Alma Mater Studiorum · University of Bologna

**School of Science**
**Department of Physics and Astronomy**
**Master Degree in Physics**

# Leptogenesis Assisted by a Scalar Singlet

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Academic Year 2022/2023

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*September 2023*

# Abstract

In this thesis, we study how thermal leptogenesis is modified by the addition of a complex scalar field $S$ to the type-I seesaw extension of Standard Model (SM) with $n_r$ right-handed sterile neutrinos $\nu_{R,i}$, with $i = 1, \ldots, n_r$. Specifically, we extend the SM by a global $U(1)_{B-L}$ symmetry under which $S$ and $\nu_{R,i}$ are charged with charges $+2$ and -1, respectively, while remaining singlets under the SM gauge group. After the $U(1)_{B-L}$ spontaneous symmetry breaking, with the real part of $S$ acquiring a non-vanishing vacuum expectation value, the coupling between $S$ and $\nu_{R,i}$ generates a Majorana mass term for the right-handed neutrinos and we recover the well known type-I seesaw Lagrangian. In addition, two real scalars $\phi$, massive, and $\theta$, massless, are left after the $U(1)_{B-L}$ breaking, which are coupled to the right-handed neutrinos and to the Higgs doublet. We discuss the contributions of the interactions involving $\phi$ and $\theta$ to the requisite CP-asymmetry, the production rates of right-handed neutrinos and the wash-out processes in the Boltzmann equations for leptogenesis. After identifying the leading contributions, we modify the ordinary Boltzmann equations and study the effects on the final baryon asymmetry of the Universe.

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# Introduction

In the field of theoretical physics of the fundamental interactions, two of the biggest open questions are: the nature of neutrino masses; and the observed asymmetry between matter and antimatter. The theoretical framework which describes almost everything that happens in particle physics, the Standard Model (SM), is not able to account for these two experimental pieces of evidence. In the last century the SM has been able to explain and predict a lot of experimental observations, culminating in the discovery of the Higgs bosons at CERN LHC in 2012 [1] [2], which led Peter Higgs and François Englert to win the Nobel Prize in 2013 for theorizing its existence. At the same time we now know that we need to go beyond it if we want to be able to account for the new physics observed in neutrino experiments and in the Universe.

The discovery of neutrino masses is related to the experimental evidence of neutrino oscillations [3], and after more than 20 years from it, it remains something for which we don't have an explanation (1.6).
Neutrinos come in three generations, as all the other SM particles: L. M. Lederman, M. Schwartz and J. Steinberger in 1962 found out the existence of two different kind of neutrinos, one associated to the electron and the other one to the muon $\mu$ [4]; evidences for a third type of neutrino, the one associated with the $\tau$ lepton, were found in 2000 by the DONUT experiment [5]. The fact that they come in different families opened the question of under which conditions there could be mixing between them.
Neutrino oscillations were discovered in 1998 when the Super-Kamiokande experiment showed that $\mu$ neutrinos were disappearing with an $L/E$ dependence, where $L$ represents the neutrino travelled distance from production to detection and $E$ its energy, in agreement with an oscillatory behavior [6]. We now know that $\mu$ neutrinos oscillate into $\tau$ neutrinos which cannot be detected by that experiment. T. Kajita for the Super-Kamiokande collaboration and A. B. McDonald for the SNO collaboration [3] received the Nobel Prize in Physics for "the discovery of neutrino oscillations, which shows that neutrinos have a mass".
Experiments also tell us that neutrinos do not posses any electromagnetic charge and so they could be indistinguishable from their antiparticles, the antineutrinos, i.e. they could be Majorana particles, as Majorana pointed out in 1937 [7]. They are the only SM

particle which can have this feature. This question is strongly related with the nature of their masses and with the conservation or not of lepton number $L$.

Observations tell us also something else very interesting: in the Universe there is more matter than antimatter [8], or equivalently, that the number of baryons, i.e. protons and neutrons, is different from the number of antibaryons, i.e. antiprotons and antineutrons. Up to our knowledge, all the structures we see in the Universe - stars, galaxies and clusters - consist of matter, while antimatter in appreciable quantities is not present. The (small) amount of antiprotons and positrons in cosmic rays can be explained by their secondary origin in cosmic particle collisions or high energetic astrophysical processes and no antinuclei, even as light as anti-deuterium or as tightly bounded as anti-$\alpha$ particles, has ever been detected. Another evidence of the absence of cosmological antimatter can be seen to be the absence of annihilation radiation $p\bar{p} \to ...\pi^0 \to ...2\gamma$; this excludes significant matter-antimatter mixtures in objects up to the size of clusters $\simeq 20$ Mpc. Consequences of having an Universe which is a patchwork of distinct regions of matter and antimatter have been studied in [9] but it can be excluded looking at the limits on anomalous contributions to the cosmic diffuse $\gamma$-ray background and thanks to the absence of distortions in the cosmic microwave background (CMB). Anyway, we cannot exclude the vanishing of the average asymmetry for super-horizons scales; this would be the case if the fundamental Lagrangian is C and CP symmetric and charge invariance is broken spontaneously.

This project aims to address these two open problems in theoretical physics: the nature of neutrino masses and the baryon asymmetry of the Universe (BAU). Between the various extension of the Standard Model able to predict the presence of neutrino masses, and explain their values, there is a class of models called seesaw models. These models are mainly based on the fact that neutrinos are Majorana particles and that the addition to the SM particle content of some new heavy degrees of freedom, e.g. the right handed neutrinos (RHNs) in the type I seesaw, predicts the existence of the light neutrino masses. Moreover, in models of this kind a mechanism which could account for the BAU is naturally embedded. This mechanism is called Leptogenesis (LG). Its easiest realization is within the framework of the type I seesaw and it is based on the $L$- $C$- and $CP$-violating[1] out-of-equilibrium decays of the RHNs in the early Universe.
The type I seesaw mechanism by itself does not give any explanation on the origin of these new degrees of freedom, they are put in by hand. Anyway, it turns out that some ultra-violet (UV) completions of the Standard Model such as Grand Unified Theories (GUTs) predicts naturally the presence of RHNs and of the seesaw mechanism.

[1] Here $L$ stands for lepton number, $C$ for the charge conjugation and $P$ for the parity.

The thesis focuses on a model where we add to the SM particle content and symmetries not only singlet RHNs $\nu_{R,i}$ with $i = 1, \dots, n_r$ but also a singlet complex scalar, called S, and a $U(1)_{B-L}$ global symmetry, under which the RHNs have charge $B - L = -1$ and the complex scalar 2. We study the implications of the $U(1)_{B-L}$ spontaneous symmetry breaking at some energy scale $v_s$ far above the electroweak scale: the symmetry breaking will give the mass to the RHNs, generating automatically the usual type I seesaw mechanism with its consequent LG.
The motivations that led us to consider such model are multiple.

1. We would like to find an origin to the heavy RHNs masses and not to simply put their direct mass terms in the Lagrangian using the fact that a term like $\propto M_R \nu_R^T C^\dagger \nu_R$ is invariant under the SM gauge symmetry group. But then, since we know that LG can be realized in the simple type I seesaw scenario, we would like to quantify the effects of adding new degrees of freedom coming from the scalar singlet S. In fact, besides the standard Yukawa terms allowing for the $\nu_R$'s decay, we will have new interactions of the kind $\propto (\mathrm{S}^\dagger \mathrm{S})(H^\dagger H)$ and $\propto \mathrm{S} \nu_R^T C \nu_R$ which can play a role in the LG process. In particular, we compute the new contributions to the CP asymmetry factor arising from the new loop diagrams involving the scalar S and the $\nu_R$'s, as well as the modifications of the Boltzmann Equations for the evolution of the RHN number densities when taking into account the possibility for the scalar of decaying into two $\nu_R$'s in the early Universe.

2. A model like this is pretty general. $U(1)_{B-L}$ is the only $U(1)$ interaction, apart from $U(1)_Y$, which could be added to the SM ones since it's non-anomalous. So, our model can also be modified in this direction, considering instead of a global $U(1)_{B-L}$, a local one. Always working in this framework, if one fixes the number of right handed neutrinos to be $n_R = 3$, i.e. the same number of generations we have in the SM, model like the one proposed here can be embedded into unified theories. In fact, some of these models, like $SO(10)$ GUTs [10] or Left-Right symmetric models [11][12], use gauged symmetries like $U(1)_{B-L}$ at some stage of their symmetry breaking pattern and their scalar sectors contain more than one complex singlet under the SM interactions.

   Moreover, many studies in the dark matter direction consider the global $U(1)_{B-L}$, not gauged, in extensions of the SM. These are the so-called dark matter Majoron models: starting from the consequences of spontaneous symmetry breaking in terms of particle spectrum, i.e. the existence of massless Goldstone bosons, one add a small explicit symmetry breaking term in the Lagrangian to give a small mass to the Goldstone boson, i.e. the Majoron. In this way, and under the right conditions, the Majoron could become a viable dark matter candidate [13] [14] [15].

Similar analyses to the one in this work have been done: in [16] the authors studied the influence of adding a real scalar to the standard seesaw type I picture, considering in the Lagrangian also direct mass terms for the RHNs; in [17] the authors looked at the modifications in the CP asymmetry factor in a very general case when an arbitrary number of RHNs, complex scalars and Higgs doublets are added to the theory; in [18] a $U(1)_{B-L}$ model has been studied but without looking at the CP asymmetry factor and focusing on the influence on the Boltzmann equations by $2 \to 2$ scattering $\nu_R \nu_R$SS in the thermal bath, i.e considering a different region for the parameter space of the singlet scalar; in [19] they considered the influence of the decay of a scalar into RHNs in the early Universe, but in the different context of GeV sterile neutrinos and freeze-in LG.

This thesis is organised as follows. In Chapter 1 we give a brief review of the Standard Model of particle physics mainly taken by [20]: we start from introducing its fundamental symmetries and its particle content (1.1) and then we pass to more advanced topics like spontaneous symmetry breaking (1.2)(1.3), the CKM matrix and its CP violation (1.4), anomalies (1.5) and open problems of the Standard Model (1.6). Then Chapter 2 focuses on neutrino masses and it is mainly based on [21]. We start by reviewing the concepts of leptonic mixing (2.1) and neutrino oscillations theory and measurements (2.2). Then, we address the problem of the nature of neutrino masses from a more theoretical point of view (2.3), introducing the seesaw mechanism in some of its different realizations (2.4). In Chapter 3 we introduce the problem of the baryon asymmetry of the Universe and we explain why we need physics beyond the Standard Model to explain it. Then we give a look into Leptogenesis and its basic ingredients like the CP asymmetry factor, Boltzmann Equations and sphalerons. Chapter 3 is mainly based on the LG reviews by [22], [8] and [23]. Chapter 4 is the one where we build our model and find the results. We first write the whole Lagrangian and study the symmetry breaking pattern of the theory, looking for the scalar mass eigenstates. We also find the mass basis for the right handed neutrinos $\nu_R$'s and finally look at the new contributions to the CP asymmetry given by the new interactions involving the scalar singlet S. Then, we study how the presence of the new scalar particle in the theory can influence thermal LG in certain region of the parameter space: we first compute the analytical contributions to the Boltzmann Equations due to the scalar decay into RHNs; then we look at their solutions for particular choices of masses and couplings, to see what happens in particular to the prediction for the BAU.

# Chapter 1

# The Standard Model of Particle Physics

The SM of particle physics is the theory which describes three of the four known fundamental forces - the electromagnetic, weak and strong ones, with the exclusion of gravity. It is based on the gauge symmetry group

$$\mathrm{G}_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{1.1}$$

In this kind of theories, i.e gauge theories, we identify interactions with gauge groups: the strong interactions is described by the $SU(3)_C$ gauge group, the weak interaction by $SU(2)_L$ one and the hypercharge interaction by $U(1)_Y$. Not all of these symmetries are manifest in the world we live, some of them are hidden. We will come back on this in Sec. (1.2). The gauge principle carries two crucial features:

i. **The existence of gauge bosons.** Gauge bosons are the mediators of the interactions, i.e. the messengers which "carry" the information about the interaction from a point in spacetime to another avoiding the action at a distance which would violate the principles of special relativity.

ii. **Renormalizability.** Renormalizability is a property of quantum field theories which allows us to reabsorb infinities coming out from loop integrals into the so-called "bare parameters" in the Lagrangian, i.e parameters which are not directly observable in experiments, and thus not physical. Renormalization gives us the possibility of expressing observables as function of other observables (or of the same observable measured at a different energy scale), i.e the possibility of making predictions out of measurements, predictions which can be then tested in experiments.

In this Chapter we are going to give a review of the SM mainly based on [20] and [24]. We will look first at its structure in terms of symmetries and particle content, arriving to the various SM pieces of the Lagrangian in Sec. (1.1). Then the important feature of spontaneous symmetry breaking (SSB), characteristic of field theories, is introduced first for the case of a global $U(1)$ symmetry breaking in Sec. (1.2) and then looking to its proper application in the SM, i.e the electroweak spontaneous symmetry breaking (EWSSB), in Sec. (1.3). An introduction to CP violation and anomalies in the Standard Model is given Sec. (1.4, 1.5) especially considering the CKM matrix and quark mixing and the $B$ and $L$ anomalies respectively. Finally, we present a fast review of theoretical and experimental problems of the SM in Sec. (1.6).

## 1.1 The particle content of the Standard Model

Differently from what happens in standard quantum field theory (QFT), where we have no distinctions between interactions and matter because they are simply viewed as different representations of the Lorentz group, in the SM we distinguish between them. Matter are represented by the fermionic fields, the so-called fermions, i.e spin-$\frac{1}{2}$ particles; interactions are represented by the bosons fields, or simply bosons, spin-0,1 particles. Charges can be assigned to particles which are related to their transformation under the gauge group actions: the charge related to $SU(3)_C$ is called color, the one related to $SU(2)_L$ is the weak isospin and that related to $U(1)_Y$ is called the hypercharge $Y$. Now we understand the meaning of the subscripts in Eq. (1.1): $C$ stands for color, $Y$ for hypercharge while $L$ stands for left, because of the nature of weak interaction.
Weak isospin and hypercharge can be related to the better known electric charge Q thanks to the relation

$$Q = T_3 + \frac{Y}{2} = \frac{\sigma_3 + Y}{2}, \tag{1.2}$$

where $T_3 = \sigma_3/2$ is the third generator of the $SU(2)$ group and $\sigma_i$ is the $i$-th Pauli matrix.

In 1956 the Wu experiment carried out by the physicist Chien-Shiung Wu showed that parity [1] was not conserved by the weak interactions, it was actually maximally violated by it. Parity was believed to be a fundamental symmetry of nature since it was already established that the electromagnetic and strong interactions preserved it. But the story was different for the weak interaction. After this discovery, as pointed out by Steven Weinberg in [25], Marshak and Sudarshan came out in 1957 with the idea that the right way to describe the weak current, and the weak interaction, was through a vector minus an axial (V-A) current [26]. Therefore, nowadays, since weak interaction only couples to left-handed particles and it is described by the $SU(2)_L$ group, we describe the particle content of the theory using different representations for particles with different chiralities.

[1] Parity is the transformation which takes the spatial coordinates $x_i \to -x_i$. Being parity invariant for a system, or a theory, means that the system, or the equation of motion of the theory, is invariant under spatial invertion.

**Leptons** Leptons in the SM come into three generations, or families: left chirality, i.e left-handed, leptons like $(\nu_i, i)_L^T$, with $i = e, \mu, \tau$ are put in the fundamental representation of $SU(2)_L$, $SU(2)_L$ doublets, while right-handed ones like $e_R$, $\mu_R$ and $\tau_R$ are put into the singlet representation of the same group.[2] The group structure of the theory implies also that all the particles in the same doublet have the same value of the hypercharge Y and color. The hypercharge of the lepton doublets is $Y_L = -1$ while for the right handed leptons $Y_R = -2$. All leptons are $SU(3)_C$ singlets.

As we can see already at this point, right handed neutrinos $\nu_R$'s are missing. This is because they would have $Q = 0$, and they would not be charged both under weak nor strong interactions, thus being completely decoupled from the SM interactions. We should be able to see them only through a mass term in the Lagrangian. But adding mass terms in a chiral theory is not a trivial issue.

**Quarks** Quarks have both a weak and an electromagnetic charge. They also carry color and they come into the same number of generations as leptons. We can divide them into $SU(2)_L$ doublets

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

and singlets,

$$\begin{matrix} u_R & d_R & c_R \\ s_R & t_R & b_R. \end{matrix}$$

Differently from the lepton case, since quarks have an electric charge, all the doublets components have their right-handed counterpart. What distinguishes quarks from leptons is that they interact strongly, i.e they carry color; all the quarks, both left- and right-handed are in fact in the fundamental representation of $SU(3)_C$, i.e $SU(3)_C$ triplets.

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian is made of some different terms.

The electroweak part for one generation of fermions is given by

$$\begin{aligned} \mathcal{L}_{EW,fermions} =& \bar{e}_R i\gamma^\mu D_\mu^R e_R + \bar{L} i\gamma^\mu D_\mu^L L + \bar{Q} i\gamma^\mu D_\mu^L Q + \\ &+ \bar{u}_R i\gamma^\mu D_\mu^R u_R + \bar{u}_R i\gamma^\mu D_\mu^R u_R + \text{h.c.}, \end{aligned} \tag{1.3}$$

where $D_\mu^i$ is the covariant derivative needed for gauge invariance and it's where the interactions with the gauge bosons come from and we distinguished left (L) from (R)

[2]The index L or R stands for left-handed and right-handed respectively.

interactions since the weak force is chiral: $D^L_\mu = \partial_\mu + ig'\frac{Y_L}{2}a_\mu + ig\frac{\vec{b}_\mu \cdot \vec{\sigma}}{2}$ and $D^R_\mu = \partial_\mu + ig'\frac{Y_R}{2}a_\mu + 0$. $g'$ is the $U(1)_Y$ gauge coupling, $g$ is the $SU(2)_L$ gauge coupling, $Y_{L,R}$ is the hypercharge, $a_\mu$, $b^i_\mu$ are the gauge bosons respectively for $U(1)_Y$ and $SU(2)_L$.
The strong part of the Lagrangian involves only quarks and can be written as

$$\mathcal{L}_{Strong,quarks} = \bar{Q}_i i\gamma^\mu D^S_\mu Q_i, \tag{1.4}$$

where $Q_i = u, d, c, s, t, b$ (the strong force does not distinguish between left- and right-handed particles) and $D^S_\mu \equiv \partial_\mu + ig_s\frac{\vec{g}_\mu \cdot \vec{\lambda}}{2}$ is the covariant derivative associated to the $SU(3)_C$ strong interaction and $\lambda_i$ are the eight Gell-Mann matrices, i.e the infinitesimal generators of $SU(3)$.
The kinetic Lagrangian for the gauge bosons,

$$\mathcal{L}_{kin,g.bos.} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}F^i_{\mu\nu}F^{\mu\nu,i} - \frac{1}{4}G^i_{\mu\nu}G^{\mu\nu,i}, \tag{1.5}$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the field strenght tensor for the Abelian $U(1)_Y$ gauge bosons while $F^i_{\mu\nu}$ and $G^i_{\mu\nu}$ are the extension to the non-Abelian case defined by [24]

$$F^i_{\mu\nu} = \partial_\mu b^i_\nu - \partial_\nu b^i_\mu - g\epsilon^{ijk}b^k_\mu b^j_\nu, \tag{1.6}$$

$$G_{\mu\nu} = \partial_\mu g^i_\nu - \partial_\nu g^i_\mu - g_s f^{ijk} g^k_\mu g^j_\nu, \tag{1.7}$$

where $\epsilon^{ijk}$ and $f^{ijk}$ are the structure functions respectively for $SU(2)$ and $SU(3)$.

**Observations**

i. Even if $a_\mu$ and $b^i_\mu$ are gauge bosons, i.e force carriers, they are not the same as the photon $\gamma$ and the massive bosons $W^\pm$ and $Z^0$. For them, we need to ESSB to happen.

ii. We have the same coupling to the gauge bosons for all the different fermions, and this is typical of gauge theories. This is called universality and it's extended also to the self coupling between gauge bosons of non-Abelian gauge theories. Universality is a consequence and a quantitative prediction of the theory.

iii. The number of gauge bosons is strictly related to the dimension of the symmetry group, i.e the number of generators of the group, to which they are related. We can see it as another strong prediction of the theory.

iv. There are no mass term in the Lagrangian so far because they would violate gauge invariance.

## 1.2 Spontaneous Symmetry Breaking

Spontaneous symmetry breaking (SSB) happens when a stable state, e.g the vacuum of the system, transforms non-trivially, i.e it is not invariant, under certain symmetries of the theory. These symmetries are then said to be spontaneously broken, or hidden, and the vacuum state is called the broken state. The Lagrangian of the system is indeed invariant under the broken symmetries, it is only the ground state which is not. Spontaneous symmetry breaking is one of the most important concepts in quantum field theory; this is because SSB has different and interesting implications depending on the nature of the symmetry which is broken. All these consequences has to do with the Goldstone Theorem.

**The Goldstone's Theorem.** *Every continuous symmetry of the Lagrangian and broken by the vacuum implies the existence of a massless mode, i.e the existence of a mass eigenstate with zero eigenvalue.*

As an example we study the case of a $U(1)$ symmetry breaking, which will be needed later for the construction of our high energy theory invariant under a global $U(1)_{B-L}$.

