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ASPECTS OF DARK BUBBLE COSMOLOGY FROM STRING THEORY

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To my parents, with all the love in the world.

Chi non spera l'insperabile non lo scoprirà, poiché è chiuso alla ricerca,
e a esso non porta nessuna strada.

Eraclito, fr.22b18 Diels-Kranz

Contents

Abstract

This work investigates thoroughly a brane world scenario called "Dark Bubble Cosmology", which differs in crucial aspects from other proposals in string phenomenology: our four-dimensional universe rides an expanding bubble whose embedding naturally induces a positive cosmological constant. This model avoids the Swampland constraints of constructing fundamental de Sitter vacua based on pure compactifications of String Theory and, at the same time, provides a concrete realization of a brane world with an asymmetric inside-outside construction. The bubble represents a Coleman-de Luccia or Brown-Teitelboim instanton which nucleates in an unstable five-dimensional Anti de Sitter spacetime. This proposal therefore involves inducing four-dimensional gravity on a brane which mediates the decay from a non-supersymmetric false AdS_5 vacuum to a lower energy vacuum.

In this thesis, brane world constructions based both on Type IIB and 0'B string theories are examined and a general mechanism to embed electromagnetic gauge fields living on the brane is found. The latter requires a non-trivial interplay between gauge fields on the brane and stringy fields in the bulk, and can be hopefully generalized to include non-abelian gauge fields, with the perspective of embedding the full Standard Model of Particle Physics in the Dark Bubble scenario. In the context of the (non-supersymmetric) Type 0'B string theory, the Dark Bubble model yields precise predictions for the evolution of the cosmological constant Λ and of the scale factor a. However, it turns out this setup is incompatible with experimental observations and has to be discarded. A Swampland approach is also justified, especially in connecting the emerging scales with hypotheses on scale separation and the dark dimension.

Introduction

String Theory is the leading candidate for a consistent theory of Quantum Gravity, namely a framework which combines together the two pillars of modern theoretical physics: Einstein's theory of General Relativity and Quantum Field Theory. To be precise, the two theories are perfectly compatible up to very high energy scales: the gravitational effective theory breaks down at the Planck scale $M_P = 1.22 \times 10^{19}$ GeV[1]. Hence, what is still lacking is a quantum theory which never ceases to be reliable in any physical regime, and reduces to an Effective Field Theory which includes GR as the leading gravitational sector. Indeed, treating the gravitational field as a quantum spin-2 field and employing the usual techniques of QFT, such as renormalization, we encounter an infinite number of divergences: the theory is said to be *non-renormalizable* at a perturbative quantum level. It is in this sense that GR and QFT are "incompatible".

The Standard Model of Particle Physics does not account for the gravitational interaction, nor does it provide a unified framework for all fundamental interactions. Gravity can therefore only be treated as an EFT whose UV-completion is negligible at energy scales far below M_P. We know, however, that quantum effects of gravity must play a role in the extreme regions and events of our universe, such as close to black hole singularities or during early cosmology (Big Bang, inflation, etc.).

String Theory represents in some sense the simplest and most conservative modification of QFT and GR. The only new dynamical input can be summarized as follows: *the fundamental objects in nature are not pointlike, but one-dimensional*. This axiom, combined with the standard kinematics of general covariance and the usual procedure of quantization, results in a consistent unified description of gravity and Yang-Mills theory coupled to matter. The two sectors arise from the fact that the string can have two possible topologies: it can be open or closed[2]. Open strings describe Yang-Mills theory, while closed strings describe gravity. Since open strings can close up and vice versa, gravity and Yang-Mills theory are automatically related dynamically. Moreover, String Theory naturally incorporates most of the theoretical ideas for physics beyond the Standard Model (gauge unification, Supersymmetry, extra dimensions and so on). Some of these ideas could be tested in the upcoming years, either in accelerator experiments or

[1]It may break down earlier, according to the species scale bounds of [1, 2].

[2]Heterotic strings also incorporate Yang-Mills interactions, albeit in a different manner.

by future cosmological and astrophysical observations.

Nevertheless, String Theory is far from being fully understood. We mostly understand the theory at the perturbative level, and we have explored only a very limited corner of string vacua in a possibly much larger landscape of possibilities.

One of the major open issues is how to build a phenomenologically realistic model of our expanding (de Sitter-like) universe. Indeed, experimental observations of supernovae, Cosmic Microwave Background, Baryon Acoustic Oscillations and Large-Scale Structure indicate that our universe is expanding at an accelerating rate. The term "dark energy" has appeared in the titles and abstracts of scientific papers in 1998 after announcements about the discovery of this accelerated expansion, and it refers to our ignorance on the very nature of this acceleration. Is it due to a cosmological constant (the famous Λ in Einstein's equations) or is it related to some kind of vacuum energy? Or else, is there a scalar field slowly rolling along its potential responsible for this acceleration, as in Quintessence [3, 4] models? Regardless of the answer, since the discovery of dark energy, String Theory has been faced with the challenge of reproducing a small positive vacuum energy. The dominant approach has been the reliance on a landscape of different vacua [5] equipped with a transition mechanism such that the anthropic principle selects our vacuum. This approach became calculable in String Theory with the construction of KKLT [6], in which one can in principle achieve a landscape of scale-separated vacua, with both positive and negative cosmological constants, by tuning flux numbers. However, the latter is still very controversial and it is the subject of ongoing debate in the community.

There have also been numerous attempts to build (meta)stable dS vacua in String Theory (see for instance [7, 8]), but they all require a great amount of fine-tuning. Moreover, these persistent difficulties in achieving (meta)stable dS solutions in String Theory have led to conjecture[3] that no such vacuum can exist[4]. Anti-de Sitter (AdS) vacua, on the other hand, are common in String Theory, but there are reasons to believe that, unless they are supersymmetric, they are unstable and must decay either perturbatively or non-perturbatively [9].

Dark Bubble Cosmology [10–20] is an alternative approach to describe our universe without dealing with standard compactification mechanisms: it is a brane world scenario in which our Universe is embedded in a higher-dimensional spacetime and the Standard Model of Particle Physics lives on the brane. Indeed, our Universe "rides" an expanding bubble in a five-dimensional Anti-de Sitter Spacetime and, since the time evolution of the brane world takes place in the fifth dimension, lower-dimensional observers stuck on the brane (us) experience a 4d expanding cosmology. The Dark Bubble therefore perfectly realizes a four-dimensional de Sitter universe on the brane and 4d gravity simply arises as an effective description on a dynamical object embedded in a higher dimensional space time.

[3] See Section 1.5.

[4] At least at the boundary of moduli space.

In Chapter 1, we will mention some basic concepts of String Theory, such as string worldsheet theory, D-branes and string compactification. We will then examine the five superstring theories and the three non-tachyonic non-supersymmetric theories, with a particular attention to Type 0'B theory. We will also make a detour to explore the conjectures of the Swampland program.

In Chapter 2, we will present the Dark Bubble model, describing its main features and focusing on the way gravity is induced on the brane world. We will see how bubbles of lower-energy vacuum nucleate from an unstable five-dimensional Anti-de Sitter spacetime, in agreement with the Weak Gravity Conjecture and the de Sitter Conjecture, and how to realize radiation and matter fields on the brane.

Chapter 3 will be devoted to describing Vilenkin's tunneling proposal and Hartle-Hawking's no-boundary proposal for a quantum beginning of the universe. We will then show how the former proposal perfectly matches with the nucleation amplitudes in the Dark Bubble model, meaning the tunneling event in 4d is just a shadow of a higher-dimensional nucleation event.

In Chapter 4, we will describe a general mechanism to embed electromagnetic gauge fields living on the brane and how the backreaction of the electromagnetic tensor on the bubble modifies the brane geometry.

Finally, in Chapter 5 we will build a brane world model similar to the Dark Bubble, using D_3-branes in the non-supersymmetric Type 0'B string theory. We will see how, in this construction, the cosmological "constant" depends logarithmically on the cosmological scale-factor a, entailing a quasi-dS evolution for the braneworld. This model will turn out to be incompatible with experimental evidence, and shall therefore be discarded. However, it is a very neat construction and it is worth studying, as it provides us with a deeper insight into Dark Bubble Cosmology itself.

Let us now set the stage for this intricate model, in which Cosmology and String Theory are deeply intertwined, with a lightning-review of String Theory and its main features.

Chapter 1

Elements of String Theory

In this Chapter, we will explore some of the main features of String Theory, emphasizing some aspects of the theory more and leaving others only briefly sketched. For a satisfactory introduction to String Theory, we refer the reader to [21–25].

1.1 Bosonic formulation

String Theory is a quantum theory of 1-dimensional objects (*strings*) moving in a D-dimensional spacetime. Strings sweep a 2d surface, the *worldsheet*, labelled by the spatial coordinate σ and the time coordinate τ ($0 \leq \sigma \leq \pi$ for open strings and $0 \leq \sigma \leq 2\pi$ for closed strings). The worldsheet is embedded into spacetime by the fields $X^\mu(\tau, \sigma)$. The intrinsic length and mass scales of the theory are the string length l_s and the string mass scale M_s, given by

$$l_s = 2\pi\sqrt{\alpha'}, \qquad M_s = \frac{1}{\sqrt{\alpha'}}. \tag{1.1.1}$$

α' is called *Regge slope* and it is the only free parameter of String Theory. In principle it can take any value in the range 10^{-33}cm (Planck length) $\leq \sqrt{\alpha'} \leq 10^{-17}$cm (TeV scale). Apart from this dimensionful parameter, there are no free dimensionless parameters.

String Theory, in its bosonic formulation, is described by Polyakov action

$$S_P = -\frac{1}{4\pi\alpha}\int d\sigma d\tau \sqrt{-\gamma}\gamma^{\alpha\beta}\eta_{MN}\partial_\alpha X^M \partial_\beta X^N, \tag{1.1.2}$$

where $M = 0, ..., D-1$ and $\alpha, \beta = \tau, \sigma$. In this equation, X^M are the functions defining the embedding of the worldsheet in spacetime, $\gamma^{\alpha\beta}$ is the worldsheet metric, η_{MN} is the spacetime Minkowski metric and the string tension is related to α' via

$$T = \frac{1}{2\pi\alpha'}. \tag{1.1.3}$$

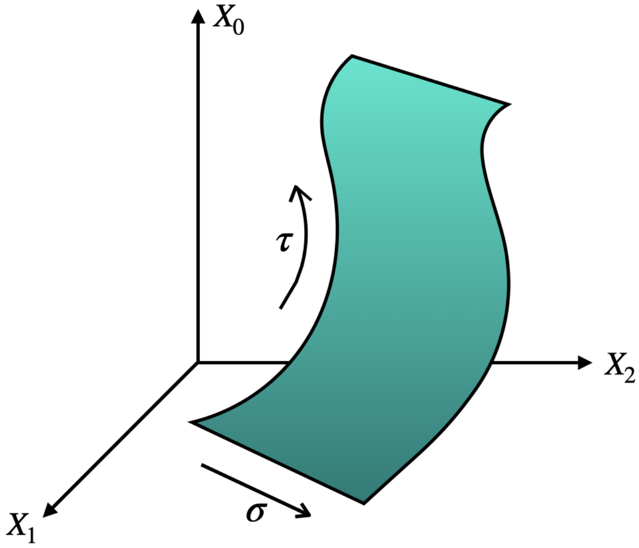

Figure 1.1: Embedding into spacetime of the string worldsheet, parameterized by the coordinates σ and τ.

Eq.(1.1.2) enjoys three important symmetries: D-dimensional Poincarè symmetry, diffeomorphisms invariance and Weyl rescaling invariance.

A deeper insight into bosonic String Theory is beyond the scope of this thesis. We will therefore just mention that applying the rules of quantisation to the worldsheet theory provides us with the Fock space of string excitations. The massless modes of the bosonic sector include (among others): the vector boson of Yang-Mills Theory $A_{\mu\nu}$ (open string) and the spin-2 graviton $h_{\mu\nu}$ (closed string). In addition, there is a tower of massive string excitations of mass

$$M^2 = \frac{1}{\alpha'}(N-1) \tag{1.1.4}$$

for the open sector and

$$M^2 = \frac{4}{\alpha'}(N-1) \tag{1.1.5}$$

for the closed sector, where $N = 0, 1, 2, ...$ The appearance of the tachyon, the lowest lying state of negative mass will be overcome by moving from the bosonic string theory to superstring theory, in which no tachyons are present.

In the low-energy limit, at energies much lower than the string scale M_s, the effective dynamics of the theory reduces to gauge interactions and gravity. Consistency of the worldsheet theory predicts explicitly that to lowest order in $\sqrt{\alpha'}$ the gravitational laws are given by Einstein's equations.

Finally, let us mention that String Theory predicts the existence of higher-dimensional objects called *D-branes*, which are hypersurfaces on which open strings can end. These objects play a crucial role for the dynamics of the theory.

1.2 Superstring Theories

In order to have fermions in the spectrum of the theory, we need to add fermionic fields to our worldsheet, employing the principle of Supersymmetry in the so-called RNS formalism: for each bosonic field X^M, there is a corresponding superpartner Ψ^M or $\tilde{\Psi}^M$ plus the gravitino ψ_a, which is the superpartner of the metric g_{ab}. The main peculiarity of Superstring Theory respect to the purely bosonic theory consists in the possibility to "project out" the tachyonic degrees of freedom and the remakable fact is that this projection, called *GSO projection* [26], can lead to a completely supersymmetric spectrum in 10d. The GSO projection is thus essential for the consistency of the theory: it eliminates the tachyons from the spectrum and, when spacetime supersymmetry is preserved, leaves an equal number of bosons and fermions at each mass level. We will discuss non-supersymmetric strings in the following sections in more detail.

The RNS action reads

$$S_{RNS} = \frac{1}{4\pi} \int d^2\sigma \eta_{MN} \left[\frac{1}{\alpha'} \partial X^M \bar{\partial} X^N + \Psi^M \bar{\partial} \Psi^N + \tilde{\Psi}^M \bar{\partial} \tilde{\Psi}^N \right], \tag{1.2.1}$$

where we have used the euclidean worldsheet by sending $\tau \to -i\tau$, and the derivatives are with respect to the holomorphic coordinate $z = e^{\tau - i\sigma}$. The equations of motion for Ψ and $\bar{\Psi}$ admit two possible boundary conditions, dubbed *Ramond* and *Neveu-Schwarz*

$$\begin{aligned} \text{R}: &\qquad \Psi^M(\tau, 0) = \Psi^M(\tau, 2\pi) \\ \text{NS}: &\qquad \Psi^M(\tau, 0) = -\Psi^M(\tau, 2\pi). \end{aligned} \tag{1.2.2}$$

and similarly for $\tilde{\Psi}$. We can thus divide the fields in two sectors, the **R-R** sector and the **NS-NS** sector. Very loosely speaking, the choice of the boundary conditions (1.2.2) leads to different formulations of Superstring Theory.

Until 1995, there were five known Superstring Theories in 10d: Type I, Type IIA and IIB, and the two heterotic theories $E_8 \times E_8$ and $SO(32)$. They seemed to be very different from each other and the only known link between these theories was a stringy symmetry called T-duality, relating Type IIA and Type IIB and the two heterotic ones. That same year, Edward Witten shed light on an intricate web of dualities among all five Superstring Theories, where an essential role was reserved for a new theory in 11d, whose low-energy limit should be 11d Supergravity. In this picture, these five theories represent various perturbatively defined limits of this 11-dimensional theory, which Witten called *M-theory*[28]. Moreover, their low-energy limit is always a Supergravity theory.

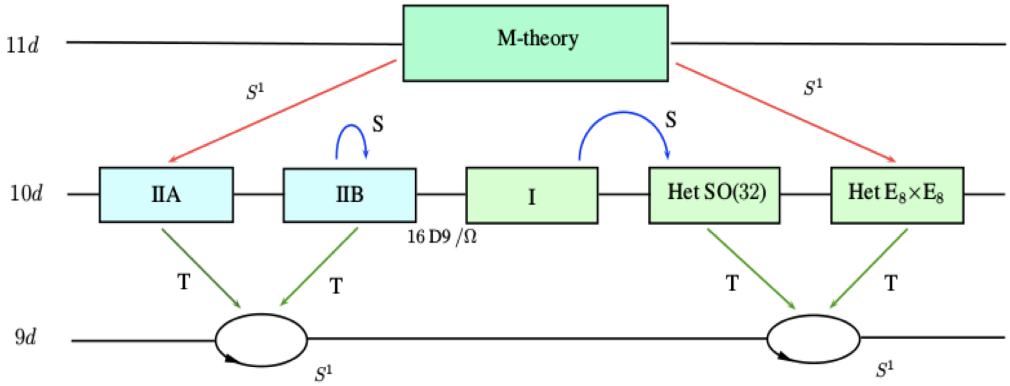

Figure 1.2: Schematic illustration taken from [27] of the intricate web of dualities among all Superstring Theories and M-theory. Type IIA and Type IIB are related via T-duality, as well as Heterotic $SO(32)$ and $E_8 \times E_8$, while Type I is related to Heterotic $SO(32)$ via S-duality.

We shall now quickly mention, for the sake of completeness and brevity, only the main features and the field content of the five superstring theories.

- **Type IIA**: It contains only closed strings.

  In the NS-NS sector, there is the metric $G_{\mu\nu}$, the antisymmetric 2-form Kalb-Ramond field $B_{\mu\nu}$ and the dilaton Φ.

  In the R-R sector, there is a 1-form field C_1 and a 3-form field C_3.

  We have to add to these bosonic fields also the RNS and NRS dilatinos λ_α and gravitinos $\psi_{M\alpha}$.

  Type IIA contains D_p-branes with p even: D0, D2, D4, D6, D8.

- **Type IIB**: It contains only closed strings.

  In the NS-NS sector, we still have $G_{\mu\nu}$, $B_{\mu\nu}$ and the dilaton Φ.

  In the R-R sector, we have a 0-form field C_0 (axion), a 2-form field C_2 and a 4-form field C_4.

  Again, we have to add to these bosonic fields also the RNS and NRS dilatinos λ_α and gravitinos $\psi_{M\alpha}$.

  Type IIB contains D_p-branes with p odd: D(-1), D1, D3, D5, D7, D9.

  Type IIB and Type IIA are related via **T-duality** (for example compactifying the 10d theory on S^1).

- **Type I**: Type I is derived by removing all the degrees of freedom from the Type IIB theory that exhibit an odd behavior under worldsheet parity. We therefore retain only those states in Type IIB theory that remain unchanged when subjected to worldsheet parity transformations. Practically, Type I$\sim$Type IIB/Ω.

In the NS-NS sector, we have $G_{\mu\nu}$ and the dilaton Φ.

In the R-R sector, we have the 2-form field C_2.

In addition to these fields, we have 32 D9-branes. In fact, projecting out the odd parity states, we generate negative RR charge which is only cancelled by adding the D9-branes. These branes give rise to $SO(32)$ gauge fields coming from open strings, meaning that the spectrum of Type I contains both open and closed strings.

- **Heterotic Theories**: The $SO(32)$ theory has 496 generators. In the NS-NS sector, we have $G_{\mu\nu}$, $B_{\mu\nu}$ and the dilaton Φ.

  In addition to this, there are 496 gauge fields $A^a_u, a = 1, ..., 496$.

  Type I and $SO(32)$ are dual to each other under **S-duality**, while $SO(32)$ and $E_8 \times E_8$ are related via T-duality. The latter theory is very interesting for a realistic phenomenology.

Let us conclude this section by stating that D-branes are more than just hyperplanes on which open strings end. They are by themselves dynamical objects which gravitate by coupling to closed strings in the NS-NS sector (they have a mass) and which are charged under RR p-form potentials. Indeed, brane dynamics can be captured in a low-energy effective action for the world-volume of the D_p-brane of the form[1]

$$S_{eff} = S_{DBI} + S_{CS}, \tag{1.2.3}$$

where the DBI part represents the coupling to the NS-NS sector, while the CS term the coupling to the R-R sector.

1.3 String compactifications

String Theory, as stated in the previous section, is consistently defined in ten dimensions[2]. In order to extract any information about 4d physics, we need to understand the way the 10d theory is compactified to these four dimensions; in other words, we need to know the precise way in which the Standard Model and Einstein's gravity reembedded as low-energy limits in String Theory. The 10d spacetime is divided into an external non-compact spacetime and an internal compact space

$$\mathcal{M}_{10} = \mathcal{M}_6 \times \mathcal{M}_4. \tag{1.3.1}$$

[1]See Chapter 4.

[2]To be more precise, non-geometric sectors also exist in the landscape. In this thesis we restrict to purely geometric settings.

The compactification scale is $M_C = 1/R$, where R is the typical length scale associated with the internal space, and it is considered to be much smaller than the string scale $M_s = 1/l_s$ Early attempts involved mostly compactification of the heterotic string on Calabi-Yau[3] manifolds [29] or on exact $(2,0)$ backgrounds, and exceptionally type I theory. With the introduction of D-branes, compactifications of the type-II String Theory involving orientifolds and intersecting D-branes became the center of attention.

The current state of the art is that one can find semi-realistic models in both frameworks, but several key issues remain open. Among them, one is the problem of *moduli stabilization*: in any of these compactifications, the 4d low-energy action has a number of massless fields with no potential. These would lead to long-range scalar forces unobserved in nature. Furthermore, the couplings of other fields (like Yukawa couplings) depend on their Vacuum Expectation Values. As a consequence, no predictions can be made in these scenarios since the VEV of the moduli can take any value. Therefore, there should be a mechanism that generates a potential for the moduli, stabilizing their VEV's.

The most studied mechanism within perturbative String Theory thus far is via fluxes: turning on fluxes for some of the field strengths available in the theory generates a non trivial potential for the moduli, which stabilize at their minima. The new issue that arises is that fluxes backreact on the geometry, and whatever manifold was allowed in the absence of fluxes, will generically be forbidden in their presence.

Much of what we know about stabilisation of moduli is done in Calabi-Yau compactifications under a certain combination of 3-form fluxes whose backreaction on the geometry just makes them conformal Calabi-Yau manifolds, where fluxes stabilize the moduli corresponding to the complex structure of the manifold, as well as the dilaton. To stabilize the other moduli, stringy corrections are invoked. The result is that one can stabilize all moduli in a regime of parameters where the approximations can be somehow trusted, but it is very hard to rigorously prove that the corrections not taken into account do not destabilize the full system. Alternative approaches include rigid constructions with no moduli besides the dilaton, although a deeper understanding of these settings, alongside supersymmetry breaking, is required.

For comprehensive reviews of compactification in String Theory, we refer to [30, 31].

1.4 Non-supersymmetric String theories

Supersymmetry is a fundamental ingredient in String Theory. However, we do not observe it in nature, at least up to the energy scales we managed to investigate at LHC ($\simeq$ 13TeV). This means that the phenomenon of SUSY-breaking plays a crucial role in a full understanding of String Theory and Quantum Gravity in general. A deeper understanding of the subtle issues related to SUSY-breaking in String Theory is the

[3] These special manifolds always allow for a Ricci-flat metric.

first step to achieve a more complete picture of its underlying foundational principles and more realistic phenomenological models. In this Section, we will briefly discuss the relevant non-supersymmetric models in String Theory which do not contain tachyons in their spectra, focusing mainly on the Type 0'B Theory, which will be of great use for the brane world scenario we want to address in Chapter 5.

1.4.1 Tachyon-free vacuum amplitudes

Let us begin by constructing the relevant 10d string models starting from the 1-loop vacuum amplitudes of their "parent" models. While Type 0'B arises as a non-tachyonic orbifold of the tachyonic type 0B, and SUSY is therefore absent at the outset, the $USp(32)$ model and the $SO(16) \times SO(16)$ heterotic models arise as projections of type IIB and $E_8 \times E_8$ superstrings respectively, meaning SUSY is broken at the string scale.

The following construction is based on the characters $(O_{2n}, V_{2n}, S_{2n}, C_{2n})$ of the level-1 affine so$(2n)$ algebra

$$
\begin{aligned}
O_{2n} &\equiv \frac{\vartheta^n \begin{bmatrix} 0 \\ 0 \end{bmatrix} (0|\tau) + \vartheta^n \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} (0|\tau)}{2\eta^n(\tau)}, \\
V_{2n} &\equiv \frac{\vartheta^n \begin{bmatrix} 0 \\ 0 \end{bmatrix} (0|\tau) - \vartheta^n \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} (0|\tau)}{2\eta^n(\tau)}, \\
S_{2n} &\equiv \frac{\vartheta^n \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} (0|\tau) + i^{-n}\vartheta^n \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (0|\tau)}{2\eta^n(\tau)}, \\
C_{2n} &\equiv \frac{\vartheta^n \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} (0|\tau) - i^{-n}\vartheta^n \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (0|\tau)}{2\eta^n(\tau)},
\end{aligned}
\tag{1.4.1}
$$

where $\eta(\tau) \equiv q^{\frac{1}{24}} \Pi_{n=1}^{\infty}(1-q^n)$ is the Dedekind η function and the Jacobi ϑ functions are defined as

$$
\vartheta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} (z|\tau) \equiv \sum_{n \in \mathrm{Z}} q^{\frac{1}{2}(n+\alpha)^2} \mathrm{e}^{2\pi i (n+\alpha)(z-\beta)}. \tag{1.4.2}
$$

These characters encode degeneracies of states in various spacetime (or gauge) representations, and can be used to express worldsheet partition functions. In particular, 1-loop vacuum amplitudes contain information about the single-particle spectrum. As anticipated, in ten dimensions there are three non-tachyonic superstrings without (unbroken) spacetime supersymmetry:

- **USp(32) model**: Introduce the relevant orientifold projections via 1-loop vacuum amplitudes, starting from the case of the Type I superstring. The torus amplitude

$$\mathcal{T}_1 = \frac{1}{2}\int_{\mathcal{T}} \frac{d^2\tau}{\tau_2^6} \frac{(V_8 - S_8)\overline{(V_8 - S_8)}}{|\eta(\tau)|^{16}} \tag{1.4.3}$$

has to be added to the amplitudes pertaining to the open string and unoriented sectors, which are associated to the Klein bottle, the annulus and the Moebius strip

$$\mathcal{K} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6} \frac{(V_8 - S_8)(2i\tau_2)}{\eta^8(2i\tau_2)}, \tag{1.4.4}$$

$$\mathcal{A} = \frac{N^2}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6} \frac{(V_8 - S_8)(i\frac{\tau_2}{2})}{\eta^8(\frac{i\tau_2}{2})}, \tag{1.4.5}$$

$$\mathcal{M} = \frac{\varepsilon N}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6} \frac{(\hat{V}_8 - \hat{S}_8)(\frac{i\tau_2}{2} + \frac{1}{2})}{\hat{\eta}^8(\frac{i\tau_2}{2} + \frac{1}{2})}. \tag{1.4.6}$$

The corresponding (loop-channel) UV divergences arise from tadpoles in the NS-NS and R-R sectors, whose cancellation requires $N = 32$, $\varepsilon = -1$. We can obtain the $USp(32)$ model introducing an O9-plane with positive tension and charge together with $\overline{D9}$-branes, yielding a vanishing R-R tadpole. However, the NS-NS tadpole is not cancelled, and thus SUSY is broken at the string scale[4]. This amounts to changing a sign in the Moebius strip amplitude, which now becomes

$$\mathcal{M}_{BSB} = \frac{\varepsilon N}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6} \frac{(\hat{V}_8 + \hat{S}_8)(\frac{i\tau_2}{2} + \frac{1}{2})}{\hat{\eta}^8(\frac{i\tau_2}{2} + \frac{1}{2})}. \tag{1.4.7}$$

In order to cancel the R-R tadpole, $\varepsilon = 1$ and $N = 32$, leading to a $USp(32)$ gauge group. Since the residual tension in the NS-NS tadpole does not cancel, the low-energy physics of this model includes the Einstein-frame runaway exponential potential

$$T\int d^{10}x\sqrt{-g}e^{\gamma\phi}, \qquad \gamma = \frac{3}{2}. \tag{1.4.8}$$

- **0'B model**: Starting from the 0B model, which is described by the torus amplitude

$$\mathcal{T}_{0'B} = \int_{\mathcal{T}} \frac{d^2\tau}{\tau_2^6} \frac{O_8\overline{O_8} + V_8\overline{V_8} + S_8\overline{S_8} + C_8\overline{C_8}}{|\eta(\tau)|^{16}} \tag{1.4.9}$$

[4]More precisely, SUSY is preserved in the closed-string sector, but it is non-linearly realized in the open-sector via "Brane Supersymmetry Breaking".

we can perform an orientifold projection to obtain the $USp(32)$ type 0'B model. This involves adding to half of (1.4.9) the open-sector amplitudes

$$\mathcal{K}_{0'B} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6}(-O_8 + V_8 + S_8 - C_8), \tag{1.4.10}$$

$$\mathcal{A}_{0'B} = \int_0^\infty \frac{d\tau_2}{\tau_2^6} n\overline{n} V_8 - \frac{n^2 + \overline{n}^2}{2} C_8, \tag{1.4.11}$$

$$\mathcal{M}_{0'B} = \int_0^\infty \frac{d\tau_2}{\tau_2^6}\frac{n + \overline{n}}{2}\hat{C}_8, \tag{1.4.12}$$

where tadpole cancellation fixes $n = \overline{n} = 32$ and a $U(32)$ gauge group. The corresponding O9-plane has vanishing tension, and therefore the relevant exponential potential reads

$$\frac{1}{2}T\int d^{10}x\sqrt{-g}e^{\gamma\phi}, \qquad \gamma = \frac{3}{2}. \tag{1.4.13}$$

- **SO(16)×SO(16) model**: Starting from the torus amplitude of the $E_8 \times E_8$ superstring

$$\mathcal{T}_{HE} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6}\frac{(V_8 - S_8)\overline{(O_{16} + S_{16})}^2}{|\eta(\tau)|^{16}}, \tag{1.4.14}$$

we can project onto the states with even total fermion number. So, we have to add to the projected torus amplitude its images under S and T modular transformations in order to restore modular invariance. The final amplitude reads

$$\begin{aligned}\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6}\frac{1}{|\eta(\tau)|^{16}}[&O_8\overline{(V_{16}C_{16} + C_{16}V_{16})} \\ &+ V_8\overline{(O_{16}O_{16} + S_{16}S_{16})} \\ &- S_8\overline{(O_{16}S_{16} + S_{16}O_{16})} \\ &- C_8\overline{(V_{16}V_{16} + C_{16}C_{16})}].\end{aligned} \tag{1.4.15}$$

While tachyons are absent from the perturbative spectrum by virtue of level matching, the 1-loop vacuum energy does not vanish and its value is of order one in string units. In the string-frame low-energy effective action it appears as a cosmological constant, and thus in Einstein frame it corresponds to a runaway exponential potential

$$T\int d^{10}x\sqrt{-g}e^{\gamma\phi}, \qquad \gamma = \frac{5}{2}. \tag{1.4.16}$$

It is therefore evident that the low-energy manifestation of gravitational tadpoles in both the orientifold models and the $SO(16)\times SO(16)$ heterotic model can be encompassed by the same type of exponential potential for the dilaton.

1.4.2 Brane content and low-energy effective description

Similarly to Type I superstring, the $USp(32)$ orientifold model contains charged D_1-branes and D_5-branes, while the remaining values of p correspond to uncharged branes whose stability can be addressed studying tachyonic excitations. In particular, a single D_3-brane and a single D_4-brane are free of tachyons, while all other D_p-branes are unstable due to the presence of tachyons.

The Type 0'B model contains charged D_p-branes with p odd, and while their world-volume excitations are free of tachyons, $D_p - D_q$ exchanges include tachyons for $|p-q| < 4$. Even values of p correspond to uncharged branes. At leading order, the D_p-D_p interaction between charged branes vanishes, but we have to take into account the presence of the $\overline{D9}$-branes and $O9$-plane, which bring along non-trivial contributions.

Let us now introduce the low-energy effective description of our three non-supersymmetric models. Both the orientifold models and the heterotic model can be described at low energies by an Einstein-frame action of the form

$$S = \frac{1}{\kappa_D^2} \int d^D x \sqrt{-g} \left(R - \frac{4}{4-D} (\partial\phi)^2 - V(\phi) - \frac{f(\phi)}{2(p+2)!} H_{p+2}^2 \right) , \tag{1.4.17}$$

where the bosonic fields include a dilaton ϕ and a $(p+2)$-form field-strength $H_{p+2} = dB_{p+1}$. In the relevant string models $D = 10$, while

$$V(\phi) = T e^{\gamma\phi}, \qquad f(\phi) = e^{\alpha\phi}. \tag{1.4.18}$$

The low-energy dynamics of the orientifold models is described by the Einstein-frame parameters $D = 10$, $p = 1$, $\gamma = \frac{3}{2}$, $\alpha = 1$, while for the heterotic model $D = 10$, $p = 1$, $\gamma = \frac{5}{2}$, $\alpha = -1$.

