On the Timing of Default Losses: when and why?

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Abstract

At the light of what happened in 2010 and 2011, a lot of European countries founded themselves in a difficult position where all the credit rating agencies were downgrading debt states. Problem of solvency and guarantees on the states' bond were perceived as too risky for a Monetary Union as Europe is. Fear of a contagion from Greece as well was threatening the other countries as Italy, Spain, Portugal and Ireland; while Germany and France asked for a division between risky and riskless bond in order to feel more safe.

Our paper gets inspiration by Roch and Uhlig (2011), it refers to the Argentinian case examined by Arellano (2008) and examine possible interventions as monetization or bailout as proposed by Cole and Kehoe (2000).

We propose a model in which a state defaults and cannot repay a fraction of the old bond; but contrary to Roch and Uhlig that where considering a one-time cost of default we consider default as an accumulation of losses, perceived as unpaid fractions of the old debts. Our contributions to literature is that default immediately imply that economy faces a bad period and, accumulating losses, government will be worse-off. We studied a function for this accumulation of debt period by period, in order to get an idea of the magnitude of this waste of resources that economy will face when experiences a default. Our thesis is that bailouts just postpone the day of reckoning (Roch, Uhlig); so it's better to default before accumulate a lot of debts. What Europe need now is the introduction of new reforms in a controlled default where the Eurozone will be saved in its whole integrity and a state could fail with the future promise of a resurrection. As experience show us, governments are not interested into reducing debts since there are ECB interventions. That clearly create a distortion between countries in the same monetary union, giving to the states just an illusion about their future debtor position.

Contents

Introduction

In 2010 fears that Greece would not be able to repay its sovereign debt start to spread on financial markets. Following Greece a lot of countries as Italy, Spain, Portugal, Ireland were downgrading by credit rating agencies (CRAs) and a lot of other countries as France were threated of a possible contagion. Fears of a possible separation within the Euro zone or a possible default of the Euro area, or possible threats to the European Monetary System were so real. The finance Minister of Europe approved a rescue package for Greece and also created the European Financial Stability Facility "to prevent a default as well as to return yield spreads to pre-crisis levels" (Roch and Uhlig, 2011).

In 2011 Italy adopted a financial and economic plan, trying to downgrade the level of its spread but this turns out to be persistently high. After the plan of Monti's government was approved spread level decreases but after some days it increases again. As Pastor and Veronesi (2010) have argued, it's profitable to use a new policy if the impact of the old one was perceived as unfavourable by markets (as was in the Italian case); but at the same time, at announcement of a policy change the discount rate rise immediately because higher uncertainty about the future. Repay the debt becomes costly more and more. On average investments are cutted and spread level rise quite rapidly. That's what happened in Italy after the reform.

Literature is so wide and it give us a huge amount of different studies to approach the default on sovereign debts.

Contributions from Lejour et al. (2010) and Arghyrou and Kontonikas (2011) give us a good representation of the issues and empirics of the situation. In the European Monetary Union (EMU) the market pricing behaviour changes totally from a convergence trade model to a fear of contagion from Greece, driven by macroeconomic fundamentals and international common risk. As it's pointed out: there was a "double shift in private expectations from a fully credible commitment to a non fully credible EMU commitment without fiscal guarantees". "EMU needs for structural and competitiveness-inducing reforms in periphery EMU countries and at EMU level monitoring and policy coordination". But "it's more a problem of trust" rather than economy. (Arghyrou, Kontonikas, 2011). According to them, main drivers of the crisis are macroeconomic fundamentals and contagion; and Greek debt crisis is just a result of a shift in market expectations, deteriorating the macroeconomic performance, as happened for Mexico (1994-'95) and Argentina (1998).

On the contrary, Ang and Longstaff (2011) focused on systemic sovereign credit risk and the role of Credit Default Swap (CDS) as insurance against default of a country on its debt. They conclude that a systemic risk may lead to a cascade of defaults but "there is more systemic risk in Europe than was expecting for U.S. and its root has to be found in the financial market, not in the macroeconomic fundamentals".

Authors, as usual, ended up with different conclusions: for example Greiner et al (2007) have calculated that current debt levels in EMU member countries are probably sustainable in principle. But they were supposing a government that takes corrective actions as result of rising debt ratio, increasing primary surplus to GDP ratio. Their conclusion is that in the long run a high debt to GDP ratio will have negative repercussion for the growth rate of the economy. As we know the easiest way to reduce the public debt is to decrease public spending, but countries as U.S. have

a percentage of public spending to GDP ratio equivalent to 20%, while countries as Italy have a percentage of 40-50%.
Cole and Kehoe (1996, 2000) affirm that debt crises may be self-fulfilling: the fear of a future default may trigger a current rise in default premia on sovereign debt and thereby raise the probability of a default in the first place. Mexico for example had its fundamentals in the so-called "crisis zone" and this was linked with a huge loss of confidence in the government. When government cares more about private consumption is more likely to be in the crisis zone.
In this case the maturity of the debt could shrink the crisis zone in which a country could be, even if a change in the maturity structure is too costly (Mexico had a high debt level with a short-term maturity structure); but credibility and other policies can just have perverse effects.
Both studies imply, however, that countries would have a strong incentive to avoid default-triggering scenarios in the first place. But Beetsma and Uhlig (1999) show that "shortsighted governments fail to internalize consequences of their debt policies for a common inflation rate".
Cooper, Kempf and Peled (2010) substained that "regional governments, anticipating central bank financing of their debt obligations, have an incentive to create excessively large deficits" but as Beetsma, Uhlig (1999) point out: "it is hard to imagine the ECB standing by idly, while the debt pileup in a member country ... leands to debt down- grading or default". The Stability Growth Pact is not still "effective" to punish countries with excessive deficits while governments fail to internalize benefits from the SGP by reducing debt for a lower future inflation level. And countries often think themselves as "too big to fail" (Cooper, Kempf, Peled, 2010).
Arellano (2008) built a model in which default equilibria occur and she shows that defaults are more likely when income is low. In the Argentinian case incompleteness of asset's markets played a huge role. As substained by Banjeree and Duflo (2010), during a recession, a risk averse borrower find more costly to repay a debt than default on it: this is why, in part, a government should find it optimal to default on its debt, while repay it in boom times is costless.
While all the countries sit in meetings trying to find a solution for the Euro zone, Cooper, Kempf and Peled (2010) argued for the impossibility of a policy to eliminate fiscal spillovers within the European Monetary Union. There are fiscal irresponsibilities by some states; there is a huge commitment problem for the European Central Bank that cannot leave members to default scenarios. They proposed different solutions as dollarization that took place in the Argentinian case and monetization that creates free-rider problems and incentives for some agents to erect impediments as debt restrictions for the future or limits on holdings. Increase of the debt in a region reduce the capital stock of a federation, impacting on wages, return on capitals and interest rates of the other regions' agents. They also tried to offer an answer to a possible situation of future taxation but without money creation.
Starting by the work of Roch and Uhlig (2011) that discuss about equilibrium default and the role of bailouts, we have built a model where government can default on its debt and this generates a huge amount of losses that can be recovered only with time. In fact if a government has a bad performance it's supposed to accumulate debt because of public expenditure, because of expenses for the management of the state, because of too much investment in the bond market. The idea is that default doesn't happen immediately but it's a consequence of this bad management: the state

starts to accumulate losses because of the shortsighted policy (as said in Beetsma, Uhlig, 1999) and it doesn't take into account the possibility that the European Central Bank cannot finance its debt. States think that they're "too big to fail" (Cooper, Kempf, Peled, 2010), so they accumulate losses until a point in which they need for an ECB intervention. Naturally ECB has to intervene but the intervention can change with different results: if ECB will intervene with monetization this will create spillovers in the other countries as Cooper, Kempf and Peled (2010) substained, but the country will get benefits and default losses will be absorbed quite rapidly. In the other case if ECB intervene but proposing a reform these losses will take more time to be recovered.
The literature is considerably larger and every author uses a different approach trying to model the real world situation. We hope to give a contribution to this field of studies with the aim that future research will benefit from this work.
As suggested by Roch, Uhlig (2011) our idea is that bailout means just "post-poning the day of reckoning". The state knows when it starts to accumulate losses because of bad management and anticipating this it has to intervene immediately (with a reform, with a change in government, with change in its policy), to avoid that losses will become too big and ECB must to intervene. Make a reform when the default is already started doesn't make any sense: it doesn't make things better as we've seen in Mexico (when the crisis occured was too late to change debt maturity structure). It's better to solve problems before that they take place, using ex-ante solutions.

We have constructed a model in which there are identical households and there is a government that is assumed to be benevolent and want to maximize the welfare of the households. We have constructed a function to measure losses that the government could accumulate if it defaults, since no one before made it. We're thinking that if a state default it will accumulate losses, period by period: we cannot talk about a one-time utility cost as sustained by Roch and Uhlig (2011). We're considering an accumulation of debt out of debt, perceived as unpaid fraction of the old bond that the government commit to repay. We're going to describe what happens in this economy when state experience a default and what could be a good solution to default.
The paper is organized as follows: Section 1 presents the theoretical model that we have built, Section 2 discuss what happens at the steady state, Section 3 presents all the graphs in order to show how this economy looks like, Section 4 concludes. There is also an Appendix that is worth to explain some computational aspects.

1 The model

The model combines the approach of Roch and Uhlig (2011), following their specification of the utility function for the government, using a utility cost of default; and add to this some variables as described in Arellano (2008) and Cole and Kehoe (2000). We assume that there is a continuum of infinitely living households that get utility from consumption of private and public goods and invests in the capital market and bonds; and there is a benevolent government that wants to maximize households' consumption of public goods, levying taxes on capital and labor market, with the promise to repay bonds.

1.1 Households

The economy is characterized by the existence of a continuum of infinitely living households supplying labor to the market. Each household gets utility from consumption of goods C_t while each household dislikes supplying labor L_t necessary for firm's production. Each household wants to maximize its utility:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{L_t^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \right] \tag{1.1}$$

where β is the discount factor of the households, $u(.)$ is a CRRA, strisctly increasing, strictly concave and twice differentiable utility function. The time-separable utility function has a coefficient of relative risk aversion $\frac{1}{\sigma}$ for the consumption; σ is the intertemporal elasticity of substitution between c_t and c_{t+1}: it describes how households shift conumption from one period to another, trying to smooth it (for more details see the Appendix 5.1.). $\frac{1}{\eta}$ is the Frisch elasticity that measures the substitution effect of a change in the wage rate on labor supply . Using this kind of utility we make things simplier than using a non-separable in time utility function. The utility of representative household is pretty similar to that in Arellano (2008) as well as Cole and Kehoe (2000); Y_t is total output (considered as GDP of the economy) and C_t is the consumption of households, where the government is assumed to maximize welfare. According to Roch and Uhlig (2011), given uncertainty of future consumption or return on bonds or other investments, a household may discount the future more steeply than it usually does.
Households' maximization problem is described as:

$$\begin{aligned} & max \sum_{t=0}^{\infty} \beta^t [u(C_t)] \\ s.t. \quad & C_t + I_t + \frac{R_t}{\Pi_t} B_{t-1} = B_t + Y_t \end{aligned} \tag{1.2}$$

where investments are described as:

$$I_t = K_{t+1} - (1-\delta) K_t \tag{1.3}$$

with δ that is the depreciation rate of capital; and households' income (revenues from labor and capital market) is given by:

$$Y_t = (1 - \tau_t^l) w_t L_t + r_t (1 - \tau_t^k) K_t \tag{1.4}$$

with:

- $\tau_t^l = \rho^l \tau_{t-1}^l + \epsilon_t^l$ is an AR1 process with $\epsilon_t^l \sim N(0, \sigma^2)$, representing taxation on labor market;
- $\tau_t^k = \rho^k \tau_{t-1}^k + \epsilon_t^k$ is an AR1 process with $\epsilon_t^k \sim N(0, \sigma^2)$, representing taxation on capital market.

