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Phenomenology of Closed String Dark Photons

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*"Murphy's law doesn't mean that something bad will happen.
It means that whatever can happen, will happen."*

— Cooper, Interstellar

Abstract

This Master Thesis project is focused on the study of the phenomenology of *Ramond-Ramond Photons* in type IIB string compactifications.
These modes are Standard Model gauge-singlets which are typically massless, and so do not develop any physical mixing with the hypercharge $U(1)$. They are instead gravitationally coupled to closed string moduli. Moreover, a generic Calabi-Yau compactification can feature hundreds of these *Dark Photons*.
It is well known that, after neutrinos decoupled from the thermal bath, they have remained as relativistic degrees of freedom constituting the cosmic neutrino background. While in the Standard Model this number of relativistic species is fixed as the effective number of neutrino species at Cosmic Microwave Background temperature and at Big Bang Nucleosynthesis temperature, any measured excess from the Standard Model value would constitute an evidence of extra *Dark Radiation* and a hint for new physics. In the context of string models of the early universe, these relativistic degrees of freedom, decoupled from the Standard Model ones, could be generated by the decay of the lightest modulus.
The main purpose of this thesis is therefore to study the cosmological production of Ramond-Ramond Dark Photons from the decay of the longest-living moduli, setting bounds from the requirement to avoid Dark Radiation overproduction.

Contents

Introduction

Although the Standard Model represents a successful milestone for the physics of the twentieth century, describing with high precision elementary particle physics and electromagnetic, weak and strong interaction, it can only be fully trusted as an Effective Field Theory (EFT) up to the electroweak energy scale (i.e. up to few TeV). In fact, there are several issues that cannot be addressed within the "classical" Standard Model of physics. The most fundamental limitation is the fact that quantum gravity is not embedded in it. One way out is to consider gravity as an Effective Field Theory itself, valid only up to a cutoff smaller than the Planck scale where quantum effects are no more negligible and every description fails to foresee what could happen. Some other known problems that the Standard Model cannot address are listed below.

- Hierarchy problem: The mass of the Higgs boson receives loop corrections that one would naturally expect to be of the order of the cutoff of the theory and therefore of the order of the Planck scale. However, it is well known that the Higgs boson mass is about $125GeV$, well below the cutoff scale of the theory. This value has no explanation in the context of Standard Model without an extreme fine tuning.

- Strong CP problem: The QCD coupling term $\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$ is a CP violating term containing a parameter, θ, which turns out to be unnaturally small experimentally. However, there is no apparent explanation in the Standard Model for this parameter to be very small. Moreover, a well known solution to this problem requires the existence of a Peccei-Quinn axion, a new particle that would only be present in Beyond the Standard Model Physics.

- Cosmological constant problem: Observations of the universe tell us that the value of the cosmological constant interpreted as vacuum energy is close to zero $\Lambda \sim 10^{-120} M_P^4$. However, once again, in the Standard Model there is no way to naturally achieve this value.

Other unanswered questions arise if one considers the cosmological aspect. In fact, taking into account the ΛCDM model with the inflationary solution to explain the flatness problem, the horizon problem and the CMB perturbations, the Standard Model of particle physics does not contain any suitable candidate neither to explain the cold Dark Matter, nor to drive inflation.

Nowadays, String Theory is considered to be a very promising framework that could provide a quantum theory combining gravity and the other fundamental interactions. In addition to this, it could offer solutions to the unanswered Standard Model problems thanks to the presence of an additional symmetry, Supersymmetry, and of a large number of degrees of freedom due to the existence of extra dimensions. Moreover, the presence of these extra dimensions, which in Type IIB string theory need to be 10, does not constitute a problem since it is possible to reduce the theory to a 4-dimensional one through the dimensional reduction in which the extra 6-dimensions are compactified on a complex manifold called Calabi-Yau. An important characteristic of the dimensional reduction is that the resulting spectrum contains massless scalar fields called moduli, which, at tree level, represent flat directions of the scalar potential and which parametrize the shape and the size of the extra dimensions. Due to the fact that these scalar fields could generate unobserved fifth forces, it is necessary to lift these flat directions by making the moduli acquire a Vacuum Expectation Value (VEV), and so a mass. The stabilization model we are going to focus on is the Large Volume Scenario (LVS) in which the complex structure moduli are stabilized by 3-form fluxes, while the Kähler moduli are stabilized by an interplay between perturbative corrections to the Kähler potential and non-perturbative corrections to the superpotential.

From a cosmological point of view, in addition to the stabilization mechanism it is interesting to study how these fields interact gravitationally with the other ordinary particles. Planck suppressed interactions imply that these scalars could live long enough to dominate the energy density of the universe, and therefore to generate a period of early moduli domination which makes them the main drivers of the reheating process.
While late-time decaying string moduli could have important effects also on Dark Matter, here we are going to consider another effect of the decay of the longest-living modulus, that is the production of hidden relativistic degrees of freedom that remain relativistic at CMB temperature and at Big Bang Nucleosynthesis (BBN) temperature. These relativistic degrees of freedom constitute the so called Dark Radiation, whose abundance is parametrized as an excess with respect to the effective number of neutrino species that, for the Standard Model, is $N_{eff,BBN} = 3$ and $N_{eff,CMB} = 3.046$ [1, 2]. Any departure from these values would constitute new physics Beyond the Standard Model. It has been experimentally observed that the excess of hidden relativistic degrees of freedom over the Standard Model predictions is bounded by $\Delta N_{eff} \sim 0.2 - 0.5$ [1].

In the framework we are going to consider in this thesis, the Large Volume Scenario with sequestered Standard Model, we obtain that reheating is driven by the decay of the modulus parametrizing the large volume. The aim of this thesis is to estimate the amount

of Dark Radiation produced by the decay of the large volume modulus into Ramond-Ramond $U(1)$s and their consequent influence on the number of effective neutrino species. These Ramond-Ramond $U(1)$s (Dark Photons) are massless modes coming from the reduction of the Ramond-Ramond C_4 form in Type IIB String Compactifications, and they could behave as relativistic degrees of freedom contributing to a potentially harmful excess of Dark Radiation.
If the amount of these Dark Photons, and therefore of Dark Radiation, is large, it could happen that observational bounds on ΔN_{eff} are not respected, ruling out those models where the reheating is driven by the decay of the volume modulus.

In Chapter 1 we are going to study the effective theory of Type IIB String Compactifications. After looking briefly at the massless spectrum of Type IIB theory and its 10-dimensional bosonic action, we are going to focus on the compactification of the extra dimensions: starting from the simple example of Kaluza-Klein dimensional reduction, we are going to introduce the more advanced concepts of Calabi-Yau manifolds and of orientifolds. In particular we are going to study the moduli space of Calabi-Yau 3-folds in Type IIB compactifications and we are going to assign to each of the relevant moduli of the theory the corresponding Kähler potential.

In Chapter 2 we are going to address the problem of the moduli flat directions by introducing the concept of flux-compactification: after obtaining the Superpotential and showing the No-Scale Structure of the scalar potential, we are going to add the corrections allowed by the Non-Renormalization Theorems (perturbative and non-perturbative). Thanks to these corrections, we will stabilize the Kähler moduli introducing the Large Volume Scenario and briefly studying the cosmological moduli problem and the problems related to the decay of the longest-living moduli. After this, we are going to focus on the sequestered Large Volume Scenario that will constitute the framework for our analysis of Dark Radiation from the decay of the large volume modulus. Then, in the final part of this Chapter, we are going to focus on the decay of the large volume modulus into axions which could constitute another possible source of Dark Radiation.

The main results of this thesis are presented in Chapter 3 where we will first introduce Ramond-Ramond Photons. In particular, we are going to study how their kinetic terms arise from the dimensional reduction of the 10-dimensional bosonic action introduced in Chapter 1, more precisely from the kinetic term of the Ramond-Ramond C_4 in Type IIB theory. Then, we are going to focus on their gauge kinetic function and we are going to show its holomorphic dependence on the complex structure moduli. Moreover, we are going to highlight the topological requirements to have such a moduli dependence in the gauge kinetic function, finally discussing the generality of this configuration while giving an explicit example of a Calabi-Yau that could embed it.

We will then continue our discussion on Dark Radiation production from the decay of the longest-living moduli considering a simple, but quite general model. Due to the dependence of the Ramond-Ramond Photons gauge kinetic function on the complex structure moduli, we will analyze the perturbative production mechanism from moduli decay. We are going to consider the case of a Kähler modulus associated to the Calabi-Yau volume and of a complex structure modulus mixed to the former one by a loop correction, in order to find a viable decay channel for the longest-living modulus (the volume one) into these Ramond-Ramond Photons. We are going to canonically normalize the fields to find the mixing of interest by explicitly computing the scalar potential, the mass matrix, and by explicitly solving the corresponding eigen-system.

Finally we are going to close the circle on Dark Radiation from Ramond-Ramond Dark Photons by explicitly computing the decay rate of the longest living modulus into these Photons. The final step will be the estimation of the effective number of neutrino species due to the presence of this additional source of Dark Radiation in the sequestered Large Volume Scenario with the Standard Model realized on $D3$-branes at singularities.

In the final Chapter of this thesis (Chapter 4), we are going to summarize the main goals and the main findings of this work and we are going to present briefly some open paths for possible developments.

Chapter 1

Effective Theory of Type IIB String Compactifications

"See that the imagination of nature is far, far greater than the imagination of man."
— Richard P. Feynman

In this chapter, we are going to review the basics of Type IIB Superstring Theories starting from the massless spectrum, through the 10-dimensional action, towards the concept of Calabi Yau (CY) compactifications.
It is well known that the inclusion of supersymmetry in the bosonic string theory and the subsequent projection (GSO projection) of the quantized sectors allow us to introduce fermions and to get rid of the tachyonic modes in the spectrum. Still, we have that at low energies there are 5 different theories related by duality transformations that constitute the 10-dimensional description of the spacetime.

For the purposes of this work, we are going to focus our attention on Type IIB Superstring Theories since the low effective action for these models are well understood and, moreover, they present many attractive features from the phenomenological point of view. First of all, the backreaction of the fluxes in type IIB is mild and so leads to internal manifolds that are conformally Calabi-Yau. Then, in these models, there exists a well defined separation between the string scale, the Kaluza-Klein scale and the masses of the moduli such that the moduli effective action could provide a good description of the dynamics at low energy. Another attractive feature of Type IIB models is that it is possible to obtain a hierarchically suppressed Supersymmetry breaking scale with the gravitino mass much smaller than the Kaluza-Klein scale (we will see that with the right construction it is possible also to obtain a supersymmetry breaking scale of the order of a few TeV which could help to address also the Standard Model hierarchy problem) [3].

1.1 Type IIB Superstring Spectrum

After GSO projection, the massless spectrum of Type IIB Superstring Theory is shown in Tab. 1.1.

Sector	**SO(8)-rep**	**SO(8)-irrep**	**10-dim. fields**
$(\mathrm{NS}_+, \mathrm{NS}_+)$	$\mathbf{8}_V \otimes \mathbf{8}_V$	$[0] \oplus [2] \oplus (2)$	$\phi, B_{[MN]}, g_{MN}$
$(\mathrm{NS}_+, \mathrm{R}_+)$	$\mathbf{8}_V \otimes \mathbf{8}_S$	$\mathbf{8}_C \oplus \mathbf{56}_S$	$\lambda^1_\alpha, \psi^1_{M\alpha}$
$(\mathrm{R}_+, \mathrm{NS}_+)$	$\mathbf{8}_S \otimes \mathbf{8}_V$	$\mathbf{8}_C \oplus \mathbf{56}_S$	$\lambda^2_\alpha, \psi^2_{M\alpha}$
$(\mathrm{R}_+, \mathrm{R}_+)$	$\mathbf{8}_S \otimes \mathbf{8}_S$	$[0] \oplus [2] \oplus [4]$	C_0, C_2, C_4

Table 1.1: *Massless spectrum of Type IIB Superstring theory. For each sector the $SO(8)$ representation and the $SO(8)$ irreducible representation are shown. The last column shows the matter content. Let us recall the notation:* $\boldsymbol{8}_V$ is the vector representation of $SO(8)$; the lower indices S and C denotes the positive chirality spinor and the negative chirality spinor of $SO(8)$.

We can see that the NS-NS sector contains a scalar field ϕ which is the dilaton, a two index antisymmetric tensor $B_{[MN]}$ corresponding to the Kalb-Ramond field, and a traceless symmetric tensor g_{MN} corresponding to the graviton. From now on, when referring to the Kalb-Ramond field, we are going to drop the notation with the square brackets. The R-R sector contains p-forms: C_0 is a scalar field, C_2 is a 2-form and C_4 is a 4-form. Finally, the NS-R sector and the R-NS sector contain a spin 1/2 dilatino λ_α and a spin 3/2 gravitino $\psi_{M\alpha}$ (with α chirality) each. Differently from Type IIA, Type IIB is a chiral theory since we have two copies $\lambda \oplus \psi$ with the same chirality.

1.2 Ten-dimensional Type IIB Supergravity

Having established the content of Type IIB Superstring theory, we are now ready to look at the bosonic part of the 10-dimensional action in the string-frame [4]

$$S_{IIB,S} = \frac{1}{2k_{10}^2} \int d^{10}x\sqrt{-g}\Big[e^{-2\phi}\Big(R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H_3^2\Big) - \frac{1}{2}F_1^2 - \frac{1}{2\cdot 3!}\tilde{F}_3^2 - \frac{1}{4\cdot 5!}\tilde{F}_5^2\Big] \tag{1.1}$$

where we have used $2k_{10}^2 = (2\pi)^7\alpha'^4$ and

$$\tilde{F}_3 = F_3 - C_0 \wedge H_3, \qquad \tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3, \tag{1.2}$$

where $\tilde{F}_5$ has the property of self-duality $\tilde{F}_5 = *\tilde{F}_5$. We recall that $H_3 = dB_2$ is the NS-NS 3-form field strength, while $F_3 = dC_2$ and $F_5 = dC_4$ are the R-R field strengths. Let us notice that the first part of the action in the round brackets is the action for the NS sector, while the remaining pieces with F_1, $\tilde{F}_3$ and $\tilde{F}_5$ constitute the kinetic terms for the p-forms. From the action above two parts are missing. The first one, usually denoted with S_{CS}, does not contain the metric and encodes topological interaction terms between the gauge fields (the p-form fields enjoy the gauge symmetry $C_p \to C_p + d\eta_{p-1}$). It is referred to as Chern-Simons action and reads as

$$S_{CS} = \frac{1}{4k_{10}^2} \int e^{\phi} C_4 \wedge H_3 \wedge F_3. \tag{1.3}$$

The second missing piece, usually denoted with S_{loc}, includes the actions of the various localized objects such as Dp-branes. For a $D3$-brane we have

$$S_{loc} \supset S_{D3} = \frac{1}{2\pi^3} \alpha'^2 \int_{D3} C_4 - \int_{D3} d^4\xi \sqrt{-g} T_3 \tag{1.4}$$

where T_3 is the tension and in general is given by

$$T_p = \frac{e^{(p-3)\phi/4}}{(2\pi)^p \alpha'^{(p+1)/2}} \tag{1.5}$$

and the coordinates ξ parametrize the world-volume of the brane. However, to get the full action for Dp-branes, we must include the gauge fields living on the branes and their superpartners getting the so called Dirac-Born-Infeld action

$$S_{DBI} = -T_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab} + B_{ab})} \tag{1.6}$$

with g_{ab} the pullback of the 10-dimensional metric, F_{ab} is the field strength for the gauge fields on the Dp-brane and B_{ab} the pullback of the Kalb-Ramond field.
The same action (1.1) can also be written in the so called Einstein-frame by performing the Weyl rescaling $g_{E,MN} \equiv e^{-\phi/2} g_{MN}$ where it looks like [5, 6, 7]

$$\begin{aligned} S_{IIB,E} =& \frac{1}{2k_{10}^2} \int d^{10}x \sqrt{-g_E} \left(R - \frac{\partial_M \bar{\tau} \partial^M \tau}{2(\mathrm{Im}\tau)^2} - \frac{|G_3|^2}{12\mathrm{Im}\tau} - \frac{1}{4 \cdot 5!} |\tilde{F}_5|^2 \right) \\ & + S_{CS} + S_{loc} \end{aligned} \tag{1.7}$$

where we have defined

$$\tau = C_0 + ie^{-\phi} \quad \text{and} \quad G_3 = F_3 - \tau H_3 \tag{1.8}$$

such that the $SL(2,\mathbb{R})$ symmetry, that leaves the metric and the 4-form invariant and acts on the other fields as

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad \text{and} \quad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R}), \tag{1.9}$$

becomes manifest. Notice that $ad - bc = 1$.

1.3 Type IIB String Compactification

Until now we have seen a theory describing the 10-dimensional $N = 2$ supergravity. However, to get a realistic and effective description of nature, we need to reduce the 10-dimensions characterizing Type IIB string theory, to the 3 spatial dimensions plus the time dimension we observe.
In particular, what we would like to achieve, is a 4-dimensional Effective Field Theory via the compactification of the remaining 6 dimensions.

1.3.1 Dimensional Reduction

The idea of a fifth extra dimension was first proposed by Theodor Kaluza in 1921 and then elaborated by Oskar Klein later in 1926 who were trying to unify gravitational and electromagnetic interactions.

In what follows, we are going to look at the simplest example of the compactification of a 5-dimensional scalar field on a circle and then we are going to consider 5-dimensional gravity [4, 8].

Let us start by considering a 5-dimensional massless scalar $\phi(x^M)$ with $M = 0, \ldots, 4$ whose action is given by

$$S = \frac{1}{2} \int d^5x \partial_M \phi \partial^M \phi. \tag{1.10}$$

We further consider our manifold to be decomposed into a 4-dimensional Minkowski and a circle $\mathcal{M} = \mathbb{R}^4 \times S^1$ such that we can identify $x^4 \equiv y$ with $y \simeq y + 2\pi r$ where r now is the radius of our circle. We can thus express the y dependence expanding in Fourier modes

$$\phi(x^\mu, y) = \sum_k \phi_k(x^\mu) e^{iky/r} = \sum_k \phi(x^\mu) \cos\left(\frac{ky}{r}\right) + \sum_k \phi_k(x^\mu) \sin\left(\frac{ky}{r}\right). \tag{1.11}$$

At this point, substituting the expansion into the original action and integrating over y we get

$$S_{4d} = 2\pi r \int d^4x \left[\frac{1}{2}\partial_\mu \phi_0 \partial^\mu \phi_0 + \sum_k \left(\partial_\mu \phi_k \partial^\mu \phi_k^* + \frac{k^2}{r^2}\phi_k \phi_k^*\right)\right]. \tag{1.12}$$

What we have obtained is a 4-dimensional theory with a massless scalar ϕ_0 and an infinite tower of massive scalars with $m_k^2 = k^2/r^2$. When dialing with energies $E \ll 1/r$ we have that only the massless mode is observable so that the EFT is a 4-dimensional theory with a massless scalar. This massless mode parametrizes a flat direction and could be referred to as *modulus*.

Now we are going to study the KK reduction of 5-dimensional gravity. In this case we consider the action for general relativity

$$S_{5d} = \frac{M_{P,5}^3}{2} \int d^5x \sqrt{-G} R_{5d} \tag{1.13}$$

with $G = \det(g_{MN})$, $M, N = 0, \ldots, 4$ and R_{5d} the 5-dimensional curvature scalar. Then, we parametrize the metric as

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} + \left(\frac{2}{M_P^2}\right)\phi^2 A_\mu A_\nu & \left(\frac{\sqrt{2}}{M_P}\right)\phi^2 A_\mu \\ \left(\frac{\sqrt{2}}{M_P}\right)\phi^2 A_\nu & \phi^2 \end{pmatrix} \tag{1.14}$$

where M, N take the same values as before, $\mu, \nu = 0, \ldots, 3$ and M_P, A_μ, ϕ are just parameters for now. As for the massless scalar case, we compactify $x^4 \equiv y$ with $y \in (0, 2\pi r)$ on S^1 and, expanding in Fourier modes the field ϕ using $g_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = 0$, $\phi^2 = G_{44} = 1$ we get

$$S_{4d} = \int d^4x \sqrt{-g}\phi \left(\frac{M_P^2}{2} R - \frac{1}{4}\phi^2 F_{\mu\nu}F^{\mu\nu} + \frac{M_P^3}{3} \frac{\partial_\mu \phi \partial^\mu \phi}{\phi^2} \right). \tag{1.15}$$

In this case the 5-dimensional degrees of freedom give rise to the usual 4-dimensional metric $g_{\mu\nu}$, a vector boson A_μ and a scalar. For what concerns the vector boson its presence should not be surprising since the $U(1)$ related gauge theory is inherited from the diffeomorphisms invariance of S^1. The scalar field ϕ corresponds to the degree of freedom of the radius $r = \frac{\langle \phi \rangle}{2\pi}$. It is called radion and it is a modulus field (a scalar degree of freedom with tree-level vanishing potential). Finally, comparing the 4-dimensional action in Eq. (1.15) with the usual Einstein-Hilbert action for gravity

$$S = \frac{M_P^2}{8\pi} \int d^4x \sqrt{-g} R \tag{1.16}$$

we can notice that the identification $M_P^2 = 2\pi r M_{P,5}^3$ suggests the possibility of having a low fundamental 5-dimensional gravity scale $M_{P,5}$ and generating a large 4-dimensional Planck scale by taking r large enough.

1.3.2 Calabi-Yau Compactification

As already stated, 10-dimensional Type IIB theory is not suitable to describe the real world that contains only 4 observed dimensions. The trick is to get rid of the extra 6-dimensions to get a low energy 4-dimensional EFT. The most appealing way to do so is through compactification on a Calabi-Yau (CY) manifold Y_6. This manifold, after orientifold projection, breaks supersymmetry in a way that allows to finally obtain an $N = 1$ supersymmetric theory that is chiral. Hence, after breaking the last supersymmetry, it is possible to reproduce the Standard Model.

Calabi-Yau 3-fold *A CY 3-fold is a compact, complex, Kähler manifold with* $SU(3)$ *holonomy and vanishing first Chern Class.*

In what follows, we are going to analyze our statement [4].

Complex Manifold A complex manifold by definition contains a map $J : T_p^* \to T_p^*$ which is called complex structure. The complex structure J is a tensor

$$J = idz^i \otimes \frac{\partial}{\partial z^i} - id\bar{z}^{\bar{i}} \otimes \frac{\partial}{\partial \bar{z}^{\bar{i}}} \tag{1.17}$$

with

$$\frac{\partial}{\partial z^i}, \quad \frac{\partial}{\partial \bar{z}^{\bar{i}}} \quad \text{and} \quad dz^i, \quad d\bar{z}^{\bar{i}} \tag{1.18}$$

local bases of the tangent and of the cotangent spaces respectively. In this case the z^i are given by $z^i = x^i + iy^i$. This map, can also be written as

$$J = \begin{pmatrix} i\mathbb{I} & 0 \\ 0 & -i\mathbb{I} \end{pmatrix}, \qquad J = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix} \tag{1.19}$$

in a complex basis and in a real basis respectively such that $J^2 = -\mathbb{I}$. Written in this way we can see that, roughly speaking, this map corresponds to a multiplication by i in the cotangent space. It is important to underline that a manifold with such a structure is an almost complex manifold. To get a complex manifold we must require also the vanishing of the Nijenhuis tensor $N^p_{mn} = \partial_{[n} J^p_{m]} - J^q_{[m} J^r_{n]} \partial_r J^p_q = 0$.

Kähler Manifold The next step is to introduce a metric in our complex manifold which must be compatible with J. In other words: a complex manifold is Kähler if the form $dJ = 0$ is closed. Let us understand the statement:

- We introduce a metric which is Kähler and has only mixed components:

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial z^i \partial \bar{z}^{\bar{j}}} \tag{1.20}$$

  where K is a real function called Kähler potential;

- The Kähler metric allows us to turn J into $J = ig_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$ that is a 2-form called Kähler form;
- $d = \partial + \bar{\partial}$ is the exterior derivative.

Let us underline that in our complex Kähler manifold the metric is real $g_{i\bar{j}} = \overline{g_{\bar{i}j}}$ and Hermitian $g_{ij} = g_{\bar{i}\bar{j}} = 0$.

Holonomy Let us introduce the concept of holonomy group. We know from differential geometry that a metric manifold comes with a connection and thus with the possibility of parallel-transporting tangent vectors. So, given a closed loop γ and considering a point x, we have that the set of all the linear maps $R(\gamma) : T_x \to T_x$ forms the holonomy group. Strictly speaking, the holonomy group is given by the set of rotations $R(\gamma)$ experienced under parallel-transport around all possible closed loops in the manifold.
At this point the question arises naturally: how is the concept of holonomy connected to the definition of a CY 3-fold?

It is possible to show that the 10-dimensional supersymmetry transformations of the two gravitini of Type IIB theory, are proportional to the covariant derivative of a Majorana-Weyl tensor ϵ [9]

$$\langle \delta\psi^1_{M\alpha} \rangle \propto \nabla_M \epsilon^1, \qquad \langle \delta\psi^2_{M\alpha} \rangle \propto \nabla_M \epsilon^2, \tag{1.21}$$

where $\langle \dots \rangle$ denotes the vacuum configuration. At this point we can embed the 10-dimensional spinor in the tensor product of a 4-dimensional spinor and a 6-dimensional spinor $\epsilon_{\beta B}(x, y) = \epsilon_\beta(x) \otimes \eta_B(y) + h.c$ ($\beta = 1, 2$ complex components of the Weyl spinor in 4-dimensions and $B = 1, \dots, 4$ complex components of a Weyl spinor in 6-dimensions) such that

$$\langle \psi^i_{\mu\alpha} \rangle \propto \nabla_\mu \epsilon^i_\beta, \qquad \langle \psi^i_{m\alpha} \rangle \propto \nabla_m \eta^i_B \tag{1.22}$$

where $\mu = 0, \dots, 3$ is the index for the usual 4-dimensional Minkowski spacetime, $m = 4, \dots, 9$ is the index for the 6 extra dimensions. For supersymmetry to be unbroken, we ask the variation of all fields in the vacuum configuration to vanish. Consequently the tensors on the right-hand sides of Eq. (1.22) must be covariantly conserved (note that we are considering the case without fluxes for simplicity). The covariant conservation in 4-dimensions leads to a vanishing Ricci scalar $R = 0$, while the covariant conservation in 6-dimensions leads to $R_{mn} = 0$, that means that Y_6 is Ricci flat. All together ($R = 0$ and $R_{mn} = 0$) they tell us that $R_{MN} = 0$ and so the Einstein equations are solved without sources.
As we can see the request of having supersymmetry conservation when compactifying the 6 extra dimensions is directly connected to the covariant conservation of a spinorial quantity, and so to the existence of a holonomy group.

Why should the holonomy group for a CY 3-fold be $SU(3)$? Since $SO(6) = Spin(6)/\mathbb{Z}_2$ and $Spin(6) = SU(4)$ the Weyl spinor representation $Spin(6)$ (in 6-dimensions) transforms as $SU(4)$. Without loss of generality we can assume that in $SU(4)$ the spinor takes the form

$$\epsilon(y) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \epsilon_3 \end{pmatrix} \tag{1.23}$$

such that, for this spinor to be covariantly constant, we must have that the holonomy group only acts on the first three components. Thus we need $SU(3)$ holonomy.

Let us highlight that the concept of holonomy is also strictly connected to another property of the CY 3-fold in exam, that is the fact that our CY manifold is Ricci flat.

First Chern Class Before introducing the concept of Chern Class and what it means for it to vanish, we state the Yau Theorem:

Yau Theorem *Let X be a Kähler manifold and J its Kähler form. If the first Chern Class vanishes, then a Ricci flat metric with Kähler form J' in the same cohomology class can be given. This is called Calabi-Yau metric and it is unique.*

Due to the presence of $U(N)$ holonomy, we can write the curvature 2-form as

$$R(T_X) = dz^i \wedge d\bar{z}^{\bar{j}} R^k_{i\bar{j}l}. \tag{1.24}$$

Having this we can write the multi-form

$$c(X) = \det(\mathbb{I} + R(T_X)) \tag{1.25}$$

which can be expanded as

$$c(X) = 1+c_1(X)+c_2(X)+\dots = 1+Tr(R(T_X))+Tr(R(T_X)\wedge R(T_X)-2(Tr(R(T_X)))^2)+\dots \tag{1.26}$$

with c_k a $2k$-form defining the k-th Chern Class. The first Chern Class is defined as

$$c_1 = \frac{[R]}{2\pi} \tag{1.27}$$

and the fact that it must be vanishing means that it is exact, i.e. zero in cohomology ($c_1 = d\omega$ for some ω).

At this point, we need one last ingredient before fully characterizing a CY manifold and before studying its moduli space.

Cohomology What we want to see is how the requirement of a vanishing first Chern Class is connected to the concept of cohomology. Before doing so for a Kähler manifold, let us recall some results for real manifolds [10, 4].
Let us consider a compact Riemannian manifold $\mathcal{M}$ of dimension m.

- An n-form A_n is closed if $dA_n = 0$.
- An n-form A_n is exact if it can be written as $A_n = dB_{n-1}$ such that $dA_n = d^2 B_{n-1} = 0$.
- We can define the Hodge star operator $* : A_n \to A_{m-n}$.
- Using the Hodge star operator, we are able to define also the inner product $(A_n, B_n) = \int_{\mathcal{M}} A_n \wedge *B_n$.
- We can define the adjoint exterior derivative acting on an n-form A_n, $d^\dagger A_n \equiv (-1)^{mn+m+1} * d(*A_n)$ such that $(d^\dagger)^2 = 0$. In this way, $d^\dagger A_n = 0$ is said to be co-closed and $A_n = d^\dagger B_{n-1}$ is co-exact.
- We can define the Laplacian $\triangle_d = dd^\dagger + d^\dagger d$.
- A form is harmonic if it satisfies $\triangle_d A_n = 0$.

At this point we can introduce the concept of cohomology and the Hodge Decomposition Theorem.

