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THERMAL INFLATION AND THE COSMOLOGICAL MODULI PROBLEM

Relatore:
Dott. Michele Cicoli

Presentata da:
Marco Giommetti

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Summary

Superstring theory is one of the most plausible candidates for a theory describing quantum gravity. Due to consistency reasons, this theory requires the existence of extra dimensions which are compactified. The low-energy effective field theory of string compactifications is populated by hundreds of moduli fields.

String moduli correspond to massless scalar fields in the 4-dimensional Minkowski spacetime, so they can act as mediators for new unobserved forces. However, they are massless at three level: taking into account quantum correction they acquire mass. The dynamics which stabilises the moduli generally also breaks supersymmetry, and so the moduli masses are related to the masses of the soft supersymmetry breaking terms. The requirement of TeV-scale supersymmetry in order to solve the hierarchy problem leads to moduli masses in the range from 1 MeV to 10^3 TeV, depending on the model. Furthermore, moduli couple only gravitationally so they have Planck suppressed interactions with Standard Model particles.

Moduli are produced by the Big Bang, and in a lesser extend, by any phase transition. Thanks to this features, after inflation the moduli come to dominate the energy density of the Universe, until they decay. Light moduli decay very late in the history of our Universe: after they decay, the reheating temperature has to be larger than $\mathcal{O}(1)$ MeV to allow successful Big Bang Nucleosynthesis (BBN). If this is not the case, the theory would suffer from a serious problem: the so-called *cosmological moduli problem* (CMP), which is the stringy version of the Polonyi problem encountered in supergravity. Furthermore, the decay of such particles generates a huge amount of entropy, which in turn could wash out any previously generated baryon-antibaryon asymmetry. This might be a welcomed effect if the mechanism generating the asymmetry in the early Universe is too efficient,

but it can also be a problem.

From the cosmological point view, the usual recipe to get rid of unwanted relic is to invoke a period of inflation. Hence, in order to solve this problem, one has to realize a short period of (low temperature) inflation. This inflation has to take place at low temperature and has to last no more than 10 e-foldings, in order not to affect density perturbation (generated in "ordinary inflation") and not to create too much CMB anisotropies. For this reason, this kind of inflation should be different by ordinary slow-roll inflation. The most successful model is *thermal inflation*. This mechanism is roughly based on finite-temperature corrections to the effective potential of a scalar field named "flaton" which drives this inflationary period. Fields with flat potential and large Vacuum Expectation Value (VEV) are very common in supersymmetric extension of the Standard Model and if they are in thermal equilibrium there can be finite temperature corrections to their effective masses making them able to develop a short period of inflation, which, under several circumstances, may solve, or at least relax, the cosmological moduli problem.

Abstract

I have studied the possibility to solve the cosmological moduli problem which affects some string compactifications by the dilution induced by a low-energy period of thermal inflation caused by finite temperature effects. I have then applied this general dilution mechanism to the study of the cosmological moduli problem in the particular case of type IIB Large Volume compactifications. The thesis in divided into five chapters.

The first chapter introduces the reader to moduli fields starting from the simplest example: the Kaluza-Klein five dimensional theory.

The second chapter is devoted to the cosmological moduli problem and other cosmological problems caused by moduli fields. Here it should be stressed that the real problem, that is the cosmological moduli problem, is that if the moduli decay after Big Bang Nucleosynthesis, their decay would change the abundances of the light nuclei. Furthermore there are also other important problems, as gravitini overproduction through moduli decay, distortion from the black-body spectrum of CMB radiation and finally stable moduli can overclose the universe unless the mass is below the eV-scale.

The third chapter contains a description of thermal inflation and something about flaton cosmology. In the past decade a lot of work have been made in the context of the cosmology with flat potentials. In particular, it has been recognized that they are able to develop a short period of inflation different from the primordial one. Since this mechanism is based on finite temperature corrections acquired by the flaton potential, it has been called *Thermal Inflation*.

The fourth chapter is devoted to the type IIB stabilization mechanism known as the Large Volume Scenario (LVS). Since all the parameters of the low energy effective field theory are tied to moduli VEVs, the moduli need to be stabilized. Moduli stabilization

is today an area of attractive research and it seems to be (at least partially) understood only in type IIB string theory. The Large Volume Scenario is a promising stabilization mechanism since it leads to a dynamical solution of the gauge hierarchy problem (why is the Higgs mass so smaller than the Planck mass?) by using a volume of the extra dimensions which is exponentially large in string units. Here there are two Kähler moduli, whose canonical normalisation leads to the moduli fields Φ and χ. It is found that the first decays rapidly in the history of the universe, while the second is extremely long-lived. Furthermore since the latter oscillates with a Planckian amplitude, it is subject to the CMP.

In the last chapter, the dilution of the moduli number density has been computed, starting from a more general case and then focusing on the light modulus χ of the LVS. In particular it is found that even if thermal inflation provides a huge dilution, there is still a large moduli reproduction due to its relatively low mass. It seems that this modulus after two stages of thermal inflation is able to reproduce as a typical modulus after one stage of thermal inflation. The conclusion is that, even if the computations have been performed by order of magnitude and there are many free parameters, this modulus seems to need a further stage of thermal inflation, because it behaves as if it had "lost" one stage of thermal inflation. Other possible way-outs would be either to increase the modulus mass by paying the price of not having anymore low-energy supersymmetry, or invoking a mechanism to suppress the original amplitude of the modulus oscillations. Another interesting option would be to consider models where the visible sector is sequestered from supersymmetry breaking, and so the modulus mass can be increased keeping still at the same time TeV-scale supersymmetry.

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Introduction

String theory is a very plausible candidate for a high energy theory beyond the Standard Model. Consistency of this theory requires ten spacetime dimensions: it is expected that this ten dimensional spacetime is the product $M^{1,3} \times X$ of the four dimensional Minkowski spacetime $M^{1,3}$ with a six dimensional space X; the latter has to be very tiny, which would explain why it has not been detected so far in high energy experiments. Each choice of X lead to a different *effective field theory* (EFT) on Minkowski spacetime which should be the theory that describes our world.

This poses a severe constraint for the space X. Indeed, this cannot be arbitrary but it has to be such that the four dimensional effective field theory admits $\mathcal{N} = 1$ supersymmetry (SUSY) which has then to be dynamically broken leading to TeV-scale soft terms in order to solve the gauge hierarchy problem. It has been shown [1] that this requires the internal space X to be a Calabi-Yau manifold of complex dimension 3.

Size and shape of Calabi-Yau are controlled by parameters called *moduli*. Compactification to four dimensions typically produces dozens of hundreds of these fields in the spectrum of the four dimensional theory.

All the five superstring theory contain moduli in the low energy EFT spectrum, so the moduli problem is an independent feature.

The moduli parameters in string theory corresponds to massless scalars in four dimensional effective supergravity (SUGRA) and this implies the possibly of long range interactions, *i.e.* they could be mediators for new forces. There are experiments searching for a "fifth force" studying apparent deviations from inverse square law of Newtonian gravity, but nowadays there is no compelling experimental evidence for such deviation, although there some anomalous results which remain to be understood [2] . Futhermore, moduli couplings to matter fields are model dependent, which implies that matter fields will experience different accelerations: this is a violation of equivalence principle, which has been tested [3] by the ratio of inertial to gravitational mass up to 10^{-13}.

The very natural consequence is that all of moduli should be massive. Indeed, they are massless at three level: taking into account quantum corrections they acquire mass-squareds proportional to the second derivative of the effective scalar potential. The latter determines the vacuum configurations of the theory as local minima, since it plays the same role of potential energy for a quantum field theory. All the parameters of the low energy effective field theory as the electron mass, the Yukawa and gauge couplings, etc... are related to the vacuum expectation value (VEV) of moduli. Therefore, to do realistic phenomenology, it is important to have models with stabilized moduli. Moduli stabilization has been subject of study for many string theorists [15,16,17]. This goal has been achieved in type IIB string, throught the Large Volume Scenario (LVS) [4]. Models with unstabilized moduli generally suffer of lack of predictability.

But what can we infer about moduli masses? Since they receive mass from SUSY breaking and non perturbative effects, one expects for them masses of the same order of the gravitino mass. Furthermore, since in gravity mediated models the mass of the soft terms is of order the gravitino mass and the solution to the gauge hierarchy problem requires supersymmetry at the TeV scale, at the first sight one expects that the gravitino and all the moduli share the same mass of $\mathcal{O}(1)$ TeV. However moduli masses also depend on the stabilization mechanism: generally speaking there is no favoured value for their masses. Put in other words, their masses are model dependent.

Constraints and bounds for their values come from cosmological observation. Indeed, it is found that heavy moduli (*i.e.* moduli with mass $\mathcal{O}(100)$ TeV) decay very rapidly while light moduli are long lived [4,12]. Because of their feature and since they behave as non relativistic matter, soon after the inflation they come to dominate energy density of the universe till they decay. After they decay, radiation era begins and the cosmological history is the usual one. If these light moduli decay after Big Bang Nucleosynthesis, they would destroy the successful predictions of the abundances of the light elements. This problem is known as *The Cosmological Moduli Problem* (for the first time pointed out in [25]). Furthermore the decay of such light particles might generate a huge amount of entropy, washing out any matter-antimatter asymmetry generated in the early universe. This is however not necessarily a problem if this original mechanism was too efficient. Also supergravity, the low energy limit of string theory, suffers from a similar cosmological problem, namely the Polonyi Problem [13,14].

This is a quite general problem: fields with “almost” flat potentials and masses of order of the soft supersymmetry breaking scale that couple only gravitationally are fatal for standard cosmology. Fields with such feature have been called *flatons* [1] and, despite they could be troublesome, it has been recognized that they may be cosmological significant [18,19,20]. They are very common in supersymmetric extension of the Standard Model and they are interesting because, under certain circumstances, they can drive a short period of inflation that could be the solution to the CMP. This kind of inflation is different from ordinary slow-roll inflation and it has been called *thermal inflation*, for the first time developed in [21]and then revisited and improved in [22]. It could be seen as a complement of ordinary inflation to diluite relics abundances. It works as follows: in the early universe, if the flaton is in thermal equilibrium it can acquire finite temperature contributions to its effective potential, forcing it to stay in a false vacuum situation. A short period of inflation develops and when the temperature drops below a critical value, the flaton rolls away from the origin and thermal inflation shuts off; then the flaton start to oscillate around its vev. This inflation lasts only for a few e-foldings, so the density perturbations accounting for CMB anisotropies and large scale structures are appoximately left unaltered. There is also the possibility of two or more stages of thermal inflation, where the second diluites the relic left over from the previous period.

[1] This name is due to the flatness of the effective potential and should not be confused with inflaton.

Chapter 1

String Compactifications and Light Scalars

One of the most important problems in string theory is to connect it with what has been measured in high-energy experiment. Toward this direction a lot of progress has been made, but we still have many problems to solve and up to now there is no direct evidence that elementary particles that we observe are strings. Around 1985 it was established that there are five different ten-dimensional string theories: type I strings, type IIA and IIB strings, $E_8 \times E_8$ strings and SO(32) heterotic strings. Since these theories are all supersymmetric, we are dealing with superstring theories. Furthermore, since all of them unifies gauge theories with gravity in a consistent quantum theory, they could be candidate for a quantum gravity theory.

In 1990 there was recognized that they are all part of a single eleven dimensional theory, and that was called M-theory. The synthesis of the five different string theories into a single underlying theory is a fascinating story that is far from fully understood and remains a major area of research in string theory today.

If we want to connect string theory with experiments we first of all have to explain why the observed space-time has only four dimensions rather than ten, that is what has happened to the other six dimensions. This is usually achieved by compactifying six of the ten dimensions on a compact six-dimensional manifold, sufficiently small to avoid detection. Since supersymmetry has not been observed so far in particle experiments, we also need to understand how it gets broken on the way from the string scale to the length scale probed by current experiments. It is often assumed that supersymmetry is preserved at the characteristic length scale of the compact manifold and is instead broken by some effect in the four-dimensional field theory at lower energies. In this case the compact manifold must satisfy rather stringent mathematical conditions which string theorists have studied in detail. In string compactifications the four dimensional physics depend not only on the string length, but also on size and shape of the compact manifold. The parameters characterizing a particular compactification are called moduli and their values, together with that of another string theory field called the dilaton, must somehow be determined in order to make contact with the observed particle physics.

Until few years ago it was not known how to stabilize the moduli because their potential was flat to each order of string perturbation theory with no particular values favored. It turns out however, that a potential can be generated for the moduli by introducing fluxes of closed string gauge fields along different directions inside the compact manifold. The minima of this potential correspond to favored values of the moduli which in turn determine couplings and particles masses in the four dimensional EFT. While this allows us in principle to predict various features of particle physics from a given string model, the moduli can be fixed in a huge numbers of ways and this lead to an enormous numbers of different predictions that are *a priori* equally valid. This looks like a big trouble for any theory, in particular for string theory, which suppose to predict from first principles the behaviour of elementary particles that we observe in high-energy experiments. The multitude of potential minima for the moduli goes under the name of *string landscape* which has been studied by many string theorists in recent years. The program of connecting string theory to particle phenomenology faces many challenges in addition to the landscape problem. There are essentially two approaches to string compactification. The first is mostly based on the heterotic string theory and assumed both string length and size of the compact manifold are of the order of the Planck length. The second one is based on the so-called *Dirichlet branes* of type I and II theories and allows much larger values for the string length and size of the compact manifold, even as large as the length scale that will be proved in the upcoming experiments at the LHC.

In the second approach, which is referred as *brane-world compactification*, the gauge theories of the Standard Model (and Minimal Supersymmetric Standard Model) are defined inside the world volume of stacks of D-branes. Gauge fields then correspond to open string with both ends attached to branes in a particular stack, while quarks and leptons correspond to open strings having their two end points attached to two different stacks of D-branes. In order to have chiral matter, the two stacks of the D-brane must be at angles or carry different magnetizations in the compact extra-dimensions. Simple toy model of this type, where the compact manifold is a flat six dimensional torus, can be studied in considerable detail and can serve as prototype for more general model of string compactifications. Several technical issues need to be addressed in order to make these models fully consistent. So called orientifold planes are introduced to enable cancellations of certain tadpoles and supersymmetry can be fully or partially broken introducing orbifold singularities into the compact geometry.

Semi-realistic models that are stringy extensions of the SM and MSSM have been constructed using both the top-down and bottom-up approaches to string compactification.

As a consequence of the compactification, a huge number of massless scalar fields appear in the spectrum of the four dimensional EFT: all the five superstring theories share this feature. This is very discouraging because these particles have not been observed yet and might induce some problems, both phenomenologically and cosmologically. To see the way these fields are tied to the compactification mechanism, it is useful to recall the pedagogical example of the Kaluza-Klein theory of gravity.

1.1 Basics of Kaluza-Klein theories

1.1.1 A bit of history

It is an old idea that unification of forces may be tied to the existence of extra space-time dimensions (EDs). Already in 1920 Oscar Kaluza developed a theory in five dimensions, unifying Maxwell's theory of electromagnetism and Einstein's General Relativity, the two theories well understood at that time. In this framework the electromagnetic field emerge as a component of gravity as a consequence of general coordinate transformation invariance. However he was faced with two important questions. Firstly, is this fifth dimension a real, physical dimension or it is only a mathematical device? Secondly, if it is a real meaningful dimension, why haven't we seen yet in high energy experiments? Kaluza himself didn't really understand if this dimension has to be considered as a physical dimension: indeed, although there are experimental phenomena that could be interpreted as a four dimensional coordinate invariance, there is no evidence for a fifth dimension. Kaluza then demanded that all the derivative with respect to this dimension had to vanish: physics, in his opinion, was to take place in the four dimensional Minkowski spacetime. This passed through the history as the *cylinder condition.*

In 1926 Oscar Klein showed that the cylinder condition is equivalent to a circular topology for the fifth dimension: the total space M^5 is factorized in the following way $M^5 = M^{1,3} \times S^1$, where S^1 is a circle of some radius R and $M^{1,3}$ is the Minkowski spacetime. It is assumed that all the fields depend on it periodically and so one can perform a Fourier expansion. In order to understand better what produces the compactification mechanism, let's see as example the behaviour of the fields in this space M^5, focusing our attention to the cases of scalar field, vector field and an antisymmetric tensor field.

1.1.2 Scalar field in M^5

Let's label the M^5 coordinate with x^A $A = 0, 1, 2, 3, 4$, the $M^{1,3}$ ones with x^μ where $\mu = 0, 1, 2, 3$ and that of S^1 with y and let's consider a five dimensional action for a massless scalar field $\Phi(x^M)$

$$S^5 = \int d^5x\, \partial_M \Phi\, \partial^M \Phi \tag{1.1}$$

Periodicity in y allow us to write

$$\Phi(x^\mu, y) = \sum_{n=-\infty}^{+\infty} \phi_n(x^\mu) \exp\left(\frac{\imath n y}{R}\right) \tag{1.2}$$

for some set of four dimensional, orthonormal and complete eigenfunction $\phi_n(x^\mu)$ (in general these are complex object). Equation of motion are easily obtained by varying the five dimensional action with respect to Φ: obviously one has

$$\partial_M \partial^M \Phi = 0 \tag{1.3}$$

where $\partial_M \partial^M = \Box_5 = \partial_\mu \partial^\mu + \partial_y \partial^y$. Plugging the normal modes expansion in (1.1) one has a relation for the eigenfunctions

$$\sum_{n=-\infty}^{+\infty} \left(\partial_\mu \partial^\mu - \frac{n^2}{R^2} \right) \phi_n(x^\mu) \exp\left(\frac{\imath n y}{R} \right) = 0$$

But the vanishing of a linear combination on a basis implies that all the coefficients of the linear combination have to vanish, that is

$$\left(\partial_\mu \partial^\mu - \frac{n^2}{R^2} \right) \phi_n(x^\mu) = 0 \tag{1.4}$$

This describes an infinite number of equations for four dimensional scalars fields whose mass squared is related to the integer n by $m_n^2 = \frac{n^2}{R^2}$. Only the zero mode is massless while non-zero modes have a mass inversely proportional to the radius of the circle. To recover the four dimensional action starting from (1.1) one has to substitute the expansion of Φ into (1.1) and integrate over the fifth coordinate y. The result is

$$\begin{aligned} S^5 &= \int d^4x \int dy \sum_{n,m=-\infty}^{+\infty} \left(\partial_\mu \phi_n(x^\mu)\, \partial^\mu \phi_m(x^\mu) - \frac{nm}{R^2}\, \phi_n(x^\mu) \phi_m^*(x^\mu) \right) \exp\left(\frac{\imath(n-m)y}{R} \right) \\ &= 2\pi R\, \delta_{nm} \int d^4x \sum_{n,m=-\infty}^{+\infty} \left(\partial_\mu \phi_n(x^\mu)\, \partial^\mu \phi_m(x^\mu) - \frac{nm}{R^2}\, \phi_n(x^\mu) \phi_m^*(x^\mu) \right) \\ &= 2\pi R \int d^4x \sum_{n=-\infty}^{+\infty} \left(\partial_\mu \phi_n(x^\mu)\, \partial^\mu \phi_n(x^\mu) - \frac{n^2}{R^2}\, |\phi(x^\mu)|^2 \right) \end{aligned}$$

We are usually interested in the limit $R \to 0$ in which ϕ_0 remains light and ϕ_n with $n \neq 0$ are heavy and can be discarded. We refer to this limit as *dimensional reduction*: under this assumption one has

$$\begin{aligned} S^5 &= 2\pi R \int d^4x \left(\partial_\mu \phi_0(x^\mu)\, \partial^\mu \phi_0(x^\mu) \right) + \dots \\ &= S^4 \quad + \quad \infty \quad \text{tower of massive state} \end{aligned}$$

The action of the five-dimensional massless scalar field is reduced to the action for a massless four-dimensional scalar field plus a tower of massive state. We restrict our attention to the zero mode, *i.e.* we discard the tower of massive fields: in this case one speaks about *dimensional reduction* and this is formally equivalent to the cylinder condition. If we keep all the massive modes we speak about *compactification*.

