Appendix A

3DoF Trim Results

Main Simulation Settings

```
# UPM.dat simulation settings

ts_size_i = 12              # initial step size
ts_size_f = 12              # final step size
r0_core = 0.8               # initial vortex core radius for wake
                              vortex sheet
roll_up = 1                 # (1:on, 0:off) activate tip
                              vortex roll-up model
r0_core_tip = 0.3           # initial vortex core radius for tip
                              vortex (roll-up model)
roll_up_start = 60.0        # azimuth angle in deg to start
                              applying the roll-up model

# UPM.dat FIRST simulation settings
stedy_toll = 0.005          # tolerance for the steady state
                              simulation

# ENVIRONMENT PARAMETERS
temp_offset = 8.0           # temperature offset [deg]
altitude = 0.0              # altitude [m]

# GEOMETRY PARAMETERS
Vtip = 120                  # tip velocity  [m/s]
CHORD = 0.168               # reference length
phase = [0 0]               # phase vector

# TRIM PARAMETERS
tolltrim_perc = 0.001       # percentage of target Thrust
```

Listing A.1: Trim script main settings parameters used for the simulations.

Hover Condition [1]
3DoF ISA SL OGE χ = [0 ... 0]

4 ROTORS

- **Control Variables:**

$$\theta_0^1 = 18.372°, \quad \theta_0^2 = 18.372°, \quad \theta_0^3 = 18.372°, \quad \theta_0^4 = 18.372°, \quad \Theta = -3.4505 \times 10^{-13} \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_z\ [N] \\ M_y\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0 \\ 2.9430 \times 10^3 \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{c} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} \\ \begin{bmatrix} -6.4768 \times 10^{-1} & 1.1388 \times 10^{-1} & 6.4768 \times 10^{-1} & -1.1388 \times 10^{-1} \\ 1.1301 & 8.7466 \times 10^{-1} & -1.1301 & -8.7466 \times 10^{-1} \\ 7.5042 \times 10^2 & 7.2090 \times 10^2 & 7.5042 \times 10^2 & 7.2090 \times 10^2 \end{bmatrix} \end{array} \begin{array}{c} F_x \\ F_y \\ F_z \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{c} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} \\ \begin{bmatrix} -1.2437 \times 10^3 & -1.1958 \times 10^3 & 1.2437 \times 10^3 & 1.1958 \times 10^3 \\ -1.2281 \times 10^3 & 1.2281 \times 10^3 & 1.1889 \times 10^3 & -1.1889 \times 10^3 \\ -7.5076 \times 10^1 & 7.4955 \times 10^1 & -7.5076 \times 10^1 & 7.4955 \times 10^1 \end{bmatrix} \end{array} \begin{array}{c} M_x \\ M_y \\ M_z \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 1.4761 \times 10^{-10} \\ -3.7900 \times 10^{-10} \\ 2.9426 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 2.3867 \times 10^{-8} \\ 8.2757 \times 10^{-9} \\ -2.4316 \times 10^{-1} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 1.4761 \times 10^{-10} \\ 3.5080 \times 10^{-1} \\ 8.2757 \times 10^{-9} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 1.1693 \times 10^{-3}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 4.1605 \times 10^{-4}$$

[1]Rotor forces and moments are reported in the trim frame (SubCh. 4.2). Consequently, the latter includes both the pure rotor moment contribution and the moment resulting from force displacement with respect to the centre of gravity.

6 ROTORS

- **Control Variables:**

$$\theta_0^1 = 18.497^\circ, \quad \theta_0^2 = 18.421^\circ, \quad \theta_0^3 = 18.345^\circ,$$
$$\theta_0^4 = 18.345^\circ, \quad \theta_0^5 = 18.421^\circ, \quad \theta_0^6 = 18.497^\circ, \quad \Theta = 1.1280 \times 10^{-2} \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_z\ [N] \\ M_y\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0 \\ 4.4145 \times 10^{3} \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{c} \begin{array}{cccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} \end{array} \\ \begin{bmatrix} -1.0608 & -9.0865 \times 10^{-2} & 4.6647 \times 10^{-1} & 6.9992 \times 10^{-1} & -1.6683 \times 10^{-1} & -6.0447 \times 10^{-1} \\ 7.2098 \times 10^{-1} & 9.0570 \times 10^{-1} & 4.3488 \times 10^{-1} & -1.2872 \times 10^{-1} & -7.0358 \times 10^{-1} & -7.3963 \times 10^{-1} \\ 7.2758 \times 10^{2} & 7.4910 \times 10^{2} & 7.3069 \times 10^{2} & 7.3133 \times 10^{2} & 7.3850 \times 10^{2} & 7.3486 \times 10^{2} \end{bmatrix} \end{array} \begin{array}{c} F_x \\ F_y \\ F_z \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{c} \begin{array}{cccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} \end{array} \\ \begin{bmatrix} -1.2818 \times 10^{3} & -2.6213 \times 10^{3} & -1.2778 \times 10^{3} & 1.2771 \times 10^{3} & 2.5856 \times 10^{3} & 1.2931 \times 10^{3} \\ -2.2034 \times 10^{3} & -7.2414 & 2.2216 \times 10^{3} & 2.2232 \times 10^{3} & -9.4378 & -2.2241 \times 10^{3} \\ -7.6825 \times 10^{1} & 7.5984 \times 10^{1} & -7.7244 \times 10^{1} & 7.5784 \times 10^{1} & -7.5791 \times 10^{1} & 7.6275 \times 10^{1} \end{bmatrix} \end{array} \begin{array}{c} M_x \\ M_y \\ M_z \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -7.5656 \times 10^{-1} \\ 4.8963 \times 10^{-1} \\ 4.4121 \times 10^{3} \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} -2.5150 \times 10^{1} \\ 5.9808 \times 10^{-1} \\ 1.8164 \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 7.5656 \times 10^{-1} \\ 2.4356 \\ 5.9808 \times 10^{-1} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 5.7710 \times 10^{-3}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 2.0813 \times 10^{-2}$$

8 ROTORS

- **Control Variables:**

$$\theta_0^1 = 18.553°, \quad \theta_0^2 = 18.553°, \quad \theta_0^3 = 18.553°, \quad \theta_0^4 = 18.553°,$$

$$\theta_0^5 = 18.553°, \quad \theta_0^6 = 18.553°, \quad \theta_0^7 = 18.553°, \quad \theta_0^8 = 18.553°, \quad \Theta = 3.4513 \times 10^{-11} \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_z\ [N] \\ M_y\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0 \\ 5.8860 \times 10^3 \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{c} \begin{array}{cccccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} & \textit{Rotor 7} & \textit{Rotor 8} \end{array} \\ \begin{bmatrix} -1.5483 \times 10^{-1} & 1.4246 \times 10^{-1} & -1.3211 \times 10^{-1} & 1.3416 \times 10^{-1} & 1.5483 \times 10^{-1} & -1.4246 \times 10^{-1} & 1.3211 \times 10^{-1} & -1.3416 \times 10^{-1} \\ 5.8334 \times 10^{-2} & 8.4373 \times 10^{-1} & 7.7813 \times 10^{-1} & -7.7618 \times 10^{-3} & -5.8334 \times 10^{-2} & -8.4373 \times 10^{-1} & -7.7813 \times 10^{-1} & 7.7618 \times 10^{-3} \\ 7.3888 \times 10^{2} & 7.3249 \times 10^{2} & 7.3550 \times 10^{2} & 7.3881 \times 10^{2} & 7.3888 \times 10^{2} & 7.3249 \times 10^{2} & 7.3550 \times 10^{2} & 7.3881 \times 10^{2} \end{bmatrix} \end{array} \begin{array}{c} F_x \\ F_y \\ F_z \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{c} \begin{array}{cccccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} & \textit{Rotor 7} & \textit{Rotor 8} \end{array} \\ \begin{bmatrix} -1.3213 \times 10^{3} & -3.1567 \times 10^{3} & -3.1695 \times 10^{3} & -1.3211 \times 10^{3} & 1.3213 \times 10^{3} & 3.1567 \times 10^{3} & 3.1695 \times 10^{3} & 1.3211 \times 10^{3} \\ -3.1852 \times 10^{3} & -1.3111 \times 10^{3} & 1.3161 \times 10^{3} & 3.1853 \times 10^{3} & 3.1852 \times 10^{3} & 1.3111 \times 10^{3} & -1.3161 \times 10^{3} & -3.1853 \times 10^{3} \\ -7.7640 \times 10^{1} & 7.8720 \times 10^{1} & -7.8632 \times 10^{1} & 7.7829 \times 10^{1} & -7.7640 \times 10^{1} & 7.8720 \times 10^{1} & -7.8632 \times 10^{1} & 7.7829 \times 10^{1} \end{bmatrix} \end{array} \begin{array}{c} M_x \\ M_y \\ M_z \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 3.4922 \times 10^{-9} \\ -5.2700 \times 10^{-11} \\ 5.8914 \times 10^{3} \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 2.6231 \times 10^{-8} \\ 7.6029 \times 10^{-8} \\ 5.5510 \times 10^{-1} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 3.4922 \times 10^{-9} \\ 5.3691 \\ 7.6029 \times 10^{-8} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 8.9484 \times 10^{-3}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 1.2515 \times 10^{-4}$$

Level Forward Flight Condition
TAS = 28.33 m/s 3DoF ISA +150m χ = [0 ... 0]

4 ROTORS

- **Control Variables:**

$$\theta_0^1 = 16.011°, \quad \theta_0^2 = 17.486°, \quad \theta_0^3 = 17.486°, \quad \theta_0^4 = 16.011°, \quad \Theta = -9.2940 \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x \; [N] \\ F_z \; [N] \\ M_y \; [N \cdot m] \end{bmatrix} = \begin{bmatrix} 3.5072 \times 10^2 \\ 2.9430 \times 10^3 \\ -1.7536 \times 10^2 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F} \; [N] = \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \\ \left[\begin{array}{c} 8.8791 \times 10^1 \\ -1.1033 \times 10^1 \\ 7.2556 \times 10^2 \end{array}\right. & \begin{array}{c} 8.6759 \times 10^1 \\ 1.5501 \times 10^1 \\ 7.4717 \times 10^2 \end{array} & \begin{array}{c} 8.6757 \times 10^1 \\ -1.5494 \times 10^1 \\ 7.4721 \times 10^2 \end{array} & \left.\begin{array}{c} 8.8798 \times 10^1 \\ 1.1036 \times 10^1 \\ 7.2558 \times 10^2 \end{array}\right] \end{array} \quad \begin{array}{c} F_x \\ F_y \\ F_z \end{array}$$

- **Moments:**

$$\mathbf{M} \; [N \cdot m] = \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \\ \left[\begin{array}{c} -1.3712 \times 10^3 \\ -1.2662 \times 10^3 \\ 9.5340 \times 10^1 \end{array}\right. & \begin{array}{c} -1.0416 \times 10^3 \\ 1.1783 \times 10^3 \\ 1.6082 \times 10^2 \end{array} & \begin{array}{c} 1.0416 \times 10^3 \\ 1.1784 \times 10^3 \\ -1.6083 \times 10^2 \end{array} & \left.\begin{array}{c} 1.3712 \times 10^3 \\ -1.2662 \times 10^3 \\ -9.5347 \times 10^1 \end{array}\right] \end{array} \quad \begin{array}{c} M_x \\ M_y \\ M_z \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}} \; [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 3.5111 \times 10^2 \\ 9.9654 \times 10^{-3} \\ 2.9455 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}} \; [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 8.1471 \times 10^{-2} \\ -1.7564 \times 10^2 \\ -1.6621 \times 10^{-2} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 3.8114 \times 10^{-1} \\ 2.5172 \\ 2.7843 \times 10^{-1} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}} \; [m/s^2] = 8.4864 \times 10^{-3}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}} \; [rad/s^2] = 6.2303 \times 10^{-4}$$

6 ROTORS

- **Control Variables:**

$$\theta_0^1 = 17.022^\circ, \quad \theta_0^2 = 17.377^\circ, \quad \theta_0^3 = 17.731^\circ,$$
$$\theta_0^4 = 17.731^\circ, \quad \theta_0^5 = 17.377^\circ, \quad \theta_0^6 = 17.022^\circ, \quad \Theta = -12.128 \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x \ [N] \\ F_z \ [N] \\ M_y \ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 7.3540 \times 10^2 \\ 4.4145 \times 10^3 \\ -3.6770 \times 10^2 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{c} \begin{array}{cccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} \end{array} \\ \begin{bmatrix} 1.2187 \times 10^2 & 1.2568 \times 10^2 & 1.2034 \times 10^2 & 1.2033 \times 10^2 & 1.2570 \times 10^2 & 1.2186 \times 10^2 \\ -1.3237 \times 10^1 & 1.3600 \times 10^1 & -1.4405 \times 10^1 & 1.4375 \times 10^1 & -1.3623 \times 10^1 & 1.3217 \times 10^1 \\ 7.2594 \times 10^2 & 7.4903 \times 10^2 & 7.3324 \times 10^2 & 7.3323 \times 10^2 & 7.4905 \times 10^2 & 7.2595 \times 10^2 \end{bmatrix} \end{array} \begin{array}{c} \\ F_x \\ F_y \\ F_z \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{c} \begin{array}{cccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} \end{array} \\ \begin{bmatrix} -1.4603 \times 10^3 & -2.4439 \times 10^3 & -1.4488 \times 10^3 & 1.4488 \times 10^3 & 2.4441 \times 10^3 & 1.4603 \times 10^3 \\ -2.2923 \times 10^3 & -7.2611 \times 10^1 & 2.1811 \times 10^3 & 2.1813 \times 10^3 & -7.2745 \times 10^1 & -2.2922 \times 10^3 \\ 1.4146 \times 10^2 & 4.7680 \times 10^2 & 2.1210 \times 10^2 & -2.1199 \times 10^2 & -4.7688 \times 10^2 & -1.4151 \times 10^2 \end{bmatrix} \end{array} \begin{array}{c} \\ M_x \\ M_y \\ M_z \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 7.3578 \times 10^2 \\ -7.3269 \times 10^{-2} \\ 4.4135 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 1.7673 \times 10^{-1} \\ -3.6741 \times 10^2 \\ -2.0338 \times 10^{-2} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 3.7583 \times 10^{-1} \\ 1.9298 \\ 2.9219 \times 10^{-1} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 4.3720 \times 10^{-3}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 2.8250 \times 10^{-4}$$