- We can define the De Rham cohomology groups $H^n_{dR}(\mathcal{M}, \mathbb{R})$ as

$$H^n_{dR}(\mathcal{M}, \mathbb{R}) = \frac{C^n_d(\mathcal{M})}{E^n_d(\mathcal{M})} \tag{1.28}$$

  where $C^n_d(\mathcal{M})$ is the set of closed n-forms and $E^n_d(\mathcal{M})$ is the set of exact n-forms. In this way two forms are considered equivalent if they differ by an exact form. The dimension of $H^n_{dR}(\mathcal{M}, \mathbb{R})$ is given by the Betti numbers $b^n = \dim_{\mathbb{R}} H^n_{dR}(\mathcal{M}, \mathbb{R})$.

- The Hodge Decomposition Theorem tells us that every n-form can be decomposed as the sum of an exact form, a co-exact form and a harmonic form in the following way

$$\alpha = d\beta + d^\dagger \gamma + h \ , \ \ h \in \mathcal{H}(\mathcal{M}) \, . \tag{1.29}$$

With all this information, we can notice that, as a consequence, a closed form can be expressed uniquely as the sum of a harmonic and an exact form. In this way there is an isomorphism between harmonic forms $\mathcal{H}(\mathcal{M})$ and $H^n_{dR}(\mathcal{M},\mathbb{R})$ meaning that every de Rham cohomology class on $\mathcal{M}$ contains a unique harmonic representative.

Everything we have seen can be transported to the case of Kähler manifolds which is the case we are interested in [10, 4]. First of all we can see that a real form can be decomposed into holomorphic and anti-holomorphic components ($2m$ real coordinates $x_1,\dots,x_m,y_1,\dots y_m$ can be divided into m holomorphic coordinates $z_i = x_i + iy_i$ and m anti-holomorphic ones $\bar{z}^{\bar{i}} = x_{\bar{i}} - iy_{\bar{i}}$ with $i,\bar{i} = 1,\dots,m$)

$$\begin{aligned} A_n &= \frac{1}{n!}A_{k_1\dots k_n}dx^{k_1}\wedge\dots\wedge dx^{k_n} \\ &\downarrow \\ A_{p,q} &= \frac{1}{p!q!}A_{i_1\dots i_p,\bar{i}_1\dots\bar{i}_q}dz^{i_1}\wedge\dots\wedge dz^{i_p}\wedge d\bar{z}^{\bar{i}_1}\wedge\dots\wedge d\bar{z}^{\bar{i}_q}. \end{aligned} \tag{1.30}$$

The exterior derivative becomes $d = \partial + \bar{\partial}$ such that $\partial^2 = \bar{\partial}^2 = 0$ with ∂ and $\bar{\partial}$ Dolbeault operators:

$$\partial : A_{p,q}(\mathcal{M}) \to A_{p+1,q}(\mathcal{M}), \qquad \bar{\partial} : A_{p,q}(\mathcal{M}) \to A_{p,q+1}(\mathcal{M}) \tag{1.31}$$

where $\mathcal{M}$ is now a complex manifold of real dimension $2m$ with Kähler metric.
The Hodge star operator is such that $* : A_{p,q}(\mathcal{M}) \to A_{m-p,m-q}(\mathcal{M})$.
We can define two Laplacian operators

$$\triangle_\partial = \partial\partial^\dagger + \partial^\dagger\partial, \quad \text{with} \quad \partial^\dagger = - * \partial * \tag{1.32}$$

$$\triangle_{\bar{\partial}} = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial}, \quad \text{with} \quad \bar{\partial}^\dagger = - * \bar{\partial}^\dagger * \tag{1.33}$$

satisfying $\triangle_\partial = \triangle_{\bar{\partial}} = \triangle_d/2$. In this way we can introduce the concept of harmonic forms in analogy with the case of a real manifold: a (p,q)-form is harmonic if $\triangle_{\bar{\partial}}A_{p,q} = 0$. We are ready again to give the concept of Dolbeault cohomology groups and to state the Hodge Decomposition Theorem.

- The Dolbeault cohomology groups $H^{p,q}_{\bar{\partial}}(\mathcal{M},\mathbb{C})$ are defined as

$$H^{p,q}_{\bar{\partial}}(\mathcal{M},\mathbb{C}) = \frac{C^{p,q}_{\bar{\partial}}(\mathcal{M})}{E^{p,q}_{\bar{\partial}}(\mathcal{M})} \tag{1.34}$$

  where $C^{p,q}_{\bar{\partial}}(\mathcal{M})$ is the set of (p,q)-forms closed under $\bar{\partial}$ and $E^{p,q}_{\bar{\partial}}(\mathcal{M})$ is the set of (p,q)-forms exact under $\bar{\partial}$. The dimension of $H^{p,q}_{\bar{\partial}}(\mathcal{M},\mathbb{C})$ is given by the Hodge numbers $h^{p,q} = \dim H^{p,q}_{\bar{\partial}}(\mathcal{M},\mathbb{C})$.

- Again, the Hodge Decomposition Theorem allows us to split any (p,q)-form as the sum of a harmonic form, an exact form and a coexact form.

The main take-away message from the analysis of Kähler manifolds is that the space of harmonic (p, q)-forms is isomorphic to the Dolbeault cohomology groups, meaning again that there is a unique harmonic representative in each Dolbeault cohomology class.

1.3.3 Calabi-Yau Moduli Spaces

As we have seen, there are various requirements for a CY manifold to be satisfied. Before going any further in our description, let us summarize what kind of conditions characterize a CY 3-fold (Y_6):

1. It is a *complex Kähler manifold*;
2. It is *Ricci flat*;
3. It admits a *vanishing first Chern Class*;
4. It has $SU(3)$ *holonomy*;
5. It features a *unique (3,0)-form* Ω.

We want now to connect the discussion on cohomology groups to the CY 3-fold case and to understand its meaning. Recalling the examples from KK-dimensional reduction, we have seen that the low-energy theory in 4-dimensions contains only massless modes called moduli and that the tower of massive KK-modes is not observable.
Considering the massless wave equations in $\mathbb{R}_{1,9}$ [11]

$$\Box\phi = 0\,, \qquad i\gamma^M D_M\psi = 0, \tag{1.35}$$

their counterparts in $\mathbb{R}_{1,3} \times Y_6$ are given by

$$(\Box_{1,3} + \triangle_y)\phi(x, y), \qquad i(\gamma^\mu D_\mu + \gamma^m D_m)\psi(x, y) = 0 \tag{1.36}$$

where x^μ ($\mu = 0, \ldots, 3$) are the coordinates for the 4-dimensional spacetime and y^m ($m = 4, \ldots, 9$) are the coordinates of Y_6. What emerges from this decomposition is that the massless modes in 4-dimensions correspond to the zero modes of the Laplace operator on Y_6 which is exactly the one defined in Eq. (1.33) (or equivalently Eq. (1.32)). This means that the zero modes are in one-to-one correspondence with the harmonic forms on Y_6 which are in one-to-one correspondence with the elements of the Dolbeault cohomology group (1.34). Roughly speaking, the Hodge numbers tell us how many harmonic forms are present in a CY, and so how many massless modes.

Typically, the Hodge numbers are arranged in the so-called Hodge diamond which, in the case of a CY 3-fold, becomes

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{1.37}$$

To get this particular form, we have used some properties of the CY 3-fold: horizontal and vertical symmetry ($h^{1,1} = h^{2,2}$), Hodge duality ($h^{p,q} = h^{m-p,m-q}$, with $\dim_{\mathbb{C}}(\mathcal{M}) = m$), $SU(3)$ holonomy ($h^{1,0} = h^{2,0} = 0$), connectedness ($h^{0,0} = h^{3,3} = 1$), Yau's Theorem and vanishing first Chern Class 1.3.2 ($h^{3,0} = h^{0,3} = 1$). This means that there is a unique holomorphic, harmonic 3-form Ω which is a characteristic feature of a CY 3-fold.

We have seen in Subsec. 1.3.2 that the metric is Hermitian such that we can write

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{i\bar{j}}dz^i d\bar{z}^{\bar{j}}, \tag{1.38}$$

and, from Yau's Theorem, we have seen the existence of a unique Ricci flat metric $g_{i\bar{j}}$. What we want to know now is if it is possible to deform such a metric without spoiling Ricci-flatness. We consider deformations of the metric of the form

$$g_{i\bar{j}}dz^i dz^{\bar{j}} \to g_{i\bar{j}}dz^i dz^{\bar{j}} + \delta g_{i\bar{j}}dz^i d\bar{z}^{\bar{j}} + \delta g_{ij}dz^i dz^j + h.c.. \tag{1.39}$$

The first deformation, $\delta_{i\bar{j}}$, corresponds directly to a change of the Kähler form $\delta g_{i\bar{j}} = -i\delta J_{i\bar{j}}$. These deformations give rise to $h^{1,1}$ scalar fields, the *Kähler moduli*, that parametrize modifications of the volume of the manifold leaving the overall shape invariant. Using a basis of the cohomology group $H^{1,1}(Y_6)$ we can expand the Kähler form as

$$J = t^{\tilde{A}}(x)\omega_{\tilde{A}} \tag{1.40}$$

with $\tilde{A} = 1, \ldots, h^{1,1}$ and ω_A harmonic $(1,1)$ forms. The second kind of deformations violates the assumption of hermiticity of the metric and therefore must come with a change in the complex structure itself to preserve the CY conditions. These deformations are in one-to-one correspondence with harmonic $(1,2)$-forms $\bar{\chi}_K$ such that

$$\delta g_{ij} = \frac{i}{||\Omega||^2}\bar{z}^K(x)(\bar{\chi}_K)_{i\bar{i}\bar{j}}\Omega^{\bar{i}\bar{j}}_j \tag{1.41}$$

with $K = 1, \ldots, h^{1,2}$ and $z^K(x)$ 4-dimensional complex scalar fields called *complex structure moduli.*

At this point we can expand the fields appearing in Eq. (1.1) in terms of harmonic forms on Y_6 [12, 13]:

- For the NS-NS sector we have that the Kalb-Ramond field gets expanded in the following way
$$B_2 = \hat{B}_2(x) + b^{\tilde{A}}(x)\omega_{\tilde{A}} \tag{1.42}$$
where again $\tilde{A} = 1, \ldots, h^{1,1}$, $\hat{B}_2(x)$ is the 4-dimensional version of B_2 which is 10-dimensional and, as already mentioned, $\omega_{\tilde{A}}$ are harmonic $(1,1)$-forms.

- In the R-R sector we get that the forms can be expanded as
$$C_2 = \hat{C}_2(x) + c^{\tilde{A}}(x)\omega_{\tilde{A}}, \tag{1.43}$$
$$C_4 = D_2^{\tilde{A}}(x) \wedge \omega_{\tilde{A}} + V^K(x) \wedge \alpha_K - U_K(x) \wedge \beta^K + \rho_{\tilde{A}}(x)\tilde{\omega}^{\tilde{A}} \tag{1.44}$$
where $K = 0, \ldots, h^{1,2}$, $\hat{C}_2(x)$ and $\hat{C}_4(x)$ are the 4-dimensional counterparts of C_2 and C_4, $\tilde{\omega}^{\tilde{A}}$ are the harmonic $(2,2)$-forms dual to the $(1,1)$-forms and (α_K, β^L) are harmonic 3-forms forming a real, symplectic basis on $H^3(Y_6)$ satisfying
$$\int \alpha_K \wedge \beta^L = \delta_K^L \quad \text{and} \quad \int \alpha_K \wedge \alpha_L = \int \beta^K \wedge \beta^L = 0. \tag{1.45}$$

In the expansions above, $b^{\tilde{A}}(x)$, $c^{\tilde{A}}(x)$ and $\rho_{\tilde{A}}(x)$ are 4-dimensional scalar fields, $V^K(x)$ and $U_K(x)$ are 1-forms corresponding to 4-dimensional gauge fields.
We can summarize the cohomology groups, their basis and the fields coming from the expansions in the following tables where we have included the Type IIB scalar fields ϕ and C_0 in the 10-dimensional spectrum and their 4-dimensional version $\hat{\phi}$ and $\hat{C}_0$ in the $4D$ one (see Tab. 1.2 and Tab. 1.3).

Cohomology Group	**Basis Elements**	**Dimension**
$H^{1,1}(Y_6)$	$\omega_{\tilde{A}}$	$h^{1,1}$
$H^{2,2}(Y_6)$	$\tilde{\omega}^{\tilde{A}}$	$h^{2,2} = h^{1,1}$
$H^3(Y_6)$	(α_K, β^L)	$2h^{1,2} + 2$
$H^{1,2}(Y_6)$	$\bar{\chi}_K$	$h^{1,2}$

Table 1.2: *Cohomology groups on Y_6 with their basis elements and dimensions.*

Multiplet	**Field Content**	**Number of Multiplets**
Gravity	$(g_{\mu\nu}, V^0)$	1
Vector	(V^K, z^K)	$h^{1,2}$
Hypermultiplets	$(t^{\tilde{A}}, b^{\tilde{A}}, c^{\tilde{A}}, \rho_{\tilde{A}})$	$h^{1,1}$
Double-Tensor	$(\hat{B}_2, \hat{C}_2, \hat{\phi}, \hat{C}_0)$	1

Table 1.3: *Type IIB String Theory massless spectrum multiplets in $N = 2$ supersymmetry.*

Let us notice that:

1. Thanks to the self-duality condition of $\tilde{F}_5$ we are able to get rid of half of the degrees of freedom in C_4. In particular we choose to eliminate U_K and $D_2^{\tilde{A}}$ in order to keep V^K and $\rho_{\tilde{A}}$;

2. Everything we have seen until now, due to the $SU(3)$ holonomy of the CY, is in $N = 2$ supersymmetry, since the $SU(3)$ holonomy reduces supersymmetry by 1/4.

1.3.4 From $N = 2$ to $N = 1$ supersymmetry: Orientifold Projection

Up to now, we have worked with $N = 2$ supersymmetry in 4-dimensions. However, $N = 2$ supersymmetry is not a realistic model of nature since the theory does not allow fermions in chiral representations of gauge groups. As it is well known, Type IIB Supergravity in 10-dimensions contains also Dp-branes (with p odd) which are objects that are electromagnetically charged under $(p + 1)$-form gauge fields C_{p+1}. Moreover, wrapping stacks of Dp-branes on the various cycles of the CY, after compactification, gives rise to different gauge sectors. One of them could in principle be the Standard Model. However, due to the R-R charge and tension these objects carry, it is impossible to obtain a zero charge compact space, which is inconsistent with the requirement of tadpole cancellation. A possible way out would be the introduction of anti-Dp-branes ($\overline{Dp}$) with opposite charge; however, this solution would fail since the two objects will attract each other and annihilate.
A better solution is offered by orientifold planes which are non-dynamical objects appearing at the fixed point loci of an involution $\mathcal{O}$ that reverses the orientation of the string worldsheet. These objects have opposite R-R charge with respect to Dp-branes and break supersymmetry consistently with the Dp-branes: together they break supersymmetry to half of the one in the 10-dimensional theory. In our case, considering Type IIB String Theory, we break supersymmetry from $N = 2$ to $N = 1$.

To perform an orientifold projection $\mathcal{O}$ ([7, 12, 13, 14]), we need to couple two operations that are the holomorphic involution σ and the worldsheet parity Ω_P. The first one is an operation on Y_6 and acts on the harmonic 3-form $\Omega \in H^{3,0}(Y_6)$ and on the Kähler form J as

$$\sigma^* J = J, \qquad \sigma^* \Omega = -\Omega \tag{1.46}$$

where σ^* is the pull-back of σ acting on forms. The second operation instead, can be combined with $(-1)^{F_L}$, where F_L is the left-moving fermion number, and together they can act as

$$\begin{aligned} &\Omega_P(-1)^{F_L} g_{MN} = g_{MN}, && \Omega_P(-1)^{F_L} B_2 = -B_2, \\ &\Omega_P(-1)^{F_L} \phi = \phi, && \Omega_P(-1)^{F_L} C_{2p} = (-1)^p C_{2p}. \end{aligned} \tag{1.47}$$

The overall orientifold projection can be written as

$$\mathcal{O} = (-1)^{F_L} \Omega_P \sigma^*. \tag{1.48}$$

Applying Eq. (1.48) we are left with a truncation of the $N = 2$ massless spectrum to $N = 1$ such that, in 4-dimensions, only the states invariant under the projection are left. Considering the action of $(-1)^{F_L}$ in Eq. (1.47), we can see that the invariant states must obey

$$\sigma^* \phi = \phi, \tag{1.49}$$
$$\sigma^* g_{MN} = g_{MN}, \tag{1.50}$$
$$\sigma^* B_2 = -B_2, \tag{1.51}$$
$$\sigma^* C_0 = C_0, \tag{1.52}$$
$$\sigma^* C_2 = -C_2, \tag{1.53}$$
$$\sigma^* C_4 = C_4. \tag{1.54}$$

Before looking at the effects of the projection on the Type IIB spectrum, let us underline two important consequences:

1. The fixed loci of such an orientifold action are points or 4-cycles on Y_6 and, since the involution leaves the 4-dimensional part invariant (acting only on Y_6), the orientifold planes are spacetime filling; the result is that the fixed loci of σ correspond to $O3/O7$-planes;

2. σ splits the cohomology groups of Y_6 into even and odd eigen-spaces under the action of the pullback σ^* such that

$$H^{p,q}(Y_6) = H^{p,q}_+(Y_6) \oplus H^{p,q}_-(Y_6). \tag{1.55}$$

   $H^{p,q}_+(Y_6)$ has dimension $h^{p,q}_+$ and $H^{p,q}_-(Y_6)$ has dimension $h^{p,q}_-$.

The invariance of the Kähler form under σ^* (first expression in Eq. (1.46)) implies that

$$J = t^{\tilde{\alpha}}(x) \omega_{\tilde{\alpha}} \tag{1.56}$$

with $\tilde{\alpha} = 1, \dots, h_+^{1,1}$ and $\omega_{\tilde{\alpha}}$ basis of $H_+^{1,1}(Y_6)$ meaning that only the even Kähler deformations are allowed in the $N = 1$ Type IIB 4-dimensional spectrum.

The invariance of the metric (1.50) and the action of σ^* on the 3-form Ω (second expression in Eq. (1.46)) tell us that

$$\delta g_{ij} = \frac{i}{||\Omega||^2} \bar{z}^k(x)(\bar{\chi}_k)_{i\bar{i}\bar{j}}\Omega_j^{\bar{i}\bar{j}} \tag{1.57}$$

where $k = 1, \dots, h_-^{1,2}$ and $\bar{\chi}_k$ is a basis of $H_-^{1,2}(Y_6)$. Thus, only odd complex structure deformations remain in the $N = 1$ Type IIB 4-dimensional spectrum.

From the expressions (1.51) and (1.53) we see that, when expanding B_2 and C_2, the even elements are projected out by the orientifold, leaving only the odd ones. In this way we get

$$B_2 = b^{\tilde{a}}(x)\omega_{\tilde{a}}, \tag{1.58}$$

$$C_2 = c^{\tilde{a}}(x)\omega_{\tilde{a}}, \tag{1.59}$$

with $\tilde{a} = 1, \dots, h_-^{1,1}$ and $\omega_{\tilde{a}}$ basis of $H_-^{1,1}(Y_6)$.

Finally, from Eq. (1.54), we have that under the orientifold projection, only the even elements are kept in the $N = 1$ Type IIB 4-dimensional spectrum such that

$$C_4 = D_2^{\tilde{\alpha}}(x) \wedge \omega_{\tilde{\alpha}} + V^\gamma(x) \wedge \alpha_\gamma + U_\gamma(x) \wedge \beta^\gamma + \rho_{\tilde{\alpha}}(x)\bar{\omega}^{\tilde{\alpha}}, \tag{1.60}$$

where $\gamma = 1, \dots, h_+^{1,2}$, $\bar{\omega}^{\tilde{\alpha}}$ is a basis of $H_+^{2,2}(Y_6)$ dual to $\omega_{\tilde{\alpha}}$ and $(\alpha_\gamma, \beta^\gamma)$ is a real symplectic basis of $H_+^3(Y_6) = H_+^{1,2}(Y_6) \oplus H_+^{2,1}(Y_6)$.
In the following table (Tab. 1.4) we show the final massless spectrum of Type IIB String Theory with $N = 1$ supersymmetry. As for the spectrum with $N = 2$ we use the duality condition of $\tilde{F}_5$ to get rid of half the degrees of freedom of C_4, eliminating $D_2^{\tilde{\alpha}}(x)$ and $U_\gamma(x)$ in favor of $\rho_{\tilde{\alpha}}(x)$ and $V^\gamma(x)$.

Multiplet	**Field Content**	**Number of Multiplets**
Gravity	$g_{\mu\nu}$	1
Vector	V^γ	$h_+^{1,2}$
Chiral	z^k	$h_-^{1,2}$
	$(\hat{\phi}, \hat{C}_0)$	1
	$(b^{\tilde{a}}, c^{\tilde{a}})$	$h_-^{1,1}$
Chiral/Linear	$(t^{\tilde{\alpha}}, \rho_{\tilde{\alpha}})$	$h_+^{1,1}$

Table 1.4: *Type IIB $N = 1$ massless spectrum of O3/O7-orientifold compactification.*

Let us now give the description of a generic model using the 4-dimensional Supergravity language in which the Lagrangian of the theory can be written as

$$\mathcal{L} = K_{i\bar{j}}(\partial X^i)(\partial \bar{X}^{\bar{j}}) + \dots \tag{1.61}$$

where, recalling the expression for the CY metric in Eq. (1.20), we have $K_{i\bar{j}} \equiv g_{i\bar{j}}$. X^i describe the Kähler moduli and the complex structure moduli, while the dots include other possible fields of the general model. As we can see, one of the main elements for the description of a 4-dimensional Supergravity theory is the Kähler potential. In what follows we are going to obtain it for each modulus belonging to the theory.
From the decomposition of the Kähler form in harmonic forms in Eq. (1.40) [4, 7, 13, 15] we can write the compactification volume $\mathcal{V}$ as

$$\mathcal{V} = \frac{1}{6}\int_{Y_6} J \wedge J \wedge J = \frac{1}{6}\mathcal{K}_{\tilde{A}\tilde{B}\tilde{C}} t^{\tilde{A}} t^{\tilde{B}} t^{\tilde{C}} \tag{1.62}$$

where $\mathcal{K}_{\tilde{A}\tilde{B}\tilde{C}}$ are the triple intersection numbers of the 4-cycles Poincaré dual[1] to the $\omega_{\tilde{A}}$, that are given by

$$\mathcal{K}_{\tilde{A}\tilde{B}\tilde{C}} = \int_{Y_6} \omega_{\tilde{A}} \wedge \omega_{\tilde{B}} \wedge \omega_{\tilde{C}}. \tag{1.64}$$

The Kähler variables $t^{\tilde{A}}$ are therefore the volumes of the 2-cycles in the CY. These 2-cycles volumes can be related to the 4-cycles volumes as

$$\tau_{\tilde{A}} = \frac{\partial \mathcal{V}}{\partial t^{\tilde{A}}} = \frac{1}{2}\mathcal{K}_{\tilde{A}\tilde{B}\tilde{C}} t^{\tilde{B}} t^{\tilde{C}}. \tag{1.65}$$

Notice that $\tau_{\tilde{A}}$ are real variables. All these definitions survive the orientifold projection and appear in the $N = 1$ theory where now instead of $\tilde{A} = 1, \dots, h^{1,1}$, we are going to use $\tilde{\alpha} = 1, \dots, h^{1,1}_+$. We can switch to a complex description using

$$\rho_{\tilde{\alpha}} = \int_{\Sigma_4^{\tilde{\alpha}}} C_4 \tag{1.66}$$

as imaginary parts such that $T_{\tilde{\alpha}} = \tau_{\tilde{\alpha}} + i\rho_{\tilde{\alpha}}$ where $\rho_{\tilde{\alpha}}$ is an axion-like particle. In this way we can express the volume $\mathcal{V}$ as a real function of the variables $T_{\tilde{\alpha}}$ and write the Kähler potential as

$$K_{K-mod} = -2\ln \mathcal{V}\,. \tag{1.67}$$

[1] The canonical isomorphism $H^p(Y_6) \cong H_{n-p}(Y_6)$ (with n dimension of the complex manifold that in our case is Y_6) between cohomology and homology is known as Poincaré duality. More explicitly, a p-form ω_p is Poincaré dual to a $(n-p)$-cycle c_{n-p} if [4]

$$\int_{c_{n-p}} \omega_{n-p} = \int \omega_p \wedge \omega_{n-p}, \qquad \forall\, \omega_{n-p}. \tag{1.63}$$

For the Kähler potential of the complex structure moduli we start from the expression involving the 3-form $\Omega \in H^3(Y_6)$ ([13, 16])

$$K_{cs} = -\ln\left(-i\int \Omega \wedge \bar{\Omega}\right) \tag{1.68}$$

where *cs* stands for complex structure. We can rewrite this Kähler potential in terms of the so called period vector. Taking (A^I, B_I) with $I = 0, \ldots, h^{1,2}$ as the canonical homology basis of $H_3(Y_6)$ and its dual (α_I, β^I) as a real, symplectic basis on $H^3(Y_6)$ (as seen before), we can write the periods of Ω and the corresponding period vector respectively as

$$Z^I = \int_{A^I} \Omega = \int_{Y_6} \Omega \wedge \beta^I, \quad \mathcal{F}_I = \int_{B_I} \Omega = \int_{Y_6} \Omega \wedge \alpha_I, \quad \Pi = \begin{pmatrix} \int_{B_I} \Omega \\ \int_{A^I} \Omega \end{pmatrix} = \begin{pmatrix} \mathcal{F}_I \\ Z^I \end{pmatrix}, \tag{1.69}$$

where $\Omega = Z^I \alpha_I - \mathcal{F}_I \beta^I$. At this point we can define the prepotential as $\mathcal{F}(Z^I) = (Z^0)^2 \mathrm{F}(z)$ where we have factored out Z^0 such that $\mathcal{F}_I = \partial \mathcal{F}/\partial Z^I$. In this way we can also use the so called special coordinates $z = Z^I/Z^0$. Notice that in the following we are going to set $Z^0 = 1$ [17].
In this way we can write the Kähler potential for the complex structure moduli as

$$K_{cs} = -\ln\left(-i\int \Omega \wedge \bar{\Omega}\right) = -\ln[i(\bar{Z}^I \mathcal{F}_I - Z^I \bar{\mathcal{F}}_I)] = -\ln(-i\Pi^\dagger \Sigma \Pi) \tag{1.70}$$

where

$$\Sigma = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix} \tag{1.71}$$

is the symplectic matrix. Let us remark that the expressions we have introduced in (1.70) are obtained for $N = 2$ supersymmetry. Again, we can project out the degrees of freedom in $H^{1,2}_+(Y_6)$ to get the corresponding Kähler potential in $N = 1$. In this case we can write $\Omega = Z^k \alpha_k - \mathcal{F}_k \beta^k$ with $k = 0, \ldots, h^{1,2}_-$ such that

$$K_{cs} = -\ln\left(-i\int \Omega \wedge \bar{\Omega}\right) = -\ln[i(\bar{Z}^k \mathcal{F}_k - Z^k \bar{\mathcal{F}}_k)]. \tag{1.72}$$

Finally, we can introduce another piece to the game, that is a non-geometric modulus related to the dilaton $\hat{\phi}$. In fact, as we have done for the Kähler modulus, we can complexify the dilaton using the imaginary part of the RR-form $\hat{C}_0$ to obtain the so called *axio-dilaton*

$$iS \equiv \tau = \hat{C}_0 + ie^{-\hat{\phi}} \tag{1.73}$$

where we have used the expression (1.8). We remark that the string coupling is $g_s = e^{\phi}$, so that we have equivalently

$$g_s = \frac{1}{\mathrm{Re}(S)} \,. \tag{1.74}$$

The Kähler potential for the axio-dilaton S can be written as

$$K_S = -\ln(S + \bar{S}) \tag{1.75}$$

such that, the overall Kähler potential for the $N = 1$ Type IIB compactification is given by

$$K = -2\ln\mathcal{V} - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S}). \tag{1.76}$$

Note that at the level of $N = 2$ supersymmetry (and so before orientifold projection) the scalar manifold $\mathcal{M}$ arises as the product of a quaternionic manifold $\mathcal{M}^{Q}$, spanned by the scalars in the $h^{1,1} + 1$ hypermultiplets, and a special Kähler manifold $\mathcal{M}^{\mathcal{SK}}$ spanned by the scalars z^K in the vector multiplets [13].

Chapter 2

Large Volume Scenarios: Moduli Stabilisation, Sequestering and Axionic Dark Radiation

We have seen that CY compactifications of Type IIB String Theory with $O3/O7$-orientifold come with many different moduli which are the way extra dimensions manifest themselves into the 4-dimensional Effective Field Theory. They are scalar degrees of freedom describing low energy excitations in the extra dimensions and usually have gravitational strength interactions. As already mentioned, however, they are massless at tree level and, coming only with the Kähler potential, they represent flat directions. This represents a problem for different reasons:

1. We get Yukawa-like long range fifth forces which have not been observed;

2. We have a lack of predictability in the 4-dimensional low energy effective theory because all the parameters of string theory (such as couplings, ratios of scales and compactification parameters) are entirely determined by the vacuum expectation values (VEV) of these moduli. Not having these VEVs would leave us with flat directions which prevent us to completely characterize the low energy phenomenology.

2.1 Moduli Stabilization

In the first part of this chapter we are going to focus on moduli stabilization trying to lift the flat directions through quantum corrections in order to get phenomenologically viable models.

2.1.1 Background Fluxes

The goal of moduli stabilization is to generate a scalar potential to fix the VEVs of the moduli and to give them large enough masses to overcome the phenomenological problems seen before. To do so we are going to introduce fluxes for the field-strength tensors, thus studying the so called *flux compactifications* for Type IIB String Theory.

