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The "FRA wheel" as a teaching tool to elaborate on the identity aspects of physics and mathematics as disciplines in interdisciplinary contexts

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To my grandparents and
to Kita, so that you may live forever.

Abstract

This thesis project is framed in the research field of *Physics Education* and aims to contribute to the reflection on the importance of disciplinary identities in addressing interdisciplinarity through the lens of the Nature of Science (NOS). In particular, the study focuses on the module on the parabola and parabolic motion, which was designed within the EU project IDENTITIES. The project aims to design modules to innovate pre-service teacher education according to contemporary challenges, focusing on interdisciplinarity in curricular and STEM topics (especially between physics, mathematics and computer science). The modules are designed according to a model of disciplines and interdisciplinarity that the project IDENTITIES has been elaborating on two main theoretical frameworks: the Family Resemblance Approach (FRA), reconceptualized for the Nature of science (Erduran & Dagher, 2014), and the boundary crossing and boundary objects framework by Akkerman and Bakker (2011). The main aim of the thesis is to explore the impact of this interdisciplinary model in the specific case of the implementation of the parabola and parabolic motion module in a context of preservice teacher education. To reach this purpose, we have analyzed some data collected during the implementation in order to investigate, in particular, the role of the FRA as a learning tool to: a) elaborate on the concept of "discipline", within the broader problem to define interdisciplinarity; b) compare the epistemic core of physics and mathematics; c) develop epistemic skills and interdisciplinary competences in student-teachers.

The analysis of the data led us to recognize three different roles played by the FRA: FRA as epistemological activator, FRA as scaffolding for reasoning and navigating (inhabiting) the complexity, and FRA as lens to investigate the relationship between physics and mathematics in the historical case.

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Introduction

The present thesis fits into the research field of *Physics Education* and aims to contribute to the reflection on the importance of disciplinary identities in addressing interdisciplinarity through the lens of the Nature of Science (NOS). The thesis contains a study carried out after the implementation of a module on parabola and parabolic motion, which was designed and developed within the EU project IDENTITIES. This project aims to create innovative teaching modules and courses for preservice teacher education to address contemporary challenges. The main focus is on the interdisciplinarity between physics, mathematics, and computer science. In dealing with interdisciplinarity, IDENTITIES supports the idea that disciplinary aspects come along with interdisciplinary ones. On one hand, the enhancement of disciplinary identities in interdisciplinary contexts allows not only to compare different disciplines, but also to establish a dialogue between them. On the other hand, interdisciplinary aspects allow us to elaborate on the single disciplines and explore their deep identity in light of their interrelation. According to this point of view, the thesis aims to contribute to explore the potentialities of the interdisciplinary model that is being elaborated within the project. The interdisciplinary model considers two specific theoretical frameworks: the Reconceptualized Family resemblance approach for the Nature of science (RFN), introduced by Erduran and Dagher, to explore the disciplinary identities; the boundary crossing and boundary objects framework by Akkerman and Bakker to describe the interdisciplinary dialogue. The RFN, and, more generally, the Family Resemblance Approach (FRA), contributes to the broader methodological issue of defining science providing a new vision of science: it avoids looking for the correct definition of what science is, since "there is no fixed set of necessary and sufficient conditions which determine the meaning of [science]" (Irzik and Nola, 2011, p. 594). Rather, this approach aims to provide a wider characterization of science, considering both the specificities and the common aspects of the scientific disciplines, which belong to the same "science family".
In particular, I participated to the analysis of classroom data to investigate the relevance and the impact of the FRA wheel on:

a. students teachers' ideas about the concept of "discipline" within the context of defining interdisciplinarity.
b. students teachers' ideas about the comparison of physics and mathematics' epistemic cores.
c. students teachers' development of epistemic and interdisciplinary skills.

The analyzed data were collected in the Master's courses in "Physics teaching: theoretical and experimental aspects" at the University of Bologna, in which students of different backgrounds were involved (e.g. Mathematics, Physics and Astronomy). In particular, the context of the study is the module on parabola and parabolic motion, which focuses on two interdisciplinary episodes in order to show the historical interplay between mathematics and physics and their development as disciplines.

The thesis is articulated in four chapters. In the first chapter, I present the IDENTITIES project and the theoretical framework of reference: the FRA framework, within the Nature of Science research topic, to reflect on disciplinary identities and the boundary objects and boundary crossing to reflect on the broader context of interdisciplinarity.

In the second chapter, I present the module on parabola and parabolic motion and the context of implementation, where we collected the classroom data analyzed for the study.

In the third chapter, I describe the methodology adopted for the study, specifying the tools of data collection, the data processing and management. I also present an overview of the evolution concerning the qualitative analysis' criteria, up to the final ones used for the thesis.

Finally, in the last chapter, I present the results of the analysis.

Chapter 1 Theoretical framework

Physics and mathematics have been deeply interrelated since the ancient world and they show mutual influence in their development, as emerges from the analysis of historical case studies. However, in educational contexts this relationship is scarcely evident, since at all levels of education the teaching and learning of mathematics is kept well separated from physics, and vice versa (Tzanakis, 2016). This leads not only to an impoverishment of the mutual relation between the disciplines, but also to distorted and unidimensional views of the disciplines' identity. Indeed, mathematics is usually perceived "as a mere tool to describe and calculate, whereas in mathematics education, physics is commonly viewed as a possible context for the application of mathematical concepts that were previously defined abstractly" (Karam, 2015). As Karam (2015) points out, this duality leads to significant learning problems for students, since for them it is hard to understand the origin of mathematical concepts and why physics has little to do with their experiential world.
Karam (2015) points out that to solve the math-physics dichotomy the contribution from experts in different fields is required. In parallel, Tzanakis considers that "mathematics and physics should be conceived (hence, taught and learnt) both as the result of intellectual enterprises and as the procedures leading to these results. Knowledge gained in their context has an evolutionary character; by its very nature, historicity is a deeply-rooted characteristic" (Tzanakis, 2016). Over the last 40 years, the educational value of history has actually been recognized, since it allows us to appreciate the physical and mathematical knowledge's evolution, and it helps to overcome the idea that these disciplines are just deductively structured intellectual products (Tzanakis, 2016).
The present thesis contains a study framed within the module on parabola and parabolic motion, recently implemented in the Master's course "Physics teaching: theoretical and experimental aspects" at the University of Bologna. In agreement with Tzanakis and Karam's points of view, the named module is centered on an interdisciplinary topic treated from an historical perspective. Indeed, in the module, the parabola's theme is reconstructed on one hand to create an interdisciplinary context where the intertwining between physics and mathematics' is made explicit, and on the other, to dive into the respective historical episodes, since they allow to uncover some key aspects of the two disciplines' epistemological nature.

1.1 The IDENTITIES project

The present study focuses on the module on parabola and parabolic motion, which has been designed within the project IDENTITIES (Integrate Disciplines to Elaborate Novel Teaching approaches to InTerdisciplinarity and Innovate pre-service teacher Education for STEM challenges), an Erasmus+ project coordinated by the University of Bologna. The project, started in September 2019, involves other 4 partners: the University of Montpellier, the University of Barcella (UB), the University of Crete and the University of Parma (https://identitiesproject.eu/it/).
The project aims to create transferable and innovative modules to rethink pre-service teacher education according to contemporary challenges. The modules focus on interdisciplinarity in curricular and STEM topics, with a particular emphasis on the interdisciplinarity between physics, mathematics and computer science.
The need to educate teachers and students on interdisciplinarity is becoming nowadays more and more important. From the educational point of view, interdisciplinarity appears to be relevant to contextualize and enhance the interest in scientific disciplines, which always more often are seen as harsh and artificial. In order to enhance the role that interdisciplinarity has in schools, a specific preparation for teachers is demanded. In fact, although teachers and teachers' educators have solid disciplinary backgrounds, they are often not aware of the interrelationship that disciplines may have, missing in this way the possibility to establish an interdisciplinary dialogue.

The IDENTITIES project centers its research within three key-aspects:

1. The focus is either on advanced interdisciplinary STEM topics (e.g. climate change) or curricular interdisciplinary topics (e.g. the parabola and the parabolic motion), which are thought as resources to make disciplines more interesting, profound and meaningful.
2. In dealing with interdisciplinarity, IDENTITIES wants to avoid seeing it as a-disciplinarity (which wants to destroy the knowledge's organization based on disciplines), transdisciplinarity (which aims to go beyond disciplines), multidisciplinarity (disciplines are simply juxtaposed, without a real communication between them) or interdisciplinarity in the sense that one discipline is instrumental to another (for example mathematics as an instrument for physics). Moreover, disciplines are conceived as knowledge's re-arrangement for teaching purposes, consistently with the meaning of the word "discipline" (it contains the latin root "discere", which means to learn).

   In order to reach an interdisciplinary dialogue, IDENTITIES considers fundamental to take into account the disciplines' single epistemological identities, meanwhile looking for an epistemology of interdisciplinarity.
3. The project aims to explore the interdisciplinary territory through a comparison of the different languages, and the recognition of the common one. To do this, a powerful tool is that of epistemological and linguistic activators, that may be thought as words and concepts able to trigger the disciplines' meta-analysis, in the sense that they foster the activation of epistemological, methodological and conceptual reflections on the nature of knowledge and science itself

As a research outcome, the project aims to elaborate a model for interdisciplinarity both for curricular and STEM topics. The model, under development, is based on the concept that disciplinarity and interdisciplinarity are mutually related: indeed, the first permits to elaborate on disciplinary identities, which allows to put them in comparison, while the interdisciplinary dialogue contributes to enrich and deepen the discipline's features. This interrelation led us to choose as a theoretical framework: the Reconceptualized Family resemblance approach for the Nature of science (RFN), introduced by Erduran and Dagher, to elaborate on the disciplinary aspects; the boundary crossing and boundary objects by Akkerman and Bakker to describe the mechanisms of interdisciplinary dialogue.

The present thesis aspires to contribute to the IDENTITIES project by studying how the "FRA wheel", which was introduced by Erduran and Dagher in the RFN, and the boundary metaphor of Akkerman and Bakker, can be used as teaching tools to explore identity aspects that characterize physics and mathematics as disciplines.

In particular, it will be analyzed how student-teachers experienced the FRA wheel to:

- elaborate the concept of "discipline" within the broader problem to define interdisciplinarity.
- compare physics and mathematics' epistemic cores.
- foster the development of epistemic and interdisciplinary skills.

In the following, I start by introducing the FRA theoretical framework within the Nature of Science research topic. After that, I present the boundary objects and boundary crossing mechanisms within the broader context of interdisciplinarity.

1.2 Nature of Science

In the context of science education, the Nature of Science (NOS) research field aims to investigate which is the nature of science and which ideas about the nature of science is advisable to teach and learn (Erduran and Dagher, 2014, p. 1). Regarding the first question, we may approach it in three possible ways (Irzik and Nola, 2014): the nature of science might be seen as an essence, where with "essence" we mean a collection of properties that the object "*must* have and without which it is *not possible* for that thing exist and to be that *kind* of thing " (Irzik and Nola, 2014); we might look for a set of necessary and sufficient properties that science possesses in order to be called such; we may simply try to list a collection of characteristics that do not pretend to be necessary or sufficient to define science. As Irzik and Nola (2014) highlight in their paper, the first two options are not satisfactory, as for the first it is doubtful to think that science has an essence just as, for example, a triangle as one, and as for the second, any attempt by philosophers and science educators vanished to delineate a precise demarcation criterion of what science is.

The proposal of Irzik and Nola (2011, 2014) is then to address the NOS issue for science education by following the third option, that is by delineating an open and not exhaustive set of features through which we can characterize scientific disciplines. The authors highlight that this approach faces the issue that not all the scientific disciplines present the same identical characteristics, and at the same time, it seems not useful to provide a list of preferred items. To solve this point, they propose to advocate the family resemblance idea, originally developed by Wittgenstein (1958).

Irzik and Nola's proposal was subsequently revised and enriched by Erduran and Dagher (2014). In the following I will present in more detail these two related frameworks since they were the fundamental theoretical basis for the thesis.

1.2.1 Family Resemblance Approach

The idea of family resemblance was firstly proposed by the philosopher Wittgenstein (1958) by recognizing the fact that it is not possible to define terms by setting a list of necessary and sufficient conditions or by specifying essences (Wittgenstein 1958, Sects. 66–71). As an example, Wittgenstein compared the two terms "triangle" and "game": for the first it is possible to propose a definition which defines a set of necessary and sufficient conditions, instead for the term "game" there is no set of conditions through which we can characterize all the possible games while excluding everything that cannot be considered a game. As a solution, Wittgenstein proposed to say that all games form a "family resemblance", that is a complex network of reciprocal similarities and differences.

Irzik and Nola (2011, 2014) transferred the idea of family resemblance to the NOS for science education. In their paper, they highlight how the problem of defining what science is can be difficult to solve, since science is rich and dynamic, and it seems impossible to find a list of characteristics that is univocally able to define all the scientific subjects (Irzik and Nola, 2011). The authors argue that what is called "the consensus view", although promising, is not enough to solve the NOS issue. The consensus view proposes to share with students only the characteristics of science that may be widely accepted in literature, and on which the controversies are at its minimum (Irzik and Nola, 2011). The proposed list of features is highly shareable, and it includes, for example the fact that: scientific knowledge is based on observation and experiments; science is dynamic since it changes with time; science is a human product, thus it is partly subjective and influenced by social and cultural contexts. Irzik and Nola (2011) think that the consensus view's list presents a too narrow picture of science, not showing for example its aims and methodological rules. Moreover, the consensus view does not explain the differences between scientific disciplines, suggesting an idea of science that is quite steady and changeless. Finally, the consensus view lacks systematicity and the pointed-out characteristics seem to be not always coherent, for example one might ask: if science is so dependent

on the cultural context, how is it possible that we somehow have the feeling that it has a valence also between different countries?
Irzik and Nola (2011) explain how the idea of family resemblance can be a fruitful alternative to characterize science. The authors follow one of the anthropologist Rodney Needham's proposals to delineate in which way the set of features may form a family:
«Consider a set of four characteristics {A, B, C, D}. Then one could imagine four individual items which share some three of these characteristics taken together such as (AandBandC) or (BandCandD) or (AandBandD) or (AandCandD); that is, the various family resemblances are represented as four disjuncts of conjunctions of three properties chosen from the original set of characteristics. This example of a polythetic model of family resemblances can be generalized as follows. Consider any set S of n characteristics; then any individual is a member of the family if and only if it has all of the n characteristics of S, or any (n - 1) conjunction of characteristics of S, or any (n - 2) conjunction of characteristics of S, or any (n - 3) conjunction of characteristics of S, and so on. How large n may be and how small (n - x) may be is something that can be left open as befits the idea of a family resemblance which does not wish to impose arbitrary limits and leaves this to a "case by case" investigation. » (Irzik and Nola, 2011, pp. 594-595).
Although it is possible to find some core characteristics that all sciences share (e.g. observing), they are not enough to define science (for example, it is not correct to say that every practice involving observing is science). In this sense, the family resemblance idea discourages looking for a common set of characteristics. Rather, by reflecting on several similarities and dissimilarities that scientific disciplines share, Irzik and Nola come to outline 8 categories that structurally characterize the NOS (see Table 1.1).

Science							
Science as a cognitive-epistemic system				Science as a social system			
1	2	3	4	5	6	7	8
Processes of inquiry	Aims and values	Methods and methodological rules	Scientific knowledge	Professional activities	Scientific ethos	Social certification and dissemination of scientific knowledge	Social values

Table 1.1: Categories that structurally characterize the NOS (Irzik and Nola, 2014).

The categories are grouped according to whether science is perceived as a cognitive-epistemic system (Process of Inquiry, Aims and Values, Methods and Methodological Rules, Scientific Knowledge) or as a social-institutional system (Professional Activities, The Scientific Ethos, The Social Certification and Dissemination of Scientific Knowledge, Social Values of Science). The Family Resemblance Approach (FRA) is obtained by applying the idea of family resemblance to these categories.
Irzik and Nola (2014) precise that each category can be modified as science progresses and its list of elements is not finite or fixed in time. Each science can be characterized with a family resemblance definition by taking a subset of characteristics from each category. Taken two different sciences, some features will be identical, and others not, just as the members in a family. Even though the common and different characteristics can vary depending on the scientific disciplines considered, if the number of overlaps and similarities between them is sufficient, they can be considered scientific, thus falling in the science's family resemblance idea.

1.2.2 Reconceptualized Family resemblance approach to NOS

The Reconceptualized Family resemblance approach for the Nature of science by Erduran and Dagher (RFN framework) is based on the FRA approach developed by Irzik and Nola (2011) described in the previous section. Erduran and Dagher have chosen the FRA approach to NOS since it showed engaging aspects: it is able to "consolidate the epistemic, cognitive and social aspects of science in a wholesome, flexible, descriptive but non-prescriptive way. […] It creates a much-needed space for conversation and dialog about science in a comprehensive way." (Erduran and Dagher, 2014, p. 24); it includes characteristics of relevant NOS frameworks (e.g. the consensus view); it provides a more inclusive characterization of science since it holds together different aspects of science (e.g. social-institutional ones), which can potentially be more appealing to a wider range of students (Erduran and Dagher, 2014, p. 24-25).
According to the RFN framework, the NOS is represented through the "FRA wheel" (displayed in Fig. 1.1), where the cognitive-epistemic system of science and social-institutional one, previously described by Irzik and Nola (2011), interact together. Erduran and Dagher have identified a total of 11 categories through which it is possible to characterize the Nature of Science.

Figure 1.1: The FRA wheel (Erduran and Dagher, 2014).

At the core of the wheel, we find the cognitive-epistemic system, which is divided in four categories: Aims and values, Methods and methodological rules, Practices, and Knowledge. The social-institutional system is represented by the outer circle, which is divided in 4 categories: Social values, Scientific ethos, Professional activities, and Social certification and dissemination. Erduran and Dagher pointed out that the social-institutional aspects were limited in Irzik and Nola work (2014). For this reason, they added three more social-institutional categories, which constitute the external circular level: Social organizations and interactions, Political power structures, and Financial systems. Erduran and Dagher think that this concentric circle model provides a more holistic, dynamic and interactive picture of science (Erduran and Dagher, 2014). Even though the categories are well divided, the boundaries between them are thought as porous, allowing communication in all directions. In particular, all categories interact with each other and influence scientific activity.
In the thesis, I will use "FRA", "FRA model" or "FRA framework" when I refer to the *family resemblance approach* used in the RFN theoretical framework of Erduran and Dagher (2014). With

"FRA categories" or "FRA wheel categories", I will refer to the categories delineated by Erduran and Dagher in the RFN.

FRA wheel's categories

In the following, the FRA wheel categories are presented in more detail, distinguishing between the cognitive-epistemic and the social-institutional ones.

Cognitive-epistemic system's categories:

- Aims and Values: they are the set of values that orient scientific practices, such as objectivity, consistency, testability, simplicity, etc. They have a powerful role in knowledge's growth and development, since values such as empirical adequacy or explanatory power may favor one theory over another, thus influencing methodological decisions or interpretations. There are different kinds of aims and values (displayed in Fig. 1.2), where the distinctions between them are rather blurry: *cognitive* aims and values highlight key values in scientific reasoning (e.g. critical examination, revising convictions); *epistemic* ones characterize the scientific knowledge (e.g. objectivity, novelty and accuracy); *social* aims and values do not want to establish moral codes, rather there are ethical principle that scientists are expected to embrace (e.g. addressing human needs and honesty).