8 ROTORS

- **Control Variables:**

$$\theta_0^1 = 18.811^\circ, \quad \theta_0^2 = 18.811^\circ, \quad \theta_0^3 = 18.986^\circ, \quad \theta_0^4 = 18.986^\circ,$$

$$\theta_0^5 = 18.986^\circ, \quad \theta_0^6 = 18.986^\circ, \quad \theta_0^7 = 18.811^\circ, \quad \theta_0^8 = 18.811^\circ, \quad \Theta = -16.352 \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x \ [N] \\ F_z \ [N] \\ M_y \ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 1.3841 \times 10^3 \\ 5.8860 \times 10^3 \\ -6.9207 \times 10^2 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{cc} \begin{array}{cccccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} & \textit{Rotor 7} & \textit{Rotor 8} \end{array} & \\ \begin{bmatrix} 1.7541 \times 10^2 & 1.7841 \times 10^2 & 1.6250 \times 10^2 & 1.7607 \times 10^2 & 1.7589 \times 10^2 & 1.6234 \times 10^2 & 1.7828 \times 10^2 & 1.7540 \times 10^2 \\ -1.6676 \times 10^1 & 1.7286 \times 10^1 & -1.7419 \times 10^1 & 1.6989 \times 10^1 & -1.6856 \times 10^1 & 1.7530 \times 10^1 & -1.7225 \times 10^1 & 1.6736 \times 10^1 \\ 7.4243 \times 10^2 & 7.5032 \times 10^2 & 7.0441 \times 10^2 & 7.4645 \times 10^2 & 7.4610 \times 10^2 & 7.0377 \times 10^2 & 7.5002 \times 10^2 & 7.4231 \times 10^2 \end{bmatrix} & \begin{array}{c} F_x \\ F_y \\ F_z \end{array} \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{cc} \begin{array}{cccccccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} & \textit{Rotor 5} & \textit{Rotor 6} & \textit{Rotor 7} & \textit{Rotor 8} \end{array} & \\ \begin{bmatrix} -1.5462 \times 10^3 & -3.0261 \times 10^3 & -3.2321 \times 10^3 & -1.1488 \times 10^3 & 1.1479 \times 10^3 & 3.2291 \times 10^3 & 3.0248 \times 10^3 & 1.5460 \times 10^3 \\ -3.3531 \times 10^3 & -1.4509 \times 10^3 & 1.2281 \times 10^3 & 3.2310 \times 10^3 & 3.2300 \times 10^3 & 1.2264 \times 10^3 & -1.4500 \times 10^3 & -3.3529 \times 10^3 \\ 2.1188 \times 10^2 & 8.3128 \times 10^2 & 6.9858 \times 10^2 & 2.7678 \times 10^2 & -2.7696 \times 10^2 & -6.9805 \times 10^2 & -8.3059 \times 10^2 & -2.1159 \times 10^2 \end{bmatrix} & \begin{array}{c} M_x \\ M_y \\ M_z \end{array} \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 1.3843 \times 10^3 \\ 3.6540 \times 10^{-1} \\ 5.8858 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} -5.4467 \\ -6.9146 \times 10^2 \\ 1.3206 \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 1.5649 \times 10^{-1} \\ 2.0138 \times 10^{-1} \\ 6.1452 \times 10^{-1} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 7.4266 \times 10^{-4}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 2.1955 \times 10^{-3}$$

Appendix B

Phase Shift Effect on Trim Results

Main Simulation Settings

```
# UPM.dat simulation settings

ts_size_i = 12            # initial step size
ts_size_f = 12            # final step size
r0_core = 0.8             # initial vortex core radius for
                            wake vortex sheet
roll_up = 1               # (1:on, 0:off) activate tip
                            vortex roll-up model
r0_core_tip = 0.3         # initial vortex core radius for
                            tip vortex (roll-up model)
roll_up_start = 60.0      # azimuth angle in deg to start
                            applying the roll-up model

# UPM.dat FIRST simulation settings
stedy_toll = 0.005        # tolerance for the steady state
                            simulation

# ENVIRONMENT PARAMETERS

temp_offset = 8.0         # temperature offset [deg]

# GEOMETRY PARAMETERS

Nrotor = 4                # number of rotors
Vtip = 120                # [m/s] tip velocity

# TRIM PARAMETERS

tolltrim_perc = 0.005     # percentage of target Thrust
```

Listing B.1: Trim script main settings parameters used for the simulations.

Hover Condition (ISA SL OGE) [1]

PHASE SHIFT VECTOR χ = [0 -45 0 -45]

3DoF

- **Control Variables:**

$$\theta_0^1 = 18.527°, \quad \theta_0^2 = 18.527°, \quad \theta_0^3 = 18.527°, \quad \theta_0^4 = 18.527°, \quad \Theta = -6.1518 \times 10^{-14} \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_z\ [N] \\ M_y\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0 \\ 2.9430 \times 10^3 \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{cc} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} & \\ \begin{bmatrix} -7.8972 \times 10^{-1} & 8.5974 \times 10^{-1} & 7.8972 \times 10^{-1} & -8.5974 \times 10^{-1} \\ 5.5124 \times 10^{-1} & 6.3153 \times 10^{-1} & -5.5124 \times 10^{-1} & -6.3153 \times 10^{-1} \\ 7.3865 \times 10^2 & 7.3728 \times 10^2 & 7.3865 \times 10^2 & 7.3728 \times 10^2 \end{bmatrix} & \begin{array}{c} F_x \\ F_y \\ F_z \end{array} \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{cc} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} & \\ \begin{bmatrix} -1.2264 \times 10^3 & -1.2257 \times 10^3 & 1.2264 \times 10^3 & 1.2257 \times 10^3 \\ -1.2156 \times 10^3 & 1.2145 \times 10^3 & 1.2156 \times 10^3 & -1.2145 \times 10^3 \\ -7.8248 \times 10^1 & 7.8285 \times 10^1 & -7.8248 \times 10^1 & 7.8285 \times 10^1 \end{bmatrix} & \begin{array}{c} M_x \\ M_y \\ M_z \end{array} \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -6.5028 \times 10^{-12} \\ 3.3196 \times 10^{-14} \\ 2.9518 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 2.2737 \times 10^{-13} \\ -1.5916 \times 10^{-12} \\ 7.3566 \times 10^{-2} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 6.5028 \times 10^{-12} \\ 8.8481 \\ 1.5916 \times 10^{-12} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 2.9494 \times 10^{-2}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 1.2587 \times 10^{-4}$$

[1] Rotor forces and moments are reported in the trim frame (SubCh. 4.2). Consequently, the latter includes both the pure rotor moment contribution and the moment resulting from force displacement with respect to the centre of gravity.

6DoF

- **Control Variables:**

$$\theta_0^1 = 18.468°, \quad \theta_0^2 = 18.532°, \quad \theta_0^3 = 18.468°, \quad \theta_0^4 = 18.532°,$$

$$\Theta = 5.8167 \times 10^{-14} \text{ deg}, \quad \Phi = -4.0738 \times 10^{-15} \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_y\ [N] \\ F_z\ [N] \\ M_x\ [N \cdot m] \\ M_y\ [N \cdot m] \\ M_z\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2.9430 \times 10^3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{c} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} \\ \begin{bmatrix} -7.8299 \times 10^{-1} & 8.5862 \times 10^{-1} & 7.8299 \times 10^{-1} & -8.5862 \times 10^{-1} \\ 5.4999 \times 10^{-1} & 6.3443 \times 10^{-1} & -5.4999 \times 10^{-1} & -6.3443 \times 10^{-1} \\ 7.3498 \times 10^2 & 7.3812 \times 10^2 & 7.3498 \times 10^2 & 7.3812 \times 10^2 \end{bmatrix} \end{array} \begin{array}{c} \\ F_x \\ F_y \\ F_z \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{c} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} \\ \begin{bmatrix} -1.2203 \times 10^3 & -1.2271 \times 10^3 & 1.2203 \times 10^3 & 1.2271 \times 10^3 \\ -1.2095 \times 10^3 & 1.2160 \times 10^3 & 1.2095 \times 10^3 & -1.2160 \times 10^3 \\ -7.7681 \times 10^1 & 7.8325 \times 10^1 & -7.7681 \times 10^1 & 7.8325 \times 10^1 \end{bmatrix} \end{array} \begin{array}{c} \\ M_x \\ M_y \\ M_z \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -3.4245 \times 10^{-13} \\ 2.0948 \times 10^{-13} \\ 2.9462 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} -2.2737 \times 10^{-13} \\ -1.1369 \times 10^{-12} \\ 1.2877 \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 3.4245 \times 10^{-13} \\ 2.0948 \times 10^{-13} \\ 3.2057 \\ 2.2737 \times 10^{-13} \\ 1.1369 \times 10^{-12} \\ 1.2877 \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 1.0686 \times 10^{-2}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 2.2033 \times 10^{-3}$$

PHASE SHIFT VECTOR χ = [0 90 0 90]

3DoF

- **Control Variables:**

$$\theta_0^1 = 18.491^\circ, \quad \theta_0^2 = 18.491^\circ, \quad \theta_0^3 = 18.491^\circ, \quad \theta_0^4 = 18.491^\circ, \quad \Theta = -1.0021 \times 10^{-14} \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_z\ [N] \\ M_y\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0.000\,00 \\ 2.9430 \times 10^3 \\ 0.000\,00 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{c} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} \\ \begin{bmatrix} -7.2552 \times 10^{-1} & 6.7525 \times 10^{-1} & 7.2552 \times 10^{-1} & -6.7525 \times 10^{-1} \\ 6.7202 \times 10^{-1} & 8.5298 \times 10^{-1} & -6.7202 \times 10^{-1} & -8.5298 \times 10^{-1} \\ 7.3136 \times 10^2 & 7.3568 \times 10^2 & 7.3136 \times 10^2 & 7.3568 \times 10^2 \end{bmatrix} \end{array} \begin{array}{c} F_x \\ F_y \\ F_z \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{c} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} \\ \begin{bmatrix} -1.2155 \times 10^3 & -1.2227 \times 10^3 & 1.2155 \times 10^3 & 1.2227 \times 10^3 \\ -1.2034 \times 10^3 & 1.2110 \times 10^3 & 1.2034 \times 10^3 & -1.2110 \times 10^3 \\ -7.7551 \times 10^1 & 7.6876 \times 10^1 & -7.7551 \times 10^1 & 7.6876 \times 10^1 \end{bmatrix} \end{array} \begin{array}{c} M_x \\ M_y \\ M_z \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -5.1315 \times 10^{-13} \\ 1.6664 \times 10^{-13} \\ 2.9341 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 2.2737 \times 10^{-13} \\ 6.5938 \times 10^{-12} \\ -1.3498 \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 5.1315 \times 10^{-13} \\ 8.9300 \\ 6.5938 \times 10^{-12} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 2.9767 \times 10^{-2}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 2.3095 \times 10^{-3}$$