Let us recall that the field strength p-form is defined as $F_p = dC_{p-1}$. It is possible to show that under certain conditions and in the presence of localized sources such as Dp-branes and $\mathcal{O}$-planes, the field strength can assume non-trivial background values called *background fluxes*. These fluxes are determined by their integrals over p-cycles in the CY Y_6.
In the case of Type IIB flux compactification [8, 7, 11, 18, 19], taking $G_3 = F_3 - \tau H_3$ as already shown in Eq. (1.8), imposing the imaginary self-duality (ISD) condition $*_6 G_3 = iG_3$ and requiring $\tilde{F}_5$ to satisfy the Bianchi identity, we have that supersymmetry is preserved. Therefore, one finds that the fluxes for Type IIB are F_3 and H_3 which must obey the Dirac quantization condition

$$\frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_a} F_3 = n_a \in \mathbb{Z}, \qquad \frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_b} H_3 = m_b \in \mathbb{Z} \tag{2.1}$$

where Σ_a and Σ_b are 3-cycles on Y_6. Moreover, the integrability request for the Bianchi identity leads to a tadpole-cancellation condition

$$Q_3^{loc} + \frac{1}{(2\pi)^4(\alpha')^2}\int_{Y_6} H_3 \wedge F_3 = 0 \tag{2.2}$$

where Q_3^{loc} is the total D3-charge associated with the local sources.
The effect of these fluxes is encoded in the so called *Gukov-Vafa-Witten (GVW) superpotential* [20]

$$W_{GVW} = \int_{Y_6} G_3 \wedge \Omega. \tag{2.3}$$

Clearly it depends only on the complex structure moduli (encoded by the holomorphic $(3,0)$-form Ω) and on the dilaton through G_3. Moreover, it is the tree-level superpotential since it does not include any possible correction. Together with the tree-level Kähler potential (1.76), the superpotential (2.3) generates the F-term scalar potential for the $N = 1$ Supergravity low energy theory

$$V_F = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) \tag{2.4}$$

where i, j runs through all the moduli present in the theory, D_i is the Kähler covariant derivative defined as $D_i W = \partial_i W + (\partial_i K)W$ and where we are considering $M_P = 1$. Let us notice that, from now on (in this Chapter), we are going to write the complex structure moduli as U_k $(k = 1, \ldots, h_-^{1,2})$ to be coherent with the notation used in some references.
Due to the fact that the superpotential does not depend on the Kähler moduli, the Kähler potential is of *No-Scale* type [21, 22] which means that (focusing for simplicity on the case with a single Kähler modulus T)

$$K^{T\overline{T}} D_T W D_{\overline{T}} \bar{W} = K^{T\overline{T}} K_T K_{\overline{T}} |W|^2 = 3|W|^2 \tag{2.5}$$

such that the scalar F-term potential is only a function of the complex structure moduli and of the dilaton

$$V_F = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W}) \tag{2.6}$$

where now $i, j = U, S$. The minimum of this potential can be obtained for $D_U W = 0$ and $D_S W = 0$ which happen to be also the condition to have supersymmetry preservation. In this way the dilaton and the complex structure moduli are stabilized. However, the outcome is completely different for the Kähler moduli. In fact, being absent in the scalar potential, they still represent flat directions that have to be lifted. Moreover, their F-term is non-vanishing.

Proof. Starting from the Kähler potential for the Kähler moduli in Eq. (1.67), recalling the relations (1.62) and (1.65), and considering for simplicity the case of a single Kähler modulus such that

$$\mathcal{V} = \frac{1}{6}\mathcal{K}t^3, \qquad \tau = \frac{1}{2}\mathcal{K}t^2, \tag{2.7}$$

we can find $\mathcal{V} \sim \tau^{3/2}$. Moreover, from the fact that $\tau = (T + \overline{T})/2$ we can find that

$$K_{K-mod} = -2\ln\mathcal{V} \sim -2\ln(\tau^{3/2}) \sim -2\ln((T+\overline{T})^{3/2}) \sim -3\ln(T+\overline{T}). \tag{2.8}$$

At this point we are ready to compute the F-term in the following way

$$F^T = e^K K^{T\overline{T}} D_{\overline{T}}\overline{W} = e^K K^{T\overline{T}} K_{\overline{T}}\overline{W} = e^K \overline{W}\frac{(T+\overline{T})^2}{3}\frac{(-3)}{T+\overline{T}} = -e^K (T+\overline{T})\overline{W} \neq 0. \tag{2.9}$$

□

The fact that the F-term for the Kähler moduli is non-vanishing tells us that supersymmetry is broken along the Kähler moduli flat directions. The name No-Scale refers to the fact that it is impossible to know the energy scales of the F-term computed above and of the gravitino mass

$$m_{3/2}^2 = e^K |W|^2 M_P^2 \tag{2.10}$$

and therefore to know the energy scale of supersymmetry breaking.
This particular structure arises from the fact that the imaginary part of the Kähler moduli $T_{\tilde{\alpha}} = \tau_{\tilde{\alpha}} + i\rho_{\tilde{\alpha}}$, being an axion-like particle, enjoys a shift symmetry which is preserved at all orders in perturbation theory; however, being the superpotential a holomorphic function, the Kähler moduli are excluded from its expression, leading to the No-Scale Structure.
Notice that we are going to expand the argument about axion-like particles later on in this chapter.

What we have seen is that turning on non-trivial fluxes in Type IIB String compactification on a CY Y_6, we are able to stabilize, and therefore to lift, the flat directions of the complex structure moduli and the dilaton. As a consequence, these fields acquire a non-zero mass [23] which is given by

$$m_{U,S} \simeq \frac{M_P}{\mathcal{V}}\,, \tag{2.11}$$

where $\mathcal{V}$ is the unfixed (up to now) volume of the internal manifold. The Kähler moduli are still massless and therefore still constitute a phenomenological problem.
From the Non-Renormalization Theorems we know that the Kähler potential receives perturbative corrections at all orders and also non-perturbative ones, while the superpotential only admits non-perturbative corrections such that

$$K = K_{tree} + K_{pert} + K_{non-pert} \quad \text{and} \quad W = W_{tree} + W_{non-pert} \tag{2.12}$$

where K_{tree} and W_{tree} are, respectively, (1.76) and (2.3). Exploiting this argument and these corrections we can overcome the problem of the flat direction for the Kähler moduli.

2.1.2 Non-Perturbative Corrections

Non-perturbative corrections to the superpotential can be generated both from gaugino condensation and from $D3$-brane instantons. In the first case we have a stack of $D7$-branes wrapping a 4-cycle Σ_4 with a Yang-Mills action for the 4-dimensional gauge fields A_μ; in the second case the 4-cycle Σ_4 is wrapped by a Euclidean $D3$-brane leading to instanton contributions to the path integral ($D3$-brane instantons). We are not going to give more details about gaugino condensation and Euclidean $D3$-branes; however, we can show the contribution these mechanisms give to the superpotential which reads as ([19, 3])

$$W_{non-pert} = \sum_i A_i\, e^{-a_i T_i} \tag{2.13}$$

where A_i is in general a function of the complex structure moduli and of the dilaton which can be considered of $\mathcal{O}(1)$ since these moduli are stabilized at tree level and so they can be integrated out; a_i is a constant which takes the value 2π when the non-perturbative correction is generated by the Euclidean $D3$-branes, while it takes the value $2\pi/N$ (with N rank of the condensing gauge group) when the non-perturbative corrections are generated by gaugino condensation.

Non-perturbative corrections to the superpotential can be used to stabilize the Kähler moduli (unfixed by fluxes) in the so called KKLT (Kachru, Kallosh, Linde, Trivedi) scenario [23] which relies on the fact that the vacuum expectation value of the flux superpotential can be tuned to small values. We are going to expand this model in the Appendix

A since this is not the one we are interested in. In fact it leads to supersymmetric vacua with negative cosmological constant which is not a realistic description.

Concerning the Kähler potential, we are not interested in its non-perturbative corrections since they are negligible with respect to the perturbative ones.

2.1.3 Perturbative Corrections

Perturbative corrections to the Kähler potential can be generated from the α' *expansion* (α' corrections) and the g_s *expansion* (string-loop corrections).

α' corrections This kind of corrections is parametrized by α' which is connected to the string length as $l_s = 2\pi\sqrt{\alpha'}$, telling us that we are dealing with fundamental objects which are different from the ordinary point-like particle ($\alpha' \to 0$).
The leading correction to the Kähler potential in 4-dimensions comes from the 10-dimensional action (1.1) and in particular is an $\mathcal{O}(\alpha'^3)$ correction to the Einstein-Hilbert part (i.e. $\propto \alpha'^3 R^4$). The whole expansion of the 10-dimensional action has been computed in [24], but we are not going to go into deep details here. The resulting contribution to the Kähler potential reads as

$$K_{K-mod} = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right), \qquad \xi = -\frac{\chi(Y_6)\zeta(3)}{2(2\pi)^3 g_s^{3/2}} \tag{2.14}$$

where $\chi(Y_6)$ is the CY Euler number and $\zeta(3)$ the Riemann zeta function. For future uses of this expression we are going to recast it in terms of the Kähler modulus T:

$$\begin{aligned} K_{K-mod} &= -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right) \sim -2\ln\left(\tau^{3/2} + \frac{\xi}{2}\right) \\ &\sim -2\ln\left((T+\overline{T})^{3/2} + \frac{\xi}{2}\right) \sim -3\ln(T+\overline{T}) - \frac{2x}{(T+\overline{T})^{3/2}} \end{aligned} \tag{2.15}$$

with $x = \xi/2$.

g_s corrections String-loop corrections to the Kähler potential can be parametrized as [15]

$$\delta K_{g_s} = \delta K_{g_s}^{KK} + \delta K_{g_s}^{W} \tag{2.16}$$

where $\delta K_{g_s}^{KK}$ comes from the exchange of closed strings carrying Kaluza-Klein momentum between $D7$ and $D3$-branes and $\delta K_{g_s}^{W}$ comes from the exchange of winding strings between intersecting stacks of $D7$-branes. The first kind of corrections read as

$$\delta K_{g_s}^{KK} \sim \sum_i \frac{c_i^{KK}(U,\bar{U})(a_{il}t^l)}{\mathrm{Re}(S)\mathcal{V}} \tag{2.17}$$

where $c_i^{KK}(U,\bar{U})$ is a function of the complex structure moduli that in general is unknown and $a_{il}t^l$ is a linear combination of the basis 2-cycles volumes t^l; the second kind of corrections is instead given by

$$\delta K_{g_s}^{W} \sim \sum_i \frac{c_i^W(U,\bar{U})}{(a_{il}t^l)\mathcal{V}} \tag{2.18}$$

where again, $c_i^W(U,\bar{U})$ is an unknown function of the complex structure moduli.
An important feature of the string-loop corrections coming from $\delta K_{g_s}^{KK}$ is that they generate the so called *Extended No-Scale Structure* for the scalar potential, which means that the cancellation of the No-Scale Structure is extended to one further order in this string loop expansion. This happens when $\delta K_{g_s}^{KK}$ is a homogeneous function of degree $n = -2$ in the 2-cycles volumes. We are going to give a quite explicit heuristic proof of this Extended No-Scale Structure which will be useful in the next chapters [15].

Extended No-Scale Structure *Let Y_6 be a Calabi-Yau three-fold and consider Type IIB $N=1$ supergravity where the Kähler potential and the superpotential in the Einstein frame take the form*

$$K = K_{tree} + \delta K\,, \tag{2.19}$$
$$W = W_0\,. \tag{2.20}$$

If the loop correction δK to K is a homogeneous function in the 2-cycles volumes of degree $n=-2$, at leading order

$$\delta V_{g_s} = 0. \tag{2.21}$$

Starting from

$$\delta V_{g_s} = (K^{ij}K_iK_j - 3)\frac{|W|^2}{\mathcal{V}^2} \tag{2.22}$$

with $K = -2\ln\mathcal{V} + \delta K_{g_s}$, we get that the expansion of the scalar potential due to string-loop corrections to the Kähler potential is given by

$$\delta V_{g_s} = V_0 + \epsilon\delta V_1 + \epsilon^2\delta V_2 + \mathcal{O}(\epsilon^2) \tag{2.23}$$

with

$$V_0 =(K_0^{ij}K_i^0K_j^0 - 3)\frac{|W|^2}{\mathcal{V}^2}, \tag{2.24}$$

$$\delta V_1 =(2K_0^{ij}K_i^0\delta K_j - K_0^{im}\delta K_{ml}K_0^{lj}K_i^0K_j^0)\frac{|W|^2}{\mathcal{V}^2}, \tag{2.25}$$

$$\begin{aligned}\delta V_2 =&(K_0^{ij}\delta K_i\delta K_j - 2K_0^{im}\delta K_{ml}K_0^{lj}K_i^0\delta K_j \\ &+ K_0^{im}\delta K_{mp}K_0^{pq}\delta K_{ql}K_0^{lj}K_i^0K_j^0)\frac{|W|^2}{\mathcal{V}^2}.\end{aligned} \tag{2.26}$$

and ϵ an expansion parameter.

Proof. For the sake of brevity, we are going to sketch only the main steps to obtain the previous expansions for the scalar potential. We introduce an expansion parameter ϵ such that we can define

$$\mathcal{K}_0 = \frac{\partial^2 K_0}{\partial\tau_i\partial\tau_j}, \qquad \delta\mathcal{K} = \frac{\partial^2(\delta K_{g_s})}{\partial\tau_i\partial\tau_j} \tag{2.27}$$

with $i, j = 1, \ldots, h_{1,1}$. Using these, we can write K^{ij} in the following way:

$$K^{ij} = (\mathcal{K}_0 + \epsilon\delta\mathcal{K})^{ij} = (\mathcal{K}_0(1 + \epsilon\mathcal{K}_0^{-1}\delta\mathcal{K}))^{ij} = (1 + \epsilon\mathcal{K}_0^{-1}\delta\mathcal{K})^{il}K_0^{lj}. \tag{2.28}$$

Using now the Neumann series

$$(1 + \epsilon\mathcal{K}_0^{-1}\delta\mathcal{K})^{il} = \delta^i_l - \epsilon K_0^{im}\delta K_{ml} + \epsilon^2 K_0^{im}\delta K_{mp}K_0^{pq}\delta K_{ql} + \mathcal{O}(\epsilon^2) \tag{2.29}$$

we find, up to second order,

$$K^{ij} = K_0^{ij} - \epsilon K_0^{im}\delta K_{ml}K_0^{lj} + \epsilon^2 K_0^{im}\delta K_{mp}K_0^{pq}\delta K_{ql}K_0^{lj} + \mathcal{O}(\epsilon^2), \tag{2.30}$$

which, substituted back into Eq. (2.22) gives Eq. (2.23). □

We still need to look at the statement of the Extended No-Scale Structure that tells us that δV_1 vanishes if K is a homogeneous function of the 2-cycles volumes of degree $n = -2$.

It has been proven in [15] that, rewriting δV_1 in terms of the 2-cycle volumes and exploiting the Euler theorem, the first correction to the scalar potential (2.25) becomes

$$\delta V_1 = -\frac{|W|^2}{\mathcal{V}^2}\frac{1}{4}(3n + n(n-1))\delta K = -\frac{|W|^2}{\mathcal{V}^2}\frac{1}{4}n(n+2)\delta K \tag{2.31}$$

if δK is of degree n. Recalling the forms of the string-loop corrections for the KK modes (2.17) and for the winding modes (2.18), we can infer that

$$\begin{cases} \delta V_1^{KK} = 0 \\ \delta V_1^{W} = -2\delta K_{g_s}^{W}\frac{|W|^2}{\mathcal{V}^2} \end{cases} \tag{2.32}$$

since $n = -2$ for the KK modes and $n = -4$ for the winding modes. Concerning instead the further order correction (2.26), it can be proven that it becomes

$$\delta V_2 = \frac{(c_a^{KK})^2}{\mathrm{Re}(S)^2}K^0_{aa}\frac{|W|^2}{\mathcal{V}^2} \tag{2.33}$$

where we are in the simple case of one Kähler modulus labeled by a.

Before moving on, it is worth stressing that the contributions given by $\delta K_{g_s}^{KK}$ are subleading with respect to the α' corrections when considering the so called Large Volume limit for the CY volume scalar potential. This is a feature that we are going to see again in Chapter 3.

2.2 Large Volume Scenario

2.2.1 Minimization

The model for Kähler moduli stabilization we are interested in is the *Large Volume Scenario (LVS)* [25, 18] whose advantage with respect to the KKLT construction is to give a non-supersymmetric minimum (which, however, is still AdS).
The most common construction of the LVS arises in "Swiss-Cheese" CY manifolds where the overall volume can be written as

$$\mathcal{V} = \alpha \left(\tau_b^{3/2} - \sum_{r=1}^{N_s} \lambda_r (\tau_s^r)^{3/2} \right) \tag{2.34}$$

with τ_b volume of the single large 4-cycle, τ_s^r volumes of the small $N_s = h_+^{1,1} - 1$ 4-cycles ($r = 1, \ldots, N_s$), $\alpha > 0$ and $\lambda_r > 0$. The necessary conditions to have such a structure are discussed in [26]; in general, one needs $h_+^{1,1} \equiv N_s + N_b > 1$ and $N_s \geq 1$ Kähler moduli corresponding to the blow-ups of points (further restrictions are required if $N_s > 1$ or if $N_b > 1$). Let us highlight that the small cycles correspond to exceptional divisors that arise from the necessity of "blowing-up" the singularities of the CY, that is replacing them with a new smooth space which, in our constructions, usually corresponds to a diagonal Del Pezzo surface.

In the LVS we take advantage of the interplay between the α' corrections to the Kähler potential and the non-perturbative corrections to the superpotential to stabilize the Kähler moduli. For the purposes of this thesis it is useful to give an explicit example of stabilization in LVS; we are going to follow mainly [27] and [18] in which compactifications on the orientifold of $\mathbb{P}^4_{[1,1,1,6,9]}$ with $h^{1,1} = 2$ are considered. In this case we have two Kähler moduli $T_b = \tau_b + i\rho_b$ and $T_s = \tau_s + i\rho_s$ where τ_b is the volume of the big 4-cycle and τ_s is the volume of the small blow-up cycle. What we are interested in is the limit $\mathcal{V} \to \infty$ (i.e. $\tau_b \to \infty$). The overall volume of the CY can be written as

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_b^{3/2} - \tau_s^{3/2} \right). \tag{2.35}$$

After integrating out the complex structure moduli and the dilaton, we can write the Kähler potential and the superpotential in the following way

$$K = -2 \ln \left(\frac{1}{9\sqrt{2}} (\tau_b^{3/2} - \tau_s^{3/2}) + \frac{\xi}{2 g_s^{3/2}} \right) = -2 \ln ((\tau_b^{3/2} - \tau_s^{3/2}) + \xi'), \tag{2.36}$$

$$W = W_0 + A_s \, e^{-a_s T_s} \tag{2.37}$$

where we have considered the non-perturbative correction supported on the small cycle (so the small cycle should be rigid) and where W_0 is the constant contribution from the flux superpotential (after integrating out the complex structure moduli and the dilaton as already mentioned). Let us notice that for the sake of simplicity we have absorbed the factor $1/(9\sqrt{2})$ since it does not affect computations. At this point we need to compute the scalar potential in Eq. (2.4) and to do so, we need to compute the inverse Kähler matrix $K^{i\bar{j}}$ and the derivatives of the Kähler potential. We start from the first derivatives:

$$K_b = \frac{1}{2}\frac{\partial K}{\partial \tau_b} = -\frac{3}{2}\frac{\sqrt{\tau_b}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')}, \tag{2.38}$$

$$K_s = \frac{1}{2}\frac{\partial K}{\partial \tau_s} = \frac{3}{2}\frac{\sqrt{\tau_s}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')}. \tag{2.39}$$

The second derivatives follow immediately from the two expressions above and constitute the Kähler matrix

$$K_{i\bar{j}} = \begin{pmatrix} \frac{3}{4\tau_b^2} & -\frac{9\tau_s^{1/2}}{8\tau_b^{5/2}} \\ -\frac{9\tau_s^{1/2}}{8\tau_b^{5/2}} & \frac{3}{8\tau_s^{1/2}\tau_b^{3/2}} \end{pmatrix} \tag{2.40}$$

where we have considered the limit $\tau_b \gg \tau_s$. The inverse Kähler matrix follows straightforwardly from Eq. (2.40) and reads as

$$K^{i\bar{j}} = \begin{pmatrix} \frac{4\tau_b^2}{3} & 4\tau_b\tau_s \\ 4\tau_b\tau_s & \frac{8\tau_b^{3/2}\tau_s^{1/2}}{3} \end{pmatrix}. \tag{2.41}$$

The scalar potential therefore reads as

$$\begin{aligned} V &= \frac{8(a_s A_s)^2\sqrt{\tau_s}e^{-2a_s\tau_s}}{3\tau_b^{3/2}} - \frac{4a_s A_s W_0 \tau_s e^{-a_s\tau_s}}{\tau_b^3} + \frac{\nu|W_0|^2}{\tau_b^{9/2}} \\ &= \frac{a_s^2\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{\tau_b^{3/2}} - \frac{\mu a_s \tau_s |W_0| e^{-a_s\tau_s}}{\tau_b^3} + \frac{\nu|W_0|^2}{\tau_b^{9/2}} \end{aligned} \tag{2.42}$$

where we have used

$$\lambda = \frac{8}{3}A_s^2, \qquad \mu = 4A_s, \qquad \nu = \frac{27\sqrt{2}\xi}{4g_s^{3/2}}. \tag{2.43}$$

The first thing we want to do is to minimize the scalar potential. To do so we compute the first derivative of Eq. (2.42) with respect to τ_s and the first derivative with respect to τ_b; imposing the first one equal to zero and inverting the expression we find that

$$e^{-a_s\tau_s} = \frac{3W_0}{a_s A_s}\frac{\sqrt{\tau_s}}{\tau_b^{3/2}}\frac{(1-a_s\tau_s)}{(1-4a_s\tau_s)} \tag{2.44}$$

while, imposing the second one equal to zero and using the volume one can obtain from Eq. (2.44), we get that

$$\tau_s^{3/2} = \frac{\nu}{8}\frac{(1-4a_s\tau_s)^2}{(1-a_s\tau_s)^2}\frac{1}{\frac{(1-4a_s\tau_s)}{(1-a_s\tau_s)}-1}. \tag{2.45}$$

The expression (2.44) is telling us that the overall volume of the CY in the LVS goes as

$$\langle\mathcal{V}\rangle = \langle\tau_b^{3/2}\rangle \propto e^{a_s\langle\tau_s\rangle} \tag{2.46}$$

meaning that the overall volume could be exponentially large depending on the value of $a_s\tau_s$; the expression (2.45) instead, recalling the form of ν shows that

$$\langle\tau_s\rangle \propto \frac{\xi^{2/3}}{g_s}. \tag{2.47}$$

Thus, in a regime where the Effective Field Theory is under control ($g_s \ll 1$), we get that the volume could become exponentially large. It is possible to show that these behaviours correspond to an AdS minimum which is not phenomenologically interesting. The mechanism for building metastable dS vacua solutions is known as uplifting. However, it should not be thought of as an additional step to the one of moduli stabilization after having obtained an AdS solution, but rather as a process which proceeds along moduli stabilization such that a dS vacuum is obtained thanks to the interplay of several contributions to the scalar potential [28]. Here we are going to mention some of the possible ways to achieve a dS solution [3, 28].

- **Anti-branes:** This method was first proposed as a part of the KKLT construction in [23]. It consists on breaking the AdS supersymmetric solution by adding anti $D3$-branes ($\overline{D3}$-branes) in the compactification. This solution generically requires the presence of 3-form fluxes such that the metric is warped [7]

$$g_{MN}dx^M dx^N = e^{2A(y)}g_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dx^m dx^n, \tag{2.48}$$

  with $e^{A(y)}$ warp factor (it is a function on Y_6), and such that the CY develops a strongly warped region (i.e. a warped throat). In this scenario the anti $D3$-branes are driven at the end of the throat where the warp factor is minimized. The need for the anti-brane to stay on the tip of the throat is due to the fact that, in such a way, its energy is redshifted enough to have a contribution that does not spoil the volume potential into a runaway and that allows for a positive contribution to the scalar potential [3]

$$V_{\overline{D3}} \sim \frac{e^{A_0}}{(T+\overline{T})^2}, \tag{2.49}$$

making it suitable to uplift the AdS vacuum (e^{A_0} is the value of the warp factor at the end of the throat).

- **Magnetized branes:** In this case there are $U(1)$ fluxes localized on a stack of $D7$-branes that wrap the volume modulus. The contribution to the scalar potential that uplifts the AdS solution comes from D-terms.
- **Kähler uplift:** In this case, if the volume is small enough, the α' correction can compete with the fluxes and with non-perturbative effects to generate solutions with positive vacuum energy. However, the dS minima, are in regions corresponding to the edge of the validity of the EFT.
- **Complex structure F-terms:** In general the complex structure moduli are stabilized such that $D_U W = 0$. However, there could exist minima in which this F-term is non vanishing leading to dS vacua once the Kähler moduli are stabilized.
- **Non-perturbative dS vacua:** In this case all the moduli are stabilized in one step thanks to background fluxes and non-perturbative contributions to the superpotential, leading to dS minima.
- **Dilaton dependent non-perturbative effects:** These effects could arise from strong dynamics on hidden sector $D3$-branes at singularities and give a positive contribution to the potential which can uplift AdS solutions.

2.2.2 Moduli mass spectrum

For the purposes of this thesis (we are going to follow the same approach when studying RR Dark Photons), it is instructive to compute also the masses of the moduli after the canonical normalization of the fields. First of all we compute the mass matrix which is given by the second derivatives of the scalar potential $M^2_{i\bar{j}} = V_{i\bar{j}}/2$. To do so, it is useful to expand Eq. (2.44) and Eq. (2.45) in powers of the quantity

$$\epsilon = \frac{1}{4a_s\tau_s} \sim \mathcal{O}\left(\frac{1}{\ln \mathcal{V}}\right) \tag{2.50}$$

up to second order: from Eq. (2.44) we get that

$$e^{-a_s\tau_s} = \frac{3W_0}{4a_sA_s}\frac{\sqrt{\tau_s}}{\tau_b^{3/2}}(1-3\epsilon-3\epsilon^2) = \frac{\mu}{2\lambda}\frac{W_0}{a_s}\frac{\sqrt{\tau_s}}{\tau_b^{3/2}}(1-3\epsilon-3\epsilon^2), \tag{2.51}$$

while from Eq. (2.45) we obtain

$$\tau_s^{3/2} = \frac{2}{3}\nu(1+2\epsilon+9\epsilon^2) \to \frac{\mu^2}{4\lambda}\tau_s^{3/2} = \nu(1+2\epsilon+9\epsilon^2). \tag{2.52}$$

At this point, the second derivatives of the scalar potential expanded in ϵ read as [27]

$$\frac{\partial^2 V}{\partial \tau_b^2} = \frac{9|W_0|^2\nu}{2\tau_b^{13/2}}(1+2\epsilon), \tag{2.53}$$

$$\frac{\partial^2 V}{\partial \tau_s \tau_b} = \frac{-3a_s|W_0|^2\nu}{\tau_b^{11/2}}(1-5\epsilon+4\epsilon^2), \tag{2.54}$$

$$\frac{\partial^2 V}{\partial \tau_s^2} = \frac{2a_s^2|W_0|^2\nu}{\tau_b^{9/2}}(1-3\epsilon+6\epsilon^2) \tag{2.55}$$

such that the mass matrix is simply given by

$$M^2_{i\bar{j}} = \begin{pmatrix} \frac{9|W_0|^2\nu}{4\tau_b^{13/2}}(1+2\epsilon) & \frac{-3a_s|W_0|^2\nu}{2\tau_b^{11/2}}(1-5\epsilon+4\epsilon^2) \\ \frac{-3a_s|W_0|^2\nu}{2\tau_b^{11/2}}(1-5\epsilon+4\epsilon^2) & \frac{a_s^2|W_0|^2\nu}{\tau_b^{9/2}}(1-3\epsilon+6\epsilon^2) \end{pmatrix}. \tag{2.56}$$

The moduli masses are given by the eigenvalues of the matrix $K^{i\bar{j}}M^2_{i\bar{j}}$, which is (dropping the indices for brevity)

$$K^{-1}M^2 = \frac{2a_s\tau_s|W_0|^2\nu}{3\tau_b^{9/2}}\begin{pmatrix} -9(1-7\epsilon) & 6a_s\tau_b(1-5\epsilon+16\epsilon^2 \\ -\frac{6\tau_b^{1/2}}{\tau_s^{1/2}}(1-5\epsilon+4\epsilon^2) & \frac{4a_s\tau_b^{3/2}}{\tau_s^{1/2}}(1-3\epsilon+6\epsilon^2) \end{pmatrix}. \tag{2.57}$$

Due to the presence of a hierarchy between the moduli masses, we can compute them using the following simplifications:

$$\begin{aligned} m_\chi^2 \simeq \mathrm{Tr}(K^{-1}M^2) &\simeq \frac{8a_s^2|W_0|^2\nu\tau_s^{1/2}}{3\tau_b^3} = \frac{8a_s^2\nu\tau_s^{1/2}}{3}m_{3/2}^2 \\ &= \left(2m_{3/2}\ln\left(\frac{M_P}{m_{3/2}}\right)\right)^2 \sim \left(\frac{\ln(\mathcal{V})}{\mathcal{V}}\right)^2, \end{aligned} \tag{2.58}$$

$$m_\phi^2 \simeq \frac{\det(K^{-1}M^2)}{\mathrm{Tr}(K^{-1}M^2)} \simeq \frac{27|W_0|^2\nu}{4a_s\tau_s\tau_b^{9/2}} \sim \frac{1}{\mathcal{V}^3\ln(\mathcal{V})}. \tag{2.59}$$

These expressions show us that $m_\chi^2 \gg m_\phi^2$, and so the volume modulus is much lighter than the small blow-up modulus.
Some comments are now in order.

