along the TSI's screen, rotating the SG magnet and changing the paths' lengths, or equivalently playing with a phase shifter, in MZ) means moving along the interference pattern.

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[6] The phase factor is multiplied only to one of the basis kets because of taking one the two complex amplitudes as positive and real, which we are free to choose

4.3.2 Distancing from wave imaginary: superposition in physics

As previously pointed out, the crucial point of the proposal stands in recognising the Stern-Gerlach apparatus as an interferometer because in students' imaginary the superposition principle tends to refer to the domain of waves (so to an object ontology), and not to the states. Thus, the ontological shift is supposed to help students to satisfy the other cognitive needs (comparability and visualisation), as it would allow to compare the SG and the MZ apparatus with the double slit experiment through the concept of superposition of states, that is also the new comprehensive idea for visualisation.

However, in order to detach the superposition principle from the classical wave picture another reflection can be proposed. As just underlined, one of the strongest points of quantum formalism is precisely its generality, as the same abstract structure can describe a great quantity of different physical phenomena. Not only: from such an abstract model a great number of physical previsions can be made, such as uncertainty principle, entanglement and statistical ensemble properties (Paty, 2003). This is particularly clear for quantum theory, but it's quite a general feature of mathematical descriptions of physical reality, even for classical physics.

In particular, *superposition principle* is the descriptive base for a great number of phenomena, as every time we have deal with a vector we do deal with superposition principle. Namely, despite being talking about an electric field, a velocity or a force, the mathematical formalism shows the same fundamental property of *linearity*, and the previsions too are based upon the identification of this mathematical structure. Took for granted the abstract ground, we can use its power entirely on moving forward to foresee other physical properties, which probably couldn't be imagined in the previous experimental researches (some classical examples could be brought out in this perspective).

Leading the attention towards some different phenomena, even the classical ones, described by the superposition principle, could become a powerful tool for understanding the nature of the quantum formalism on one hand, and for distancing it from the only waves' domain on the other.

4.3.3 Sketch of a possible scheme

Below, a possible scheme for a group activity is briefly outlined.

1. After the interpretation of the SG and MZ results with the introduction of superposition state expression, an in-depth reflection on *superposition principle* and the concept of *vector* can be led in the activity, through the review of some known examples of physical quantities described by this very mathematical structure, as can be for the angular momentum, electro-magnetic fields or mechanical waves. As underlined, this could allow for previously distancing the concept of superposition from the waves domain.

   One critic point to be dealt here is the concept of *abstract space*, as to give an intuitive idea of what kind of vector space are the Hilbert's ones. This could be made by writing down the general expression for a two-component vector (for example for the force, $\vec{F} = |F|cos\theta\hat{n}_x + |F|sin\theta\hat{n}_y$) and gradually substituting the axes with different variables, as x and y, $\overrightarrow{E_x}$ and $\overrightarrow{E_y}$, or even with not 'physical' magnitudes, till pointing out that the properties of any system, or object, can be expressed as a point in an abstract space with the appropriate axes. This is also for the quantum state, defined as a vector in a particular complex space, the Hilbert space.

2. After this brief review on the superposition principle, the effective formal equivalence between TSI, SG and MZ can be constructed. Some calculations with simple values of the angle θ and ϕ can be made, so as to show the change of probability in dependence of the latters. An interactive tool, as an *applet*, could be developed in order to move easily along the interference patterns by changing (or sliding) angle's values.

3. Further, an active discussion on the physical meaning of formalism and on its predictive role can be led, also with the help of historical and metaphorical texts.

Conclusions

The analysis reported in this dissertation aimed at interpreting a specific phenomenon we observed in classes of secondary schools exposed to Bologna's teaching proposal on quantum physics: the phenomenon of *non-acceptance* of quantum physics, as we called it. Namely, we systematically observed cases of students who didn't accept the theory as a personally reliable and convincing description of physical reality, even though seeming to understand the basic concepts of QP as they were proposed.