The gross return to capital $R_t = 1 + r_t$ is the gross interest rate and we assume it's described as:

$$R_{t+1} = Q_{t+1} + \Pi_{t+1} \tag{1.5}$$

where $Q_t$is the marginal rate of return to capital and Π_t is the inflation.
Inflation is assumed to follow a Taylor rule structure:

$$\beta E_t \left(\Pi_{t+1} \right) = \Pi_t - k_y Y_t \tag{1.6}$$

where k_y is the coefficient that determines the impact of inflation on households' income.
Budget constraint of a representative household (HBC=Households Budget Constraint) is expressed in real terms: it describes which resources are available at time t and how is possible to transfer wealth across periods using the capital market and investing on state's bonds. Each households buy a bond in order to repay interests on the old one and has a income Y_t obtained both selling labor to firms and getting revenues on investments on the capital market. With their incomes each household has a private consumption C_t , invests in the capital market, buy bond of the government; but has to pay the taxes levied by the government on labor and capital markets

We also assume that the production function of firms where households supply their labor takes the form of a Cobb-Douglas where:

$$Y_t = A_t L_t^{\alpha} K_t^{1-\alpha} \tag{1.7}$$

with:

- $w_t = \frac{\partial Y_t}{\partial L_t} = \alpha A_t \left(\frac{K_t}{L_t} \right)^{1-\alpha}$ the real wage, as marginal product of labor;
- $Q_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha) A_t \left(\frac{L_t}{K_t} \right)^{\alpha}$ the rental rate on capital stock, as marginal product of capital;
- $A_{t+1} = \rho^a A_t + \varepsilon_{t+1}^a$ follows an AR1 process with $\epsilon_t^a \sim N(0, \sigma^2)$, representing the technology process;
- α the fraction of labor in the production function (and consequently $1 - \alpha$ is the fraction of capital in the production function)

Notice also that:

$$R_t = \left[(1-\alpha)A_t\left(\frac{L_t}{K_t}\right)^{\alpha} + (1-\delta)\right]$$

HBC can be re-written as:

$$C_t + K_{t+1} - (1-\delta)K_t + \frac{R_t}{\Pi_t}B_{t-1} = B_t + [(1-\tau_t^l)w_tL_t + r_t(1-\tau_t^k)K_t] \tag{1.8}$$

1.1.1 Inter-temporal maximization problem for households

The intertemporal maximization problem of the government can be written as:

$$\begin{aligned}\mathcal{L} =& E_t\Bigg\{\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{L_t^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}\right) + \sum_{t=0}^{\infty}\beta^{t+1}\left(\frac{C_{t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{L_{t+1}^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}\right) + \sum_{t=0}^{\infty}\beta^t\lambda_t[B_t+ \\ &+ (1-\tau_t^l)w_tL_t + r_t(1-\tau_t^k)K_t - C_t - K_{t+1} + (1-\delta)K_t - \frac{R_t}{\Pi_t}B_{t-1}]+ \\ &+ \sum_{t=0}^{\infty}\beta^{t+1}\lambda_{t+1}[B_{t+1} + (1-\tau_{t+1}^l)w_{t+1}L_{t+1} + r_{t+1}(1-\tau_{t+1}^k)K_{t+1}+ \\ &- C_{t+1} - K_{t+2} + (1-\delta)K_{t+1} - \frac{R_{t+1}}{\Pi_{t+1}}B_t]\Bigg\}\end{aligned} \tag{1.9}$$

Now we can proceed solving household's intertemporal optimization problem.
F.O.C.s
for private consumption:

$$\tfrac{\partial\mathcal{L}}{\partial C_t} = E_t\left(\beta^tC_t^{-\frac{1}{\sigma}} - \beta^t\lambda_t\right) = 0$$

$$\lambda_t = C_t^{-\frac{1}{\sigma}} \tag{1.10}$$

for labor:

$$\tfrac{\partial\mathcal{L}}{\partial L_t} = E_t\left[(-)\beta^tL_t^{-\frac{1}{\eta}} + \beta^t\lambda_t(1-\tau_t^l)w_t\right] = 0$$

$$L_t^{-\frac{1}{\eta}} = \lambda_t(1-\tau_t^l)w_t$$

$$\lambda_t = \frac{L_t^{-\frac{1}{\eta}}}{w_t(1-\tau_t^l)} \tag{1.11}$$

for bond:

$$\tfrac{\partial\mathcal{L}}{\partial B_t} = E_t\left[\beta^t\lambda_t - \beta^{t+1}\lambda_{t+1}\tfrac{R_{t+1}}{\Pi_{t+1}}\right] = 0$$

$$\lambda_t = \beta E_t(\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}}) \tag{1.12}$$

for capital:

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = E_t \left\{(-)\beta^t \lambda_t + \beta^{t+1}\lambda_{t+1}[r_{t+1}(1-\tau^k_{t+1}) + (1-\delta)]\right\} = 0$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1}[r_{t+1}(1-\tau^k_{t+1}) + (1-\delta)]\right\} \tag{1.13}$$

where $\frac{P_{t+1}}{P_t}$ is the gross inflation rate and is equal to $\Pi_{t+1} = 1 + \pi_{t+1}$.
Now, using F.O.C.s of bond (eqn.(1.12)) and capital (eqn.1.13)), that are:

$$\lambda_t = \beta E_t(\lambda_{t+1}\tfrac{R_{t+1}}{\Pi_{t+1}})$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1}[r_{t+1}(1-\tau^k_{t+1}) + (1-\delta)]\right\}$$

We observe that:

$$\frac{R_{t+1}}{\Pi_{t+1}} = [r_{t+1}(1-\tau^k_{t+1}) + (1-\delta)] \tag{1.14}$$

$$R_{t+1} = \Pi_{t+1}[r_{t+1}(1-\tau^k_{t+1}) + (1-\delta)] \tag{1.15}$$

Plugging the F.O.C. for consumption into that of bonds, that means using eqn. (1.10) we get:

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left[C_{t+1}^{-\frac{1}{\sigma}}\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)\right] \tag{1.16}$$

This is the Euler equation; it describes the behaviour of consumption that obviously depends on interest rate and inflation.
Now substituing for R_{t+1} we get:

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left[C_{t+1}^{-\frac{1}{\sigma}}\tfrac{[r_{t+1}(1-\tau^k_{t+1})+(1-\delta)]}{\Pi_{t+1}}\right]$$

Since we know that $\lambda_t = C_t^{-\frac{1}{\sigma}}$ and $\lambda_t = \frac{L_t^{-\frac{1}{\eta}}}{w_t(1-\tau^l_t)}$, then:

$$C_t^{-\frac{1}{\sigma}} = \frac{L_t^{-\frac{1}{\eta}}}{w_t(1-\tau^l_t)} \tag{1.17}$$

from which we can get the relationship between consumption and labor, that we will plug later in our system of equations after having log-linearized it.

Plugging this into Euler equation:

$$\frac{L_t^{-\frac{1}{\eta}}}{w_t(1-\tau^l_t)} = \beta E_t \left[\frac{L_{t+1}^{-\frac{1}{\eta}}}{w_{t+1}(1-\tau^l_{t+1})}\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)\right]$$

$$L_t^{-\frac{1}{\eta}} = \beta E_t \left[L_{t+1}^{-\frac{1}{\eta}}\frac{w_t(1-\tau^l_t)}{w_{t+1}(1-\tau^l_{t+1})}\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)\right] \tag{1.18}$$

and substituting for w_t the equation becomes:

$$L_t^{-\frac{1}{\eta}} = \beta E_t \left[L_{t+1}^{-\frac{1}{\eta}} \frac{(1-\tau_t^l)}{(1-\tau_{t+1}^l)} \frac{\alpha A_t^\alpha \left(\frac{K_t}{L_t}\right)^{1-\alpha}}{\alpha A_{t+1}^\alpha \left(\frac{K_{t+1}}{L_{t+1}}\right)^{1-\alpha}} \left(\frac{R_{t+1}}{\Pi_{t+1}}\right) \right] \tag{1.19}$$

$$L_t^{-\frac{1}{\eta}} = \beta E_t \left[L_{t+1}^{-\frac{1}{\eta}} \frac{(1-\tau_t^l)}{(1-\tau_{t+1}^l)} \frac{A_t^\alpha \left(\frac{K_t}{L_t}\right)^{1-\alpha}}{A_{t+1}^\alpha \left(\frac{K_{t+1}}{L_{t+1}}\right)^{1-\alpha}} \left(\frac{R_{t+1}}{\Pi_{t+1}}\right) \right] \tag{1.20}$$

$$L_t^{-\frac{1}{\eta}} = \beta E_t \left[L_{t+1}^{-\frac{1}{\eta}} \mathrm{T} \frac{A_t^\alpha \left(\frac{K_t}{L_t}\right)^{1-\alpha}}{A_{t+1}^\alpha \left(\frac{K_{t+1}}{L_{t+1}}\right)^{1-\alpha}} \left(\frac{R_{t+1}}{\Pi_{t+1}}\right) \right] \tag{1.21}$$

having defined $\mathrm{T} = \frac{(1-\tau_t^l)}{(1-\tau_{t+1}^l)}$.

1.2 Government

The Government is benevolent and is assumed to maximize welfare, providing all public goods that households need. Its utility function is what Roch and Uhlig (2011) called "felicity function", according to the interpretation that utility represents preferences of the policy maker. The government wants to maximize:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{G_t^{1-\gamma}}{1-\gamma} - \chi_t z \right] \tag{1.22}$$

where β is the discount factor and $u(.)$ is a CRRA strictly increasing, strctly concave and twice differentiable function, whereγ is the coefficient of relative risk aversion into provide public goods for households. G_t stands for public expenditure, since the government provides public goods and services to the households and we want to examine better their idea to model the cost of a default. Changing the interpretation, we talk about ***costs of default***, having constructed a function χ_t to model losses from default, where z is a binary variable that takes values of 0 in case of non-default and 1 in case of default. While in Roch and Uhlig there was a one-time utility cost of default that was depending on past defaults of the state, here we're taking into consideration a series of losses that government starts to accumulate over time, given a bad management of the state. We cannot talk about a one-time utility cost as Roch and Uhlig (2011): it's not correct. We're talking about costs: if a state defaults it will not recover its loss in the next period but it must need more time. In fact if a state is going to default it will not be able to repay at least a fraction of its old debt. In this mechanism the state will accumulate interests that it has to pay on its bonds, and the future ones won't be enough to recover previous losses. We will have a look in the following section. Government maximization problem is:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\frac{G_t^{1-\gamma}}{1-\gamma} - \chi_t z\right]$$
$$s.t. \quad G_t + \frac{R_t}{\Pi_t} B_{t-1} = B_t + T_t(1-\varphi_t) - \chi_t z \tag{1.23}$$

where:

- $T_t = [\tau_t^l w L_t + \tau_t^k (r_t - \delta) K_t]$ is the output that government gets from taxation;
- $\chi_t = s(\frac{R_t}{\Pi_t} B_{t-1} - B_t)$ is the amount of losses that government must minimize if a default scenario takes place;

whit:

- s the fraction of the old bond that government cannot repay if default takes place;
- τ_t^l,$\tau_t^k$are AR1 processes of taxation, as anticipated before;
- $\varphi_t = \rho^\varphi \varphi_{t-1} + \epsilon_t^\varphi$is an AR1 process with $\epsilon_t^\varphi \sim N(0, \sigma^2)$, representing the fraction of households' output that escape from taxation;

In the budget constraint of the government that is expressed in real terms (GBC=Government Budget Constraint) G_t is the level of expenses for providing public goods that are financed both through taxes and also bonds B_t that are offered to households through institutional investors. Government extracts money from households levying taxes T_t on labor (τ_t^l) and capital (τ_t^k) markets and we have introduced a parameter φ_t to take into account the shadow economy: in fact there is a fraction of labor and capital that, escaping from taxation, makes the government worse-off. s is the fraction of the debt on which the government defaults and cannot repay it. Clearly in case of a non default $z = 0 \;\; s = 0$, while if default happens we can set different values of s (i.e. s=0.05, s=0.1..). We will see later that for a default scenario the government won't be able to provide the same level of public goods as in the non-default case.
GBC can be re-written as:

$$G_t + \frac{R_t}{\Pi_t} B_{t-1} = B_t + T_t(1-\varphi_t) - sz(\frac{R_t}{\Pi_t} B_{t-1} - B_t) \tag{1.24}$$

1.2.1 A function for default losses

Our contribution to literature is a default function, or better, a function to measure default losses. Contrary to Roch and Uhlig (2011) we don't think that the cost of default is a one-time utility cost; we think that is better to imagine as cost of default an amount of losses that government accumulate through time: before the crucial time, in which default is declared and after, until all losses are recovered. To make clear our idea that losses are too big when a state experiences a default. To simplify let's think about two cases: the first one, when the government represents a good management of the state and in this case it will not get in a financial turmoil; and the second,

when the government represents a bad management and the state will get in troubles incurring in huge losses, for itself and for its citizens.
The function to measure these default losses is:

$$\chi_t z \tag{1.25}$$

with:

- $\chi_t = s(\frac{R_t}{\Pi_t}B_{t-1} - B_t)$; as measure of the debt level that the government cannot repay even if issuing a new debt
- $z \in [0,1]$; where 0 stands for the non-default in case of a good management, while 1 is the default case if bad management of the state.

Default has an history: it doesn't happen immediately and losses start to accumulate from a certain period until they explode. So we're saying that a government could have losses both before the default date and also after this time.
In fact:

$$\sum_{t=0}^{\infty} \chi_t = \sum_{t=0}^{\infty} s(\frac{R_t}{\Pi_t}B_{t-1} - B_t)$$

where s is the fraction of the debt on which the government default.
Our point is that each state needs to default before accumulate losses: the government should know it and ex-ante it can save itself and its citizens from a disaster. This in part support the idea that Argentina made well in the past; defaulting when necessaire, re-covering losses during time and starting to grow again. In fact we want to show that since losses could be too big it's better to default immediately without "postponing the day of reckoning" (Uhlig 2011). When losses occur they will accumulate over time until a point in which the state needs for an ECB intevention; through monetization, bailouts, or a programme of the ECB to buy each state's bond. If a kind of intervention occurs the state will cover all the losses and debts, starting a new cycle of its economy. The point in fact, as sustained by many authors, is that government knows that is going to default, but since it's expecting ECB intervention it won't default accumulating losses and becoming unsolvent. In this way in every period the state need for a transfer in order to cover the fraction of its debt that it cannot repay. We support the idea that reforms are needed in order for a state to feel more safe without fear of contagion (that we're not delve into more depth here).

Using Matlab we have a constructed a function, that represents how these government' s losses could be when the state experience a default. This is our contribution to the topic of default. Our idea is so innovative because no one examined this aspect before: in the computational part we represent losses as unpaid fractions of old debt, even if mathematically we can make a parametrization of this function, that help us to get an idea of the waste of resources in our economy when default takes place. Future works are needed in order to explore this details and all the consequences that follows from all possible interventions. For the moment we can construct an idea of how these losses could be and how it's possible for a government to accumulate debts out of debts, being unable to guarantee a fraction of the old ones.
The function to measure these default losses is:

$$\chi_t z \tag{1.26}$$

with χ_t that takes different values according to different intervals of time as follows:

- $\chi_t = ln(t-k+1)$ for $t \in (k-1, t_1)$, increasing and concave in t
- $\chi_t = -c(t-t_1)^2 + \overline{\chi}$ for $t \in (t_1, t_3)$, decreasing and convex in t

and $z \in [0,1]$; where 0 stands for the non-default in case of a good management, while 1 is the default case if bad management of the state.
The parameter c indicates how much faster the function decreases, hence how many time it takes for the state to recover all the debt losses in case of a bad performance from the government: for small values of c it decreases slowly, that means the state needs more time to recover losses; while for big values of c losses are covered more rapidly. (See Appendix 5.2 for computations and properties of the function).
Default has an history: it doesn't happen immediately and losses start to accumulate from a certain period k and in a certain moment $t_1$will explode. So we're saying that a government could have losses both before the default date and also after this time.