### 1.2.1 The Case of $U(1)$ Symmetry Breaking

The Lagrangian of the $U(1)$ theory can be written as [20],

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + m^2 \phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2. \tag{1.8}$$

The symmetry is given by $\phi(x) \to e^{i\alpha}\phi(x)$ for constant $\alpha$. The potential is $V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$. To find the vacuum state of the theory we need to minimize the potential, i.e to study its first and second derivatives. In this case, if $m^2 > 0$, then the theory is unstable around $\phi = 0$ and the minimum condition is given by $|\phi|^2 = \frac{2m^2}{\lambda}$. This means that we have an infinite number of equivalent vacua $|\Omega_\theta\rangle$ parametrized by $\theta$ with $\langle\Omega_\theta|\,\phi\,|\Omega_\theta\rangle = \sqrt{\frac{2m^2}{\lambda}}e^{i\theta}$. The structure of the vacua reflects the $U(1)$ invariance. Usually one chooses $|\Omega\rangle$ such that $\langle\Omega_\theta|\,\phi\,|\Omega_\theta\rangle$ is real and equal to $v = \sqrt{\frac{2m^2}{\lambda}}$. Then the physical degrees of freedom are represented by the oscillations around the true vacuum $|\Omega\rangle$; to find them we parametrize the complex field $\phi(x)$ in terms of two real fields $\sigma(x)$ and $\pi(x)$, both with zero vacuum expectation value. So,

$$\phi(x) = \left(\sqrt{\frac{2m^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma(x)\right)e^{i\frac{\pi(x)}{F_\pi}}, \tag{1.9}$$

with $F_\pi$ a real number. The potential then depends only on $\sigma(x)$ and not on $\pi(x)$ and the Lagrangian becomes

$$\begin{aligned}\mathcal{L} =&\frac{1}{2}(\partial_\mu \sigma(x))^2 + \left(\sqrt{\frac{2m^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma(x)\right)^2 \frac{1}{F_\pi^2}(\partial_\mu \pi)^2 + \\ &- \left(-\frac{m^4}{\lambda} + m^2\sigma^2 + \frac{1}{2}\sqrt{\lambda} m \sigma^3 + \frac{1}{16}\lambda\sigma^4\right).\end{aligned} \tag{1.10}$$

$F_\pi$ is chosen in a way that the kinetic terms are canonically normalized. This Lagrangian (1.10) describes a massive mode $\sigma$ and a massless particle $\pi$, as expected for the Goldostone's Theorem, as pictured in Fig. (1.1).

It is important to note that the presence of the Goldstone Boson has nothing to do with how we parametrize $\phi(x)$: writing

$$\phi(x) = \frac{2m}{\sqrt{\lambda}} + \tilde{\phi}(x), \tag{1.11}$$

would have led to a mass matrix with a zero eigenvalue for one of the two components of $\tilde{\phi}(x)$.

In this model $\sigma$ has a mass of $m_\sigma = \sqrt{2}m$ and represents radial oscillations in the potential, while the Goldstone boson has a mass $m_\pi = 0$ and represents oscillations in the flat direction of the potential. Goldstone bosons are also associated to *shift symmetry*, which forbid them to acquire a mass and it's related to the invariance of the Lagrangian under

$$\pi(x) \to \pi(x) + F_\pi \theta. \tag{1.12}$$

So far we have only talked about spontaneously broken *global* symmetries. The same mechanism works also for *gauge symmetry* breaking, but with different features. In gauge theories spontaneous symmetry breaking is described by the *Brout-Englert-Higgs mechanism* (BEH mechanism), and the same mechanism which represents the EWSSB in the SM. What happens there is that the Goldstone bosons seem to have very similar properties to the gauge bosons of the theory. In fact, it turns out that Goldstone bosons in gauge theories are gauge-dependent and so we call them *pseudo-Goldstone bosons*. In particular there is a gauge choice, the so-called *unitary gauge*, where the pseudo-Goldstones completely disappear from the Lagrangian; they are absorbed by the gauge bosons and give a mass to it.

The *Brout-Englert-Higgs mechanism* is able to explain gauge bosons masses with spontaneous symmetry breaking and the Goldstone's Theorem: the massless pseudo-Goldstones are swallowed by the massless gauge bosons becoming their longitudinal

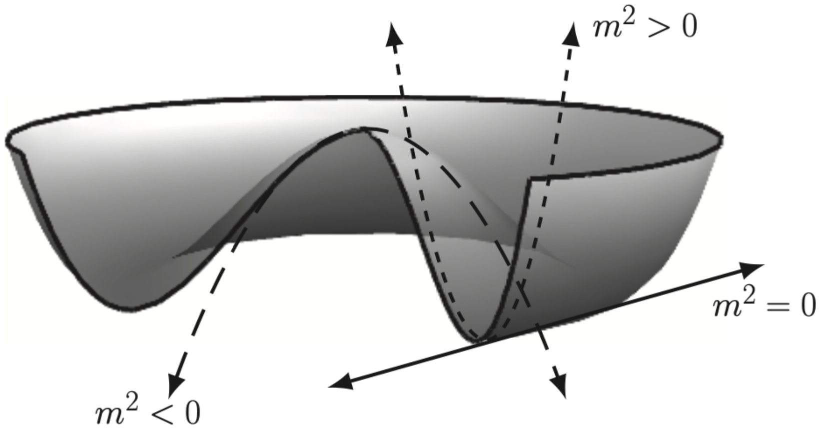



FIGURE 1.1: Mexican hat potential often present in spontaneously broken theory. Masses squared are related to second derivatives of the potential: oscillations around the origin represents two tachyonic (negative $m^2$) modes; radial oscillations around a minimum are described by a positive mass-squared mode while oscillations along the symmetry direction are related to the massless mode, i.e the Goldstone boson. The picture is taken from [20].

polarization, and giving them the mass. In fact, a massless spin-1 particle has two degrees of freedom, i.e two helicity states, while a massive spin-1 particle has three degrees of freedom: the pseudo-Goldstone is the third one.

## 1.3 Electroweak Symmetry Breaking and Masses in the Standard Model

At this point we can apply the notion of Sec. (1.2) to solve some of the problems left from (1.1).

We know that:

**Weak interaction is short-range.** This means that the related gauge bosons are massive. We have to find a way to give them a mass and we have to pay attention in particular to the fact that $SU(2)$ is a non-Abelian group and a direct mass term in the Lagrangian is violating gauge invariance.

**Leptons and quarks are massive.** We know from experimental evidence that fermions and quarks are massive, so we have to find a way to give them a mass; this can't be done adding terms like $m_e \bar{L}_e R_e$ in the Lagrangian since they would violate gauge invariance.

We need a mass generation mechanism for both bosons and leptons masses.

**Where is electromagnetism?** There is an other experimentally and theoretically well-known interaction, the electromagnetic one, which is based on the $U(1)_{EM}$ gauge group. It's a long-range interaction and so the associated vector bosons, i.e the photon $\gamma$ is massless. In the SM there seems to be no trace of electromagnetism. Can we find a way to embed $U(1)_{EM}$ in the SM?

In turns out that in the Standard Model we can solve all these issues using one single mechanism, the Brout-Englert-Higgs Mechanism of SSB. Weinberg, Salam and Glashow were able to apply this mechanism to the SM and showed that it was possible, with the help of *only* a scalar particle, called the Higgs doublet $H$, to go from the $SU(2)_L \times U(1)_Y$ symmetry at high energies, to the better known $U(1)_{EM}$ of electromagnetism. The last one is indeed a manifest symmetry of the low-energy world we live, while the others are hidden.

The astonishing fact is that, thanks to the symmetry breaking, also gauge bosons and fermions acquire their masses. What we need to do is to add in the Lagrangian all the possible terms compatible with the symmetries and including the scalar $SU(2)_L$ doublet $H = (H^+, H^0)^T$, where $H^+$ has an electromagnetic charge equals to $+1$ and $H^0$ is the neutral component.[3] The new pieces in the Lagrangian are

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger (D^\mu H) - V(H^\dagger H), \tag{1.13}$$

where $D_\mu H = \partial_\mu H + ig' \frac{Y_H}{2} a_\mu H + ig \frac{\vec{b}_\mu \cdot \vec{\sigma}}{2} H$, $Y_H = 1$ and the Higgs potential is given by

$$V(H^\dagger H) = m^2 H^\dagger H + \lambda (H^\dagger H)^2. \tag{1.14}$$

The symmetry breaking happens for $m^2 < 0$. With the Higgs doublet there are new invariant terms which can be added to our theory as

$$\mathcal{L}_{Yukawa} = -\mathrm{Y}_i^L (\bar{L}_i H) e_{R,i} - \mathrm{Y}_{ij}^D (\bar{Q}_i \tilde{H}) d_{R,j} - \mathrm{Y}_{ij}^U (\bar{Q}_i \tilde{H}) u_{R,j} + \text{h.c.}, \tag{1.15}$$

,for all the three generations of leptons and quarks: $i, j$ run over the generations and so $e_R$ stands for $e$,$\mu$ and $\tau$, while $u_R$ and $d_R$ stands in general for the up and low component of the doublet (not only for the up and down quark).
After the EWSSB, the Higgs field acquires a vacuum expectation value $\neq 0$ along its neutral component. It must be this way because $U(1)_{EM}$ remains unbroken and so to not break gauge invariance in the Lagrangian only neutral components under the preserved symmetries can acquire a vacuum expectation value $\neq 0$. We then $H_0 = \frac{1}{\sqrt{2}}(0, v)^T$ where

[3] Both these components are complex fieds with two real degrees of freedom, so we have four real degrees of freedom in the Higgs doublet.

the value for $v = \sqrt{\frac{-m^2}{\lambda}}$ is obtained minimizing the potential. In the unitary gauge we can write

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \tag{1.16}$$

with $h$ such that $h_0 = 0$. This new field $h$ is the degree of freedom describing the oscillations around the true vacuum at low energies. Inserting (1.16) in Eq. (1.13), Eq. (1.14), Eq. (1.15) we obtain the wanted mass terms for the fermions and for the gauge bosons.

Since we are breaking $SU(2)_L \times U(1)_Y \to U(1)_{EM}$, the Goldstone Theorem, when applied to gauge theories (1.2.1), predicts the existence of three pseudo-Goldstone bosons, which in our case, i.e working in the unitary gauge, are the three degrees of freedom swallowed by the massless gauge bosons to gain their mass. Three massive gauge bosons and a massless one come out after EWSSB from the covariant derivative in Eq. (1.13); they are respectively the $W^{\pm}$, $Z^0$ bosons and the photon $\gamma$, the last one corresponding to the unbroken $U(1)_{EM}$ gauge invariance. Fermion masses come from Eq. (1.15); since we have no $\nu_R$'s we cannot predict the masses of the neutrinos, on the contrary, we are predicting them to be massless. It is worth to point out that it's not trivial to be able to give mass to both the component of the $SU(2)_L$ doublets only through one Higgs field: this is peculiar in the SM and it's a feature of minimality related to the nature of the $SU(2)$ group, for which $\tilde{H} \equiv i\sigma_2 H^*$ transform as a doublet, but has opposite hypercharge $Y$ with respect to $H$ and can be used to give mass to the upper part of the $SU(2)_L$ doublets. Otherwise we whould have needed an additional Higgs doublet.
The predictions of the EWSSB are not finished here. We expect also the existence of a massive neutral scalar, a physical $h$ field, with a mass of $m_h^2 = 2\lambda v^2$. This particle is the one which has been discovered at CERN LHC in 2012 with a mass of $m_h = 125.35 \pm 0.15$ GeV [1][2]. Also the relations between the $SU(2)_L$ and $U(1)_Y$ bosons $b^i_\mu$ and $a_\mu$ with the physical $W^{\pm}_\mu$,$Z^0_\mu$ and $A_\mu (= \gamma)$ come out from the EWSSB. They can be written as,

$$W^+_\mu \equiv \frac{b^1_\mu - i b^2_\mu}{\sqrt{2}}, \tag{1.17}$$

$$W^-_\mu \equiv \frac{b^1_\mu + i b^2_\mu}{\sqrt{2}}, \tag{1.18}$$

$$Z^0_\mu \equiv \frac{-g' a_\mu + g b^3_\mu}{\sqrt{g'^2 + g^2}} = -\sin\theta_W a_\mu + \cos\theta_W b^3_\mu, \tag{1.19}$$

$$A_\mu \equiv \frac{g a_\mu + g' b^3_\mu}{\sqrt{g'^2 + g^2}} = \cos\theta_W a_\mu + \sin\theta_W b^3_\mu, \tag{1.20}$$

where $\cos\theta_W \equiv g/\sqrt{g'^2+g^2}$ and $\sin\theta_W \equiv g'/\sqrt{g'^2+g^2}$. The masses for the $W^{\pm}$ and $Z^0$ bosons are predicted to be

$$M_W = \frac{gv}{2}, \tag{1.21}$$

$$M_Z = \frac{v}{2}\sqrt{g^2+g'^2}. \tag{1.22}$$

This is another strong prediction of the SM of particle physics: even if they belong to the description of the same interaction, the weak interaction, the $W$ bosons have different mass with respect to the $Z$ boson, in agreement with the experimental observations.

## 1.4 The CKM matrix and CP violation in the SM

In the Standard Model it happens that the quark mass eigenstates, i.e the physical states, are not the ones which participate to the weak interaction [27]. The transformation between the two different basis of mass and weak eigenstates can be described with a unitary matrix, the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix. If we choose by convention the charge +2/3 quarks (u,c,t) to be the pure states, then the flavour mixing can be described by a $3\times3$ matrix operating on the states $d.s$ and $b$, i.e

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = V_{CKM} \begin{bmatrix} d \\ s \\ b \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}. \tag{1.23}$$

The states *d',s',b'* are the partners of $u, c, t$ in the weak isospin doublets.

The CKM model allow us to preserve universality of the weak coupling while explaining also the different transition rates inside a single quark family, e.g up and down, and those connecting different families, e.g up and strange. In the CKM matrix is also included the GIM mechanism which suppresses flavour-changing neutral-current (FCNC) processes, which are experimentally non-observed. The Lagrangian density term for charge current processes which involves quarks depends on the CKM matrix and can be written as

$$\mathcal{L}_{CC,CKM} \propto G_F[\bar{u}\ \bar{c}\ \bar{t}]\gamma^{\mu}(1-\gamma^5)V_{CKM}\begin{bmatrix} d \\ s \\ b \end{bmatrix} W_{\mu} + \text{h.c.} \tag{1.24}$$

The CKM matrix can account for CP violation (CPV) in the Standard Model, if it is complex regardless of the phase convention of the quark fermionic fields. Since it is a $3\times3$ matrix, it can be parametrized in terms of three Euler angles and one complex phase, which is responsible for weak CPV in the SM. Several parametrization are possible for this kind of matrices; we write here the "canonical" one adopted by the Particle Data

Group collaboration (PDG) [28] and proposed by Chau and Keung combining notations already used by Maiani and Wolfenstein,

$$V_{CKM} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}, \tag{1.25}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $\theta_{ij}$ being the mixing angle between the $i$th and the $j$th generation and $\delta_{13}$ is the phase for CPV. Using the matrices of the Yukawa couplings $Y_u$ and $Y_d$, which can be related to the diagonal mass matrices $M_u$ and $M_d$ by $Y_d = U_d M_d U_d^\dagger$ and $Y_u = U_u M_u U_u^\dagger$, CPV here can be encoded in the commutator [20]

$$-iC = [Y_u, Y_d] = \left[ U_u M_u U_u^\dagger, U_d M_d U_d^\dagger \right] = U_u[M_u, V M_d V^\dagger]. \tag{1.26}$$

The matrix $C$ is traceless and Hermitian, and its determinant is basis-invariant and given by

$$\det C = -\frac{16}{v^6}(m_t - m_c)(m_t - m_u)(m_c - m_u)(m_b - m_s)(m_b - m_d)(m_s - m_d)J, \tag{1.27}$$

where, for any $i, j, k$ and $l$

$$\mathrm{Im}\big\{(V_{ij}V_{kl}V_{il}^*V_{kj}^*)\big\} = J\sum_{m,n}\epsilon_{ikm}\epsilon_{jln}. \tag{1.28}$$

In terms of the "canonical" parameterization,

$$J = s_{12}s_{23}s_{31}c_{12}c_{23}c_{31}^2\sin\delta, \tag{1.29}$$

where $J$ is called the Jarlskog invariant and is fundamental because all weak CP violation in the SM is proportional to $\mathrm{Im}\{\det[Y_u, Y_d]\}$, i.e. the Jarlskog invariant vanishes if and only if there is no CP violation. From Eq. (1.25) is clear that if the CKM matrix is real then $J = 0$ and we have no CPV. But it is not the only case; there is no CPV if we have degenerate quark masses, as seen from Eq. (1.27): in fact, what happens in that case is that we get an extra free phase rotation and then the $\delta$-phase becomes non-physical and can be removed.

## 1.5 Global Anomalies in The Standard Model

Anomalies are symmetries of the classical theory which are not symmetries of the quantum theory based on the same Lagrangian. This can happen because the quantum theory is not only determined by the action $\mathcal{S}$ but also by the measure of the associated Path Integral (PI), and if a symmetry of the action is not a symmetry of the PI measure, then it is anomalous. From Noether's theorem [20] we know that continuous global symmetries imply conserved currents, and so for the same reasons if a symmetry is anomalous, then the associated Noether current will not be conserved. This can lead to problems with Ward identities in gauge theories where massless spin-1 particles are coupled to these currents and, as a consequence, to unitarity violation. So, to preserve unitarity, gauged symmetries must be anomaly free. This is not the case for global symmetries, i.e global anomalies don't imply theoretical inconsistencies. Fortunately, the Standard Model is free from gauge anomalies, but not from global ones. Examples of global anomalies in the SM are baryon and lepton numbers; these global anomalies turn out to be relevant for Leptogenesis.

### 1.5.1 B and L anomalies

Both baryon and lepton numbers are accidental symmetries of the Standard Model, i.e. global symmetry of the Lagrangian which are not "imposed" by the symmetry structure of the model itself. Baryon number is $B = \frac{1}{3}$ for quarks and $B = 0$ for leptons, while lepton number is $L = 0$ for quarks and $L = 1$ for leptons. For the respective currents we have that,

$$\partial^\mu J^B_\mu = \partial^\mu J^L_\mu = \frac{3g^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F^a_{\alpha\beta}, \tag{1.30}$$

where as in Eq. (1.5) $F^a_{\mu\nu}$ is the $SU(2)$ field strength. It is worth notice that the same Eq. (1.30) implies that $B - L$ is not anomalous, i.e. one cannot have a gauge boson associated to $B$ and $L$, while it is possible to associate one to $B - L$, which is what happens in Grand Unified Theories [11] [10]. What is the physical consequence of having $\partial_\mu J^B_\mu \neq 0$? It is the existence of instantons and sphalerons, which play an important role in the context of baryogenesis and/or leptogenesis. Sphalerons will be treated in Subsec. (3.3.7).

## 1.6 Problems of the Standard Model

Even if the Standard Model is one of the cornerstones of physics and one of the great triumphs of the $20^{th}$ century, being experimentally verified in many ways, e.g its predictions for the electron anomalous magnetic moment, or the precision measurements of the $W^\pm$ and $Z^0$ masses, there are some questions it's not able to answer [29] [30]:

- *Neutrino Masses:* As already said, in the Standard Model one cannot account for neutrino masses [21], since there are not right-handed neutrinos $\nu_R$'s in the theory and a mass term of the form $\propto \nu_L^T C^\dagger \nu_L$ would violate gauge invariance. But, as it will be explained in Chapter (2), we have experimental evidence of the fact that neutrinos have a mass, and we would like to find a theoretical explanation for that.

- *The Baryon Asymmetry of The Universe:* The Standard Model by itself is not able to explain the amount of asymmetry between the relic density of baryons and of antibaryons, also known as matter-antimatter asymmetry [31]. This is one of the "great mysteries in physics" and we are going to say more about it in Chapter (3).

- *Dark Matter:* We know that a significant fraction of matter in the Universe is non-baryonic [32]. The first strong indication for this came from rotational velocity of stars in galaxy [33][34]: evidences shows that the circular velocity flattens out at high distances from the galaxy centre, a different behavior from the expected one $v_c \propto 1/\sqrt{r}$. Since that, a huge amount of progress has been made on both the theoretical and experimental fronts in understanding this missing matter [35] [36] [14].

- *The Hierarchy Problem:* The hierarchy problem can be stated in different ways and related to the concept of *naturalness* in scientific thought [37] and has to do with renormalization in quantum field theories and the mass of the Higgs. Since the Higgs is a scalar, there is no symmetry protecting its mass as in the case of leptons with the presence of chiral symmetry, so what happens is that the mass of the Higgs boson receives quadratic (and not logarithmic) corrections from the heavy masses in the theory, e.g the top mass or heavier degrees of freedom added in UV completion or beyond Standard Model (BSM) models [38].

- *The Strong CP Problem:* The problem refers to the CP violation term in the QCD Lagrangian $\propto \theta G^{\mu\nu}\tilde{G}_{\mu\nu}$, where $\theta$ is a parameter while $G^{\mu\nu}$ is the QCD field strength. We have no explanation for the $\theta$-parameter being so small as measured, i.e $\theta < 10^{-10}$ [39]. There is a concrete proposal due to Peccei and Quinn fore the introduction of a new particle, the axion $a$, to solve this problem [40].