The field equations derived from Eq. (1.4.17) read

$$\begin{aligned} R_{MN} &= \tilde{T}_{MN}, \\ \Box\phi - V'(\phi) - \frac{f'(\phi)}{2(p+2)!} H_{p+2}^2 &= 0, \\ d * (f(\phi) H_{p+2}) &= 0, \end{aligned} \tag{1.4.19}$$

where the trace-reversed stress-energy tensor is

$$\tilde{T}_{MN} = T_{MN} - \frac{1}{D-2} T_A^A g_{MN}, \qquad T_{MN} \equiv \frac{\delta S_{matter}}{\delta g^{MN}}. \tag{1.4.20}$$

For the action in Eq.(1.4.17), this implies

$$\tilde{T}_{MN} = \frac{4}{D-2} \partial_M \partial_N \phi + \frac{f(\phi)}{2(p+1)!} (H_{p+2}^2)_{MN} \tag{1.4.21}$$

$$+ \frac{g_{MN}}{D-2} \left(V - \frac{p+1}{2(p+2)!} f(\phi) H_{p+2}^2 \right) . \tag{1.4.22}$$

Let us now study the interactions between these branes. There are some complementary regimes in which computations seem to be under control. In particular, considering two parallel stacks of N_p D_p-branes and N_q D_q-branes, some interesting cases are

- The probe regime $N_p \gg N_q$, in which we can replace the heavy stack of N_p D_p-branes with the corresponding back-reacted geometry probed by the D_q-branes.
- The string-amplitude regime $N_p, N_q = \mathcal{O}(1)$, which needs to be described by perturbative string amplitudes
- The holographic regime, which is negligible for the purpose of this work.

Extremal branes of equal dimension strictly repel, realizing the WGC[5] in the absence of Supersymmetry, while the NS-NS interactions in the presence of at least one uncharged stack are repulsive or attractive depending on the values of p and q [32].

1.4.3 Probe potentials and Weak Gravity

We will focus only on the probe regime, thus replacing the heavy brane stack with its backreacted geometry. We will consider a string-frame world-volume action of the form

$$S_p = -T_p \int d^{p+1}\zeta \sqrt{-j^* g_S} e^{-\sigma\phi} + \mu_p \int B_{p+1}, \tag{1.4.23}$$

where j is the embedding of the world-volume coordinates ζ in space-time. Its Einstein-frame expression reads

$$S_p = -T_p \int d^{p+1}\zeta \sqrt{-j^* g_S} e^{\left(\frac{2(p+1)}{D-2} - \sigma\right)\phi} + \mu_p \int B_{p+1}, \tag{1.4.24}$$

where $\sigma = 1, 2$ for D-branes and $NS5$-branes respectively.

The crucial point is that the heavy stack sources the $AdS \times S$ throat probed by the light stack. Consider the dynamics of an extremal D_p-brane moving in an $AdS_{p+2} \times S^q$ geometry. The dynamics at stake emerge spontaneously, since bubble nucleation entails separation of pairs and antipairs of branes. It can be shown that like-charge branes are repelled, while anti-branes are attracted, leading to brane-flux annihilation.

For convenience, let us work in Poincaré-like coordinates, where the Einstein-frame metric of the $AdS \times S$ throat reads

$$ds^2 = \frac{L^2}{z^2}(dz^2 + dx_{1,p}^2) + R^2 d\Omega_q^2. \tag{1.4.25}$$

[5]See section 1.5

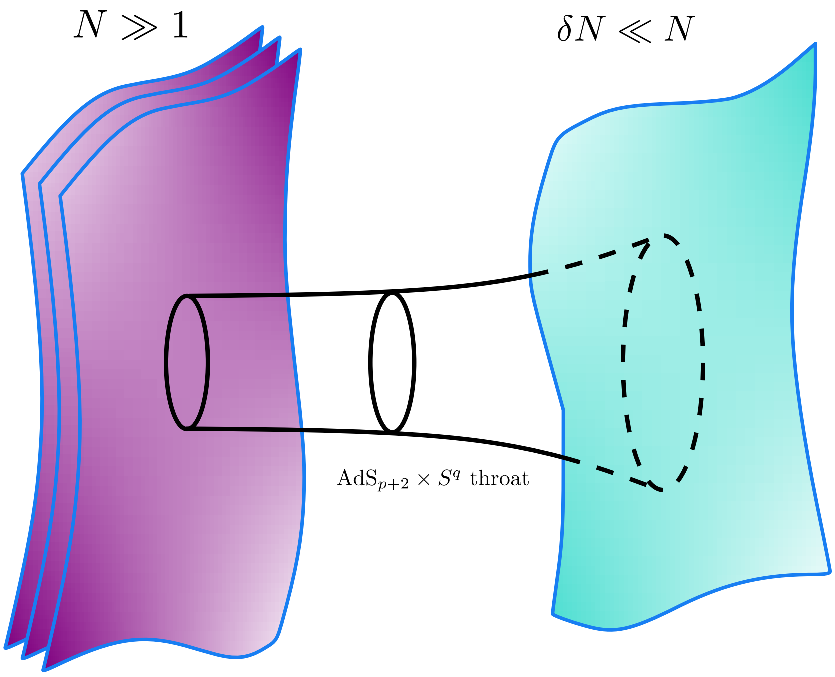

Figure 1.3: Schematic representation taken from [32] of the interaction between a heavy stack of $N \gg 1$ branes and $\delta N \ll N$ probe branes. The heavy stack sources the $AdS \times S$ throat probed by the light stack.

We can choose the world-volume embedding $j : x^\mu = \zeta^\mu, z = Z(\zeta), \theta^i = \theta_0^i$, with θ_0^i fixed coordinates on S^q. The action (1.4.24) then becomes

$$S_p = -\tau_p \int d^{p+1}\zeta \left(\frac{L}{Z}\right)^{p+1} \left[\sqrt{1 + \eta^{\mu\nu}\partial_\mu Z \partial_\nu Z} - \frac{cL}{p+1}\frac{\mu_p}{\tau_p}\right], \tag{1.4.26}$$

with the dressed tension $\tau_p \equiv T_p g_s^{-\frac{\alpha}{2}}$. Therefore, rigid branes are subject to the potential

$$\begin{aligned} V_{probe}(Z) &= \tau_p \left(\frac{L}{Z}\right)^{p+1} \left[1 - \frac{cLg_s^{\frac{\alpha}{2}}}{p+1}\frac{\mu_p}{T_p}\right] \\ &= \tau_p \left(\frac{L}{Z}\right)^{p+1} \left[1 - \nu_0 \frac{\mu_p}{T_p}\right]. \end{aligned} \tag{1.4.27}$$

The presence of such a potential will reveal to be essential for the Dark Bubble construction in Type 0'B String Theory.

1.5 The Swampland Program

We conclude this chapter by introducing one of the main approaches to investigate the nature of Quantum Gravity: the so-called *Swampland program*.

As we discussed, String Theory is a theory of Quantum Gravity. Seeing things from a bottom-up perspective, we can start from QFT and build all kinds of low-energy EFTs. Going up with the energy scales, gravitational quantum effects start to be relevant and we have observed that many – actually, most – of these effective theories turn out to be inconsistent when coupled to gravity. In other words, *not everything is possible in Quantum Gravity.*

One might have hoped that String Theory would have been sufficiently constrained to single out some very specific low-energy EFT as the only one compatible with Quantum Gravity. This may still be the case, but we do not yet *fully* understand String Theory. Within our current understanding, there is no principle which we know that can pick out a specific such theory. Rather, it appears that the range of low-energy effective theories that can arise in String Theory is huge. This is the so-called *Landscape* of String Theory. Ironically, while the Landscape is huge, there is still not a single known way to embed the Standard Model, of particle physics and cosmology, in String Theory. So while our universe is in principle consistent with String Theory, in practice we still do not know how its embedding could work in detail. It is therefore an important and interesting task to work out the 'details' of this embedding, and this is a large part of the research field of String Phenomenology.

The so-called Swampland program aims to determine the constraints that an EFT *must* satisty in order to be consistent with a UV embedding in a Quantum Gravity theory. These constraints are formulated in the form of Swampland conjectures. The theories which do not respect these conjectures are said to be inconsistent or *in the Swampland*, while the "good" ones are said to belong to the Landscape.

The goal of the Swampland program is to identify Quantum Gravity constraints, gather evidence to prove (or discard) them in a Quantum Gravity framework, try to explain them in a model-independent way (microscopic interpretation) and understand their phenomenological implications for low-energy physics. Even if the notion of the Swampland is not restricted to String Theory only, the various conjectures are often motivated by or checked in String Theory setups. In fact, String Theory represents the perfect setup to rigorously test these conjectures, improving our understanding of new possible String Theory compactifications.

A remarkable insight of the Swampland program is that it breaks with the logic of naturalness, which is based on scale separation. Indeed, if Quantum Gravity imposes non-trivial constraints on the IR (the low-energy physics), this constitutes the UV/IR mixing which could explain the hierarchy problems that we observe in nature.

We will now briefly examine the most relevant conjectures, putting aside even the slightest attempt of being exhaustive. For a detailed review of the Swampland program, we refer the reader to [33], [34] and [35].

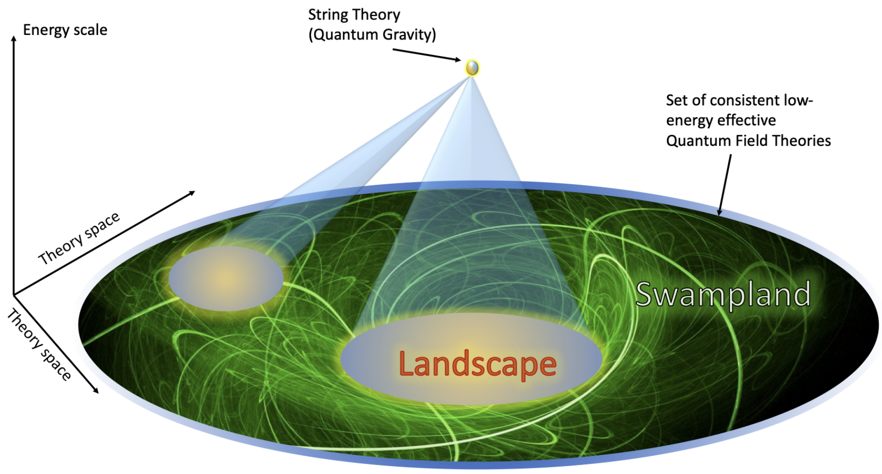

Figure 1.4: *Image taken from [33], which shows the set of low-energy EFTs which are inconsistent with Quantum Gravity and are therefore in the Swampland. As we increase the energy scale, only the theories which obey the Swampland constraints are consistent and connected to Quantum gravity.*

- **No global symmetries**: there are no global symmetries in Quantum Gravity (any symmetry is either broken or gauged).

  We define a global symmetry as a transformation described by a unitary local operator $U(g), g \in G$ such that:

  1) satisfies a group law: $U(g)U(g') = U(gg')$,

  2) acts non-trivially on the Hilbert space: there exists a charged local operator $\mathcal{O}(x)$ such that $U^\dagger O(x)U(g) \neq O(x)$,

  3) commutes with the Hamiltonian (and thus the stress-energy tensor is neutral) $U^\dagger T_{\mu\nu} U = T_{\mu\nu}$,

  4) maps local operators into local operators.

- **Completeness of spectrum**: A gauge field coupled to gravity must contain physical states with all possible gauge charges consistent with Dirac quantisation. Quantum Gravity allows for a spectrum which contains all charged states of a given representation (according to Dirac's quantisation). As a consequence of this conjecture, all continuous gauge groups must be compact.

  There is a generalisation of this conjecture to include BPS states, which goes under **BPS completeness**: if a charge can be populated by BPS states, then a BPS state with this charge should be part of the physical spectrum.

- **Cobordism conjecture**: It unifies the previous two. Two manifolds are said to be *cobordant* (or in the same cobordism class) if their union is the boundary of another compact manifold one dimension higher. Consider a (D-d)-dimensional theory, where d is the number of compactified dimensions. Then all the cobordism classes Ω_d^{QG} of the d-dimensional theory must vanish, otherwise they would give rise to a (D-d-1)-form global symmetry with charges $M \in [\Omega_d^{QG}]$. It is equivalent to say that Quantum Gravity is topologically trivial.

- **Weak Gravity Conjecture (WGC)**: It provides phenomenological evidence by providing an upper bound on the mass of the light charged states which appear when trying to restore a global symmetry. Considering Maxwell theory coupled to Einstein gravity (no massless scalar fields), the bound implies that gravity acts weaker than the gauge force over this state (the charge is greater than the mass). This is no longer true in the presence of massless scalar fields.

  Electric WGC: given a gauge theory, weakly coupled to Einstein gravity, there exists an electrically charged state with

$$\frac{Q}{m} \geq \frac{\mathcal{Q}}{M}|_{extremal} = \mathcal{O}(1) \tag{1.5.1}$$

  in Planck units. $\mathcal{Q}$ and M are the charge and the mass of an extremal BH, and $\mathcal{Q} = qg$, where q is the quantized charge and g the gauge coupling.

  Magnetic WGC: The EFT cutoff Λ is bounded from above by the gauge coupling

$$\Lambda \leq g M_P^{(d-2)/2}, \tag{1.5.2}$$

  so that, given some EFT coupled to gravity, its cutoff is smaller than M_P if the gauge coupling is small. For a p-form gauge field

$$\Lambda \leq (g^2 M_P^{(d-2)})^{\frac{1}{2(p+1)}}., \tag{1.5.3}$$

  We will make great use of the WGC for the purpose of the Dark Bubble model: generalizing to a p-form gauge field, we automaticsally derive the existence of a $(p-1)$-brane of tension T satistfying

$$f^{ab} q_a q_b \geq g^{ij} (\partial_{\phi^i} T)(\partial_{\phi^j} T) + \frac{p(d-p-2)}{d-2} T^2, \tag{1.5.4}$$

  where the gauge charge is written explicitly in terms of the gauge kinetic function f_{ab} and gauge quantized charges q_a.

- **Distance conjecture**: The moduli space is non-compact. Given a point P in the moduli space, there always exsits a point Q such that it is at infinite-distance from the P. Moreover, if we try to approach any infinite field distance limit, we get an infinite tower of states becoming exponentially light:

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi}, \tag{1.5.5}$$

as $\Delta\phi \to \infty$. Moreover, as we approach the long distance limits, our EFTs break down. We can therefore express the distance conjecture in terms of the Quantum Gravity cutoff, which goes exponentially to zero as $\Lambda_{QG} = \Lambda_0 e^{-\lambda\Delta\phi}$. As a consequence, we get that EFTs are only valid for finite scalar field variations. Considering that $\Lambda_0 \leq M_p$, we get $\Delta\phi \leq \frac{1}{\lambda}\log\frac{M_p}{\Lambda}$, we see that the maximum field variation actually depends on the cutoff of the EFT. So the higher the cutoff of the process changing the vev of the scalar, the smaller is the maximum field distance that can be described within the effective field theory.

- **Emergent String Conjecture**: Any infinite distance limit is either a decompactification limit or a limit in which there is a weakly coupled string becoming tensionless. This means that the leading tower becoming light is either a KK tower or some string excitation modes, such that the resolution in Quantum Gravity of the EFT breaking down is given by growing an extra dimension or by considering a string perturbation theory.

- **AdS distance conjecture**: It's a generalization of the Distance conjecture. It states that any AdS vacuum has an infinite tower of states becoming light in the flat space limit $\Lambda \to 0$, satisfying

$$m \sim |\Lambda|^{\alpha}. \tag{1.5.6}$$

  A strong version of this conjecture implies that $\alpha = \frac{1}{2}$ if the vacuum is supersymmetric, $\alpha \geq \frac{1}{2}$ for non-supersymmetric AdS and $\alpha \leq \frac{1}{2}$ for dS space.

- **Emergence Proposal**: All the kinetic terms in an EFT emerge from integrating out the massive states up to some Quantum Gravity cut-off. This implies that all fields are non-dynamical at the Quantum Gravity scale where gravity becomes strongly coupled, hinting at some sort of UV topological description. This means that there is no kinetic term to start with; it is only upon going to the IR and integrating out the tower of states predicted by the Distance conjecture, that we get some finite kinetic terms.

- **AdS Instability Conjecture**: This conjecture, originally applied to non-SUSY AdS vacua, has been generalized to the fact that any non-SUSY vacuum is at best metastable and has to decay eventually. This means SUSY is the only mechanism to protect a vacuum from decaying in Quantum Gravity. Instabilities typically arise when SUSY is broken, and can be perturbative or non-perturbative.

This conjecture is closely related to the WGC, as the latter implies the presence of a codimension-1 brane with a tension smaller than its charge in non-SUSY vacua, which describes a bubble instability. According to this argument, *any AdS non-SUSY vacuum with fluxes should be at best metastable.* Useless to say, a metastable non-SUSY AdS vacuum will be the starting point of our brane world construction.

- **dS conjecture** A scalar potential of an EFT weakly coupled to Einstein gravity must satisfy

$$M_P \frac{|\nabla V|}{V} \geq c, \tag{1.5.7}$$

where c is some $\mathcal{O}(1)$ constant. This implies that no dS vacua can exist in a weakly coupled regime.

This is the most controversial and less accepted conjecture in the scientific community. It was actually further refined by stating that the previous bound only needs to be imposed if the following condition on the second derivative of the potential is violated

$$\min(\nabla_i \nabla_j V) \leq -\frac{c' V}{M_P^2}, \tag{1.5.8}$$

with c' another $\mathcal{O}(1)$ constant. This way, only dS minima (and not critical points in general) are ruled out.

The dS conjecture is the reason to seek alternatives to describe our expanding universe without the need to compactify to a stable or metastable dS vacua in String Theory.

- **Transplanckian Censorship Conjecture (TCC)**: The expansion of the universe must slow down before all Planckian modes are stretched beyonf the Hubble size. This has two implications:

1) No dS minima can exist at the asymptotic boundaries of the moduli space. In the asymptotic regimes, one recovers a bound constraining the asymptotic behaviour of the potential

$$\frac{|\nabla V|}{V} \geq \frac{2}{\sqrt{(d-1)(d-2)}}. \tag{1.5.9}$$

2) A dS minimum can exist deep in the bulk, but it must be short-lived. The lifetime τ for a metastable dS vacuum is bounded from above by

$$\tau \leq \frac{1}{H} \log \frac{M_P}{H}, \tag{1.5.10}$$

where H is the Hubble scale.

The TCC is weaker than the dS conjecture and does not completely forbid the existence of dS vacua, but only does so asymptotically.

As more and more evidence for the Swampland conjectures is gathered from String Theory setups, black hole physics and AdS/CFT, researchers are discovering an unexpected interplay of connections between these conjectures, which perhaps stems from some kind of *fundamental Swampland principle* pointing directly at Quantum Gravity. Finally, let us pose to ourselves a very important question: how universal is String Theory? This question is coupled to the Swampland program, since much of the evidence that we have for the conjectures comes from String Theory. Therefore, at some point we must ask whether our results are just an artifact of the lamppost we are looking under, or whether the conjectures are more general and are actually reflecting inconsistencies in Quantum Gravity. If we can show that the anomaly free theories which do not appear in String Theory, do not appear for some underlying QG reason, namely they are inconsistent with a Swampland conjecture which is expected to be more general, this could imply that we really get everything we can get in String Theory. At present, this seems to be the case for highly supersymmetric setups. The idea that everything that can possibly happen (i.e. that is not inconsistent with QG) does happen in String Theory is called **String Universality** or the **String Lamppost Principle**. Certainly, it would be very interesting indeed if this is the case and any consistent Quantum Gravity theory is somehow connected to String Theory!

Chapter 2

Dark Bubble Cosmology

In this Chapter, we thoroughly examine the Dark Bubble scenario, explaining its most relevant features and mechanisms. In particular, the focus will be on the way gravity behaves on the brane world and on the induced cosmology on the bubble.

2.1 Introduction to the model

As stated in Chapter 1, consistency of the worldsheet theory requires that the total number of dimensions in Superstring Theory be $d = 10$. The standard approach to deal with the six extra dimensions that we do not perceive is to find a time-independent compactification and extract low-energy four-dimensional effective field theories to describe our universe, which we know is undergoing a phase of accelerated expansion (de Sitter phase). However, the persistent difficulties in constructing a stable dS vacuum in String Theory have led to conjecture that no (meta)stable dS vacuum can exist, at least at the boundary of moduli space (see Section 1.5). AdS vacua, on the other hand, are common in String Theory but there are reasons to believe that, unless they are supersymmetric, they are unstable and must decay either perturbatively or non-perturbatively.

An alternative approach to describe our universe without dealing with the compactification of these extra dimensions are the so-called brane world scenarios, in which our Universe is embedded in a higher-dimensional spacetime (the bulk) and the Standard Model of Particle Physics (or a generalization thereof) lives on a stack (D-)branes in a the bulk.

There are many interesting brane world models which deserve to be studied and investigated (see [36, 37] as an example). In this thesis, we will focus on the recent model of Dark Bubble Cosmology, first proposed in 2019 by Ulf Danielsson, Souvik Banerjee, Giuseppe Dibitetto, Suvendu Giri and Marjorie Schillo [11].

In this scenario, which combines together String Theory and cosmology in a beautiful and intricate interplay, our universe rides a 4d bubble in a 5d Anti de Sitter spacetime.

The latter, indeed, is unstable and decays non-perturbatively[1], nucleating bubbles of true[2] vacuum. The Dark Bubble (DB) proposal therefore involves inducing 4d gravity on a D_3-brane, which corresponds to an expanding Coleman-de Luccia bubble [38] and mediates the decay from a non-supersymmetric false AdS_5 vacuum to a lower energy vacuum. The DB scenario thus comes naturally whenever there is a codimension-1 brane present in the theory that can mediate the decay of the false AdS_5.

The DB model differs from the Randall-Sundrum (RS) construction [36], in which two insides of the bubble are glued together. Indeed, the Dark Bubble has an inside and an outside. This changes the way gravity is realized on the brane, and needs the presence of a growing mode in the bulk spacetime outside the bubble. This leads to 4d matter being endpoints of strings stretching along the 5th dimension, whereas in the RS model the 4d matter is localized on the brane.

Lower-dimensional observers (us) are confined on a (3+1)-dimensional brane and perceive Einstein gravity as an effective 4d theory.

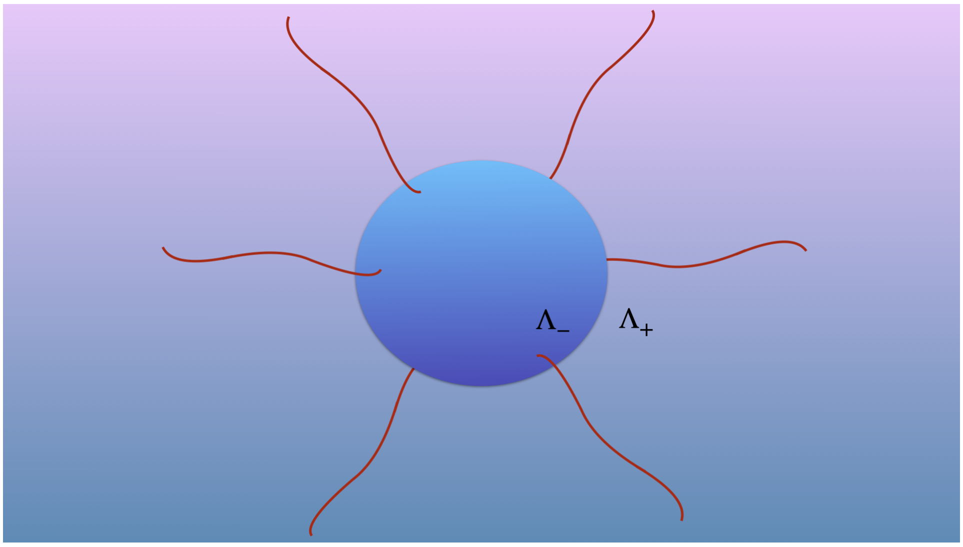

Figure 2.1: *Pictorial representation of the Dark Bubble, with an inside/outside construction such that $\Lambda_- < \Lambda_+$. Strings extending all the way to the UV at infinity correspond to massive particles in 4d.*

Gravity on the bubble is spin-2 and *not* spin-0. The confusion may arise from the fact that one may wrongly consider that gravity is localized, and not considering that sources, such as stretched strings, holographically extend in the 5th direction (which is transverse to the DB). These extended sources generate growing modes in the bulk outside of the

[1]By the WGC, all non-supersymmetric AdS vacua must decay.

[2]There is no guarantee that the nucleated bubble contains a true vacuum; the best we can say is that the nucleated bubbles are bubbles of lower energy vacuum.

bubble, which then yields a spin-2 graviton. Strings are essential to extract a consistent effective theory of gravity on the brane. The value of G_4 is finite and independent of the possibly infinite volume of the extra dimensions[3]. Sources extend all the way to the UV at infinity, and affect the spacetime all the way through non-normalizable modes. It is possible to introduce a cutoff to restore normalizability, and low-energy physics is not affected by this. Indeed, the mass of the matter fields turns out to be independent of the size of the extra dimension through mass renormalization as the bubble traverses along the extra dimension.

Uplifting gravitational waves on the DB to 5d and computing their backreaction on 4d is equivalent to the backreaction considered purely in 4d. The crucial point we need to emphasize is that 4d gravity arises as an effective description on a dynamical object embedded in a higher dimensional space time. Indeed, the 4d Einstein equation follows from the junction condition across the brane. The brane geometry is sourced by the energy-momentum tensor of the brane itself (which for empty branes acts as a cosmological constant) and by contributions from higher dimensional geometry. When adding matter to the brane, the full back-reacted solution needs to be considered. This leads to a net *positive* energy density, taking into account the extrinsic curvature through the junction condition.

Stretched strings pulling on the brane look like particles in 4d. We can express the mass of a particle in terms of the string tension. If we put a mass on top of the brane, ignoring the backreaction, we would expect the brane to bend down towards the inside. In RS there are two insides glued to each other, while in the DB model we have an inside and an outside. This means the bending of the brane can be neatly explained by a pulling string. If we have a string cloud (corresponding to dust in 4d), the pulling is fully accounted for by the extrinsic curvature, reproducing FRLW cosmology with a positive mass density.

The DB model realizes an effective dS vacuum[4] with lower dimensional observers confined to the bubble boundary, where they perceive an expanding FLRW cosmology with a positive cosmological constant. Furthermore, an observer living on the DB sees that it lasts longer than the Hubble time. The time scale relevant is the proper time of the brane. For a large bubble there is a huge blue-shift due to the metric factor, which ensures that cosmological times will pass while the proper radius of the bubble only increases by the AdS radius.

Finally, we notice that the dynamics of the DB is perfectly consistent with Vilenkin's tunneling proposal in quantum cosmology, with the bubble nucleation in 5d AdS spacetime being identified with a 4d Big Bang event. The amplitude of bubble nucleation in 5d matched identically with Vilenkin's tunneling amplitude in 4d quantum cosmology.

[3]There is no simple way to render G_4 finite without the stringy sources of the DB.

[4]In the DB model, as soon as we have a tension of the brane that is smaller than the critical value, we automatically get dS.

This tunneling dynamics was embedded in a stringy model in order to obtain a small, positive cosmological constant compatible with observations. What a lower-dimensional observer would call the Big Bang has the bulk interpretation of a well-understood nucleation event à la Brown-Teitelboim [39]. From the higher dimensional perspective, the Big Bang does *not* appear as a singularity.

Let us now jump onto the Dark Bubble and, holding tightly to stretched strings, prepare to embark on a cosmic ride.

2.2 Gravity on the Braneworld

In order for this composite spacetime to be a solution of the Einstein equations, the stress-energy tensor on the shell needs to source a jump in the extrinsic curvature. The Israel junction conditions [40] imply

$$\sigma = \frac{3}{8\pi G_5}\left(\sqrt{k_-^2 + \frac{1+\dot{a}^2}{a^2}} - \sqrt{k_+^2 + \frac{1+\dot{a}^2}{a^2}}\right), \tag{2.2.1}$$

where σ is the tension of the brane supporting the shell, while the cosmological constants inside and outside the shell are given by $\Lambda_\pm = -6k_\pm^2 = -6/L_\pm^2$ and $\Lambda_- < \Lambda_+ < 0$ (or $k_- > k_+$). The vacuum with higher energy can then decay through the nucleation of a spherical Brown-Teitelboim instanton. In terms of the proper time τ on the shell located at $r = a(\tau)$, the induced metric on the shell has a FLRW form:

$$ds^2_{shell} = -d\tau^2 + a(\tau)^2 d\Omega_3^2. \tag{2.2.2}$$

For large $k_\pm$, we find that

$$\frac{\dot{a}^2}{a^2} \simeq -\frac{1}{a^2} + \frac{8\pi G_4}{3}\Lambda_4, \tag{2.2.3}$$

which is just the Friedmann equation in the presence of a positive cosmological constant Λ_4 given by

$$\Lambda_4 = \frac{3(k_- - k_+)}{8\pi G5} - \sigma. \tag{2.2.4}$$

Moreover, we can perform the following identification

$$G_4 = \frac{2k_-k_+}{k_- - k_+} G_5. \tag{2.2.5}$$

Gravity on the shell is described by 4d Einstein's equations plus high energy corrections. In general, the resulting Friedmann equation will be non-linear in the shell tension. However, when the 5d cosmological constants $k_\pm$ are large compared to the 4d Hubble parameter, the tension of the shell approaches from below the extremal(critical) tension, which results in a flat shell

$$\sigma_{cr} = \frac{3(k_- - k_+)}{8\pi G_5}. \tag{2.2.6}$$

Expanding in $\epsilon = 1 - \sigma/\sigma_{cr}$, we recover the the usual Friedmann equation plus small corrections which are independent of $a(\tau)$

$$\frac{\dot{a}^2}{a^2} = -\frac{1}{a^2} + \frac{8\pi G_4}{3}\Lambda_4 + O(\epsilon^2), \tag{2.2.7}$$

where $\Lambda_4 = \sigma_{cr} - \sigma$. This represents a dS universe with positive spatial curvature.

This shows that in order to have an expanding dS bubble, the tension must be *subcritical* ($\sigma < \sigma_{cr}$). The bubble nucleates with $\dot{a} = 0$, with its radius set by the 4d cosmological constant. As a consequence, the universe starts with a size comparable to the horizon scale, with the subsequent expansion further reducing the curvature.

2.2.1 Friedmann Cosmology

Let us now examine in detail how to obtain a dS_4 cosmology as the induced 4d metric on a codimension-1 bubble in AdS_5. The 5d bulk geometries inside and outside the bubble correspond to AdS_5 vacua:

$$ds^2_\pm = g^\pm_{\mu\nu} dx^\mu dx^\nu = -f(z)_\pm dt^2 + \frac{dz^2}{f(z)_\pm} + z^2 d\Omega_3^2, \tag{2.2.8}$$

where $+$ or $-$ refer to the inside and the outside of the bubble respectively, $d\Omega_3^2 = \gamma_{ij} dx^i dx^j$ is the metric on S^3 and $f_\pm$ for pure AdS_5 is given by

$$f(z)_\pm = 1 + k_\pm^2 z^2. \tag{2.2.9}$$

The constant k defines the AdS_5 scale as $L_{AdS} = 1/k$ and the 5d cosmological constant is given by $\Lambda_5 = -6k^2$. A false (outside) AdS_{5_+} vacuum can decay to a true (inside) AdS_{5_-} vacuum via the nucleation of a spherical Brown-Teitelboim instanton provided $k_- > k_+$. Once nucleated, the bubble expands rapidly eating all of AdS_{5_+} in finite time.

The bubble can be described by specifying its radius $z = a(\tau)$, where τ is some time parameter on the bubble (proper time on the shell). We will assume the bubble to be sufficiently large: $ka \gg 1$. The induced metric on the bubble wall is exactly of the FRLW form:

$$ds^2_{ind} = -N^2(\tau) d\tau^2 + a(\tau)^2 d\Omega_3^2, \tag{2.2.10}$$

where we introduced the lapse-function N to make time reparametrization invariance manifest. The relation between the bulk time t and the brane time τ is

$$N^2(\tau) = f(a)\dot{t}^2 - \frac{\dot{a}^2}{f(a)} \qquad (2.2.11)$$
$$= f(a)\left(\frac{\partial t}{\partial \tau}\right)^2 - \left(\frac{\partial a}{\partial \tau}\right)^2 \frac{1}{f(a)}.$$

Assuming only spherical symmetry of the instanton and a brane of constant tension σ, Israel's junction condition gives a I order equation for the evolution of the radius, $a(\tau)$ (expansion of the bubble):

$$\sigma = \frac{3}{\kappa_5}\left(\sqrt{\frac{f_-(a)}{a^2} + \frac{\dot{a}^2}{N^2 a^2}} - \sqrt{\frac{f_+(a)}{a^2} \frac{\dot{a}^2}{N^2 a^2}}\right), \qquad (2.2.12)$$

where σ is the tension of the bubble wall. Expanding the square root, the first Friedmann equation becomes

$$\frac{1}{N^2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_4}{3}\rho_\Lambda - \frac{1}{a^2}, \qquad (2.2.13)$$

where the 4d gravitational constant is simply

$$\kappa_4 = \frac{2k_- k_+}{k_- - k_+}\kappa_5, \qquad (2.2.14)$$

and where the 4d cosmological constant is determined by $\rho_\Lambda = \sigma_{cr} - \sigma$.

σ_{cr} is the critical brane tension at which the bubble remains static

$$\sigma_{cr} = \frac{3}{\kappa_5}(k_- - k_+). \qquad (2.2.15)$$

The critical tension σ_{cr} can be obtained in the limit that all 4d energy scales (set by curvature and the Hubble scale) are small compared to 5d scales ($k \gg 1/a, k \gg \dot{a}/a$): the brane appears as a spatially Minkowski space with a critical tension σ_{cr}. The Friedmann equation only admits real solutions if $\sigma < \sigma_{cr}$. From the 5d perspective, this means that bubbles with a tension greater than the critical one simply cannot nucleate.