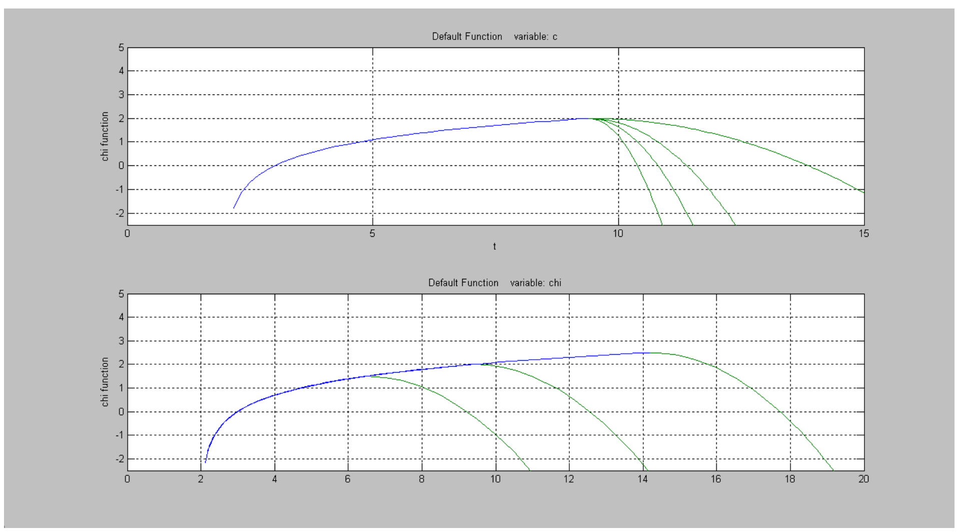


Figure 1:

As we can see in Figure 1 consequences of a default could be too big! The government could faces a waste of resources that will affect all the economy.
In the first graph we are assuming that there is an ECB intervention as monetization: in this case the government will cover all the unpaid fraction of old bonds through a transfer that it receives. Clearly, as pointed out by many authors as Beetsma and Uhlig (1999), Greiner et al (2007), Ang and Longstaff (2011) this kind of intervention creates a distortion as an inflationary burden on all the other countries (governments) since we're talking about a monetary union but our work is not

interested on the inflationary consequences, as in the overlapping generation model proposed by Cole and Kehoe(2000).
In the second graph we were supposing no intervention from the ECB and this implies that the government must act immediately through reforms (as was the case for Italy and other countries as Greece as well) trying to re-cover all the losses with its own resources. In other words, the government cannot repay its debts using a transfer; this will force the government to acknowledge its limits and act in order to reach a break-even point in its balance-sheet. Also we can see that in this case government could notice earlier that is going to default and it will take remedies before losses become too big. This could have no distortion on the other countries except for the fact that in a Stability Growth Pact the states "can punish each other if somebody raises too much debt" (Beetsma, Uhlig 1999), but as pointed out the Stability Growth Pact is not really effective.
Making a comparison between the two graph it's possible to understand two things: the first one is that with ECB intervention the government has no incentive to reduce its debt level since it's waiting for a monetization that surely occurs. In this case losses are rapidly absorbed and our function is able to capture this realization. The second thing to notice is that without monetization the government has more incentive to recover losses before they become too big but for sure this process will take more time since the state needs reforms that takes place as earliest as possible.

1.2.2 Inter-temporal maximization problem for the government

The intertemporal maximization problem of the government can be written as:

$$\begin{aligned}\mathcal{L} =&E_t\Bigg\{\sum_{t=0}^{\infty}\beta^t\left(\frac{G_t^{1-\gamma}}{1-\gamma}-s(\frac{R_t}{\Pi_t}B_{t-1}-B_t)z\right)+\sum_{t=0}^{\infty}\beta^{t+1}\left(\frac{G_{t+1}^{1-\gamma}}{1-\gamma}-s(\frac{R_{t+1}}{\Pi_{t+1}}B_t-B_{t+1})z\right)+\\ &+\sum_{t=0}^{\infty}\beta^t\lambda_t[B_t+(1-\varphi_t)T_t-sz(\frac{R_t}{\Pi_t}B_{t-1}-B_t)-G_t-\frac{R_t}{\Pi_t}B_{t-1}]+\\ &+\sum_{t=0}^{\infty}\beta^{t+1}\lambda_{t+1}[B_{t+1}-sz(\frac{R_{t+1}}{\Pi_{t+1}}B_t-B_{t+1})-G_{t+1}-\frac{R_{t+1}}{\Pi_{t+1}}B_t+\\ &+(1-\varphi_{t+1})T_{t+1}]\Bigg\}\end{aligned} \tag{1.27}$$

Now we can solve it deriving F.O.C.s;
for public expenditure:

$$\tfrac{\partial\mathcal{L}}{\partial G_t}=E_t\left(\beta^tG_t^{-\gamma}-\beta^t\lambda_t\right)=0$$

$$\lambda_t=G_t^{-\gamma} \tag{1.28}$$

for bonds:

$$\tfrac{\partial\mathcal{L}}{\partial B_t}=E_t\left[\beta^tsz-\beta^{t+1}\tfrac{R_{t+1}}{\Pi_{t+1}}sz+\beta^t\lambda_t+\beta^t\lambda_tsz-\beta^{t+1}\lambda_{t+1}\tfrac{R_{t+1}}{\Pi_{t+1}}sz-\beta^{t+1}\lambda_{t+1}\tfrac{R_{t+1}}{\Pi_{t+1}}\right]=0$$

$$\beta^t\lambda_t\left(1+sz\right)=E_t\left[\beta^{t+1}\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}}\left(1+sz\right)\right]-\beta^tsz+E_t\left(\beta^{t+1}\frac{R_{t+1}}{\Pi_{t+1}}sz\right)$$

$$\lambda_t\left(1+sz\right)=\beta E_t\left[\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}}\left(1+sz\right)\right]-sz+\beta E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}sz\right)$$

$$\lambda_t=\beta E_t\left[\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}}\right]-\frac{sz}{(1+sz)}+\frac{\beta sz}{(1+sz)}E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)$$

$$\lambda_t=\beta E_t\left[\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}}\right]-\frac{sz}{(1+sz)}\left(1-\beta E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)\right)$$

$$\lambda_t=\beta E_t\left[\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}}\right]+\frac{sz}{(1+sz)}\left(\beta E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)-1\right) \tag{1.29}$$

where we can make a distinction between the two cases:

- for $z=0\Rightarrow\lambda_t=\beta E_t(\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}})$, as was the Euler for households;
- for $z=1\Rightarrow\lambda_t=\beta E_t\left[\lambda_{t+1}\frac{R_{t+1}}{\Pi_{t+1}}\right]+\frac{s}{(1+s)}\left(\beta E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)-1\right)$, considering losses from default.

We have to remember that the government faces the same gross interest rate of households, that is:

$$R_{t+1}=[r_{t+1}(1-\tau^k_{t+1})+(1-\delta)]$$

Plugging eqn. (1.28) into eqn. (1.29) we will get:

$$G_t^{-\gamma}=\beta E_t\left[G_{t+1}^{-\gamma}\frac{R_{t+1}}{\Pi_{t+1}}\right]+\frac{sz}{(1+sz)}\left(\beta E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)-1\right) \tag{1.30}$$

$$G_t^{-\gamma}=\beta E_t\left[G_{t+1}^{-\gamma}\frac{R_{t+1}}{\Pi_{t+1}}\right]+\frac{sz}{(1+sz)}\left(\beta E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)-1\right) \tag{1.31}$$

that is the Euler equation for the government; according to which it provides public goods to households.

Notice that it can be re-written as:

$$G_t^{-\gamma}=\beta E_t\left[G_{t+1}^{-\gamma}\frac{R_{t+1}}{\Pi_{t+1}}\right]+\frac{sz}{(1+sz)}\beta E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right)-\frac{sz}{(1+sz)} \tag{1.32}$$

Eqn. (1.32) is the Euler equation, that shows us how the government shifts its level of public expenditure between different periods, given the possibility of a default. At the steady state it could be the case that, if default takes place, the level of public expenditure in the future will decrease since the previous level of expenditure cannot be substained at all. The government in fact won't be able to repay a fraction of its old debt.

A lot of author as Cole and Kehoe (1996), Roch and Uhlig (2011) at the light of the recent crisis talked about inflation equilibria, default equilibria, equilibria in the crisis zone.

Our contribution is focused on losses that comes from a default scenario. We will see what happens when default occurs, trying to model different situations in order to approach the behavior of the real world economy.

2 What happens in the economy at the steady state

To describe what happens at the steady state, before detrending and log-linearizing all the equations, we first define an equilibrium concept.

The equilibrium of our economy is an equilibrium in which:

· households maximize their expected utilities shifting consumption and investments from one period to the following one, investing in states' bonds in order to finance their private expenditure.

· government is benevolent and maximize the level of public expenditure in order to provide services and goods to the households; but at the same time government is subject to a default possibility in case of a bad management of the state, and if it's not able to repay the fraction of the old debts this will have strong consequences on future provision of public goods (we're not considering consequences as the impossibility to borrow in the future as other models did).

2.1 Detrending the model

We're assuming as in Campbell (1994) that capital, consumption, output and technology all grow at a constant common rate. So there is a trend that we describe as:

$$\begin{aligned} G^r &= \frac{G^r_{t+1}}{G^r_t} \\ &\quad (1+g^r_t) \end{aligned} \tag{2.1}$$

where g^r is the net growth level. This trend is assumed to be present also in the public expenditure that government substain in order to provide public goods to households. Now we proceed detrending the model in order both to remove the trend's effect and also to show absolute changes in values, to allow potential cyclical patterns to be identified. Most of the computational part is omitted here (for a deep analysis see the Appendix).
Using the resource constraint of households we can rearrange all the equation to:

$$G^r k_{t+1} = (1-\delta)k_t + y_t - \frac{R_t}{\Pi_t} b_{t-1} + b_t - c_t \tag{2.2}$$

And using the fact that:

· $Y_t = A_t L_t^{\alpha} K_t^{1-\alpha} = a_t l_t^{\alpha} k_t^{1-\alpha}$,

· $a\left(\frac{l}{k}\right) = A\left(\frac{L}{K}\right)^{\alpha} \equiv \left(\frac{r+\delta}{1-\alpha}\right)$

we can simplify eqn.(2.2) to:

$$(g^r+\delta)\frac{k}{y} = 1+\frac{b}{y}(1-\frac{R_t}{\Pi_t})-\frac{c}{y} \tag{2.3}$$

$$\frac{c}{y} = 1-(g^r+\delta)\frac{1-\alpha}{r+\delta}+\frac{b}{y}(1-\frac{R_t}{\Pi_t})$$

$$\frac{c}{y} = 1-\frac{(1-\alpha)(g^r+\delta)}{(r+\delta)}+\frac{b}{y}(1-\frac{R_t}{\Pi_t}) \tag{2.4}$$

Plugging the values of parameters (that we will explain later) inside we can re-write eqn.(2.4) as:

$$\frac{c}{y} = 1-\frac{(0.667)(0.041)}{(0.076)}+\frac{b}{y}(1-\frac{1.04}{1.004})$$

and substituting for the bond to GDP ratio $\frac{b}{y}=0.9$, using values obtained through Datastream 2011, we get:

$$\frac{c}{y} = 1-0.36-0.036(0.9) = 0.60$$

where, according RBC (real business cycle) literature, households' consumption is the 60% of their GDP.

The detrended Euler Equation is:

$$c_t = \beta E_t \left\{ c_{t+1} G^{(-\frac{1}{\sigma}),r} \frac{\left[(1-\alpha)a_{t+1}\left(\frac{l_{t+1}}{k_{t+1}}\right)^{\alpha}+(1-\delta)\right]}{\Pi_{t+1}} \right\} \tag{2.5}$$

Using the detrended equations for labor and consumption, getting rid of the inflation, and using the fact that at steady state:

- $c_t = c_{t+1}$
- $R_{t+1} = \left[(1-\alpha)a_{t+1}\left(\frac{l_{t+1}}{k_{t+1}}\right)^{\alpha}+(1-\delta)\right] = 1+r_{t+1}$

it follows that:

$$\beta^{\star} = \frac{1}{G^{(-\frac{1}{\sigma}),r}R} \tag{2.6}$$

Detrending the equation of labor, that we get by comparing F.O.C.s the result is the following:

$$l_t^{-\frac{1}{\eta}} = \beta E_t \left\{ l_{t+1}^{-\frac{1}{\eta}} G_t^{-\frac{1}{\eta},r} T \frac{a_t\left(\frac{k_t}{l_t}\right)^{1-\alpha}}{a_{t+1}\left(\frac{k_{t+1}}{l_{t+1}}\right)^{1-\alpha}} \frac{R_{t+1}}{\Pi_{t+1}} \right\} \tag{2.7}$$

where $T = \frac{(1-\tau_t^l)}{(1-\tau_{t+1}^l)}$.

Since at the steady state:

- $l_t^{-\frac{1}{\eta}} = l_{t+1}^{-\frac{1}{\eta}}$,
- $\tau_t^l = \tau_{t+1}^l \Rightarrow 1-\tau_t^l = 1-\tau_{t+1}^l \Rightarrow T=1$,
- $a_t = a_{t+1}$,

- $l_t^{\alpha-1} = l_{t+1}^{\alpha-1}$,
- $k_t^{1-\alpha} = k_{t+1}^{1-\alpha}$,
- $\left[(1-\alpha)a_{t+1}\left(\frac{l_{t+1}}{k_{t+1}}\right)^{\alpha} + (1-\delta)\right] = R = 1+r$

then the detrended equation for labor can be re-written in the following terms:

$$\beta E_t \left\{ G_t^{-\frac{1}{\eta},r} \left[(1-\alpha)\left(\frac{\hat{l}_{t+1}}{k_{t+1}}\right)^{\alpha} + (1-\delta) \right] \right\} = 1 \tag{2.8}$$

so that:

$$\beta E_t \left\{ G_t^{-\frac{1}{\eta},r} R \right\} = 1$$

$$\beta^{\circ} = \frac{1}{G_t^{-\frac{1}{\eta},r} R} \tag{2.9}$$

As pointed above, later all the parameters will be examined one by one.