- *Quantum Gravity:* The Standard Model describes three of the four known fundamental interaction at the quantum level [29]. Gravity is only treated classically since it's very weakly coupled at the SM relevant energy, it can be considered as an effective (classical) field theory valid at scales smaller than the Planck scale $\left(M_{pl} = \sqrt{\frac{Gh}{c^3}} \simeq 10^{19}\, GeV\right)$.

# Chapter 2

# Neutrino Masses

In this Chapter, which is mainly based on [21], we are going to review first the concept of leptonic mixing Sec. (2.1) which leads directly to the phenomenology of neutrino oscillations in vacuum Sec. (2.2). Then we look to the problem of neutrino masses more deeply and in particular to one of its possible solutions which is the so-called seesaw mechanism Sec. (2.3) Sec. (2.4).

## 2.1 Leptonic Mixing

In this section we will show why the discovery of neutrino oscillations is a clear evidence that neutrinos have a mass. If we assume that neutrinos have masses, we can describe them using two different basis [1]: the flavour basis, the one related to the $SU(2)$ doublet and weak interaction where each neutrino is associated to the correspondent charged lepton, $\nu_\alpha$, with $\alpha = e, \mu, \tau$, and the mass basis, $\nu_i$, with $i = 1, 2, 3$, where each neutrino has a definite mass. The two basis are related by a unitary transformation, very similarly to what happens with the CKM matrix, which in this case is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$$\nu_{L,\alpha} = \sum_{i=1}^{3} U_{\alpha i} \nu_{L,i}, \tag{2.1}$$

The PMNS matrix enters in the charged-current (CC) Lagrangian, which can also be expressed in terms of the mass eigenstates as,

$$\mathcal{L}_{CC} \supset -\frac{g}{\sqrt{2}} \sum_{\alpha,i} \bar{\nu}_i U^*_{\alpha i} \gamma^\mu P_L l_\alpha W_\mu + \text{h.c.}, \tag{2.2}$$

where $P_L \equiv \frac{1}{2}(1-\gamma^5)$ is the left projector and we are in basis were the charged lepton $l_\alpha$ mass matrix is diagonal.

[1]We are also assuming that the two basis, i.e. the mass and the flavour ones, are different, as happen in the quark sector of the Standard Model.

Similar arguments to the ones for the CKM matrix holds for the PMNS one, but with an important difference: if neutrinos are Dirac particles, i.e. their mass term in the Lagrangian don't violate lepton number, as happens for the charged leptons, the physical degree of freedom of the PMNS matrix can be parameterized using three angles and one complex phase, but if neutrinos are Majorana particles, then the physical complex CPV phases become more than one. This will result in three physical phases, two of which only entering in lepton number violating processes, e.g. the double beta decay. As in the case of the quark mixing, CP violation can be parametrized by the Jarlskog invariant and it is related to the complex nature of the PMNS matrix. As before, we cannot have CPV with only two generations involved, and so Dirac CP violation becomes a genuine 3-neutrino mixing effect, whose physical impact depends on all the three mixing angles.

It can be shown that if neutrinos have a non-degenerate mass spectrum and there is leptonic mixing, then neutrino oscillations happens. This effect is a manifestation of quantum mechanics on macroscopic distances: in production and detection neutrinos are well described by flavour states since the interacting Hamiltonian is diagonal on the flavour basis. Every flavour state, as described by Sec. (2.1), is a coherent superposition of states with different masses[2]. Then every massive component of the initial state propagate with a different phase over long distances; this happens because during the propagation the mass states are the eigenstates of the Hamiltonian. This leads to the possibility that at the time of detection, when we have to project the flavour components out, a different flavour is found with respect to the one of production.

## 2.2 Neutrino Oscillations (in Vacuum)

The oscillation probability can be derived in different ways, we follow the one in [21] where plane-wave approximation is used. This approximation neglect the momentum uncertainty necessary for coherence and so we need to assume that the initial state is in a coherent superposition of massive states. A neutrino coming from a CC interaction can be described as $\nu_\alpha$, for some flavour $\alpha$, but it can also be described as

$$|\nu, t=0\rangle = |\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \,. \tag{2.3}$$

[2]We are assuming that we can have *coherence* in the superposition thanks to the uncertainty on the neutrino momentum at its production.

The propagation of the neutrinos is better described through the eigenstates of the free Hamiltonian, i.e $|\nu_i\rangle$, which has eigenvalues $E_i = \sqrt{\mathbf{p}^2 + m_i^2}$. We obtain

$$|\nu, t\rangle = e^{-i\hat{H}t} |\nu_\alpha\rangle = \sum_i U_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle . \tag{2.4}$$

The probability of having a transition $|\nu_\alpha\rangle \to |\nu_\beta\rangle$ can be obtained by

$$| \langle \nu_\beta | \nu, t\rangle |^2 = \left| \sum_i U_{\beta i} U_{\alpha i}^* e^{-iE_i t} \right|^2 \equiv P(\nu_\alpha \to \nu_\beta, t), \tag{2.5}$$

where we used that $\langle \nu_j | \nu_i \rangle = \delta_{ij}$. Since in all relevant situations neutrinos are highly relativistic, we can make the following approximations:

$$E_i - E_j \simeq \frac{m_i^2 - m_j^2}{2p}, \tag{2.6}$$

for commmon momentum $p$, and also $L \simeq t$. Considering also $E \simeq p$ and defining $\Delta m_{i1}^2 \equiv m_i^2 - m_1^2$ we can rewrite Eq. (2.5) as,

$$P(\nu_\alpha \to \nu_\beta, t) = | \langle \nu_\beta | \nu, t\rangle |^2 = \left| \sum_i U_{\beta i} U_{\alpha i}^* \exp\left\{ -i \frac{\Delta m_{i1}^2 t}{2E} \right\} \right|^2 . \tag{2.7}$$

It is evident from Eq. (2.7) that neutrino oscillations between different flavours are possible only if there is leptonic mixing, i.e. if the PMNS matrix is $\neq 1$, and if neutrinos have masses. This is why the discovery of neutrino oscillations is a strong evidence of the fact that neutrinos have non-zero masses and that the SM is not a complete theory. We can notice that neutrino oscillations conserve lepton number $L$ (but not lepton flavour number $L_e$, $L_\mu$ or $L_\tau$) and that the Majorana phases do not enter in the oscillation formula, as expected since they are related to lepton number violating processes. Neither the overall mass scale for neutrinos enters in the oscillation probability, only their squared mass differences. Neutrino oscillations parameters are currently under measure using atmospheric, accelerator, reactor and solar neutrinos; every experiment is sensible to some of the PMNS matrix parameters. Different experiments are needed to check consistency in measurements and to try to catch all the different parameters. From [41] we know that

- The $\Delta m_{21}^2$ mass splitting is known to be $> 0$ and with a value of $7.50(6.94-8.14) \times 10^{-5}$ eV$^2$ with a $3\sigma$ range.
- The $\Delta m_{31}^2$ is known less precisely and its sign is not established; we talk about normal (NO) or inverted (IO) ordering. For NO we have $\Delta m_{31}^2 = 2.55(2.47 -$

2.63) $\times 10^{-3}$ eV$^2$ while for IO $\Delta m^2_{32} = -2.45(2.37 - 2.53) \times 10^{-3}$ eV$^2$ with again considering a $3\sigma$ range.

For what concern the mixing angles we have:

- $\theta_{12} = 34.3(31.4 - 37.4)$ for both NO and IO,
- $\theta_{23} = 49.26(41.20 - 51.33)$ for NO and $\theta_{23} = 49.46(41.16 - 51.25)$ for IO,
- $\theta_{13} = 8.53(8.13 - 8.92)$ for NO and $\theta_{13} = 8.58(8.17 - 8.96)$ for IO,
- Hints for leptonic CP violation have been reported, in fact the measued value for the $\delta$ phase are $\delta = 194(128 - 359)$ for NO and $\delta = 194(128 - 359)$ for IO. No information on the Majorana phases is currently available.

Future neutrino oscillations experiments aim to a better understanding of neutrino mass ordering, CP violating processes and to the precise determination of the oscillation parameters. As we are going to show below, oscillation parameter, i.e low-energy parameter, can be related to high energy ones coming from BSM theories, e.g seesaw Lagrangian parameters.

## 2.3 Dirac or Majorana particles?

Neutrinos are neutral fermions, so they could be either Dirac or Majorana particles, as pointed out by E. Majorana in 1937 [7]. In the first case particles and antiparticles are different as in the case for electrons and positrons, in the second one there is no distinction between a particle and its own antiparticle. In the SM only neutrinos can be Majorana particles and this open an important window on the issue of lepton number conservation. Lepton number is an accidental symmetry in the Standard Model Lagrangian, it holds at a perturbative level but it's not known if it is a property of an ultimate theory of particles. Experiments are going on to put bounds on lepton number violating processes, the most sensitive of which is the so-called [42] *neutrinoless double beta decay* [43][44]. Understanding the nature of neutrinos is necessary since we want to add to the SM Lagrangian their mass term in the right way making a big step forward in the comprehension of Nature.

### 2.3.1 Dirac masses

If neutrinos are Dirac particles, to describe their masses we need to add to the theory new degrees of freedom, the SM singlets right-handed neutrinos $\nu_R$'s and a term in the Lagrangian of the kind

$$\mathcal{L}_{y,\nu} = \bar{L} Y_D \tilde{H} \nu_R + \text{h.c.}. \tag{2.8}$$

After EWSSB this term generates a Dirac mass for the light neutrinos, given by

$$\mathcal{L}_{Dirac,\nu} = \frac{v}{\sqrt{2}} \bar{\nu}_L Y_D \nu_R + \text{h.c.} \tag{2.9}$$

This Yukawa coupling and the resulting Dirac mass conserve lepton number and in general they are not diagonal. But it is worth notice that a Majorana mass term for $\nu_R$ would not violate any fundamental symmetry once added right handed neutrinos to the theory; not including this kind of term means that we are promoting lepton number $L$ from an accidental symmetry to a fundamental one. We can also give an estimate of the order of magnitude of the needed Yukawa couplings: if we suppose $m_v$ to be sub-eV, we get that $Y_D \simeq 10^{-12}$. This is a very small number and there is no explanation here for the strong hierarchy between lepton masses.

### 2.3.2 Majorana Masses and the Weinberg Operator

In principle neutrinos can also have a Majorana mass term without the need of adding the new particles to the theory. But we have to pay attention since $\nu_L^T C^\dagger \nu_L$ is not a gauge invariant term. We can use the fact that $\bar{L}\tilde{H}$ is gauge invariant and construct a singlet combination through,

$$\mathcal{L}_{Maj,EFF} = \frac{\lambda}{\Lambda} L^T \tilde{H}^* C^\dagger \tilde{H}^\dagger L + \text{h.c.}. \tag{2.10}$$

In this way we avoided to introduce new particles, but we came out with a dimension 5 operator, the so-called Weinberg operator, which can only be an effective operator, normalized by a mass scale $\Lambda$ in the denominator and suggesting the presence of new physics at that scale. The analogy here is with the Fermi theory as low-energy realisation of the $W$-mediated weak interaction. It is worth saying that the Weinberg operator is the only D=5 operator admitted by the Standard Model, while the other effective operators are higher dimensional. It also violates lepton number and after EWSSB leads to a mass term of the kind,

$$\mathcal{L}_{Maj,M} = \frac{\lambda v^2}{2\Lambda} \nu_L^T C^\dagger \nu_L + \text{h.c.} \tag{2.11}$$

But, what gives rise to the Weinberg operator at high energies?

## 2.4 The Seesaw Mechanism

That of the seesaw is a mechanism which can explain the existence of neutrino masses and also their smallness in terms of the presence and the heavy masses of new degrees of freedom, for example the RHNs $\nu_R$'s [45] [46]. This mechanism is naturally present in various extension and UV completion of the SM, e.g. unified theories, a fact that makes it very appealing from a theoretical point of view [47]. Its advantage is that large Yukawa couplings are allowed because the suppression of neutrino masses is due to the heavy mass of the RHNs. The seesaw mechanism can be realized in different ways, which can be classified in terms of the exchanged particle. The three main options are [21]:

- Type I seesaw for a singlet fermion $N_R$, left panel in Fig. (2.1),
- Type II seesaw using heavy triplet scalars $\Delta$, central panel in Fig. (2.1),
- Type III seesaw for triplet fermions $\Sigma_R$, right panel in Fig. (2.1).

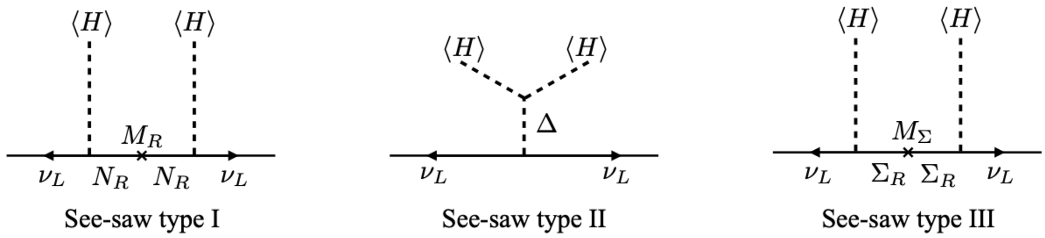



FIGURE 2.1: This picture, taken from [21], shows the diagrams contributing to light neutrino masses in the three seesaw realizations. $\langle H\rangle$ indicates the vev of the Higgs field neutral component, $\Delta$ is the scalar triplet and $\Sigma$ is the neutral component of a fermion triplet with mass $M_\Sigma$.

The problem with seesaw models is that having such heavy new particles is very difficult to test nowadays. And also from the theoretical point of view, if we do not have some mechanism which can stabilize the electroweak scale, like supersymmetry, then the new physics scale will induce quadratic corrections $\propto M_{NEW}^2$ to the Higgs mass [48], as we have seen in Sec. (1.6). Lowering the mass scale is possible, if we lower the Yukawa couplings; a lot of attention has been devoted to the TeV scale for new physics since it's the one accessible at LHC, where the presence of new particles, i.e. scalar and fermion triplet as well as sterile neutrinos, would leave some characteristic signature. Allowing for even small couplings lowers the scale further to GeV, MeV and even eV heavy neutrinos. These low energy seesaw realizations have interesting features, keV right-handed neutrinos can be a candidate for dark matter if they are stable on cosmological timescales [49], while GeV sterile neutrinos could be at the origin of the baryon asymmetry of the Universe via the LG via oscillations mechanism [50][51].

### 2.4.1 Type I Seesaw

The type I seesaw mechanism is the simplest extension of the Standard Model which can account for neutrino masses and also for their smallness. It is naturally predicted in different UV completion of the SM, as in $SO(10)$ GUT models or in left-right symmetric theories. It also has the added features that LG can be embedded in it to provide an explanation for the BAU, thanks to the L-, C- and CP-violating out-of-equilibrium RHNs decays. We have to introduce at least 2 sterile neutrinos $N_{R,j}$ with $j > 1$ to reproduce the two squared mass differences observed in neutrino oscillations experiments. The $N_{R,j}$ are fermions with no SM gauge numbers, i.e they are singlet of the SM. We choose to work in this section in the mass basis for right-handed neutrinos, i.e the basis where the mass matrix $M_N$ is diagonal, without loss of generality .

The most general Lagrangian which respects the Standard Model gauge symmetries is

$$\mathcal{L}_{\text{type-I}} = -\sum_{j,\alpha} \bar{L}_\alpha Y^D_{\alpha j} \tilde{H} N_{R,j} + \sum_j \frac{1}{2} N^T_{R,j} C^\dagger M_{N,j} N_{R,j} + \text{h.c.}, \tag{2.12}$$

where $y$ is a $3 \times j$, in general non diagonal, matrix. As we can see, this seesaw Lagrangian breaks lepton number by two units and predicts Majorana neutrinos. After EWSSB, the Yukawa term induces a Dirac mass term

$$m_D \equiv \frac{Y^D v}{\sqrt{2}} \tag{2.13}$$

and to find the mass eigenstates of the system we need to diagonalize a Lagrangian of the form

$$\mathcal{L}_{type-I,mass} = \frac{1}{2} \begin{bmatrix} (\nu^c_L)^T & N^T_R \end{bmatrix} C^\dagger \begin{bmatrix} 0 & m_D \\ m^T_D & M_N \end{bmatrix} \begin{bmatrix} \nu^c_L \\ N_R \end{bmatrix} + \text{h.c.} \tag{2.14}$$

which is of the Dirac+Majorana type. If we consider the limit of $m_D \ll M_N$, typical of the seesaw situation, then one set of the mass eigenstates will be given by nearly-sterile neutrinos, i.e the heavy neutrinos remain mainly in the sterile neutrino direction and have a mass $\simeq M_j$, the other set will be given by the active light neutrinos, which acquire a small mass of the order,

$$m_\nu \simeq -m_D \frac{1}{M_N} m^T_D. \tag{2.15}$$

We see from Eq. (2.15) that the larger $M_N$ the smaller are the neutrino masses (for fixed Yukawas); so the smallness of neutrino masses is due here to the large hierarchy between the two energy, or mass, scales. Notice that we have a mixing between the heavy neutrinos and the active ones, but since it's of the order $\sin^2\theta = m_\nu/M_N$. For instance, if we take the new scale $M_N$ to be the GUT scale, i.e $10^{12} - 10^{14}$ GeV, we are able to predict neutrino masses of order $m_\nu = 0.1 - 1$ eV.

### 2.4.2 Casas-Ibarra Parameterization

We just saw that with the type I seesaw mechanism we can explain the small masses of active neutrinos through the introduction of new heavy degrees of freedom, which introduce a new mass scale $M_N$ and which are coupled to the Standard Model Higgs and left-handed neutrinos through Yukawa couplings. These parameters are all related to the high-energy theory but they can be connected to the low-energy ones, i.e the ones appearing in neutrino oscillations: charged lepton masses, neutrino masses, the angles and the phases of the PMNS matrix.

Parameterizing the seesaw means finding a way to relate the high energy parameters to the low energy ones. It is worth noticing here that for 3 sterile neutrinos, a type I see-saw model presents 3 heavy masses and a non-diagonal complex Yukawa matrix with 9 real parameters and 6 phases [52]; of all these parameters we can in principle measure only 3 light neutrino masses, 3 mixing angles and 3 phases, if we also assume to be able to experimentally reach the Majorana phases. So we see that there are some "unknown" parameters, which turns out to be relevant for LG.
The seesaw can be parameterized in various way. One can adopt a "top-down parameterization", where the input parameters are chosen to be all the high-energy ones present in Eq. (2.12) and in such a way that they can reproduce the known neutrino oscillations data. Another way is to use a "bottom-up" point of view, starting from experimental data about neutrino masses and then finding out a proper form for the Yukawa matrix $\mathrm{Y}_\nu$ able to reproduce these data.
A very general approach to the "bottom-up" parameterization has been given in [53] by Casas and Ibarra and it goes under the name of Casas-Ibarra parameterization. To find it one starts from Eq. (2.15) and define the $\mathcal{K}$ matrix to extract the Higgs vev $v$ as

$$\mathcal{K} = m_\nu / v^2. \tag{2.16}$$

Both Eq. (2.15) and (2.16) are not defined at low energy but at the "Majorana scale", $M_N$ and so in order to compare them to the experiment values one has to run them down to low energies through RGE. Working in the basis where the charged lepton Yukawa matrix is diagonal (also gauge interactions are flavour-diagonal), we can diagonalize the $\mathcal{K}$ matrix by the PMNS matrix $U$,

$$U^T \mathcal{K} U = \mathrm{diag}(k_1, k_2, k_3) \equiv D_{\mathcal{K}}, \tag{2.17}$$

where $U$ is the unitary matrix which relates flavour to mass eigenstates, i.e.

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}. \tag{2.18}$$

If we choose $k_i \geq 0$, then $U$ can be written as

$$U = V \cdot \mathrm{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1), \tag{2.19}$$

where $\phi$ and $\phi'$ are CP violating phases and V has the ordinary form in Eq. (1.25). On the other hand, since RHNs are Majorana particles, we can always choose to diagonalize their mass matrix $M_N$ and to work in the eigenstates basis, i.e.

$$U_R^\dagger M_N U_R^* = D_N = \mathrm{diag}(M_1, M_2, M_3), \tag{2.20}$$

where $M_i \geq 0$. From Eqs. (2.16) and (2.17) we have

$$D_\mathcal{K} = U^T (Y^D)^T \frac{1}{\sqrt{D_N}} \frac{1}{\sqrt{D_N}} Y^D U, \tag{2.21}$$

or

$$\frac{1}{\sqrt{D_\mathcal{K}}} D_\mathcal{K} \frac{1}{\sqrt{D_\mathcal{K}}} = 1 = \frac{1}{\sqrt{D_\mathcal{K}}} U^T (Y^D)^T \frac{1}{\sqrt{D_N}} \frac{1}{\sqrt{D_N}} Y^D U \frac{1}{\sqrt{D_\mathcal{K}}}, \tag{2.22}$$

which can be rewritten as

$$1 = \left[ \frac{1}{\sqrt{D_N}} Y^D U \frac{1}{\sqrt{D_\mathcal{K}}} \right]^T \left[ \frac{1}{\sqrt{D_N}} Y^D U \frac{1}{\sqrt{D_\mathcal{K}}} \right], \tag{2.23}$$

whose solution can be written in terms of an orthogonal matrix $R$[3], i.e

$$R \equiv \frac{1}{\sqrt{D_N}} Y^D U \frac{1}{\sqrt{D_\mathcal{K}}}. \tag{2.24}$$

In order to reproduce the low energy parameters, i.e. the light neutrino masses, mixing angles and CPV phases, contained respectively in $D_\mathcal{K}$ and $U$, the most general Yukawa matrix $Y^D$ is given by [53]

$$Y^D = \sqrt{D_N} R \sqrt{D_\mathcal{K}} U^\dagger. \tag{2.25}$$

This parameterization, besides being very general, also collect the additional nine parameters present in the high energy theory and not imprinted in the low energy one in these two matrices: $D_N$ and $R$ (three unknown positive mass eigenvalues for the RHN and three complex parameters defining $R$). Anyway, it can be shown that in practical

[3] $R$ can be also complex provided that it satisfies $R^T R = 1$

cases the number of relevant free parameters can be drastically reduced.
Some particular choices for the $R$-matrix can be to put it directly to $R = 1$ or to select $Y^D$ such that it can be written as $Y^D = WD_Y$ with $W$ unitary and $D_Y$ diagonal.