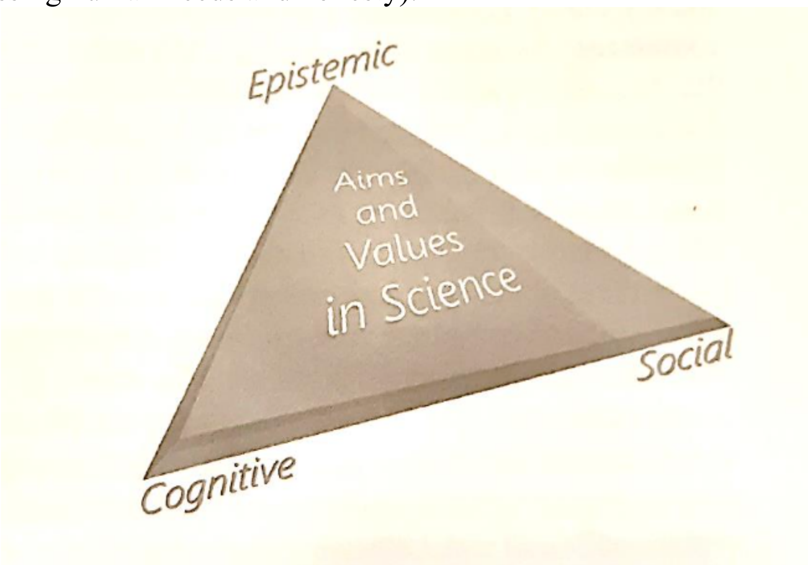


Figure 1.2: Aims and values in science (Erduran and Dagher, 2014, p. 49).

- Methods and methodological rules: they refer to the many observational, investigative and analytical methods used by scientists for their inquiry, and to the methodological rules that guide scientific practices to produce reliable knowledge (e.g. construct testable hypotheses, avoid *ad hoc* changes to theories).
- Practices: it comprehends the cognitive, epistemic, and discursive practices that characterize scientific activities, such as observation, experimentation, reasoning, etc.
- Knowledge: it refers to knowledge perceived as "an interrelated network of theories, laws, and models", TLM (Erduran and Dagher, 2014, p.5). Knowledge is seen as a "product of a collective human enterprise to which scientists make individual contributions which are purified and extended by mutual criticism and intellectual co-operation" (Ziman, 1991, p. 3). TLM is conceptualized as a coherent and holistic network, not as discrete and disconnected fragments of scientific knowledge (Erduran and Dagher 2014a adapted from Yeh *et al,* 2019, p.295).

Social-institutional system's categories:

- Social values: refer to social values present in science, such as social utility, decentralizing power, improving people's quality of life, etc.

- Scientific ethos: are norms that scientists follow during their scientific inquiry, such as skepticism, universalism, intellectual honesty, etc.
- Professional activities: dissemination activities proposed by scientists to communicate their research (e.g. reviewing papers, presenting the work in conferences).
- Social certification and dissemination: peer review process to revise and validate new scientific knowledge.
- Social organizations and interaction: social organizations in which scientists meet and work (e.g. universities, research centers).
- Political power structures: political environments that influence scientific enterprise.
- Financial systems: economic factors (e.g. funding) on which science relays.

Final remarks

The FRA and RFN framework were so far used in science education to: develop teaching strategies in teacher education (Cullinane 2018), as an analytical tool for analyzing textbooks (McDonald and Abd-El-Khalick 2017), and science and physics curriculum (Kaya and Erduran, 2016; Yeh, Erduran and Hsu, 2019; Park, Wu and Erduran, 2020).

In the present thesis, the FRA framework, along with the FRA wheel, played a central and innovative role to investigate the identity aspects of physics and mathematics as disciplines and their mutual relation. In fact, the FRA wheel has been used together with the framework of boundary objects and boundary crossing mechanisms as teaching tools to design activities for the module on the parabola and parabolic motion.

In the following I will present the other fundamental framework for the study, that is meta-theory of Akkerman and Bakker (2011) on the boundary objects and crossing mechanisms. To do this, I will firstly introduce the broader context of interdisciplinarity and what do we mean with this word.

1.3 Interdisciplinarity

In the first major interdisciplinary typology, written in 1972 by Apostel and co-sponsored by the Organisation for Economic Co-operation and Development (OECD), the author makes extended use of the terms: multidisciplinary, interdisciplinary, and transdisciplinary, which are terms often used interchangeably (Thompson Klein, 2010; Alvargonzález, 2011). As Thompson Klein (2010) points out, other labels soon followed, leading to a confused set of characterizations.

Choi and Pak conducted a literature review on the use of the terms multidisciplinary, interdisciplinary, transdisciplinary, and proposed the following characterization:

«Multidisciplinarity draws on knowledge from different disciplines but stays within their boundaries. Interdisciplinarity analyzes, synthesizes and harmonizes links between disciplines into a coordinated and coherent whole. Transdisciplinarity integrates the natural, social and health sciences in a humanities context, and transcends their traditional boundaries… The common words for multidisciplinary, interdisciplinary and transdisciplinary are additive, interactive, and holistic, respectively.» (Choi and Pak 2007, p. 351)

Consistently with this description, Thompson Klein (2010) proposed a classification of the different types of interdisciplinarity, which are displayed in Fig. 1.2. According to Thompson Klein (2010), multidisciplinarity is intended as an additive juxtaposition of disciplines that maintains a precise separation between them. Disciplines in this case lack intercommunication, and they are at most aligned in sequencing-coordinating fashion. On the other hand, interdisciplinarity involves an active and fruitful interaction between the different disciplines, which are linked and integrated in a blended space. As Peek and Guikema highlight: “The key defining concept of interdisciplinarity is *integration*, a blending of diverse inputs that differs from and is more than the sum of the parts.” (Peek and Guikema, 2021, p. 1048).

Transdisciplinary is again different, since it goes beyond disciplines, leading to a transformative synthesis of the subjects involved (Thompson Klein, 2010).

Multidisciplinarity	Interdisciplinarity	Transdisciplinarity
• juxtaposing	• integrating	• transcending
• sequencing	• interacting	• transgressing
• coordinating	• linking	• transforming
	• focusing	
	• blending	

Figure 1.3: Classification of the different types of interdisciplinarity (Thompson Klein, 2010).

In this thesis, in accordance with the definitions presented above, I will refer to the concept of interdisciplinarity as a mutual exchange relationship between the parties involved, where each discipline's structure is respected.

1.3.1 Disciplinarity, interdisciplinarity and complexity

Specialized domains of knowledge have characterized the Western culture since the late nineteenth century (Thompson Klein, 2010). This disciplinary compartmentalization is currently found also in schools and universities, whose aim is to form disciplinary communities of professionals. Although this may be driven by the necessity to form experts before integrating the different perspectives, the risk is that the boundaries between disciplines become even more invisible and steadier. When boundaries become barriers, people from different fields find it hard to communicate and are prevented from establishing fruitful collaborations.

Another side effect of the persistent disciplinary separation is that the boundaries are felt by systems that cannot be treated through the specific conventions of each traditional field. This happens more and more often since modern societies are affected by issues (e.g. environmental problems, pandemic) that cannot be confined to just one specific domain of knowledge (Barelli et al., in press.). As a result, these problems are not easily resolvable by simply applying new tools to existing models (Thompson Klein, 2004), since they inhabit "indeterminate zones of practice" (Schön, 1987, pp. 3-6).
At the same time, modern issues are complex in nature since they "are emergent phenomena with nonlinear dynamics. Effects have positive and negative feedback to causes, uncertainties continue to arise, and unexpected results occur." (Thompson Klein, 2004).
Consequently, there is the need to maintain the advantages of disciplinary organization, while forming professional figures able to manage complexity and integrate the different points of view (Thompson Klein, 2004, Barelli et al., in press.).
The interdisciplinarity and complexity's themes were already linked in 1972, when at the first international conference on interdisciplinarity, Eric Jantsch invoked the need to find a new approach able to manage "complex and dynamically changing situations" (Jantsch, 1972). Later on, in 1994, Ursula Hübenthal talked about the need to establish interdisciplinary collaboration since "problems are much too complex to be judged appropriately, much less solved, merely with the subject-knowledge of a single discipline." (Hübenthal, 1994).

Consequently, Thompson Klein (2004) suggests that knowledge itself should be rethought in light of nowadays complexity:
«Once described as a *foundation* or *linear structure*, knowledge today is depicted as a *network* or a *web* with multiple nodes of connection, and a *dynamic system*. The metaphor of *unity*, with its accompanying values of universality and certainty, has been replaced by metaphors of *plurality relationality* in a *complex* world. Images of *boundary crossing* and *cross-fertilization* are superseding images of disciplinary *depth* and *compartmentalization*. Isolated modes of work are being supplanted by, and *alliances*. And, older values of *control, mastery*, and *expertise* are being reformulated as *dialogue, interaction,* and *negotiation*. Changes in the spatial and temporal structures of knowledge also call into question traditional images of knowledge as a cognitive *map* with distinct *territories and borders* or a *tree with different branches.* They are too linear. In their place, images of *fractals*, a *kaleidoscope*, or a wildly growing *rhizome* without a central root have been proposed (Klein, 1999).» (Thompson Klein, 2004)
New terms for the organization of knowledge are thus required to consider the increasing importance that interdisciplinarity is having in our world. New taxonomies should not be intended as new, fixed parameters through which our present knowledge is reorganized, rather they should be conceived as fluid and mobile with change (Thompson Klein, 2010).
The reorganization of knowledge is linked to disciplines. Indeed, disciplines can be seen as classificatory means that allow us to recognize which aspects of knowledge vary in time, excluding those that are not relevant anymore (Weingart, 2010). Firstly, we should clarify what we mean with "discipline", since it is a term usually discussed in a stereotyped and not proper way (Linn, Eylon and Davis, 2004, p. 41). The term "discipline" contains the Latin root *discere*, which means "to learn". Thus, with discipline we intend a collection of knowledge or skills based on the social relationship between disciple and teacher (Alvargonzález, 2011). Accordingly, disciplines ground their roots into the educational necessity of reorganizing knowledge for the sake of teaching and learning it (Alvargonzález, 2011). This reorganization should lead students to develop epistemic skills such as problem-solving, representing, arguing, explaining, testing and sharing (Authors, 2019).
The FRA framework helps in this sense to enlarge the notion of discipline, characterizing it in a multidimensional perspective: a discipline is not only a bunch of knowledge and peculiar individual cognitive skills, but an organization with its own aims and values, practices, methods, that necessitate to consider a wider interaction within people and/or within institutions. At the same time, the FRA allows collaboration between people identifying in disciplines because of their background and personal attitudes. For this reason, the FRA has been a fundamental theoretical framework for this thesis, since it permits to elaborate the concept of interdisciplinarity by comparing the epistemic core of different disciplines, reflect on their identities, and investigate the complex dynamics that involve the dialogue between people with different attitudes and backgrounds (Authors, 2021). The interaction between people coming from different communities is framed in the theory of boundary crossing and boundary objects by Akkerman and Bakker (2011), to which the next section is dedicated.

1.3.2 The boundary metaphor

Compartmentalization of disciplines most of the time leads us to inhabit single domains, staying within boundaries of practices and domains of expertise. However, since the latter half of last century, more and more interdisciplinary fields are arising from cross-fertilizations of different subjects (Thompson Klein, 2004). This process is driven by: the intrinsically complex identity of society and nature; the exploration of problems or issues that cannot be easily delimited to one single discipline; the need to solve societal problems and the developing of new technologies (Thompson Klein, 2010). As a consequence, we find that many professional activities require multiple competences at the same time, challenging the concept of singular bounded domains and opening to the possibility of inhabiting the boundary (Akkerman and Bakker, 2011).

As Akkerman and Bakker (2011) explain in their paper, when we choose to place ourselves at the boundary we are in between different worlds, in a zone that belongs *neither* to one world *nor* the other. This nobody's land is characterized by boundary people, objects, and mechanisms that are ambiguous in nature. The intrinsic ambiguity of the boundary suggests its ability to link the different sides and, at the same time, mark a border between the different worlds. People living at the boundary carry points of view of the respective worlds, and at the same time can overcome the border since they have "an unspecified quality of their own (neither nor)" (Akkerman and Bakker, 2011). As Akkerman and Bakker (2011) highlight, it is the boundary's ambiguous nature that is appealing for education, since it incorporates multivoicedness (both-and) and an unspecified quality (neither-nor) at the same time. These two characteristics need to be discussed and renegotiated through a dialogue which potentially can lead to new knowledge.

1.3.3 Boundary people, boundary objects and boundary crossing mechanisms

Boundary people

People who inhabit the boundary zone are called boundary people, brokers, boundary crossers or boundary workers. Tanggaard defines them as marginal strangers "who sort of belong and sort of don't" (Tanggaard, 2007, p. 460). This ambiguity reflects the fact that people at the boundary may act as bridges between different worlds, with the privilege to introduce new valuable elements from one practice to another, but at the same time they are able to maintain the characteristics and divisions of the single worlds. To be able to enact as a boundary person, one should be able to sustain dialogues with different experts, meanwhile having inner dialogues where the different perspectives coexist and are explored (Akkerman and Bakker, 2011).

Boundary crossers' double nature is the reason why they are often seen by others as marginal people not belonging to any world and whose identity is not fully defined.

Boundary objects

Objects may also have a crucial role in crossing boundaries. These artifacts are characterized with an ambiguity similar to that of boundary people: usually they are adaptable enough to respect local needs of the singular worlds articulating different perspectives, but at the same time they are sufficiently robust to be positioned within different overlapping worlds maintaining a shared identity (Akkerman and Bakker, 2011). Boundary objects are not only ambiguous artifacts that can be viewed from different perspectives. On the contrary, they are "a means of translation" (Star and Griesemer, 1989, p 393), carrying the potential to create a common space in which different experts can encounter, have a dialogue and negotiate perspectives.

An interesting metaphor to understand the concept of boundary is to view it as a black box. The black box term comes from cybernetics, and it is used: "Whenever a piece of machinery or a set of commands is too complex. In its place they draw a little black box about which they need to know nothing but its input and output." (Latour, 1987, p. 2, 3). In a similar way, boundary objects are usually conceived and manipulated as invisible mediators between different worlds, without the need of questioning their meaning. But when we open and examine them by taking a closer look, we find that they hide many learning opportunities (Williams & Wake, 2007). The process of breaking down the black box is described by Latour:

«Take for instance an overhead projector, it is a point in a sequence of action (in a lecture say), a silent and mute intermediary, taken for granted, completely determined by its function. Now suppose the projector breaks down. The crisis reminds us of the projector's existence. As the repairmen swarm around it… we remember that the projector is made of several parts, each with its role and function and its relatively independent goals. Whereas a moment before the 'projector' scarcely existed, now even its parts have individual existence, each its own ‘black box’. In an instant, our projector grew from being composed of zero parts to one to many. » (Latour, 2009, p. 161).

Boundary crossing mechanisms

Boundary crossing happens when we enter a nobody’s land, a “territory in which we are unfamiliar and, to some significant extent therefore unqualified” (Suchman, 1994, p. 25). In this situation, we “face the challenge of negotiating and combining ingredients from different contexts to achieve hybrid situations” (Engeström et al., 1995, p. 319).

Akkerman and Bakker (2011) have studied the literature concerning learning processes at the boundary, and they recognized four *learning mechanisms* (displayed in Fig. 1.3): *identification*, *coordination*, *reflection*, and *transformation*. Excluding *identification*, the other three *learning mechanisms* are related to the effort of rethinking disciplines’ relationships and exchange, which possibly leads to innovation. In the following, I will describe in more detail each learning mechanism.

Identification

In the identification process, people encounter and reassemble boundaries between different practices. This may happen because the different domains are questioned due to feelings of threat (process of *othering*) or because they are showing increasing common aspects (process of *legitimating coexistence*). In the *othering* process people try to identify one practice by analyzing how it differs with respect to the other one. This may happen, for example, when people are part of different institutionalized domains, such as school and home, and are called to participate and compromise these worlds when they intersect, as it happens, for example, when a mother helps her son in doing homework (Akkerman and Bakker, 2011). In the *legitimating coexistence* process instead, people are involved in an overlap between different organizations, and they need to request their role in the intersecting site to be accepted by the other groups (Akkerman and Bakker, 2011).

Although the outcome of identification processes is not always to overcome the friction points, the processes lead people to acquire a new perception of the disciplines’ differences and identities.

Coordination

In coordination mechanisms, people allow practices to communicate and exchange through the boundary. This process does not aim to rebuild the boundary by confronting the differences between the practices, as it happens for the identification mechanism. Rather, it wants to facilitate the exchanges between the different worlds by smoothing the border and establishing a sense of continuity.

The coordination process requires a *communicative connection* between different practices and *efforts of translation* between the boundary worlds, which can both be achieved for example through boundary objects. At the same time, *enhancing boundary permeability* allows to not be aware of the differences between the practices at the boundary, since the passage at the border is made smooth and easy to pass. To enhance the permeability, the process of *routinization* seems to be particularly relevant, since it tries to smooth procedures at the border by automatizing practices (Akkerman and Bakker, 2011).

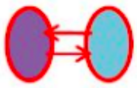
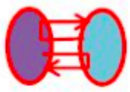
Visualisation of the learning mechanism	Aim of the learning mechanism	What questions to ask yourself to stimulate the learning mechanism
	Identification Gaining insight into complementarity and added value of the different practices around the boundary	• What expertise do I have? • What expertise do I lack in the context of the sustainability problem at hand? • Who are the stakeholders? • What is their expertise, stake and perspective? • How to they relate to each other?
	Coordination Collaboration to deal with the problem, but geared towards efficiency and working along each other (e.g. task division)	• How can I involve the different stakeholders? • How do I approach the different stakeholders? • How can we communicate and collaborate effectively? • What agreements do we make with each other? • What object can I use or develop to facilitate mutual communication
	Reflection Learning to see the problem through the eyes of another. Both defining and exchanging perspectives focused on mutual meaning making and connecting different perspectives and expertise.	• How do I help other stakeholders understand my perspective? • What can I learn from the perspectives of the other stakeholders involved? • What can we learn from each other?
	Transformation Development of new knowledge/practices; an end result that could not have been developed without actual collaboration and integration of perspectives.	• What is my vision on the new practice? • How can we combine our knowledge and perspectives into a (innovative, but realistic) solution? • How can I get others enthusiastic for this new practice? • How can I stimulate follow-up to build on the new practice (towards a sustainable new practice)?

Figure 1.4: Boundary crossing learning mechanisms (Gulikers and Oonk, 2019).

Reflection

When boundary crossing happens through reflection mechanisms, people are led to take a different perspective than one's own on the same issue. Thus, they return from the boundary zone with an enriched awareness of their own and others' practices. As for the coordination mechanism, boundary objects can be useful artifacts through which reflection may take place. Indeed, they lead to express one's knowledge and assumptions in the moment of explicit interpretation of the object (Akkerman and Bakker, 2011).

The outcome of reflection is not only comprehension, but also enrichment of one's perspectives and identity, in a process known as *perspective making* (Boland and Tenkasi, 1995). To collect different points of view, people are encouraged to express their understanding and knowledge on a certain topic, for example through a narrative. By sharing each personal perspective, it is possible to "reflexively give access to the implicit and unstated assumptions" (Boland and Tenkasi, 1995, p. 364). In order for the dialogue to be fruitful, the juxtaposition of the different perspectives is not enough. Rather, the attitude should be creative and lead to knowledge generation on the topic.

Another learning process that characterizes the *reflection* mechanisms is that of *perspective taking* (Boland and Tenkasi, 1995), which leads people to see themselves through the viewpoint of another person, who belongs to a different world. This process is particularly important for example in cross-cultural business negotiations, where people may perceive negotiation in a way that negatively affects the outcome if perspective taking is not present (Akkerman and Bakker, 2011).

Transformation

Transformation mechanisms are activated when discontinuities between the different worlds emerge and are not immediate to be overcome. These situations carry the opportunity of *confrontation* with the issue, which leads boundary domains to renegotiate their current practices and relationships, reinforcing back the identity of the respective worlds. As a consequence, transformation is different from other mechanisms, since it carries deep changes for the practices, that may be profound enough to make a new "boundary practice" emerge. For this reason, it is probably the most desirable learning mechanism at the boundary (Akkerman and Bakker, 2011).