6DoF

- **Control Variables:**

$$\theta_0^1 = 18.457°, \quad \theta_0^2 = 18.563°, \quad \theta_0^3 = 18.457°, \quad \theta_0^4 = 18.563°,$$

$$\Theta = -1.7416 \times 10^{-15} \text{ deg}, \quad \Phi = 1.4683 \times 10^{-14} \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_y\ [N] \\ F_z\ [N] \\ M_x\ [N \cdot m] \\ M_y\ [N \cdot m] \\ M_z\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2.9430 \times 10^3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{cc} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} & \\ \begin{bmatrix} -8.7624 \times 10^{-1} & 7.0115 \times 10^{-1} & 8.7624 \times 10^{-1} & -7.0115 \times 10^{-1} \\ 4.2823 \times 10^{-1} & 5.8978 \times 10^{-1} & -4.2823 \times 10^{-1} & -5.8978 \times 10^{-1} \\ 7.3530 \times 10^2 & 7.4155 \times 10^2 & 7.3530 \times 10^2 & 7.4155 \times 10^2 \end{bmatrix} & \begin{array}{c} F_x \\ F_y \\ F_z \end{array} \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{cc} \begin{array}{cccc} \textit{Rotor 1} & \textit{Rotor 2} & \textit{Rotor 3} & \textit{Rotor 4} \end{array} & \\ \begin{bmatrix} -1.2207 \times 10^3 & -1.2327 \times 10^3 & 1.2207 \times 10^3 & 1.2327 \times 10^3 \\ -1.2102 \times 10^3 & 1.2219 \times 10^3 & 1.2102 \times 10^3 & -1.2219 \times 10^3 \\ -7.7887 \times 10^1 & 7.8233 \times 10^1 & -7.7887 \times 10^1 & 7.8233 \times 10^1 \end{bmatrix} & \begin{array}{c} M_x \\ M_y \\ M_z \end{array} \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -8.9781 \times 10^{-14} \\ -7.5704 \times 10^{-13} \\ 2.9537 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} -3.3629 \times 10^{-10} \\ 0 \\ 6.9031 \times 10^{-1} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 8.9781 \times 10^{-14} \\ 7.5704 \times 10^{-13} \\ 1.0706 \times 10^1 \\ 3.3629 \times 10^{-10} \\ 0 \\ 6.9031 \times 10^{-1} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 3.5688 \times 10^{-2}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 1.1811 \times 10^{-3}$$

Appendix C

NOISE-Chain Results

NR4 : Four Rotors Configuration

Caption	Zone (phi)	Hemisphere (χ)	Ground (χ)
(a)	[-20, 20]	[0, -45, -85, 7]	[0, -14, -86, 57]
(b)	[20, 60]	[0, 55, 57, 33]	[0, -37, 81, 11]
(c)	[60, 100]	[0, -30, 30, 1]	[0, 67, 48, -75]
(d)	[100, 140]	[0, -27, 28, 19]	[0, 66, 42, -45]
(e)	[140, 180]	[0, -46, 62, 61]	[0, 24, 62, -65]
(f)	[180, 220]	[0, 5, 72, -60]	[0, -38, -87, 75]
(g)	[220, 260]	[0, 10, -45, -17]	[0, 82, -66, 41]
(h)	[260, 300]	[0, 28, -32, -1]	[0, -63, -38, 61]
(i)	[300, 340]	[0, -29, -28, -85]	[0, 68, -48, -12]

Table C.1: NR4 phase (χ) optimization results.

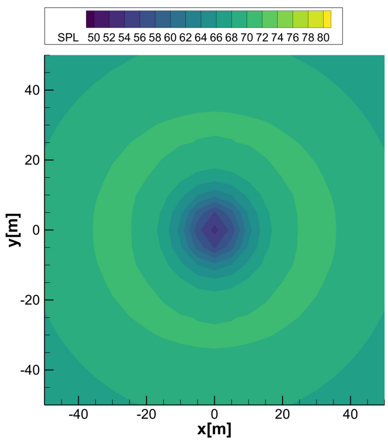


Figure C.1: Unsynchronized noise emission for the 4 rotor configuration.

Hemisphere Optimization

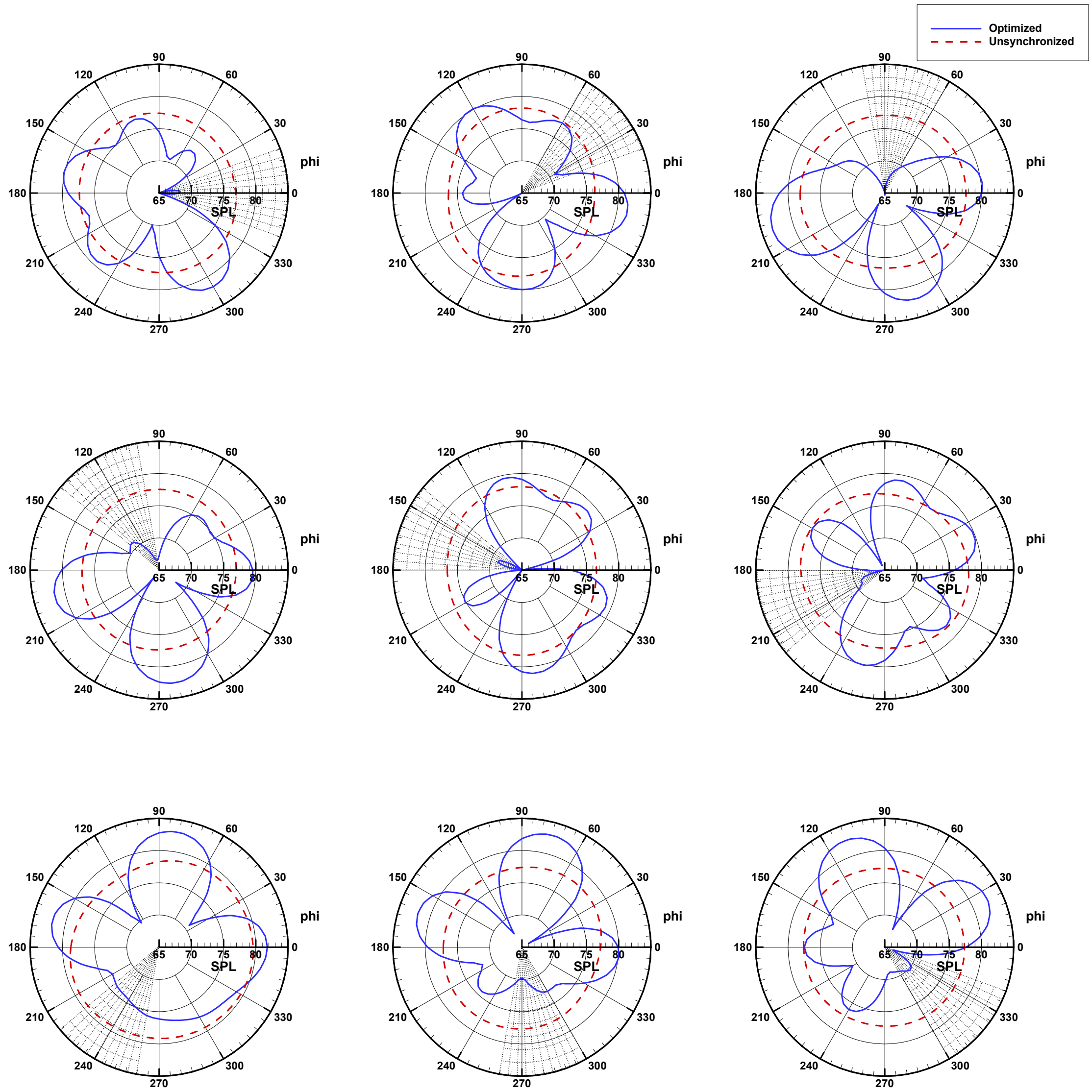


Figure C.2: NR4 Hemisphere SPL [dB] at -45 deg elevation.

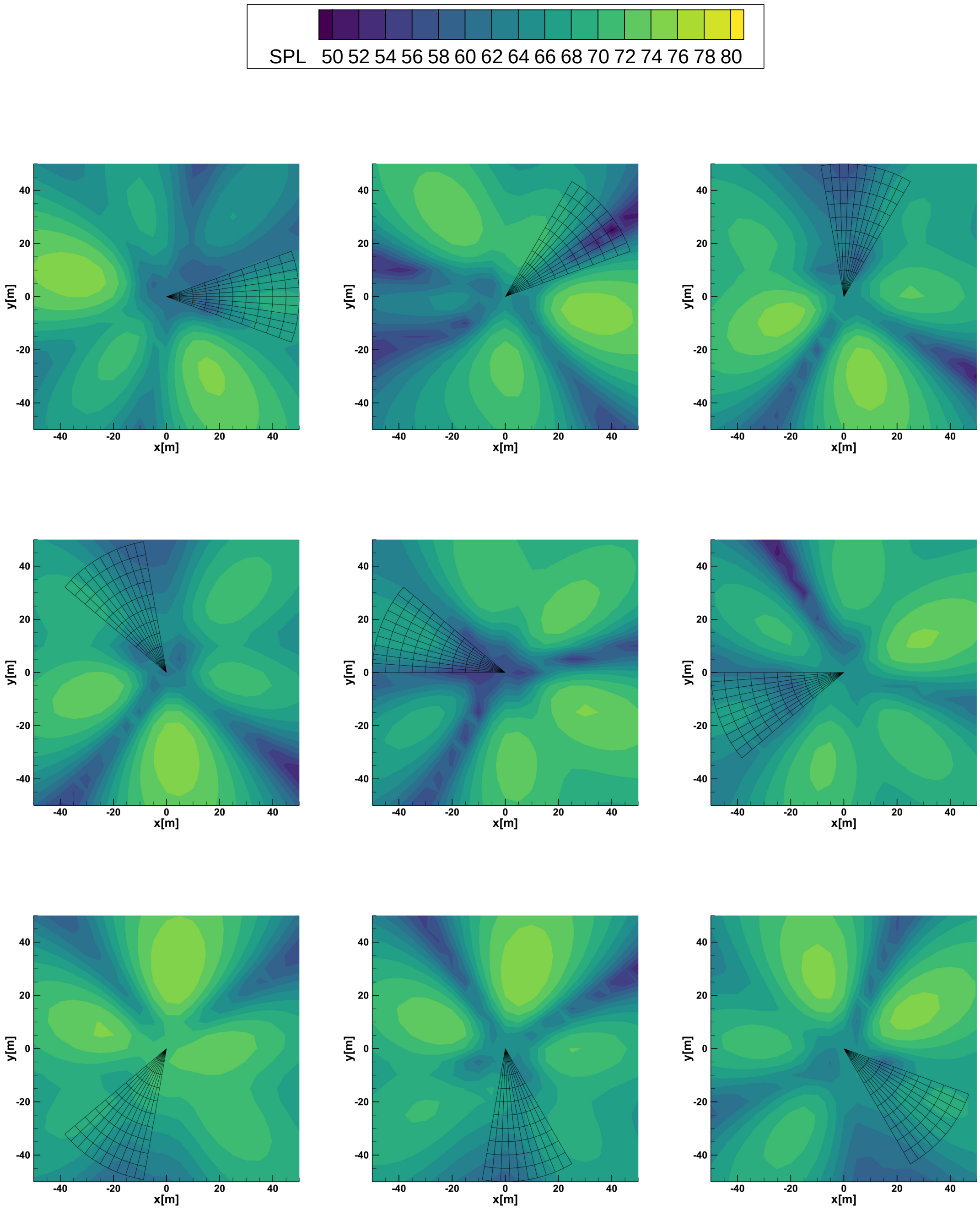


Figure C.3: NR4 Ground SPL [dB] 25m below the configuration.

Ground Optimization

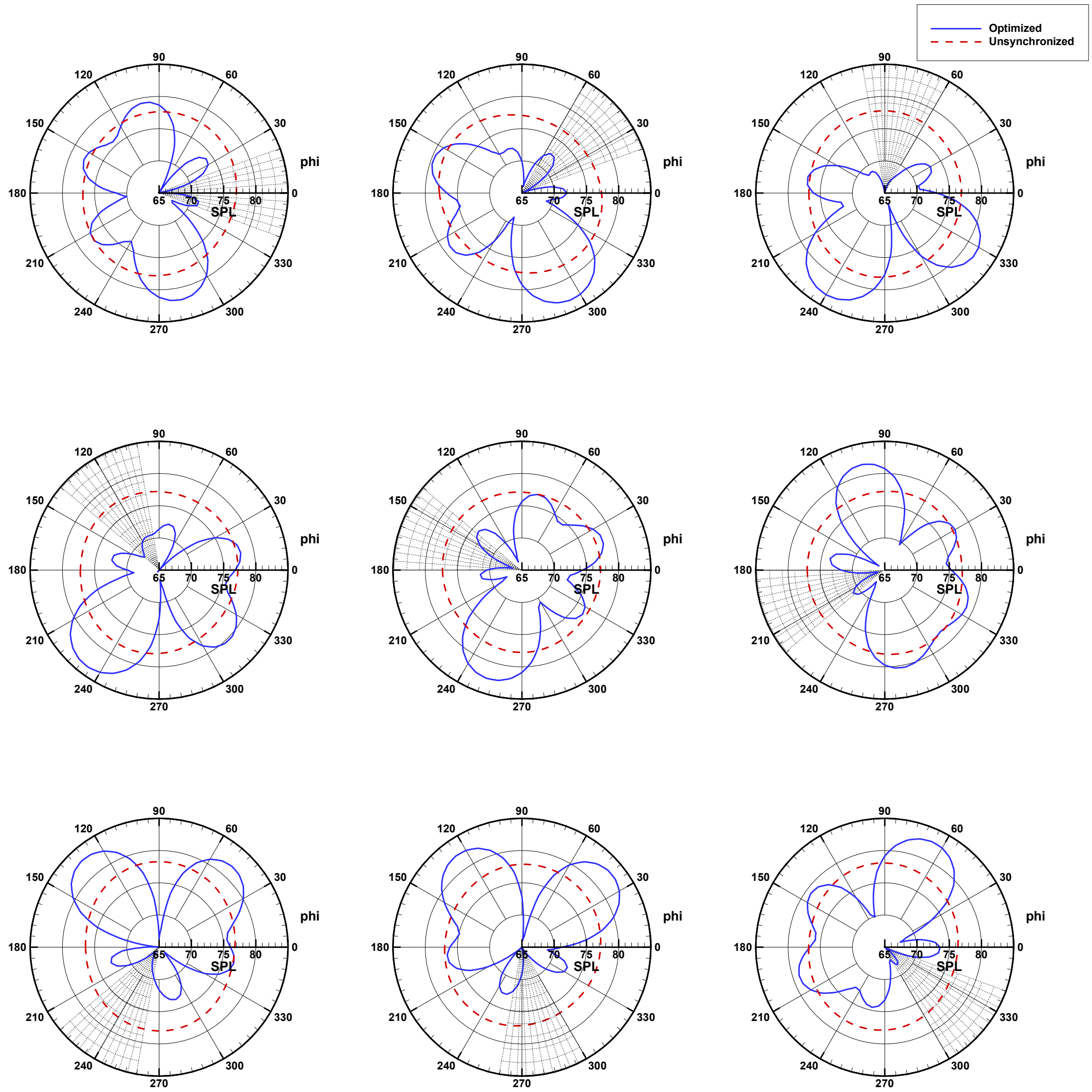


Figure C.4: NR4 Hemisphere SPL [dB] at -45 deg elevation.