### 2.4.3 Variants of the Seesaw

#### Triplet Scalars (Type II)

Neutrino masses can be generated also by tree level exchange of an $SU(2)$-triplet scalars [54]. These scalars must be color-singlet and carry hypercharge $Y_T = +2$. The minimal model is with a single triplet $T$ and the relevant part in the Lagrangian are given by

$$\mathcal{L}_{Type-II} = -M_T^2|T|^2 + \frac{1}{2}(y_{T,\alpha\beta}L_\alpha^T TL_\beta + M_T\lambda_H H^T i\sigma_2 T^\dagger H + \text{h.c.}), \tag{2.26}$$

where $M_T$ is a real mass parameter, $\lambda_L$ a symmetryc $3 \times 3$ matrix of dimensionless complex Yukawa couplings and $\lambda_H$ is the dimensionless complex coupling between the Higgs and the triplet. The contribution to neutrino masses from this model is

$$[m_{II}]_{\alpha\beta} = y_{\alpha\beta}^T \frac{\lambda_H v^2}{M_T} \tag{2.27}$$

Also this model involves lepton number violation because the co-existence of $y^T$ and $\lambda_H$ does not allow a consistent way of assigning a lepton charge to $T$. The complex nature of these two couplings can also provide a new source of CPV in the theory, leaving space for explanation for the BAU [55].

We can give a look to the degrees of freedom of this model: there are eleven parameters beyond those of the Standard Model, 8 real and 3 imaginary ones. As before in Subsec. (2.4.2), 9 of these can be determined from the light neutrino parameters, while the other 2 ($M_T$ and $|\lambda_H|$) are related to the full energy theory.

#### Triplet Fermions (Type III)

One can also generate neutrino masses by the tree level exchange of an $SU(2)$-triplet fermion $\Psi_i^a$[4], color-singlets and with hypercharge 0 [56] [57]. The relevant part of the Lagrangian takes the form

$$\mathcal{L}_{Type-III} = -\frac{1}{2}M_i\Psi_i^a\Psi_i^a + y_{T,\alpha k}\sigma_{\rho\lambda}^a L_\alpha^\rho H^\lambda \Psi_k^a + \text{h.c.} \tag{2.28}$$

Here, $M_i$ are real mass parameters and the $y_{T,\alpha k}$ are the complex Yukawa couplings.

[4] $i$ denotes a heavy mass eigenstates while $a$ is an $SU(2)$ index.

The mass contribution to the neutrino masses in this case is

$$[m_{III}]_{\alpha\beta} = y_{T,\alpha k}\frac{v^2}{M_k}y_{T,\beta k}. \tag{2.29}$$

This model violate again lepton number because the co-existence of $\lambda_T$ and $M_T$ does not allow a consistent way of assigning a lepton number to $\Psi_i$ and provide for new sources of CPV via the $y_T$ couplings.

As in the standard type I seesaw model, here we have 18 parameters beyond those of the Standard Model: 12 real and 6 imaginary.

# Chapter 3

# Why Leptogenesis?

That of the Baryon Asymmetry of the Universe (BAU) is a beautiful and very challenging mystery in theoretical physics because we do not have an unique and verified theory to explain it. In Sec. (3.1) we review its experimental evidences, in Sec. (3.2) why the SM by itself is not enough, and then we go deeply in the study of Leptogenesis, in particular defining and computing in detail the contribution to the CP asymmetry factor $\epsilon_{i\alpha}$, in Sec. (3.3.1), and the equations needed to study the lepton asymmetry evolution during the history of the Universe, the Boltzmann equations for LG, in Sec. (3.3.6).

## 3.1 The Baryon Asymmetry of the Universe

To explain the baryon asymmetry of the Universe we can argue either that the asymmetry was already there as an initial condition or that it has been generated dinamically during the evolution of the Universe. There are some reasons to believe that the baryon asymmetry of the Universe must have been generated dinamilcally in the Early Universe. For example, if a baryon asymmetry were an initial condition, it must have been a highly fine-tuned one, i.e. for every $\sim 2 \cdot 10^9$ antibaryons we needed $\sim 2 \cdot 10^9 + 1$ baryons. Also, if we believe that inflation is the reason why we have a CMB radiation with such an uniform temperature, then we also have to recognize that any primordial baryon asymmetry would have been exponentially diluted away by inflation. Two questions can arise at this point: how much baryon asymmetry do we observe in our Universe? Are we able to explain it with our current knowledge or is new physics required?

The baryon asymmetry of the Universe can be defined in two equivalent ways:

$$\eta \equiv \left. \frac{n_B - n_{\bar{B}}}{n_\gamma} \right|_0 = (6.21 \pm 0.16) \times 10^{-10}, \tag{3.1}$$

$$Y_{\Delta B} \equiv \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.75 \pm 0.23) \times 10^{-11}, \tag{3.2}$$

where $n_B$, $n_{\bar{B}}$, $n_\gamma$, $s$ are the number densities of baryons, antibaryons, photons and entropy, the subscript 0 stands for "at present time", while the numerical value comes from combined CMB and large scale structure data (WMAP 5 year data, Baryon Acoustic Oscillations and Type Ia Supernovae). Since the entropy density is given by

$$s = g_*(2\pi^2/45)T^3, \tag{3.3}$$

where $g_*$ is the number of degrees of freedom in the plasma and $T$ is its temperature, and is conserved during the expansion of the Universe, it is convenient to work with $Y_{\Delta B}$. Eq. (3.1) and Eq. (3.2) differ only by a numerical factor, i.e. $Y_{\Delta B} = (n_{\gamma 0}/s_0)\eta \simeq \eta/7.04$. A third way to express the asymmetry is in terms of the baryonic fraction of the critical energy density,

$$\Omega_B \equiv \rho_B/\rho_{crit}, \tag{3.4}$$

where $\rho$ are energy densities and the relation with $\eta$ can be written in terms of the present Hubble parameter $h \equiv H_0/100$ km $s^{-1}$ Mpc$^{-1}$ $= 0.701 \pm 0.013$.

$$\eta = 2.74 \times 10^{-8}\Omega_B h^2. \tag{3.5}$$

The value of the baryon asymmetry of the Universe can be inferred from observations in two independent ways. The first way is from big bang nucleosynthesis (BBN), because deuterium $D$ and $^3He$ abundances are very sensitive to $\eta$ and can be inferred by various observations. It turns out that there is a range for $\eta$ which is consistent with all the abundances of these four light elements, which is (at 95% CL) [58]

$$4.7 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}, \tag{3.6}$$

or

$$0.017 \leq \Omega_B h^2 \leq 0.024. \tag{3.7}$$

The second way to determine $\Omega_B$ is from looking at the CMB anisotropies [59] [60]. To a very good approximation, the CMB spectrum is that of a blackbody radiation with a constant temperature $T$. So, the most interesting observables become the temperature fluctuations $\boldsymbol{\Theta}(\hat{n}) \equiv \Delta T/T$, where $\hat{n}$ represents the direction in the sky. One can study the main features of the CMB applying fluid mechanics to the cosmological plasma and treating it as a perfect photon-baryon fluid, neglecting dynamical effects due to gravity and the baryons. If $c_s$ is the speed of sound in the baryon-free fluid, $\rho$ the photon energy density and $p$ the photon pressure, then it holds

$$\ddot{\boldsymbol{\Theta}} + c_s^2 k^2 \boldsymbol{\Theta} = 0, \tag{3.8}$$

where for photons $c_s \equiv \sqrt{\dot{p}/\dot{\rho}} = \sqrt{1/3}$. In the anisotropy spectrum the relevant features are: the existence of peaks and troughs, the location of the first peak and the spacing between adjacent peaks. To study it we need to include baryons and gravity correction in our analysis, since the physical effect of baryons will be to provide extra gravity and to enhance the compressional phase. This will translate into enhancement of the odd peaks in the spectrum and so we can measure the odd/even peak disparity to constrain the baryon energy density. The modifications from baryons and gravity can be understood from the addition of their effects to Eq. (3.8),

$$\ddot{\mathbf{\Theta}} + c_s^2 k^2 \mathbf{\Theta} = F, \tag{3.9}$$

where $F$ is the forcing term due to gravity and now $c_s = 1/\sqrt{3(1+3\rho_B/4\rho_\gamma)}$. Assuming a ΛCDM model with a scale-free power spectrum for the primordial density fluctuations, the fit to the observations from WMAP5 gives (at $2\sigma$) [61]

$$0.02149 \leq \Omega_B h^2 \leq 0.02397. \tag{3.10}$$

The crucial time for primordial nucleosynthesis (BBN) is when the thermal bath temperature drops below $T \simeq 1$ MeV, while the crucial time for CMB is that of recombination, i.e. when the temperature fell below $T \simeq 1$ eV and neutral hydrogen can be formed. The non-trivial fact that we have consistency between these two independent predictions is a triumph of the hot big-bang cosmology.

## 3.2 Can the SM Explain the BAU?

We would like to have a theory which can explain such a value for $\Omega_B$. As being pointed out by Sakharov in [62], to generate dynamically a baryon asymmetry we need to satisfy three conditions:

I. Baryon number violation: This condition is required if we want to go from a symmetric Universe, i.e. with $Y_{\Delta B} = 0$ to a non-symmetric state with $Y_{\Delta B} \neq 0$.

II. C and CP violation: If either C or CP are conserved, then processes which involves baryons would have the same rate as their C- or CP-conjugate ones involving antibaryons. This will result in no net baryon asymmetry generation.

III. Out of equilibrium dynamics: Equilibrium distribution functions $n_{eq}$ are determined only by the particle energy E, its chemical potential $\mu$ and its mass. The mass is the same for particles and antiparticles thanks to the CPT theorem; when charges are not conserved, e.g the baryon number (B), the corresponding chemical potentials vanish, leading to $n_B = \int \frac{d^3p}{(2\pi)^3} n_{eq} = n_{\bar{B}}$. Equilibrium forces us to have the same number of baryons and antibaryons.

These three ingredients are all present in the SM but what happens is that it's not able to reproduce quantitavely the observed BAU.

I. B is violated in the SM by the triangle anomaly. The triangle anomaly leads to processes that involve nine left-handed quarks (three of each generation) and three left-handed leptons (one from each generation), with the selection rule $\Delta B = \Delta L = \pm 3$, which implies that the sphalerons do not mediate proton decay. It has been showed in [63] that, if at zero temperature the rate for this kind of processes is highly suppressed, at a sufficiently high temperatures it can exceed the expansion rate of the Universe, becoming non-negligble and possibly giving origin to the BAU if also the Weinberg-Salam phase transition (electroweak phase transition) is strongly first order.

II. The weak interactions of the SM violates C maximally and trace of CPV can be find in the Kobayashi-Maskawa mechanism. This CPV can be parametrized and quantified by the Jarlskog invariant and turns out to be of order $10^{-20}$. Since there are also no kinematic enhancements in the thermal bath [64][65], it is not possible to generate the baryon asymmetry observed in the Universe, i.e $Y_{\Delta B} \simeq 10^{-10}$ with such small CPV. We need a new source of CPV, beyond the Kobayashi-Maskawa phase, if we want to account for baryogenesis.

III. Departure from thermal equilibrium in the Standard Model happens with the EWSSB [66] [67]. We need a strong first order phase transition for succesful baryogenesis, but lower bounds on the Higgs mass imply the electroweak phase transition (EWPT) is not enough. This is why we need a different kind of departure from thermal equilibrium in the early Universe, both due to new BSM physics or to a modification of EWPT itself.

Baryogenesis requires new BSM physics. We need to extend the SM accounting for new sources of CPV and a different departure from thermal equilibrium either through new symmetry breaking processes, e.g $U(1)_L$, $U(1)_{B-L}$, L-R breaking or through a modification of the EWPT.
There are various models and mechanisms for baryogenesis, some of them are:

**GUT baryogenesis** [68] [69] [70]. It tries to explain the baryon asymmetry in the out-of-equilibrium decays of heavy bosons in Grand Unified Theories (GUTs). These heavy bosons come from the gauge nature of Unified Theories and their pattern of symmetry breakings. GUT baryogenesis has some problems with the non-observation of proton decay, which puts a lower bound on the decaying boson mass and as a consequence on the reheat temperature after inflation. Not only, since the heavy bosons generate in their decays a B+L asymmetry, and since the SM sphalerons, which violates B+L, are in equilibrium at high temperatures they can destroy the asymmetry present. GUT leptogenesis can be a solution [31][71].

**Electroweak baryogenesis** [72] [73]. It tries to explain the BAU using the EWPT as departure from equilibrium. Since we know that in the SM the EWPT is not enough first order for having succesful baryogenesis, we have to modify in some way the scalar potential to change the nature of the symmetry breaking (also we need to provide new sources of CPV). Examples of this are the 2HDM (two Higgs doublets model), where the Higgs potential has more parameters and violates CP or the MSSM (minimal supersymmetric SM), where the presence of a light stop, i.e. the supersymmetric partner of the top, modifies the Higgs potential in the required way and there are new CP violating phases [74]. Attention must be paid for example in the case of MSSM where LHC constraints on the stop mass as well as those on EDM essentially rule out this scenario [75][76].

**Leptogenesis.** It was invented by Fukugita and Yanagida in [77]. New particles - singlet RHNs - are introduced to explain neutrino masses via the seesaw mechanism. Their Yukawa couplings can provide the necessary new source of CPV. Departure from thermal equilibrium occurs if the Yukawa interactions are slow enough, i.e. $\Gamma \leq H$ when $T \simeq M$ of the RHNs. L-violation comes naturally from the Majorana nature of the new particles and the Standard Model sphalerons can convert it into a baryon asymmetry.

## 3.3 Leptogenesis

LG by itself can be realized in different ways.

**Thermal LG** is the scenario where the lepton asymmetry comes from heavy Majorana neutrinos in the thermal bath. Usually two different pictures are considered: the RHNs are produced by scatterings in the thermal bath starting from a vanishing initial abundance, i.e $Y_{N_i}(0) = 0$; they are already at equilibrium with the thermal bath at the time when LG begins, i.e $Y_{N_i}(0) = Y_{N_i}^{eq}$. Then the evolution of the RHNs densities can be determined only as a function of the seesaw parameters and of the reheat temperature of the Universe. One can then consider some limits for the RHN masses, e.g the **hierarchical singlet neutrinos**, where $M_1 \ll M_2 \ll M_3$ or the opposit limit of non-hierarchical RHNs, arriving to the so-called **Resonant Leptogenesis** scenario [78], for which the heavy Majorana neutrinos are quasi-degenerate, i.e $M_2 - M_1 \simeq \Gamma_{D,2}/2$. In this situation there are resonances in the CP asymmetry factor which can take it to $\mathcal{O}(1)$ allowing us to lower the scale of LG.

**Supersymmetric (SUSY) LG** is the study of LG in the case of supersymmetric models. SUSY can give origin to new mechanism, e.g Affleck-Dine LG or soft LG. From the point of view of SUSY Thermal LG there are only some small qualitative and quantitative differences, e.g we have to consider also the sneutrino $\tilde{N}_1$ decay, there are twice the number of states running in the loops, and so on. In the soft LG scenario [79] [80] some small SUSY breaking terms are added in the Lagrangian which provide additional sources of lepton number and CP violation.

One can consider **Non-Thermal Leptogenesis**, where the RHNs (or only some of the RHNs generation) are assumed to be produced in the early Universe but not at equilibrium with the thermal bath, e.g thanks to the inflaton decay [81], or in preheating [82]. In the SUSY scenario the singlet sneutrino $\tilde{N}$ could be the inflaton, producing a lepton asymmetry in its decay [83].

**LG via oscillations** is the idea proposed by Akhmedov, Rubakov and Smirnov (ARS) that if the RHNs are at the electroweak scale (or below) then CPV effects leading to LG can be induced by the coherent superposition of different RHNs mass eigenstates.

Depending on the temperature at which LG occurs one can consider also flavour effects and solve the matrix density equations instead of working in the one-flavour regime approximation. In this thesis we will work in the framework of thermal LG with hierarchical singlet neutrinos and we will mainly consider the case with vanishing initial abundance for the $N_i$ number densities. Also, we are going to consider the one-flavour regime.

A potential drawback of thermal LG with hierarchical masses $M_i$'s is the lower bound on $M_1$ [84] given in Subsec. (3.3.5), which makes the model difficult to test and which gives a lower bound on the reheat temperature [85][86].

### 3.3.1 CP Asymmetry

We consider the CP asymmetry for the $N_i$ decay in lepton flavour $\alpha$ and in zero temerature field theory, since thermal corrections can be neglected in first approximation. It can be defined in terms of the right-handed neutrinos width as, [22]

$$\epsilon_{i\alpha} \equiv \frac{\Gamma(N_i \to L_\alpha H) - \Gamma(N_i \to \bar{L}_\alpha \bar{H})}{\sum_\alpha \Gamma(N_i \to L_\alpha H) + \Gamma(N_i \to \bar{L}_\alpha \bar{H})}. \tag{3.11}$$

It turns out that at tree level we cannot have CP violation: this can be shown using unitarity but also simply looking at the decay rates. In fact, the tree level decay $\Gamma(N_i \to L_\alpha H)$ can be written in terms of [87]

$$\mathcal{M}(N_i \to L_\alpha H) = \mathrm{Y}^*_{\alpha i} \bar{u}_\alpha P_R u_i, \tag{3.12}$$

where $\mathrm{Y}_{\alpha i}$ is the Yukawa coupling of the seesaw Lagrangian. The CP conjugate process, i.e $\Gamma(N_i \to \bar{L}_\alpha H^\dagger)$ will be related to

$$\mathcal{M}(N_i \to \bar{L}_\alpha H^\dagger) = -\mathrm{Y}_{\alpha i} \bar{u}_\alpha P_L u_i. \tag{3.13}$$

The two amplitudes in Eqs. (3.12) and (3.13) differs only by the Yukawa coupling, i.e $\mathrm{Y}^*_{\alpha i}$ vs $\mathrm{Y}_{\alpha i}$, and by the projectors, i.e $P_L$ vs $P_R$. However, these differences give the same contribution after squaring the amplitude and properly taking the usual sum/average (trace) over internal states, i.e $1/2 \sum_{spins} |\mathcal{M}|^2$. Since decay widths are proportional to this factor, we obtain $\Gamma(N_i \to L_\alpha H) = \Gamma(N_i \to \bar{L}_\alpha H^\dagger)$. We need to look for CPV in the contribution given by the loop diagrams.