1.1.3 Vector in M^5

Now we move to the simpler next case: the abelian vector field in five dimensions $A^M(x^M)$. Upon reduction to four dimensions, this object became equivalent to a vector $A^\mu(x^\mu)$ (the abelian gauge potential) and a scalar $A^4 \equiv \rho$. The Fourier modes expansion for both these fields reads

$$A^\mu = \sum_{n=-\infty}^{+\infty} A^\mu_n \exp\left(\frac{\imath n y}{R}\right)$$
$$\rho = \sum_{n=-\infty}^{+\infty} \rho_n \exp\left(\frac{\imath n y}{R}\right)$$

The five dimensional action is given by

$$S^5 = \frac{1}{g_5^2}\int d^5x\, F_{MN}F^{MN}$$

where the field strength F^{MN} is related to A^M via $F^{MN} = \partial^M A^N - \partial^N A^M$, implying $\partial^M\partial_M A_N - \partial^M\partial_N A_M = 0$. Choosing a gauge such that $\partial^M A_M = 0$ and $A_0 = 0$, one has $\Box_5 A_N = 0$. In this way this situation in the same of the massless scalar field for each component of A_M: indeed the latter implies both $\Box_5 A_\mu = 0$ and $\Box_5\rho = 0$. To each massless state in five dimensions correspond a massless state plus a tower of massive states in four dimensions. Plugging the normal modes expansion into the lagrangian and integrating over the fifth variable y one readily has

$$S^4 = \int d^4x\left(\frac{2\pi R}{g_5^2}F_{(0)\,\mu\nu}F^{\mu\nu}_{(0)} + \frac{2\pi R}{g_5^2}\partial_\mu\rho_0\partial^\mu\rho_0 + \dots\right)$$

So we have obtained a four dimensional theory of massless gauge potential, a massless scalar field and an infinite tower of massive states.

Comment

The relation between the gauge coupling of the five dimensional and four dimensional action is given by

$$\frac{1}{g_4^2} = \frac{2\pi R}{g_5^2}$$

This can be immediately generalized to the case of a D-dimensional compact manifold (say a D-sphere of radius R)

$$\frac{1}{g_4^2} = \frac{V_{D-4}}{g_5^2}$$

being V_D the volume of the D-dimensional compact manifold.

1.1.4 Antisymmetric tensor field

Up to now we have considered a scalar field and a vector field defined in a five dimensional manifold. Now we turn to the case of an antisymmetric tensor field F_{MN}. First of all we have to clarify the matter content of this object. Technically one has to study the decomposition of $SO(1,4)$ under $SO(1,3) \times SO(2)$: the result is $F_{\mu\nu}$ (antisymmetric tensor in four dimension), $F_{4\nu}$ and $F_{\mu 4}$ (four components vectors) and F_{55} (scalar). In the language of group theory this decomposition is written as $\mathbf{5} \otimes \mathbf{5} = \mathbf{16} \oplus \mathbf{4} \oplus \mathbf{4} \oplus \mathbf{1}$. The fact that the five dimensional antisymmetric tensor field is also equivalent to scalar and four vectors in four dimensions is due to a particular symmetry known as *duality*. The simplest example of duality can be found in Maxwell's equations of electromagnetism: indeed employing the covariant formalism these are described by the Maxwell tensor $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$. The latter is derived from the Levi-Civita symbol

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{even permutation of 0123} \\ -1 & \text{odd permutation of 0123} \\ 0 & \text{two or more index are equal} \end{cases}$$

$\epsilon^{0123} = 1$ in the following way

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \tag{1.5}$$

where the pre-factor 1/2 takes the antisymmetric properties of both $\epsilon^{\mu\nu\rho\sigma}$ and $F_{\mu\nu}$ into account. Furthermore, also $\tilde{F}^{\mu\nu}$ is antisymmetric with respect to its index μ and ν and the electromagnetic field equation in vacuo are

$$\partial_\mu F^{\mu\nu} = 0 \qquad \text{Maxwell equations}$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad \text{Bianchi identities}$$

The exchange $F \leftrightarrow \tilde{F}$ corresponds to the swap $E \leftrightarrow B$ in the equations above. Notice that in four dimensions both the Maxwell tensor $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$ have the same number of index; in a generic D-dimensions this could not necessarily be true. In the language of differential geometry an antisymmetric $(p+1)$-tensor $A_{M_1 \ldots M_{(p+1)}}$ that is called $(p+1)$-*form* and with this one, a field strength tensor can be constructed

$$F_{M_1 \ldots M_{p+2}} = \partial_{[M_1} A_{M_2 \ldots M_{p+2}]}$$

and the latter is a $(p+2)$-form. Let's see what we can say about the dual: since the dimension is fixed to D, the dual of $F_{M_1 \ldots M_{p+2}}$ must have $D-(p+2)$ index, indeed

$$\tilde{F}_{M_1 \ldots M_{D-p-2}} = \epsilon_{M_1 \ldots M_D} F^{D-p-1 \ldots M_D}$$

Example in $D = 4$

We have just seen in four dimensions how we can derive a field strength tensor $F^{\mu\nu}$ starting from the gauge potential A^μ. Indeed we found $F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and associated to $F^{\mu\nu}$ there exists its dual $\tilde{F}^{\mu\nu}$ given by (1.5). What can we say about a third rank tensor? And what about its dual? Let's consider a third rank field strength tensor $F_{\mu\nu\rho}$ and suppose it can be derived from a potential $B_{\mu\nu}$ in this way $F_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$. It is easy to construct its dual, indeed $F_{\mu\nu\rho}\epsilon^{\mu\nu\rho\sigma} = \tilde{F}^\sigma = \partial^\sigma a$. From these simple considerations we find that the dual potentials that yield the fields strength have a different number of index: indeed we have a two rank tensor $B_{\mu\nu}$ and a scalar potential a.

Example in $D = 6$

Now let's consider a six dimensional space. Suppose we have a third dimensional field strength tensor F_{IJK} derived from a two index potential B_{JK} in the usual way $F_{IJK} = \partial_{[I} B_{JK]}$. We can construct $\tilde{F}_{LMN}$, that is the dual of F_{IJK}, employing the six dimensional Levi-Civita symbol ϵ^{IJKLMN}: one has $\epsilon^{IJKLMN} F_{IJK} = \tilde{F}^{LMN}$ and $\tilde{F}_{LMN} = \partial_L \tilde{B}_{MN]}$. Here the potential B_{IJ} and its dual $\tilde{B}_{IJ}$ have the same number of index. Furthermore they both have 15 degrees of freedom. This can understand as follows: an antisymmetric tensor of rank two in a generic D dimension has $D(D-1)/2$ independent component, so since $D = 6$ a generic two rank tensor B_{IJ} has 15 independent component. Generalization to a generic tensor with M_{p+1} index immediately follows. If we want to find the degrees of freedom of a generic $B_{M_1,\ldots,M_{p+1}}$ tensor we have to consider its decomposition under the little group $B_{M_1,\ldots,M_{p+1}} \to B_{i_1,\ldots,i_{p+1}}$ where $i_k = 1, \ldots, D-2$. These are $\binom{D-2}{p+1}$ independent components: since in this particular case $D = 6$ and $p = 1$ (because $p + 1 = 2$) we have $\frac{4!}{2!\,2!} = 3 \cdot 2 = 6$ degrees of freedom.

1.2 Gravity in Kaluza-Klein theory

Here we recall some basic facts of Kaluza-Klein theory. Since the very last goal of this section is to point out how the moduli emerge in higher dimensional theories, we do not explain Kaluza-Klein theory in detail but we briefly summarize the main results. Consider a five dimensional Minkowski spacetime described by the metric tensor

$$g_{AB} = \phi^{-1/3} \begin{pmatrix} g_{\mu\nu} - \kappa^2 \phi A_\mu A_\nu & -\kappa\phi A_\mu \\ -\kappa\phi A_\nu & \phi \end{pmatrix}$$

Here κ is a constant, ϕ is a scalar field and A_μ is a yet-undefined vector. A Fourier expansions reads

$$g_{AB} = \phi^{(0)\,-1/3} \begin{pmatrix} g^{(0)}_{\mu\nu} - \kappa^2 \phi^{(0)} A^{(0)}_\mu A^{(0)}_\nu & -\kappa\phi^{(0)} A^{(0)}_\mu \\ -\kappa\phi^{(0)} A^{(0)}_\nu & \phi^{(0)} \end{pmatrix} + \quad \infty \quad \text{tower of massive state} \tag{1.6}$$

Now let's consider the so called minimal extension of General Relativity: the five dimensional Hilbert Einstein action is

$$S^5 = \int \sqrt{-g_5}\; {}^{(5)}\mathcal{R}\, d^5x$$

and inserting (1.6) and reducing to four dimensions we arrive at

$$S^4 = \int d^4x \sqrt{-g_4} \left[M^2_{Pl}\; {}^{(4)}\mathcal{R} - \frac{1}{4} F^{(0)}_{\mu\nu} F^{(0)\,\mu\nu} + \frac{1}{6} \frac{\partial_\mu \phi^{(0)} \partial^\mu \phi^0}{(\phi^{(0)})^2} + \ldots \right]$$

that is a unified theory of electromagnetism, gravity and scalar fields.

1.2.1 Symmetries

The five dimensional theory is defined on $M^5 = M^{1,3} \times S^1$ where $M^{1,3}$ is the Minkowski spacetime and S^1 is a circle of radius R. The coordinate on S^1 is denoted by y. It is assumed that the radius of the fifth dimension is very small [1] in order to explain why this dimension has not be seen in high energy experiments. The five dimensional theory is invariant under general coordinate transformation

$$g'_{AB} = \frac{\partial x^C}{\partial x'^A} \frac{\partial x^D}{\partial x'^B} g_{CD}$$

Furthermore the field equation are scale invariant, that is if g_{AB} is a solution then also λg_{AB} with λ constant is a solution. However it is assumed that the fifth dimension is compactified so as to have the geometry of a circle of very small radius. Then there is a residual four dimensional general coordinate invariance, an abelian gauge invariance associated with the transformation of the compact manifold and the overall rescaling. In other words, the original five dimensional general coordinate invariance is spontaneously broken in the ground state in four dimensional coordinate invariance plus an abelian gauge invariance: this last feature allows the identification of A_μ with the gauge potential. Let's see these two last features in more detail. Recall that the line element can be written employing (1.6) as

$$ds^2 = \phi^{(0)\,-1/3} \left[g^{(0)}_{\mu\nu} dx^\mu dx^\nu - \phi^{(0)} (dy - \kappa A^{(0)}_\mu dx^\mu)^2 \right] \qquad (1.7)$$

y transformation

The most general transformation for the y variable is expected to be of the form

$$y \to y' = F(x^\mu, y)$$

In order to leave (1.7) invariant, the F dependence of x^μ and y cannot be arbitrary. Indeed the latter must be

$$y' = F(x^\mu, y) = y + f(x^\mu)$$

[1] For simplicity the radius is assumed to be of the same order of the Planck length, so that an energy equal to the Planck energy is needed to resolve it

so that differential on y' leads to

$$dy' = dy + \frac{\partial f(x^\mu)}{\partial x^\mu}\, dx^\mu$$

and since we want (1.7) invariant

$$A'^{(0)}_\mu = A^{(0)}_\mu + \frac{1}{\kappa}\frac{\partial f(x^\mu)}{\partial x^\mu}\, dx^\mu \tag{1.8}$$

It is well known that (1.8) represents the abelian gauge transformation associated to the vector potential $A^{(0)}_\mu$ (modulo a constant κ irrelevant for this purpose).

Overall rescaling

Consider the line element (1.7). Clearly the transformations

$$\begin{aligned} y &\to y' = \lambda y \\ A^{(0)}_\mu &\to A'^{(0)}_\mu = \lambda A^{(0)}_\mu \\ \phi^{(0)} &\to \phi'^{(0)} = \frac{1}{\lambda^2}\phi^{(0)} \end{aligned}$$

imply

$$ds^2 \to ds'^2 = \lambda^{2/3} ds^2$$

This means that if ds^2 is a solution and λ is a constant, then $ds'^2 = \lambda^{2/3} ds^2$ is also a solution. This reflects the fact that classical gravity is a scale invariant theory.

Comments

We have just pointed out that the price to pay to built a five dimensional theory unifying gravity and electromagnetism is the appearance of a massless scalar field $\phi^{(0)}$ in the spectrum of the four dimensional theory. In the original Kaluza-Klein theory $\phi^{(0)}$ was called *radion* while in the string theory context it has been called *modulus*. $\phi^{(0)}$ is a massless modulus field that parameterize the flat direction in the potential and so $\langle \phi^{(0)} \rangle$ and then the size of the fifth dimension is arbitrary and the theory does not provide any way to fix it. In other words, *it looks like all the values of the radius are equally good.* This is a manifestation of the fact that the theory cannot prefer a flat five dimensional Minkowski spacetime over $M^{1,3} \times S^1$ or over $M^{1,2} \times S^1 \times S^1$ as a solution.

Now one can asks what kind of manifold can produce a theory "roughly similar" to the SM one with SUSY at the TeV scale (and a little but non vanishing value of the cosmological constant in order to justify the recent observation of the accelerated expansion). The answer was found in [1]: the internal manifold must be Calabi-Yau one of complex dimension three. Size and shape (and therefore the volume) of the Calabi-Yau are controlled by moduli and the compactification on this manifold leads to the appearence of dozen of hundreds of these parmeters in the spectrum of the low energy EFT. The geometrical moduli can be divided into complex structure and Kähler moduli and since all the parameters of the low energy theory are tied to their VEV, moduli need to be stabilized. As said in the introduction, a theory with unstabilized moduli will suffer of lack of predictability.

Recent developments (fluxes, perturbative and non perturbative effects) allows to fix the volume and the shape of EDs leading to a large but discrete set of solutions. In a typical model the latter are estimated to be of the order of 10^{500}, leading to the so-called *string landscape* of solutions. From a mathematical point of view all of them are equally good, but from the physical it is expected that only one will describe the world we live in.

1.3 Scales and hierarchies

The aim of this section is to point out the scale of energy of the fundamental theory. We work in natural unit $\hbar = c = 1$ and the only free parameter is assumed to be the string tension α'. The string length l_s is tied to the tension by $l_s = 2\pi\sqrt{\alpha'}$ and the string mass is $M_s = l_s^{-1}$. Now consider the Einstein-Hilbert action in a D dimensional spacetime

$$S \sim M_*^{D-2} \int d^D x \sqrt{-g}\, \mathcal{R} \tag{1.9}$$

where M_* is the D dimensional (or fundamental) Planck mass and $\mathcal{R}$ is the D dimensional Ricci scalar. In the example of type II string compactification the $D = 10$ string frame action takes the form

$$S \sim M_s^8 \int d^{10}x \sqrt{-g}\, e^{-2\phi}\, \mathcal{R} \tag{1.10}$$

being ϕ the dilaton and the string coupling is $g_s = e^{\langle\phi\rangle}$. Comparing (1.9) and (1.10) we e find the relation between the fundamental Planck scale and the string scale. Setting $D = 10$ in (1.9) one has

$$M_* \sim M_s\, g_s^{-1/4}$$

Now we focus our attention to the case $D = 4$ and we call the four dimensional Planck mass simply as M_{Pl}. Comparing now the four dimensional Einstein - Hilbert action and the ten dimensional string action we find

$$M_{Pl}^2 = M_s^8 \,\mathrm{Vol}\,(X_6) \tag{1.11}$$

where $\mathrm{Vol}\,(X_6)$ denotes the overall volume of the internal manifold. The latter can be written in terms of the string length and an dimensionless quantity $\mathcal{V}$ as

$$\mathrm{Vol}\,(X_6) = \mathcal{V}\, l_s^6 = \frac{\mathcal{V}}{M_s^6}$$

Finally from (1.11) follows the relation

$$M_s = \frac{M_{Pl}}{\mathcal{V}^{1/2}} \tag{1.12}$$

To estimate Kaluza-Klein mass we first recall the toroidal compactification. A stringy ground state of Kaluza-Klein and winding integers n and w has mass[2]

$$m_{KK}^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} \tag{1.13}$$

where R is the dimensionful Kaluza-Klein radius, that can be written in terms of the string length as $R = R_s l_s$ where $R_s \gg 1$. If this holds, then we can estimate Kaluza-Klein mass as

$$m_{KK} \sim \frac{M_s}{R_s}$$

and if we assume $\mathcal{V} \sim R_s^6$ we readily get

$$m_{KK} \sim \frac{M_s}{\mathcal{V}^{1/6}} \sim \frac{M_{Pl}}{\mathcal{V}^{2/3}} \tag{1.14}$$

where the last relation follows immediately from (1.12).

[2]Strictly speaking (1.13) holds only for toroidal compactification, but it suffices to estimate the relevant mass scale.

How large can EDs be?

So far we have pointed out all the relevant mass scale for a typical theory. These can be written as a function of the (four dimensional) Planck mass and an dimensionless volume, as in (1.12) and (1.14). Furthermore, the Planck mass M_{Pl} can be written in terms of the fundamental Planck mass M_* and the volume of the D-dimensional internal manifold X as

$$M_{Pl}^2 = M_*^{D-2}\,\mathrm{Vol}(X_{D-4}) \tag{1.15}$$

$$\sim M_*^{D-2} R^{D-4} \tag{1.16}$$

Since in high energy experiments we have explored regions near $\mathcal{O}(10^{-16})$ cm, consistency requires $R \leq \mathcal{O}(10^{-16})$ and so $M_* \geq \mathcal{O}(1)$ TeV. In Kaluza-Klein theories there are no reasons to suppose a large value of the volume and it has usually been assumed that $M_* \sim M_{Pl}$. However the actual value of M_* has to be determined dynamically by moduli stabilization.

Chapter 2

The Cosmological Moduli Problem

Despite the differences between the various type of string theory, the presence of a moduli sector is a generic and model independent feature. String moduli are fields that interact only *via* gravitational strength interactions and hence have Planck-suppressed couplings to Standard Model's fields. Their potential is exactly flat in the supersymmetric limit but become curved due to supersymmetry breaking and non-perturbative effects, obtaining then a defined vev. Moduli are produced by Big Bang and, in a lesser extend, by any phase transition. After (ordinary) inflation, they are expected to be far from the low energy minimum and they begin to oscillate. Energy stored in the oscillations redshifts as $a^{-3}(t)$ ($a(t)$ being the scale factor) so they behave as non relativistic matter, opposed to radiation. This, in turn, implies that moduli can dominate the energy density of the universe until they decay: thanks to their long lifetime this occur late in history of the universe, spoiling the successful predictions of Big Bang Nucleosynthesis (BBN). This happens because they decay at a temperature too low to allow nucleosynthesis; furthermore decay product may destroy hydrogen leaving an overproduction of helium and entropy release will reset the baryon/antibaryon asymmetry. These are the main problems, but there are also other as overproduction of gravitini and dark matter. Let's see this in more formal term.