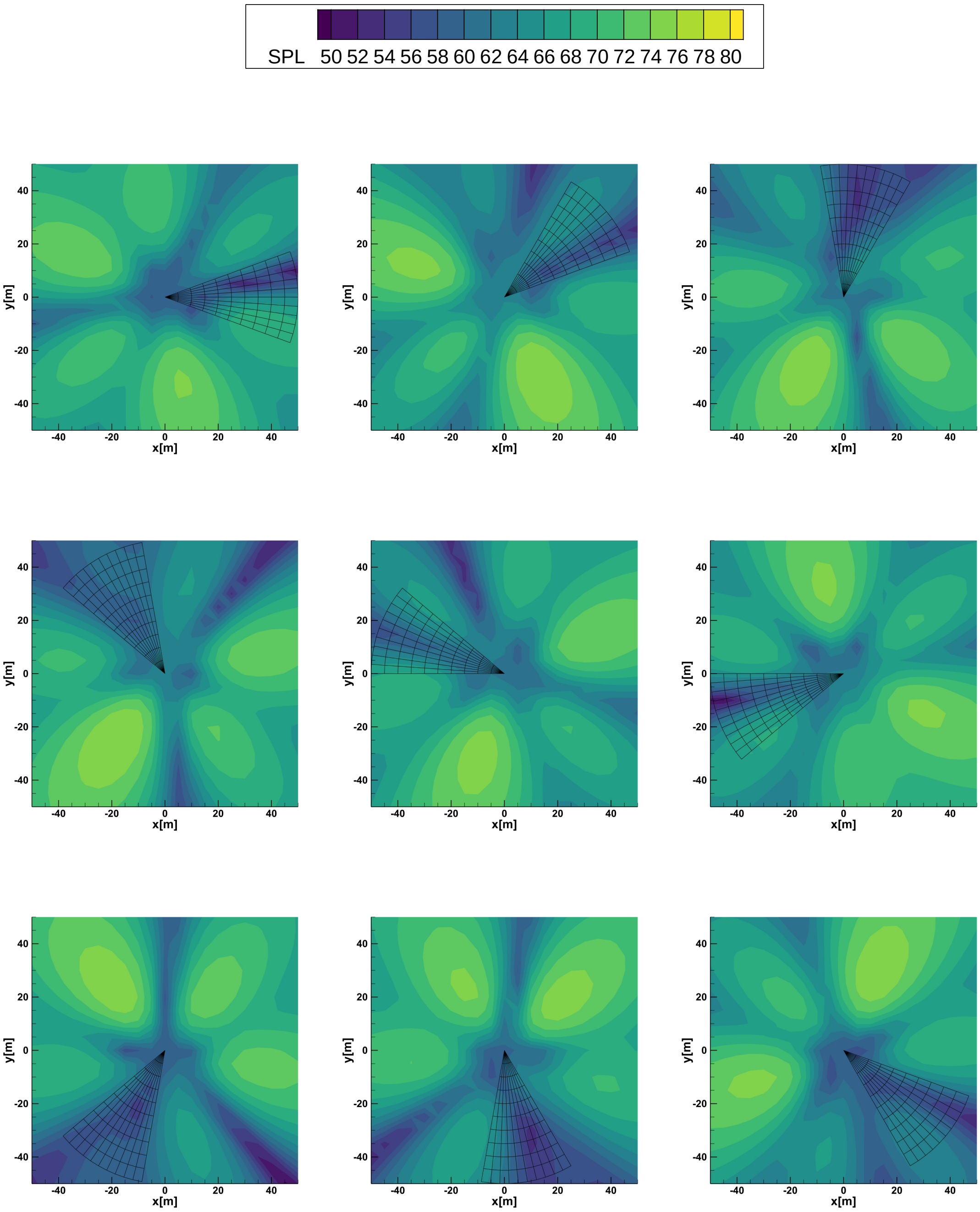


Figure C.5: NR4 Ground SPL [dB] 25m below the configuration.

NR6 : Six Rotors Configuration (HALO)

Caption	Zone (phi)	Hemisphere (χ)	Ground (χ)
(a)	[-20, 20]	[0, 37, -28, -25, -32, -5]	[0, 40, -41, 34, -42, 70]
(b)	[20, 60]	[0, 8, -26, -39, 3, 6]	[0, -56, -28, 11, 41, 23]
(c)	[60, 100]	[0, -17, 39, 8, 28, 31]	[0, -27, 41, -16, 45, 29]
(d)	[100, 140]	[0, 27, -17, 54, 27, 12]	[0, 50, 22, 34, 48, -69]
(e)	[140, 180]	[0, -67, 29, -20, 22, -22]	[0, -7, -27, 2, 76, -64]
(f)	[180, 220]	[0, 45, 1, 53, -44, 23]	[0, -70, -2, 19, -48, 37]
(g)	[220, 260]	[0, 13, 41, -29, 16, -12]	[0, -40, 19, 26, -29, -29]
(h)	[260, 300]	[0, 53, -4, 57, -32, 6]	[0, 15, -40, 8, -59, -36]
(i)	[300, 340]	[0, 4, -44, -26, 4, 2]	[0, 16, -51, 44, 29, -54]

Table C.2: NR6 phase (χ) optimization results.

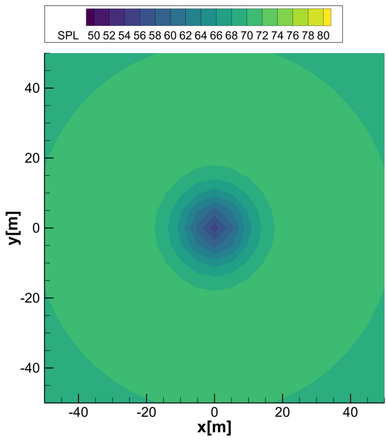


Figure C.6: Unsynchronized noise emission for the HALO configuration.

Hemisphere Optimization

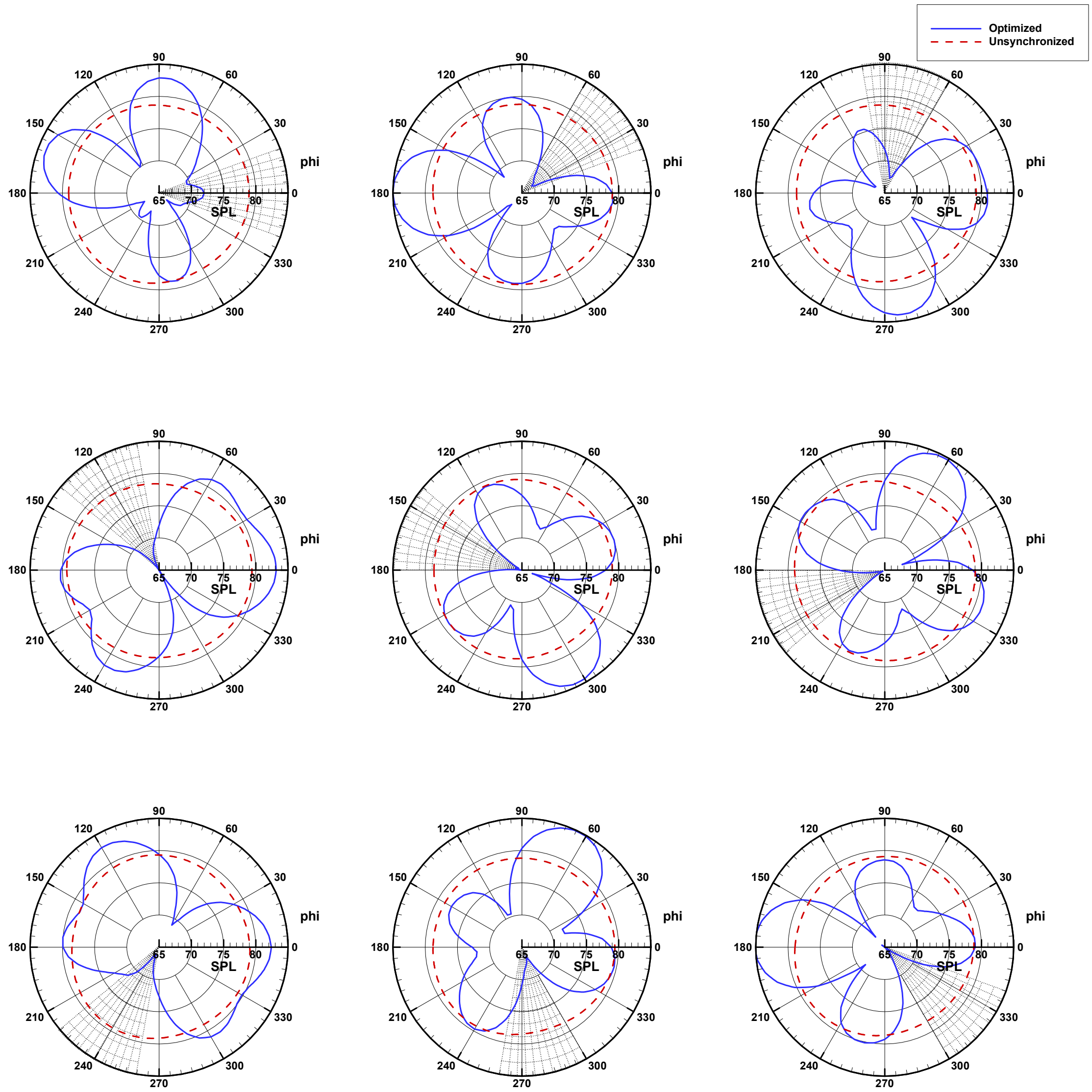


Figure C.7: NR6 Hemisphere SPL [dB] at -45 deg elevation.

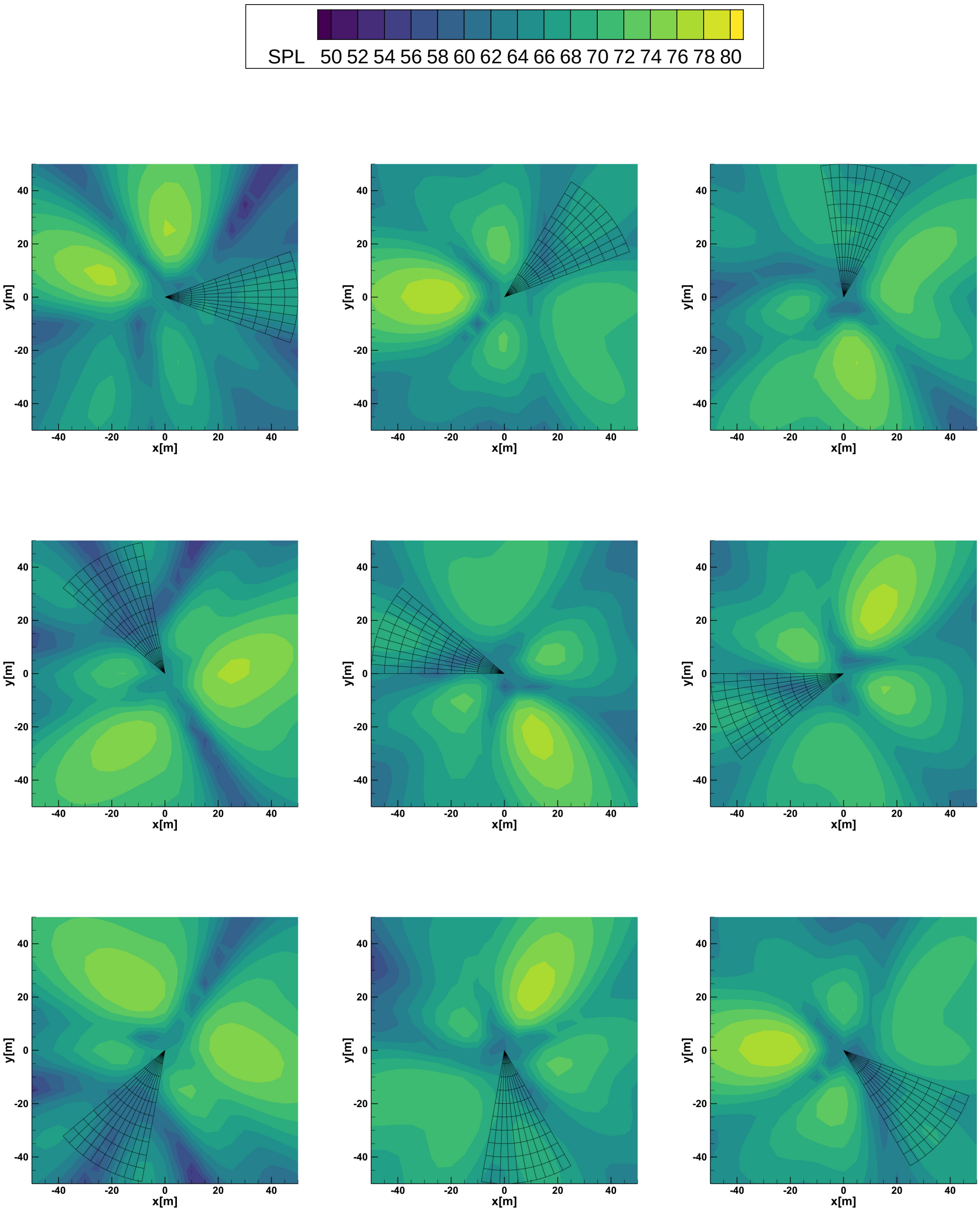


Figure C.8: NR6 Ground SPL [dB] 25m below the configuration.