### 3.3.2 Implications of Unitarity in CP Violation in Decays

We can use unitarity and CPT invariance to show where the CP asymmetry come from and to obtain constraints on it. Simply starting from the unitarity of the S-matrix, written as $S = 1 + iT$, we can write

$$1 = S^\dagger S = (1 - iT^\dagger)(1 + iT) = 1 - iT^\dagger + iT + T^\dagger T, \tag{3.14}$$

where $T$ is the transition matrix, defined as $\langle f| T |i\rangle \equiv (2\pi)^4 \delta^4(p_i - p_f) \mathcal{M}(i \to f)$ [20] . From Eq. (3.14)

$$T^\dagger T = i(T^\dagger - T) \tag{3.15}$$

which, when sandwiched between the initial and final states $|i\rangle$, $|f\rangle$, i.e considering the matrix elements, gives

$$\langle f| T^\dagger T |i\rangle = \langle f| i(T^\dagger - T) |i\rangle . \tag{3.16}$$

Since we are dealing with Hilbert spaces, it holds that

$$\langle f| T^\dagger |i\rangle = \langle i| T |f\rangle^* , \tag{3.17}$$

and so, from Eq. (3.16)

$$iT^*_{if} - iT_{fi} = \left[T^\dagger T\right]_{fi} \tag{3.18}$$

Taking then modulus square of $iT_{fi} = iT^*_{if} - \left[T^\dagger T\right]_{fi}$ and considering that $T_{fi}$, $T^*_{if}$ and $\left[T^\dagger T\right]_{fi}$ are complex numbers with a real and an imaginary part, e.g $T_{fi} = T^R_{fi} + iT^I_{fi}$, we can write

$$|T_{fi}|^2 = \left|iT^{*R}_{if} - T^{*I}_{if} - [T^\dagger T]^R_{fi} - i[T^\dagger T]^I_{fi}\right|^2, \tag{3.19}$$

Performing some algebra and collecting some terms,

$$|T_{fi}|^2 = \left|(-T^{*I}_{if} - \left[T^\dagger T\right]^R_{fi}) + i(T^{*R}_{if} - \left[T^\dagger T\right]^I_{fi})\right|^2, \tag{3.20}$$

$$|T_{fi}|^2 = (T^{*I}_{if})^2 + (\left[T^\dagger T\right]^R_{fi})^2 + 2T^{*I}_{if}\left[T^\dagger T\right]^R_{fi} +$$
$$+ (T^{*R}_{if})^2 + (\left[T^\dagger T\right]^I_{fi})^2 - 2T^{*R}_{if}\left[T^\dagger T\right]^I_{fi}$$

we obtain

$$|T_{fi}|^2 = |T_{if}|^2 + \left|\left[T^\dagger T\right]_{fi}\right|^2 + 2T^{*I}_{if}\left[T^\dagger T\right]^R_{fi} - 2T^{*R}_{if}\left[T^\dagger T\right]^I_{fi}, \tag{3.21}$$

Considering then that $(T^{*R}_{fi}) = (T^R_{fi})$ and $(T^{*I}_{fi}) = -(T^I_{fi})$ we can rewrite the relation above as

$$|T_{fi}|^2 - |T_{if}|^2 = \left|\left[T^\dagger T\right]_{fi}\right|^2 - 2Im\left\{\left[T^\dagger T\right]_{fi} T_{if}\right\}, \tag{3.22}$$

How is this result related with CP asymmetry factor? We need to use CPT invariance to show it: from CPT we have that $|\mathcal{M}(f \to i)|^2 = \left|\mathcal{M}(\bar{i} \to \bar{f})\right|^2$, which in the case of a Majorana state $|i\rangle$ becomes

$$|\mathcal{M}(f \to i)|^2 = \left|\mathcal{M}(i \to \bar{f})\right|^2.$$

And so, since

$$|T_{fi}|^2 \simeq |\mathcal{M}(i \to f)|^2,$$

and

$$|T_{if}|^2 \simeq |\mathcal{M}(f \to i)|^2 \simeq \left|\mathcal{M}(\bar{i} \to \bar{f})\right|^2 \simeq \left|\mathcal{M}(i \to \bar{f})\right|^2.$$

The relation

$$|T_{fi}|^2 - |T_{if}|^2 = |T_{fi}|^2 - \left|T_{\bar{f}i}\right|^2 = \left|\left[T^\dagger T\right]_{fi}\right|^2 - 2Im\left\{\left[T^\dagger T\right]_{fi} T_{if}\right\}, \tag{3.23}$$

tells us that the CP violation in a process like an $N_i$ decay can first arise in the loop corrections: the first non-zero contribution is given by the term $-2Im\left\{\left[T^\dagger T\right]_{fi} T_{if}\right\}$, while the $\left|\left[T^\dagger T\right]_{fi}\right|^2$ is higher order in the couplings. We can also write

$$\left[T^\dagger T\right]_{fi} = \sum_k T^\dagger_{fk} T_{ki} = \sum_k \langle f| T^\dagger |k\rangle \langle k| T |i\rangle = \sum_k \langle k| T |f\rangle^* \langle k| T |i\rangle. \tag{3.24}$$

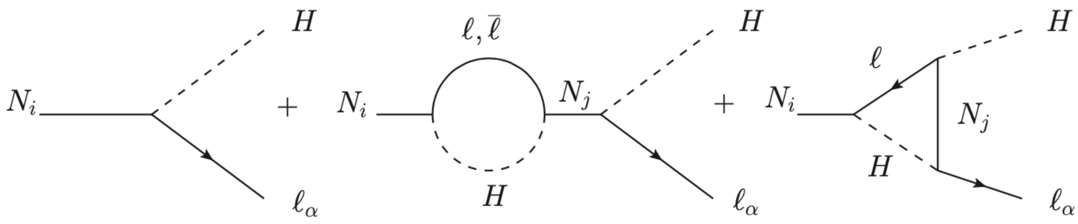



FIGURE 3.1: In this figure, taken from [8], the diagrams contributing to the CP asymmetry factor $\epsilon_{i\alpha}$ are shown. On the left we have the tree-level amplitude, in the centre the wave diagram contributions and on the right the vertex contribution.

This important result, known under the name of Cutkosky Rules, or Optical Theorem, shows that the loop amplitude in Eq. (3.23) has an imaginary part when there are branch cuts corresponding to intermediate on-shell particles.

### 3.3.3 Standard Contribution to the CP asymmetry

We want now study the CP asymmetry factor in standard Thermal Leptogenesis. This computation was first carried out in [88] and [89]. We can first rewrite Eq. (3.11) in terms of Eq. (3.23): writing the decay width $\Gamma(N_i \to l_\alpha \phi, \bar{l}_\alpha \phi^\dagger)$ in terms of the amplitude $|\mathcal{M}|^2$ and the phase-space integration factors $d\Pi_{l\phi,\bar{l}\bar{\phi}}$, we obtain

$$\epsilon_{i\alpha} = \frac{\int d\Pi_{l\phi}\tilde{\delta}|\mathcal{M}|^2 - \int d\Pi_{\bar{l}\bar{\phi}}\tilde{\delta}|\bar{\mathcal{M}}|^2}{\sum_\alpha \int d\Pi_{l\phi}\tilde{\delta}|\mathcal{M}|^2 + \int d\Pi_{\bar{l}\bar{\phi}}\tilde{\delta}|\bar{\mathcal{M}}|^2}, \tag{3.25}$$

and considering that

$$d\Pi_{l\phi} = \frac{d^3p_l d^3p_\phi}{2E_l 2E_\phi} = \frac{d^3p_{\bar{l}} d^3p_{\bar{\phi}}}{2E_{\bar{l}} 2E_{\bar{\phi}}} = d\Pi_{\bar{l}\bar{\phi}}, \tag{3.26}$$

the relation between $\mathcal{M}$ and $T$ given in (3.3.2), the CP asymmetry factor becomes,

$$\epsilon_{i\alpha} = \frac{\int d\Pi_{l_\alpha\phi}(2\pi)^4\delta^4(P_i - P_f)\Big(-2Im\Big\{\left[T^\dagger T\right]_{fi} T_{if}\Big\}\Big)}{2\sum_\alpha \int d\Pi_{l_\alpha\phi}(2\pi)^4\delta^4(P_i - P_f)|\mathcal{M}(i \to f)|^2}. \tag{3.27}$$

In the one flavour regime, one has to sum over the flavour index $\alpha$ in Eq. (3.27),

$$\epsilon_i \equiv \sum_\alpha \epsilon_{i\alpha}. \tag{3.28}$$

The denominator is the common part of all the different contribution, shown in Fig. (3.1), to the asymmetry factor $\epsilon_{i\alpha}$. The factor of 2 in front comes from the fact that at tree level the decay rates are the same both in the $N_i$ decay and in its CP conjugate one. The details are shown in Append. (B.1). The decay rate for the right handed neutrinos is

$$\Gamma_{N_i} = \frac{(\mathrm{Y}^\dagger \mathrm{Y})_{ii} M_i}{8\pi} \tag{3.29}$$

while the denominator of Eq. (3.27) is given by

$$\frac{1}{8\pi} \sum_\alpha |\mathrm{Y}_{\alpha i}|^2 M_i^2. \tag{3.30}$$

We can define here a useful dimensionful parameter which represents the decay rate $\Gamma_D$ and will be used later; it has to do with the washout scenario of the LG process, and in the literature is usually referred to as

$$\tilde{m} \equiv \sum \tilde{m}_{\alpha\alpha} \equiv \sum_\alpha \frac{\mathrm{Y}^*_{\alpha 1} \mathrm{Y}_{\alpha 1} v^2}{M_1} = \frac{(\mathrm{Y}^\dagger \mathrm{Y})_{ii} v^2}{M_i} = 8\pi \frac{v^2}{M_1^2} \Gamma_D. \tag{3.31}$$

The contribution to the CP asymmetry in the numerator of Eq. (3.27) are given by the interference between the tree-level diagram and the one loop diagrams in Fig. (3.1): the two diagrams in the centre (one with $l$ and the other with $\bar{l}$) will give the wave function corrections to the CP asymmetry while the one the left the vertex correction. The details of the computations for this contributions can be found in Append. (B.1.1), (B.1.2) and (B.1.3). After having defined $z \equiv M_j^2/M_i^2$, what we obtain is

$$\epsilon_{i\alpha}^{wave-1} = \frac{1}{8\pi} \frac{1}{(\mathrm{Y}^\dagger \mathrm{Y})_{ii}} \sum_{j \neq i} \mathrm{Im}\Big\{ \big(\mathrm{Y}^\dagger \mathrm{Y}\big)_{ji} \mathrm{Y}_{\alpha i} \mathrm{Y}^*_{\alpha j} \Big\} \frac{\sqrt{z}}{1-z} \tag{3.32}$$

for the wave contribution with the antilepton $\bar{l}$ running in the loop;

$$\epsilon_{i\alpha}^{wave-2} = \frac{1}{8\pi} \frac{1}{(\mathrm{Y}^\dagger \mathrm{Y})_{ii}} \sum_{j \neq i} \mathrm{Im}\Big\{ \big(\mathrm{Y}^\dagger \mathrm{Y}\big)_{ij} \mathrm{Y}_{\alpha i} \mathrm{Y}^*_{\alpha j} \Big\} \frac{1}{1-z} \tag{3.33}$$

for the wave contribution with the lepton $l$ running in the loop;

$$\epsilon_{i\alpha}^{vertex} = \frac{1}{8\pi} \frac{1}{\big(\mathrm{Y}^\dagger \mathrm{Y}\big)_{ii}} \mathrm{Im}\Big\{ \big(Y^\dagger Y\big)_{ji} \mathrm{Y}^\star_{\alpha j} \mathrm{Y}_{\alpha i} \Big\} \sqrt{z} \left[ 1 - (1+z) \ln\left(1 + \frac{1}{z}\right)\right], \tag{3.34}$$

for the vertex diagram contribution.

So, summing all Eqs. (3.32), (3.33) and (3.34) we obtain the total contribution

$$\begin{aligned}\epsilon_{i\alpha} =& \frac{1}{8\pi}\frac{1}{(\mathrm{Y}^\dagger\mathrm{Y})_{ii}}\sum_{j\neq i}\mathrm{Im}\Big\{\left(\mathrm{Y}^\dagger\mathrm{Y}\right)_{ij}\mathrm{Y}_{\alpha i}\mathrm{Y}^*_{\alpha j}\Big\}\frac{1}{1-z}+\\ &+\frac{1}{8\pi}\frac{1}{(\mathrm{Y}^\dagger\mathrm{Y})_{ii}}\mathrm{Im}\Big\{\left(Y^\dagger Y\right)_{ji}\mathrm{Y}^\star_{\alpha j}\mathrm{Y}_{\alpha i}\Big\}g(z),\end{aligned} \tag{3.35}$$

where $g(z)$ is a loop function defined as

$$g(z) = \sqrt{z}\left[\frac{1}{1-z} + 1 - (1+z)\ln\left(1+\frac{1}{z}\right)\right]. \tag{3.36}$$

From (3.35) we can notice two different things:

I. Resonances can appear from the terms of the form $(1-z)^{-1} \propto (M_i^2 - M_j^2)^{-1}$ if $M_i \sim M_j$, this is the case of the so-called resonant leptogenesis, where a strong enhancement in the value of $\epsilon_{i\alpha}$ can lead to lowering significantly the RHN mass scale.[78] There is also a singularity in the resonant term for $M_i = M_j$. Anyway this singularity can be regulated by using for example effective field theory approaches based on resummation [90].

II. At least two right handed neutrinos are needed, otherwise the combination of Yukawa couplings in (3.35) becomes real and the CP asymmetry vanishes.

In this work we are mainly going to use the one flavour regime approximation, where the asymmetry factor becomes,

$$\epsilon_i \equiv \sum_\alpha \epsilon_{i\alpha} = \frac{1}{8\pi}\frac{1}{(\mathrm{Y}^\dagger\mathrm{Y})_{ii}}\sum_{j\neq i}\mathrm{Im}\Big\{\left(Y^\dagger Y\right)^2_{ji}\Big\}g(z). \tag{3.37}$$

### 3.3.4 CP Violation in Scatterings

CP violation also occurs in $2 \to 2$ scattering processes, the leading ones are those involving the top quark, and with an Higgs exchange in $s$- or $t$-channel. Scattering can also involve the gauge bosons, with processes like $NL \to \bar{H}A$ or $NH \to \bar{l}A$ [91] . The relevant part of the Lagrangian for these scattering in the mass basis of the $N_i$'s, the charged leptons and the quarks is,

$$\mathcal{L}_{scatt.} = -y_{i\alpha}\bar{N}_i\tilde{H}^\dagger L_\alpha - y_t\bar{Q}\tilde{H}t + \text{h.c.}. \tag{3.38}$$

As in the case of the CP asymmetry from the $N_i$ decay, also with scatterings it arises from the interference between the tree level amplitude and the one-loop ones, as shown in Fig. (3.2).

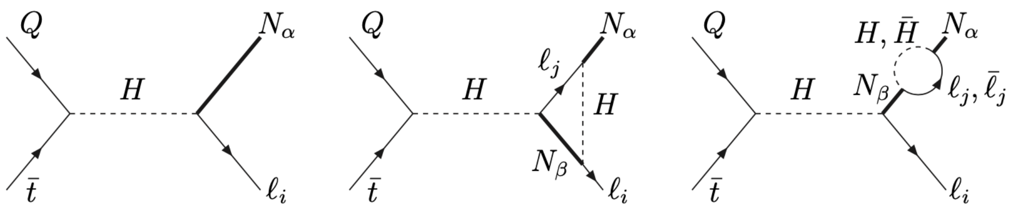



FIGURE 3.2: s-channel diagrams $Q\bar{t} \longleftrightarrow N_\alpha l_i$ scatterings which contributes to the CP asymmetry. The Higgs can also be exchanged in a t-channel process $QN_\alpha \longleftrightarrow tl_i$. The picture is taken from [91].

What one finds out is that this source of CPV gives an important contribution to the generation of a lepton asymmetry only at high temperatures and that the approximation of the CP asymmetry factor from scattering being the same of the one from the RHNs decay is good in the case with hierarchical Majorana neutrino masses. What happens is that, for example, in a strong washout regime, where the final lepton asymmetry is almost independent from early times conditions, the final results are essentially unaffected by the inclusion of new sources of CPV, while in the weak washout the effects of the scatterings CP violation is to reduce the final asymmetry.

### 3.3.5 The Davidson-Ibarra Bound

If we assume a hierarchical spectrum for the right handed neutrinos and that the dominant contribution to the lepton asymmetry comes from the $N_1$ decay, i.e. the lightest RHN [8], then

$$\epsilon_1 \equiv \sum_\alpha \epsilon_{1\alpha} \simeq \frac{3}{16\pi}\frac{1}{(\mathrm{Y}^\dagger\mathrm{Y})_{11}}\sum_{j\neq 1}\mathrm{Im}\big\{(\mathrm{Y}^\dagger\mathrm{Y})^2_{j1}\big\}\frac{M_1}{M_j}, \tag{3.39}$$

and, working with three generations of RHNs and the Casas-Ibarra parameterization Eq. (2.25) for the Yukawas, Eq. (3.39) becomes,

$$\epsilon_1 \simeq \frac{3}{16\pi}\frac{M_1}{v^2}\frac{\sum_i m_{\nu_i}\,\mathrm{Im}\{R^2_{1i}\}}{\sum_i m_{\nu_i}|R_{1i}|^2}. \tag{3.40}$$

Using the orthogonality condition $\sum_i R^2_{1i} = 1$ we obtain the Davidson-Ibarra (DI) bound [84]

$$|\epsilon_1| \leq \epsilon^{DI} = \frac{3}{16\pi}\frac{M_1}{v^2}(m_{\nu_3} - m_{\nu_1}) = \frac{3}{16\pi}\frac{M_1}{v^2}\frac{\Delta m^2_{atm}}{m_{\nu_1} + m_{\nu_3}} \tag{3.41}$$

where $m_{\nu_1}$ ($m_{\nu_3}$) is the lightest (heaviest) light neutrino mass. Considering $Y_{\Delta B}(\infty) \geq Y_B^{CMB} \simeq 10^{-10}$ we can obtain bounds on $M_1$ and $m_{\nu_1}$; for succesful LG under these assumptions we need $M_1 \geq 10^9$ GeV for $m_{\nu_1} \leq 0.1$ eV which in turn implies a reheating temperature after inflation $T_{RH} \geq 10^9$ GeV. It is worth to say that the violation of one of the assumption allows us to lower someway the RHNs scale.

### 3.3.6 Boltzmann Equations for Leptogenesis

If we want to study thermal LG we need also to use thermodynamics in the early Universe [92]. In fact, the evolution of the hot plasma in time happens while mantaining thermal equilibrium, but the cosmologically important events focus on those epochs where some species decouple from the thermal bath, e.g BBN or Recombination. Qualitatively, the decoupling happens when the interaction rate of the particle species and the expansion rate of the Universe are of the same order, i.e $\Gamma \sim H$, but for making predictions we need a more quantitative approach which is given by the Boltzmann Equation, a very useful tool to describe the out-of-equilibrium dynamics of the species in the thermal bath. The Boltzmann Equation can be written in terms of operators as

$$\hat{L}[f] = \hat{C}[f], \tag{3.42}$$

where $f$ is thermal distribution function which can be of the Fermi-Dirac type or pf the Maxwell-Boltzmann one, $\hat{L}[f]$ is the Liouville operator and $\hat{C}[f]$ is the collision operator, and both operators applies to $f$.
The Liouville operator is defined as

$$\hat{L} = p^{\mu}\frac{\partial}{\partial x^{\mu}} - \Gamma^{\mu}_{\nu\rho}p^{\nu}p^{\rho}\frac{\partial}{\partial p^{\mu}}, \tag{3.43}$$

where $x^{\mu}$ is the 4-vector of the spatial coordinates, $p^{\mu}$ is the 4-momentum and $\Gamma^{\mu}_{\nu\rho}$ are the Christoffel symbols defined in Append. (A.1). The collision operator $\hat{C}[f]$ describes instead the scattering amplitude considering also the thermal distribution function of the particles. The Liouville operator depends instead on the metric used for describing the physical situation; in this context we are considering the Robertson-Walker metric, defined in Append. (A.2). If also the distribution function $f$ only depends on the energy and the time, i.e $f = f(E, t)$ the Boltzmann Equation becomes,

$$\hat{L}[f(E,t)] = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\vec{p}|^2\frac{\partial f}{\partial E}. \tag{3.44}$$

The expansion of the Universe can be described in this context by the scale factor $a = a(t)$, or using the Hubble parameter defined as,

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}, \tag{3.45}$$

which, as a function of the temperature $T$ of the thermal bath becomes

$$H(T) = \sqrt{\frac{8\pi G}{3}\rho_r} = \sqrt{\frac{8\pi G}{3}}\sqrt{\frac{\pi g_\star}{30}}T^2 \simeq 1.66\sqrt{g_\star}\frac{T^2}{M_{PL}}, \tag{3.46}$$

or in terms of $z = M_1/T$

$$H(z) \simeq 1.66\sqrt{g_\star}\frac{M_1^2}{M_{PL}}1/z^2. \tag{3.47}$$

It's often more useful working with the particle yelds $Y_i$, which are related to the particle number density $n_i$ and the entropy density of the Universe $s$ through

$$Y_i = \frac{n_i}{s}. \tag{3.48}$$

Writing also the collision operator $\hat{C}[f(E,t)]$ as

$$\hat{C}[f(E,t)] = \gamma_{ab...}^{ij...}\left(\frac{n_a n_b ...}{n_a^{eq} n_b^{eq} ...} - \frac{n_i n_j ...}{n_i^{eq} n_j^{eq} ...}\right), \tag{3.49}$$

where $\gamma_{ab...}^{ij...}$ is the thermal cross-section defined in Append. (E.1), we can rewrite the Boltzmann Equation in Eq. (3.42) for the case of a process $(i + j \to a + b)$ where the particles $a$ and $b$ are in equilibrium with the thermal bath, i.e $n_a = n_a^{eq}$ and $n_b = n_b^{eq}$, as

$$\frac{dY_i}{dz} = -D_i(Y_i^2 - (Y_i^{eq,})^2), \tag{3.50}$$

where $D_i$ is some factor which depends on $z$, $H(M_i)$ and the thermal cross section $\gamma_{ab}^{ij}$.