2.1 Moduli dynamics

It is usually assumed that the history of the universe begin with a period of inflation. This period is invoked to solve the problems left over by the Hot Big Bang theory, such as flatness, horizons and also provide an explanation for Large Scale Structures and CMB anisotropies. However inflation can't provide an exhaustive explanation about topological defects and relics (this problem was noted in [21]). Since universe had undergone to many phase transitions, one expects topological defect may be produced: so why haven't we seen them yet? The usual answer to this question is that these objects were diluited by the inflation, so even if universe is populated by a huge amount of them, they can't be seen because they are diluited too much.

However, the problem is not so simple, especially when one tries to conciliate the Standard Cosmology Theory with SUSY and SUGRA. The problem essentially lies in the fact that relics and other potentially dangerous fields may be produced after the end of the inflation, and typically this is the case. Let's focus our attention on moduli. In the previous chapter we learnt that the price to pay to have a theory unifying both electromagnetism and gravity was the appearance of a scalar, non-physical field in the spectrum of the four dimensional effective theory; this was due to the compactification on the circle S^1. Then after the discovery of strong and weak forces, and later of SUSY, physicists were faced with this question: what is the manifold that, after compactification, can give a four dimensional SUSY theory? As pointed out in [1], the manifold must be a Calabi-Yau one of complex dimension 3. Compactification on this manifold typically produces dozen of hundred of these scalar fields, in modern language are called *moduli*, which parametrize size and shape of the Calabi-Yau.

Moduli are produced in the first universe as coherent oscillation and, in a lesser extend, by any phase transition. Inflation cannot address the moduli problem because an excessive number of these is produced after the inflation has ended. Let's see this in detail. Consider a modulus Φ; its evolution is governed by the effective potential, which depends not only on Φ, but also on other scalar fields ϕ and the temperature T. An useful parametrization is [35]

$$V_{\text{eff}}(\Phi, \phi, T) = V_0(\phi, T) + V_1(\phi, T) + V_2(\phi, T) + \dots$$

where ellipses denotes higher order irrelevant terms. Referring to the present, the V_n's are given by

$$V_0(\phi_0, T_0) = V_1(\phi_0, T_0) = 0 \tag{2.1}$$
$$V_2 = m_0^2/2 \tag{2.2}$$

where ϕ_0, T_0 and m_0 denotes the present VEV of ϕ, the temperature and the present mass of Φ. The vanishing of V_0 is due to the extremely small value of the cosmological constant. In the early universe this values are significantly different: it is expected that $V_2(\phi, T) \sim \alpha^2 H^2$ and so the effective mass m_Φ is thought to be different from the present value m_0. The expected form is

$$V_{\text{eff}}(\Phi, T) = \frac{m_0^2}{2}(\Phi - \Phi_0)^2 + \frac{\alpha^2 H^2}{2}(\Phi - \Phi_1)^2 + \dots \tag{2.3}$$

Here Φ_1 is the VEV of Φ in the early universe and generally it is different from Φ_0. The displacement from the true VEV is quantified by $\delta\Phi = \Phi - \Phi_0$ and introducing the variable Φ_2 defined as $\Phi_2 = \Phi_1 - \Phi_0 \sim M_{Pl}$ we can rewrite the effective potential as

$$V_{\text{eff}}(\Phi, T) = \frac{m_0^2}{2}(\delta\Phi)^2 + \frac{\alpha^2 H^2}{2}(\delta\Phi - \Phi_2)^2 + \dots \tag{2.4}$$
$$= \frac{(m_0^2 + \alpha^2 H^2)}{2}\left(\delta\Phi - \frac{\alpha^2 H^2}{m_\Phi^2 + \alpha H^2}\Phi_2\right)^2 + \dots \tag{2.5}$$

To obtain a more accurate results one has to solve the equation of motion for Φ

$$\ddot{\Phi} + 3H\dot{\Phi} + (m_0^2 + \alpha^2 H^2)\Phi = \alpha^2 H^2 \Phi_1 \tag{2.6}$$

We can treat both α and Φ_1 as time independent constant and we set as initial conditions

$$\Phi(t_i) = \Phi_i \tag{2.7}$$
$$\dot{\Phi}(t_i) = 0$$

This equation has to be solved during inflation, where $H(t)$ can be treated as time independent constant, and both during radiation-dominated era (when $H = 1/2t$) and matter-dominated era (for which $H = 2/3t$).

2.1.1 Moduli dynamics during inflation

In this section we study the behaviour of the moduli fields during inflation. Computation can be found in A.1 so we limit our discussion to some comments.

Comments

We can consider some interesting limit for the solutions of (A.12). Let's consider β and suppose $\alpha \ll 1$ and $m_0/H \ll 1$ and set $x = \alpha^2 + m_0^2/H^2$. Clearly $x \ll 1$ and a Taylor expansion of the square roots yields

$$\beta \sim 1 - \frac{1}{2}x^2 = 1 - \frac{2}{9}\left(\alpha^2 + \frac{m_0^2}{H^2}\right)$$

Calling Φ_f the modulus's value after inflation, this is given in terms of the e-folding number $\mathcal{N}_{\rm ef}$ by

$$\Phi_f \simeq \Phi_i - \frac{\mathcal{N}_{\rm ef}}{3}(\Phi_i - \Phi_{\rm min})\left(\frac{H^2}{m_0^2 + \alpha^2 H^2}\right)$$

and in this case $\mathcal{N}_{\rm ef} \ll \frac{H^2}{m_0^2+\alpha^2 H^2}$. This is the behaviour during inflation: before considering the role of H due to expansion, Φ is frozen at some initial value Φ_i.

If $m_0 \ll H$ but $\alpha \sim 1$ the final modulus value is

$$\Phi_f \simeq \Phi_{\rm min} + (\Phi_i - \Phi_{\rm min}) \times \mathcal{O}\left(e^{\frac{-3\mathcal{N}_{\rm ef}}{2}}\right)$$

showing that independently from the initial value Φ_i the modulus tends to reach the temporal minimum $\Phi_{\rm min} \simeq \Phi_i$. Finally we consider the limit $m_0 \gg H$. The modulus exponentially approaches to the minimum $\Phi_{\rm min} \simeq \frac{\alpha^2 H^2}{m_0^2}\Phi_1$ with exponentially decreasing oscillations.

2.1.2 Post-inflationary dynamics

In the post-inflationary dynamics, the equation of motion (2.6) get a further complication due to the time dependence of $H(t)$ encoded by $H = p/t$ where $p = \frac{1}{2}$ (RD) and $p = \frac{2}{3}$ (MD). Introducing a new variable $z = m_0 t$, the solution is [35]

$$\Phi(z) = p^2\alpha^2\Phi_1 \frac{S_{\theta-1,\nu}(z)}{z^\theta} + C_1\frac{J_\nu(z)}{z^\theta} + C_2\frac{Y_\nu(z)}{z^\theta} \tag{2.8}$$

where $\nu^2 = \theta^2 - p^2\alpha^2 \geq 0$ for $\theta = \frac{3p-1}{2}$, $J_\nu(z)$ and $Y_\nu(z)$ are Bessel functions and $S_{\mu,\nu}(z)$ is the Lommel function

$$S_{\mu,\nu}(z) = \frac{\pi}{2}\left[Y_\nu(z)\int_0^z y^\mu J_\nu(y)\,dy - J_\nu(z)\int_0^z y^\mu Y_\nu(y)\,dy\right]$$
$$+ 2^{\mu-1}\Gamma\left(\frac{\mu-\nu+1}{2}\right)\Gamma\left(\frac{\mu+-\nu+1}{2}\right)\left[\sin\left(\frac{\mu-\nu}{2}\,\pi\right)J_\nu(z) - \cos\left(\frac{\mu-\nu}{2}\,\pi\right)Y_\nu(z)\right]$$

From the initial condition follows the values of the constants C_1 and C_2

$$C_1 = A_1\Phi_i + B_1\alpha^2\Phi_1$$
$$C_2 = A_2\Phi_i + B_2\alpha^2\Phi_1$$

where

$$A_1 = \frac{\pi}{2}z_i^\theta\left[z_i\,Y_\nu'(z_i) - \theta\,Y_\nu(z_i)\right]$$
$$A_2 = -\frac{\pi}{2}z_i^\theta\left[z_i\,J_\nu'(z_i) - \theta\,J_\nu(z_i)\right]$$
$$B_1 = -\frac{\pi}{2}z_i^\theta\left[Y_\nu'(z_i)S_{\theta-1,\nu}(z_i) - Y_\nu(z_i)S_{\theta-1,\nu}'(z_i)\right]p^2$$
$$B_2 = \frac{\pi}{2}z_i^\theta\left[J_\nu'(z_i)S_{\theta-1,\nu}(z_i) - J_\nu(z_i)S_{\theta-1,\nu}'(z_i)\right]p^2$$

and the primes denotes the differentiation with respect to z. We are interesting in moduli abundance coming from coherent oscillations. For $z \gg 1$, $\Phi(z)$ is dominated by the oscillatory tail

$$\Phi(z) \sim \left(\frac{2}{\pi}\right)^{\frac{1}{2}} z^{-\frac{3p}{2}}\left\{C_1\cos\left[z - \frac{(\nu+1/2)\pi}{2}\right] + C_2\sin\left[z - \frac{(\nu+1/2)\pi}{2}\right]\right\}$$

From this last relation we can estimate moduli number density n_Φ as

$$n_\Phi = \frac{m_0}{2}\Phi^2 \sim \left(\frac{1}{\pi}\right)z^{-3p}m_0(C_1^2 + C_2^2) \tag{2.9}$$

and the energy density as $\rho_\Phi = n_\Phi m_0$. We can introduce the quantity Y_Φ defined as

$$Y_\Phi = \frac{n_\Phi}{s} \tag{2.10}$$

where s is the entropy density of the radiation

$$s = \frac{2\pi^2}{45}\,g_*(T)T^3$$

Unless some entropy is produced, the quantity Y_Φ remains constant during the history of the universe and so it has a crucial role in the calculation of moduli abundances. Its value is

$$Y_\Phi = \frac{45}{2\pi^2 g_*}\left(C_1^2 + C_2^2\right)\frac{m_0}{z^{3p}T^3} \tag{2.11}$$

Most of the cosmological implications are associate to the amplitude of the oscillations

$$\delta\Phi \equiv \left(\frac{C_1^2 + C_2^2}{\pi}\right)^{1/2} \tag{2.12}$$

which represent the initial moduli misalignment. If this value gets too large, moduli are able to oscillate to a relatively long epoch, leading to a matter dominated-era before the radiation one and spoiling the predictions of the Big Bang Nucleosynthesis.

2.2 Cosmological Moduli Problem

Consider a modulus Φ with effective mass m_Φ moving in a Freedman-Robertson-Walker background: oscillations' amplitude fulfill

$$\ddot{\Phi} + (3H + \Gamma_\Phi)\,\dot{\Phi} + V_{,\Phi} = 0 \tag{2.13}$$

where $H = H(t) = \frac{\dot{a}}{a}(t)$ is the Hubble parameter, $\Gamma_\Phi \sim m_\Phi^3/M_{Pl}^2$ is the decay rate and $V_{,\Phi}$ denotes the derivative of the potential energy respect to Φ. When $H \geq m_\Phi$, at some time $t \leq t_{in}$, the friction term dominates in the evolution equation forcing Φ to stay at some initial value, say $\Phi = \Phi_{in}$. Then, when Hubble parameter become of the same order of modulus mass ($H \sim m_\Phi$), at $t > t_{in}$, modulus starts to oscillate around the minimum and soon dominates the energy density. This occurs because at $t > t_{in}$, the moduli energy density at temperature $T_{in} \sim (m_\Phi M_{Pl})^{1/2}$ is $\rho_\Phi \sim m_\Phi^2 \Phi_{in}^2$ while, since Friedmann equations implies $H \sim T^2/M_{Pl}$ for radiation, its energy density is $\rho_{\rm rad} \sim H^2 M_{Pl}^2$. Since for a modulus we expect $\Phi_{in} \sim M_{Pl}$, one has $\rho_{\rm rad} \sim \rho_\Phi$. Moduli coherent oscillations will soon dominate energy density of the universe because oscillations energy decrease as a^{-3} while radiation energy density decrease as a^{-4}: then we are entering in a moduli dominated universe. Then, consider the relation

$$\frac{\rho_\Phi(T)}{\rho_\Phi(T_{in})} = \left(\frac{T}{T_{in}}\right)^3 \tag{2.14}$$

If the field Φ is stable, oscillations may overclose the universe. Imposing $\rho_\Phi(T) < \rho_{crit} \sim (10^{-3}\text{eV})^4$ one finds that a scalar fields of mass $m_\Phi > 10^{-26}\text{eV}$ will overclose the universe. But generally moduli decay and so many other problems arise. The scalar field Φ decays at temperature T_D for which $H(T_D) \sim \Gamma_\Phi$; evaluating (2.14) at $T = T_D$ when moduli energy density is $\rho_\Phi(T_D) = (\Gamma_\Phi M_{Pl})^2$ one can find the decay temperature, that is

$$\begin{aligned} T_D &= T_{in}\left(\frac{\rho_\Phi(T_D)}{\rho_\Phi(T_{in})}\right)^{1/3} \\ &= (m_\Phi M_{Pl})^{1/2}\left(\frac{(\Gamma_\Phi M_{Pl})^2}{(m_\Phi \Phi_{in})^2}\right)^{1/3} \\ &= m_\Phi^{11/6} M_{Pl}^{-1/6} \Phi_{in}^{-2/3} \end{aligned}$$

Moduli decay reheat the universe. We can estimate the reheating temperature T_{RH} supposing the decay product promptly thermalize $T_{RH} \simeq (\rho_\Phi(T_D))^{1/4} \sim (\Gamma_\Phi M_{Pl})^{1/2}$, that is

$$T_{RH} \sim \left(\frac{m_\Phi^3}{M_{Pl}}\right)^{1/2} \tag{2.15}$$

In order not to upset nucleosynthesis is required $T_{RH} \geq \mathcal{O}(10)$ MeV and this put a lower bound on modulus mass. Indeed it has to be $m_\Phi^3/M_{Pl} \sim 10^{-4}\,(\text{GeV})^2$ and so $m_\Phi^3 \sim 10^{14}\,(\text{GeV})^3$: we conclude that moduli whose mass is $m_\Phi \geq \mathcal{O}(100)$ TeV are not dangerous for Standard Cosmology because they decay before BBN. However moduli with mass lower than this bound have a reheating temperature too low to allow successful nucleosynthesis: this is *The Cosmological Moduli Problem* of string theories. In fact, if in this case the moduli decay into photons and their energy exceeds the binding energy of light nuclei, photo-dissociation process are allowed and abundance of light elements may be profoundly altered, causing a significant discrepancy between theory and observation. Knowing the abundances of Hydrogen, Deuterium and Helium today we can infer bounds of moduli number density, masses and lifetimes [27].

There are also others problems, as we can see in the next section.

2.3 Other cosmological problems

2.3.1 Gravitino overproduction

If $m_\Phi \gg m_{3/2}$, the decay of a modulus into gravitini is allowed. This, together with the fact that gravitini may also be produced by scattering process caused by thermal radiation after moduli decay, leads to an overproduction of gravitini at low energies whose decay products can destroy light nuclei produced in early universe. As an example, for gravitino mass 10^{2-3} GeV, scalar masses must be large than $\mathcal{O}(100)$ TeV to ensure the validity of BBN [6] (here scalar masses denotes any scalar field that interact only gravitationally).

2.3.2 Baryogenesis

Moduli decay generate a huge amount of entropy: this is quantified by

$$\begin{aligned}\Delta = \frac{s(T_{RH})}{s(T_D)} &\sim \left(\frac{T_{RH}}{T_D}\right)^3 = \left(\frac{(m_\Phi^3/M_{Pl})^{1/2}}{m_\Phi^{11/6} M_{Pl}^{-1/6} \Phi_{in}^{-2/3}}\right)^3 \\ &\sim \frac{\Phi_{in}^2}{m_\Phi M_{Pl}}\end{aligned}$$

Since for a modulus $\Phi_{in} \sim M_{Pl}$ and $m_\Phi \sim m_{3/2} \sim O(1)$ TeV, it is expected that $\Delta \sim 10^{15}$. This is an enormous increase of entropy that can erase the previous baryon-antibaryon asymmetry. At high temperature there are mechanisms to generate this asymmetry: for example the electro-weak baryogenesis [23]which uses electroweak phase transition and sphalerons. However, the reheat temperature after flaton decay will be too low to make this mechanism works. Maybe the most efficient mechanism could be the Affleck-Dine (AD) baryogenesis [24], because it can generate huge asymmetries which can survive to the full entropy production of the thermal inflation needed to diluite moduli to acceptable levels.

Chapter 3

Thermal Inflation

In the previous chapter we have noticed the cosmological difficulties associated to string moduli. From a cosmological point of view this problem could be solved by a short period of low energy inflation, such to leave unaffected the large scale density perturbation accounting for the CMB anisotropies and the large scale structure.

Indeed following [21]the problems of flatness and horizon as well as the formation of large scale structures are solved by the (ordinary) inflation while a short second period of weak-scale energy inflation diluites relics left over and solves the CMP.

Let's see why this kind of inflation should be different from the slow-roll one. This essentially lies on the bound imposed by the slow-roll conditions: necessary condition for the slow-roll inflation is that the inflaton mass must be less than the Hubble parameter. Since to avoid too much moduli reproduction there must be

$$V_0^{1/4} \sim 10^7 \text{ to } 10^8 \,\mathrm{GeV} \tag{3.1}$$

one has a severe constraint on the inflaton mass: knowing $H \sim V_0^{1/2}/M_{Pl}$ one has $H \sim \mathcal{O}(1)$ MeV and to have inflation is needed $m_{\mathrm{inf}} \ll \mathcal{O}(1)$ MeV, *i.e.* the inflaton should have a very low mass. But thermal inflation naturally occurs at the energy scale displayed in (3.1) and since at these scales $H \ll m_{\mathrm{s}}$ one can expect that under optimistic circumstances the thermal inflation to solve the CMP. Indeed a generic modulus is expected to have a mass of the same order of m_{s}, defined as the mass of the supersymmetric partners of the Standard Model's particles: slow-roll inflation can't solve the problem because it occurs at $H \gg m_{\mathrm{s}}$ while moduli are generate at $H \sim m_{\mathrm{s}}$ and in a lesser extend by any phase transition at $H < m_s$. Therefore, in order to address the problem it is needed inflation at $H \ll m_s$ and thermal inflation is the most plausible candidate. Since flatons have central role in thermal inflation, before going further it is useful to examine carefully their properties and dynamics.

3.1 Flaton's dynamics

Flaton's dynamics is determined by the form of its effective potential. The effective potential in early universe is generally expected to be different from the effective potential today: while the first is fundamental for the dynamic at high energy, the latter play a central role in the low-energy dynamic. It is necessary to know both them to have a complete picture of the flaton's dynamic. First of all we need to clarify what is meant for high (resp.) low energy effective potential.