A comparison with boundary zones may arise if there is a discontinuity in the flow of work, or if a third perspective appears. Confrontation may lead to *recognizing a shared problem space*, which needs to be solved collaboratively by the different people sharing the space.

When transformative processes take place, *hybridization* may happen, that is the creation of a new cultural form arising from the confrontation of different expertise. This hybrid form can be for example the creation of a new concept, an analytical model or an interdisciplinary field of science (Akkerman and Bakker, 2011). When the new hybrid is created, the process of *crystallization* may happen if it reifies, that is "congeal this experience into 'thingness'" (Wenger 1998, p. 58). The reification may lead, for example, to the development of new procedures that reflect the new creation. Akkerman and Bakker (2011) highlight that although *crystallization* is cited in many studies, it is rarely realized, proving that transformation at the boundary should not be considered as an obvious mechanism.

Hybridization does not necessarily mean that transformation leads to a completely new product that cannot be traced back to its sources. On the contrary, usually transformed practices help to reinforce back the source practices. Indeed, studies often denote the relevance of *maintaining uniqueness of the intersecting practices*, which stands for conserving the link with the initial fields. The balance between the creation of new practices and the conservation of their characteristics is described by Palmer:

«Interdisciplinary research requires a balance between established core knowledge and the infusion of new knowledge. As researchers explore new problem areas, they do not necessarily abandon their disciplinary concentrations. Most have dual or multiple agendas, building on a core research specialization as they transit into a newer hybrid area. » (Palmer, 1999, p. 250)

Finally, for the boundary crossing to be productive, a *continuous joint work at the boundary* is suggested, that is a process of continuous negotiation of meaning. Edwards and Mutton, in discussing how assessment schemes can be considered as boundary objects between different educational grades, give an idea of what may happen then joint work is lacking:

«Once the scheme has been worked on and it enters each system as a tool to be used within the system rather than a joint object to be worked on [between the systems], its potential to reconfigure practices may diminish. » (Edwards and Mutton, 2007, p. 508)

Final remarks

Kapon and Erduran (2021) used Akkerman and Bakker framework to confront three approaches to interdisciplinarity for science education, which were presented in the symposium "Crossing boundaries - Examining and problematizing interdisciplinarity in science education" at the ESERA 2019 conference. In the paper, three different cases where interdisciplinarity boundary crossing can be relevant for science education are presented:

- Schvartzer et al. (2019) problematize the engagement of learners when they are learning or using disciplinary knowledge in diverse interdisciplinary contexts and problems.
- Levi et al. (2019) studied the explanatory potential of interdisciplinarity for disciplinary-based problems.
- Authors (2019) focused on the inherent interdisciplinarity of STEM disciplines in discipline-based educational systems from a historical and curriculum development perspective.

In the present thesis, the boundary crossing framework is investigated in a hybrid context of teacher-students education, where the Master students have different backgrounds (physics, mathematics and natural sciences). In particular, it is used a networked framework, where the FRA and the boundary crossing mechanisms cooperate. It will be investigated whether and how the FRA triggers in teachers-students a process of recognition of disciplinary identities in their actual representation in institutions and in history. Moreover, it will be studied whether the networked framework helps to unpack, clarify and keep under control the process of construction and crossing of disciplinary boundaries.

Chapter 2 The module and the context of implementation

2.1 Module on the parabola and parabolic motion

The context where we investigated the effectiveness of the FRA framework in teacher-students education is the module on parabola and parabolic motion, which was implemented in the Master's course "Physics teaching: theoretical and experimental aspects" at the University of Bologna. The named module was designed within the EU project IDENTITIES, since it focuses on an interdisciplinary topic able to show the historical interplay between mathematics and physics and their development as disciplines.
The module was implemented for the first time in a course for in-service teacher education from November to December 2019, in collaboration with the project Piano Lauree Scientifiche (PLS) of Bologna. The course was entitled "Strumenti di analisi e comprensione del testo scientifico per l'interdisciplinarità: un confronto tra fonti e manuali su temi di fisica e matematica" (tr. "Tools of scientific text analysis and comprehension for interdisciplinarity: a comparison of sources and textbooks on physics and mathematics topics"). The course consisted of 4 meetings and involved experts in different disciplines from physics (professor Olivia Levrini and Paola Fantini) to mathematics education (professor Alessia Cattabriga and Laura Branchetti), from linguistics (professor Matteo Viale and Veronica Bagaglini) to linguistic philosophy (professor Sebastiano Moruzzi and Carlotta Capuccino).
The structure of the course was:

1. Introduction (Olivia Levrini). Presentation of the topic and the selected texts; their disciplinary analysis (Laura Branchetti, Alessia Cattabriga, Paola Fantini).
2. Lecture on "Linguistic tools for text analysis" (Veronica Bagaglini, Matteo Viale).
3. Lecture on "Epistemological tools for the argumentative analysis of texts" (Sebastiano Moruzzi, Carlotta Capuccino; tutor: Enrico Liverani, Alessia Marchetti, Elena Tassoni, Luca Zanetti).
4. Conclusive workshop with a debate on the possible contributions and implementations of the proposed analyses in the classroom (tutor: Eleonora Barelli, Enrico Liverani, Alessia Marchetti, Sara Satanassi, Elena Tassoni, Luca Zanetti).

From the first implementation the module was refined with IDENTITIES: the target group became future teachers, while the focus on the physics-mathematics interdisciplinarity and linguistic aspects was maintained. The module has been implemented four times: two times within the course "Physics teaching: theoretical and experimental aspects" of the Master's Degree in Physics, University of Bologna (in the years 2020, 2021, from October to November), one time within the course "Mathematics Education" of the Master Degree in Mathematics, University of Milan (from November to December 2021) and within the first international Summer School of the IDENTITIES project (from June 28th to July 2nd, 2021).
The entire module is structured in six sub-modules and was designed according to the Study and Research Path to Teacher Education (SRP-TE model; Barquero, Bosch & Romo, 2018), framed within the Anthropological Theory of Didactics (ATD) developed by Chevallard and colleagues (Chevallard, 1985; Bosch & Gascon, 2006). To innovate the mathematics teaching, Chevallard (2015) proposed the "questioning the world" paradigm, whose main point is that knowledge is associated with a dynamic exploration of relevant questions. In particular, the SRP model for teacher education aims to equip teachers with inquiry-based study processes, meanwhile encouraging them to question the taught knowledge.
In the parabola and parabolic motion module, this approach was chosen for its innovative aspects, and to break consolidated narratives on physics and mathematics. Throughout the module, the

participants are asked to play different roles: interdisciplinary explorers, students and analysts (Fig. 2.1).

In the first part of introduction to the module, student-teachers play the role of "interdisciplinary explorers" since they are asked to reflect on the boundary metaphor and to share personal experiences of boundary crossing with the other participants. Afterwards, in the following sub-modules, the participants assume the role of "students" since they are presented to two historical episodes that were fundamental for the development of physics and mathematics as disciplines: the discovery of the parabolic shape of a bullet motion's trajectory, and the reclassification of conic sections by Kepler. The latter allowed to describe the parabola through her focal properties, likewise all the other conics, by considering parallelism as a special case of incidence. The historical episodes were chosen as exemplary cases of interdisciplinarity: on one hand, it is possible to see how mathematics begins to structure the physical reasoning, and on the other, it emerges how physics helped to overcome an epistemological barrier in mathematics.

In the fourth and fifth sub-modules student-teachers play the role of "analysts" and are guided to analyze textbook excerpts on the theme "parabola and parabolic motion", both from a linguistic and epistemological perspective. The analysis' purpose is to meditate both on disciplinary and interdisciplinary aspects, and to reflect on the epistemological core of physics and mathematics as disciplines. Finally, the last sub-module includes a wrap-up of the entire module, with the aim of rethinking the initial open questions.

In the following, I will present a summary of the sub-modules' content about the history of mathematics and physics.

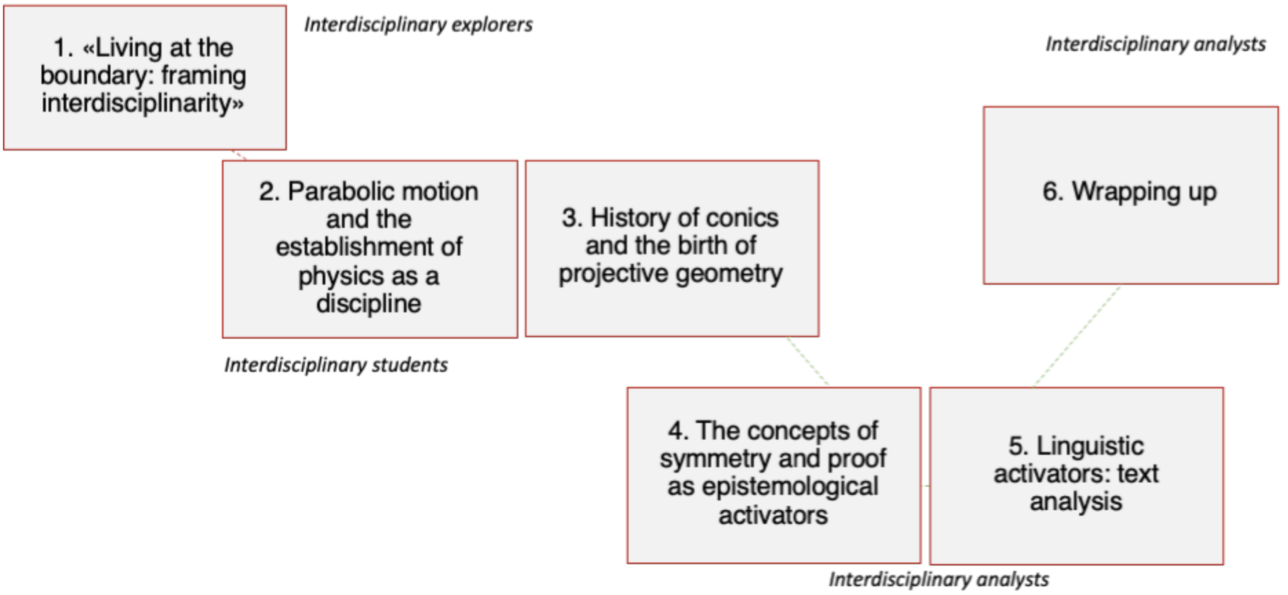


Figure 2.1: The module on the parabola and parabolic motion.

Parabolic motion and the establishment of physics as a discipline
The submodule on the "Parabolic motion and the establishment of physics as a discipline" aims to analyze the role played by the parabola in the birth of modern physics, which is framed within the scientific revolution of the XVII Century (Renn et al., 2001). Indeed, the parabolic motion had a fundamental role in changing the cosmological vision (from medieval to modern), in the process of unification of "the earth and the skies", and in the mathematization of the "sublunary world" (Koyré,

1939; Renn et al., 2001). In particular, in the submodule it emphasizes the revolutionary aspect of the bullet motion's decomposition into two more basic motions: the uniform rectilinear motion and the uniformly accelerated motion. This distinction represented a deep ontological change in the description of the world. Indeed, the decomposition overcame the medieval distinction between the natural and violent motions and led to a new conceptualization of the relationship between matter, space and time (for a summary of the historical episode for educational purposes see Gilbert & Zylbersztajn, 1985).

The Aristotelian paradigm has been at the basis of the scientific and philosophical theories, presented to society and intellectuals, for many centuries. At the end of the XVI century, the inquiry on the bullet's motion was still ongoing, and the mathematization of terrestrial phenomena had just begun. In this context, the distinction between violent and natural motions has been progressively affected, and consequently, the previous interpretative schemes were reconsidered.

According to the Aristotelian vision, the allowed interpretative schemes for the bullet's motion were the irreducible figures of the straight and circular line. Moreover, natural and violent motions could not overlap: one of the two would always win with respect to the other, conforming to the dominance principle. Consistently, in Tartaglia's representation, the projectile's trajectory is composed of three parts: an initial straight part, followed by a section of a circle, which ends in a straight vertical line (displayed in Fig. 2.2). The shape of this trajectory derives from the conception of an initial dominant violent motion (AB), followed by a mixed one (BC), compounded of a decreasing violent motion in the original direction together with the natural downward vertical motion, and a final part (CD) in which the bullet's weight wins over the violent motion, leading the projectile to direct towards Earth's center (Renn et al., 2001). The projectile's motion is asymmetric and sequential: the violent and natural motion hinder each other, according to a typical medieval idea of "force fight". The lack of symmetry suggests that Tartaglia's trajectory is not a sketch derived from experience. On the contrary, it is as a projection on paper of the Aristotelian theories on motion. Moreover, the asymmetric trajectory conceals the idea that the sublunar world could not be described through a mathematical language: the two motions could not be compounded to form a geometric curve.

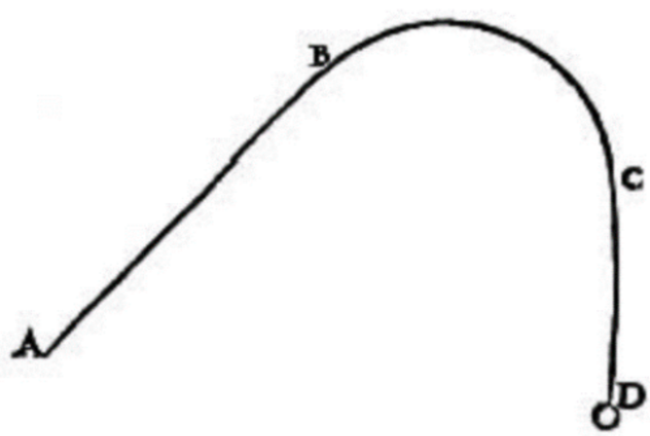


Figure 2.2: Tartaglia's representation of the projectile motion in Nova Scientia (1537).

The discovery of the decomposition of the projectile's motion in two distinct ones views as main protagonists Guidobaldo del Monte and Galileo Galilei, whose texts were analyzed in the module to recognize the epistemological core elements of their discourses.

A first analysis concerns the inspection of the underlying extract from Guidobaldo's notebook (1952), which contains an objective and experience-based description of the bullet's motion, accompanied by the explanatory drawing represented in Fig. 2.3:

«If one throws a ball with a catapult or with artillery or by hand or by some other instrument above the horizontal line, it will take the same path in falling as in rising, and the shape is that which, when inverted under the horizon, a rope makes which is not pulled, both being composed of the natural and the forced, and it is a line which in appearance is similar to a parabola and hyperbola. And this can be seen better with a chain than with a rope, since [in the case of] the rope ABC, when AC are close to each other, the part B does not approach as it should because the rope remains hard in itself, while a chain or little chain does not behave in this way. The experiment of this movement can be made by taking a ball colored with ink, and throwing it over a plane of a table which is almost perpendicular to the horizontal. Although the ball bounces along, yet it makes points as it goes, from which one can clearly see that as it rises so it descends, and it is reasonable this way, since the violence it has acquired in its ascent operates so that in falling it overcomes, in the same way, the natural movement in coming down so that the violence that overcame [the path] from B to C, conserving itself, operates so that from C to D [the path] is equal to CB, and the violence which is gradually lessening when descending operates so that from D to E [the path] is equal to BA, since there is no reason from C towards DE that shows that the violence is lost at all, which, although it lessens continually towards E, yet there remains a sufficient amount of it, which is the cause that the weight never travels in a straight line towards E.»[1] (Renn et al., 2001, pp. 15−16)

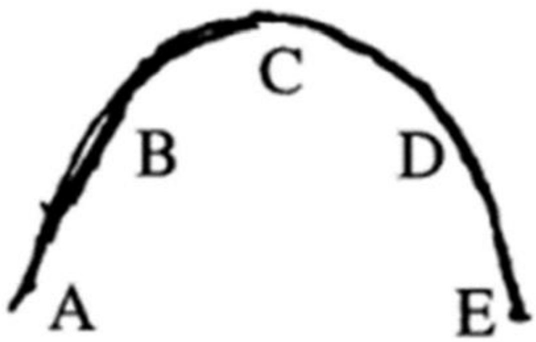


Figure 2.3: Guidobaldo's representation of the projectile motion (1592).

This excerpt represents an important step towards a paradigm shift from the medieval vision. Firstly, Guidobaldo makes use of an objective method based on experience, looking for regularity aspects and opening to the possibility that the mathematical language can be used to investigate nature. Indeed, he recognizes that the trajectory is symmetric, and he looks for the mathematical curve able

[1] Tr. «Se si tira una palla o con una balestra o con artiglieria, o con la mano, o con altro instrumento, sopra la linea dell'horizonte, il medesimo viaggio fa nel callar che nel montare e la figura è quella che rivoltata sotto la linea horizontale fa una corda che non stia tirata, essendo l'un e l'altro composto di naturale e di violento et è una linea in vista simile alla parabola et hyperbole e questo si vide meglio con una catena che con una corda, poiché la corda ABC, quando AC sono vicini la parte B non si accosta come dovrebbe perché la corda resta in sé dura. Che non fa così una catena, o catenina. La esperienza di questo moto si po' far pigliando una palla tinta d'inchiostro, e tirandola sopra un piano di una tavola, il qual stia quasi perpendicolare all'horizonte, che se ben la palla va saltando, va però facendo li punti, dalli quali si vede chiaro che sicome ella ascende così anco descende ... la violentia che ella ha acquistato nell'andar sù, fà che nel callar vada medesimamente: superando il moto naturale nel venir giù ... essendo che non ci è ragione che dal C verso DE mostri che si perda a fatto la violentia ... La violenza che ha superato [il percorso] da B a C, conservandosi, opera in modo che [il percorso] da C a D sia uguale a CB, e la violenza che diminuisce gradualmente quando si discende opera in modo che [il percorso] da D a E sia uguale a BA, poiché non c'è motivo da C verso DE che dimostri che la violenza sia completamente persa, ma che, sebbene diminuisca continuamente verso E, tuttavia ne rimanga una quantità sufficiente, che è la causa per cui il peso non viaggia mai in linea retta verso E.» (from Cerreta, 2019)

to best fit the motion. Although Guidobaldo identifies a similarity with the parabola and the hyperbola, he seems not ready to exactly juxtapose the physical trajectory of the object with the mathematical curve. Indeed, on an ontological level the geometric curve materializes in a catenary, the latter becoming a concept able to link mathematics with physical experience. To explain the observed symmetry, Guidobaldo hypotheses that during the object's falling and rising, natural and violent motion overlap without hindering each other, leading in this way to a symmetric curve. To test this hypothesis, he designs an experiment to collect data and register the trajectory's curve, which confirms his initial thoughts.

The above analysis aims to show that the mathematical representation of the parabolic motion played a central role in developing a scientific method based on experiments and empirical evidence, paving the way to modern physics' birth.