1. We have canonically normalized τ_b and τ_s using the eigenvectors v_1 and v_2 of the matrix $K^{i\bar{j}}M^2_{i\bar{j}}$, and solving the eigensystem such that $K^{i\bar{j}}M^2_{i\bar{j}}v_1 = m_\phi^2 v_1$ and $K^{i\bar{j}}M^2_{i\bar{j}}v_2 = m_\chi^2 v_2$
$$\begin{pmatrix} \hat{\tau}_b \\ \hat{\tau}_s \end{pmatrix} = \begin{pmatrix} v_1 \end{pmatrix} \frac{\phi}{\sqrt{2}} + \begin{pmatrix} v_2 \end{pmatrix} \frac{\chi}{\sqrt{2}}, \tag{2.60}$$
where we have expanded the moduli in the minimum $\tau_i = \langle \tau_i \rangle + \hat{\tau}_i$ with $\langle \tau_i \rangle$ the vacuum expectation value in the minimum and $\hat{\tau}_i$ a small fluctuation. ϕ is the canonical normalized field corresponding to the big volume modulus and χ is the canonical normalized field corresponding to the volume of the small blow-up cycle. The final expression of (2.60) turns out to be
$$\hat{\tau}_b = \left(\sqrt{\frac{4}{3}} \langle \tau_b \rangle \right) \frac{\phi}{\sqrt{2}} + \left(\sqrt{6} \langle \tau_b \rangle^{1/4} \langle \tau_s \rangle^{3/4} (1 - 2\epsilon) \right) \frac{\chi}{\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{2/3})\phi, \tag{2.61}$$
$$\hat{\tau}_s = \left(\frac{\sqrt{3}}{a_s} (1 - 2\epsilon) \right) \frac{\phi}{\sqrt{2}} + \left(\frac{2\sqrt{6}}{3} \langle \tau_b \rangle^{3/4} \langle \tau_s \rangle^{1/4} \right) \frac{\chi}{\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{1/2})\chi. \tag{2.62}$$
We can see that, although there is a small mixing between the two moduli, τ_b is mostly in the direction of ϕ and τ_s is mostly in the direction of χ.

2. Using Eq. (2.10) we find that the mass of the gravitino is given by
$$m_{3/2}^2 \simeq \frac{W_0^2}{\mathcal{V}^2} M_P^2. \tag{2.63}$$
Compared to Eq. (2.58) and Eq. (2.59), we can see that the mass of the small blow-up cycle volume modulus is higher than the gravitino mass and therefore it is out of the supersymmetry breaking scale, while the mass of the large volume modulus, being smaller than the gravitino mass, could cause the so called cosmological moduli problem (that we are going to address briefly in the following).

3. Thanks to LVS it is possible to get a hierarchically suppressed supersymmetry breaking scale which can be achieved by an exponentially large volume $\mathcal{V} \gg 1$ by naturally keeping $W_0 \sim \mathcal{O}(1 - 10)$, instead of having to fine tune it to small values (as in the KKLT construction).

2.2.3 Cosmological Moduli Problem

The fact that the volume modulus τ_b acquires a mass (2.59) which is smaller than the mass of the gravitino (2.63) and of the other moduli constitutes a problem which is well known in the literature and goes under the name of *cosmological moduli problem* [29]. The problem is related to the fact that these light moduli outlive all the other moduli which are much more massive, and also other particles. This is due to the fact that these moduli interact through gravitationally suppressed couplings and their decay rate can be estimated as [3] (more on the decay rate of the large volume modulus will be discussed in the next chapters)

$$\Gamma_{mod} = \frac{\lambda}{16\pi}\frac{m_{mod}^3}{M_P^2} \tag{2.64}$$

with $\lambda \sim \mathcal{O}(1)$. Particles with renormalizable perturbative decays, on the other hand, have decay rates which go as [3]

$$\Gamma_p \sim \frac{y^2}{16\pi} m_p, \tag{2.65}$$

with m_p mass of the particle and y Yukawa coupling, which are larger and much less suppressed than the previous ones. As long as there is some initial displacement in the moduli fields, the most common expectation is the one of a moduli-dominated era in which the longest-living modulus reheats the universe.

Assuming a scalar field ϕ (canonically normalized) with gravitational strength interaction in a FLRW background, we have that its time evolution is given by [29]

$$\ddot{\phi} + (3H + \Gamma_\phi)\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \tag{2.66}$$

with $H = \dot{a}/a$ the Hubble parameter, a the scale factor, V the scalar potential and Γ_ϕ the decay rate given by Eq. (2.64). We expect the moduli fields to be initially shifted from their zero-temperature minimum after inflation due to the effects of thermal fluctuations or quantum fluctuations during inflation and due to their initial supersymmetric flat potential. It is possible to show that, if the initial shift of the scalar fields is $\phi_{in} \sim M_P$, coherent oscillations of the fields will bring to an energy density domination of ϕ since radiation decreases as a^{-4} while, for these scalar fields, their energy density decreases as a^{-3} [27]:

$$\rho_\phi(T) = \rho_\phi(T_{in})\left(\frac{T}{T_{in}}\right)^3 \tag{2.67}$$

with $a \propto 1/T$. Using the values of the temperature and the Hubble parameter today, it is possible to obtain two constraints for $\Gamma_\phi \sim 0$ from the requirement of avoiding an overclosed universe [27, 29]:

$$\phi_{in} < 10^{-10} M_P \left(\frac{m_\phi}{100\, GeV} \right)^{-1/4}, \tag{2.68}$$

$$m_\phi > 10^{-26}\, eV; \tag{2.69}$$

the first one is a upper bound for the initial displacement of the scalar field, while the second one is a lower bound on its mass. If the scalar field is not stable, i.e. $\Gamma_\phi \neq 0$, and long-lived, its decay may spoil the products of Big Bang Nucleosynthesis (BBN). The hard cosmological moduli problem requirement is that the temperature at which the longest-living modulus reheats the universe should be bigger than the BBM temperature: $T_{rh} > T_{BBN}$. It can be shown that a temperature $T_{rh} \leq \mathcal{O}(1)\, MeV$ will spoil BBN giving a lower bound for $m_\phi \geq \mathcal{O}(30)\, TeV$ [27].

2.2.4 Moduli Decays

Another issue associated with the longest-living moduli is their branching ratio to other particles. The first concern is about the decay of the longest-living moduli into the Minimal Supersymmetric Standard Model particles (MSSM). It is well known that, once gravity mediation is assumed, the chiral multiplets split into a part corresponding to the MSSM particles, the *visible sector*, and into a part constituting the so called *hidden sector* (corresponding to the moduli) in which supersymmetry is broken. This breaking reflects into the visible sector through the *soft terms* (scalar masses, gaugino masses, trilinear couplings) such that, in general LVS constructions [30]

$$m_{soft} \sim m_{3/2} \sim \frac{M_P}{\mathcal{V}}. \tag{2.70}$$

However, the form and the scale of the soft terms depend on how the MSSM is realized in the LVS such that we can have two possibilities [3, 30, 31]:

$$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim m_{soft}^{geom} \gg m_\phi \sim \frac{M_P}{\mathcal{V}^{3/2}}, \tag{2.71}$$

$$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \gg m_\phi \sim \frac{M_P}{\mathcal{V}^{3/2}} \gg m_{soft}^{seq} \sim \frac{M_P}{\mathcal{V}^2}. \tag{2.72}$$

In the first case (2.71) we have that the mass of the longest-living modulus is smaller or, at most, of the order of the soft masses and so the branching ratio between these two is kinematically forbidden. In the second case (2.72), however, the mass of the longest-living modulus is bigger than the soft terms which, in turn, are far below the gravitino mass; the longest-living modulus can thus decay into the MSSM particles. This second

possibility arises in the context of the *sequestered Large Volume Scenario* which will be the background of the study of dark radiation from RR Photons.
Before studying sequestering in LVS, it is worth noticing that the possibility of the decay of the longest-living modulus into other non-Standard Model particles, could be a problem if these decay products affect the effective number of neutrino species bounding the amount of relativistic degrees of freedom and thus Dark Radiation. After having introduced the sequestered LVS, we are going to also address the topic of Dark Radiation production.

2.3 Sequestered Large Volume Scenario

2.3.1 Moduli stabilisation and supersymmetry breaking

To obtain phenomenologically viable models with a TeV scale soft terms, we need them to be hierarchically smaller than the gravitino mass as in Eq. (2.72); this configuration can be achieved in the context of the sequestered LVS.
It is important to highlight that the scale of the soft terms is determined by the position and the type of the SM Dp-brane in the CY orientifold compactification.

In sequestered scenarios the SM degrees of freedom are localized in the extra dimensions. This setup can be realized when the visible sector sits on $D3$-branes at singularities. The attractive feature of this construction is that the F-term of the SM cycle vanishes and the dominant one is associated with the moduli. Moreover, the moduli are feebly coupled to the visible sector due to their bulk separation, leading to the hierarchy in Eq. (2.72) [32]. Differently, if the F-term of the modulus associated to the cycle wrapped by the SM breaks supersymmetry, we recover the case in which the masses of the soft terms are of the order of the gravitino mass, or they present a little suppression with respect to it.

The general setup considered in [32] consists of a big 4-cycle controlling the CY volume with $T_b = \tau_b + i\rho_b$ associated Kähler modulus, a small blow-up cycle associated to $T_s = \tau_s + i\rho_s$ which supports the non-perturbative effects to the superpotential, a cycle supporting the visible sector with $T_{SM} = \tau_{SM} + i\rho_{SM}$ and its orientifold image $G = b + ic$; τ_b, τ_s and $\tau_{SM} \to 0$ are the 4-cycles volumes, ρ_b, ρ_s and ρ_{SM} are the axions resulting from the reduction of C_4 over the corresponding 4-cycles and b, c are the reduction of B_2 and C_2 on the 2-cycle dual to the 4-cycle. The first two moduli constitute therefore the classical Large Volume Scenario studied in Sec. 2.2, while the other two are associated to two diagonal del Pezzo divisors collapsing to zero due to D-term fixing.[1]

[1]See [33] for a more detailed analysis of Dp-branes at del Pezzo singularities.

The Kähler potential can be written as [32]

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) - \ln(S+\bar{S}) + \lambda_{SM}\frac{\tau_{SM}^2}{\mathcal{V}} + \lambda_b\frac{b^2}{\mathcal{V}} + K_{cs}(U) + K_{dS} + K_{matter} \quad (2.73)$$

with λ_{SM}, λ_b coefficients of order $\mathcal{O}(1)$, $K_{cs}(U)$ Kähler potential for the complex structure moduli at tree level, K_{dS} Kähler potential containing terms to uplift the AdS vacuum and

$$K_{matter} = \tilde{K}_\alpha(M,\bar{M})\bar{C}^{\bar{\alpha}}C^\alpha + [Z(M,\bar{M})H_uH_d + h.c.] \quad (2.74)$$

where M denotes a moduli dependence, C^α are MSSM superfields, H_u and H_d are the two Higgs doublets of the MSSM and Z a function of the axio-dilaton S or of the complex structure moduli U parametrizing the Higgs bilinear presence (this is called Giudice-Masiero coupling term). $\tilde{K}_\alpha$ is the matter metric of the visible sector that is parametrized as [32]

$$\tilde{K}_\alpha = \frac{f_\alpha(U,S)}{\mathcal{V}^{2/3}}\left(1 - c_s\frac{\hat{\xi}}{\mathcal{V}} + \tilde{K}_{dS} + c_{SM}\tau_{SM}^p + c_b b^p\right), \quad (2.75)$$

where $p > 0$ to have a well-behaved metric in the singular limit $\tau_{SM}, b \to 0$, the parameters c_s, c_{SM} and c_b are taken to be constant and they take particular values in the local and ultra-local limits (which we are going to briefly explain in a few). Notice that the functions $f_\alpha(U,S)$ could be non-universal generating different soft terms masses. We can also write the superpotential for this setting that is [32]

$$W = W_{flux}(U,S) + A_s(U,S)e^{-a_sT_s} + W_{dS} + W_{matter} \quad (2.76)$$

where $W_{flux}(U,S)$ is the tree level superpotential (2.3), the second term is the well known non-perturbative correction we have already discussed in Subsec. 2.1.2, W_{dS} is the contribution to uplift the AdS vacuum and

$$W_{matter} = \mu(M)H_uH_d + \frac{1}{6}Y_{\alpha\beta\gamma}(M)C^\alpha C^\beta C^\gamma + \ldots, \quad (2.77)$$

where $Y_{\alpha\beta\gamma}$ are the Yukawa couplings for the MSSM superfields and the dots denote higher dimensional operators.

At this point we need to stabilize the the moduli of the theory. It can be shown that the Kähler moduli where the visible sector brane is located are stabilized using D-terms; since the moduli T_{SM} and G are charged under $U(1)$ symmetries, their D-term scalar potential reads as [32, 33]

$$V_D = \frac{1}{2\mathrm{Re}(f_1)}\left(\sum_\alpha q_{1\alpha}\frac{\partial K}{\partial C^\alpha}C^\alpha - \xi_1\right)^2 + \frac{1}{2\mathrm{Re}(f_2)}\left(\sum_\alpha q_{2\alpha}\frac{\partial K}{\partial C^\alpha}C^\alpha - \xi_2\right)^2 \quad (2.78)$$

with q_1 and q_2 charges, f_1 and f_2 gauge kinetic functions of the two $U(1)$s and ξ_1 and ξ_2 Fayet-Iliopoulos terms. The remaining flat directions, instead, are stabilized by subleading F-terms. If all the matter fields acquire vanishing VEVs from these F-term contributions, the D-term potential admits a supersymmetric minimum for $\tau_{SM} = b = 0$. Roughly speaking, it means that the SM modulus T_{SM} does not break supersymmetry since its F-term is proportional to τ_{SM} that goes to zero due to the D-term stabilization. In this way the visible sector does not feel any supersymmetry breaking effect and it is said to be *sequestered*. Moreover, the axions ρ_{SM} and c are eaten up by the two $U(1)$s. Supersymmetry is broken in the bulk due to the moduli stabilization we have already seen in the LVS discussion in Sec. 2.2 [34]. The associated F-terms can be computed using the general supergravity formula

$$F^i = e^{K/2} K^{i\bar{j}} D_{\bar{j}} \overline{W} \tag{2.79}$$

and using the expressions (2.38) and (2.39) for the first derivatives of the Kähler potential and Eq. (2.41) for the inverse Kähler matrix; performing the simple computations they reads as

$$\frac{F^{T_b}}{\tau_b} \simeq -2m_{3/2}, \qquad \frac{F^{T_s}}{\tau_s} \simeq -6m_{3/2}\epsilon \tag{2.80}$$

where ϵ is the one in Eq. (2.50), and so they enter in the computations of the soft terms values. Let us highlight once more that, performing the computations, one gets

$$F^G = F^{T_{SM}} = 0 \tag{2.81}$$

that reflects the sequestered behavior of the model in exam.
If we now take into account also non-perturbative effects at singularities (as gaugino condensation on $D3$-branes or E(-1) instantons) we can uplift the AdS vacuum[2] to a dS one. These contributions enter in both the Kähler potential and the superpotential reading as [32]

$$K_{dS} = \lambda_{dS} \frac{\tau_{dS}^2}{\mathcal{V}}, \tag{2.82}$$

$$W_{dS} = A_{dS}(U,S)\, e^{-a_{dS}(S + k_{dS} T_{dS})} \tag{2.83}$$

where we have introduced a new blow-up mode $\tau_{dS} = \mathrm{Re}(T_{dS})$. Assuming these expressions and taking into account the fact that the physical Yukawa couplings do not depend on the volume $\mathcal{V}$ nor at leading order, neither at subleading order (ultra-local limit where $c_s = c_{dS} = 1/3$)[3], we obtain universal gaugino masses [32, 34]

$$M_{1/2} \sim \frac{m_{3/2}}{\mathcal{V}} (\ln \mathcal{V})^{3/2} \tag{2.84}$$

[2]To see more about the AdS vacuum uplift see [32].

[3]More about the subject of locality of the SM construction can be found in [31]. For the soft terms in the ultra-local limit see [32]

and non-universal scalar masses scaling roughly as [32, 34]

$$m_\alpha^2 \sim c_\alpha M_{1/2}^2 \sim \frac{M_P^2}{\mathcal{V}^4} \tag{2.85}$$

where $c_\alpha = c_\alpha(U, S)$ is a function involving derivatives of $f_\alpha(U, S)$. With the construction we have just introduced we are able to get the requested hierarchy between the masses

$$m_{soft} \sim \frac{M_P}{\mathcal{V}^2} \ll m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}} \ll m_{3/2} \sim \frac{M_P}{\mathcal{V}}. \tag{2.86}$$

Before moving on, let us mention that such a hierarchy is desirable to avoid the gravitino problem which arises when considering KKLT construction for moduli stabilization as in Appendix A.

2.3.2 Dark Radiation

It is well known that after neutrinos decouple from the thermal bath at a temperature of roughly 1 MeV, they were left as a cosmic background which is called Dark Radiation due to impossibility of a direct detection. This cosmic neutrino background contributes to the total energy density of the universe ρ_{tot} as [2]

$$\rho_{tot} = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right) \tag{2.87}$$

where N_{eff} is the *effective number of neutrino species*; in the Standard Model we have that $N_{eff,BBN} = 3$ at BBN time, while $N_{eff,CMB} = 3.046$ at CMB time. If $\Delta N_{eff} = N_{eff} - N_{eff,SM} \neq 0$, there would be extra Dark Radiation, which cannot be explained with SM physics, giving hints about the presence of new physics beyond the SM. Present observational bounds from Planck measurements [1] show that $\Delta N_{eff} \leq 0.2 - 0.5$.

In string models for the early universe we have seen that the lightest modulus, whose coupling is gravitationally suppressed, can outlive other moduli and other particles giving rise to a moduli-driven reheating of the universe. As already mentioned, this modulus must decay before BBN and its mass should be big enough to overcome the gravitino problem.

In generic compactification models, the most common outcome is that the gravitino mass, the modulus mass and the soft terms are of the same magnitude; however, this is a problem from the point of view of viable string phenomenology, since the lower bound on the modulus mass due to BBN constraint, would give soft terms which are above the TeV scale spoiling the solution of the hierarchy problem. A solution is to consider sequestered LVS for string compactification where the mass hierarchy in Eq. (2.86) is such that we can obtain soft masses around the TeV scale considering the mass of the

modulus around $10^7\,GeV$ (which is safe from the point of view of the cosmological moduli problem) [34].
The question we need to answer before directly studying Dark Radiation production is: which is the lightest modulus and therefore the longest-living one? Recalling our discussion about LVS and about its sequestering, we know that the bulk moduli τ_b and τ_s are stabilized through the mechanism already described in Sec. 2.2. After the canonical normalization of these fields we obtain their masses which are given by Eq. (2.59) for the large volume modulus and by Eq. (2.58) for the blow-up volume modulus. Rewriting them in terms of the gravitino mass we obtain that

$$m_\phi \simeq m_{3/2}\sqrt{\frac{m_{3/2}}{M_P}} \ll m_{3/2}, \tag{2.88}$$

$$m_\chi \simeq m_{3/2}\ln\left(\frac{M_P}{m_{3/2}}\right) > m_{3/2} \tag{2.89}$$

showing that the last modulus to decay is the one associated to the large volume.

At this point we are ready to study the decay of the large volume modulus and the consequent production of Dark Radiation. In fact, the lightest modulus could in principle decay into light hidden sector degrees of freedom which could behave as Dark Radiation.

2.3.3 Axionic Dark Radiation

One of the most interesting, and perhaps most studied, sources of Dark Radiation is the decay of the lightest modulus into light axions.
Axions enter the expression of the moduli as their imaginary part and one of their most compelling feature is that they enjoy a shift symmetry that is preserved at all orders in perturbation theory, while it could be broken at non-perturbative level. Thus, if the moduli are stabilized by non-perturbative effects, they get a mass which is as big as the one of the corresponding axion; in this case the decay is kinematically forbidden. However, if moduli are stabilized by perturbative effects, the axions remain massless due to the shift symmetry and therefore the moduli decay channel into axions is allowed with the consequent potential formation of Dark Radiation[4]. Our goal in what follows is to show how Dark Radiation arises from the decay of the longest-lived modulus. In particular we are going to work in the sequestered LVS where the large volume modulus τ_b is the one that reheats the universe. To estimate the contribution of this extra source of Dark Radiation to the effective number of neutrino species we need the branching

[4]More about the QCD axion, about how axions arise in String Theory and about their relation to moduli stabilization can be found in Appendix B

ratio between the decay of the volume modulus into Dark Radiation degrees of freedom and the decay of the volume modulus into the visible sector since

$$\Delta N_{eff} = \frac{43}{7}\left(\frac{10.75}{g_*(T_{rh})}\right)^{1/3}\frac{\Gamma_{\tau_b\to DR}}{\Gamma_{\tau_b\to SM}} \tag{2.90}$$

where $g_*(T_{rh}) = 106.75$ is the number of relativistic degrees of freedom at reheating temperature. It has been shown in [2] that the decay rates of the large volume modulus into MSSM gauge bosons, MSSM matter scalars, MSSM matter fermions are highly suppressed with respect to the decay rate of the volume modulus into its corresponding axion. The dominant channel for the decay into the visible sector is given by Higgs bosons.

We start our analysis from the decay into volume axions. Considering the Kähler potential (2.36) we can see that, since the small blow-up cycle modulus is more massive than the big one and since any mixing between small and big moduli is suppressed, we can inspect only the part of the Kähler potential containing

$$K = -3\ln(T_b + \overline{T}_b) \tag{2.91}$$

which gives us a Kähler matrix similar to the one in Eq. (2.40) without the off diagonal terms. The Lagrangian can be written as in Eq. (1.61) and reads as

$$\mathcal{L} = \frac{3}{4\tau_b^2}\partial_\mu\tau_b\partial^\mu\tau_b + \frac{3}{4\tau_b^2}\partial_\mu\rho_b\partial^\mu\rho_b. \tag{2.92}$$

To find the coupling term we are interested in, we need to canonically normalize the fields. We start from the volume modulus and we solve the following differential equation for τ_b

$$\frac{3}{4\tau_b^2}\partial_\mu\tau_b\partial^\mu\tau_b = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \quad\rightarrow\quad \tau_b = e^{\sqrt{\frac{2}{3}}\phi}. \tag{2.93}$$

Then we expand $\tau_b = \langle\tau_b\rangle + \hat{\tau}_b$ and $\phi = \langle\phi\rangle + \hat{\phi}$ such that the quantities between the brackets are the VEVs of the fields, while the hatted quantities are small fluctuations. Imposing

$$\langle\tau_b\rangle + \hat{\tau}_b = e^{\sqrt{\frac{2}{3}}(\langle\phi\rangle+\hat{\phi})} \tag{2.94}$$

we obtain that

$$\frac{\hat{\tau}_b}{\langle\tau_b\rangle} = \sqrt{\frac{2}{3}}\hat{\phi} \tag{2.95}$$

which will be useful in a few passages. At this point we substitute the expansion for τ_b into the Lagrangian term for the axion and we canonically normalize also the axion ρ_b to find the interaction term between the two fields, which turns out to be

$$\mathcal{L}_{int} = -\sqrt{\frac{2}{3}}\hat{\phi}\partial_\mu a_b\partial^\mu a_b \tag{2.96}$$

where a_b is the canonically normalized axion.

Proof. To derive the interaction term between the volume modulus and the corresponding axions, as already said, we start by substituting the expansion for τ_b in the second term of Eq. (2.92) such that

$$\begin{aligned}\frac{3}{4(\langle\tau_b\rangle+\hat{\tau}_b)^2}\partial_\mu\rho_b\partial^\mu\rho_b &= \frac{3\partial_\mu\rho_b\partial^\mu\rho_b}{4\langle\tau_b\rangle^2\left(1+\frac{\hat{\tau}_b}{\langle\tau_b\rangle}\right)^2} = \frac{3}{4\langle\tau_b\rangle^2}\left(1+\frac{\hat{\tau}_b}{\langle\tau_b\rangle}\right)^{-2}\partial_\mu\rho_b\partial^\mu\rho_b \\ &\simeq \frac{3}{4\langle\tau_b\rangle^2}\left(1-\frac{2\hat{\tau}_b}{\langle\tau_b\rangle}\right)\partial_\mu\rho_b\partial^\mu\rho_b \\ &= \frac{3}{4\langle\tau_b\rangle^2}\partial_\mu\rho_b\partial^\mu\rho_b - \frac{3}{2\langle\tau_b\rangle^2}\sqrt{\frac{2}{3}}\hat{\phi}\partial_\mu\rho_b\partial^\mu\rho_b, \qquad (2.97)\end{aligned}$$

where the last term is exactly the coupling term. To get the full coupling we must canonically normalize also the axion ρ_b with a similar procedure used for τ_b. We impose

$$\frac{3}{4\tau_b^2}\partial_\mu\rho_b\partial^\mu\rho_b = \frac{1}{2}\partial_\mu a_b\partial^\mu a_b \qquad (2.98)$$

that solved for ρ_b gives us

$$\rho_b = \sqrt{\frac{2}{3}}a_b e^{\sqrt{\frac{2}{3}}\langle\phi\rangle} = \sqrt{\frac{2}{3}}a_b\langle\tau_b\rangle \qquad (2.99)$$

from which Eq. (2.96) follows straightforwardly. □

Having the coupling term, it is easy to compute the Feynman rule associated with it, that reads as $\sim i\sqrt{\frac{2}{3}}\delta^{(4)}(p_3+p_2-p_1)(p_3\cdot p_2+p_2\cdot p_3)$ where p_1 is taken to be the in-going momentum of ϕ and p_2, p_3 are the out-going momenta of the axions. The decay rate follows from the well known expression

$$\Gamma = \frac{|M|^2}{2E}\frac{1}{8\pi} \qquad (2.100)$$

with M the amplitude and E the energy of the center of mass. Proceeding in the standard way, the decay rate of the volume modulus into its volume axion is

$$\Gamma_{\phi\to a_b a_b} = \frac{1}{48\pi}\frac{m_\phi^3}{M_P^2} \qquad (2.101)$$

where of course, the canonical normalizations of the fields are in units of M_P.
To study the decay into visible sector particles we focus on the Higgs bosons. The Kähler potential in this case admits a Giudice-Masiero coupling for the Higgses and reads as

$$K = -3\ln(T_b + \overline{T}_b) + \frac{H_u\bar{H}_u + H_d\bar{H}_d}{(T_b + \overline{T}_b)} + \left(\frac{ZH_uH_d}{(T_b + \overline{T}_b)} + h.c.\right) \tag{2.102}$$

with Z an undetermined constant[2]. Our goal is the same as before: finding the coupling term to compute the decay rate. To do so we first need to find the Lagrangian of our theory (at least the pieces that will help us to find the canonical normalizations for the fields). We start by computing the Kähler matrix starting from the Kähler potential we already have: we compute first and second derivatives with respect to τ, H_u and H_d that read as

$$K_{T\overline{T}} = \frac{1}{2}\frac{\partial}{\partial\tau_b}\left(-\frac{3}{2\tau_b} - \frac{H_u\bar{H}_u + H_d\bar{H}_d}{4\tau_b^2} - \frac{ZH_uH_d}{4\tau_b^2} + h.c.\right) \sim \frac{3}{4\tau_b^2}, \tag{2.103}$$

$$K_{u\bar{u}} = \frac{\partial}{\partial H_u}\left(\frac{H_u}{2\tau_b}\right) = \frac{1}{2\tau_b} = K_{d\bar{d}}, \tag{2.104}$$

$$K_{u\overline{T}} = \frac{\partial}{\partial H_u}\left(-\frac{3}{2\tau_b} - \frac{H_u\bar{H}_u + H_d\bar{H}_d}{4\tau_b^2} - \frac{ZH_uH_d}{4\tau_b^2} + h.c.\right) = -\frac{\bar{H}_u}{4\tau_b^2} - \frac{ZH_d}{4\tau_b^2}, \tag{2.105}$$

$$K_{d\overline{T}} = \frac{\partial}{\partial H_d}\left(-\frac{3}{2\tau_b} - \frac{H_u\bar{H}_u + H_d\bar{H}_d}{4\tau_b^2} - \frac{ZH_uH_d}{4\tau_b^2} + h.c.\right) = -\frac{\bar{H}_d}{4\tau_b^2} - \frac{ZH_u}{4\tau_b^2} \tag{2.106}$$

where for $K_{T\overline{T}}$ we have neglected terms suppressed by higher orders of the volume. Our kinetic Lagrangian now reads as

$$\begin{aligned}\mathcal{L} = &\frac{3}{4\tau_b^2}\partial_\mu\tau_b\partial^\mu\tau_b + \frac{1}{2\tau_b}\partial_\mu H_u\partial^\mu\bar{H}_u + \frac{1}{2\tau_b}\partial_\mu H_d\partial^\mu\bar{H}_d \\ &- \frac{\bar{H}_u}{4\tau_b^2}\partial_\mu H_u\partial^\mu\tau_b - \frac{ZH_d}{4\tau_b^2}\partial_\mu H_u\partial^\mu\tau_b - \frac{\bar{H}_d}{4\tau_b^2}\partial_\mu H_d\partial^\mu\tau_b - \frac{ZH_u}{4\tau_b^2}\partial_\mu H_d\partial^\mu\tau_b.\end{aligned} \tag{2.107}$$

For the volume modulus the canonical normalization is equivalent to the one computed before in Eq. (2.93) and the relation (2.95) is valid. The fields H_u and H_d, having similar kinetic terms, will have the same canonical normalization:

$$\frac{1}{2\tau_b}\partial_\mu H_u\partial^\mu\bar{H}_u = \partial_\mu h_u\partial^\mu\bar{h}_u \quad \rightarrow \quad H_u = \sqrt{2\tau_b}h_u \tag{2.108}$$

and the same holds for H_d and its canonical counterpart h_d. To find the interaction term between the two Higgses and the volume modulus we focus on the terms of the

Lagrangian that contain the mixed derivatives and the Giudice-Masiero constant Z such that

$$\mathcal{L}_{int} \sim -\frac{Z}{4\tau_b^2}\Big(H_d\partial_\mu H_u\partial^\mu\tau_b - H_u\partial_\mu H_d\partial^\mu\tau_b\Big). \tag{2.109}$$

We expand $\tau_b = \langle\tau_b\rangle + \hat{\tau}_b$ and we substitute it in the denominator of $\mathcal{L}_{int}$, while we substitute all the fields in the parentheses with the corresponding canonical normalization to get

$$\mathcal{L}_{int} \sim \frac{Z}{4\langle\tau_b\rangle^2}\left(1-2\frac{\hat{\tau}}{\langle\tau_b\rangle}\right)\sqrt{\frac{2}{3}}\langle\tau_b\rangle\Big[2\langle\tau_b\rangle(h_d\partial_\mu h_u\partial^\mu\hat{\phi} + h_u\partial_\mu h_d\partial^\mu\hat{\phi})\Big] \tag{2.110}$$

where we have used again the Taylor expansion for the denominator we have already used in (2.97). At this point, expanding the computations and performing an integration by parts, we are able to find the final coupling term that is

$$\mathcal{L}_{int} = \frac{1}{\sqrt{6}}(Zh_u h_d\Box\hat{\phi} + h.c.); \tag{2.111}$$

the corresponding Feynman rule comes straightforwardly from this term and is roughly given by $\sim \frac{Z}{\sqrt{6}}\delta^{(4)}(-ip_1^2)(p_3+p_2-p_1)$ where we have used the same conventions on the momenta as before. The decay rate of the volume modulus into the Higgses is

$$\Gamma_{\phi\to h_u h_d} = \frac{2Z^2}{48\pi}\frac{m_\phi^3}{M_P^2} \tag{2.112}$$

which is comparable to the decay rate into volume axions and therefore is not suppressed. The excess of effective number of neutrino species in Eq. (2.90) is therefore proportional to

$$\Delta N_{eff} = \frac{43}{7}\left(\frac{10.75}{106.75}\right)^{1/3}\frac{1}{2Z^2}. \tag{2.113}$$

Imposing now the observational bound for ΔN_{eff} we get that [35]

$$\Delta N_{eff} \leq 0.5 \quad\rightarrow\quad Z \geq 1.7 \tag{2.114}$$

if we allow for one pair of Higgs doublets.
To complete our discussion we are going to show the suppressed decay rates into the other visible sector fields and into other possible axions [2].