3.1.1 High energy Effective Potential

In early universe, the interactions of a given field σ with other fields ϕ, ψ modify the form of its effective potential $V(\sigma)$. So we have to clarify the statement "effective potential of σ". There exists only one effective potential V, and this is a function of all the scalar fields $V(\sigma, \phi, \psi, \dots)$. In early universe it is reasonable to assume the fields are displaced from their VEVs, so saying effective potential of a given field we have in mind the full effective potential where all the others fields are taken with their current time average, so that terms like $\sigma^2\phi^2$ gets replaced by $\sigma^2\langle\phi^2\rangle_t$ (here $\langle\dots\rangle_t$ denotes a temporal average). Even if the effective potential $V(\sigma)$ changes with the history of the universe, we can always assume a vanishing gradient at the origin: this is due to the invariance respect with one ore more $\mathbb{Z}_n$. Indeed if we expand the full potential $V(\sigma, \phi, \psi, \dots)$ as a power series of its fields, each term is expected to be invariant upon one (or more) $\mathbb{Z}_n$ symmetry, unless it consist in just the first power of of one field. As an example, a term like $\sigma^2\phi^2$ is $\mathbb{Z}_2$ invariant with respect both σ and ϕ. Since only a few leading terms are important, it is reasonable to assume the full effective potential contain one or more $\mathbb{Z}_n$ symmetry and so a vanishing gradient at the origin. Now let's say what we can infer about the effective mass squared $V''(0)$ in early universe. During inflation, all the fields are expected to acquire contribution of order $\pm H^2$ due to expansion to their effective mass square: this is precisely what happens for the moduli, because they feel only gravitational strength interaction. For other fields one can think to a stronger contribution, that is $\pm\alpha^2 H^2$ with $\alpha \gg 1$: this is true for flatons σ with a smaller VEV. If we set $\langle\sigma\rangle \equiv M$ then we can identify $\alpha = M_{Pl}/M$. This is what we think to happened during inflation. After inflation it is not clear what the mass-squared will be. If the interaction is of gravitational strength, one expect contributions of the same order of $\pm H^2$. We can say that near the origin the flaton can have unsuppressed interactions with other fields. Let's consider an interaction term like $\lambda|\sigma|^2\phi^2$: when the flaton σ is in the vev it gives a contribution $2\lambda\langle\sigma\rangle = 2\lambda M$ to m_ϕ^2. Since M is large, if m_ϕ is small then λ must be small. If instead m_ϕ is of the same order of M and it is generated by this interaction, then $\lambda \sim 1$ is expected for the flaton near the origin and the field ϕ becomes light. This address the fact that the flaton near the origin can have *unsuppressed interaction with light fields*. If these fields have an effective mass of order $|\sigma|$, the flaton will be in thermal equilibrium in the regime $|\sigma| \leq T$, because fields with effective mass greater than T are too rare to be maintained in thermal equilibrium. If this is the case, one can consider the finite temperature correction to the effective potential, that in turn gives a contribution $(T^2 - m_0^2)$ to the effective mass. The effective potential acquires a local minimum in the origin for some T bigger than $T_C \sim m_0 \sim m$, being m_0 the effective $T = 0$ mass squared. In this situation the flaton is forced to stay at the origin. At $T \sim T_C$ the phase transition occurs and the flaton moves from the origin towards its true VEV, that is the true minimum of the effective potential.

3.1.2 Low energy Effective Potential

Consider a flaton σ. In the limit of absolutely flat potential there is a U(1) symmetry, so the effective potential depends on σ only through $|\sigma|$. In reality one cannot speak about "low energy effective potential of σ": there exists only one effective potential V and this is a function of all the scalar fields $V(\phi, \psi, \sigma, \dots)$. Saying low energy effective potential one has in mind $V(\phi, \psi, \sigma, \dots)$ where all the fields ϕ, ψ except the flaton are evaluated at their VEVs, so terms like $\psi^2\phi^2$ get replaced by $\langle\psi^2\rangle\langle\phi^2\rangle$. The U(1) symmetry may survive or may be broken: if it remains exact, the Goldstone boson corresponding to the angular direction is massless; if it gets broken, the Goldstone boson will acquire mass. Its mass depends from how the symmetry is broken: if the symmetry is slightly broken the Goldstone boson is light, while if it is strongly broken the Goldstone boson became just another flaton. In what follows we consider the case when the symmetry survive.

Global U(1) symmetry

We consider the case where the symmetry survive. The effective potential along the flat direction can be written as

$$V(\,|\sigma|\,) = V_0 - m_0^2\,|\sigma|^2 + \sum_{n=1}^{\infty} \lambda_n \,\frac{|\sigma|^{2n+4}}{M_{Pl}^{2n}} \tag{3.2}$$

where $m_0 \sim 10^2$ to 10^3 GeV is the true effective mass of the flaton and higher order non renormalizable terms make the effective potential "almost flat" near the VEV. We have in mind the case where the true mass squared at the origin is negative: this assigns a non vanishing VEV, but rather $\langle\sigma\rangle \equiv M \gg m_0$ and we can safely assume $M \geq 10^{10}$GeV (see appendix A.6 how to reach this conclusion). V_0 is tuned to have a vanishing cosmological constant at the VEV $V_0 = m_\sigma^2 M^2$ and

$$\begin{aligned} m_\sigma^2 &= 2(n+1)m_0^2 \\ M^{2n+2}M_{Pl}^{-2n} &= [2(n+1)(n+2)\lambda_n]^{-1}m_\sigma^2 \\ V_0 &= [2(n+2)]^{-1}m_\sigma^2 M^2 \end{aligned}$$

where $m_\sigma = V''(M)/2$ has been used. Observe that the potential (3.2) does not contain the term $\lambda|\sigma|^4$: this term is forbidden by discrete or continuous gauge symmetry in combination with SUSY. SUSY breaking generate this term with suppressed coupling $\lambda \sim (m_0/M_{Pl})^2$ and it is negligible for all flatons that are not moduli.

3.2 Cosmology with flatons

3.2.1 Flaton initially held at the origin

Suppose that the flaton is trapped at the origin because of finite temperature correction, giving to it a positive effective mass squared. The energy density is

$$\rho = V_0 + \frac{\pi^2}{30} g_* T^4 \tag{3.3}$$

where g_* is the effective number of species in thermal equilibrium. When the flaton is held at the origin, the vacuum energy V_0 dominates and a short period of inflation develops. This era starts at $T_{\rm in} \sim V_0^{1/4} \sim (m_0 M)^{1/2}$ and ends at $T_{\rm end} \sim m_0$ when the flaton rolls away from the origin and move towards the true VEV M and start oscillating around it. The e-folding number is estimated as

$$\mathcal{N}_{\rm ef} \sim \ln\left(\frac{T_{\rm in}}{T_{\rm end}}\right) \sim \frac{1}{2}\ln\left(\frac{M}{m_0}\right)$$

and we can safely assume $\mathcal{N}_{\rm ef} \lesssim 10$, so thermal inflation can never replace ordinary inflation. Recall that the latter takes place at a very high energy scale: in most of the models $V_0^{1/4} \sim 10^{16}$ GeV and the lowest value proposed is $V_0^{1/4} \sim 10^{12}$ GeV while thermal inflation follows the bound displayed in (3.1). After the end of thermal inflation we enter in a matter dominated era by flaton particles.

Conditions for the trapping

In the previous section we speak about a trapping due to the finite temperature correction: this has to be meant as a contribution to the scalar potential of the flaton. Let's see how this is made possible. Consider a flaton σ: in order to be held at the origin it has to interact rapidly with the fields in the thermal bath of the universe. Suppose there is a very massive scalar field ψ that interact with the flaton through the interaction $g|\sigma|^2\psi$ (here g is a coupling constant). Recalling (3.2), the one loop thermal corrections associated to ψ will alter the effective potential, that in turn looks as

$$V_{\rm eff}(|\sigma|) = V_0 + (gT^2 - m_0^2)|\sigma|^2 + \dots \tag{3.4}$$

At high temperature the flaton has a positive effective mass squared that forces it to stay in a false vacuum situation. When the temperature drops below the critical value $T_C \sim \frac{m_0}{g^{1/2}}$ the phase transition occurs and the effective potential develops two minima. The flaton then starts rolling towards the $T = 0$ minimum and begins to oscillate around it. Once the minimum has been reached, the mass terms for ψ is generated and we can say that $m_\psi \sim M$.

Thermal correction to the flaton itself

One can also consider thermal correction to the flaton itself. However, as pointed out in [31], these are irrelevant because they can neither trap the field nor cause a phase transition. Let's consider one loop thermal correction: these describe an ideal gas of non interacting particles and they have the standard form

$$V_1(m_0, T) = \pm \frac{T^4}{2\pi^2} \int_0^\infty dx x^2 \ln\left(1 \mp e^{-\sqrt{x^2 + m_0^2/T^2}}\right) \tag{3.5}$$

where the upper (lower) signs are for bosons (fermions). Since we are interested in high temperature regime $T \gg m_0$, we have to look for solution in the approximation $m_0/T \ll 1$. It is found that

$$V_1(m_0, T) = -\frac{\pi^2 T^4}{90}\alpha + \frac{T^2 m_0^2}{24} + \mathcal{O}(T m_0^3) \tag{3.6}$$

where $\alpha = 1$ for bosons and $\alpha = 7/8$ for fermions. These corrections can be interpreted as a σ independent shift in the potential: this corresponds to add a constant to the energy density and so equation of motion are left unaltered. But one can go beyond the one loop approximation, hoping to find a correction to m_0^2 proportional to σ^2: it happens that this correction occurs at the $(n+1)$-loop and it is of the order $T(T^2/M_{Pl})^{2n}$, so it is negligible.

This can also be understood in a simpler way as follows: since one loop thermal correction goes as

$$V_T \sim T^2 m_0^2 = T^2 \frac{dV(|\sigma|)}{d|\sigma|} \tag{3.7}$$

and in (3.2) there isn't the term $\lambda\sigma^4$ we cannot have a term proportional to σ^2 and then we cannot give to the flaton the effective mass required to be held at the origin.

3.2.2 Flaton initially displaced from the origin

Now suppose that the flaton field is displaced from the origin, that is it has a large value in early universe. In this case the flaton cannot be held in thermal equilibrium because we know its interactions are too weak. The potential energy can be parameterize as

$$V(|\sigma|) = m_0^2(|\sigma| - M)^2 + \alpha^2 H^2(|\sigma| - \sigma_0)^2 \tag{3.8}$$

Here M denotes the true VEV, *i.e.* the minimum of the effective potential at small H while σ_0 is the minimum of the effective potential at large H. To simplify our analysis let's suppose σ_0 time-independent, so the only time dependence is encoded in H. Then we have to distinguish both the cases $\alpha \sim 1$ and $\alpha \gg 1$.

Case $\alpha \sim 1$

For flatons such as moduli is expected $\alpha \sim 1$. Furthermore since moduli feel only gravitational strength interaction we set $M \sim M_{Pl}$. For large H the minimum is σ_0 and when H drops below m_0 the flaton moves towards the true minimum and start to oscillate around it with large amplitude, since $|\sigma_0 - M| \sim M$.

Case $\alpha \gg 1$

If $\alpha \gg 1$, things drastically changes. Indeed, as pointed out by Linde [37]the flaton reaches the true minimum without appreciable oscillations. Put in other words, the flaton is all times near to the small H minimum. If this is the case, cosmological production of flaton fields is strongly suppressed.

3.2.3 The flaton decay rates and reheating temperature

After the thermal inflation has ended, the flaton moves toward its true vev and starts to oscillate around it. We enter in a matter dominated era by flaton particles because they behave as matter (opposed to the radiation) and so they redshifts as $a^{-3/2}$. It is commonnly believed that the interactions with other fields take away some of the oscillations energy, so oscillations amplitude decrease faster. If oscillations amplitude is sufficiently small and interactions are sufficiently weak, each field decay at a single particle decay rate Γ. The decay temperature T_D can be estimated setting $\Gamma^{-1} \sim H^{-1}$: recalling $H = g_*^{1/2} T^2 / M_{Pl}$ we find

$$T_D = \left(\frac{\Gamma M_{Pl}}{g_*^{1/2}} \right)^{\frac{1}{2}} \mathrm{GeV} \tag{3.9}$$

The assumption that each flaton decay at a single particle decay rate is thought to be incorrect because one has also to take into account non linear-relaxation effect as parametric resonance. As soon as oscillations begin, parametric resonance can drain off much of the oscillations energy, converting it in marginally relativistic scalar particles (also spin 1 particles may be produced while fermions cannot be produced in a significant number because of the Pauli exclusion principle). If the decay product thermalize they get converted into relativistic radiation while if nothing happened to the produced scalar particles they are expected to decay after few Hubble times at one particle decay rate. Nowadays it is not clear how parametric resonance can create particles which thermalize successfully; it is however clear that the flaton components of the produced particles cannot thermalize because the interactions are too weak to be maintained in thermal equilibrium. Any radiation produced by parametric resonance will redshift away in few Hubble times, so after the end of thermal inflation the energy density is dominated by non relativistic scalar particles, including the flatons. Each particle will decay at a single particle decay rate, so we are expected to find only the long-lived particles, that dominate the energy density until they decay.

To simplify our analysis we assume each flaton decay at a single particle decay rate. To estimate the decay temperature T_D we need the relation between the decay rate Γ and the VEV M of the flaton. From a naïf dimensional analysis it is expected that $\Gamma \sim m_0^3/M^2$ where $m_0 \sim 10^3$GeV and set $g_*^{-1/4} \sim 1$ [1]

$$T_D \sim \frac{10^{14}}{M} \mathrm{GeV}^2 \tag{3.10}$$

Now we are going to point out some bonds on T_D: in particular, since T_D and M are inversely proportional, it follows that the larger is M, the smaller gets T_D.

[1] According to the Standard Model, $g_*^{1/4}$ range from 1 to 2 if $T \gtrsim 100\,\mathrm{MeV}$ and amounts to 4 when $T \gtrsim 100\,\mathrm{GeV}$ in supersymmetric extension of the Standard Model

Electro-weak baryogenesis

Electro-weak baryogenesis requires $T \gtrsim 100\,\mathrm{GeV}$ so from (3.10) it follows that $M \lesssim 10^{12}\,\mathrm{GeV}$

Thermalization of stable LSP

Thermalization of stable LSP requires $T \gtrsim 1\,\mathrm{GeV}$ so from (3.10) it follows that $M \lesssim 10^{14}\,\mathrm{GeV}$

Successful nucleosynthesis

Successful nucleosynthesis requires $T \gtrsim 10\,\mathrm{MeV}$ so from (3.10) it follows that $M \lesssim 10^{16}\,\mathrm{GeV}$

Comment

The most serious problem lies in the nucleosynthesis: this because thermal inflation can provide itself a mechanism for baryogenesis [36]. Otherwise baryogenesis can also be implemented through Affleck-Dine mechanism [24].

If the decay product promptly thermalize, the reheating temperature T_R is equal to the decay temperature T_D. From the discussion above it should be clear that a modulus with a Planckian VEV is nothing but a disaster for standard cosmology. Indeed the reheating temperature (in this approximation) associated to a modulus with the same mass of the flaton amounts to $T_R \sim \mathcal{O}(10^{-4})\mathrm{MeV}$, that is five order of magnitude below the bound required to allow nucleosynthesis.

Chapter 4

Moduli stabilization

One of the mayor problem facing the past decade was to find a well-defined vacuum solution with all moduli stabilized. This is a very important task because values of low energy parameters, such as coupling constants, fine-structure constant are tied to moduli VEVs. Having a model with moduli stabilized we are able to do realistic phenomenology and compute all the relevant scales: Kaluza-Klein mass, gravitino mass and also masses of different particles in moduli sector. This issue has been successfully attempt in the context of type IIB string theory (for exhaustive reviews see [8,9]). In this framework there are Kähler moduli, complex structure moduli and the dilaton. Most of the geometric moduli are stabilized by fluxes and for the remainig moduli was at first proposed the KKLT scenario, then ameliorated and extended in the Large Volume Scenario. Here there is a simple overview.

4.1 KKLT Mechanism

String theory type IIB take place in 10 dimensions and has 32 supercharges. The ten dimensional bosonic massless field consist of the metric (g_{MN}), the dilaton (ϕ), RR antisymmetric forms (C_0, C_2, C_4 with the self-dual field strength) and NS-NS antisymmetric tensor (B_2). To obtain the four dimensional model we compactify on Calabi-Yau orientifold. Fluxes for the RR 3-form $F_3 = dC_2$ and NS-NS 3-form $H_3 = dB_2$ can be turned on and quantisation conditions must be imposed

$$\frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_a} F_3 = n_a \in \mathbb{Z} \qquad\qquad \frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_b} H_3 = m_b \in \mathbb{Z}$$

where $\Sigma_{a,b}$ represent the 3-cycles of Calabi-Yau manifold. Furthermore, fluxes should satisfy tadpole condition.

The superpotential at three level is independent of the Kähler moduli and is given by the Gukov-Vafa-Witten superpotential [10]

$$W = \int_{CY} G_3 \wedge \Omega = W(S, U)$$

where $G_3 = F_3 - \imath S H_3$, being S the dilaton-axion field and Ω the holomorphic $(3, 0)$ form of CY and the last equality enforce the superpotential's dependence of the dilaton, as it appears in G_3 and of the complex structure moduli U through Ω.

Kähler potential is the sum of three terms, depending on different moduli: it is given by

$$\mathcal{K} = -2\ln[\mathcal{V}] + \ln\left[-\imath\int_{CY}\Omega\wedge\bar{\Omega}\right] - \ln(S + \bar{S})$$

where the first term depends on Kähler moduli *via* CY volume $\mathcal{V}$, the second on complex structure moduli U and the last on dilaton- axion field. CY volume is given by

$$\mathcal{V} = \int_{CY} J \wedge J \wedge J = \frac{\kappa_{ijl}\, t^i t^j t^l}{6}$$

Here J represent the Kähler class and t_i are moduli measuring the size of 2-cycles. The corresponding 4-cycles moduli τ_i are defined by

$$\tau_i = \frac{\partial\mathcal{V}}{\partial t_i} = \frac{\kappa_{jl}\, t^j t^l}{2}$$

The complexified Kähler moduli are

$$T_j = \tau_j + \imath b_j$$

where the real parts are 4-cycles volumes and the imaginary parts b_j are axionic fields coming from RR four-form. The standard $\mathcal{N} = 1$ SUGRA scalar potential is given by

$$V = e^{\mathcal{K}}(\mathcal{K}^{A\bar{B}}\, D_A W\, D_{\bar{B}}\bar{W} - 3\,|W|^2)$$

where the index A, B run over all moduli fields, $D_A W = \partial_A W + W \partial_A \mathcal{K}$ is the Kähler-covariant derivative and $\mathcal{K}^{A\bar{B}} = (\mathcal{K}_{A\bar{B}})^{-1}$, being $\mathcal{K}_A = \partial_A \mathcal{K}$. Scalar potential at three level has the important property that the sum over Kähler moduli and $-3\,|W|^2$ exactly vanishes: it is *no-scale type*. Let a, b denote the dilaton and complex structure moduli and i, j the Kähler moduli, so one has

$$
\begin{aligned}
V &= e^{\mathcal{K}}(\mathcal{K}^{a\bar{b}}\, D_a W\, D_{\bar{b}} \bar{W} + \mathcal{K}^{i\bar{j}}\, D_i W\, D_{\bar{j}} \bar{W} - 3\,|W|^2) \\
&= e^{\mathcal{K}}(\mathcal{K}^{a\bar{b}}\, D_a W\, D_{\bar{b}} \bar{W}) \\
&\equiv V_{no-scale}
\end{aligned}
$$

Since $V_{noscale}$ is positive definite, one can stabilize complex structure moduli and dilaton by solving $D_a W = 0 = D_{\bar{b}} \bar{W}$. The W satisfying this constraint is then set to W_0, and from now regard as fixed. To stabilize Kähler moduli, non perturbative correction to superpotential have to be included. The full non-perturbative superpotential is expected to be

$$
W = W_0 + \sum_i A_i\, e^{-a_i T_i}
$$

Here, a_i's and A_i's are model-dependent constant. No-scale structure is broken and this non perturbative effects allow T moduli to be stabilized by solving $D_T W = 0$. To understand better the situation, let's consider only one modulus [11], denoted by τ and the corresponding axion set to zero. Kähler potential, superpotential and scalar potential are given by

$$
\begin{aligned}
\mathcal{K} &= -3\ln(T + \bar{T}) \\
W &= W_0 + A e^{-aT} \\
V &= e^{\mathcal{K}}(\mathcal{K}^{T\bar{T}}\, |D_T W|^2 - 3|W|^2)
\end{aligned}
$$

The condition of unbroken SUSY allows to find W_0, whose expression is

$$
W_0 = -A e^{-a\tau}\left(1 + \frac{2}{3}\, a\tau\right)
$$

and in a straightforward way the scalar potential minimum

$$
V = -3 e^{\mathcal{K}} |W|^2 = -\frac{a^2 A^2 e^{-2a\tau}}{6\tau}
$$

This is a SUSY, AdS (Anti-de Sitter) minimum. The important feature is that no scale is broken by non perturbative contribution W_{np} to superpotential W. Since each term in W_{np} is exponentially suppressed on Kähler moduli, we generally expect a similar suppression occurs in scalar potential. However, this is not consistent with the neglect of α' and g_s correction because these go as some powers of Kähler moduli and so dominate exponentially suppressed terms coming from W_{np}. Their neglect can be justified if complex structure and dilaton moduli are stabilized at a very small value of W_0, so one has to fine-tune W_0 to a very small value, that is the stabilization only works for a small parameters range.