The presence of mass in the bulk modifies the AdS_5 metric, resulting in a Schwarzschild-AdS solution where $f(r)$ becomes

$$f(r) = 1 + k_\pm^2 r^2 - \frac{8G_5 M_\pm}{(3\pi r^2)}, \qquad (2.2.16)$$

where M_- and M_+ are the masses measured inside and outside the bubble, respectively. Through the space-space component of the junction condition, we see that matter in the bulk contributes with a radiation term to the Friedmann equations

$$\frac{\dot{a}^2}{a^2} \simeq -\frac{1}{a^2} + \frac{8\pi G_4}{3}\left[\Lambda_4 + \frac{1}{2\pi^2 a^4}\left(\frac{M_+}{k_+} - \frac{M_-}{k_-}\right)\right], \qquad (2.2.17)$$

where we drop terms higher order in ϵ and $M_\pm/k_\pm$. The time-time component of the junction condition (i.e. the second Friedmann equation) can be combined with the first Friedmann equation to reproduce the 4d continuity equation on the shell. We can therefore identify the gravitational backreaction of bulk matter as the source of an effective energy density with a radiation equation of state on the shell. If $M_+ > 0$ on the outside but $M_- = 0$ on the inside, we get a positive density of radiation.

Adding matter is more complicated. As we have seen, 5d matter confined to the shell yields a $1/a^4$ contribution to the Friedmann equation. The way to get a matter contribution that goes like $1/a^3$ is to construct massive particles as strings ending on the shell. The metric is then given by $m_-(r) = 0$ for $r < a$ and $m_+(r) = 0$ for $r > a$, where η is the effective tension of the strings. This gives an effective 4d matter with density $\rho = \eta/a^3 k_+$ on the shell. As the shell climbs up the throat, it eats the strings and the massive particles represented by the endpoints are supplied with the required potential energy to keep a constant rest mass.

If all of these massive particles annihilate into massless radiation on the shell, by 5d energy conservation, m_+ just outside the shell would be equal to the total mass of the strings that vanished. Moreover, m_-, evaluated on the shell, will increase dramatically to represent the mass that was captured by the shell. The 4d observer only feels the difference $(m_+ - m_-)$, which will be determined via 4d energy conservation.

In this way, all processes on the shellworld will be like shadows of processes taking place in 5d involving much larger energies.

To get a massive particle, we need a string that pulls upwards from the brane. With a homogeneous distribution of these strings, one reproduces the Friedmann equations in the presence of dust. The DB "eats" the strings as it expands. The energy from the strings is used so that the effective energy density on the bubble (and thus H^2), decays only as $1/a^3$ rather than $1/a^4$, as in the case of radiation:

$$H^2 = \frac{\dot{a}^2}{a^2} \simeq -\frac{1}{a^2} + \frac{8\pi G_4}{3}\left[\Lambda_4 + \frac{1}{2\pi^2 a^4}\left(\frac{M_+}{k_+} - \frac{M_-}{k_-}\right) + \frac{3}{8\pi a^3}\frac{\tau}{k_+}\right]. \tag{2.2.18}$$

Thus, 4d physics is not localized to the shell; instead, it is described by the full 5d bulk in which the strings stretch. The effective mass of a particle is given by τL_{AdS}, where τ is the string tension and $L_{AdS} = \frac{1}{k_+}$. therefore, the mass is independent of the length of the string.

Putting everything together, if we consider a more general metric corresponding to a gas of strings in Schwarzschild-AdS space, $f(r)$ will be given by

$$f(r)_\pm = 1 + k_\pm^2 r^2 - \frac{\kappa_5 M_\pm}{3\pi^2 r^2} - \frac{\kappa_5 \alpha_\pm}{4\pi r}, \tag{2.2.19}$$

and, using again the junction condition, we can identify several different contributions to Friedmann equation

$$\frac{1}{N^2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_4}{3}\rho_\Lambda + \frac{\kappa_4}{3}\rho_r a^{-4} + \frac{\kappa_4}{3}\rho_m a^{-3} - \frac{1}{a^2}, \tag{2.2.20}$$

where the vacuum energy ρ_Λ, the radiation density ρ_r and the matter density ρ_m find their origin in the bulk geometry:

$$\rho_\Lambda \approx \sigma_{cr} - \sigma \qquad \rho_r \approx \frac{1}{2\pi^2}\left(\frac{M_+}{k_+} - \frac{M_-}{k_-}\right) \qquad \rho_m \approx \frac{3}{8\pi}\left(\frac{\alpha_+}{k_+} - \frac{\alpha_-}{k_-}\right). \tag{2.2.21}$$

We conclude that a bulk black hole with mass M gives rise to radiation in the 4d world, while a gas of stretched strings with average density α gives rise to dust.

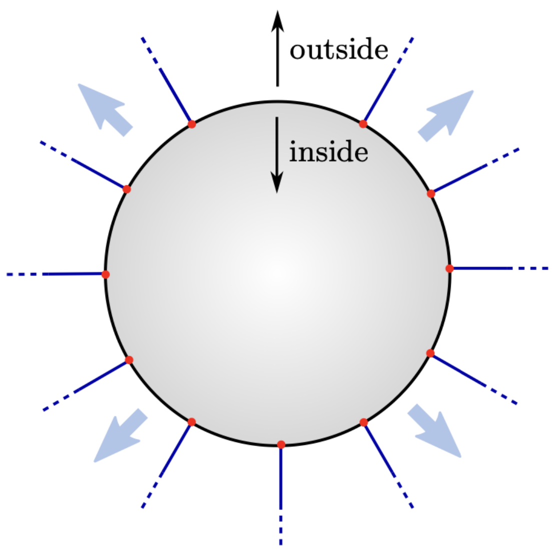

Figure 2.2: *Radially stretched strings attached to the expanding bubble correspond to massive particles in the 4d Universe on the Dark Bubble.*

Examining the size of the cosmological constant relative to our 4d reduced Planck mass, we find

$$\frac{\Lambda_4}{M_4^4} = 256\pi^2 G_5^2 \sigma_{cr}\epsilon\left(\frac{k_- k_+}{k_- - k_+}\right)^2 \sim \epsilon k^3 M_5^{-3}, \tag{2.2.22}$$

where we used for simplicity to assume no large hierarchy between the interior and the exterior AdS radii, leading to the approximation $k_- \sim k_+ \sim (k_- - k_+) \sim k$. Thus, we see that as long as the 5d spacetime is weakly curved ($k < M_5$), the 4d cosmology will be weakly curved as well. Moreover, we are interested in the case of a phenomenologically relevant 4d cosmology, i.e. one with a cosmological constant problem, but we would like the fundamental AdS_5 vacuum to be natural. In this case, a modest hierarchy $k < M_5$ implies a tuning of the brane tension to be very nearly critical with $\epsilon \sim 10^{-120}$. This precise tuning, where the 4d cosmology presents the observed cosmological constant problem but the 5d vacuum has no extreme hierarchies, guarantees that the lifetime of a typical bubble before collisions with another bubble is longer than the 4d Hubble time.

We will consider collisions in a metastable AdS_+ vacuum with initial conditions defined such that at some time $t = 0$ the entire space is in the false vacuum. Then, not only is a bubble guaranteed to nucleate as long as the decay rate is finite, but due to the infinite spatial volume of the $t = 0$ surface, *an infinite number of decays will occur.* Choosing the bubble where our universe lives to be at the center of AdS means that at the same time another bubble will nucleate near the boundary of AdS. because the bubbles undergo constant proper acceleration, asymptoting to a lightcone, they will collide in a time of order the AdS radius: $t_{collision} \sim L_+$, measured in the global time coordinate (2.2.8).

It is interesting to compare this to the Hubble time, given in a 4d observer's proper time $\tau_H = H^{-1}$. The relation between these two coordinates can be found by insisting that a point on the brane follows a timelike trajectory and results in

$$\frac{dt}{d\tau} = \frac{\sqrt{f(r) + (\partial_r r)^2}}{f(r)}. \tag{2.2.23}$$

Neglecting the spatial coordinate so that H is constant and $r(\tau) = r_0 e^{H\tau}$, one readily finds global time as a function of proper time. Again, neglecting the decaying spatial curvature, we have $H \approx \sqrt{\Lambda_4/M_4} \approx \sqrt{\epsilon}k$. Since the bubble nucleates at rest, we also have $r_0 = H^{-1} = \tau_H$. Then, converting to Hubble time in global coordinates, we find $t_H = (e-1)/ek + O(\epsilon)$. Therefore

$$\frac{t_H}{t_{collision}} \approx \frac{e-1}{e} \approx 0.6, \tag{2.2.24}$$

which indicates that we expect the braneworld cosmology to remain relatively uneventful for longer than the age of the universe, but one cannot wait an arbitrarily long time before a collision.

This result strongly depends on our choice of boundary conditions. Without the assumption that the entire space is in the false vacuum at $t = 0$, we would find that the lifetime of any point before the collision is zero due to the infinite volume of the past lightcone. The boundary conditions considered here correspond to a *cutoff in time*, before which the decay of the false vacuum is "turned off". We could alternatively replace this cutoff in time with a spatial cutoff, such as a cutoff brane at large radial coordinate similar to the two-brane RS scenario. A spatial cutoff also presents the opportunity for tuning the lifetime before collision relative to the Hubble rate, exponentially increasing the expected time until collision. Therefore, the possibility of constructing a finite-volume

AdS may be more relevant for model building. On the other hand, brane collisions may give rise to interesting phenomenology related to inflation and reheating.

Finally, we can estimate the number of d.o.f. on the shellworld centered in AdS-Schwarzschild by considering how thermal equilibrium is established. For an AdS-Schwarzschild metric outside the bubble with $m = m_+$, the effective temperature just outside the shell as measured by a distant observer, at $r \gg 1/k_+$, scales as $T_+^4 \sim m_+k_+^2/r^4$. in the interior of the bubble, all mass is in the form of a black hole with mass m_- (not necessarily equal to m_+) with temperature $T_-^4 \sim m_-k_-^2/r^4$. For the shell to be in equilibrium we need $T_- = T_+ = T$, leading to $m_-k_-^2 = m_+k_+^2$. Since $k_- > k_+$, we get that $m_- < m_+$, meaning that the black hole has lost some mass due to the presence of the shell.

Therefore, the energy density for radiation on the shell at $a = r$ can be estimated as $\rho \sim (m_+L_+ - m_-L_-)/a^4 = (L_+^3 - L_-^3)T^4/a^4$, implying the number of d.o.f. is $\propto L_+^3 - L_-^3$.

2.2.2 General case

A purely gravitational way of understanding the 4d theory on the bubble wall is obtained using Gauss-Codazzi equations to study the induced Riemann curvature. We will use Greek indices to refer to 5d (bulk) geometry and Latin indices for quantities associated to the induced one. We finally find that it gives rise to 4d gravity.

In general, the different bulk metrics across the bubble's wall cause the presence of an energy-momentum tensor S_b^a on the brane. The second Isreael's junction condition implies

$$\kappa_5 S_{ab} = [K_{ab} - Kh_{ab}]|_+^-, \tag{2.2.25}$$

where $[A]_-^+ = A_- - A_+$ and $K_{ab} = \nabla_\beta n_\alpha e_a^\alpha e_b^\beta$, with n_α being a unit normal vector to the wall and e_a^α its tangent vector. h_{ab} is the induced metric on the wall. K_{ab} (with trace K) represents the extrinsic curvature, which tells us about the bubble's embedding in the bulk geometry. For simplicity, we will consider the case where the wall is a simple empty brane with $S_{ab} = -\sigma h_{ab}$.

Gauss-Codazzi equations read:

$$R^{(5)}_{\alpha\beta\gamma\delta} e_a^\alpha e_b^\beta e_c^\gamma e_d^\delta = R^{(4)}_{abcd} + K_{ad}K_{bc} - K_{ac}K_{bd}. \tag{2.2.26}$$

Inserting this equation (and its contractions) into the thin-shell junction condition eliminates the extrinsic curvature in favor of the energy momentum tensor. Therefore:

$$8\pi G_5 S_{ab} = \Delta K_{ab} - \Delta K h_{ab}, \tag{2.2.27}$$

where S_{ab} is the total stress-energy tensor on the shell, which includes T_{brane} in addition to the matter, radiation and cosmological constant induced from the 5d bulk. Assuming that the extrinsic curvature on the brane is dominated by the 5d cosmological constant, we can write $K_{ab} = kh_{ab} + \tau_{ab}$, where h_{ab} is the induced metric on the brane and τ_{ab} is subleading compared to k. Therefore

$$G^{(4)}_{ab} = h_{ab}\left[16\pi G_5\sigma\left(\frac{k_+k_-}{k_--k_+}\right) - 3k_+k_-\right] + \left(\frac{k_+k_-}{k_--k_+}\right)\left[\left(\frac{\mathcal{J}^+_{ab}}{k_+} - \frac{\mathcal{J}^-_{ab}}{k_-}\right) - \frac{1}{2}h_{ab}\left(\frac{\mathcal{J}^+}{k_+} - \frac{\mathcal{J}^-}{k_-}\right)\right], \tag{2.2.28}$$

with $\mathcal{J}_{ab}$ defined as

$$\mathcal{J}_{ab} = R^{(5)}_{\alpha\beta\gamma\delta}e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d h^{cd}. \tag{2.2.29}$$

$$\mathcal{J}_{ab} - R^{(4)}_{ab} = -3k^2h_{ab} - k(2\tau_{ab} + \tau h_{ab}) + \mathcal{O}\left(\frac{\tau^2}{k^2}\right). \tag{2.2.30}$$

Therefore, the 4d geometry is sourced by the bulk geometry through the tensor $\mathcal{J}_{ab}$: in addition to the expected contributions to the Einstein equations coming from the stress tensor on the brane, we have to include also contributions from 5d geometry via the $\mathcal{J}$ tensor.

We can verify that this expression reproduces the FLRW case above: choose the bulk metric to be asympotically AdS_5 with matter and a uniformly dense cloud of strings

$$ds^2_\pm = -f(r)_\pm dt^2 + \frac{dr^2}{f(r)_\pm} + r^2d\Omega_3^2, \tag{2.2.31}$$

where

$$f(r)_\pm = 1 + k_\pm^2r^2 - \frac{8G_5M_\pm}{3\pi r^2} - \frac{2G_5\alpha_\pm}{r}. \tag{2.2.32}$$

With this, the 4d Einstein tensor becomes

$$(G^{(4)})^a_b = -\underbrace{2k_+k_-\left(3 - \frac{8\pi G_5}{k_--k_+}\sigma\right)}_{\equiv 8\pi G_4(\sigma_{crit}-\sigma)\equiv 8\pi G_4\Lambda_4}\delta^a_b - \frac{8G_5}{\pi a(\tau)^4}\left(\frac{M_+k_- - M_-k_+}{k_--k_+}\right)(\delta^a_0\delta^0_b - \frac{1}{3}\Sigma^3_{i=1}\delta^a_i\delta^i_b) \tag{2.2.33}$$

$$- \frac{6G_5}{a(\tau)^3}\left(\frac{\alpha_+k_- - \alpha_-k_+}{k_--k_+}\right)(\delta^a_0\delta^0_b) - 16\pi G_5\left(\frac{k_+k_-}{k_--k_+}\right)(T_{brane})^a_b$$

$$= -8\pi G_4\Lambda_4 - \frac{4G_4}{\pi a(\tau)^4}\left(\frac{M_+}{k_+} - \frac{M_-}{k_-}\right)(\delta^a_0\delta^0_b - \frac{1}{3}\Sigma^3_{i=1}\delta^a_i\delta^i_b) \tag{2.2.34}$$

$$- \frac{3G_4}{a(\tau)^3}\left(\frac{\alpha_+}{k_+} - \frac{\alpha_-}{k_-}\right)(\delta^a_0\delta^0_b) - 8\pi G_4(T_{brane})^a_b$$

$$= 8\pi G_4(T^{(4)})^a_b, \tag{2.2.35}$$

which are precisely the 4d Einstein equations, corresponding to a positive cosmological constant, radiation, matter and additional world volume matter on the brane respectively. Notice that the last term T_{brane} contributes to the Einstein tensor with a negative sign to the energy density (in fact, it contributes with the same sign as the components of *inside* of the bubble, M_- and α_-). The added matter onto the brane gives rise to a negative energy density in 4d, but *it also backreacts on the 5d spacetime*, thus yielding a contribution to the extrinsic curvature which gives an overall positive 4d cosmological constant.

Moreover, notice that also in pure AdS_5, $\mathcal{J}_{ab}$ has a contribution $-3k^2 h_{ab}$, which contributes to a net cosmological constant given by

$$\Lambda_4 = 6k_+k_- - \kappa_4\sigma = \kappa_4 \left(\frac{3}{\kappa_5}(k_- - k_+) - \sigma \right). \tag{2.2.36}$$

In the 5d Einstein equation above we see how the bulk geometry induces matter in the effective 4d theory, which then sources the 4d Einstein equations.

A localized matter source in 4d (such as a massive particle) is uplifted into a string that stretches into the bulk. Contrary to the RS model, neither gravity nor matter is localized to the brane but extends holographically into the bulk. The gravitational attraction between two stretched strings in the bulk projects down to the gravitational attraction between two point particles in 4d.

2.3 Differences with RS model

In the scenario developed by Randall and Sundrum [36], two identical AdS_5 vacua are glued together across a D_3-brane. The 5d graviton has a zero mode confined on the brane that gives rise to an effective 4d gravity despite the existence of large extra dimensions: this solves the issue of finding scale-separated vacua. The DB model is a variation of this scenario that starts with a metastable false AdS_5 vacuum that non-perturbatively decays to a supersymmetric true AdS_5 vacuum through bubble nucleation. Here, a spherical brane separates the two phases with an inside and an outside, and 4d observers confined to the brane experience an effective dS_4.

Consider an effective theory of 5d Einstein gravity with matter:

$$S = \frac{1}{2\kappa_5^2} \int dx^5 \sqrt{-g^{(5)}}(R - 2\Lambda + \mathcal{L}_m), \tag{2.3.1}$$

where $\kappa_5^2 = 8\pi G_5$ is the reduced Newton's constant in 5d. There are two relevant cases:

- $\mathcal{L}_m = 0$: the maximally symmetric spacetime solution is AdS_5

$$ds^2 = e^{2A(z)}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2, \tag{2.3.2}$$

where $A(z) = \pm 2kz$ is the warp factor and $k = \sqrt{\frac{-\Lambda}{6}}$ is the AdS curvature. The holographic coordinate $z \in (-\infty, \infty)$ increases monotonically from the center to the boundary of AdS.

- $\mathcal{L}_m$ corresponds to a shell of matter located at $z = 0$, with AdS_5 having curvature $k_\pm$ on the two sides:

$$A(z) = \epsilon_+ k_+ \Theta(z) z + \epsilon_- k_- \Theta(-z) z. \tag{2.3.3}$$

This corresponds to a shell with constant tension

$$\mathcal{L}_m = \frac{3}{\kappa_5^2}(\epsilon_+ k_+ - \epsilon_- k_-)\delta(z). \tag{2.3.4}$$

Choosing $\epsilon_- = -\epsilon_+ =1$ and $k_- = k_+ = k$ corresponds to RS. On the other hand, if we do not have this symmetry on k and and $\epsilon_+ = \epsilon_- = 1$, we have the DB.

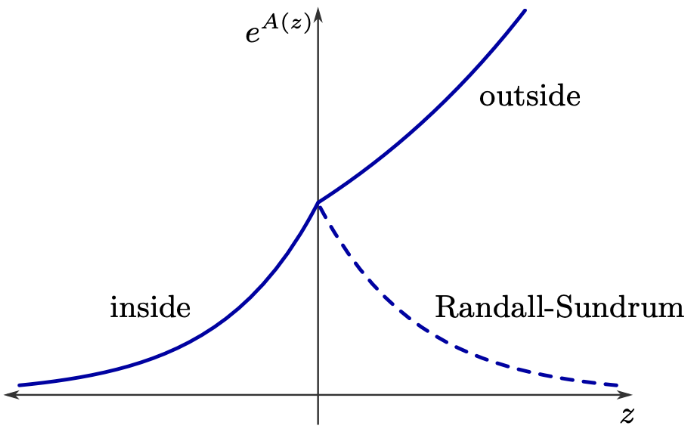

Figure 2.3: *Figure taken from [15] plotting the warp factor for RS and the DB. For the DB, the volume of radial slices decreases inward and increases outward, defining the inside and the outside of the bubble respectively.*

Dimensional reduction of the 5d action along the z direction gives G4:

$$G_5 = G_4 \int_{-\infty}^{z_0} dz e^{2(\epsilon_+ k_+ \Theta(z) + \epsilon_- k_- \Theta(-z))z} \tag{2.3.5}$$

$$= \frac{G_4}{2}\left(\frac{1}{\epsilon_- k_-} - \frac{1}{\epsilon_+ k_+}\right) + G_4 \frac{1}{2\epsilon_+ k_+} e^{2\epsilon_+ k_+ z_0}\,. \tag{2.3.6}$$

For RS, the second piece vanishes as $z_0 \longrightarrow \infty$, so we get $G_4^{RS} = G_5 k$. For DB, instead, the divergent second term is just the divergent volume of AdS as one approaches the boundary that has to be regulated, while the first term is regular and gives the correct G_4. The Gauss-Codazzi Eq. (2.2.26) describes the backreacted embedding of the brane and automatically produces the correct, regulated value of the brane.

Matter in the brane contributes with a negative sign to the effective energy density in 4d, however, backreaction from the bulk, contributing to the extrinsic curvature, yields a net positive energy density. We arrive to:

$$G_4^{DB} = 2G_5 \left(\frac{1}{k_+} - \frac{1}{k_-} \right)^{-1} = G_5 \frac{2k_- k_+}{k_- - k_+} \tag{2.3.7}$$

For DB, we get finite results only when the k_+ and k_- are different. It is exactly a phase transition, with the nucleated brane separating two vacua with different cosmological constant. Also, the presence of the outside AdS_5 drives G_4 to zero (our sources are strings extending in the 5th direction, which produce non-normalizable gravitational modes in the exterior).

In RS, spacetime has an exact $\mathbb{Z}_2$ symmetry across the bubble, but the bubble nucleation process demands an inside/outside construction where there can be no $\mathbb{Z}_2$ symmetry across the bubble.

Finally, as stated before, stretched strings pulling on the brane look like particles in 4d. Putting a mass on top of the brane and ignoring backreaction should make the brane bend down towards the inside. For RS, due to the difference in sign, the bending is in the other direction, away from the inside. In the DB there is an inside and an outside, meaning that the bending of the brane can be explained by a pulling string.

2.4 Graviton propagator in momentum space

We now turn our attention to local gravitational physics on the 4d shell and compute like in [12] the graviton propagator between 2 point particles on the brane to confirm the presence of Einstein's (spin-2) gravity on the bubble. We can show that, while the massless graviton propagates in 5d, it has a 0-mode in its KK reduction that mediates an effective gravitational theory at low energy. Thus, while scattering at high momenta will probe the radial direction, low energy physics will appear 4-dimensional on the brane.

For simplicity, consider a bubble at late times, when the curvature is negligible, in the Poincaré patch, written in domain-wall coordinates

$$ds^2 = dz^2 + a(z)^2 \eta_{\mu\nu} dx^\mu dx^\nu, \tag{2.4.1}$$

where $a(z) = e^{zk_\pm}$. In the DB model there an inside and an outside, so the warp factor increases towards the boundary. This results in configurations of a potential such that the 0-mode of the graviton is not normalizable due to divergence at the boundary of

AdS. If we place a cutoff brane near the boundary, the non-normalizable[5] mode causes both branes to bend. This effect can be interpreted as localized sources induced on the branes by the bulk modes. We identify these sources as the endpoints of the stretched strings between the brane. The strings result in a relation between these two sources which ensures continuity of the 0-mode across the branes (for example, there could be a RS brane at $\tilde{a}(z) = \tilde{a}_{RS}$ which plays the role of a cut-off brane).

Consider perturbations h_{ab} of the bulk metric

$$ds^2 = dz^2 + a(z)^2(\eta_{ab} + h_{ab}(z, x^a))dx^a dx^b, \tag{2.4.2}$$

with $a(z) = e^{kz}$, k being the AdS_5 curvature. Now choose h_{ab} to be in the Lorentz gauge $\partial^a h_{ab} = \frac{1}{2}\partial_b h$ and define $\bar{\gamma}_{ab} = \gamma_{ab} - \frac{1}{2}\eta_{ab}\gamma$, where $\gamma_{ab} \equiv a(z)^2 h_{ab}$.

After a Fourier transformation

$$\left(-\frac{p^2}{a^2} + \partial_z^2 - 4k^2\right)\tilde{\chi}_{ab}(p, z) = -2\kappa_5^2\tilde{\Sigma}_{ab}, \tag{2.4.3}$$

where χ_{ab} and Σ_{ab} are the traceless parts of $\bar{\gamma}_{ab}$ and the stress tensor T_{ab} respectively, and the tildes are their Fourier transformations in the transverse directions. We can solve for the corresponding Green's functions $\Delta_{\tilde{\chi}}(p; a_+, a_-)$[6] piece-wise, inside and outside the shell in terms of modified Bessel functions K_2 and I_2, which diverge at large and small a respectively.

Applying the same considerations as in [12], and working in the small momentum limit, the Green's function becomes

$$\Delta_{\tilde{\chi}}^{shell} = \frac{a^2}{p^2}\left(\frac{2k_- k_+}{k_- - k_+}\right) + \mathcal{O}(p^0) \tag{2.4.4}$$

The leading order term gives the 0-mode of the 5d graviton with subleading corrections in finite momentum. This 5d 0-mode corresponds to the 4d graviton, confirming Einstein's gravity on the DB.

The 0-mode of momentum-space graviton propagators, χ, with 3-momentum p, satisfies the equation

$$\left(-\frac{p^2}{\tilde{a}^2} + \partial_z^2 - 4k^2\right)\chi(p, z) = 0, \tag{2.4.5}$$

which has solutions

$$\chi(p, z) = A(p)I_2\left[\frac{p}{(k\tilde{a})}\right] + B(p)K_2\left[\frac{p}{(k\tilde{a})}\right]. \tag{2.4.6}$$

[5]In AdS/CFT, normalizable modes are the bulk modes that decay with radial distance near the boundary and can be interpreted as the expectation value of the corresponding operator on the QFT on the boundary. Non-normalizable modes, instead, have the interpretation of sources. For instance, for the graviton propagator normalizable modes correspond to the stress tensor, while non-normalizable modescorrespond to the metric on the boundary.

[6]$a_\pm$ corresponds to the scale factor outside/inside the shell.

Variation of the extrinsic curvature across the shell gives an additional boundary condition

$$\tilde{\chi}'_-(p,\tilde{a}_b) - \chi'_+(p,\tilde{a}_b) - \left(\frac{\sigma}{3}\right)\chi(p,\tilde{a}_b) = \Sigma_b, \tag{2.4.7}$$

where ' denotes derivative with respect to $\tilde{a}$ and Σ_b is a source on the shell.

The presence of stretched strings results in $\chi(p,z) \propto K_2$ near the cut-off. This gives

$$\chi_b = \frac{\Sigma_b \tilde{a}_b}{p}\left[\frac{K_1(\frac{p}{\tilde{a}_b k_-})}{K_2(\frac{p}{\tilde{a}_b k_-})} - \frac{K_1(\frac{p}{\tilde{a}_b k_+})}{K_2(\frac{p}{\tilde{a}_b k_+})}\right]^{-1} \tag{2.4.8}$$

$$= -G_4 \frac{\Sigma_b \tilde{a}_b^2}{p^2 + \mathcal{O}(p/a_b k_\pm)^3}. \tag{2.4.9}$$

Therefore, we have reproduced the correct $1/p^2$ interaction for Newtonian gravity at small p, including the correct constant G_4, providing a consistency check for our scenario.

The tensor Σ_b appearing in the above equation is made of two parts:

$$\Sigma_b = \Sigma_b^{brane} + \Sigma_b^{string}. \tag{2.4.10}$$

The first term $\Sigma_b^{brane} = T_{\mu\nu} - T\gamma_{\mu\nu}/3$ contains the contribution from the world-volume fields confined to the shell with induced metric $\gamma_{\mu\nu}$. This gives a negative contribution to the equation above, just as in Friedmann equation.

The other piece, Σ_b^{string}, arises from the bending effect of the strings on the shell. Viewing the effect of mass as a localized deformation, imparted by the endpoint of the stretched string, the contribution from strings is of the form

$$\Sigma_b^{string} \sim -(\alpha_+/k_+ - \alpha_-/k_-), \tag{2.4.11}$$

where $\alpha_\pm$ is the energy carried by the strings. This follows from consistency with Friedmann equation when we identify $\alpha_\pm$ with $m_\pm$. Therefore Σ_b^{string} yields a positive contribution to χ_b. Therefore, strings are fundamental for the existence of a non-vanishing 0-mode and to ensure a well defined propagator realizing localized gravitational effects on the shell-world.

Fourier transforming (2.4.4) back into position space gives

$$\Delta_5 = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-\tilde{x})} \frac{\Delta_{\tilde{\chi}}}{a^2} = \frac{\kappa_4^2}{\kappa_5^2}\frac{1}{r} + ..., \tag{2.4.12}$$

where r is the radial coordinate in 4d. χ_{ab} is the convolution of the scalar Green's function with the source:

$$\chi_{ab} = -2\kappa_4^2 \int \sqrt{g}\Sigma_{ab}\Delta_5 \tag{2.4.13}$$

So far we assumed the brane sits at $\xi = 0$. However, the brane bends in response to a source: placing matter directly on the brane (or having a string pulling from inside) contributes with a negative sign to the stress tensor. As a result, the brane sits at $\xi = f(x^a)$ instead of $\xi = 0$. Performing a coordinate transformation

$$\xi \longrightarrow \xi - f(x^a), \tag{2.4.14}$$

and a corresponding change in the worldvolume coordinates x^a, brings back the brane to $\xi = 0$ in these new coordinates. *This contributes with additional terms to the metric perturbation χ_{ab} as well as to the stress-energy tensor Σ_{ab}.* These extra contributions are not traceless. Demanding that the full stress tensor, including the contribution from bending, be traceless then determines the amount of bending. If we perform the computation in position space, this leads to

$$h_{ab} = -2\kappa_4^2 \int \sqrt{g}(T_{ab} - \frac{1}{2}\eta_{ab}T)\frac{1}{r} + ... \tag{2.4.15}$$

With the negative sign as discussed above, this perfectly reproduces 4d Einstein gravity. The above result crucially depends from the choice of boundary conditions.

2.5 Gravitational waves in Dark Bubble Cosmology

In the presence of spin-2 gravity on the 4d bubble, we expect to find tensor gravitational waves. Let us now briefly review the construction of [17] gravitational waves on the Dark Bubble, along with their uplift from 4d to 5d.

Since we want to write down the junction condition for our brane world, we are obliged to find the backreaction of the waves on the bulk geometry. For this purpose, we will work perturbatively in the metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \xi g^{(1)}_{\mu\nu} + \xi^2 g^{(2)}_{\mu\nu} + O(\xi^3), \tag{2.5.1}$$

where ξ is a formal expansion parameter. The standard procedure is to solve Einstein's equation order by order in χ. Plugging the previous expression into Einstein equation, we find:

$$\begin{aligned} G_{\mu\nu} + \Lambda g_{\mu\nu} =& (G^{(0)}_{\mu\nu}[g^{(0)}] + \Lambda g^{(0)}_{\mu\nu}) + \xi(G^{(1)}_{\mu\nu}[g^{(1)}] + \Lambda g^{(1)}_{\mu\nu}) \\ &+ \xi^2(G^{(2)}_{\mu\nu}[g^{(1)}] + G^{(1)}_{\mu\nu}[g^{(2)}] + \Lambda g^{(2)}_{\mu\nu}) + O(\xi^3) = 0, \end{aligned} \tag{2.5.2}$$

where $G^{(i)}_{\mu\nu}[g^{(j)}]$ denotes the i-th order variation of the Einstein tensor evaluated on the j-th order metric perturbation, which is a quantity of order $\max(i, j)$ in ξ.

- At 0-th order, the Einstein Eq. simply yields the background geometry $g^{(0)}_{\mu\nu}$. We will construct, for simplicity, a background of pure AdS.

- Gravitational waves appear at first order in χ through the linearized Einstein equation, which can be understood as solutions to

$$(G^{(1)}_{\mu\nu}[g^{(1)}] + \Lambda g^{(1)}_{\mu\nu}) = 0. \tag{2.5.3}$$

  Gravitational waves in the DB model must satisfy specific requirements that the 5d bulk induces a 4d metric on the DB constrained by junction conditions.

- The second order Einstein equation can now be written as

$$G^{(1)}_{\mu\nu}[g^{(2)}] + \Lambda g^{(2)}_{\mu\nu} = -G^{(2)}_{\mu\nu}[g^{(1)}]. \tag{2.5.4}$$

  This can in principle be solved to give $g^{(2)}$. *This tells us how the geometry reacts in the presence of the gravitational wave* $g^{(1)}$. The right-hand side can be interpreted as an effective energy momentum tensor

$$\langle T_{\mu\nu}\rangle = -\kappa_D^{-1}\left\langle G^{(2)}_{\mu\nu}[g^{(1)}]\right\rangle, \tag{2.5.5}$$

  where κ_D is the gravitational constant in D dimensions. This energy-momentum tensor is quadratic in $g^{(1)}$. The averaging procedure has been done over several wavelengths. Note an overall minus sign in the definition of $\langle T_{\mu\nu}\rangle$.