Ground Optimization

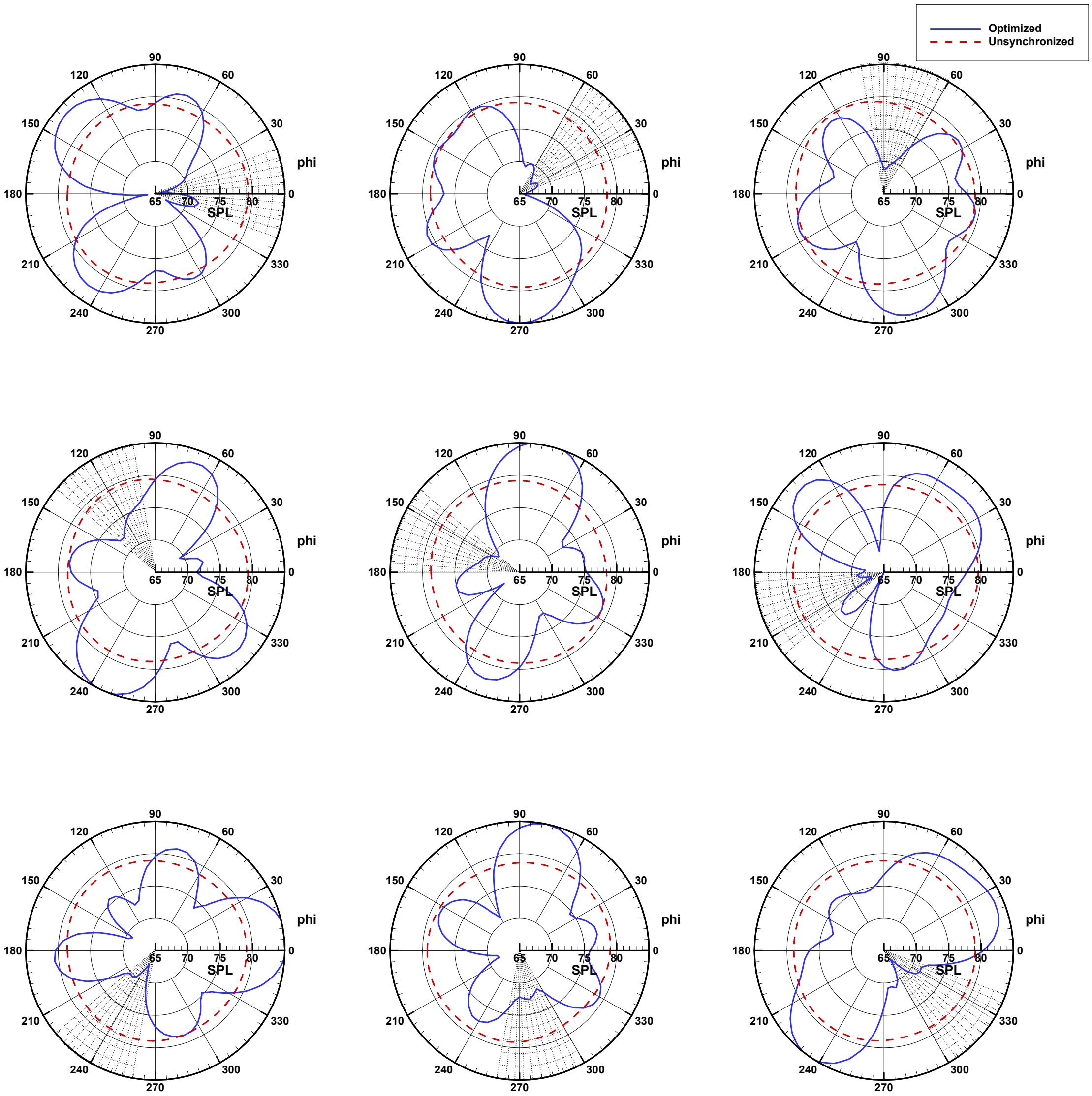


Figure C.9: NR6 Hemisphere SPL [dB] at -45 deg elevation.

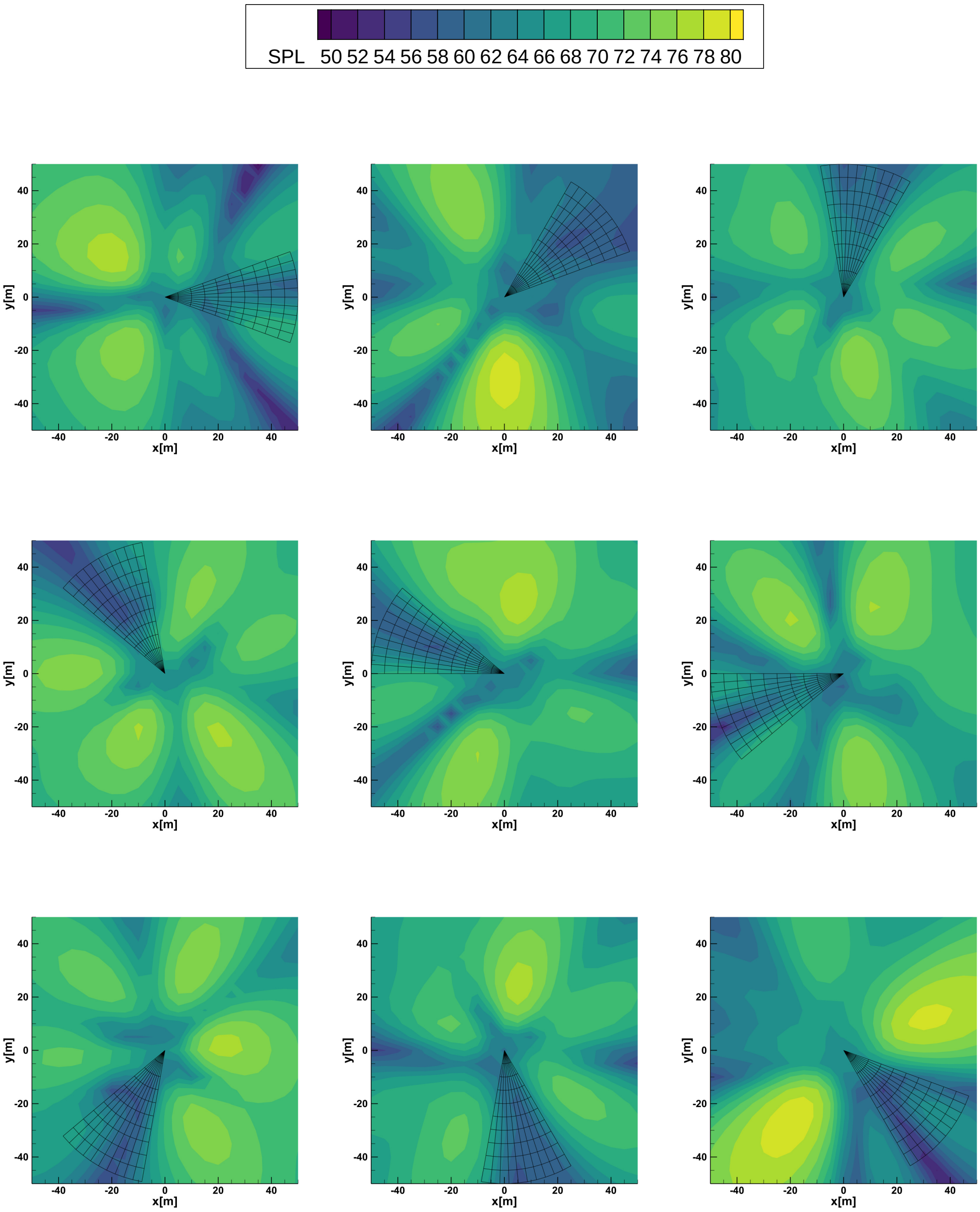


Figure C.10: NR6 Ground SPL [dB] 25m below the configuration.

NR8 : Eight Rotors Configuration

Caption	Zone (phi)	Hemisphere (χ)	Ground (χ)
(a)	[-20, 20]	[0, 14, -6, -22, 43, 15, 18 , 3]	[0, 13,-47,-17, 45,-62,-62,-25]
(b)	[20, 60]	[0, -28, -13, -2, -15, 18, -15, -3]	[0, 44, -60, -2, -13, -16, 35, 23]
(c)	[60, 100]	[0, -22, 50, 22, -14, 5, 1, 37]	[0, 43, -29, -35, 18, -48, 26, 5]
(d)	[100, 140]	[0, 33, 11, -56, 5, -13, 3, 13]	[0, -23, 55, -14, 27, 10, -14, 20]
(e)	[140, 180]	[0, 0, 0, -8, 20, -10, 12, -45]	[0, -69, 33, -39, 33, -23, 20, -26]
(f)	[180, 220]	[0, -23, 1, 0, 2, 19, -4, 44]	[0, -47, 13, 9, -15, 13, -26, 32]
(g)	[220, 260]	[0, 0, 1, -18, -3, -28, -9, -15]	[0, -48, 0, -13, -24, 22, -54, -41]
(h)	[260, 300]	[0, -12, -16, 23, -27, 39, 9, -27]	[0, 43, -52, 12, -39, -44, 34, 3]
(i)	[300, 340]	[0, 14, 1, -2, 6, 14, 18, 39]	[0, -13, -13, -15, -8, -49, 37, -7]

Table C.3: NR8 phase (χ) optimization results.

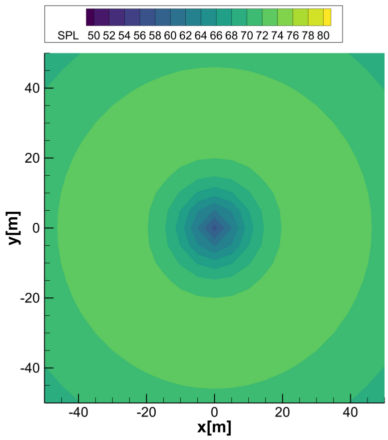


Figure C.11: Unsynchronized noise emission for the 8 rotor configuration.