Now, one needs to uplift this minimum to a de Sitter one (introducing positive energy density) and the lifting term has to be choose in a way to give vanishing cosmological constant.

If one consider more than one Kähler modulus, the expression of the scalar potential is more complicated. In particular one has to check that the minimum is a true minimum and not only a saddle point (minimum respect one variable).

4.2 Large Volume Scenario

KKLT Scenario presents some difficulties:

- consistency requires $W_0 \ll 1$ while fluxes prefers $W_0 \sim \mathcal{O}(1)$;
- moduli are stabilized in two steps;
- AdS and SUSY minimum;
- SUSY broken by uplifting mechanism, so it is not well controlled.

Large Volume Scenario goes along the line of KKLT, with the difference that perturbative α' corrections are now included to Kähler potential and no-scale structure is broken

$$\mathcal{K} = -2\ln\left[\mathcal{V} + \frac{\xi(S+\bar{S})^{3/2}}{2}\right] + \ln\left[-\imath\int_{CY}\Omega\wedge\bar{\Omega}\right] - \ln(S+\bar{S})$$

where $\xi = -\chi/2(2\pi)^3$ with χ the Euler number of the Calabi-Yau three-fold. For large volume, corrections go as inverse powers in the volume

$$\ln\left[\mathcal{V} + \frac{\xi(S+\bar{S})^{3/2}}{2}\right] \sim \ln\mathcal{V} + \frac{\xi(S+\bar{S})^{3/2}}{\mathcal{V}} - \frac{\xi^2(S+\bar{S})^2}{2\mathcal{V}^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right)$$

and will dominate in the scalar potential the exponentially suppressed terms coming from non perturbative contribution to superpotential. Using the superpotential one finds that scalar potential is split into three terms

$$V = e^{\mathcal{K}}(V_{np1} + V_{np2} + V_{\alpha'}) \tag{4.1}$$

where the explicit expression are

$$\begin{aligned} V_{np1} &= \mathcal{K}^{i\bar{j}}\partial_i W_{np}\partial_{\bar{j}}\bar{W}_{np} \\ V_{np2} &= \mathcal{K}^{i\bar{j}}[\partial_i W_{np}\mathcal{K}_{\bar{j}}(\bar{W}_0 + \bar{W_{np}}) + \mathcal{K}_i(\bar{W}_0 + \bar{W_{np}})\partial_{\bar{j}}\bar{W}_{np}] \\ V_{\alpha'} &= (\mathcal{K}^{i\bar{j}}\mathcal{K}_i\mathcal{K}_{\bar{j}} - 3)|W|^2 \end{aligned}$$

Inserting these relations in (4.1) one has a full analytic expression of scalar potential. Since we are interested only on the solutions at large volume, we can take only the leading terms in the scalar potential. For concrete calculations one can use the $\mathbb{P}_{[1,1,1,6,9]}$ Calabi-Yau with two Kähler moduli: $T_b = \tau_b + \imath b_b$ and $T_s = \tau_s + \imath b_s$. Their name suggest that τ_b modulus is stabilized big and τ_s is stabilized small. The Calabi-Yau volume can be written in terms of Kähler moduli yielding

$$\mathcal{V} = \frac{1}{9\sqrt{2}}(\tau_b^{3/2} - \tau_s^{3/2}) \tag{4.2}$$

In terms of these we can write Kähler potential and superpotential

$$\mathcal{K} = -2\ln\left(\frac{1}{9\sqrt{2}}(\tau_b^{3/2} - \tau_s^{3/2}) + \frac{\xi}{2g_s^{3/2}}\right) \tag{4.3}$$

$$W = W_0 + A_s e^{-a_s\tau_s} \tag{4.4}$$

where ξ is the term that take into account perturbative α' correction and g_s is the string coupling. After extremizing the axionic field one has the supergravity scalar potential at large volume. The latter at the leading order is given by

$$V = \frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{3\tau_b^{3/2}} - \frac{\mu a_s|W_0|\tau_s e^{-a_s\tau_s}}{\tau_b^3} + \frac{\nu|W_0|^2}{\tau_b^{9/2}} \tag{4.5}$$

with $\lambda = 8(a_s A_s)^2$ and $\mu = 4A_s$. This potential has a non-SUSY AdS minimum at $\mathcal{V} \sim e^{a_s \tau_s} \gg 1$ with $\tau_s = \xi^{2/3}/g_s$. This minimum has a negative cosmological constant and there exists various method to introduce positive energy and uplift this to a de Sitter one (for the details of this construction see [11,26]). The stabilized exponentially large volume can generate hierarchies because to small variations of $a_s \tau_s$ correspond large variations of $\mathcal{V}$. The gravitino mass $m_{3/2}$ is given by

$$m_{3/2} = e^{\mathcal{K}/2}|W_0| = \frac{|W_0|}{\mathcal{V}} M_{Pl}$$

Phenomenological reasons require $m_{3/2} \sim \mathcal{O}(\text{TeV})$ from which $\mathcal{V} \sim 10^{15}$ in string unit and the string scale is related to the volume by

$$M_s = \frac{M_{Pl}}{\mathcal{V}^{1/2}}$$

From (4.5) we can compute moduli masses. These are given by $m_b^2 \sim \mathcal{K}^{bb} V_{bb}$ and $m_s^2 \sim \mathcal{K}^{ss} V_{ss}$ with

$$m_{\tau_b} \sim \frac{M_{Pl}}{\mathcal{V}^{3/2}} \tag{4.6}$$

$$m_{\tau_s} \sim \frac{M_{Pl} \ln \mathcal{V}}{\mathcal{V}} \tag{4.7}$$

Also the axionic partners b_b and b_s of τ_b and τ_s receive masses after stabilization: b_s has the same mass of τ_s while b_b is essentially massless.[1]

4.3 Canonical normalization

Once the minimum has been located (A.17) (A.18) we can expand the lagrangian around it. Setting

$$\begin{pmatrix} \tau_b \\ \tau_s \end{pmatrix} = \boldsymbol{\tau} = \langle \boldsymbol{\tau} \rangle + \boldsymbol{\delta\tau} = \begin{pmatrix} \langle \tau_b \rangle + \delta\tau_b \\ \langle \tau_s \rangle + \delta\tau_s \end{pmatrix} \tag{4.8}$$

where $\langle \boldsymbol{\tau} \rangle = \langle \tau_i \rangle$ $(i = b, s)$ represent the VEVs and $\boldsymbol{\delta\tau} = (\delta\tau)_i$ $(i = b, s)$ are the real fields, one has the following lagrangian

$$\mathcal{L}_{free} = \partial_\mu \boldsymbol{\delta\tau}^T \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu \boldsymbol{\delta\tau} - V_0 - \boldsymbol{\delta\tau}^T \cdot \boldsymbol{M}^2 \cdot \boldsymbol{\delta\tau} - O(\boldsymbol{\delta\tau})^3 \tag{4.9}$$

[1] We shall not analyse in depth the cosmological role played by axion fields.

To write (4.9) in terms of the canonical normalized fields Φ and χ, related to τ_b and τ_s *via*

$$\boldsymbol{\delta\tau} = \boldsymbol{v}_\Phi \frac{\Phi}{\sqrt{2}} + \boldsymbol{v_\chi} \frac{\chi}{\sqrt{2}} \tag{4.10}$$

one has to impose the normalization condition for the kinetic terms

$$\boldsymbol{v}_\Phi \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{v}_\chi = \delta_{\Phi,\chi} \tag{4.11}$$

and the eigenvalues equations for $\boldsymbol{v_\Phi}$ and $\boldsymbol{v_\chi}$

$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2\boldsymbol{v_\Phi} = m_\Phi^2 \boldsymbol{v_\Phi} \tag{4.12}$$

$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2\boldsymbol{v_\chi} = m_\chi^2 \boldsymbol{v_\chi} \tag{4.13}$$

being

$$\boldsymbol{v_\Phi} = \begin{pmatrix} (v_\Phi)_b \\ (v_\Phi)_s \end{pmatrix} \qquad \boldsymbol{v_\chi} = \begin{pmatrix} (v_\chi)_b \\ (v_\chi)_s \end{pmatrix}$$

As shown in appendix A.5 the lagrangian is terms of Φ and χ has the following form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V_0 - \frac{1}{2}m_\Phi^2\Phi^2 - \frac{1}{2}m_\chi^2\chi^2 \tag{4.14}$$

We can also consider and interaction term between the small modulus and the electromagnetic field, described by the interaction energy

$$V_{int} = \tau_s F_{\mu\nu}F^{\mu\nu}$$

and add this to the lagrangian, which become

$$\mathcal{L} = \underbrace{\partial_\mu\boldsymbol{\delta\tau}^T \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu\boldsymbol{\delta\tau} - V_0 - \boldsymbol{\delta\tau}^T \cdot \boldsymbol{M}^2 \cdot \boldsymbol{\delta\tau} - O(\boldsymbol{\delta\tau})^3}_{=\mathcal{L}_{free}} - \underbrace{\tau_s F_{\mu\nu}F^{\mu\nu}}_{=\mathcal{L}_{int}} \tag{4.15}$$

that is, in terms of Φ and χ

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V_0 - \frac{1}{2}m_\Phi^2\Phi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{(\Phi(v_\Phi)_s + \chi(v_\chi)_s)}{4\sqrt{2}\langle\tau_s\rangle M_{Pl}}F_{\mu\nu}F^{\mu\nu} \tag{4.16}$$

The coupling of the two moduli to photons, denoted by λ, is

$$\lambda_{\Phi\gamma\gamma} = \frac{(v_\Phi)_s}{\sqrt{2}\langle\tau_s\rangle}$$
$$\lambda_{\chi\gamma\gamma} = \frac{(v_\chi)_s}{\sqrt{2}\langle\tau_s\rangle}$$

To go further we need explicit expression, eigenvalues and normalized eigenvectors of $\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2$. This is given by

$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2 = \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}} \begin{pmatrix} -9(1-7\epsilon) & 6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2) \\ -\frac{6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(1-5\epsilon+4\epsilon^2) & \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(1-3\epsilon+6\epsilon^2) \end{pmatrix} \tag{4.17}$$

where $\epsilon = (4a_s\langle\tau_s\rangle)^{-1}$. To obtain the eigenvalues m_Φ^2 and m_χ^2 one can observe since we have $m_\Phi^2 \gg m_\chi^2$ in first approximation

$$m_\Phi^2 \simeq \mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) \simeq \frac{8a_s^2|W_0|^2\langle\tau_s\rangle^{1/2}\nu}{3\langle\tau_b\rangle^3} \sim \left(\frac{\ln\mathcal{V}}{\mathcal{V}}\right)^2 M_{Pl}^2 \tag{4.18}$$

$$m_\chi^2 \simeq \frac{\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)}{\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)} \simeq \frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}} \sim \frac{M_{Pl}^2}{\mathcal{V}^3\ln\mathcal{V}} \tag{4.19}$$

Finding the eigenvectors of $\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2$ we can write $\delta\tau_b$ and $\delta\tau_s$ in terms of the canonical normalized fields Φ and χ (see A.5) [4,12]

$$\delta\tau_b = \left(\sqrt{6}\langle\tau_b\rangle^{1/4}\langle\tau_s\rangle^{3/4}\right)\frac{\Phi}{M_{Pl}\sqrt{2}} + \left(\sqrt{\frac{4}{3}}\langle\tau_b\rangle\right)\frac{\chi}{M_{Pl}\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{1/6})\frac{\Phi}{M_{Pl}} + \mathcal{O}(\mathcal{V}^{2/3})\frac{\chi}{M_{Pl}} \tag{4.20}$$

$$\delta\tau_s = \left(\frac{2\sqrt{6}}{3}\langle\tau_b\rangle^{3/4}\langle\tau_s\rangle^{1/4}\right)\frac{\Phi}{M_{Pl}\sqrt{2}} + \left(\frac{\sqrt{3}}{a_s}\right)\frac{\chi}{M_{Pl}\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{1/2})\frac{\Phi}{M_{Pl}} + \mathcal{O}(1)\frac{\chi}{M_{Pl}} \tag{4.21}$$

From this we deduce that τ_b is mostly χ while τ_s is mostly χ. There is however an important mixing which is subleading and coefficients depending on different powers of $\mathcal{V}$.

The dimensionful χ lagrangian is

$$\mathcal{L}_\chi = \frac{1}{2}\,\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\,m_\chi^2\chi^2 - \frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\left(\frac{\sqrt{6}}{2a_s\langle\tau_s\rangle}\right)\frac{\chi}{M_{Pl}}\,F_{\mu\nu}F^{\mu\nu}$$

which show that the coupling of χ to photons is not only suppressed by M_{Pl}, but there is a further suppression factor proportional to

$$a_s\langle\tau_s\rangle \sim \ln\mathcal{V} \sim \ln(M_{Pl}/m_{3/2})$$

The dimensionful coupling of χ to photons is

$$\lambda_{\chi\gamma\gamma} = \left(\frac{3}{2}\right)^{1/2}\frac{1}{M_{Pl}\ln\mathcal{V}}$$

different from the naïf expectation $\lambda_{\chi\gamma\gamma} \sim 1/M_{Pl}$ while the dimensionful coupling of Φ is

$$\lambda_{\Phi\gamma\gamma} \sim \left(\frac{2}{\sqrt{3}}\frac{\langle\tau_b\rangle^{3/4}}{\langle\tau_s\rangle^{3/4}M_{Pl}}\right) \sim \frac{\mathcal{V}^{1/2}}{M_{Pl}} \sim \frac{1}{M_s}$$

This shows that the interactions of Φ with photons are suppressed by the string scale and therefore the decay rates are much faster than is usually assumed for moduli fields.

4.3.1 Decay rates and lifetimes

the discussion of the previous section allows to estimate decay rates and lifetimes of the two fields. From (4.18) and (4.19)

$$m_\Phi = \left(\frac{\ln\mathcal{V}}{\mathcal{V}}M_{pl}\right)\text{GeV} \sim 10^5\,\text{GeV} \tag{4.22}$$

$$m_\chi = \left(\frac{M_{Pl}}{(\mathcal{V}^3\ln\mathcal{V})^{1/2}}\right)\text{GeV} \sim 10^{-3}\,\text{GeV} \tag{4.23}$$

Since we know the coupling constant $\lambda_{\Phi\gamma\gamma}$ and $\lambda_{\chi\gamma\gamma}$ we can estimate the decay rates $\Gamma_{\Phi\longrightarrow\gamma\gamma}$ and $\Gamma_{\chi\longrightarrow\gamma\gamma}$ of Φ and χ into photons

$$\Gamma_{\Phi\longrightarrow\gamma\gamma} = \left(\frac{\lambda_{\Phi\gamma\gamma}}{64\pi}\frac{m_\Phi^3}{M_s^2}\right)\text{GeV} \sim 10^{-8}\,\text{GeV} \tag{4.24}$$

$$\Gamma_{\chi\longrightarrow\gamma\gamma} = \left(\frac{\lambda_{\chi\gamma\gamma}}{64\pi}\frac{m_\chi^3}{M_{Pl}^2}\right)\text{GeV} \sim 10^{-50}\,\text{GeV} \tag{4.25}$$

The lifetimes are related to the decay mode by $\Gamma^{-1} = \tau$. From the relation $M_{Pl}^{-1} \sim 10^{-18}\,\text{GeV}^{-1} \sim 10^{-43}\,\text{sec}$ we can write

$$\tau_\Phi \sim 10^{-17}\,\text{sec} \tag{4.26}$$

$$\tau_\chi \sim 10^{25}\,\text{sec} \tag{4.27}$$

As expected, the heavy modulus decay suddenly in the history of the universe while the light modulus has a lifetimes longer than the age of the universe[2] and their lifetimes differs of a factor 10^{42} sec.

[2]Remember that the age of the universe is estimated about 10^{17} sec.

Chapter 5

Cosmology with Thermal Inflation

We are going to see how the thermal inflation can provide a solution for the CMP. Before going further it is instructive to summarize the main results of the previous chapters.

We have learnt that compactification leads to an enormous number of massless scalar fields (called moduli) in the spectrum of the low energy EFT. These fields are gauge singlet and interact only via gravitational strength interaction, so they are expected to have Planck-suppressed couplings to Standard Model's particles.

Classically moduli are massless, so they could mediate new, non-physical forces. This happens because their potential is flat to all order in the SUSY limit. However SUSY, if it is realized in Nature, can't be an exact symmetry, otherwise s-particles would have been observed long time ago. So SUSY must be broken at some low energy [1] and taking into account SUSY breaking and quantum corrections, moduli acquire mass. At a first sight, their mass is expected to be of the same order of the gravitino mass. Furthermore, since they feel only gravitational strength interactions, they have a very long lifetime: indeed in the most general case one expects that $\tau_\Phi \sim N^{-1}\frac{M_{Pl}^2}{m_\Phi^3}$ (here N denotes the decay channel). Since in gravity mediated models the gravitino mass is estimated as $m_{3/2} \sim \mathcal{O}(1)\,\mathrm{TeV}$, modulus lifetime amounts to $\tau_\Phi \sim 10^{17}\,\mathrm{sec}\,N^{-1}\left(\frac{100\,\mathrm{MeV}}{m_\Phi}\right)^3$, that is much more than the age of the universe. Thanks to their relatively weak interactions, they came to dominate the energy density of the universe, until they decay. When the

[1] Phenomenological reasons and stabilization of Higgs' mass require a SUSY theory at the TeV scale

decay occurs, the reheating temperature is very low and nucleosynthesis cannot take place. This is the CMP in the context of (super)string theories.

Then we describe the thermal inflation and we say that this mechanism could provide a solution to the CMP: this essentially lies in the fact that the decay of the flaton release a huge amount of entropy. Let's see how this can happened.

5.1 Cosmology with Thermal Inflation and CMP

Now we explain how the cosmological history can be recast if thermal inflation really takes place.