Our y is the average of GDP in real terms weighted for the number of civilians, in order to get GDP pro-capita.

The ratio $\frac{b}{y}$ is obtained through Datastream (2011), taking the public debt that is our B and dividing it by GDP in real terms in order to approach a real economy setting. Approximately this ratios for European countries is on average close to 90%.

From the resource constraint of the government we get:

$$g_t + \frac{R_t}{\Pi_t} b_{t-1} = \frac{b_t}{p_t} + t_t(1-\varphi_t) - sz\left(\frac{R_t}{\Pi_t} b_{t-1} - b_t\right) \tag{2.10}$$

$$\frac{g}{y} = (1+sz)\frac{b}{y} - (1+sz)\frac{Rb}{\Pi y} + \frac{t(1-\varphi_t)}{y}$$

$$\frac{g}{y} = \frac{t(1-\varphi_t)}{y} + (1+sz)\frac{b}{py} - (1+sz)\frac{(1+r)b}{\Pi y}$$

$$\frac{g}{y} = \frac{t(1-\varphi_t)}{y} + (1+sz)\frac{b}{y}\left(1 - \frac{R}{\Pi}\right)$$

$$\frac{g}{y} = (1-\varphi_t)\frac{t}{y} + (1+sz)\frac{b}{py}\left(1 - \frac{R}{\Pi}\right)$$

then plugging the value of bond to GDP ratio and substituting for taxation to GDP ratio we get the final expression for government public expenditure (g) to income ratio; as expenses to provide public goods, burocracy, transfers and subsidies of income:

$$\frac{g}{y} = (1-\varphi_t)(0.45) - (1+sz)0.9\,(0.036)$$

$$\frac{g}{y} = (1-\varphi_t)(0.45) - 0,032(1+sz)$$

$$\frac{g}{y} = \frac{t}{y}(1-\varphi) - 0.032(1+zs) \tag{2.11}$$

so that for z=0:

$$\frac{g}{y} = (0.85)0.45 - 0.032(1 + sz) \approx 0.36$$

and for z=0 and no tax evasion (i.e. $\varphi = 0$):

$$\frac{g}{y} = 0.45 - 0.032(1 + sz) \approx 0.42$$

while for z=1, s=0.05:

$$\frac{g}{y} = (0.85)0.45 - 0.032(1 + 0.05) \approx 0.35$$

while for z=1, s=0.05 and no tax evasion (i.e. $\varphi = 0$):

$$\frac{g}{y} = 0.45 - 0.032(1 + 0.05) \approx 0.41$$

Both if there is a default or not, the level of public expenditure will be:

$$\frac{g}{y} = 0.382 - 0.032(1 + sz)$$

.

At the steady state this fits pretty well to a real world economy. The only thing to notice is that the level of public expenditure is lower than that of RBC (real business cycle) literature, just because we're considering tax evasion as percentage of taxes that are unpaid by households, both on capital and labor market.

According to literature, for a case of no evasion the public expenditure to GDP ratio is close to 40%, precisly 42%. The detrended Euler equation of the government is:

$$g_t = \beta E_t \left[g_{t+1} G^{-\gamma,r} \frac{R_{t+1}}{\Pi_{t+1}} \right] - \frac{sz}{(1+sz)} + \beta \frac{sz}{(1+sz)} E_t \left(\frac{R_{t+1}}{\Pi_{t+1}} \right) \tag{2.12}$$

or:

$$g_t = \beta E_t \left[\frac{R_{t+1}}{\Pi_{t+1}} \left(g_{t+1} G^{-\gamma,r} + \frac{sz}{(1+sz)} \right) \right] - \frac{sz}{(1+sz)} \tag{2.13}$$

Given that at the steady state $g_t = g_{t+1}$:

$$\beta R \left(G^{-\gamma,r} + \frac{sz}{g^{-\gamma}(1+sz)} \right) = 1 + \frac{sz}{g^{-\gamma}(1+sz)}$$

$$\beta^{\triangleright} = \frac{1 + \frac{sz}{g^{-\gamma}(1+sz)}}{R^{ss} \left(G^{-\gamma,r} + \frac{sz}{g^{-\gamma}(1+sz)} \right)}$$

where we can notice that, if a default hit government provision of public goods, the intertemporal discount factor will change.

As said before, we obtained $\frac{g}{y}$ as net public expenditure calculated with interests, while for $\frac{b}{y}$ we use values taken from Datastream 2011 as discussed before. The ratio of bond to GDP is B/Y=0.9, close to the real situation faced by some countries before the Greek crisis took place.

2.2 Log-linearizations

Now we proceed log-linearizing the model around the steady state, in order to check what happens around the equilibrium. We refer to the equilibrium defined before to see what happens in a context in which households and a benevolent government want to maximize their economic activities while the government can default on its debt or not. We are going to describe both situation: when default takes place and when not.

2.2.1 Households'equations

In order to be clear we specify each equation for each variable of our interest, after having log-linearized it:

· Production Function:

$$\widetilde{y}_{t+1} = \widetilde{a}_{t+1} + \alpha\widetilde{l}_{t+1} + (1-\alpha)\widetilde{k}_{t+1} \tag{2.14}$$

· Wage:

$$\widetilde{w}_{t+1} = \widetilde{a}_{t+1} + (\alpha-1)\,\widetilde{l}_{t+1} + (1-\alpha)\widetilde{k}_{t+1} \tag{2.15}$$

· Gross Interest Rate:

$$\widetilde{R}_{t+1} = \lambda_5\widetilde{Q}_{t+1} + \lambda_6\widetilde{\Pi}_{t+1} \tag{2.16}$$

· Marginal Rate of Return to Capital:

$$\widetilde{Q}_{t+1} = \widetilde{a}_{t+1} + \alpha\widetilde{l}_{t+1} - \alpha\widetilde{k}_{t+1} \tag{2.17}$$

· Capital Accumulation Constraint:

$$\widetilde{k}_{t+1} = \lambda_7\widetilde{k}_t + \lambda_8\widetilde{I}_t \tag{2.18}$$

· Households budget constraint (we explicitate for investments):

$$\begin{aligned}\widetilde{I}_{t+1} = {} & \lambda_9\left(\widetilde{R}_{t+1} + \widetilde{b}_t - \widetilde{\Pi}_{t+1}\right) + \lambda_{10}\left(\widetilde{w}_{t+1} + \widetilde{l}_{t+1}\right) - \lambda_{11}\widetilde{\tau}^l_{t+1} + \lambda_{12}\left(\widetilde{r}_{t+1} + \widetilde{k}_{t+1}\right) + \\ & - \lambda_{13}\widetilde{\tau}^k_{t+1} - \lambda_{14}\widetilde{c}_{t+1} - \lambda_{15}\widetilde{b}_{t+1}\end{aligned} \tag{2.19}$$

· Inflation:

$$\widetilde{\Pi}_{t+1} = \frac{1}{\beta}\left(\widetilde{\Pi}_t - k_y\widetilde{Y}_t + \eta^{\Pi}_{t+1}\right) \tag{2.20}$$

with η^{Π}_{t+1} that is the endogenous error term that follows from the expectational term present in the original equation;

· Euler equation:

$$\widetilde{c}_{t+1} = \widetilde{c}_t + \sigma\widetilde{R}_{t+1} - \sigma\widetilde{\Pi}_{t+1} + \eta^c_{t+1} \tag{2.21}$$

with η^c_{t+1} is the endogenous error that follows from expectation as before;

· Labor:

$$\widetilde{l}_{t+1} = \lambda_3 \widetilde{c}_{t+1} - \eta \widetilde{w}_{t+1} + \lambda_4 \widetilde{\tau}^l_{t+1} \tag{2.22}$$

The coefficients are:

- $\lambda_3 = \frac{\eta}{\sigma}$
- $\lambda_4 = \frac{\eta \tau^l}{(1-\tau^l)}$
- $\lambda_5 = \frac{Q}{R}$
- $\lambda_6 = \frac{\Pi}{R}$
- $\lambda_7 = \frac{(1-\delta)}{(1+g^r)}$
- $\lambda_8 = \frac{I}{K(1+g^r)}$
- $\lambda_9 = \frac{RB}{\Pi I}$
- $\lambda_{10} = \frac{(1-\tau^l)wL}{I}$
- $\lambda_{11} = \frac{\tau^l wL}{I}$;
- $\lambda_{!2} = \frac{r(1-\tau^k)K}{I}$
- $\lambda_{!3} = \frac{r_t \tau^k K}{I}$
- $\lambda_{14} = \frac{C}{I}$
- $\lambda_{15} = \frac{B}{I}$

We use data taken from World Bank (2011), to approximate the bond to capital ratio, the public expenditure to capital ratio and private expenditure to capital ratio for the households. To obtain the capital to income ratio we refer to r as in Lucke, Wurzel (2011). While we use a $g^r = 0.005$ as in Campbell (1994). The discount factor clearly depends exogenously by the gross interest rate.

2.2.2 Government's equations

- Government Resource Constraint:

$$\widetilde{b}_{t+1} = \lambda_{19} \left(\widetilde{R}_{t+1} + \widetilde{b}_t - \widetilde{\Pi}_{t+1} \right) \lambda_{16} \widetilde{g}_{t+1} \lambda_{17} \widetilde{t}_{t+1} + \lambda_{18} \widetilde{\varphi}_{t+1} \tag{2.23}$$

- Equation for the taxation level:

$$\widetilde{t}_{t+1} = \lambda_1 \left(\widetilde{\tau}^l_{t+1} + \widetilde{w}_{t+1} + \widetilde{l}_{t+1} \right) + \lambda_2 \left(\widetilde{\tau}^k_{t+1} + \widetilde{k}_{t+1} \right) \tag{2.24}$$

- Euler Equation:

$$\widetilde{g}_{t+1} = \widetilde{g}_t + \lambda_{20} \left(\widetilde{R}_{t+1} - \widetilde{\Pi}_{t+1} \right) + \eta^g_{t+1} \tag{2.25}$$

where η^g_{t+1} is the endogenous error term (that follows from the expectational term) in the government Euler equation (to provide public goods).

The coefficients are:

- $\lambda_1 = \frac{\tau^l wL}{T}$
- $\lambda_2 = \frac{\tau^k (r-\delta)K}{T}$
- $\lambda_{16} = \frac{G}{B(1+sz)}$
- $\lambda_{17} = \frac{T(1-\varphi)}{B(1+sz)}$
- $\lambda_{18} = \frac{\varphi T}{B(1+sz)}$
- $\lambda_{19} = \frac{R}{\Pi}$
- $\lambda_{20} = \frac{\beta R}{\gamma \Pi}\left(G^{-\gamma,r} + \frac{sz}{(1+sz)g^{-\gamma}}\right)$

All the AR1 processes are rewritten as:

- Technology:
$$\widetilde{A}_{t+1} = \rho^a \widetilde{A}_t + \varepsilon^a_{t+1} \tag{2.26}$$
- Taxation on Labor:
$$\widetilde{\tau}^l_{t+1} = \rho^l \widetilde{\tau}^l_t + \varepsilon^l_{t+1} \tag{2.27}$$
- Taxation on Capital:
$$\widetilde{\tau}^k_{t+1} = \rho^k \widetilde{\tau}^k_t + \varepsilon^k_{t+1} \tag{2.28}$$
- Tax Evasion:
$$\widetilde{\varphi}_{t+1} = \rho^\varphi \widetilde{\varphi}_t + \varepsilon^\varphi_{t+1} \tag{2.29}$$

In order to solve the model through the gensys procedure in Matlab we will use the following set of log-linearized equation, with the following order:

1. $\widetilde{y}_{t+1} = \widetilde{A}_{t+1} + \alpha \widetilde{l}_{t+1} + (1-\alpha)\widetilde{k}_{t+1}$
2. $\widetilde{t}_{t+1} = \lambda_1 \left(\widetilde{\tau}^l_{t+1} + \widetilde{w}_{t+1} + \widetilde{l}_{t+1}\right) + \lambda_2 \left(\widetilde{\tau}^k_{t+1} + \widetilde{k}_{t+1}\right)$
3. $\widetilde{w}_{t+1} = \widetilde{A}_{t+1} + (\alpha - 1)\,\widetilde{l}_{t+1} + (1-\alpha)\widetilde{k}_{t+1}$
4. $\widetilde{l}_{t+1} = \lambda_3 \widetilde{c}_{t+1} - \eta \widetilde{w}_{t+1} + \lambda_4 \widetilde{\tau}^l_{t+1}$
5. $\widetilde{Q}_{t+1} = \widetilde{A}_{t+1} + \alpha \widetilde{l}_{t+1} - \alpha \widetilde{k}_{t+1}$
6. $\widetilde{R}_{t+1} = \lambda_5 \widetilde{Q}_{t+1} + \lambda_6 \widetilde{\Pi}_{t+1}$
7. $\widetilde{A}_{t+1} = \rho^a \widetilde{A}_t + \varepsilon^a_{t+1}$
8. $\widetilde{\tau}^l_{t+1} = \rho^l \widetilde{\tau}^l_t + \varepsilon^l_{t+1}$
9. $\widetilde{\tau}^k_{t+1} = \rho^k \widetilde{\tau}^k_t + \varepsilon^k_{t+1}$
10. $\widetilde{\varphi}_{t+1} = \rho^\varphi \widetilde{\varphi}_t + \varepsilon^\varphi_{t+1}$
11. $\widetilde{k}_{t+1} = \lambda_7 \widetilde{k}_t + \lambda_8 \widetilde{I}_t$