Through a comparative analysis of students' own words we were allowed for individuating some *cognitive needs* which students are not disposed to renounce for accepting QP. We them grouped in three categories: one, the *need of visualisation*, concerning the requirement for a comprehensive image of phenomena and demanding for a 'clearer' knowledge; another, the *need of comparability*, gathering the attempt for comparing the quantum world with the classical one, and asking for a 'broader' knowledge; and the *need of reality*, grounded on an ontological plane and searching for a 'deeper' knowledge. Also, we found that the strength of such needs depends on idiosyncratic aspects of each student.

A review of the literature about students' main difficulties in dealing with QP pointed out that behind the cognitive need of reality could be found the very problematic "conceptual shift needed in order to form a new ontology" (Mannila & Koponen, 2001). Our analysis showed that this ontological issue does not concern so much philosophical arguments about QP's interpretations, but it's rather a 'cognitive ontology'. The ontological shift that learning QP seems to require for its acceptance, appears to have, by several students, the same nature as the one suggested by Levrini & diSessa (2008) for special relativity. There the needed ontological shift was between thinking in terms of 'objects' and thinking in terms of 'events'. For QP the new ontology has to refer to the 'state', as the basic ontological entity on which the imaginary for interpreting phenomena should be grounded.

In force of these and other findings about critical points of Bologna's teaching proposal (namely the uncertainty principle and the superposition state), some suggestions were developed in order to improve the path and facilitate the acceptance of QP.

In particular, for the *uncertainty principle*, it has been proposed (1) to abandon the Heisenberg's gamma rays microscope thought experiment, and (2) to not follow a destructive logic for polishing an obsolete view upon uncertainty, but to give from the beginning a synthetic image (as to gather the need of visualisation) of it with the analysis of the single slit apparatus, and only after possibly compare this image (need of comparability) with classical paradigms.

As to accomplish the ontological shift, instead, a more detailed and precise work has been proposed for polishing the language and better focus on a 'state' ontology. Another proposal developed in this perspective is to reread the Stern-Gerlach apparatus as an interferometer, as to re-give, on one hand, to the superposition state its space, detached form wave formalism, and on the other hand, to compare it with more 'familiar' phenomena as the double slits experiment.

We can conclude by pointing out some possible future directions:

- To deepen the relations between the acceptance dynamics, learning and appropriation in the light of the proposed cognitive needs;
- To apply the cognitive needs in further analyses to check their effectiveness in a broader and broader empirical basis and, vice versa, to refine their description through their test against new data;
- To further explore the ontological shift as inner mechanism for accepting QP and for supporting deep processes of conceptual changes;
- To develop new materials about 'which way?' apparatuses in a more detailed and formal way, as to reinforce the concept that measurements can be carried out without disturbing in a classical sense the system (almost always addressed by students as unconceivable).

REFERENCES

AMALDI U. (2015), *L'Amaldi per i licei scientifici*, Zanichelli Ed., second edition

AYENE M., KRIEK J., DAMTIE B. (2011), *Wave-particle duality and uncertainty principle: Phenomenographic categories of description of tertiary physics students' depictions*, Physical Review Special Topics-Physics Education Research, 7(2), 020113

BAILY C., FINKELSTEIN N.D. (2010), *Teaching and understanding of quantum interpretations in modern physics courses*, Physics Education Research 6, 010101-11

BELL J. (1987), *Beables for Quantum Field Theory*, in *Speakable and unspeakable in quantum mechanics*, Cambridge University Press.

BERMAN P. (1997), *Atom Interferometry*, Academic Press, New York

BROOKES D.T., ETKINA E. (2007), *Using conceptual metaphor and functional grammar to explore how language used in physics affects student learning*, PHYS. REV. ST PHYS. EDUC. RES. 3, The American Physical Society

CUTNELL J. D., JOHNSON K. W. (2007), *Physics*, John Willey and Sons, NY

DE REGT H.W. (1997), *Erwin Schro¨dinger, Anschaulichkeit, and quantum theory*, Studies in History and Philosophy of Modern Physics, 28(4), 461–481.