The mathematization of physical processes continues in Galileo Galilei's "Discorsi e dimostrazioni matematiche intorno a due nuove scienze" (tr. "Dialogues concerning two new sciences" by Crew & De Salvio (1914)). This work develops as a dialogue within three characters: Salviati, Simplicio and Sagredo. Salviati argues for the Copernican vision and is the spokesman of Galileo's thoughts; Simplicio presents the traditional, medieval point of view on scientific topics; Sagredo is the mediator between Salviati and Simplicio and presents a neutral point of view.
The book is structured in four days:

1. First day: *First new science, treating of the resistance which solid bodies offer to fracture.*
2. Second day: *Concerning the cause of the cohesion.*
3. Third day: *Second new science, treating of motion.*
4. Fourth day: *Violent motions. Projectile.*

At the end of the second day, Galileo describes a rigorous method to draw a parabola:

«There are many ways of tracing these curves; I will mention merely the two which are the quickest of all. One of these is really remarkable; because by it I can trace thirty or forty parabolic curves with no less neatness and precision, and in a shorter time than another man can, by the aid of a compass, neatly draw four or six circles of different sizes upon paper. I take a perfectly round brass ball about the size of a walnut and project it along the surface of a metallic mirror held in a nearly upright position, so that the ball in its motion will press slightly upon the mirror and trace out a fine sharp parabolic line; this parabola will grow longer and narrower as the angle of elevation in- creases. The above experiment furnishes clear and tangible evidence that the path of a projectile is a parabola […]. In the execution of this method, it is advisable to slightly heat and moisten the ball by rolling in the hand in order that its trace upon the mirror may be more distinct. »[2] (Crew and De Salvio, 1914, pp. 148−149)

[2] Tr. «Modi di disegnar tali linee ce ne son molti, ma due sopra tutti gli altri speditissimi glie ne dirò io: uno de i quali è veramente maraviglioso, poiché con esso, in manco tempo che col compasso altri disegnerà sottilmente sopra una carta quattro o sei cerchi di differenti grandezze, io posso disegnare trenta e quaranta linee paraboliche, non men giuste sottili e pulite delle circonferenze di essi cerchi. Io ho una palla di bronzo esquisitamente rotonda, non più grande d'una noce; questa, tirata sopra uno specchio di metallo, tenuto non eretto all'orizonte, ma alquanto inchinato, sì che la palla nel moto vi possa camminar sopra, calcandolo leggiermente nel muoversi, lascia una linea parabolica sottilissimamente e pulitissimamente descritta, e più larga e più stretta secondo che la proiezzione si sarà più o meno elevata. Dove anco abbiamo chiara e sensata esperienza, il moto de i proietti farsi per linee paraboliche [...]. La palla poi, per descrivere al modo detto le parabole, bisogna, con maneggiarla alquanto con la mano, scaldarla ed alquanto inumidirla, ch́e così lascerà più apparenti sopra lo specchio i suoi vestigii.» (from Galilei, 1638, pp. 145−146)

From a methodological point of view, Galileo carefully describes the drawing procedure, in order to make it easily replicable. Actually, more than describing, Galileo is defending with rigor his remarkable method to trace a parabola.
Reading the excerpt, we can perceive a shift with respect to Guidobaldo: Galileo does not need to materialize the geometric curve through a catenary, and he makes the object's trajectory directly coincide with that of a geometric curve. For this reason, the trajectory is no more a rough sketch, but a fine and sharp parabolic line, legitimizing in this way mathematics to describe sublunar phenomena likewise celestial ones.

At the beginning of the third day, Galilei presents with great precision the physical procedure (represented in Fig. 2.4) through which it is possible to obtain a semi parabolic curve:

«Let us imagine an elevated horizontal line or plane ab along which a body moves with uniform speed from a to b. Suppose this plane to end abruptly at b; then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular bn. Draw the line be along the plane ba to represent the flow, or measure, of time; divide this line into a number of segments, bc, cd, de, representing equal intervals of time; from the points b, c, d, e, let fall lines which are parallel to the perpendicular bn. On the first of these lay off any distance ci, on the second a distance four times as long, df; on the third, one nine times as long, eh; and so on, in proportion to the squares of cb, db, eb, or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from b to c with uniform speed, it also falls perpendicularly through the distance ci, and at the end of the time-interval bc finds itself at the point i. In like manner at the end of the time-interval bd, which is the double of bc, the vertical fall will be four times the first distance ci; for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space eh traversed during the time be will be nine times ci; thus it is evident that the distances eh, df, cl will be to one another as the squares of the lines be, bd, bc. Now from the points i, f, h draw the straight lines io, fg, hl parallel to be; these lines hl, fg, io are equal to eb, db and cb, respectively; so also are the lines bo, bg, bl respectively equal to ci, df, and eh. The square of hl is to that of fg as the line lb is to bg; and the square of fg is to that of io as gb is to bo; therefore the points i, f, h, lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola. Q.E.D.»[3] (Crew and De Salvio, 1914, pp. 248−250)

[3] Tr. from Latin: «Si intenda la linea orizzontale ossia il piano ab posto in alto, e un mobile si muova su di esso da a in b di moto equabile; mancando ora il sostegno del piano in b, sopravvenga al medesimo mobile, per la propria gravità, un moto naturale deorsum secondo la perpendicolare bn. Si intenda inoltre che la linea be, la quale prosegue il piano ab per diritto, rappresenti lo scorrere del tempo, ossia [ne costituisca] la misura, e su di essa si segnino ad arbitrio un numero qualsiasi di porzioni di tempo eguali, bc, cd, de; inoltre dai punti b, c, d, e si intendano condotte linee equidistanti dalla perpendicolare bn: sulla prima di esse si prenda una parte qualsiasi ci; sulla [linea] successiva se ne prenda una quattro volte maggiore, df; [sulla terza,] una nove volte maggiore, eh; e così di séguito sulle altre linee secondo la proporzione dei quadrati delle [porzioni di tempo] cb, db, eb, o vogliam dire in duplicata proporzione delle medesime. Se poi intendiamo che al mobile, il quale si muove oltre b verso c con moto equabile, si aggiunga un movimento di discesa perpendicolare secondo la quantità ci, nel tempo bc [esso mobile] si troverà situato nell'estremo i. Ma continuando a muoversi, nel tempo db, cioè [in un tempo] doppio di bc, sarà disceso per uno spazio quattro volte maggiore del primo spazio ci; abbiamo infatti dimostrato nel primo trattato, che gli spazi percorsi da un grave, con moto naturalmente accelerato, sono in duplicata proporzione dei tempi: e parimenti, il successivo spazio eh, percorso nel tempo be, sarà nove [volte maggiore del primo spazio]: sì che risulterà manifesto che gli spazi eh, df, ci stanno tra di loro come i quadrati delle linee eb, db, cb. Si conducano ora dai punti i, f, h le rette io, fg, hl, equidistanti dalla medesima eb: le linee hl, fg, io saranno eguali, ad una ad una, alle linee eb, db, cb; e così pure le linee bo, bg, bl saranno eguali alle linee ci, df, eh; inoltre il quadrato di hl starà al quadrato di fg come la linea lb sta alla bg, e il quadrato di fg starà al quadrato di io come gb sta a bo; dunque, i punti i, f, h si trovano su un unica e medesima linea parabolica. Similmente si dimostrerà che, preso un numero qualsiasi di particole di tempo eguali di qualunque grandezza, i punti, che il mobile mosso di un simile moto composto occuperà in quei tempi, si troveranno su una medesima linea parabolica. È dunque manifesto quello che ci eravamo proposti.» (Galilei, 1638, pp. 241−242)

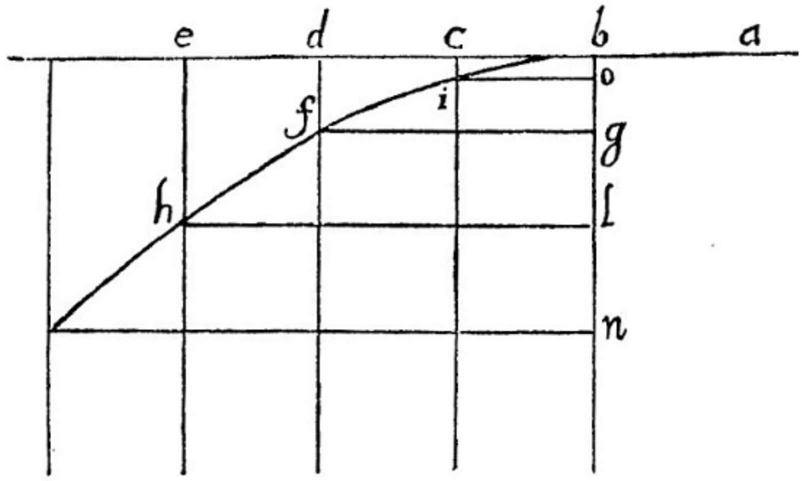


Figure 2.4: Geometrical structure of Galileo's proof (Galilei, 1638, p. 242).

What emerges from this excerpt is that Galileo uses as a fundament for his reasoning a rigorous mathematical proof, asserting that the projectile's motion is compounded by the equable (uniform rectilinear) on the horizontal, and the uniformly accelerated motion on the vertical. In this way, the distinction between natural and violent motions inevitably disappears. Moreover, as it emerges from the structure of Galileo's proof, mathematics appears to have a foundational role in the development of physics as a discipline, contrarily to the approach of many contemporary physics textbooks, where mathematics is treated as a "mere tool to describe and calculate" (Karam, 2015).
From the above analyses, we realize how symmetry and proof activated a new way to look at terrestrial phenomena. Indeed, the named terms emerge as fundamental concepts through which modern physics started to develop. The term "proof" and its different meanings are studied in more detail in the third submodule, where the history of conics and the birth of projective geometry is presented. The description of this part is the object of the next section. Taking a step further, in the fourth and fifth submodule, a comparative analysis of Galileo's historical text and a textbook treatment on the parabolic motion is carried out. In particular, the concepts of symmetry (firstly observed by Guidobaldo) and proof (realized by Galileo) are here conceived as boundary objects and activators of the linguistic and epistemological dimension. The activation of the latter permits to extract and compare the epistemic disciplinary cores of physics and mathematics, recognizing the interplay the two disciplines had in the historical case. The role of symmetry and proof as epistemological activators emerges by comparing the previously presented pictures (Fig. 2.5).

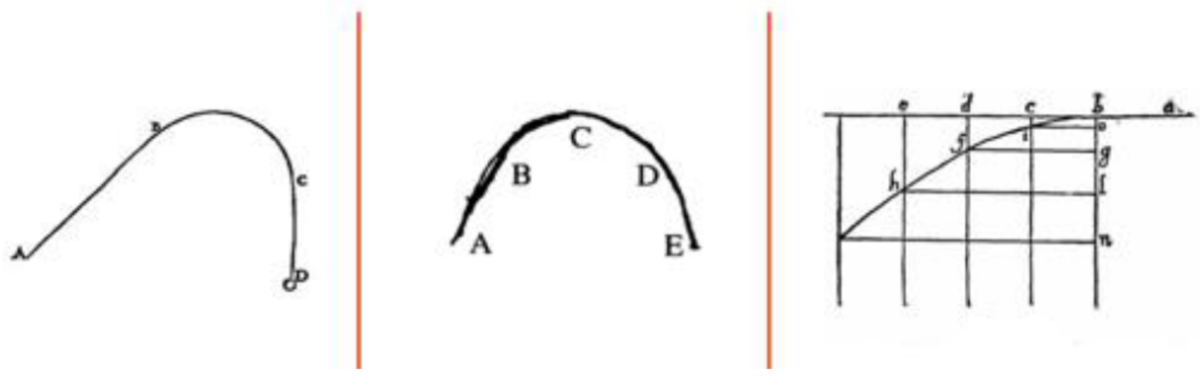


Figure 2.5: Symmetry and proof as epistemological activators.

From left: Tartaglia's representation of the projectile motion in his Nova Scientia (1537); the sketch from Guidobaldo's notebook (1592); the geometrical structure of Galileo's proof (Galilei, 1638).

History of conics and the birth of projective geometry

The third submodule focuses on the parabola from a mathematical historical perspective. Firstly, an analysis of the Pythagorean theorem is carried out, to delineate criteria for the analysis of Galileo's proof on the parabolic motion. This point should help disrupt the common prejudice that mathematics is just an instrument, by emphasizing its identity as a discipline and its structural role for physics.

In this submodule, the development of the definitions and theorems concerning conic sections is also presented, highlighting the contributions that now belong to the physics' discipline. Conic sections were considered the most important curves, after straight lines and circles, used to solve geometrical problems. The reference for their study until the XIX century has been the work "Conics" by Apollonius of Perga (262-190 b.C.), which is conceived as a masterpiece in the history of geometry. Apollonius begins his work with two sets of geometrical objects' definitions that will be used to demonstrate the succeeding sixty propositions and corollaries. The latter refer to the construction of the different curves (e.g. the parabola, the hyperbola, the ellipse and the circumference) using a secant plane to the cone. Apollonius presents the generation of the different curves by explaining their properties and clarifying his demonstrations through figures. His work can be considered the generalization of the results that Euclid collected in the 11th book of his "Elements". Similarly to Euclid, Apollonius makes use of an argumentative and rigorous structure, which reflects the characteristics of ancient Greek geometry. This structure will contribute to the development of the mathematical and physical knowledge. However, Apollonius also added many innovative features to Euclid's work and presents a different approach. For example, Euclid defines the conics by taking different cones generated by different triangles, while Apollonius constructs the conics by taking different section of the same cone (which is a double cone with a circular base).

In the submodule, it is presented the construction of the parabola contained in Apollonius' work. The parabola is built by intersecting a cone with a parallel plane that passes through the directrix (Fig. 2.6). This construction is consistent with the definition of the term parabola, which derives from the Greek word parabolé, and means "to flank, to parallel".

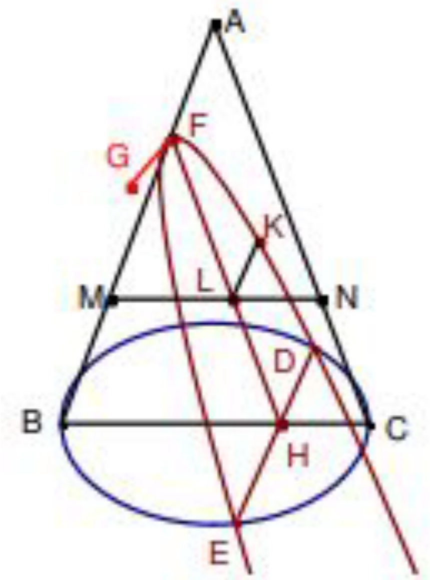


Figure 2.6: The parabola's sign.

It is also pointed out that the construction of conics is achievable with the use of mathematical machines too, which is linked to the characterization of conics in terms of focal properties. In this respect, Apollonius treats the parabola as a special case, and he does not include it in the discussion on focal properties, since he considers "the lines that, reflected, converge towards the focus" as parallel. To consider that those parallel lines intersect in a "point at infinity" is not a trivial passage, since it touches the epistemological barrier of the concept of infinity, an issue known since the

Ancient Greeks. As a consequence, no mathematical machine allowed the construction of the parabola and centuries passed until this issue could be solved.

Approaching the 13th century, the conics had been increasingly studied since they were fundamental for many researches in the physics' field (e.g. optics). As an example, Witelo in his "Optics" (1270) studies the conics' focal properties in the context of burning mirrors (whose surfaces have the shape of a rotating paraboloid). In this context, the contribution of Kepler was fundamental since he found a way to define the parabola similarly to other conics while studying optical phenomena. Kepler considered analogies as fundamental tools to investigate nature. Indeed, he was looking for an analogy between refraction and reflection using Witelo's idea of perceiving the conic section as a mirror. His analogical reasoning led him to associate parallel lines with the special case of incident lines intersecting at infinity, thus recognizing the parabola as a link between the ellipse and the hyperbola and unifying all the conics under the focal properties' definition (Fig. 2.7).

This historical narrative once again shows the mutual interaction between mathematics and physics. In particular, it highlights the role that physics had for mathematics in overcoming the epistemological barrier on the concept of infinity.

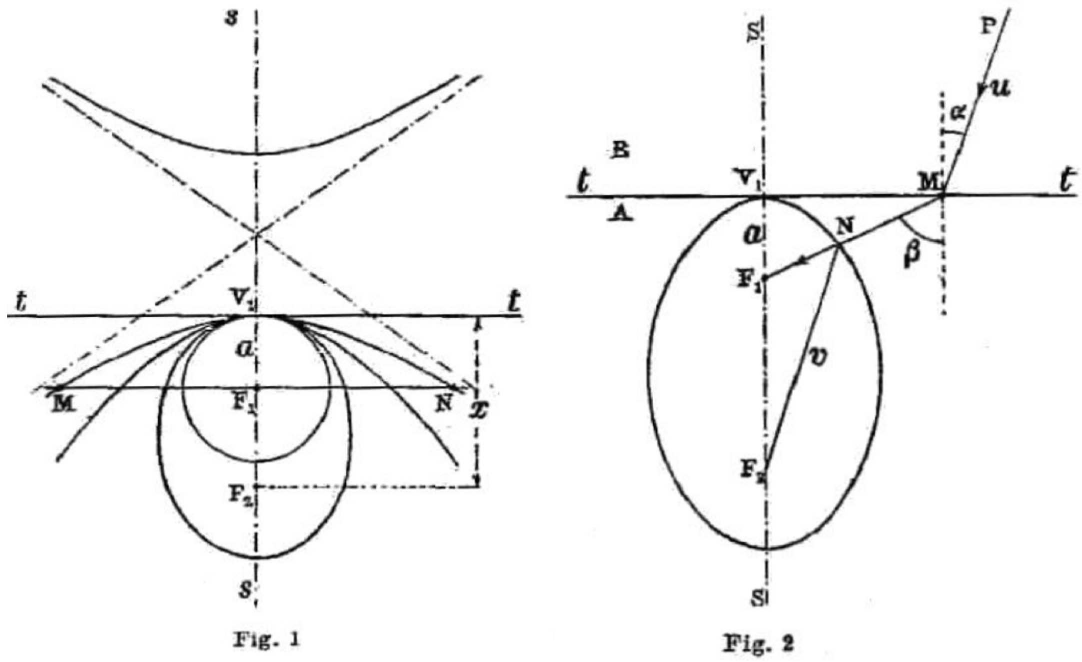


Figure 2.7: Kepler's contribution to conic's theory.

On the left: The different conics can be obtained through a continuous plane deformation (from Viola). On the right: Rotating ellipsoid with analogy between reflection and refraction.

The FRA framework in the module's design

In implementing the module, the FRA and the boundary crossing framework were used as teaching tools to regenerate the potential and richness of the previously described historical episodes, which have been progressively impoverished through school narratives. Both the frameworks helped in triggering a reflection on epistemological aspects of mathematics and physics as disciplines and on their mutual interaction.

In particular, the use of the FRA perspective in the texts' analyses allowed to highlight characterizing traits of physics as a discipline, thus helping its epistemic core to emerge. Indeed, both Guidobaldo and Galilei take particular care not only in reporting the results of their investigation, but also in explicating the practices they implement in their approach, the aims and values that guide their inquiry, and the methodological rules they follow. As a result, the FRA framework allows to keep track of the various dimensions that emerge in the historical case, differently from what usually happens by reading physics textbooks, where the processes of the scientific inquiry are often hidden, and a unidimensional handling prevails. Moreover, the FRA wheel was used to recognize the innovative aims and values that characterize the legitimation of mathematical descriptions for the sublunar world. Finally, the FRA wheel was used in discussing practices and methods, in reflecting on the different kinds of knowledge (such as beliefs, evidence-based, mathematical engines that lead to knowledge…) and it triggered reflections on the different kinds of knowledge's organization (e.g. the Euclidean model of theory, models…).

2.2 Context of implementation and the FRA in the module's activities

Context of implementation

The classroom data analyzed for the study were collected in the Master's course in "Physics teaching: theoretical and experimental aspects", in particular within the module on parabola and parabolic motion for student-teachers, which was designed and implemented for the EU project IDENTITIES, as previously described.

The module was implemented between October and November 2021, and the implementation lasted 24 hours. The classroom was composed of 57 university students with different backgrounds (Fig. 2.8). The majority of them were undergraduate students in physics (20) and students of the curriculum in history of physics and physics education (17) and of the curriculum in mathematics education (15). Other participants were enrolled in curricula of natural sciences, in the physics master or in didactics and communication of natural sciences (DICONS).