12. $\widetilde{I}_{t+1} = \lambda_9 \left(\widetilde{R}_{t+1} + \widetilde{b}_t - \widetilde{\Pi}_{t+1}\right) + \lambda_{10} \left(\widetilde{w}_{t+1} + \widetilde{l}_{t+1}\right) - \lambda_{11} \widetilde{\tau}^l_{t+1} + \lambda_{12} \left(\widetilde{r}_{t+1} + \widetilde{k}_{t+1}\right) - \lambda_{13} \widetilde{\tau}^k_{t+1} - \lambda_{14} \widetilde{c}_{t+1} - \lambda_{15} \widetilde{b}_{t+1}$

13. $\widetilde{b}_{t+1} = \lambda_{19} \left(\widetilde{R}_{t+1} + \widetilde{b}_t - \widetilde{\Pi}_{t+1}\right) \lambda_{16} \widetilde{g}_{t+1} \lambda_{17} \widetilde{t}_{t+1} + \lambda_{18} \widetilde{\varphi}_{t+1}$

14. $\beta \widetilde{\Pi}_{t+1} = \widetilde{\Pi}_t - k_y \widetilde{Y}_t + \eta^{\Pi}_{t+1}$

15. $\widetilde{c}_{t+1} = \widetilde{c}_t + \sigma \widetilde{R}_{t+1} - \sigma \widetilde{\Pi}_{t+1} + \eta^c_{t+1}$

16. $\widetilde{g}_{t+1} = \widetilde{g}_t + \lambda_{20} \left(\widetilde{R}_{t+1} - \widetilde{\Pi}_{t+1}\right) + \eta^g_{t+1}$

We specify again all the coefficients:

· $\lambda_1 = \frac{\tau^l wL}{T}$

· $\lambda_2 = \frac{\tau^k (r-\delta) K}{T}$

· $\lambda_3 = \frac{\eta}{\sigma}$

· $\lambda_4 = \frac{\eta \tau^l}{(1-\tau^l)}$

· $\lambda_5 = \frac{Q}{R}$

· $\lambda_6 = \frac{\Pi}{R}$

· $\lambda_7 = \frac{(1-\delta)}{(1+g^r)}$

· $\lambda_8 = \frac{I}{K(1+g^r)}$

· $\lambda_9 = \frac{RB}{\Pi I}$

· $\lambda_{10} = \frac{(1-\tau^l) wL}{I}$

· $\lambda_{11} = \frac{\tau^l wL}{I}$;

· $\lambda_{!2} = \frac{r(1-\tau^k)K}{I}$

· $\lambda_{!3} = \frac{r_t \tau^k K}{I}$

· $\lambda_{14} = \frac{C}{I}$

· $\lambda_{15} = \frac{B}{I}$

· $\lambda_{16} = \frac{G}{B(1+sz)}$

· $\lambda_{17} = \frac{T(1-\varphi)}{B(1+sz)}$

· $\lambda_{18} = \frac{\varphi T}{B(1+sz)}$

· $\lambda_{19} = \frac{R}{\Pi}$

· $\lambda_{20} = \frac{\beta R}{\gamma \Pi} \left(G^{-\gamma, r} + \frac{sz}{(1+sz) g^{-\gamma}}\right)$

Now we clarify all the parameters, that are defined as follows:

· $\gamma = 0.333$ as in Walsh Monetary Policy and Theory (2003)

- $\sigma = 0.5$ as in Campbell (1994)
- $\eta = 3.56$ as in Chori, Kehoe, McGrattan (1999)
- $\alpha = 0.333$ labor share ratio
- $1 - \alpha = 0.667$ capital share ratio
- $R = 1.04 \Rightarrow r = 0.04$ as in Lucke and Wurzel (2011)
- $\frac{c}{y} = 0.604$ as close to RBC literature (where it is 0.60)
- $\frac{b}{y} = 0.9$ according to data observed in Datastream (2011)
- $\delta = 0.036$
- $\frac{I}{Y} = 0.23$
- $\frac{I}{K} = 0.15$
- $\frac{G}{Y} = 0.38$ as in the RBC literature, except for the fact that we consider tax evasion
- $\frac{T}{Y} = 0.42$ close to RBC literature
- $k_y = 0.3$
- $\tau^l = 0.3$ at steady state
- $\tau^k = 0.2$ at the steady state
- β is endogenously determined and its value depends on values that the interest rate takes
- $\rho^k = 0.95$
- $\rho^l = 0.95$
- $\rho^a = 0.95$
- $\rho^\varphi = 0.90$
- $Var(\varepsilon^a) = 0.005$, according to Campbell (1994)
- $Var(\varepsilon^k) = 0.0001$
- $Var(\varepsilon^l) = 0.0005$
- $Var(\varepsilon^\varphi) = 0.0003$

2.3 System in Matrix notation

$$\begin{bmatrix}
1 & 0 & 0 & -\alpha & 0 & 0 & -1 & 0 & 0 & 0 & (\alpha-1) & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -\lambda_1 & -\lambda_1 & 0 & 0 & 0 & -\lambda_1 & -\lambda_2 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & (1-\alpha) & 0 & 0 & -1 & 0 & 0 & 0 & (\alpha-1) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \eta & 1 & 0 & 0 & 0 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_3 & 0 \\
0 & 0 & 0 & -\alpha & 1 & 0 & -1 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\lambda_5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\lambda_{10} & -\lambda_{10} & 0 & -\lambda_9 & 0 & \lambda_{11} & \lambda_{13} & 0 & -\lambda_{12} & 1 & \lambda_{15} & \lambda_9 & \lambda_{14} & 0 \\
0 & \lambda_{17} & 0 & 0 & 0 & -\lambda_{19} & 0 & 0 & 0 & -\lambda_{18} & 0 & 0 & 1 & \lambda_{19} & 0 & -\lambda_{16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\sigma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -\lambda_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{20} & 0 & 1
\end{bmatrix}
\begin{bmatrix} Y_{t+1} \\ T_{t+1} \\ W_{t+1} \\ L_{t+1} \\ Q_{t+1} \\ R_{t+1} \\ A_{t+1} \\ \tau^l_{t+1} \\ \tau^k_{t+1} \\ \varphi_{t+1} \\ K_{t+1} \\ I_{t+1} \\ B_{t+1} \\ \Pi_{t+1} \\ C_{t+1} \\ G_{t+1} \end{bmatrix} =$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho^a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^k & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^\varphi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_7 & \lambda_8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{19} & 0 & 0 & 0 \\
\frac{k_y}{\beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} Y_t \\ T_t \\ W_t \\ L_t \\ Q_t \\ R_t \\ A_t \\ \tau^l_t \\ \tau^k_t \\ \varphi_t \\ K_t \\ I_t \\ B_t \\ \Pi_t \\ C_t \\ G_t \end{bmatrix} +$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \varepsilon^a_{t+1} & \varepsilon^l_{t+1} & \varepsilon^k_{t+1} & \varepsilon^\varphi_{t+1} \end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{\beta} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \eta^\Pi_{t+1} & \eta^c_{t+1} & \eta^g_{t+1} \end{bmatrix}$$

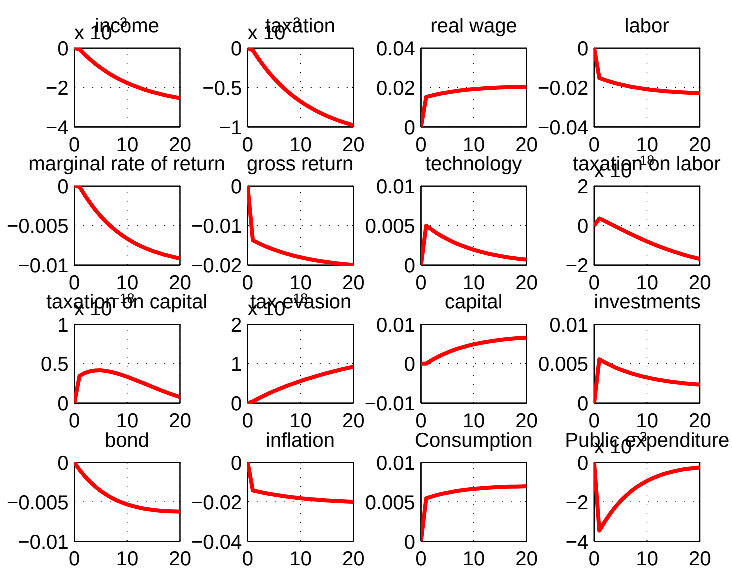
3 Results

After the gensys procedure on Matlab, now we can make some comments about our model; we're now able to describe households' and government's behaviour when default happens and when not. In our steady state we're expecting that the government will mantain the same level of public expenditure, just smoothing it between periods. While in a default scenario all levels of public expenditure and bonds will decrease since the government won't be able to repay a fraction of its old debt. For what concerns the fraction of the old bond that the government is no more able to repay, for z=1 we can see what happens using s=0.05 or 0.1, thinking that it's a huge loss if a government is not able to repay 10% of its old debt. As anticipated this losses becomes really big if there are no government interventions, since the economy needs more money in its attempt to mantain high levels of cosumption in all the economic activities. A regime in which a country continue to borrow more and more in order to repay debts is no longer sustainable. Our point is that Europe needs reforms. Europe needs also for a fight against tax evasion that reaches higher percentages in countries as the Est-Europeans, Greece, Italy but it's still present in Germany and France as well. Some reforms have to be made by european authorities while other has to be made within each country, with the help of each government. If a state experience a default it will take time to recover all the losses.

3.1 Impulse Response Functions

We are going to comment how our impulse response functions look like, checking for exogenous shocks.

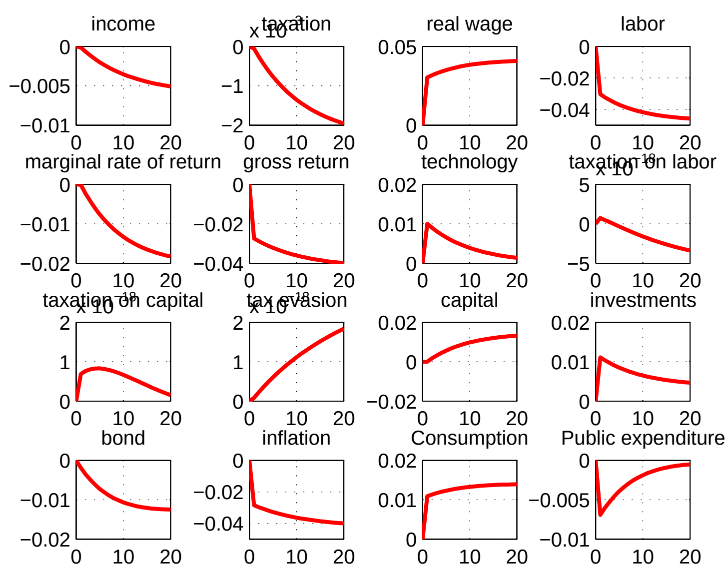
Technology shock $\rho^a = 0.005$, $z = 0 \Rightarrow no\,default$



We have introduce a positive technology shock of the same magnitude of $Var(\varepsilon^a)$. At the time the shock is introduced it has effect on output through the production function. Indeed it raises the

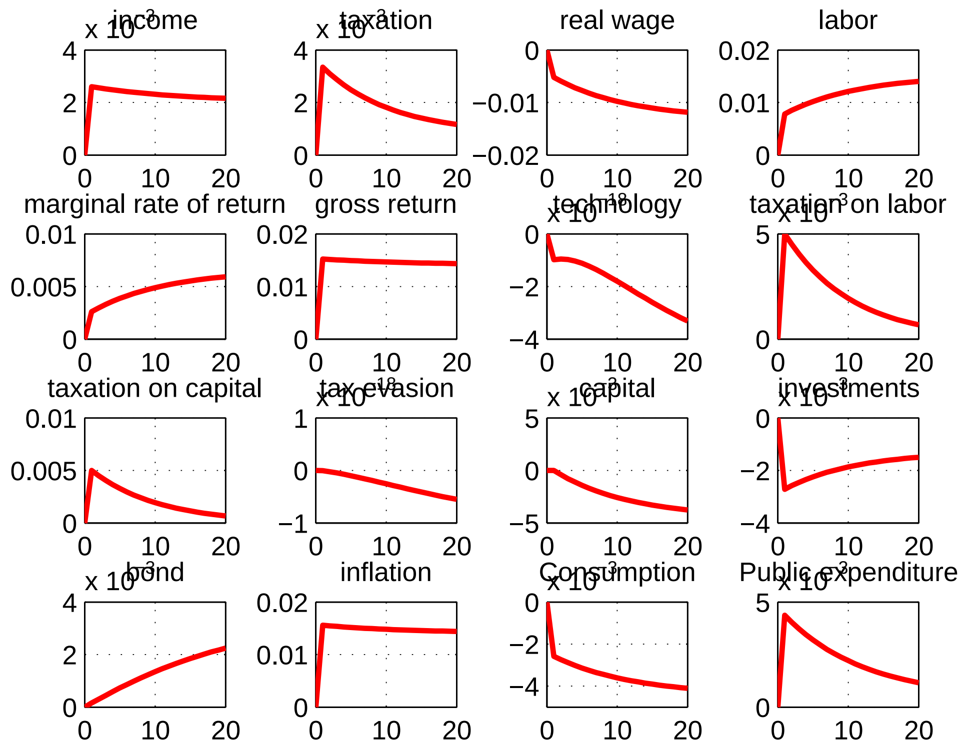
marginal productivity of labor, leading firms to be able to get the same output with a smaller amount of labor input. The representative household feels itself more richer because of wage increasing and it will supply a lower level of labor. This effect can be noticed in the plots, where labor is subject to a sharp downfall as the shock is introduced. The effect of the shock on inflation is persistent given that it takes a lot of time to came back at the initial level; as can be seen inflation is pushed down. The behaviour of consumpion can be better understood interpreting the Euler Equation in terms of consumption. The decrease in the marginal rate of return makes future consumption less profitable (the magnitude is determined by the intertemporal elasticity of substitution), inducing household to increase their current consumption.
The government has more difficulty to provide public goods also because of a sharp downfall in the taxation level, that is undirectly affected by technology through wage and labor. The rapidly increase in the level of investments on the capital market will bring the economy to invest less resources on state's bonds and the government itself find difficulties to commit itself to a debt repayment. As the shock effects vanishes after some quarters, with duration determined by the persistence coefficient, the economy faces problem to come back to steady state levels.