DIRAC P.A.M (1947), *The principles of Quantum Mechanics*, Oxford University Press, Amen House, London E.C.4, third edition (first edition in 1930)

DOBSON K., LAWRENCE I., BRITTON P. (2000), *The A to B of quantum Physics*, Insitute of Physics Advancing Physics Project, London, UK

DREYFUS B. W., SOHR E. R., GUPTA A., ELBY A. (2015), *'Classicalish': Negotiating the boundary between classical and quantum particles*, arXiv preprint arXiv:1507.00668.

DUBSON M., GOLDHABER S., POLLOCK S, PERKINS K. (2009), *Faculty disagreement about the Teaching of Quantum Mechanics*, PERC Proceedings, Melville, NY

EINSTEIN A., INFELD L. (1938), *L'evoluzione della fisica*, Bollati Boringhieri Ed., pp. 236-241.

EINSTEIN A. (1953), *Einleitende Bemerkungen über Grundbegriffe*, reported in Popper K.R. (1959) *The Logic of Scientific Discovery*, London Hutchinson

ENGLERT B.G. (1999), *Remarks on some basic issues in quantum mechanics*, Zeitschrift f¨ur Naturforschung 54a, 11–32

FISCHLER H., LICHTFELDT M. (1992), *Modern physics and students' conceptions*, International Journal of Science Education, 14(2), 181

FEYNMAN R,, HIBBS A.R. (1965), *Quantum mechanics and path integrals*, emended edition by Styer D.F., McGraw-Hill Companies, Inc., New York

FEYNMAN R. (1985), *QED: The strange theory of light and matter*, Princeton University Press, 0-691-08388-6

FRAYN M. (2009), *Copenhagen*, Sironi Ed.

GHIRARDI G. (1997), *Un'occhiata alle carte di dio*, Il Saggiatore

GRECA I.M., FREIRE O. JR. (2003), *Does an Emphasis on the Concept of Quantum States Enhance Students' Understanding of Quantum Mechanics?*, Science & Education 12: 541–557

GREENE B. (2004), *The fabric of the cosmos. Space, time, and the texture of reality*, Random House, Inc., New York

HABER-SCHAIM U. (1975), *On the Teaching of Quantum Physics in the Senior High School*, in Seminar on the theaching of physics in schools 2, Loria A., Thomsen P., Gylendal Ed.: GIREP.

HADZIDAKI P. (2006), *The Heisenberg microscope: a powerful Instructional tool for promoting meta-cognitive and meta-scientific thinking on quantum mechanics and the 'nature of science*', Science&Education, DOI 10.1007/s11191-006-9057-3

HALLIDAY D., RESNICK R., WALKER J. (2001), *Fondamenti di Fisica*, Zanichelli Ed.

HALLIDAY D., RESNICK R., WALKER J. (2010), *Fundamentals of physics extended*, John Wiley and Sons

HEISENBERG W. (1927), *The actual content of quantum theoretical kinematics and mechanics*, English translation of the paper Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik, Wheeler and Zurek, 1983

HEISENBERG W. (1962), *Physics and philosophy, The revolution in modern science*, Harper & Row Publishers, Inc., New York

IRESON G. (2000), *The quantum understanding of pre-university physics students*, Physics Education 35(1)

JOHANSSON K.E., MILSTEAD D. (2008), *Uncertainty in the classroom - teaching quantum physics*, Physics Education, 43(2), 173

JOHNSTON I. D., CRAWFORD K., AND FLETCHER P. R. (1998), *Student diffculties in learning quantum mechanics*, International Journal of Science Education, 20(4), 427-446.

LAWRENCE I. (1996), *Quantum Physics in School*, Physics Education, 31, 5, pp. 278-286

LEVRINI O., DISESSA A. (2008), *How students learn from multiple contexts and definitions: Proper time as a coordination class*, Phys. Rev. ST Phys. Educ. Res. **4**, 010107

LEVRINI O., FANTINI P., PECORI B. (2008), *''The problem is not understanding the theory, but accepting it'': a study on students' difficulties in coping with quantum physics*, GIREP-EPEC conference, frontiers of physics education (2007, Opatija), selected contributions (319–324). Rijeka: Zlatni rez.