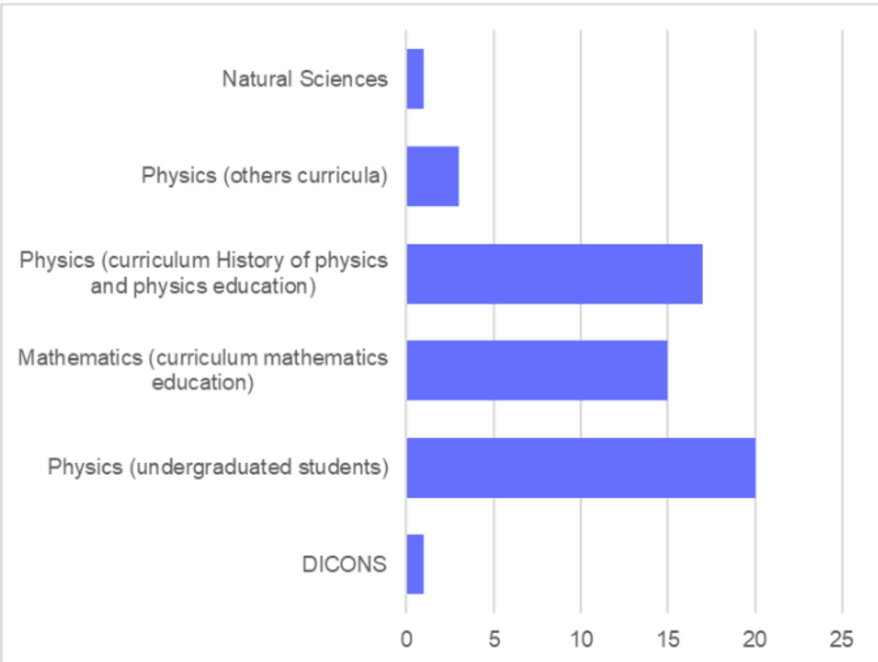


Figure 2.8: Students' background.

The FRA and NOS in the module's activities

The FRA and the NOS have been a constant presence throughout the previously described module. In order to make visible their contribution to the lectures, a selection of data was carried out, by isolating the moments in which the FRA framework or the NOS were explicated to the students. In Table 2.1 are shown the data collected during the module's implementation, chronologically ordered. For each activity (row) it is present:

- An ID abbreviation (e.g. F1).
- The type of data collected.
- The main requests or prompts.
- The number of items collected.
- The contribution to the analysis part studies.

Table 2.1: Data collected during the implementation.

ID	Data type	Main requests or prompts	Number of items	Contribution to part studies
B1	Closed and open-ended responses to the questions	What emotions does the word "epistemology" trigger? (perplexity/curiosity/nothing specifically) If you are perplexed about it, which words do you associate with the word epistemology? If you are curious about it, which words do you associate with the word epistemology?	46	/
B1_cd	Transcript of collective discussion	Discussion about the word clouds of the previous activity	1	(2)
B2	Items on a virtual board	How can we inquire about the issue "what is science?"? What kind of questions (simpler and more addressable) can we ask to investigate the issue?	34	
F1	Open-ended responses to a questionnaire organized in a word cloud	Which values and aims of science do you feel close to and motivate your choice for a scientific curriculum? Which activities/practices do you think belong to science? Which methodological aspects characterize science?	38	(1)
F1_cd	Transcript of collective discussion	Collective discussion about the word clouds obtained in the previous activity.	1	(1)
T1.1	Essays	Choose a criterion that is suitable to you and reorganize in clusters the responses of the virtual board (B2).	33	(2)
T1.2	Closed and open-ended responses to a questionnaire	Before this experience, had you ever reflected on the NOS? To what extent do you think you understood the following aspects of the FRA that characterize its way of dealing with the NOS (the choice of characterizing and not defining what science is; the articulation of the epistemic core; the meaning of aims and values, methods, practices, and knowledge); the external wheels)? Which aspects did you find most interesting or touched you most?	43	(1)
T2	Essays	• What structure of the argument emerges? Reformulate the underlying reasoning of the chapter by highlighting the structure of the argument.	37	/

		• What role is attributed to mathematics in the text? Are there typical features of mathematics as a discipline? If so, which ones? That is, referring to the FRA model, do aims & values, methods, practices, knowledge emerge? If so, which ones? • Which characteristics of physics (referring to the FRA model: aims & values, methods, practices, knowledge) as a discipline emerge?		
F2	Transcript of collective discussion	How was the FRA wheel useful to analyze the textbook? Which aspects of physics as a discipline did it contribute to identify?	1	/
F3	Transcript of collective discussion	Do you have any doubts about the FRA wheel? What is the contribution of its external parts (regarding the socio-institutional systems) to the characterization of science?	1	/
T3	Essays	• What added value to knowledge and reasoning can the collocation of the study of motions and conics in the "Great history of physics and mathematics" give? • What added value to knowledge and reasoning can give looking at the study of motions and conics in an interdisciplinary perspective? • What aspects and characteristics of physics and mathematics as disciplines can be brought to light if this "historical case" is analyzed by means of FRA? Consider the internal FRA wheel - both the core represented by the "cognitive-epistemic system" (aims and values, practices, methods and methodological rules and knowledge) and the "socio-institutional system". If you want, you can use a table to report the FRA aspects that you have recognized as characterizing physics and mathematics. • In what sense can the curve and the proof be cases of "boundary objects" and / or cases of "epistemological activators"? What border "crossing mechanisms" can be put in place? How?	37	(3)

Chapter 3 Methods

3.1 Tools of data collection

To analyze the process of appropriation of the FRA framework in the module, different kinds of data were collected throughout the activities. These include:

- videorecording of all the lectures.
- open questionnaire.
- team works products and surveys (e.g. Wooclap word clouds, jamboards).
- written essays.

The variety of tools for the data collection was useful to check them one another in order to validate evidence, and to evaluate the extent to which all evidence converges (Anfara et al., 2002).

3.2 Data processing

After the dataset collection, the data were pseudo-anonymized: this allowed us to analyze the data objectively, while being able to resume the correspondence with the students if necessary.
According to the research questions, we were interested in investigating the impact of the FRA framework in student-teachers, thus, as a criterion for the selection of data, we chose the module's activities where the reference to the FRA or NOS was explicit. The data were then checked and revised using the professor's syllabus of the daily lectures. We organized the dataset according to the date, the topic, and the aim of the activity. Particular care has been taken in checking all the recordings and listing them according to the data of creation and the content. The recordings' transcription was performed during the analysis only for the lectures and activities that seemed to us particularly rich in terms of students' engagement and discussion. The first product of these processes is reported in Table 3.1, which represents a first raw timeline of the FRA and NOS thread throughout the module. The timeline was later refined in the already presented table 2.1 (chapter 2). Each activity is identified through an ID (e.g. F1), in order to make it easily and univocally trackable inside the module. The ID letters' specific meaning will be introduced in the next section.

Table 3.1: Initial dataset organization.

Date and ID	Topic	Aim	Dataset
23.09.2021 B1	Brainstorming on the word epistemology	Collect students' starting point on epistemology. (impressions, previous knowledge)	Epistemologia (INS 21-22).pdf Same results, in Excel file: ZTDJGE_EPISTEMOLOGIA_insegnamento_21-22-results.xlsx Recording 4
27.09.2021 B2	Brainstorming on how to think about NOS	Main point: the distinction between "defining something or searching for a definition" and "characterizing an issue, searching for features"	Sulla scienza (corso di INS 2021-22).pdf No recording

30.09.2021 F1	Introduction of the FRA approach to NOS	Collect students' initial thoughts and feelings on the FRA approach	NOS e Introduzione alla ruota FRA 21-22.pdf Recording 7
Kinematics pause			
11.10.2021 T1.1 T1.2 T2	Clustering and questioning, FRA questionnaire, textbook analysis	Collect data on the initial state: epistemological ideas (clustering); FRA comprehension; Analysis of the textbook chapter on parabolic motion (Walker)	Task 1.1- Clustering and questioning Task 1.2- FRA questionnaire Task 2- Text analysis No recording
8.11.2021 F2	FRA wheel to analyze a text, final tasks	Collect final feelings/thoughts on the module. How to use FRA wheel to analyze a text (example: Walker)	Recording 19
11.11.2021 F3	External parts of FRA wheel	Collecting final doubts on FRA wheel, reasoning on its external parts	Recording 20
End of module-exam T3	Last task	Final analysis of the historical case through the FRA, searching for main characteristics of physics and math as disciplines	Task 3

3.3 Data management

Before stepping into the analysis, the activities reported in Table 3.1 were translated into something more visual, that could guide the research throughout the module. To achieve this, the reflection on the NOS and the FRA was articulated using the spiral shape displayed in Fig. 3.1. This shape was chosen since it suggested the idea that the FRA framework gradually supported students to delve into the surface of physics, explore its epistemological core and its profound disciplinary characteristics.

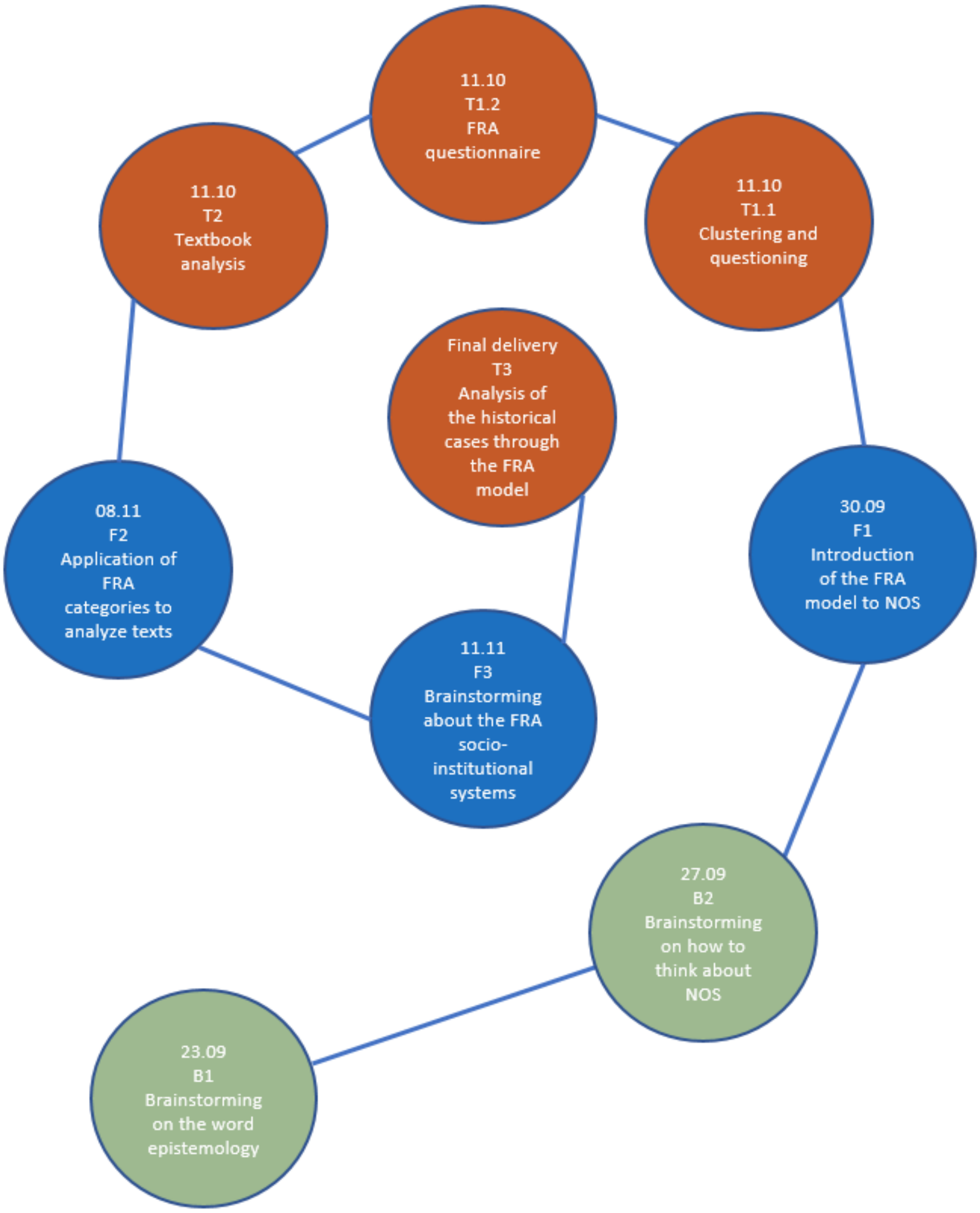


Figure 3.1: The thread of the NOS and FRA activities in the module.

The different activities are chronologically ordered from the outer arm of the spiral to its center. Each activity, represented through a circle, is characterized with an ID abbreviation and a specific color:

- B- Brainstorming, green color. Represents the two initial brainstorming on the NOS.
- F- FRA, blue color. Represents the moments in which there was an explicit reference to the FRA wheel's features.
- T- task, orange color. Represents the tasks students had to perform in order to get acquainted with the FRA wheel.

Starting from the outer arm of the spiral, we find the two initial activities that aimed to provide a background on the NOS topic before the introduction of the FRA framework. In particular, in B1 students were asked to brainstorm on the word epistemology using a virtual board. A collective and open discussion within the students followed, by commenting on the various answers on the board. In particular, the central focus of the discussion was whether the word "objectivity" could be conceived as a criterion to define what science is, with respect to other disciplines. In B2, the students

were involved in an activity of questioning (Winsløw, Matheron, and Mercier, 2013; Winsløw, Barquero, De Vleeschouwer, and Hardy, 2014), where they were asked to articulate the question "What is science?" in more specific and addressable questions.
In F1, there was the introduction to the FRA framework and a real-time survey, which aimed to collect the first students' reactions about the presented approach. In the survey, participants were asked to write aims and values, scientific practices, methods and methodological rules to characterize science through shared word clouds. Meanwhile, with the professor's guidance, the students collectively discussed the results appearing on the boards. In F2 students had to analyze an excerpt of the Walker textbook using the FRA wheel, to highlight disciplinary aspects of physics as a discipline. In F3, there was a brief class discussion about the socio-institutional system of the FRA wheel.
To promote the appropriation of the FRA wheel, individual tasks were given to students to help them to get acquainted with the approach. In task 1.1 students had to look at the virtual board elaborated in the B2 activity and choose a clustering criterion for the displayed questions. The criterion's choice had to be explained and justified through a brief text. In T1.2 the participants were given a questionnaire that aimed to collect students' initial impressions about the FRA model. In T2 students were invited to analyze a textbook through the FRA categories and produce a written essay. Finally, in the T3 activity the students were asked to produce an essay that contained their reflections on: the added value that an interdisciplinary and historical perspective can give to the parabola and parabolic motion topic; which aspects of physics and mathematics as disciplines can be brought to light if the interdisciplinary historical case is analyzed through the FRA framework; in which sense the curve and proof can be seen as boundary objects/ epistemological activators and which crossing mechanisms can be recognized.

3.4 Process of data analysis

Hereafter I present a diary which aims to show the evolution in time of the qualitative analysis' criteria used for the present study. The intention is to show that the markers for the analysis continued to change and improve, thanks to triangulations and collective discussions within the research team. Participating in the process was a very enriching experience for me, since it made me understand how to carry out a qualitative analysis, and how to balance between personal ideas (sometimes too abstract), and concrete, solid results. To reach the latter, I undoubtedly recognized the value of working in a team, since the contribution of each member was valuable and essential to solve the puzzle and get the final picture.

1.03.2022 The beginning
From an initial discussion with the research group, two conjectures were proposed, in order to outline a first work plan:

- The FRA wheel helped to structure and promote a process of refinement of the epistemological dimension in students, thus helping them to characterize physics as a discipline.
- The FRA acted as a tool to reflect on disciplinary identities in contexts that do not explicitly address NOS or FRA discussions. Through these contexts, it was indeed possible on one hand to delve into the disciplinary identities, allowing a comparison between the different disciplines, and, on the other, to enrich disciplinary identities thanks to the interdisciplinary dialogue.

In the following, I will refer to the evolution of the first conjecture since it was later chosen to be the principal and more promising one for our purposes.
To begin with, we started by selecting the activities with explicit references to the NOS or FRA. In particular, from a previous glance at the dataset, two activities seemed of particular relevance: the first brainstorming on the word epistemology (B1), and the survey carried out after the FRA's

introduction (T1.2). Guided by the initial impressions we had on these two activities, we pointed out some row markers to look for possible patterns within the data:

a. students' kind of judgments on science (from delineate positions to blurred points of view).
b. categories and epistemological language used by the students (from a bunch of terms to an enlargement).
c. progressive discovery of the key aspects that characterize science: for example, the values' dimension, the difference between practices and methods, and the importance of external wheels.

10.03.2022 Look for patterns

During the week, the B1 activity was transcribed, and from the transcript a sense of polarization emerged: there was indeed the perception that students opposed each other's comments in discussing whether objectivity could be conceived as a criterion to define what science is, as if they were trying to find the correct definition by discarding the other proposals. This feeling emerged for example from expressions like: "*Maybe the <u>right word</u> is invariance*"; "*in my opinion objectivity is a <u>wrong word</u> because…*"; "*the <u>right term</u> for science is universal, <u>more than</u> objective, in the sense that […]*". Moreover, in T1.2 students admitted the difficulty of accepting the fact that science cannot be uniquely defined. To take into consideration these ideas, the previous three markers were revised:

a. students' kind of judgments on science: from polarized to soft-edged opinions.
b. categories and epistemological language used by the students: from a suffered lack of definition of what science is, to the presence of more ways to characterize science.
c. Enlargement of FRA and refinement of the language: show the progress of discovery of the values' dimension, the difference between practices and methods, and the importance of external wheels.

To explore the above markers, while checking out if they can be used to indicate a progress in students' epistemological ideas, we decided to choose three moments in the spiral that can give us an idea about the possible evolution of students' perceptions of disciplinary identities choosing an initial, intermediate, and final state. We chose B1 as the initial activity, since it represented a stage where the FRA was not yet introduced, F1 as the intermediate, since it coincides with the FRA's presentation, and F2 as the final one, since it contained an interesting discussion, and it was placed near the end of the module (Fig. 3.2). The recordings of the three lectures were transcribed in order to analyze them.

Figure 3.2: The first three stages of interest were B1, F1 and F2.

16.03.2022 Sharing the first results

With respect to each of the above markers, a first analysis of the B1, F1 and F2 transcripts led to the following results:

a. In B1 the students' attitude is "oppositional" since their different opinions seem to oppose one another, creating polarities. However, dichotomies seem to fade away along with the discussion, thanks to the presence of "mediators", that is students that embrace different points of view and, changing the focus of the discussion, allow the presence of hybrid opinions. In F2 no polarities emerge, and the attitude is constructive since each student tries to contribute to the collective discussion without opposing others' opinions.
b. In B1 students seem to try to find the right word for describing science, thus looking for a definition. Instead in the F2 transcript many sentences suggest the tendency of characterizing rather than defining science. This second marker can be possibly integrated with the previous one, since the polarities may suggest the tendency to think that there is the right and the wrong, and a definition to find out, rather than a discipline's characterization to construct.
c. Comparing the B1 and F2 transcripts, the introduction of the FRA framework marked the students' progress on their initial perception of disciplinary identities and a refinement of the language. For example, in F2 an interesting and fine-grained discussion on the difference between methods and practices is carried out.

A fundamental issue readily emerged during the sharing of the results: is it reasonable to compare the B1 and F2 activities? Indeed, the two lectures are very different on a double level:

1. for the content, since in B1 a discussion concerning the word epistemology is carried out, while in F2 a collective analysis on the excerpt of a textbook is performed using the FRA wheel;
2. for the modality of lecture's conduction, since in B1 the classroom brainstorming is rather free and spontaneous, while in F2 the teacher guides much more the participant's dialogue.