  This effective energy-momentum term can also be captured by a backreacted background metric. This fact is of crucial importance as, once the backreaction is accounted for, *the junction condition will dictate how gravitational waves in the bulk will affect the evolution of the 4d bubble.*

2.5.1 Uplift of Gravitational Waves from 4d to 5d

Let us discuss gravitational waves and their backreaction on the metric, reviewing first gravitational waves in an expanding flat or spherical FRLW topology. In the DB model, the AdS scale k is assumed to be a UV scale that lies somewhere in between the scales of particle physics and M_P. Let us use for simplicity $H \ll k$, meaning the Hubble scale is much smaller than any such UV scale.

When the wavelength is sufficiently small, waves in a flat universe serve serve as proxy for those in a spherical universe. In fact, high frequency waves only probe small regions and do not feel the curvature at larger scales.

Gravitational waves are described by transverse-tracefree (TT) perturbations to the metric $ds^2_{ind} = N^2(\tau)d\tau^2 + a(\tau)^2 d\Omega_3^2$. In conformal time gauge, these are:

$$ds^2 = a^2(\eta)[-d\eta^2 + (\gamma_{ij} + \xi h_{ij}(\eta, x))dx^i dx^j], \tag{2.5.6}$$

where η is the conformal time, ξ is the perturbation parameter, γ_{ij} is the metric on a spatial slice in x^i-coordinates and h_{ij} corrseponds to transverse and trace-free perturbations.

For simplicity, let us ignore matter and radiation contributions and focus solely on a pure dS cosmology with positive cosmological constant Λ_4 only. We then have that the 4d Hubble constant is given by $H^2 = \kappa_4\rho_\Lambda/3 = \Lambda_4/3$.

- In a **flat dS universe**, the scale factor is $a(\eta) = -\frac{1}{H\eta}$ with $-\infty < \eta < 0$. For concreteness, we will consider a GW travelling in the x_1 direction with either a $+$ or a $\times$ polarization. The perturbation can be expanded into harmonics on the spatial manifold. The I-order Einstein equation then yields a wave equation for each mode separately. For a single mode $h_{4d}(\eta, x_1) = e^{iqx_1}h_{4d}(\eta)$, labelled by some continuous wave number q, we find

$$\frac{d^2h_{4d}}{d\eta^2} + 2\mathcal{H}\frac{dh_{4d}}{d\eta} + q^2h_{4d} = 0, \tag{2.5.7}$$

  where $\mathcal{H} = -\frac{1}{\eta}$ is the conformal Hubble rate. Solutions are easily found and read

$$h_{4d}(\eta) = -\eta\cos(q\eta + \phi_0) + \frac{1}{q}\sin(q\eta + \phi_0), \tag{2.5.8}$$

  where ϕ_0 is an arbitrary phase. The wave h freezes out to a constant at late times.

- A **closed dS universe** is a bouncing cosmology with scale factor $a(\eta) = -\frac{1}{H\sin\eta}$, where $-\pi/2 \leq \eta < 0$. The moment $\eta = -\pi/2$ is the bounce, which coincides with the moment of nucleation. In a transverse-traceless gauge, 4d gravitational waves on the DB (conformally $R^1 \times S^3$) are given by $h_{ij} = h^{4d}(\eta)Y_{ij}$, where η is the conformal time on the DB and Y_{ij} are tensor spherical harmonics on S^3. The time-dependent solution was found in [17] and is a linear combination of

$$\begin{aligned} h_1^{4d}(\eta) &= \frac{\cos((n+1)\eta)}{n+1} + \sin\eta\sin(n\eta), \\ h_2^{4d}(\eta) &= \frac{\sin((n+1)\eta)}{n+1} - \sin\eta\cos(n\eta), \end{aligned} \tag{2.5.9}$$

  where n labels the wave number and $3 - n^2$ is the eigenvalue of the laplacian on S^3.

The gravitational waves, which perturb the metric at the first order, source an energy-momentum tensor. At late times, this corresponds to curvature and radiation. The right procedure to deal with gravitational waves is to solve Einstein's equations to first order, and then to identify the energy of the first order waves eith the second order piece representing the failure of the first order waves to solve the vacuum equations. The isotropic and wavelength-averaged energy-momentum tensor in 4d becomes

$$\langle T^a_b\rangle_{iso} = \frac{7}{8\kappa_4 a^2}\begin{pmatrix}1&0&0&0\\0&\frac{1}{3}&0&0\\0&0&\frac{1}{3}&0\\0&0&0&\frac{1}{3}\end{pmatrix} + \frac{q^2}{4\kappa_4 H^2 a^4}\begin{pmatrix}-1&0&0&0\\0&\frac{1}{3}&0&0\\0&0&\frac{1}{3}&0\\0&0&0&\frac{1}{3}\end{pmatrix}. \tag{2.5.10}$$

Notice how the first term goes like $\sim 1/a^2$, behaving like curvature in the Friedmann equations, while the second one has an equation of state $p = -\rho/3$ and goes like $\sim 1/a^4$, thus behaving like radiation.When the wavelength of the waves is larger than the horizon, it freezes and the only remaining contribution is curvature.

Gravitational waves in AdS_5 which are the uplift of the 4d waves are of the form $h_{ij} = h^{5d}(t,z)Y_{ij}$, where t and z are the global time coordinate and the radial direction in AdS_5 respectively[7].

This wave is required to behave like the induced 4d one (2.5.9) and to be sourceless ($h^{5d}(t,r) = 0$ when $r \to 0$). Imposing these boundary conditions, the resulting bulk wave h^{5d} is a linear combination of

$$h_1^{5d}(t,z) = \frac{(kz)^{n-1}}{(1+k^2z^2)^{\frac{n-1}{2}}}\left[\frac{\frac{1}{2}(1+n)(2-n)+k^2z^2}{(n+1)(1+k^2z^2)}\cos((n+1)kt)+\sin(kt)\sin(nkt)\right], \tag{2.5.11}$$

$$h_2^{5d}(t,z) = \frac{(kz)^{n-1}}{(1+k^2z^2)^{\frac{n-1}{2}}}\left[\frac{\frac{1}{2}(1+n)(2-n)+k^2z^2}{(n+1)(1+k^2z^2)}\sin((n+1)kt)-\sin(kt)\sin(nkt)\right].$$

Similar to 4d, this perturbation induces an energy-momentum tensor in AdS_5 and has components corresponding to radiation and flux

$$\langle T^\mu_\nu\rangle_{rad} = \frac{k^2n^2t^2}{4\kappa_5 r^2}\begin{pmatrix} -1 & 0 & 0 & 0 & 0\\ 0 & \frac{1}{3} & 0 & 0 & 0\\ 0 & 0 & \frac{1}{3} & 0 & 0\\ 0 & 0 & 0 & \frac{1}{3} & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix}, \qquad \langle T^\mu_\nu\rangle_{flux} = \frac{n^2}{8\kappa_5 r^2}\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{-2t}{k^2r^3}\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 2k^2tr & 0 & 0 & 0 & 0\end{pmatrix}, \tag{2.5.12}$$

$$\langle T^\mu_\nu\rangle_{curv} = \frac{1}{8\kappa_5 r^2}\left(7-\frac{n^4}{2k^4r^4}\right)\begin{pmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & \frac{1}{3}+\mathcal{O}(\frac{n^2}{k^2r^2}) & 0 & 0 & 0\\ 0 & 0 & \frac{1}{3}+\mathcal{O}(\frac{n^2}{k^2r^2}) & 0 & 0\\ 0 & 0 & 0 & \frac{1}{3}+\mathcal{O}(\frac{n^2}{k^2r^2}) & 0\\ 0 & 0 & 0 & 0 & 1+\mathcal{O}(\frac{n^2}{k^2r^2})\end{pmatrix}$$

Here, the energy-momentum tensor has been averaged and made isotropic, analogously to the 4d case (2.5.10). The first line corresponds to gravitational radiation. Notice that, in particular, the t, r components of the flux different from zero indicate a flow of energy in the direction in which the bubble expands, as expected. The second line corresponds to waves that are frozen as they become larger than the horizon scale.

The energy-momentum (2.5.12) sources a backreaction on the 5d bulk metric, which in turn induces a second order correction to the stress tensor on the bubble. The back-

[7] This is analogous to $ds^2_\pm = -f(r)_\pm dt^2 + \frac{dr^2}{f(r)_\pm} + r^2 d\Omega_3^2$ but with $M_\pm = \alpha_\pm = 0$, thus $f(r)_\pm = 1 + k^2_\pm r^2$

reacted metric is given by

$$ds^2_{backreacted} \simeq -[1 + k^2r^2 + \epsilon^2(q_1 - q_2k^2t^2)]dt^2 + [1 + k^2r^2 + \epsilon^2(q_1 - q_2k^2t^2)]^{-1}dr^2 + r^2d\Omega_3^2, \quad (2.5.13)$$

where $|\epsilon| \ll 1$ and the coefficients q_i read $q_1 = -\frac{7}{24}$, $q_2 = -\frac{q^2}{6}$, $q_3 = -\frac{q^2}{12}$.

This second order correction induced on the DB from the 5d backreaction is exactly the same as what would have been obtained by directly solving Einstein's equations in 4d directly at second order. This result further establishes the consistency of the DB model perturbatively in response to gravitational wave fluctuations.

2.6 AdS decay

We start from a metastable false AdS_5 vacuum that non-perturbatively decays to a stable supersymmetric AdS_5 vacuum of lower energy through bubble nucleation[8].

Recalling the conjectures of Section 1.5, let us make explicit the argument which concludes that the WGC applied to a higher form gauge field implies that non-supersymmetric AdS must decay. We can show that an extension of the higher form WGC to codimension-1 branes is equivalent to the charge-to-tension relation required for AdS (supported by flux) to be unstable to the nucleation of charged D_p-branes first derived by Brown-Teitelboim (see [39, 41]). The Brown-Teitelboim instanton describes the nucleation of a spherical, charged, codimension-1 membrane in a vacuum whose cosmological constant is sourced (at least in part) by a top-form flux (i.e. a d-form field strength in d-dimensions). *The membrane nucleates and discharges a unit of flux, thus lowering the value of the cosmological constant.*

The instanton solution describes an energy-conserving configuration where the sphere nucleates at rest at a finite size. The radius of the instanton is determined by the condition that the energy freed-up in reducing the cosmological constant exactly balances the energy cost of the tension of the membrane. Moreover, it was shown in [39] that charged bubbles cannot nucleate in AdS for any arbitrary value of the membrane charge-to-tensor ratio. This is because if the tension becomes large relative to the charge, the critical bubble will have to grow so that the energy liberated in the volume of the bubble is enough to "pay" for the energy cost of nucleating the membrane. However, in AdS, at large radial coordinate, the volume and the area of a sphere grow with the same power of radius: *making the bubble larger no longer balances the energy budget.*

[8] The vacuum that nucleates could in principle be non-supersymmetric and unstable, but the process will eventually with the nucleation of a stable supersymmetric AdS. Regardless of the stability of the nucleating vacuum, since the bubble is a spacelike surface that asympotes to the lightcone, subsequent decays on the interior of the bubble will never collide or interact with the original domain wall and are irrelevant for our construction.

The condition for being able to nucleate a bubble arises from the condition that the radius of the critical bubble is real:

$$R_* = \left[\frac{2\tilde{\Lambda}_+}{(d-1)(d-2)} + \sigma_p^{-2} \left(\frac{1}{d-1} \left(Q_p E_+ - \frac{1}{2} Q_p^2 \right) - \frac{M_d^{2-d} \sigma_p^2}{2(d-1)} \right)^2 \right]^{-1/2}, \qquad (2.6.1)$$

where d is the spacetime dimension, M_d is the d-dimensional reduced Planck mass, $\sigma_p(Q_p)$ is the tension (charge) of the D_p-brane, $\tilde{\Lambda}_+$ is the cosmological constant exterior to the bubble and $\Lambda_d \equiv \tilde{\Lambda}_d M_d^{d-2}$.

E_+ is the electric field associated to the top-form flux ($F^{\mu_1 \cdots \mu_d} \propto E_+ \varepsilon^{\mu_1 \cdots \mu_d}$). Since this holds for membrane nucleation with codimension-1, in this result $p = d-2$.

Let us consider the case that the cosmological constant outside the bubble *is sourced entirely by flux*

$$\tilde{\Lambda}_+ = -\frac{1}{2} M_d^{2-d} E_+^2. \qquad (2.6.2)$$

Furthermore, we assume that there are a large number of units of flux

$$E_+ = Q_p N, \qquad N \gg 1. \qquad (2.6.3)$$

This is the limit where higher curvature corrections can be neglected in supergravity.

Solving the condition that the radius of nucleation is positive yields the inequality:

$$\sigma_{d-2}^2 \leq \frac{d-2}{d-1} Q_{d-2}^2 M_d^{d-2} - \frac{(d-2)(2d-3)}{(d-1)^2} Q_{d-2}^2 M_d^{d-2} \frac{1}{N} + \mathcal{O}\left(\frac{1}{N^2}\right), \qquad (2.6.4)$$

where we have included the sub-leading correction in $1/N$ to illustrate that it is negative, lowering the tension relative to the charge, and therefore will not provide a loophole for violating the WGC.

All to all, the WGC for higher form fields can be written as

$$Q_p^2 M_d^{d-2(p+2)} \geq \frac{(p+1)(d-p-3)}{d-2} \left(\frac{\tau_p}{M_d^{(p+1)}} \right)^2. \qquad (2.6.5)$$

This does not apply easily to the our case ($d = 5$, $p = 3$), because the right-hand side becomes negative. This makes the inequality a trivial statement; however, a non-trivial *extension* of this result, effectively replacing $(d-p-3) \longrightarrow |d-p-3|$ for the case $p = d-2$, could be the correct guess. This is in perfect agreement with the $N \longrightarrow \infty$ limit of the previous σ_{d-2}^2 inequality.

The saturation of this inequality corresponds to an extremal, flat, domain wall. This flat domain wall does not have a finite Euclidean action and thus cannot nucleate in a decay process, defining the boundary of stability.

2.7 Embedding in String Theory

Finally, let us review a first attempt to embed the Dark Bubble model in String Theory [17]. For an explicit and more accurate embedding of the DB model in String Theory, we refer to the involved construction in [16], which makes use of rotating branes in the 5th dimension. In this embedding, an unstable AdS configuration with large enough R-charge chemical potential emits color D_3-branes and tunnels through a potential barrier to a low energy stable configuration. At lowest order, the universe living on the worldvolume of the emitted D_3-brane has a vanishing cosmological constant, and keeps growing up to a maximal size before it starts contracting again. Considering also $1/N$ corrections, in line with the WGC, we obtain a small cosmological constant compatible with observations.

We start by considering the decay of the non-supersymmetric Romans vacuum to the supersymmetric $AdS_5 \times S^5$ vacuum via the nucleation of a spherical (p, q)-5 brane. Reducing this picture to 5d, where the (p, q)-5 brane is an S^3, will result in an EFT with gravitational interactions described in the previous sections.

The Romans vacuum is given by the reduction of Type IIB over a 5d Sasaki-Einstein manifold seen as a $U(1)$ fibration over a $\mathbb{CP}^3$ base. The vacuum results from a non-trivial relative stretching of the fiber to the base and is supported by 3-form flux. This solution has been identified with the $SU(3) \times U(1)$ invariant critical point of 5d supergravity.

While the consistent truncation to 5d supergravity is perturbatively stable at the $SU(3) \times U(1)$ critical point, it is still possible that there are tachyonic modes which have been truncated out. Due to the lack of supersymmetry, the instability is likely and has been taken as a foregone conclusion. However, it is also possible that we remove the the annoying tachyonic modes by taking an orbifold. The orbifold of the Romans vacuum will decay into $AdS_5 \times S^5/\mathbb{Z}_k$, which is in general not supersymmetric and has a bubble of nothing non-perturbative instabilty. Nevertheless, bubbles of nothing nucleating within the true vacuum must remain inside the lightcone describing the braneworld trajectory and will therefore not affect our scenario.

In addition to the self-dual 5-form flux present in the supersymmetric vacuum, the Romans vacuum has non-zero 3-form flux $G_3 = F_3 + \tau H_3$, which *breaks all supersymmetry* and leads to the squashing of the S^5.

The 10d metric is given by the product $AdS_5 \times \mathcal{M}_5$, where $\mathcal{M}_5$ is a specific fibration over $\mathbb{CP}^2$

$$ds^2_{10} = \xi^2 ds^2_{dS_4} + \frac{d\xi^2}{1 + \xi^2/L^2_{AdS}} + L^2_5(\nu ds^2_{\mathbb{CP}^2} + \nu^{-4} e^2_5), \tag{2.7.1}$$

where L_{AdS} parametrizes the the AdS scale, (L_5) parametrizes the KK scale, ν controls the relative fiber/base stretching and e_5 is a 1-form satisfying $de_5 = J$, where J is the Kähler form for the unit $\mathbb{CP}^2$.

The axion C_0 is set to zero, while other fluxes read

$$F_5 = \alpha_5(1+*)\text{vol}_5, \tag{2.7.2}$$
$$G_3 = \alpha_3 F_3 + \beta_3 H_3 = dK = L_5 e^{-3i\phi} K \wedge e_5, \tag{2.7.3}$$

where K is a holomorphic 2-form on $\mathbb{CP}^2$ and ϕ is the fiber coordinate. Solving the 10d field equations yields:

$$SUSY: \qquad L_{AdS} = L_5, \qquad \nu = 1, \qquad \alpha_5 = \frac{4}{g_s L_5}, \tag{2.7.4}$$
$$\not{SUSY}: \qquad L_{AdS} = \frac{2^{11/10}}{3^{3/5}} L_5, \quad \nu = (\frac{2}{3})^{1/5}, \tag{2.7.5}$$
$$\alpha_5 = \frac{108^{1/5}}{g_s L_5}, \quad \beta_3 = g_s \alpha_3 = \frac{3^{4/5}}{2^{3/10}} \frac{1}{L_5}.$$

In order to compare them, let us now measure the curvature in the two vacua in 5d Planck units:

$$M_P^3 = \frac{L_5^5 \pi^3}{(g_s^2 (2\pi)^7 \alpha'^4)}. \tag{2.7.6}$$

Furthermore, we are interested in decay via nucleation of 5-branes which remove $G_{(3)}$ but leave $F_{(3)}$ untouched. The flux quantisation condition thus reads

$$\int_{\mathcal{M}_5} F_5 = (4\pi^2 \alpha')^2 N_5 = \alpha_5 L_5^5 \pi^3. \tag{2.7.7}$$

Thus, holding N_5 fixed and using Planck units, we have

$$L_{AdS} = \left(\frac{N_5}{\pi}\right)^{2/3} M_P^{-1} \times \begin{cases} \frac{1}{2}, SUSY \\ \frac{2^{7/6}}{3}, \not{SUSY} \end{cases}. \tag{2.7.8}$$

The important result of these calculations is the *hierarchy* of the potential energy ($V \propto L_{AdS}^{-2}$) between the two vacua:

$$10\text{d} \longrightarrow \frac{V_{\not{SUSY}}}{V_{SUSY}} = \frac{9}{2^{13/3}} < 1, \tag{2.7.9}$$

which indicates that *the supersymmetric vacuum lies below the non-supersymmetric one.*

In order for a perturbative decay channel to exist, the $(p,q)-5$ brane in the theory which realizes the desired flux shift should have a tension $\sigma < \sigma_{ext}$. Using the previous relation, we can compute σ_{ext} from

$$\sigma_{ext} = M_P^4 \left(\frac{\pi}{N_5}\right)^{2/3} \left(6 - \frac{9}{2^{7/6}}\right). \tag{2.7.10}$$

While the precise embedding of the five-branes which mediate this decay depends on the orbifold needed to ensure perturbative stability (again, refer to [16]), we are able to demonstrate the existence of such a decay channel by using SUGRA to obtain the tension of the desired 5-brane. If we use the following superpotential and potential

$$W(\rho,\chi) = \frac{1}{4\rho^2}\left(\cosh(2\chi)(\rho^6-2)-(3\rho^6+2)\right), \tag{2.7.11}$$

$$V(W) = g^2\left(\frac{1}{8}\left|\frac{\partial W}{\partial \chi}\right|^2 + \frac{1}{48}\left|\rho\frac{\partial W}{\partial \rho}\right|^2 - \frac{1}{3}|W|^2\right). \tag{2.7.12}$$

The maximally symmetric critical point is at $\rho = 1$, $\chi = 0$, and the Romans vacuum is located at $\rho = 1$, $\chi_* = \text{arccosh}(2)/2$.

However, we notice that the hierarchy of thew vacua is reversed with respect to Eq.(2.7.9), i.e.

$$\frac{V_{\not{SUSY}}}{V_{SUSY}} = \frac{9}{8} > 1. \tag{2.7.13}$$

This is because moving in the χ direction corresponds to a *deformation of the internal manifold* that does not preserve N_5. However, since the superpotential is linear in the fluxes, rescaling the superpotential such that the hierarchy (2.7.9) is recovered will also amount to holding the 5-form flux fixed. Thus, we should use the superpotential of (2.7.11) at the supersymmetric critical point, and $\tilde{W} = 2^{-\frac{2}{3}}W$ at the critical point corresponding to the Romans vacuum.

To deduce the tension of the fundamental (p,q)-5 brane that can mediate the decay, we can notice that there should be a BPS brane that sources the $G_{(3)}$ flux, the tension of which will be given by the junction condition for an extremal brane, where $k_\pm$ are associated to $V_+ = -g^2\tilde{W}^2\frac{(1,\chi_*)}{3}$ and $V_- = -g^2\tilde{W}^2\frac{(1,0)}{3}$. Again, since W is linear in flux, and we have rescaled $\tilde{W}$ such that the difference relative to W is entirely due to the change in 3-form flux, using the supersymmetric values for the potential gives the effective 5d tension for the BPS brane that sources this change in flux numbers.

If we compare the tension of the BPS brane to the extremal tension (2.7.10), we find $\sigma_{BPS} < \sigma_{ext}$, so that the fundamental brane which sources the correct charge can also facilitate the decay via a spherical bubble with finite Euclidean action.

2.8 Hierarchies from an explicit stringy embedding

We will now only mention the hierarchy of energy scales of the Dark Bubble model as realized in the explicit embedding of [16] in Type IIB string theory, without paying attention to numerical factors.

Starting from the 10d Planck length

$$l_{10}^4 \sim g_s l_s^4, \tag{2.8.1}$$

and then reducing to 5d, we get

$$l_5^3 \sim l_{10}^8/L^5. \tag{2.8.2}$$

Indeed, the background we consider is a deformation of $AdS_5 \times S_5$. There is no scale separation and the length L sets the size of the S_5 as well as of the curvature scale of the non-compact AdS_5.

The next step is to go down to 4d. In a standard dimensional reduction on a compact dimension of size L, we would get $l_4^2 \sim l^5/L$. This means that the lower-dimensional Planck length is smaller than the higher-dimensional one: in other words, gravity is weaker in the lower-dimensional theory.

In the DB model, however, there is an inside and an outside, leading to a 4d Planck scale given by

$$l_4^2 \sim \frac{k_- k_+}{k_- - k_+} l_5^3, \tag{2.8.3}$$

where $k_\pm = 1/L_\pm$. Moreover, if k_- is close to k_+, then l_4 can be *larger* than l_5. Indeed, the hierarchy that we will find is

$$l_{10} \gg l_4 \gg l_5. \tag{2.8.4}$$

The reason is that, in the model of [16], the background $AdS_5 \times S^5$ is dual to a background of N D3-branes. *One of these branes can nucleate and start to expand.* By studying its junction condition and imposing $k_- \sim k_+ \sim k$, this was found to correspond to

$$(k_- - k_+) \sim \frac{k}{N} \sim \frac{1}{NL} \longrightarrow l_4^2 \sim \frac{N}{L} l_5^3. \tag{2.8.5}$$

It is a large N what ensures that $l_4 \gg l_5$. Furthermore, the scale L of the background is given by $L^2 \sim l_s^2 \sqrt{g_s N}$, which, using (2.8.1) and (2.8), is equivalent to

$$l_5^3 \sim \frac{L^3}{N^2}. \tag{2.8.6}$$

There is only one remaining relation to be fixed, which will determine N in terms of the cosmological constant. The nucleated brane is supposed to be critical[9], meaning its tension is given by

$$T_3 = \frac{1}{(2\pi)^3 g_s l_s^4}. \tag{2.8.7}$$

As argued in [16], we expect there to be corrections of order $1/N$, in line with the WGC, which will *reduce the value of the cosmological constant.* This will shift the tension from its critical value and generate an effective 4d cosmological constant of magnitude

$$\rho_\Lambda \sim \frac{1}{g_s l_s^4} \frac{1}{N} \sim \frac{1}{L^4}. \tag{2.8.8}$$

[9] if all numerical coefficients are restored, it *exactly* solves the conditions for a critical bubble $\sigma = \frac{3}{8\pi G_5}(k_- - k_+)$.

From this, we get that $L \sim 10^{-5}$ m, which has the size of the Dark Dimension discussed in [42]. This is a nice result, but we should keep in mind that in the DB model the scalings do not arise from a hierarchy between a single mesoscopic dimension and the other (scale-separated) ones. Instead, (2.8.8) arises from the peculiar relation between the 5d and the 4d Planck scales in (2.8.3).

Using the previous relations, we can now express the 5d and the 4d Planck scales, the 4d Hubble length R_H and the length scale L of AdS_5 in terms of N and the 4d Planck length as

$$R_H \sim N l_4, \quad L \sim N^{\frac{1}{2}} l_4, \quad l_{10} \sim N^{\frac{1}{4}} l_4, \quad l_5 \sim N^{-\frac{1}{6}} l_4. \tag{2.8.9}$$

Finally, we have $l_s \sim l_{10}/g_s^{\frac{1}{4}}$, which fixes $N \sim 10^{60}$. From there, we find the non-trivial prediction that the 10d Planck scale should sit at around 10 TeV, with the string scale just below and a string coupling g_s that is less than one but not too small.

Chapter 3

Bubble Nucleation in Quantum Cosmology

In this Chapter, we shall examine how Dark Bubble Cosmology perfectly matches with Vilenkin's nucleation from nothing in a higher dimensional perspective. We will start by reviewing the two main proposals for a quantum beginning of the universe, namely Vilenkin's and Hartle-Hawking's proposal. We will then argue that only the former is compatible with the Dark Bubble model.

3.1 Introduction

Quantum cosmology (see [43] for a review) aims at computing the wave function of a closed universe

$$\Psi[h_{ij}(\mathbf{x}), \Phi(\mathbf{x}), \Sigma]. \tag{3.1.1}$$

This is the amplitude that the universe contains a three-surface Σ on which the three-metric is $h_{ij}(\mathbf{x})$ and the matter field configuration is $\phi(\mathbf{x})$. From this amplitude, then, one would hope to extract various predictions concerning the outcome of large scale observations. To fix the amplitude (3.1.1), we first needs a theory of dynamics, such as General Relativity. From this, we can derive an equation analogous to the Schrödinger equation, the so-called Wheeler-DeWitt equation, which the wave function of the universe must satisfy.

This is clearly an ambitious program and several aspects of its basic principles remain a source of debate in the literature. One particular issue concerns the choice of boundary conditions on the wave function. Just as in ordinary quantum mechanics, boundary conditions (and normalisation) are needed in order to uniquely fix solutions to Schrödinger's equation. Such boundary conditions are often argued for by physical principles or by more formal principles such as the self-adjointness of the Hamiltonian. In quantum cosmology this is highly non-trivial to implement for several reasons. First of all, we do not know the full Hamiltonian since we lack the complete Hamiltonian of quantum gravity. The common attitude is to ignore the UV completion of gravity and to work in a semi-classical approximation, which is then further approximated by a mini-superspace approach (a drastic cut in the number of degrees of freedom). Given this, one can debate the choice of boundary conditions and reach various conclusions. Two natural choices are the Vilenkin choice (i.e. the tunneling wave function) and the Hartle-Hawking (HH) choice (i.e. no-boundary proposal). We will examine both proposals, focusing mainly on the former and giving only the essential details of the latter, emphasizing how the Dark Bubble perfectly agrees with Vilenkin's one.

3.2 Vilenkin's tunneling proposal

Standard cosmology gives a successful description of many features of the evolution of the universe. However, many theoretical issues are not yet solved. One of these puzzles is the fact that we would need very fine-tuned initial conditions at the Big Bang: we have to postulate that the universe started in a perfectly homogeneous and isotropic state with tiny density fluctuations which then evolve into galaxies. This fact, together with the observation that the Universe appears to be very closely spatially flat, gives rise to the so-called flatness and horizon problems [44]. These can be resolved if we take into account a period of rapid expansion similar to the de Sitter solution, inflation: the Universe we see today started as a very small bubble, enough so that all of its parts had time to come into causal contact; it then underwent a period of (almost) exponential expansion, during which the initial content of matter and radiation was almost entirely wiped out; this fast growth then ended, leaving an essentially empty Universe (thus homogeneous and isotropic) which also looked spatially flat inside the Hubble horizon; finally, matter was produced again as a quantum process called *reheating*, which occurred in a radiation dominated locally FLRW space-time. Nevertheless, the inflationary scenario does not resolve the initial singularity problem of the Universe. Therefore, some proposals for removing the singularity were presented using the Wheeler-deWitt equation.

In 1982, Vilenkin proposed a cosmological model [45], [46] in which the universe is spontaneously created by quantum tunneling from nothing into a de Sitter space, where by *nothing* we mean a state with no classical space-time. The Universe therefore entered a de Sitter regime with finite initial scale factor as a quantum transition from a 4d geometry.

This model does not have any Big bang singularity and does not require any initial or boundary conditions. Moreover, it does not require any changes in the fundamental equations of physics: it only gives a new interpretation to a well-known cosmological solution.

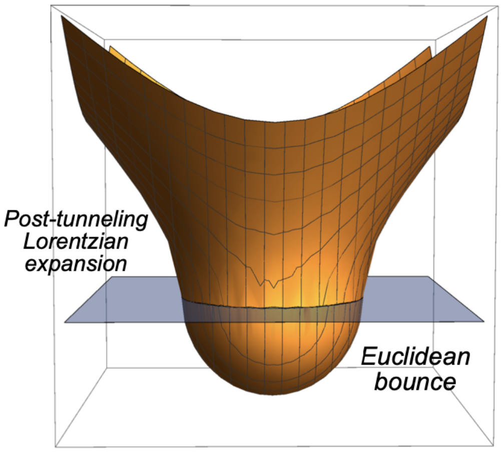

Figure 3.1: *Graphic realization of Vilenkin's proposal. An expanding de Sitter universe is born via quantum tunneling from a compact Euclidean geometry.*

Let us consider a model of matter fields interacting with gravity, where for simplicity the matter fields are represented by a single Higgs field ϕ with an effective potential $V(\phi)$. If $\phi = \eta$ is the true minimum of the effective potential, then we require that $V(\eta) \sim 0$, so that the cosmological constant is small today.

Besides $\phi = \eta$, $V(\phi)$ can have other extrema. If $\phi = \phi_0$ is such an extremum, $V'(\phi_0) = 0$, and $\phi = \phi_0 = \text{const}$ is a solution of the classical field equation for ϕ

$$\Box\phi + V'(\phi) = 0. \tag{3.2.1}$$

The vacuum energy density at $\phi = \phi_0$ will be non-vanishing (and positive) in general

$$\rho_V = V(\phi_0) > 0. \tag{3.2.2}$$

Suppose that the universe starts in the symmetric vacuum state and is described by a closed ($k = 1$) FLRW metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{(1-r^2)} + r^2 d\Omega^2\right]. \tag{3.2.3}$$

Then, the scale factor $a(t)$ can be derived from the evolution equation

$$\dot{a}^2(t) + 1 = \frac{8}{3}\pi G_N \rho_\Lambda a^2, \tag{3.2.4}$$

whose solution is the de Sitter space

$$a(t) = H_\Lambda^{-1}\cosh(H_\Lambda t), \tag{3.2.5}$$

where $\dot{a} = \frac{da}{dt}$ and $H_\Lambda = (\frac{8\pi G_N \rho_\Lambda}{3})$. This solution describes a universe which is contracting at $t < 0$, reaches its minimum size ($a_{min} = H_\Lambda^{-1}$) at $t = 0$, and is expanding at $t > 0$. This behaviour is similar to that of a bubble of true vacuum surrounded by a false vacuum. The radius of the bubble is given by [47]

$$R = (R_0^2 + t^2)^{1/2}. \tag{3.2.6}$$

In the actual history of the universe, however, the $t < 0$ part is absent. In fact, the bubble tunnels quantum mechanically from $R = 0$ to $R = R_0$, and then evolves according to (3.2.6) with $t > 0$. This suggests that the birth of the universe might be a quantum tunneling effect: the universe might have emerged at the "bounce point" having a finite size ($a = H_\Lambda^{-1}$) and zero "velocity" ($\dot{a} = 0$), while its following evolution is described by (3.2.5) with $t > 0$.

It is known that a semiclassical description of quantum tunneling is given by the bounce solution of Euclidean field equations, which can be obtained by replacing $t \longrightarrow -i\tau$ in the field equations. Normally, bounce solutions are used to describe a decay of a quasi-stable state. If the decaying state is at the bottom of a potential well at $x = x_1$, then the bounce solution starts with $x = x_1$ at $t \longrightarrow -\infty$, bounces off the classical turning point at the end of the barrier and returns to $x = x_1$ at $t \longrightarrow +\infty$.