LEVRINI O., FANTINI P. (2013), *Encountering Productive Forms of Complexity in Learning Modern Physics*, Sci & Educ, 22:1895–1910.

LEVRINI O., LULLI G., BERTOZZI E., ERCOLESSI E., MATTEUCCI G., MONZONI V., PECORI B. (2014), Laboratorio PLS: "L'esperimento più bello della fisica", in Anzellotti G., Catena L. M., Catti M., Cosentino U., Immé J., Vittorio N. (a cura di), L'insegnamento della matematica e delle scienze nella società della conoscenza. Mondadori Università, 197-201.

LEVRINI O., FANTINI P., TASQUIER G., PECORI B., LEVIN M. (2015b), *Defining and Operationalizing Appropriation for Science Learning*, The Journal of the Learning Sciences, 24:1, 93-136

LEVRINI O., BRANCHETTI L., ERCOLESSI E., FANTINI P., GARAGNANI E., LULLI G., MONZONI V., TASQUIER G. (2015b), *Fisica quantistica a scuola tra innovazione curricolare, vincoli scolastici e obiettivi di ricerca: l'esperienza di Bologna*, SIF, Roma

LÉVY-LEBLOND J.M., BALIBAR F. (1990), *Quantique: rudiments of quantum physics*, North Holland Ed., p. 68.

LÉVY-LEBLOND J.M. (2003), *On the Nature of Quantons*, Science & Education 12: 495–502

LODOVICO L. (2016), *Processi di appropriazione nello studio della fisica quantistica: analisi di una sperimentazione didattica in una quinta liceo scientifico*, Master degree dissertation, University of Bologna, Supervisor: Levrini O., Co-supervisor: Tasquier G.

LULLI G. (2013), *L'esperimento più bello. L'interferenza di elettroni singoli e il mistero della meccanica quantistica*, Apogeo education

MALGIERI M., ONORATO P., DE AMBROSIS A. (2014), *Teaching quantum physics by the sum over paths approach and GeoGebra simulations*, European Journal of Physics, 35(5), 055024.

MALGIERI M. (2015), *Teaching quantum physics at introductory level: a sum over paths approach*, Doctorate dissertation, University of Pavia, Supervisor: De Ambrosis A.

MANNILA K., KOPONEN I.T., JOUNI A.N. (2001), *Building a picture of students' conceptions of wave- and particle-like properties of quantum entities*, European Journal of Physics 23 (2002), 45-53

MATTEUCCI G., FERRARI L., MIGLIORI A. (2010), *The Heisenberg uncertainty principle demonstrated with an electron diffraction experiment*, European Journal of Physics 31, 1287-1293, doi: 10.1088/0143-0807/31/5/027

MCINTYRE D.H., MANOGUE C.A., TATE J. (2013), *Paradigms in Physics: Quantum Mechanics*, Pearson Ed.

MERMIN D. (1989), *What's wrong with this pillow?*, Physics Today, 42, 9.

MICHELINI M., RAGAZZONI R., SANTI L., STEFANEL A. (2000), *A Proposal for quantum physics in secondary school*, Phys. Educ. 35, 6, pp. 406-410.

MICHELINI M., SANTI L., STEFANEL A. (2010), *Impostazione alla Dirac. La proposta didattica di Udine per la fisica quantistica*, progetto IDIFO, Unità di Ricerca in Didattica della Fisica, Università di Udine

MIFFRE A., JACQUEY M., BÜCHNER M., TRÉNEC G., VIGUÉ J. (2008), *Atom interferometry,* Laboratoire Collisions, Agrégats, Réactivité (UMR 5589 CNRS-UPS), IRSAMC, Université Paul Sabatier Toulouse

MUIÑO P.L. (2000), *Introducing the Uncertainty Principle Using Diffraction of Light Waves*, Journal of Chemical Education, Vol. 82 No.8

MÜLLER R., WIESNER H. (2002) Teaching quantum mechanics on an introductory level," American Journal of Physics, 70(3), 200-209

POSPIECH G. (1999), *Teaching EPR paradox at high school?*, Phys. Educ. 34 311-6.

