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Subleading corrections to F-terms in the Large Volume Scenario

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Abstract

In Large Volume Scenarios, the axio-dilaton and complex-structure F -terms receive non-vanishing contributions from several subleading effects, some of which have already been extensively studied in the literature [1–5]. For instance, the inverse of the corrected Kähler metric induces mixing between the axio-dilaton S and the Kähler directions T_i . Since $D_{T_i}W$ is non-zero at the LVS AdS minimum, this mixing generates non-vanishing F -terms for both S and z_a . In addition, the perturbative correction to the Kähler potential directly modifies the covariant derivatives D_{T_i} , D_S , and D_{z_a} . Also the dependence of W_{np} on z_a and S via its prefactor: $W_{\text{np}} = A(z_a, S)e^{-a_i T^i}$ leads the F -terms to acquire non-zero contributions.

In this thesis, we investigate a distinct contribution arising from the displacement of the axio-dilaton and complex-structure moduli away from their supersymmetric tree-level flux vacuum. Although these fields are conventionally stabilised at leading order and subsequently treated as frozen in the analysis of Kähler-moduli stabilisation, the perturbative and non-perturbative effects responsible for the Large Volume Scenario generate non-vanishing gradients along the (S, z_a) directions. As a consequence, the full minimum is shifted by amounts δS and δz_a .

We compute these displacements perturbatively in the large-volume and weak-coupling regime, organising the results according to their parametric scaling in inverse powers of the compactification volume $\mathcal{V}$ and powers of the string coupling g_s . We then determine how the shifted vacuum modifies the corresponding F -terms. The analysis is first performed in a simplified model with rigid complex-structure and subsequently extended to a model containing a dynamical complex-structure modulus.

We find that the shift-induced contribution to the complex-structure sector F -term is enhanced by an additional factor of g_s^{-1} relative to the previously known contributions [2, 6] and can therefore become parametrically dominant in the weak-coupling regime. By contrast, the corresponding contribution to the axio-dilaton F -term is generically of the same parametric order in g_s and $\mathcal{V}^{-1}$ as the contributions already discussed in the literature. We also discuss the phenomenological implications of these corrections for soft supersymmetry-breaking terms in sequestered LVS scenarios [3]. In particular, while soft terms controlled by F^S remain parametrically unchanged, the modified complex-structure F -terms can lead to parametrically different contributions to scalar masses in the ultra-local regime and to the masses of hidden RR photini [4].

Contents

Abstract	i
1 Supersymmetry and Supergravity	5
1.1 The supersymmetry algebra	6
1.2 Superspace and Superfields in $\mathcal{N} = 1$ Supersymmetry	10
1.3 $\mathcal{N} = 1$ matter actions	15
1.4 Supersymmetry breaking	24
1.5 $\mathcal{N} = 1$ Supergravity	26
1.6 Phenomenological Aspects	33
2 Superstrings	38
2.1 Type II string theory	39
2.2 Physical States-Massless spectrum of type II string theory	43
2.3 Type II Compactifications	48
2.4 Type II orientifolds	54
3 Large Volume Scenario	58
3.1 Flux Compactifications and Moduli Stabilization	59
3.2 Large Volume Scenario	63
3.3 Sequestered LVS scenarios	66
4 Corrections to the F-terms	71
4.1 General considerations	73
4.2 Vacuum Displacement and Induced F -term Corrections	77
4.3 First Application: A Toy Model	87
4.4 Shift-induced corrections to the F -terms in a minimal model	91
4.5 Physical relevance	96
5 Conclusions and Outlook	100
Acknowledgements	103
A Conventions	104
A.1 Conventions of the toy model	104
A.2 Extension to a dynamical complex-structure modulus	107
B Explicit Computations	108
B.1 Power counting	113

Introduction

One of the central challenges in string theory is to determine whether it admits phenomenologically viable four-dimensional vacua. A realistic compactification should reproduce, at sufficiently low energies, the essential features of observed particle physics. A widely studied possibility is that the resulting four-dimensional theory contains an $\mathcal{N} = 1$ supersymmetric matter sector, with supersymmetry spontaneously broken at an energy scale compatible with phenomenological constraints. The properties of the four-dimensional effective theory are determined by the vacuum configuration of the compactification. In particular, quantities such as gauge couplings, Yukawa couplings, particle masses, and supersymmetry-breaking parameters depend on the vacuum expectation values of scalar fields known as moduli. These fields parametrise continuous deformations of the internal geometry, as well as other background data characterising the compactification.

For a long time, the presence of unstabilised moduli represented one of the main obstacles to connecting string theory with low-energy phenomenology. Massless moduli would mediate long-range fifth forces and, since their vacuum expectation values enter directly into physical observables, would prevent the theory from making definite predictions. Although perturbative and non-perturbative quantum effects are generically expected to generate a potential for the moduli in four-dimensional $\mathcal{N} = 1$ theories, a systematic and parametrically controlled mechanism for stabilising all of them remained unavailable for many years.

A major advance was provided by the development of flux compactifications and by the systematic study of their role in moduli stabilisation [7–9]. Type IIB string compactifications provide a particularly suitable setting for this programme. In this framework, background three-form fluxes generate a superpotential for the axio-dilaton S and the complex-structure moduli z_a , allowing these fields to be stabilised at tree level. The Kähler moduli T_α , by contrast, remain unfixed at this order as a consequence of the no-scale structure of the tree-level effective theory. Their stabilisation therefore requires effects that are absent at the classical level. In particular, the interplay between non-perturbative contributions to the superpotential [10] and perturbative α' corrections to the Kähler potential [11, 12] generates a non-trivial scalar potential for the Kähler sector. This potential can be analysed within a controlled expansion at large internal volume and leads to the Large Volume Scenario [1], in which the overall volume of the compactification is stabilised at an exponentially large value. Such vacua provide a particularly rich framework for studying supersymmetry breaking and its possible phenomenologi-

cal consequences.

The resulting hierarchy naturally suggests a two-step treatment of moduli stabilisation. One first determines the supersymmetric tree-level vacuum of the axio-dilaton and complex-structure sector by imposing the supersymmetric condition, and subsequently, treats the fields (S, z_a) as effectively frozen while analysing the subleading dynamics responsible for the stabilisation of the Kähler moduli. This procedure captures the leading structure of the vacuum and forms the basis of the conventional four-dimensional LVS effective theory. However, the separation between the two sectors is not exact. The perturbative and non-perturbative effects responsible for Kähler-moduli stabilisation depend, in general, on the axio-dilaton and complex-structure moduli. Once these contributions are included, the complete scalar potential develops non-vanishing gradients along the (S, z_a) directions. Consequently, the original tree-level flux vacuum is displaced from its supersymmetric position [2, 6]. The assumption that the flux-stabilised fields remain exactly frozen therefore neglects their backreaction on the full vacuum.

The purpose of this thesis is to investigate this effect systematically. In particular, we determine explicitly the displacement of the axio-dilaton and complex-structure vacuum induced by the subleading corrections responsible for Kähler-moduli stabilisation. The corrected minimum is computed perturbatively around the tree-level flux vacuum, organising the analysis according to the parametric dependence on the compactification volume $\mathcal{V}$ and the string coupling g_s . Our results are derived in the controlled regime of exponentially large volume and weak string coupling,

$$\mathcal{V} \gg 1, \quad g_s \ll 1. \quad (0.1)$$

This allows us to identify the leading contributions to the vacuum shifts and to determine their scaling in the simultaneous large-volume and weak-coupling expansions. Once the corrected vacuum configuration has been determined, one can compute the corresponding F -terms, which provide the relevant order parameters for supersymmetry breaking in the four-dimensional effective theory.

A central result of this analysis is that the contribution induced by the displacement of the vacuum is not parametrically negligible. For the axio-dilaton, the shift-induced correction enters at the same order in the simultaneous expansions in g_s and $\mathcal{V}^{-1}$ as the contribution obtained by evaluating the corrected F -terms at the unshifted tree-level vacuum. In the complex-structure direction, the effect is even more pronounced:

the shift-induced contribution is enhanced by an additional factor of g_s^{-1} relative to the corresponding unshifted term. Therefore, despite the smallness of the vacuum displacement itself, its effect on the F -terms can be of leading-order relevance, in the weak-coupling regime.

We first study this mechanism in a simplified model with a rigid complex-structure sector, in which only the axio-dilaton S is displaced. We then extend the analysis to a model containing a dynamical complex-structure modulus and explicitly account for the mixing between the S and z directions. This allows us to determine both the parametric behaviour and the explicit form of the shifts δS and δz in the large-volume and weak-coupling expansions.

As previously stated, the corrections to the axio-dilaton and complex-structure F -terms can have direct phenomenological consequences. In sequestered LVS scenarios, soft supersymmetry-breaking terms depend sensitively on the auxiliary fields of the flux-stabilized sector [3]. We find that the shift of the vacuum does not parametrically modify F^S , so that soft terms whose leading contribution is controlled by the axio-dilaton auxiliary field retain the standard LVS scaling. By contrast, the modified complex-structure F -terms can give parametrically different contributions to observables which depend directly on F^z , including scalar masses in the ultra-local regime.

A further application concerns the supersymmetric partners of closed-string hidden photons. In type IIB orientifold compactifications, RR photons arise from the dimensional reduction of the RR four-form $C_{(4)}$ on orientifold-even three-cycles, and their gauge kinetic functions are controlled by the complex-structure moduli [4, 13]. Consequently, the corresponding hidden-photino masses depend directly on the complex-structure F -terms. Therefore, any parametric modification of F^{z^a} induced by the displacement of the flux vacuum is directly inherited by the hidden-photino mass, up to the model-dependent complex-structure dependence of the gauge kinetic function. These examples provide a direct connection between the corrections studied in this work and the low-energy phenomenology of sequestered string compactifications.

This thesis is organised as follows. The first two chapters provide the theoretical background required for the subsequent analysis. After reviewing the essential elements of four-dimensional supergravity, we introduce the main features of superstring theory and of Type IIB compactifications. Particular attention is devoted to the field content and effective description of Type IIB orientifolds with O3/O7-planes, with the aim of defining the quantities and conventions needed throughout the rest of the thesis. Chapter 3 is devoted to the problem of moduli stabilisation. We first discuss the role of background fluxes in fixing the axio-dilaton and complex-structure moduli and then introduce the Large Volume Scenario as a mechanism for stabilising the Kähler sector.

In this context, we emphasise the conventional two-step procedure in which the flux sector is stabilised at tree level and subsequently treated as effectively frozen when analysing the subleading Kähler-moduli dynamics. The final chapter contains the original results of this work. We compute explicitly the displacement of the flux-stabilised vacuum induced by the perturbative and non-perturbative corrections responsible for LVS stabilisation. The analysis is first carried out in a simplified toy model with rigid complex structure and is then extended to a more general setting including a dynamical complex-structure modulus. We finally determine the corresponding corrections to the axio-dilaton and complex-structure F -terms and analyse their parametric behaviour in the large-volume and weak-coupling regime.

1 Supersymmetry and Supergravity

Supersymmetry is a spacetime symmetry relating bosonic and fermionic degrees of freedom and organizing them into common supermultiplets. It plays a fundamental role in superstring theory, whose development was historically motivated by the attempt to incorporate fermionic excitations into the bosonic string.

The low-energy dynamics of string compactifications is naturally described in terms of supersymmetric field theories. In four-dimensional compactifications preserving four supercharges, the relevant framework is $\mathcal{N} = 1$ supersymmetry. We therefore begin this chapter by reviewing the basic structure of four-dimensional rigid supersymmetry, introducing its algebra, the corresponding supermultiplets, and the supersymmetric actions.

We then turn to local supersymmetry, obtained by promoting the parameters of supersymmetry transformations to spacetime-dependent spinors. Such a local symmetry cannot be consistently realized without introducing gravity: the corresponding gauge theory necessarily contains the graviton, together with its fermionic superpartner, the gravitino. The resulting framework is $\mathcal{N} = 1, D = 4$ supergravity.

Finally, we present the supergravity action for chiral and vector multiplets coupled to gravity, emphasizing the role of the Kähler potential, the superpotential, and the gauge kinetic functions. Particular attention will be devoted to the scalar potential and to the auxiliary F -terms, which provide the essential language for describing moduli stabilization and supersymmetry breaking in four-dimensional effective theories arising from string compactifications.

1.1 The supersymmetry algebra

Since the supersymmetry algebra is a graded extension of the Poincaré algebra, we begin by reviewing the Lorentz and Poincaré groups, their Lie algebras, and their finite-dimensional representations.

1.1.1 The Lorentz and Poincaré groups

The Lorentz group is the group of linear transformations that preserve the Minkowski metric. With the mostly-minus convention,

$$\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1), \quad (1.1)$$

the full Lorentz group is defined as

$$O(1, 3) = \{ \Lambda \in GL(4, \mathbb{R}), \mid \Lambda^T \eta \Lambda = \eta \}. \quad (1.2)$$

The subgroup of proper Lorentz transformations is

$$SO(1, 3) = \{ \Lambda \in O(1, 3), \mid \det \Lambda = 1 \}. \quad (1.3)$$

In the following, we shall mainly consider the proper orthochronous Lorentz group,

$$SO^+(1, 3) = \{ \Lambda \in SO(1, 3), \mid \Lambda^0_0 \geq 1 \}, \quad (1.4)$$

which is the connected component of the identity.

The Lorentz algebra has six generators: three generators (J_i) of spatial rotations and three generators (K_i) of Lorentz boosts. Their commutation relations are

$$\begin{aligned} [J_i, J_j] &= i\varepsilon_{ijk} J_k, \\ [J_i, K_j] &= i\varepsilon_{ijk} K_k, \\ [K_i, K_j] &= -i\varepsilon_{ijk} J_k. \end{aligned} \quad (1.5)$$

It is useful to combine these generators into an antisymmetric Lorentz tensor ($M_{\mu\nu}$), defined by

$$M_{\mu\nu} = -M_{\nu\mu}, \quad M_{0i} = K_i, \quad M_{ij} = \varepsilon_{ijk} J_k. \quad (1.6)$$

Equivalently,

$$J_i = \frac{1}{2} \varepsilon_{ijk} M_{jk}, \quad K_i = M_{0i}. \quad (1.7)$$

In this notation, the Lorentz algebra takes the manifestly covariant form

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i \left(\eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho} \right). \quad (1.8)$$

To classify the finite-dimensional representations of the Lorentz algebra, one introduces the complex linear combinations

$$J_i^{(+)} = \frac{1}{2} (J_i + iK_i), \quad J_i^{(-)} = \frac{1}{2} (J_i - iK_i). \quad (1.9)$$

Using Eq. (1.5), one finds

$$\begin{aligned} [J_i^{(+)}, J_j^{(+)}] &= i\varepsilon_{ijk} J_k^{(+)}, \\ [J_i^{(-)}, J_j^{(-)}] &= i\varepsilon_{ijk} J_k^{(-)}, \\ [J_i^{(+)}, J_j^{(-)}] &= 0. \end{aligned} \quad (1.10)$$

Therefore, the complexified Lorentz algebra decomposes as

$$\mathfrak{so}(1, 3)_{\mathbb{C}} \simeq \mathfrak{sl}(2, \mathbb{C})_L \oplus \mathfrak{sl}(2, \mathbb{C})_R. \quad (1.11)$$

Equivalently, at the level of commutation relations, it consists of two mutually commuting angular-momentum algebras.

An important relation one should bear in mind is the relation between the Lorentz group and $\mathrm{SL}(2, \mathbb{C})$, namely

$$\mathrm{SO}^+(1, 3) \simeq \frac{\mathrm{SL}(2, \mathbb{C})}{\mathbb{Z}_2}. \quad (1.12)$$

Thus, $\mathrm{SL}(2, \mathbb{C})$ is the double cover of the proper orthochronous Lorentz group. This distinction is essential when introducing spinorial representations.

The Poincaré group is obtained by adjoining spacetime translations to the Lorentz group. Restricting again to the component connected to the identity, it is given by the semidirect product

$$\mathrm{ISO}^+(1, 3) = \mathbb{R}^{1,3} \rtimes \mathrm{SO}^+(1, 3). \quad (1.13)$$

An element of the Poincaré group is represented by a pair (a, Λ) , acting on Minkowski spacetime as

$$x^\mu \longmapsto x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu. \quad (1.14)$$

The group multiplication law is

$$(a, \Lambda)(b, \Sigma) = (a + \Lambda b, \Lambda \Sigma). \quad (1.15)$$

In addition to the Lorentz generators $M_{\mu\nu}$, the Poincaré algebra contains the four generators P_μ of spacetime translations. Its complete commutation relations are

$$\begin{aligned} [M_{\mu\nu}, M_{\rho\sigma}] &= -i \left(\eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho} \right), \\ [M_{\mu\nu}, P_\rho] &= -i \left(\eta_{\mu\rho} P_\nu - \eta_{\nu\rho} P_\mu \right), \\ [P_\mu, P_\nu] &= 0. \end{aligned} \quad (1.16)$$

1.1.2 The supersymmetry algebra

The Coleman–Mandula theorem [14] states that, under a set of physically reasonable assumptions, the most general symmetry algebra of a non-trivial interacting relativistic quantum field theory has to be a direct product of the Poincaré algebra with an internal symmetry algebra. In particular, the theorem assumes that the full symmetry algebra is an ordinary Lie algebra, whose product is given only by commutators. This assumption can be relaxed by allowing also anticommutators between fermionic generators. The resulting structure is not an ordinary Lie algebra, but a $\mathbb{Z}_2$ -graded Lie algebra, or Lie superalgebra. The corresponding extension of the Coleman–Mandula theorem is the Haag–Lopuszanski–Sohnius theorem, which shows that the only possible additional spacetime symmetries are generated by spinorial charges, namely the supersymmetry generators.

Let us first recall the definition of a $\mathbb{Z}_2$ -graded Lie algebra.

Definition 1.1. A $\mathbb{Z}_2$ -graded Lie algebra is a vector space

$$L = L_0 \oplus L_1, \quad (1.17)$$

where L_0 is called the even subspace and L_1 the odd subspace. The algebra is equipped with a graded bracket

$$[\cdot, \cdot] : L \times L \longrightarrow L \quad (1.18)$$

such that, for homogeneous elements $X, Y, Z \in L$, the following properties hold:

$$[L_i, L_j] \subseteq L_{i+j \pmod{2}} \quad (1.19)$$

$$[L_i, L_j] = -(-1)^{ij} [L_j, L_i], \quad (1.20)$$

and

$$(-1)^{ik} [L_i, [L_j, L_k]] + (-1)^{ij} [L_j, [L_k, L_i]] + (-1)^{jk} [L_k, [L_i, L_j]] = 0. \quad (1.21)$$

The even part L_0 contains the Poincaré generators, together with possible internal symmetry generators. The odd part L_1 contains the supersymmetry generators. In four dimensions, for $\mathcal{N}$ -extended supersymmetry, these generators are Weyl spinors $(Q_\alpha^I, \bar{Q}_{\dot{\alpha}, I})$, with $I = 1, \dots, N$ where $\alpha, \dot{\alpha}$ are spinor indices. The charges Q_α^I transform in the $(1/2, 0)$ representation of the Lorentz group, while $\bar{Q}_{\dot{\alpha}, I}$ transform in the $(0, 1/2)$ representation.

The bosonic part of the algebra contains the Poincaré algebra (1.16). The supersymmetry generators commute with translations,

$$[P_\mu, Q_\alpha^I] = 0, \quad [P_\mu, \bar{Q}_{\dot{\alpha}, I}] = 0, \quad (1.22)$$

and transform as spinors under Lorentz transformations:

$$[M_{\mu\nu}, Q_\alpha^I] = i(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta^I, \quad (1.23)$$

$$[M_{\mu\nu}, \bar{Q}^{I\dot{\alpha}}] = i(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{Q}^{I\dot{\beta}}. \quad (1.24)$$

The defining part of the supersymmetry algebra is given by the anticommutation relations

$$\{Q_\alpha^I, \bar{Q}_{\dot{\beta}}^J\} = 2\delta^{IJ}\sigma_{\alpha\dot{\beta}}^\mu P_\mu, \quad (1.25)$$

$$\{Q_\alpha^I, Q_\beta^J\} = 2\epsilon_{\alpha\beta}Z^{IJ}, \quad (1.26)$$

$$\{\bar{Q}_{\dot{\alpha}}^I, \bar{Q}_{\dot{\beta}}^J\} = 2\epsilon_{\dot{\alpha}\dot{\beta}}(Z^{IJ})^*. \quad (1.27)$$

Here $Z^{IJ} = -Z^{JI}$ are central charges. They commute with all generators of the super-Poincare algebra and can appear only for extended supersymmetry, namely for $\mathcal{N} > 1$.

1.2 Superspace and Superfields in $\mathcal{N} = 1$ Supersymmetry

The usual space-time Lagrangian formulation is not the most convenient one for describing supersymmetric field theories. This is because in ordinary space-time supersymmetry is not manifest. In fact, an extension of ordinary space-time, known as superspace, happens to be the best and most natural framework in which to formulate supersymmetric theories.

1.2.1 Superspace as a coset

The superspace formulation provides a geometric framework in which supersymmetry is realized as a translation in both bosonic and fermionic directions. In four-dimensional $\mathcal{N} = 1$ supersymmetry, ordinary Minkowski spacetime, with coordinates x^μ , is enlarged by introducing two complex Grassmann-valued Weyl spinors,

$$\theta^\alpha, \quad \bar{\theta}^{\dot{\alpha}} = (\theta^\alpha)^\dagger, \quad \alpha, \dot{\alpha} = 1, 2. \quad (1.28)$$

These coordinates satisfy

$$\theta^\alpha \theta^\beta = -\theta^\beta \theta^\alpha, \quad \bar{\theta}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = -\bar{\theta}^{\dot{\beta}} \bar{\theta}^{\dot{\alpha}}, \quad \theta^\alpha \bar{\theta}^{\dot{\beta}} = -\bar{\theta}^{\dot{\beta}} \theta^\alpha. \quad (1.29)$$

The resulting superspace is therefore parametrized by

$$z^M = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}), \quad (1.30)$$

and has dimension $(4|4)$: four commuting bosonic coordinates and four anticommuting fermionic coordinates. It will be denoted by $\mathbb{M}^{4|4}$.

The fermionic directions are associated with the supersymmetry generators Q_α and $\bar{Q}_{\dot{\alpha}}$, while the bosonic directions are associated with the translation generators P_μ . In the conventions adopted here, the non-vanishing anticommutator of the supersymmetry algebra is

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu. \quad (1.31)$$

Let $G_{\text{sp}}^{(\mathcal{N}=1)}$ denote the four-dimensional $\mathcal{N} = 1$ super-Poincaré group. A generic group element can locally be written as

$$g(x, \theta, \bar{\theta}, \omega) = \exp [ix^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}] \exp \left[\frac{i}{2} \omega^{\mu\nu} M_{\mu\nu} \right], \quad (1.32)$$

where $M_{\mu\nu}$ are the generators of the Lorentz group.

Four-dimensional $\mathcal{N} = 1$ Minkowski superspace is defined as the coset

$$\mathbb{M}^{4|4} \simeq \frac{G_{\text{sp}}^{(\mathcal{N}=1)}}{SO^+(1, 3)}. \quad (1.33)$$

The quotient by the Lorentz group removes the Lorentz transformations from the coset coordinates, leaving only the supertranslations. A convenient coset representative is therefore

$$\Omega(x, \theta, \bar{\theta}) = \exp [ix^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}] . \quad (1.34)$$

Consequently, each point of superspace is identified with a supertranslation parametrized by $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$. The Grassmann coordinates should not be interpreted as ordinary numerical coordinates: they belong to the odd sector of a Grassmann algebra and encode the fermionic directions of the superspace.

1.2.2 Superfields as fields in superspace

A superfield is a field defined on superspace, namely a function of the superspace coordinates,

$$Y = Y(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) . \quad (1.35)$$

Since the fermionic coordinates are Grassmann-valued, they satisfy $(\theta^\alpha)^2 = (\bar{\theta}^{\dot{\alpha}})^2 = 0$. Since a Weyl spinor has only two independent components, every monomial containing more than two factors of θ vanishes, and the same holds for monomials containing more than two factors of $\bar{\theta}$. Therefore, unlike an ordinary Taylor expansion, the expansion of a superfield in the fermionic coordinates terminates after finitely many terms.

The most general complex scalar superfield can be written as

$$\begin{aligned} Y(x, \theta, \bar{\theta}) = & f(x) + \theta^\alpha \psi_\alpha(x) + \bar{\theta}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta} n(x) \\ & + \theta\sigma^\mu\bar{\theta} v_\mu(x) + \theta\theta \bar{\theta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}(x) + \bar{\theta}\bar{\theta} \theta^\alpha \rho_\alpha(x) + \theta\theta \bar{\theta}\bar{\theta} d(x), \end{aligned} \quad (1.36)$$

where

$$\theta\theta \equiv \theta^\alpha \theta_\alpha, \quad \bar{\theta}\bar{\theta} \equiv \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}, \quad \theta\sigma^\mu\bar{\theta} \equiv \theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}}. \quad (1.37)$$

The coefficient functions in this expansion are ordinary spacetime fields. The fields f , m , n , and d are Lorentz scalars, ψ_α , ρ_α , $\bar{\chi}^{\dot{\alpha}}$, and $\bar{\lambda}^{\dot{\alpha}}$ are Weyl spinors, while v_μ transforms as a Lorentz vector. Thus, a general superfield packages a finite collection of bosonic and fermionic component fields into a representation of supersymmetry, which is generally reducible.

Under a supertranslation with Grassmann-valued parameters ϵ^α and $\bar{\epsilon}^{\dot{\alpha}}$, the superspace coordinates transform as

$$\theta'^\alpha = \theta^\alpha + \epsilon^\alpha, \quad \bar{\theta}'^{\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}} + \bar{\epsilon}^{\dot{\alpha}}, \quad (1.38)$$

and

$$x'^\mu = x^\mu + i(\theta\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\theta}). \quad (1.39)$$

Defining the supertranslation operator as

$$U(\epsilon, \bar{\epsilon}) = \exp [i(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})], \quad (1.40)$$

an operator-valued scalar superfield transforms according to

$$U(\epsilon, \bar{\epsilon}) Y(x, \theta, \bar{\theta}) U^{-1}(\epsilon, \bar{\epsilon}) = Y(x', \theta', \bar{\theta}'). \quad (1.41)$$

Equivalently, when the superfield is regarded as a function on superspace, the supersymmetry generators are represented by differential operators acting on its superspace coordinates. Infinitesimally, one may write

$$\delta_\epsilon Y = i (\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}) Y. \quad (1.42)$$

The transformation of the superfield therefore induces transformations among its component fields, showing explicitly that they form a supersymmetry multiplet.

1.2.3 Supersymmetry-invariant actions

Since supersymmetry transformations are realized as translations in superspace, supersymmetric actions can be constructed by integrating suitable superfields over superspace. Let $Y(x, \theta, \bar{\theta})$ be a Lorentz-scalar superfield. Its integral over the full superspace,

$$S_D = \int d^4x d^2\theta d^2\bar{\theta} Y(x, \theta, \bar{\theta}), \quad (1.43)$$

is invariant under supersymmetry transformations, up to a spacetime boundary term.

Indeed, adopting the infinitesimal transformation law

$$\delta_\epsilon Y = i (\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}) Y, \quad (1.44)$$

one finds

$$\begin{aligned} \delta_\epsilon S_D &= \int d^4x d^2\theta d^2\bar{\theta} (\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}) Y \\ &= 0, \end{aligned} \quad (1.45)$$

where the last equality follows from Berezin integration in the fermionic directions and from the assumption that spacetime boundary terms vanish.

The integral over the Grassmann coordinates extracts the highest component of the superfield. Consequently, a supersymmetric action can be written as

$$S = \int d^4x d^2\theta d^2\bar{\theta} \mathcal{A}(x, \theta, \bar{\theta}) = \int d^4x \mathcal{L}(\phi(x), \psi(x), A_\mu(x), \dots), \quad (1.46)$$

where $\mathcal{A}$ is an appropriate Lorentz-scalar superfield, generally constructed from products of elementary superfields. For the action to be real, the integrand must be real, or the corresponding Hermitian-conjugate contribution must be included.

An unconstrained scalar superfield contains several component fields and, in general, furnishes a reducible representation of the supersymmetry algebra. Elementary

supersymmetry multiplets are obtained by imposing constraints that are compatible with supersymmetry. To formulate such constraints, one introduces the superspace covariant derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \partial_\mu, \quad \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^\beta \sigma_{\beta\dot{\alpha}}^\mu \partial_\mu. \quad (1.47)$$

They satisfy

$$\{D_\alpha, \bar{D}_{\dot{\beta}}\} = 2i\sigma_{\alpha\dot{\beta}}^\mu \partial_\mu, \quad \{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0. \quad (1.48)$$

Moreover, the covariant derivatives anticommute with the supersymmetry generators,

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = \{\bar{D}_{\dot{\alpha}}, Q_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0. \quad (1.49)$$

Constraints written in terms of D_α and $\bar{D}_{\dot{\alpha}}$ are therefore preserved by supersymmetry transformations.

Chiral superfields. A chiral superfield Φ is defined by the covariant constraint

$$\bar{D}_{\dot{\alpha}} \Phi = 0. \quad (1.50)$$

This constraint can be solved by

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) = & \phi(x) + \sqrt{2} \theta \psi(x) + \theta \theta F(x) + i\theta \sigma^\mu \bar{\theta} \partial_\mu \phi(x) \\ & - \frac{i}{\sqrt{2}} \theta \theta \partial_\mu \psi(x) \sigma^\mu \bar{\theta} - \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \square \phi(x). \end{aligned} \quad (1.51)$$

Here ϕ is a complex scalar field, ψ_α is a Weyl spinor, and F is a complex auxiliary scalar field.

Under an infinitesimal supersymmetry transformation, its component fields transform as

$$\delta_\epsilon \phi = \sqrt{2} \epsilon \psi, \quad (1.52)$$

$$\delta_\epsilon \psi_\alpha = -\sqrt{2} \epsilon_\alpha F + \sqrt{2} i \sigma_{\alpha\dot{\alpha}}^\mu \bar{\epsilon}^{\dot{\alpha}} \partial_\mu \phi, \quad (1.53)$$

$$\delta_\epsilon F = i\sqrt{2} \partial_\mu \psi_\alpha \bar{\sigma}^{\mu\alpha\dot{\alpha}} \epsilon_{\dot{\alpha}}. \quad (1.54)$$

Real superfields. A real superfield V is defined by the condition

$$V = V^\dagger. \quad (1.55)$$

Real superfields are used to describe vector multiplets. The reality condition alone, however, does not reduce the general superfield to an irreducible vector multiplet. In the Abelian case, one must also identify superfields related by the gauge transformation

$$V \longrightarrow V + \Lambda + \Lambda^\dagger, \quad (1.56)$$

where Λ is an arbitrary chiral superfield,

$$\bar{D}_{\dot{\alpha}}\Lambda = 0. \quad (1.57)$$

By exploiting this gauge freedom, several components of the general real superfield can be eliminated. In the Wess–Zumino gauge, the vector superfield takes the form

$$\begin{aligned} V_{\text{WZ}}(x, \theta, \bar{\theta}) = & \theta\sigma^{\mu}\bar{\theta} v_{\mu}(x) + i\theta\theta \bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta} \theta\lambda(x) \\ & + \frac{1}{2}\theta\theta \bar{\theta}\bar{\theta} D(x). \end{aligned} \quad (1.58)$$

The component field v_{μ} is a real gauge field, λ_{α} and $\bar{\lambda}_{\dot{\alpha}}$ are the gaugino fields, and D is a real auxiliary scalar field.