Hemisphere Optimization

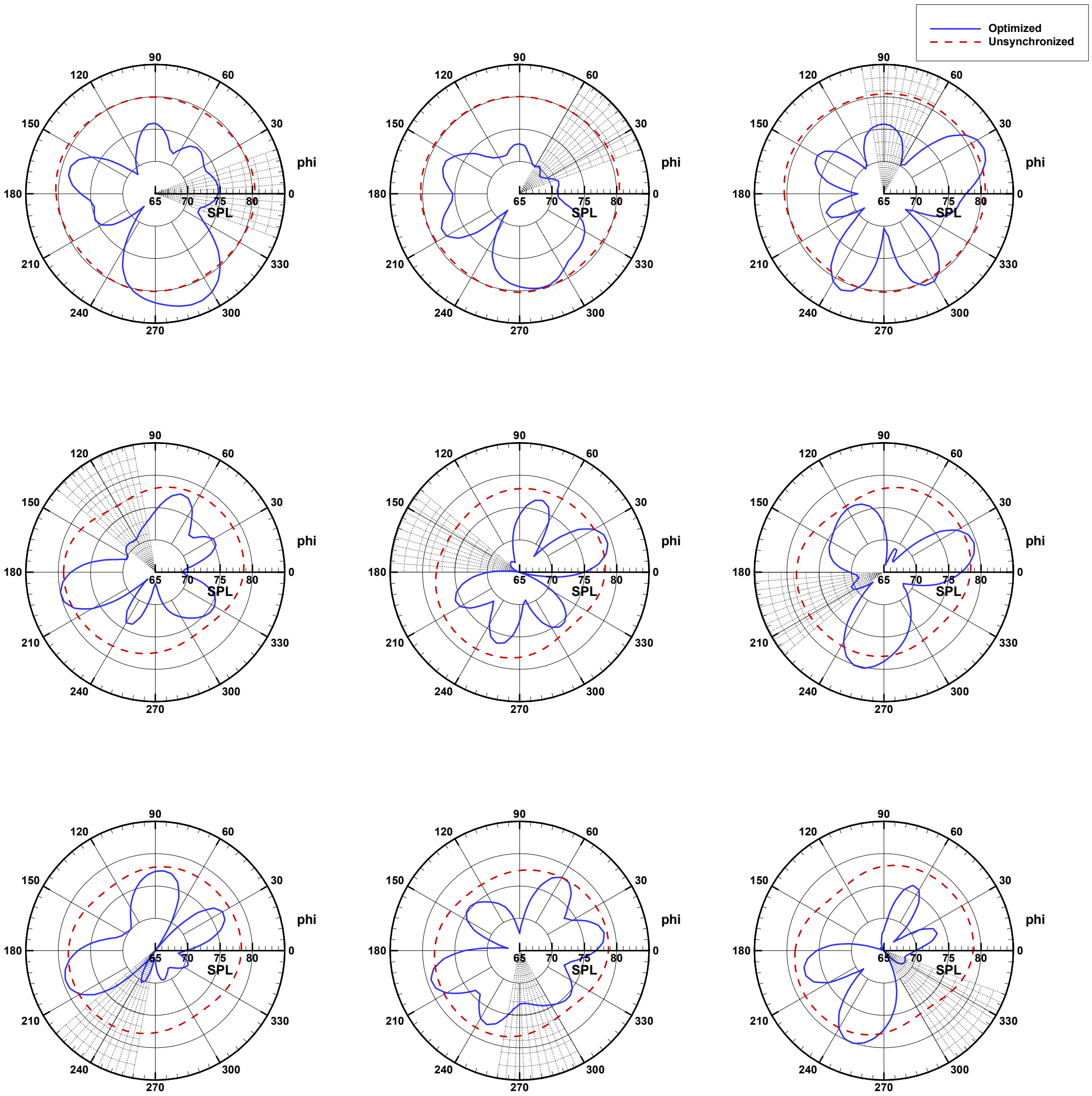


Figure C.12: NR8 Hemisphere SPL [dB] at -45 deg elevation.

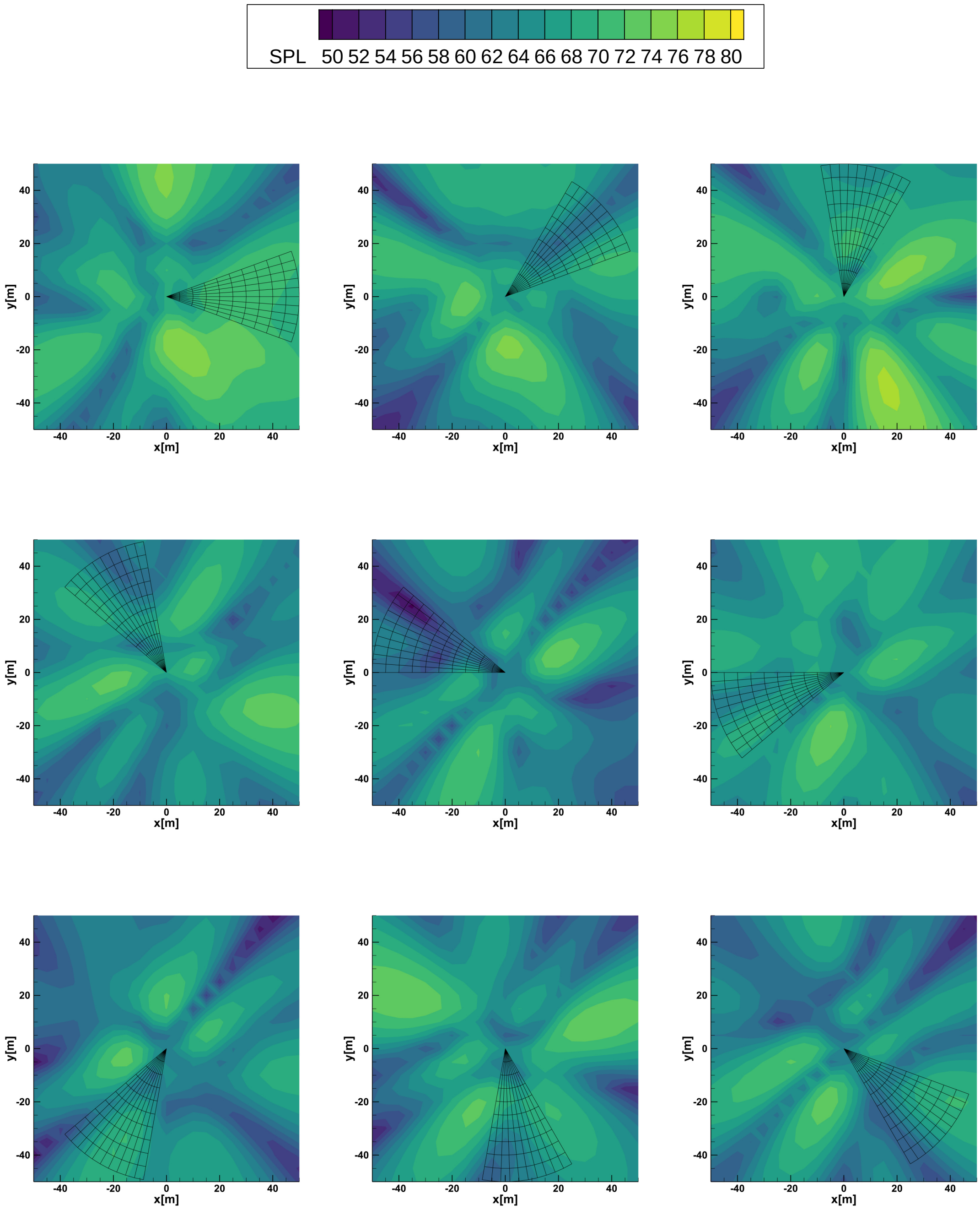


Figure C.13: NR8 Ground SPL [dB] 25m below the configuration.

Ground Optimization

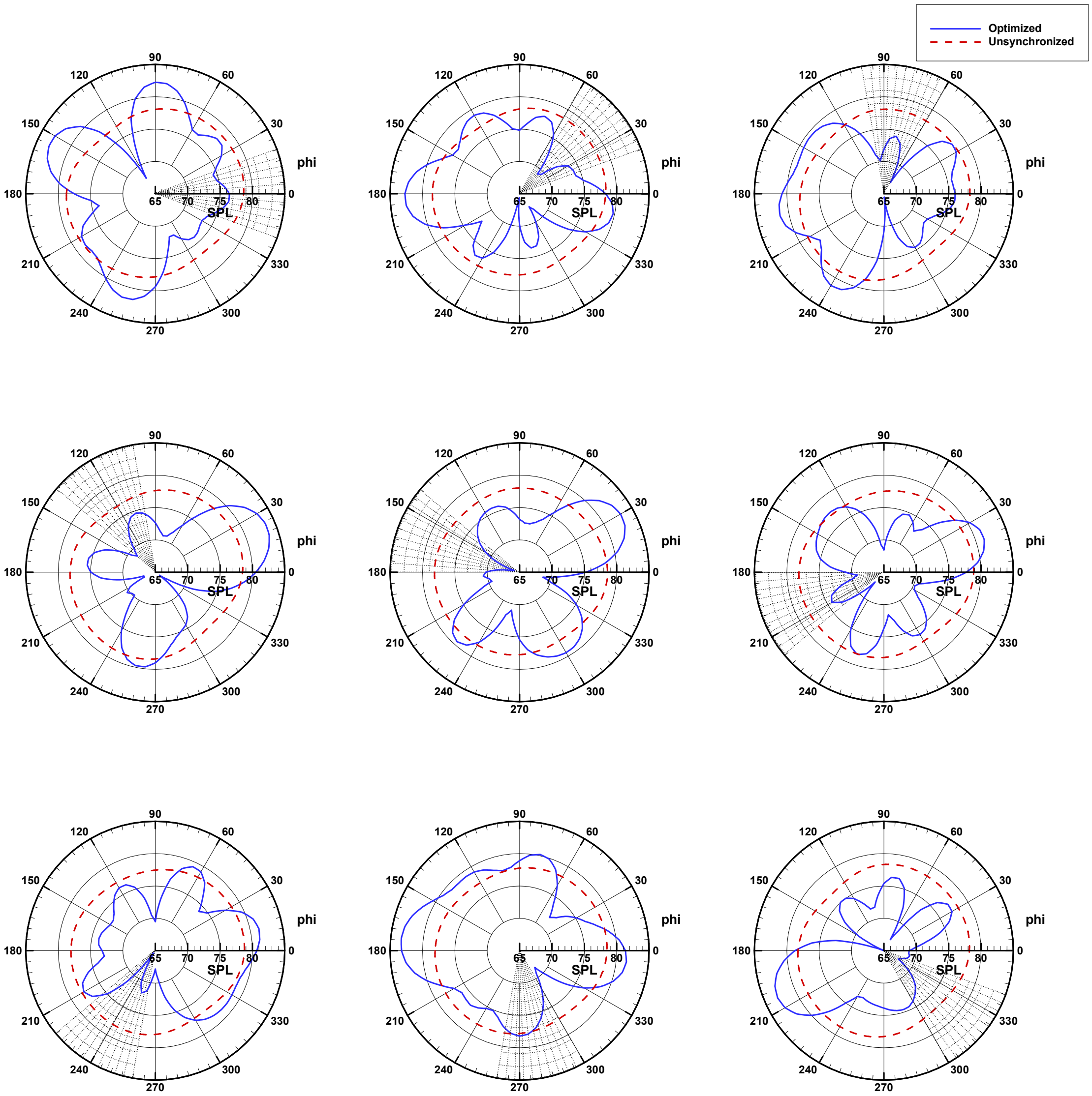


Figure C.14: NR8 Hemisphere SPL [dB] at -45 deg elevation.

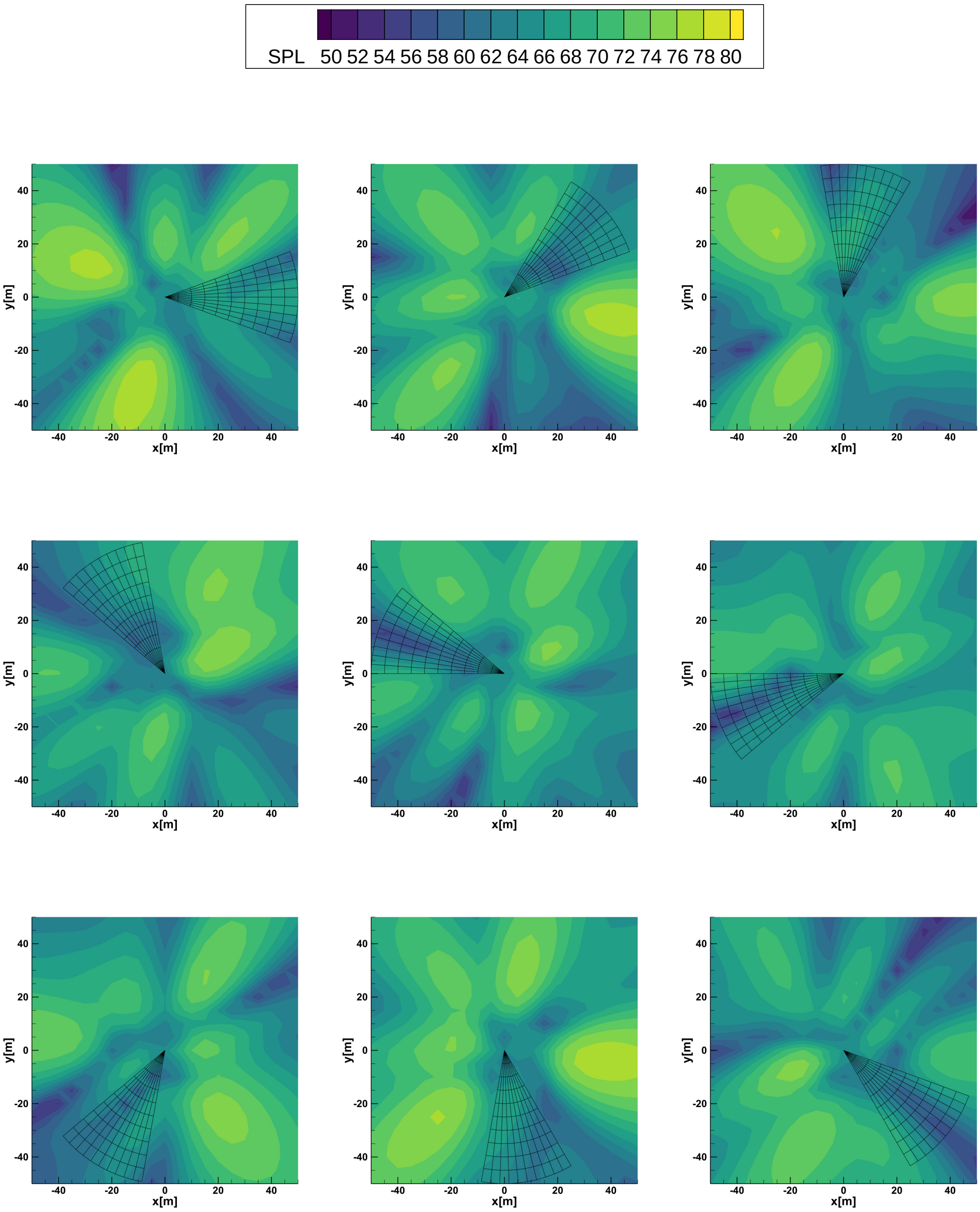


Figure C.15: NR8 Ground SPL [dB] 25m below the configuration.