With thermal LG what happens is that we have as initial condition for our Universe some abundance of RHNs, which can be $Y_{N_i}(0) = 0$ or $Y_{N_i}(0) = Y_{N_i}^{eq}$, which is then modified in time thanks to the interacting terms in the Lagrangian with the leptons and the Higgs and can be studied with Boltzmann equations. At the same time, since processes involving RHNs $N_i$'s (decay and scattering) are CP violating, they create a lepton asymmetry yields $\equiv Y_{\Delta L}$ which can again be tracked with Boltzmann Equations. In LG, one usually works directly with the $\Delta_\alpha \equiv \frac{B}{3} - L_\alpha$, since the sphalerons conserve $B - L$.
We always have to solve a system of coupled Boltzmann equations, but we can use different approximations depending on the case of study.[8] For example, if LG occurs at $T \geq 10^{12}$ $GeV$, then the charged lepton Yukawa interactions are out of equilibrium, i.e we are not able to distinguish flavours, and this defines the one flavour regime. We can also reduce the number of equations in the system if we assume that the reheating temperature after inflation is such that $N_i$ with $i > 1$ are not produced [93] and then only the dynamics of $N_1$ is relevant. Then the evolution of the $N_1$ density and lepton asymmetry can be described by the following (classical) BEs

$$\frac{dY_{N_1}}{dz} = -D_1(Y_{N_1} - Y_{N_1}^{eq}), \tag{3.51}$$

$$\frac{dY_{\Delta_\alpha}}{dz} = \epsilon_1 D_1 (Y_{N_1} - Y_{N_1}^{eq}) - W_1 Y_{\Delta_\alpha}, \tag{3.52}$$

where $z \equiv M_1/T$ as always,

$$D_1(z) = \frac{\gamma_{N_1} z}{sH(M_1)} = K_1 z \frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)}, \tag{3.53}$$

is the decay term which depends on the n-th order modified Bessel function of second kind $\mathcal{K}_n(z)$ and on the Hubble parameter evaluated at the masses of $N_1$, i.e

$$H(M_1) \equiv H_1 = z^2 H(z) \simeq 1.66 \sqrt{g_\star} \frac{M_1^2}{M_{PL}}. \tag{3.54}$$

The Boltzmann Equation (3.51) depend also on the adimensional parameter [94]

$$K_1 \equiv \frac{\tilde{m}_1}{m^*}, \tag{3.55}$$

which determines the washout regime (weak or strong). The parameter $m^*$ is the analogous of $\tilde{m}$ with the Hubble parameter and it's defined as

$$m^* \equiv 8\pi \frac{v^2}{M_1^2} H(T = M_1) \simeq 1.1 \times 10^{-3} \text{eV}. \tag{3.56}$$

As we can see, the equation for the lepton asymmetry, Eq. (3.52) depends on two different kind of contributions, which acts as opponents: there is a term which tends to increase the lepton asymmetry and it's proportional to the $N_1$ thermal decay width, with the CP asymmetry factor $\epsilon_1 \equiv \sum_\alpha \epsilon_{1\alpha}$ as proportionality constant, and an other term which tends to erase the asymmetry created given by

$$W_1(z) = \frac{1}{2} D_1(z) \frac{Y_{N_1}^{eq}(z)}{Y_l^{eq}}. \tag{3.57}$$

The parameter defined in Eq. (3.55) helps to distinguish two different regimes:

**Strong washout regime.** In this regime, defined by $K_1 \gg 1$, at $T \sim M_1$ the $N_1$ number density is at thermal equilibrium and the total lepton asymmetry is $Y_L \sim 0$ since any asymmetry created has been washed out thanks to the equilibrium condition for the decay and its inverse process. Only when the temperature drops and the inverse $N_1$ decay gets out of equilibrium, i.e their rate $\Gamma_{ID} < H$, the asymmetry created through the decay survives contributing to $Y_{\Delta B}$.

**Weak washout regime.** In this scenario, the the total decay rate, i.e $\Gamma(N_1 \to L_\alpha H)$, is small when compared to $H(M_1)$. As a consequence the $N_1$ number density does not reach its equilibrium distribution during the evolution of the Universe

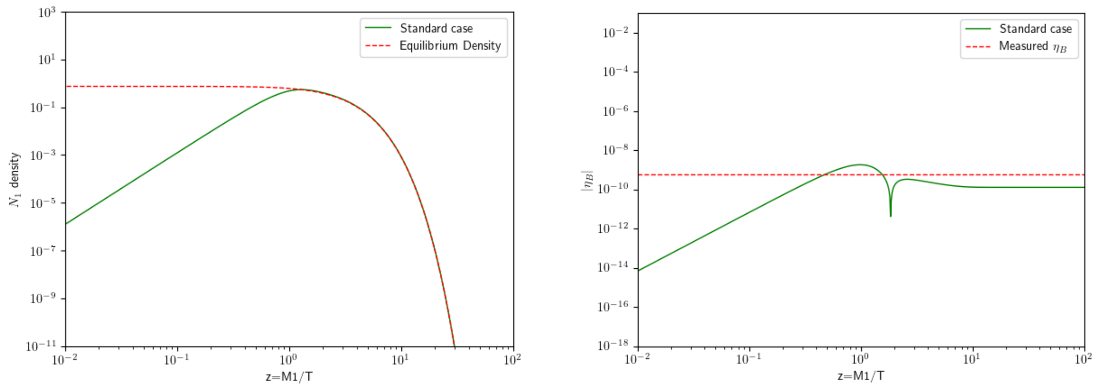



FIGURE 3.3: Left and right panels shows respectively the lightest RHN number density and the absolute value of the lepton asymmetry, as a function of $z = M_1/T$, in the case of vanishing initial RHN density. Here $M_1 = 10^{14}$ GeV and the washout factor is $K_1 = 10$.

but at the same time any lepton asymmetry which is created for $z < 1$ will not be erased, differently from what happens in the strong washout case. An example of the weak washout scenario is shown in Fig. (3.3).

Fig. (3.3) shows the $N_1$ number density (left panel) and the absolute value of the baryon asymmetry (right panel) both as a function of $z = M_1/T$ in the case of vanishing initial abundance, i.e. $Y_{N_1}(0) = 0$. As one can see from the left panel, in this situation the RHN number density slowly increases with time thanks to its interactions with the SM particles, reaching its equilibrium distribution. Then, if we look to the right panel, which represents the BAU, we can notice the presence of a dip for $z \simeq 1$: it represents a change of sign in the baryon asymmetry. This behaviour is peculiar of the vanishing initial abundance case, i.e. the case when $Y_{N_1}(0) = 0$, an it is due to the fact that, for $z \ll 1$, is the inverse $N_1$ decay which populates the thermal bath of the RHN species, generating in this way an asymmetry in some "direction", e.g. destroying more antileptons than leptons, but then, when the RHNs reach their equilibrium distribution, it is their decay which dominate the production of the asymmetry, symmetry which now will have the opposite sign with respect to the one created before. The presence of the dip will not be present in the case of thermal initial abundance for the RHNs density studied in Sec. (4.14). It is worth to say here that we have to obtain a positive value for the BAU, i.e. we must have more baryons than antibaryons; in our scenario, i.e. thermal LG, this can always be done thanks to the freedom in the choice of some phases in the Casas-Ibarra parameterization, given in Eq. (2.25).

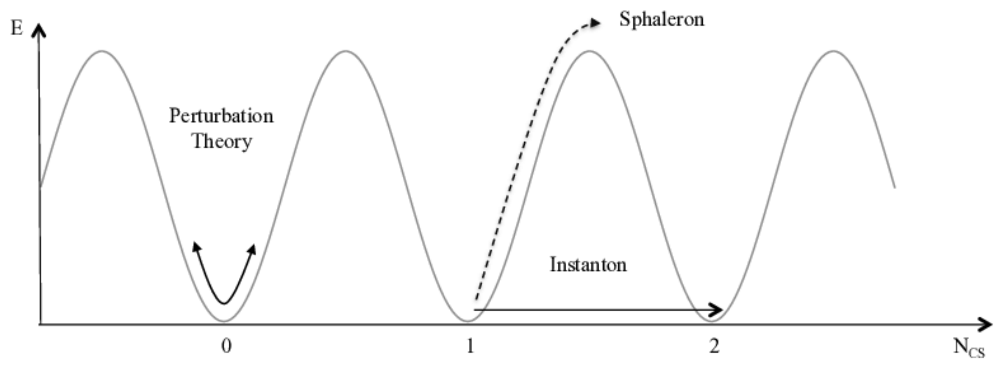



FIGURE 3.4: Instanton and Sphaleron processes in the topology of a Yang-Mills vacuum; on the y-axis is shown the energy density of the gauge field while on the x-axis is present the winding number (or Chern-Simons) $N_{CS}$, which represents the topological charge of the system. These processes leads to a $\Delta N_{CS} = 1$. The figure is taken from [95].

### 3.3.7 Sphalerons

Since what is observed in the Universe is a baryon asymmetry, it is important for LG to have a mechanism which can translate the lepton asymmetry into the BAU. Here we want to give a qualitative introduction [22] of the mechanism which is used to account for this conversion. This mechanism has to do with the anomalous $B + L$ violation introduced in Sec. (1.5). One can show, from Eq. (1.30), that the anomaly term is a total derivative,

$$\epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta} = \partial_\mu K^\mu, \tag{3.58}$$

so it cannot contribute at any order in perturbation theory. However, it can give a contribution to the path integral through a field configuration which is locally gauge equivalent to 0, but which is topologically stable. These configurations violate B and L, i.e act as sources for $B + L \equiv B + L_e + L_\mu + L_\tau$, at the same time preserving $B - L \equiv B - L_e - L_\mu - L_\tau$, which is non-anomalous. If such configurations are suppressed at low energies, they are very frequent in the early Universe, where the temperature $T$ is far above the Electroweak scale, i.e the scale of Electroweak phase transition, leading to rapid $B + L$ violation and to important consequences for LG.
In fact, if we consider the rate of this kind of solutions, what happens is that, at zero temperature, their action is given by:

$$\left| \frac{1}{4g^2} \int d^4x F^A_{\mu\nu} F^{\mu\nu A} \right| \geq \left| \frac{1}{4g^2} \int d^4x F^A_{\mu\nu} \tilde{F}^{\mu\nu A} \right| \geq \frac{64\pi^2 N}{4g^2}, \tag{3.59}$$

where $N$ is the integer change in the fermion number. Since the rate of the process is,

$$\Gamma_{inst} \propto e^{-Action} \sim e^{-4\pi/\alpha_W}, \tag{3.60}$$

it's too small to be observed. This processes correspond to tunneling configurations and are called instantons. We can have a glimpse of what this non-perturbatuive solutions look like considering the ground state of the gauge field as a periodic potential, like in Fig.(3.4): then the instantons represents the vacuum fluctuations between different minima with a $\Delta N_{CS} = 1$ ($\Delta B = \Delta L = n_f \Delta N_{CS}$, with $n_f = 3$ being the number of flavours).
With the same analogy, one can imagine that at a finite temperature, thanks to thermal effects, there can be fluctuations able to climb over the energy barrier; these configurations, in the presence of the Higgs vacuum expectation value, are the sphalerons [96]. The rate of the sphaleron is, differently from the instanton one, Boltzmann suppressed:

$$\Gamma_{sph} \propto e^{-E_{sph}/T}, \tag{3.61}$$

where $E_{sph} = 2Bm_W/\alpha_W$ is the height of the barrier at $T = 0$ and $1.5 \leq B \leq 2.75$ is a parameter related to the Higgs mass. We are interested in the rate at the Leptogenesis temperatures [97], which are far above the EWPT. This rate has been estimated as [98] [99]

$$\Gamma_{B \not{+} L} \simeq 250\alpha_W^5 T. \tag{3.62}$$

So, at temperatures below $10^{12}$ GeV and above EWPT, $B + L$ violating processes are in equilibrium, i.e. faster than the Hubble expansion rate $H$. So, what happens is that the lepton asymmetry produced in the $N_1$ decay implies a baryon excess of

$$Y_{\Delta B} \simeq A \sum_\alpha Y_{\Delta_\alpha}, \tag{3.63}$$

where $A$ depends on the model we are considering, in particular $A = 12/37$ in the Standard Model and $A = 10/31$ in the MSSM [22] [100].

# Chapter 4

# Scalar-Singlet Assisted Leptogenesis

In this chapter we are going to study how the standard picture of thermal LG is modified in a model where, in addition to the SM particle content and symmetry group, an arbitrary number $n_r$ of RHNs $\nu_R$'s, a complex scalar singlet S and a $U(1)_{B-L}$ global symmetry are added. Thermal LG depends is usually studied within the formalism of Boltzmann equations. In our model these equations will be influenced by the presence of these new scalars $\phi$ and $\theta$ coupled to the right handed neutrinos $N_i$. A similar analysis has been done in [18] where the contribution to leptogenesis Boltzmann equations given by the presence of new interactions between the Majorana heavy neutrinos and scalars coming from a dynamically broken $U(1)_{B-L}$ is considered. What differs from our case is the parameter space considered: the focus there is on $N_i$'s annihilation into the two scalars, i.e contributions coming from scatterings, and not on the scalar decay into $N_1$'s. Moreover, they consider mainly the case where both $\phi$ and $\theta$ are massless, or at least massive but with masses $m_\phi, m_\theta \leq M_1$. A similar idea was investigated in [19] where the influence of new scalar degrees of freedom in generating RHNs at the GeV scale and their consequent freeze-in is studied in the scenario of LG via oscillations (ARS leptogenesis). First we write in detail all the different parts of the Lagrangian of the model Sec. (4.1). Then we analyze the symmetry breaking chain, considering first the $U(1)_{B-L}$ spontaneous symmetry breaking Sec. (4.2), Sec. (4.3) and then looking at the vacuum structure of the system after EWSSB Sec. (4.6) finding the scalar mass eigenstates. Also the mass basis for the RHNs is found Sec. (4.5). Then the contribution of the two new scalars degrees of freedom $\phi$ and $\theta$ to the CP asymmetry factor Eq. (3.27) are computed in Sec. (4.7). In Sec. (4.8) we compute the analytical contributions to the Boltzmann equations for LG given by the new interactions between the scalars $\phi$, $\theta$ and the RHNs; in particular we consider the situation where the scalar can decay in a couple of the lightest RHNs, i.e. when the condition $m_\phi > 2M_1$ holds. Then a study of the relation between the Yukawa parameters, the washout/decay factor and the $N_1$ mass is done in Sec. (4.9). We obtain then the results for different values of the washout $K_1$ parameter and initial condition $Y_{N_1}(0)$ in Sec. (4.11), Sec. (4.12), Sec. (4.13) and Sec. (4.14).

## 4.1 The Lagrangian of The Model

The Lagrangian with the global $U(1)_{B-L}$ symmetry, $n_r$ $\nu_R$'s (SM singlets w/ $B-L=-1$) and 1 S SM singlet complex scalar w/ $B-L=2$ will be made by,

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{kin(\nu_R,\mathrm{S})} + \mathcal{L}_{seesaw} + \mathcal{L}_{potential}. \tag{4.1}$$

To write it expicitly we need to consider all the symmetries respecting renormalizable terms; in particular we cannot neglect the couplings between the Standard Model Higgs $H$ and the new scalar singlet. Since S is a complex field, it can be written as

$$\mathrm{S}(x) = \frac{\phi(x)+i\theta(x)}{\sqrt{2}} = \frac{\rho(x)}{\sqrt{2}}e^{i\pi(x)/f}. \tag{4.2}$$

At very high energies, if we suppose that both the $U(1)_{B-L}$ and the electroweak gauge $SU(2)_L \times U(1)_Y$ are unbroken, i.e. if high-energy symmetry restoration happens, we can write the potential in the following form:

$$\begin{aligned} -\mathcal{L}_{potential} = V_{scalars}(H,\mathrm{S}) = m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \\ + \Lambda(H^\dagger H)(\mathrm{S}^\dagger \mathrm{S}) + \mu^2(S^\dagger S) + \lambda_s(\mathrm{S}^\dagger\mathrm{S})^2. \end{aligned} \tag{4.3}$$

Having added to our model $n_r$ SM singlet RHNs $\nu_R$'s, we will have in our Lagrangian also a seesaw part which is going to account for the RHNs masses after the $U(1)_{B-L}$ SSB.
The seesaw Lagrangian is given by

$$-\mathcal{L}_{see-saw} = \bar{L}_L \mathrm{Y}_D^* \tilde{H}\nu_R + \frac{1}{2}\nu_R^T C \mathrm{Y}_R \mathrm{S}\nu_R + \bar{\nu}_R \mathrm{Y}_D^T \tilde{H}^\dagger L_L + \frac{1}{2}\bar{\nu}_R \mathrm{Y}_R^\dagger \mathrm{S}^* \nu_R^c, \tag{4.4}$$

where $\mathrm{Y}_D$ and $\mathrm{Y}_R$ are respectively a $3\times n_R$ and a $n_R \times n_R$ (in general non-diagonal) matrices. Notice that we cannot add a direct mass term for $\nu_R$'s because it would violate the $U(1)_{B-L}$ symmetry of the model.

## 4.2 Symmetry Breaking and Goldstone Theorem

### 4.2.1 The Unbroken Phase

The high-energy potential invariant under the

$$(SU(3)_C \times SU(2)_L \times U(1)_Y)_{gauge} \times U(1)_{B-L,global}$$

symmetry group is given by Eq. (4.3). The minimum energy configurations of the system can be obtained studying first and second derivatives of the potential with respect to the various fields in it. Writing $H^\dagger H = |H|^2$ and $\mathrm{S}^\dagger\mathrm{S} = |\mathrm{S}|^2$ we can clearly see that the potential is a function only of the modulus squared of the scalar fields, as it should be thanks to the symmetries. The potential then becomes

$$V_{scalars}(H,\mathrm{S}) = m^2|H|^2 + \lambda|H|^4 + \Lambda|H|^2|\mathrm{S}|^2 + \mu^2|\mathrm{S}|^2 + \lambda_s|\mathrm{S}|^4. \tag{4.5}$$

The conditions for the critical points can be written as

$$\frac{\partial V}{\partial|H|} = 2m^2|H| + 4\lambda|H|^3 + 2\Lambda|H||\mathrm{S}|^2 = 0, \tag{4.6}$$

$$\frac{\partial V}{\partial|\mathrm{S}|} = 2\mu^2|\mathrm{S}| + 4\lambda_s|\mathrm{S}|^3 + 2\Lambda|H|^2|\mathrm{S}| = 0, \tag{4.7}$$

which are satisfied by

$$|H| = 0, \qquad |H|^2 = -\frac{m^2 + \Lambda|\mathrm{S}|^2}{2\lambda}, \tag{4.8}$$

$$|\mathrm{S}| = 0, \qquad |\mathrm{S}|^2 = -\frac{\mu^2 + \Lambda|\mathrm{H}|^2}{2\lambda_s}, \tag{4.9}$$

and we can see that in the region of the parameter where $m^2$,$\mu^2$, $\lambda$, $\lambda_s > 0$ and also $\Lambda > 0$, the only viable situation is the one for which $|H| = 0$ and $|\mathrm{S}| = 0$ .

Is this a maximum or a minimum for our potential?
To understand it we need to look at the Hessian, the second derivatives matrix, given by

$$\frac{\partial^2 V}{\partial|H|^2} = 2m^2 + 12\lambda|H|^2 + 2\Lambda|\mathrm{S}|^2, \tag{4.10}$$

$$\frac{\partial^2 V}{\partial|\mathrm{S}|^2} = 2\mu^2 + 12\lambda_s|\mathrm{S}|^2 + 2\Lambda|H|^2, \tag{4.11}$$

$$\frac{\partial^2 V}{\partial|\mathrm{S}|\partial|H|} = \frac{\partial^2 V}{\partial|H|\partial|\mathrm{S}|} = 4\Lambda|H||\mathrm{S}|. \tag{4.12}$$

When we evaluate the Hessian matrix in our stationary point, i.e. $|H| = 0$ and $|\mathrm{S}| = 0$, we obtain

$$\frac{\partial^2 V}{\partial|H|^2} = 2m^2, \qquad \frac{\partial^2 V}{\partial|\mathrm{S}|^2} = 2\mu^2, \tag{4.13}$$

$$\frac{\partial^2 V}{\partial|\mathrm{S}|\partial|H|} = \frac{\partial^2 V}{\partial|H|\partial|\mathrm{S}|} = 0, \tag{4.14}$$

which means that it's positive definite in that specific point and this identifies such a point as a (at least local) minimum.

So, this shows that at sufficiently high energies we can perform perturbation theory around the origin, i.e around $\{|H| = 0, |\mathrm{S}| = 0\}$, which represents the vacuum of the system and no spontaneous symmetry sreaking has happened so far.

### 4.2.2 The $U(1)_{B-L}$ Spontaneous Symmetry Breaking

Spontaneous symmetry breaking of the $U(1)_{B-L}$ global symmetry happens if for some physical reasons, e.g. thermal effects while the temperature $T$ of the thermal bath drops down, the values of $\mu^2$ in Eq. (4.3) becomes negative. If this happen, then the potential becomes,after sending $\mu^2 \to -\mu^2$,[1]

$$\begin{aligned} V_{scalars}(H,\mathrm{S}) &= m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \Lambda(H^\dagger H)(\mathrm{S}^\dagger \mathrm{S}) - \mu^2(S^\dagger S) + \lambda_s(\mathrm{S}^\dagger \mathrm{S})^2 \\ &= m^2|H|^2 + \lambda|H|^4 + \Lambda|H|^2|\mathrm{S}|^2 - \mu^2|\mathrm{S}|^2 + \lambda_s|\mathrm{S}|^4. \end{aligned} \tag{4.15}$$

We now can repeat the procedure in Subsec. (4.2.1) to see what has happened to the old vacuum state, i.e. if the minimum configuration of the potential has changed. The entire procedure is carried on in section Append. (C.1). What we find is that the minus sign in front of $\mu^2$ makes a great difference; the point $\{|H| = 0, |\mathrm{S}| = 0\}$ is no more a minimum, moreover, being the Hessian matrix indefinite when evaluated there, we know that it represents a saddle point for the potential. In fact, it is still representing a minimum in the "$|H|$-direction" while it has become a maximum in the "$|\mathrm{S}|$-direction". But we find also a minimum configuration represented by the condition

$$\left\{ |H| = 0,\ |\mathrm{S}|^2 = \frac{\mu^2}{2\lambda_s} \equiv \frac{v_s^2}{2} \right\}. \tag{4.16}$$

We point out here that this condition represents a set of points of equivalent minima which reflects the $U(1)_{B-L}$ symmetry of the theory even if, when we choose one point between all of them as vacuum for the system, the original symmetry seems to be lost. This is the meaning of broken, or hidden, symmetries.