1.3 $\mathcal{N} = 1$ matter actions

Let Φ^i be a collection of chiral superfields and $\bar{\Phi}^{\bar{i}}$ their Hermitian conjugates. A general supersymmetric contribution obtained by integration over the full superspace is

$$S_D = \int d^4x d^2\theta d^2\bar{\theta} K(\Phi^i, \bar{\Phi}^{\bar{j}}), \quad (1.59)$$

where K is a real Lorentz-scalar function. The function K is called the Kähler potential. Since

$$[d^2\theta] = 1, \quad [d^2\bar{\theta}] = 1, \quad (1.60)$$

the Kähler potential must have mass dimension

$$[K] = 2 \quad (1.61)$$

in order for the component Lagrangian density to have mass dimension four.

Restricting to theories with at most two spacetime derivatives, the Kähler potential is taken to depend algebraically on the chiral and antichiral superfields, without explicit superspace covariant derivatives. For a single chiral superfield, it may be expanded as

$$K(\Phi, \Phi^\dagger) = \sum_{m,n=0}^{\infty} c_{mn} (\bar{\Phi})^m \Phi^n, \quad (1.62)$$

with the reality condition

$$c_{mn} = c_{nm}^*. \quad (1.63)$$

Assuming the canonical mass dimension $[\Phi] = 1$, the coefficients have dimension

$$[c_{mn}] = 2 - m - n. \quad (1.64)$$

In an effective field theory with cutoff scale Λ , one therefore expects parametrically

$$c_{mn} \sim \Lambda^{2-m-n}, \quad (1.65)$$

up to dimensionless coupling constants.

The terms in K which are purely holomorphic or purely antiholomorphic do not contribute to the full superspace integral. Consequently, in global supersymmetry, the Kähler potential is defined only up to the transformation

$$K(\Phi, \bar{\Phi}) \longrightarrow K(\Phi, \bar{\Phi}) + F(\Phi) + \bar{F}(\bar{\Phi}), \quad (1.66)$$

where F is an arbitrary holomorphic function.

The simplest choice is the canonical Kähler potential,

$$K_{\text{can}} = \Phi^\dagger \Phi. \quad (1.67)$$

A second class of supersymmetric invariants is obtained by integrating a chiral superfield over chiral superspace. If Σ satisfies

$$\bar{D}_{\dot{\alpha}}\Sigma = 0, \quad (1.68)$$

then

$$\int d^4x d^2\theta \Sigma \quad (1.69)$$

is invariant under supersymmetry transformations up to a spacetime boundary term.

Since any holomorphic function of chiral superfields is itself chiral, one may introduce a function

$$W = W(\Phi^i), \quad \frac{\partial W}{\partial \Phi^{\bar{i}}} = 0. \quad (1.70)$$

The function W is called the superpotential. Its contribution to the action is

$$S_F = \int d^4x d^2\theta W(\Phi^i) + \int d^4x d^2\bar{\theta} \bar{W}(\bar{\Phi}^{\bar{i}}). \quad (1.71)$$

Equivalently,

$$S_F = \int d^4x d^2\theta W(\Phi^i) + \text{h.c.} \quad (1.72)$$

Since the chiral measure has mass dimension one, the superpotential must have dimension

$$[W] = 3. \quad (1.73)$$

For a single chiral superfield, the most general holomorphic expansion is

$$W(\Phi) = \sum_{n=0}^{\infty} a_n \Phi^n, \quad (1.74)$$

where

$$[a_n] = 3 - n \quad (1.75)$$

for $[\Phi] = 1$.

In a perturbatively renormalizable theory, only terms up to cubic order are allowed,

$$W(\Phi) = a_0 + a_1\Phi + \frac{1}{2}m\Phi^2 + \frac{1}{3!}y\Phi^3. \quad (1.76)$$

Higher powers are instead interpreted as non-renormalizable operators suppressed by an appropriate ultraviolet scale.

The most general globally supersymmetric action for chiral superfields, at the two-derivative level and in the absence of gauge interactions, is therefore

$$\begin{aligned} S = & \int d^4x d^2\theta d^2\bar{\theta} K(\Phi^i, \bar{\Phi}^{\bar{j}}) \\ & + \int d^4x d^2\theta W(\Phi^i) + \int d^4x d^2\bar{\theta} \bar{W}(\bar{\Phi}^{\bar{i}}). \end{aligned} \quad (1.77)$$

The Kähler potential determines the kinetic terms and the geometry of the scalar-field space, while the superpotential determines the non-derivative interactions and, after eliminating the auxiliary fields, the scalar potential.

1.3.1 Non-linear sigma models

In a supersymmetric effective field theory, one should allow for the most general interactions compatible with supersymmetry and with the other symmetries of the system. In particular, higher-dimensional operators may appear in the low-energy description even when the underlying theory is perturbatively renormalizable. For chiral multiplets, such interactions are encoded in a non-canonical Kähler potential and naturally lead to a supersymmetric non-linear sigma model.

Let ϕ^i , with $i = 1, \dots, n$, denote the complex scalar components of a collection of chiral multiplets, and let $\bar{\phi}^{\bar{i}}$ denote their complex conjugates. We introduce the differential operators

$$\partial_i \equiv \frac{\partial}{\partial \phi^i}, \quad \partial_{\bar{i}} \equiv \frac{\partial}{\partial \bar{\phi}^{\bar{i}}}.$$

More generally, derivatives of the Kähler potential will be denoted by

$$K_{i_1 \dots i_p \bar{j}_1 \dots \bar{j}_q} \equiv \partial_{i_1} \dots \partial_{i_p} \partial_{\bar{j}_1} \dots \partial_{\bar{j}_q} K, \quad (1.78)$$

while derivatives of the holomorphic superpotential and its complex conjugate are written as

$$W_{i_1 \dots i_p} \equiv \partial_{i_1} \dots \partial_{i_p} W, \quad \bar{W}_{\bar{i}_1 \dots \bar{i}_p} \equiv \partial_{\bar{i}_1} \dots \partial_{\bar{i}_p} \bar{W}. \quad (1.79)$$

The integration of the superpotential over chiral superspace gives

$$\int d^2\theta W(\Phi) + \text{h.c.} = -W_i F^i - \frac{1}{2} W_{ij} \psi^i \psi^j + \text{h.c.}, \quad (1.80)$$

where all functions on the right-hand side are understood to be evaluated on the scalar fields.

The component expansion of the Kähler-potential contribution is more involved. After discarding total derivatives, one obtains

$$\begin{aligned} \int d^2\theta d^2\bar{\theta} K(\Phi, \bar{\Phi}) &= K_{i\bar{j}} \left(F^i \bar{F}^{\bar{j}} + \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - \frac{i}{2} \psi^i \sigma^\mu \partial_\mu \bar{\psi}^{\bar{j}} + \frac{i}{2} \partial_\mu \psi^i \sigma^\mu \bar{\psi}^{\bar{j}} \right) \\ &+ \frac{i}{4} K_{ij\bar{k}} \left(\psi^i \sigma^\mu \bar{\psi}^{\bar{k}} \partial_\mu \phi^j + \psi^j \sigma^\mu \bar{\psi}^{\bar{k}} \partial_\mu \phi^i - 2i \psi^i \psi^j \bar{F}^{\bar{k}} \right) + \text{h.c.} \\ &+ \frac{1}{4} K_{ij\bar{k}\bar{l}} \psi^i \psi^j \bar{\psi}^{\bar{k}} \bar{\psi}^{\bar{l}}. \end{aligned} \quad (1.81)$$

The matrix

$$g_{i\bar{j}} \equiv K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K \quad (1.82)$$

is Hermitian because K is real. A physically consistent theory requires $g_{i\bar{j}}$ to be non-degenerate and positive definite, so that the scalar and fermionic kinetic terms have the correct sign.

The scalar fields may consequently be interpreted as local complex coordinates on a manifold $\mathcal{M}$ of complex dimension n . More precisely, a field configuration defines a map

$$\phi : \mathbb{R}^{1,3} \longrightarrow \mathcal{M}, \quad x^\mu \longmapsto \phi^i(x).$$

The Hermitian metric on $\mathcal{M}$ is locally determined by the Kähler potential through (1.82). The associated two-form is

$$\omega = i K_{i\bar{j}} d\phi^i \wedge d\bar{\phi}^{\bar{j}}. \quad (1.83)$$

Since the metric is locally given by mixed derivatives of K , the two-form is closed,

$$d\omega = 0.$$

The target space $\mathcal{M}$ is therefore a Kähler manifold, and K is a local Kähler potential for its metric.

We denote the inverse Kähler metric by $K^{i\bar{j}}$, defined by

$$K_{k\bar{j}} K^{i\bar{j}} = \delta_k^i, \quad K_{i\bar{k}} K^{i\bar{j}} = \delta_{\bar{k}}^{\bar{j}}. \quad (1.84)$$

The only non-vanishing components of the Levi-Civita connection in complex coordinates are

$$\Gamma_{jk}^i = K^{i\bar{l}} K_{j\bar{k}\bar{l}}, \quad \Gamma_{\bar{j}\bar{k}}^{\bar{i}} = K^{\bar{l}i} K_{l\bar{j}\bar{k}}. \quad (1.85)$$

The two sets of components are related by complex conjugation.

The pullback of the target-space covariant derivative to spacetime acts on the fermions according to

$$\mathcal{D}_\mu \psi^i = \partial_\mu \psi^i + \Gamma_{jk}^i \partial_\mu \phi^j \psi^k, \quad (1.86)$$

and

$$\mathcal{D}_\mu \bar{\psi}^{\bar{i}} = \partial_\mu \bar{\psi}^{\bar{i}} + \Gamma_{\bar{j}\bar{k}}^{\bar{i}} \partial_\mu \bar{\phi}^{\bar{j}} \bar{\psi}^{\bar{k}}. \quad (1.87)$$

The non-vanishing components of the Riemann tensor are

$$R_{i\bar{j}k\bar{l}} = K_{ik\bar{j}\bar{l}} - K^{m\bar{n}} K_{ik\bar{n}} K_{m\bar{j}\bar{l}}. \quad (1.88)$$

The remaining components follow from complex conjugation and the usual symmetries of the curvature tensor of a Kähler manifold.

It is also useful to introduce the target-space covariant derivatives of the superpotential,

$$\nabla_i W_j = W_{ij} - \Gamma_{ij}^k W_k, \quad (1.89)$$

and

$$\nabla_{\bar{i}} \bar{W}_{\bar{j}} = \bar{W}_{\bar{i}\bar{j}} - \Gamma_{\bar{i}\bar{j}}^{\bar{k}} \bar{W}_{\bar{k}}. \quad (1.90)$$

The fields F^i and $\bar{F}^{\bar{i}}$ enter the action without spacetime derivatives and are therefore non-dynamical auxiliary fields. Their algebraic equations of motion give

$$F^i = K^{i\bar{j}} \bar{W}_{\bar{j}} - \frac{1}{2} \Gamma_{j\bar{k}}^i \psi^j \psi^{\bar{k}}, \quad (1.91)$$

together with its complex conjugate,

$$\bar{F}^{\bar{i}} = K^{j\bar{i}} W_j - \frac{1}{2} \Gamma_{j\bar{k}}^{\bar{i}} \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{k}}. \quad (1.92)$$

Substituting these solutions back into the component action gives the on-shell Lagrangian

$$\begin{aligned} \mathcal{L} = & K_{i\bar{j}} \left(\partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} + \frac{i}{2} \mathcal{D}_\mu \psi^i \sigma^\mu \bar{\psi}^{\bar{j}} - \frac{i}{2} \psi^i \sigma^\mu \mathcal{D}_\mu \bar{\psi}^{\bar{j}} \right) \\ & - K^{i\bar{j}} W_i \bar{W}_{\bar{j}} - \frac{1}{2} \nabla_i W_j \psi^i \psi^j - \frac{1}{2} \nabla_{\bar{i}} \bar{W}_{\bar{j}} \bar{\psi}^{\bar{i}} \bar{\psi}^{\bar{j}} \\ & + \frac{1}{4} R_{i\bar{k}j\bar{l}} \psi^i \psi^j \bar{\psi}^{\bar{k}} \bar{\psi}^{\bar{l}}. \end{aligned} \quad (1.93)$$

The scalar potential is therefore

$$V(\phi, \bar{\phi}) = K^{i\bar{j}} W_i \bar{W}_{\bar{j}}. \quad (1.94)$$

Since the inverse Kähler metric is positive definite, the scalar potential is non-negative,

$$V(\phi, \bar{\phi}) \geq 0.$$

A supersymmetric vacuum satisfies

$$W_i = 0 \quad \text{for every } i,$$

and consequently has vanishing vacuum energy in global supersymmetry.

The complete two-derivative component theory is thus determined by the Kähler geometry of the target manifold and by the holomorphic superpotential. The scalar and fermionic kinetic terms are governed by the Kähler metric, the fermionic interactions involve the associated connection and curvature, and the non-derivative interactions are fixed by W .

1.3.2 General $\mathcal{N} = 1$ Lagrangian

We now construct a supersymmetric action describing the dynamics and interactions of chiral and vector multiplets. For simplicity, we restrict to canonical kinetic terms and a constant gauge coupling. The resulting theory is the most general renormalizable globally supersymmetric gauge theory with this field content.

The Lagrangian can be decomposed as

$$\mathcal{L} = \mathcal{L}_{\text{SYM}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{FI}}, \quad (1.95)$$

where the three contributions describe, respectively, the gauge sector, the matter sector and the Fayet-Iliopoulos terms.

Let us discuss separately the different contributions appearing in (1.95), starting from the supersymmetric generalization of Yang-Mills theory.

Let $G = U(N)$ be the gauge group, with Hermitian generators $\{T^a\}$ satisfying

$$[T^a, T^b] = if^{abc}T^c. \quad (1.96)$$

The vector superfields V_a can be assembled into the Lie-algebra-valued superfield

$$V = V_a T^a, \quad a = 1, \dots, \dim G. \quad (1.97)$$

Under a superfield gauge transformation, it transforms as

$$e^V \longrightarrow e^{i\bar{\Lambda}} e^V e^{-i\Lambda}, \quad (1.98)$$

where Λ is a Lie-algebra-valued chiral superfield.

The corresponding gauge field-strength superfields are defined by

$$W_\alpha = -\frac{1}{4}\bar{D}^2 (e^{-V} D_\alpha e^V), \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2 (e^V \bar{D}_{\dot{\alpha}} e^{-V}). \quad (1.99)$$

and transform covariantly under gauge transformations:

$$W_\alpha \longrightarrow e^{i\Lambda} W_\alpha e^{-i\Lambda}. \quad (1.100)$$

Similarly,

$$\bar{W}_{\dot{\alpha}} \longrightarrow e^{i\bar{\Lambda}} \bar{W}_{\dot{\alpha}} e^{-i\bar{\Lambda}}. \quad (1.101)$$

Consequently, $\text{Tr}(W^\alpha W_\alpha)$ is a gauge-invariant chiral superfield.

The supersymmetric Yang-Mills Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{SYM}} &= \frac{1}{32\pi} \text{Im} \left[\tau \int d^2\theta \text{Tr} (W^\alpha W_\alpha) \right] \\ &= \text{Tr} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\lambda\sigma^\mu D_\mu \bar{\lambda} + \frac{1}{2} D^2 \right] \\ &\quad + \frac{\theta_{\text{YM}}}{32\pi^2} g^2 \text{Tr} (F_{\mu\nu} \tilde{F}^{\mu\nu}). \end{aligned} \quad (1.102)$$

Here g is the gauge coupling and

$$\tau = \frac{\theta_{\text{YM}}}{2\pi} + \frac{4\pi i}{g^2} \quad (1.103)$$

is the complexified gauge coupling. The non-abelian field strength and its dual are defined by

$$F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - ig[v_\mu, v_\nu], \quad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \quad (1.104)$$

The covariant derivative acting on the gaugino is

$$D_\mu \lambda = \partial_\mu \lambda - ig[v_\mu, \lambda]. \quad (1.105)$$

Let us now consider the coupling between the gauge and matter sectors. Let Φ be a chiral superfield transforming in a representation R of the gauge group G . Its gauge transformation is

$$\Phi \longrightarrow e^{i\Lambda} \Phi, \quad \bar{\Phi} \longrightarrow \bar{\Phi} e^{-i\bar{\Lambda}}. \quad (1.106)$$

The gauge-invariant matter Lagrangian is

$$\mathcal{L}_{\text{matter}} = \int d^2\theta d^2\bar{\theta} \bar{\Phi} e^V \Phi + \int d^2\theta W(\Phi) + \int d^2\bar{\theta} \bar{W}(\bar{\Phi}). \quad (1.107)$$

The first term provides the gauge-covariant kinetic terms of the matter fields, together with their interactions with the vector multiplet. The superpotential $W(\Phi)$ must be a holomorphic gauge-invariant function of the chiral superfields. Infinitesimally, gauge invariance requires

$$W_i(T^a)^i_j \phi^j = 0. \quad (1.108)$$

Finally, if the gauge group contains abelian factors, one may introduce Fayet–Iliopoulos terms. Let V^A , with $A = 1, \dots, m$, denote the vector superfields associated with the m abelian factors. The corresponding contribution is

$$\mathcal{L}_{\text{FI}} = \sum_{A=1}^m \xi_A \int d^2\theta d^2\bar{\theta} V^A = \frac{1}{2} \sum_{A=1}^m \xi_A D^A, \quad (1.109)$$

where ξ_A are real constants. Such a term is gauge invariant for an abelian vector multiplet because its gauge variation is the sum of a chiral and an anti-chiral superfield, whose integral over the full superspace vanishes.

In components, the on-shell Lagrangian takes the form

$$\begin{aligned} \mathcal{L} = & \text{Tr} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\lambda\sigma^\mu D_\mu \bar{\lambda} \right] + \frac{\theta_{\text{YM}}}{32\pi^2} g^2 \text{Tr} \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \\ & + \overline{D_\mu \phi} D^\mu \phi - i\psi\sigma^\mu D_\mu \bar{\psi} + i\sqrt{2}g \bar{\phi}\lambda\psi - i\sqrt{2}g \bar{\psi}\bar{\lambda}\phi \\ & - \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \psi^i \psi^j - \frac{1}{2} \frac{\partial^2 \bar{W}}{\partial \bar{\phi}_i \partial \bar{\phi}_j} \bar{\psi}_i \bar{\psi}_j - V(\phi, \bar{\phi}). \end{aligned} \quad (1.110)$$

The scalar potential is

$$\begin{aligned} V(\phi, \bar{\phi}) &= W_i W^i + \frac{g^2}{2} \sum_a |\bar{\phi}_i (T^a)^i_j \phi^j + \xi^a|^2 \\ &= \bar{F}_i F^i + \frac{1}{2} D^a D^a. \end{aligned} \quad (1.111)$$

The two terms are conventionally denoted by

$$V_F = W_i W^i, \quad V_D = \frac{1}{2} D^a D^a, \quad (1.112)$$

and are referred to as the F -term and D -term potentials, respectively. Since both contributions are sums of squares, the scalar potential is positive semi-definite:

$$V(\phi, \bar{\phi}) \geq 0. \quad (1.113)$$

Expressing the potential in terms of the auxiliary fields provides a direct way to determine whether a supersymmetric theory admits supersymmetric vacua. We restrict to Poincaré-invariant vacuum configurations. Lorentz invariance requires the expectation values of fermionic fields and gauge field strengths to vanish, while translational invariance requires the scalar expectation values to be constant. The energy density of such a configuration is therefore entirely determined by the scalar potential.

The supersymmetry algebra implies that the energy of any state is non-negative. Indeed, in the rest frame one has

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\delta_{\alpha\dot{\beta}} H, \quad (1.114)$$

and therefore

$$\langle \Omega | H | \Omega \rangle = \frac{1}{4} \sum_{\alpha=1}^2 \left[|Q_\alpha | \Omega \rangle|^2 + |Q_\alpha^\dagger | \Omega \rangle|^2 \right] \geq 0. \quad (1.115)$$

The vacuum energy vanishes if and only if the vacuum is annihilated by all the supercharges. Hence, a vacuum is supersymmetric if and only if

$$F^i = 0, \quad D^a = 0 \quad (1.116)$$

for every chiral and vector multiplet.

Using the auxiliary-field equations of motion, the conditions for an unbroken supersymmetric vacuum become

$$W_i(\phi) = 0, \quad (1.117)$$

and

$$\bar{\phi}_i (T^a)^i_j \phi^j + \xi^a = 0. \quad (1.118)$$

These are known as the F -flatness and D -flatness equations, respectively. If they admit a simultaneous solution, the theory has a supersymmetric vacuum with vanishing energy. Conversely, if they cannot be solved simultaneously, the minimum of the potential has positive energy and supersymmetry is spontaneously broken.

The classical moduli space of supersymmetric vacua is consequently

$$\mathcal{M}_{\text{cl}} = \frac{\{\phi, \mid W_i(\phi) = 0, \quad \bar{\phi}_i (T^a)^i_j \phi^j + \xi^a = 0\}}{G}, \quad (1.119)$$

where G is the gauge group.

1.4 Supersymmetry breaking

If supersymmetry is realized in Nature, it cannot be an exact symmetry at the energy scales currently accessible to experiments. Indeed, exact supersymmetry would require the bosonic and fermionic states belonging to the same supermultiplet to have identical masses. No such mass degeneracy has been observed in the spectrum of elementary particles.

Supersymmetry must therefore be broken at some characteristic scale, which we denote by M_{SUSY} . At energies much larger than this scale, the effects responsible for the breaking may become negligible, and the theory approaches its supersymmetric limit. This statement is particularly natural when supersymmetry is broken by soft terms.

There are two conceptually distinct ways in which supersymmetry may be broken:

- *Spontaneous supersymmetry breaking.* The Lagrangian is exactly supersymmetric, but the vacuum is not invariant under the action of the supercharges. In a globally supersymmetric theory, this occurs when at least one auxiliary field acquires a non-vanishing vacuum expectation value.
- *Explicit supersymmetry breaking.* The Lagrangian contains terms which are not invariant under supersymmetry transformations. A generic explicit breaking would spoil the ultraviolet cancellations which motivate supersymmetry. One therefore usually restricts to *soft supersymmetry-breaking terms*, which do not reintroduce quadratic ultraviolet divergences. Their coefficients have positive mass dimension, so their effects become progressively less important at energies much larger than the supersymmetry-breaking scale.

Typical soft-breaking operators include scalar mass terms, gaugino masses and scalar trilinear and bilinear interactions. Although these terms explicitly break supersymmetry, they preserve the improved ultraviolet behaviour of the supersymmetric theory.

1.4.1 Vacua in supersymmetric theories

We now consider the conditions under which a globally supersymmetric theory admits supersymmetric vacua. For a theory of chiral and vector multiplets with canonical kinetic terms, the scalar potential is

$$V(\phi, \bar{\phi}) = \bar{F}_i F^i + \frac{1}{2} D^a D^a, \quad (1.120)$$

and is therefore positive semi-definite,

$$V(\phi, \bar{\phi}) \geq 0. \quad (1.121)$$

For a Poincaré-invariant bosonic vacuum, the scalar fields are constant, while the vacuum expectation values of fermions, gauge field strengths and spacetime derivatives vanish.

It follows that a vacuum preserves supersymmetry if and only if all the auxiliary fields vanish:

$$F^i(\phi) = 0, \quad D^a(\bar{\phi}, \phi) = 0. \quad (1.122)$$

Using their equations of motion, these conditions are equivalently written as

$$W_i(\phi) = 0, \quad \bar{\phi}_i(T^a)^i_j \phi^j + \xi^a = 0. \quad (1.123)$$

They are referred to as the F -flatness and D -flatness conditions, respectively.

Because the scalar potential is a sum of non-negative terms, a configuration satisfying (1.122) has vanishing vacuum energy:

$$V_{\text{vac}} = 0. \quad (1.124)$$

Conversely, in global supersymmetry, a vacuum with vanishing energy must satisfy all the F -term and D -term equations and therefore preserves supersymmetry.

If no set of scalar vacuum expectation values simultaneously satisfies (1.122), supersymmetry is spontaneously broken. In this case, every vacuum necessarily has strictly positive energy,

$$V_{\text{min}} > 0. \quad (1.125)$$

The non-vanishing expectation values of the auxiliary fields,

$$\langle F^i \rangle \neq 0 \quad \text{and/or} \quad \langle D^a \rangle \neq 0, \quad (1.126)$$

act as order parameters for supersymmetry breaking.

When a continuous global symmetry is spontaneously broken, Goldstone's theorem implies the existence of a massless mode whose quantum numbers are related to those of the broken symmetry. Since supersymmetry is a fermionic symmetry, its spontaneous breaking gives rise to a massless spin-1/2 fermion, called the *goldstino*. The goldstino is a linear combination of the fermions belonging to the multiplets whose auxiliary fields acquire non-zero expectation values. Up to an overall normalization and convention-dependent phases, its direction in field space is

$$\tilde{G} \propto \langle F^i \rangle \psi_i + \frac{i}{\sqrt{2}} \langle D^a \rangle \lambda^a. \quad (1.127)$$

In a theory with global supersymmetry, the goldstino is a physical massless fermion. When supersymmetry is promoted to a local symmetry, the goldstino is instead absorbed by the gravitino, which becomes massive through the super-Higgs mechanism.

1.5 $\mathcal{N} = 1$ Supergravity

Supergravity is the supersymmetric extension of general relativity. Its defining property is that supersymmetry is realized as a local symmetry, so that the spinorial transformation parameter becomes an arbitrary function of the spacetime coordinates,

$$\epsilon = \epsilon(x). \quad (1.128)$$

Since the anticommutator of two supersymmetry transformations generates a spacetime translation, promoting supersymmetry to a local symmetry necessarily introduces invariance under local spacetime transformations and therefore gravity.

In four-dimensional $\mathcal{N} = 1$ supergravity, the gravity multiplet consists of the vierbein e_μ^a , describing the graviton, and a single Majorana vector-spinor ψ_μ , called the gravitino,

$$(e_\mu^a, \psi_\mu). \quad (1.129)$$

The gravitino is the gauge field associated with local supersymmetry. On shell, the massless graviton and gravitino carry the same number of physical degrees of freedom.

In this section, we first introduce the conventions used for four-component spinors and review pure $\mathcal{N} = 1$ supergravity. We then discuss its Minkowski and anti-de Sitter vacua before introducing the couplings to chiral multiplets that are relevant for four-dimensional string compactifications.

1.5.1 Pure $\mathcal{N} = 1$ Supergravity

Flat-space gamma matrices satisfy the Clifford algebra

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}\mathbf{1}_4. \quad (1.130)$$

In the Weyl representation, they may be written as

$$\gamma^a = \begin{pmatrix} 0 & \sigma^a \\ \bar{\sigma}^a & 0 \end{pmatrix}, \quad (1.131)$$

where σ^i are the Pauli matrices.

Curved-space gamma matrices are defined through the vierbein as

$$\gamma^\mu = e_a^\mu \gamma^a, \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbf{1}_4. \quad (1.132)$$

Antisymmetrized products are defined with unit weight,

$$\gamma^{a_1 \dots a_p} \equiv \gamma^{[a_1} \dots \gamma^{a_p]}, \quad (1.133)$$

where the brackets include the factor $1/p!$.

A four-component Majorana spinor may be constructed from a two-component Weyl spinor and its complex conjugate according to

$$\chi = (\chi_\alpha \bar{\chi}^{\dot{\alpha}}). \quad (1.134)$$

Its Dirac conjugate is

$$\bar{\chi} \equiv \chi^\dagger \gamma^0. \quad (1.135)$$

We also define

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \quad P_L = \frac{1 - \gamma_5}{2}, \quad P_R = \frac{1 + \gamma_5}{2}. \quad (1.136)$$

At the two-derivative level, pure $\mathcal{N} = 1$ supergravity is obtained by combining the Einstein–Hilbert action with the Rarita–Schwinger action for the gravitino. Up to terms quartic in the gravitino, the action is

$$S_{\text{SUGRA}} = \int d^4x e \left[\frac{M_P^2}{2} R - \frac{1}{2} \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho \right] + \mathcal{O}(\psi^4), \quad (1.137)$$

where

$$e \equiv \det(e_\mu{}^a) = \sqrt{-g}. \quad (1.138)$$

The Lorentz- and spacetime-covariant derivative acting on the gravitino is

$$D_\nu \psi_\rho = \partial_\nu \psi_\rho - \Gamma_{\nu\rho}^\sigma \psi_\sigma + \frac{1}{4} \omega_\nu{}^{ab} \gamma_{ab} \psi_\rho. \quad (1.139)$$

To lowest order in the fermions, the local supersymmetry transformations are

$$\delta e_\mu{}^a = \frac{1}{2M_P} \bar{\epsilon} \gamma^a \psi_\mu, \quad \delta \psi_\mu = M_P D_\mu \epsilon + \mathcal{O}(\psi^2). \quad (1.140)$$

The spacetime dependence of $\epsilon(x)$ makes explicit the interpretation of the gravitino as the gauge field of local supersymmetry.

Maximally symmetric backgrounds. Consider the Einstein–Hilbert action with a cosmological constant,

$$S_\Lambda = \frac{M_P^2}{2} \int d^d x \sqrt{-g} (R - 2\Lambda). \quad (1.141)$$

Its equations of motion are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0. \quad (1.142)$$

A maximally symmetric spacetime has Riemann tensor

$$R_{\mu\nu\rho\sigma} = \frac{2\Lambda}{(d-1)(d-2)} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}). \quad (1.143)$$

Positive, vanishing and negative values of Λ correspond, respectively, to de Sitter, Minkowski and anti-de Sitter spacetime. In terms of the curvature radius ℓ , in four dimensions,

$$\Lambda_{\text{dS}_4} = \frac{3}{\ell^2}, \quad \Lambda_{\text{AdS}_4} = -\frac{3}{\ell^2}. \quad (1.144)$$

The corresponding homogeneous spaces are

$$\text{dS}_d = \frac{SO(1, d)}{SO(1, d-1)}, \quad \text{AdS}_d = \frac{SO(2, d-1)}{SO(1, d-1)}. \quad (1.145)$$

Anti-de Sitter supergravity. A cosmological constant cannot be introduced in pure supergravity independently of the gravitino sector. Local supersymmetry requires adding both a gravitino mass-like term and a constant contribution to the scalar potential. In four-component Majorana notation, and for a real parameter $m_{3/2}$, the resulting Lagrangian is

$$\mathcal{L}_{\text{AdS}} = e \left[\frac{M_P^2}{2} R - \frac{1}{2} \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho - \frac{m_{3/2}}{2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu + 3M_P^2 m_{3/2}^2 \right] + \mathcal{O}(\psi^4). \quad (1.146)$$

The relative coefficients in (1.146) are fixed by local supersymmetry. The corresponding transformations are

$$\delta e_\mu{}^a = \frac{1}{2M_P} \bar{\epsilon} \gamma^a \psi_\mu, \quad (1.147)$$

and

$$\delta \psi_\mu = M_P \left(D_\mu \epsilon + \frac{m_{3/2}}{2} \gamma_\mu \epsilon \right) + \mathcal{O}(\psi^2). \quad (1.148)$$

Comparing the last term of (1.146) with the cosmological term in (1.141), one obtains

$$\Lambda = -3m_{3/2}^2, \quad m_{3/2} = \frac{1}{\ell}. \quad (1.149)$$

Thus, a nonvanishing supersymmetric gravitino mass parameter is associated with a negative cosmological constant. Although $m_{3/2}$ appears in a term resembling a mass term, it does not signal supersymmetry breaking: in the AdS_4 vacuum the gravitino is the massless gauge field of local supersymmetry and belongs to a shortened AdS representation.