Technology shock $\rho^a = 0.01$, $z = 0 \Rightarrow no\, default$



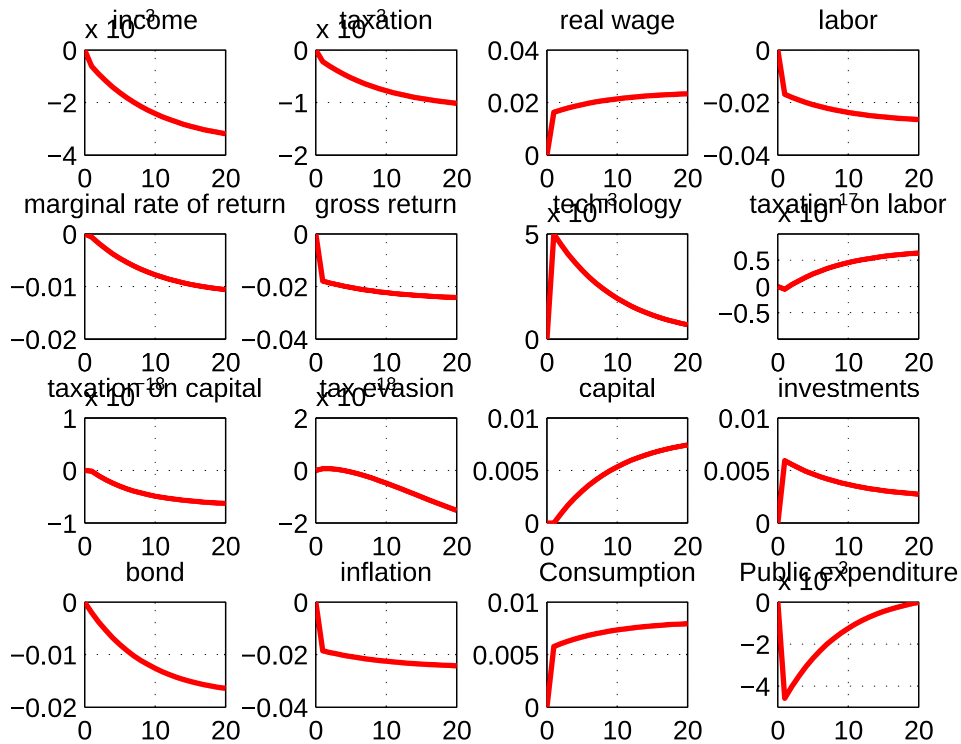
What we can observe here is that for a bigger shock on technology clearly all the economy is hitten more: real wage has a greater increase while labor a greater decrease. This bring households to make more investments because they feel richer. The level of capital in the market also increases, even if there is a downfall both in the marginal and the gross rate of return. Strangely, even if a huge fall in the taxation level, public expenditure is decreasing but not that much. Clearly for households the consumption level increases since they feel future consumption less attractive.

Shock on taxation on labor and taxation on capital



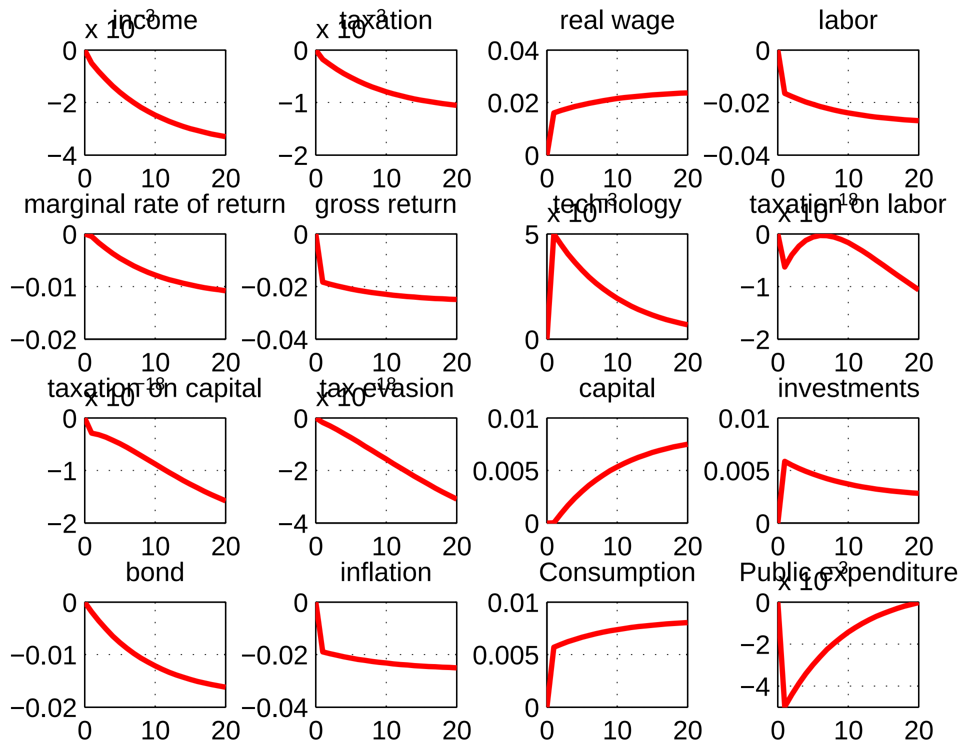
For an increase in the taxation level the economy faces a huge increase in public goods. The level of investment, consumption, wage has a sharp decrease and also the level of capital invested in the economy. Since the shock positively hit the marginal rate of return, households reduce their consumption and save resources for the future: future consumption is felt more attractive. Households are forced to work more in order get a wage (even if it will be lower). As a consequence the economy will suffer but the productivity of firms will be higher. In a period of high taxation households consume less resources and they reduce their level of investments on the capital market; because now they're perceived as too costly. The government support investments on its bonds, since raise in taxation and now it feels able to commit itself to a future bond repayment.

Technology shock $\rho^a = 0.005$ and default takes place $z = 1$, $s = 0.05$



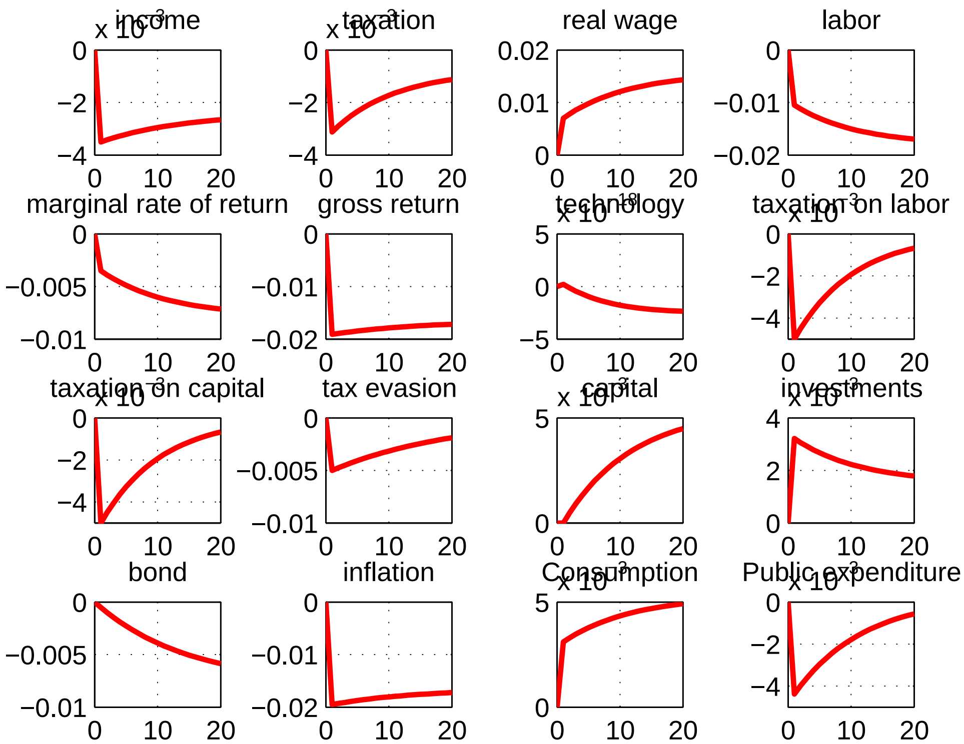
For a default scenario the situation could become terrible even if there is a positive shock on technology. The real wage is directly hitten by the shock and this will bring to a sharp rise on the investment level. Invest today is more profitable because of the downfall in the marginal rate of return. According to this fact, since the real wage increases also the taxation on labor will increase since the government could try to extract more resources from workers. Consequently households, feeling richer than before the shock, will increase their consumption level even if for a small amount. Again, for a decrease in the marginal rate of return to capital, future consumption is perceived as less attractive. All the rest of the economy faces a huge downfall in all the activities: there will be a sharp decrease of taxation and consequently on public expenditure, bond, GDP, inflation. The economy will be hitten forever. This support our thesis that Europe needs interventions and reforms as we have seen before for the case of positive shock on taxation. An economy like this is no more sustainable.

Technology shock $\rho^a = 0.005$ and default takes place $z = 1$, $s = 0.1$



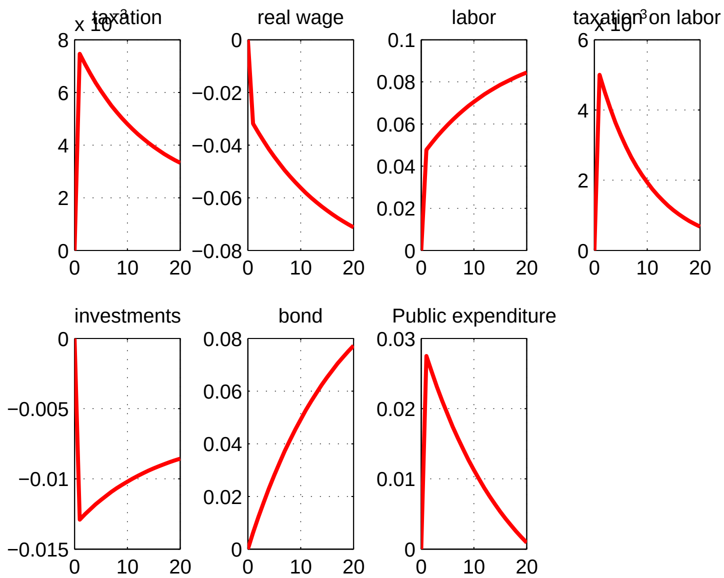
In the case of a bigger shock and bigger losses the situation appears clearly as before, except for the fact that public expenditure will sharply decrease more and more. Strangely, in this case even if there is a rise in the real wage, taxation on labor immediately decreases but after some periods it came back to its initial level and finally it has a downfall.

Negative shock on taxes as a decrease in taxation



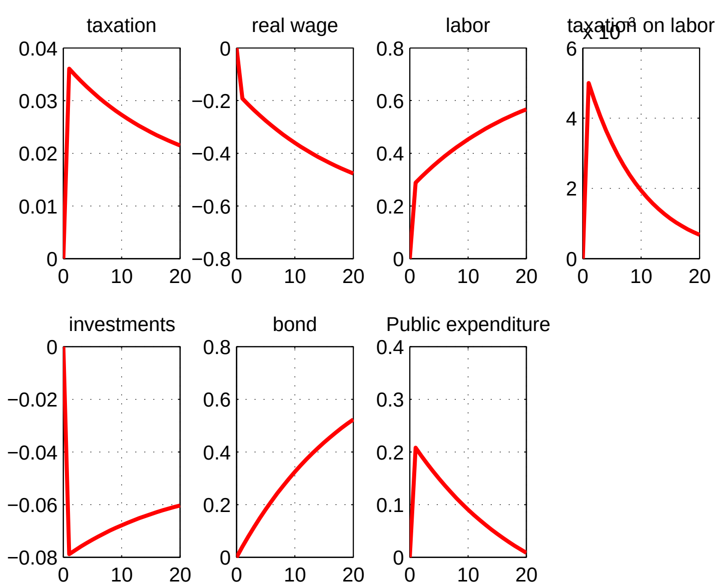
In this case there is a negative shock on taxation, both on labor and capital and we're also assuming not so much control on tax evasion. As it is for a government that didn't want to levy taxes on its households, even if it has experienced a default (similar to the Italian scenario before the change of government). Consequences of a policy like this could be really dangerous. Since there is less taxation, even if the state experienced a default, immediately the provision of public goods from the government has a downfall. It will take a lot of time to come back at the initial level. This policy gives incentives for households to invest on the capital market (since it's costless) and to consume more, because of a downfall in the rates of return. Clearly the government cannot commit itself to repay its debts and that's why the level of its bond is decreasing. In a situation like this is so difficult to came back at all the initail levels for all the economic activities. If the government won't intervene this economy surely will fail.