5.1.1 Before Thermal Inflation

It is commonly believed that the history of the universe begin with a period of inflation. This period is typically invoked in order to solve the problems of flatness and horizons. After the inflation, the inflaton decay and (supposing that the decay product promptly thermalize) the universe gets reheated.

Depending on the specific model of inflation, moduli can oscillate either after or before the end of the primordial inflation. Indeed there are some models of inflation with a low reheating temperature and moduli oscillations begin before the end of the inflation. If this is the case, one can also consider a dilution of moduli abundance due to inflation. Note however that the moduli problem could not be solved by choosing the model of the primordial inflation, even if one assumes an extremely low reheating temperature $\mathcal{O}(10)\,\mathrm{MeV}$.

If φ is the inflaton and Γ_φ its decay rate, moduli oscillations begin after (before) the end of the inflation if $\Gamma_\varphi > m_\Phi$ ($\Gamma_\varphi < m_\Phi$).

As said in 2.1, when $H \sim m_\Phi$ moduli begin to oscillate with amplitude $\Phi_{\rm in} \sim M_{Pl}$. Moduli number density amounts to $n_\Phi = \frac{1}{2} m_\Phi \Phi_{\rm in}^2$ and the energy stored in the oscillations is $\rho_\Phi = m_\Phi n_\Phi$. Moduli abundance is encoded in Y_Φ defined in 2.1. Suppose first that the oscillations begin after the full reheating of the inflation: if this is the case, then the cosmic temperature is

$$T_{\rm osc} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{m_\Phi M_{Pl}} \tag{5.1}$$

Setting $g_* \simeq 200$ we have $T_{\rm osc} \sim \mathcal{O}(10^8)$ GeV. The ratio between moduli number density to the radiation entropy is

$$Y_\Phi = \frac{n_\Phi}{s} = \frac{\frac{1}{2} m_\Phi \Phi_{\rm in}^2}{\frac{2\pi^2}{45} g_* T_{\rm osc}^3} \sim \frac{m_\Phi \Phi_{\rm in}^2}{(m_\Phi M_{Pl})^{3/2}} = \left(\frac{M_{Pl}}{m_\Phi}\right)^{1/2} \left(\frac{\Phi_{\rm in}}{M_{Pl}}\right)^2 \tag{5.2}$$

Since for a modulus $\Phi_{\rm in} \sim M_{Pl}$, we have $Y_\Phi \sim \mathcal{O}(\frac{M_{Pl}}{m_\Phi})^{1/2}$. In the opposite case moduli oscillations begin before the end of the inflation. The reheating temperature at the end of the inflation $T_{R\varphi}$ is

$$T_{R\varphi} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma_\varphi M_{Pl}} \tag{5.3}$$

and moduli abundance is

$$Y_\Phi = \frac{3T_{R\varphi}}{8} \left(\frac{\Phi_{\rm in}}{M_{Pl}}\right) \tag{5.4}$$

5.1.2 Moduli dilution from Thermal Inflation

When the cosmic temperature is in the range $10^8\,\text{GeV} \lesssim T \lesssim 10^3\,\text{GeV}$ we suppose the universe experience the thermal inflation. The entropy released in the decay of the flaton amounts to

$$\Delta = \frac{s_{\rm after}}{s_{\rm before}} = \frac{\frac{4V_0}{3T_D}}{\frac{2\pi^2}{45} g_* T_{\rm end}^3} \tag{5.5}$$

where T_D denotes the decay temperature [2] associated to the flaton σ and $T_{\rm end}$ is the cosmic temperature at the end of the thermal inflation . The decay temperature is related to the decay rate Γ_σ of the flaton: indeed $\Gamma_\sigma \sim m_\sigma^3/M^2$, where as usual $M = \langle\sigma\rangle$. To estimate the order of magnitude of Δ, set $m_\sigma \sim \mathcal{O}(10^3)$ GeV, $M \sim \mathcal{O}(10^{12})$ GeV and recall $V_0^{1/4} \sim 10^7$ to 10^8 GeV and $T_{\rm end} \sim m_\sigma$: one has $\Delta \sim 10^{16}$. Moduli abundance get a huge dilution: at the end of the thermal inflation the moduli abundance drastically changes

$$Y_\Phi \longrightarrow Y'_\Phi = \frac{Y_\Phi}{\Delta} = \frac{\frac{1}{2} m_\Phi \Phi_{\rm in}^2}{\frac{2\pi^2}{45} g_* T_{\rm osc}^3} \times \frac{1}{\Delta} \tag{5.6}$$

[2]We suppose that decay product promptly thermalize, so that the decay temperature is equal to the reheating temperature

However, as pointed out in 2.1, during the thermal inflation the moduli are a bit displaced from the minimum (in 2.1 this displacement was called misalignment). This displacement is quantified by $\delta\Phi \sim (V_0/m_\Phi^2 M_{Pl}^2)\Phi_{\rm in} \sim V_0/m_\Phi^2 M_{Pl}$ and this causes a further oscillation for the modulus. Usually moduli produced during thermal inflation are called *Thermal Inflation Moduli*, in order to distinguish them for the *Big Bang Moduli*. Clearly there is not a huge reproduction but this may still be dangerous. The abundance of thermal inflation moduli is

$$Y_{\Phi\, TI} = \frac{\frac{1}{2} m_\Phi \delta\Phi^2}{\frac{2\pi^2}{45} g_* T_{\rm end}^3} \times \frac{1}{\Delta} \sim \frac{V_0^2}{m_\sigma^3 m_\Phi^3 M_{Pl}^2} \times 10^{-16} \tag{5.7}$$

while the total moduli number density is

$$Y_{\Phi\, \rm TOT} = \frac{Y_\Phi + Y_{\Phi\, TI}}{\Delta} \tag{5.8}$$

If moduli oscillations begin before the full reheating of the inflation, then $T_{\rm osc} \geq T_{R\varphi}$

$$Y_\Phi = \frac{3T_{R\varphi}}{8}\left(\frac{\Phi_{\rm in}}{M_{Pl}}\right) \times \frac{1}{\Delta} \sim 10^{-2}\Gamma_\varphi^{1/2} M_{Pl}^{1/2}\left(\frac{\Phi_{\rm in}}{M_{Pl}}\right)^2 \times \frac{1}{\Delta} \tag{5.9}$$

However moduli can be produced by the decay of the flaton, so in order to have a further dilution of moduli abundance, a second stage of thermal inflation can be implemented.

5.1.3 Double Thermal Inflation

We have just seen that even after the thermal inflation, the moduli oscillations can still be dangerous and this is due to the fact that during the thermal inflation the moduli are expected to be displaced from the low energy minimum: this distance is quantified by $\delta\Phi$. We can consider a second stage of thermal inflation to dilute moduli abundance left over from the first stage. In the simplest model one can consider two non interacting flatons σ_i with $i = 1, 2$ and assume their potential is of the form (3.2),

$$V(|\sigma_1|, |\sigma_2|) = V_1 + V_2 - m_{\sigma_1}^2|\sigma_1|^2 - m_{\sigma_2}^2|\sigma_2|^2 + \dots \tag{5.10}$$

where the ellipses denote the higher order terms that stabilize each flaton near the VEV $\langle\sigma_i\rangle = M_i$, $V_i \sim m_{\sigma_i}^2 M_i^2$ are the value of the potentials energies at the origin. The temperature for which the phase transitions occurs are set to $T_{C,i}$. The mechanism works as follows: when the background temperature drops below the value $T_{C,1}$ the first flaton is destabilized from the origin and rolls towards its true VEV M_1. Meanwhile the other flaton is still held at the origin and when the background temperature drops below the critical value $T_{C,2}$ also the second flaton is destabilized from the origin and rolls towards its true VEV M_2. Since we have implicitly supposed $T_{C,1} > T_{C,2}$, it follows that $m_{\sigma_1} > m_{\sigma_2}$. Finally suppose that $T_{\text{end , i}} \sim m_{\sigma_i}$ denotes the temperature for which the i-th thermal inflation shuts off. From (5.5) the entropy production coming from both the stage of thermal inflation can be estimated

$$\Delta_1 \sim \frac{T_{D,1}^{3/2}}{T_{\text{end},1}^3} \frac{V_1}{V_2^{5/8}}$$
$$\Delta_2 \sim \frac{V_2}{T_{\text{end},2}^3 T_{D,2}}$$

In order to make the mechanism work we suppose $V_1 \gg V_2$.

5.1.4 Moduli dilution from double Thermal Inflation

From the result of the previous section we can estimate moduli abundance during the overall history of the universe. Big Bang Moduli experienced a double thermal inflation, so they get a huge dilution

$$Y_\Phi \longrightarrow Y'_\Phi = \frac{Y_\Phi}{\Delta_1 \Delta_2} \sim \frac{\Phi_{\text{in}}^2}{m_\Phi^{1/2} M_{Pl}^{3/2}} \times \frac{1}{\Delta_1} \times \frac{1}{\Delta_2} \sim \frac{\Phi_{\text{in}}^2 T_{\text{end , 1}}^3 T_{\text{end , 2}}^3}{m^{1/2_\Phi} V_1^{3/4} V_2^{3/4} M_{Pl}^{3/2}} \tag{5.11}$$

This clearly depends upon the VEVs M_i, the vacuum energies V_i, the temperature at the end of each inflationary stage $T_{\text{end},i}$ and the modulus mass m_Φ. For typical values it has been found that $Y_\Phi \sim \mathcal{O}(10^{-18})$ and thus Big Bang Moduli abundance is diluted to a safer level. Recall that when nucleosynthesis begin there must be $Y_\Phi \sim 10^{-12}\,\text{to}\,10^{-15}$. Moduli produced at the first stage of thermal inflation. Now we focus our attention on moduli produced by the first stage of thermal inflation: their abundance is estimated as

$$Y_\Phi \sim \frac{\Phi_{\text{in}}^2 V_2^2 / m_\Phi^3 M_{Pl}^4}{V_1^{3/4} \Delta_2} \tag{5.12}$$

that amounts to $\mathcal{O}(10^{-15})$. Finally the abundance of moduli produced at the end of the second stage of thermal inflation is

$$Y_\Phi \sim \frac{\Phi_{\rm in}^2 V_2^2/m_\Phi^3 M_{Pl}^4}{V_2^{3/4}} \sim \frac{\Phi_{\rm in}^2 V_2^{5/4}}{m_\Phi^3 M_{Pl}^4} \sim \frac{V_2^{5/4}}{m_\Phi^3 M_{Pl}^2} \tag{5.13}$$

This shows that the reproduction of moduli after a double stage of thermal inflation is strongly suppressed, because the Y_Φ goes as the inverse of the second power of M_{Pl}.

5.2 CMP in LVS models

As we have already pointed out, in LVS we are faced with two kind of Kähler moduli, whose canonical normalization leads to the fields Φ and χ. The first is the modulus controlling the volume of the small 4-cycle τ_s: it has mass $m_\Phi \sim \mathcal{O}(10^5)$GeV and in early universe it starts to oscillate with stringy amplitude, so it has a very short lifetime. Indeed it is found that $\tau_\Phi \sim 10^{-17}$sec: its decay occurs before the BBN, hence it is harmless and we don't have to worry about it. Instead we can consider entropy production from its decay as a dilution source for dangerous moduli [28].

The canonically normalized modulus associated with the 4-cycles τ_b controlling the overall volume has mass $m_\chi \sim \mathcal{O}(1)$MeV and in early universe stars to oscillate with Planckian amplitude so this is subject to CMP. The modulus abundance follows directly from the previous discussion: indeed it amounts to $Y_\chi \sim \mathcal{O}(10^{11})$. Such a huge number of χ moduli is a cosmological disaster! If we try to dilute its abundance with a single thermal inflation we find that the χ abundance after thermal inflation is reduced to $\mathcal{O}(10^{-5})$ but there is a huge reproduction due to its low mass: indeed thermal inflation moduli are reproduced with abundance

$$Y_{\chi TI} \sim \mathcal{O}(10^{-5})\left(\frac{\rm GeV}{m_\chi}\right)^3 \tag{5.14}$$

where (5.7) was used with $m_\sigma \sim 10^3$ GeV. So if by one side thermal inflation gives a huge dilution of the χ moduli coming from Big Bang, by the other it creates a reproduction that yields $Y_{\chi TI} \sim \mathcal{O}(10^4)$ for $m_\chi \sim 1$ MeV. So a further stage of thermal inflation is needed to relax this problem. But instead consider the χ moduli reproduction after the second stage of thermal inflation: even if this quantity goes as M_{Pl}^{-2}, there is still a non negligible reproduction, indeed for typical values one has

$$Y_{\Phi TI_2} \sim \mathcal{O}(10^{-2}) \tag{5.15}$$

Also this is due to the fact that this modulus has a mass well below the GeV. This is the result expected for a typical modulus *at the end of the first thermal inflation*: so this modulus seems to need a further sources of dilution. Clearly we have take into account only entropy coming from the thermal inflation and computations have been taken by order of magnitude, however it looks as thermal inflation in LVS fails in diluting the light modulus abundance.

There are however three possible way-outs:

1. If one does not insist on low-energy supersymmetry, then the mass of χ can be increased so to make this modulus decay before BBN. Of course, one would then have to rely on tuning in order to solve the gauge hierarchy problem.

2. In the presence of a primordial mechanism that suppresses the initial amplitude of the modulus oscillations, χ would initially store much less energy, resulting in a very suppressed original production of Big Bang moduli. See [37] for an example of such a mechanism which would relax the CMP that could be then completely solved by a late period of thermal inflation.

3. In this analysis, we considered models where the soft terms acquire masses of order the gravitino mass: $M_{soft} \sim m_{3/2}$. However, in models where the visible sector is sequestered from supersymmetry breaking, the soft terms can be hierarchically lighter than the gravitino: $M_{soft} \ll m_{3/2}$. In this case, the modulus mass could be increased above 100 TeV evading the CMP but still keeping TeV-scale supersymmetry for the solution of the hierarchy problem.

Appendix A

Computational details

A.1 Moduli equation of motion

Let's consider the equation (2.6) treating $H(t)$ as time independent constant with the initial conditions provided by (2.7). First of all we have to locate the minimum of Φ solving

$$V'_{\text{eff}}(\Phi, T) = \frac{V_{\text{eff}}(\Phi, T)}{\partial \Phi} = 0$$

This implies

$$m_0^2 \Phi^* + \alpha^2 H^2 (\Phi^* - \Phi_1) = 0$$

whose solution for Φ^* is

$$\Phi^* = \frac{\alpha^2 H^2}{m_0^2 + \alpha^2 H^2} \Phi_1$$

Since $V''_{\text{eff}}(\Phi^*, T) > 0$, Φ^* clearly defines a minimum for the effective potential, so we set

$$\Phi^* = \frac{\alpha^2 H^2}{m_0^2 + \alpha^2 H^2} \Phi_1 \equiv \Phi_{\text{min}} \tag{A.1}$$

Then let's consider the secular equation associated to (2.6)

$$\lambda^2 + 3H\lambda + (m_0^2 + \alpha^2 H^2) \; = \alpha^2 H^2 \Phi_1 \tag{A.2}$$

The most general solution for this second order differential equation is the sum of the homogeneous solution and a particular one. The homogeneous equation is

$$\lambda^2 + 3H\lambda + (m_0^2 + \alpha^2 H^2) = 0 \tag{A.3}$$

The discriminant is

$$\Delta = 9H^2 - 4(m_0^2 + \alpha^2 H^2) \tag{A.4}$$

and we have to consider the case $\Delta > 0$ and $\Delta < 0$ separately.

Case $\Delta > 0$

Here we consider the case $\Delta > 0$. The relation (A.4) can be recast to

$$\left(\frac{3H}{2}\right)^2 > m_0^2 + \alpha^2 H^2 \tag{A.5}$$

and the solution of (A.3) can be written as

$$\begin{aligned}
\lambda &= \frac{-3H \pm \sqrt{9H^2 - 4(m_0^2 + \alpha^2 H^2)}}{2} \\
&= \frac{-3H \pm \sqrt{9H^2 \left[1 - \frac{4(m_0^2+\alpha^2H^2)}{9H^2}\right]}}{2} \\
&= \frac{-3H \pm 3H\sqrt{1 - \frac{4(m_0^2+\alpha^2H^2)}{9H^2}}}{2} \\
&= -\frac{3H}{2}\left[1 \mp \sqrt{1 - \frac{4(m_0^2 + \alpha^2 H^2)}{9H^2}}\right] \\
&= -\frac{3H}{2}\left[1 \mp \sqrt{1 - \frac{4}{9}\left(\alpha^2 + \frac{m_0^2}{H^2}\right)}\right]
\end{aligned}$$

Setting $\beta = \sqrt{1 - \frac{4}{9}\left(\alpha^2 + \frac{m_0^2}{H^2}\right)}$ one has

$$\lambda = -\frac{3H}{2}[1 \mp \beta] \tag{A.6}$$

From the theory of differential equation we can write the solution as

$$\Phi(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \tag{A.7}$$

To avoid a cumbersome notation we impose (2.7) directly on (A.7). This leads to

$$\Phi_i = c_1 e^{\lambda_1 t_i} + c_2 e^{\lambda_2 t_i} \tag{A.8}$$

$$0 = c_1 \lambda_1 e^{\lambda_1 t_i} + c_2 \lambda_2 e^{\lambda_2 t_i} \tag{A.9}$$

where Φ_i is fixed. From the second one obtains

$$c_1 e^{\lambda_1 t_i} = -c_2 \frac{\lambda_2}{\lambda_1} e^{\lambda_2 t_i} \tag{A.10}$$

so that

$$\Phi_i = \left(1 - \frac{\lambda_2}{\lambda_1}\right) c_2 e^{\lambda_2 t_i} \tag{A.11}$$

Since

$$\begin{aligned} \frac{\lambda_2}{\lambda_1} &= \frac{1+\beta}{1-\beta} \\ 1 - \frac{\lambda_2}{\lambda_1} &= -\frac{2\beta}{1-\beta} \end{aligned}$$

one has

$$\begin{aligned} c_2 e^{\lambda_2 t_i} &= -\frac{1+\beta}{2\beta}\Phi_i \\ c_1 e^{\lambda_1 t_i} &= -\frac{1-\beta}{1+\beta}\left(-\frac{1+\beta}{2\beta}\right)\Phi_i = \frac{1-\beta}{2\beta}\Phi_i \end{aligned}$$

and so the value of the two integrations constants

$$\begin{aligned} c_1 &= \frac{1-\beta}{2\beta}\Phi_i e^{-\lambda_1 t_i} \\ c_2 &= -\frac{1+\beta}{2\beta}\Phi_i e^{-\lambda_2 t_i} \end{aligned}$$

The solution of the homogeneous equation is then

$$\Phi(t) = \frac{1-\beta}{2\beta}\Phi_i e^{-\frac{3H(1-\beta)}{2}(t-t_i)} - \frac{1+\beta}{2\beta}\Phi_i e^{-\frac{3H(1+\beta)}{2}(t-t_i)}$$

and recalling the expression of Φ_{min} we can write the most general solution as

$$\Phi(t) - \Phi_{\text{min}} = (\Phi_i - \Phi_{\text{min}})\left[\frac{1+\beta}{2\beta}\, e^{\frac{-3(1-\beta)}{2}H(t-t_i)} - \frac{1-\beta}{2\beta}\, e^{\frac{-3(1+\beta)}{2}H(t-t_i)}\right] \tag{A.12}$$