Unbroken supersymmetry on a bosonic background requires the existence of a non-vanishing spinor satisfying

$$\delta \psi_\mu = 0, \quad (1.150)$$

and therefore the Killing-spinor equation

$$\left(D_\mu + \frac{m_{3/2}}{2} \gamma_\mu \right) \epsilon = 0. \quad (1.151)$$

The integrability condition of (1.151) gives

$$R_{\mu\nu} = -3m_{3/2}^2 g_{\mu\nu}, \quad (1.152)$$

which reproduces (1.149). Consequently, the maximally supersymmetric vacuum of pure $\mathcal{N} = 1$ supergravity is Minkowski spacetime when $m_{3/2} = 0$, and AdS_4 when $m_{3/2} \neq 0$.

The AdS superalgebra. The isometry algebra of AdS_4 is $\mathfrak{so}(2, 3)$. Defining the AdS translation generators P_a in terms of the generators of $\mathfrak{so}(2, 3)$, one finds

$$[P_a, P_b] = -\frac{1}{\ell^2} M_{ab}. \quad (1.153)$$

Unlike in the Poincaré algebra, translations therefore do not commute.

The minimal supersymmetric extension of $\mathfrak{so}(2, 3)$ is the superalgebra $\mathfrak{osp}(1|4)$. Its structure requires the commutator between translations and supercharges to be non-vanishing:

$$[P_a, Q] = -\frac{1}{2\ell} \gamma_a Q. \quad (1.154)$$

Moreover, the supercharge anticommutator contains both translations and Lorentz transformations:

$$\{Q_\alpha, \bar{Q}_\beta\} = 2(\gamma^a)_{\alpha\beta} P_a + \frac{1}{\ell} (\gamma^{ab})_{\alpha\beta} M_{ab}, \quad (1.155)$$

up to convention-dependent signs and normalizations. The additional terms proportional to ℓ^{-1} are required by the graded Jacobi identities and vanish in the flat-space limit

$$\ell \longrightarrow \infty. \quad (1.156)$$

In this limit, $\mathfrak{osp}(1|4)$ contracts to the ordinary $\mathcal{N} = 1$ super-Poincaré algebra.

Equations (1.149) and (1.151) also show why a maximally supersymmetric de Sitter vacuum is not possible in standard unitary $\mathcal{N} = 1$ supergravity. A positive cosmological constant is incompatible with the integrability condition for a physical Majorana Killing spinor. Therefore a de Sitter vacuum, whenever generated through additional matter sectors or uplifting contributions, necessarily breaks supersymmetry.

1.5.2 Matter Couplings in Supergravity

We now consider the coupling of chiral multiplets to four-dimensional $\mathcal{N} = 1$ supergravity. A chiral multiplet contains a complex scalar ϕ^I , a Weyl fermion χ^I and an auxiliary complex scalar F^I ,

$$\Phi^I = (\phi^I, \chi^I, F^I). \quad (1.157)$$

The index I labels the different chiral multiplets.

At the two-derivative level, a general theory of chiral multiplets coupled to supergravity is determined by two functions:

- a real Kähler potential

$$K(\phi, \bar{\phi}), \quad (1.158)$$

which determines the geometry of the scalar manifold;

- a holomorphic superpotential

$$W(\phi), \quad (1.159)$$

which determines the interactions and masses of the chiral multiplets.

When vector multiplets are included, the theory also depends on a holomorphic gauge kinetic function $f_{AB}(\phi)$. Since the moduli sector relevant for the present discussion consists only of chiral multiplets, we initially omit gauge fields and D -term contributions.

The scalar manifold is Kähler (1.3.1), with metric

$$K_{I\bar{J}} \equiv \partial_I \partial_{\bar{J}} K, \quad (1.160)$$

The corresponding holomorphic Christoffel symbols are

$$\Gamma^I_{JK} = K^{I\bar{L}} \partial_J \partial_{\bar{K}} K_{L\bar{L}}. \quad (1.161)$$

The Kähler potential and superpotential are not separately physical. The theory is invariant under Kähler transformations

$$K \longrightarrow K + F(\phi) + \bar{F}(\bar{\phi}), \quad W \longrightarrow e^{-F(\phi)/M_P^2} W, \quad (1.162)$$

where $F(\phi)$ is an arbitrary holomorphic function.

The Kähler-covariant derivative of the superpotential is

$$D_I W \equiv \partial_I W + \frac{K_I}{M_P^2} W, \quad K_I \equiv \partial_I K. \quad (1.163)$$

Its second covariant derivative is

$$D_I D_J W = \left(\partial_I + \frac{K_I}{M_P^2} \right) D_J W - \Gamma^K_{IJ} D_K W. \quad (1.164)$$

Restricting to the bosonic sector, the Lagrangian is

$$e^{-1} \mathcal{L}_{\text{bos}} = \frac{M_P^2}{2} R + K_{I\bar{J}} \partial_\mu \phi^I \partial^\mu \bar{\phi}^{\bar{J}} - V_F. \quad (1.165)$$

After integrating out the auxiliary fields, the F -term scalar potential is

$$V_F = e^{K/M_P^2} \left[K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - \frac{3}{M_P^2} |W|^2 \right]. \quad (1.166)$$

Here

$$D_{\bar{J}}\bar{W} = \partial_{\bar{J}}\bar{W} + \frac{K_{\bar{J}}}{M_P^2}\bar{W}. \quad (1.167)$$

The first term in (1.166) is the supergravity generalization of the globally supersymmetric F -term potential. Ordinary derivatives of the superpotential are replaced by Kähler-covariant derivatives. The second term,

$$-3e^{K/M_P^2}\frac{|W|^2}{M_P^2}, \quad (1.168)$$

is a genuinely gravitational contribution and has no analogue in global supersymmetry. As a consequence, the supergravity scalar potential is not positive definite.

The on-shell auxiliary fields are

$$F^I = e^{K/(2M_P^2)}K^{I\bar{J}}D_{\bar{J}}\bar{W}. \quad (1.169)$$

In terms of these fields, the scalar potential can be written as

$$V_F = K_{I\bar{J}}F^I\bar{F}^{\bar{J}} - 3M_P^2m_{3/2}^2, \quad (1.170)$$

where the gravitino mass is

$$m_{3/2} = e^{K/(2M_P^2)}\frac{|W|}{M_P^2}. \quad (1.171)$$

The quantities F^I are order parameters for supersymmetry breaking. On a vacuum with constant scalar fields, unbroken supersymmetry requires

$$F^I = 0, \quad (1.172)$$

or equivalently

$$D_I W = 0 \quad \text{for every } I. \quad (1.173)$$

At a supersymmetric critical point, the scalar potential therefore reduces to

$$V_{\text{susy}} = -3e^{K/M_P^2}\frac{|W|^2}{M_P^2} = -3M_P^2m_{3/2}^2. \quad (1.174)$$

Equation (1.173) leads to two possibilities. If

$$D_I W = 0, \quad W = 0, \quad (1.175)$$

then

$$m_{3/2} = 0, \quad V_{\text{susy}} = 0, \quad (1.176)$$

and the vacuum is supersymmetric Minkowski spacetime. By contrast, if

$$D_I W = 0, \quad W \neq 0, \quad (1.177)$$

then

$$m_{3/2} \neq 0, \quad V_{\text{susy}} < 0, \quad (1.178)$$

and the vacuum is supersymmetric AdS₄.

A de Sitter vacuum has positive vacuum energy and therefore cannot satisfy (1.178). Consequently, a de Sitter vacuum in standard four-dimensional $\mathcal{N} = 1$ supergravity necessarily breaks supersymmetry.

1.6 Phenomenological Aspects

It is particularly interesting to investigate the interplay between supersymmetry, space-time geometry and the matter sector. Consider a theory containing n_S real scalar fields $\varphi^i(x)$. For a field configuration in which the scalars are constant,

$$\varphi^i(x) = \varphi_c^i, \quad (1.179)$$

and all the remaining non-gravitational fields vanish, the scalar equations of motion reduce to

$$\partial_i V|_{\varphi=\varphi_c} = 0. \quad (1.180)$$

Therefore, a vacuum configuration must correspond to a critical point of the scalar potential.

In the following, quantities evaluated at the critical point $\varphi^i = \varphi_c^i$ will be denoted by angular brackets. For example, Eq. (1.180) can be written as

$$\langle \partial_i V \rangle = 0. \quad (1.181)$$

The value of the scalar potential at the critical point,

$$\langle V \rangle \equiv V(\varphi_c), \quad (1.182)$$

represents the vacuum energy density,

$$\rho_{\text{vac}} = \langle V \rangle. \quad (1.183)$$

It enters the Einstein equations as an effective cosmological constant,

$$\Lambda = \frac{\rho_{\text{vac}}}{M_P^2} = \frac{\langle V \rangle}{M_P^2}. \quad (1.184)$$

Consequently,

$$\langle V \rangle = 0, \quad \langle V \rangle > 0, \quad \langle V \rangle < 0, \quad (1.185)$$

correspond, respectively, to Minkowski, de Sitter and anti-de Sitter spacetime.

The condition $\langle \partial_i V \rangle = 0$ guarantees only that the configuration is a stationary point. Its stability is determined by the spectrum of scalar fluctuations around the vacuum, and hence by the eigenvalues of the Hessian matrix

$$\mathcal{H}_{ij} = \langle \partial_i \partial_j V \rangle. \quad (1.186)$$

In Minkowski and de Sitter spacetime, perturbative stability requires the absence of negative mass-squared eigenvalues. In anti-de Sitter spacetime, negative mass-squared eigenvalues may instead be allowed, provided that they satisfy the Breitenlohner–Freedman bound.

The stability of the vacuum, together with several other phenomenologically relevant properties, depends crucially on whether supersymmetry is preserved or spontaneously broken at the critical point.

1.6.1 Spontaneous supersymmetry breaking in local SUSY

In matter-coupled four-dimensional $\mathcal{N} = 1$ supergravity, the scalar potential is given by the F -term contribution in Eq. (1.166), together with a possible D -term contribution. For later convenience, we introduce the decomposition

$$V = V_F - V_G + V_D, \quad (1.187)$$

where

$$V_F = e^{K/M_P^2} |\mathcal{D}W|^2, \quad (1.188)$$

$$V_G = 3e^{K/M_P^2} \frac{|W|^2}{M_P^2}, \quad (1.189)$$

and

$$V_D = \frac{1}{2} \text{Re } f_{IJ} D^I D^J. \quad (1.190)$$

- Unlike the scalar potential of global supersymmetry, the supergravity potential is not positive semi-definite. Depending on the relative magnitude of V_F , V_G and V_D , its value at a critical point may be positive, negative or zero. This is one of the most important differences between global and local supersymmetry.
- Supersymmetry is preserved by a bosonic background if the supersymmetry variations of all fermionic fields vanish. For a Lorentz-invariant vacuum with constant scalar fields and vanishing gauge field strengths, the vanishing of the chiral-fermion and gaugino variations requires

$$\langle D_m W \rangle = 0, \quad (1.191)$$

and

$$\langle D^I \rangle = 0. \quad (1.192)$$

- At a supersymmetric vacuum, both V_F and V_D vanish, and the scalar potential reduces to

$$\langle V \rangle = -3 \left\langle e^{K/M_P^2} \frac{|W|^2}{M_P^2} \right\rangle \leq 0. \quad (1.193)$$

Thus, the vacuum energy of an unbroken supersymmetric configuration is necessarily non-positive. This differs from the globally supersymmetric case, in which unbroken supersymmetry implies a vanishing vacuum energy.

- A supersymmetric Minkowski vacuum is obtained only if the superpotential also vanishes at the critical point,

$$\langle W \rangle = 0. \quad (1.194)$$

Since

$$D_m W = \partial_m W + \frac{K_m}{M_P^2} W, \quad (1.195)$$

when $\langle W \rangle = 0$ the conditions for an unbroken supersymmetric Minkowski vacuum become

$$\langle \partial_m W \rangle = 0, \quad \langle W \rangle = 0, \quad \langle D^I \rangle = 0. \quad (1.196)$$

By contrast, if

$$\langle D_m W \rangle = 0, \quad \langle D^I \rangle = 0, \quad \langle W \rangle \neq 0, \quad (1.197)$$

the vacuum energy is strictly negative,

$$\langle V \rangle = -3 \left\langle e^{K/M_P^2} \frac{|W|^2}{M_P^2} \right\rangle \leq 0. \quad (1.198)$$

The resulting supersymmetric vacuum is therefore anti-de Sitter.

The converse statement, however, does not hold: a vanishing or negative vacuum energy does not imply that supersymmetry is preserved. Indeed, supersymmetry may be broken by non-vanishing F - or D -terms, while the positive contributions to the scalar potential are partially or exactly cancelled by the negative gravitational contribution $-V_G$. For example, one may have

$$\langle D_m W \rangle \neq 0, \quad \langle W \rangle \neq 0, \quad (1.199)$$

and nevertheless obtain

$$\langle V \rangle = 0, \quad (1.200)$$

if

$$\langle V_F + V_D \rangle = \langle V_G \rangle. \quad (1.201)$$

No-scale models in string theory provide an important realization of this cancellation and will be discussed in Sec. 3.1.2.

The spontaneous breakdown of susy in a phenomenologically realistic model also defines a number of important mass scales.

The Vacuum-Energy Scale M_{vac} . It is useful to define the supersymmetry-breaking scale through the positive contributions to the scalar potential,

$$M_{\text{susy}}^4 \equiv \langle V_F + V_D \rangle, \quad (1.202)$$

and the vacuum-energy scale as

$$M_{\text{vac}}^4 \equiv \langle V \rangle. \quad (1.203)$$

Using the decomposition introduced above, one obtains

$$M_{\text{vac}}^4 = M_{\text{susy}}^4 - \langle V_G \rangle = M_{\text{susy}}^4 - 3M_P^2 m_{3/2}^2. \quad (1.204)$$

This equation shows that, in supergravity, a high supersymmetry-breaking scale can in principle coexist with a very small vacuum energy. The observed dark-energy density is of order

$$\rho_{\text{vac}} \sim (10^{-3} \text{eV})^4, \quad (1.205)$$

which is extremely small compared with any phenomenologically relevant particle-physics scale. However, Eq. (1.204) does not provide an explanation for this hierarchy: it requires an extraordinarily precise cancellation between the positive supersymmetry-breaking contribution and the negative gravitational contribution.

The Gravitino Mass $m_{3/2}$. After spontaneous supersymmetry breaking, the goldstino is absorbed by the gravitino through the super-Higgs mechanism, providing the longitudinal degrees of freedom of a massive spin-3/2 field. The gravitino mass is

$$m_{3/2} = \left\langle \frac{e^{K/(2M_P^2)} |W|}{M_P^2} \right\rangle. \quad (1.206)$$

For a vacuum with a negligibly small cosmological constant, Eq. (1.204) implies

$$M_{\text{susy}}^4 \simeq 3M_P^2 m_{3/2}^2, \quad (1.207)$$

and therefore

$$m_{3/2} \simeq \frac{M_{\text{susy}}^2}{\sqrt{3}M_P}. \quad (1.208)$$

Because the fundamental supersymmetry-breaking scale is not directly fixed by the electroweak scale,

$$M_{\text{EW}} \simeq 246, \text{ GeV}, \quad (1.209)$$

the gravitino mass may vary over a very wide range.

The Soft-Mass Scale M_{soft} . The masses of the Standard Model superpartners arise from soft supersymmetry-breaking operators. These operators break supersymmetry explicitly in the low-energy effective theory without reintroducing quadratic ultraviolet divergences. At the fundamental level, however, they are generally understood as originating from the spontaneous breaking of supersymmetry in a hidden sector.

The symbol M_{soft} denotes a characteristic scale of the soft parameters in the visible sector. It should not be interpreted as a unique mass scale, since the superpartner spectrum may contain several hierarchically different masses. Experimental constraints

depend strongly on the particle spectrum, decay channels and assumptions concerning the lightest supersymmetric particle. Moreover, requiring supersymmetry to stabilize the electroweak hierarchy favours soft parameters not too far above the electroweak scale. This is, however, a naturalness criterion rather than a consistency condition.

The relation between M_{soft} and the fundamental supersymmetry-breaking scale depends on the mediation mechanism. If supersymmetry breaking is characterized by an auxiliary-field expectation value F , dimensional analysis gives

$$M_{\text{soft}} \sim \frac{F}{M_{\text{med}}}, \quad (1.210)$$

up to dimensionless couplings and possible loop-suppression factors, where M_{med} denotes the characteristic mediation scale.

In gravity-mediated scenarios, the mediation scale is typically of order the Planck scale, so that

$$M_{\text{soft}} \sim \frac{F}{M_P}. \quad (1.211)$$

For a nearly Minkowski vacuum in which supersymmetry breaking is dominated by F -terms,

$$F \sim m_{3/2} M_P, \quad (1.212)$$

and therefore one typically finds

$$M_{\text{soft}} \sim m_{3/2}. \quad (1.213)$$

The precise relation, however, depends on the Kähler potential, the gauge kinetic function and the coupling between the supersymmetry-breaking and visible sectors. In particular, sequestering or cancellations may suppress the soft terms relative to the gravitino mass. Consequently, the same value of $m_{3/2}$ may correspond to very different visible-sector spectra.

2 Superstrings

In this chapter, we review the main ingredients of type II superstring theory that are required for the compactification framework developed in the remainder of this thesis [15]. We begin with the Ramond–Neveu–Schwarz formulation of superstrings in ten-dimensional flat Minkowski spacetime, emphasizing the role of worldsheet fermions, the Neveu–Schwarz and Ramond sectors, and the GSO projection. The latter removes the tachyonic state and leads to a spectrum containing both spacetime bosons and fermions, organized into supersymmetric multiplets.

Among the five consistent superstring theories in ten dimensions, type IIA, type IIB, type I, and the two heterotic theories with gauge groups $SO(32)$ and $E_8 \times E_8$, we focus on the type II theories, and in particular on type IIB string theory, which provides the ten-dimensional starting point for the compactifications considered in this work. We therefore review its massless spectra and its corresponding low-energy supergravity descriptions.

We then turn to compactifications on Calabi–Yau threefolds. After introducing the geometric and cohomological properties of Calabi–Yau manifolds, we discuss how the ten-dimensional massless fields give rise to four-dimensional fields through Kaluza–Klein reduction. Compactification of type II string theory on a Calabi–Yau threefold preserves $\mathcal{N} = 2$ supersymmetry in four dimensions. To obtain the $\mathcal{N} = 1$ effective theory relevant for moduli stabilization, we subsequently introduce orientifold projections and specialize to type IIB Calabi–Yau orientifolds with $O3/O7$ -planes. This construction determines the surviving moduli fields and their organization into four-dimensional chiral and vector multiplets, providing the setting in which fluxes, the effective Kähler potential, and the corrections to the moduli F -terms will be studied.

2.1 Type II string theory

Type II string theories describe oriented closed strings. In the Ramond–Neveu–Schwarz formulation, their worldsheet degrees of freedom consist of the bosonic embedding coordinates $X^M(\tau, \sigma)$ and their fermionic superpartners $\psi^M(\tau, \sigma)$. In the light-cone gauge, only the transverse components are physical, and we denote them by

$$X^i(\tau, \sigma), \quad \psi^i(\tau, \sigma), \quad i = 2, \dots, D-1.$$

For a closed string, the spatial worldsheet coordinate is periodic. Denoting its period by ℓ , the bosonic fields satisfy

$$X^i(\tau, \sigma + \ell) = X^i(\tau, \sigma). \quad (2.1)$$

As in the bosonic closed string, the equations of motion allow the fields to be decomposed into left- and right-moving components. The two sectors can be quantized independently, subject to the level-matching condition imposed on physical closed-string states. In the following, we describe explicitly the left-moving sector; the construction of the right-moving sector is completely analogous.

For the worldsheet fermions, consistency on the spatial circle allows two possible spin structures. Introducing the left-moving coordinate

$$\sigma^+ \equiv \tau + \sigma, \quad (2.2)$$

the fermionic fields may obey either antiperiodic or periodic boundary conditions:

$$\begin{aligned} \text{Neveu–Schwarz sector:} \quad & \psi_L^i(\sigma^+ + \ell) = -\psi_L^i(\sigma^+), \\ \text{Ramond sector:} \quad & \psi_L^i(\sigma^+ + \ell) = \psi_L^i(\sigma^+). \end{aligned} \quad (2.3)$$

The boundary conditions of the left- and right-moving fermions can nevertheless be chosen independently. Consequently, the closed-string Hilbert space decomposes into four sectors:

$$\text{NS–NS}, \quad \text{NS–R}, \quad \text{R–NS}, \quad \text{R–R}. \quad (2.4)$$

We now construct the fermionic Hilbert spaces associated with the Neveu–Schwarz and Ramond boundary conditions.

2.1.1 Neveu–Schwarz sector

The antiperiodic boundary condition implies that the fermionic modes are half-integer moded. The left-moving field therefore admits the expansion

$$\psi_L^i(\sigma^+) = i\sqrt{\frac{\alpha'}{2}} \sum_{r \in \mathbb{Z} + \frac{1}{2}} \psi_r^i \exp\left(-\frac{2\pi i r \sigma^+}{\ell}\right). \quad (2.5)$$

Since $r \in \mathbb{Z} + \frac{1}{2}$, the Neveu–Schwarz sector contains no fermionic zero modes. Canonical quantization gives

$$\{\psi_r^i, \psi_s^j\} = \delta^{ij} \delta_{r,-s}. \quad (2.6)$$

The left-moving bosonic and fermionic number operators are defined by

$$N_B = \sum_{i=2}^{D-1} \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i, \quad (2.7)$$

and

$$N_{F,NS} = \sum_{i=2}^{D-1} \sum_{\substack{r \in \mathbb{Z} + \frac{1}{2} \\ r > 0}} r \psi_{-r}^i \psi_r^i. \quad (2.8)$$

The zero-point energies of the transverse bosonic and Neveu–Schwarz fermionic oscillators are

$$E_0^B = -\frac{D-2}{24}, \quad E_0^{F,NS} = -\frac{D-2}{48}. \quad (2.9)$$

Their sum determines the Neveu–Schwarz normal-ordering constant,

$$a_{NS} = \frac{D-2}{16}. \quad (2.10)$$

The Neveu–Schwarz vacuum is defined by

$$\psi_r^i |0\rangle_{NS} = 0 \quad \text{for all } r > 0, \quad (2.11)$$

together with

$$\alpha_n^i |0\rangle_{NS} = 0 \quad \text{for all } n > 0. \quad (2.12)$$

The remaining states are obtained by acting on $|0\rangle_{NS}$ with the negative-mode oscillators.

Quantum consistency requires the cancellation of the worldsheet conformal anomaly, fixing the critical dimension to

$$D = 10. \quad (2.13)$$

The transverse little group is therefore $SO(8)$, and

$$a_{NS} = \frac{1}{2}. \quad (2.14)$$

For a closed string, the mass-shell and level-matching conditions can be written as

$$\frac{\alpha' M^2}{4} = N_L - a_L = N_R - a_R. \quad (2.15)$$

The lowest left-moving states in the Neveu–Schwarz sector are consequently

State	$\alpha' M_{NS,L}^2/2$	$SO(8)$
$ 0\rangle_{NS}$	$-1/2$	1
$\psi_{-1/2}^i 0\rangle_{NS}$	0	8_V

The Neveu–Schwarz ground state is tachyonic before the GSO projection, whereas the first excited states are massless and transform in the vector representation $\mathbf{8}_V$ of $SO(8)$.

2.1.2 Ramond sector

In the Ramond sector, the fermionic fields are periodic and therefore integer-moded:

$$\psi_L^i(\sigma^+) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \psi_n^i \exp\left(-\frac{2\pi i n \sigma^+}{\ell}\right). \quad (2.16)$$

The oscillator modes satisfy

$$\{\psi_m^i, \psi_n^j\} = \delta^{ij} \delta_{m,-n}. \quad (2.17)$$

The corresponding fermionic number operator is

$$N_{F,R} = \sum_{i=2}^{D-1} \sum_{n=1}^{\infty} n \psi_{-n}^i \psi_n^i. \quad (2.18)$$

The zero-point energy of the Ramond fermions is

$$E_0^{F,R} = \frac{D-2}{24}, \quad (2.19)$$

which exactly cancels the bosonic zero-point energy. The Ramond normal-ordering constant is therefore

$$a_R = 0, \quad (2.20)$$

and the mass-shell condition becomes

$$\frac{\alpha' M^2}{4} = N_L = N_R \quad (2.21)$$

when both left- and right-moving sectors are of Ramond type.

The positive-frequency modes annihilate the Ramond ground states:

$$\psi_n^i |A\rangle_R = 0, \quad \alpha_n^i |A\rangle_R = 0, \quad n > 0. \quad (2.22)$$

In contrast to the Neveu–Schwarz sector, however, the Ramond expansion contains fermionic zero modes. These satisfy

$$\{\psi_0^i, \psi_0^j\} = \delta^{ij}, \quad (2.23)$$

and hence generate the Clifford algebra associated with the transverse group $SO(8)$. It follows that the Ramond ground state is degenerate and transforms as a spinor of $SO(8)$.

To construct this representation explicitly, we introduce four pairs of fermionic creation and annihilation operators,

$$A_a^\pm = \frac{1}{\sqrt{2}} (\psi_0^{2a} \pm i\psi_0^{2a+1}), \quad a = 1, \dots, 4. \quad (2.24)$$

They satisfy

$$\{A_a^-, A_b^+\} = \delta_{ab}, \quad (2.25)$$

with all remaining anticommutators vanishing. Defining a reference state by

$$A_a^- |0\rangle_R = 0, \quad a = 1, \dots, 4, \quad (2.26)$$

the sixteen Ramond ground states are obtained by acting with all possible products of the four operators A_a^+ .

The states containing an even number of creation operators are

$$|0\rangle_R, \quad A_{a_1}^+ A_{a_2}^+ |0\rangle_R, \quad A_1^+ A_2^+ A_3^+ A_4^+ |0\rangle_R, \quad (2.27)$$

and form a space of dimension 8. The states containing an odd number of creation operators are

$$A_a^+ |0\rangle_R, \quad A_{a_1}^+ A_{a_2}^+ A_{a_3}^+ |0\rangle_R, \quad (2.28)$$

and form a second space of dimension 8.

The even- and odd-fermion-number states have opposite chiralities and therefore furnish the two inequivalent chiral spinor representations of $SO(8)$,

$$\mathbf{8}_S \quad \text{and} \quad \mathbf{8}_C. \quad (2.29)$$

Since $a_R = 0$, all Ramond ground states lie at the massless level. Before imposing the GSO projection, the lowest states are therefore

State	$\alpha' M_{NS,L}^2/2$	$SO(8)$
$ \mathbf{8}_C\rangle$	0	$\mathbf{8}_C$
$ \mathbf{8}\rangle_S$	0	$\mathbf{8}_S$

The GSO projection selects one of the two chiral Ramond representations. The relative choice of chirality in the left- and right-moving sectors distinguishes type IIA from type IIB string theory.

2.2 Physical States-Massless spectrum of type II string theory

The physical spectrum of type II string theory is obtained by imposing the GSO projection independently on the left- and right-moving sectors. In the Neveu-Schwarz sector, this projection removes the tachyonic ground state and retains states with an odd number of fermionic excitations. The lowest surviving state is therefore

$$\psi_{-\frac{1}{2}}^i |0\rangle_{\text{NS}},$$

which is massless and transforms in the vector representation 8_V of the transverse little group $SO(8)$.

In the Ramond sector, the GSO projection selects one of the two chiral spinor representations, 8_S or 8_C . Type IIB retains Ramond ground states of the same chirality in the two sectors, whereas type IIA retains states of opposite chirality.

The closed-string spectrum is obtained by taking tensor products of the GSO-projected left- and right-moving states. This produces the four sectors

$$\text{NS-NS}, \quad \text{NS-R}, \quad \text{R-NS}, \quad \text{R-R}. \quad (2.30)$$

At the massless level, both the left- and right-moving contributions satisfy

$$N_L - a_L = N_R - a_R = 0, \quad (2.31)$$

so that the level-matching condition is automatically fulfilled.

2.2.1 Type IIB string theory

In type IIB string theory, the left- and right-moving Ramond sectors have the same chirality. The massless spectrum has the following structure:

sector	$ \rangle_L \otimes \rangle_R$	$SO(8)$	10d field
NS-NS	$8_V \otimes 8_V$	$1 + 28_V + 35_V$	ϕ, B_{MN}, G_{MN}
NS-R	$8_V \otimes 8_C$	$8_S + 56_S$	$\lambda_\alpha^1, \psi_{M\alpha}^1$
R-NS	$8_C \otimes 8_V$	$8_S + 56_S$	$\lambda_\alpha^2, \psi_{M\alpha}^2$
R-R	$8_C \otimes 8_C$	$1 + 28_C + 35_C$	a, C_{MN}, C_{MNPQ}

The NS-NS sector contains the dilaton ϕ , the antisymmetric two-form potential B_{MN} , and the spacetime metric G_{MN} . Their physical polarizations transform respectively as the singlet 1, the antisymmetric representation 28, and the symmetric traceless representation 35_V of $SO(8)$.

The mixed NS–R and R–NS sectors contain the spacetime fermions. Each sector provides a spin- $\frac{3}{2}$ Rarita–Schwinger field ψ_M , interpreted as a gravitino, together with a spin- $\frac{1}{2}$ dilatino λ . Since the Ramond chiralities of the two sectors are equal, type IIB is a chiral ten-dimensional theory.

The R–R sector contains differential-form gauge potentials of even degree:

$$C_0, \quad C_2, \quad C_4. \quad (2.32)$$

Here C_0 is the R–R scalar, often referred to as the axion, C_2 is an antisymmetric two-form, and C_4 is a four-form potential.

The complete massless spectrum forms the unique ten-dimensional chiral supergravity multiplet with 32 real supercharges.

2.2.2 Type IIA string theory

In type IIA string theory, the Ramond ground states retained in the left- and right-moving sectors have opposite chiralities. The massless spectrum is

sector	$ \rangle_L \otimes \rangle_R$	$SO(8)$	10d field
NS-NS	$\mathbf{8}_V \otimes \mathbf{8}_V$	$\mathbf{1} + \mathbf{28}_V + \mathbf{35}_V$	ϕ, B_{MN}, G_{MN}
NS-R	$\mathbf{8}_V \otimes \mathbf{8}_S$	$\mathbf{8}_C + \mathbf{56}_C$	$\lambda_{\dot{\alpha}}^1, \psi_{M\dot{\alpha}}^1$
R-NS	$\mathbf{8}_C \otimes \mathbf{8}_V$	$\mathbf{8}_S + \mathbf{56}_S$	$\lambda_{\alpha}^2, \psi_{M\alpha}^2$
R-R	$\mathbf{8}_C \otimes \mathbf{8}_S$	$\mathbf{8}_V + \mathbf{56}_V$	C_M, C_{MNP}

The NS–NS sector is identical to that of type IIB and contains the metric G_{MN} , the two-form B_{MN} , and the dilaton ϕ . The NS–R and R–NS sectors contain two gravitini and two dilatini. In contrast to type IIB, the corresponding fermions occur with opposite ten-dimensional chiralities, so type IIA is non-chiral.

The R–R sector contains differential-form potentials of odd degree,

$$C_1, \quad C_3. \quad (2.33)$$

The complete spectrum forms the ten-dimensional non-chiral type IIA supergravity multiplet, again containing 32 real supercharges.

2.2.3 Local Symmetries of Type IIA and IIB string theories

Type IIA and IIB string theories exhibit several fundamental local spacetime symmetries:

1. invariance under spacetime diffeomorphisms, with the metric G_{MN} describing the graviton;

2. gauge invariance of the p -form potentials,

$$C_p \rightarrow C_p + d\Lambda_{p-1}, \quad (2.34)$$

so that these fields behave as generalized gauge potentials;

3. local supersymmetry, with 32 supercharges organized into two ten-dimensional chiral spinors of 16 components each. The spectra correspond to the gravity multiplet of chiral or non-chiral 10d $\mathcal{N} = 2$ superalgebra, for type IIB or type IIA theories, respectively.