Shock on $\rho^l = 0.005$



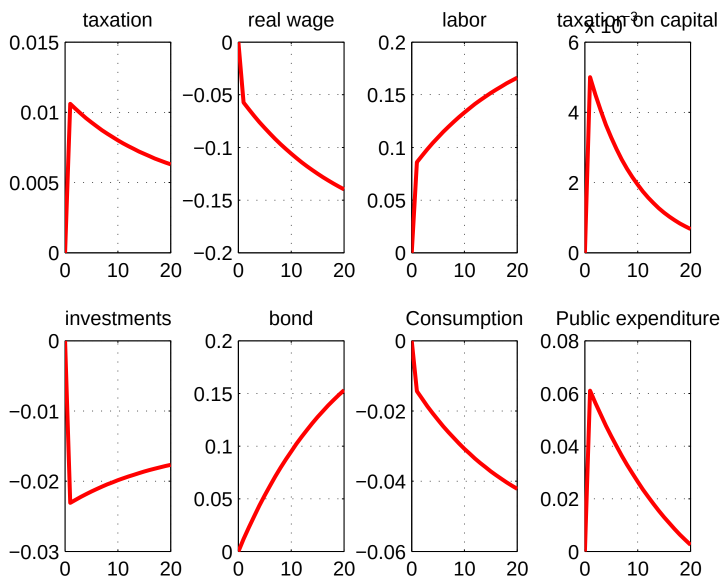
Isolating just the function that are directly hitten, we can see better the consequences of an increase of taxation on labor. The real wage fall down immediately and households need to supply more labor force to firms. Clearly for a higher level of taxes investments on the capital market will be reduced while the state will provide more public goods financed also through a greater bond with the possibility of a commitment to a repayment.

Shock on $\rho^l = 0.005$; what happens if a default takes place, $z = 1$, $s = 0.1$



Clearly all the economic activities are hitten more: there will be a lower level of public expenditure that will be financed through a lower bond and a lower taxation level. There will be an immediate downfall in the level of investments and also the real wage will decrease by a lower amount since losses from default impact all the activities making both households and government worse-off.

Shock $\rho^k = 0.005$, $z = 1$, $s = 0.1$



What happens when there is a shock on taxation on capital is similar to that of taxation on labor. But when default takes place public good provision is lower than before and consumption decreases according to a downfall in the wage. But the government cannot commit itself to repay a huge fraction of its bond and this will bring him to issue a lower bond. Investments will decrease but less than it was for tax on labor, since households feel themselves poorer because of the effect of taxes.

3.2 Possible interventions and implications

As we have anticipated is so clear that without any intervention the economy, after experienced a default, cannot come back at the initial level of its activities. After a default took place the government needs to make reforms and it has to convince itself that it cannot borrow more and more accumulating debts out of debts. Our function to measure default losses makes clear this idea and help us to understand how much times it takes for a government to recover all the losses that it accumulates. From the impulse response functions we can observe that just for an increases in taxation the government will be able to commit itself to repay the old debt and to provide public goods. But this in turn has strong implications on households activities as consumption, investments, wages, labor. What Europe needs now, in order to face the actual fear of a default, it's a wave of reforms. If a state faces a default, it's clearly difficult to emerge and maybe what it needs is not a monetization as pointed out by many authors as Cole and Kehoe (2000), Cooper, Kempf and Peled (2009,2010). Monetization and bailouts "just postpone the day of reckoning" as sustained by Roch and Uhlig (2011). At most some states need for a controlled default where each defaulting state receives help to apply new reforms, standards and regulations to its activities.

4 Conclusions

At the light of the recent crisis we have built a model to analyze what happens when default takes place. This topic could be examined in a lot of different ways: we focused precisely on default losses and we have analyzed how the economy could be shocked if a government default because of a waste of resources.

We have constructed a function to measure these losses in order to understand the magnitude of the impact that a default could have on the economy.

No one before have examined this aspect and in particular we have three important findings: first at all, if a state experience a default and there are no interventions from the government, consequences could be really dangerous and the economy won't reduce these losses in a small amount of time. In particular there could be the case that losses will not be absorbed.

Second, if there are reforms as in the labor market and in the capital market (as we have considered) as increase in taxation, the government can provide public goods and all the necessaire to its households; but they will face a bad period since lower level of wages and investment, decline in consumption and GDP, because of downfall in the marginal rate of return. But this is the only case in which the government can commit itself to a debt repayment.

Third, when a state experience a default, increasing taxation, it will be able to finance a new debt emission but this will bring the state in a hole since it will be forced to commit itself to repay a new debt in order to repay interests on the previous one. And because of the experienced default the government could not be sure about this commitment.

More works are needed in order to explore all this details and in particular to focus on which kind of intervention could be considered the first best to exit from the so-called "crisis zone". We hope our work is a good contribution to the literature on debt crises, thinking that it will be useful for future research on this topic.

5 Appendix

5.1 CRRA utility function

The Constant Relative Risk Averse utility function is:

$U(c) = \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{L_t^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$ for $\frac{1}{\sigma} > 0$ and $\frac{1}{\sigma} \neq 1$; for $\frac{1}{\eta} > 0$ and $\frac{1}{\eta} \neq 1$
$= ln(C_t) - ln(L_t)$ iff $\frac{1}{\sigma} = 1$ and $\frac{1}{\eta} = 1$.

Takin' the derivative we get:

$$U'(C) = C_t^{-\frac{1}{\sigma}}$$

hence,

$$\frac{U'(C_t)}{U'(C_{t+1})} = \frac{C_t^{-\frac{1}{\sigma}}}{C_{t+1}^{-\frac{1}{\sigma}}} = \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1}{\sigma}}$$

or solving for $\left(\frac{C_{t+1}}{C_t}\right)$:

$$\left(\frac{C_{t+1}}{C_t}\right) = \left(\frac{U'(C_t)}{U'(C_{t+1})}\right)^{\sigma}$$

Here σ is the elasticity of the ratio between consumption in two different periods with respect to the marginal rate of substitution. By definition is then the elasticity of substitution, which is constant for the CRRA utility function. σ is a measure of the strength of the substitution effect that uncertainty induces: households in fact shift consumption from today to tomorrow in order to smooth it over time.
The same could be said for the Frisch Elasticity:
Takin' the derivative we get:

$$U'(L) = L_t^{-\frac{1}{\eta}}$$

hence,

$$\frac{U'(L_t)}{U'(L_{t+1})} = \frac{L_t^{-\frac{1}{\eta}}}{L_{t+1}^{-\frac{1}{\eta}}} = \left(\frac{L_{t+1}}{L_t}\right)^{\frac{1}{\eta}}$$

or solving for $\left(\frac{L_{t+1}}{L_t}\right)$:

$$\left(\frac{L_{t+1}}{L_t}\right) = \left(\frac{U'(L_t)}{U'(L_{t+1})}\right)^{\eta}$$

Here η is the elasticity of substitution: it measures the magnitude according to which households shift their supply of labor between different periods. Frisch elasticity measures the substitution effect of a change in the wage rate on labor supply.

5.2 Function for Default Losses

Our function to measure default losses is given by:

$$\chi_t z$$

with:

- $\chi_t = ln(t - k + 1)$ for $t \in (k-1, t_1)$, increasing and concave in t
- $\chi_t = -c(t - t_1)^2 + \overline{\chi}$ for $t \in (t_1, t_3)$, decreasing and convex in t
- $z \in [0, 1]$; where 0 stands for the non-default in case of a good management, while 1 is the default case if bad management of the state
- c measure how much faster the function is going to decrease: for small values it decreases slowly, while for big values it decreases quite rapidly.

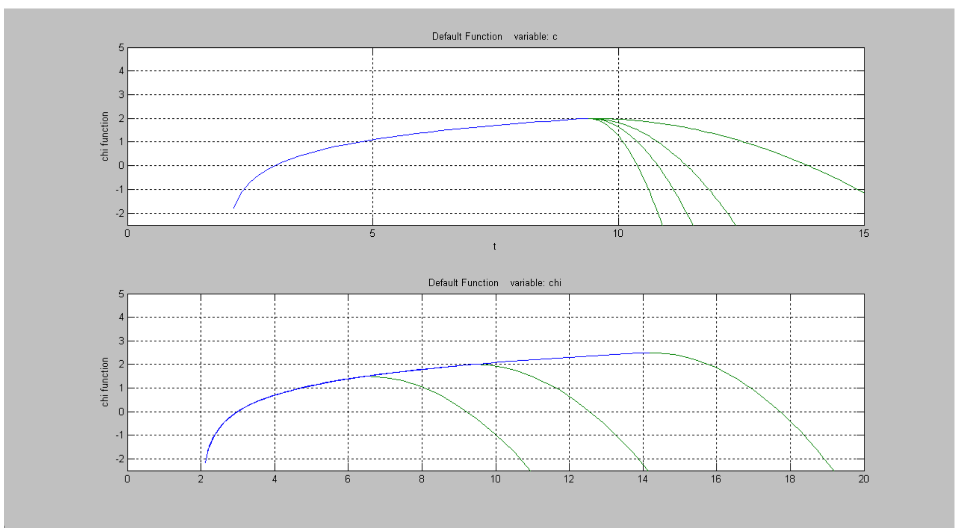


Figure 2:

In particular, for $t \in (k-1, t_1)$, the function is increasing and concave in t:
$\frac{\partial ln(t-k+1)}{\partial t} = \frac{1}{t-k+1} > 0$
$\frac{\partial^2 ln(t-k+1)}{\partial t^2} = (t - k + 1)^{-2} > 0.$
$t_1$is the point such that:

$$ln(t_1 - k + 1) = \overline{\chi}$$

$$t_1 - k + 1 = e^{\overline{\chi}}$$

$$t_1 = e^{\overline{\chi}} + k - 1.$$

In order to calculate the area below the function, that give us a measure of losses before the critical date, we use the integral:

$$\int_k^{e^{\overline{\chi}}+k-1} ln(t_1-k+1)dt$$

and we solve it by substitution, acting as follows:

$$t_1-k+1=x, \text{ with } dt=dx.$$

What we get is the following:

$$\begin{aligned}\int_1^{e^{\overline{\chi}}} ln(x)dx &= [xln(x)]_1^{e^{\overline{\chi}}} - \int_1^{e^{\overline{\chi}}} x\left(\frac{1}{x}\right)dx \\ & e^{\overline{\chi}}\overline{\chi}-e^{\overline{\chi}}+1 \\ & e^{\overline{\chi}}(\overline{\chi}-1)+1\end{aligned}$$

For $t\in(t_1,t_3)$, the function is decreasing and convex in t:
$\frac{\partial[-c(t-t_1)^2+\overline{\chi}]}{\partial t}=-2c(t-t_1)<0$
$\frac{\partial^2[-c(t-t_1)^2+\overline{\chi}]}{\partial t^2}=-2c<0.$
We want to find the point in which the function intersects again the axis and reach a value of 0 (i.e. all the losses are covered), that is exactly in t_2. In fact $t_2$is the point such that:

$$-c(t-t_1)^2+\overline{\chi}=0$$

$$-ct^2+2ctt_1-ct_1^2+\overline{\chi}=0.$$

$$t=\frac{ct_1\pm\sqrt{ct_1^2+c(-ct_1^2+\overline{\chi})}}{c}=t_1\pm\frac{[ct_1^2+c(\overline{\chi}-ct_1^2)]^{\frac{1}{2}}}{c}.$$

$$t=t_1+\frac{(c^2t_1^2+c\overline{\chi}-c^2t_1^2)^{\frac{1}{2}}}{c}=t_1+\sqrt{\frac{\overline{\chi}}{c}}.$$

Notice that we're interested just in positive values of t_2.
Finally, I can measure the area below the function, according to the values that it takes, using again an integral as:

$$\int_{t_1}^{t_1+\sqrt{\frac{\overline{\chi}}{c}}}[-c(t-t_1)^2+\overline{\chi}]dt=\int_{t_1}^{t_1+\sqrt{\frac{\overline{\chi}}{c}}}(-ct^2+2ctt_1-ct_1^2+\overline{\chi})dt$$

$$=\left[-c\frac{t^3}{3}-ctt_1^2+ct_1t^2+\overline{\chi}t\right]_{t=t_1}^{t=t_1+\sqrt{\frac{\overline{\chi}}{c}}}$$

$$=\left[-c\frac{t_1+\sqrt{\frac{\overline{\chi}}{c}}}{3}-ct_1^2(t_1+\sqrt{\frac{\overline{\chi}}{c}})+ct_1(t_1+\sqrt{\frac{\overline{\chi}}{c}})^2+\overline{\chi}(t_1+\sqrt{\frac{\overline{\chi}}{c}})+c\frac{t_1^3}{3}+ct_1^3-ct_1^3+\overline{\chi}t_1\right]$$

$$=\left[-c\frac{t_1+\sqrt{\frac{\overline{\chi}}{c}}}{3}-ct_1^2(t_1+\sqrt{\frac{\overline{\chi}}{c}})+ct_1(t_1+\sqrt{\frac{\overline{\chi}}{c}})^2+\overline{\chi}(t_1+\sqrt{\frac{\overline{\chi}}{c}})+c\frac{t_1^3}{3}+\overline{\chi}t_1\right].$$

I can also continue to solve it by substitution (i.e. assuming $t_1+\sqrt{\frac{\overline{\chi}}{c}}=x$ but things won't be reduced in a easier form. A good calculator as Matlab is able to compute it. By the way now we can measure the amount of losses: obviously these depend on the magnitude of c and the level of $\overline{\chi}$ that is reached by the government.