Decay into Gauge Bosons The coupling between the volume modulus and the gauge bosons is present at loop level and is of the form

$$\frac{\lambda_a\alpha_{SM}}{4\pi}\phi F_{\mu\nu}F^{\mu\nu} \tag{2.115}$$

such that the decay rate is suppressed by a loop factor with respect to the one in Eq. (2.101)

$$\Gamma_{\phi \to A^{\mu}_{SM} A^{\mu}_{SM}} \sim \left(\frac{\alpha_{SM}}{4\pi} \right)^2 \frac{m_\phi^3}{M_P^2}. \tag{2.116}$$

Decay into Matter Scalars The starting point is the Kähler potential

$$K = -3 \ln(T_b + \overline{T}_b) + \frac{C\bar{C}}{(T_b + \overline{T}_b)} \tag{2.117}$$

where the C are matter scalars. Following the same steps we have done for the decay into volume axions and the decay into Higgs bosons (derivatives of the Kähler potential to find the Kähler matrix, canonical normalization of the fields) we find the coupling term

$$\mathcal{L} = \frac{1}{2}\sqrt{\frac{2}{3}} \phi (\bar{\eta} \Box \eta + \eta \Box \bar{\eta}), \tag{2.118}$$

where ϕ, η are the canonically normalized fields, such that

$$\Gamma_{\phi \to \eta\bar{\eta}} \sim \frac{m_\eta^2 m_\phi}{M_P^2}. \tag{2.119}$$

Decay into Local Closed String Axions The Kähler potential for these axions reads as

$$K = -3 \ln(T_b + \overline{T}_b) + \frac{Z_{ax}}{2} \frac{(X + \bar{X})^\gamma}{(T_b + \overline{T}_b)^\lambda} \tag{2.120}$$

where $\mathrm{Im}(X)$ is the axion, $\mathrm{Re}(X)$ is the saxion, $\lambda = 3/2$ and $\gamma = 2$ in the case of a saxion at the singularity. Notice that the existence of closed string axions is model dependent. Writing $X = (A + iB)/\sqrt{2}$ and proceeding with the usual canonical normalization we obtain a coupling term of the form

$$\mathcal{L}_{int} = \sqrt{\frac{2}{3}} \frac{\lambda}{4M_P} (A^2 - B^2) \Box \phi \tag{2.121}$$

such that the decay rate is

$$\Gamma_{\phi \to BB} = \left(\frac{\lambda}{3/2} \right)^2 \frac{9 m_\phi^3}{768 \pi M_P^2}. \tag{2.122}$$

In this case the decay rate is not suppressed with respect to the one in Eq. (2.101) and so it could be an additional source of Dark Radiation.

Decay into Open String Axions Decays into open string axions come from coupling terms of the form

$$\mathcal{L}_{int} = -\sqrt{\frac{2}{3}}\left(\frac{\langle\rho\rangle}{M_P}\right)^2 \phi\theta\Box\theta + \frac{4}{3}\left(\frac{\langle\rho\rangle}{M_P}\right)^2 \phi\partial_\mu a_b \partial^\mu\theta \tag{2.123}$$

where $\langle\rho\rangle$ is the VEV of the scalar matter fields C and θ is the canonically normalized open string axion. Due to the presence of the suppressed term $(\langle\rho\rangle/M_P)^2$ the decay rate is also suppressed.

An important decay mentioned in [2] is the decay of the volume modulus into the bulk closed string $U(1)$s coming from the reduction of C_4 along 3-cycles. In the remaining part of our work we are going to focus on these particular RR Photons as a possible source of Dark Radiation.

Chapter 3

Ramond-Ramond Photons in LVS

"Not only does God play dice, but He sometimes throws them where they cannot be seen."
— Stephen Hawking

One of the peculiarities of Type IIB string compactifications is that they come with some additional $U(1)$ gauge symmetries with respect to the Standard Model hypercharge one. The $U(1)$ gauge symmetries we are interested in for the purposes of this thesis, are the ones coming from the dimensional reduction of Ramond-Ramond (RR) forms on the CY. Their gauge bosons are closed string Dark Photons. Some of these $U(1)$ symmetries could acquire a mass through the Stückelberg mechanism which happens in the presence of torsional p-cycles in the compactification manifold[1]. Some of these RR Photons, the ones which are not related to torsional cycles, remain massless and therefore could behave as Dark Radiation, potentially constituting a harmful contribution to the effective number of neutrino species.
In what follows we are going to focus our attention on the massless RR Photons. In particular, before looking at their contribution to Dark Radiation, we are going to show their action and their kinetic mixing.

3.1 RR Photons: Origin and Gauge Kinetic Function

The starting point to study closed string Dark Photons is to recall the dimensional reduction of the Ramond-Ramond C_4 form in Eq. (1.60) after the orientifold projection where we have the one-forms $V^\gamma(x)$ and $U_\gamma(x)$ ($\gamma = 1, \ldots, h_+^{1,2}$). As already mentioned in Subsec. 1.3.4, using the self duality condition for $\tilde{F}_5$ we can get rid of $U_\gamma(x)$ keeping only $V^\gamma(x)$. This choice corresponds to express the action in terms of an electric gauge potential instead of a magnetic one. These gauge bosons $V^\gamma(x)$ appearing in the $N = 1$ vector multiplet (see Tab. 1.4) will be the RR Dark Photons.

To find the kinetic term for such gauge bosons we need to start from the 10-dimensional

[1]For a complete discussion on torsional p-cycles and Stückelberg mechanism for hidden $U(1)$ symmetries see [36].

Type IIB supergravity action (1.1) that we are going to recast in the following way

$$S^{10D}_{IIB,S} = \frac{1}{2k^2_{10}} \int d^{10}x\sqrt{-g}[e^{-2\phi}(R + 4(\partial\phi)^2 - h_3 H_3^2) - f_1 F_1^2 - f_3 \tilde{F}_3^2 - f_5 \tilde{F}_5^2] \tag{3.1}$$

where we have absorbed the numerical factors in h_3, f_1, f_3, f_5 and where the square is intended again with respect to the wedge star product. What we are interested in, is the piece containing the 5-form field strength $\tilde{F}_5$ defined as (1.2); containing $F_5 = dC_4$, it depends on the RR 4-form we want to study. At this point, we could in principle obtain the $N = 2$ 4-dimensional effective action by substituting the expansions (1.40), (1.41), (1.42), (1.43) and (1.44) into the action (3.1). However, we have seen that the $N = 2$ theory is not phenomenologically useful and therefore we must perform an orientifold projection to get $N = 1$. Using the orientifolded forms (1.58), (1.59) and (1.60) and substituting them into the definition of $\tilde{F}_5$ (recalling that $H_3 = dB_2$ and $F_3 = dC_2$), we obtain the following expression for the 5-form field strength [13]

$$\tilde{F}_5 = dD_2^{\tilde{\alpha}}(x)\wedge\omega_{\tilde{\alpha}} + dV^\gamma(x)\wedge\alpha_\gamma - dU_\gamma(x)\wedge\beta^\gamma + d\rho_{\tilde{\alpha}}(x)\bar{\omega}^{\tilde{\alpha}} - \frac{1}{2}(c^{\tilde{a}}db^{\tilde{b}} - b^{\tilde{a}}dc^{\tilde{b}})\wedge\omega_{\tilde{a}}\wedge\omega_{\tilde{b}}, \tag{3.2}$$

where $\tilde{a}, \tilde{b} = 1, \ldots, h^{1,1}_-$, $\tilde{\alpha} = 1, \ldots, h^{1,1}_+$ and $\gamma = 1, \ldots, h^{1,2}_+$. We need now to substitute the obtained field strength into the action (3.1) to get [13]

$$S^{4D}_{IIB,E} \supset \int \frac{1}{4}\mathrm{Re}(M_{\gamma\lambda})F_2^\gamma \wedge F_2^\lambda + \frac{1}{4}\mathrm{Im}(M_{\gamma\lambda})F_2^\gamma \wedge *F_2^\lambda \tag{3.3}$$

where we have performed a Weyl rescaling to obtain the action in Einstein frame and we have considered only the terms involving the RR Photon ($F_2^\gamma \equiv dV^\gamma$). In this action we have introduced the $N = 1$ gauge kinetic matrix that is defined in terms of the symplectic basis (α_K, β^L) as [13]

$$\left(\int \alpha_K \wedge *\alpha_L\right)\Big|_{z^\gamma=\bar{z}^\gamma=0} = -(\mathrm{Im}(M) + \mathrm{Re}(M)(\mathrm{Im}(M))^{-1}\mathrm{Re}(M))_{KL}|_{z^\gamma=\bar{z}^\gamma=0}, \tag{3.4}$$

$$\left(\int \beta^K \wedge *\beta^L\right)\Big|_{z^\gamma=\bar{z}^\gamma=0} = -((\mathrm{Im}(M))^{-1})^{KL}|_{z^\gamma=\bar{z}^\gamma=0}, \tag{3.5}$$

$$\left(\int \alpha_K \wedge *\beta^L\right)\Big|_{z^\gamma=\bar{z}^\gamma=0} = -(\mathrm{Re}(M)(\mathrm{Im}(M))^{-1})_K^L|_{z^\gamma=\bar{z}^\gamma=0}, \tag{3.6}$$

where evaluating everything at $z^\gamma = \bar{z}^\gamma = 0$ means removing from the spectrum the fields projected out by the orientifold involution. The same matrix can also be expressed in terms of the periods defined in (1.69) as

$$M_{\gamma\lambda} = \left(\bar{\mathcal{F}}_{\gamma\lambda} + 2i\frac{(\mathrm{Im}(\mathcal{F}))_{\gamma\mu}Z^\mu(\mathrm{Im}(\mathcal{F}))_{\lambda\nu}Z^\nu}{Z^\nu(\mathrm{Im}(\mathcal{F}))_{\nu\mu}Z^\mu}\right)\Big|_{z^\gamma=\bar{z}^\gamma=0} \tag{3.7}$$

with $\gamma, \lambda = 1, \ldots, h_+^{1,2}$ and [12, 13]

$$Z^\lambda(z) = \int_{Y_6} \Omega \wedge \beta^\lambda = 0, \tag{3.8}$$

$$\mathcal{F}_\lambda(z)\Big|_{z^\lambda=0} = \frac{\partial \mathcal{F}(z)}{\partial Z^\lambda}\Big|_{z^\lambda=0} = \int_{Y_6} \Omega \wedge \alpha_\lambda = 0, \tag{3.9}$$

$$\mathcal{F}_{\gamma\lambda}(z) = \frac{\partial^2 \mathcal{F}(z)}{\partial Z^\gamma(z) \partial Z^\lambda(z)} \tag{3.10}$$

where $\mathcal{F}_{\gamma\lambda}$ is the period matrix and $Z^\lambda(z)$, $\mathcal{F}_\lambda(z)$ depend holomorphically on the complex structure moduli of the chiral multiplet. The CY 3-form Ω is still defined as

$$\Omega(z^k) = Z^k(z^k)\alpha_k - \mathcal{F}_k(z^k)\beta^k \tag{3.11}$$

with $k = 0, \ldots, h_-^{1,2}$, since everything else must vanish under orientifold.
Now, to get the final expression of the action for the RR Photons, which will give us the kinetic terms, we have to switch to the Kähler coordinates [13] such that we can recast everything in terms of the Kähler potential, the superpotential and the gauge kinetic function. In this way the standard $N = 1$ action can be written as

$$S^{4D}_{IIB,E} \supset \int \frac{1}{2}\text{Re}(f_{\gamma\lambda}) F_2^\gamma \wedge *F_2^\lambda + \frac{1}{2}\text{Im}(f_{\gamma\lambda}) F_2^\gamma \wedge F_2^\lambda + V(z^k) \tag{3.12}$$

where V is the scalar potential and where we can define the gauge kinetic function as

$$f_{\gamma\lambda} = -\frac{i}{2}\bar{M}_{\gamma\lambda}\Big|_{z^\gamma=\bar{z}^\gamma=0}. \tag{3.13}$$

From here, using the relations (3.4), (3.5) and (3.6), it is possible to show that the gauge kinetic matrix M is block diagonal, meaning that $M_{\gamma k} = 0$ for $\gamma = 1, \ldots, h_+^{1,2}$ and $k = 0, \ldots, h_-^{1,2}$. This property implies that [12]

$$M_{\gamma\lambda}|_{z^\gamma=\bar{z}^\gamma=0} = \bar{\mathcal{F}}_{\gamma\lambda}|_{z^\gamma=\bar{z}^\gamma=0} \tag{3.14}$$

such that the final gauge kinetic function for the RR Photons is

$$f_{\gamma\lambda}(z^k) = -\frac{i}{2}\mathcal{F}_{\gamma\lambda}(z^k)\Big|_{z^\gamma=\bar{z}^\gamma=0}. \tag{3.15}$$

The important property of this gauge kinetic function is that is is holomorphic in the complex structure moduli z^k, $k = 0, \ldots, h_-^{1,2}$ due to the holomorphy of the prepotential $\mathcal{F}^k$.

3.1.1 Evaluation of the gauge kinetic function

The main result of the previous section is the form of the gauge kinetic function for the RR Photons. Its dependence on the complex structure moduli tells us that, for the volume modulus to decay into such Dark Photons, there must be a mixing between the complex structure moduli and and the Kähler moduli. The Kähler potential we are going to use from now on is of the form

$$K = -3\ln(T+\overline{T}) - 3\ln(U+\overline{U}) + K_{\alpha'} + K_{loop} \tag{3.16}$$

where we are focusing for simplicity on a single Kähler modulus T and a single complex structure modulus U (following the notation of [37], from now on we denote the complex structure moduli as U). The interesting part for now is the Kähler potential for the complex structure modulus:

$$K_{cs} = -3\ln(U+\overline{U}) = -3\ln(2u) \tag{3.17}$$

with $u = \mathrm{Re}(U)$. In the next sections we are going to show where the additional mixing between the Kähler modulus and the complex structure modulus comes from, and we are going to perform a more detailed analysis. For now, let us stick to the form of the gauge kinetic function and of the prepotential, which are going to give us important information about the topology needed for the coupling of the RR Photons with the closed string moduli.

To evaluate the gauge kinetic function of the theory, the first step is to compute the second derivative of the $N=2$ prepotential. Following [37] we write the perturbative part of the prepotential as

$$\mathcal{F}_{pert}(U) = -\frac{1}{3!}\mathcal{K}_{IJK}U^I U^J U^K + \frac{1}{2}a_{IJ}U^I U^J + b_I U^I + \xi \tag{3.18}$$

with $\mathcal{K}_{IJK}$ triple intersection numbers of the mirror Calabi-Yau, a_{IJ} and b_I rational and $I, J, K \in \{0, h^{1,2}\}$ [38]. Because of the choice to use U instead of z, we need to adjust the notation used in the previous chapters to be able to use the expressions (3.15) and (3.18). First of all we take the Kähler potential for the complex structure moduli to be of the form (1.68). We have seen in Subsec. 1.3.4 that, taking (A^I, B_I) with $I = 0, \ldots, h^{1,2}$ as the canonical homology basis of $H_3(Y_6)$ and its dual (α_I, β^I) as a real, symplectic basis on $H^3(Y_6)$, we can write the periods of Ω and the corresponding period vector respectively as (1.69). At this point we can define the prepotential also as $\mathcal{F}(Z^I) = (Z^0)^2\mathrm{F}(z)$ where we have factored out Z^0 such that $\mathcal{F}_I = \partial\mathcal{F}/\partial Z^I$ and $z = Z^I/Z^0$ (these are the special coordinates we have already shown in Sec. 1.3.4 without showing how to use them). Notice that in the following we are going to set $Z^0 = 1$ [17].

Since we are considering the case with only one complex structure modulus, the period can finally be written as

$$\Pi = \begin{pmatrix} \mathcal{F}_0 \\ \mathcal{F}_1 \\ Z^0 \\ Z^1 \end{pmatrix} = Z^0 \begin{pmatrix} 2\mathrm{F} - \mathrm{F}_z \\ \mathrm{F}_z \\ 1 \\ z \end{pmatrix} \tag{3.19}$$

where we have used the chain rule to compute the derivatives of the prepotential in the following way

$$\mathcal{F}_0 = 2Z^0\mathrm{F} + (Z^0)^2 \frac{\partial \mathrm{F}}{\partial z}\frac{\partial z}{\partial Z^0}, \qquad \mathcal{F}_1 = (Z^0)^2 \frac{\partial \mathrm{F}}{\partial z}\frac{\partial z}{\partial Z^1}. \tag{3.20}$$

Having this we compute the Kähler potential of the complex structure moduli using the period vector as

$$K_{cs} = -\ln(-i\Pi^\dagger \Sigma \Pi) \tag{3.21}$$

where the symplectic matrix is

$$\Sigma = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix} \tag{3.22}$$

Inserting now Eq. (3.20) into Eq. (3.19) and using the expression (3.21) we obtain

$$K_{cs} = -\ln(2i(\mathrm{F} - \bar{\mathrm{F}}) - i(z - \bar{z})(\mathrm{F}_z + \bar{\mathrm{F}}_z)). \tag{3.23}$$

Identifying $z = Z^I/Z^0$ with the U in the formula for the prepotential (3.18) and $\mathcal{F}_{pert}(U)$ with $\mathrm{F}(z)$, the next step is to compute the derivatives of the prepotential to find the final form for the Kähler potential of the complex structure moduli to match the notations. We focus on the case with 2 complex structure moduli $z^\alpha, z^a, \alpha = h^{2,1}_+ = a = h^{2,1}_- = 1$, one modded out by the orientifold projection, $z^\alpha = \zeta$, and the one invariant under orientifolding, $z^a = z$, in such a way that the prepotential can be written as

$$\mathrm{F}(z, \zeta) = -\frac{1}{6}\mathcal{K}_{zzz} z^3 + \frac{a}{2} z^2 + bz + \xi - \frac{1}{6}\mathcal{K}_{\zeta\zeta z}\zeta^2 z. \tag{3.24}$$

We can compute then the derivatives appearing in the Kähler potential for the complex structure moduli, denoting $\mathcal{K}_{\zeta\zeta z}$ as $\mathcal{K}$:

$$\mathrm{F}_\zeta = -\frac{1}{3}\mathcal{K}\zeta z^2, \qquad \mathrm{F}_{\zeta\zeta} = -\mathcal{K} z \tag{3.25}$$

$$\mathrm{F}_z = -\frac{1}{2}\mathcal{K}_{zzz} z^2 + az + b - \frac{1}{6}\mathcal{K}\zeta^2, \qquad \mathrm{F}_{zz} = -\mathcal{K}_{zzz} z + a \tag{3.26}$$

and the same for the complex conjugate. Inserting these derivatives into K_{cs} and projecting out the complex structure deformation $\zeta^\alpha = \zeta$, we find

$$K_{cs} = -\ln\left(\frac{i\mathcal{K}_{zzz}}{6} z^3 - \frac{i\mathcal{K}_{zzz}}{6}\bar{z}^3 + \frac{i\mathcal{K}_{zzz}}{2} z\bar{z}^2 - \frac{i\mathcal{K}_{zzz}}{2}\bar{z} z^2\right). \tag{3.27}$$

Writing $z = p + iq$ and $\bar{z} = p - iq$, after a brief computation, we finally obtain

$$K_{cs} = -\ln\left(\frac{4}{3}\mathcal{K}_{zzz}q^3\right) \sim -\ln(8q^3) - \ln\left(\frac{\mathcal{K}_{zzz}}{6}\right) \sim -3\ln(2q). \tag{3.28}$$

Having this, we can compare our expressions for the Kähler potential of the complex structure moduli (3.17) with the one obtained above. What we can see is that, in order to finally match all the notations, we need the following map

$$q = \mathrm{Im}(z) \to u = \mathrm{Re}(U), \qquad p = \mathrm{Re}(z) \to \sigma = \mathrm{Im}(U) \tag{3.29}$$

such that $z \to i\overline{U}$ (we have parametrized the complex structure modulus as $U = u + i\sigma$). At this point we are ready to fully compute the gauge kinetic function for our theory using Eq. (3.15). Rewriting it in special coordinates we get that

$$\mathrm{f}_{\gamma\lambda}(z^k) = -\frac{i}{2}\mathrm{F}_{\gamma\lambda}\bigg|_{z^\gamma=\bar{z}^\gamma=0}, \tag{3.30}$$

such that the final expression reads

$$\mathrm{f} = -\frac{i}{2}\mathrm{F}_{\zeta\zeta}\bigg|_{z=i\overline{U}} = -\frac{i}{2}(-\mathcal{K}z)\bigg|_{z=i\overline{U}} = -\frac{\mathcal{K}}{2}\overline{U} = -\frac{\mathcal{K}}{2}u + \frac{\mathcal{K}}{2}i\sigma. \tag{3.31}$$

This formula has a clear meaning: the Dark Photons are $h^{1,2}_+$ and they appear in the decomposition of C_4 as wedging elements of the $H^{1,2}_+$ group (see Eq. (1.60))

$$C_4 \supset V^\gamma \wedge \alpha_\gamma, \qquad \gamma = 1, \cdots, h^{1,2}_+ . \tag{3.32}$$

Hence they are geometrically related to the even 3-cycles Σ^+_γ which are mirror dual to the odd 2-cycles D^-_i in the mirror Calabi-Yau manifold, and whose Poincaré dual are the projected out complex structure moduli z^α. The fact that the gauge kinetic function f for the Dark Photons is proportional to $\mathcal{K} = \mathcal{K}_{\zeta\zeta z}$ implies that, in order for f to be non-zero, in the mirror CY the odd 2-cycle D^- should intersect with the even 2-cycle D^+ (mirror and Poincaré dual to the odd complex structure modulus). This construction is shown in Fig. 3.1.

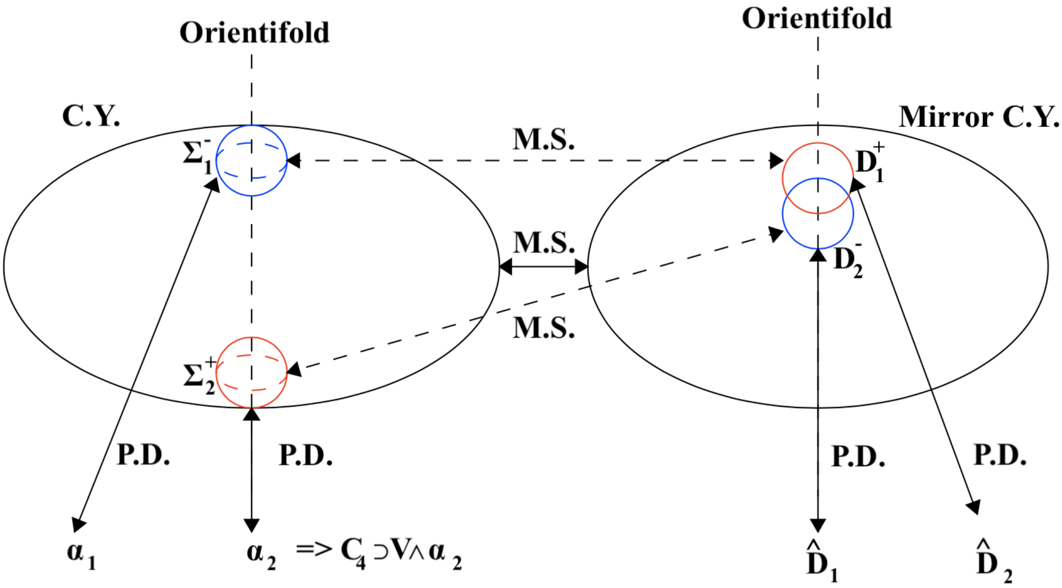

Figure 3.1: *Geometric construction that leads to a complex structure - RR Dark Photons coupling. The 2-cycles $D_i^{\mp}$ are mirror dual to the 3-cycles $\Sigma_i^{\pm}$ Poincaré dual to the 3-forms α_i. The even cycles under the orientifold involution are red while the odd ones are blue. The Dark Photon V^γ arises from the reduction of C_4 over the 3-form α_2 which is Poincaré dual to the even 3-cycle Σ_2^+. The intersection between the 2-cycles D_1^+ and D_2^- leads to a non-zero coupling between the Dark Photon and the complex structure modulus.*

It is possible to show an explicit construction with non-trivial intersection between such cycles for a typical example in the sequestered LVS.

3.1.2 An explicit Calabi-Yau Example

We have seen how, in order to obtain a non vanishing gauge kinetic function for the Dark Photons, we need a mixing between even and odd complex structure moduli in the prepotential. Such a mixing is geometrically interpreted as the intersection between orientifold odd and even 2-cycles which are mirror symmetric with respect to the 3-cycles

$$\mathcal{K}_{IJK} = \int_{C.Y.} \hat{D}_I \wedge \hat{D}_J \wedge \hat{D}_K \, , \tag{3.33}$$

$$\hat{D}_I \overset{P.D.}{\leftrightarrow} D_I \overset{M.S.}{\leftrightarrow} \Sigma_I \overset{P.D.}{\leftrightarrow} \alpha_I \, , \tag{3.34}$$

where $M.S.$ stands for "Mirror Symmetry" and $P.D.$ for "Poincaré Duality".

Before showing an explicit example, let us recall that, under the action of the mirror symmetry we get

$$h^{1,2} \overset{M.S.}{\leftrightarrow} h^{1,1}\,, \tag{3.35}$$

$$h_-^{1,1} \overset{M.S.}{\leftrightarrow} h_+^{1,2}\,, \tag{3.36}$$

$$h_+^{1,1} \overset{M.S.}{\leftrightarrow} h_-^{1,2}\,, \tag{3.37}$$

where we can see that the orientifold even complex structure moduli are mapped into the orientifold odd Kähler moduli.

We now proceed by giving an example of a CY which is mirror to the one we need, taking it from [33]. Such a CY is characterized by $h^{1,1} = 4, h^{1,2} = 112$ and a Swiss-Cheese structure with one big divisor D_1, one $\mathbb{P}^2$ divisor D_2 invariant under the orientifold involution, and 2 dP_0 quiver loci D_3, D_4 with $D3$-branes on them which are exchanged under the orientifold. This construction is shown in Fig. 3.2.

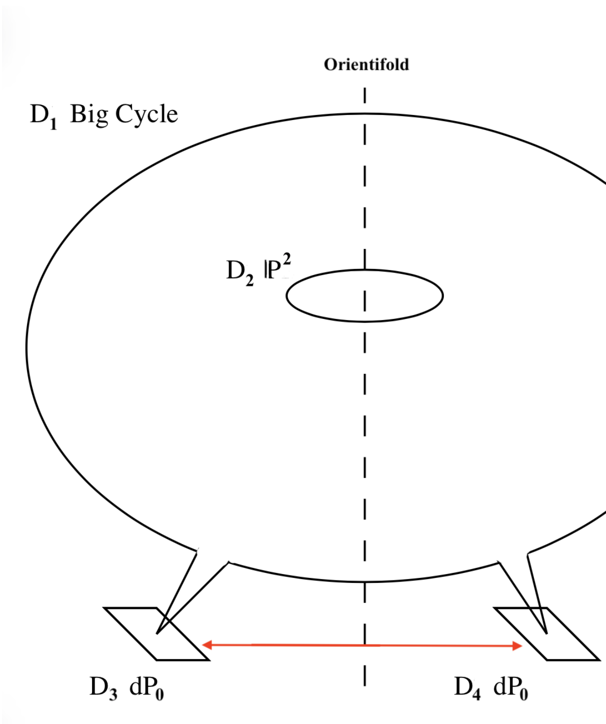

Figure 3.2: *Calabi-Yau with four divisors D_1, D_2, D_3, D_4. The D3-branes supporting the Standard Model sit at D_3, D_4 which are exchanged under the orientifold involution $D_3 \leftrightarrow D_4$; the invariant divisors are D_1 and D_2.*

The Kähler form J can be expanded in a basis of $(1,1)$-forms as:

$$J = t_1 \hat{D}_1 + t_2 \hat{D}_2 + t_3 \hat{D}_3 + t_4 \hat{D}_4 \,. \tag{3.38}$$

Consequently, the Calabi-Yau volume looks like:

$$\mathcal{V} = \frac{1}{6} \int_{CY} J \wedge J \wedge J = \frac{1}{6} \left(\mathcal{K}_{111}\, t_1^3 + \mathcal{K}_{222}\, t_1^3 + \mathcal{K}_{333}\, t_3^3 + \mathcal{K}_{333}\, t_4^3 \right) , \tag{3.39}$$

since the only non-zero intersection numbers are $\mathcal{K}_{111}$, $\mathcal{K}_{222}$ and $\mathcal{K}_{333} = \mathcal{K}_{444}$.