2.2.4 Ten-dimensional low-energy effective actions

In the low-energy regime,

$$\alpha' E^2 \ll 1, \quad (2.35)$$

the massive string excitations can be integrated out, and the dynamics of the massless modes is described, at leading order in the derivative and string-loop expansions, by ten-dimensional supergravity. The two type II string theories give rise to the two maximal supergravity theories in ten dimensions, both possessing 32 real supercharges. Type IIA supergravity is non-chiral and has $\mathcal{N} = (1, 1)$ supersymmetry, whereas type IIB supergravity is chiral and has $\mathcal{N} = (2, 0)$ supersymmetry.

When regarded as stand-alone quantum field theories, these supergravity theories are non-renormalizable and must therefore be interpreted as effective field theories. The corresponding type II string theories provide their ultraviolet completion through the infinite tower of massive string states and the extended nature of string interactions.

The distinction between the two theories is particularly relevant for the discussion of quantum anomalies. Since type IIB supergravity is chiral, its fermionic fields and its self-dual tensor field can in principle generate a purely gravitational anomaly. Consistency requires the total twelve-form anomaly polynomial to vanish. For the massless field content of type IIB supergravity, the contributions of the two gravitini, the two dilatini and the self-dual four-form potential cancel exactly. This cancellation is a highly non-trivial consistency property of the theory. By contrast, type IIA supergravity is non-chiral: its fermionic degrees of freedom occur with opposite chiralities, and the theory is therefore free of local perturbative gravitational anomalies.

In the following, we restrict attention to the bosonic two-derivative actions in string frame. Fermionic terms, localized sources, higher-derivative α' corrections and string-loop corrections will be omitted.

Type IIB supergravity The bosonic sector of type IIB supergravity is described by the pseudo-action

$$S_{\text{IIB}}^{(0)} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[e^{-2\phi} \left(R + 4 \partial_M \phi \partial^M \phi - \frac{1}{2} |H_3|^2 \right) - \frac{1}{2} |F_1|^2 - \frac{1}{2} |\tilde{F}_3|^2 - \frac{1}{4} |\tilde{F}_5|^2 \right] - \frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3. \quad (2.36)$$

Where $2\kappa_{10}^2 = (2\pi)^7 \alpha'^4$ and

$$H_3 = dB_2 \quad (2.37)$$

is the field strength of the NS-NS two-form. The Ramond-Ramond field strengths are

$$F_1 = dC_0, \quad F_3 = dC_2, \quad F_5 = dC_4, \quad (2.38)$$

while the gauge-invariant combinations entering the pseudo-action are

$$\tilde{F}_3 = F_3 - C_0 H_3, \quad (2.39)$$

and

$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3. \quad (2.40)$$

For a p -form field strength, we use the convention

$$|F_p|^2 \equiv \frac{1}{p!} F_{M_1 \dots M_p} F^{M_1 \dots M_p}, \quad (2.41)$$

so that

$$F_p \wedge \star_{10} F_p = |F_p|^2 \star_{10} 1. \quad (2.42)$$

The action (2.36) must be supplemented by the self-duality constraint

$$\tilde{F}_5 = \star_{10} \tilde{F}_5. \quad (2.43)$$

This condition must be imposed only after deriving the equations of motion from the pseudo-action. Indeed, a standard manifestly covariant action written solely in terms of the usual type IIB fields does not simultaneously reproduce the dynamics and impose the self-duality condition. The factor $1/4$ in the kinetic term of $\tilde{F}_5$, rather than the usual factor $1/2$, accounts for the halving of its independent degrees of freedom.

Type IIA supergravity The bosonic two-derivative action of massless type IIA supergravity in string frame is

$$\begin{aligned}
S_{\text{IIA}}^{(0)} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[e^{-2\phi} \left(R + 4 \partial_M \phi \partial^M \phi - \frac{1}{2} |H_3|^2 \right) \right. \\
\left. - \frac{1}{2} |F_2|^2 - \frac{1}{2} |\tilde{F}_4|^2 \right] \\
- \frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4.
\end{aligned} \tag{2.44}$$

The Ramond–Ramond potentials are the one-form C_1 and the three-form C_3 , with field strengths

$$F_2 = dC_1, \quad F_4 = dC_3, \tag{2.45}$$

and gauge-invariant four-form

$$\tilde{F}_4 = F_4 - C_1 \wedge H_3. \tag{2.46}$$

Unlike type IIB, type IIA does not contain a self-dual field strength, and (2.44) is therefore an ordinary action rather than a pseudo-action. The expression above describes massless type IIA supergravity; the Romans deformation, characterized by a non-vanishing zero-form field strength F_0 , is not included.

2.3 Type II Compactifications

We now discuss how the ten-dimensional supergravity theories arising as the low-energy limits of type II string theories can be reduced to four dimensions. In the simplest class of compactifications, one considers an unwarped product geometry of the form

$$M_{10} = M_4 \times X_6, \quad (2.47)$$

where M_4 is a four-dimensional spacetime and X_6 is a compact six-dimensional internal manifold. At this stage, we assume that the dilaton is constant and that all background form-field strengths vanish, so that the only non-trivial background field is the ten-dimensional metric.

If the compactification is required to preserve part of the ten-dimensional supersymmetry, the geometry of X_6 is strongly constrained. Compactification of either type IIA or type IIB supergravity on a Calabi–Yau threefold with holonomy exactly $SU(3)$ preserves eight real supercharges and therefore gives an $\mathcal{N} = 2$ supersymmetric theory in four dimensions. Additional ingredients, such as orientifold projections, are required to reduce the supersymmetry further to $\mathcal{N} = 1$.

2.3.1 Calabi–Yau manifolds

Let us first explain why Calabi–Yau manifolds naturally arise in supersymmetry-preserving compactifications. In a purely geometric background, with vanishing fluxes and constant dilaton, the supersymmetry variation of the gravitino requires the ten-dimensional supersymmetry parameter ϵ to be covariantly constant,

$$\nabla_M \epsilon = 0. \quad (2.48)$$

Upon decomposing the ten-dimensional spinor into four- and six-dimensional components, this condition implies the existence of a non-vanishing spinor η on X_6 satisfying

$$\nabla_m \eta = 0, \quad m = 1, \dots, 6. \quad (2.49)$$

The existence of such a covariantly constant spinor can be expressed in terms of the holonomy group of the internal manifold. The holonomy group $\text{Hol}(X_6)$ is generated by the parallel transport of tensors and spinors around closed curves in X_6 . For a generic orientable six-dimensional Riemannian manifold, the holonomy group is contained in $SO(6)$. A spinor satisfying (2.49) must be invariant under the action of the holonomy group. Since

$$SU(3) \subset SO(6), \quad (2.50)$$

and $SU(3)$ leaves a chiral six-dimensional spinor invariant, the condition

$$\text{Hol}(X_6) \subseteq SU(3) \quad (2.51)$$

ensures the existence of at least one covariantly constant internal spinor.

For an irreducible compactification manifold with holonomy exactly $SU(3)$, each ten-dimensional supersymmetry parameter gives rise to one four-dimensional supersymmetry parameter. Since type II theories possess two ten-dimensional Majorana-Weyl supersymmetry parameters, compactification on such a manifold leads to four-dimensional $\mathcal{N} = 2$ supersymmetry. If the holonomy is contained in a proper subgroup of $SU(3)$, additional covariantly constant spinors may exist and the resulting supersymmetry is enhanced.

We shall use the following characterization of a Calabi-Yau manifold.

Definition 2.1. A Calabi-Yau N -fold is a compact Kähler manifold of complex dimension N with vanishing first Chern class,

$$c_1(X) = 0. \quad (2.52)$$

Yau's theorem states that every Kähler class on a compact Kähler manifold with vanishing first Chern class contains a unique Ricci-flat Kähler metric. The holonomy of this metric is contained in $SU(N)$. It is equal to $SU(N)$ when the manifold is irreducible and does not admit any further holonomy reduction.

To clarify these statements, let us briefly recall the relevant geometric structures. A real $2N$ -dimensional manifold is a complex manifold if it admits an integrable complex structure, namely a globally defined tensor I^m_n satisfying

$$I^m_p I^p_n = -\delta^m_n. \quad (2.53)$$

Integrability guarantees the existence of local complex coordinates

$$z^i = x^i + iy^i, \quad \bar{z}^{\bar{i}} = x^i - iy^i, \quad i = 1, \dots, N. \quad (2.54)$$

A metric compatible with the complex structure is called Hermitian. In complex coordinates, its only non-vanishing components are of mixed type,

$$ds^2 = 2g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}}. \quad (2.55)$$

The corresponding fundamental two-form, usually called the Kähler form, is defined by

$$J = i g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}. \quad (2.56)$$

A Hermitian manifold is Kähler if

$$dJ = 0. \quad (2.57)$$

The deformations of a Calabi-Yau metric can be classified in terms of the cohomology of the internal manifold. On a complex manifold, differential forms decompose

according to their numbers of holomorphic and antiholomorphic indices. A differential form of type (p, q) is locally of the form

$$\omega^{(p,q)} = \frac{1}{p!q!} \omega_{i_1 \dots i_p \bar{j}_1 \dots \bar{j}_q} dz^{i_1} \wedge \dots \wedge dz^{i_p} \wedge d\bar{z}^{\bar{j}_1} \wedge \dots \wedge d\bar{z}^{\bar{j}_q}. \quad (2.58)$$

Accordingly, the complexified de Rham cohomology decomposes as

$$H^k(X, \mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}(X). \quad (2.59)$$

For example, on a compact Kähler manifold, the Hodge decomposition gives

$$H^3(X, \mathbb{C}) = H^{3,0}(X) \oplus H^{2,1}(X) \oplus H^{1,2}(X) \oplus H^{0,3}(X). \quad (2.60)$$

The dimensions

$$h^{p,q}(X) = \dim_{\mathbb{C}} H^{p,q}(X) \quad (2.61)$$

are known as the Hodge numbers. The Euler characteristic can be expressed in terms of them as

$$\chi(X) = \sum_{p,q=0}^3 (-1)^{p+q} h^{p,q}(X). \quad (2.62)$$

For a Calabi–Yau threefold with holonomy exactly $SU(3)$, this reduces to

$$\chi(X) = 2 (h^{1,1}(X) - h^{2,1}(X)). \quad (2.63)$$

The Hodge diamond of such a Calabi–Yau threefold is

$$\begin{array}{ccccccc} & & & & 1 & & \\ & & & & & & \\ & & 0 & & & 0 & \\ & & & & & & \\ 0 & & h^{1,1} & & 0 & & \\ & & & & & & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & & & & & & \\ 0 & & h^{1,1} & & 0 & & \\ & & & & & & \\ & & 0 & & 0 & & \\ & & & & & & \\ & & & & 1 & & \end{array} . \quad (2.64)$$

Here we have used the Hodge symmetries

$$h^{p,q} = h^{q,p} = h^{3-p,3-q}. \quad (2.65)$$

Infinitesimal deformations of the Kähler structure are parametrized by $H^{1,1}(X)$, while infinitesimal deformations of the complex structure are parametrized by $H^{2,1}(X)$. Consequently, $h^{1,1}$ counts the independent Kähler deformations, whereas $h^{2,1}$ gives the complex dimension of the complex-structure moduli space.

Finally, a Calabi–Yau threefold admits a nowhere-vanishing holomorphic $(3, 0)$ -form,

$$\Omega \in H^{3,0}(X), \quad (2.66)$$

which is unique up to an overall complex normalization.

Infinitesimal deformations of the Ricci-flat metric are therefore parametrized by $h^{(1,1)}$ real Kähler moduli and $h^{(2,1)}$ complex-structure moduli, corresponding to a total of $h^{(1,1)} + 2h^{(2,1)}$ real parameters.

2.3.2 Calabi–Yau compactification of type II string theories

The four-dimensional massless spectrum arising from a Calabi–Yau compactification can be determined by means of a Kaluza–Klein decomposition. The basic idea is to expand every ten-dimensional field in eigenmodes of the appropriate differential operator on the compact manifold X_6 . From the four-dimensional perspective, non-zero eigenvalues correspond to massive Kaluza–Klein excitations, while zero modes give rise to massless fields.

Consider first a ten-dimensional scalar field $\Phi(x^\mu, y^m)$ on $M_4 \times X_6$. It can be expanded as

$$\Phi(x, y) = \sum_n \phi_n(x) Y_n(y), \quad (2.67)$$

where the internal wavefunctions satisfy

$$-\nabla_{X_6}^2 Y_n = \lambda_n Y_n, \quad \lambda_n \geq 0. \quad (2.68)$$

The corresponding four-dimensional fields obey

$$(\square_4 - \lambda_n) \phi_n = 0, \quad (2.69)$$

and therefore have masses

$$m_n^2 = \lambda_n. \quad (2.70)$$

Massless fields are associated with harmonic scalar functions,

$$\nabla_{X_6}^2 Y_0 = 0. \quad (2.71)$$

On a connected compact manifold, every harmonic scalar is constant. Hence, a single ten-dimensional scalar gives rise to one four-dimensional massless scalar.

The same argument applies to differential forms. A ten-dimensional p -form potential may be expanded schematically as

$$C_p(x, y) = \sum_{q=0}^p \sum_I C_{p-q}^I(x) \wedge \omega_q^I(y), \quad (2.72)$$

where ω_q^I are q -forms on X_6 . The internal operator governing their Kaluza–Klein masses is the Hodge–de Rham Laplacian,

$$\Delta_q = dd^\dagger + d^\dagger d. \quad (2.73)$$

A four-dimensional field is massless when

$$\Delta_q \omega_q^I = 0. \quad (2.74)$$

Such forms are harmonic and satisfy

$$d\omega_q^I = 0, \quad d^\dagger \omega_q^I = 0. \quad (2.75)$$

By the Hodge theorem, harmonic q -forms are in one-to-one correspondence with elements of the de Rham cohomology group $H^q(X_6, \mathbb{R})$. Their number is therefore the Betti number

$$b_q(X_6) = \dim H^q(X_6, \mathbb{R}). \quad (2.76)$$

Thus, a ten-dimensional p -form produces massless four-dimensional $(p - q)$ -forms with multiplicities determined by the topology of X_6 .

For a ten-dimensional spinor, one similarly writes

$$\Psi(x, y) = \sum_n \psi_n(x) \otimes \eta_n(y). \quad (2.77)$$

On an unwarped product geometry, the ten-dimensional Dirac operator decomposes schematically as

$$\not{D}_{10} = \not{D}_4 \otimes \mathbf{1} + \gamma_{(4)} \otimes \not{D}_6. \quad (2.78)$$

Consequently, massless four-dimensional fermions arise from internal spinors satisfying

$$\not{D}_6 \eta = 0. \quad (2.79)$$

On a compact Ricci-flat manifold, the Lichnerowicz formula implies that such zero modes are covariantly constant. For a Calabi–Yau threefold with holonomy exactly $SU(3)$, the covariantly constant internal spinor is responsible for the supersymmetry preserved by the compactification. In particular, each ten-dimensional gravitino gives rise to one four-dimensional gravitino, and type II compactifications therefore preserve $\mathcal{N} = 2$ supersymmetry.

The ten-dimensional metric decomposes into the components

$$G_{MN} \longrightarrow \{G_{\mu\nu}, G_{\mu m}, G_{mn}\}. \quad (2.80)$$

Their four-dimensional interpretation is as follows:

- $G_{\mu\nu}$ contains the unique four-dimensional graviton associated with the constant scalar zero mode on X_6 ;
- $G_{\mu m}$ would give rise to four-dimensional vector fields associated with continuous isometries of X_6 . A compact Calabi–Yau threefold with holonomy exactly $SU(3)$ has no continuous isometries, and therefore no massless vectors arise from these components;
- G_{mn} gives rise to scalar fields describing infinitesimal deformations of the Ricci-flat Calabi–Yau metric.

The metric deformations divide into Kähler and complex-structure deformations. The former are parametrized by

$$H^{1,1}(X_6) \quad (2.81)$$

and give $h^{1,1}$ real scalar fields, while the latter are parametrized by

$$H^{2,1}(X_6) \quad (2.82)$$

and give $h^{2,1}$ complex scalar fields. These massless scalars are the geometrical moduli of the compactification.

Therefore, for type IIB, its KK reduction is encoded in the following table:

IIB	Gravity	$h^{2,1}$ Vector	$h^{1,1}$ Hyper	Hyper
$G \rightarrow$	$G_{\mu\nu}$	$2h^{2,1}$ Cmplx.Str.	$h^{1,1}$ Kähler	
$B \rightarrow$			$B_{i\bar{j}}$	$B_{\mu\nu}$
$\phi \rightarrow$				ϕ
$a \rightarrow$				a
$C_2 \rightarrow$			$C_{i\bar{j}}$	$C_{\mu\nu}$
$C_4 \rightarrow$	$C_{ijk\mu}$	$C_{ij\bar{k}\mu}$	$C_{i\bar{j}\mu\nu}$	

The massless spectrum is then given by the $4d \mathcal{N} = 2$ supergravity multiplet, $h^{(2,1)}$ vector multiplets and $(h^{(1,1)} + 1)$ neutral hypermultiplets.

2.4 Type II orientifolds

Orientifold projections of Type II Calabi–Yau compactifications provide a standard mechanism for reducing four-dimensional supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$. They introduce orientifold planes and require the inclusion of D-branes for Ramond–Ramond charge cancellation, thereby also giving rise to gauge sectors and charged matter in the four-dimensional effective theory.

Starting from Type II string theory compactified on a Calabi–Yau threefold X_3 , one quotients the theory by a discrete orientifold action involving worldsheet parity and an involution of the internal manifold. Depending on the Type II theory and on the class of orientifold, the orientifold operator takes the form

$$\mathcal{O} = (-1)^{F_L} \Omega \sigma. \quad (2.83)$$

Here Ω reverses the orientation of the string worldsheet, F_L is the spacetime fermion number in the left-moving sector, and σ is an involutive isometry of the internal Calabi–Yau manifold,

$$\sigma^2 = \text{id}. \quad (2.84)$$

The precise form of $\mathcal{O}$, as well as the action of σ on the Calabi–Yau geometry, depends on the orientifold construction under consideration.

The geometric fixed loci of σ support orientifold planes. An Op -plane extends along the four non-compact spacetime dimensions and wraps an internal cycle of real dimension $p-3$. In the standard orientifold constructions considered here, these objects carry negative Ramond–Ramond charge and negative tension. Their total Ramond–Ramond charge must be cancelled by appropriate configurations of D-branes and, when relevant, background fluxes. This requirement is encoded in the corresponding Ramond–Ramond tadpole-cancellation conditions.

Spacetime-filling D-branes extend along the four non-compact dimensions and wrap internal cycles of X_3 . They need not wrap the same cycles as the orientifold planes. Rather, the total charge carried by the D-branes and their orientifold images must cancel the charges of the orientifold planes, together with any flux-induced contributions, in the appropriate homology or cohomology class. Open strings ending on the D-branes give rise to four-dimensional gauge fields and charged matter multiplets.

Preservation of four-dimensional $\mathcal{N} = 1$ supersymmetry imposes further conditions on the involution σ . In Type IIB orientifolds, σ acts holomorphically on the Calabi–Yau manifold, whereas in the standard Type IIA constructions it acts antiholomorphically. Its action assigns definite parities to the Kähler form J , the holomorphic three-form Ω , and the ten-dimensional form fields. The orientifold projection retains only the degrees of freedom invariant under the combined action $\mathcal{O}$, thereby truncating the original four-dimensional $\mathcal{N} = 2$ spectrum to an $\mathcal{N} = 1$ spectrum.

2.4.1 $\mathcal{N} = 1$ in Type IIB orientifolds with $O3/O7$ -planes

We begin by reviewing aspects of Type IIB string theory compactified on a Calabi–Yau threefold X_3 , quotiented by the orientifold action [13]. For $O3/O7$ -orientifold compactifications one requires

$$\sigma^* J = J, \quad \sigma^* \Omega = -\Omega, \quad (2.85)$$

where J is the Kähler form (2.56) and Ω is the holomorphic $(3, 0)$ -form (2.66). The action of σ^* splits the cohomology groups into even and odd eigenspaces,

$$H^{p,q}(X_3) = H_+^{p,q}(X_3) \oplus H_-^{p,q}(X_3), \quad (2.86)$$

with dimensions denoted by $h_+^{p,q}$ and $h_-^{p,q}$, respectively. The cohomology groups relevant for the four-dimensional massless spectrum are summarised in Table 1.

Cohomology group	Dimension	Basis
$H_+^{1,1}$	$h_+^{1,1}$	ω_α
$H_-^{1,1}$	$h_-^{1,1}$	ω_a
$H_+^{2,2}$	$h_+^{1,1}$	$\tilde{\omega}^\alpha$
$H_-^{2,2}$	$h_-^{1,1}$	$\tilde{\omega}^a$
$H_+^{2,1}$	$h_+^{2,1}$	χ_κ
$H_-^{2,1}$	$h_-^{2,1}$	χ_k
H_+^3	$2h_+^{2,1}$	$(\alpha_\kappa, \beta^\kappa)$
H_-^3	$2h_-^{2,1} + 2$	$(\alpha_{\hat{k}}, \beta^{\hat{k}})$

Table 1: Cohomology groups and basis elements relevant for $O3/O7$ -orientifold compactifications.

The Kähler form is even under the orientifold involution. Therefore only the even Kähler deformations survive the projection, and one expands

$$J = v^\alpha(x) \omega_\alpha, \quad \alpha = 1, \dots, h_+^{1,1}. \quad (2.87)$$

Here the real fields v^α parametrise the volumes of the even two-cycles. The complex structure deformations which are compatible with the orientifold projection are instead associated with $H_-^{2,1}(X_3)$. We denote the corresponding complex scalar fields by

$$z^k(x), \quad k = 1, \dots, h_-^{2,1}. \quad (2.88)$$

Equivalently, the metric fluctuations induced by complex structure deformations are expanded in a basis $\chi_k \in H_-^{2,1}(X_3)$.

The NS-NS and R-R two-forms are odd under the orientifold projection and hence are expanded in the odd basis of $H_-^{1,1}(X_3)$,

$$B_2 = b^a(x) \omega_a + b^\alpha(x) \omega_\alpha, \quad C_2 = c^a(x) \omega_a, \quad (2.89)$$

with $a = 1, \dots, h_-^{1,1}$, $b^\alpha \in \{0, 1/2\}$. The R-R four-form gives rise to four-dimensional two-forms, vectors and scalars, and can be expanded as ¹

$$C_4 = C_\alpha(x) \tilde{\omega}^\alpha + A^\kappa(x) \wedge \alpha_\kappa, \quad \kappa = 1, \dots, h_{2,1}^+, \quad \alpha = 1, \dots, h_{1,1}^+. \quad (2.90)$$

where b^a and c^a are four-dimensional scalars, while A^κ are $U(1)$ gauge bosons. It is useful to organise the surviving scalar fields into $\mathcal{N} = 1$ chiral multiplets. The axio-dilaton multiplet is

$$S \equiv iC_0 + e^{-\phi}, \quad (2.91)$$

where C_0 is the R-R axion and ϕ is the ten-dimensional dilaton. The odd two-form moduli combine into chiral fields of the schematic form

$$G^a = iC^a - Sb^a. \quad (2.92)$$

The Kähler moduli are obtained by combining the volumes of the orientifold-even four-cycles with the axions descending from the R-R four-form. Introducing

$$\tau_\alpha \equiv \frac{1}{2} \kappa_{\alpha\beta\gamma} v^\beta v^\gamma, \quad \rho_\alpha \equiv \int_{X_3} C_4 \wedge \omega_\alpha, \quad (2.93)$$

we define the corresponding chiral coordinates as

$$T_\alpha \equiv \tau_\alpha + i\rho_\alpha. \quad (2.94)$$

Here

$$\kappa_{\alpha\beta\gamma} \equiv \int_{X_3} \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma \quad (2.95)$$

are the triple-intersection numbers of the Calabi-Yau threefold.

The Kähler potential is generally not known as an explicit closed function of the chiral coordinates T_α , G^a , and S . This is because the Calabi-Yau volume is naturally expressed in terms of the two-cycle volumes v^α , whereas the relation between v^α and the chiral fields T_α is in general implicit. In the truncation in which the odd moduli G^a are absent or fixed, the tree-level Kähler potential takes the form

$$k_4^2 K_{IIB} = -\log \left(-i \int \Omega_3 \wedge \bar{\Omega}_3 \right) - \log(S + \bar{S}) - 2 \log(\mathcal{V}), \quad (2.96)$$

¹The self-duality of F_5 allows us to eliminate half of the field of C_4

Multiplet	Number	Fields
Gravity multiplet	1	$g_{\mu\nu}$
Complex-structure multiplets	$h_-^{2,1}$	z^k
Odd axionic multiplets	$h_-^{1,1}$	$G^a = (c^a, b^a)$
Vector multiplets	$h_+^{2,1}$	A^κ
Axio-dilaton multiplet	1	$S = (C_0, \phi)$
Kähler moduli multiplets	$h_+^{1,1}$	$T_\alpha = (\tau_\alpha, \rho_\alpha)$

Table 2: Massless closed-string spectrum of Type IIB Calabi–Yau orientifold compactifications with O3/O7-planes.

in the Einstein frame. The dimensionless Calabi–Yau volume in string units is

$$\mathcal{V} = \frac{1}{6} \int_{X_3} J \wedge J \wedge J. \quad (2.97)$$

The expressions above are valid at leading order in the α' expansion. The first perturbative correction relevant for Kähler-moduli stabilization arises at order α'^3 . It originates from the dimensional reduction of the ten-dimensional higher-derivative R^4 interaction and is inherited by the four-dimensional $\mathcal{N} = 1$ orientifold theory. The corrected Kähler potential is

$$\kappa_4^2 K_{\text{IIB}} = -\log \left(-i \int_{X_3} \Omega \wedge \bar{\Omega} \right) - \log(S + \bar{S}) - 2 \log \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right), \quad (2.98)$$

where

$$\hat{\xi} \equiv \xi \left(\frac{S + \bar{S}}{2} \right)^{3/2}, \quad (2.99)$$

and

$$\xi = -\frac{\zeta(3) \chi(X_3)}{16\pi^3}. \quad (2.100)$$

Here $\chi(X_3)$ denotes the Euler characteristic of the Calabi–Yau threefold and $\zeta(3)$ is the Riemann zeta function. This correction breaks the tree-level no-scale structure and plays a central role in the stabilization of the Kähler moduli in the Large Volume Scenario.

3 Large Volume Scenario

The four-dimensional massless spectrum arising from string compactifications generally contains a large number of scalar fields, known as moduli. In Type IIB Calabi–Yau orientifold compactifications with O3/O7-planes, whose field content is summarized in Table 2, these fields parametrise continuous deformations of the internal geometry and determine several effective four-dimensional couplings. If they remain unstabilized, the moduli correspond to flat directions of the scalar potential and may give rise to light scalar degrees of freedom. Such fields can mediate long-range interactions and induce phenomenological effects that are incompatible with current observations. A realistic string compactification must therefore provide a mechanism that lifts these flat directions, fixes the vacuum expectation values of the moduli, and generates sufficiently large masses for them.

In this chapter, we review the Large Volume Scenario (LVS), which provides a systematic framework for moduli stabilization in Type IIB flux compactifications. The stabilization mechanism proceeds hierarchically. At leading order, background three-form fluxes stabilize the axio-dilaton and the complex-structure moduli at a supersymmetric minimum, while the Kähler moduli remain unfixed as a consequence of the no-scale structure. Their stabilization is subsequently achieved by including perturbative α' corrections to the Kähler potential together with non-perturbative contributions to the superpotential. This two-step structure will play a central role in the analysis developed in the remainder of this thesis.

3.1 Flux Compactifications and Moduli Stabilization

A mechanism to stabilize the moduli consists in turning on background fluxes for the field strengths of the various p -form gauge fields of the ten-dimensional theory.

More precisely, let us consider Type IIB string theory compactified on a Calabi–Yau orientifold in the presence of non-trivial NSNS and RR three-form fluxes, H_3 and F_3 . In this case, the Type IIB supergravity action (2.36) takes the form

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[R - \frac{\partial_M S \partial^M \bar{S}}{2 (\text{Re } S)^2} - \frac{|G_3|^2}{2 \text{Re } S} - \frac{1}{4} |\tilde{F}_5|^2 \right] + \frac{1}{8i\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\text{Re } S} + S_{\text{local}}, \quad (3.1)$$

where

$$S = iC_0 + e^{-\phi}, \quad G_3 = F_3 - iSH_3, \quad (3.2)$$

with

$$F_3 = dC_2, \quad H_3 = dB_2, \quad (3.3)$$

and

$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3. \quad (3.4)$$

The three-form fluxes satisfy the Dirac quantization condition,

$$\frac{1}{(2\pi)^2 \alpha'} \int_{\gamma} F_3 = m_{\gamma} \in \mathbb{Z}, \quad \frac{1}{(2\pi)^2 \alpha'} \int_{\gamma} H_3 = n_{\gamma} \in \mathbb{Z}, \quad (3.5)$$

for every three-cycle γ of the internal Calabi–Yau manifold.

The effective action receives corrections in two independent perturbative expansions: the string loop expansion, controlled by the string coupling

$$g_s = \langle e^{\phi} \rangle, \quad (3.6)$$

and the α' expansion, which corresponds to an expansion in higher-derivative operators. The full ten-dimensional effective action can therefore be written schematically as

$$S_{10} = \sum_{m,n=0}^{\infty} (\alpha')^m g_s^n S_{10}^{(m,n)}. \quad (3.7)$$

The individual terms can be organized as [16]

$$S_{10}^{(m,n)} \equiv S_{10}^{(p,r,n)} \propto \int \sqrt{-g} \left(\frac{1}{\text{Im } S} \right)^{\frac{2n-p+r+1}{2}} (g^{**} R_{***})^p (g^{**} g^{**} g^{**} G_{***} G_{***})^r + \dots, \quad (3.8)$$

where the asterisks denote the appropriate index contractions and the integers satisfy

$$m = p + r - 1. \quad (3.9)$$

In particular, supersymmetry forbids corrections at lower orders in α' , so that the first non-vanishing higher-derivative contribution appears at order α'^3 , starting with the well-known R^4 term [17].

Another important ingredient is the superpotential W . Recall from Paragraph 1.3 that the superpotential is holomorphic. Holomorphy, together with the perturbative axionic shift symmetries and the non-renormalization properties of the four-dimensional $\mathcal{N} = 1$ theory, forbids a perturbative dependence of the superpotential on the Kähler moduli

Thus, we can generically write

$$W = W_{\text{tree}} + W_{\text{np}}, \quad K = K_{\text{tree}} + K_{\text{pert}} + K_{\text{np}} \quad (3.10)$$

Given K and W one can compute the F-term scalar potential 1.166, which is the key quantity to compute in order to find the vacua.

3.1.1 Superpotential

Background three-form fluxes generate the classical Gukov–Vafa–Witten superpotential [18]

$$W_{\text{tree}} \equiv W_{\text{flux}} = \sqrt{\frac{2}{\pi}} \int_{X_6} G_3 \wedge \Omega, \quad (3.11)$$

where Ω is the holomorphic $(3, 0)$ -form introduced in (2.66).

To express (3.11) in terms of the flux quanta and the complex structure moduli, let α_A, β^A , with $A = 0, \dots, h^{2,1}$, denote a symplectic basis of $H^3(X_6, \mathbb{Z})$. The periods of the holomorphic three-form can be assembled into the symplectic vector

$$\Pi = \begin{pmatrix} X^A \\ \mathcal{F}_A \end{pmatrix}, \quad X^A = \int_{A^A} \Omega, \quad \mathcal{F}_A = \int_{B_A} \Omega, \quad (3.12)$$

where the X^A define projective coordinates on the complex structure moduli space.