### 4.2.3 $U(1)_{B-L}$ and Goldstone Theorem

As pointed out in Sec. (1.2), it is a very important result for field theories the one known as the Goldstone Theorem, which tells us that every time we have a spontaneous breaking of a continuous and global symmetry, then a massless particle, the Goldstone

[1] In this way we continue working with $\mu^2 > 0$ and factorize out its change of sign in the minus in front.

Boson, enters the game. More precisely, the number of independent Goldstone modes will be the same as the number of the generators broken by the vacuum configuration.

Should we expect any Goldstone bosons after the $U(1)_{B-L}$ symmetry breaking? And how many of them? And, if so, where are they, since in the Hessian matrix in Append. (C.13) has only positive eigenvalues which means no massless particles?
We have broken a global $U(1)_{B-L}$ symmetry and being $U(1)$ a continuous group, we expect our model to predict the presence of Goldstone bosons. More precisely, the breaking of the $U(1)$ symmetry is complete and since its dimension is one, we expect the presence of one Goldstone boson. To find it, we must look more deeply into the different degrees of freedom of the scalar which is driving the symmetry breaking, i.e the complex field S.
We can parameterize the most general complex field through its real and imaginary parts, using its radius and a phase, i.e.

$$\mathrm{S}(x) = \frac{1}{\sqrt{2}}(\phi(x) + i\theta(x)) = \frac{\rho(x)}{\sqrt{2}} e^{i\frac{\pi(x)}{f}}. \tag{4.17}$$

We choose to work with the $\phi(x)$ and $\theta(x)$ parameterization and the potential becomes

$$\begin{aligned} V_{scalars}(H, \phi, \theta) = m^2|H|^2 + \lambda|H|^4 + \frac{\Lambda}{2}|H|^2(\phi^2 + \theta^2) \\ -\frac{\mu^2}{2}(\phi^2 + \theta^2) + \frac{\lambda_s}{4}(\phi^2 + \theta^2)^2. \end{aligned} \tag{4.18}$$

The minima are found in Append. (C.1). First, we find the following stationarity condition: $|H|_0 = 0$, since the electroweak symmetry is not broken, and

$$\phi = 0, \qquad \phi^2 + \theta^2 = \frac{\mu^2}{2\lambda_s} \equiv v_s^2, \tag{4.19}$$

$$\theta = 0, \qquad \phi^2 + \theta^2 = \frac{\mu^2}{2\lambda_s} \equiv v_s^2, \tag{4.20}$$

which reflects the ones obtained for $|\mathrm{S}|$ in Sec. (4.2.2). Notice the $U(1)$ symmetry of the theory reflected in the fact the stationarity condition are only given in terms of $\phi^2 + \theta^2$, which represents a circle. Looking then at the Hessian we find again that the origin is no more a minimum and that instead the minimum condition is the one given by

$$\phi^2 + \theta^2 = \frac{\mu^2}{2\lambda_s} \equiv v_s^2. \tag{4.21}$$

But we find also something new: if we choose as configuration the one given by $\phi_0 = v_s$ and $\theta_0 = 0$, then the second derivatives evaluated at the minimum will give

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 + \Lambda v_s^2, \qquad \frac{\partial^2 V}{\partial \phi^2} = 2\mu^2, \tag{4.22}$$

$$\frac{\partial^2 V}{\partial \theta^2} = -\mu^2 + \lambda_s v_s^2 = 0, \qquad \frac{\partial^2 V}{\partial |H| \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial |H|} = 0, \tag{4.23}$$

$$\frac{\partial^2 V}{\partial |H| \partial \phi} = \frac{\partial^2 V}{\partial \phi \partial |H|} = 0, \qquad \frac{\partial^2 V}{\partial \phi \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial \phi} = 0. \tag{4.24}$$

The eigenvalue correspondent to the $\theta$ field is 0. This means that if we Taylor expand around that minimum configuration, the quadratic term in $\theta$ vanishes, telling us that oscillations along the $\theta$ direction will be described by massless modes. We found the Goldstone boson. As shown in Append. (C.2), choosing a different vacuum between the ones respecting $\phi^2 + \theta^2 = v_s^2$ only corresponds to a rotation of the $\phi$ field into the $\theta$ one, creating a mixing which can always be taken back to a diagonal form looking for the scalar mass eigenstates and so there is no loss of generality in considering the case $\{|H| = 0;\ \theta = 0;\ \phi^2 = v_s^2\}$, as we should have expected from the beginning thanks to the $U(1)_{B-L}$ symmetry.

## 4.3 The Potential After $U(1)_{B-L}$ Symmetry Breaking

Having found out that after $U(1)_{B-L}$ Symmetry Breaking the field $\phi(x)$ takes a vacuum expectation value $v_s$ we can now proceed in writing the potential using a different parameterization for the S field, i.e. sending $\phi \to \phi + v_s$, so that $S = \frac{1}{\sqrt{2}}(\phi + v_s + i\theta)$. In this way both $\phi$ and $\theta$ are describing dynamical oscillations around the true vacuum of the theory, i.e $\phi_0 = 0$ and $\theta_0 = 0$ and the potential can be written as,

$$\begin{aligned} V(H, \phi, \theta) &= m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \Lambda(H^\dagger H)(S^\dagger S) + \lambda_s \left( S^\dagger S - \frac{v_s^2}{2} \right)^2 = \\ &= m^2|H|^2 + \lambda|H|^4 + \frac{\Lambda}{2}|H|^2\phi^2 + \frac{\Lambda}{2}|H|^2\theta^2 + \frac{\Lambda}{2}|H|^2 v_s^2 + \Lambda v_s |H|^2 \phi + \\ &\quad + \lambda_s v_s^2 \phi^2 + \frac{\lambda_s}{4}\phi^4 + \frac{\lambda_s}{4}\theta^4 + \lambda_s v_s \phi^3 + \lambda_s v_s \phi \theta^2 + \frac{\lambda_s}{2}\phi^2\theta^2. \end{aligned}$$

Our model predicts:

I. A mass term for the scalar field $\phi$ of the value $m_\phi^2 = 2\lambda_s v_s^2$;

II. A CP odd massless particle $\theta$, i.e. the Goldstone boson, known also as the Majoron;

III. Self-interactions for both the $\phi$ and the $\theta$ fields;

IV. Interaction between the two degrees of freedom of the scalar S, $\phi$ and $\theta$;

IV. Interactions between the SM Higgs $H$ and the new degrees of freedom $\phi$ and $\theta$; these terms will be important in our following analysis on Leptogenesis. Notice that if it's present an interaction term of the form $|H|^2\phi$, we don't have the correspondent $|H|^2\theta$ term.

But what is going to happen after EWSSB?

## 4.4 The Seesaw Lagrangian

After the $U(1)_{B-L}$ symmetry breaking, i.e. when the oscillation around the vacuum state are described by $\phi$ and $\theta$ in $\mathrm{S} = \frac{1}{\sqrt{2}}(v_s + \phi + i\theta)$, something very interesting happens also in the seesaw Lagrangian

$$-\mathcal{L}_{see-saw} = \bar{L}_L \mathrm{Y}_D^* \tilde{H} \nu_R + \frac{1}{2}\nu_R^T C \mathrm{Y}_R \mathrm{S} \nu_R + \bar{\nu}_R \mathrm{Y}_D^T \tilde{H}^\dagger L_L + \frac{1}{2}\bar{\nu}_R \mathrm{Y}_R^\dagger \mathrm{S}^* \nu_R^c, \tag{4.25}$$

which becomes,

$$\begin{aligned} -\mathcal{L}_{see-saw} = \bar{L}_L \mathrm{Y}_D^* \tilde{H} \nu_R + \nu_R^T C \frac{\mathrm{Y}_R \phi}{2\sqrt{2}} \nu_R + \nu_R^T C \frac{\mathrm{Y}_R v_s}{2\sqrt{2}} \nu_R + \nu_R^T C \frac{\mathrm{Y}_R i\theta}{2\sqrt{2}} \nu_R + \\ + \bar{\nu}_R \mathrm{Y}_D^T \tilde{H}^\dagger L_L + \bar{\nu}_R \frac{\mathrm{Y}_R^\dagger \phi}{2\sqrt{2}} \nu_R^c + \bar{\nu}_R \frac{\mathrm{Y}_R^\dagger v_s}{2\sqrt{2}} \nu_R^c - \bar{\nu}_R \frac{\mathrm{Y}_R^\dagger i\theta}{2\sqrt{2}} \nu_R^c. \end{aligned} \tag{4.26}$$

We can notice that:

I. The $\nu_R$ become massive with $\mathrm{M}_R = \frac{\mathrm{Y}_R v_s}{\sqrt{2}}$ mass matrix and a Majorana mass term in the Lagrangian

$$-\mathcal{L}_{\nu_R, Maj.} = \frac{1}{2}\nu_R^T C \frac{\mathrm{Y}_R v_s}{\sqrt{2}} \nu_R + \frac{1}{2}\bar{\nu}_R \frac{\mathrm{Y}_R^\dagger v_s}{\sqrt{2}} \nu_R^c; \tag{4.27}$$

II. Interactions between the $\nu_R$'s and the scalars $\phi$,$\theta$ which will be the relevant ones for our studies on LG;

III. We can use the fact that $\mathrm{M}_R$ is symmetric, being a Majorana mass term, i.e. $\mathrm{M}_R = \mathrm{M}_R^T$, which means $\mathrm{M}_R^\dagger = \frac{\mathrm{Y}_R^\dagger v_s}{\sqrt{2}} = \mathrm{M}_R^* = \frac{\mathrm{Y}_R^* v_s}{\sqrt{2}}$. This implies $\mathrm{Y}_R^\dagger = \mathrm{Y}_R^*$. So,

$$-\mathcal{L}_{\nu_R, Maj.} \supset \bar{L}_L \mathrm{Y}_D^* \tilde{H} \nu_R + \bar{\nu}_R \mathrm{Y}_D^T \tilde{H}^\dagger L_L + \frac{1}{2}\nu_R^T C \mathrm{M}_R \nu_R + \frac{1}{2}\bar{\nu}_R \mathrm{M}_R^* \nu_R^c. \tag{4.28}$$

## 4.5 The Right Handed Neutrinos Mass Basis

So far we have written only the scalar potential in its proper mass basis; the Majorana mass matrix in Eq. 4.28 is in general not diagonal and so we have to look for the right-handed neutrinos mass eigenstates which we are going to call $N$'s. Since $\mathrm{M}_R$ is symmetric, we can diagonalize it through $\mathrm{M}_R = U_R \mathrm{D_N} U_R^T$ where $U_R$ is an unitary matrix and $\mathrm{D_N}$ is the diagonal matrix with the right-handed neutrinos mass (real) eigenvalues. The mass eigenvalues for $\mathrm{D}_N$ can also be chosen to be real.
The Lagrangian in the mass basis for the RHNs will be given by

$$\begin{aligned} -\mathcal{L}_{\nu_R,Maj.} &= \frac{1}{2}\nu_R^T C U_R \mathrm{D_N} U_R^T \nu_R + \frac{1}{2}\bar{\nu}_R (U_R \mathrm{D_N} U_R^T)^* \nu_R^c = \\ &= \frac{1}{2}\nu_R^T U_R C \mathrm{D_N} (U_R^T \nu_R) + \frac{1}{2}\bar{\nu}_R U_R^* \mathrm{D_N} U_R^\dagger \nu_R^c \end{aligned} \tag{4.29}$$

If we define $N_R \equiv U_R^T \nu_R$, the mass terms can be finally rewritten as,

$$-\mathcal{L}_{N_R,Maj.} = \frac{1}{2} N_R^T C \mathrm{D_N} N_R + \frac{1}{2}\bar{N}_R \mathrm{D_N} N_R^c \tag{4.30}$$

Considering the inverse transformation, i.e. $\nu_R = U_R^* N_R$ (and $\nu_R^\dagger = N_R^\dagger U_R^T$), we can write the entire Lagrangian in terms of the scalar mass eigenstates and the right-handed neutrino ones.

$$\begin{gathered} \mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}(\partial_\mu \theta)(\partial^\mu \theta) - \bar{l}_L \mathbf{Y}_D^* \tilde{H} U_R^* N_R - \bar{N}_R U_R^T \mathbf{Y}_D^T \tilde{H}^\dagger l_L + \\ -\frac{1}{2} N_R^T C \mathcal{D}_{\mathcal{M}} N_R - \frac{1}{2}\bar{N}_R \mathcal{D}_{\mathcal{M}} N_R^c - \frac{1}{2} N_R^T C \frac{U_R^\dagger \mathbf{Y}_R U_R^*}{\sqrt{2}} \phi N_R - \frac{1}{2}\bar{N}_R \frac{U_R^T \mathbf{Y_R}^* U_R}{\sqrt{2}} \phi N_R^c + \\ -\frac{1}{2} N_R^T C \frac{U_R^\dagger \mathbf{Y}_R U_R^*}{\sqrt{2}} i\theta N_R + \frac{1}{2}\bar{N}_R \frac{U_R^T \mathbf{Y_R}^* U_R}{\sqrt{2}} i\theta N_R^c - \left(m^2 + \frac{\Lambda v_s}{2}\right)(H^\dagger H) + \\ -\lambda (H^\dagger H)^2 - \frac{\Lambda}{2}(H^\dagger H)\phi^2 - \frac{\Lambda}{2}(H^\dagger H)\theta^2 - \Lambda v_s (H^\dagger H)\phi + \\ -\frac{\lambda_s}{4}\phi^4 - \frac{m_s^2}{2}\phi^2 - \lambda_s v_s \phi^3 - \frac{\lambda_s}{4}\theta^4 - \frac{\lambda_s}{2}\phi^2\theta^2 - \lambda_s v_s \phi \theta^2 \end{gathered}$$

Defining now

I. $\mathbf{Y}_D U_R \equiv \mathrm{Y_D} \longrightarrow \mathbf{Y}_D^* U_R^* \equiv \mathrm{Y_D}^*$ and $U_R^T \mathbf{Y_D}^T = (\mathrm{Y_D}^*)^\dagger = \mathrm{Y_D}^T$

II. $U_R^\dagger \mathbf{Y_R} U_R^* \equiv \mathrm{Y_R} \longrightarrow U_R^T {\mathbf{Y_R}}^\dagger U_R = U_R^T {\mathbf{Y_R}}^* U_R \equiv {\mathrm{Y_R}}^\dagger = {\mathrm{Y_R}}^\star$ and, since we are working with real masses, the Yukawa couplings $\mathrm{Y}_R$ will be real too and so $\mathrm{Y}_R = \mathrm{Y}_R^\dagger$.

After the electroweak symmetry breaking the seesaw Lagrangian will give origin to neutrino masses in the usual way, but with the difference that now we have a relation also for the masses of the right handed neutrinos, i.e $D_N = (v_s Y_R)/\sqrt{2}$. The masses of the light neutrinos will be instead given by Eq. (2.15),

$$m_\nu \simeq -\frac{1}{\sqrt{2}} \mathrm{Y}_D \frac{v^2}{v_s Y_R} \mathrm{Y}_D^T, \tag{4.31}$$

where we used that

$$m_D = \frac{\mathrm{Y}_D v}{\sqrt{2}}. \tag{4.32}$$

We can then rewrite the whole Lagrangian as

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \bar{l}_L\mathcal{Y_D}^*\tilde{H}N_R - \bar{N}_R\mathcal{Y_D}^T\tilde{H}^\dagger l_L - \frac{1}{2}N_R^T C\mathcal{D_M}N_R + \\ -\frac{1}{2}\bar{N}_R\mathcal{D_M}N_R^c - \frac{1}{2}N_R^T C\frac{\mathcal{Y_R}}{\sqrt{2}}\phi N_R - \frac{1}{2}\bar{N}_R\frac{\mathcal{Y_R}^\star}{\sqrt{2}}\phi N_R^c - \frac{1}{2}N_R^T C\frac{\mathcal{Y_R}}{\sqrt{2}}i\theta N_R + \frac{1}{2}\bar{N}_R\frac{\mathcal{Y_R}^\star}{\sqrt{2}}i\theta N_R^c + \\ -\left(m^2 + \frac{\Lambda v_s}{2}\right)(H^\dagger H) - \lambda(H^\dagger H)^2 - \frac{\Lambda}{2}(H^\dagger H)\phi^2 - \frac{\Lambda}{2}(H^\dagger H)\theta^2 - \Lambda v_s(H^\dagger H)\phi + \\ -\frac{\lambda_s}{4}\phi^4 - \frac{m_s^2}{2}\phi^2 - \lambda_s v_s\phi^3 - \frac{\lambda_s}{4}\theta^4 - \frac{\lambda_s}{2}\phi^2\theta^2 - \lambda_s v_s\phi\theta^2.\end{aligned}$$

Defining the Majorana fermion $N = N_R + N_R^c$ such that $N = N^c$ and writing left and right projectors explicitly, we obtain the following form for the Lagrangian,

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \bar{L}_L\mathrm{Y_D}^* P_R\tilde{H}N + \\ -\bar{N}\mathrm{Y_D}^T P_L\tilde{H}^\dagger L_L - \frac{1}{2}\mathrm{D_N}\bar{N}N - \frac{\mathrm{Y_R}}{2\sqrt{2}}\bar{N}N\phi - i\frac{\mathrm{Y_R}}{2\sqrt{2}}\bar{N}\gamma^5 N\theta + \\ -\left(m^2 + \frac{\Lambda v_s}{2}\right)(H^\dagger H) - \lambda(H^\dagger H)^2 - \frac{\Lambda}{2}(H^\dagger H)\phi^2 - \frac{\Lambda}{2}(H^\dagger H)\theta^2 + \\ -\Lambda v_s(H^\dagger H)\phi - \frac{\lambda_s}{4}\phi^4 - \frac{m_s^2}{2}\phi^2 - \lambda_s v_s\phi^3 - \frac{\lambda_s}{4}\theta^4 - \frac{\lambda_s}{2}\phi^2\theta^2 - \lambda_s v_s\phi\theta^2.\end{aligned} \tag{4.33}$$

## 4.6 The Breaking of Electroweak Symmetry

We need to find from our model the physics we see at low energy and from the Standard Model we know that at low energy the electroweak symmetry is broken down to $U(1)_{EM}$, the electromagnetic gauge symmetry group. So, also in our model EWSSB must be viable. It is also interesting to see what happens in our with EWSSB because it is more difficult than in the usual SM case: the potential contains an allowed interaction between the Higgs $H$ and the new scalar S, i.e. the new scalars $\phi$ and $\theta$, which makes things not trivial. What one can do, starting from Eq. (4.3), is to write the Higgs field as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h \end{pmatrix}, \tag{4.34}$$

in a way that $H^\dagger H = \frac{1}{2}h^2$, then perform derivatives of the potential $V(h, \phi, \theta)$ with respect to the various field, looking for the stationarity conditions: $\partial_\phi V = 0$, $\partial_\theta V = 0$ and $\partial_h V = 0$ and at the end study the Hessian matrix to find the minima.
We parameterize the Higgs field in this way because we know that the component which can take a vacuum expectation value after the symmetry breaking can only be along the neutral component with respect to the unbroken symmetry, i.e the component with electric charge $Q = 0$, since $U(1)_{EM}$ is the unbroken symmetry of the theory. We are also already working in the unitary gauge so that we don't see the pseudo-Goldstones bosons appearing after symnmetry breaking.
It turns out that an easier way to study the minimum configurations of the potential after EWSSB is to start directly from Eq. (4.18) and to consider that for having EWSSB there must be also $m^2 < 0$. Then we send $m^2 \to -m^2$ and consider the parameterization for the Higgs field in Eq. (4.34), so that the potential becomes,

$$V(h, \phi, \theta) = -\frac{m^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{\Lambda}{4}h^2(\phi^2 + \theta^2) - \frac{\mu^2}{2}(\phi^2 + \theta^2) + \frac{\lambda_s}{4}(\phi^2 + \theta^2)^2. \tag{4.35}$$

The study of the stationary configurations is done in Append. (C.3). The minimum of the potential is given by the system of conditions,

$$h_0^2 = \frac{m^2 - \frac{\Lambda}{2}(\phi_0^2 + \theta_0^2)}{\lambda}, \tag{4.36}$$

$$\phi_0^2 + \theta_0^2 = \frac{\mu^2 - \frac{\Lambda}{2}h_0^2}{\lambda_s}, \tag{4.37}$$

where we are again free to choose and work with $\theta_0 = 0$ without loss of generality, obtaining

$$h_0^2 = \frac{\lambda_s m^2 - \frac{\Lambda}{2}\mu^2}{\lambda\lambda_s - \frac{\Lambda^2}{4}}, \qquad \phi_0^2 = \frac{\lambda\mu^2 - \frac{\Lambda}{2}m^2}{\lambda\lambda_s - \frac{\Lambda^2}{4}}, \qquad \theta_0 = 0. \tag{4.38}$$

Notice that in the limit where $\Lambda \to 0$, i.e in the limit where the Higgs and the new scalar fields are decoupled, we recover the vacuum expectation values $\theta$ and $\phi$ took after only the $U(1)_{B-L}$ symmetry breaking. The EWSSB, because of the term $\Lambda|H|^2|S|^2$ present in the potential, is modifying the vacuum also in the $\theta$ and $\phi$ direction. We can also look at the Hessian matrix of the system evaluated in the selected minimum to study the mass spectrum of the system:

$$\partial^2_{hh}V = \frac{2\lambda\lambda_s m^2 - \Lambda\lambda\mu^2}{\lambda\lambda_s - \frac{\Lambda^2}{4}}, \qquad \partial^2_{\phi\phi}V = \frac{2\lambda\lambda_s \mu^2 - \Lambda\lambda_s m^2}{\lambda\lambda_s - \frac{\Lambda^2}{4}}, \qquad \partial^2_{\theta\theta}V = 0,$$
$$\partial^2_{h\phi}V = \partial^2_{\phi h}V = \frac{\Lambda\sqrt{\lambda\lambda_s m^2\mu^2 - \frac{\Lambda}{2}(\lambda_s m^4 + \lambda\mu^4) + \frac{\Lambda^2}{4}m^2\mu^2}}{\lambda\lambda_s - \frac{\Lambda^2}{4}}, \tag{4.39}$$
$$\partial^2_{h\theta}V = \partial^2_{\theta h}V = 0, \qquad \partial^2_{\phi\theta}V = \partial^2_{\theta\phi}V = 0.$$

At this point, we can note two things:

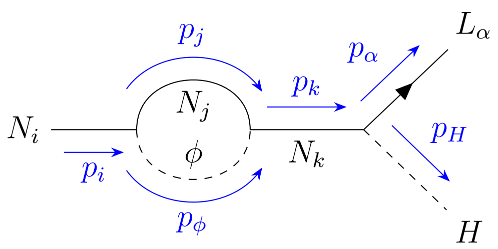
I. We have also here a zero eigenvalue in correspondence of the derivatives with respect to the $\theta$ field. As in Subsec. (4.2.3), this means that $m_\theta = 0$ even after the EWSSB, as expected from the Goldstone theorem.