What we are searching for is a non-vanishing intersection in the mirror Calabi-Yau, following [38], between one orientifold-odd and one orientifold-even cycle dual to the Kähler moduli. The orientifold involution acts on the cycles of our model as follows

$$D_1' = D_1, \tag{3.40}$$

$$D_2' = D_2, \tag{3.41}$$

$$D_3' = D_4, \tag{3.42}$$

$$D_4' = D_3, \tag{3.43}$$

It is convenient to switch to another basis, $\{\hat{D}_1, \hat{D}_2, \hat{D}_3, \hat{D}_4\} \to \{\hat{D}_1, \hat{D}_2, \hat{D}_+, \hat{D}_-\}$, defined as

$$\hat{D}_+ = \frac{\hat{D}_3 + \hat{D}_4}{2}, \qquad \hat{D}_- = \frac{\hat{D}_3 - \hat{D}_4}{2} \,. \tag{3.44}$$

Using the expressions (3.42) and (3.43), we can show that the new $(1,1)$-forms $\hat{D}_+$ and $\hat{D}_-$ are odd and even under the orientifold respectively:

$$\hat{D}_+' = \frac{\hat{D}_3' + \hat{D}_4'}{2} = \frac{\hat{D}_4 + \hat{D}_3}{2} = \hat{D}_+, \tag{3.45}$$

$$\hat{D}_-' = \frac{\hat{D}_3' - \hat{D}_4'}{2} = \frac{\hat{D}_4 - \hat{D}_3}{2} = -\hat{D}_-. \tag{3.46}$$

In this way we end up with

$$\hat{D}_1, \hat{D}_2, \hat{D}_+ \in H_+^{1,1}(Y_6), \tag{3.47}$$

$$\hat{D}_- \in H_-^{1,1}(Y_6). \tag{3.48}$$

The Kähler form can be expanded in this new basis as

$$J = \sum_{i=1}^{4} t_i \hat{D}_i \tag{3.49}$$

$$= t_1\hat{D}_1 + t_2\hat{D}_2 + t_3\left(\frac{\hat{D}_+ + \hat{D}_-}{2}\right) + t_4\left(\frac{\hat{D}_+ - \hat{D}_-}{2}\right)$$

$$= t_1\hat{D}_1 + t_2\hat{D}_2 + \hat{D}_+\left(\frac{t_3 + t_4}{2}\right) + \hat{D}_-\left(\frac{t_3 - t_4}{2}\right)$$

$$= t_1\hat{D}_1 + t_2\hat{D}_2 + \hat{D}_+ t_+ + \hat{D}_- t_-, \tag{3.50}$$

where t_+ and t_- have been defined as

$$t_+ \equiv \frac{t_3 + t_4}{2} \quad \text{and} \quad t_- \equiv \frac{t_3 - t_4}{2}. \tag{3.51}$$

However, we have seen that under the orientifold involution $\hat{D}_3 \leftrightarrow \hat{D}_4$, and so we have that $t_3 = t_4$ as a consequence of cohomological equivalence. Thus, the Kähler modulus associated to the odd cycle is projected out ($t'_- = 0$), leaving us with

$$J' = t_1\hat{D}_1 + t_2\hat{D}_2 + \hat{D}_+ . J' = t_1\hat{D}_1 + t_2\hat{D}_2 + \hat{D}_+. \tag{3.52}$$

At this point we are finally ready to compute the intersection numbers remembering that we are taking the divisors D_1, D_2, D_3, D_4 to be diagonal and so the only non-vanishing intersection numbers between them are $\mathcal{K}_{iii}$ with $i \in \{1, 2, 3, 4\}$:

$$\mathcal{K}_{+++} = \int_{CY} \hat{D}_+ \wedge \hat{D}_+ \wedge \hat{D}_+ = \int_{CY} (\hat{D}_3^3 + \hat{D}_4^3)$$

$$= \mathcal{K}_{333} + \mathcal{K}_{444} = 2\mathcal{K}_{333} \neq 0, \tag{3.53}$$

$$\mathcal{K}_{---} = \int_{CY} \hat{D}_- \wedge \hat{D}_- \wedge \hat{D}_- = \int_{CY} (\hat{D}_3^3 - \hat{D}_4^3)$$

$$= \mathcal{K}_{333} - \mathcal{K}_{444} = 0, \tag{3.54}$$

$$\mathcal{K}_{++-} = \int_{CY} \hat{D}_+ \wedge \hat{D}_+ \wedge \hat{D}_- = \int_{CY} \left[(\hat{D}_3 + \hat{D}_4)^2(\hat{D}_3 - \hat{D}_4)\right]$$

$$= \mathcal{K}_{333} - \mathcal{K}_{444} = 0, \tag{3.55}$$

$$\mathcal{K}_{--+} = \int_{CY} \hat{D}_- \wedge \hat{D}_- \wedge \hat{D}_+ = \int_{CY} \left[(\hat{D}_3 - \hat{D}_4)^2(\hat{D}_3 - \hat{D}_4)\right]$$

$$= \mathcal{K}_{333} + \mathcal{K}_{444} = 2\mathcal{K}_{333} \neq 0\,. \tag{3.56}$$

The expression we are more interested in is Eq. (3.56) since it shows us that, within this particular setting of the CY, $\mathcal{K}_{--+} \propto \mathcal{K} \equiv \mathcal{K}_{\zeta\zeta z}$ is non-vanishing. This means that the gauge kinetic function of the Dark Photon is indeed non-zero.

We finally compute the volume form that, under mirror symmetry ($t_i^+ \overset{M.S.}{\leftrightarrow} z^a, t_i^- \overset{M.S.}{\leftrightarrow} \zeta^\alpha$), is mapped into the prepotential which gives us the gauge kinetic function

$$\mathcal{V} \overset{M.S.}{\leftrightarrow} \mathrm{F}(z,\zeta). \tag{3.57}$$

The volume form becomes

$$\mathcal{V} = \frac{1}{6}\int_{CY} J' \wedge J' \wedge J' \supset \frac{1}{6}(t_+\hat{D}_+ + t_-\hat{D}_-)^3 \tag{3.58}$$

$$= \frac{1}{6}(\mathcal{K}_{+++}t_+^3 + 3\mathcal{K}_{--+}t_-^2 t_+) = \frac{1}{3}\,\mathcal{K}_{333}(t_+^3 + 3t_-^2 t_+), \tag{3.59}$$

which is mapped, through mirror symmetry, to a prepotential of the form

$$\mathrm{F}(z,\zeta) \supset \frac{1}{3}\mathcal{K}_{333}(z^3 + 3\zeta^2 z) = \gamma\frac{1}{6}\mathcal{K}_{zzz}z^3 + \gamma\frac{1}{6}\mathcal{K}_{\zeta\zeta z}\zeta^2 z, \tag{3.60}$$

with γ a sign factor of $\mathcal{O}(1)$ coming from the mirror map and where we can identify the intersection numbers as

$$\mathcal{K}_{zzz} \simeq \mathcal{K}_{+++} = 2\,\mathcal{K}_{333}, \tag{3.61}$$

$$\mathcal{K}_{\zeta\zeta z} \equiv \mathcal{K} \simeq 3\mathcal{K}_{--+} = 6\,\mathcal{K}_{333}. \tag{3.62}$$

This last result shows us the mapping between the intersection numbers on the two mirror Calabi-Yau manifolds.

With the knowledge of the gauge kinetic function of the Dark Photons, we are now ready to proceed to the final part of our analysis which consists into the study of these Dark Photons as Dark Radiation.

3.2 Moduli Mixing

In this section and in the next one we are going to expand our analysis on Dark Radiation produced by the decay of the volume modulus. In particular, after having seen the known possible sources of Dark Radiation in Subsec. 2.3.3, we are going to inspect RR Dark Photons as one of them.

Before starting our description, let us make a brief summary of the ingredients we have already seen:

- We have seen that RR Photons come from the dimensional reduction of the RR C_4 form.
- We have seen that the gauge kinetic function of these Dark Photons (3.15) depends holomorphically on the complex structure moduli that are not projected out by the orientifold involution. Recall that we are going to use the notation $U = u + i\sigma$ for the complex structure moduli such that $z \to i\overline{U}$ and the gauge kinetic function we are going to use is (3.31).
- To have a non-zero gauge kinetic function for the RR Photons, the structure of the CY must be such that the intersection number between orientifold-even and orientifold-odd complex structure moduli is non-vanishing.
- Complex structure moduli and Kähler moduli can in principle couple in the Kähler potential. This can induce a kinetic mixing between the two fields that, upon canonical normalization, can enter the gauge kinetic function leading to a Kähler modulus - Dark Photon coupling. Such a coupling gives all the ingredients needed to have a post-inflationary production mechanism of such RR bulk $U(1)s$.

In what follows we are going to show the chosen form of Kähler potential and to show how the mixing between the volume modulus and the complex structure moduli arises in the construction.

For the purposes of this work we are going to use a Kähler potential of the form

$$K = -3\ln(T+\overline{T}) - 3\ln(U+\overline{U}) - \frac{2x}{(T+\overline{T})^{3/2}} + \frac{f(U+\overline{U})}{(T+\overline{T})} \tag{3.63}$$

where $T = \tau + i\rho$ is the large volume modulus and $U = u + i\sigma$ is the complex structure modulus. The moduli are stabilized in the LVS as highlighted by the presence of the α' correction term (the third one) with $x = \xi/(2g_s^{3/2})$. The last piece, corresponding to K_{loop} in Eq. (3.16), is a loop correction. In particular, this Kaluza-Klein loop correction is of the form (2.17) where $f(U+\overline{U})$ is a generic function of the complex structure moduli. Let us mention that, without loss of generality, we are going to consider only one Kähler modulus related to the exponentially large volume and one complex structure modulus to have simpler computations in the canonical normalization of the fields.
The Kähler potential can be recast in terms of the real parts of the moduli as

$$K = -3\ln(2\tau) - 3\ln(2u) - \frac{2x}{(2\tau)^{3/2}} + \frac{f(2u)}{(2\tau)}. \tag{3.64}$$

Before moving to the explicit canonical normalization of the fields we want to focus on two features of this potential: the presence of an Extended No-Scale Structure and the fact that indeed it allows for a mixing between the Kähler modulus and the complex structure modulus.

3.2.1 Extended No-Scale structure

The Kähler potential in exam, at tree level,

$$K = -3\ln(T+\overline{T}) - 3\ln(U+\overline{U}) \tag{3.65}$$

features a No-Scale Structure. In fact, when computing the F-term supergravity scalar potential

$$V_F = e^K(K^{i\bar{j}}D_iWD_{\bar{j}}\overline{W} - 3|W|^2), \tag{3.66}$$

since the Gukov-Vafa-Witten superpotential

$$W_{GVW} = \int_X G_3 \wedge \Omega \tag{3.67}$$

depends only on the axio-dilaton S (through $G_3 = F_3 + iSH_3$ where F_3 and H_3 are respectively the RR and the NSNS 3-form fluxes) and on the complex structure moduli U (through Ω which is the holomorphic $(3,0)$-form), then the term proportional to $(K^{T\overline{T}}K_TK_{\overline{T}} - 3)|W|^2$ is equal to zero. This is something we have already described in Sec. 2.1. For the Non-Renormalization theorems however, we know that K could get corrections order by order in perturbation theory plus non-perturbative corrections, while the superpotential could only receive non-perturbative corrections. Focusing on the perturbative ones such that

$$K = K_{tree} + K_p, \tag{3.68}$$

$$W = W_0, \tag{3.69}$$

one can show that the No-Scale model extends to one further order in what is called Extended No-Scale Structure. The precise statement has been given in Subsec. 2.1.3. Let us now go back to our case: our aim is to prove that our Kähler potential written as

$$K = K_{tree} + \delta K_{(g_s)} = -3\ln(T+\overline{T}) - 3\ln(U+\overline{U}) + \frac{f(U+\overline{U})}{(T+\overline{T})} \tag{3.70}$$

enjoys an Extended No-Scale Structure which makes the loop contribution to the scalar potential vanish at leading order. To compute the corrections δV_1 and δV_2 to the scalar potential we need to find the first derivatives of the Kähler potential, the Kähler matrix and its inverse. The first derivatives of the Kähler potential are given by

$$K_T = \frac{1}{2}\frac{\partial K}{\partial \tau} = -\frac{3}{2\tau} - \frac{f(2u)}{4\tau^2}, \tag{3.71}$$

$$K_U = \frac{1}{2}\frac{\partial K}{\partial u} = -\frac{3}{2u} + \frac{f_u(2u)}{2\tau}, \tag{3.72}$$

from which the second derivatives follow straightforwardly; the lower index u denotes derivatives with respect to it. It is therefore easy to compute the Kähler matrix

$$K_{i\bar{j}} = \frac{1}{4}\begin{pmatrix} \frac{3}{\tau^2} + \frac{f(2u)}{\tau^3} & \frac{-f_u(2u)}{\tau^2} \\ \frac{-f_u(2u)}{\tau^2} & \frac{3}{u^2} + \frac{2f_{uu}(2u)}{\tau} \end{pmatrix}. \tag{3.73}$$

Now we want to use the approximation of Eq. (2.30) to compute the inverse Kähler matrix where K_0 is identified with K_{tree}. We can split the expression (3.73) into K_0 and δK in the following way

$$K^0_{i\bar{j}} = \begin{pmatrix} \frac{3}{4\tau^2} & 0 \\ 0 & \frac{3}{4u^2} \end{pmatrix}, \qquad \delta K_{i\bar{j}} = \begin{pmatrix} \frac{f(2u)}{4\tau^3} & \frac{-f_u(2u)}{4\tau^2} \\ \frac{-f_u(2u)}{4\tau^2} & \frac{f_{uu}(2u)}{2\tau} \end{pmatrix}. \tag{3.74}$$

Then we must invert $K^0_{i\bar{j}}$ obtaining

$$K_0^{ij} = \begin{pmatrix} \frac{4\tau^2}{3} & 0 \\ 0 & \frac{4u^2}{3} \end{pmatrix}. \tag{3.75}$$

At this point, using Eq. (2.30) to first order in the parameter ϵ we obtain the full inverse Kähler matrix, that is:

$$K^{i\bar{j}} = \begin{pmatrix} \frac{4\tau^2}{3} - \frac{4\tau f(2u)}{9} & \frac{4u^2 f_u(2u)}{9} \\ \frac{4u^2 f_u(2u)}{9} & \frac{4u^2}{3} - \frac{8u^4 f_{uu}(2u)}{9\tau} \end{pmatrix}. \tag{3.76}$$

Now we are ready to evaluate the corrections to the scalar potential. Let us first look at δV_1. From Eq. (2.25) we have that

$$\begin{aligned} \delta V_1 &= (2K_0^{T\overline{T}} K^0_T \delta K_T - K_0^{T\overline{T}} \delta K_{T\overline{T}} K_0^{T\overline{T}} K^0_T K^0_T) \frac{|W|^2}{\mathcal{V}^2} \\ &= \left[2\frac{4\tau^2}{3}\left(\frac{-3}{2\tau}\right)\left(\frac{-f(2u)}{4\tau^2}\right) - \frac{4\tau^2}{3}\left(\frac{f(2u)}{4\tau^3}\right)\frac{4\tau^2}{3}\left(\frac{-3}{2\tau}\right)^2\right]\frac{|W|^2}{\mathcal{V}^2} = 0, \end{aligned} \tag{3.77}$$

as expected. Looking instead at Eq. (2.26) we have

$$\begin{aligned}\delta V_2 &= (K_0^{T\overline{T}}\delta K_T\delta K_T - 2K_0^{T\overline{T}}\delta K_{T\overline{T}}K_0^{T\overline{T}}K_T^0\delta K_T \\ &+ K_0^{T\overline{T}}\delta K_{T\overline{T}}K_0^{T\overline{T}}\delta K_{T\overline{T}}K_0^{T\overline{T}}K_T^0K_T^0)\frac{|W|^2}{\mathcal{V}^2} \\ &= \left[\frac{4\tau^2}{3}\left(\frac{-f(2u)}{4\tau^2}\right)\left(\frac{-f(2u)}{4\tau^2}\right) - 2\frac{4\tau^2}{3}\left(\frac{f(2u)}{4\tau^3}\right)\frac{4\tau^2}{3}\left(\frac{-3}{2\tau}\right)\left(\frac{-f(2u)}{4\tau^2}\right)\right. \\ &\left. + \frac{4\tau^2}{3}\left(\frac{f(2u)}{4\tau^3}\right)\frac{4\tau^2}{3}\left(\frac{f(2u)}{4\tau^3}\right)\frac{4\tau^2}{3}\left(\frac{-3}{2\tau}\right)\left(\frac{-3}{2\tau}\right)\right]\frac{|W|^2}{\mathcal{V}^2} \\ &= \frac{f^2(2u)}{12\tau^2}\frac{|W|^2}{\mathcal{V}^2} = \frac{f^2(2u)|W|^2}{12\mathcal{V}^{10/3}}\end{aligned} \tag{3.78}$$

This result implies that the loop correction that induces a mixing between the Kähler modulus and the complex structure modulus is suppressed with respect to the LVS contribution (2.42). However, even if it looks subleading as we were expecting, it is important to study the presence of Dark Radiation in our model.

3.2.2 Kinetic Mixing

Before proceeding with the complete canonical normalization following in the footsteps of what we have done in Sec. 2.2, we are going to briefly show where this mixing between the moduli is and what we would like to find.
We know that in general the Lagrangian is given by Eq. (1.61) and in our case it can be expanded as

$$\mathcal{L} = K_{T\overline{T}}\partial_\mu\tau\partial^\mu\tau + K_{U\overline{U}}\partial_\mu u\partial^\mu u + K_{T\overline{U}}\partial_\mu\tau\partial^\mu u + K_{U\overline{T}}\partial_\mu u\partial^\mu\tau. \tag{3.79}$$

As already mentioned we are going to consider a Large Volume approximation, for which we have that $K_{T\overline{T}} \sim 3/(4\tau^2)$ and $K_{U\overline{U}} \sim 3/(4u^2)$: in fact, looking at the expression (3.73), $1/\tau^3$ is more suppressed with respect to $1/\tau^2$ and $u \sim \mathcal{O}(1)$. We can therefore split our Lagrangian into a first piece containing classical kinetic terms at leading order $\mathcal{L}_{kin}^{LO}$, and a second piece which contains the kinetic mixing between τ and u, namely $\mathcal{L}_{kin}^{mix}$:

$$\mathcal{L}_{kin}^{LO} = \frac{3}{4\tau^2}(\partial_\mu\tau)^2 + \frac{3}{4u^2}(\partial_\mu u)^2, \tag{3.80}$$

and

$$\mathcal{L}_{kin}^{mix} = -\frac{2f_u(2u)}{\tau^2}\partial_\mu\tau\partial^\mu u. \tag{3.81}$$

At this point we need to canonical normalize the kinetic terms: first of all we assume that

$$\frac{3}{4\tau^2}(\partial_\mu \tau)^2 = \frac{1}{2}(\partial_\mu \phi)^2 \tag{3.82}$$

such that $\tau = e^{\sqrt{\frac{2}{3}}\phi}$. We suppose we can expand around the minimum as $\tau = \langle\tau\rangle + \hat{\tau}$ and $\phi = \langle\phi\rangle + \hat{\phi}$. In this way we find

$$\frac{\hat{\tau}}{\langle\tau\rangle} = \sqrt{\frac{2}{3}}\hat{\phi}. \tag{3.83}$$

Analogously, we set

$$\frac{3}{4u^2}(\partial_\mu u)^2 = \frac{1}{2}(\partial_\mu \chi)^2 \tag{3.84}$$

such that $u = e^{\sqrt{\frac{2}{3}}\chi}$. Expanding around the minimum ($u = \langle u\rangle + \hat{u}$, $\chi = \langle\chi\rangle + \hat{\chi}$) we find

$$\hat{u} = \sqrt{\frac{2}{3}}\langle u\rangle\hat{\chi}. \tag{3.85}$$

Note from the Lagrangian (3.81) that the loop correction leads to a non-vanishing kinetic mixing between $\hat{\tau}$ and $\hat{u}$ for which we want to derive the volume scaling. Following the guess of [39], we suppose that we can write

$$\hat{\tau} = \sqrt{\frac{2}{3}}\langle\tau\rangle\hat{\phi} + k_{\tau u}\langle\tau\rangle^{c_{\tau u}}\hat{\chi} \tag{3.86}$$

and

$$\hat{u} = k_{u\tau}\langle\tau\rangle^{c_{u\tau}}\hat{\phi} + \sqrt{\frac{2}{3}}\langle u\rangle\hat{\chi} \tag{3.87}$$

where we used some coefficients k_{ij} with $i \neq j$ and other coefficients c_{ij} which tells us how the mixing terms scale with the volume.
What we are going to do in the following is to consider all the terms in the Kähler matrix (and not only the leading ones) to get a precise evaluation on how $\hat{\tau}$ and $\hat{u}$ are mixed and how this mixing scales with the volume.

3.2.3 Scalar Potential and Mass Mixing

Before looking at the canonical normalized fields we need to search for the expressions of the scalar potential and of the mass matrix. In fact, from Sec. 2.2 we know that the canonically normalized fields ϕ and χ can be found through the eigenvectors of the mass matrix (more precisely of the matrix $K^{-1}M^2$) which depends on the second derivatives of the scalar potential.

The Kähler potential we are going to consider is the one of the full LVS theory with the α' corrections included (3.63). In terms of the real parts of the moduli we have already seen that it could be written as (3.64).

By definition, we know that the F-term scalar potential is given by

$$V = e^K(K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2) \tag{3.88}$$

where the Kähler covariant derivative acts on the superpotential W such that $D_i W = \partial_i W + W \partial_i K$. It can be expanded by making explicit the indices i and j:

$$\begin{aligned} V =& e^K(K^{T\overline{T}} D_T W D_{\overline{T}} \overline{W} + K^{T\overline{U}} D_T W D_{\overline{U}} \overline{W} + K^{U\overline{T}} D_U W D_{\overline{T}} \overline{W} \\ &+ K^{U\overline{U}} D_U W D_{\overline{U}} \overline{W} - 3|W|^2). \end{aligned} \tag{3.89}$$

Since we are going to consider a superpotential of the form $W = W_0(U)$ which is only a function of the complex structure moduli, the scalar potential can be recast in its final form:

$$\begin{aligned} V =& e^K(K^{T\overline{T}} K_T K_{\overline{T}} |W|^2 + K^{T\overline{U}} D_T W D_{\overline{U}} \overline{W} + K^{U\overline{T}} D_U W D_{\overline{T}} \overline{W} \\ &+ K^{U\overline{U}} D_U W D_{\overline{U}} \overline{W} - 3|W|^2). \end{aligned} \tag{3.90}$$

From this expression we can see that before going any further, we need to specify the Kähler matrix. By definition, its form is the following

$$K_{i\bar{j}} = \begin{pmatrix} K_{U\overline{U}} & K_{U\overline{T}} \\ K_{T\overline{U}} & K_{T\overline{T}} \end{pmatrix} \tag{3.91}$$

where the labels T and U denote derivatives of the Kähler potential. Starting from Eq. (3.63) we can compute the following first derivatives

$$K_T = K_{\overline{T}} = -\frac{3}{(T+\overline{T})} + \frac{3x}{(T+\overline{T})^{5/2}} - \frac{f(U+\overline{U})}{(T+\overline{T})^2}, \tag{3.92}$$

$$K_U = K_{\overline{U}} = \frac{-3}{(U+\overline{U})} + \frac{f_U(U+\overline{U})}{(T+\overline{T})}, \tag{3.93}$$

from which the second derivatives come straightforward:

$$K_{T\overline{T}} = \frac{3}{(T+\overline{T})^2} - \frac{15x}{2(T+\overline{T})^{7/2}} + \frac{2f(U+\overline{U})}{(T+\overline{T})^3} = \frac{3}{4\tau^2} - \frac{15x}{16\sqrt{2}\tau^{7/2}} + \frac{f(2u)}{4\tau^3}, \tag{3.94}$$

$$K_{U\overline{U}} = \frac{3}{(U+\overline{U})^2} + \frac{f_{U\overline{U}}(U+\overline{U})}{(T+\overline{T})} = \frac{3}{4u^2} + \frac{2f''(2u)}{4\tau}, \tag{3.95}$$

$$K_{T\overline{U}} = -\frac{f_{\overline{U}}(U+\overline{U})}{(T+\overline{T})^2} = -\frac{f_u(2u)}{4\tau^2}, \tag{3.96}$$

$$K_{U\overline{T}} = -\frac{f_U(U+\overline{U})}{(T+\overline{T})^2} = -\frac{f_u(2u)}{4\tau^2}, \tag{3.97}$$

where in the second set of equalities we have used the definitions of T and U. It immediately follows that the Kähler matrix is

$$K_{i\bar{j}} = \begin{pmatrix} \frac{3}{4u^2} + \frac{2f_{uu}(2u)}{4\tau} & -\frac{f_u(2u)}{4\tau^2} \\ -\frac{f_u(2u)}{4\tau^2} & \frac{3}{4\tau^2} - \frac{15x}{16\sqrt{2}\tau^{7/2}} + \frac{f(2u)}{4\tau^3} \end{pmatrix}. \tag{3.98}$$

We now want to compute the inverse of the Kähler matrix whose form is given, as usual, by

$$K^{i\bar{j}} = \begin{pmatrix} K^{U\overline{U}} & K^{U\overline{T}} \\ K^{T\overline{U}} & K^{T\overline{T}} \end{pmatrix}; \tag{3.99}$$

the entries have been obtained numerically and they can be written as

$$K^{U\overline{U}} = \frac{\mathcal{N}(\tau,u)}{\mathcal{D}(\tau,u)}, \tag{3.100}$$

$$K^{U\overline{T}} = \frac{f_u(2u)}{4\tau^2\mathcal{D}(\tau,u)} = K^{T\overline{U}}, \tag{3.101}$$

$$K^{T\overline{T}} = \frac{1}{\mathcal{D}(\tau,u)}\left(\frac{f_{uu}(2u)}{2\tau} + \frac{3}{4u^2}\right), \tag{3.102}$$

where we have defined the following quantities to get more readable expressions

$$\mathcal{N}(\tau,u) = \frac{1}{4}\left(\frac{f(2u)}{\tau^3} - \frac{15x}{4\sqrt{2}\tau^{7/2}}\right) + \frac{3}{4\tau^2}, \tag{3.103}$$

$$\begin{aligned}\mathcal{D}(\tau,u) = & \frac{9}{16\tau^2u^2} + \frac{3f(2u)}{16\tau^3u^2} + \frac{3f_{uu}(2u)}{8\tau^3} - \frac{45x}{64\sqrt{2}\tau^{7/2}u^2} \\ & - \frac{f_u(2u)^2}{16\tau^4} + \frac{f(2u)f_{uu}(2u)}{8\tau^4} - \frac{15xf_{uu}(2u)}{32\sqrt{2}\tau^{9/2}},\end{aligned} \tag{3.104}$$

$$\begin{aligned}\mathcal{P}(U,\overline{U}) &= (\partial(D_UW|_{f=0})|_{min})(\partial(D_{\overline{U}}\overline{W}|_{f=0})|_{min}) \\ &= \left(W_{uu}(U) - \frac{3W(U)}{4u^2}\right)\left(\overline{W}_{uu}(\overline{U}) - \frac{3\overline{W}(\overline{U})}{4u^2}\right),\end{aligned} \tag{3.105}$$

$$\mathcal{S}(U,\overline{U}) = W(U)\left(u^2\partial(D_{\overline{U}}\overline{W}|_{f=0}) + 3\overline{W}(\overline{U})\right) + u^2\overline{W}(\overline{U})\partial(D_UW|_{f=0}). \tag{3.106}$$

However, in order to obtain an even better approximation, we can decompose the inverse using the expansion (2.30); truncating at the first order we get

$$K^{i\bar{j}} = \begin{pmatrix} \frac{4u^2}{3} - \frac{8u^4 f_{uu}(2u)}{9\tau} & \frac{4u^2 f_u(2u)}{9} \\ \frac{4u^2 f_u(2u)}{9} & \frac{4\tau^2}{3} + \frac{5x\tau^{1/2}}{3\sqrt{2}} - \frac{4\tau f(2u)}{9} \end{pmatrix}. \tag{3.107}$$

At this point we can start by computing the pieces which are going to constitute our scalar potential. Let us take Eq. (3.90) and write it as

$$V = e^K(V_1 + V_2 + V_3 + V_4 - 3|W|^2) \tag{3.108}$$

where V_1, V_2, V_3, V_4 correspond to each of the terms in the original expression. Let us start with $V_1 - 3|W|^2$ which turns out to be

$$\begin{aligned}V_1 - 3|W|^2 = & \frac{3\,x\,W(U)\overline{W}(\overline{U})}{4\sqrt{2}\,\tau^{3/2}} - \frac{f^2(2u)\,W(U)\overline{W}(\overline{U})}{4\,\tau^2} + \frac{7\,x\,f(2u)\,W(U)\overline{W}(\overline{U})}{4\sqrt{2}\,\tau^{5/2}} \\ & - \frac{3\,x^2\,W(U)\overline{W}(\overline{U})}{2\,\tau^3} - \frac{f^2(2u)\,W(U)\overline{W}(\overline{U})}{36\,\tau^3} + \frac{13\,x\,f^2(2u)\,W(U)\overline{W}(\overline{U})}{48\,\sqrt{2}\,\tau^{7/2}} \\ & - \frac{7\,x^2\,f(2u)\,W(U)\overline{W}(\overline{U})}{16\,\tau^4} + \frac{15\,x^3\,W(U)\overline{W}(\overline{U})}{32\,\sqrt{2}\,\tau^{9/2}}.\end{aligned} \tag{3.109}$$