Similarly, the RR and NSNS three-form fluxes are characterized by the corresponding flux vectors

$$\mathbf{f} = \left(\int_{A^A} F_3 \quad \int_{B_A} F_3 \right), \quad \mathbf{h} = \left(\int_{A^A} H_3 \quad \int_{B_A} H_3 \right), \quad (3.13)$$

whose components are quantized as a consequence of Dirac quantization. Introducing the symplectic matrix

$$\Sigma = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}, \quad (3.14)$$

the flux superpotential takes the compact form

$$W_{\text{flux}} = \sqrt{\frac{2}{\pi}} \Pi^T \Sigma (\mathbf{f} - iS\mathbf{h}). \quad (3.15)$$

It follows that the tree-level superpotential depends on the complex structure moduli through the period vector Π and on the axio-dilaton through G_3 . By contrast, the Kähler moduli do not appear perturbatively in W_{flux} . This property is a consequence of their perturbative axionic shift symmetries and plays a central role in the moduli-stabilization mechanism discussed below.

Non-perturbative effects break these continuous shift symmetries to discrete subgroups and can therefore generate an explicit dependence on the Kähler moduli. One important source of such corrections is provided by Euclidean D3-branes wrapping divisors $D \subset X_6$ [13]. In the absence of world-volume fluxes, the semiclassical action of an ED3-brane wrapping the divisor D can be written, in the conventions adopted here, as

$$\frac{S_{\text{ED3}}}{2\pi} = \frac{1}{2} \int_D J \wedge J + i \int_D C_4 \equiv T_D = c^a T_a, \quad (3.16)$$

where J is the Kähler form, C_4 is the RR four-form potential and

$$[D] = c^a [D_a] \quad (3.17)$$

denotes the expansion of the divisor class in a basis of $H_4(X_6, \mathbb{Z})$. The corresponding instanton contribution is therefore exponentially suppressed as

$$e^{-S_{\text{ED3}}} = e^{-2\pi c^a T_a}. \quad (3.18)$$

An ED3-brane contributes to the superpotential only when its unlifted fermionic zero-mode spectrum contains precisely the two universal Goldstino modes required by four-dimensional $\mathcal{N} = 1$ supersymmetry. Additional unlifted zero modes cause the contribution to vanish or generate higher-derivative and multi-fermion F -terms rather than a superpotential correction.

Under these conditions, an ED3-brane wrapping D generates a non-perturbative contribution of the form

$$W_{\text{ED3}}^{(D)} = A_D(z^i, z_{\text{D3}}, z_{\text{D7}}, S) e^{-2\pi c^a T_a}. \quad (3.19)$$

The prefactor A_D arises from the one-loop determinant of fluctuations around the instanton configuration. It may depend on the complex structure moduli z^i , the axio-dilaton and the positions of spacetime-filling D3- and D7-branes. More precisely, A_D is generally a section of a non-trivial line bundle over moduli space and may vanish on special subloci, for instance when additional charged zero modes become massless.

More generally, after including all relevant non-perturbative contributions, the four-dimensional superpotential takes the schematic form

$$W = W_{\text{flux}}(z^i, S) + \sum_D A_D(z^i, z_{\text{D3}}, z_{\text{D7}}, S) e^{-2\pi c_D^a T_a}. \quad (3.20)$$

3.1.2 Kähler potential

At tree level in both the string coupling and the α' expansion, the moduli space of a type IIB Calabi–Yau orientifold with $O3/O7$ -planes factorizes locally as

$$\mathcal{M} = \mathcal{M}_{\text{cs}}(X_6) \times \mathcal{M}_{\text{K}}(X_6) \times \mathcal{M}_S, \quad (3.21)$$

where the three factors respectively describe the complex-structure moduli, the Kähler moduli and the axio-dilaton. Consequently, the corresponding moduli-space metric is block diagonal at this order, and the Kähler potential takes the form (2.96)

$$K_{\text{tree}} = -\log(S + \bar{S}) - \log \left[-i \int_{X_6} \Omega(z) \wedge \bar{\Omega}(\bar{z}) \right] - 2 \log \mathcal{V}(T, \bar{T}). \quad (3.22)$$

The real two-cycle volumes t^a are related to the four-cycle volumes through

$$\tau_a = \frac{\partial \mathcal{V}}{\partial t^a} = \frac{1}{2} \kappa_{abc} t^b t^c, \quad (3.23)$$

with $\tau_a = \text{Re } T_a$, up to convention-dependent normalizations.

A crucial property of the tree-level Kähler sector is the identity

$$K_{\text{tree}}^{a\bar{b}} K_{\text{tree},a} K_{\text{tree},\bar{b}} = 3, \quad (3.24)$$

known as the no-scale condition [19]. Here $\{a, b\}$ label the Kähler moduli T_a . Since the flux superpotential (3.11) is independent of these fields, their Kähler-covariant derivatives are

$$D_a W_{\text{flux}} = K_a W_{\text{flux}}. \quad (3.25)$$

The contribution of the Kähler moduli to the F-term scalar potential therefore satisfies

$$K_{\text{tree}}^{a\bar{b}} D_a^{\text{tree}} W_{\text{flux}} D_{\bar{b}}^{\text{tree}} \bar{W}_{\text{flux}} = 3 |W_{\text{tree}}|^2, \quad (3.26)$$

and exactly cancels the universal negative contribution $-3|W|^2$. The tree-level scalar potential consequently reduces to

$$V_{\text{tree}} = e^{K_{\text{tree}}} K_{\text{tree}}^{M\bar{N}} D_M W_{\text{flux}} D_{\bar{N}} \bar{W}_{\text{flux}}, \quad (3.27)$$

where $\{M, N\}$ run over the axio-dilaton and the complex-structure moduli only.

Equation (3.27) is manifestly positive semidefinite. Its minima satisfy

$$D_S W_{\text{flux}} = 0, \quad D_{z^i} W_{\text{flux}} = 0, \quad (3.28)$$

and therefore stabilize the axio-dilaton and the complex-structure moduli through the background fluxes. At such minima,

$$V_{\text{tree}} = 0. \quad (3.29)$$

Nevertheless, when $W_{\text{flux}} \neq 0$, the Kähler-modulus F-terms are generically non-vanishing,

$$D_{T_a} W_{\text{flux}} = K_{T_a} W_{\text{flux}} \neq 0, \quad (3.30)$$

so supersymmetry is broken while the vacuum energy remains zero. These solutions are therefore referred to as no-scale Minkowski vacua (see 1.6.1).

The Kähler moduli remain exactly flat directions at this order, since they do not appear in the flux superpotential and their dependence in the scalar potential cancels as a consequence of the no-scale identity. Their stabilization therefore requires effects that break the tree-level no-scale structure.

Thus, the leading contribution to the Kähler-moduli potential arises from corrections that break the tree-level no-scale structure. The dominant perturbative effect is the α'^3 correction induced by the ten-dimensional R^4 interaction discussed in Eq. (2.98). Including this contribution, the Kähler potential becomes

$$K = -\log[(S + \bar{S})] - \log\left[-i \int_{X_6} \Omega \wedge \bar{\Omega}\right] - 2 \log\left(\mathcal{V} + \frac{\xi}{2} \left(\frac{S + \bar{S}}{2}\right)^{3/2}\right). \quad (3.31)$$

Where ξ is defined in (2.100). Equivalently, in the large-volume regime,

$$K = K_{\text{tree}} + \delta K, \quad \delta K = -\frac{\xi}{g_s^{3/2} \mathcal{V}} + \mathcal{O}(\mathcal{V}^{-2}). \quad (3.32)$$

This correction breaks the no-scale identity and generates a non-vanishing scalar potential for the Kähler moduli.

3.2 Large Volume Scenario

We have now introduced the main ingredients of type IIB flux compactifications on Calabi-Yau orientifolds with O3/O7-planes. We can therefore discuss how the moduli may be stabilized in isolated four-dimensional vacua.

At leading order in the α' and string-loop expansions, the effective theory is determined by the flux superpotential W_{flux} , given in Eq. (3.11), and by the tree-level Kähler potential K_{tree} , given in Eq. (3.22). For suitable choices of background fluxes, there exist points (z_0^i, S_0) in complex-structure and axio-dilaton moduli space satisfying

$$D_{z^i} W_{\text{flux}}|_{(z_0, S_0)} = 0, \quad D_S W_{\text{flux}}|_{(z_0, S_0)} = 0. \quad (3.33)$$

The vacuum expectation value of the flux superpotential is denoted by

$$W_0 \equiv \langle W_{\text{flux}} \rangle = W_{\text{flux}}(z_0^i, S_0). \quad (3.34)$$

Because of the tree-level no-scale structure, the Kähler moduli T_a remain unstabilized at this order. A non-trivial potential for these fields is generated only after including corrections to the tree-level effective data,

$$W = W_{\text{flux}} + W_{\text{np}}, \quad K = K_{\text{tree}} + \delta K_{\text{pert}}, \quad (3.35)$$

where W_{np} denotes non-perturbative contributions to the superpotential, while δK_{pert} contains perturbative α' and string-loop corrections to the Kähler potential. Non-perturbative corrections to K may also be present, but they are subleading in the regime considered here and will not be included.

The scalar potential can then be organized schematically as

$$V = V_{\text{np}^2} + V_{\text{np-flux}} + V_{\alpha'} + \dots, \quad (3.36)$$

where the three displayed terms arise respectively from contributions quadratic in W_{np} , from interference between W_{np} and W_0 , and from the breaking of the no-scale structure by the leading α'^3 correction. The tree-level term proportional to $|W_0|^2$ vanishes identically because of the no-scale cancellation.

The Large Volume Scenario arises from a competition between the exponentially suppressed non-perturbative effects and the inverse-volume-suppressed α' correction. In a generic compactification, the overall volume controls the α' expansion, while individual four-cycle volumes control the non-perturbative expansion. For example, an Euclidean D3-brane wrapping a rigid divisor D_a generates a contribution of the form

$$W_{\text{np}} \supset A_a e^{-2\pi T_a}, \quad (3.37)$$

or, more generally,

$$W_{\text{np}} = \sum_a A_a e^{-a_a T_a}, \quad (3.38)$$

where $a_a = 2\pi$ for a single Euclidean D3-brane instanton.

The essential observation is that a balance can occur between terms scaling as inverse powers of the overall volume and terms proportional to $e^{-a_s \tau_s}$. At the minimum one finds parametrically

$$e^{-a_s \tau_s} \sim \frac{1}{\mathcal{V}}, \quad (3.39)$$

and hence

$$\mathcal{V} \sim e^{a_s \tau_s}. \quad (3.40)$$

For $a_s \tau_s \gg 1$, the stabilized volume is therefore exponentially large. This is the origin of the name Large Volume Scenario [12].

To make this mechanism explicit, consider a Swiss-cheese compactification whose volume takes the form

$$\mathcal{V} = \tau_b^{3/2} - c \tau_s^{3/2}, \quad c > 0, \quad (3.41)$$

where τ_b controls the overall size of the compactification and τ_s is the volume of a rigid blow-up divisor. In the regime

$$\tau_b \gg \tau_s \gg 1, \quad (3.42)$$

the volume is large while the non-perturbative expansion associated with τ_s remains under control.

We take the superpotential and Kähler potential to be

$$W = W_0 + \mathcal{A}_s e^{-a_s T_s}, \quad (3.43)$$

and

$$K = K_{\text{cs}} - \log [S + \bar{S}] - 2 \log \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right). \quad (3.44)$$

After integrating out the complex-structure moduli and the axio-dilaton at their flux-stabilized values, the Kähler-modulus scalar potential can be written as

$$V = e^K \left[K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right]. \quad (3.45)$$

Expanding this expression at large volume and retaining the leading contributions gives

$$V_{\text{LVS}} \simeq e^{K_{\text{cs}}} V_{0,F} \left[\frac{\alpha \sqrt{\tau_s} e^{-2a_s \tau_s}}{c \mathcal{V}} - \frac{\beta |W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{3\xi |W_0|^2}{4g_s^{3/2} \mathcal{V}^3} \right], \quad (3.46)$$

where the axionic component of T_s has been minimized so that the interference term is negative, and

$$V_{0,F} = \frac{g_s}{2}, \quad \alpha = \frac{8a_s^2 |\mathcal{A}_s|^2}{3}, \quad \beta = 4a_s |\mathcal{A}_s|. \quad (3.47)$$

The first term in Eq. (3.46) is quadratic in the non-perturbative contribution, the second arises from its interference with W_0 , and the third originates from the leading α'^3 correction to the Kähler potential.

In the limit $a_s \tau_s \gg 1$, the stationary conditions

$$\partial_{\mathcal{V}} V_{\text{LVS}} = 0, \quad \partial_{\tau_s} V_{\text{LVS}} = 0, \quad (3.48)$$

admit the approximate solution [12]

$$\tau_s \simeq \frac{1}{g_s} \left(\frac{\xi}{2c} \right)^{2/3}, \quad (3.49)$$

and

$$\mathcal{V} \simeq \frac{\beta |W_0| c}{2\alpha g_s^{1/2}} \left(\frac{\xi}{2c} \right)^{1/3} \exp \left[\frac{a_s}{g_s} \left(\frac{\xi}{2c} \right)^{2/3} \right]. \quad (3.50)$$

Thus, the blow-up modulus is stabilized at moderately large size, whereas the overall volume is exponentially large. The resulting vacuum is generically a non-supersymmetric anti-de Sitter minimum, with supersymmetry broken predominantly in the Kähler-modulus sector.

3.3 Sequestered LVS scenarios

We now turn to the phenomenological implications of moduli stabilisation in sequestered Large Volume Scenario compactifications. In these constructions, the visible sector is localised at a singularity and is geometrically separated from the dominant sources of supersymmetry breaking in the bulk. As a consequence, the soft supersymmetry-breaking terms can be parametrically suppressed with respect to the gravitino mass [3, 20–22].

A minimal realisation in type IIB Calabi–Yau orientifold compactifications with $O3/O7$ -planes contains four Kähler-sector moduli: a large four-cycle T_b controlling the overall volume, a small blow-up mode T_s supporting the non-perturbative effects responsible for LVS stabilisation, the visible-sector modulus T_{SM} , and an orientifold-odd modulus G . The latter two are associated with a pair of divisors exchanged by the orientifold involution and collapsing to zero size at the singular locus. The resulting topology is characterised by

$$h_+^{1,1} = 3, \quad h_-^{1,1} = 1, \quad (3.51)$$

with

$$T_b = \tau_b + i\rho_b, \quad T_s = \tau_s + i\rho_s, \quad T_{\text{SM}} = \tau_{\text{SM}} + i\rho_{\text{SM}}, \quad G = b + ic. \quad (3.52)$$

Here τ_b , τ_s and τ_{SM} denote four-cycle volumes, while ρ_b , ρ_s and ρ_{SM} arise from the reduction of C_4 . The fields b and c arise instead from the reduction of B_2 and C_2 on the orientifold-odd two-cycle dual to the shrinking divisor.

The four-dimensional $\mathcal{N} = 1$ superpotential can be written schematically as

$$W = W_{\text{flux}}(S, z) + W_{\text{np}}(T_s, S, z) + W_{\text{dS}} + W_{\text{matter}}, \quad (3.53)$$

where the first two contributions have been analysed in the previous chapters. The term W_{dS} contains the contribution associated with the mechanism used to uplift the LVS AdS minimum, while the visible-sector superpotential is

$$W_{\text{matter}} = \mu(M)H_u H_d + \frac{1}{6}Y_{\alpha\beta\gamma}(M)C^\alpha C^\beta C^\gamma + \dots, \quad (3.54)$$

where M collectively denotes the closed-string moduli and C^α the visible-sector matter superfields.

Close to the singular locus, the Kähler potential can be parametrised as [2, 3, 23]

$$K = -\log(S + \bar{S}) - \log\left(-i \int_{X_6} \Omega \wedge \bar{\Omega}\right) - 2 \log\left[\mathcal{V} + \frac{\xi}{2} \left(\frac{S + \bar{S}}{2}\right)^{3/2}\right] + \lambda_{\text{SM}} \frac{\tau_{\text{SM}}^2}{\mathcal{V}} + \lambda_b \frac{b^2}{\mathcal{V}} + K_{\text{dS}} + K_{\text{matter}}. \quad (3.55)$$

We shall use

$$s \equiv \text{Re } S = \frac{1}{g_s}, \quad \hat{\xi} \equiv \xi s^{3/2}. \quad (3.56)$$

For a diagonal matter metric, the leading visible-sector contribution to the Kähler potential can be written as

$$K_{\text{matter}} = \tilde{K}_\alpha(M, \bar{M}) \bar{C}^{\bar{\alpha}} C^\alpha + [Z(M, \bar{M}) H_u H_d + \text{h.c.}], \quad (3.57)$$

where repeated matter indices are understood to be summed over. We parametrise the matter metric as

$$\tilde{K}_\alpha = \frac{f_\alpha(S, z)}{\mathcal{V}^{2/3}} \left[1 - c_s \frac{\hat{\xi}}{\mathcal{V}} + \tilde{K}_{\text{dS}} + c_{\text{SM}} \tau_{\text{SM}}^p + c_b b^p + \dots \right], \quad p > 0. \quad (3.58)$$

For D-branes at singularities, the gauge kinetic function takes the form

$$f_a = \delta_a S + \kappa_a T_{\text{SM}}, \quad (3.59)$$

where the coefficients δ_a are universal for $\mathbb{Z}_n$ singularities. D-term stabilisation fixes the visible-sector cycle at the singular locus, $\tau_{\text{SM}} \rightarrow 0$, and implies at leading order

$$F^{T_{\text{SM}}} = F^G = 0. \quad (3.60)$$

Supersymmetry breaking is therefore mediated to the visible sector through the auxiliary fields of the bulk moduli.

In four-dimensional $\mathcal{N} = 1$ supergravity, and setting $M_P = 1$, the moduli F -terms are

$$F^I = e^{K/2} K^{I\bar{J}} D_{\bar{J}} \bar{W}, \quad (3.61)$$

where

$$D_{\bar{J}} \bar{W} = \partial_{\bar{J}} \bar{W} + K_{\bar{J}} \bar{W}. \quad (3.62)$$

The dominant supersymmetry-breaking auxiliary field is associated with the large Kähler modulus T_b , whereas the dilaton and complex-structure F -terms are parametrically suppressed in the large-volume expansion. Nevertheless, as we shall see, these suppressed F -terms can become phenomenologically relevant whenever the leading contribution from F^{T_b} cancels.

3.3.1 Local and ultra-local matter metrics

The distinction between local and ultra-local models follows from the volume dependence of the physical Yukawa couplings. The canonically normalised Yukawa couplings are

$$\hat{Y}_{\alpha\beta\gamma} = e^{K/2} \frac{Y_{\alpha\beta\gamma}(S, z)}{\sqrt{\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma}}. \quad (3.63)$$

Since the Yukawa interactions are local quantities determined by the open-string sector at the singularity, their leading behaviour is expected to be insensitive to the overall Calabi–Yau volume. For a diagonal matter metric, this motivates the relation

$$\tilde{K}_\alpha = h_\alpha(S, z) e^{K/3}. \quad (3.64)$$

Expanding this relation at large volume gives

$$\tilde{K}_\alpha \simeq h_\alpha(S, z) \frac{e^{K_{\text{cs}}/3}}{(2s)^{1/3} \mathcal{V}^{2/3}} \left[1 - \frac{\hat{\xi}}{3\mathcal{V}} + \frac{1}{3} K_{\text{dS}} + \dots \right]. \quad (3.65)$$

This reproduces the universal leading scaling $\tilde{K}_\alpha \sim \mathcal{V}^{-2/3}$ in Eq. (3.58).

Following [3], we distinguish two classes of models:

- **Local models:** Eq. (3.64) holds only at leading order in the inverse-volume expansion.
- **Ultra-local models:** Eq. (3.64) holds exactly, order by order in the inverse-volume expansion. In this case the cancellation is exact, and the leading soft terms can become directly sensitive to the dilaton and complex-structure F -terms.

3.3.2 Soft supersymmetry-breaking terms

We now review the soft terms which are directly relevant for the present analysis.

Gaugino masses. The gravity-mediated gaugino masses are

$$M_a = \frac{1}{2 \text{Re } f_a} F^I \partial_I f_a. \quad (3.66)$$

Using Eq. (3.59), together with $\tau_{\text{SM}} = 0$ and $F^{T_{\text{SM}}} = 0$, one obtains

$$M_a \simeq \frac{F^S}{2s} \equiv M_{1/2}, \quad (3.67)$$

up to subleading corrections. Hence the gaugino masses provide a particularly direct probe of the dilaton F -term. Any correction to F^S induced by the displacement of the flux-stabilised minimum therefore translates directly into a correction to $M_{1/2}$.

Scalar masses. For a diagonal matter metric, the F -term contribution to the canonically normalised scalar masses is

$$m_\alpha^2 = m_{3/2}^2 + V_0 - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \log \tilde{K}_\alpha, \quad (3.68)$$

where

$$m_{3/2} = e^{K/2} |W| \quad (3.69)$$

and V_0 denotes the vacuum energy after moduli stabilisation and uplifting.

In the local limit, the leading no-scale cancellation is not exact at subleading order. The residual contribution is dominated by the large-volume modulus T_b and gives universal scalar masses,

$$m_\alpha^2 = m_0^2, \quad (3.70)$$

with the characteristic scaling

$$m_0^2 \sim m_{3/2} M_{1/2} \sim \frac{m_{3/2}^2}{\mathcal{V}}. \quad (3.71)$$

Thus the contribution involving F^{T_b} dominates over the terms generated by the dilaton and complex-structure F -terms.

The situation is qualitatively different in the ultra-local limit. Using Eq. (3.64), one has

$$\partial_I \partial_{\bar{J}} \log \tilde{K}_\alpha = \frac{1}{3} K_{I\bar{J}} + \partial_I \partial_{\bar{J}} \log h_\alpha(S, z). \quad (3.72)$$

Together with

$$V_F = F^I \bar{F}^{\bar{J}} K_{I\bar{J}} - 3m_{3/2}^2, \quad (3.73)$$

this yields

$$m_{\alpha,F}^2 = V_0 - \frac{1}{3} V_{F,0} - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \log h_\alpha(S, z). \quad (3.74)$$

Since h_α depends only on the axio-dilaton and complex-structure moduli, the indices in the last term of Eq. (3.74) can be restricted to

$$I, J \in \{S, z^a\}. \quad (3.75)$$

For a vacuum whose cosmological constant is tuned to be negligible, $V_0 \simeq 0$, Eq. (3.74) reduces to

$$m_{\alpha,F}^2 \simeq -\frac{1}{3} V_{F,0} - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \log h_\alpha(S, z). \quad (3.76)$$

This expression makes clear why the suppressed F -terms of S and z^a become important in the ultra-local limit. The contribution from the bulk Kähler modulus cancels, while the dependence of the matter metric on S and z^a survives. In particular, if

$$F^S, F^{z^a} \sim \mathcal{O}(\mathcal{V}^{-2}), \quad (3.77)$$

their contribution to the scalar masses squared is generically of order

$$F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \log h_\alpha \sim \mathcal{O}(\mathcal{V}^{-4}), \quad (3.78)$$

up to the corresponding powers of g_s and flux-dependent coefficients.

Trilinear terms. The trilinear soft terms are

$$A_{\alpha\beta\gamma} = F^I \left[K_I + \partial_I \log Y_{\alpha\beta\gamma} - \partial_I \log \left(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma \right) \right]. \quad (3.79)$$

In the local case, the imperfect cancellation between the bulk Kähler potential and the matter metrics leaves a contribution generated by F^{T_b} , in addition to the terms involving F^S and F^{z^a} . Schematically,

$$A_{\alpha\beta\gamma}^{\text{local}} = A_{\alpha\beta\gamma}^{(T_b)} + A_{\alpha\beta\gamma}^{(S,z)}. \quad (3.80)$$

Both contributions are parametrically suppressed with respect to $m_{3/2}$ and are typically of order $M_{1/2}$.

In the ultra-local limit the dependence on the Kähler moduli cancels exactly. Using Eq. (3.64), one obtains

$$A_{\alpha\beta\gamma}^{\text{ultra-local}} = \sum_{I=S,z^a} F^I \partial_I \log \left[\frac{Y_{\alpha\beta\gamma}(S, z)}{h_\alpha(S, z) h_\beta(S, z) h_\gamma(S, z)} \right]. \quad (3.81)$$

The ultra-local trilinear terms are therefore controlled directly by the dilaton and complex-structure F -terms and are generically flavour dependent through the functions $Y_{\alpha\beta\gamma}$ and h_α .

The sequestered setup therefore provides a direct phenomenological application of the subleading corrections to the auxiliary fields studied in this thesis. In particular, the shift of the dilaton F -term affects the gaugino masses already linearly through Eq. (3.67), while in ultra-local models both the scalar masses and the trilinear terms depend explicitly on the corrected F^S and F^{z^a} . Hence, although these auxiliary fields are parametrically suppressed with respect to F^{T_b} , they can control the leading visible-sector soft terms whenever the contribution of the bulk Kähler modulus is removed by sequestering.

These expressions show that the soft parameters are controlled not only by the moduli F -terms, but also by the dependence of the gauge kinetic functions, Yukawa couplings, and matter Kähler metrics on the closed-string moduli.

Soft terms have been computed in several D-brane realizations of the Standard Model [6, 24, 25]. Their structure depends crucially on the localization of the visible sector, on the mechanism used to uplift the vacuum to de Sitter space, and on perturbative and non-perturbative corrections to the matter Kähler metric.

4 Corrections to the F-terms

In Large Volume Scenarios, the F -terms F^S and F^{z_a} receive non-vanishing contributions from several subleading effects, some of which have already been extensively studied in the literature [1–5]. For instance, the inverse of the corrected Kähler metric, $K_0 + \delta K$, induces mixing between the (S, z_a) sector and the T_i directions. Since $D_{T_i} W$ is non-zero at the LVS AdS minimum, this mixing generates non-vanishing F -terms for both S and z_a . In addition, the correction δK directly modifies the covariant derivatives D_{T_i} , D_S , and D_{z_a} . The dependence of W_{np} on z_a and S via its prefactor: $W_{np} = A(z_a, S)e^{-a_i T^i}$ also leads the F -terms to acquire non-zero contributions.

In this work, a different source of correction is addressed: the displacement of the expectation values of S and z_a away from their leading-order vacuum (3.33). More precisely, how the shift δS and δz feeds into the F -terms is here studied.

Usually the dilaton is stabilized at leading order by background fluxes (3.1), however, perturbative and non-perturbative effects can shift it away from its original minimum. In this final chapter, we implement the mechanism described above explicitly. After reviewing the corrections to the moduli F -terms previously discussed in the literature, we identify and estimate the additional contributions induced by the displacement of the flux-stabilized fields from their tree-level supersymmetric values.

We first analyse the effect perturbatively in a simplified Large Volume Scenario model, keeping track of both the inverse-volume and weak-coupling expansions. The toy model is chosen to have a rigid complex-structure sector, $h^{2,1} = 0$, together with the minimal number of Kähler moduli required to realise an LVS vacuum, namely $h^{1,1} = 2$. This setting allows us to isolate the shift of the axio-dilaton and to determine its contribution to the corresponding auxiliary field F^S .

The most significant result emerges when the analysis is extended to a model containing one complex-structure modulus z , corresponding to $h^{2,1} = 1$. In this setting, the subleading effects responsible for LVS stabilisation displace both the axio-dilaton and the complex-structure modulus from their supersymmetric flux vacuum. We find that the resulting shift-induced contribution to F^z is parametrically enhanced in the weak-coupling regime. More precisely, within the perturbative expansion and assumptions adopted here, this contribution carries one additional inverse power of g_s relative to the standard estimate reported in Ref. [3].

For the axio-dilaton F -term, by contrast, the contribution generated by the displacement of S is generically of the same parametric order as the corrections previously identified in Ref. [2]. It therefore competes with the standard LVS contribution at the same order in both the inverse-volume and weak-coupling expansions. Within the perturbative regime considered here, the displacement does not introduce a new parametric enhancement in F^S , but rather modifies the numerical coefficient of a contribution already present at the same order.

Finally, we discuss the phenomenological implications of these corrected auxiliary fields. In sequestered LVS scenarios, soft supersymmetry-breaking terms are directly controlled by the F -terms of the moduli sector [3]. Since the displacement of the axio-dilaton does not modify the parametric scaling of F^S , soft terms whose leading contribution is determined by F^S retain the standard LVS behaviour, up to numerical coefficients. By contrast, the enhanced complex-structure F -term can lead to parametrically different contributions to observables which depend directly on F^z , including scalar masses in the ultra-local regime.

A further application concerns the supersymmetric partners of closed-string RR photons. In type IIB orientifold compactifications, the gauge kinetic functions of these hidden $U(1)$ sectors depend on the complex-structure moduli [4, 13]. Consequently, the corresponding hidden photino masses are controlled by

$$M_X \sim \frac{F^z}{M_P} \partial_z f_X, \quad (4.1)$$

so that the shift-induced modification of F^z can be directly inherited by the hidden-photino spectrum, up to the model-dependent factor $\partial_z f_X$

4.1 General considerations

Let us remember the definition of the F -term (3.61),

$$F^I = e^{K/2} K^{I\bar{J}} D_{\bar{J}} \bar{W} \quad (4.2)$$

At tree level, the auxiliary fields associated with the axio-dilaton and the complex-structure modulus vanish as a direct consequence of the supersymmetry conditions in Eq. (3.33). Explicitly,

$$\langle F_{\text{tree}}^S \rangle = e^{\hat{K}/2} \hat{K}^{S\bar{S}} \hat{D}_{\bar{S}} \bar{W}_{\text{flux}} \Big|_{S_0, z_0} = 0, \quad (4.3)$$

and

$$\langle F_{\text{tree}}^z \rangle = e^{\hat{K}/2} \hat{K}^{z\bar{z}} \hat{D}_{\bar{z}} \bar{W}_{\text{flux}} \Big|_{S_0, z_0} = 0. \quad (4.4)$$

Here, for notational convenience, we have defined

$$\hat{D}_I \equiv \partial_I + \hat{K}_I \quad (4.5)$$

and $\hat{K} \equiv K_{\text{tree}}$. Moreover, owing to the factorised structure of the tree-level Kähler potential in Eq. (3.22), the corresponding Kähler metric is block diagonal in the axio-dilaton, complex-structure and Kähler-moduli sectors. Consequently, the mixed components of its inverse vanish at this order.

Equations (4.3) and (4.4) express the fact that supersymmetry is preserved along the axio-dilaton and complex-structure directions at the tree-level flux vacuum. This does not imply that the complete vacuum is supersymmetric once subleading effects are included. In the Large Volume Scenario, perturbative and non-perturbative corrections stabilise the Kähler moduli in a non-supersymmetric AdS vacuum and induce non-vanishing contributions to F^S and F^z .

More precisely, the axio-dilaton and complex-structure F -terms can receive contributions through five conceptually distinct mechanisms:

- The inverse corrected Kähler metric develops mixed components between the (S, z_a) sector and the Kähler-moduli directions. Since the Kähler-moduli covariant derivatives $D_{T_i} W$ do not vanish at the LVS minimum, these mixed components induce non-zero contributions to F^S and F^{z_a} .
- The correction δK modifies the Kähler-covariant derivatives

$$D_I W = \partial_I W + K_I W, \quad (4.6)$$

and therefore generates additional contributions even when the corresponding tree-level covariant derivatives vanish.

- The non-perturbative superpotential may depend explicitly on the axio-dilaton and complex-structure moduli through its prefactor,

$$W_{\text{np}} = \sum_i \mathcal{A}(S, z) e^{-a_i T_i}. \quad (4.7)$$

Consequently, $\partial_S W_{\text{np}}$ and $\partial_z W_{\text{np}}$ can contribute directly to the corresponding F -terms.