Being this different, we agreed to first convey whether they can be compared, and if so, decide in which way we want to compare them. Thanks to a discussion in the research team, we agreed that the

lectures can hardly be compared on a content level, but they may be studied by focusing on how the student's knowledge organization varies from B1 to F2 (how they organize their ideas), and which attitude students have during the discussion (for example, whether they have a constructive attitude and are open to accept different points of view). According to this idea, we agreed to look at the three stages B1, F1 and F2, with the following markers in mind: epistemological architecture (how the students structure the discourse), attitude (how do students talk in the discussion), FRA wheel's role (which role had the FRA wheel in that specific activity).
The pointed-out characteristics of each of the three stages are shown in Tab. 3.2.

Table 3.2: Analysis of the B1, F1 and F2 stages.

	FRA wheel's role	Epistemological architecture	Attitude
B1	-	The classroom discussion focuses on three words to define science: objective, subjective, intersubjective. These three topics suggest a triangular architecture.	Many polarities are present, along with the presence of mediators that try to find intermediate positions. The polarities may suggest that the students are trying to find the "right" definition of what science is. The overall discussion appears to be destructive more than constructive, since the above cited polarities tend to break the conversion, rather than allowing to reach a point of view that embraces all the different comments.
F1	Epistemological activator: the word clouds produced during the activity show that the FRA immediately activates students in expressing their epistemological knowledge.	At the beginning of the lecture, the emerging architecture seems to be again a triangle, since while commenting the word clouds, students highlight once more the presence of three points of interest: objective, intersubjective, subjective.	-
F2	Lens	Students scaffold their reasoning using the FRA wheel's categories. However, this might be simply due to the fact that the assignment explicitly asked to do so.	Student's attitude visibly changed with respect to B1. The discourse does not present polarities, the exchange between the participants is rich, and not destructive: each participant contributes to the conversion, with the aim of reaching a shared opinion. The tendency is no more to define what science is, but to characterize it.

However, from a first internal triangulation with the research team, there was no complete agreement on the B1 analysis as regards the marker "epistemological architecture". In particular, the triangular architecture cited in Table 3.2 simply seemed to represent the three topics of interest during the B1 discussion. What seemed more remarkable from the B1 activity is rather that students were trying to find the right word to define what science is, discussing whether "objective" could be the right one or not.
From the triangulation, also the F2 analysis seemed to not be robust enough: indeed, the transcript was rather implicit, and it was not so easy to let results emerge in a clear and uncontestable way. As a result, we dismissed the F2 analysis and the epistemological architecture's marker. We were looking for other stages for the analysis, and we found that the task T1.1 presented interesting features: some essays contained complex and rich reflections, meanwhile suggesting a very different attitude with respect to B1. So, in order to validate our latest findings, we decided to organize a triangulation with the complete research group and volunteers from the second year of the curriculum in History of Physics and Physics Education. In organizing the triangulation activity, we decided to not include F1, since it seemed enough explicit in showing that, as soon as it was introduced, the FRA acted as an epistemological activator.

31.03.2022 Triangulation
To prepare the triangulation activity, we decided to focus on the attitude marker, and compare the degree of richness and complexity of the reflections contained in the B1 and T1.1 activities (Fig. 3.3).

Figure 3.3: The triangulation focused on the B1 and T1.1 activities.

Participants were given an excerpt with particularly meaningful parts of the B1 transcript, and five essays' excerpts from the T1.1 activity. For the T1.1 assignment, students were free to choose their favorite criterion, thus we decided to select three essays in which the FRA was used as a clustering criterion, and that presented interesting comments from the attitude's point of view. Moreover, we chose two essays which were significant for the attitude's marker in a negative way (in the sense that the texts did not suggest any progress with respect to the attitude emerging from the B1 activity).

While introducing the triangulation activity, we asked participants to analyze the excerpts using these markers:

- Attitude: try to highlight in the excerpts expressions or words that suggest that students have a particular attitude with respect to the classroom discussion.
- Complexity and richness: by comparing B1 and T1.1, can you find differences as regards the complexity and the richness of the content in the student's contributions to the discussion? Which elements suggest a change? Try to highlight them.

We then let participants work in groups and we collected some comments at the end of the activity. All the observations seemed to be aligned with our thoughts, and some of them raised interesting issues. Hereafter I report the key aspects of the individual comments (each one is from a different participant):

1. Even though students talk in different ways, they all seem to believe that science is a human product based on the sharing and exchange of ideas. In particular, in T1.1 the concept that science is a human endeavor emerges, thanks to the FRA wheel.
2. In B1 students are looking for a specific definition of what science is. The discourse is messy and lacks the multidimensionality that characterizes science. Some verbs suggest the presence of personal points of view (e.g. I think). Instead, in T1.1 more levels and features emerge.
3. In B1 there is more exploration and spontaneity with respect to T1.1, probably due the fact that the T1.1 was a written task, and it had a precise assignment.
4. In T1.1 the FRA supports the students, in the sense that it seems to structure their explanation, that is which words they use to articulate their thoughts, how they manage complexity, and the way they talk about science's characteristics (without the need to define it). So, there is a remarkable change between B1 and T1.1 regarding the student's ability to inhabit complexity.
5. The proposed excerpts from B1 and T1.1 activities can hardly be compared also because we cannot retrace the students who participated in B1 discussion. For this reason, we cannot use these data as an indicator of a "before" and "after" introduction of the FRA framework.
6. From B1 to T1.1 there is a change in the student's attitude, probably triggered by the FRA wheel.

The various comments and critical observations triggered many reflections in the research group, allowing to delineate the final criteria for the analysis. In particular, we decided to focus on the different roles the FRA played throughout the module:

- FRA as an epistemological activator (F1).
- FRA as a scaffolding to widen perspectives and inhabit complexity (T1.1).
- FRA as a lens for investigating the historical case of the interdisciplinary topic of the parabola and parabolic motion (T3).

As regards the attitude marker, we decided to use it as an indicator of a progress in the students' ability to scaffold a multidimensional and systemic view of discipline's identities. In the following, the definitive criteria will be explained, along with the methods used for the data analysis.

3.5 Final criteria for data analysis

As anticipated, after a long process of initial dataset analysis, we decided to focus on the data in which the impact of the FRA framework is more visible. For each FRA's role we conducted a part study, which is introduced, along with their principal goals and methods, in the following.

Part study 1- FRA as an epistemological activator

To investigate the FRA in the role of epistemological activator, we focused on the initial students' reaction to the FRA model. In particular, we decided to focus on F1 and T1.2 since they are the two stages that follow the FRA model's introduction. As the two activities are subsequent in time, we could compare the datasets to countercheck our impressions and hypothesis.

In particular, we studied the F1 activity by analyzing whether the FRA model is easy to understand for students and activates them in expressing their epistemological knowledge. To reach this goal, we considered the richness of the word clouds produced during the F1 lecture, together with the activity's follow-up discussion. We also exploited the T1.2 survey to study the student's initial impressions after the FRA had been introduced, by focusing on to what extent students feel familiar with the FRA model, how well they think to have understood the FRA's features, and the most interesting aspects of the approach, along with the ones they need to clarify.

By looking at the word clouds and the FRA survey's answers, we argue that students find the FRA framework easy to understand, even if the majority had not reflected on the nature of science in a systematic way before. Moreover, students show interest towards the social-institutional aspects of the FRA wheel and instantly catch the main point of the approach, which is to characterize science instead of defining it.

Part study 2- FRA as a scaffolding for reasoning and inhabiting the disciplinary complexity

To analyze whether the FRA could be used to scaffold reasoning and help to inhabit disciplinary complexity, we decided to focus on the B1 and T1.1 activities, to see whether there was any progress before and after the FRA wheel's introduction. As emerged from a comment during the triangulation, B1 and T1.1 cannot be strictly compared by seeing them as before-and-after stages of the FRA wheel's introduction. Moreover, the two activities are very different on a double level: for the related assignments, since in the B1 activity students were asked to brainstorm on the word epistemology, while in T1.1 they were asked to find a clustering criterion to organize the B2 virtual boards; for the kind of activity, since on one hand we have a spontaneous class discussion (B1), while on the other a written assignment (T1.1). In the latter, some sort of spontaneity was present anyway since students had the freedom to choose the clustering criterion they preferred. Even though we recognize that the two activities cannot be precisely put in comparison, we think it is possible to focus on the differences in attitude that students show before and after they have ever heard about the FRA model.

The different attitudes were recognized by observing the focus and articulations of students' dialogue (B1 transcript) and writings (T1.1) and the sort of language they present in the two cases (for example the verbs and words). Finally, with the use of T1.1 we explore to what extent students used (explicitly or implicitly) the FRA categories as a clustering criterion. In the B1 transcript, the students are numbered by following the order through which they enter in the discussion, while in T1.1 the enumeration refers to the pseudo-anonymization realized with the creation of the dataset.

Part study 3- FRA as lens to investigate the relationship between physics and mathematics in the historical case

In the last part study, we aim to study how the students used the FRA and the boundary objects as lenses to reflect on the historical case of the parabola and parabolic motion from an interdisciplinary perspective. To reach this goal, we analyzed the essays of the T3 activity, since its assignment led many students to explicitly use the FRA to describe physics and mathematics as disciplines and their historical interrelation. By checking the essays, we found three main approaches the students followed to use the FRA as a lens to characterize the disciplines in an interdisciplinary context, with respect to the creation of boundaries, identification and interaction between disciplines. In particular, three essays were prototypical examples of each approach, since they contained a very clear interpretation of the role of the FRA and boundary objects. The named essays permitted us to delineate three categories that were used to re-analyze data and represent possible attitudes towards the process of building-recognizing boundaries and the disciplines' identity, while allowing us to connect the

different disciplines. The categories are hybrid data-driven, but we progressively considered them from a more theoretical point of view. These categories wish to contribute to the research on the FRA model and boundary crossing and boundary objects framework. Apart from the three prototypical essays, the others did not contribute particularly to the development of the theoretical categories. Indeed, their positions were close to the three selected ones and their argument was not so focused on the role of the FRA and boundary objects. To identify the categories, we focused on the words used in the essays. In particular the predominant use of words like “distinguish”, “separate”, “divide”, “demarcate”, and in the emphasis on “differences”, “distinctions” between physics and mathematics suggested us that the attitude was to focus mainly on what distinguishes the disciplines. Whereas, words like “intertwine”, “unify”, “blend”, “similarities”, “connections”, “resemblances” and “integrations” suggested the tendency to overcome traditional disciplinary boundaries. In particular, we observed two different ways to achieve this: one is to look for similarities and resemblances, and the other is to describe how the disciplines integrate and intersect.
After that the three attitudes were recognized through the bottom-up analysis, three case-studies were selected in order to construct markers to characterize the different tendences. To do this, the framework of Akkerman and Bakker (2011) played a fundamental role. Indeed, each recognized attitude can be associated with a specific way of emerging disciplinary boundaries and the mechanisms exploited to move from one territory to another. As regards the first attitude, the FRA is used to delimit the physics discipline with respect to mathematics and vice versa. In particular, the aims, values, practices, and other elements of the FRA wheel are used to construct the boundary that separated the two disciplinary territories. Whereas, in the second case-study the FRA was used to overcome the differences between physics and mathematics by focusing on the aspects that permit to link the two disciplines and allow a continuous transition across the domains. In particular, the FRA categories allowed the characterization of the boundary zone at the interface between the disciplines.
Finally, in the third case-study the FRA allowed us to recognize the boundary crossing mechanisms between the disciplines in the historical case, which can be represented by thinking of a back and forth dynamic between the disciplinary identification and the recognition of resemblances. Thus, in this specific case, the FRA wheel not only supported the construction of the boundary zone between mathematics and physics, but also helped to cross the boundary which, on the way back, allowed to re-characterize the disciplinary identities.
The Fig. 3.4 represents a map of the disciplinary territories that emerge from the application of the FRA lens.

Figure 3.4: Representation of the disciplinary territories that emerge from the application of the FRA wheel.

From left to right: a) FRA as a lens to delimit physics with respect to mathematics and vice versa; b) FRA as a lens to recognize similarities and resemblances within and across the disciplines; c) FRA as a lens to move back and forth between the disciplinary separation and the recognition of resemblances.

Chapter 4 Results

4.1 FRA as epistemological activator

As anticipated, the first part study aims to investigate the FRA model in the role of epistemological activator. Our feeling was that the FRA immediately triggered students to delve into their repertoires and helped them to express their epistemological knowledge. To explore this perception, we decided to focus on the F1 and T1.2 activities, since they are two stages at the beginning of the module that follow the FRA's introduction. The answers to the T1.2 questionnaire were useful to countercheck the results emerging in the F1 analysis. As regards the latter, we focused on the word-clouds produced during the lecture, together with some excerpts of the follow-up discussion, to investigate whether the FRA model is easy to understand for students and activates them to elaborate on the nature of science.

4.1.1 Description of the dataset

In the F1 lecture the FRA framework was introduced to students, and to get acquainted with the approach, a wooclap activity was proposed, followed by a collective discussion. The wooclap activity aimed to explore the different aspects of the epistemic core, by answering the following questions: "Which aims and values of science do you share and influenced you to choose a scientific degree curriculum?"; "Which do you think are examples of practices that characterize science?"; "Which methods characterize science?"; "Which characteristics of scientific knowledge are remarkable to characterize science?". A collection of answers for the first three questions will be presented through word clouds; the knowledge cluster is not present since it was only orally discussed.
On the other hand, in stage T1.2 we asked students to fill in a FRA questionnaire which aimed to collect initial feelings and doubts regarding the FRA wheel (43 out of 57 answered it). We asked students: whether they have ever reflected on the nature of science; to what extent they think to have understood the main aspects of the family resemblance approach; the aspects of the framework that they find most interesting and which ones they would like to understand better.

4.1.2 Data analysis

Focusing on the collective discussion that followed the introduction of the FRA framework in the F1 lecture, we noticed the presence of two peculiar processes, that we called *alignment* and *enlargement*. In particular, in the *alignment* process, students contributed to reformulate the framework with respect to their personal knowledge. For example, S3 comments:

"*It could be said in a certain sense that this approach to attempting to define science is phenomenological, to the extent that a definition cannot be given ... that is, a compartmentalized definition, but we can say what science is not or how it should behave. So... [...] This is the thing that is striking me the most because actually not only it is already difficult, in our case, to frame physics [...], it is also difficult to do it by understanding physics at a certain historical moment. The last century could be said that it was a moment in which even physics itself has changed radically, in many aspects. So it is already an impossible task to do if you have a certain* [idea of] *physics in mind. I dare not think about doing it at all [...] Attempting to define precisely, [...], strictly in quotation marks what physics is, what science is and so on.* " (S3)

From the above excerpt we can perceive that the student is trying to tell with his own words the main aspects he grasped in the introduction, by stressing how the approach is phenomenological and how it is impossible to define physics through a *compartmentalized definition*. The student also underlines

how the research of the right description for physics is made even more impossible if its historical evolution is taken into consideration.
On the other hand, a process of *enlargement* (of words, dimensions and vocabulary) was noticed in the F1 wooclap activity and follow-up discussion, in which students were asked to think about the aims and values (Fig. 4.1), scientific practices (Fig. 4.2), methods and methodological rules (Fig 4.3) that characterize science. The word clouds produced by the students are very rich in content, as we can see from the figures below.

Figure 4.1: Word cloud answering the question "Which aims and values of science do you share and influenced you to choose a scientific degree curriculum?".

Figure 4.2: Word cloud answering the question "Which do you think are examples of practices that characterize science?".

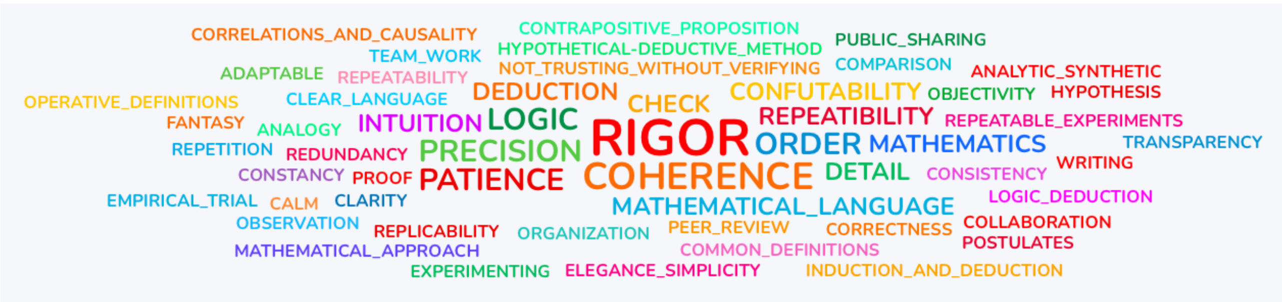


Figure 4.3: Word cloud answering the question "Which methods characterize science?".

Students pointed out a rich variety of words regarding scientific aims and values, practices, and methods. Moreover, in discussing the word-clouds, students started to recognize a plurality of dimensions, which is a typical feature of the FRA model itself. For example, S5 commented:
"*What strikes me most is that the biggest word is curiosity* [referring to Fig. 4.1] *[...] and curiosity refers to a more emotional sphere, while the others [...] refer to a rational sphere*".

Other students recognized other dimensions: S6 mentioned "*the rational approach to science*", S8 the "*sphere of learning and understanding*", S7 the *"social dimension"*. Continuing on the social dimension, S10 added "*I particularly like [the idea of] peer review. That is the idea that in science our work must be verified by other people who have the same skills as us [...]*".

In this regard, it is interesting to observe that in each word cloud aspects of the socio-institutional system are mentioned: in the aims and values "social utility", "social inclusion" and "teaching desire" are present; in the practices we read "dissemination", "teamwork", "teaching" and "publication"; in the methods we find "public sharing", "teamwork", "collaboration" and "peer review".
This interest towards the social-institutional dimension of science is confirmed by the questionnaire's answers to the question (multiple choice is possible): *Which of the following aspects of the "Family Resemblance Approach" that characterize his particular way of dealing with the theme of the "Nature of Science" did you find most interesting or surprising?*

- *The choice of characterizing and not defining what science is*
- *The articulation of the epistemic nucleus in the four categories (aims and values, practices, methods, knowledge)*
- *The presence of the two outer wheels enlightening the social institutional dimension of science, which goes beyond the cognitive-epistemic core*
- *Other…*

The count of students for each selected answer is displayed in Fig. 4.4. A consistent number of students (23) were indeed intrigued by the outer circular levels of the FRA wheel, proving that their interest towards the FRA wheel also includes its social-institutional parts. However, the most interesting aspect of the FRA seems to be the choice of characterizing instead of defining science, which is at the heart of the FRA. This aspect turned out to be also the most understandable one for students, as we will see while commenting Fig. 4.5.