Before going on, let us look at the Kähler covariant derivative of the superpotential with respect to U. By definition we have that the covariant derivative acts as

$$D_UW = \partial_UW(U) + W(U)K_T = \partial_UW(U) - \frac{3W(U)}{2u} + \frac{W(U)f_u(2u)}{2\tau}. \tag{3.110}$$

Moreover, we can state that the first two terms of the Kähler covariant derivative of the superpotential, are the only pieces one would have obtained if the loop correction was not present ($f(2u) = 0$), and so we can abbreviate the previous expression as:

$$D_U W = D_U W|_{f=0} + \frac{W(U) f_u(2u)}{2\tau}. \tag{3.111}$$

and the same for the complex conjugate. Having clarified this, we can proceed with the computation of V_2, V_3 and V_4:

$$\begin{aligned} V_2 = &- \frac{2u^2 W(U) f_u(2u) D_{\overline{U}}\overline{W}|_{f=0}}{3\tau} - \frac{u^2 W(U) f(2u) f_u(2u) D_{\overline{U}}\overline{W}|_{f=0}}{9\tau^2} \\ &- \frac{u^2 W(U)\overline{W}(\overline{U}) f_u^2(2u)}{3\tau^2} + \frac{u^2 x W(U) f_u(2u) D_{\overline{U}}\overline{W}|_{f=0}}{3\sqrt{2}\tau^{5/2}} \\ &- \frac{u^2 W(U)\overline{W}(\overline{U}) f(2u) f_u^2(2u)}{18\tau^3} + \frac{u^2 x W(U)\overline{W}(\overline{U}) f_u^2(2u)}{6\sqrt{2}\tau^{7/2}}, \end{aligned} \tag{3.112}$$

$$\begin{aligned} V_3 = &- \frac{2u^2 \overline{W}(\overline{U}) f_u(2u) D_U W|_{f=0}}{3\tau} - \frac{u^2 \overline{W}(\overline{U}) f(2u) f_u(2u) D_U W|_{f=0}}{9\tau^2} \\ &- \frac{u^2 W(U)\overline{W}(\overline{U}) f_u^2(2u)}{3\tau^2} + \frac{u^2 x \overline{W}(\overline{U}) f_u(2u) D_U W|_{f=0}}{3\sqrt{2}\tau^{5/2}} \\ &- \frac{u^2 W(U)\overline{W}(\overline{U}) f(2u) f_u^2(2u)}{18\tau^3} + \frac{u^2 x W(U)\overline{W}(\overline{U}) f_u^2(2u)}{6\sqrt{2}\tau^{7/2}}, \end{aligned} \tag{3.113}$$

$$\begin{aligned} V_4 = &\frac{4}{3} u^2 D_{\overline{U}}\overline{W}|_{f=0} D_U W|_{f=0} + \frac{2u^2 W(U) f_u(2u) D_{\overline{U}}\overline{W}|_{f=0}}{3\tau} \\ &+ \frac{2u^2 \overline{W}(\overline{U}) f_u(2u) D_U W|_{f=0}}{3\tau} - \frac{8u^4 f_{uu}(2u) D_{\overline{U}}\overline{W}|_{f=0} D_U W|_{f=0}}{9\tau} \\ &- \frac{4u^4 W(U) f_u(2u) f_{uu}(2u) D_{\overline{U}}\overline{W}|_{f=0}}{9\tau^2} - \frac{4u^4 \overline{W}(\overline{U}) f_u(2u) f_{uu}(2u) D_U W|_{f=0}}{9\tau^2} \\ &+ \frac{u^2 W(U)\overline{W}(\overline{U}) f_u^2(2u)}{3\tau^2} - \frac{2u^4 W(U)\overline{W}(\overline{U}) f_u^2(2u) f_{uu}(2u)}{9\tau^3}. \end{aligned} \tag{3.114}$$

It is easy to prove that

$$e^K \sim \frac{1}{(2\tau)^3}\frac{1}{(2u)^2} \propto \frac{1}{\mathcal{V}^2}. \tag{3.115}$$

Putting all the pieces together and taking only the leading terms, we get the following form for the scalar potential

$$V \sim V_{flux} + V_{LVS} + V_{loop} \sim \frac{h(u,\tau)}{\mathcal{V}^2} + \frac{|W(U)|^2}{\mathcal{V}^3} + \frac{|W(U)|^2}{\mathcal{V}^{10/3}} \tag{3.116}$$

where V_{loop} is given by the Extended No-Scale structure of the scalar potential (as we have shown in (3.78)) and, due to this property, it is much more suppressed than V_{LVS}, while V_{flux} has been expressed in terms of a function $h(u,\tau)$ which is given roughly by $(V_2+V_3+V_4)/(8u^3)$.
Before computing the mass matrix, we need to evaluate the minimum of the scalar potential. It is easy to show that, if we do not consider the loop correction to the Kähler potential ($f(U+\overline{U})=0$), since V_1 cancels with $-3|W|^2$ due to the No-Scale structure, we must look at $h(u,\tau)$. Taking it one can easily conclude that the scalar potential has its minimum for $D_UW=0$ which, in this case, corresponds to have $D_UW|_{f=0}=0$ and the same for the complex conjugate. We can notice that, if $f(U+\overline{U})\neq 0$, the minimum is shifted to $D_UW=(W_0(U)f_u(2u))/\tau$.

At this point, we can proceed with the computation of the mass matrix which, by definition, is given by $M_{i\bar{j}}=V_{i\bar{j}}/2$ where

$$V_{ij}=\begin{pmatrix} V_{uu} & V_{u\tau} \\ V_{\tau u} & V_{\tau\tau}\end{pmatrix}\,. \tag{3.117}$$

In what follows we are going to take $V=AB$, where $A=e^K$ and $B=(V_1+V_2+V_3+V_4-3|W|^2)$ to treat all the derivatives properly.
Let us start with the element $V_{U\overline{U}}$. Applying the chain rule we get

$$V_{U\overline{U}}=A_{U\overline{U}}B+A_UB_{\overline{U}}+A_{\overline{U}}B_U+AB_{U\overline{U}}. \tag{3.118}$$

Having written all in terms of τ and u, this is equivalent to

$$V_{uu}=A_{uu}B+2A_uB_u+AB_{uu} \tag{3.119}$$

being aware of the factor 1/2 when switching from one derivative to another. The derivatives of the exponential factor (3.115) are easy to compute:

$$e^K_U=\frac{-3}{128\tau^3u^4},\qquad e^K_{U\overline{U}}=\frac{3}{64\tau^3u^5}, \tag{3.120}$$

$$e^K_T=\frac{-3}{64\tau^4u^3},\qquad e^K_{T\overline{T}}=\frac{3}{16\tau^5u^3}, \tag{3.121}$$

$$e^K_{U\overline{T}}=\frac{9}{128\tau^4u^4},\qquad e^K_{T\overline{U}}=\frac{9}{128\tau^4u^4}. \tag{3.122}$$

Computing instead the derivatives of the scalar potential requires some effort. Doing so for all the addends in B and evaluating everything at the minimum, we obtain

$$V_{uu}|_{min}\propto\frac{1}{\tau^3}=\frac{1}{\mathcal{V}^2}. \tag{3.123}$$

Note that, when evaluating the derivatives with respect to u, one should also evaluate the derivatives of the superpotential: we can use the fact that $\partial_u W(U) = -W(U)K_U|_{f=0}$ at the minimum and the same for the complex conjugate.
For the other diagonal element $V_{T\overline{T}}$ we use the same chain rule as before

$$V_{T\overline{T}} = A_{T\overline{T}}B + A_T B_{\overline{T}} + A_{\overline{T}} B_T + AB_{T\overline{T}} \tag{3.124}$$

equivalent to (up to the proper 1/2 factor when taking the derivatives)

$$V_{\tau\tau} = A_{\tau\tau}B + 2A_\tau B_\tau + AB_{\tau\tau}. \tag{3.125}$$

In this case, the derivation of all the pieces in B evaluated in the minimum, leads us to

$$V_{\tau\tau}|_{min} = \frac{297xW(U)\overline{W}(\overline{U})}{1024\sqrt{2}u^3\tau^{13/2}} \propto V_{LVS,\tau\tau} \sim \frac{1}{\mathcal{V}^{13/3}}. \tag{3.126}$$

The off diagonal terms $V_{U\overline{T}}$ and $V_{T\overline{U}}$ obey the following chain rules

$$V_{U\overline{T}} = A_{U\overline{T}}B + A_{\overline{T}}B_U + A_U B_{\overline{T}} + AB_{U\overline{T}}, \tag{3.127}$$

$$V_{T\overline{U}} = A_{T\overline{U}}B + A_{\overline{U}}B_T + A_T B_{\overline{U}} + AB_{T\overline{U}}, \tag{3.128}$$

which can be written as

$$V_{u\tau} = V_{\tau u} = A_{u\tau}B + A_\tau B_u + A_u B_\tau + AB_{u\tau}. \tag{3.129}$$

Evaluating again all the derivatives at the minimum of the scalar potential, we obtain

$$V_{u\tau}|_{min} \propto \frac{1}{\tau^6} = \frac{1}{\mathcal{V}^4} \tag{3.130}$$

and similarly for the other term.

Computing explicitly all the derivatives we finally find:

$$
\begin{aligned}
V_{uu} &= \frac{1}{96\langle u\rangle\langle\tau\rangle^3}\left((\partial(D_U W|_{f=0}))|_{min}(\partial(D_{\overline{U}}\overline{W}|_{f=0}))|_{min}\right)\\
&= \frac{1}{96\langle u\rangle\langle\tau\rangle^3}\left(W_{uu}(\langle U\rangle) - \frac{3W(\langle U\rangle)}{4\langle u\rangle^2}\right)\left(\overline{W}_{uu}(\langle\overline{U}\rangle) - \frac{3\overline{W}(\langle\overline{U}\rangle)}{4\langle u\rangle^2}\right), \qquad (3.131)\\
V_{u\tau} &= -\frac{1}{1152\tau^6 u^3}\left(5f(2\langle u\rangle)f_u(2\langle u\rangle)W(\langle U\rangle)\left(u^2\partial(D_{\overline{U}}\overline{W}|_{f=0}))|_{min} + 3\overline{W}(\langle U\rangle)\right)\right)\\
&\quad - \frac{1}{1152\tau^6 u^3}\left(5f(2\langle u\rangle)f_u(2\langle u\rangle)u^2\overline{W}(\langle U\rangle)(\partial(D_U W|_{f=0}))|_{min}\right) \qquad (3.132)\\
&= -\frac{1}{1152\tau^6 u^3}\left(5f(2\langle u\rangle)f_u(2\langle u\rangle)W(\langle U\rangle)\left(u^2\left(\overline{W}_{uu}(\langle\overline{U}\rangle) - \frac{3\overline{W}(\langle\overline{U}\rangle)}{4u^2}\right) + 3\overline{W}(\langle\overline{U}\rangle)\right)\right)\\
&\quad - \frac{1}{1152\tau^6 u^3}\left(5f(2\langle u\rangle)f_u(2\langle u\rangle)u^2\overline{W}(\langle\overline{U}\rangle)\left(W_{uu}(\langle U\rangle) - \frac{3W(\langle U\rangle)}{4u^2}\right)\right) = V_{\tau u}, \qquad (3.133)
\end{aligned}
$$

$$
V_{\tau\tau} = \frac{297xW(\langle U\rangle)\overline{W}(\langle\overline{U}\rangle)}{1024\sqrt{2}\langle u\rangle^3}\left(\frac{1}{\langle\tau\rangle}\right)^{13/2}. \qquad (3.134)
$$

The minimum condition we have used is $(D_U W|_{f=0})|_{min} = 0$, from which it follows that $\partial_U W = -\frac{3}{2u}W$ and so

$$
\partial(D_U W|_{f=0})|_{min} = W_{uu}(\langle U\rangle) - \frac{3W(\langle U\rangle)}{4u^2}. \qquad (3.135)
$$

The same holds for the complex conjugate.

We can now compute the mass matrix from the usual formula $\tilde{M}_{ij} = \frac{1}{2}K_{il}^{-1}V_{lj}$ which give us for the complete inverse Kähler metric (3.99):[2]

$$
\tilde{M}_{ij} = \begin{pmatrix} m_{uu} & m_{u\tau} \\ m_{\tau u} & m_{\tau\tau} \end{pmatrix} \qquad (3.136)
$$

[2]From now on we will call the dynamical perturbations of the fields $\tau \to \hat{\tau}$ and $u \to \hat{u}$, while the vacuum expectation values as $\langle\tau\rangle \to \tau$ and $\langle u\rangle \to u$ to make the notation lighter

with

$$m_{uu} = \frac{1}{2}\left(\frac{\mathcal{P}(U,\overline{U})\mathcal{N}(\tau,u)}{96\tau^3 u\mathcal{D}(\tau,u)} - \frac{5f(2u)f_u(2u)^2\mathcal{S}(U,\overline{U})}{4608\tau^8u^3\mathcal{D}(\tau,u)}\right), \tag{3.137}$$

$$m_{u\tau} = \frac{1}{2}\left(\frac{297xW(U)\overline{W}(\overline{U})f_u(2u)}{4096\sqrt{2}\tau^{17/2}u^3\mathcal{D}(\tau,u)} - \frac{5f(2u)f_u(2u)\mathcal{N}(\tau,u)\mathcal{S}(U,\overline{U})}{1152\tau^6u^3\mathcal{D}(\tau,u)}\right), \tag{3.138}$$

$$m_{\tau u} = \frac{1}{2}\left(\frac{\mathcal{P}(U,\overline{U})f_u(2u)}{384\tau^5 u\mathcal{D}(\tau,u)} - \frac{5f(2u)f_u(2u)\left(\frac{f_{uu}(2u)}{2\tau}+\frac{3}{4u^2}\right)\mathcal{S}(U,\overline{U})}{1152\tau^6u^3\mathcal{D}(\tau,u)}\right), \tag{3.139}$$

$$m_{\tau\tau} = \frac{1}{2}\left(\frac{297xW(U)\overline{W}(\overline{U})\left(\frac{f_{uu}(2u)}{2\tau}+\frac{3}{4u^2}\right)}{1024\sqrt{2}\tau^{13/2}u^3\mathcal{D}(\tau,u)} - \frac{5f(2u)f_u(2u)^2\mathcal{S}(U,\overline{U})}{4608\tau^8u^3\mathcal{D}(\tau,u)}\right). \tag{3.140}$$

Using now this mass matrix we can compute its eigenvectors $\vec{x}, \vec{y}$ which give us the canonical normalization of our fields as we have done in Sec. 2.2 following [27]

$$\begin{pmatrix}\hat{u}\\ \hat{\tau}\end{pmatrix} = \begin{pmatrix}v_1\\ v_2\end{pmatrix}\frac{\phi}{\sqrt{2}} + \begin{pmatrix}w_1\\ w_2\end{pmatrix}\frac{\chi}{\sqrt{2}}, \tag{3.141}$$

where v_1, v_2 and w_1, w_2 are the entries of the normalized eigenvectors

$$\begin{pmatrix}v_1\\ v_2\end{pmatrix} = \frac{1}{||\vec{x}||}\begin{pmatrix}x_1\\ x_2\end{pmatrix}, \qquad \begin{pmatrix}w_1\\ w_2\end{pmatrix} = \frac{1}{||\vec{y}||}\begin{pmatrix}y_1\\ y_2\end{pmatrix}, \tag{3.142}$$

with

$$||\vec{b}|| = \sqrt{\vec{b}^T K\vec{b}}, \tag{3.143}$$

for $\vec{b} = \{\vec{x}, \vec{y}\}$.

Due to the existing hierarchy of the masses of the normalized fields, $m_\chi^2 \gg m_\phi^2$, we can compute the eigenvalues of the mass matrix as:

$$m_\chi^2 \simeq \mathrm{Tr}(\tilde{M}_{ij}) \simeq \frac{u\mathcal{P}(U,\overline{U})}{144\tau^3}, \tag{3.144}$$

$$m_\phi^2 \simeq \frac{\mathrm{Det}(\tilde{M}_{ij})}{\mathrm{Tr}(\tilde{M}_{ij})} \simeq \frac{99xW(U)\overline{W}(\overline{U})}{512\sqrt{2}u^3}\left(\frac{1}{\tau}\right)^{9/2}. \tag{3.145}$$

This approximation is consistent with the eigenvalues of the total matrix computed at leading order in τ expansion. In addition to this, the x dependence of the mass tells us that the volume modulus is stabilized by α' corrections.

The eigenvectors which satisfy $\tilde{M}_{ij}\vec{x} = m_\phi^2\vec{x}$ and $\tilde{M}_{ij}\vec{y} = m_\chi^2\vec{y}$ finally read as

$$\vec{x} \simeq \begin{pmatrix} \frac{5f(2u)f_u(2u)\mathcal{S}(U,\overline{U})}{12\tau^3u^2\mathcal{P}(U,\overline{U})} \\ 1 \end{pmatrix}, \qquad \vec{y} \simeq \begin{pmatrix} \frac{3}{f_u(2u)} + \frac{f(2u)\left(15\mathcal{S}(U,\overline{U})u^2+4u^4\partial(D_{\overline{U}}\overline{W}|_{f=0})\right)}{4u^4f_u(2u)\mathcal{P}(U,\overline{U})\tau} \\ 1 \end{pmatrix}. \tag{3.146}$$

The norms of the eigenvectors can be easily computed using the expression (3.143) and turn out to be

$$\frac{1}{||\vec{x}||} \simeq \frac{2\tau}{\sqrt{3}} - \frac{f(2u)}{3\sqrt{3}}\,, \tag{3.147}$$

$$\frac{1}{||\vec{y}||} \simeq \frac{2}{3\sqrt{3}\sqrt{\frac{1}{u^2f_u(2u)^2}}} - \frac{1}{18\sqrt{3}\tau u^4\mathcal{P}(U,\overline{U})\sqrt{\frac{1}{u^2f_u(2u)^2}}}\Big(4u^6f_{uu}(2u)+4u^4f(2u))\mathcal{P}(U,\overline{U})$$
$$+15u^2f(2u)(W(u)\partial(D_{\overline{U}}\overline{W}|_{f=0})+\overline{W}(\overline{U})\partial(D_UW|_{f=0}))+45f(2u)W(U)\overline{W}(\overline{U})\Big). \tag{3.148}$$

The canonically normalized eigenvectors are finally given by

$$\vec{v} \simeq \begin{pmatrix} \frac{5f(2u)f_u(2u)\mathcal{S}(U,\overline{U})}{6\sqrt{3}\tau^2u^2\mathcal{P}(U,\overline{U})} \\ \frac{2\tau}{\sqrt{3}} - \frac{f(2u)}{3\sqrt{3}} \end{pmatrix}, \tag{3.149}$$

$$\vec{w} \simeq \begin{pmatrix} \frac{2u}{\sqrt{3}} - \frac{2(u^3f_{uu}(2u))}{3\sqrt{3}\tau} \\ \frac{2uf_u(2u)}{3\sqrt{3}} - \frac{f_u(2u)(f(2u)(15\mathcal{S}(U,\overline{U})u^2+4u^4\partial(D_{\overline{U}}\overline{W}|_{f=0}))+4u^6\mathcal{P}(U,\overline{U})f_{uu}(2u))}{18\tau(\sqrt{3}u^3\mathcal{P}(U,\overline{U}))} \end{pmatrix}, \tag{3.150}$$

such that the canonically normalized fields finally read as

$$\frac{\hat{u}}{u} \simeq \left[\frac{5ff_u\mathcal{S}(U,\overline{U})}{6\sqrt{6}u^3\mathcal{P}(U,\overline{U})}\frac{1}{\tau^2}\right]\phi + \left[\frac{2}{\sqrt{6}}\left(1-\frac{u^2f_{uu}}{3\tau}\right)\right]\chi\,, \tag{3.151}$$

$$\frac{\hat{\tau}}{\tau} \simeq \left[\frac{2}{\sqrt{6}}\left(1-\frac{f}{6\tau}\right)\right]\phi + \left[\frac{2uf_u}{3\sqrt{6}}\frac{1}{\tau}\left(1+\mathcal{O}\left(\frac{f}{\tau}\right)\right)\right]\chi\,. \tag{3.152}$$

Looking at these canonical normalizations for the fields, we can see that $\hat{u}$ is mostly in the direction of χ, while $\hat{\tau}$ is mostly in the direction of ϕ. We can conclude then that the canonically normalized field ϕ is almost totally $\hat{\tau}$ and χ is almost totally $\hat{u}$. However, as we were expecting, there is a volume suppressed kinetic mixing between the volume modulus and the complex structure modulus. In fact,

$$\frac{\hat{u}}{u} \propto \frac{f f_u}{u^2 \tau^2}\, \phi \,, \tag{3.153}$$

meaning that, even if suppressed by a factor $1/\mathcal{V}^{4/3}$, the kinetic mixing is present and it is generated by the string loop corrections due to the dependence on $f(2u)$. If $f(2u) = 0$ (i.e. no loop correction), we would not have had the desired mixing term.
The presence of the mixing term between the volume modulus and the complex structure modulus ensures the existence of the decay channel of the volume modulus into RR Dark Photons opening the possibility of an additional source of Dark Radiation.

A double check of the correct scaling of the couplings can be obtained by plugging (3.151) and (3.152) inside the Lagrangian

$$\mathcal{L} \supset K_{T\overline{T}} \partial_\mu \hat{\tau} \partial^\mu \hat{\tau} + K_{U\overline{U}} \partial_\mu \hat{u} \partial^\mu \hat{u} + 2K_{U\overline{T}} \partial_\mu \hat{u} \partial^\mu \hat{\tau} - V_{uu} \hat{u}^2 - V_{\tau\tau} \hat{\tau}^2 - V_{u\tau} \hat{u} \hat{\tau} \tag{3.154}$$

where we can use the Kähler matrix (3.98) and the expressions (3.131), (3.133) and (3.134) for the second derivatives of the scalar potential. What we obtain are the leading terms for the two canonically normalized fields ϕ and χ

$$\mathcal{L}_{kin} \supset \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + \mathcal{O}\left(\frac{1}{\tau}\right) , \tag{3.155}$$

$$\mathcal{L}_{pot} \supset - \frac{99 x W(U) \overline{W}(\overline{U})}{512\sqrt{2} \tau^{9/2} u^3} \phi^2 - \frac{u \mathcal{P}(U, \overline{U})}{144 \tau^3} \chi^2 + \mathcal{O}\left(\frac{1}{\tau^4}\right) , \tag{3.156}$$

This tells us that:

1) both the kinetic and the mass leading order terms are diagonal now, and so mixing is only subdominant;

2) the leading order mass terms are exactly the eigenvalues (3.144), (3.145);

showing that the result is indeed consistent.

3.3 Moduli couplings and Decay Rates into RR Photons

The final step to see if the RR Photons Dark Radiation production is harmful for the current observational bounds on the effective number of neutrino species, is to evaluate the decay rate of the large volume modulus into RR Photons and then to evaluate the branching ratio with respect to the dominant decay channel into the visible sector. Thus, in this section we are going to compute first the decay rate of interest and then we are going to see how it affects the effective number of neutrino species in an explicit model of sequestered LVS.

Before starting our analysis let us recap one last time what we have found and what we need.

- We have seen that the action for the RR Dark Photons is given by Eq. (3.12), from which we can extract the kinetic Lagrangian

$$\mathcal{L}_{kin} \supset -\frac{1}{2}\mathrm{Re}(\mathrm{f})_{\gamma\lambda} F_2^{\gamma} \wedge *F_2^{\lambda} - \frac{1}{2}\mathrm{Im}(\mathrm{f})_{\gamma\lambda} F_2^{\gamma} \wedge F_2^{\lambda}. \tag{3.157}$$

- In Eq. (3.31) we have found that the gauge kinetic function of the RR Dark Photons is directly dependent on the complex structure moduli.
- In Eq. (3.151) we have seen that the complex structure modulus has a suppressed component in the direction of ϕ due to the loop correction, which is the one that allows the decay of the volume modulus into the Dark Photons.
- In Sec. 2.3.2 we have seen how the number of the effective neutrino species is affected by the decay of the longest-living modulus and which are the actual observational bounds on it.

3.3.1 Volume mode coupling to RR Photons

Since we are interested in the coupling between the volume modulus and the Ramond-Ramond Photons, we look at the first term of the Lagrangian (3.157) and we use the expression of the gauge kinetic function (3.31) to obtain

$$\mathcal{L} \supset -\frac{1}{2}\mathrm{Re}(\mathrm{f}) F_{\mu\nu}F^{\mu\nu} = \frac{\mathcal{K}}{4} u F_{\mu\nu}F^{\mu\nu}. \tag{3.158}$$

To obtain the coupling of the canonically normalized modulus, we expand $u = \langle u \rangle + \hat{u}$ so that

$$\frac{\mathcal{K}}{4} u F_{\mu\nu}F^{\mu\nu} = \frac{\mathcal{K}}{4}\langle u \rangle F_{\mu\nu}F^{\mu\nu} + \frac{\mathcal{K}}{4}\hat{u} F_{\mu\nu}F^{\mu\nu}. \tag{3.159}$$

To canonical normalize the field strength, we suppose the first term of the previous expression with the vacuum expectation value of the complex structure modulus to be roughly

$$\frac{\mathcal{K}}{4}\langle u \rangle F_{\mu\nu}F^{\mu\nu} \sim \frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}, \tag{3.160}$$

where $\bar{F}_{\mu\nu}$ is the canonically normalized field strength tensor, such that $\bar{F}_{\mu\nu} = \sqrt{\mathcal{K}\langle u\rangle}F_{\mu\nu}$. Performing the substitution we obtain

$$\mathcal{L} \supset \frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{\hat{u}}{4\langle u\rangle}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}. \tag{3.161}$$

Finally, to get the fully canonically normalized coupling, we substitute $\hat{u}$ with the expression (3.151) where we have made explicit the scaling of the complex structure modulus with the volume. Since we are interested in the coupling of the Ramond-Ramond photons with the volume modulus, we use only the first term with ϕ which is the canonically normalized Kähler modulus:

$$\mathcal{L} \supset \left[\frac{5ff_u\mathcal{S}(U,\bar{U})}{24\sqrt{6}\langle u\rangle^3\mathcal{P}(U,\bar{U})\tau^2}\right]\phi\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}, \tag{3.162}$$

where we have defined the functions $\mathcal{P}(U,\bar{U})$ and $\mathcal{S}(U,\bar{U})$ respectively as (3.105) and (3.106).

3.3.2 Volume Mode Decay Rate into RR Photons

To evaluate the decay rate we are going to use the well known expression

$$\Gamma = \frac{|M|^2}{2|E|}\frac{1}{8\pi} \tag{3.163}$$

where M is the amplitude and E is the energy of the center of mass as we have already done in Subsec. 2.3.3. In our case this expression becomes (in Planck units)

$$\Gamma_{\phi\to\bar{A}_\mu\bar{A}_\nu} = \frac{25f^2f_u^2\mathcal{S}^2(U,\bar{U})}{432\mathcal{P}^2(U,\bar{U})\langle u\rangle^6\tau^4}\frac{m_\phi^3}{16\pi} \tag{3.164}$$

where, to simplify the notation, we have dropped the dependence of the function f on the real part of the complex structure moduli and where we have used the following Feynman rule to compute the amplitude $i(-4p_3p_2g_{\mu\nu} + 4p_{3\mu}p_{2\nu})$ with p_2 and p_3 outgoing momenta. Recalling that $\tau = \mathcal{V}^{2/3}$ and setting

$$c \equiv \frac{ff_u\mathcal{S}(U,\bar{U})}{\mathcal{P}(U,\bar{U})\langle u\rangle^3}, \tag{3.165}$$

the expression (3.164) can be rewritten as (restoring the appropriate power of M_P)

$$\Gamma_{\phi\to\bar{A}_\mu\bar{A}_\nu} = \frac{25c^2}{6912\pi\mathcal{V}^{8/3}}\frac{m_\phi^3}{M_P^2} \sim \frac{25c}{6912\pi(\ln\mathcal{V})^{3/2}}\frac{M_P}{\mathcal{V}^{43/6}} \tag{3.166}$$

where we have used the fact that $m_\phi \sim M_P/\mathcal{V}^{3/2}$ as we have found in Eq. (3.145) during the procedure of the canonical normalization and as we have shown when discussing the sequestered LVS. Note that c is expected to be an $\mathcal{O}(1)$ factor for $f \sim f_u \sim u \sim \mathcal{O}(1)$ such as $\mathcal{S}(U,\bar{U}) \sim \mathcal{P}(U,\bar{U}) \sim \mathcal{O}(1)$.
Before going any further, let us notice that the decay rate we have found does not depend either on the number of moduli of the theory, or on the string inflationary scenario considered. The only fact we are assuming as in Sec. 2.3.2, is that the volume modulus is the one that reheats the universe.

3.4 RR Photons Dark Radiation

At this point we need to compare the decay rate we have found with the dominant decay channel for the volume modulus into the visible sector, in order to find the branching ratio needed to compute the contribution of Dark Radiation to the effective number of neutrino species. To do so we can distinguish two different models: Loop Blow-Up Inflation [35] and Non-Perturbative Blow-Up Inflation [40].

Loop Blow-Up Inflation In this case the volume has the form

$$\mathcal{V} = \tau_b^{3/2} - \lambda_s \tau_s^{3/2} - \lambda_\Phi \tau_\Phi^{3/2} \tag{3.167}$$

where, on top of the standard LVS big cycle and small cycle, there is an additional blow-up cycle τ_Φ. Non-perturbative corrections to the superpotential are present not only for the small cycle τ_s, but also for the additional blow-up cycle τ_Φ. The model includes also loop corrections that make the potential of τ_Φ (that behaves as the inflaton) flat enough to drive slow-roll inflation under certain conditions[3]. The information we are interested in concerns the value of the overall volume of the model. To match the CMB cosmological data on the amplitude of scalar perturbations, it is necessary to consider a volume of order $\mathcal{V} \sim \mathcal{O}(10^4)$.