- The inclusion of the LVS scalar potential shifts the expectation values of S and z away from their tree-level supersymmetric values,

$$S = S_0 + \delta S, \quad z = z_0 + \delta z. \quad (4.8)$$

Evaluating the tree-level quantities at the shifted vacuum then produces an additional, indirect contribution to F^S and F^z .

- the uplift to dS generically induces non-zero F -terms

The contributions arising from the first three mechanisms have been investigated in [2, 3]. The contribution induced by the displacement of the axio-dilaton and complex-structure vacuum expectation values constitutes the main focus of the present analysis. By contrast, the additional corrections associated with the uplift sector are model dependent and will not be considered here; their systematic analysis is left for future work. In the following, we first review the previously known contributions and determine their parametric dependence on g_s and the Calabi–Yau volume $\mathcal{V}$. This provides the appropriate benchmark against which the vacuum-shift contribution derived in this work can be compared.

4.1.1 F -term corrections: contributions at the unshifted flux minimum

We first review the corrections to the axio-dilaton and complex-structure F -terms that are already present when these fields are evaluated at their tree-level flux minimum 3.33,

$$S = S_0, \quad z = z_0, \quad (4.9)$$

without yet including the displacement of the vacuum induced by the LVS scalar potential. Once the perturbative correction (3.32) to the Kähler potential and the non-perturbative contribution (3.20) to the superpotential are included, these relations no longer imply the vanishing of the corresponding covariant derivatives.

At the LVS minimum, the leading corrected covariant derivatives can be parametrized as [3]

$$D_S W|_{S_0, z_0} = -\frac{3}{4} \frac{\xi W_0}{\mathcal{V} \sqrt{g_s}} + \mathcal{O}\left(\frac{g_s W_0}{\mathcal{V}}\right), \quad (4.10)$$

and

$$D_z W|_{S_0, z_0} = -\frac{3}{4} \frac{cW_0}{\mathcal{V}\sqrt{g_s}} \chi_z(z_0) + \mathcal{O}(\mathcal{V}^{-2}). \quad (4.11)$$

Here $\chi_z(z_0)$ denotes a model-dependent dimensionless function, which is generically expected to be of order one. Its precise form depends on the complex-structure dependence of the non-perturbative prefactor

$$\mathcal{A} = \mathcal{A}(S, z) \quad (4.12)$$

and on that of the complex-structure normalization $\mathcal{I}$.

The two expressions above have conceptually different origins. The leading contribution to $D_S W$ is generated by the perturbative correction to the Kähler potential. Indeed, even though the tree-level relation

$$\partial_S W_{\text{flux}} + \hat{K}_S W_{\text{flux}} = 0 \quad (4.13)$$

holds at (S_0, z_0) , the corrected derivative contains the additional term as the first non-trivial contribution in the g_s expansion,

$$D_S W \simeq K_{\text{per}, S} W_0. \quad (4.14)$$

By contrast, the leading contribution to $D_z W$ is associated with the non-perturbative sector. In particular, the dependence of the prefactor $\mathcal{A}(S, z)$ on the complex-structure modulus gives

$$\partial_z W_{\text{np}} = \partial_z \mathcal{A}(S, z) e^{-a_s T_s}, \quad (4.15)$$

while the term $K_z W_{\text{np}}$ provides an additional contribution of the same non-perturbative origin.

The corrected F -terms are then evaluated at the unshifted flux minimum gives, at leading order,

$$\delta F_{(1)}^S \equiv F^S|_{S_0, z_0} = e^{K/2} \left(K^{S\bar{I}} D_{\bar{I}} \bar{W} \right) = -\frac{3}{\sqrt{2I}} \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s^2} + \mathcal{O} \left(\frac{1}{\mathcal{V}^2 g_s^{3/2}} \right). \quad (4.16)$$

This correction is perturbative in origin and is enhanced in the weak-coupling limit by the factor g_s^{-2} .

Similarly, for the complex-structure modulus one obtains

$$\delta F_{(1)}^z \equiv F^z|_{S_0, z_0} = e^{K/2} \left(K^{z\bar{I}} D_{\bar{I}} \bar{W} \right) = -\frac{3K_0^{z\bar{z}}}{4\sqrt{2I}} \frac{c\bar{W}_0 \bar{\chi}_z(\bar{z}_0)}{\mathcal{V}^2} + \mathcal{O}(\mathcal{V}^{-3}), \quad (4.17)$$

where

$$K_0^{z\bar{z}} \equiv (\partial_z \partial_{\bar{z}} K_{\text{tree}})_{S_0, z_0}^{-1}. \quad (4.18)$$

For a single complex-structure modulus, this notation is unambiguous. In the presence of several moduli, $K_0^{z\bar{z}}$ must instead be interpreted as the appropriate component of the inverse tree-level complex-structure metric.

Equations (4.16) and (4.17) show that both F -terms are suppressed by two powers of the overall volume,

$$\delta F_{(1)}^S \sim \frac{\bar{W}_0}{\mathcal{V}^2 g_s^2}, \quad \delta F_{(1)}^z \sim \frac{\bar{W}_0}{\mathcal{V}^2}. \quad (4.19)$$

Their dependence on the string coupling is, however, substantially different. The axio-dilaton contribution carries a strong inverse power of g_s , reflecting the perturbative α' correction and the axio-dilaton dependence of the Kähler metric. The leading complex-structure contribution is instead of non-perturbative origin and does not exhibit the same weak-coupling enhancement.

This distinction will be important in the following analysis. Since the previously known contribution to F^z is comparatively suppressed in the weak-coupling expansion, additional effects, such as those induced by the displacement of the axio-dilaton and complex-structure fields away from their tree-level supersymmetric values, may be relatively more significant in the complex-structure direction.

4.2 Vacuum Displacement and Induced F -term Corrections

The perturbative α' correction to the Kähler potential and the non-perturbative contribution to the superpotential modify not only the values of the F -terms evaluated at the tree-level flux vacuum, but also the position of the vacuum itself. In particular, once these effects are included, the axio-dilaton and the complex-structure moduli are generically displaced from the supersymmetric values defined by (3.33). In the standard leading-order LVS treatment, the fields S and z are first stabilized by the flux potential and are subsequently integrated out at their tree-level values. The Kähler moduli are then stabilized by the subleading perturbative and non-perturbative effects. This procedure is justified at leading order by the hierarchy between the flux potential and the LVS potential. However, the subleading terms responsible for Kähler-moduli stabilization also exert a non-vanishing force along the axio-dilaton and complex-structure directions. Therefore, the point (S_0, z_0) does not, in general, remain an exact stationary point of the complete scalar potential.

To determine the corrected vacuum consistently, one should retain the dependence of the scalar potential on all the relevant moduli before integrating out the heavy sector. The full $\mathcal{N} = 1$ F-term scalar potential can be organized as

$$\begin{aligned} V = & e^K K^{A\bar{B}} D_A W D_{\bar{B}} \bar{W} \\ & + e^K \left(K^{A\bar{T}_j} D_A W D_{\bar{T}_j} \bar{W} + K^{T_i\bar{B}} D_{T_i} W D_{\bar{B}} \bar{W} \right) \\ & + e^K \left(K^{T_i\bar{T}_j} D_{T_i} W D_{\bar{T}_j} \bar{W} - 3|W|^2 \right), \end{aligned} \quad (4.20)$$

where

$$A, B \in \{S, z\}, \quad (4.21)$$

while T_i denote the Kähler moduli. The second line of (4.20) contains the mixed contributions between the (S, z) sector and the Kähler-moduli sector. These terms vanish at tree level because the corresponding components of the inverse Kähler metric are absent, but they are generated once the corrected Kähler potential is used.

For the purpose of the present discussion, it is useful to decompose the scalar potential schematically as

$$V = V_{\text{flux}} + \delta V, \quad (4.22)$$

where V_{flux} is the leading tree-level flux potential responsible for stabilizing S and z , whereas δV contains the perturbative and non-perturbative corrections. In the large-volume expansion, their parametric behaviour at the minimum is

$$V_{\text{flux}} \sim \frac{1}{\mathcal{V}^2}, \quad \delta V \sim \frac{1}{\mathcal{V}^3}. \quad (4.23)$$

The suppression of δV by one additional inverse power of the volume implies that the displacement of the heavy fields is perturbatively small. Nevertheless, such a dis-

placement can induce contributions to the F -terms of the same parametric order as the direct corrections discussed in the previous section.

Let us denote collectively the fields in the heavy sector by

$$\phi^A = (S, \bar{S}, z, \bar{z}) . \quad (4.24)$$

The use of both holomorphic and anti-holomorphic coordinates allows us to treat the stationarity equations as a single matrix problem. Equivalently, one could work in terms of the real and imaginary parts of S and z .

We introduce a formal perturbative parameter ε and write

$$V(\phi) = V_0(\phi) + \varepsilon V_1(\phi) + \mathcal{O}(\varepsilon^2), \quad (4.25)$$

where

$$V_0 \equiv V_{\text{flux}}, \quad \varepsilon V_1 \equiv \delta V. \quad (4.26)$$

The parameter ε should not necessarily be interpreted as a single physical coupling. It is instead a bookkeeping parameter representing the combined expansion in inverse powers of the volume and in powers of the string coupling. At the end of the calculation, each term will be classified according to its explicit dependence on $\mathcal{V}$ and g_s .

The tree-level minimum ϕ_0^A satisfies

$$\partial_A V_0|_{\phi_0} = 0. \quad (4.27)$$

Once V_1 is included, the corrected minimum can be parametrized as

$$\phi_{\text{min}}^A = \phi_0^A + \varepsilon \delta \phi^A + \mathcal{O}(\varepsilon^2). \quad (4.28)$$

The stationarity condition for the complete potential is

$$0 = \partial_A V|_{\phi_{\text{min}}} . \quad (4.29)$$

Expanding around the tree-level minimum gives

$$0 = \partial_A V_0|_{\phi_0} + \varepsilon \partial_A \partial_B V_0|_{\phi_0} \delta \phi^B + \varepsilon \partial_A V_1|_{\phi_0} + \mathcal{O}(\varepsilon^2). \quad (4.30)$$

The first term vanishes by virtue of (4.27). Defining the tree-level Hessian by

$$H_{AB} \equiv \partial_A \partial_B V_0|_{\phi_0} , \quad (4.31)$$

the first-order stationarity equations become

$$H_{AB} \delta \phi^B = - \partial_A V_1|_{\phi_0} . \quad (4.32)$$

Provided that the Hessian is invertible in the sector under consideration, the displacement is therefore

$$\boxed{\delta\phi^A = -(H^{-1})^{AB} \partial_B V_1|_{\phi_0}} \quad (4.33)$$

at first order in the perturbative expansion.

Equation (4.33) is the basic relation underlying the analysis developed in this work. It shows that the displacement of the vacuum is controlled by two distinct ingredients. The first is the gradient of the correcting potential,

$$\partial_A V_1|_{\phi_0}, \quad (4.34)$$

which represents the force generated by the perturbative and non-perturbative effects at the original flux vacuum. The second is the inverse tree-level Hessian,

$$(H^{-1})^{AB}, \quad (4.35)$$

which measures the response of the stabilized fields to this force.

The displacement is therefore not determined by the correcting potential alone. It also depends on the curvature of the leading potential in field space. A direction associated with a large tree-level mass is comparatively rigid and undergoes a small displacement, whereas a less strongly stabilized direction may be more sensitive to the same perturbing force. Moreover, if the Hessian is not diagonal, a force acting predominantly along one field-space direction can induce a displacement along another one. In the present context, the mixing between the axio-dilaton and the complex-structure modulus implies that

$$\delta S = -(H^{-1})^{SB} \partial_B V_1, \quad (4.36)$$

and

$$\delta z = -(H^{-1})^{zB} \partial_B V_1. \quad (4.37)$$

Consequently, the shift in the complex-structure direction is not determined solely by $\partial_z V_1$, but can also receive contributions from derivatives of the correcting potential with respect to the axio-dilaton.

The shift of the vacuum induces a corresponding correction to any field-dependent quantity. In particular, the complete F -term evaluated at the corrected minimum is

$$F^I(\phi_0 + \varepsilon \delta\phi). \quad (4.38)$$

Expanding to first order gives

$$\delta F_{(2)}^I = F_{\text{tree}}^I(\phi_0) + \varepsilon \delta\phi^A \partial_A F_{\text{tree}}^I|_{\phi_0} + \mathcal{O}(\varepsilon^2). \quad (4.39)$$

Written explicitly in complex coordinates, the displacement-induced contribution is

$$\boxed{\delta F_{(2)}^I = \delta\phi^A \partial_A F_{\text{tree}}^I|_{\phi_0}} \quad (4.40)$$

in the doubled complex-coordinate notation introduced above.

Substituting the expression for the displacement, one finds

$$\boxed{\delta F_{(2)}^I = - \partial_A F_{\text{tree}}^I|_{\phi_0} (H^{-1})^{AB} \partial_B V_1|_{\phi_0}} \quad (4.41)$$

at leading order. This formula summarizes the mechanism studied in the remainder of this work.

It is useful to distinguish this contribution from the direct corrections reviewed in the previous section. Schematically, the full result can be written as

$$F_{\text{full}}^I = F_{\text{tree}}^I(\phi_0) + \delta F_{(1)}^I + \delta F_{(2)}^I + \mathcal{O}(\varepsilon^2), \quad (4.42)$$

Here, F_{tree}^I denotes the tree-level F -terms, which vanish at the supersymmetric flux vacuum, as shown in Eqs. (4.3) and (4.4). The correction $\delta F_{(1)}^I$ is obtained by evaluating the corrected Kähler potential, superpotential, and inverse Kähler metric at the unperturbed tree-level vacuum, as discussed in Eq. (4.19). By contrast, $\delta F_{(2)}^I$ originates from the displacement of the vacuum induced by these corrections. The computation of this additional contribution constitutes the main focus of the present work.

For the axio-dilaton and complex-structure directions,

$$F_{\text{tree}}^S(\phi_0) = F_{\text{tree}}^z(\phi_0) = 0. \quad (4.43)$$

Nevertheless, their derivatives with respect to the moduli need not vanish,

$$\partial_A F_{\text{tree}}^S|_{\phi_0} \neq 0, \quad \partial_A F_{\text{tree}}^z|_{\phi_0} \neq 0. \quad (4.44)$$

Therefore, a parametrically small displacement of the vacuum can generate non-zero F -terms.

The central question is not only whether the shifts δS and δz are small, but whether the induced quantities $\delta F_{(2)}^S$ and $\delta F_{(2)}^z$ are subleading with respect to the direct corrections already known (4.19). Since the latter display different dependences on g_s , the answer can differ significantly between the axio-dilaton and complex-structure sectors. In particular, the direct contribution to F^z does not exhibit the same weak-coupling enhancement as the corresponding contribution to F^S . This makes the complex-structure direction especially sensitive to additional terms generated by the displacement of the vacuum.

The perturbative treatment is valid as long as

$$\left| \frac{\delta S}{S_0} \right| \ll 1, \quad \left| \frac{\delta z}{z_0} \right| \ll 1, \quad (4.45)$$

and the correcting potential does not significantly alter the eigenvalues of the leading-order mass matrix. Moreover, the Hessian H_{AB} must be non-degenerate in the stabilized sector. If an exactly flat or parametrically light direction is present, the inverse-Hessian expansion is not reliable and the corresponding field must be treated separately.

In the following sections, equation (4.41) will be implemented explicitly. We will first determine the gradient of the correcting potential and the inverse tree-level Hessian in a controlled expansion in $\mathcal{V}^{-1}$ and g_s . We will then compute the displacements of S and z and use them to evaluate the induced corrections to the corresponding F -terms.

4.2.1 Controlled large-volume and weak-coupling expansion

We now determine the leading parametric order of the contribution to the F -terms induced by the displacement of the axio-dilaton and complex-structure vacuum. As derived above, this contribution can be written as

$$\delta F_{(2)}^I = - \partial_A F_{\text{tree}}^I|_{\phi_0} (H^{-1})^{AB} \partial_B V_1|_{\phi_0}, \quad (4.46)$$

Our purpose in this subsection is not yet to determine the precise numerical coefficients appearing in (4.46), because it will be the goal of the next chapters, but rather to organize the different contributions according to their dependence on the two perturbative parameters of the effective theory,

$$\frac{1}{\mathcal{V}^m g_s^n}, \quad m, n \in \mathbb{Q}. \quad (4.47)$$

The expansion in inverse powers of $\mathcal{V}$ controls the hierarchy between the leading flux potential and the subleading corrections responsible for LVS stabilization. The expansion in g_s , on the other hand, determines the relative importance of the axio-dilaton and complex-structure directions.

Throughout this section, we assume that W_0 , the relevant flux derivatives and the geometric quantities associated with the complex-structure moduli space are of order one, unless an explicit dependence on g_s is displayed. We also assume that no accidental cancellations occur among independent flux-dependent coefficients.

Tree-level Hessian. At the supersymmetric flux minimum, the scalar potential responsible for stabilizing S and z has the characteristic scaling

$$V_{\text{tree}} \sim \frac{g_s}{\mathcal{V}^2}. \quad (4.48)$$

The dependence on the axio-dilaton can be estimated by using

$$g_s \sim \frac{1}{\text{Re } S}. \quad (4.49)$$

Consequently, a derivative with respect to S or $\bar{S}$ introduces one additional power of g_s . More precisely, for any power p ,

$$\partial_S g_s^p \sim g_s^{p+1}, \quad \partial_{\bar{S}} g_s^p \sim g_s^{p+1}, \quad (4.50)$$

up to numerical factors depending on the convention used for the normalization of S . Derivatives with respect to z or $\bar{z}$, by contrast, do not generically modify the explicit g_s -dependence.

In the complex field basis (4.24) the tree-level Hessian therefore has the schematic parametric structure ²

$$H_{AB} \equiv \partial_A \partial_B V_{\text{tree}}|_{\phi_0} \sim \frac{1}{\mathcal{V}^2} \begin{pmatrix} 0 & g_s^3 & g_s^2 & g_s^2 \\ g_s^3 & 0 & g_s^2 & g_s^2 \\ g_s^2 & g_s^2 & g_s & g_s \\ g_s^2 & g_s^2 & g_s & g_s \end{pmatrix}. \quad (4.51)$$

This expression specifies only the scaling of the different entries. Each non-vanishing component is multiplied by a distinct flux- and geometry-dependent coefficient.

The vanishing entries

$$H_{SS} = H_{\bar{S}\bar{S}} = 0 \quad (4.52)$$

are exact at the supersymmetric tree-level flux vacuum. They follow from the linear dependence of the flux superpotential on the axio-dilaton, together with the tree-level supersymmetry conditions.

Equation (4.51) exhibits a non-trivial weak-coupling hierarchy. The complex-structure block scales as

$$H_{z\bar{z}} \sim \frac{g_s}{\mathcal{V}^2}, \quad (4.53)$$

the mixed axio-dilaton–complex-structure entries behave as

$$H_{Sz}, H_{S\bar{z}} \sim \frac{g_s^2}{\mathcal{V}^2}, \quad (4.54)$$

whereas the non-vanishing axio-dilaton component scales as

$$H_{S\bar{S}} \sim \frac{g_s^3}{\mathcal{V}^2}. \quad (4.55)$$

The axio-dilaton is therefore associated with parametrically smaller entries of the Hessian at weak coupling. As a consequence, the inverse Hessian is enhanced along directions containing an axio-dilaton index.

Provided that the order-one coefficients do not generate a parametrically small determinant or accidental cancellations, inversion of (4.51) gives

$$(H^{-1})^{AB} \sim \mathcal{V}^2 \begin{pmatrix} g_s^{-3} & g_s^{-3} & g_s^{-2} & g_s^{-2} \\ g_s^{-3} & g_s^{-3} & g_s^{-2} & g_s^{-2} \\ g_s^{-2} & g_s^{-2} & g_s^{-1} & g_s^{-1} \\ g_s^{-2} & g_s^{-2} & g_s^{-1} & g_s^{-1} \end{pmatrix}. \quad (4.56)$$

²The power counting displayed in Eq. (4.51) follows directly from Eqs. (4.48) and (4.50).

Notice that the vanishing of H_{SS} and $H_{\bar{S}\bar{S}}$ does not imply the vanishing of the corresponding entries of H^{-1} . The inverse matrix depends on the complete coupled system and cannot be obtained by taking the reciprocal of each individual component.

The axio-dilaton block of H^{-1} is thus enhanced as g_s^{-3} , the mixed block as g_s^{-2} , and the complex-structure block as g_s^{-1} . This hierarchy will determine the relative size of the vacuum displacements.

Gradient of the correcting potential. We next estimate the gradient of the subleading potential V_1 . In full generality, V_1 contains several contributions and its complete dependence on the axio-dilaton and complex-structure moduli can be rather involved. Since our present purpose is only to obtain a preliminary parametric estimate, we restrict attention to the standard LVS potential,

$$V_1 \supset V_{\text{LVS}}(S, z), \quad (4.57)$$

whose explicit dependence on g_s and $\mathcal{V}$ is known. This is sufficient to identify the parametric orders that can potentially dominate the displacement of the tree-level vacuum.

This approximation must, however, be interpreted with some care. In the conventional derivation of the LVS potential, the axio-dilaton and complex-structure moduli are frozen at their supersymmetric tree-level values by imposing

$$D_S W = D_z W = 0. \quad (4.58)$$

As a consequence, all terms proportional to these covariant derivatives are set to zero before the effective LVS potential is written. Such terms are therefore not included in the standard expression for V_{LVS} , even though they are part of the full correcting potential V_1 once the heavy fields are allowed to fluctuate away from ϕ_0 ³

In principle, the omitted contributions could contain additional inverse powers of g_s , arising for instance from derivatives of the flux superpotential or from contractions with inverse moduli-space metrics. They could therefore compete with, or even dominate over, the terms inferred directly from V_{LVS} . For this reason, the following estimates should be regarded as a first power-counting guide rather than as a complete proof of the leading behaviour of $\partial_A V_1$.

The terms that vanish in the standard LVS reduction because they are proportional to $D_S W$ or $D_z W$ will be retained and analyzed explicitly in the subsequent calculation.

After imposing the leading LVS relations for the Kähler moduli, the characteristic magnitude of the correcting potential is

$$V_{\text{LVS}} \sim \frac{1}{\sqrt{g_s} \mathcal{V}^3}. \quad (4.59)$$

³More precisely, the terms in Eq. (4.20) that are proportional to the covariant derivatives in Eq. (4.58) are subleading in the large-volume expansion, since their first non-vanishing contributions arise at order $\mathcal{V}^{-4}$. However, the relevant quantities for the computation of the shifts are the derivatives of the subleading potential, for which we will find $\partial_{S,z} V_1 \sim \mathcal{V}^{-3}$.

A derivative with respect to the axio-dilaton acts on the explicit dependence on g_s . Using (4.50), one finds

$$\partial_S V_{\text{LVS}} \sim \frac{\sqrt{g_s}}{\mathcal{V}^3}, \quad \partial_{\bar{S}} V_{\text{LVS}} \sim \frac{\sqrt{g_s}}{\mathcal{V}^3}. \quad (4.60)$$

The complex-structure derivative does not, in general, introduce an additional explicit power of g_s . Thus, we have

$$\partial_z V_{\text{LVS}} \sim \frac{1}{\sqrt{g_s} \mathcal{V}^3}, \quad \partial_{\bar{z}} V_{\text{LVS}} \sim \frac{1}{\sqrt{g_s} \mathcal{V}^3}. \quad (4.61)$$

The gradient of the correcting potential consequently takes the schematic form

$$\partial_A V_1|_{\phi_0} \sim \frac{1}{\mathcal{V}^3} \left(g_s^{1/2} g_s^{1/2} g_s^{-1/2} g_s^{-1/2} \right). \quad (4.62)$$

Displacement of the flux vacuum. At first order in the correcting potential, the displacement of the heavy fields is

$$\delta\phi^A = -(H^{-1})^{AB} \partial_B V_1|_{\phi_0}. \quad (4.63)$$

Combining (4.56) with (4.62), the axio-dilaton displacement receives contributions of the form

$$\boxed{\delta S \sim \frac{1}{\mathcal{V} g_s^{5/2}}}. \quad (4.64)$$

Similarly, the complex structure,

$$\boxed{\delta z \sim \frac{1}{\mathcal{V} g_s^{3/2}}}. \quad (4.65)$$

Both shifts are suppressed by one inverse power of the volume. This is the expected consequence of the hierarchy

$$V_{\text{tree}} \sim \mathcal{V}^{-2}, \quad V_1 \sim \mathcal{V}^{-3}. \quad (4.66)$$

However, the inverse Hessian introduces a substantial enhancement at weak coupling. The axio-dilaton displacement is larger than the complex-structure displacement by one inverse power of g_s ,

$$\frac{\delta S}{\delta z} \sim \frac{1}{g_s}. \quad (4.67)$$

Induced shift of the covariant derivatives. We can now determine how the displacement of the vacuum modifies the covariant derivatives of the tree-level superpotential. Since

$$D_I W_{\text{tree}}|_{\phi_0} = 0, \quad I = S, z, \quad (4.68)$$

their values at the displaced vacuum begin at first order in $\delta\phi^A$:

$$D_I W_{\text{tree}}(\phi_0 + \delta\phi) = \delta\phi^A \partial_A D_I W_{\text{tree}}|_{\phi_0} + \mathcal{O}(\delta\phi^2). \quad (4.69)$$

Under the flux-scaling assumptions introduced above and under the assumption that $W_0 \sim \mathcal{O}(1)$, the derivatives relevant for $D_S W$ behave as

$$\partial_S D_S W = 0, \quad \partial_{\bar{S}} D_S W \sim g_s^2, \quad \partial_z D_S W \sim g_s, \quad \partial_{\bar{z}} D_S W = 0. \quad (4.70)$$

Thus,

$$D_S W_{\text{tree}}(\phi_0^I + \delta\phi^I) \sim \frac{1}{\mathcal{V}\sqrt{g_s}}. \quad (4.71)$$

The axio-dilaton and complex-structure displacements contribute at the same parametric order to $D_S W$. This is a direct consequence of the compensation between the larger displacement δS and the additional power of g_s carried by $\partial_{\bar{S}} D_S W$.

For the complex-structure covariant derivative, we instead have

$$\partial_S D_z W \sim g_s, \quad \partial_{\bar{S}} D_z W = 0, \quad \partial_z D_z W \sim \mathcal{O}(1), \quad \partial_{\bar{z}} D_z W \sim \mathcal{O}(1). \quad (4.72)$$

Consequently,

$$D_z W_{\text{tree}}(\phi^I + \delta\phi^I) \sim \frac{1}{\mathcal{V}g_s^{3/2}}. \quad (4.73)$$

Comparing (4.71) and (4.73) with the corresponding unshifted contributions reported in (4.10) and (4.11), we find that the displacement-induced correction along the complex-structure direction is enhanced by one inverse power of the string coupling,

By contrast, the correction along the axio-dilaton direction remains of the same parametric order as its unshifted counterpart. Since the F -terms depend linearly on the corresponding Kähler-covariant derivatives, this relative enhancement is directly inherited by the complex-structure F -term.

We immediately conclude that,

$$\delta F_{(2)}^S \sim \frac{1}{\mathcal{V}^2 g_s^2}. \quad (4.74)$$

and

$$\delta F_{(2)}^z \sim \frac{1}{\mathcal{V}^2 g_s}. \quad (4.75)$$

Comparing these expressions with the corrections obtained by evaluating the corrected supergravity quantities at the unshifted flux vacuum, discussed in (4.19), we find that the displacement-induced contribution to the axio-dilaton F -term is of the same parametric order as the previously known correction. The complex-structure F -term, by contrast, is enhanced by one inverse power of the string coupling:

$$\delta F_{(2)}^z \sim \frac{1}{g_s} \delta F_{(1)}^z, \quad (4.76)$$

at the level of the parametric weak-coupling expansion.

The estimates derived in this subsection are parametric rather than numerical statements. They rely on generic order-one flux and geometric coefficients, on $W_0 \sim \mathcal{O}(1)$, and on the absence of accidental cancellations or parametrically small eigenvalues of the tree-level Hessian. In the following sections, however, we will show that this is exactly what happens when you consider explicit realizations.

4.3 First Application: A Toy Model

We are now in a position to illustrate explicitly how the general mechanism described above operates in a concrete setting. As a first application, we consider a simplified model with a rigid complex structure,

$$h^{2,1} = 0, \quad (4.77)$$

and two Kähler moduli,

$$h^{1,1} = 2. \quad (4.78)$$

The latter are assumed to consist of one large modulus and one small blow-up modulus, so that the hierarchy required for the realization of the Large Volume Scenario can be achieved (see Chapter 3). Our conventions are summarized in A.1; here we only recall the ingredients that are directly relevant for the computation.

We include the leading perturbative α'^3 correction to the Kähler potential at string tree level. Expanding at large volume, one finds

$$K = \hat{K} + \delta K = -2 \log \mathcal{V} - \log(S + \bar{S}) - \frac{\xi}{g_s^{3/2} \mathcal{V}} + \mathcal{O}\left(\frac{\xi^2}{g_s^3 \mathcal{V}^2}\right), \quad (4.79)$$

The superpotential contains both the flux-induced contribution and a non-perturbative term depending on the small Kähler modulus,

$$W = W_{\text{flux}}(S) + W_{\text{np}}(T_s) = A + BS + \mathcal{A}e^{-a_s T_s}, \quad (4.80)$$

where A , B , and $\mathcal{A}$ are complex constants.

At tree level, the axio-dilaton is stabilized supersymmetrically by the flux superpotential. Indeed,

$$\begin{aligned} D_S W_{\text{flux}} &= \partial_S W_{\text{flux}} + \hat{K}_S W_{\text{flux}} \\ &= \frac{B\bar{S} - A}{S + \bar{S}}. \end{aligned} \quad (4.81)$$

The supersymmetry condition therefore gives

$$D_S W_{\text{flux}}|_{S_0} = 0 \quad \implies \quad \bar{S}_0 = \frac{A}{B}. \quad (4.82)$$

We denote the value of the flux superpotential at this vacuum by

$$W_0 \equiv W_{\text{flux}}(S_0) = A + BS_0. \quad (4.83)$$

Using $A = B\bar{S}_0$, one obtains

$$W_0 = B(S_0 + \bar{S}_0) = \frac{2B}{g_s}, \quad (4.84)$$

and hence

$$\frac{|W_0|^2}{|B|^2} = (S_0 + \bar{S}_0)^2 = \frac{4}{g_s^2}. \quad (4.85)$$

At this order, the corresponding F -term vanishes, as shown in Eq. (4.3).

Our aim is to determine how the perturbative and non-perturbative corrections displace the axio-dilaton from the tree-level supersymmetric vacuum. We therefore do not integrate out S by imposing Eq. (4.82) from the outset. Instead, we minimize the corrected scalar potential and compute the induced displacement of its expectation value.

The full F -term scalar potential can be organized as

$$\begin{aligned} V = e^K & \left[K^{S\bar{S}} D_S W D_{\bar{S}} \bar{W} + K^{S\bar{T}_j} D_S W D_{\bar{T}_j} \bar{W} + K^{T_i \bar{S}} D_{T_i} W D_{\bar{S}} \bar{W} \right] \\ & + e^K \left[K^{T_i \bar{T}_j} D_{T_i} W D_{\bar{T}_j} \bar{W} - 3|W|^2 \right]. \end{aligned} \quad (4.86)$$

An exact minimization of this expression would be unnecessarily cumbersome. We instead decompose the potential perturbatively as

$$V = V_0 + V_1 + \dots, \quad (4.87)$$

where V_0 is the tree-level flux potential and V_1 contains the first perturbative and non-perturbative corrections.