The euclidean version of (3.2.4) is $-\dot{a}^2 + 1 = H_\Lambda^2 a^2$, and the solution is

$$a(t) = H_\Lambda^{-1} \cos(H_\Lambda t). \tag{3.2.7}$$

Equations (3.2.3) and (3.2.7) describe a four-sphere (S^4) of radius H_Λ^{-1}. This is the well-known de Sitter instanton [41].

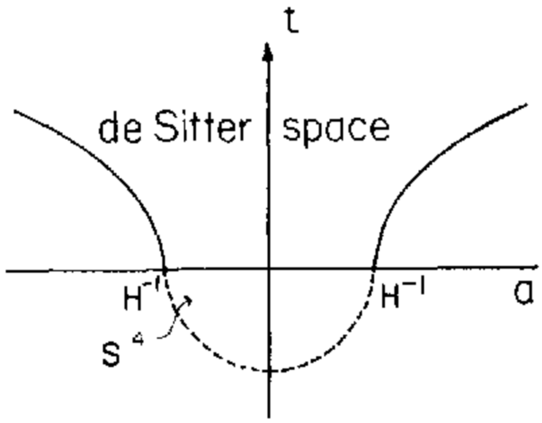

Figure 3.2: *Figure taken from [45], which schematically shows the birth of the inflationary universe at a finite size from a 4d geometry without time. the classical evolution starts at $t = 0$.*

The solution (3.2.7) bounces at the classical turning point ($a = H_\Lambda^{-1}$), but it does not approach any initial state at $t \to \pm\infty$. In fact, S^4 is a compact space and the solution (3.2.7) is defined only for $|t| < \frac{\pi}{2} H_\Lambda$. This compact instanton can therefore be interpreted as describing the tunneling to the de Sitter space (3.2.5) with finite initial scale factor from literally *nothing*.

This process is analogous to the creation of electron-positron pairs in a constant electric field E, which involves the creation of a compact instanton, as is shown in Fig.3.3.

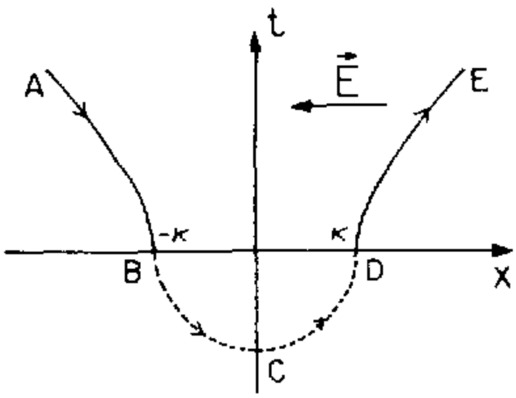

Figure 3.3: *Figure taken from [45], which schematically shows pair creation in the electric field. The classically allowed trajectories, DE and AB, describe, respectively, an electron moving forward in time and an electron moving backwards in time (a positron); the semicircle BCD represents the instanton.*

The instanton solution can be used to estimate the semiclassical probability P of pair creation per unit length per unit time

$$P \propto \exp(-S_E), \tag{3.2.8}$$

where S_E is the euclidean action for the corresponding instanton. Of course, the evaluation of the probability P makes sense because the pair creation takes place in a background flat space; the instanton solution contributes to the imaginary part of the vacuum energy.

For the de Sitter instanton, instead, it would be pointless to try to evaluate the imaginary parts of the energy of nothing. Therefore, the only relevant question is whether or not the spontaneous creation of universes is possible. The existence of the instanton (3.2.7) suggests that it is.

The euclidean action of a de Sitter instanton is negative [48], $S_E = -\frac{3}{8G_N^2 \rho_\Lambda}$. A natural conclusion would then be

$$P \propto \exp\left(\frac{3}{8G_N^2 \rho_\Lambda}\right). \tag{3.2.9}$$

Nonetheless, the correct answer seems to be $P \propto \exp(-|S_E|)$. The basic reason is that the under-barrier wave function contains growing and decreasing exponentials with roughly equal coefficients, but the growing exponential dominates. In the usual case of bubble nucleation, the Euclidean action is positive definite, thus $|S_E| = S_E$.

The problem of determining the tunneling amplitude can be approached by solving the Wheeler-deWitt equation, which is a functional differential equation on superspace for the wave function of the Universe, $\Psi[h_{ij}, \phi]$. In the following, we will work with a minisuperspace model, in which we restrict the 3-geometry to be homogenoeus, isotropic and closed, so that it is described by a single scale factor a. The scalar field ϕ is restricted to constant value at one of the extrema of the effective potential: $\phi = \phi_0$. Then, the Wheeler-deWitt equation for $\Psi(a)$ takes the form

$$\left[a^{-p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} - \left(\frac{3\pi}{2G}\right)^2 a^2(1-H^2a^2)\right]\Psi(a) = 0, \tag{3.2.10}$$

where the parameter p depends on one's choice of factor ordering. Variation of p affects Ψ only for $a < G^{1/2}$. Since these values of a are unimportant for our discussion, we can just set $p = 0$, so that eq. (3.2.10) takes the form of a one-dimensional Schrodinger equation for a "particle" described by coordinate $a(t)$, having zero energy and moving in a potential

$$U(a) = \frac{1}{2}\left(\frac{3\pi}{2G}\right)^2 a^2(1-H^2a^2). \tag{3.2.11}$$

The WKB solutions of Eq.(3.2.10) in the classically allowed region ($a > H^{-1}$) are

$$\Psi_\pm^{(1)}(a) = \exp\left(\pm i\int_{H^{-1}}^{a} p(a')da' \mp \frac{i\pi}{4}\right) \tag{3.2.12}$$

and the under-barrier ($0 < a < H^{-1}$) solutions are

$$\Psi_\pm^{(2)}(a) = \exp\left(\pm i\int_{a}^{H^{-1}} |p(a')|da'\right), \tag{3.2.13}$$

where $p(a) = (-2U(a))^{-1/2}$. Tunneling through the barrier corresponds to the choice of the "outgoing" wave for $a > H^{-1}$

$$\Psi(a > H^{-1}) \sim \Psi_+^{(1)}(a). \tag{3.2.14}$$

The WKB connection formula gives the under-barrier wave function of the form

$$\Psi(a < H^{-1}) \sim \Psi_+^{(2)}(a) + \frac{i}{2}\Psi_-^{(2)}(a). \tag{3.2.15}$$

Except in the immediate vicinity of $a = H^{-1}$, the second term in Eq. (3.2.15) is negligible, and $\Psi(a > H^{-1}) \approx \Psi_+^{(1)}(a)$. The wave function grows exponentially towards $a = 0$. The tunneling amplitude is therefore proportional to

$$\exp\left(-\int_0^{H^{-1}} |p(a')|da'\right) = \exp\left(-\frac{3}{16G^2\rho_\Lambda}\right) \tag{3.2.16}$$

which means the tunneling probability is with $S_E = -\frac{3}{8G^2\rho_\Lambda}$.

Notice that the semiclassical approximation is justified if $|S_E| \ll 1$ or $\rho_\Lambda \ll G^{-2}$ (a condition which is satisfied in most Grand Unified Theories).

We have found that the tunneling probability is $P \propto \exp\left(-\frac{3}{8G^2\rho_\Lambda}\right)$, where $\rho_\Lambda = V(\phi_0)$ and ϕ_0 is an extremum of the effective action. But what happens to the universe *after* tunneling? The symmetric vacuum state is absolutely unstable. It can decay via quantum tunneling or can be destabilized by quantum fluctuations of the Higgs (or inflaton) field. This field starts rolling down the effective potential towards the ending of the inflationary scenario, when reheating takes place.

3.3 Hartle-Hawking no-boundary proposal

In 1983, Hartle and Hawking proposed a new approach [49] to the definition of the wave function of the Universe: they suggested that the wave function $\Psi[h_{ij}, \phi]$ is given by a path integral over all compact Euclidean four-geometries and scalar field histories bounded by the configuration $[h_{ij}, \phi]$

$$\Psi[h_{ij}, \phi] = \int_C [dg][d\phi] \exp(-S_E[g, \phi]). \tag{3.3.1}$$

In particular, they put forward a proposal for the wave function of the "ground state" or state of minimum excitation: the ground state amplitude for a three-geometry is given by the path integral over all compact positive-definite four-geometries which have the three-geometry as a boundary. The requirement that the Hamiltonian be hermitian, then, defines the boundary conditions for the Wheeler-deWitt equation and the spectrum of possible excited states. The ground state corresponds to de Sitter space in the classical limit. There are excited states which represent universes which expand from zero volume, reach a maximum size and then recollapse, but which have a finite (though very small) probability of tunneling through a potential barrier to a de Sitter-type state of continual expansion. The path integral approach allows us to handle situations in which the topology of the three-manifold changes.

Let us briefly only mention some key ideas contained in [49]. In quantum mechanics, the state of a system is determined by giving its wave function on an appropriate configuration space. The possible wave functions can be constructed from the fundamental quantum-mechanical amplitude for a complete history of the system which may be regarded as the starting point for quantum theory. For instance, in the case of a single particle, a history is a path $x(t)$ and and the amplitude for a particular path is proportional to $\exp(iS[x(t)])$, where $S[x(t)]$ is the classical action. The amplitude for more restricted observations can be constructed by superposition; in particular, the amplitude that the particle, having been prepared in a certain way, is located at position x and nowhere else in time, is

$$\psi(x,t) = N \int_C [dx(t)] \exp(iS[x(t)]), \tag{3.3.2}$$

where N is a normalizing factor and the sum is over a class of paths which intersect x at time t and which are weighted accordingly to the preparation of the system. The oscillatory integral in Eq. (3.3.2) is not well defined, but can be made so by rotating the time to imaginary values ($t \to -i\tau$).

The ground state, or state of minimal excitation of the theory, is defined by the path integral over the class of paths which have vanishing action in the far past. Thus, for the ground state at $t = 0$ one would write

$$\psi_0(x,0) = N \int [dx(\tau)] \exp(-S_E[x(\tau)]), \tag{3.3.3}$$

where $S_E[x(\tau)]$ is the Euclidean action obtained from S sending $t \to -i\tau$. If in the theory there is a well-defined time and a corresponding time-independent Hamiltonian, this definition of ground state coincides with the lowest eigenfunction of the Hamiltonian.

The case of quantum fields is a straightforward generalization of quantum particle mechanics: the wave function is a functional of the field configuration on a space-like surface of constant time, $\Psi = \Psi[\phi(\mathbf{x}), t]$, and the functional Ψ gives the amplitude that a particular field distribution $\phi(\mathbf{x})$ occurs on this space-like surface. The rest of the formalism is easily generalized. For instance, the ground-state wave functional is given by

$$\Psi_0[\phi(\mathbf{x}), 0)] = N \int [d\phi(x)] \exp(-S_E[\phi(x)]), \tag{3.3.4}$$

where the integral is over all Euclidean field configurations for $\tau < 0$ which match $\phi(\mathbf{x})$ on the surface $\tau = 0$ and leave the action finite at Euclidean infinity.

When gravity enters the game, there are some modifications to be done. Focusing only on spatially closed universes, there is no well-defined intrinsic measure of the location of a space-like surface in the spacetime beyond that contained in the intrinsic or extrinsic geometry of the surface itself. One therefore labels the wave-function by the three-metric h_{ij}, writing $\Psi = \Psi[h_{ij}]$. Quantum dynamics is given by the functional integral

$$\Psi[h_{ij}] = N \int_C [dg(x)] \exp(iS[g]), \tag{3.3.5}$$

where S now is the classical action for gravity including a cosmological constant Λ, and the functional integral is over all four-geometries with a space-like boundary on which the induced metric is h_{ij} and which to the past of that surface satisfy some appropriate condition to define the state.

Eq. (3.3.5) implies a differential equation on the wave function: the Wheeler-deWitt equation. the problem of specifying cosmological states then becomes the same as specifying boundary conditions for the solution of the Wheeler-deWitt equation. This said, we can investigate which boundary conditions specify the ground state. In the quantum treatment of closed universes, there is no well defined notion of ground state as a state of lowest energy; it is still reasonable, however, to define a state of minimum excitation corresponding to the classical notion of geometry of high symmetry.

The proposal of HH is the following: extend to gravity the Euclidean functional integral construction of quantum mechanics and field theory. So, we can write for the ground-state wave function

$$\Psi_0[h_{ij}] = N \int [dg] \exp(-S_E[g]), \tag{3.3.6}$$

where S_E is the Euclidean action for gravity including a cosmological constant Λ. The Euclidean four-geometries summed over must have a boundary on which the induced metric is h_{ij}. The remaining specification of the class of the geometries which are summed over determines the ground state: HH suggested that the sum should be taken over compact geometries, meaning that the Universe does not have any boundaries in space or time (at least in the Euclidean regime). *There is thus no problem of boundary conditions. We can interpret the functional integral over all compact four-geometries bounded by a given three-geometry as giving the amplitude for that three-geometry to arise from a zero three-geometry (a single point).* Put differently, the ground state is the amplitude for the Universe to appear from nothing.

The specification of the ground state wave function is a constraint on the other states allowed in the theory (they must be such, for example, as to make the Wheeler-deWitt equation hermitian in an appropriate norm). Moreover, we are able to use these constraints to extrapolate the boundary conditions which determine the excited states of the theory from those fixed for the ground state by Eq. (3.3.5). Therefore, we could in principle determine all the allowed cosmological states. The wave functions which result from this specification will not vanish on the singular, zero-volume three-geometries which correspond to the Big Bang singularity. We can build a complete spectrum of excited states which show that a closed universe similar to ours and endowed with a cosmological constant can escape the Big Crunch and tunnel through an eternal de Sitter expansion.

Representing, for simplicity, the matter degrees of freedom by a single scalar field ϕ, we recover the definition (3.3.1) for the wave function Ψ. The sum is over a class C of spacetimes with a compact boundary on which the induced metric is h_{ij} and field configurations which match ϕ on the boundary. The remaining specification of the class C is the specification of the state.

The main difference with Vilenkin's proposal is that the universe tunnels into a Lorentzian state with no initial space-like boundary, since final geometries with $a = 0$ at the beginning are included, and no Lorentzian stage is assumed before the tunneling. Recalling what we found for Vilenkin's choice in Eq.s (3.2.14) and (3.2.15), we can compare it with HH results. Starting from Eq. (3.3.1), the HH wave function is $\Psi(a < H^{-1}) \sim \Psi_-^{(2)}(a)$ and $\Psi(a > H^{-1}) \sim \Psi_+^{(1)}(a) + \Psi_-^{(1)}(a)$. This wave function corresponds to a "particle" bouncing off the potential barrier at $a = H^{-1}$; under the barrier, $\Psi(a)$ is exponentially suppressed. Indeed, it describes a contracting and expanding Universe. Thus, the HH approach automatically gives a time-symmetric picture of the Universe: a contracting and reexpanding Universe in the case of de Sitter space and an oscillating universe in more complicated mini-superspace models.

3.4 Comparing the two wave functions

Let us now examine both proposals and try to understand their difference in mini-superspace. Consider, for simplicity, cosmologies driven by a pure positive cosmological constant. Then the Friedmann equation for a 4d cosmology in case of positive spatial curvature is

$$\dot{a}^2 = -1 + \frac{a^2}{R^2}, \tag{3.4.1}$$

where we are now introducing the de Sitter radius R and $R^{-2} \equiv \frac{8\pi G_N}{3}\rho_\Lambda = \Lambda$. The mini-superspace reduction of the Einstein-Hilbert action leading to the Friedmann equation is given by

$$S = \frac{6\pi^2}{8\pi G_N}\int d\tau N\left(-\frac{a\dot{a}^2}{N^2} + a - \frac{a^3}{R^2}\right). \tag{3.4.2}$$

N is the lapse function from the FLRW metric

$$ds^2 = -N^2(\tau)d\tau^2 + a^2(\tau)d\Omega_3^2, \tag{3.4.3}$$

which acts as a Lagrange multiplier whose constraint reproduces (3.4.1). From the action, we derive the canonical momentum $p = -(12\pi^2 a\dot{a})/N$; quantizing using $p \longrightarrow -i\frac{d}{da}$, the Hamiltonian constraint becomes the Wheeler-deWitt equation

$$\frac{N}{a}\left(-\frac{1}{24\pi^2}\frac{d^2}{da^2} + 6\pi^2 V(a)\right)\Psi(a) = 0, \tag{3.4.4}$$

where the effective potential is given by $V(a) = a^2 - R^{-2}a^4$. The plot of $V(a)$ makes explicit the analogy with tunneling through a barrier.

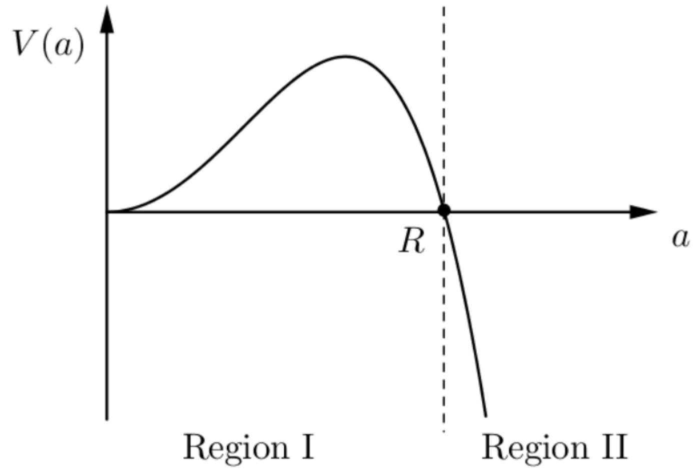

Figure 3.4: *Effective potential with two turning points: $a = 0$ and $a = R$*

There are two turning points: $a = 0$ and $a = R$. The region between these turning points is the so-called Euclidean region (I), while the region $a > R$ (II) is classical.

The WKB solution is given by

$$\Psi_I^{(a)} = \frac{1}{|V(a)|^{1/4}}(ce^{S(a,0)+de^{-S(a,0)}}),$$
$$\Psi_{II}^{(a)} = \frac{1}{|V(a)|^{1/4}}(Ae^{S(a,R)+Be^{-S(a,R)}}), \tag{3.4.5}$$

where c, d, A, B are complex constants and where we defined

$$S(a, a_i) \equiv \frac{12\pi^2}{8\pi G_N}\int_{a_i}^{a}\sqrt{|V(a')}da'. \tag{3.4.6}$$

A choice (c, d) or (A, B) reflects the choice of boundary conditions. We also need a normalization to fix them completely, and a common one is

$$\lim_{a\to\infty}|V(a)|^{1/4}\Psi(a) = 1. \tag{3.4.7}$$

The HH choice selects the growing exponential in region I by taking $(c, d) = (1, 0)$:

$$\Psi_{HH}(a) = \frac{1}{|V(a)|^{1/4}}\begin{cases} e^{S(a,0)}, & \text{Region I} \\ 2e^{S_0}\cos[S(a,R) - \frac{\pi}{4}] & \text{Region II,} \end{cases} \tag{3.4.8}$$

where $S_0 \equiv S(R, 0) = \frac{4\pi^2 R^2}{8\pi G_N}$. This leads to the following nucleation probability

$$P_{HH} \propto e^{2S_0}. \tag{3.4.9}$$

Vilenkin's choice, instead, is defined selecting only outgoing waves in region II, namely $(A, B) = (0, B)$:

$$\Psi_V(a) \approx \frac{1}{|V(a)|^{1/4}} \begin{cases} e^{S_0} e^{-S(a,0)+i\frac{\pi}{4}}, & \text{Region I} \\ e^{-iS(a,R)} & \text{Region II,} \end{cases} \tag{3.4.10}$$

where we approximated $c \approx 0$ since it is exponentially suppressed. Therefore

$$P_V \propto e^{-2S_0}. \tag{3.4.11}$$

Summarizing, the difference of the two proposals in mini-superspace can roughly be described in the following way: *at large scale-factor, the tunneling wave function can be seen as purely "outgoing" waves, just as a wave function of a particle escaping a radioactive nucleus. The no-boundary proposal instead has fine-tuned ingoing and outgoing waves such that the wave function decreases towards the Big Bang singularity.*

The amplitudes of the two wave functions behave very differently when we consider their dependence on the cosmological constant Λ (in Planck units)

$$\Psi_V \approx e^{-c/\Lambda} \qquad\qquad \Psi_{HH} \approx e^{c/\Lambda}, \tag{3.4.12}$$

with c a positive numerical constant. The amplitude for the no-boundary proposal peaks at a small positive cosmological constant, while the opposite is true for the tunneling wave function. This would suggest that the most naive interpretations of the wave function might then be at odds with the tunneling wave function is one wants to address the cosmological constant problem, but the discussion is still open.

The higher-dimensional Big-Bang interpretation in the Dark Bubble scenario is particularly interesting as it provides a connection with quantum cosmology [14]. What a lower-dimensional observer would call the Big Bang has the bulk interpretation of a well-understood nucleation event à la Brown-Teitelboim. From the higher dimensional perspective, therefore, the Big Bang does not appear as a singularity. The Dark Bubble model has the advantage of providing a UV completion of a 4d gravity on the bubble, making it possible to explore the issue of boundary conditions. Keeping the scale factor as the only dynamical variable, we can show that the amplitude of bubble nucleation in 5d perfectly matches Vilenkin's tunneling amplitude in 4d quantum cosmology.

3.5 Matching the amplitudes

We will now argue that the choice of boundary conditions for the wave function in Quantum Cosmology depends on the UV completion of General Relativity. Using the Dark Bubble model, we will see that the corresponding boundary conditions are unambiguously fixed by demanding consistency with the known physics of bubble nucleation and this selects Vilenkin's tunneling amplitude from a 4d point of view.

Let us briefly describe the quantum nucleation of such a bubble in five dimensions. The 5d action is given by

$$S = \frac{1}{\kappa_5}\int d^5x\sqrt{|g|}(R^{(5)} - 2\Lambda) - \sigma\int d^4\zeta\sqrt{|\eta|} + \frac{1}{\kappa_5}\oint d^4x\sqrt{|h|}K. \tag{3.5.1}$$

The second term describes the brane-shell with tension σ and induced metric η with brane-coordinates ζ. The 5d metric is given by

$$ds^2_\pm = -A_\pm(r)dt^2_\pm + \frac{dr^2}{A_\pm(r)} + r^2 d\Omega_3^2, \tag{3.5.2}$$

where $A_\pm(r) = 1 - \frac{\Lambda_\pm}{6}r^2$. The shell glues the two spacetimes together at a radial coordinate $r = a(\tau)$ and its metric coincides with (3.4.3). From (3.5.2) and (3.4.3) we can then deduce that

$$\dot{t}_\pm = \frac{dt_\pm}{d\tau} = \frac{\beta_\pm}{A_\pm}, \qquad \beta_\pm = (A_\pm N^2 + \dot{a}^2)^{1/2} \tag{3.5.3}$$

The on-shell action receives three contributions: the bulk piece, the shell contribution and the boundary term. Summing all the terms (and neglecting those which are not relevant for the dynamics of the shell), we find the mini-superspace Lagrangian to be

$$L = \frac{6\pi^2}{\kappa_5}\left[-a^2\dot{a}\tanh^{-1}\frac{\dot{a}}{\beta} + a^2\beta\right]^-_+ - 2\pi^2 a^3\sigma N. \tag{3.5.4}$$

Expanding to quadratic order in $\dot{a}$ and using (2.2.1), Eq.(3.4.2) can be recovered. Let us therefore study nucleation, where R will be the radius of the nucleated bubble.

We now argue that the physics of bubble nucleation in 5D then fixes the amplitude to be of the tunneling type. The goal is simple: verify that Vilenkin's tunneling amplitude exactly matches the known Coleman-de Luccia amplitudes, as expected from the physical picture of tunneling. There are a few different ways to calculate the nucleation probability $P = e^{-B}$ that all yield the same result. Using the approach of Brown and Teitelboim [39], for instance, we recover Vilenkin's choice. The Euclidean instanton is then obtained by integrating (3.5.1) over a $O(5)$ symmetric ball of radius R, with a 4D sphere as boundary, and corresponds to a bounce. Evaluating expression (6.4) in [39] with $R^{(5)}_\pm = -20k^2_\pm$, which follows from the equations of motion, we find

$$B = \sigma A_4 + \frac{1}{\kappa_5}\left[4k^2 V_5(R,k) - \frac{4}{R}\beta A_4\right]^-_+ \tag{3.5.5}$$

with $A_4 = \frac{8\pi^2 R^4}{3}$, and $\frac{dV}{dR} = \frac{A_4}{\beta}$. Extremizing using $\frac{dB}{dR} = 0$ implies the junction condition and fixes R to the critical value. Expanding in large k, we then have

$$V_5 = \frac{A_4}{4k}\left(1 - \frac{1}{k^2R^2}\right). \tag{3.5.6}$$

Inserting this, and replacing κ_5 by κ_4 using (2.2.1), we recover Vilenkin's choice

$$B = \frac{24\pi^2}{\kappa_4}\int_0^R da\sqrt{a^2 - \frac{a^4}{R^2}} = \frac{8\pi^2 R^2}{\kappa_4}. \tag{3.5.7}$$

Going back to the full expression, the canonical momentum is given by

$$\cosh\left(\frac{\kappa_5 p}{6\pi^2 a^2}\right) = \frac{\beta_-\beta_+ - \dot{a}^2}{N^2\sqrt{A_-A_+}}, \tag{3.5.8}$$

and the Hamiltonian plays the role of a constraint imposing the junction condition:

$$H = 2\pi^2 N a^3\left(\sigma - \frac{3(\beta_- - \beta_+)}{a\kappa_5}\right) = 0. \tag{3.5.9}$$

Expressed in terms of the canonical momentum the Hamiltonian constraint becomes

$$H = -\frac{6\pi^2}{\kappa_5}\left(A_- + A_+ - 2\sqrt{A_-A_+}\cosh\left(\frac{\kappa_5 p}{6\pi^2 a^2}\right)\right)^{1/2} + 2\pi^2 N a^3\sigma = 0 \tag{3.5.10}$$

We can quantize the system to obtain the WdW-equation by making the replacement $p \to -\frac{i}{a^{3/2}}\frac{d}{da}a^{3/2}$. For general p the equation is of infinite order in p and turns into a difference equation. We focus on the limit of small p, where the Hamiltonian becomes quadratic in p. This is the limit that is relevant for the case of a small cosmological constant compared to fundamental scales. In this limit, we recover (3.4.4)

$$\left(-\frac{1}{24\pi^2}\frac{1}{a^{3/2}}\frac{d^2}{da^2}(a^{3/2}\psi) + 6\pi^2 V(a)\right)\psi = 0, \tag{3.5.11}$$

with a different normalization of the wave function. Here, in fact, the wave function ψ is supported in four spatial dimensions, and is related to the wave function in mini-superspace through $\Psi = a^{3/2}\psi$. Note that $\int a^3|\psi|^2 = \int|\Psi|^2$ is used for normalization.

Discussion

We conclude that Vilenkin's amplitude in quantum cosmology can be understood as the nucleation probability of a bubble of true vacuum in an unstable AdS_5 space. Our understanding of the physics of Coleman-de Luccia bubbles translates to an understanding of more involved issues in quantum cosmology such as the choice of boundary conditions, which affect the amplitudes. In this model there is no Big-Bang singularity to worry about and all physics at all length scales that are involved in the process are essentially understood. Indeed, the Big Bang singularity is not present in 5d and would correspond to the zero size of the bubble, which is not a physical solution to worry about. The undetermined coefficients in the mini-superspace wave function are then completely fixed, since it has to be consistent with the standard story of thin wall Coleman-de Luccia tunneling (or the particular Brown-Teitelboim incarnation in this case). From the point of view of a 4d observer, the nucleation of the closed bubble universe is a creation out of nothing. From a 5d point of view, however, one still needs to explain the origin of the AdS space time.

It is useful to compare the above results with the model of Karch-Randall [37]. There, a closed dS universe is represented by a bubble with its inside identified with itself across its boundary. Such a bubble has no outside and cannot nucleate into a pre-existing space time. Interestingly, [50] concluded that the quantum creation of such a universe would be described by the Hartle-Hawking amplitude. This is in contrast with the nucleating bubbles we have considered, which tunnel into existence as a false vacuum decays and thus need to be described by Vilenkin's tunneling amplitude. The Swampland conjectures and the difficulty to construct stable de Sitter vacua suggest that it is the latter possibility that has a chance of being realized in String Theory [51].

Chapter 4

Embedding Electromagnetism in the Bubble

One of the most important tasks we have to achieve in order to connect the Dark Bubble model with phenomenology is trying to embed the Standard Model of Particle Physics into our braneworld construction. A first step in realizing this consists in describing how the electromagnetic gauge field "lives" on the brane and induces a backreaction in the bulk, which in turn will affect the motion of the brane. This has been realized in [18], exploiting the action of the B field. In fact, the B field mediates the interplay between the brane and the bulk: worldvolume fields backreact on the ambient universe in which the bubble expands, which in turn affects the energy-momentum distribution and the effective gravity induced on the brane. Let us briefly summarize this construction and then move on to a more general setup.

4.1 DBI action with B field

Let us start from the Dirac-Born-Infeld action for the D_3 brane

$$S_{D_3} = -T_3 \int d^4x \sqrt{-\det(g_4 + \tau \mathcal{F})}, \tag{4.1.1}$$

where $\alpha' \equiv l_s^2$ is the string tension, $T_3 = \frac{1}{(2\pi)^3 \alpha'^2 g_s}$ and $\tau = 2\pi\alpha'$. Expanding for $\alpha' \ll 1$, we find

$$\begin{aligned} S_{DBI} &= -T_3 \int d^4x \sqrt{-\det(g_4)} \left(1 + \frac{\tau^2}{4} \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \mathcal{O}(\mathcal{F}^2)\right) \\ &= -\int d^4x \sqrt{-\det(g_4)} \left(T_3 + T_3 \frac{\tau^2}{4} \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \mathcal{O}(\mathcal{F}^2)\right) \\ &= -\int d^4x \sqrt{-\det(g_4)} \left(T_3 + \frac{1}{4g^2} \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \mathcal{O}(\mathcal{F}^2)\right), \end{aligned} \tag{4.1.2}$$

where $g^2 \equiv 2\pi g_s$ is the gauge coupling. The second piece in this action corresponds to the action of electromagnetism in 4d.

Now, recall that the gauge field on the brane can be written as $\tau\mathcal{F} = \tau F + B$. Notice that the conventions are such that $\tau\mathcal{F}$ and B are dimensionless. Obviously, B sources a 3-form field $H = dB$. By varying the DBI action with respect to B, we get a source term for H given simply by $\mathcal{F}$. Since we are working with a shell of co-dimension one, this means there is a jump in H across the shell. We assume that H vanishes inside the shell and that the non-vanishing value outside is purely due to the presence of the electromagnetic field on the brane.

The bulk equations we need to solve are obtained from the following bulk action, with the DBI contribution included

$$S_5 = \frac{1}{2\kappa_5}\int d^5x\sqrt{-\det(g_5)}\left(R - \frac{1}{12g_s}H^2\right) - T_3\int d^5x\delta(r - a[\eta])\sqrt{-\det(g_4 + \tau\mathcal{F})}. \tag{4.1.3}$$

We are working in the 5d Einstein frame, where the dilaton coupling of the B field is fixed to $e^{-\phi} = g_s^{-1}$ and, since the bulk metric is of the form $ds_5^2 = k^2r^2(-dt^2 + dx^2 + dy^2 + dz^2) + \frac{dr^2}{k^2r^2}$, the following relation holds

$$\sqrt{-g_5} = \frac{1}{kr}\sqrt{-g_4}, \tag{4.1.4}$$

where we are using the notation $\sqrt{-\det(g_d)} \equiv \sqrt{-g_d}$. We also assume that internal moduli and the dilaton are stabilized.

If we expand for small α', as we did in (4.1), we can write

$$\begin{aligned} S_5 &= \frac{1}{2\kappa_5}\int d^5x\sqrt{-g_5}\left(R - \frac{1}{12g_s}H^2\right) - T_3\int d^5x\delta(r - a[\eta])\sqrt{-\det(g_4 + \tau\mathcal{F})} \\ &= \int d^5x\sqrt{-g_4}\left[\frac{1}{2\kappa_5 kr}\left(R - \frac{1}{12g_s}H^2\right) - \left(T_3 + T_3\frac{\tau^2}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right)\delta(r - a[\eta])\right]. \end{aligned} \tag{4.1.5}$$

Varying the action with respect to B we get

$$\delta_B S_5 = \int d^5x\sqrt{-g_4}\left[\frac{1}{2\kappa_5 kr}\left(-\frac{1}{12g_s}\delta_B(H^2)\right) - \frac{T_3\tau^2}{4}\delta_B(\mathcal{F}^2)\delta(r - a[\eta])\right]. \tag{4.1.6}$$

Noticing that $\delta_B(\mathcal{F}^2) = 2\mathcal{F}\delta_B B_{\mu\nu}$ and $\delta_B(H^2) \equiv \delta_B(H_{r\mu\nu}H^{r\mu\nu}) = 2H^{r\mu\nu}\delta_B H_{r\mu\nu}$ and integrating by parts, we find

$$\partial_r H^{r\mu\nu} = \frac{2\kappa_5 kr}{\pi^2\alpha'}\mathcal{F}_{\mu\nu}\delta(r - a[\eta]). \tag{4.1.7}$$

Integrating across the brane at $r = a(\eta)$, we find

$$\Delta H^{r\mu\nu}\big|_{r=a} = H^{r\mu\nu}\big|_{r=a} = \frac{2\kappa_5 ka}{\pi^2\alpha'}\mathcal{F}_{\mu\nu}\big|_{r=a}, \tag{4.1.8}$$

which means that the electromagnetic waves on the brane source the H-field in the bulk above the brane.