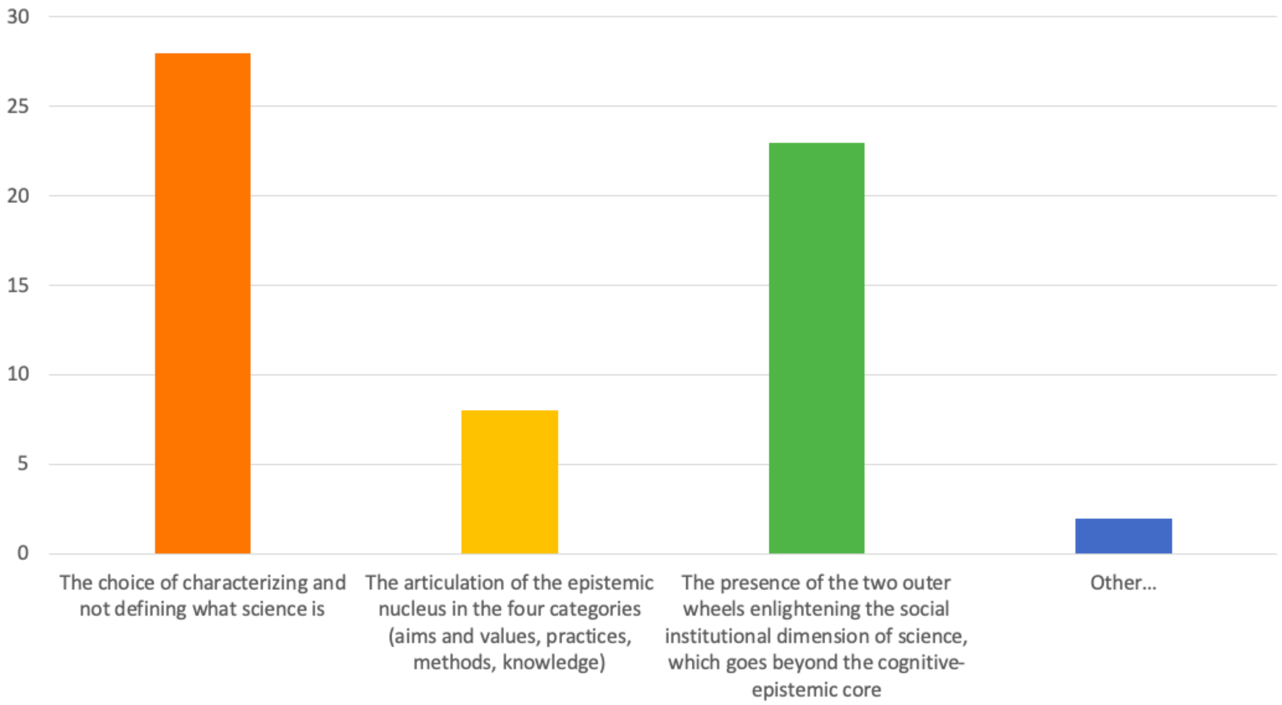


Figure 4.4: Students' distribution along the FRA aspects that they find most interesting.

Looking at the word clouds presented above (Fig. 4.1, Fig. 4.2 and Fig. 4.3), the feeling was that as it was introduced, the FRA framework helped students in voicing their knowledge. In particular, the FRA wheel seemed to immediately activate and support the creation of a students' systemic and multidimensional view of science even though they had just been introduced to the framework. To confirm this idea, we checked the answers to the following question in the FRA survey: *To what extent do you think you understand the following aspects of the "Family Resemblance Approach" that* *characterize his particular way of dealing with the theme of the "Nature of Science"? (1 not at all, 6 definitely):*

- *The choice of "characterizing" and "not defining" what science is*
- *The articulation of the epistemic nucleus into the four categories (aims & values, practices, methods, knowledge)*
- *The meaning of "aims & values"*
- *The meaning of "scientific practices"*
- *The meaning of "methods and methodological rules"*
- *The presence of the socio-institutional system*
- *The meaning of "knowledge"*

From Fig. 4.5, which shows the average values that the students associated with each aspect, we can see that the scores are very high. In particular, students felt to have understood quite well the difference between characterizing and defining science (5,42), the articulation of the epistemic core into the four categories (5,02), the meaning of the aims and values (5,07), the presence of the socio-institutional system (4,98), and the meaning of "knowledge" (4,95). Although the "delta" is not particularly significant, the average value is lower for the meaning of "methodological rules" (4,84), and the meaning of "scientific practices" (4,70). This issue emerged also during the word-clouds' collective discussion. To investigate this small decrease in the average value, we looked to the answers to the question "*Can you describe what you would like to understand better and what kind of difficulty you have encountered and/or are experiencing?*": 13 students wrote that they did not completely understand the articulation of the epistemic nucleus in the four categories and, in particular, the difference between scientific practices and methods and methodological rules. For example, one student admits that "*The main difficulty I encountered in interpreting the FRA wheel is the difference between the categories that make up the epistemic core, in particular between the "methods and methodological rules" and "practices" categories.*".

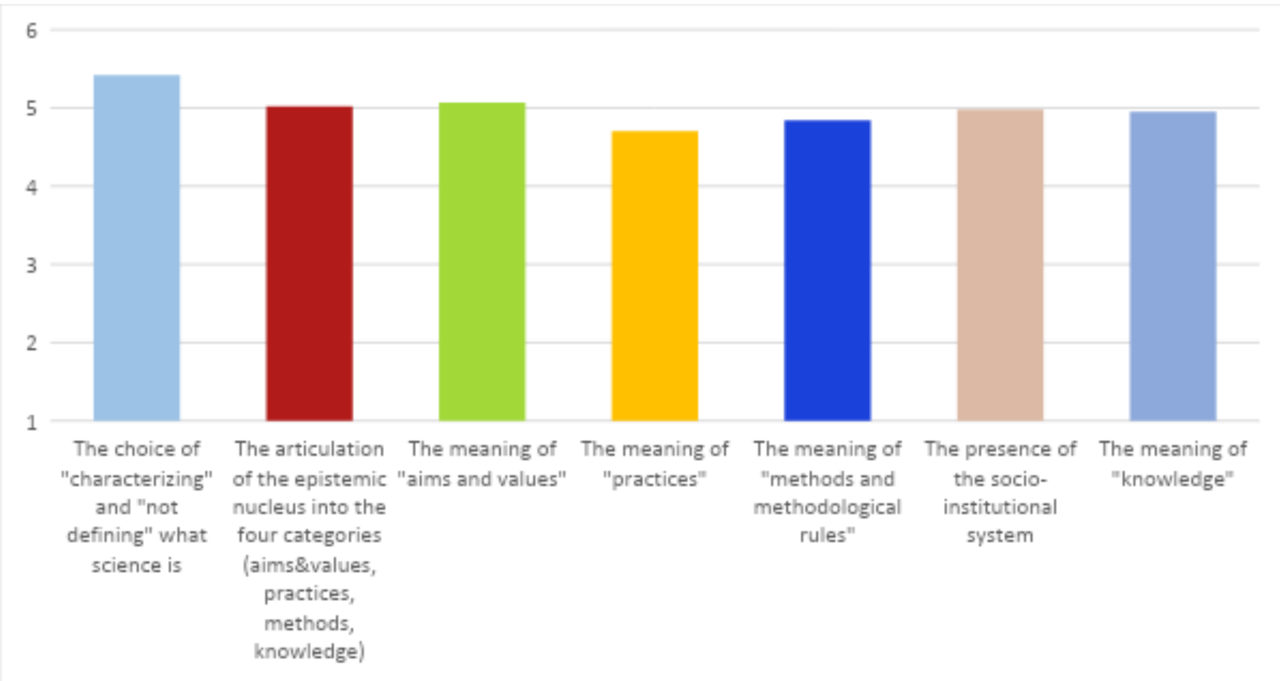


Figure 4.5: Understanding of the different FRA aspects. Average values attributed by students to each item.

It is worth stressing that most students never thought about the nature of science in a systematic way before the course. In fact, as presented in Fig. 4.6, at the question "*Have you ever thought about the nature of science (what is meant by science) before this course?*" the majority (23 out of 43) answered that they heard about the topic sporadically and that they never thought about it systematically, 11 out of 43 students answered that they occasionally thought about it and that they have read something, and the remaining 9 students answered instead that it is a topic they like and on which they have reflected in depth.

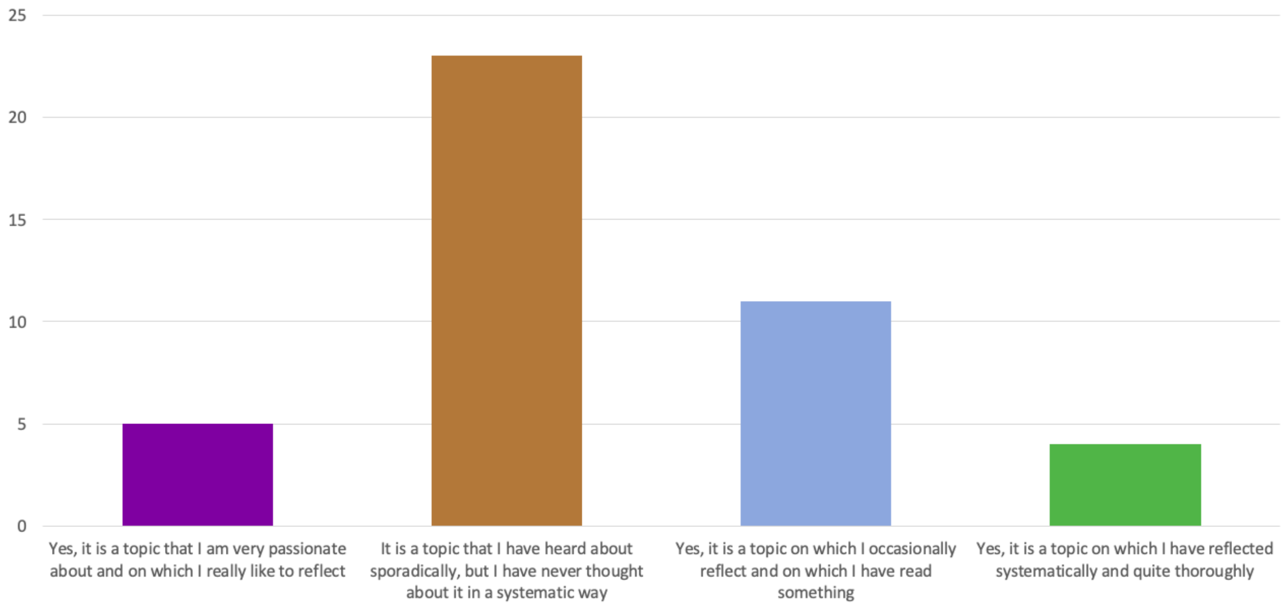


Figure 4.6: Count of students for each selected answer to the question "Have you ever thought about the nature of science (what is meant by science) before this course?".

So, even though the majority had never thought about the nature of science in a systematic way, the new approach seems to activate the knowledge they might implicitly have on the topic.

4.1.3 Partial discussion

From the above analysis, it emerges that the FRA framework acted as an epistemological activator, providing students a vocabulary through which they could express their personal ideas and disciplinary knowledge. From the immediate reaction students had after the FRA wheel's introduction and the answers to the T1.2 survey, we argue that students find the FRA framework easy to understand, even if the majority had not reflected on the nature of science in a systematic way before. The processes of *alignment* and *enlargement* recognized in the F1 activity, attest that students caught the essence of the FRA framework in a very natural way, catching the difference between defining and characterizing, recognizing the multi-dimensional nature of science (e.g. its social-institutional aspects). In particular, the *enlargement* of the dimensions that emerges from the activity led us to the conclusion that the FRA model is able to scaffold students' reasoning in an authentic way, allowing them to delve into their knowledge and to look for a characterization of science, rather than a definition.
Finally, we believe that the FRA's introduction alone is not enough to exploit all of its potential. Indeed, the FRA's anchoring to a specific topic can help students deepen the nature of science and help them, for example, to effectively recognize the differences between the cognitive-epistemic nucleus' categories and clear-out residual doubts.

4.2 FRA as scaffolding for reasoning and navigating (inhabiting) the complexity

The second part study aims to investigate whether the FRA helped students in scaffolding their reasoning and inhabiting disciplinary complexity. To reach this goal, we analyzed the B1 and T1.1 activities looking for a possible change in students' attitude before and after they have ever heard about the FRA framework. In particular, we looked at how students articulate their contributions to the dialogue (B1 transcript) and to the written assignment (T1.1), and which personal ideas on the nature of science emerge. In particular, the interest was in checking whether in the T1.1 assignment students feel more inclined to embrace a multidimensional perspective on the nature of science topics. Finally, we explored to what extent students used (explicitly or implicitly) the FRA categories as a clustering criterion for the T1.1 task.

4.2.1 Description of the dataset

The B1 activity was carried out at the beginning of the module, at a time when the FRA had not yet been introduced, while the task T1.1 was given after the FRA's introduction. In the B1 activity students were asked to brainstorm on the word epistemology using a virtual board. A spontaneous debate within the students started during the activity about the meaning of objectivity in science. In particular, the students were discussing the possible criteria to define "science".
Meanwhile, in the individual task T1.1 students were asked to group the questions collected through a virtual board during a previous activity (B2), in which the issue "*what is science?*" was unpacked into simpler and more addressable questions. In particular, students were free to choose the clustering criterion they preferred, which had to be justified through a brief text. We found out that some students decided to reorganize the B2 virtual board by highlighting the main questions or key points that they considered important (e.g. S6, S23, S24, S25), while others described through a discursive text their clustering of the questions (e.g. S17, S20).

4.2.2 Data analysis

Analyzing the B1 discussion concerning the word "objective", we noticed the emerging of students' personal ideas about the nature of science. In particular, we observed three different ways of intending the word "objective":

1. that concerns the object of study itself, and is independent on who thinks it (for example, S13 comments: "*I would associate objective to something that is independent from those who think it*").
2. that concerns an idea that is shareable among a group of people (S10: "*science is the tool that connects people on a common idea and then [science] becomes objective when everyone thinks it*").
3. that concerns the invariance of results while replicating an experiment (S12: *"when I do an experiment, I see that some results do not depend on other conditions. I ask another to do it again and I see that he [finds it] too...*").

Moreover, during the discussion, students were trying to decide whether the word objectivity was the *right* one to define what science is. For example, for a student the *best term* was "universal" (S12: *"rather than the term objective for science, or in our case for physics, is better the term universal"*), for another one the best word was "inter-subjective" (S13: "*I thought maybe talking about inter-subjectivity rather than objectivity*"). The interesting point here is that students seemed to not allow

the coexistence of all these different terms at once. Indeed, students were frequently trying to discard the previous proposals while intervening, in the attempt to find a better one (S10: "*in my opinion objectivity is a wrong word because…*"; S11: "*Maybe the right word is invariance*"; S12: "*the right term for science is universal, more than objective, in the sense that […]*").
Finally, during the debate, students were usually trying to define science with respect to non-science. This tendency is highlighted from the use of expressions like "the science is…", "the science may be…", "science is not…". This approach was also present in some texts submitted for the T1.1 assignment, for example a student wrote: "*to be able to understand and define what science is it is essential to clarify what [science] is not*" (S8).
In the T1.1 task students chose different criteria for the clustering, for example:

- S23 decided to group together the questions that aimed to investigate similar aspects of science ("*The subdivision was based on what characteristic aspects of science the questions were probing*").
- S24 reorganized the questions in seven groups according to their affinity ("*I grouped the questions on the jamboard based on the affinity between them into seven groups. To each group I have assigned a title, followed by a main question that seems to me to grasp the essence of the group's questions*").
- S6 exploited the FRA model to cluster the questions ("*We begin to sketch the questions and answers for each of the identified clusters, starting from the questions related to the science's epistemic nucleus* [of FRA wheel]").

In particular, 10 students out of 33 used the FRA model's categories but did not explicitly refer to it, while 16 students explicitly stated that they used the FRA framework as a clustering criterion. For example, S22 wrote: "*While I was carrying out the [clustering] process I had in mind the FRA model: more than looking for strict definitions or criteria to build hermetically sealed compartments, I was chasing the similarities between the questions I was looking at, and I was clustering them in this phenomenological way*".
Moreover, many students explicitly stated that, in clustering the questions, their aim was to characterize and not define what science is. Some students also referred to the impossibility of finding a definition of science: "*All the questions try in some way to characterize the science circumscribing the attempt, established impossible, to define it*" (S23); *"to look for a definition of science able to include all its different types is very difficult. To characterize necessary and sufficient conditions to be a science while being able to bring justice to sciences' complexity and diversity is impossible, from here it comes the FRA approach."* (S10).
Consistently with the tendency of not looking for the right definition for science, they also used verbs like "*to characterize*" and "*to delimit*" while describing science, and they used words like "features", "characteristics":

- "*The characteristics: What is peculiar to science?*" (S24).
- "*A greater awareness of what science is, what it is characterized by, how it proceeds and how it is built, can be an important key to…*" (S20).
- "*Delimiting it [science] may prove to be limiting: we could instead understand how best to characterize it*" (S6).
- "*Let's try to give an order to all this, always keeping in mind how unattainable the concept in question is and therefore trying to find a way to delimit a concept without drawing its boundaries*" (S17).

Overall, the majority of students that explicitly used the FRA, together with the ones that used other criteria, were able to explore the nature of science topics embracing its complexity and multidimensional structure. This emerges for example from the following excerpts:

- *"Science is an expanding city, with blurred boundaries and mutable inside"* (S6).
- *"science [...] obviously is not a finite and universal product or a simple method that we can apply to every social and cultural environment from which the reality around us is made"* (S12).
- *"the question "What do we mean with science?" shifts the issue's center of gravity towards itself; [...] the risk is to lose the many nuances of the topic under study, with the consequence of not being entirely able to comprehend what science is and how it is possible to observe it from different points of view"* (S17).

Among the students that decided not to choose the FRA as a criterion, a few did not seem to recognize the nature of science's complexity and seemed to undertake a unidimensional approach. This perception comes from their description of the scientific method, which refers to a stereotyped and simplified procedure based on empirical evidence:

- "*the scientific method is divided in three parts: the phenomenon's observation, the formulation of a hypothesis, the experimental proof. On these three simple passages the scientific knowledge is built and its reliability is based*". (S24)
- "*The typical instrument used by science to understand is the "scientific method", based on observation and in particular on the experiments and measures, on the formulation of predictive theories and models that must then be confirmed or refused from empirical data*" (S30).
- *"Science follows what it is known as scientific method, a bunch of rules and procedures that modify and evolve with time. The principal instrument available to the scientist is the observation, through which it is possible to get empirical data that can later be analyzed and compared with preexisting theories"* (S11).

4.2.3 Partial discussion

From the above analysis, we believe that most students show a different *attitude* before and after the FRA's introduction. Firstly, we saw how in the B1 collective discussion on the term "objective", students projected their beliefs and personal ideas while trying to give a meaning to the word. In particular, from the discussion's dynamic, it emerged how they discarded each other's proposals, with the aim of finding the right word to define science (e.g. "universal", "objective", "intersubjective" were discussed terms). Overall, the tendency seems to be that of not being able to embrace all the different words, and to advocate to the research of an univocal definition of what science is.
Meanwhile, in the T1.1 individual task, students presented richer and more complex descriptions of science. In our opinion, the introduction of the FRA framework (which preceded the T1.1 task) helped students in changing their *attitude* towards the nature of science issues. In particular, many students supported the idea that it is not possible to define what science is, while a characterization is achievable, the latter aspect being a value at the core of the FRA framework.
From the way they described the scientific method, a few students seemed to not present a very different attitude with respect to B1. This may be due to diverse reasons, for example the presence of a personal difficulty in accepting the new framework (since it may be very dissimilar from their own idiosyncratic view on the nature of science), the need of additional time to feel familiar with the approach, or the necessity to first explore the FRA framework by anchoring it to a specific topic, like the parabola and parabolic motion one. However, the majority of students showed to accept and

support the idea that the nature of science topic is complex and multidimensional in structure, adding many times personal nuances to this belief.
The overall change in *attitude* (from looking for the right word to define science, to allowing the presence of more features in science's description) in our opinion may depend not only on the kind of activity carried out (on one hand, a collective discussion, on the other a written individual assignment), but also on the FRA framework's introduction, which supported students with a new vocabulary and a perspective that triggered them to rethink their personal vision of science by placing it into a broader and multi-faced frame, in which science can be characterized through multiple categories and it is not reducible to a definition.
As regards the clustering criterion's choice, we noticed how most students (26 out of 33) decided to choose the FRA categories (both the cognitive-epistemic and the socio-institutional system), suggesting that they deeply grasped the essence of the model and felt confident enough to use it for the assignment.