NR8b : Eight Rotors Configuration with pair coupled direction of rotation

Ground Optimization

Caption	Zone (phi)	Ground (χ)
(a)	[-20, 20]	[0, 67, -22, -24, 51, -63, 4, -37]
(b)	[20, 60]	[0, -14, -25, 15, -58, -11, 36, 43]
(c)	[60, 100]	[0, 28, -9, 64, -7, -45, 30, 65]
(d)	[100, 140]	[0, -10, 20, 60, 0, 19, -1, 28]
(e)	[140, 180]	[0, -43, -8, 10, -27, -12, 26, -42]
(f)	[180, 220]	[0, -37, 29, 54, -19, 2, 9, -37]
(g)	[220, 260]	[0, -20, 49, 28, -34, 48, 25, 68]
(h)	[260, 300]	[0, 18, -6, -21, -42, 15, -23, 25]
(i)	[300, 340]	[0, 1, 4, 28, -24, -31, 18, 8]

Table C.4: NR8b phase (χ) optimization results.

Rotors direction of rotation:	
ROTOR 1	counter-clockwise
ROTOR 2	clockwise
ROTOR 3	clockwise
ROTOR 4	counter-clockwise
ROTOR 5	counter-clockwise
ROTOR 6	clockwise
ROTOR 7	clockwise
ROTOR 8	counter-clockwise

Table C.5: NR8b rotors direction of rotation definition.

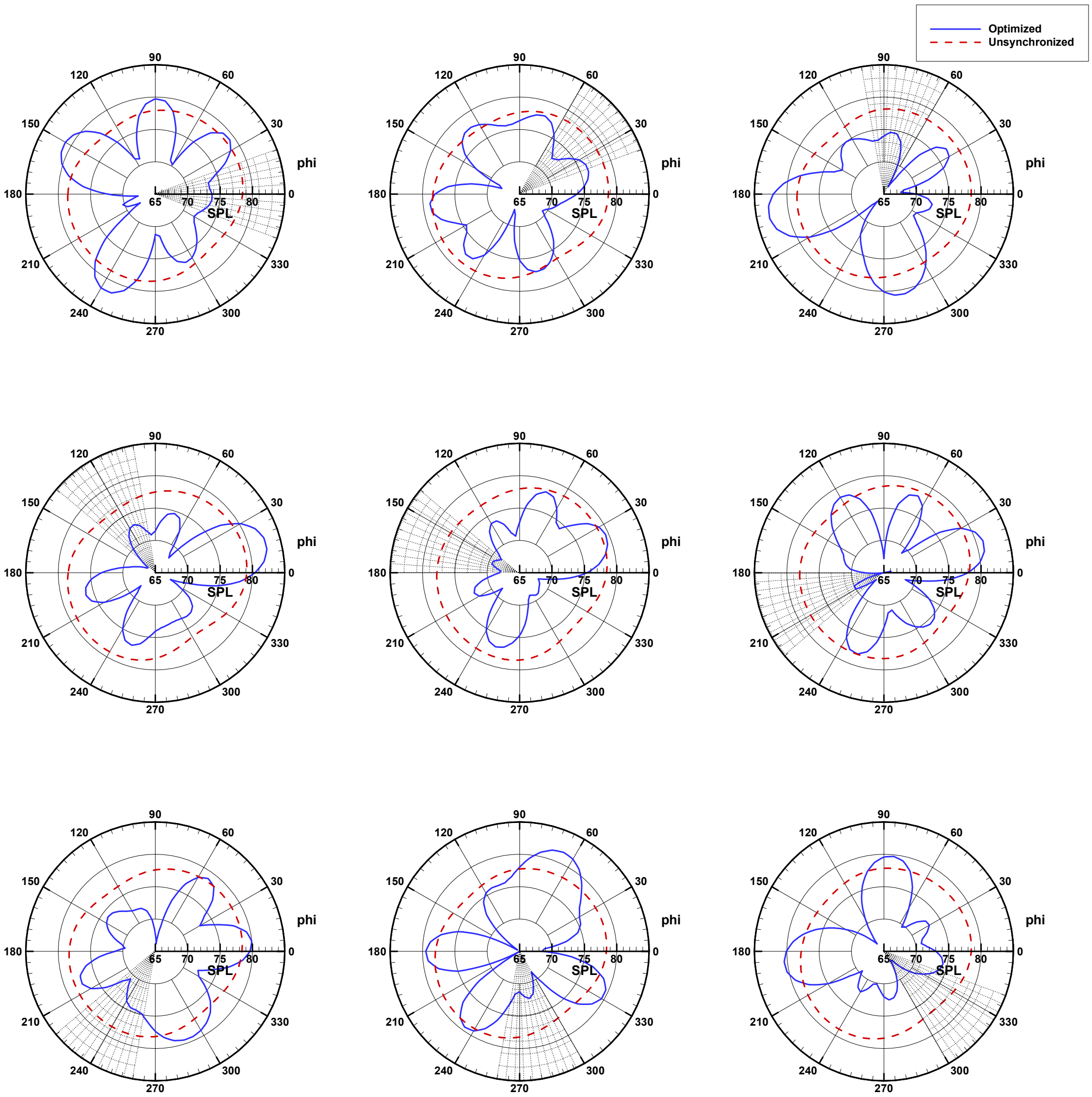


Figure C.16: NR8b Hemisphere SPL [dB] at -45 deg elevation.

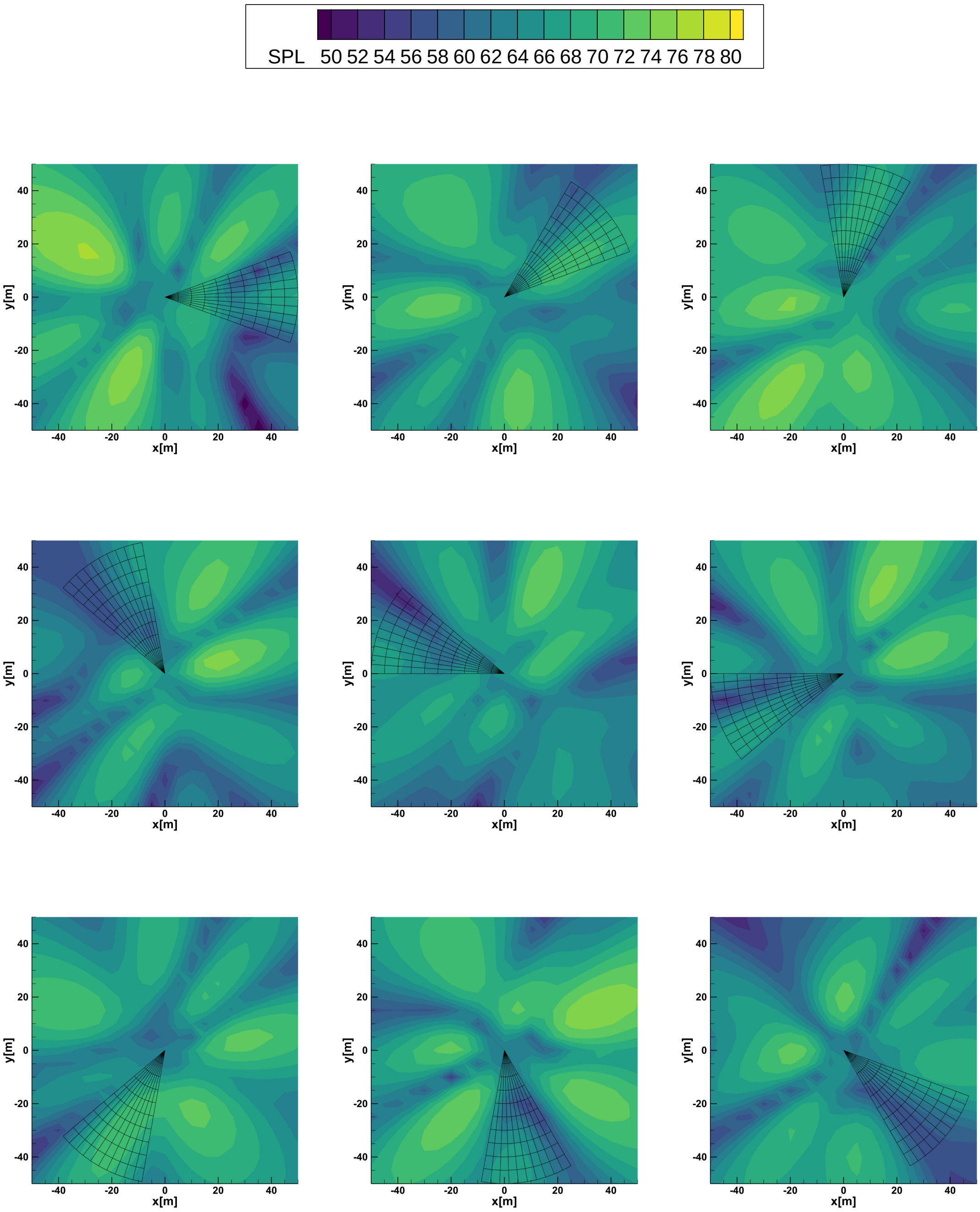


Figure C.17: NR8b Ground SPL [dB] 25m below the configuration.

Appendix D

DUALO Results

Trim

Main Simulation Settings

```
# UPM.dat simulation settings

ts_size_i = 12              # initial step size
ts_size_f = 12              # final step size
r0_core = 1                 # initial vortex core radius for wake
                              vortex sheet
roll_up = 1                 # (1:on, 0:off) activate tip vortex
                              roll-up model
r0_core_tip = 0.3           # initial vortex core radius for tip
                              vortex (roll-up model)
roll_up_start = 60.0        # azimuth angle in deg to start
                              applying the roll-up model

# UPM.dat FIRST simulation settings
stedy_toll = 0.005          # tolerance for the steady state
                              simulation

# ENVIRONMENT PARAMETERS
temp_offset = 8.0           # temperature offset [deg]
altitude = 25.0 / 150       # altitude [m]

# GEOMETRY PARAMETERS
Vtip = 120                  # tip velocity  [m/s]
CHORD = 0.2                 # reference length
phase = [0 0]               # phase vector

# TRIM PARAMETERS
tolltrim_perc = 0.01      # percentage of target Thrust
```

Listing D.1: Trim script main settings parameters used for the simulations.

Hover Condition [1]
3DoF ISA +25m OGE, χ = [0 0]

- **Control Variables:**

$$\theta_0^1 = 12.612°, \quad \theta_0^2 = 12.612°, \quad \theta_C^1 = 0.022°, \quad \theta_C^2 = 0.022°, \quad \Theta = 0 \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_z\ [N] \\ M_y\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 0 \\ 1.4715 \times 10^3 \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{cc} \begin{array}{cc} \textit{Rotor 1} & \textit{Rotor 2} \end{array} & \\ \begin{bmatrix} 4.4480 \times 10^{-1} & 4.9982 \times 10^{-1} \\ 9.1162 \times 10^{-1} & -9.3962 \times 10^{-1} \\ 7.2806 \times 10^2 & 7.3136 \times 10^2 \end{bmatrix} & \begin{array}{c} F_x \\ F_y \\ F_z \end{array} \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{cc} \begin{array}{cc} \textit{Rotor 1} & \textit{Rotor 2} \end{array} & \\ \begin{bmatrix} -1.6750 \times 10^3 & 1.6826 \times 10^3 \\ 2.2240 \times 10^{-1} & 2.4991 \times 10^{-1} \\ -8.4590 \times 10^1 & 8.4580 \times 10^1 \end{bmatrix} & \begin{array}{c} M_x \\ M_y \\ M_z \end{array} \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 9.4462 \times 10^{-1} \\ -2.8008 \times 10^{-2} \\ 1.4594 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 7.6013 \\ 4.7231 \times 10^{-1} \\ -1.0354 \times 10^{-2} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 9.4462 \times 10^{-1} \\ 1.2074 \times 10^1 \\ 4.7231 \times 10^{-1} \end{bmatrix}$$

- **Residual Translational Acceleration:**

$$a_{\text{res,transl}}\ [m/s^2] = 8.0739 \times 10^{-2}$$

- **Residual Rotational Acceleration:**

$$a_{\text{res,rot}}\ [rad/s^2] = 3.3209 \times 10^{-2}$$

[1] Rotor forces and moments are reported in the trim frame (SubCh. 4.2). Consequently, the latter includes both the pure rotor moment contribution and the moment resulting from force displacement with respect to the centre of gravity.