Let us now understand how to relate the 4d and the 5d field strengths to each other. In particular, let us focus on electromagnetic waves propagating along z, with the electric field polarized along y. Given a wave propagating in the z direction, and **E,B** respectively polarized along x, y directions, we have

$$\mathcal{F}^{tx} = \frac{\mathcal{E}(t,z)}{k^2 a^4}, \qquad \mathcal{F}^{xz} = \frac{\mathcal{E}(t,z)}{k^2 a^4}, \tag{4.1.9}$$

where $\mathcal{E}$ is a dimensional function which determines the amplitude of the wave. We can therefore identify the behavior of the H field strength on the brane, assuming there is no H-field turned on inside the bubble. Imposing this boundary condition to (4.1.8) gives us

$$\begin{aligned} \Delta H^{rtx}\big|_{r=a} \equiv h_1(t,a,z) &= \frac{2\kappa_5 ka}{\pi^2\alpha'}\mathcal{F}_{tx} = \frac{2\kappa_5 ka}{\pi^2\alpha' k}\frac{\mathcal{E}(t,z)}{a^3}, \\ \Delta H^{rtx}\big|_{r=a} \equiv h_2(t,a,z) &= \frac{2\kappa_5 ka}{\pi^2\alpha'}\mathcal{F}_{xz} = \frac{2\kappa_5 ka}{\pi^2\alpha' k}\frac{\mathcal{E}(t,z)}{a^3}. \end{aligned} \tag{4.1.10}$$

The H-field must solve the equations of motion, as well as the Bianchi identities away from the brane:

$$\begin{aligned} *d*H = 0 &\Rightarrow \nabla_\alpha H^{\alpha\beta\gamma} = 0, \\ *dH = 0 &\Rightarrow \epsilon_\kappa^{\alpha\mu\nu\lambda}\partial_\alpha H^{\mu\nu\lambda} = 0, \end{aligned} \tag{4.1.11}$$

where * is the 5d Hodge star. However, it is not enough to turn on these components of the bulk field: any non-trivial time dependence also requires the presence of $H^{txz} \equiv h_3(t,r,z)$.

For the AdS background, in the limit of flat 4d spacetime, the equations of motion become

$$\begin{aligned} \left(\partial_r + \frac{3}{r}\right) h_1 + \partial_z h_3 &= 0, \\ \left(\partial_r + \frac{3}{r}\right) h_1 + \partial_z h_3 &= 0, \\ \partial_z h_2 - \partial_t h_1 &= 0, \end{aligned} \tag{4.1.12}$$

while the Bianchi identity reads

$$r^2(\partial_t h_2 - \partial_z h_1) + k^4\partial_r(r^6 h_3) = 0. \tag{4.1.13}$$

Let us choose a general ansatz for the H field strength, accounting for the aforementioned H^{txz} component that represents such a flux:

$$\begin{aligned} h_1 &= f_1(t,z,x)k^{-2}r^{-3}, \\ h_2 &= f_2(t,z,x)k^{-2}r^{-3}, \\ h_3 &= f_3(t,z,x)k^{-3}r^{-4}, \end{aligned} \tag{4.1.14}$$

with $x = 2k^3tr^2$ and $f_i(t,z,x) \equiv \alpha_i(t,x)\sin(kn(t+z)) + \beta_i(t,x)\cos(kn(t+z))$, where α_i and β_i are dimensionless functions. With this ansatz, we find the solution to the equations of motion and the Bianchi identity to order $(knt)^{-1}$ but all orders in x, given by

$$h_1(t,z,x) = \frac{\alpha}{k^2r^3}\sin(kn(t+z)) + \frac{2\beta}{k^2r^3}\left[\sin\left(\frac{x}{2}+kn(t+z)\right) - \sin(kn(t+z))\right] \tag{4.1.15}$$

$$h_2(t,z,x) = h_1(t,z,x) - \frac{\beta x}{k^2ntr^3}\cos\left(\frac{x}{2}\right)\sin(kn(t+z)), \tag{4.1.16}$$

$$h_3(t,z,x) = \frac{2\beta x}{nk^3r^4}\sin\left(\frac{x}{2}+kn(t+z)\right), \tag{4.1.17}$$

where α and β are now constants.

4.1.1 Energy-momentum tensor in the bulk

Varying the bulk part of the action (4.1.5) with respect to $g_{\mu\nu}$, we obtain the energy-momentum tensor for the electromagnetic fields

$$T_{\mu\nu} = \frac{1}{\kappa_5}\left(-\frac{1}{2}g_{\mu\nu}H^2 + 3H_{\mu\alpha\beta}H_\nu^{\alpha\beta}\right). \tag{4.1.18}$$

So far, our computations have been for a single electromagnetic wave travelling in the z direction, electric and magnetic fields accordingly polarized along x, y directions. A uniform background of electomagnetic waves can be realized by accounting for all ingoing and outgoing waves along some given distribution. Moreover, we should average over waves travelling isotropically along x, y, z, with any possible polarization. The isotropic energy-momentum tensor thus reads

$$\langle T^\mu_\nu\rangle_{iso} = \langle T^\mu_\nu\rangle_{rad} + \langle T^\mu_\nu\rangle_{flux}, \tag{4.1.19}$$

where the radiation and flux contributions to the 5d energy-momentum tensor are given by

$$\langle T^\mu_\nu\rangle_{rad} = \frac{3\alpha^2}{\kappa_5k^2r^4}\begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \langle T^\mu_\nu\rangle_{flux} = \frac{3\alpha^2}{\kappa_5k^2r^4t}\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{-\beta}{\alpha k^4r^3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\beta r}{\alpha} & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.1.20}$$

Off-diagonal terms in the flux correspond to the energy flow in the radial direction, which corresponds to a non-vanishing T^t_r component. This energy flow is analogous to the one along the radial direction in Eq. (2.5.12). Covariant conservation of this energy-momentum tensor is guaranteed, as each component h_i of the bulk field strength H solve both their equation of motion (4.1.12) and the Bianchi identity (4.1.13).

Let us now calculate the backreaction and move on to matching parameters with the 4d Dark Bubble cosmology.

4.1.2 Backreaction

If there were no net flux along the tr direction, the metric would be very similar to an AdS-Schwarzschild one, where $f(r) = 1 + k^2r^2 - \frac{1}{r^2}$ should be replaced by $\hat{f}(r) = 1 + k^2r^2 - \frac{\log r}{r^2}$. In 4d, this would correspond to a logarithmic increase of energy with time on the brane, compared to the expected $1/a^4$ decay as the universe expands. This is because the expanding bubble scoops up matter from bulk that is added to the brane. To avoid this, we need the bulk matter to expand up along the throat of AdS, just as it does in the case of gravitational waves.

The energy momentum tensor in the bulk which does the job has $\frac{\log r}{r^2} \longrightarrow \frac{\log tr}{r^2}$. Since $kt = -\frac{1}{Hr}$ on the bubble[1], this gives the correct behavior in 4d. To be precise, the 5d metric, in the flat 4d limit, is given by

$$ds^2_{backreacted} \simeq -f_1(t,r)dt^2 + f_2(t,r)^{-1}dr^2 + r^2 d\Omega_3^2, \tag{4.1.21}$$

where

$$f_i(t,r) \equiv k^2r^2 - \frac{\epsilon^2(\log(-\xi k^2 tr) + q_i)}{k^2r^2}, \tag{4.1.22}$$

with q_i undetermined numbers. This line invariant yields an energy momentum tensor of the form

$$\langle T^\mu_\nu \rangle_{rad} = \frac{3\epsilon^2}{2\kappa_5 k^2 r^4} \begin{pmatrix} -1 & 0 & 0 & 0 & \frac{-1}{k^4r^3t} \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{r}{t} & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4.1.23}$$

provided $q_1 - q_2 = \frac{1}{4}$. Note that ξ is a free parameter that does not affect the energy momentum tensor. Tuning it corresponds to changing a piece of the metric that is the vacuum AdS-Schwarzschild background.

Let us now compare both the energy momentum tensor (4.1.19) and (4.1.23) to relate the parameters α, β, ϵ. It is easy to see that

$$\alpha = \beta = \frac{\epsilon}{\sqrt{2}}. \tag{4.1.24}$$

[1] On the brane $\eta = kt = -\frac{1}{Hr}$.

We can also express these coefficients in terms of the amplitude of the 4d electromagnetic wave: comparing to Eq. (4.1.10), we can see that

$$\alpha = \frac{2\kappa_5 k}{\alpha' \pi^2}\mathcal{E}. \tag{4.1.25}$$

Finally, the covariant conservation of the energy momentum tensor (4.1.23) is still respected, as it coincides with that derived for the H field in (4.1.19) and it obeys the Einstein equation.

4.1.3 Induced 4d energy momentum tensor on the bubble

Let us now study the backreaction of the bulk geometry, due to the presence of H field strength, changes the motion the motion of the bubble wall, and hence the induced energy momentum tensor.

We can equivalently arrive to the same conclusion by computing Israel's second junction condition (2.2.25) or the Gauss-Codazzi equation. There will be two main contributions to the 4d energy momentum tensor:

- A positive contribution coming from the extrinsic curvature,
- A piece that comes from the gauge field on the brane, which adds to the tension and thus gives a negative contribution.

Israel's second junction condition (2.2.25) yields an induced Friedmann equation of the form

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 \delta^a_b = \frac{\Lambda_4}{3}\delta^a_b + \epsilon^2 \frac{\kappa_4}{6\kappa_5 k^3 a^4}(3\delta^a_0\delta^0_b - \delta^a_i\delta^i_b)\left(q_2 + \log\left[-\xi\frac{k_+}{H}\right]\right) \\ + \frac{\kappa_4}{3}\left(\mathcal{F}^{ac}\mathcal{F}_{bc} - \frac{1}{4}\delta^a_b\mathcal{F}_{ij}\mathcal{F}^{ij}\right).\end{aligned} \tag{4.1.26}$$

The left hand side is the usual geometrical evolution of the scale factor for late time cosmologies (there is no contribution of the curvature term) in Friedmann equations, while the right hand side corresponds to the energy momentum tensor associated with the induced cosmology.

For indices $a = b = 0$, we obtain the first Friedmann equation, and the gauge field on the brane (in the second line), gives a negative contribution to the energy density, just like the brane tension does. *This needs to be corrected to a positive contribution by the first term due to the extrinsic curvature.*

Restricting the right hand side to be that of 4d electromagnetism in an expanding cosmology[2] and choosing the free parameter ξ so that $-\xi\frac{k}{H}=1$. This implies

$$\epsilon^2\frac{q_2}{2a^4\kappa_5 k_+^3}(3\delta_0^a\delta_b^0-\delta_i^a\delta_b^i)+\underbrace{\left(\mathcal{F}^{ac}\mathcal{F}_{bc}-\frac{1}{4}\delta_b^a\mathcal{F}_{ij}\mathcal{F}^{ij}\right)}_{\langle T_b^a\rangle_{iso}}\equiv-\langle T_b^a\rangle_{iso}. \tag{4.1.27}$$

Using the relations (4.1.24), (4.1.25) and the fact that[3]

$$l_4^2\sim\frac{N}{L}l_5^3, \tag{4.1.28}$$

we can fix q_2 to be

$$q_2=\frac{k_+^2M_P^2}{2^7T_{D_3}g_s}N\pi, \tag{4.1.29}$$

where N is the number of D_3-branes in the $10d$ background.

Summarizing what we found, *for the electromagnetic field the backreaction happens through the H field that sources the term q_2 in the metric. Electromagnetic radiation is therefore accompanied by a non-trivial H field, with non-vanishing energy density in the bulk. This yields a different bulk geometry.*

Let us now consider electrostatic field configurations.

4.1.4 Un pliàge electrique

Consider a constant electric field $\mathcal{E}$, pointing in the z-direction[4]. Such constant electric field cannot be supported over an extended region. In practice, it could be realized as a piece of an electric field far away from a localized source. In this case, the solutions of the equations (4.1.12) are very simple, yielding

$$H^{rtx}\big|_{r=a}\equiv h_1=\frac{\gamma}{k^2r^3},\quad H^{rxz}\big|_{r=a}\equiv h_2=0,\quad H^{txz}\big|_{r=a}\equiv h_3=0, \tag{4.1.30}$$

where $\gamma\approx\mathcal{E}$ is a dimensionless constant. This sources a $5d$ energy-momentum tensor

$$\langle T_\nu^\mu\rangle\simeq\frac{\gamma^2}{\kappa_5k^2}\begin{pmatrix}-\frac{1}{r^4}&0&0&0&0\\0&\frac{1}{r^4}&0&0&0\\0&0&\frac{1}{r^4}&0&0\\0&0&0&\frac{1}{r^4}&0\\0&0&0&0&-\frac{1}{r^4}\end{pmatrix}, \tag{4.1.31}$$

[2]We exclude for simplicity dark radiation contributions generated by adding a 5d black hole in the background.

[3]See Section 2.8

[4]For simplicity, we ignore the expansion of the universe.

which leads to the following backreaction in the bulk

$$ds^2_{backreacted} \simeq A(r)(-dt^2 + dz^2) + \frac{1}{A(r)^2}dr^2 + B(r)(dx^2 + dy^2), \tag{4.1.32}$$

with

$$A(r) = (kr)^2 - \frac{2}{3}\gamma^2 \frac{\log(k\zeta r)}{(kr)^2}, \tag{4.1.33}$$

$$B(r) = (kr)^2 + \gamma^2 \frac{\log(k\zeta r)}{(kr)^2}, \tag{4.1.34}$$

where ζ is a constant. The next step consists in finding the embedding of the brane that gives the correct induced metric in 4d corresponding to a constant electric field. To do this, we must construct the constant electric field as a local approximation in a setup containing a physical source.

Let us use the 4d metric of a point source, where we are far enough from the source that the electric field can be considered constant in a big enough region[5]. The induced 4d metric we need in our embedded brane is thus

$$\begin{aligned} ds^2 &= -\left(1 + \frac{Q^2}{\rho^2}\right)dt^2 + \left(1 + \frac{Q^2}{\rho^2}\right)^{-1} d\rho^2 + \rho^2 d\Omega_2 \\ &\longrightarrow -\left(1 + \frac{Q^2}{\rho^2}\right)dt^2 + \left(1 + \frac{Q^2}{\rho^2}\right) d\tilde{\rho}^2 + \rho^2 d\Omega_2 \\ &\longrightarrow -\left(1 + \frac{Q^2}{z^2}\right)dt^2 + \left(1 + \frac{Q^2}{z^2}\right) d\tilde{z}^2 + dx^2 + dy^2 \\ &\sim \left(1 + \frac{Q^2}{\tilde{z}^2}\right)dt^2 + \left(1 + \frac{Q^2}{\tilde{z}^2}\right) d\tilde{z}^2 + dx^2 + dy^2. \end{aligned} \tag{4.1.35}$$

The first arrow represents a coordinate transformation in the ρ direction, introducing the new coordinate ρ, through $\frac{d\tilde{\rho}}{d\rho} = (1 + \frac{Q^2}{\rho^2})^{-1}$. For convenience, we express the result using $\rho(\tilde{\rho})$. The second arrow means that we zoom in on $x = \rho\phi$ and $y = \rho\theta$ small, with the angles ϕ and θ small. We can then express the metric using Cartesian coordinates where we write $\tilde{z} = \tilde{\rho}$. Similarly, it is natural to write $z = \rho$. Finally, since $\frac{Q^2}{z^2}$ is small, we can put $z \sim \tilde{z}$. Now let us see if we can match this to the induced metric on a brane embedded into Eq. (4.1.4).

Since the electric field giving rise to the metric in (4.1.4) is assumed to be constant (for simplicity), we can only hope to match the induced metric in (4.1.36) over a short interval. We choose to study the embedding close to $\tilde{z} = \tilde{z}_1$, where $\tilde{z}_1$ is far away from

[5]We will not take into account any gravitational forces, so $1 \gg \frac{Q^2}{r^2} \gg \frac{M}{r}$. This is the relevant situation for any realistic experiment.

the source such that $Q/\tilde{z}$ is small. This means that $r(\tilde{z}) = r_1 + \alpha(\tilde{z} - \tilde{z}_1)$ is almost constant with α small. We set the asymptotic value of r at infinity to r_0. Noting that the dr^2 piece of the bulk metric only contributes at quadratic order in α to the induced metric, we see that the induced metric can be put in the form (4.1.36) provided that

$$k^2 r^2 - \frac{2\gamma^2 \log(k\zeta r)}{3k^2 r^2} = k^2 r_0^2 \left(1 + \frac{Q^2}{z^2}\right), \tag{4.1.36}$$

$$k^2 r^2 + \frac{\gamma^2 \log(k\zeta r)}{3k^2 r^2} = k^2 r_0^2. \tag{4.1.37}$$

The first equation comes from matching dt^2 and dz^2, while the second comes from dx^2 and dy^2. Here we have already made a conformal rescaling of x, y, z to match the equations at 0-th order. This gives

$$\frac{r^2}{r_0^2} = 1 + \frac{Q^2}{3z^2}, \tag{4.1.38}$$

$$\frac{\gamma^2 \log(k\zeta r)}{3k^2 r^2} = -\frac{k^2 r_0^2 Q^2}{3z^2}. \tag{4.1.39}$$

Due to our approximation for the bulk metric, using a constant electric field, these equations are only valid close to a given point such as $z = z_1$. The first of the two equations give us r, for a given z, while the second equation determines ζ (changing z_1 leads to a different ζ as a consequence of the approximation). We immediately see from the first equation that r grows in the direction towards the charge sourcing the field.

The interpretation is simple. In order to capture the blue shift of the induced metric as we approach the source, the brane must bend upwards to make use of the bulk blue shift due to AdS. In this way the induced metric comes out right. The brane bends upwards in the direction of increasing electric field but now it is caused by the presence of the bulk B field. This gives rise to a backreaction in the bulk that makes the brane bend upwards.

4.2 New fields in the game

If we also include to the brane action the contribution of the Chern-Simons term to the brane action, we have to deal with new potentials and thus new fields in the bulk. The full action for a D_p-brane is

$$S_{D_p} = -T_p \int_{W_{p+1}} d^{p+1}x \sqrt{-\det(g_{\mu\nu} + \tau \mathcal{F}_{\mu\nu})} + \mu_p \int_{W_{p+1}} (\Sigma_n c_n) \wedge \sqrt{\frac{\hat{A}(TW)}{\hat{A}(NW)}} \wedge e^{(\tau \mathcal{F})}, \tag{4.2.1}$$

where W_{p+1} is the worldvolume and the brane coupling μ_p isrelated to the charge of the brane

$$\mu_p = \frac{(\alpha')^{-(p+1)/2}}{(2\pi)^p}. \tag{4.2.2}$$

The last term in the second piece of the action can be expanded as follows

$$e^{\tau\mathcal{F}} = 1 + \tau\mathcal{F} + \frac{\tau^2}{2}\mathcal{F}\wedge\mathcal{F} + ... \tag{4.2.3}$$

In our case, we can write

$$S_{D_3} = -T_3\int_{W_4} d^4x\sqrt{-\det(g_{\mu\nu}+\tau\mathcal{F}_{\mu\nu})} + \mu_3\int_{W_3}(C_4 + \tau\mathcal{F}\wedge C_2), \tag{4.2.4}$$

where C_2 and C_4 are respectively a 2-form and a 4-form potential which correspond to a 3-form field $F_{(3)}$ and a 5-form field $F_{(5)}$ in the bulk. The total action which contains also the contribution from the bulk now has additional contributions

$$S_5 = \frac{1}{2\kappa_5}\int d^5x\sqrt{-g_5}\left(R - \frac{1}{12g_s}H^2 - \frac{1}{12g_s}F^2_{(3)} - \frac{1}{480g_s}F^2_{(5)}\right) + S_{D_3}. \tag{4.2.5}$$

Let us now study case by case what happens if we play with these potentials: first, we will consider the case where $B = 0$ and $C_2 \neq 0$, and then what happens when both B and C_2 are present in the game.

- **CASE 1:** $B = 0$ **and** $C_2 \neq 0$

  In this case $\tau\mathcal{F} = \tau F$. First of all, the exterior product between F and C_2 can be expanded as

$$F\wedge C_2 = \frac{1}{4}F_{\mu\nu}C_{\rho\sigma}dx^\mu\wedge dx^\nu\wedge dx^\rho\wedge dx^\sigma = \frac{1}{4}F_{\mu\nu}C_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\sqrt{-g_4}d^4x. \tag{4.2.6}$$

  Therefore, the variation of the action is

$$\delta S_5 = \frac{1}{2\kappa_5}\int d^5x\sqrt{-g_5}\left(-\frac{1}{12g_s}\delta(F^2_{(3)})\right) + \mu_3\tau\int d^4x\sqrt{-g_4}\delta\left(\frac{1}{4}F_{\mu\nu}C_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\right). \tag{4.2.7}$$

  Recalling the relation in Eq. (4.1.4) and using $d^5x = d^4x\delta(r-a[\eta])$, we can write

$$\int d^5x\sqrt{-g_4}\left(-\frac{1}{24g_s\kappa_5 kr}\delta(F_{r\rho\sigma}F^{r\rho\sigma}) + \frac{\mu_3\tau}{4}\delta(F_{\mu\nu}C_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma})\delta(r-a[\eta])\right). \tag{4.2.8}$$

  If we proceed in the same way as in Eq. (4.1.6), we find that $\mu_3 = -T_3$ and

$$\partial_r F^{r\rho\sigma} = \frac{\kappa_5 kr}{4\pi^2\alpha'}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\delta(r-a[\eta]). \tag{4.2.9}$$

Therefore, being $\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ equal to the dual field-strength for the electromagnetic tensor, $\tilde{F}^{\mu\nu}$,

$$\Delta F^{r\rho\sigma}\big|_{r=a} = F^{r\rho\sigma}\big|_{r=a} = \frac{\kappa_5 ka}{4\pi^2\alpha'}\tilde{F}^{\mu\nu}\big|_{r=a}. \tag{4.2.10}$$

This means that the $F_{(3)}$ field in the bulk is sourced by the dual electromagnetic field on the brane.

- **CASE 2: $B \neq 0$ and $C_2 \neq 0$**

  In this case, $\tau\mathcal{F} = \tau F + B$, so, expanding the exterior product between $\mathcal{F}$ and C_2 as

$$\tau\mathcal{F} \wedge C_2 = (\tau F + B) \wedge C_2 = \frac{1}{4}(\tau F_{\mu\nu} + B_{\mu\nu})C_{\rho\sigma}dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma, \tag{4.2.11}$$

  we see there is a new coupling between B and C_2. The equations of motion for B imply

$$\begin{aligned}\partial_r H^{r\mu\nu} &= \frac{\kappa_5 kr}{8\pi^3\alpha'^2}\left[\epsilon^{\mu\nu\rho\sigma}C_{\rho\sigma} + 2\tau\mathcal{F}^{\mu\nu}\right]\delta(r - a[\eta]),\\ &= \frac{\kappa_5 kr}{8\pi^3\alpha'^2}\left[\tilde{C}^{\mu\nu} + 2\tau\mathcal{F}^{\mu\nu}\right]\delta(r - a[\eta]).\end{aligned} \tag{4.2.12}$$

  while for C_2 we obtain

$$\begin{aligned}\partial_r F^{r\rho\sigma} &= \frac{\kappa_5 kr}{4\pi^2\alpha'}\mathcal{F}_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\delta(r - a[\eta]),\\ &= \frac{\kappa_5 kr}{4\pi^2\alpha'}\tilde{\mathcal{F}}^{\rho\sigma}\delta(r - a[\eta])\end{aligned} \tag{4.2.13}$$

Now, assuming $\tilde{C}\big|_{brane}$ vanishes on the brane, we recover this nice pair of equations

$$\Delta H^{r\mu\nu} = \frac{2\kappa_5 ka}{2\pi^2\alpha'}\mathcal{F}^{\mu\nu}, \tag{4.2.14}$$

$$\Delta F^{r\rho\sigma} = \frac{\kappa_5 ka}{4\pi^2\alpha'}\tilde{\mathcal{F}}^{\rho\sigma}. \tag{4.2.15}$$

All to all, if B and C vanish on the brane, the big picture is

$$\Delta H \propto F, \qquad \Delta F^{(3)} \propto \tilde{F}, \tag{4.2.16}$$

which means that H in the bulk is sourced by the electromagnetic field-strength on the brane, while $F^{(3)}$ in the bulk is sourced by the dual electromagnetic tensor.

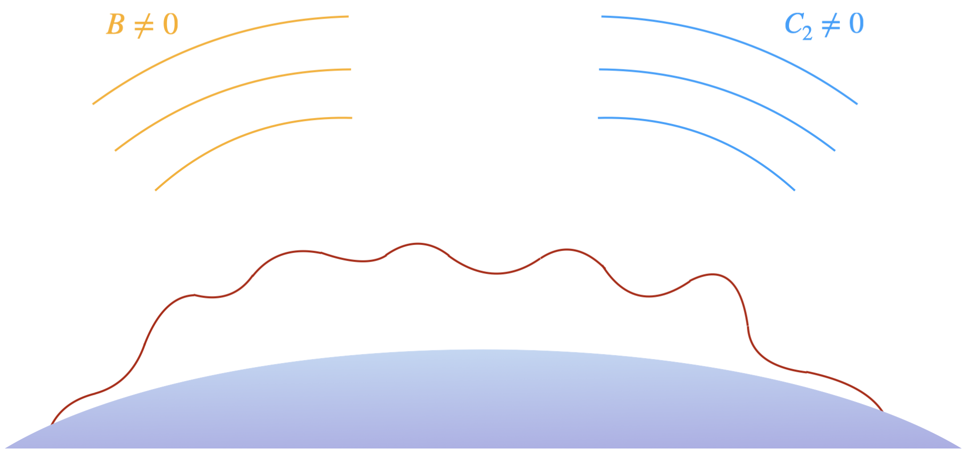

Figure 4.1: Representation of the mechanism induced by the electromagnetic field. Electromagnetic waves on the brane source both the B and the C_2-field in the bulk.

4.2.1 Same stress, same energy

To verify the consistency of the above result, we need to compute the stress-energy tensor for the case of the dual field-strength. All the derivation of the backreacted metric and of the backreacted stress-energy tensor we performed in Section 4.1.2 and 4.1.3 is still perfectly valid if $T_{\mu\nu}$ is invariant. In particular, we need the term

$$\left(\mathcal{F}^{ac}\mathcal{F}_{bc} - \frac{1}{4}\delta^a_b \mathcal{F}_{ij}\mathcal{F}^{ij}\right) \tag{4.2.17}$$

to be invariant, in order for the mechanism to be truly valid both for the B field and the C_2 field.

First, let us prove the invariance of the term $\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \subset T_{\mu\nu}$. Indeed,

$$\begin{aligned} \tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} &= \left(\frac{1}{2}\epsilon_{\alpha\beta\mu\nu}F^{\mu\nu}\right)\left(\frac{1}{2}\epsilon^{\alpha\beta\rho\sigma}F_{\rho\sigma}\right), \\ &= \frac{1}{4}4F^{\mu\nu}\delta^\rho_\mu\delta^\sigma_\nu F_{\rho\sigma}, \\ &= F_{\mu\nu}F^{\mu\nu}, \end{aligned} \tag{4.2.18}$$

where in the second passage we have used the fact that $\epsilon_{\rho\sigma\mu\nu}\epsilon^{\rho\sigma\alpha\beta} = 2!\delta^\nu_{[\mu}\delta^\beta_{\nu]}$. We only have to check that the term $\tilde{F}_{\mu\alpha}\tilde{F}^\alpha_\nu \subset T_{\mu\nu}$ is equivalent to $F_{\mu\alpha}F^\alpha_\nu$. Indeed, using the same steps as before,

$$\tilde{F}_{\mu\alpha}\tilde{F}^{\alpha}_{\nu} = \left(\frac{1}{2}\epsilon_{\mu\alpha\rho\sigma}F^{\rho\sigma}\right)\left(g_{\nu\mu}\tilde{F}^{\mu\alpha}\right), \tag{4.2.19}$$

$$= \frac{1}{4}\epsilon_{\mu\alpha\rho\sigma}\epsilon^{\mu\alpha\nu\lambda}F^{\rho\sigma}g_{\nu\mu}F_{\nu\lambda},$$

$$= g_{\rho\mu}F_{\rho\sigma}F^{\rho\sigma},$$

$$= F_{\rho\sigma}F^{\sigma}_{\mu}. \tag{4.2.20}$$

We have therefore proven the full validity of our results.

The appearance of the dual electromagnetic tensor in (4.2.15) is a clear indication that we can have configurations of constant magnetic field. We can therefore check that the exact same considerations of Section 4.1.4 apply, allowing us to realize a configuration of constant magnetic field on the braneworld. In this case, the brane bends upwards in the direction of increasing magnetic field, but now it is caused by the presence of the bulk C_2-field. This gives rise to a backreaction in the bulk that makes the brane bend upwards.

Chapter 5

Construction from 0'B String Theory

In Chapter 2, we have vastly discussed the Dark Bubble model and seen how to realize it and embed it into Type IIB String Theory. However, recalling the non-supersymmetric theories of Section 1.4, we can try to reproduce a braneworld model which is very similar to the DB. The most natural theory which allows us to do so is Type 0'B, which contains charged D3-branes whose near-horizon geometry in the probe regime is close to $\text{AdS}_5 \times S^5$. We will see, however, that this new setup entails a drastic difference in the behavior of the cosmological constant and eventually reveals to be incompatible with experimental data. Anyway, it is a very neat construction, and we shall explore it with great detail.

5.1 D3-branes in the type $0'$B model

For D3-branes in the type $0'$B model, the relevant near-horizon geometry was studied in [52–54]. More specifically, the model of [52, 53] involves an O'3-plane, but its contribution is sub-leading for large fluxes. In [53] the authors found non-homogeneous deviations from $\text{AdS}_5 \times \mathbb{RP}^5$ which are suppressed, but not uniformly so, in the large-flux limit[1]. In detail, in coordinates in which the (string-frame) metric takes the form[2] [53]

$$ds^2 = R^2(u)\,\frac{du^2}{u^2} + \frac{\alpha'^2\,u^2}{R^2(u)}\,dx_{1,3}^2 + \widetilde{R}^2(u)\,d\Omega_5^2\,, \tag{5.1.1}$$

[1] Similar results in tachyonic type 0 strings were obtained in [55].

[2] The local expression in Eq. (5.1.1) does not account for the global distinction between S^5 and $\mathbb{RP}^5$.

the would-be AdS_5 and $\mathbb{RP}^5$ curvature radii $R(u)\,,\widetilde{R}(u)$ and the dilaton $\phi(u)$ acquire a dependence on the energy scale u that, in the large-flux limit, behaves as

$$\frac{R^2(u)}{R^2_\infty} \sim 1 - \frac{3}{16}\, g_s\, \alpha' T\, \log\left(\frac{u}{u_0}\right)\,, \tag{5.1.2}$$

$$\frac{\widetilde{R}^2(u)}{R^2_\infty} \sim 1 - \frac{3}{16\sqrt[4]{8}}\, g_s^2\, N\, \alpha' T\, \log\left(\frac{u}{u_0}\right)\,, \tag{5.1.3}$$

$$\frac{1}{N}\, e^{-\phi} \sim \frac{1}{g_s\, N} + \frac{3}{8\sqrt[4]{8}}\, g_s\, \alpha' T\, \log\left(\frac{u}{u_0}\right)\,, \tag{5.1.4}$$

where u_0 is a reference scale, $R^2_\infty = \sqrt{4\pi g_s N}\alpha'$ is the supersymmetric value of the radii and $N \gg 1$ ought to be interpreted as the number of D3-branes sourcing the geometry. In the large-N limit the 't Hooft coupling $\lambda = 4\pi\, g_s\, N$ in the absence of the tadpole T ought to be fixed, but the validity of the EFT description also requires $\lambda \gg 1$ [56], while on account of the second of Eq. (5.1.2) $g_s^2\, N \ll 1$.

Let us therefore consider probe-regime interactions between D_3-branes, whose corresponding near-horizon throat deviates from $AdS \times S$. Let us start by the solution in Eq.s (5.1.1) and (5.1.2), embedding the probe world-volume parallel to the x^μ according to $j : x^\mu = \zeta^\mu, u = U(\zeta)$, $\theta^i = \theta^i_0$, where the coordinate u is again an energy scale.