4.3 FRA as lens to investigate the relationship between physics and mathematics in the historical case

With this last part study, we aim to analyze the FRA and boundary objects in the role of lenses to reflect on the interrelation between physics and mathematics in the historical case of the parabola and parabolic motion. We analyzed the essays written by students for the T3 activity and found three main ways through which they used the FRA as a lens to characterize the disciplines in an interdisciplinary context. In particular, the three approaches differ regarding: the creation of boundaries, the identification and interaction within disciplines. We selected three essays that can be used as case studies to illustrate each approach, since they were particularly clear and evident in explicating the role of the FRA and boundary objects. Moreover, these essays allow us to point out three categories that aim to describe the possible attitudes towards the process of building-recognizing boundaries and the disciplines' identity. For the latter process, we referred to the Akkerman and Bakker's framework (2011), since each attitude can be linked to a peculiar way of designing the disciplines' boundaries and of choosing the mechanisms to move from one side to the other.

4.3.1 Description of the dataset

In the following, the analysis of the T3 essays produced by the students at the end of the module is reported. The T3 task asked students to answer the following questions:

1. What added value to knowledge and reasoning can the collocation of the study of motions and conics in the "Great history of physics and mathematics" give?
2. What added value to knowledge and reasoning can, look at the study of motions and conics from an interdisciplinary perspective, give?
3. What aspects and characteristics of physics and mathematics as disciplines can be brought to light if this "historical case" is analyzed by means of FRA? Consider the internal FRA wheel - both the core represented by the "cognitive-epistemic system" (aims and values, practices, methods and methodological rules and knowledge) and the "socio-institutional system". If you want, you can use a table to report the FRA aspects that you have recognized as characterizing physics and mathematics.
4. In what sense can the curve and the proof be cases of "boundary objects" and / or cases of "epistemological activators"? What border "crossing mechanisms" can be put in place? How?

For our purposes, we will focus on the answers to the third question.

4.3.2 Data analysis

Hereafter the three case studies are presented, with the aim to illustrate the different ways through which students used the FRA model as a lens to investigate the relationship between physics and mathematics in the historical case of parabola and parabolic motion. Apart from the first case study, the other two selected essays were written by female students who received the maximum mark from the three independent examiners during the evaluation. For this reason, they can be considered prototypical examples of analyses of the historical case through the FRA lens, from a conceptual, epistemological, and methodological point of view. Interestingly, the three essays present many differences when put in comparison, showing how the same analytical instrument can convey many diverse personal ideas and can be used in different ways.

1) FRA as a lens of separation (S29)

Figure 4.7: Mark Rothko (from https://www.guggenheim-venice.it/it/arte/opere/untitled-red/).

The role of FRA as lens of separation emerged while reading the essay written by S29. The student begins her essay by introducing the historical case of the parabola and parabolic motion, focusing on the different interplays that historically occurred between mathematics and physics. In particular, she highlights how the historical case permits to show many cases of interdisciplinarity, which are most of the times absent in nowadays physics (or mathematics) textbooks:

"*Placing parabolic motion in a historical context highlights how historically mathematics and physics were much more intertwined than they are today in textbooks, providing, with the work "Discourses and mathematical demonstrations around two new sciences" by Galilei, an example of interdisciplinarity. In fact, the text respects the characteristics of an interdisciplinary approach, highlighting the references to the disciplines, but at the same time explaining and motivating their intertwining.*"

However, when S29 gets to answer the question "What aspects and characteristics of physics and mathematics as disciplines can be brought to light if this historical case is analyzed through FRA?", her approach definitely changes. In particular, no more reference to interdisciplinarity, disciplinary contaminations or intertwining is present. She indeed exploits the FRA wheel's categories (specifically, aims, cognitive and epistemic practices, methodological rules, scientific knowledge, and socio-institutional system) to establish a clear demarcation line between the disciplinary territories of physics and mathematics. She explicitly states this shift of focus and refers to the FRA as a lens through which it is possible to disentangle the disciplines' characteristics from Galilei's text:

"*Although Galilei's text is a valid example of interdisciplinarity, the references to the disciplines are evident and appear in an epistemologically significant way. In particular, referring to the framework of the Family Resemblance Approach, and also considering the analyzes of the textbooks, the following characteristic aspects of the two disciplines emerged.*"

This sentence is followed by two separate bullet lists of characteristics, one for physics and one for mathematics, structured according to the FRA categories. The bullet lists' graphic choice suggests the tendency to distinguish the two disciplines, and to draw a clear-cut boundary between their territories (this idea visually reminds Rothko's art, represented in Fig. 4.7):

"Physics:

- *Aims: Criterion of simplicity that manifests itself through the separation of the variables, that is, the independence of the motions; intelligibility, as we speak of common experiences.*
- *Practices: Experiment (epistemic practice) and dialogue (cognitive practice).*
- *Methodological rules: Construction of models starting from the observation of phenomena.*
- *Scientific knowledge: Mathematics provides logical and consistent explanations to develop understanding.*
- *Socio-institutional system: The dissemination of concepts that revolutionize the vision of the world inevitably leads to a cultural and social revolution.*

Mathematics:

- *Aims: Rationality and consistency of axiomatic-deductive reasoning and objective value and truth of proof.*
- *Methodological rules: Model building with a solid logical argument.*
- *Socio-institutional system: A rationally structured argument has a social utility in that it produces answers to human needs and guarantees the equality of intellectual authority."*

To conclude, in the above case study, we saw an example of how the FRA model can be used as a lens to highlight the disciplinary characteristics of physics and mathematics in the historical case. In particular, S29 used the FRA as a lens of separation to disentangle and recognize each disciplinary identity. As we will see in the following case studies, other students used the FRA lens in a totally different way.

2) FRA as a lens to recognize similarities and resemblances within and across the disciplines (S18)

Figure 4.8: Philip Guston (from https://www.artribune.com/arti-visive/arte-moderna/2018/04/monet-americani-parigi-orangerie/).

This second case study focuses on the essay written by S18. Throughout the whole essay, she looks for "contact points" between mathematics and physics that are defined as "inseparable". She indeed highlights how the historical case was a profound interdisciplinary moment, that shows the partial loss of disciplinary boundaries, which are described as "blurred and not very clear-cut":

"The birth of conics and of motions are highly interdisciplinary events, presenting blurred and not very clear-cut boundaries between mathematics and physics [...]."

The image of blurred boundaries evoked by S18 made us develop the representation in Fig. 3.4.b, which represents a continuum between the lands, where each side is not clearly definable with respect to the other (this idea also visually reminds Guston's paintings, one of which is reported in Fig. 4.8). After the student stated the "inseparability of the disciplines", as it emerges from the historical case, in the subsequent section of the essay she focuses on using the FRA (in red) as lens to recognize the elements that characterize each discipline (in green) or their intersection (in blue):

"In the inseparability of the disciplines that have emerged in these historical cases, we find some of their constituent and most significant characteristics. If we put ourselves in a FRA perspective, we would immediately notice rigor as the basis of knowledge and value and objective of mathematics, and mathematics itself as an element of knowledge for physics, with the role of language plus reasoning, language more logic, and not only language as is commonly thought. Again in the physical field, various experimental practices are presented: sensory observation (e.g. the thrown ball seen by Guidobaldo) must generate hypotheses (e.g. the catenary as a curve) that allow other experiments (e.g. the ball soiled with ink) and lead to a model that must always maintain a logical and sensible structure (we can consider it as a value and aim shared by the disciplines) to explain its functioning. Another physical value is reproducibility, in this case, understood as validation of the method and the object of knowledge analyzed (e.g. the "wondrous way" is actually wondrous for this reason). The evolution of both concepts, and therefore the steps they have taken, show us various common aspects of mathematics and physics: they outline a method of non-static hypothetical-deductive approach,

both for the kind of activity to carry out one's studies, both with regard to work ethics and the certification/validation of one's work."

In the above excerpt the FRA has a completely different role with respect to the previously illustrated case study. In particular, while S29 uses the FRA as a separation lens to recognize disciplinary features and listing them according to the FRA wheel's categories, S18 carries out a complementary operation. Indeed, she uses the FRA to analyze blurred areas, such as the disciplinary domain and their boundary zone, the "*common aspects of mathematics and physics*".

We believe that here the *blurred* transition between the disciplinary domains does not implicate a *blurry* characterization of interdisciplinarity. On the contrary, S18 uses the FRA as a tool to, on one hand, characterize the disciplines, and on the other, to recognize where and how the disciplinary intertwining takes place.

3) FRA as a lens to move back and forth between the disciplinary separation and the recognition of resemblances (S20)

Figure 4.9: William Turner (from https://www.milanoplatinum.com/lestetica-del-sublime-william-turner.html).

For the last case study, we focused on the essay written by S20. In the essay, she tackles the issue from a personal viewpoint. She starts by highlighting how the disciplinary separation between mathematics and physics is one of the deepest dichotomies of our times, in her words:

"The disciplinary division between mathematics and physics embodies one of the deepest and most deeply rooted dichotomies of our culture: that between theory and experimentation, between abstract knowledge and real, concrete knowledge of the world. It brings with it, in this accelerated, complex, uncertain present, characterized by the fusion of knowledge, where disciplines are called to merge, intersect, reflect each other (climatology, AI, big data science, ...), prejudices of form and thought which, perhaps, we no longer need."

In particular, she believes that the study of the historical case helps students and teachers to overcome this dichotomy:

"[The boundary zone between the disciplines] *has been explored with a good compass and through narratives coming from the 'great history of physics and mathematics', that were able, first of all, to break down the dichotomic prejudice."*

She continues by introducing two elements (the FRA wheel and the boundary crossing mechanisms) that allow to change the way through which students and teachers characterize the disciplinary identities:

"[To explore interdisciplinarity we have] *a model for disciplinary identification (the FRA wheel) and a theoretical framework to investigate the boundary territory between mathematics and physics (boundary theory* [in English]*)."*

In particular, her approach to navigate the boundary zone is to unpack the four boundary-crossing mechanisms, with the aim of finding a suitable position for the FRA wheel, which will be used as a tool to establish the role of the disciplines' identities. She finds out that this position corresponds to the *identification mechanism*:

"The mechanisms of boundary investigation and crossing identified by Akkerman and Bakker are four. The first is the disciplinary identification (identification [in English]*) with which the specificities of the disciplines are highlighted. For this purpose, the FRA wheel (Family Resemblance Approach* [in English]*) is chosen as the theoretical framework, a tool that helps and guides us in defining both the epistemic heart of the two disciplines divided into aims and values, practices, methods, and methodological rules and knowledge (core), and the relationship of the community of reference with society, their being part of society (first circle), and their more general relationships with citizenship and the economic and political decision-making powers that govern it (second circle)."*

She then carefully underlines how the process of identifying disciplinary features using the FRA wheel does not lead to the disciplinary separation present in textbooks:

"*The trend is no longer to separate and distinguish but to unite."*

She also explains how the textbooks' analysis through the FRA model allowed to realize an articulated *back and forth dynamics* between mathematics and physics (this process visually recalls paintings like Turner's one, reported in Fig. 4.9). In particular, it was possible to highlight *how much mathematics* can be recognized at the foundation of physics. Moreover, in the historical case it is possible to notice how a mechanism of reflection allowed to overcome the boundaries, leading to a collaboration between the disciplines. In this way, entities like proof, models, theorems (which are usually conceived as typical methodological elements of mathematics) became boundary objects that, through a cooperation process, played a fundamental role in the establishing of physics as a discipline:

"[We have realized] *how much mathematics begins to characterize physics (reflection* [in English]*). We talk about proof and mathematical model in the methods, we talk about identifying the basic assumptions in the practices (cooperation* [in English]*), [...] and in the knowledge. Mathematics, with demonstration, with models, thus becomes an argumentative structure that keeps reasoning under control in a physical problem, but there is more."*

"[...] The experience of IDENTITIES, [...] has therefore brought back to light [...] an implicit foundation of physics, which had become so implicit that we forget that sometimes mathematics is more "physical" [that is there is much more mathematics in the foundations of physics] *than it may seem."*

Finally, the student restates the sense of her argumentation, that is to recognize how the process of characterizing the disciplines and their identity is fundamental in order to being able to recognize and unpack the boundary crossing mechanisms and breaking down the dichotomous prejudice:

"What is interesting to see, therefore, is how the identity characterizations of the two disciplines change before and after the interdisciplinary approach, before and after, therefore, having explored that boundary territory with a good compass and through narratives coming from the "great history of physics and mathematics" which, above all, succeeded in breaking down the dichotomous prejudice."

"The placement of the study of motions and conics in the "great history of physics and mathematics" has certainly led to bringing the two disciplines closer together, eliminating the dichotomous prejudice from which we started: therefore, greater awareness has grown that mathematics and physics are close and that the aims and values of one also guide the other."

4.3.3 Partial discussion

This last part study aimed to show whether and how students used the FRA framework (Irzik and Nola, 2011; Erduran and Dagher, 2014) and the boundary object and boundary crossing framework (Akkerman and Bakker, 2011) as lenses to investigate the historical case. In particular, by reading the students' essays, we identified three main ways to use the FRA model in an interdisciplinary context. These different approaches were particularly evident in three essays, which were studied as prototypical examples.
Interestingly, all the three essays used the FRA model to carry out a process that "entail a questioning of the core identity of each of the intersecting sites" (Akkerman and Bakker, 2011, p. 142), which led students to define "one practice in light of another, delineating how it differs from the other practice".
The three dynamics (schematically pictured in Fig. 3.4) recalled us the idea of Plato's dialectic in Phaedrus:

"*Phaedrus: What are these features?*
Socrates: First, bringing things which are scattered all over the place into a single class by gaining a comprehensive view of them, so that one can define any given thing and so clarify the topic one wants to explain at any time. That's what we did just now, when we were trying to explain what love is by defining it first: whether or not we were right, our speech did at least achieve clarity and internal consistency thanks to this procedure.
Phaedrus: And what was the other feature you meant, Socrates?
Socrates: Being able to cut things up again, class by class, e according to their natural joints, rather than trying to break them up as an incompetent butcher might. [...]" (*Phaedrus*, p.55)

What Phaedrus calls features are the two opposite but complementary processes that we recognized in the first two dynamics. Indeed, in the first two case studies the FRA is used in opposite ways to analyze the historical case. In particular, for S29 (Fig. 4.7) the FRA was a lens through which it was possible to separate the disciplines and draw a clear-cut demarcation line. The disciplines are in this case kept well separated, also while listing their peculiar identity aspects in terms of the categories of the cognitive-epistemic and of the socio-institutional systems. This approach recalls the cutting of the disciplinary worlds at *their natural joints*, "to dismember the object of study".
On the other hand, in the second case study, S18 (Fig. 4.8) uses the FRA model complementary, that is as a lens to recognize the similarities and resemblances within and across the disciplines, to recognize how elements of mathematics become elements for physics and vice versa, to "*encompass multiplicities from an overarching perspective and trace them back to the same shape*".

Finally, in the third dynamics (Fig. 4.9) the FRA model was used as a tool to re-elaborate and articulate the historical case through a *back and forth dynamic* illuminating the boundary-crossing mechanisms between disciplines, that is a back and forth dynamic between the disciplinary identification and the recognition of resemblances, between the *dismembering the object following the "natural joints"* and the embracing in an overall view and bringing multiplicity back to a single form. Thus, in this case the FRA model helps to build a boundary territory between mathematics and physics, and allows to cross the boundaries to re-characterize the disciplinary identities in the light of the interdisciplinary case.

Conclusion

Personal considerations

During my bachelor's in physics, I irremediably suffered the excessive technicalities through which professors were teaching, that left most of the time no room to include an historical or cultural perspective. As a result, although I was proficient with the exams, I have always felt trapped within the borders of the physics' courses. Only a few of them were intellectually satisfying for me. This probably is not so surprising since I chose to study physics not for the object of study itself, but for the difficult and elegant reasonings it requires to adopt.
After the bachelor's degree, I went to Belgium to study biophysics, since I really needed to integrate my physics' studies with something else. Eventually, six months later I returned to Italy, but during that period abroad I really enjoyed the freedom to study in an interdisciplinary environment, and to interact with people who had a different background with respect to mine.
After this experience, I registered for the Master in physics at the University of Bologna, and during the first year I attended the course in "Physics teaching: theoretical and experimental aspects", where I experienced the parabola and parabolic motion module from a student's perspective. I still remember that I felt invigorated each time I attended the lectures, since I loved to see how a physics' topic can be studied from so many points of view. I was also introduced to epistemology for the first time, and I remember how much this word scared me at first, since I could not understand what it really meant. At the end, I appreciated the approach of the module so much that I decided to work on this topic for my thesis.
Unfortunately, from my personal experience, I think that today interdisciplinarity is still seen by many physicists in a bad light, as something that is "neither-nor". What I truly wish for in the future, is that interdisciplinary considerations and identity aspects of physics as a discipline will be treated in the bachelor or high school's programs, in parallel with the more technical aspects. It took years for me to understand that my interests were not wrong, and that in physics there is room for everyone, not just for "theorists" or "experimental" people. I hope that boundary people will not have to wait too long to feel welcomed to study physics.

Final remarks

In the thesis I presented an empirical-based qualitative analysis which wishes to contribute to the reflection on the importance of disciplinary identities to address the interdisciplinarity theme within the Nature of Science research field. In particular, the study focused on a module designed within the EU project IDENTITIES on the parabola and parabolic motion. In the thesis, it was adopted the peculiar stance that IDENTITIES takes to deal with interdisciplinarity, which should not be conceived as a-disciplinarity, multidisciplinarity, or transdisciplinarity. Indeed, interdisciplinarity means that on one hand, the identity of the single disciplines is fundamental to reach an interdisciplinary dialogue, and, on the other hand, that the interdisciplinary exchange allows to enrich the discipline's identity. Grounding on this concept, within the project two specific theoretical frameworks were chosen: the FRA framework to elaborate on disciplinary identities, the boundary objects and boundary crossing framework to describe the boundary crossing mechanisms. Through these frameworks, we investigated whether and how the FRA wheel triggered teacher-students in recognizing the disciplinary identities of physics and mathematics and supported them in distinguishing and highlighting the processes of construction and crossing of disciplinary boundaries.
From the analysis, we identified three main roles played by the FRA model: as an epistemological activator, as a scaffolding for reasoning and inhabiting the disciplinary complexity, and as a lens to investigate the relationship between physics and mathematics in the historical case. Each role was studied through an empirical, qualitative analysis, and contributed to answer the initial research questions.
In the first part study, we saw how the FRA wheel acted as an epistemological activator, in the sense that it immediately triggered students providing a rich and familiar vocabulary that supported them

to investigate their personal ideas on the nature of science and disciplinary identities. In particular, the FRA framework was able to scaffold students' reasoning and to promote an *enlargement* of the dimensions, which led students to look for a characterization of science, rather than a definition.

In the second part study, we also saw how the FRA fostered students to change their *attitude* towards the NOS. In particular, students were gradually supported to take into account and embrace the complex and multidimensional nature of the scientific disciplines. In the end, most of them seemed to accept the concept that science cannot be uniquely defined.

Finally, in the third part study we pointed out three dynamics, three ways through which students used the FRA to characterize physics and mathematics in an interdisciplinary context. We were able to recognize in these ways three different attitudes through which students tackled the processes of building, recognizing, and crossing boundaries in the historical case. In the first case, the FRA was used as a tool to distinguish physics from mathematics, cutting the disciplinary territories at *their natural joints*. In the second case, complementary to the first one, the FRA allowed to highlight the common aspects between physics and mathematics, overcoming the disciplinary boundaries and *gaining a comprehensive view*. In the third case, the FRA allowed to shed light on the boundary-crossing mechanisms (Akkerman and Bakker, 2011) between physics and mathematics, namely a *back and forth dynamic* from the identification and recognition of resemblances to embracing a comprehensive perspective. This latter case showed how the FRA can support the crossing of the disciplinary boundaries, allowing to re-characterize the disciplinary identities considering the interdisciplinary dialogue that took place.

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