The corrected vacuum is parametrized by

$$S = S_0 + \delta S. \quad (4.88)$$

Applying the general vacuum-shift formula derived in Eq. (4.33), the leading displacement is

$$\delta S = -H^{S\bar{S}} \partial_{\bar{S}} V_1|_{S_0}, \quad (4.89)$$

where $H^{S\bar{S}}$ denotes the relevant component of the inverse tree-level Hessian.

At leading order, the no-scale cancellation removes the Kähler-moduli contribution to the scalar potential. The remaining term is

$$\begin{aligned} V_0 &= e^{K_0} K_0^{S\bar{S}} D_S W_{\text{flux}} D_{\bar{S}} \bar{W}_{\text{flux}} \\ &= \frac{|B\bar{S} - A|^2}{(S + \bar{S})\mathcal{V}^2}. \end{aligned} \quad (4.90)$$

Since $B\bar{S}_0 - A = 0$, the non-vanishing mixed component of the Hessian is

$$H_{S\bar{S}} = \partial_S \partial_{\bar{S}} V_0|_{S_0} = \frac{|B|^2}{(S_0 + \bar{S}_0)\mathcal{V}^2}. \quad (4.91)$$

The purely holomorphic and anti-holomorphic components vanish at the supersymmetric vacuum,

$$H_{SS} = H_{\bar{S}\bar{S}} = 0. \quad (4.92)$$

It follows that

$$H^{S\bar{S}} = \frac{(S_0 + \bar{S}_0)\mathcal{V}^2}{|B|^2} = \frac{2 \operatorname{Re}(A/B)\mathcal{V}^2}{|B|^2}. \quad (4.93)$$

We now turn to the subleading potential. Besides the conventional LVS contribution, one must retain the corrections involving the axio-dilaton F -term and the mixed components of the inverse Kähler metric. We therefore write

$$V_1 = V_{S\bar{S}} + V_{S\bar{T}} + V_{\bar{T}S} + V_{\text{LVS}}, \quad (4.94)$$

where

$$V_{S\bar{S}} \equiv \left[e^K K^{S\bar{S}} D_S W D_{\bar{S}} \bar{W} \right]_{\text{sub}}, \quad (4.95)$$

$$V_{S\bar{T}} \equiv \left[e^K K^{S\bar{T}_j} D_S W D_{\bar{T}_j} \bar{W} \right]_{\text{sub}}, \quad (4.96)$$

and

$$V_{\bar{T}S} \equiv \left[e^K K^{T_i \bar{S}} D_{T_i} W D_{\bar{S}} \bar{W} \right]_{\text{sub}}. \quad (4.97)$$

Here the subscript “sub” indicates that the tree-level contribution already included in V_0 has been removed.

The quantities relevant for the vacuum shift are not the individual values of these terms, but their derivatives with respect to the axio-dilaton (4.89). Evaluating them at the tree-level vacuum and using the leading LVS relations for the Kähler moduli, the detailed computation presented in Appendix B gives

$$\partial_{\bar{S}} V_{\text{LVS}}|_{S_0} = \frac{9\xi}{32} \frac{\sqrt{g_s}}{\mathcal{V}^3} |W_0|^2 + \mathcal{O}\left(\frac{g_s^{3/2}}{\mathcal{V}^3}\right), \quad (4.98)$$

$$\partial_{\bar{S}} V_{S\bar{S}}|_{S_0} = -\frac{3\xi}{8} \frac{\sqrt{g_s}}{\mathcal{V}^3} |W_0|^2 + \mathcal{O}\left(\frac{g_s^{3/2}}{\mathcal{V}^3}\right), \quad (4.99)$$

and

$$\partial_{\bar{S}} V_{S\bar{T}}|_{S_0} = \frac{9\xi}{16} \frac{\sqrt{g_s}}{\mathcal{V}^3} |W_0|^2 + \mathcal{O}\left(\frac{g_s^{3/2}}{\mathcal{V}^3}\right). \quad (4.100)$$

The conjugate mixed contribution does not enter at the order retained here,

$$\partial_{\bar{S}} V_{\bar{T}S}|_{S_0} = \mathcal{O}(\mathcal{V}^{-4}). \quad (4.101)$$

Combining Eqs. (4.98)–(4.101), we obtain

$$\partial_{\bar{S}} V_1|_{S_0} = \frac{15\xi}{32} \frac{\sqrt{g_s}}{\mathcal{V}^3} |W_0|^2 + \mathcal{O}\left(\frac{g_s^{3/2}}{\mathcal{V}^3}\right). \quad (4.102)$$

Substituting Eqs. (4.93) and (4.102) into Eq. (4.89), and using Eq. (4.85), the shift can be written as

$$\delta S = -\frac{15\xi}{4} \frac{1}{g_s^{5/2} \mathcal{V}} + \mathcal{O}\left(\frac{1}{\mathcal{V} g_s^{3/2}}\right). \quad (4.103)$$

The corresponding shift-induced contribution to the axio-dilaton F -term follows directly from Eq. (4.41). With the conventions adopted above, one finds

$$\delta F_{(2)}^S = -\frac{15\sqrt{2}}{8} \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^2 g_s^{3/2}}\right). \quad (4.104)$$

This result induces a genuinely new contribution to the axio-dilaton F -term. To compare it with the correction obtained by evaluating the corrected supergravity quantities at the unshifted tree-level vacuum, we recall Eq. (4.16),

$$\delta F_{(1)}^S = -\frac{3}{\sqrt{2}} \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^2 g_s^{3/2}}\right). \quad (4.105)$$

The shift-induced correction $\delta F_{(2)}^S$ derived above is of the same parametric order in both the large-volume and weak-coupling expansions. Moreover, the two contributions exhibit the same functional dependence on ξ , $\bar{W}_0$, $\mathcal{V}$, and g_s . Their relative importance is therefore determined entirely by their numerical coefficients.

Combining the two contributions, we find that, in the present toy model, the axio-dilaton F -term is given at leading order by

$$F^S = \delta F_{(1)}^S + \delta F_{(2)}^S = -\frac{27}{4\sqrt{2}} \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^2 g_s^{3/2}}\right). \quad (4.106)$$

Thus, the vacuum displacement does not alter the parametric scaling of the axio-dilaton F -term, but provides an additional contribution with the same sign as the unshifted one. Consequently, the two terms add constructively, and the leading coefficient of F^S is enhanced by a factor of $9/4$ relative to the result obtained at the unshifted vacuum.

4.4 Shift-induced corrections to the F -terms in a minimal model

In this final section, we explicitly compute the shift-induced contributions $\delta F_{(2)}^I$ and compare them with the corrections $\delta F_{(1)}^I$ obtained by evaluating the corrected theory at the unperturbed supersymmetric vacuum 4.19, in a minimal model. The purpose of this comparison is to determine whether the displacement of the axio-dilaton and complex-structure moduli from their tree-level supersymmetric minimum generates contributions that are parametrically subleading, comparable to, or larger than those discussed in Section 4.1.

In Section 4.2.1, we performed a parametric analysis in powers of g_s and $\mathcal{V}^{-1}$. This analysis suggested that the shift-induced correction to the complex-structure F -term may be enhanced by one inverse power of the string coupling with respect to the corresponding unshifted contribution. More precisely, the estimates in Eqs. (4.74) and (4.75) indicate that, while the axio-dilaton contribution is expected to be of the same parametric order as the standard correction, the complex-structure contribution may be larger by a factor of $1/g_s$.

Such a scaling argument, however, does not exclude cancellations among the different terms entering the explicit computation. Similar cancellations are known to occur in string compactifications, as in the case of the extended no-scale structure [26]. The purpose of the present calculation is therefore to verify whether the parametric estimates survive in an explicit model. We will find that no additional cancellation modifies the leading scaling and that the explicit results are consistent with the estimates derived previously.

The simpler toy model considered in Section 4.3 allowed us to verify this mechanism in the axio-dilaton direction. Since that model did not contain dynamical complex-structure moduli, it could not provide any information about the corresponding complex-structure F -term. We therefore consider a minimal extension containing one complex-structure modulus z , the axio-dilaton S , and two Kähler moduli T_b and T_s , as required for the realization of the Large Volume Scenario.

The superpotential is taken to be

$$W = W_{\text{flux}}(S, z) + W_{\text{np}}(z, T_s) = A(z) + B(z)S + \mathcal{A}(z)e^{-a_s T_s}. \quad (4.107)$$

Here $A(z)$ and $B(z)$ encode the dependence of the flux superpotential on the complex-structure modulus (3.15), while $\mathcal{A}(z)$ denotes the one-loop determinant multiplying the non-perturbative contribution.

More generally, the non-perturbative prefactor may depend on both the axio-dilaton and the complex-structure modulus,

$$\mathcal{A} = \mathcal{A}(S, z), \quad (4.108)$$

as discussed, for instance, in Ref. [27]. In the present analysis, we neglect its explicit dependence on S and retain only the dependence on z . This is justified by the fact that

the dependence of the axio-dilaton appears only in the exponent of the perturbative contribution, and so, it can be neglected when $g_s \ll 1$.

We recall that the shift-induced correction to the F -terms is given by

$$\delta F_{(2)}^I = - \partial_A F_{\text{tree}}^I|_{\phi_0} (H^{-1})^{AB} \partial_B V_1|_{\phi_0}, \quad (4.109)$$

where ϕ^A is the usual basis (4.24).

The explicit evaluation of Eq. (4.109) therefore requires three ingredients:

$$H_{AB}, \quad \partial_A V_1|_{\phi_0}, \quad \partial_A F_{\text{tree}}^I|_{\phi_0}. \quad (4.110)$$

We begin by computing the tree-level Hessian and organizing it in block-matrix form. This representation will make its inversion considerably more transparent and will allow us to isolate the components that contribute to the axio-dilaton and complex-structure shifts. We organize the tree-level Hessian as

$$H = \frac{1}{\mathcal{V}^2} \begin{pmatrix} \mathbb{A} & \mathbb{B} \\ \mathbb{B}^T & \mathbb{C} \end{pmatrix}, \quad (4.111)$$

The matrices $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{C}$ are 2×2 matrices whose explicit entries are given in Appendix B.

Since H is the Hessian of the tree-level scalar potential, its overall volume dependence is inherited from

$$V_{\text{tree}} \sim \frac{1}{\mathcal{V}^2}. \quad (4.112)$$

Derivatives with respect to the axio-dilaton and complex-structure moduli do not modify this scaling. We can therefore factor out the universal dependence $\mathcal{V}^{-2}$, while all the remaining dependence on S , z , and the flux parameters is contained in the block matrices appearing in Eq. (4.111).

The inverse Hessian can then be obtained using the Schur-complement formula. Assuming that $\mathbb{C}$ and the corresponding Schur complement are invertible, we define

$$\mathbb{E} \equiv \mathbb{A} - \mathbb{B}\mathbb{C}^{-1}\mathbb{B}^T. \quad (4.113)$$

The inverse of the Hessian is therefore

$$H^{-1} = \mathcal{V}^2 \begin{pmatrix} \mathbb{E}^{-1} & -\mathbb{E}^{-1}\mathbb{B}\mathbb{C}^{-1} \\ -\mathbb{C}^{-1}\mathbb{B}^T\mathbb{E}^{-1} & \mathbb{C}^{-1} + \mathbb{C}^{-1}\mathbb{B}^T\mathbb{E}^{-1}\mathbb{B}\mathbb{C}^{-1} \end{pmatrix}. \quad (4.114)$$

This representation is particularly useful because it reduces the inversion of the full 4×4 Hessian to the inversion of the 2×2 matrices $\mathbb{C}$ and $\mathbb{E}$.

We now turn to the second ingredient entering the shift formula, namely the gradient of the subleading potential evaluated at the tree-level vacuum,

$$\partial_A V_1|_{\phi_0} \quad (4.115)$$

Keeping the leading terms in the simultaneous large-volume and weak-coupling expansion, one finds (Appendix B),

$$\partial_S V_1|_{\phi_0} = \frac{15}{32} \frac{\xi |W_0|^2}{\mathcal{I} \mathcal{V}^3} \sqrt{g_s} + \mathcal{O} \left(\frac{g_s^{3/2}}{\mathcal{V}^3} \right), \quad (4.116)$$

and

$$\partial_z V_1|_{\phi_0} = \frac{3}{4} \frac{\xi \bar{W}_0}{\mathcal{I} g_s^{3/2} \mathcal{V}^3} P + \mathcal{O} \left(\frac{\sqrt{g_s}}{\mathcal{V}^3} \right), \quad (4.117)$$

where

$$P \equiv D_z B|_{z_0}, \quad P = \mathcal{O}(g_s). \quad (4.118)$$

The remaining components follow from the reality of the scalar potential,

$$\partial_{\bar{S}} V_1|_{\phi_0} = \left(\partial_S V_1|_{\phi_0} \right)^*, \quad \partial_{\bar{z}} V_1|_{\phi_0} = \left(\partial_z V_1|_{\phi_0} \right)^*. \quad (4.119)$$

It is convenient to introduce the two-component vectors

$$\delta \mathbf{S} \equiv \begin{pmatrix} \delta S \\ \delta \bar{S} \end{pmatrix}, \quad \delta \mathbf{z} \equiv \begin{pmatrix} \delta z \\ \delta \bar{z} \end{pmatrix}, \quad (4.120)$$

together with

$$\nabla_S V_1 \equiv \begin{pmatrix} \partial_S V_1 \\ \partial_{\bar{S}} V_1 \end{pmatrix}_{\phi_0}, \quad \nabla_z V_1 \equiv \begin{pmatrix} \partial_z V_1 \\ \partial_{\bar{z}} V_1 \end{pmatrix}_{\phi_0}. \quad (4.121)$$

Using the block decomposition of the tree-level Hessian introduced above and the Schur complement (4.113), the axio-dilaton displacement is

$$\delta \mathbf{S} = -\mathcal{I} \mathcal{V}^2 \mathbb{E}^{-1} \left(\nabla_S V_1 - \mathbb{B} \mathbb{C}^{-1} \nabla_z V_1 \right). \quad (4.122)$$

The corresponding complex-structure displacement is

$$\delta \mathbf{z} = \mathcal{I} \mathcal{V}^2 \mathbb{C}^{-1} \left[\mathbb{B}^T \mathbb{E}^{-1} \nabla_S V_1 - (\mathbb{I}_2 + \mathbb{B}^T \mathbb{E}^{-1} \mathbb{B} \mathbb{C}^{-1}) \nabla_z V_1 \right]. \quad (4.123)$$

Where $\mathbb{I}_2$ is the 2×2 identity matrix. Equivalently, making use of Eq. (4.122), this expression can be written in the more compact form

$$\delta \mathbf{z} = -\mathbb{C}^{-1} \mathbb{B}^T \delta \mathbf{S} - \mathcal{I} \mathcal{V}^2 \mathbb{C}^{-1} \nabla_z V_1. \quad (4.124)$$

Substituting Eqs. (4.116) and (4.117) into the preceding expressions, and expanding the inverse matrices in the weak-coupling regime, gives

$$\delta S = \frac{15\xi}{4} \frac{m(S_0, z_0)}{g_s^{5/2}\mathcal{V}} + \mathcal{O}\left(\frac{1}{g_s^{3/2}\mathcal{V}}\right), \quad (4.125)$$

and

$$\delta z = \frac{3\xi}{4} \frac{n(S_0, z_0)}{g_s^{3/2}\mathcal{V}} + \mathcal{O}\left(\frac{1}{\sqrt{g_s}\mathcal{V}}\right). \quad (4.126)$$

Here $m(S_0, z_0)$ and $n(S_0, z_0)$ are dimensionless complex functions encoding the dependence on the flux data and on the tree-level vacuum. Their explicit expressions are given in Appendix B, and they are generically of order unity,

$$m(S_0, z_0), n(S_0, z_0) = \mathcal{O}(1). \quad (4.127)$$

Although both displacements are suppressed by one inverse power of the volume, the axio-dilaton shift carries one additional inverse power of g_s relative to the complex-structure shift.

We can now determine the contribution to the F -terms arising exclusively from the displacement of the vacuum. Recalling the general expression

$$\delta F_{(2)}^I = \delta\phi^A \partial_A F_{\text{tree}}^I|_{\phi_0}, \quad (4.128)$$

we obtain, for the complex-structure direction,

$$\delta F_{(2)}^z = \delta S \partial_S F_{\text{tree}}^z|_{\phi_0} + \delta \bar{S} \partial_{\bar{S}} F_{\text{tree}}^z|_{\phi_0} + \delta z, \partial_z F_{\text{tree}}^z|_{\phi_0} + \delta \bar{z}, \partial_{\bar{z}} F_{\text{tree}}^z|_{\phi_0}. \quad (4.129)$$

At leading order this gives

$$\delta F_{(2)}^z = \frac{3}{4\sqrt{2\mathcal{I}}} \frac{\xi \bar{W}_0 K_0^{z\bar{z}}}{\mathcal{V}^2 g_s} \omega_z(S_0, z_0) + \mathcal{O}\left(\frac{\bar{W}_0}{\mathcal{V}^2 \sqrt{g_s}}\right). \quad (4.130)$$

Similarly, the shift-induced axio-dilaton F -term is

$$\delta F_{(2)}^S = \delta S \partial_S F_{\text{tree}}^S|_{\phi_0} + \delta \bar{S} \partial_{\bar{S}} F_{\text{tree}}^S|_{\phi_0} + \delta z \partial_z F_{\text{tree}}^S|_{\phi_0} + \delta \bar{z} \partial_{\bar{z}} F_{\text{tree}}^S|_{\phi_0}, \quad (4.131)$$

from which one finds

$$\delta F_{(2)}^S = \frac{3}{\sqrt{2\mathcal{I}}} \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s^2} \omega_S(S_0, z_0) + \mathcal{O}\left(\frac{\bar{W}_0}{\mathcal{V}^2 g_s^{3/2}}\right). \quad (4.132)$$

The functions $\omega_S(S_0, z_0)$ and $\omega_z(S_0, z_0)$ collect the vacuum-dependent combinations entering the derivatives of the tree-level F -terms and are generically of order unity,

$$\omega_S(S_0, z_0), \omega_z(S_0, z_0) = \mathcal{O}(1). \quad (4.133)$$

The resulting parametric behaviour is therefore

$$\delta F_{(2)}^S \sim \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s^2}, \quad \delta F_{(2)}^z \sim \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s}. \quad (4.134)$$

The axio-dilaton contribution is of the same parametric order as the standard LVS correction to F^S (4.19). By contrast, the shift-induced complex-structure contribution is enhanced by one inverse power of the string coupling relative to the corresponding result evaluated at the unshifted tree-level vacuum. Consequently, in the weak-coupling regime $g_s \ll 1$, the displacement of the vacuum provides the parametrically dominant contribution to F^z .

4.5 Physical relevance

In section 3.3, we reviewed the soft supersymmetry-breaking terms arising in sequestered LVS scenarios. We can now determine how their parametric behaviour is affected by the displacement of the axio-dilaton and complex-structure moduli derived in this work.

For comparison with the standard treatment, let us schematically denote by F_{std}^I the F-terms obtained by using the conventional estimate of the shift, and by F_{sh}^I our shifted result. The main results of our analysis can be summarised as

$$F_{\text{sh}}^S \sim \mathcal{O}(1) F_{\text{std}}^S, \quad F_{\text{sh}}^z \sim \frac{1}{g_s} F_{\text{std}}^z, \quad (4.135)$$

where the first relation means that the dilaton F-term is modified only by an order-one coefficient, whereas the complex-structure F-term is parametrically enhanced at weak coupling.

Consider first the gaugino masses. Since the gauge kinetic function depends on the dilaton and $F^{T_{\text{SM}}} = 0$, their leading contribution is

$$M_{1/2} = \frac{g_s F^S}{2}. \quad (4.136)$$

Therefore,

$$\frac{M_{1/2}^{\text{sh}}}{M_{1/2}^{\text{std}}} = \frac{F_{\text{sh}}^S}{F_{\text{std}}^S} \sim \mathcal{O}(1). \quad (4.137)$$

The displacement of the vacuum thus does not change the parametric dependence of the gaugino masses on either g_s or $\mathcal{V}$, but only their flux-dependent numerical coefficient. In particular, the volume scaling found in [3],

$$M_{1/2} \sim \frac{m_{3/2}}{\mathcal{V}} \quad (4.138)$$

up to logarithmic corrections, remains unchanged.

The situation is more interesting for scalar masses. In the local limit their leading contribution is generated by F^{T_b} and scales as

$$m_\alpha^2|_{\text{local}} \sim \frac{m_{3/2}^2}{\mathcal{V}}, \quad (4.139)$$

whereas the contributions involving F^S and F^z are further volume suppressed,

$$m_\alpha^2|_{S,z} \sim \frac{m_{3/2}^2}{\mathcal{V}^2}. \quad (4.140)$$

Our enhancement of F^z therefore does not alter the leading large-volume behaviour of scalar masses in the local limit.

In the ultra-local limit, instead, the T_b -dependent contribution cancels. For a single complex-structure modulus, the remaining F-term contribution can be written schematically as

$$m_{\alpha,F}^2 = -F^S \bar{F}^{\bar{S}} \partial_S \partial_{\bar{S}} \ln h_\alpha - F^S \bar{F}^{\bar{z}} \partial_S \partial_{\bar{z}} \ln h_\alpha - F^z \bar{F}^{\bar{S}} \partial_z \partial_{\bar{S}} \ln h_\alpha - F^z \bar{F}^{\bar{z}} \partial_z \partial_{\bar{z}} \ln h_\alpha.$$

Using (4.135), the four contributions are modified parametrically according to

$$SS : \mathcal{O}(1), \quad Sz, ; zS : \mathcal{O}(g_s^{-1}), \quad zz : \mathcal{O}(g_s^{-2}), \quad (4.141)$$

relative to the standard estimate. Hence, the zz contribution is enhanced by g_s^{-2} relative to its standard estimate and can become phenomenologically important, depending on the moduli dependence of the matter metric.

Finally, consider the trilinear A -terms. Their general structure is linear in the moduli F-terms,

$$A_{\alpha\beta\gamma} = F^I \partial_I \left[K + \ln \left(\frac{Y_{\alpha\beta\gamma}}{\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma} \right) \right]. \quad (4.142)$$

In the local limit, the leading contributions arise from F^{T_b} , F^S , and F^z . The shift therefore enhances only the complex-structure contribution,

$$A_z^{\text{sh}} \sim g_s^{-1} A_z^{\text{std}}, \quad (4.143)$$

while the S -dependent term remains parametrically unchanged.

The result is particularly transparent in the ultra-local limit, where the contribution proportional to F^{T_b} cancels. Defining

$$\mathcal{Y}_{\alpha\beta\gamma} \equiv \frac{Y_{\alpha\beta\gamma}}{h_\alpha h_\beta h_\gamma}, \quad (4.144)$$

one obtains

$$A_{\alpha\beta\gamma}^{\text{UL}} = F^S \partial_S \ln \mathcal{Y}_{\alpha\beta\gamma} + F^z \partial_z \ln \mathcal{Y}_{\alpha\beta\gamma}. \quad (4.145)$$

Consequently,

$$A_{\alpha\beta\gamma}^{\text{UL,sh}} \sim F_{\text{std}}^S \partial_S \ln \mathcal{Y}_{\alpha\beta\gamma} + \frac{1}{g_s} F_{\text{std}}^z \partial_z \ln \mathcal{Y}_{\alpha\beta\gamma}. \quad (4.146)$$

Whether this contribution becomes comparable to, or dominates over, the axio-dilaton contribution depends on the detailed S - and z -dependence of the physical Yukawa couplings and matter metrics entering $\mathcal{Y}_{\alpha\beta\gamma}$. In contrast to the scalar masses, where the zz contribution is quadratic in F^z and therefore receives a relative enhancement of order g_s^{-2} , the complex-structure contribution to the trilinear terms is only linearly enhanced, by a factor g_s^{-1} .

Another phenomenologically interesting consequence concerns the soft masses of hidden photini, namely the supersymmetric partners of closed-string hidden photons

[4]. In type IIB Calabi–Yau orientifold compactifications with background fluxes and $O3/O7$ -planes, a class of hidden $U(1)$ gauge bosons arises from the dimensional reduction of the RR four-form potential $C_{(4)}$ on orientifold-even three-cycles. In particular, expanding $C_{(4)}$ on a symplectic basis $(\alpha_\kappa, \beta^\kappa)$ of $H_+^3(X)$ gives

$$C_{(4)} = V_{(1)}^\kappa(x) \wedge \alpha_\kappa + U_{(1)\kappa}(x) \wedge \beta^\kappa + \dots, \quad \kappa = 1, \dots, h_+^{2,1}. \quad (4.147)$$

Because of the self-duality constraint on $F_{(5)}$, the electric and magnetic vector fields appearing in this expansion are not independent. The resulting four-dimensional gauge fields are usually referred to as RR photons.

The corresponding photini can acquire soft masses once supersymmetry is broken. Following [4], the relevant part of the four-dimensional Lagrangian can be written schematically as

$$e^{-1} \mathcal{L} = -\frac{1}{4} \text{Re} \mathcal{F}_{ab} F_{\mu\nu}^a F^{b\mu\nu} - i \text{Re} \mathcal{F}_{ab} \lambda^a \sigma^\mu \partial_\mu \bar{\lambda}^b + (M_{ab} \lambda^a \lambda^b + \text{h.c.}) + \dots, \quad (4.148)$$

where

$$F_{\mu\nu}^a \equiv (F_{\mu\nu}^{\text{SM}}, X_{\mu\nu}), \quad \lambda^a \equiv (\lambda_{\text{SM}}, \lambda_X), \quad (4.149)$$

and $\mathcal{F}_{ab}$ denotes the matrix of holomorphic gauge kinetic functions,

$$\mathcal{F} = \begin{pmatrix} f_{\text{SM}} & \epsilon \\ \epsilon & f_X \end{pmatrix}. \quad (4.150)$$

For a visible sector realized on D7-branes one has, parametrically,

$$f_{\text{SM}} \propto T_{\text{SM}}, \quad (4.151)$$

whereas the gauge kinetic function of an RR photon is a holomorphic function of the complex-structure moduli,

$$f_X = f_X(z^a), \quad (4.152)$$

as follows from the dimensional reduction of type IIB supergravity [13]. Therefore, the RR-photon gauge coupling and the corresponding photino mass are directly sensitive to the complex-structure sector.

In the conventions of [4], the gaugino mass matrix before canonical normalization is controlled by

$$M_{ab} = \frac{F^I}{M_P} \partial_I \mathcal{F}_{ab}, \quad (4.153)$$

where the index I runs over the moduli which participate in supersymmetry breaking. After diagonalizing the gauge kinetic terms, and working to leading order in the kinetic

mixing parameter ϵ , one obtains parametrically

$$\widehat{M} \simeq \frac{1}{M_P} \begin{pmatrix} F^{T_{\text{SM}}} & F^{S,z^a} \partial_{S,z^a} \epsilon - \epsilon F^{S,z^a} \partial_{S,z^a} f_X \\ F^{S,z^a} \partial_{S,z^a} \epsilon - \epsilon F^{S,z^a} \partial_{S,z^a} f_X & F^{S,z^a} \partial_{S,z^a} f_X \end{pmatrix}. \quad (4.154)$$

For RR photons, since

$$f_X = f_X(z^a), \quad (4.155)$$

the diagonal hidden-photino mass reduces to

$$M_X \simeq \frac{1}{M_P} F^{z^a} \partial_{z^a} f_X. \quad (4.156)$$

In a model with a single relevant complex-structure modulus z , this simply becomes

$$M_X \simeq \frac{F^z}{M_P} \partial_z f_X. \quad (4.157)$$

Equation (4.156) makes the phenomenological relevance of the complex-structure F -terms particularly transparent. Any modification of F^z induced by the displacement of the flux-stabilized vacuum feeds directly into the soft mass of the RR photino, up to the model-dependent factor $\partial_z f_X$. Therefore, if the shifted vacuum changes the parametric size of F^z , the hidden-photino mass inherits the same parametric modification, provided that $\partial_z f_X$ does not introduce an additional hierarchy.

After supersymmetry and electroweak symmetry breaking, the bino, wino and neutral higgsinos mix to form the MSSM neutralinos. Since the off-diagonal entry of $\widehat{M}$ is generically non-vanishing, the RR photino λ_X can in turn mix with the neutralino sector. This opens the possibility of collider signatures and, if M_X lies below the MSSM superpartner masses, can also have important cosmological consequences [4]. Hence, determining the complex-structure F -terms is directly relevant for estimating both the hidden photino mass scale and its mixing with the visible sector.

5 Conclusions and Outlook

The Large Volume Scenario is commonly analysed through a two-step procedure. At tree level, background fluxes stabilise the axio-dilaton and the complex-structure moduli at a supersymmetric vacuum, while the Kähler moduli remain unfixed because of the no-scale structure. Perturbative and non-perturbative effects are then included to stabilise the Kähler sector, treating the flux-stabilised fields as effectively frozen at their tree-level values.

This separation provides a controlled description of the vacuum at leading order in the inverse-volume expansion. It is, however, not exact. The effects responsible for Kähler-moduli stabilisation depend in general on the axio-dilaton and complex-structure moduli and therefore generate non-vanishing gradients along these directions. As a consequence, the tree-level flux vacuum is displaced once the complete scalar potential is taken into account.

The purpose of this thesis has been to study this displacement systematically and to determine how it modifies the auxiliary fields associated with the flux-stabilised sector. Particular attention has been devoted to the simultaneous large-volume and weak-coupling expansions,

$$\mathcal{V} \gg 1, \quad g_s \ll 1, \quad (5.1)$$

which provide the parametric control required by the four-dimensional effective description.

The corrected vacuum was determined perturbatively by separating the scalar potential into its leading flux contribution and the subleading terms. This procedure makes explicit that the magnitude of the displacement is determined not only by the correcting potential, but also by the inverse Hessian of the tree-level flux vacuum. The latter carries a non-trivial dependence on the string coupling and plays an essential role in the final power counting.

In the first simplified model, the only displaced flux-stabilised field is the axio-dilaton. The calculation showed that the shift of S generates an additional contribution to its auxiliary field F^S . This contribution scales parametrically as

$$\delta F_{\text{shift}}^S \sim \frac{\xi \overline{W}_0}{\mathcal{V}^2 g_s^2}. \quad (5.2)$$

It is therefore of the same parametric order, in both the inverse-volume and weak-coupling expansions, as the correction obtained by evaluating the corrected supergravity quantities at the unshifted tree-level flux vacuum. The displacement does not generate a new weak-coupling enhancement in the axio-dilaton direction. Nevertheless, it contributes to the numerical coefficient of F^S at the first non-vanishing order and must consequently be included in a complete computation at this accuracy.

The analysis was subsequently extended to a model containing one dynamical complex-structure modulus z , together with the axio-dilaton and the two Kähler moduli.

For generic flux data, the resulting displacements take the form

$$\delta S = \frac{15\xi}{4} \frac{m(S_0, z_0)}{g_s^{5/2}\mathcal{V}} + \mathcal{O}\left(\frac{1}{g_s^{3/2}\mathcal{V}}\right), \quad \delta z = \frac{3\xi}{4} \frac{n(S_0, z_0)}{g_s^{3/2}\mathcal{V}} + \mathcal{O}\left(\frac{1}{\sqrt{g_s}\mathcal{V}}\right), \quad (5.3)$$

where $m(S_0, z_0)$ and $n(S_0, z_0)$ are dimensionless functions determined by the flux data and by the geometry of the tree-level vacuum. Under the genericity assumptions adopted throughout the thesis, these functions are of order one.

Both displacements are suppressed by one inverse power of the compactification volume, consistently with the perturbative treatment. The axio-dilaton displacement is nevertheless enhanced by one additional inverse power of g_s relative to the complex-structure displacement.