In this case we obtain a value for the decay rate (3.166) that is

$$\Gamma_{\phi \to \bar{A}_\mu \bar{A}_\nu} \sim 10^{-14}. \tag{3.168}$$

Non-Perturbative Blow-Up Inflation This case, as stated by the authors in [35], turns out to be a variant of Loop Blow-Up Inflation where Loop effects can be neglected. In fact, the model proposed in [40], features a large volume modulus as in the LVS construction and τ_i blow-ups that appear non-perturbatively in the superpotential of the

[3]More details on how slow-roll inflation can be achieved in this construction are given in [35]. A complete discussion of this topic is beyond the scope of this thesis.

theory[4]. In this case, to match the CMB cosmological data, a volume $\mathcal{V} \sim \mathcal{O}(10^7)$ is needed.

The decay rate (3.166) has now a value of order

$$\Gamma_{\phi\to\bar{A}_\mu\bar{A}_\nu} \sim 10^{-36}. \tag{3.169}$$

We are now going to work in the context of sequestered LVS models where the Standard Model is built on $D3$-branes at singularities. The hierarchy of energy scales in this case is shown in Fig. 3.3 and it corresponds to the one found in Sec. 2.3.

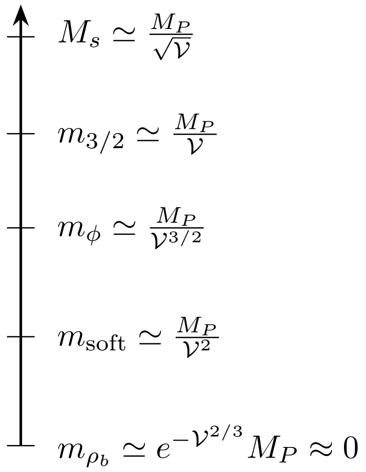

Figure 3.3: *Typical energy scales of LVS models with sequestered Standard Model on $D3$-branes at singularities. M_s is the string scale mass, $m_{3/2}$, m_ϕ and m_{soft} are the gravitino mass, the canonically normalized volume modulus mass and the mass of the soft terms. Finally m_{ρ_b} is the mass of the axion associated to the volume modulus.*

If we consider the Standard Model to be on $D3$-branes at singularities, in both inflationary models described above the latest modulus to decay is always the large volume modulus (corresponding to the canonically normalized field ϕ) [35]. In this case the dominant decay channel into the visible sector is the one into Higgs bosons we have already computed in Eq. (2.112), which is given by [35]

$$\Gamma_{\phi\to H_u H_d} = \frac{Z^2}{24\pi}\frac{m_\phi^3}{M_p^2} \simeq \left(\frac{Z^2 W_0^3}{2\pi(\ln\mathcal{V})^{3/2}}\right)\frac{M_p}{\mathcal{V}^{9/2}}. \tag{3.170}$$

Having this, we can now compute the contribution of RR Dark Radiation to the effective number of neutrino species using the expression (2.90) as follows

$$\Delta N_{eff}^{RR} \simeq \frac{43}{7}\frac{\Gamma_{\phi\to\bar{A}_\mu\bar{A}_\nu}}{\Gamma_{\phi\to H_u H_d}}\left(\frac{10.75}{g_*(T_{rh})}\right)^{1/3} \tag{3.171}$$

[4]A detailed discussion on how slow-roll inflation is realized in this framework can be found in [40].

where $g_*(T_{rh}) = 106.75$ is the number of relativistic species at the reheating temperature and where we have ignored the production of bulk ultralight axions. Considering an arbitrary number of hidden RR Photons $N_{\rm hp}$, the contribution to the effective number of neutrino species becomes

$$\Delta N_{eff}^{RR} \simeq \left(\frac{c}{Z}\right)^2 \frac{0.25}{\mathcal{V}^{8/3}} N_{\rm hp}. \tag{3.172}$$

For Loop Blow-Up Inflation with $\mathcal{V} \gtrsim 10^3$, and for $c \sim Z \sim \mathcal{O}(1)$, we finally obtain

$$\Delta N_{eff}^{RR} \lesssim 2.5 N_{\rm hp} \times 10^{-9}, \tag{3.173}$$

that is an upper bound on the effective number of neutrino species found with the lowest value on the overall volume of the CY. This upper bound becomes much smaller for Non-Perturbative Blow-Up Inflation which requires a larger volume. Even considering a number of RR Dark Photons $N_{\rm hp} \lesssim 400$, we obtain

$$\Delta N_{eff}^{RR} \lesssim 10^{-6} \ll \Delta N_{eff} \sim 0.2 - 0.5 \tag{3.174}$$

which is well below the observational bounds set by [1].

Chapter 4

Conclusions

In this thesis we have studied how moduli driven reheating could be potentially harmful from a phenomenological point of view. In particular, we have seen that, if the longest-living modulus is the one associated to the large volume, it could decay into light hidden sector degrees of freedom that could behave as Dark Radiation. We were interested in studying this Dark Radiation production in the sequestered Large Volume Scenario which, due to the fact that the Standard Model is sitting on $D3$-branes at singularities, produces a hierarchy between the masses that is phenomenologically interesting. In fact, in this case one obtains that the mass of the large volume modulus (the longest-living one) is safe from both the gravitino and the cosmological moduli problem. Moreover the scale of the soft terms is of the order of a few TeV which is desirable to solve the Standard Model hierarchy problem.
The main goal of this thesis was therefore to study the decay of the large volume modulus into degrees of freedom that behave as Dark Radiation and to compare our results with observational bounds. The degrees of freedom of our interest were Ramond-Ramond Dark Photons.

In Chapter 1 and in Chapter 2 we have reviewed the basics of Type IIB String Compactifications and of moduli stabilization. In particular, we have focused our attention on the Large Volume Scenario for moduli stabilization and its sequestered version. Then, we have analyzed the decay of the large volume modulus into axionic Dark Radiation to study another potential source of Dark Radiation and to furnish a guide for the work of the next Chapter.

In Chapter 3 we have obtained the main results of this thesis. We have first introduced Ramond-Ramond Dark Photons and we have found their action from the dimensional reduction of the Type IIB bosonic action (1.1). Most importantly, we have seen that their gauge kinetic function is a holomorphic function of the complex structure moduli (the ones that are not projected out by the orientifold). This implies that, in the presence of a mixing with the complex structure moduli, the volume mode can decay into Ramond-Ramond Photons. To search for a mixing term between the large volume modulus and the complex structure moduli, we have considered a Kähler potential containing only one Kähler modulus and one complex structure modulus with a loop correction term (3.63). Working in the LVS we have canonically normalized the fields finding the scaling with the overall volume of the expected mixing between the volume modulus and the complex structure modulus.

We have then used this mixing term (appearing in the canonical normalization of the complex structure modulus) in the gauge kinetic function of the Ramond-Ramond Dark Photons to find the coupling between the Dark Photons and the volume modulus. In turn, we managed to compute the resulting decay rate.
Finally, we have considered an inflationary string model, Loop Blow-Up Inflation, as a phenomenological background in which we have evaluated the contribution to the effective number of neutrino species due to the decay of the volume modulus into Ramond-Ramond Dark Photons.
What we have found in this scenario is a contribution to the effective number of neutrino species

$$\Delta N_{eff}^{RR} \lesssim 2.5 N_{\mathrm{hp}} \times 10^{-9} \tag{4.1}$$

which is well below the observational bounds $\Delta N_{eff} \sim 0.2 - 0.5$ even if we consider a large amount of hidden sector Dark Photons (N_{hp}).

Some clarifications are now in order. We have seen that the mixing term between the volume modulus and the complex structure modulus comes from a KK loop correction; however, we have only used one cycle, the one associated to the large volume. In our computations we have considered just one Kähler modulus and one complex structure modulus to simplify the canonical normalization but, when considering the sequestered Large Volume Scenario and inserting an inflationary background to compute the effective number of neutrino species, it is well known that at least three different moduli are needed. The question is: how can we claim that our construction is consistent? We are safe with our construction and our analysis on Dark Radiation from Ramond-Ramond $U(1)$s because all the computations we have done before the effective number of neutrino species (and so the canonical normalization of the fields, the gauge kinetic function for the Dark Photons and the decay rate), do not depend on the number of moduli we consider and are independent from the inflationary model, and thus the analysis is quite general.

Regarding further developments of this thesis work, it would be interesting to explore a few different paths about RR Dark Photons:

- The most immediate extension of our work could be an analogous study to the one we have already performed, where the Standard Model is no more on $D3$-branes at singularities, but on $D7$-brane stacks in the so called *geometric regime*. In this case one could expect to have an even lower amount of Dark Radiation contributing to the effective number of neutrino species since the decay of the volume modulus into Standard Model Higgses is enhanced with respect to the one seen in the sequestered case.

- Regarding the phenomenology of bulk Dark Photons it could be interesting to study the case in which these RR Photons become massive, inspecting the possibility of having regions of the moduli space where this mass can be below the string scale entering the EFT, and thus inducing visible effects through the mixing with the Standard Model $U(1)$s.

- It could be interesting to inspect also the case of warped geometries where the mixing between Kähler and complex structure moduli is induced by the warp factor, resulting potentially in a less suppressed production of Dark Radiation, and so to a more harmful contribution to the effective number of neutrino species.

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Appendix A

KKLT Construction

The KKLT scenario is based on the presence of non-perturbative corrections to uplift the flat directions of the Kähler moduli. Focusing for simplicity on the case of a single Kähler modulus we have that the resulting Kähler potential and superpotential are [3, 4, 8]

$$K = -3\ln(T + \overline{T}), \tag{A.1}$$

$$W = W_0 + Ae^{-aT}, \tag{A.2}$$

where we recall that $a = 2\pi$ in the case of non-perturbative corrections generated by Euclidean $D3$-branes and $a = 2\pi/N$ if non-perturbative corrections are generated by gaugino condensation.
Using the expression (2.4) we have that, in this case, the scalar potential reads as

$$V = e^K(K^{T\overline{T}}D_TWD_{\overline{T}}\overline{W} - 3|W|^2). \tag{A.3}$$

To compute it we need the inverse Kähler metric, the first derivatives of the Kähler potential and the covariant derivative of the superpotential; it is easy to show that they are given by

$$K_T = -\frac{3}{(T+\overline{T})} = -\frac{3}{2\tau} = K_{\overline{T}}, \tag{A.4}$$

$$K_{T\overline{T}} = \frac{3}{(T+\overline{T})^2} = \frac{3}{4\tau^2}, \tag{A.5}$$

$$K^{T\overline{T}} = \frac{(T+\overline{T})^2}{3} = \frac{4\tau^2}{3}, \tag{A.6}$$

$$D_TW = \partial_TW + WK_T = -aAe^{-aT} - \frac{3}{(T+\overline{T})}(W_0 + Ae^{-aT}). \tag{A.7}$$

At this point, setting $\mathrm{Im}(T) = 0$ and taking W and A real [23], we can compute the full scalar potential which turns out to be

$$V = \frac{(aA)^2}{6\tau}e^{-2a\tau} + \frac{aA^2}{2\tau^2}e^{-2a\tau} + \frac{aAW_0}{2\tau^2}e^{-a\tau}; \tag{A.8}$$

this passage is justified by the fact that the axion will enter the scalar potential with a cosine and so, minimizing V with respect to it, will lead to an aligned or an anti-aligned phase for the axion which can be reabsorbed making the scalar potential dependent only on τ. The minimum of the resulting scalar potential is a supersymmetric minimum since the F-term for the Kähler modulus is equal to zero; it can be computed by imposing $D_T W = 0$ and therefore, by imposing (A.7) equal to zero. Doing so and inverting the relation we can find

$$W_0 = -Ae^{-a\tau}\left(1 + \frac{2a\tau}{3}\right) \tag{A.9}$$

from which we can see that, to be able to compare the two sides of the expression, the value of W_0 must be fine tuned to be exponentially small. Inverting this relation for τ we get that

$$\tau \sim \frac{1}{a}\ln\left(\frac{1}{|W_0|}\right) \tag{A.10}$$

where we have neglected the subleading terms proportional to $1/\tau$. This expression strengthens the request of a small flux superpotential W_0 to have parametric control of the theory (i.e. to have $R_{CY} \gg 1$) [4]. Using this minimum we have that

$$V|_{min} = -3e^K|W|^2 \tag{A.11}$$

that tells us that the vacuum obtained with this construction is supersymmetric and AdS and therefore it needs to be uplifted.
Using the expression (2.10) we can compute the mass of the gravitino which, in the case of KKLT stabilization, reads as

$$m_{3/2} \simeq \frac{aA}{3(2\tau)^{1/2}}e^{-a\tau}. \tag{A.12}$$

Taking the case $a\tau > 1$ we can recast the previous formula as

$$m_{3/2} \simeq \frac{W_0}{\tau^{3/2}} = \frac{W_0}{\mathcal{V}}. \tag{A.13}$$

Of course everything is in Planck mass units. It has been shown in [41] that the mass of the modulus is proportional to

$$m_\tau \simeq 2a\tau m_{3/2} \tag{A.14}$$

which shows that the mass of the modulus is greater (even if not so much) than the gravitino mass. If $m_\tau > 2m_{3/2}$, then the modulus could decay into the gravitini giving rise to the so called gravitino problem. In this scenario, if the produced gravitini are long-lived, they could decay after the start of BBN spoiling the observational predictions; moreover, their decay could bring to an overproduction of Dark Matter.

Appendix B

QCD Axion and String Theory Axions

In what follows, we are going to briefly introduce the QCD axion as a solution to the well known strong CP problem and then we are going to see how axions arise in Type IIB string compactification.

B.1 QCD Axion

The QCD Lagrangian contains a CP violating term of the form

$$\mathcal{L} = \frac{\bar{\theta}}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{B.1}$$

with $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2$ dual field strength tensor. In principle, this term would not contribute to the field equations because it contains a total derivative. However, due to the presence of the term $\bar{\theta} = \theta + \arg(\det m_q)$ with $\theta \to \theta + 2\pi$ a periodic parameter under shift symmetry and m_q quark mass matrix, we have that (B.1) affects the physics at the quantum level. Experimental bounds on the neutron electric dipole moment, $d_n < 2.9 \cdot 10^{-26} e \cdot cm$, give a value of the parameter $\bar{\theta}$ which can be different from zero, but very small $\bar{\theta} \leq 10^{-10}$. The strong CP problem lies in the fact that this parameter should be extremely fine-tuned to get such a small value since naively, one would expect $\bar{\theta}$ to be of order one between 0 and 2π [8, 42, 43].
A well known solution to this problem is the introduction of the so called Peccei-Quinn axion by promoting θ to a dynamical field (i.e. the axion a) such that the Lagrangian reads as

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{32\pi^2 f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{B.2}$$

where f_a is the axion decay constant. The axion a is invariant under shifts of the shape $a \to a + const.$ and so at the classical level it is a Nambu-Goldstone boson of a broken global symmetry that is the Peccei-Quinn shift symmetry (PQ). Although this symmetry is preserved at perturbative level in the quantum theory, it is broken by QCD instanton effects that generates a periodic potential for the axion [8, 42]

$$V \sim \Lambda^4_{QCD}\left(1 - \cos\left(\frac{a}{f_a}\right)\right). \tag{B.3}$$

where Λ_{QCD} is the QCD energy scale. This potential gets a minimum for $\theta + a/f_a = 0$ which allows to cancel the parameter $\bar{\theta}$ solving the CP problem. As a consequence the axion gets a mass [43]

$$m_a \sim 6 \cdot 10^{-10} eV \left(\frac{10^{16} GeV}{f_a} \right) \tag{B.4}$$

which depends on the axion decay constant. The lower bound on this constant comes from astrophysical constraints and comes from supernovae axion emission $f_a \geq 10^9 GeV$, while the upper bound has cosmological origin and it comes from the request that axions do not overclose the universe $f_a \leq 10^{12} GeV$ [8, 42].

B.2 String Theory Axions

In this section we want to focus on axions arising in Type IIB compactification. We have seen in Subsec. 1.3.3 that after dimensional reduction we obtain a low energy spectrum that contains KK massless modes related to the number of harmonic forms in the CY. These massless modes could be axion-like particles which are the ones we are interested in. Let us notice that these are closed string particles since open string axions live on branes and their presence is model dependent. Of course, under orientifold projection, some of these axion-like particles would be projected out from the EFT.

In Type IIB compactification on CY with $O3/O7$ orientifolds, we have seen that the RR forms C_2 and C_4 can be reduced as

$$C_2 = c^{\tilde{a}}(x)\omega_{\tilde{a}} \quad \text{and} \quad C_4 = \rho_{\tilde{\alpha}}(x)\bar{\omega}^{\tilde{\alpha}} + \dots \tag{B.5}$$

where $\tilde{a} = 1, \dots, h_-^{1,1}$, $\tilde{\alpha} = 1, \dots, h_+^{1,1}$ and $c^{\tilde{a}}(x)$, $\rho_{\tilde{\alpha}}(x)$ are pseudo-scalar axion-like fields. They come from the integration of C_2 and C_4 over 2-cycles and 4-cycles respectively

$$c^{\tilde{a}} = \int_{\Sigma_2} C_2 \quad \text{and} \quad \rho_{\tilde{\alpha}} = \int_{\Sigma_4} C_4. \tag{B.6}$$

Due to the gauge symmetry enjoyed by the C_p forms, these axion-like particles inherit a shift symmetry.

We are not going into details about the discussion on how to get a QCD axion in Type IIB string compactification here, but in [44] it has been argued that $f_a \ll M_P$ to avoid Dark Matter overproduction and to avoid non-QCD corrections spoiling the axion potential. Moreover, in the context of LVS moduli stabilization, it has been shown that a consistent suppression of the axion decay constant can be obtained thanks to the large volume limit. If the QCD axion arises from a brane stack directly wrapping a small blow-up cycle, then we get [44]

$$f_a \propto \frac{1}{\mathcal{V}^{1/2}}. \tag{B.7}$$

More suppression can be obtained if the SM arises in different ways.

Another issue we would like to touch is what happens to the massless axions when we stabilize the other moduli [45]. We know that the shift symmetry could be broken only at non-perturbative level, while in perturbation theory it is preserved. This means that, if the flat directions of the moduli are lifted by non-perturbative corrections as in the KKLT scenario, then also the axions get stabilized by the same effects and develop a mass which is of the same order of the mass of the moduli [46]. However, if moduli are stabilized by perturbative effects they acquire a mass which is greater than the one of the axions which could be uplifted only at subleading non-perturbative order. Considering the LVS scenario with $T_b = \tau_b + i\rho_b$ the big volume modulus and $T_s = \tau_s + i\rho_s$ the small blow-up cycle volume modulus, and performing the same computations as in Sec. 2.2, one finds the expressions (2.59) and (2.58) for the masses of the moduli. However, including in the computations also ρ_s, which appears as the argument of a cosine, and minimizing the scalar potential with respect to it, it is possible to find [46]

$$m_{\rho_s} \sim \frac{M_P}{\mathcal{V}} \tag{B.8}$$

which is exactly of the same order of the mass of the small volume modulus since they are stabilized by the same effect. For what concerns the big-cycle axion, subleading non-perturbative corrections for the big volume modulus of the form $W_{non-pert} = A_b e^{-a_b T_b}$ induce a small mass for the axion ρ_b which is given by [46]

$$m_{\rho_b} \sim \sqrt{|W_0|} M_P e^{-\frac{a_b}{2}\mathcal{V}^{2/3}} \tag{B.9}$$

that is exponentially suppressed at large volume. These very light axions could behave as QCD axions under appropriate conditions. What it is important is that not all of them are proper candidates for Dark Radiation since they could still be eaten up by anomalous $U(1)$ gauge bosons. The D-term potential[1] for each anomalous $U(1)$ is given by [45]

$$V_D \simeq g^2 \left(\sum_\alpha q_\alpha |C_\alpha|^2 - \xi \right)^2 \tag{B.10}$$

where C_α are the open string matter fields charged under $U(1)$ (with charge q_α) already mentioned in Sec. 2.3 and ξ the Fayet-Iliopoulos term given by [45]

$$\xi = -q_i \frac{\partial K}{\partial T_i} M_P^2. \tag{B.11}$$

[1]More about the D-term potential can be found in [32]

Through D-term stabilization a combination of closed[2] and open[3] string axions gets eaten up by the anomalous $U(1)$s that acquire a mass of the form [45]

$$M_{U(1)}^2 \simeq g^2[(f_a^{open})^2 + (f_a^{closed})^2] \tag{B.12}$$

with $(f_a^{open})^2 = |C|^2$ and $(f_a^{closed})^2 = M_P^2 \partial^2 K/(\partial T^2)$. This means that if $f_a^{open} \gg f_a^{closed}$ the axions that are eaten up are mostly the ones from open strings while $f_a^{open} \ll f_a^{closed}$ the eaten up axions are the ones from closed strings.

Taking into account the case of sequestered LVS described in Sec. 2.3 we have that ρ_s is ruled out acquiring a mass of the order of the mass of τ_s while ρ_b has a highly suppressed mass with respect to the one of τ_b. It can be shown that this axion is not eaten up and therefore it could be a product of the decay of the large volume modulus behaving as Dark Radiation. However, being sequestered from the SM sector, it cannot be the QCD axion. On the other hand, still in the case of sequestered LVS, also the open string axion ρ_{SM} is not eaten up by anomalous $U(1)s$ and moreover, due to the fact that its decay constant depend on $|C|^2$, it can be shown that if $|C|$ acquires a non-vanishing VEV, it could behave as a QCD axion (this is not true in the ultra-local limit, but only in the local one where the Yukawa couplings do not depend on $\mathcal{V}$ at leading order) [45].

[2]Those are "bulk" axions arising from the dimensional reduction of the p-forms, like the previous $\rho_{\tilde{\alpha}}, c^{\tilde{\alpha}}$.

[3]Those are axions arising as phase of the complex scalar fields living on the brane like the matter ones.

Bibliography

[1] N. Aghanim *et al.*, "Planck2018 results: Vi. cosmological parameters," *Astronomy amp; Astrophysics*, vol. 641, p. A6, Sep. 2020. [Online]. Available: http://dx.doi.org/10.1051/0004-6361/201833910

[2] M. Cicoli, J. P. Conlon, and F. Quevedo, "Dark radiation in LARGE volume models," *Phys. Rev. D*, vol. 87, no. 4, p. 043520, 2013.

[3] M. Cicoli, J. P. Conlon, A. Maharana, S. Parameswaran, F. Quevedo, and I. Zavala, "String cosmology: From the early universe to today," *Phys. Rept.*, vol. 1059, pp. 1–155, 2024.

[4] A. Hebecker, *Naturalness, String Landscape and Multiverse: A Modern Introduction with Exercises*, 3 2021, vol. 979.

[5] S. B. Giddings, S. Kachru, and J. Polchinski, "Hierarchies from fluxes in string compactifications," *Phys. Rev. D*, vol. 66, p. 106006, 2002.

[6] B. Valeixo Bento, D. Chakraborty, S. Parameswaran, and I. Zavala, "A guide to frames, 2π's, scales and corrections in string compactifications," *Int. J. Mod. Phys. D*, vol. 34, no. 10, p. 2530003, 2025.

[7] D. Baumann and L. McAllister, *Inflation and String Theory*, ser. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 5 2015.

[8] L. E. Ibáñez and A. M. Uranga, *String theory and particle physics: an introduction to string phenomenology.* Cambridge University Press, 2012.

[9] T. Weigand, "Introduction to string theory," Heidelberg University, winter term 2011/12, Institut für Theoretische Physik, Universität Heidelberg. [Online]. Available: https://www.thphys.uni-heidelberg.de/courses/weigand/Strings11-12.pdf

[10] V. Bouchard, "Lectures on complex geometry, Calabi-Yau manifolds and toric geometry," 2 2007.

[11] J. Louis, “Preliminary lecture notes: Introduction to string phenomenology,” Lectures given at the University of Hamburg, winter term 2014/15, Fachbereich Physik der Universität Hamburg, 2015. [Online]. Available: https://www.physik.uni-hamburg.de/en/th2/ag-louis/lecture-notes/ws-14-15.html

[12] H. Jockers and J. Louis, “The Effective action of D7-branes in N = 1 Calabi-Yau orientifolds,” *Nucl. Phys. B*, vol. 705, pp. 167–211, 2005.

[13] T. W. Grimm and J. Louis, “The Effective action of N = 1 Calabi-Yau orientifolds,” *Nucl. Phys. B*, vol. 699, pp. 387–426, 2004.

[14] P. Betzler and E. Plauschinn, “Type IIB flux vacua and tadpole cancellation,” *Fortsch. Phys.*, vol. 67, no. 11, p. 1900065, 2019.

[15] M. Cicoli, J. P. Conlon, and F. Quevedo, “Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications,” *JHEP*, vol. 01, p. 052, 2008.

[16] M. Grana, “Flux compactifications in string theory: A Comprehensive review,” *Phys. Rept.*, vol. 423, pp. 91–158, 2006.

[17] M.-S. Seo, “Bounds on complex structure moduli values for perturbative control,” 4 2025.

[18] J. P. Conlon, F. Quevedo, and K. Suruliz, “Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking,” *JHEP*, vol. 08, p. 007, 2005.

[19] M. Cicoli, C. Mayrhofer, and R. Valandro, “Moduli Stabilisation for Chiral Global Models,” *JHEP*, vol. 02, p. 062, 2012.

[20] S. Gukov, C. Vafa, and E. Witten, “CFT’s from Calabi-Yau four folds,” *Nucl. Phys. B*, vol. 584, pp. 69–108, 2000, [Erratum: Nucl.Phys.B 608, 477–478 (2001)].

[21] E. Cremmer, S. Ferrara, C. Kounnas, and D. V. Nanopoulos, “Naturally Vanishing Cosmological Constant in N=1 Supergravity,” *Phys. Lett. B*, vol. 133, p. 61, 1983.

[22] J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos, and K. Tamvakis, “No-Scale Supersymmetric Standard Model,” *Phys. Lett. B*, vol. 134, p. 429, 1984.

[23] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev. D*, vol. 68, p. 046005, 2003.

[24] K. Becker, M. Becker, M. Haack, and J. Louis, “Supersymmetry breaking and alpha-prime corrections to flux induced potentials,” *JHEP*, vol. 06, p. 060, 2002.

[25] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," *JHEP*, vol. 03, p. 007, 2005.

[26] M. Cicoli, J. P. Conlon, and F. Quevedo, "General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation," *JHEP*, vol. 10, p. 105, 2008.

[27] J. P. Conlon and F. Quevedo, "Astrophysical and cosmological implications of large volume string compactifications," *JCAP*, vol. 08, p. 019, 2007.

[28] M. Cicoli, S. De Alwis, A. Maharana, F. Muia, and F. Quevedo, "De Sitter vs Quintessence in String Theory," *Fortsch. Phys.*, vol. 67, no. 1-2, p. 1800079, 2019.

[29] B. de Carlos, J. Casas, F. Quevedo, and E. Roulet, "Model-independent properties and cosmological implications of the dilaton and moduli sectors of 4d strings," *Physics Letters B*, vol. 318, no. 3, p. 447–456, Dec. 1993. [Online]. Available: http://dx.doi.org/10.1016/0370-2693(93)91538-X

[30] J. P. Conlon, S. S. Abdussalam, F. Quevedo, and K. Suruliz, "Soft SUSY Breaking Terms for Chiral Matter in IIB String Compactifications," *JHEP*, vol. 01, p. 032, 2007.

[31] J. P. Conlon, D. Cremades, and F. Quevedo, "Kahler potentials of chiral matter fields for Calabi-Yau string compactifications," *JHEP*, vol. 01, p. 022, 2007.

[32] L. Aparicio, M. Cicoli, S. Krippendorf, A. Maharana, F. Muia, and F. Quevedo, "Sequestered de Sitter String Scenarios: Soft-terms," *JHEP*, vol. 11, p. 071, 2014.

[33] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo, and R. Valandro, "D-Branes at del Pezzo Singularities: Global Embedding and Moduli Stabilisation," *JHEP*, vol. 09, p. 019, 2012.

[34] M. Cicoli and F. Muia, "General Analysis of Dark Radiation in Sequestered String Models," *JHEP*, vol. 12, p. 152, 2015.

[35] S. Bansal, L. Brunelli, M. Cicoli, A. Hebecker, and R. Kuespert, "Loop blow-up inflation," *JHEP*, vol. 07, p. 289, 2024.

[36] P. G. Camara, L. E. Ibanez, and F. Marchesano, "RR photons," *JHEP*, vol. 09, p. 110, 2011.

[37] M. Demirtas, M. Kim, L. Mcallister, and J. Moritz, "Vacua with Small Flux Superpotential," *Phys. Rev. Lett.*, vol. 124, no. 21, p. 211603, 2020.

[38] S. Hosono, A. Klemm, and S. Theisen, "Lectures on mirror symmetry," *Lect. Notes Phys.*, vol. 436, pp. 235–280, 1994.

[39] M. Cicoli and A. Mazumdar, "Reheating for Closed String Inflation," *JCAP*, vol. 09, p. 025, 2010.

[40] J. P. Conlon and F. Quevedo, "Kahler moduli inflation," *JHEP*, vol. 01, p. 146, 2006.

[41] A. Linde, Y. Mambrini, and K. A. Olive, "Supersymmetry Breaking due to Moduli Stabilization in String Theory," *Phys. Rev. D*, vol. 85, p. 066005, 2012.

[42] J. P. Conlon, "The QCD axion and moduli stabilisation," *JHEP*, vol. 05, p. 078, 2006.

[43] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, and J. March-Russell, "String Axiverse," *Phys. Rev. D*, vol. 81, p. 123530, 2010.

[44] M. Cicoli, A. Hebecker, J. Jaeckel, and M. Wittner, "Axions in string theory — slaying the Hydra of dark radiation," *JHEP*, vol. 09, p. 198, 2022.

[45] R. Allahverdi, M. Cicoli, B. Dutta, and K. Sinha, "Correlation between Dark Matter and Dark Radiation in String Compactifications," *JCAP*, vol. 10, p. 002, 2014.

[46] M. Cicoli, K. Dutta, and A. Maharana, "N-flation with Hierarchically Light Axions in String Compactifications," *JCAP*, vol. 08, p. 012, 2014.