Case $\Delta < 0$

Now we move to the case $\Delta < 0$, that is $9H^2 - 4(m_0^2 + \alpha^2 H^2) < 0$ and this means that $4(m_0^2 + \alpha^2 H^2) > 9H^2$. A little algebra yields

$$4m_0^2\left(1 + \frac{\alpha^2 H^2}{4m_0^2} - \frac{9H^2}{4m_0^2}\right) > 0$$

that is

$$4m_0^2\left[1-\left(\frac{9}{4}-\alpha^2\right)\frac{H^2}{m_0^2}\right]>0$$

In this case, setting $\beta'=\sqrt{1-\left(\frac{9}{4}-\alpha^2\right)\frac{H^2}{m_0^2}}$ we can write the solution as

$$\Phi(t)-\Phi_{\min}=\Phi_i-\Phi_{\min}\,e^{-\frac{3}{2}H(t-t_i)}\left[\cos[\beta' m_0(t-t_i)-\frac{3H}{2\beta' m_0}\sin[\beta' m_0(t-t_i)]\right] \quad (A.13)$$

A.2 Kähler metric components

Starting from the expression (4.3) of Kähler potential let us calculate the matrix

$$\mathcal{K}_{i\bar{j}}=\begin{pmatrix}\mathcal{K}_{b\bar{b}} & \mathcal{K}_{b\bar{s}}\\ \mathcal{K}_{s\bar{b}} & \mathcal{K}_{s\bar{s}}\end{pmatrix}$$

where

$$\mathcal{K}_{i\bar{j}}\equiv\frac{\partial^2\mathcal{K}}{\partial T_i\partial\bar{T}_{\bar{j}}}$$

i, j running over Kähler moduli. Let's start with the first element: one has

$$\begin{aligned}\mathcal{K}_b=\frac{\partial\mathcal{K}}{\partial T_b}&=\frac{\partial\mathcal{K}}{\partial\tau_b}\frac{\partial\tau_b}{\partial T_b}=\frac{1}{2}\frac{\partial\mathcal{K}}{\partial\tau_b}\\ &=\frac{1}{2}(-2)\frac{3/2\,\tau_b^{1/2}}{\tau_b^{3/2}-\tau_s^{3/2}+\xi'}\\ &=-\frac{3}{2}\frac{\tau_b^{1/2}}{\tau_b^{3/2}-\tau_s^{3/2}+\xi'}\end{aligned}$$

Deriving now with respect to $\bar{T}_{\bar{b}}$

$$\begin{aligned}\mathcal{K}_{b\bar{b}}&=\frac{\partial\mathcal{K}_b}{\partial\bar{T}_{\bar{b}}}=\frac{\partial\mathcal{K}_b}{\partial\tau_b}\frac{\partial\tau_b}{\partial\bar{T}_{\bar{b}}}=\frac{1}{2}\frac{\partial\mathcal{K}_b}{\partial\tau_b}\\ &=-\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left[\frac{(1/2)\tau_b^{-1/2}(\tau_b^{3/2}-\tau_s^{3/2}+\xi')-\tau_b^{1/2}(3/2)\tau_b^{1/2}}{(\tau_b^{3/2}-\tau_s^{3/2}+\xi')^2}\right]\\ &\sim-\left(\frac{3}{4}\right)\frac{(1/2)\tau_b-(3/2)\tau_b}{\tau_b^3}\\ &=\frac{3}{4\tau_b^2}\end{aligned}$$

Now let's calculate the off-diagonal elements. Since the Kähler metric is symmetric, one needs to calculate only element.

$$\begin{aligned}\mathcal{K}_{s\bar{b}} &= \frac{\partial \mathcal{K}_{\bar{b}}}{\partial T_s} = \frac{\partial \mathcal{K}_{\bar{b}}}{\partial \tau_s}\frac{\partial \tau_s}{\partial T_s} = \frac{1}{2}\frac{\partial \mathcal{K}_{\bar{b}}}{\partial \tau_s} \\ &= \left(\frac{1}{2}\right)\left(-\frac{3\tau_b^{1/2}}{2}\right)(-1)(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^{-2}\left(-\frac{3}{2}\tau_s^{1/2}\right) \\ &= -\left(\frac{9}{8}\right)\frac{\tau_b^{1/2}\tau_s^{1/2}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^2} \\ &\sim -\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}\end{aligned}$$

It is easy to see that

$$\mathcal{K}_s = \frac{3}{2}\frac{\tau_s^{1/2}}{\tau_b^{3/2} - \tau_s^{3/2} + \xi'}$$

and

$$\begin{aligned}\mathcal{K}_{s\bar{s}} &= \frac{\partial \mathcal{K}_s}{\partial \bar{T}_{\bar{s}}} = \frac{\partial \mathcal{K}_s}{\partial \tau_s}\frac{\partial \tau_s}{\partial \bar{T}_{\bar{s}}} = \frac{1}{2}\frac{\partial \mathcal{K}_s}{\partial \tau_s} \\ &= \left(\frac{3}{4}\right)\frac{(1/2)\tau_s^{-1/2}(\tau_b^{3/2} - \tau_s^{3/2} + \xi') - \tau_s^{1/2}(-3/2)\tau_s^{1/2}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^2} \\ &\sim \frac{3}{8}\frac{\tau_b^{3/2}\tau_s^{-1/2}}{(\tau_b^{3/2} - \tau_s^{3/2} + \xi')^2} \\ &\sim \frac{3}{8\tau_b^{3/2}\tau_s^{1/2}}\end{aligned}$$

Kähler metrics components are

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \frac{3}{4\tau_b^2} & -\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}} \\ -\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}} & \frac{3}{8\tau_b^{3/2}\tau_s^{1/2}} \end{pmatrix} \tag{A.14}$$

Now, since we are interested in $\mathcal{K}^{i\bar{j}} = (\mathcal{K}_{i\bar{j}})^{-1}$, we have to invert (A.14). Kähler metric is non singular, so the inverse of (A.14) exists. It can be calculated from

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} (A - BD^{-1}C)^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{pmatrix}$$

One has

$$D - CA^{-1}B = \frac{3}{8\tau_b^{3/2}\tau_s^{1/2}} - \left(-\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}\right)\left(\frac{3}{4\tau_b^2}\right)^{-1}\left(-\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}\right) = \frac{3}{8\tau_b^{3/2}\tau_s^{1/2}} - \frac{27\tau_s}{8\tau_b^3}$$
$$= \frac{3}{8\tau_b^{3/2}\tau_s^{1/2}} + \mathcal{O}\left(\frac{1}{\tau_b^3}\right)$$
$$(D - CA^{-1}B)^{-1} \sim \frac{8\tau_b^{3/2}\tau_s^{1/2}}{3}$$

The second and the third elements are given by

$$-A^{-1}B(D - CA^{-1}B)^{-1} \sim -\left(\frac{3}{4\tau_b^2}\right)^{-1}\left(-\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}\right)\left(\frac{8\tau_b^{3/2}\tau_s^{1/2}}{3}\right)$$
$$= 4\tau_s\tau_b$$
$$-(D - CA^{-1}B)^{-1}CA^{-1} \sim -\left(\frac{8\tau_b^{3/2}\tau_s^{1/2}}{3}\right)\left(-\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}\right)\left(\frac{3}{4\tau_b^2}\right)^{-1}$$
$$= 4\tau_s\tau_b$$

Then we have to calculate the first element of (A.14). One has

$$A - BD^{-1}C = \frac{3}{4\tau_b^2} - \left(-\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}\right)\left(\frac{8\tau_b^{3/2}\tau_s^{1/2}}{3}\right)\left(-\frac{9\,\tau_s^{1/2}}{8\,\tau_b^{5/2}}\right) = \frac{3}{4\tau_b^2} - \frac{27\tau_s^{3/2}}{8\tau_b^{7/2}} = \frac{3}{4\tau_b^2} + \mathcal{O}\left(\frac{1}{\tau_b^{7/2}}\right)$$
$$(A - BD^{-1}C)^{-1} \sim \frac{4\tau_b^2}{3}$$

At the leading order in τ_b we find

$$\mathcal{K}^{i\bar{j}}\begin{pmatrix} \frac{4\tau_b^2}{3} & 4\tau_b\tau_s \\ 4\tau_b\tau_s & \frac{8\tau_b^{3/2}\tau_s^{1/2}}{3} \end{pmatrix}$$

A.3 Minimum of the scalar potential

The explicit expression of the scalar potential is

$$V = \frac{\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}{\tau_b^{3/2}} - \frac{\mu |W_0| a_s \tau_s e^{-a_s\tau_s}}{\tau_b^3} + \frac{\nu |W_0|^2}{\tau_b^{9/2}} \tag{A.15}$$

and it has to be minimized with respect to τ_b, τ_s, so we look at the solutions of

$$\frac{\partial V}{\partial \tau_b} = 0 = \frac{\partial V}{\partial \tau_s}$$

Let's start with the first of these relations

$$\begin{aligned}
0 = \frac{\partial V}{\partial \tau_b} &= -\frac{3\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}{2\tau_b^{5/2}} + \frac{3\mu|W_0|a_s\tau_s e^{-a_s\tau_s}}{\tau_b^4} - \frac{9\nu|W_0|^2}{2\tau_b^{11/2}} \\
&= -\left(\frac{3\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}{2\tau_b^{11/2}}\right)\left(\tau_b^3 - \frac{2\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\tau_b^{3/2} + \frac{3\nu|W_0|^2}{\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}\right)
\end{aligned}$$

This can be recast in a second order equation: to this purpose, set $x = \tau_b^{3/2}$, then this relation is equivalent to

$$x^2 - \frac{2\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}x + \frac{3\nu|W_0|^2}{\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}} = 0$$

whose solutions are

$$\begin{aligned}
x &= \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}} \pm \sqrt{\left(\frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\right)^2 - \frac{3\nu|W_0|^2}{\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}} \\
&= \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}} \pm \sqrt{\left(\frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\right)^2 \left(1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}\right)} \\
&= \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)
\end{aligned}$$

Now let's minimize with respect the other variable τ_s

$$\begin{aligned}
0 = \frac{\partial V}{\partial \tau_s} &= \frac{\lambda a_s^2}{\tau_b^{3/2}} e^{-2a_s\tau_s}\left[(1/2)\tau_s^{-1/2} - 2a_s\tau_s^{1/2}\right] - \frac{\mu|W_0|a_s}{\tau_b^3} e^{-a_s\tau_s}(1 - a_s\tau_s) \\
&= \frac{\lambda a_s^2}{\tau_b^{3/2}} e^{-2a_s\tau_s}\left[\frac{1}{2\tau_s^{1/2}}(1 - 4a_s\tau_s) - \frac{\mu|W_0|}{\lambda a_s \tau_b^{3/2} e^{-a_s\tau_s}}(1 - a_s\tau_s)\right]
\end{aligned}$$

This requirement is equivalent to

$$\begin{aligned}
\frac{1 - 4a_s\tau_s}{2\tau_s^{1/2}} &= \frac{\mu|W_0|}{\lambda a_s \tau_b^{3/2} e^{-a_s\tau_s}}(1 - a_s\tau_s) \\
e^{-a_s\tau_s} &= \frac{2\mu|W_0|\tau_s^{1/2}}{\lambda a_s \tau_b^{3/2}} \frac{1 - a_s\tau_s}{1 - 4a_s\tau_s} \qquad \text{(A.16)}
\end{aligned}$$

We can simplify the last factor observing that $\tau_s \gg 1$. For computations, set $y = a_s\tau_s$ and perform a Taylor expansion in the limit $y \gg 1$. One has [1]

$$\frac{1-y}{1-4y} = \frac{1}{4} - \frac{3}{16y^2} - \frac{3}{64y^2} + \mathcal{O}\left(\frac{1}{y^3}\right)$$

Inserting this result in (A.16)

$$e^{-a_s\tau_s} = \frac{\mu|W_0|\tau_s^{1/2}}{2\lambda a_s\tau_b^{3/2}}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right] \tag{A.17}$$

Now, the previous result

$$\tau_b^{3/2} = \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s e^{-a_s\tau_s}}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)$$

allows us to obtain an implicit equation for τ_s: combining these relations one finds

$$\tau_b^{3/2} = \frac{\mu|W_0|\tau_s^{1/2}}{\lambda a_s}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)\frac{2\lambda a_s\tau_b^{3/2}}{\mu|W_0|\tau_s^{1/2}}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]^{-1}$$
$$1 = 2\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]^{-1}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right)$$

Multiplying both sides of this equation for the square-bracket term

$$\frac{1}{2}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right] = 1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}$$
$$\frac{1}{2}\left[1 - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right] - 1 = \pm\sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}$$
$$-\frac{1}{2} - \frac{3}{4a_s\tau_s} - \frac{3}{16a_s^2\tau_s^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right) = \pm\sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}$$

Squaring both sides

$$\frac{1}{4}\left[1 + \frac{3}{2a_s\tau_s} + \frac{9}{(4a_s\tau_s)^2} + \frac{6}{(4a_s\tau_s)^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right] = 1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}$$

[1]For this result *Wolfram Alpha* has been used

A little algebra yields

$$\frac{\mu^2}{4\lambda}\,\tau_s^{3/2} = \nu\left[1 - \frac{1}{2a_s\tau_s} - \frac{5}{(4a_s\tau_s)^2} + \mathcal{O}\left(\frac{1}{a_s^3\tau_s^3}\right)\right]^{-1}$$

Since $y = a_s\tau_s \gg 1$ we can expanding the term in square bracket

$$\left[1 - \frac{1}{2y} - \frac{5}{16y^2}\right]^{-1} = 1 + \frac{1}{2y} + \frac{9}{16y^2} + \mathcal{O}\left(\frac{1}{y^3}\right)$$

and finally we obtain an implicit relation defining the minimum for τ_s, that is

$$\frac{\mu^2}{4\lambda}\,\tau_s^{3/2} = \nu\left(1 + \frac{1}{2a_s\tau_s} + \frac{9}{(4a_s\tau_s)^2} + \dots\right) \tag{A.18}$$

A.4 Mass matrix elements

This section is devoted to the calculation of mass matrix elements. This is given by

$$M_{ij}^2 = \frac{1}{2}\begin{pmatrix} \frac{\partial^2 V}{\partial\tau_b^2} & \frac{\partial^2 V}{\partial\tau_b\partial\tau_s} \\ \frac{\partial^2 V}{\partial\tau_s\partial\tau_b} & \frac{\partial^2 V}{\partial\tau_s^2} \end{pmatrix} \tag{A.19}$$

We need the second derivative of the scalar potential evaluated at the minimum. Remember this is characterized by (A.17), (A.18). The first derivative of the scalar potential with respect to τ_b and τ_s has been calculated in the previous section; using the previous results one has

$$
\begin{aligned}
\frac{\partial^2 V}{\partial^2 \tau_b} &= \frac{\partial}{\partial \tau_b}\left[-\frac{3\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}{2\tau_b^{5/2}} + \frac{3\mu|W_0|a_s\tau_s e^{-a_s\tau_s}}{\tau_b^4} - \frac{9\nu|W_0|^2}{2\tau_b^{11/2}}\right] \\
&= \frac{15\lambda a_s^2 \tau_s^{1/2} e^{-2a_s\tau_s}}{4\tau_b^{7/2}} - \frac{12\mu|W_0|a_s\tau_s e^{-a_s\tau_s}}{\tau_b^5} + \frac{99\nu|W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15\lambda a_s^2 \tau_s^{1/2}}{4\tau_b^{7/2}}\left[\frac{\mu|W_0|\tau_s^{1/2}}{2\lambda a_s\tau_b^{3/2}}\left(1-\frac{3}{4a_s\tau_s}-\frac{3}{16a_s^2\tau_s^2}\right)\right]^2 \\
&- \frac{12\mu|W_0|a_s\tau_s}{\tau_b^5}\left[\frac{\mu|W_0|\tau_s^{1/2}}{2\lambda a_s\tau_b^{3/2}}\left(1-\frac{3}{4a_s\tau_s}-\frac{3}{16a_s^2\tau_s^2}\right)\right] + \frac{99\nu|W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15\mu^2|W_0|^2\tau_s^{3/2}}{16\lambda\tau_b^{13/2}}\left(1-\frac{3}{2a_s\tau_s}+\frac{3}{(4a_s\tau_s)^2}\right) - \frac{12\mu^2|W_0|^2\tau_s^{3/2}}{2\lambda\tau_b^{13/2}}\left(1-\frac{3}{2a_s\tau_s}-\frac{3}{(4a_s\tau_s)^2}\right) \\
&+ \frac{99\nu|W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15|W_0|^2\nu}{4\tau_b^{13/2}}\left(1+\frac{1}{2a_s\tau_s}+\frac{9}{(4a_s\tau_s)^2}\right)\left(1-\frac{3}{2a_s\tau_s}+\frac{3}{(4a_s\tau_s)^2}\right) \\
&- \frac{24|W_0|^2\nu}{\tau_b^{13/2}}\left(1+\frac{1}{2a_s\tau_s}+\frac{9}{(4a_s\tau_s)^2}\right)\left(1-\frac{3}{2a_s\tau_s}-\frac{3}{(4a_s\tau_s)^2}\right) \\
&+ \frac{99\nu|W_0|^2}{4\tau_b^{13/2}} \\
&= \frac{15|W_0|^2\nu}{4\tau_b^{13/2}}\left(1-\frac{1}{2a_s\tau_s}\right) - \frac{24|W_0|^2\nu}{\tau_b^{13/2}}\left(1-\frac{1}{4a_s\tau_s}\right) + \frac{99\nu|W_0|^2}{4\tau_b^{13/2}}
\end{aligned}
$$

The first element of (A.19) is given by

$$\frac{\partial^2 V}{\partial \tau_b^2} = \frac{9|W_0|^2\nu}{2\tau_b^{13/2}}\left(1+\frac{1}{2a_s\tau_s}\right) \tag{A.20}$$

in agreement with [4]. Others elements are easy calculated: they are given by

$$\frac{\partial^2 V}{\partial \tau_s^2} = \frac{2a_s^2|W_0|^2\nu}{\tau_b^{9/2}}\left(1-\frac{3}{4a_s\tau_s}+\frac{6}{(4a_s\tau_s)^2}\right) \tag{A.21}$$

$$\frac{\partial^2 V}{\partial \tau_b \tau_s} = -\frac{3a_s|W_0|^2\nu}{\tau_b^{11/2}}\left(1-\frac{5}{4a_s\tau_s}+\frac{4}{(4a_s\tau_s)^2}\right) \tag{A.22}$$

so together with (A.20) and recalling (A.19) one has

$$M_{ij}^2 = \begin{pmatrix} \frac{9|W_0|^2\nu}{4\tau_b^{13/2}}\left(1+\frac{1}{2a_s\tau_s}\right) & -\frac{3a_s|W_0|^2\nu}{2\tau_b^{11/2}}\left(1-\frac{5}{4a_s\tau_s}+\frac{4}{(4a_s\tau_s)^2}\right) \\ -\frac{3a_s|W_0|^2\nu}{2\tau_b^{11/2}}\left(1-\frac{5}{4a_s\tau_s}+\frac{4}{(4a_s\tau_s)^2}\right) & \frac{a_s^2|W_0|^2\nu}{\tau_b^{9/2}}\left(1-\frac{3}{4a_s\tau_s}+\frac{6}{(4a_s\tau_s)^2}\right) \end{pmatrix} \tag{A.23}$$