POSPIECH G. (2000), *Uncertainty and complementarity: the heart of quantum physics?*, Phys. Educ. 35 (6).

POSPIECH G. (2010), *Insegnare la Fisica Quantistica dal punto di vista filosofico*, in Michelini M. (a cura di), Fisica moderna per la scuola, LithoStampa, Udine, pp. 31-42.

RINAUDO G. (2010), *Il metodo di Feynman della somma su molti cammini per l'introduzione della Meccanica Quantistica*, in Michelini M. (ed.) *Formazione a distanza degli insegnanti all'innovazione didattica in fisica moderna e orientamento*, Udine (Italy): LithoStampa

RIOUX F. (2005), *Single-Slit Diffraction and the Uncertainty Principle*, Journal of Chemical Education, Vol. 82 No.8

ROBERTSON H.P. (1929), *The uncertainty principle*, Phys. Rev., 34, 163-164

SCHLOSSHAUER M., KOFLER J., ZEILINGER A. (2013), *A snapshot of foundational attitudes toward quantum mechanics*, Elsevier Ltd., Studies in History and Philosophy of Modern Physics 44 (2013) 222–230

SCHWINDT P. D. D., KWIAT P. G., ENGLERT B.-G. (1999), Phys. Rev. A 60, 4285

SCULLY M. O., ENGLERT B. G., H. WALTHER (1991), *Quantum optical tests of complementarity*, Nature 351:111–116

SJØBERG S. (2001), *Why don't they love us anymore? Science and Technology Education: A European high priority political concern!*, Proceedings Third International E.S.E.R.A. Conference. Thessaloniki. Greece. 19-20.

STEFANEL A. (2007), *La meccanica quantistica nella scuola secondaria per formare al pensiero teoretico*, in "Il mondo dei quanti", Enciclopedia Italiana Treccani, accessibile in rete all'indirizzo: www.treccani.it/site/Scuola/nella scuola/area_fisica/index.htm

STEFANEL A. (2008), *Impostazioni e percorsi per l'insegnamento della meccanica quantistica nella scuola secondaria*, Giornale di fisica, 49(1), 15-53

STEFANINI L. (2013), *Corso-laboratorio PLS "l'esperimento più bello": analisi di un'esperienza di insegnamento/apprendimento della fisica quantistica*, Bachelor's degree's dissertation, University of Bologna, Supervisor: Levrini, O.; Co-supervisors: Bertozzi E., Ercolessi E.

STYER D.F (1996), *Common Misconceptions Regarding Quantum Mechanics*, American Journal of Physics 64, 31-34

PATY M. (2003), *The concept of quantum state: new views on old phenomena*, Kluwer Academic Publishers, Netherlands

TAROZZI F. (2005), *Un progetto di insegnamento della meccanica quantistica a livello di scuola secondaria superiore: alla ricerca di un formalismo possibile*, dissertation for master degree in Physics, supervisor: Levrini O., co-supervisors: Grimellini N., Fantini P.

TAYLOR E.F., VOKOS S., O'MEARA J.M., THORNBER N.S. (1998), *Teaching Feynman's sum-over-paths quantum theory*, Computers in Physics, 12, 190

VELENTZAS A., AND HALKIA K. (2011), *The 'Heisenberg's Microscope' as an example of using thought experiments in teaching physics theories to students of the upper secondary school*, Research in Science Education, 41(4), 525-539.

WALBORN S. P., TERRA CUNHA M. O, PÁDUA S., MONKEN C. H. (2002). Double-slit quantum eraser. Physical Review A 65:033818.