Consequently, the leading shift-induced contributions to the auxiliary fields scale as

$$\delta F_{\text{shift}}^S \sim \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s^2}, \quad \delta F_{\text{shift}}^z \sim \frac{\xi \bar{W}_0}{\mathcal{V}^2 g_s}. \quad (5.4)$$

The first expression confirms the result already observed in the rigid complex-structure model: the displacement-induced correction to F^S is of the same parametric order as the previously known LVS contribution,

$$\delta F_{\text{shift}}^S \sim \delta F_{\text{unshifted}}^S. \quad (5.5)$$

The result for the complex-structure auxiliary field is qualitatively different. The standard contribution evaluated at the unshifted flux vacuum scales as

$$\delta F_{\text{unshifted}}^z \sim \frac{\xi \bar{W}_0}{\mathcal{V}^2}. \quad (5.6)$$

The displacement-induced contribution therefore satisfies

$$\boxed{\delta F_{\text{shift}}^z \sim \frac{1}{g_s} \delta F_{\text{unshifted}}^z}. \quad (5.7)$$

It is parametrically enhanced by one inverse power of the string coupling and becomes the leading contribution in the weak-coupling expansion. Finally, we have discussed some phenomenological consequences of the corrected axio-dilaton and complex-structure F -terms. In sequestered LVS scenarios, the shift of the vacuum does not parametrically modify F^S , so that soft terms whose leading contribution is controlled by the axio-dilaton retain the standard LVS scaling [3]. On the other hand, the modified complex-structure F -terms can affect observables that depend directly on F^{z^a} . In particular,

they can lead to parametrically different contributions to scalar masses in the ultra-local regime.

A particularly direct example is provided by the supersymmetric partners of closed-string RR photons. Since the corresponding gauge kinetic functions depend on the complex-structure moduli, their soft masses scale as

$$M_X \sim \frac{F^{z^a}}{M_P} \partial_{z^a} f_X, \quad (5.8)$$

so that any parametric modification of F^{z^a} is directly inherited by the hidden-photino mass, up to the model-dependent factor $\partial_{z^a} f_X$ [4]. These observations show that the vacuum displacement studied in this work can have direct consequences for the low-energy spectrum, even when the parametric structure of the axio-dilaton sector remains unchanged.

The most immediate continuation of this work would be to extend the analysis to the complete moduli space of Type IIB Calabi–Yau orientifold compactifications. One should consider

$$\phi^A = (S, z^a, T_\alpha), \quad a = 1, \dots, h_-^{2,1}, \quad \alpha = 1, \dots, h_+^{1,1}, \quad (5.9)$$

and determine the simultaneous displacement of all the flux-stabilised fields S and z_a induced by Kähler-moduli stabilisation.

At the technical level, this requires generalising the block-matrix treatment of the Hessian from the 4×4 system studied here to the full (S, z^a) sector. The quantities $m(S_0, z_0)$, $n(S_0, z_0)$, $\omega_S(S_0, z_0)$, and $\omega_z(S_0, z_0)$ would be replaced by tensors and vectors on complex-structure moduli space. An important question is whether the g_s^{-1} enhancement found for one complex-structure modulus persists for generic linear combinations of the z^a , or whether it is restricted to particular directions selected by the flux superpotential and by the spectrum of the Hessian.

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A Conventions

A.1 Conventions of the toy model

In this appendix, we collect the conventions and the main formulae employed in the analysis of the two-modulus LVS toy model. We assume that the complex-structure sector has already been stabilized by background fluxes and neglect its fluctuations. The light scalar fields retained in the effective theory are therefore the axio-dilaton S and two Kähler moduli,

$$T_b = \tau_b + i\theta_b, \quad T_s = \tau_s + i\theta_s, \quad (\text{A.1})$$

where τ_b controls the overall size of the compactification manifold, while τ_s measures the volume of a rigid blow-up divisor supporting non-perturbative effects.

The Einstein-frame volume is assumed to have a Swiss-cheese form,

$$\mathcal{V} = \tilde{\mathcal{V}}(\tau_b) - c\tau_s^{3/2}, \quad c > 0, \quad (\text{A.2})$$

with

$$\tilde{\mathcal{V}}(\tau_b) \propto \tau_b^{3/2}. \quad (\text{A.3})$$

We adopt the convention

$$S = \frac{1}{g_s} + iC_0, \quad \text{Re } S = \frac{1}{g_s}, \quad (\text{A.4})$$

where C_0 is the Ramond–Ramond axion (2) and $g_s \ll 1$ is the string coupling. At tree level in both g_s and the α' expansion, the Kähler potential for the axio-dilaton and Kähler-moduli sectors is

$$K_{\text{tree}} = -\log(S + \bar{S}) - 2\log \mathcal{V}. \quad (\text{A.5})$$

The constant contribution from the stabilized complex-structure sector has been absorbed into the normalization of the four-dimensional theory.

The leading perturbative α'^3 correction modifies the Kähler potential according to

$$K = -\log(S + \bar{S}) - 2\log \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right), \quad (\text{A.6})$$

where

$$\hat{\xi} \equiv \frac{\xi}{g_s^{3/2}}, \quad \xi = -\frac{\chi(X_6)\zeta(3)}{2(2\pi)^3}. \quad (\text{A.7})$$

Here $\chi(X_6)$ denotes the Euler characteristic of the internal Calabi–Yau , while $\zeta(3)$ is the value of the Riemann zeta function at 3.

In the large-volume regime,

$$\frac{\widehat{\xi}}{\mathcal{V}} \ll 1, \quad (\text{A.8})$$

Eq. (A.6) can be expanded as

$$K = K_{\text{tree}} + \delta K = -2 \log \mathcal{V} - \log(S + \bar{S}) - \frac{\xi}{g_s^{3/2} \mathcal{V}} + \mathcal{O}\left(\frac{\xi^2}{g_s^3 \mathcal{V}^2}\right). \quad (\text{A.9})$$

In the absence of dynamical complex-structure moduli, the flux superpotential is linear in the axio-dilaton and can be written as

$$W_{\text{flux}}(S) = A + BS, \quad A, B \in \mathbb{C}. \quad (\text{A.10})$$

The small Kähler modulus T_s supports a non-perturbative contribution of the form

$$W_{\text{np}} = \mathcal{A} e^{-a_s T_s}, \quad (\text{A.11})$$

where $\mathcal{A} \in \mathbb{C}$ is the non-perturbative prefactor. In the present toy model, $\mathcal{A}$ is treated as independent of S .

The complete superpotential is therefore

$$W = W_{\text{flux}}(S) + W_{\text{np}}(T_s) = A + BS + \mathcal{A} e^{-a_s T_s}. \quad (\text{A.12})$$

Combining the corrected Kähler potential (A.6) with the superpotential (A.12), the $\mathcal{N} = 1$ F-term scalar potential is

$$V_F = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right). \quad (\text{A.13})$$

After minimizing the axionic component θ_s , the relative phase between W_0 and the non-perturbative contribution is fixed so that their interference term is negative. In the regime

$$\mathcal{V} \gg 1, \quad a_s \tau_s \gg 1, \quad (\text{A.14})$$

the resulting scalar potential takes the standard LVS form

$$V_{\text{LVS}} \simeq V_{0,F} \left[\frac{\alpha \sqrt{\tau_s} e^{-2a_s \tau_s}}{c \mathcal{V}} - \frac{\beta |W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{3 \xi |W_0|^2}{4 g_s^{3/2} \mathcal{V}^3} \right]. \quad (\text{A.15})$$

With the conventions adopted here,

$$V_{0,F} = \frac{g_s}{2}, \quad \alpha = \frac{8a_s^2 |\mathcal{A}|^2}{3}, \quad \beta = 4a_s |\mathcal{A}|. \quad (\text{A.16})$$

The normalization $V_{0,F} = g_s/2$ assumes that the constant complex-structure contribution to e^K has been set to unity.

The minimum is determined by

$$\partial_{\mathcal{V}} V_{\text{LVS}} = 0, \quad \partial_{\tau_s} V_{\text{LVS}} = 0. \quad (\text{A.17})$$

Differentiating Eq. (A.15) with respect to τ_s gives

$$0 = \frac{\alpha}{c\mathcal{V}} e^{-2a_s\tau_s} \left(\frac{1}{2\sqrt{\tau_s}} - 2a_s\sqrt{\tau_s} \right) - \frac{\beta|W_0|}{\mathcal{V}^2} e^{-a_s\tau_s} (1 - a_s\tau_s). \quad (\text{A.18})$$

Solving for $\mathcal{V}$, one obtains

$$\mathcal{V} = \frac{\beta|W_0|c}{\alpha} \sqrt{\tau_s} e^{a_s\tau_s} \frac{1 - a_s\tau_s}{\frac{1}{2} - 2a_s\tau_s}. \quad (\text{A.19})$$

In the controlled regime $a_s\tau_s \gg 1$, this becomes

$$\mathcal{V} = \frac{\beta|W_0|c}{2\alpha} \sqrt{\tau_s} e^{a_s\tau_s} \left[1 + \mathcal{O}\left(\frac{1}{a_s\tau_s}\right) \right]. \quad (\text{A.20})$$

The derivative with respect to the overall volume gives

$$0 = -\frac{\alpha\sqrt{\tau_s} e^{-2a_s\tau_s}}{c\mathcal{V}^2} + 2\frac{\beta|W_0|c\tau_s e^{-a_s\tau_s}}{\mathcal{V}^3} - \frac{9}{4} \frac{\xi|W_0|^2}{g_s^{3/2}\mathcal{V}^4}. \quad (\text{A.21})$$

Substituting Eq. (A.20) into Eq. (A.21) yields

$$\tau_s^{3/2} = \frac{3\alpha}{\beta^2 c} \frac{\xi}{g_s^{3/2}} \left[1 + \mathcal{O}\left(\frac{1}{a_s\tau_s}\right) \right]. \quad (\text{A.22})$$

Finally,

$$\tau_s = \frac{1}{g_s} \left(\frac{\xi}{2c} \right)^{2/3} \left[1 + \mathcal{O}\left(\frac{1}{a_s\tau_s}\right) \right]. \quad (\text{A.23})$$

Inserting this result into Eq. (A.20), we finally obtain

$$\mathcal{V} = \frac{\beta|W_0|c}{2\alpha\sqrt{g_s}} \left(\frac{\xi}{2c} \right)^{1/3} \times \exp \left[\frac{a_s}{g_s} \left(\frac{\xi}{2c} \right)^{2/3} \right] \left[1 + \mathcal{O}\left(\frac{1}{a_s\tau_s}\right) \right]. \quad (\text{A.24})$$

These expressions reproduce the standard LVS minimum [12].

The result displays the characteristic LVS hierarchy:

$$\tau_s \sim \frac{1}{g_s}, \quad \mathcal{V} \sim \exp \left(\frac{\text{const.}}{g_s} \right). \quad (\text{A.25})$$

A.2 Extension to a dynamical complex-structure modulus

We now extend the toy model introduced in Appendix A.1 by including a single dynamical complex-structure modulus z . All conventions concerning the axio-dilaton, the Kähler moduli and the Calabi–Yau volume remain unchanged.

The first modification concerns the tree-level Kähler potential, which now includes the complex-structure contribution,

$$K_{\text{tree}} = -2 \log \mathcal{V} - \log(S + \bar{S}) + K_{\text{cs}}(z, \bar{z}). \quad (\text{A.26})$$

We define

$$K_{\text{cs}}(z, \bar{z}) = -\log \mathcal{I}(z, \bar{z}), \quad \mathcal{I}(z, \bar{z}) \equiv -i \int_{X_6} \Omega(z) \wedge \bar{\Omega}(\bar{z}), \quad (\text{A.27})$$

where $\Omega(z)$ is the holomorphic $(3, 0)$ -form of the Calabi–Yau manifold (2.66).

The leading α'^3 correction is the same as in Eq. (A.6), since it modifies only the Kähler-moduli and axio-dilaton sectors. The second modification concerns the flux superpotential. The constants A and B appearing in the rigid model (A.10) are promoted to holomorphic functions of the complex-structure modulus,

$$W_{\text{flux}}(S, z) = A(z) + SB(z). \quad (\text{A.28})$$

The sign multiplying $SB(z)$ is conventional and may be absorbed into the definition of $B(z)$. Including the non-perturbative contribution associated with the small Kähler modulus, the complete superpotential becomes

$$W = A(z) + SB(z) + \mathcal{A}(z)e^{-a_s T_s}. \quad (\text{A.29})$$

After fixing S and z at their tree-level values, the large-volume scalar potential retains precisely the same functional dependence on $\mathcal{V}$ and τ_s as in Eq. (A.15). The only change is the overall normalization

$$V_{0,F} = \frac{g_s}{2\mathcal{I}} \quad (\text{A.30})$$

Thus, introducing the complex-structure modulus does not modify the leading LVS stabilization of the Kähler sector when S and z are held at their tree-level supersymmetric values.

B Explicit Computations

In this appendix, we collect the explicit computations required to determine the displacement of the axio-dilaton and complex-structure moduli induced by the subleading scalar potential. Unless otherwise stated, all flux-dependent quantities are evaluated at the supersymmetric tree-level vacuum

$$\hat{D}_S W_{\text{flux}}|_{\phi_0} = \hat{D}_z W_{\text{flux}}|_{\phi_0} = 0. \quad (\text{B.1})$$

In the complex-coordinate basis

$$(S, \bar{S}, z, \bar{z}), \quad (\text{B.2})$$

the tree-level Hessian can be written in block form as

$$H = \frac{1}{\mathcal{IV}^2} \begin{pmatrix} \mathbb{A} & \mathbb{B} \\ \mathbb{B}^T & \mathbb{C} \end{pmatrix}, \quad (\text{B.3})$$

where,

$$\mathbb{A} = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \quad \mathbb{B} = \begin{pmatrix} u & v \\ w & t \end{pmatrix} \quad \mathbb{C} = \begin{pmatrix} b & c \\ c & d \end{pmatrix} \quad (\text{B.4})$$

The corresponding entries are

$$a = \frac{|B|^2 + K_0^{z\bar{z}}|P|^2}{2} g_s, \quad b = \bar{W}_0 g_s Q, \quad d = g_s W_0 \bar{Q} \quad (\text{B.5})$$

$$c = \frac{g_s}{2} (K_0^{z\bar{z}}|Q|^2 + K_{0,z\bar{z}}|W_0|^2) + \frac{2}{g_s}|P|^2 \quad (\text{B.6})$$

$$\begin{aligned} u &= g_s \bar{W} P, & t &= g_s W_0 \bar{P} \\ v &= \frac{g_s}{2} (K_0^{z\bar{z}} \bar{Q} P), & w &= \frac{g_s}{2} (K_0^{z\bar{z}} Q \bar{P}) \end{aligned} \quad (\text{B.7})$$

where

$$P \equiv D_Z B, \quad Q \equiv \partial_z D_z W. \quad (\text{B.8})$$

The inverse Hessian can be obtained by means of the Schur-complement formula. Assuming that $\mathbb{C}$ and the associated Schur complement are invertible, we define

$$\mathbb{E} \equiv \mathbb{A} - \mathbb{B} \mathbb{C}^{-1} \mathbb{B}^T. \quad (\text{B.9})$$

The inverse of the Hessian is then

$$H^{-1} = \mathcal{IV}^2 \begin{pmatrix} \mathbb{E}^{-1} & -\mathbb{E}^{-1}\mathbb{B}\mathbb{C}^{-1} \\ -\mathbb{C}^{-1}\mathbb{B}^T\mathbb{E}^{-1} & \mathbb{C}^{-1} + \mathbb{C}^{-1}\mathbb{B}^T\mathbb{E}^{-1}\mathbb{B}\mathbb{C}^{-1} \end{pmatrix}. \quad (\text{B.10})$$

This representation is particularly useful because it reduces the inversion of the full 4×4 Hessian to the inversion of the 2×2 matrices $\mathbb{C}$ and $\mathbb{E}$.

We first invert the complex-structure block. Defining

$$\Delta_C = bd - c^2 \quad (\text{B.11})$$

we obtain

$$\mathbb{C}^{-1} = \frac{1}{\Delta_C} \begin{pmatrix} d & -c \\ -c & b \end{pmatrix}. \quad (\text{B.12})$$

It follows that

$$\mathbb{B}\mathbb{C}^{-1} = \frac{1}{\Delta_C} \begin{pmatrix} ud - vc & vb - uc \\ wd - tc & tb - wc \end{pmatrix}. \quad (\text{B.13})$$

Writing

$$\mathbb{E} = \begin{pmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{pmatrix}, \quad (\text{B.14})$$

the entries of the Schur complement are,

$$\begin{aligned} E_{11} &= -\frac{1}{\Delta_C}(du^2 - 2cuv + bv^2) \\ E_{12} &= a - \frac{1}{\Delta_C}(duw - c(ut + vw) + bvt) \\ E_{22} &= -\frac{1}{\Delta_C}(dw^2 - 2cwt + bt^2) \end{aligned} \quad (\text{B.15})$$

Defining

$$\Delta_E \equiv \det \mathbb{E} = E_{11}E_{22} - E_{12}^2, \quad (\text{B.16})$$

we have

$$\mathbb{E}^{-1} = \frac{1}{\Delta_E} \begin{pmatrix} E_{22} & -E_{12} \\ -E_{12} & E_{11} \end{pmatrix}. \quad (\text{B.17})$$

Using the upper block of Eq. (B.10), the axio-dilaton displacement can be written as

$$\begin{pmatrix} \delta S \\ \delta \bar{S} \end{pmatrix} = -\frac{\mathcal{IV}^2}{\Delta_E} \begin{pmatrix} E_{22} & -E_{12} \\ -E_{12} & E_{11} \end{pmatrix} \left\{ \begin{pmatrix} \partial_S V_1 \\ \partial_{\bar{S}} V_1 \end{pmatrix} - \frac{1}{\Delta_C} \begin{pmatrix} ud - vc & cu + bu \\ wd - ct & -cw + tb \end{pmatrix} \begin{pmatrix} \partial_z V_1 \\ \partial_{\bar{z}} V_1 \end{pmatrix} \right\} \quad (\text{B.18})$$

For convenience, we introduce

$$\eta(z_0, S_0) = \partial_S V - \frac{(ud - vc)\partial_z V + (vb - uc)\partial_{\bar{z}} V}{\Delta_c} \quad (\text{B.19})$$

and

$$\sigma(z_0, S_0) = \partial_{\bar{S}} V_1 - \frac{(wd - tc)\partial_z V_1 + (tb - wc)\partial_{\bar{z}} V_1}{\Delta_c} \quad (\text{B.20})$$

The displacement of S is therefore,

$$\delta S = -\mathcal{I}\mathcal{V}^2 \left\{ (E^{-1})_{11}\eta(z_0, S_0) + (E^{-1})_{12}\sigma(z_0, S_0) \right\}. \quad (\text{B.21})$$

It remains to determine the derivatives of V_1 with respect to the axio-dilaton and complex-structure fields. To organize the different contributions, we introduce

$$V_{I\bar{J}} \equiv e^K K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} \quad (\text{B.22})$$

No summation over the displayed indices I and $\bar{J}$ is implied in Eq. (B.22). When a Kähler-modulus index (T) appears, instead, it denotes the sum over the Kähler moduli. In particular,

$$V_{I\bar{T}} \equiv \sum_i e^K K^{I\bar{T}_i} D_I W D_{\bar{T}_i} \bar{W}, \quad (\text{B.23})$$

with analogous definitions for $V_{T\bar{I}}$.

We retain the leading contributions in the simultaneous large-volume and weak-coupling expansion.

The relevant contributions to the complex-structure gradient are

$$\begin{aligned} \partial_z V_{S\bar{S}} &\simeq -\frac{3}{2\mathcal{I}} \frac{\xi \bar{W}_0 P}{g_s^{3/2} \mathcal{V}^3}, \\ \partial_z V_{S\bar{z}} &= 0, \\ \partial_z V_{S\bar{T}} &\simeq \frac{9}{4\mathcal{I}} \frac{\xi \bar{W}_0 P}{g_s^{3/2} \mathcal{V}^3}, \\ \partial_z V_{z\bar{S}} &= 0, \\ \partial_z V_{z\bar{z}} &= \mathcal{O}\left(\frac{1}{\sqrt{g_s} \mathcal{V}^3}\right), \\ \partial_z V_{z\bar{T}} &= 0, \\ \partial_z V_{T\bar{S}} &= \mathcal{O}(\mathcal{V}^{-5}), \\ \partial_z V_{T\bar{z}} &= 0, \\ \partial_z V_{\text{LVS}} &= \mathcal{O}\left(\frac{\sqrt{g_s}}{\mathcal{V}^3}\right). \end{aligned} \quad (\text{B.24})$$

In particular, the contribution from $V_{z\bar{z}}$ follows from

$$\begin{aligned}\partial_z V_{z\bar{z}} = & e^K (K_z K^{z\bar{z}} + \partial_z K^{z\bar{z}}) D_z W D_{\bar{z}} \bar{W} \\ & + e^K K^{z\bar{z}} (\partial_z D_z W) D_{\bar{z}} \bar{W} + e^K D_z W \bar{W},\end{aligned}\quad (\text{B.25})$$

where we used

$$\partial_z D_{\bar{z}} \bar{W} = K_{z\bar{z}} \bar{W}, \quad K^{z\bar{z}} K_{z\bar{z}} = 1. \quad (\text{B.26})$$

At the flux minimum,

$$\hat{D}_z W = \hat{K}_z W_{\text{np}}, \quad (\text{B.27})$$

and therefore

$$\partial_z V_{z\bar{z}} = e^K K^{z\bar{z}} (\partial_z D_z W_{\text{flux}})_0 K_{\bar{z}} \bar{W}_{\text{np}} + e^K K_z W_{\text{np}} \bar{W}_0 + \mathcal{O}(\mathcal{V}^{-4}). \quad (\text{B.28})$$

Using

$$e^K \sim \frac{g_s}{\mathcal{V}^2}, \quad (\partial_z D_z W_{\text{flux}})_0 \sim \mathcal{O}(1), \quad W_{\text{np}} \sim \frac{W_0}{\sqrt{g_s} \mathcal{V}}, \quad (\text{B.29})$$

one obtains the scaling reported in Eq. (B.24).

The leading weak-coupling contribution to the complete complex-structure gradient is thus

$$\partial_z V_1 = \frac{3}{4\mathcal{I}} \frac{\xi \bar{W}_0 P}{g_s^{3/2} \mathcal{V}^3} + \mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right). \quad (\text{B.30})$$

Since V_1 is real,

$$\partial_{\bar{z}} V_1 = (\partial_z V_1)^*. \quad (\text{B.31})$$

The corresponding contributions to the axio-dilaton gradient are

$$\begin{aligned}\partial_S V_{S\bar{S}} & \simeq -\frac{3}{8\mathcal{I}} \frac{\xi \sqrt{g_s} |W_0|^2}{\mathcal{V}^3}, \\ \partial_S V_{S\bar{z}} & = 0, \\ \partial_S V_{S\bar{T}} & = \mathcal{O}(\mathcal{V}^{-4}), \\ \partial_S V_{z\bar{S}} & = 0, \\ \partial_S V_{z\bar{z}} & = \mathcal{O}\left(\frac{g_s^{3/2}}{\mathcal{V}^3}\right), \\ \partial_S V_{z\bar{T}} & = 0, \\ \partial_S V_{T\bar{S}} & \simeq \frac{9}{16\mathcal{I}} \frac{\xi \sqrt{g_s} |W_0|^2}{\mathcal{V}^3}, \\ \partial_S V_{T\bar{z}} & = 0, \\ \partial_S V_{\text{LVS}} & \simeq \frac{9}{32\mathcal{I}} \frac{\xi \sqrt{g_s} |W_0|^2}{\mathcal{V}^3}.\end{aligned}\quad (\text{B.32})$$

Keeping the leading terms in the weak-coupling expansion, we therefore find

$$\partial_S V_1 = \frac{15}{32} \frac{\xi \sqrt{g_s} |W_0|^2}{\mathcal{I} \mathcal{V}^3} + \mathcal{O} \left(\frac{g_s^{3/2}}{\mathcal{V}^3} \right). \quad (\text{B.33})$$

Thus,

$$\boxed{\partial_S V_1 = \frac{15}{32} \frac{\xi \sqrt{g_s} |W_0|^2}{\mathcal{I} \mathcal{V}^3}} \quad (\text{B.34})$$

at the order relevant for the computation of the leading vacuum displacement.

At this stage, all the ingredients required to compute the shift δS (B.21) explicitly have been determined. For convenience, we introduce the following quantity,

$$m(S_0, z_0) \equiv -g_s^3 |W_0|^2 \left\{ (E^{-1})_{11} + (E^{-1})_{12} - \frac{(E^{-1})_{11} \nabla_z^\eta V - (E^{-1})_{12} \nabla_z^\sigma V}{\partial_S V} \right\} \quad (\text{B.35})$$

where,

$$\nabla_z^\eta \equiv \frac{(ud - vc) \partial_z + (vb - uc) \partial_{\bar{z}}}{\Delta_c} \quad (\text{B.36})$$

$$\nabla_z^\sigma \equiv \frac{(wd - tc) \partial_z + (tb - wc) \partial_{\bar{z}}}{\Delta_c} \quad (\text{B.37})$$

Then,

$$\boxed{\delta S = \frac{15\xi}{4} \frac{m(z_0, S_0)}{\mathcal{V} g_s^{5/2}}} \quad (\text{B.38})$$

We can now use Eq. (4.124) to compute the complex-structure modulus shift δz .

$$\delta z = -\frac{(du - cv) \delta S + (dw - ct) \delta \bar{S}}{\Delta_C} - \frac{\mathcal{I} \mathcal{V}^2}{\Delta_C} (d \partial_z V_1 - c \partial_{\bar{z}} V) \quad (\text{B.39})$$

Let us define

$$n(z_0, S_0) = \frac{3}{8g_s} (mX + m^* Y) - \frac{d\bar{W}_0 P - cW_0 \bar{P}}{\Delta_c} \quad (\text{B.40})$$

where

$$X \equiv -\frac{du - cv}{\Delta_C}, \quad Y \equiv -\frac{dw - ct}{\Delta_C} \quad (\text{B.41})$$

and, finally,

$$\boxed{\delta z = \frac{3\xi}{4} \frac{n(S_0, z_0)}{g_s^{3/2} \mathcal{V}}} \quad (\text{B.42})$$

We are now in a position to compute the corrections to the F -terms explicitly. We shall present the detailed calculation only for the complex-structure direction, since the derivation for the axio-dilaton proceeds in exactly the same way.

$$\delta F_{(2)}^z = \delta z \partial_z F_{\text{tree}}^z|_{\phi_0} + \delta S \partial_S F_{\text{tree}}^z|_{\phi_0} + \delta \bar{z} \partial_{\bar{z}} F_{\text{tree}}^z|_{\phi_0} \delta \bar{S} \partial_{\bar{S}} F_{\text{tree}}^z|_{\phi_0} \quad (\text{B.43})$$

Where,

$$\begin{aligned} \partial_{\bar{z}} F_{\text{tree}}^z &= \frac{1}{\mathcal{V}} \sqrt{\frac{g_s}{2\mathcal{I}}} K_0^{z\bar{z}} f(z_0, S_0), & \partial_z F_{\text{tree}}^z &= \frac{1}{\mathcal{V}} \sqrt{\frac{g_s}{2\mathcal{I}}} \bar{W}_0 \\ \partial_S F_{\text{tree}}^S &= 0 & \partial_{\bar{S}} F_{\text{tree}}^z &= \frac{1}{\mathcal{V}} \sqrt{\frac{g_s}{2\mathcal{I}}} K_0^{z\bar{z}} \bar{P} \end{aligned} \quad (\text{B.44})$$

Where

$$f(z_0, S_0) \equiv \bar{W}_{0,\bar{z}\bar{z}} + K_{0,\bar{z}\bar{z}} \bar{W}_0 + K_{0,\bar{z}} \bar{W}_{0,\bar{z}} \quad (\text{B.45})$$

Then we define,

$$\omega_z(S_0, z_0) \equiv K_{0,\bar{z}\bar{z}} n(S_0, z_0) \bar{W}_0 + n^*(S_0, z_0) f(S_0, z_0) + \frac{5}{8g_s} m^*(S_0, z_0) \bar{P} \quad (\text{B.46})$$

And we obtain,

$$\delta F_{(2)}^z = \frac{3}{4\sqrt{2\mathcal{I}}} \frac{\xi K_0^{z\bar{z}}}{g_s \mathcal{V}^2} \omega(S_0, z_0) \quad (\text{B.47})$$

B.1 Power counting

We are now interested in estimating the dependence on g_s of the quantities obtained above. In particular, we focus on Eq. (B.47). The quantity whose scaling we ultimately want to determine is $\omega_z(S_0, z_0)$, defined in Eq. (B.46).

Under the working assumption

$$W_0 \sim \mathcal{O}(1), \quad (\text{B.48})$$

we need to determine the g_s -dependence of the functions

$$n(S_0, z_0), \quad f(S_0, z_0), \quad m(S_0, z_0), \quad (\text{B.49})$$

as well as that of P . Our final goal is to show that

$$\omega_z(S_0, z_0) \sim \mathcal{O}(1). \quad (\text{B.50})$$

We begin with $m(S_0, z_0)$, whose definition is reported here for convenience:

$$m(S_0, z_0) = -g_s^3 |W_0|^2 \left[(E^{-1})_{11} + (E^{-1})_{12} - \frac{(E^{-1})_{11} \nabla_z^\eta V - (E^{-1})_{12} \nabla_z^\sigma V}{\partial_S V} \right]. \quad (\text{B.51})$$

The relevant components of the inverse matrix are

$$(E^{-1})_{11} = \frac{E_{22}}{\Delta_E}, \quad (E^{-1})_{12} = -\frac{E_{12}}{\Delta_E}. \quad (\text{B.52})$$

From the definitions of the matrix elements (B.5), (B.6), we have

$$E_{22} \sim -\frac{dw^2}{\Delta_c}, \quad \Delta_c \sim c^2. \quad (\text{B.53})$$

Moreover, recalling the definitions in Eqs. (B.5), (B.6), and (B.8), we find

$$Q \sim \mathcal{O}(1), \quad P \sim g_s. \quad (\text{B.54})$$

It follows that

$$d \sim g_s, \quad w \sim g_s^2, \quad c \sim g_s. \quad (\text{B.55})$$

Therefore,

$$E_{22} \sim -\frac{dw^2}{c^2} \sim \frac{g_s^2 g_s^4}{g_s^2} \sim g_s^4. \quad (\text{B.56})$$

On the other hand,

$$E_{12} \sim a \sim g_s^3. \quad (\text{B.57})$$

Assuming that the leading contribution to the determinant is

$$\Delta_E \sim E_{12}^2 \sim a^2 \sim g_s^6, \quad (\text{B.58})$$

we obtain

$$(E^{-1})_{11} \sim \frac{dw^2}{c^2 a^2} \sim g_s^{-3}, \quad (\text{B.59})$$

while

$$(E^{-1})_{12} \sim \frac{1}{a} \sim g_s^{-3}. \quad (\text{B.60})$$

Furthermore,

$$\nabla_z^\eta V_{(1)} \sim \frac{ud, \partial_z V_{(1)}}{\Delta_c} \sim \sqrt{g_s}. \quad (\text{B.61})$$

Hence, at leading order in the weak-coupling expansion,

$$m(S_0, z_0) \sim \mathcal{O}(1). \quad (\text{B.62})$$

Using the same logic, it is possible to show that

$$n(S_0, z_0) \sim \mathcal{O}(1) \quad (\text{B.63})$$

and finally,

$$\omega_z(S_0, z_0) \sim \mathcal{O}(1) \quad (\text{B.64})$$

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