Recall the low-energy effective action of Eq. (1.4.17) for D_3-branes in the 0'B model

$$S = \frac{1}{2\kappa^2_{10}} \int d^{10}x \sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - T e^{\frac{3}{2}\phi} - \frac{1}{120} F^2_{(5)}\right)\,. \tag{5.1.5}$$

The five-form R-R field strength F_5 is self-dual, closed[3], and reads

$$F_5 = (1 + \star)\, f_5\, N\, \mathrm{vol}_{S^5} \tag{5.1.6}$$

$$= f_5\, N\, \mathrm{vol}_{S^5} + \frac{f_5\, N}{\widetilde{R}(u)^5}\left(\frac{\alpha'\, u}{R(u)}\right)^3 d(\alpha' u) \wedge d^4x \tag{5.1.7}$$

with vol_{S^5} the volume form of the unit 5-sphere (neglecting the global distinction with the projective plane, which can be easily reinstated). The flux quantization condition

$$\frac{1}{2\kappa^2_{10}} \int_{S^5} F_5 = \mu_3\, N\,, \tag{5.1.8}$$

where μ_3 is the charge of a single D3-brane, then fixes

$$f_5 = \frac{2\kappa^2_{10}\, \mu_3}{\Omega_5}\,, \tag{5.1.9}$$

[3]Since the orientifold projection removes the Kalb-Ramond form, no additional terms appear in the Bianchi identity.

and the relevant contribution to the potential C_4, to be pulled back on the probe world-volume, takes the form

$$C_4 = c_4(u)\, d^4x + \dots \tag{5.1.10}$$

where $dC_4 = F_5$ implies

$$\frac{c_4'(u)}{\alpha'} = \frac{f_5\, N}{\widetilde{R}(u)^5} \left(\frac{\alpha'\, u}{R(u)}\right)^3 . \tag{5.1.11}$$

Collecting all the ingredients, and using the string-frame worldvolume action for $N_3 \ll N$ static D3-branes parallel to the sources, the probe potential evaluates to

$$V_{\text{probe}}^{\text{D3}}(U) = N_3 T_3 \left(\frac{\alpha'\, U}{R(U)}\right)^4 e^{-\phi(U)} - N_3 \mu_3\, c_4(U)\,, \tag{5.1.12}$$

where once again T_3 ensures the tension of a single D3-brane. The dominant contribution in the EFT limit $g_s\,, g_s^2\, N \ll 1$, $g_s\, N \gg 1$ is

$$\begin{aligned} \frac{V_{\text{probe}}^{\text{D3}}(U)}{N_3 U^4} \sim \frac{16\,\pi\,\alpha'^2\, T_3 - f_5\,\mu_3}{64\,\pi^2\, g_s^2\, N} + \frac{15\, f_5\,\mu_3\,\alpha' T}{8192\sqrt[4]{8}\,\pi^2} \qquad (5.1.13)\\ + \frac{3\,(64\,\pi\,\alpha'^2 T_3 - 5\, f_5\,\mu_3)\,\alpha' T}{2048\sqrt[4]{8}\,\pi^2}\log\left(\frac{U}{u_0}\right) . \end{aligned}$$

As expected, substituting the supersymmetric values [54]

$$2\kappa_{10}^2 = (2\pi)^7\,\alpha'^4\,, \qquad T_3 = \mu_3 = \frac{1}{(2\pi)^3\,\alpha'^2} \tag{5.1.14}$$

for $N_3 \ll N$ probes, and using Eq. (5.1.9), the leading term vanishes, on account of the BPS property, while the remaining sub-leading terms reflect supersymmetry breaking and their U-dependence simplifies to

$$V_{\text{sub-leading}}^{\text{D3}}(U) \propto U^4 \left[5 - 4\,\log\left(\frac{U}{u_0}\right)\right] . \tag{5.1.15}$$

This potential is repulsive. As depicted in fig. (5.1), the potential in Eq. (5.1.15) features a maximum at $U = e^{\frac{5}{4}} u_0$, and the height of the potential barrier scales according to u_0^4. Therefore, even if the probe stack were initially located in the classically attractive region, it would eventually tunnel to the repulsive region.

In order to compute all numerical factors, we need the NLO effective DBI-CS-WZ Lagrangian (density), which takes the form

$$\mathcal{L}_{\text{NLO}} = \frac{N_3 N U^4}{2\pi^2\lambda^2}\Bigg[- \left(1 + \frac{3\lambda^2\alpha' T}{128\sqrt[4]{8}\pi^2 N}\,\log\frac{U}{u_0}\right)\sqrt{1 - \frac{\lambda \dot{U}^2}{U^4}} \tag{5.1.16}$$

$$+ 1 - \frac{15\lambda^2\alpha' T}{2048\sqrt[4]{8}\pi^2 N}\left(1 - 4\,\log\frac{U}{u_0}\right)\Bigg]\,, \tag{5.1.17}$$

where once again $\lambda \equiv 4\pi g_s N$ and, in the 0'B model, $\alpha' T = 8/\pi^2$.

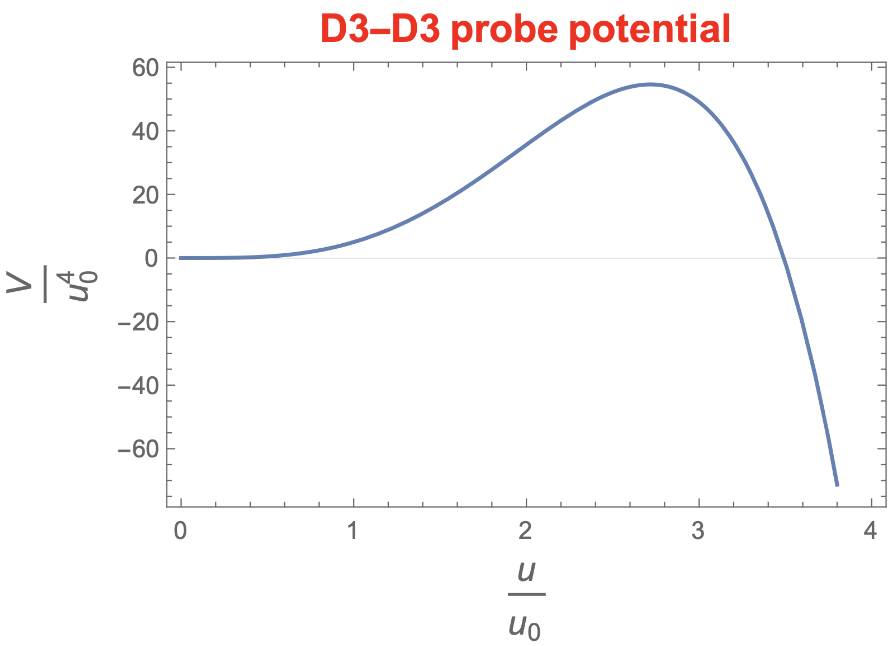

Figure 5.1: Normalized probe potential in Eq. (5.1.13) in units of the reference scale u_0.

5.2 Braneworld model

The worldvolume action can also be used to obtain Friedmann equations for the braneworld model, casting the induced metric on the brane in a cosmological guise. For the Poincaré-like patch, the appropriate form is $ds^2_{\text{brane}} = -d\tau^2 + a(\tau)^2 d\mathbf{x}^2$ with flat spacelike slices.

From the equation of motion of the NLO brane Lagrangian one can recover the Friedmann equation with the precise numerical factors (to NLO). The pullback of the 10d metric gives a 4d cosmology with scale factor

$$a(t) = \frac{\alpha' U(t)}{R(U(t))}, \tag{5.2.1}$$

while the cosmological time satisfies

$$\left(\frac{d\tau}{dt}\right)^2 = -\frac{R(U)^2}{U^2}\dot{U}^2 + \frac{\alpha'^2 U^2}{R(U)^2}. \tag{5.2.2}$$

The Lagrangian yields the on-shell conserved Hamiltonian

$$\mathcal{H} \sim \frac{N_3 N U^4}{2\pi^2\lambda^2}\left[\frac{1}{\sqrt{1-\frac{\lambda\dot{U}^2}{U^4}}} - 1 + \frac{3\,\lambda^2\,\alpha' T}{2048\sqrt[4]{8}\pi^2 N}\left(5 - 4\log\frac{U}{u_0}\right)\right], \tag{5.2.3}$$

once again re-expanded to NLO. Defining now

$$\epsilon \equiv \frac{3\,\lambda^2\,\alpha' T}{256\sqrt[4]{8}\pi^2 N} = \frac{3\,\alpha' T}{16\sqrt[4]{8}}\,g_s^2 N \overset{\text{0'B}}{=} \frac{3}{2^{\frac{7}{4}}\pi^2}\,g_s^2 N \approx 0.09\,g_s^2 N\,, \tag{5.2.4}$$

one finds

$$\mathcal{H} \sim \frac{N_3 N U^4}{2\pi^2\lambda^2}\left[\frac{1}{\sqrt{1-\frac{\lambda \dot{U}^2}{U^4}}} - 1 + \frac{\epsilon}{2}\left(\frac{5}{4} - \log\frac{U}{u_0}\right)\right]\,. \tag{5.2.5}$$

By conservation, at late times the term in square brackets vanishes, since the prefactor $U \to \infty$ as the brane expands (in more standard coordinates, $Z \to 0^+$). Thus, to leading order,

$$a \sim \frac{\alpha' U}{R_\infty}\,, \qquad H^2 \sim \frac{\lambda \dot{U}^2}{R_\infty^2 U^4} \sim \epsilon R_\infty^{-2}\log\left(\frac{a}{a_*}\right)\,, \tag{5.2.6}$$

including the numerical factor in ϵ and absorbing the $5/4$ in the reference scale factor a_*. All in all, to NLO

$$H^2 = \epsilon M^2 \log\left(\frac{a}{a_*}\right)\,, \tag{5.2.7}$$

where $M \equiv R_\infty^{-1}$ denotes the EFT cutoff (if there is an EFT description on the braneworld) and $\epsilon \equiv \mathcal{O}(1) g_s^2 N \ll 1$ with a calculable positive order-one prefactor. The arbitrary reference scale a_* disappears by fixing the initial time so that $a(0) = a_0$ is the scale factor today, and similarly $H(0) = H_0$. The resulting (today-accelerating branch of the) solution takes the form

$$a(\tau) = a_0\,\exp\left(H_0\,\tau + \frac{\epsilon M^2 \tau^2}{4}\right)\,, \tag{5.2.8}$$

whereas the branch that decelerates at present time would have a minus sign in front of the linear term in τ. The link between the Hubble scale $H_0^2 \propto \Lambda$ and the EFT scale M is then captured by the slow-roll parameter $\epsilon_V = \epsilon_H \equiv \frac{\dot{H}}{H^2}$, where $\eta_H = 0$ since H is linear in τ and $\eta_H \propto \ddot{H}$.

5.2.1 Scales in the game

The 10d solution should be reliable for bulk coordinates $g_s^2 N \log\frac{u}{u_0} \lesssim 1$, which, to leading order, translates into $\epsilon \log\frac{a}{a_*} \lesssim 1$ Since $a_* = a_0 \exp\left(-\frac{H_0^2}{\epsilon M^2}\right)$ is fixed by the initial conditions, one finds $\tau \lesssim \frac{2}{\epsilon M}\left(1 - \frac{H_0}{M}\right)$ to leading order in ϵ. Correspondingly, the Hubble rate in the regime of validity is bounded by $H \lesssim M$, namely the EFT is expected to be reliable. This result is a nice, but unsurprising, cross-check. Imposing however that the maximal cosmological time for the EFT validity be greater than the age of the universe, one has $\frac{1}{H_0} \lesssim \frac{2}{\epsilon M}\left(1 - \frac{H_0}{M}\right)$, so that $H_0 \lesssim M \lesssim H_0/\epsilon$, always up to order-one factors. Actually, the upper bound on M arises without using a particular value of a_* at leading order in ϵ, as expected.

In string units $1 = M_s \propto \alpha'^{-\frac{1}{2}}$, the scalings with g_s and N of the various scales in the game are given by

$$M \sim \frac{1}{R_\infty} \sim g_s^{-\frac{1}{4}} N^{-\frac{1}{4}}\,, \tag{5.2.9}$$

$$M_{\text{Pl}} \sim \left(\frac{R^6}{g_s^2 N}\right)^{\frac{1}{2}} \sim g_s^{-\frac{1}{4}} N^{\frac{1}{4}}\,, \tag{5.2.10}$$

$$M_5 \sim \left(\frac{R^5}{g_s^2}\right)^{\frac{1}{3}} \sim g_s^{-\frac{1}{4}} N^{\frac{5}{12}}\,, \tag{5.2.11}$$

where M_{Pl} denotes the 4d Planck scale. It derives from the dark bubble relation $M_{\text{Pl}}^2 \sim M_5^3 R/N$, where $M_5^3 = M_s^8 R^5/g_s^2$ as above is the 5d Planck scale. In 4d Planck units,

$$\frac{M}{M_{\text{Pl}}} \sim N^{-\frac{1}{2}} \ll 1\,, \tag{5.2.12}$$

$$\frac{M_5}{M_{\text{Pl}}} \sim N^{\frac{1}{6}} \gg 1\,. \tag{5.2.13}$$

Since $g_s N \gtrsim 1$ for perturbative control, $\epsilon \gtrsim \frac{1}{N}$, so that the above bounds can be extended to

$$\frac{H_0}{M_{\text{Pl}}} \lesssim \frac{M}{M_{\text{Pl}}} \lesssim \frac{H_0}{\epsilon M_{\text{Pl}}} \lesssim \frac{H_0}{M_{\text{Pl}}}\, N\,. \tag{5.2.14}$$

Hence, in 4d Planck units one finds

$$\Lambda^{\frac{1}{2}} \sim H_0 \lesssim M \lesssim H_0^{\frac{1}{3}} \sim \Lambda^{\frac{1}{6}}\,, \tag{5.2.15}$$

where this time we left the 4d Planck scale implicit. According to the considerations in [42], the lower bound on M is consistent with the Higuchi bound [57], while the upper bound in our case is $M \sim \Lambda^{\frac{1}{6}}\, M_{\text{Pl}} \approx 100$ MeV instead of $\Lambda^{\frac{1}{4}}\, M_{\text{Pl}} \approx 3$ meV.

The validity of these arguments for a braneworld model, as opposed to a compactification, should be assessed with more care. For instance, the argument for the upper bound with exponent $\frac{1}{d} = \frac{1}{4}$ stems from considerations on the effective potential generated by a tower of light states, whose appearance, according to the distance conjecture(s) generally affects the bulk rather than a braneworld. The smallness of the quasi-dS dark energy in our construction is not necessarily related to the smallness of the bulk AdS scale, although in this case they both are and one can apply the AdS version of the distance conjecture. In particular, in the bulk (quasi-)AdS$_5$ there is a light tower of KK states with masses $m_{\text{KK}} \sim R_\infty^{-1} \sim M$, and the bound of [42] would then take the form

$M/M_5 \lesssim (\Lambda_{\rm AdS}/M_5^2)^{\frac{1}{5}} \sim (M/M_5)^{\frac{2}{5}}$, which is satisfied since $M/M_5 \ll 1$. On the other hand, if this tower dominates the species bound, one obtains the species scale

$$\Lambda_{\rm sp} = \frac{M_{\rm pl}}{\sqrt{N(\Lambda_{\rm sp})}} \sim M^{\frac{5}{7}} \lesssim \Lambda^{\frac{5}{42}} \approx 100 \text{ TeV} \tag{5.2.16}$$

as a QG cutoff in the spirit of [42].

The slow-roll parameter today then reads $\epsilon_H = \frac{\dot{H}}{H_0^2} = \frac{\epsilon M^2}{2H_0^2} \sim \epsilon \frac{M^2}{\Lambda}$, so that $\Lambda \lesssim \epsilon_H \lesssim \Lambda^{-\frac{2}{3}}$ 4d in Planck units. In particular, in the controlled regime

$$10^{-122} \lesssim \epsilon_H \lesssim 10^{81} \,, \tag{5.2.17}$$

a very large window of possible values In fact, imposing that $\epsilon_H \lesssim 1$ to be compatible with the standard cosmological models, one finds $M \lesssim \frac{H_0}{\sqrt{\epsilon}} \lesssim \Lambda^{\frac{1}{2}} N^{\frac{1}{2}} \sim \Lambda^{\frac{1}{2}}/M$, thus $M \lesssim \Lambda^{\frac{1}{4}}$. Within the restricted window $\Lambda^{\frac{1}{2}} \lesssim M \lesssim \Lambda^{\frac{1}{4}}$, which coincides with that of [42], the species scale driven by KK modes of the S^5 is then bounded according to $M^{\frac{5}{7}} \lesssim \Lambda^{\frac{5}{28}} \approx 10$ MeV.

5.3 Cosmological interpretation

The behavior of the scale factor (5.2.8) indicates a quasi-dS evolution. We would like to understand if it is possible to reproduce this behavior considering the equations of motion of some homogeneous scalar field ϕ subject to an effective potential $V(\phi)$.

Consider a scalar field $\phi(\mathbf{x}, t)$. Using FLRW metric in flat space

$$ds^2 = -c^2 dt^2 + a^2(t) d\mathbf{x}^2, \tag{5.3.1}$$

the action for the scalar field reads

$$S = \int d^4x a^3(t) \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]. \tag{5.3.2}$$

Since we are interested only in spatially homogeneous solutions, $\phi(\mathbf{x}, t) = \phi(t)$ and

$$S = \int d^4x a^3(t) \left[\frac{1}{2}\dot{\phi}^2 - V(\phi)\right]. \tag{5.3.3}$$

The equations of motion are

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \tag{5.3.4}$$

which look like a harmonic oscillator, where the extra term $3H\dot{\phi}$ looks like a friction term.

Computing the Hamiltonian, we get that the density of the scalar field is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{5.3.5}$$

ρ_ϕ determines the evolution of $a(t)$ through the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho_\phi = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{5.3.6}$$

Using the continuity equation $\dot{\rho} + 3H(\rho + p) = 0$, we find the pressure of the scalar field to be

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{5.3.7}$$

If we define the state parameter as $\omega = \frac{P_\phi}{\rho_\phi}$, we see clearly that this does not fit into our usual classification of fluids with $P = \omega\rho$ for some constant ω. Instead, we have something more dynamical to deal with.

After some manipulation of the above equations, we get

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}(\dot{\phi}^2 - V(\phi)). \tag{5.3.8}$$

Notice that, in the limit $V(\phi) \gg \dot{\phi}^2$, $P_\phi \approx -\rho_\phi$, which is exactly the case of dark energy.

Imposing the slow-roll condition $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$ and requiring this exponential expansion to last long enough

$$\ddot{\phi} \ll H\dot{\phi}, \tag{5.3.9}$$

the Eq.s (5.3.4) and (5.3.6) become

$$H^2 \approx \frac{8\pi G}{3}V(\phi), \tag{5.3.10}$$

$$3H\dot{\phi} \approx -\frac{\partial V}{\partial \phi}. \tag{5.3.11}$$

These are now straightforward to solve and the problem simply boils down to picking the right choice of the potential $V(\phi)$ in order to reproduce the quasi-de Sitter evolution (5.2.8).

However, there is no guarantee that we can always use the slow-roll approximation. Indeed, let us try to compute the potential in the most general case. Taking the time derivative of (5.3.6), we get

$$2\dot{H}H = \frac{8\pi G}{3}\ddot{\phi}\dot{\phi} + \frac{8\pi G}{3}\frac{\partial V}{\partial \phi}\dot{\phi}. \tag{5.3.12}$$

Substituting the expression for $\frac{\partial V}{\partial \phi}$ in (5.3.4), the second derivatives of ϕ w.r.t. time cancel out and we end up with

$$4\pi G\dot{\phi}^2 = -\dot{H}, \tag{5.3.13}$$

which has no real solutions.

This result holds even if we have a multi-field scalar potential. Consider the following lagrangian density

$$\mathcal{L} = \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi), \tag{5.3.14}$$

where g_{ij} is a metric in field space and the spacetime metric $g_{\mu\nu}$ is encoded in the partial derivatives of ϕ. Euler-Lagrange equations read

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} &= \partial_\mu(g_{ij}(\phi)\partial^\mu\phi^j) \\ &= \partial_\mu g_{ij}(\phi)\partial^\mu\phi^j + g_{ij}(\phi)\Box\phi.\end{aligned} \tag{5.3.15}$$

We can rewrite Eq.s (5.3.4) and (5.3.6) in a more general manner

$$\Box\phi^k + \Gamma^k_{ij}\partial_\mu\phi^i\partial^\mu\phi^j = -g^{kj}(\phi)\frac{\partial V}{\partial\phi^j}, \tag{5.3.16}$$

$$H^2 = \frac{8\pi G}{3}(V(\phi) + \frac{1}{2}g_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j), \tag{5.3.17}$$

where the Christoffel symbols are defined as $\Gamma^k_{ij} = \frac{1}{2}g^{kl}(\partial_j g_{il} + \partial_i g_{jl} - \partial_l g_{ij})$. Since we are interested only in spatially homogeneous solutions, $\phi^i(\mathbf{x}, t) = \phi^i(t)$, (5.3.16) becomes

$$\ddot{\phi}^k + 3H\dot{\phi}^k + \Gamma^k_{ij}\dot{\phi}^i\dot{\phi}^j = -g^{kj}(\phi)\frac{\partial V}{\partial\phi^j}. \tag{5.3.18}$$

(5.3.18) can be recast as

$$\ddot{\phi}^k\dot{\phi}_k + 3H\dot{\phi}^2 + g_{li}\Gamma^l_{kj}\dot{\phi}^i\dot{\phi}^j\dot{\phi}^k = -g^{kj}(\phi)\frac{\partial V}{\partial\phi^j}\dot{\phi}^k \tag{5.3.19}$$

On the other hand, taking the time derivative of (5.3.17), we get

$$\frac{\partial V}{\partial\phi_k}\dot{\phi}^k + g_{ij}\ddot{\phi}^i\dot{\phi}^j = \frac{3}{8\pi G}(2H\dot{H}) - g_{li}\Gamma^l_{kj}\dot{\phi}^i\dot{\phi}^j\dot{\phi}^k. \tag{5.3.20}$$

Finally, substituting the expression for $\frac{\partial V}{\partial\phi^k}\dot{\phi}^k$ in (5.3.19), we get

$$\frac{3}{4\pi G}\dot{H} - \frac{\Gamma\dot{\phi}\dot{\phi}\dot{\phi}}{H} + 3\dot{\phi}^2 + \frac{\Gamma\dot{\phi}\dot{\phi}\dot{\phi}}{H} = 0. \tag{5.3.21}$$

The terms with Γ cancel-off and we are left with the same expression as in Eq.(5.3.13).

We have therefore proved a **no-go for scalar potentials**: no effective scalar potential is able to reproduce the time evolution of the braneworld (5.2.8).

5.3.1 Evoking phantom scalars

In this Section, we claim that there might be a way to get around the no-go for scalar potentials. Indeed, for models of scalar driven accelerated expansion, the first derivative of the Hubble radius must necessarily decrease. This can be directly seen from the equation of state

$$w = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \geq -1. \tag{5.3.22}$$

We recall how different cosmological solutions depend on the equation of state as

- $w > -1$: $\quad a \simeq \tau^{\frac{2}{3(w+1)}}$,
- $w = -1$: $\quad a \simeq e^{H_0 \tau}$,
- $w < -1$: $\quad a \simeq (\tau_s - \tau)^{\frac{2}{3(w+1)}}$,

where τ_s denote the time of the singularity.

As we can see, models with $w < -1$ lead to space-time singularities. In such a universe, the comoving particle horizon shrinks to zero size in finite time, and all objects in the universe would be causally disconnected. This is usually referred to as Big Rip. From the Friedmann equations

$$H^2 = \frac{\rho}{3M_{Pl}^2}, \qquad \dot{H} = -\frac{\rho}{2M_{Pl}^2}(1+w) \tag{5.3.23}$$

we can derive the relation

$$w = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} \tag{5.3.24}$$

and we can then compute the equation of state from Eq. (5.2.8) as

$$w_{0'B} = -1 - \frac{\epsilon M^2}{3\left(H_0 + \frac{\epsilon M^2}{2}\tau\right)^2}. \tag{5.3.25}$$

We can note that the equation of state always satisfies $w < -1$, but approaches pure dS ($w = -1$) at late times $\tau \to \infty$. In fact, the logarithmic dependence on the scale factor in Eq.(5.2.7) denotes that the space time singularity sits at infinite time, and further more, that the derivatives of the Hubble scale vanish at infinite time. This corresponds to a specific scenario of Big Rip often dubbed Little Sibling of Big Rip (LSBR). This behaviour corresponds to an abrupt event rather than a future space-time singularity. At this event, the Hubble rate and the scale factor blow up but the cosmic derivative of the Hubble rate does not [58]. Consequently, this abrupt event takes place at an infinite cosmic time where the scalar curvature diverges.

A simple realization of such scenario is by introducing a *phantom* scalar field, as canonically normalized scalar field with a scalar potential, but with the "wrong" sign in the kinetic energy. In this sense, the phantom scalar field can be viewed as a particular case of the K-essence models [59] with $K = -X$. Its Lagrangian is then

$$S = \int d^4x a^3(t) \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \right], \tag{5.3.26}$$

As described in [60], this can also be described by a $d = 4$ theory including only gravity and a 3-form field with a potential. Such a theory does not lead to a loss of unitarity, but can, in some regimes, violate the Wheeler-DeWitt equation. For cosmological solutions this simplifies to

$$S = \int d^4x a^3(t) \left[-\frac{1}{2}\dot{\phi}^2 - V(\phi) \right], \tag{5.3.27}$$

leading to an equation of state

$$w = \frac{P_\phi}{\rho_\phi} = \frac{-\frac{1}{2}\dot{\phi}^2 - V(\phi)}{-\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{5.3.28}$$

We can see that for dominant potential energy, $V(\phi) > \dot{\phi}^2/2$, the equation of state always satisfies $w < -1$. For a Hubble scale $H = H_0 + \epsilon M^2 \frac{\tau}{2}$, the Friedmann equations for the phantom scalar lead to

$$V(\phi) = 3M_{Pl}^2 \left(H_0 + \epsilon M^2 \frac{\tau}{2} \right)^2 + \frac{1}{2}\dot{\phi}^2, \qquad \dot{\phi} = \sqrt{\frac{\epsilon M^2 M_{Pl}^2}{4}}. \tag{5.3.29}$$

Appropriately choosing the integration constant for ϕ as

$$\phi = \frac{2H_0 M_{Pl}}{\sqrt{\epsilon M^2}} + \sqrt{\frac{\epsilon M^2 M_{Pl}^2}{4}}\tau, \tag{5.3.30}$$

we can finally recast our results as a theory of cosmic acceleration driven by a phantom scalar with a quadratic potential

$$V(\phi) = \epsilon \frac{M^2}{2} M_{Pl}^2 + \epsilon \frac{3M^2}{4}\phi^2, \tag{5.3.31}$$

recovering the results found in slowroll approximation. This was previously noted in [61].

Now, it is clear that a solution to Eq. (5.3.13) exists for non slow-roll solutions precisely for the phantom scalar considered. Within slow-roll approximation we have as equation of motion

$$\ddot{\phi} + 3H\dot{\phi} - V' \simeq 3H\dot{\phi} - V' = 0 \tag{5.3.32}$$

Using this, we can then write the slow roll parameters for the phantom scalar

$$\varepsilon = -\frac{\dot{H}}{H^2} \simeq -\frac{1}{2}\frac{\dot{\phi}^2}{H^2} \simeq \varepsilon_V = -\frac{1}{2}\left(\frac{V'}{V}\right)^2, \tag{5.3.33}$$

$$\eta = -\frac{\dot{\varepsilon}}{H\varepsilon} \simeq -2\left(\frac{V'}{V}\right)^2 + 2\left(\frac{V''}{V}\right) \simeq 4\varepsilon_V - 2\eta_V. \tag{5.3.34}$$

After redefining the fields in (5.3.31) and substituting the latter in (5.3.33) and (5.3.34), we get

$$\epsilon = -\frac{1}{2}\left(\frac{2\phi}{\frac{2}{3}+\phi^2}\right)^2, \tag{5.3.35}$$

$$\eta = -2\left(\frac{2\phi}{\frac{2}{3}+\phi^2}\right)^2 + \frac{4}{\frac{2}{3}+\phi^2}, \tag{5.3.36}$$

where for simplicity we take $M_{Pl} = 1$.

5.4 Inflating the bubble

One could ask is there is any place for inflation in the Dark Bubble Model. Let us focus on the braneworld model in Type 0'B String Theory. From Eq. (5.2.8), we see that the Hubble parameter H contains a logarithmic dependence on the scale factor a. This could be phenomenologically interesting to embed the inflationary paradigm in this scenario.

Consider a scalar field ϕ with the quadratic potential we derived before, which we write for simplicity as

$$V(\phi) = V_0 + V_1\phi^2 \tag{5.4.1}$$

We now try to do cosmological perturbation theory with the phantom scalar. For this we express the field as a background term, satisfying the Friedmann equations plus a perturbation

$$\phi(x,t) = \phi_0(t) + \frac{\delta\phi(t,x)}{a} \tag{5.4.2}$$

Upon computing the equation of motion for the perturbation we arrive at

$$\delta\phi'' + \left(k^2 - \frac{a}{a''}\right)\delta\phi = 0, \tag{5.4.3}$$

where we have assumed spatially flat gauge, as well as slow-roll for the background field. We can immediately note that the equation for the perturbation is independent on the sign of the kinetic energy, this allows us to immediately use the usual results for scalar inflation, while keeping in mind the sign of the slow roll parameter. At horizon crossing we find the corresponding scalar and tensor power spectra. *For isotropic perturbations, the result is the same even if one chooses a different effective realization* (e.g. vector field inflation), as the equations for the perturbations remain the same.

$$P_\zeta = \left(\frac{H^2}{2\pi\dot{\phi}_0^2}\right)^2 = -\frac{H^2}{8\pi^2\varepsilon} \tag{5.4.4}$$

$$P_T = 2\frac{H^2}{\pi^2}. \tag{5.4.5}$$

It is then straightforward to compute the spectral tilt and tensor-to-scalar ratio

$$n_s - 1 = \frac{dP_\zeta/dt}{HP_\zeta} = -2\epsilon - \eta \tag{5.4.6}$$

$$r = \frac{P_T}{P_\zeta} = -16\epsilon. \tag{5.4.7}$$

With experimentally constrained values given by

$$\begin{aligned} n_s - 1 &= -0.035 \pm 0.004, \\ r &< 0.03 \end{aligned} \tag{5.4.8}$$

We can immediately notice that unless $1 \gg \eta \gg -\epsilon$, the scalar spectrum will be blue shifted, corresponding to $n_s > 1$, which is experimentally ruled out. In order to relate the relevant quantities to experimental data, we must compute such quantities some number of e-folds N_e (around 60) before the end of inflation. For the case of the bubble, which leads to eternal inflation, one must introduce an additional mechanism for inflation to end. We keep the assumption of such mechanism general, and assume some field value ϕ_* N_e e-folds before the end of inflation. For the potential considered in (5.4.1), we have

$$n_s - 1 = -\frac{4V_1\left(V_0 - 2V_1\phi_*^2\right)}{\left(V_0 + V_1\phi_*^2\right)^2} \tag{5.4.9}$$

$$r = \frac{32V_1^2\phi_*^2}{\left(V_0 + V_1\phi_*^2\right)^2} \tag{5.4.10}$$

In order to satisfy the experimental bounds we require $V_1\phi^2 \ll V_0$, meaning that ϕ_* must be close to the minimum of the potential. This is typically not possible in scalar inflation, since the inflaton rolls down towards the minimum. For a phantom field, the negative kinetic energy instead makes the field roll up, away from the minimum. With this we obtain

$$n_s - 1 \simeq -\frac{4V_1}{V_0} \tag{5.4.11}$$

$$r = \frac{32V_1^2\phi_*^2}{V_0^2}. \tag{5.4.12}$$

If we fit these expressions with the experimental data (5.4.8), we notice that the regime proposed for fitting experimental data is away from the full solution. For the full solution, we have $\dot{H} = \text{const}$, which is valid in the regime $V_1\phi^2 > V_0$. As long as we remain within slow-roll $\varepsilon \ll 1$ there should be a scalar realization, so we can try to compute ε,η (and with that n_s and r) directly from the full expression for a in (5.2.8).

$$\varepsilon = -\frac{\frac{\epsilon M^2}{2}}{(H_0 + \frac{\epsilon M^2}{2}\tau)^2} \tag{5.4.13}$$

$$\eta = -\frac{\epsilon M^2}{(H_0 + \frac{\epsilon M^2}{2}\tau)^2} \tag{5.4.14}$$

We can see that both slow-roll parameters are negative at all times, this will translate into a **blue-shifted spectral tilt, incompatible with observation**. We would also like to comment that, in the case of a 3-form realization, the spectral tilt is given by [62]

$$n_s \simeq -2\varepsilon - \frac{3}{2}\eta, \tag{5.4.15}$$

and as such the spectrum is also blue-shifted.

Finally, let us comment that it could be interesting to explore inflation in the braneworld using the tools of $f(R)$ inflation. The $f(R)$ model hypothesizes that, during the inflationary epoch, the energy density is not dominated by a scalar field (i.e. the inflaton), but that the exponential expansion is due to the way gravity behaves at very high energies, and the effective action takes the form

$$S = \int d^4\sqrt{-g}\left[\frac{M_P^2}{2}f(R) + \mathcal{L}_m\right], \tag{5.4.16}$$

Computations seem to work in the slow-roll approximation. However, if we consider a more general scenario, since $f(R)$ can always be recast as a theory of EH gravity plus a canonically normalized scalar field, it can never lead to accelerated expansion with $w < -1$ and this framework has to be discarded as well.