5.3 Detrending the model

Starting from eqn. (6) that is HBC we proceed detrending in order to obtain an equation for the capital accumulation:

$$C_t + K_{t+1} - (1-\delta)K_t + \tfrac{R_t}{\Pi_t}B_{t-1} = B_t + Y_t$$

Then we plug the values of w_t and Q_t and what we get is:

$$C_t + K_{t+1} - (1-\delta)K_t + \frac{R_t}{\Pi_t}B_{t-1} = B_t + Y_t \tag{5.1}$$

that can be re-written as:

$$K_{t+1} = (1-\delta)K_t + (A_t L_t^{\alpha} K_t^{1-\alpha}) - C_t - \frac{R_t}{\Pi_t}B_{t-1} + B_t \tag{5.2}$$

Supposing that the trend in the capital growth rate is described by:

$$\begin{aligned} G^r &= \frac{G_{t+1}^r}{G_t^r} \\ & \quad (1+g_t^r) \end{aligned} \tag{5.3}$$

Now we start detrending as follows:

$$\frac{K_{t+1}}{G_{t+1}^r}\frac{G_{t+1}^r}{G_t^r} = (1-\delta)\frac{K_t}{G_t^r} + \frac{A_t}{G_t^r}\frac{L_t^{\alpha}}{G_t^{\alpha,r}}\frac{K_t^{1-\alpha}}{G_t^{(1-\alpha),r}} - \frac{C_t}{G_t^r} - \frac{R_t B_{t-1}}{\Pi_t}\frac{1}{G_{t-1}^r} + \frac{B_t}{G_t^r} \tag{5.4}$$

$$G^r k_{t+1} = (1-\delta)k_t + \hat{l}_t^{\alpha} k_t^{1-\alpha} - \frac{R_t}{\Pi_t}b_{t-1} + b_t - c_t$$

$$(1+g^r)k_{t+1} = (1-\delta)k_t + \hat{l}_t^{\alpha} k_t^{1-\alpha} - \frac{R_t}{\Pi_t}b_{t-1} + b_t - c_t$$

$$(1+g^r)\frac{k}{y} = (1-\delta)\frac{k}{y} + \frac{y}{y} - \frac{Rb}{\Pi y} + \frac{b}{y} - \frac{c}{y}$$

$$(1+g^r)\frac{k}{y} = (1-\delta)\frac{k}{y} + 1 - \frac{Rb}{\Pi y} + \frac{b}{y} - \frac{c}{y}$$

$$(g^r+\delta)\frac{k}{y} = 1 + \frac{b}{py}(1-\frac{R}{\Pi}) - \frac{c}{y} \tag{5.5}$$

And using the fact that:

$$R = 1 + r = \left[(1-\alpha)a\left(\frac{l}{k}\right)^{\alpha} + (1-\delta)\right]$$

$$1 + r = \left[(1-\alpha)A\left(\frac{L}{K}\right)^{\alpha} + (1-\delta)\right]$$

$$A\left(\frac{L}{K}\right)^{\alpha} = \frac{(1+r)-(1-\delta)}{(1-\alpha)} = \left(\frac{r+\delta}{1-\alpha}\right)$$

so it follows that:

$$\frac{y}{k} = \frac{r+\delta}{1-\alpha}$$

and consequently:

$$\frac{k}{y} = \frac{1-\alpha}{r+\delta}$$

so we can plug it inside eqn. (5.6) and we obtain:

$$(g^r+\delta)\frac{1-\alpha}{r+\delta} = 1 + \frac{b}{py}(1-\frac{R}{\Pi}) - \frac{c}{y}$$

$$\frac{c}{y} = 1-(g^r+\delta)\frac{1-\alpha}{r+\delta} + \frac{b}{py}(1-\frac{R}{\Pi}) \tag{5.6}$$

Now using parameters according to Campbell (1994):

$g^r = 0.005$

$\alpha = 0.333$

while for the interest rate we refer to Lucke and Wurzel (2011):

$r = 0.04$

setting $\delta = 0.036$ according to literature and normalizing prices in the unit simplex, eqn. (5.6) becomes:

$$\frac{c}{y} = 1 - \frac{(0.041)(0.667)}{(0.076)} - 0.036\frac{b}{y}$$

$$\frac{c}{y} = 1 - 0.36 - 0.036(0.9) = 0.60$$

using data taken from Datastream (2011) we can approximate the bond to GDP ratio as $\frac{b}{y} = 0.9$ and we obtain as consumption to GDP ratio $\frac{c}{y} = 0.60$.

Now we start detrending the Euler equation:

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left[C_{t+1}^{-\frac{1}{\sigma}} \frac{R_{t+1}}{\Pi_{t+1}} \right]$$

that can be re-written as:

$$C_t^{-\frac{1}{\sigma}} = \beta E_t \left[C_{t+1}^{-\frac{1}{\sigma}} \frac{[(1-\alpha)\left(A_{t+1}L_{t+1}^{\alpha}K_{t+1}^{-\alpha}\right)+(1-\delta)]}{\Pi} \right]$$

we start detrending getting rid of inflation as:

$$\frac{C_t^{-\frac{1}{\sigma}}}{G_t^{(-\frac{1}{\sigma}),r}} = \beta E_t \left\{ \frac{C_{t+1}^{-\frac{1}{\sigma}}}{G_{t+1}^{(-\frac{1}{\sigma}),r}} \frac{G_{t+1}^{(-\frac{1}{\sigma}),r}}{G_t^{(-\frac{1}{\sigma}),r}} \left[(1-\alpha)A_{t+1}\left(\frac{L_{t+1}^{\alpha}}{K_{t+1}^{\alpha}}\right)\frac{G_{t+1}^{\alpha,r}}{G_{t+1}^{\alpha,r}} + (1-\delta)\right]\right\} \tag{5.7}$$

$$c_t = \beta E_t \left\{ c_{t+1} G^{(-\frac{1}{\sigma}),r} \left[(1-\alpha)a_{t+1}\left(\frac{l_{t+1}}{k_{t+1}}\right)^{\alpha} + (1-\delta)\right]\right\} \tag{5.8}$$

where:

- $\left[(1-\alpha)a_{t+1}\left(\frac{l_{t+1}}{k_{t+1}}\right)^{\alpha} + (1-\delta)\right]$ is the gross return to capital (that is equal to R_{t+1}),
- $(1-\alpha)a_{t+1}\frac{l_{t+1}^{\alpha}}{k_{t+1}^{\alpha}}$ is the rental rate on capital (that is equivalent to Q_{t+1}).

Since at the steady state $c_t = c_{t+1}$,
so it follows that eqn.(5.8) can be re-written as:

$$\beta E_t \left(G^{(-\frac{1}{\sigma}),r} R \right) = 1 \tag{5.9}$$

or, explicitating for the intertemporal discount factor:

$$\beta^{\star} = \frac{1}{G^{(-\frac{1}{\sigma}),r} R} \tag{5.10}$$

and it turns out to be:
$\beta^{\star} = 0.956$
Detrending eqn. (1.24) we get:

$$\frac{L_t^{-\frac{1}{\eta}}}{G_t^{-\frac{1}{\eta},r}} = \beta E_t \left[\frac{L_{t+1}^{-\frac{1}{\eta}}}{G_{t+1}^{-\frac{1}{\eta},r}} \frac{G_{t+1}^{-\frac{1}{\eta},r}}{G_t^{-\frac{1}{\eta},r}} \frac{(1-\tau_t^l)}{(1-\tau_{t+1}^l)} \frac{A_t \left(\frac{K_t}{L_t}\right)^{1-\alpha}}{A_{t+1} \left(\frac{K_{t+1}}{L_{t+1}}\right)^{1-\alpha}} \frac{G_{t+1}^r}{G_t^r} \frac{G_{t+1}^{(\alpha-1),r}}{G_t^{(\alpha-1),r}} \frac{G_{t+1}^{(1-\alpha),r}}{G_t^{(1-\alpha),r}} \left[(1-\alpha) A_{t+1} \left(\frac{L_{t+1}}{K_{t+1}}\right)^{\alpha} \frac{1}{G_{t+1}^r} \frac{G_{t+1}^{\alpha,r}}{G_{t+1}^{\alpha,r}} + (1-\delta) \right] \right] \tag{5.11}$$

$$l_t^{-\frac{1}{\eta}} = \beta E_t \left\{ l_{t+1}^{-\frac{1}{\eta}} G_t^{-\frac{1}{\eta},r} T \frac{a_t \left(\frac{k_t}{l_t}\right)^{1-\alpha}}{a_{t+1} \left(\frac{k_{t+1}}{l_{t+1}}\right)^{1-\alpha}} R \right\} \tag{5.12}$$

where $T = \frac{(1-\tau_t^l)}{(1-\tau_{t+1}^l)}$.
Since at the steady state:

- $l_t^{-\frac{1}{\eta}} = l_{t+1}^{-\frac{1}{\eta}}$,
- $\tau_t^l = \tau_{t+1}^l \Rightarrow 1 - \tau_t^l = 1 - \tau_{t+1}^l \Rightarrow T = 1$,
- $a_t = a_{t+1}$,
- $l_t^{\alpha-1} = l_{t+1}^{\alpha-1}$,
- $k_t^{1-\alpha} = k_{t+1}^{1-\alpha}$,
- $\left[(1-\alpha)\left(\frac{\hat{l}_{t+1}}{k_{t+1}}\right)^{\alpha} + (1-\delta)\right] = R = 1 + r$

then the detrended equation for labor can be re-written in the following terms:

$$\beta E_t \left\{ G_t^{-\frac{1}{\eta},r} \left[(1-\alpha) a_{t+1} \left(\frac{l_{t+1}}{k_{t+1}}\right)^{\alpha} + (1-\delta) \right] \right\} = 1 \tag{5.13}$$

so that:

$$\beta E_t \left\{ G_t^{-\frac{1}{\eta},r} R \right\} = 1$$

$$\beta^{\circ} = \frac{1}{G_t^{-\frac{1}{\eta},r} R} \tag{5.14}$$

that means that the intertemporal discount factor of each household depends on gross return to capital and the magnitude of the trend inside our economy. From the resource constraint of the government we want to obtain the public expenditure to GDP ratio:

$$G_t = (B_t - \frac{R_t}{\Pi_t} B_{t-1}) + (1-\varphi_t) T_t - s(\frac{R_t}{\Pi_t} B_{t-1} - B_t) z$$

$$G_t = (1-\varphi_t) T_t + B_t - \frac{R_t}{\Pi_t} B_{t-1} - s(\frac{R_t}{\Pi_t} B_{t-1} - B_t) z$$

$$G_t = (1-\varphi_t)T_t + B_t - \frac{R_t}{\Pi_t}B_{t-1} - s(\frac{R_t}{\Pi_t}B_{t-1} - B_t)z$$

$$G_t = (1-\varphi_t)T_t + B_t - \frac{R_t}{\Pi_t}B_{t-1} - sz(\frac{R_t}{\Pi_t}B_{t-1} - B_t)$$

$$g_t = (1-\varphi_t)t_t + (1+sz)\left[b_t - \frac{R_t b_{t-1}}{\Pi_t}\right]$$

$$\frac{g}{y} = (1-\varphi)\frac{t}{y} + \frac{b}{y}[1+zs] - \frac{Rb}{\Pi y}[1+zs]$$

Now substituting for capital to GDP ratio as before, and plugging inside values of the bond to GDP ratio we can get the final expression for government public expenditure (g) to income ratio; as expenses on burocracy, transfers and subsidies:

$$\frac{g}{y} = \frac{t}{y}(1-\varphi) + \frac{b}{y}(1+sz)(1-\frac{R}{\Pi}) \tag{5.15}$$

so that for z=0:

$$\frac{g}{y} = (0.85)0.45 - 0.036(0.9)(1+sz) \approx 0.36$$

and for z=0 and no tax evasion (i.e. $\varphi = 0$):

$$\frac{g}{y} = 0.45 - 0.036(0.9)(1+sz) \approx 0.42$$

while for z=1, s=0.05:

$$\frac{g}{y} = (0.85)0.45 - 0.036(0.9)(1+0.05) \approx 0.35$$

while for z=1, s=0.05 and no tax evasion (i.e. $\varphi = 0$):

$$\frac{g}{y} = 0.45 - 0.036(0.9)(1+0.05) \approx 0.43$$

Both if there is a default or not, the level of public expenditure will be:

$$\frac{g}{y} = 0.382 - 0.032(1+sz)$$

.

At the steady state this fits pretty well to a real world economy. The only thing to notice is that the level of public expenditure is lower than that of RBC literature, just because we're considering tax evasion as percentage of taxes that are unpaid by households, both on capital and labor market.

Detrending the Euler equation of the government:

$$G_t^{-\gamma} = \beta E_t\left[G_{t+1}^{-\gamma}\frac{R_{t+1}}{\Pi_{t+1}}\right] - \frac{sz}{(1+sz)} + \beta\frac{sz}{(1+sz)}E_t\left(\frac{R_{t+1}}{\Pi_{t+1}}\right) \tag{5.16}$$

$$G_t^{-\gamma} = \beta E_t\left[\frac{R_{t+1}}{\Pi_{t+1}}\left(G_{t+1}^{-\gamma} + \frac{sz}{(1+sz)}\right)\right] - \frac{sz}{(1+sz)} \tag{5.17}$$

we start de-trending as follows getting rid of inflation just for this moment:

$$\frac{G_t^{-\gamma}}{G_t^{-\gamma,r}} = \beta E_t\left\{\left[(1-\alpha)\frac{A_{t+1}}{G_t^{\alpha,r}}\frac{L_{t+1}^{\alpha}}{G_{t+1}^{\alpha,r}}\frac{K_{t+1}^{-\alpha}}{G_{t+1}^{-\alpha,r}} + (1-\delta)\right]\left(\frac{G_{t+1}^{-\gamma}}{G_{t+1}^{-\gamma,r}}\frac{G_{t+1}^{-\gamma}}{G_t^{-\gamma,r}} + \frac{szP_t}{(1+sz)}\right)\right\} - \frac{szP_t}{(1+sz)} \tag{5.18}$$

it follows that:

$$g_t = \beta E_t\left\{R_{t+1}\left(g_{t+1}G^{-\gamma,r} + \frac{sz}{(1+sz)}\right)\right\} - \frac{sz}{(1+sz)} \tag{5.19}$$

where clearly the government faces the same interest rates of households.
Given that at the steady state $g_t = g_{t+1}$:

$$1 + \frac{sz}{(1+sz)g_t} = \beta E_t\left\{R_{t+1}\left(G^{-\gamma,r} + \frac{sz}{(1+sz)g_t}\right)\right\}$$

$$\beta^{\triangleleft} = \frac{1 + \frac{sz}{g(1+sz)}}{R\left\{G^{-\gamma,r} + \frac{sz}{g(1+sz)}\right\}}$$

For a non-default case the intertemporal discount factor is the same as for households. The government discount factor is decreasing when the gross interest rate arise and for a default scenario it increases because of losses. Provision of public goods will be more costly for the government and the old level of public provision cannot be sustained at all.

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