Level Forward Flight Condition
3DoF ISA +150m 18 m/s, χ = [0,0]

- **Control Variables:**

$$\theta_0^1 = 9.945°, \quad \theta_0^2 = 9.945°, \quad \theta_C^1 = 1.279°, \quad \theta_C^2 = 1.279°, \quad \Theta = 2.5525 \text{ deg}$$

- **Target:**

$$\mathbf{Y}_{\text{target}} = \begin{bmatrix} F_x\ [N] \\ F_z\ [N] \\ M_y\ [N \cdot m] \end{bmatrix} = \begin{bmatrix} 6.9018 \times 10^1 \\ 1.4715 \times 10^3 \\ 0 \end{bmatrix}$$

- **Forces:**

$$\mathbf{F}\ [N] = \begin{array}{cc} \begin{array}{cc} \textit{Rotor 1} & \textit{Rotor 2} \end{array} & \\ \begin{bmatrix} 3.0447 \times 10^1 & 3.0433 \times 10^1 \\ 5.6949 & -5.9097 \\ 7.3594 \times 10^2 & 7.3569 \times 10^2 \end{bmatrix} & \begin{array}{c} F_x \\ F_y \\ F_z \end{array} \end{array}$$

- **Moments:**

$$\mathbf{M}\ [N \cdot m] = \begin{array}{cc} \begin{array}{cc} \textit{Rotor 1} & \textit{Rotor 2} \end{array} & \\ \begin{bmatrix} -1.6975 \times 10^3 & 1.6971 \times 10^3 \\ -1.1794 & -1.1805 \\ 2.4751 \times 10^1 & -2.4792 \times 10^1 \end{bmatrix} & \begin{array}{c} M_x \\ M_y \\ M_z \end{array} \end{array}$$

- **Total Forces:**

$$\mathbf{F}_{\text{total}}\ [N] = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 6.0880 \times 10^1 \\ -2.1477 \times 10^{-1} \\ 1.4716 \times 10^3 \end{bmatrix}$$

- **Total Moments:**

$$\mathbf{M}_{\text{total}}\ [N \cdot m] = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} -4.6036 \times 10^{-1} \\ -2.3599 \\ -4.0982 \times 10^{-2} \end{bmatrix}$$

- **Absolute Errors:**

$$\mathbf{e} = |Y_{trim} - Y_{target}| = \begin{bmatrix} 8.1378 \\ 1.3042 \times 10^{-1} \\ 2.3599 \end{bmatrix}$$

- **Residual Translational Acceleration:**

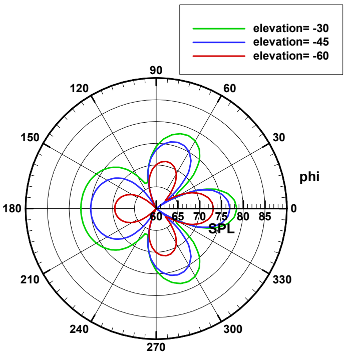
$$a_{\text{res,transl}}\ [m/s^2] = 5.4278 \times 10^{-2}$$

- **Residual Rotational Acceleration:**

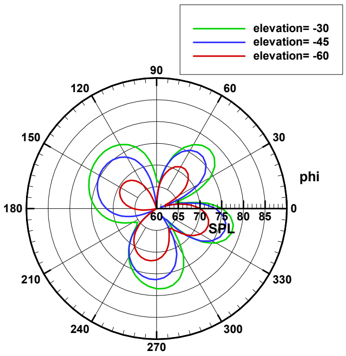
$$a_{\text{res,rot}}\ [rad/s^2] = 1.0485 \times 10^{-2}$$

Aeroacoustic Parametric Study

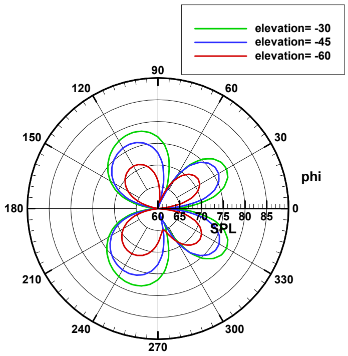
Hover



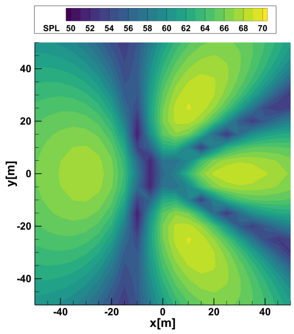
(a) χ = [0,0]: Hemisphere.



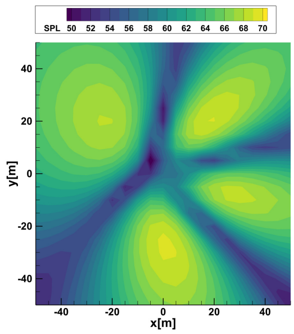
(b) χ = [0,45]: Hemisphere.



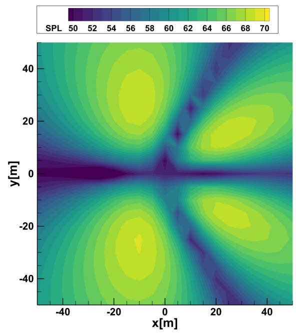
(c) χ = [0,90]: Hemisphere.



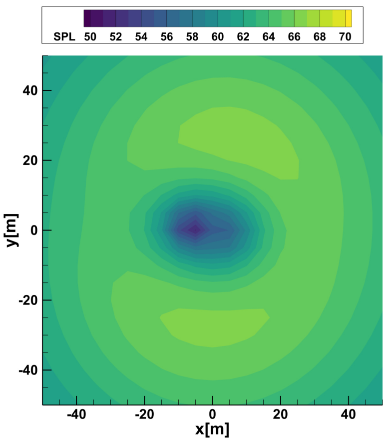
(d) χ = [0,0]: Ground footprint.



(e) χ = [0,45]: Ground footprint.



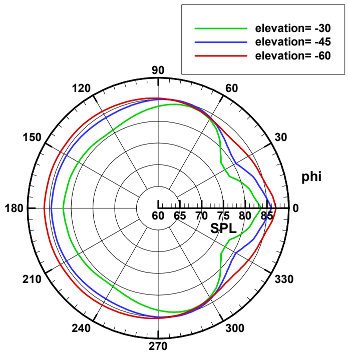
(f) χ = [0,90]: Ground footprint.



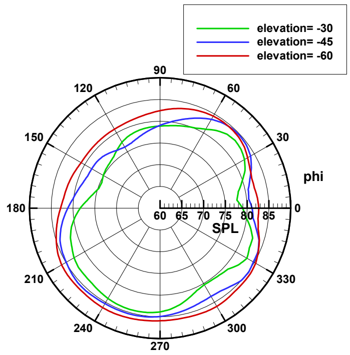
(g) Unsynchronized footprint.

Figure D.1: Plots for hover flight at 25m above sea level. First row: Hemisphere polar plots. Second row: Ground acoustic footprints. Columns correspond to different χ angles.

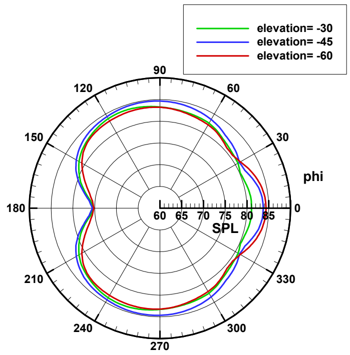
Level Forward Flight TAS = 18 m/s



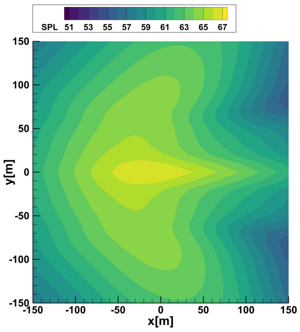
(a) χ = [0,0]: Hemisphere.



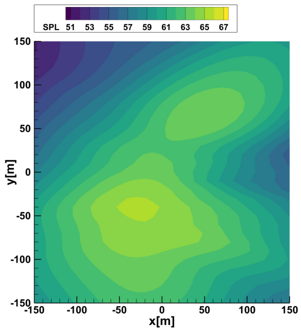
(b) χ = [0,45]: Hemisphere.



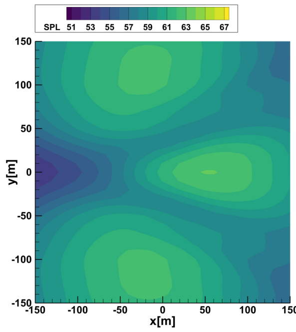
(c) χ = [0,90]: Hemisphere.



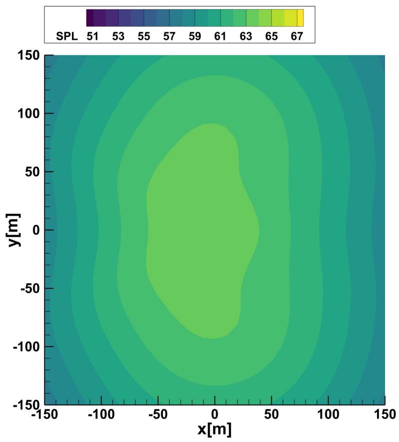
(d) χ = [0,0]: Ground footprint.



(e) χ = [0,45]: Ground footprint.



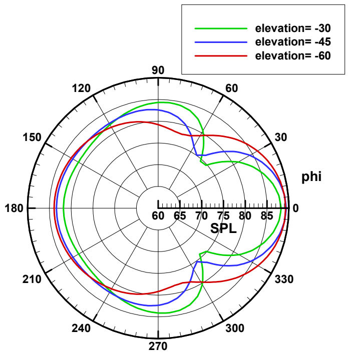
(f) χ = [0,90]: Ground footprint.



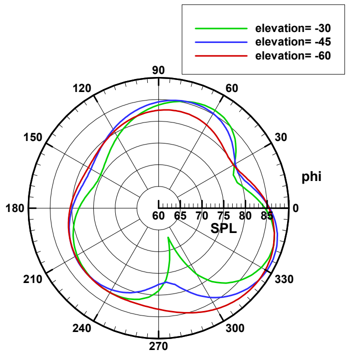
(g) Unsynchronized footprint.

Figure D.2: Plots for level forward flight at TAS = 18 m/s, 150m above sea level. First row: Hemisphere polar plots. Second row: Ground acoustic footprints. Columns correspond to different χ angles.

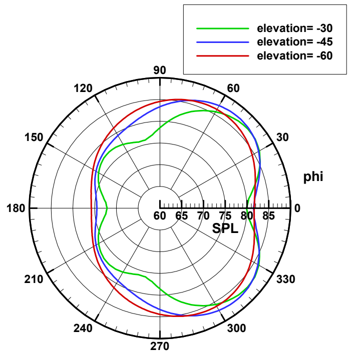
Forward Flight TAS = 28 m/s



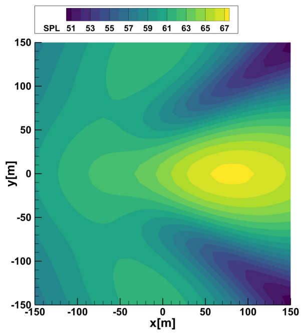
(a) χ = [0,0]: Hemisphere.



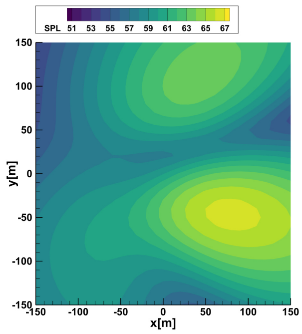
(b) χ = [0,45]: Hemisphere.



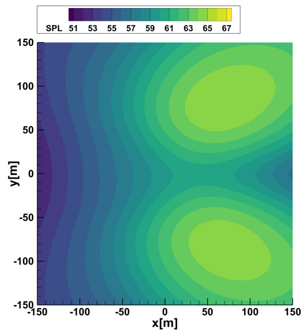
(c) χ = [0,90]: Hemisphere.



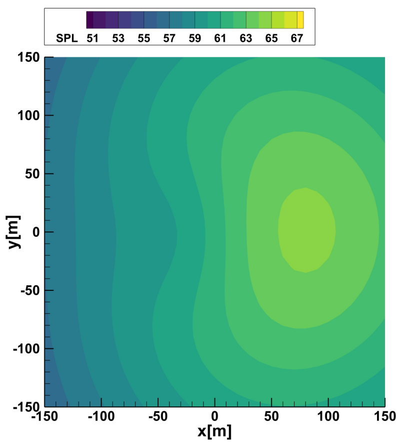
(d) χ = [0,0]: Ground footprint.



(e) χ = [0,45]: Ground footprint.



(f) χ = [0,90]: Ground footprint.



(g) Unsynchronized footprint.

Figure D.3: Plots for forward flight at TAS = 28 m/s, 150m above sea level. First row: Hemisphere polar plots. Second row: Ground acoustic footprints. Columns correspond to different χ angles.