A.5 Lagrangian in terms of canonically normalized fields

In this section we show how to write (4.16). Let's begin with the kinetic terms

$$\begin{aligned} \partial_\mu \boldsymbol{\delta\tau}^T \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu \boldsymbol{\delta\tau} &= \partial_\mu \left[\frac{\Phi}{\sqrt{2}}(\boldsymbol{v}_\Phi)^T + \frac{\chi}{\sqrt{2}}(\boldsymbol{v}_\chi)^T\right] \cdot \boldsymbol{\mathcal{K}} \cdot \partial^\mu \left[\frac{\Phi}{\sqrt{2}}\boldsymbol{v}_\Phi + \frac{\chi}{\sqrt{2}}\boldsymbol{v}_\chi\right] \\ &= \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi \end{aligned}$$

where the normalization condition (4.11) was used. Now let us see the potential energy, in particular the mass term has

$$\begin{aligned} \boldsymbol{\delta\tau}^T \cdot \boldsymbol{M}^2 \cdot \boldsymbol{\delta\tau} &= \boldsymbol{\delta\tau}^T \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2 \cdot \boldsymbol{\delta\tau} \\ &= \left[\frac{\Phi}{\sqrt{2}}(\boldsymbol{v}_\Phi)^T + \frac{\chi}{\sqrt{2}}(\boldsymbol{v}_\chi)^T\right] \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2 \left[\frac{\Phi}{\sqrt{2}}\boldsymbol{v}_\Phi + \frac{\chi}{\sqrt{2}}\boldsymbol{v}_\chi\right] \\ &= \left[\frac{\Phi}{\sqrt{2}}(\boldsymbol{v}_\Phi)^T + \frac{\chi}{\sqrt{2}}(\boldsymbol{v}_\chi)^T\right] \cdot \boldsymbol{\mathcal{K}} \cdot \left[m_\Phi^2\frac{\Phi}{\sqrt{2}}\boldsymbol{v}_\Phi + m_\chi^2\frac{\chi}{\sqrt{2}}\boldsymbol{v}_\chi\right] \\ &= \frac{1}{2}m_\Phi^2\Phi^2 + \frac{1}{2}m_\chi^2\chi^2 \end{aligned}$$

where eigenvalues equations (4.12) were used. Then we have to recover the Maxwell lagrangian and the interaction term; for this purpose set $M_{Pl} = 1$

$$\begin{aligned} \kappa\,\tau_s\,F_{\mu\nu}F^{\mu\nu} &= \kappa\left(\langle\tau_s\rangle + \delta\tau_s\right)F_{\mu\nu}F^{\mu\nu} \\ &= \kappa\,\langle\tau_s\rangle\,F_{\mu\nu}F^{\mu\nu} + \kappa\,\delta\tau_s\,F_{\mu\nu}F^{\mu\nu} \end{aligned}$$

Setting

$$\kappa\,\langle\tau_s\rangle\,F_{\mu\nu}F^{\mu\nu} = \frac{1}{4}\,G_{\mu\nu}G^{\mu\nu}$$

and evidently

$$F_{\mu\nu}F^{\mu\nu} = \frac{1}{4\kappa\,\langle\tau_s\rangle}\,G_{\mu\nu}G^{\mu\nu}$$

one obtains

$$\begin{aligned}
\kappa\,\langle\tau_s\rangle\,F_{\mu\nu}F^{\mu\nu}+\kappa\,\delta\tau_s\,F_{\mu\nu}F^{\mu\nu} &= \frac{1}{4}\,G_{\mu\nu}G^{\mu\nu}+\kappa\,\delta\tau_s\frac{1}{4\kappa\,\langle\tau_s\rangle}\,G_{\mu\nu}G^{\mu\nu}\\
&= \frac{1}{4}\,G_{\mu\nu}G^{\mu\nu}+\frac{\delta\tau_s}{\langle\tau_s\rangle}\,G_{\mu\nu}G^{\mu\nu}
\end{aligned}$$

Renaming G with F and recalling the expression of $\delta\tau_s$ in terms of Φ and χ

$$\kappa\,\tau_s\,F_{\mu\nu}F^{\mu\nu}=\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{(\Phi(v_\Phi)_s+\chi(v_\chi)_s)}{4\sqrt{2}\langle\tau_s\rangle}$$

The matrix $\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2$ of (4.17) is obtained by a simply multiplication of matrix. This is a tedious calculation, but there are no difficulties in having the expression (4.17). Its eigenvalues are m_Φ^2 and m_χ^2 and we know that $m_\Phi^2\gg m_\chi^2$, then at the leading order in ϵ

$$\begin{aligned}
\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) &= (m_\Phi^2+m_\chi^2)\simeq m_\Phi^2\\
\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) &= m_\Phi^2 m_\chi^2\\
\frac{\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)}{\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)} &\simeq m_\chi^2
\end{aligned}$$

These quantities are easy calculated

$$\begin{aligned}
\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) &= \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\Bigg[-9\,(1-7\epsilon)+\frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}\,(1-3\epsilon+6\epsilon^2)\Bigg]\\
&\simeq \left(\frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\right)\left(\frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}\right)\\
&= \frac{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}{3\langle\tau_b\rangle^3}\\
\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2) &= \frac{4a_s^2\langle\tau_s\rangle^2|W_0|^4\nu^2}{9\langle\tau_b\rangle^9}\Bigg[-9\,(1-7\epsilon)\cdot\frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}\,(1-3\epsilon+6\epsilon^2)-\\
&\quad+6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2)\cdot(1-5\epsilon+16\epsilon^2)\,\frac{-6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}\Bigg]\\
&= \frac{18a_s\langle\tau_s\rangle^2|W_0|^4\nu^2}{\langle\tau_b\rangle^{15/2}\langle\tau_s\rangle^{1/2}}\\
\frac{\mathrm{Det}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)}{\mathrm{Tr}\,(\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2)} &= \left(\frac{18a_s\langle\tau_s\rangle^2|W_0|^4\nu^2}{\langle\tau_b\rangle^{15/2}\langle\tau_s\rangle^{1/2}}\right)\left(\frac{3\langle\tau_b\rangle^3}{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}\right)\\
&= \frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}
\end{aligned}$$

Next we want the eigenvectors relatives to these eigenvalues. In other words we have to solve

$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2\begin{pmatrix}(v_\Phi)_b\\(v_\Phi)_s\end{pmatrix}=m_\Phi^2\begin{pmatrix}(v_\Phi)_b\\(v_\Phi)_s\end{pmatrix}\tag{A.24}$$

$$\boldsymbol{\mathcal{K}}^{-1}\boldsymbol{M}^2\begin{pmatrix}(v_\chi)_b\\(v_\chi)_s\end{pmatrix}=m_\chi^2\begin{pmatrix}(v_\chi)_b\\(v_\chi)_s\end{pmatrix}\tag{A.25}$$

Let's start with (A.24). Recall that $\tau_b \gg \tau_s \gg 1$, $\epsilon = (4a_s\langle\tau_s\rangle)^{-1}$ and we are interested at the leading order in τ_b.

$$\frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\begin{pmatrix}-9(1-7\epsilon) & 6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2)\\ -\frac{6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(1-5\epsilon+4\epsilon^2) & \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(1-3\epsilon+6\epsilon^2)\end{pmatrix}\begin{pmatrix}(v_\Phi)_b\\(v_\Phi)_s\end{pmatrix}=m_\Phi^2\begin{pmatrix}(v_\Phi)_b\\(v_\Phi)_s\end{pmatrix}$$

This is equivalent to the following relations

$$\frac{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}{3\langle\tau_b\rangle^3}(v_\Phi)_b=\frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\Big[-9(v_\Phi)_b+6a_s\langle\tau_b\rangle(v_\Phi)_s\Big]\tag{A.26}$$

$$\frac{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}{3\langle\tau_b\rangle^3}(v_\Phi)_s=\frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\Big[-\frac{6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(v_\Phi)_b+\frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(v_\Phi)_s\Big]\tag{A.27}$$

From the first of these

$$\frac{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}{3\langle\tau_b\rangle^3}(v_\Phi)_b\sim\frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}6a_s\langle\tau_b\rangle(v_\Phi)_s$$

while the second tells us nothing new. We have the relation between $(v_\Phi)_b$ and $(v_\Phi)_s$

$$\begin{aligned}(v_\Phi)_s&=\frac{8a_s^2\langle\tau_s\rangle^{1/2}|W_0|^2\nu}{3\langle\tau_b\rangle^3}\frac{3\langle\tau_b\rangle^{9/2}}{2a_s\langle\tau_s\rangle|W_0|^2\nu}\frac{1}{6a_s\langle\tau_b\rangle}\\&=\frac{2\langle\tau_b\rangle^{1/2}}{3\langle\tau_s\rangle^{1/2}}(v_\Phi)_b\end{aligned}$$

Employing the normalization condition

$$
\begin{aligned}
1 = \boldsymbol{v}_\Phi^T \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{v}_\Phi &= \begin{pmatrix} (v_\Phi)_b & (v_\Phi)_s \end{pmatrix} \begin{pmatrix} \mathcal{K}_{b\bar{b}} & \mathcal{K}_{b\bar{s}} \\ \mathcal{K}_{s\bar{b}} & \mathcal{K}_{s\bar{s}} \end{pmatrix} \begin{pmatrix} (v_\Phi)_b \\ (v_\Phi)_s \end{pmatrix} \\
&= (v_\Phi)_b^T \begin{pmatrix} 1 & \frac{2\langle\tau_b\rangle^{1/2}}{3\langle\tau_s\rangle^{1/2}} \end{pmatrix} \begin{pmatrix} \frac{3}{4\langle\tau_b\rangle^2} & -\frac{9}{8}\frac{\langle\tau_s\rangle^{1/2}}{\langle\tau_b\rangle^{5/2}} \\ -\frac{9}{8}\frac{\langle\tau_s\rangle^{1/2}}{\langle\tau_b\rangle^{5/2}} & \frac{3}{8\langle\tau_b\rangle^{3/2}\langle\tau_s\rangle^{1/2}} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{2\langle\tau_b\rangle^{1/2}}{3\langle\tau_s\rangle^{1/2}} \end{pmatrix} (v_\Phi)_b \\
&= (v_\Phi)_b^T \begin{pmatrix} 0 & \frac{1}{4\langle\tau_b\rangle\langle\tau_s\rangle} + \mathcal{O}(\tau_b^{-5/2}) \end{pmatrix} \begin{pmatrix} 1 \\ \frac{2\langle\tau_b\rangle^{1/2}}{3\langle\tau_s\rangle^{1/2}} \end{pmatrix} (v_\Phi)_b \\
&= \left[\frac{1}{6\langle\tau_b\rangle^{1/2}\langle\tau_s\rangle^{3/2}} + \mathcal{O}(\tau_b^3) \right] (v_\Phi)_b^2
\end{aligned}
$$

from which it is easy to see that

$$
\boldsymbol{v_\Phi} = \begin{pmatrix} (v_\Phi)_b \\ (v_\Phi)_s \end{pmatrix} \begin{pmatrix} \sqrt{6}\langle\tau_b\rangle^{1/4}\langle\tau_s\rangle^{3/4} \\ \frac{2\sqrt{6}}{3}\langle\tau_b\rangle^{3/4}\langle\tau_s\rangle^{1/4} \end{pmatrix} \tag{A.28}
$$

Now we have to solve (A.25)

$$
\frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}} \begin{pmatrix} -9(1-7\epsilon) & 6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2) \\ -\frac{6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(1-5\epsilon+4\epsilon^2) & \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(1-3\epsilon+6\epsilon^2) \end{pmatrix} \begin{pmatrix} (v_\chi)_b \\ (v_\chi)_s \end{pmatrix} = m_\chi^2 \begin{pmatrix} (v_\chi)_b \\ (v_\chi)_s \end{pmatrix}
$$

so we have

$$
\frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}(v_\chi)_b = \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\left[-9(v_\chi)_b + 6a_s\langle\tau_b\rangle(v_\chi)_s \right] \tag{A.29}
$$

$$
\frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}(v_\chi)_s = \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\left[-\frac{6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(v_\Phi)_b + \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(v_\chi)_s \right] \tag{A.30}
$$

From the first of these

$$
\begin{aligned}
(v_\chi)_s &\sim \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}}\left(6a_s\langle\tau_b\rangle\right)\frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}(v_\chi)_s \\
&= \frac{27}{16a_s^3\langle\tau_s\rangle^2\langle\tau_b\rangle}(v_\chi)_s = \frac{27\epsilon^2}{a_s\langle\tau_b\rangle}(v_\chi)_b
\end{aligned}
$$

Normalization requires

$$
\begin{aligned}
1 = \boldsymbol{v}_\chi^T \cdot \boldsymbol{\mathcal{K}} \cdot \boldsymbol{v}_\chi &= \begin{pmatrix}(v_\chi)_b & (v_\chi)_s\end{pmatrix} \begin{pmatrix}\mathcal{K}_{b\bar{b}} & \mathcal{K}_{b\bar{s}} \\ \mathcal{K}_{s\bar{b}} & \mathcal{K}_{s\bar{s}}\end{pmatrix} \begin{pmatrix}(v_\chi)_b \\ (v_\chi)_s\end{pmatrix} \\
&= (v_\chi)_b^T \begin{pmatrix}1 & \frac{27\epsilon^2}{a_s\langle\tau_b\rangle}\end{pmatrix} \begin{pmatrix}\frac{3}{4\langle\tau_b\rangle^2} & -\frac{9}{8}\frac{\langle\tau_s\rangle^{1/2}}{\langle\tau_b\rangle^{5/2}} \\ -\frac{9}{8}\frac{\langle\tau_s\rangle^{1/2}}{\langle\tau_b\rangle^{5/2}} & \frac{3}{8\langle\tau_b\rangle^{3/2}\langle\tau_s\rangle^{1/2}}\end{pmatrix} \begin{pmatrix}1 \\ \frac{27\epsilon^2}{a_s\langle\tau_b\rangle}\end{pmatrix} (v_\chi)_b \\
&= (v_\chi)_b^T \begin{pmatrix}\frac{3}{4\langle\tau_b\rangle^2} - \frac{243\epsilon^2\langle\tau_s\rangle^{1/2}}{8a_s\langle\tau_b\rangle^{7/2}} & -\frac{9}{8}\frac{\langle\tau_s\rangle^{1/2}}{\langle\tau_b\rangle^{5/2}} + \frac{81\epsilon^2}{8a_s\langle\tau_b\rangle^{5/2}\langle\tau_s\rangle^{1/2}}\end{pmatrix} \begin{pmatrix}1 \\ \frac{27\epsilon^2}{a_s\langle\tau_b\rangle}\end{pmatrix} (v_\chi)_b \\
&= (v_\chi)_b^T \begin{pmatrix}\frac{3}{4\langle\tau_b\rangle^2} + \mathcal{O}(\langle\tau_b\rangle^{-7/2}) & \mathcal{O}(\langle\tau_b\rangle^{-5/2})\end{pmatrix} \begin{pmatrix}1 \\ \frac{27\epsilon^2}{a_s\langle\tau_b\rangle}\end{pmatrix} (v_\chi)_b \\
&\sim \left[\frac{3}{4\langle\tau_b\rangle^2} + \mathcal{O}(\langle\tau_b\rangle^{-7/2})\right] (v_\chi)_b^2
\end{aligned}
$$

from which one has the expression $(v_\chi)_b$, that is

$$
(v_\chi)_b = \sqrt{\frac{4}{3}}\langle\tau_b\rangle
$$

Remembering the first of (A.29) we obtain

$$
\frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}\frac{3\langle\tau_b\rangle^{9/2}}{2a_s\langle\tau_s\rangle|W_0|^2\nu}\sqrt{\frac{4}{3}}\langle\tau_b\rangle = \left[-9\sqrt{\frac{4}{3}}\langle\tau_b\rangle + 6a_s\langle\tau_b\rangle(v_\chi)_s\right]
$$

but

$$
\frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}}\frac{3\langle\tau_b\rangle^{9/2}}{2a_s\langle\tau_s\rangle|W_0|^2\nu} = \frac{81}{8a_s^2\langle\tau_s\rangle^2}\sqrt{\frac{4}{3}}\langle\tau_b\rangle = (162\epsilon^2)\sqrt{\frac{4}{3}}\langle\tau_b\rangle
$$

and since $\epsilon \ll 1$ we are left with

$$
9\sqrt{\frac{4}{3}}\langle\tau_b\rangle \sim 6a_s\langle\tau_b\rangle(v_\chi)_s
$$

The correctly normalized eigenvector is

$$
\boldsymbol{v_\chi} = \begin{pmatrix}(v_\chi)_b \\ (v_\chi)_s\end{pmatrix} = \begin{pmatrix}\sqrt{\frac{4}{3}}\langle\tau_b\rangle \\ \frac{\sqrt{3}}{a_s}\end{pmatrix} \tag{A.31}
$$

A.6 The VEV of the flaton

In this section we want to justify the assumption $\langle\sigma\rangle \equiv M \geq 10^{10}$ GeV. Keeping $|\sigma|$ as variable, the critical points are the solution of $V'(\,|\sigma|\,) = 0$. This implies

$$|\,\sigma|^{2n+2} = \frac{m_0^2 M_{Pl}^{2n}}{(n+2)\lambda_n} \tag{A.32}$$

We now verify that this is a minimum for the effective potential. The second derivative is

$$V''(|\,\sigma|) = -2m_0^2 + 2(n+2)(2n+3)\lambda_n M_{Pl}^{-2n}|\,\sigma|^{2n+2} \tag{A.33}$$

and using the result (A.32)

$$\begin{aligned} V''(|\,\sigma|_*) &= -2m_0^2 + 2(n+2)(2n+3)\lambda_n M_{Pl}^{-2n}\frac{m_0^2 M_{Pl}^{2n}}{(n+2)\lambda_n} \\ &= (2n+1)m_0^2 \end{aligned}$$

Since $(2n{+}1)m_0^2 > 0$, the critical point found in (A.32) is a minimum, so setting $\langle\sigma\rangle = M$, one has the following relation

$$M^{2n+2} = \frac{m_0^2 M_{Pl}^{2n}}{(n+2)\lambda_n} \tag{A.34}$$

and the flaton has an effective mass squared $m_\sigma^2 = V''(M)/2$, that is

$$m_\sigma^2 = 2(n+1)m_0^2 \tag{A.35}$$

We can rewrite (A.34) as a function of m_σ^2 instead of m_0^2

$$M^{2n+2}M_{Pl}^{-2n} = \frac{m_\sigma^2}{2(n+1)(n+2)\lambda_n} \tag{A.36}$$

To estimate the VEV, suppose that only the first term of (3.2) dominates and $\lambda_1 \sim 1$, then it is easy to see that $M = (3\lambda_1)^{-1/4}(m_0 M_{Pl})^{1/2} \sim (3\lambda_1)^{-1/4} \times 10^{10}$ to 10^{11} GeV. If there are two terms and the second is dominating with $\lambda_2 \sim 1$, then $M = (4\lambda_2)^{-1/6}(m_0)^{1/3}(M_{Pl})^{2/3} \sim (4\lambda_1)^{-1/6} \times 10^{13}$ GeV. So the claim $M > 10^{10}$ GeV is justified. The height of the barrier

V_0 follows from $V(M) = 0$

$$
\begin{aligned}
0 &= V_0 - m_0^2 M^2 + \lambda_n M_{Pl}^{-2n} M^2 \left(\frac{m_0^2 M_{Pl}^{2n}}{(n+2)\lambda_n} \right) \\
V_0 &= m_0^2 M^2 \left(1 - \frac{1}{n+2} \right) \\
&= \frac{m_\sigma^2 M^2}{2(n+2)}
\end{aligned}
$$

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