5.5 Bounds on late time acceleration

Late time cosmology refers to the current epoch of the universe, in which an accelerated expansion rate is observed. Bounds depend on the specific model, for time independent dark energy (w_0CDM), the present day equation of motion is bounded as [63]

$$w_0 = -1.028 \pm 0.031. \tag{5.5.1}$$

This can in principle correspond to (5.3.25), evaluated at late times, such that $w_{0'B} \simeq -1$. Perhaps a more interesting scenario is given by $w_0 w_a CDM$, where the scale dependence of dark energy is given by be parameterized as

$$w = w_0 + (1-a)w_a, \tag{5.5.2}$$

with

$$w_0 = -0.957 \pm 0.08, \qquad w_a = -0.29^{+0.32}_{-0.26} \tag{5.5.3}$$

Appropriately choosing H_0 as the present day Hubble constant and $\tau = 0$ as present time, we can expand the equation of state as

$$w_{0'B} \simeq -1 - \frac{\epsilon M^2}{3H_0^2} - (1-a)\frac{\epsilon^2 M^4}{3H_0^4}. \tag{5.5.4}$$

Given the value of Hubble today

$$H_0 \simeq 5.9 \times 10^{-61} M_{Pl}, \tag{5.5.5}$$

this imposes the bound

$$\epsilon M^2 \lesssim 3.87 \times 10^{-122} M_{Pl} \tag{5.5.6}$$

In other words, ϵM^2 has to be, at most, of the order of the cosmological constant. This can then be used to impose bounds on the string coupling g_s and the number of stacked $D3$-branes sourcing the geometry, N, as

$$\epsilon M^2 \simeq g_s^2 M_{Pl}^2 < 10^{-122} M_{Pl}^2, \tag{5.5.7}$$

with then

$$g_s \lesssim 10^{-61}\,, \qquad N \gtrsim 10^{61}. \tag{5.5.8}$$

This sets the scales in the model as

$$M < M_s < M_{10} < M_{Pl} \ll M_5, \tag{5.5.9}$$

with

$$M \lesssim \text{meV}\,, \qquad M_{10} \lesssim \text{TeV}. \tag{5.5.10}$$

Such scales could then in principle realize the dark dimension scenario, of a single mesoscopic extra dimension of order microns, similar to the results of [18]. The key difference in this work, is the non zero ϵ, coming from the uncancelled dilaton tadpole of $0'B$ String Theory. This then strongly constraints the value of the string coupling, and makes it unfeasible to realize SM gauge couplings on the brane.

As a toy model, we explore the possible scales coming from realizing a similar scenario, with the SM localized on a stack of NS5 branes. This requires replacing the S_5 with another suitable Einstein manifold with non trivial $H^{1,1}$ cohomology class, such that one can wrap the additional two dimensions on a 2-cycle Σ_2 as part of its worldvolume Σ_{5+1}. In this case, the brane action is given by

$$\mathcal{L}_{\rm NS5} \simeq -N_5 T_{\rm NS5} \left(\frac{\alpha' U}{R(U)}\right)^4 e^{-2\phi(U)} \tilde{R}(U)^2 \sqrt{1 - \frac{\lambda \dot{U}^2}{U^4}}. \tag{5.5.11}$$

$$\mathcal{L}_{\rm NS5} \simeq -\frac{N_5 N^2 U^4}{2\pi^3 \lambda^{5/2}} \left(1 + \frac{9\lambda^2 \alpha' T}{256\sqrt[4]{8}\pi^2 N} \log\frac{U}{u_0}\right) \sqrt{1 - \frac{\lambda \dot{U}^2}{U^4}} \tag{5.5.12}$$

From this expression we can read ϵ as

$$\epsilon \simeq g_s^2 N, \tag{5.5.13}$$

which is the same scaling as for the $D3$ branes. In fact, by looking at the DBI action for p-branes one could expect the dependence on g_s, N to be generic since the leading contribution coming from Eq. (5.1.2) is of the form $g_s^2 N$. In the case of $NS5$ branes there is the further complication of lacking a CS term. Since the equation implied by energy conservation at large U is purely algebraic, this suggests that it is not an inflating solution.

It was however argued in [64] that S-dualising the Wess-Zumino action of a stack of D5 branes, including corrections to order α', leads to an action for NS5 branes containing the term

$$S_{WZ}^{NS5} \subset N_5 \mu_5 \int_{\Sigma_{5+1}} {\rm Tr}\left(2\pi\alpha' C_4 \wedge \mathcal{F}\right), \tag{5.5.14}$$

where $\mathcal{F}$ is the field strength on the NS5 brane. This is not vanishing if there is induced charge on the brane, such that

$$Q_{eff} \simeq \int_{\Sigma_2} \mathcal{F}. \tag{5.5.15}$$

The charge should then be chosen in a way such that the leading order contributions from the DBI and CS actions cancel out. In this case we can repeat the procedure detailed in Section 5.2 to again obtain $LSRB$ cosmological solutions.

5.6 Electromagnetism in 0'B

The mechanism we have seen to embed electromagnetic gauge fields on the brane was developed in a Type IIB embedding, which involves D_3-branes, the B-field and the C_2-field. All our considerations apply to the braneworld model built in the non-supersymmetric $0'B$ Theory, with the exception that, given that there is no B-field in this theory, the only relevant field is C_2. The only equation which governs the interplay between the brane and the bulk is therefore (4.2.15).

Of course these considerations do not really apply, since the braneworld model in Type 0'B was ruled out due to incompatibility with experimental observations.

Conclusion

Outlook

The Dark Bubble model is an intricate but dynamically natural construction which represents an alternative to the paradigm of standard compactification in String Theory. It is a cosmological scenario which makes extensive use of elements from String Theory, such as strings and branes, and can be very well embedded in Type IIB and in Type 0'B string theories. Whereas in the former case there is no conflict with phenomenology (at least thus far), the latter construction, albeit very elegant, is incompatible with cosmological observations and has to be discarded.

In this thesis work, we found a general mechanism to embed electromagnetic gauge fields living on the brane. The two fields responsible for this interplay are the Kalb-Ramond field $B_{\mu\nu}$ and the Ramond-Ramond field $C^{(2)}_{\mu\nu}$. This represents a first step towards a complete description of the Standard Model in the Dark Bubble scenario.

Moreover, we provided a no-go result for D_3-brane worlds arising from Type 0'B Theory: the evolution of the scale-factor $a(\tau)$ indicates a quasi-dS evolution of the brane world, which seems promising for embedding inflation in the picture; however, we find a blue-shifted spectral tilt, incompatible with observations.

Bubbles and the Swampland

We have largely discussed how some of the conjectures of the Swampland program perfectly resonate with the Dark Bubble model. Although they are not *a priori* required to hold on the brane world itself, they apply to the ambient bulk spacetime. Indeed, the de Sitter Conjecture (even in its milder formulation) was a main reason for other attempts to realize a dS universe, which do not involve (approximately) time-independent string compactifications. Moreover, the Weak Gravity Conjecture turned out to be crucial for the decay of the initial non-supersymmetric AdS vacuum and the realization of a codimension-1 brane with a tension smaller than the critical one, thus realizing a small and positive cosmological constant.

However, the Dark Bubble model does not have much to say about the Distance

Conjecture. In fact, moduli corresponding to the five internal dimensions beyond AdS_5 should be stabilized, while the radius of the brane bubble expanding in the fifth dimension corresponds to the scale factor in the 4d cosmology. Regarding the Trans-Planckian Censorship Conjecture, we notice that the bubble wall moves an infinitesimal distance (of order AdS length) in the fifth dimension over large e-folds of expansion in 4d. Since the distance traversed never exceeds a few AdS lengths, questions of runaway moduli or approaching the boundaries of moduli space never arise. Indeed, remember that, according to the Dark Bubble scenario, we live on the brane! We cannot therefore say much about infinite distance limits or global symmetries, since gravity in 4d is "fake".

Finally, let us mention that, in agreement with the Cobordism Conjecture, the two AdS spacetimes (the "inside" and the "outside" of the bubble) are indeed connected – in this case, by a codimension-1 brane –, since otherwise a global symmetry associated to the flux would be present. As should happen in Quantum Gravity, this symmetry is broken by these branes.

Future directions

We have made a first attempt in constructing a general mechanism to embed EM gauge fields on the brane world, but much work remains to be done in embedding the full Standard Model of Particle Physics. Generalizing the electromagnetic mechanism to other non-abelian gauge fields is not trivial at all, and may be either the turning point for this model or spell its own doom. Reproducing all the ingredients of the Standard Model along with all known mechanisms, such as Spontaneous Symmetry Breaking, and the correct energy scales is the only way to determine whether this model can be phenomenologically viable or not.

We unraveled the brane world construction of the Dark Bubble model in Type 0'B. This setup was particularly promising to account for a natural description of inflation on the bubble, but eventually had to be discarded for being incompatible with experimental observations. Nonetheless, the question of whether inflation can be realized in the Dark Bubble remains. While inflation is a quasi-dS phase, an exact dS period of inflation is now completely ruled out by $n_s = 0.968 \pm 0.006$ [65]. Therefore, additional ingredients will be necessary to alter the spectral tilt and find an instance of slow-roll inflation consistent with observations. Interestingly, the eventuality of bubble collisions provides potentially promising pathways for inflationary model building. It was shown in [66] that brane collisions can result in free passage for the branes for a certain range of relativistic scattering velocity. Additionally, open strings stretching between the branes will be created in the collision. The model [67] makes use of repeated collisions of this sort to provide an explanation for certain anomalies in the CMB power spectrum. Finally, due to the creation of massive particles, collisions potentially provide a transition from a cosmological constant to a matter domination era. Matter interactions and the realization of the Standard Model of Particle Physics is another important issue. The presence of branes and stretched strings suggest that the familiar D-brane constructions of gauge theories will be relevant.

An important new feature is that it is now obvious that we need more than just fundamental strings. The endpoint of a string of tension T appears as a particle of mass T/k in the four-dimensional shell-world. Assuming a large five-dimensional cosmological constant (with no hierarchy across the shell) and setting a lower limit on the tension of strings at $(\mathrm{TeV})^2$, sets a lower limit on the particle mass to be of the order of magnitude of 0.1 eV, which is, interestingly enough, of the same order of magnitude as the estimated mass of neutrinos. This is an encouraging and non-trivial observation. The low tension excludes fundamental strings and suggests that the five-dimensional stretched strings need to be either topological defects or gauge strings. To derive four-dimensional effective field theories matching the Standard Model will be an interesting, but challenging, problem.

Moreover, it would be equally interesting to see if the Dark Bubble allows for the presence of axions, whose couplings are of the type

$$C_0 \wedge F \wedge F. \tag{5.6.1}$$

The way axions behave in this setup is yet to be determined, and they might not be localized on the brane (see [68] for an example).

Finally, the incorporation of black holes into our model is an interesting direction for future research. The formation of a four-dimensional black hole is expected to proceed through the collision of stretched strings in five dimensions. Hence, one would expect a black hole to correspond to a five-dimensional black string ending on the brane world. A problem with such a solution is that it may suffer from a Gregory-Laflamme instability [69]. In four dimensions, it was argued [70] that black holes can be replaced by horizonless black shells. If there is an instability of Minkowski space towards the formation of a bubble of AdS-space, it can be argued that a transition can be stimulated to occur if matter threatens to form a black hole. It was also argued that such black shells can be stable and thus correspond to a viable alternative to a black hole. The uplift of such a transition, with the black string replaced by a black tube, seems to be a natural outcome in our model.

Finally, what about holography? Even if great progress has been made in [19], a full understanding of the Dark Bubble model from the holographic perspective is yet missing. Among the open issues, a deeper understanding of the relation between the field theory on the bubble and the one on the holographic boundary is required. Furthermore, non-supersymmetric AdS cannot be dual to a CFT [9], although it may be dual to an RG flow [71]. We leave these ideas and questions for future work.

Appendix

Tools of Differential Geometry

Let us briefly introduce some concepts of differential geometry which are very useful in the study of String Theory. A useful concept is that of differential forms. A **p-form** ω is a totally antisymmetric tensor of rank p

$$C^{(p)} = \frac{1}{p!} C_{\mu_1 \dots \mu_p} dx^{\mu 1} \wedge \dots \wedge dx^{\mu_p}, \tag{5.6.2}$$

where the wedge product $\wedge$ between a p-form and a q-form is defined by

$$A^{(p)} \wedge B^{(q)} = C^{(p+q)} \tag{5.6.3}$$

and

$$C^{(p+q)}_{\mu_1 \dots \mu_{p+q}} = \frac{(p+q)!}{p!q!} A_{[\mu_1 \dots \mu_p} B_{\mu_{p+1} \dots \mu_{p+q}]}. \tag{5.6.4}$$

Consequently, $A^{(p)} \wedge B^{(q)} = (-1)^{pq} B^{(q)} \wedge A^{(p)}$ and $[A^{(p)}, B^{(q)}] = -(-1)^{pq}[B^{(q)}, A^{(p)}]$.

We can define the **exterior derivative** as an operator d which maps a p-form into a (p+1)-form

$$C^{(p)} = \frac{1}{p!} C_{\mu_1 \dots \mu_p} dx^{\mu 1} \wedge \dots \wedge dx^{\mu_p} \longrightarrow dC^{(p)} = \frac{1}{p!} dC_{\mu_1 \dots \mu_p} dx^{\mu 1} \wedge \dots \wedge dx^{\mu_p} \tag{5.6.5}$$

$$= \frac{1}{p!} \partial_{\mu_1} C_{\mu_2 \dots \mu_{p+1}} dx^{\mu 1} \wedge \dots \wedge dx^{\mu_{p+1}}, \tag{5.6.6}$$

such that $C^{(p)}_{\mu_1 \dots \mu_{p+1}} = (p+1) \partial_{[\mu_1} C_{\mu_2 \dots \mu_{p+1}]}$. Here the square brackets denote normalized antisymmetrization

$$[\mu_1 \dots \mu_p] = \frac{1}{p!}(\text{even permutations of (12...p)} - \text{odd permutations of (12...p)}). \tag{5.6.7}$$

Finally, the exterior derivative is nilpotent $d^2 C^{(p)} = 0$.

A p-form is $C^{(p)}$ exact if there exists a (p-1)-form $B^{(p-1)}$ such that $C^{(p)} = dB^{(p-1)}$ and it is closed if $dC^{(p)} = 0$. We can define the **p-th cohomology group** of the n-dimensional manifold $\mathcal{M}$ as the quotient

$$H^p(\mathcal{M}) = \frac{\text{closed p-forms}}{\text{exact p-forms}}. \tag{5.6.8}$$

Exact forms are said to be cohomologically trivial. The dimension of $H^p(\mathcal{M})$ is called the p-th Betti number and is an important topological invariant of $\mathcal{M}$ related to Euler's characteristic χ.

We can write a version of **Stoke's theorem** for differential forms. If $\Gamma_{(p+1)}$ is a $(p+1)$-dimensional submanifold of $\mathcal{M}$ and $\partial\Gamma_{(p+1)}$ denotes its p-dimensional boundary, then

$$\int_{\Gamma_{(p+1)}} dC^{(p)} = \int_{\partial\Gamma_{(p+1)}} C^{(p)}. \tag{5.6.9}$$

The operation of taking the boundary of a p-dimensional submanifold of $\mathcal{M}$ is therefore dual to taking the exterior derivative of a p-form. A *p-fold* with $\partial\Gamma_p = 0$ is called a p-cycle, and a p-fold which is the boundary of another (p+1)-dimensional submanifold $\Gamma_p = \partial\Omega_{p+1}$ is called a p-boundary. The object object Ω_{p+1} is a (p+1)-chain. One can then define the **p-th homology group** of $\mathcal{M}$ as the set

$$H^p(\mathcal{M}) = \frac{\text{p-cycles}}{\text{p-boundaries}}, \tag{5.6.10}$$

with dimension $b_p = \text{dim} H_p(\mathcal{M})$.

The de Rahm dual of a p-fold Γ_p is defined as the (n-p)-form $\delta^{(n-p)}\Gamma_p$ such that for each p-form $\omega^{(p)}$ the following relation holds

$$\int_{\Gamma_{(p+1)}} \omega^{(p)} = \int_{\mathcal{M}} \omega^{(p)} \wedge \delta^{(n-p)}(\Gamma_p). \tag{5.6.11}$$

This gives rise to a one-to-one correspondence between homology and cohomology. In particular, $b_p = b^p$.

A crucial notion to understand duality is the **Hodge * operator**, which maps a p-form into its dual (n-p)-form

$$\begin{aligned} &* : \text{p-form} \longrightarrow \text{(n-p)-form} \\ &C^{(p)} \longrightarrow *C^{(p)} = \frac{\sqrt{|g|}}{p!(n-p)!} C_{\mu_1 \ldots \mu_p} \epsilon^{\mu_1 \ldots \mu_p}_{\nu_{p+1} \ldots \nu_n} dx^{\nu_{p+1}} \wedge \ldots \wedge dx^{\nu_n}, \end{aligned} \tag{5.6.12}$$

where we defined the totally antisymmetric tensor $\epsilon_{\mu_1 \ldots \mu_n}$ to be ± 1 if $(\mu_1 \ldots \mu_n)$ are even/odd permutations of $(1, \ldots, n)$ and 0 in the other cases.

Finally, let us make contact to String Theory and consider some examples of p-form fields. in the bosonic string, there is only the Kalb-Ramond 2-form potential $B = \frac{1}{2}B_{\mu\nu}dx^\mu dx^\nu$. In superstring theory, there are the R-R form potentials $C^{(1)}$ and $C^{(3)}$ in Type IIA and $C^{(0)}$, $C^{(2)}$ and $C^{(4)}$ in Type IIB.

These higher-rank form fields can be interpreted as generalisations of the electromagnetic 1-form gauge potential $A = A_\mu dx^\mu$. Indeed, the field strength

$$F^{(p+1)} = dC^{(p)} \tag{5.6.13}$$

is invariant under the abelian gauge transformation

$$C^{(p)} \to C^{(p)} + d\chi^{(p-1)}, \tag{5.6.14}$$

due to nilpotency of the exterior derivative. The so-defined field-strength is closed because

$$dF^{(p+1)} = d(dC^{(p)}) = 0, \tag{5.6.15}$$

a relation dubbed Bianchi identity.

The canonical kinetic term of the field-strength can compactly be written as

$$S_{kin} = -\frac{1}{2(p+1)!}\int d^n x\sqrt{|g|}F_{\mu_1\dots\mu_{p+1}}F^{\mu_1\dots\mu_{p+1}} = -\frac{1}{2}\int F \wedge *F. \tag{5.6.16}$$

The following relation holds

$$F^{(p+1)} = *\tilde{F}^{(n-p-1)}, \tag{5.6.17}$$

meaning that a p-form potential is dual to an (n-p-2)-form potential. This is the generalization of electric-magnetic duality in 4d.

Finally, a p-form couples naturally to a p-fold via

$$S_{coupled} = \mu_p \int_{\Gamma_p} C^{(p)}. \tag{5.6.18}$$

where μ_p is the charge of Γ_p. This is the natural generalisation of the coupling of the 1-form potential $A^{(1)}$ to a point particle.

Brief Review of General Relativity and Cosmology

Let us start by defining some important quantities in the study of GR.

- We can define the connection between the Christoffel symbols and the metric as

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\alpha} \left[g_{\mu\alpha,\nu} + g_{\nu\alpha,\mu} - g_{\mu\nu,\alpha} \right] . \tag{5.6.19}$$

- The mathematical quantity which allows us to define the curvature of a manifold is called the Riemann tensor

$$R_{\mu\nu} = R^{\rho}_{\mu\rho\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\alpha}_{\mu\alpha} . \tag{5.6.20}$$

  The Riemann tensor will vanish if and only if the metric is perfectly flat. Moreover, it allows us to define other quantities

- the Ricci tensor

$$R^{\lambda}_{\mu\lambda\nu} = R_{\mu\nu}, \tag{5.6.21}$$

- the Ricci scalar

$$R = R^{\mu}_{\nu} = g^{\mu\nu} R_{\mu\nu} , \tag{5.6.22}$$

- the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{5.6.23}$$

Einstein equations

The theory of General Relativity is governed by the Einstein-Hilbert action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R(g_{\mu\nu}, \partial_\rho g_{\mu\nu}, \partial_\sigma \partial_\rho g_{\mu\nu}) - 2\Lambda \right] + S_M , \tag{5.6.24}$$

The S_M part of the action contains all the matter sources, which in the standard model of cosmology come down to baryons, dark matter, photons and neutrinos. S_M is usually written from a Lagrangian

$$S_M = \int d^4x \sqrt{-g} \mathcal{L}_M(g_{\mu\nu}) . \tag{5.6.25}$$

Applying the variational principle on this action with respect to the metric $g_{\mu\nu}$, we obtain the Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} , \tag{5.6.26}$$

which are ten (with only six independent) partial differential equations. $T_{\mu\nu}$ is the *energy-momentum* tensor, which comes from the functional derivative of the S_M part of the Einstein-Hilbert action with respect to the metric $g_{\mu\nu}$

$$T^{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{\mu\nu}} \,. \tag{5.6.27}$$

For a perfect fluid with four-velocity u^μ, $T_{\mu\nu}$ is given by

$$T^{\mu\nu} = \rho u^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu) = (\rho + p)u^\mu u^\nu + p g^{\mu\nu} \,, \tag{5.6.28}$$

where ρ and P are, respectively, the density and the pressure of the fluid. Moreover, in the rest frame of the fluid the four-velocity $u^\mu = (1, 0, 0, 0)$ and $d\tau = dt$, which means that

$$T^{\mu\nu} = \begin{bmatrix} \rho & 0 \\ 0 & pg^{ij} \end{bmatrix} \,, \tag{5.6.29}$$

and

$$T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p) \,. \tag{5.6.30}$$

Moreover, the following *continuity equation* holds:

$$\nabla_\mu T^\mu_\nu = 0 \,. \tag{5.6.31}$$

Einstein equations contain an interpretation of gravity as the geometry of space-time: the energy-momentum tensor determines the curvature of space-time, which in turn affects the matter motion. Finally, the cosmological constant Λ that enters in the Einstein equations entails the contribution of *vacuum energy*, an energy density characteristic of empty space.

FLRW metric

The metric which describe an isotropic and homogeneous Universe in expansion is identified by the Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2(\theta)d\phi^2\right] \tag{5.6.32}$$

$$\equiv -dt^2 + a(t)^2 d\Sigma^2 \,, \tag{5.6.33}$$

where the origin $r = 0$ is totally arbitrary and t is the proper time of an observer moving along with the cosmic fluid at constant r, θ and ϕ. These coordinates are known as *comoving coordinates*, and only a comoving observer will think that the Universe looks isotropic.

The function $a(t)$ is called the *cosmic scale factor*, and describes the expansion of the Universe starting from the initial value $a_0 = 0$ to a conventional value of $a(t_0) = 1$, corresponding to the present day (t_0 is the age of the Universe). The parameter k is called the *curvature constant* and describes the curvature, and therefore the size, of the spatial surfaces. k could indeed take any real value, but it is common to normalize it to $k = 0, \pm 1$ and absorb the physical size of the manifold into the scale factor $a(t)$. Depending on the value of the curvature scalar, it is possible to introduce new coordinates such that the topology of the hypersurface Σ is apparent from the line element $d\sigma^2$

- The $k = 0$ case corresponds to no curvature on Σ

$$d\sigma^2 = dr^2 + r^2 d\Omega^2 = dx^2 + dy^2 + dz^2 \,, \tag{5.6.34}$$

  and Σ is called **flat** (euclidean geometry).

- The $k = +1$ case corresponds to constant positive curvature on Σ

$$r = \sin(X) \Rightarrow d\sigma^2 = dX^2 + \sin^2(X) d\Omega^2 \,, \tag{5.6.35}$$

  and Σ is a **three-dimensional sphere** (spherical geometry).

- The $k = -1$ case corresponds to constant negative curvature on Σ

$$r = \sinh(\varphi) \Rightarrow d\sigma^2 = d\varphi^2 + \sinh^2(\varphi) d\Omega^2 \,, \tag{5.6.36}$$

  and Σ is a **three-dimensional hyperboloid** (hyperbolic geometry).

Friedmann equations

The Einstein equations

$$T_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{5.6.37}$$

evaluated on the FLRW metric are the so-called *Friedmann equations*

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] = 8\pi G_N \rho \quad \Rightarrow \quad H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} \,, \tag{5.6.38}$$

$$3\frac{\ddot{a}}{a} = -4\pi G_N(\rho + 3P) \quad \Rightarrow \quad \dot{H} + H^2 = -\frac{4\pi G_N}{3}(\rho + 3P) \,, \tag{5.6.39}$$

where $\rho = \rho(t)$ is the *total* energy density of all components of the cosmic fluid and $P = P(t)$ the total pressure. The Friedmann equations should be solved to find $a(t)$, which depends on the components considered through ρ and P. It is useful to define the *density parameter*

$$\Omega = \frac{8\pi G_N}{3H^2}\rho = \frac{\rho}{\rho_{critical}} \,, \tag{5.6.40}$$

where the *critical density* is

$$\rho_{critical} = \frac{3H^2}{8\pi G_N} \,. \tag{5.6.41}$$

The Friedmann equation (5.6.38) can therefore be written as

$$\Omega - 1 = \frac{k}{H^2 a^2}. \tag{5.6.42}$$

The density parameter Ω contains the information on the topology of the Universe. Indeed

- $\rho < \rho_{critical} \iff \Omega < 1 \iff k = -1 \iff$ Open Universe
- $\rho = \rho_{critical} \iff \Omega = 1 \iff k = 0 \iff$ Flat Universe
- $\rho > \rho_{critical} \iff \Omega > 1 \iff k = 1 \iff$ Closed Universe .

The density parameter, then, tells us which of the three FLRW geometries describes our Universe. Determining it observationally is crucial: recent measurements of the CMB anisotropy suggest that Ω is very close to 1.

Cosmic fluids

Assuming again that the Universe is permeated with an ideal cosmic fluid of matter and energy, the energy-momentum tensor has the form (5.6.30). The 0-component of the continuity eq.(5.6.31) equation yields

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{5.6.43}$$

We can assume our fluid is barotropic, meaning it obeys the following equation of state

$$p = \omega \rho \,, \tag{5.6.44}$$

where the parameter ω is called **state parameter** and is a constant value independent of time. Energy conservation then reads

$$\frac{\dot{\rho}}{\rho} = -3(1+\omega)\frac{\dot{a}}{a} \implies \quad \rho \propto a^{-3(1+\omega)} \,. \tag{5.6.45}$$

Until not too long ago, the present Universe was believed to be dominated by ordinary matter (dust), while the primordial one by radiation. However, the expansion of the Universe is accelerating ($\ddot{a} > 0$), a fact which is incompatible with the gravitational effect of matter. Among the possible sources responsible of this acceleration, it has been hypothesised the so-called *dark energy*, encompassed in the cosmological constant Λ.

The components of the cosmic fluid therefore are

- **Dust**: pressureless matter, or non-relativistic matter almost exactly at rest with the cosmic frame. In this case other forces beside gravity are absent, and $\omega = 0$ (so that $p = 0$). Eq.(5.6.45) therefore yields

$$\rho_{matter} = \frac{E}{V} \propto a^{-3}\,, \tag{5.6.46}$$

which is interpreted as the decrease in the number density of particles as the Universe expands

- **Radiation**: pure electromagnetic radiation or highly-relativistic matter. Since the mass is totally negligible, so is the energy-momentum trace

$$T = -\rho + 3p = 0\,, \tag{5.6.47}$$

which implies

$$p = \frac{1}{3}\rho \quad \Rightarrow \quad \omega = \frac{1}{3}\,. \tag{5.6.48}$$

Eq. (5.6.45) yields

$$\rho_{rad} = \frac{E}{V} \propto a^{-4}\,. \tag{5.6.49}$$

This is because the number density of photons decreases in the same way as the number density of non-relativistic particeles, but individual photons lose energy as $E \propto a^{-1}$ as they redshift.

- **Vacuum energy**: a fluid with equation of state

$$\rho = -p = \frac{\Lambda}{8\pi G}, \quad \omega = -1\,. \tag{5.6.50}$$

The energy density of this fluid is therefore constant

$$\rho_\Lambda \propto 1\,. \tag{5.6.51}$$

Let us now examine the behaviour of the scale factor $a(t)$ in a flat ($k = 0$) universe dominated by these cosmic fluids.

- For a flat, matter dominated Universe, we have

$$\rho_m \sim \frac{1}{a^3} \quad \Rightarrow \quad \frac{\dot{a}^2}{a^2} \sim \frac{1}{a^3} \quad \Rightarrow \quad \sqrt{a}da \sim dt \quad \Rightarrow \quad t \sim a^{3/2}\,, \tag{5.6.52}$$

thus $a \sim t^{2/3}$.

- For a flat, radiation dominated Universe, instead

$$\rho_r \sim \frac{1}{a^4} \;\Rightarrow\; \frac{\dot{a}^2}{a^2} \sim \frac{1}{a^4} \;\Rightarrow\; a da \sim dt \;\Rightarrow\; t \sim a^2 \,, \tag{5.6.53}$$

  thus $a \sim t^{1/2}$.

- Finally, for a flat and empty Universe, with only a positive vacuum energy present, we obtain the extact solution

$$\rho_\Lambda \sim \Lambda \;\Rightarrow\; \frac{\dot{a}^2}{a^2} \sim \frac{\Lambda}{3} \;\Rightarrow\; \sqrt{\frac{\Lambda}{3}} \sim \frac{\dot{a}}{a} \equiv H_0 \;\Rightarrow\; a \sim e^{H_0 t} \,, \tag{5.6.54}$$

  where H_0 is now a true cosmological constant. This universe, subject to a constant exponential expansion, is dubbed de Sitter (dS) Universe.

The ΛCDM model

The solutions of the Friedmann equations for dust and radiation show a common behaviour. According to the inflation theory, the Universe started from a *singularity* and is expanding ever since (at least until a maximum scale factor). The standard model of cosmology is divided into two parts

- a primordial stage, behind the Last Scattering Surface, very well described by the theory of inflation [44],
- a later stage during which large scale structures such as clusters and galaxies came into being, described by the ΛCDM model. This model can be summarized as following: the dark matter is cold and the dark energy has a constant energy density, which means $\omega = -1$.

The observations collected so far denote the fact that the present Universe is spatially flat with $\Omega \simeq 1$, which corresponds to an average density

$$\rho_0 = \rho_{critical} \simeq 10^{-29} g/cm^3 \,, \tag{5.6.55}$$

equivalent to about 6 protons per square cubic meter. Three sources that contribute to ρ_0 have been identified as

- **Regular baryonic matter**, approximated by a fluid made of dust and estimated through the luminosity of galaxies,

$$\frac{\rho_{matter}}{\rho_0} \simeq 5\% \,, \tag{5.6.56}$$

- **Nonbaryonic dark matter**, which behaves like dust (gravitationally), but can't be directly detected
$$\frac{\rho_{DM}}{\rho_0} \simeq 25\% \,. \tag{5.6.57}$$

- **Dark energy**, with the equation of state of the vacuum,
$$\frac{\rho_{DE}}{\rho_0} \simeq 70\%. \tag{5.6.58}$$

In terms of the fractional densities, we can estimate that $\Omega_M^0 = (0.3 \pm 0.1)$, $\Omega_{rad}^0 = 9 \cdot 10^{-5}$ and $\Omega_{DE}^0 = (0.7 \pm 0.1)$. Note that the value of Ω_m^0 is given by the sum of the baryonic matter and the dark matter contribution $\Omega_M^0 = \Omega_{matter}^0 + \Omega_{DM}^0 = (0.0486 + 0.2589)$.

The Universe has gone through three different epochs, each one governed by a different component.

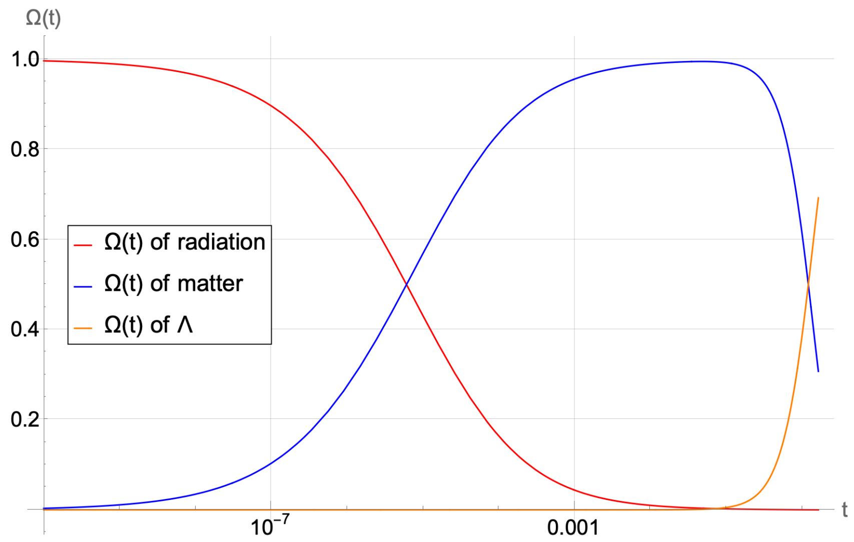

Figure 5.2: *Evolution of Ω_r, Ω_m and Ω_Λ for ΛCDM as functions of the cosmological time t. The scale factor is $a = 1$ at final value of time in the plot.. The dark energy density is increasing at late times, while radiation density becomes negligible.*

- **Radiation dominated epoch**: the radiation is prevalent up to redshift $z \simeq 10^4$; given the high speed with which it decreases, its density becomes soon negligible.
- **Matter dominated epoch**: baryonic matter and dark matter are the dominant components from $z \simeq 10^4$ until $z \simeq 0.7$.
- **Dark energy epoch**: the dark energy density is very small but fixed, as it is constantly generated by the vacuum itself, and becomes dominant when the expansion of the Universe has decreased the density of the other components.

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[4] Siiiiii?

[5] Heisenberg disapproves.

[6] I give myself some credit for that, too.

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