II. After EWSSB we obtain also a mixing between the two scalars $h$ and $\phi$. This means that if we want to find the mass eigenstates of the potential, they will be linear combinations of the two fields and, consequently, there will be some mixing between the SM Higgs and this new singlet scalar.

Anyway since in our case LG happens far above the electroweak scale, we are going to work with $H, \phi$ and $\theta$ as scalar degrees of freedom.

## 4.7 The Scalar Contributions To The Asymmetry

In this section, we give the results for the new contributions to the CP asymmetry factor given by the interaction between the $N$'s and the two scalars $\phi$ and $\theta$. These are computed in detail in Append. (D).
The first contribution to be considered is the interference between the tree-level $N_i$ decay and the following 1-loop diagram, with the scalar $\phi$ running in the loop




It is important to notice that since we need to produce on-shell loop particles to have a non-zero CP asymmetry injection, the condition $M_i > M_j + m_\phi$ must hold. For example,

we won't have CP injection in the decay of the lightest RHN, since it is not able to produce a heavier $N_j$.
The general result for the first wave function correction is obtained in Append. (D.0.1) and yields:

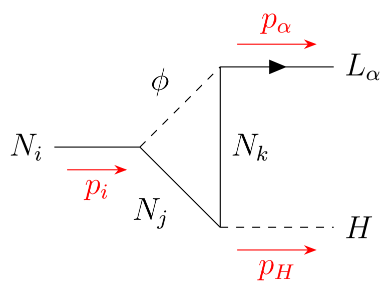
$$\begin{aligned}\epsilon_{i\alpha}^{wave,\phi} = &\left(\frac{1}{128\pi}\right)\frac{1}{\sum_\alpha |\mathrm{Y}_D^{\alpha i}|^2}\sum_{j,k\neq i}\mathrm{Im}\Big\{\mathrm{Y}_R^{jk\star}\mathrm{Y}_D^{k\alpha\star}\mathrm{Y}_R^{ji}\mathrm{Y}_D^{i\alpha}\Big\}\frac{1}{1-r_{ki}}\times\\ &\times\left[\sqrt{r_{ji}}(1+\sqrt{r_{ki}})+\frac{1}{2}(1+r_{ji}-\sigma_i)(1-\sqrt{r_{ki}})\right]\sqrt{\rho_{ij}}.\end{aligned} \tag{4.40}$$

where the following quantities has been defined:

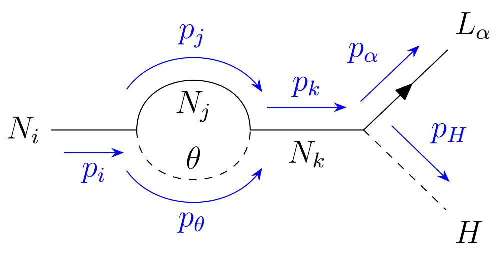
$$\sigma_i = \frac{m_\phi^2}{M_i^2}, \qquad\qquad r_{ji} = \frac{M_j^2}{M_i^2}, \tag{4.41}$$

$$\rho_{ij} = (1-r_{ji}-\sigma_i)^2 - 4r_{ji}\sigma_i. \tag{4.42}$$

In our model we are considering the addition of a SM complex scalar singlet S to the usual RHN extension of the SM particle content. So, since we have a very simple relation as Eq. (4.26) between $M_N$ and $\mathrm{Y}_R$, and since we want to work in the RHNs mass basis and $M_N$ is also symmetric being a Majorana mass matrix, what happens is that when we diagonalize the matrix $M_N$ we are automatically diagonalizing also the RHNs Yukawa matrix $\mathrm{Y}_R$. This means that $\mathrm{Y}_R^{jk} = \delta_k^j \mathrm{Y}_R^{jk} \equiv \mathrm{Y}_R^j$, and that we cannot induce a RHNs mixing through the new scalar degrees of freedom $\phi$. The main consequence is that we cannot satisfy the fundamental kinematical condition to put on-shell the particles in the loop of Append. (D.0.1), i.e we cannot have $M_i > M_j + m_\phi$ for a massive $\phi$. So, in our model $\epsilon_{i\alpha}^{wave,\phi} = 0$. For the same reason also the vertex contribution given by the interference with the diagram




vanishes. Let's see if something different happens when we consider the CP-odd degrees of freedom of the $U(1)_{B-L}$ scalar, i.e the Majoron $\theta$. This particle is massless and so in principle the kinematical condition for the on-shell loop is satisfied. Anyway, we have to say that we don't expect a $\epsilon_{i\alpha}^{wave,\phi} \neq 0$ because the lack of mixing in the RHNs-$\theta$ interaction leads also to a combination of Yukawa couplings which is real. The 1-loop diagram we have to consider for this case is,




In this case the kinematical condition becomes $M_i > M_j + m_\theta$, but with $m_\theta = 0$, so in principle itcan be satisfied even without mixing. Performing the computation for the CP asymmetry Eq. (3.27) and before performing phase space integrations, we obtain from Append. (D.0.2),

$$\epsilon_{i\alpha}^{wave,\theta} \propto \frac{1}{8} \frac{\mathrm{Im}\Big\{ \mathrm{Y}_D^{\alpha i} \mathrm{Y}_R^{kj,*} \mathrm{Y}_D^{k\alpha,*} \mathrm{Y}_R^{ij} \Big\}}{p_k^2 - M_k^2} \mathrm{Tr}\Big\{ -P_L \not{p}_\alpha \Big( \not{p}_k - M_k \Big) \Big( \not{p}_j + M_j \Big) \Big( -\not{p}_i + M_i \Big) \Big\}. \tag{4.43}$$

If we then consider diagonal RHNs Yukawa couplings, i.e if $\mathrm{Y}_R^{ij} = \delta_j^i \mathrm{Y}_R^{ij} = \mathrm{Y}_R^i$; when we start with an $|N_i\rangle$ initial state, then Eq. (4.43) takes the form

$$\epsilon_{i\alpha}^{wave,\theta} \propto \frac{1}{8} \mathrm{Im}\Big\{ \big|\mathrm{Y}_D^{i\alpha}\big|^2 \big|\mathrm{Y}_R^i\big|^2 \Big\} \mathrm{Tr}\Big\{ -P_L \not{p}_\alpha \Big( -\not{p}_i + M_i \Big) \Big\}. \tag{4.44}$$

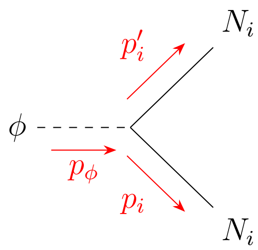
This contribution is again identically 0 because $|\mathrm{Y}_D^{i\alpha}|^2 |\mathrm{Y}_R^i|^2$ has no imaginary part.

At this point we can say that, in some sense, adding this new complex scalar singlet S and a $U(1)_{B-L}$ global symmetry is natural and do not modify the value of the CP-asymmetry $\epsilon_{i\alpha}$. But we have to pay attention saying that the new scalar does not modify the whole Leptogenesis process, since it can modify through its interactions the Boltzmann Equation for the RHNs yields $Y_{N_i}$ which are another fundamental block in thermal Leptogenesis.

It is also worth noting that modifications to the CP asymmetry factor occurs in more complex situations, like where one add more than one complex scalar singlets to the theory, or has more than one Higgs doublets, or both, as shown in [17]. Modifications can also be present if one put by hand a direct mass term for the RHNs, as done in [16] where they consider not a complex, but a real scalar singlet. We repeat here that our idea was that of studying a minimal model where the new scalar degrees of freedom $\phi$ and $\theta$ and the $U(1)_{B-L}$ symmetry have the power of generating dinamically the seesaw type-I mechanism and giving a UV explanation for it. At the same time we wanted to focus on a model which can be embedded into other theories, e.g Majoron dark matter models where the presence of the $U(1)_{B-L}$ Goldstone boson becomes a candidate for dark matter when there are small symmetry-violating terms in the Lagrangian which gives to it a small mass, [14][13] or theories which aim to be UV completion of the Standard Model, as GUT theories.

## 4.8 Boltzmann Equations Again

After having showed that $\epsilon_{i\alpha}$ is not modified in its form by the addition of one new complex scalar singlet S, which drives, after symmetry breaking, the mass mechanism for neutrinos, we can go on studying how Boltzmann equations for the right handed neutrinos are modified by the new terms in the Lagrangian. The leading order effects will be the one due to the decay of the massive scalar $\phi$, if kinematics conditions are verified, i.e. $m_\phi > 2M_i$. We need to include the contribution to the $N_1$'s density in the thermal bath due to the process




The decay of the new degrees of freedom $\phi$ can help to thermalize the $N_i$ density, i.e taking the $N_1$ number density to its equilibrium distrubution, even in the case of a weak washout regime for example, giving an enhancement in the creation of the lepton (and then of the baryon) asymmetry.
We study the case where $m_\phi > 2M_1$ and the scalar can decay into a couple of the lightest RHN $N_1$'s. The formalism used is the one presented in [22]. It holds that

$$\left(\dot{Y}_{N_1}\right)_{\phi-decay} = [\phi \longleftrightarrow N_1 N_1], \tag{4.45}$$

where

$$[\phi \longleftrightarrow N_1 N_1] \equiv \frac{Y_\phi}{Y_\phi^{eq}}\gamma^{\phi}_{N_1 N_1} - \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}}\right)^2 \gamma_\phi^{N_1 N_1} = y_\phi \gamma^{\phi}_{N_1 N_1} - y_{N_1}^2 \gamma_\phi^{N_1 N_1}, \tag{4.46}$$

$\gamma^{ab...}_{ij...}$ is the thermal cross section defined in Append. (E.1) and also $\left(\dot{Y}_{N_1}\right)_{\phi-decay}$ is defined as

$$\left(\dot{Y}_{N_1}\right)_{\phi-decay} \equiv \frac{sH_1}{z}\left(\frac{dY_{N_1}}{dz}\right)_{\phi-decay} \tag{4.47}$$

.

In our situation $\gamma_\phi^{N_1 N_1} = \gamma^{\phi}_{N_1 N_1}$ and if we also assume that the scalar $\phi$ is in thermal equilibrium with the bath, thanks for example to its interactions with the Higgs boson in Eq. (4.33), then $y_\phi \simeq 1$ and the equation we need to solve is,

$$\left(\dot{Y}_{N_1}\right)_{\phi-decay} = [\phi \longleftrightarrow N_1 N_1] \simeq (1 - y_{N_1}^2)\gamma_\phi^{N_1 N_1}. \tag{4.48}$$

The complete computation is carried out in Appendix E and gives an analytical result in the limit of $m_\phi \gg M_1$[2], which is, in terms of the $N_1$ yield, $Y_{N_1}$,

$$\left(\frac{dY_{N_1}}{dz}\right)_{\phi-decay} = \frac{1}{Y_{eq,N_1}^2}\left(Y_{eq,N_1}^2 - Y_{N_1}^2\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{sH_1} \tag{4.49}$$

This results can be rewritten in terms of the particle number density $N_1 \equiv n_1 a^3$, using the relation between the entropy density $s$ and the comoving volume $a^3$,

$$s = \frac{2\pi^2}{45}\frac{1}{a^3}, \tag{4.50}$$

giving

$$\left(\frac{dN_1}{dz}\right)_{\phi-decay} = \frac{1}{N_{eq,1} n_{eq,1}}\left(N_{eq,1}^2 - N_1^2\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{H_1}. \tag{4.51}$$

Substituting then in Eq. (4.51) the value of $n_{eq,1}$ from Appendix E and writing all in terms of the decay parameter $K_\phi$, defined as:

$$K_\phi = \frac{\Gamma_D^\phi}{H_1} = \left(\frac{1}{32\pi}\right)(Y_R^1)^2 \frac{m_\phi M_{PL}}{1.66\sqrt{g_\star} M_1^2}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)^{\frac{3}{2}}, \tag{4.52}$$

we obtain the result we were looking for:

$$\left(\frac{dN_1}{dz}\right)_{\phi-decay} = \frac{1}{\sqrt{1 - \frac{4M_1^2}{m_\phi^2}}}\left(\frac{m_\phi}{2M_1}\right)^2 K_S z \frac{\mathcal{K}_1(z\frac{m_\phi}{M_1})}{\mathcal{K}_2(z)}\frac{\left(N_{eq,1}^2 - N_1^2\right)}{N_{eq,1}}. \tag{4.53}$$

The decay parameter $K_\phi$ defined in Eq. (4.52) is an adimensional parameter which tends to describe the strength of the scalar decay width $\Gamma_D^\phi$, found in Append. (E.15), with respect to the Hubble parameter evaluated at $T = M_1$. It is somewhat similar to the washout parameter $K_1$, but in this case we are not describing any kind of washout because the $\phi$ decay into $N_1$'s and its inverse decay are not creating or deleting any lepton asymmetry. $K_\phi$ helps us to understand if the scalar decay is at equilibrium during the evolution of the Universe, especially at $T = M_1$. What we expect is that if $K_\phi \gg 1$

[2]The approximation is good already from $m_\phi \geq 3M_1$

then the scalar $\phi$ is playing an important role in taking the $N_1$ number density at its equilibrium distribution independent from what is the value of $K_1$, while if $K_\phi \ll 1$ then the $\phi$ decay is weak and if the $N_1$'s are able to reach equilibrium depends only on their standard Yukawa interactions, parameterized by $K_1$.

## 4.9 The Washout and Decay Parameters

In this section we are going to make an analysis of both the washout parameter $K_1$ and the decay parameter $K_\phi$. In particular we plot couplings versus masses for different fixed values of these two parameters, to understand which are the different choices in the parameter space which can lead to the same value for the washout or decay parameter.

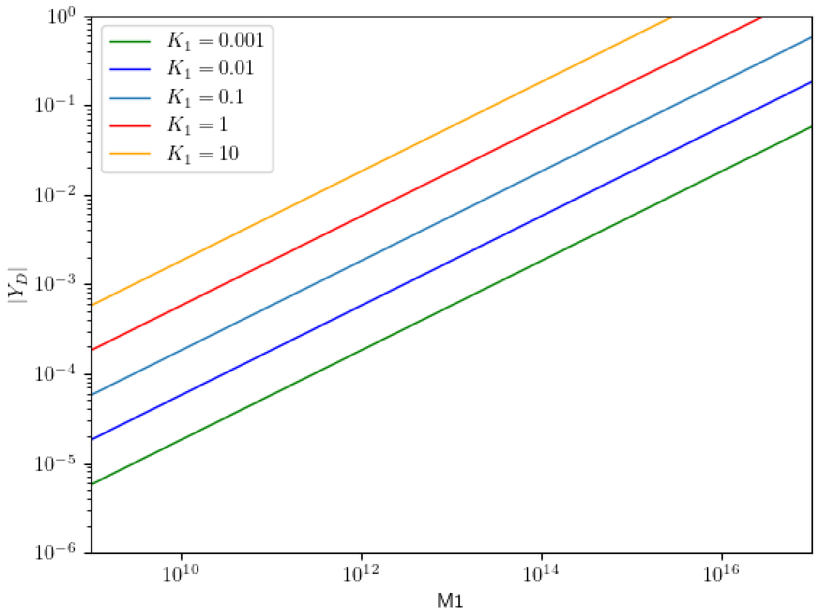



FIGURE 4.1: The panel shows the behaviour of the Yukawa coupling, in particular $|\mathrm{Y}_D|$, as a function of the mass scale $M_1$ (GeV) for a fixed value of the parameter $K_1$. The different lines represent different values for $K_1$: growing values of the washout from down to top, and with the red line ($K_1 = 1$) standing for the boundary between strong and weak washout regimes.

The results for the washout parameter $K_1$ are shown in Fig. (4.1). Recalling the definition given in Eqs. (3.55) and (3.31),

$$K_1 = \frac{\tilde{m}_1}{m^*} = \frac{(\mathrm{Y}^\dagger \mathrm{Y})_{11} v^2}{m^* M_1}, \tag{4.54}$$

we can see that to plot the results in a two-dimensional plane we need to work with some approximation, since $(\mathrm{Y}^\dagger\mathrm{Y})_{11} = \sum_\alpha \mathrm{Y}^\dagger_{\alpha 1}\mathrm{Y}_{\alpha 1}$ involves more than one Yukawa parameter due to the presence of the three flavours $\alpha$. One can choose the approximation where the three different Yukawa couplings are of the same order, i.e $\mathrm{Y}_{\alpha \mathrm{i}} \simeq \mathrm{Y}_{\beta \mathrm{i}}$ for $\alpha$, $\beta = e, \mu, \tau$, ore one can consider the case when one of them is bigger than the other two, allowing us to neglect them. Lowering the value of the washout means, for fixed values of $M_1$, lowering the Yukawa couplings, e.g for $M_1 = 10^{14}$ having a washout factor of $K_1 = 0.001$ or of $K_1 = 0.01$ means Yukawa couplings of order $\mathrm{Y}_D \sim 10^{-3}, 10^{-2}$.

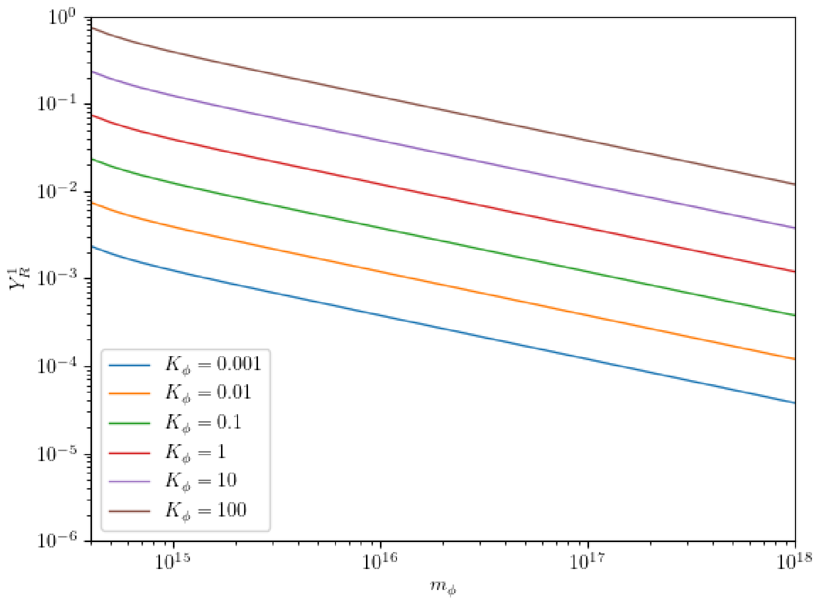



FIGURE 4.2: The panel shows the behaviour of the $N_1$-$\phi$ coupling $\mathrm{Y}^1_{\mathrm{R}}$, as a function of the mass scale of the scalar $m_\phi$ (GeV) and for a fixed value of the decay parameter $K_\phi$. The different lines represents, from down to top, increasing values for the decay parameter, with the red line ($K_1 = 1$) standing for the boundary between strong and weak decay regimes.

For what concern the decay parameter $K_\phi$, the results are shown in Fig. (4.2). From the definition in Eq. (4.52)

$$K_\phi = \left(\frac{1}{32\pi}\right)(\mathrm{Y}^1_R)^2 \frac{m_\phi M_{PL}}{1.66\sqrt{g_\star} M_1^2}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)^{\frac{3}{2}}, \tag{4.55}$$

we can see that the relation between $\mathrm{Y}^1_R$ and $m_\phi$ for fixed $K_\phi$ is non-linear.

We have seen before that in the regime of $\phi$ masses between $m_\phi = 4M_1$ and $m_\phi = 10M1, 100M_1$ the region of more interest are the ones for $K_\phi \gg 1$. From Fig. (4.2) we can translate this into the correspondent region of parameter space for the right handed neutrino-scalar coupling, obtaining $\mathrm{Y}^1_R \geq 0.1$.

## 4.10 Numerical Analysis of the Boltzmann Equations

Using numerical simulations, we want to compare what is different in the evolution of the $N_1$ density and of the BAU $\eta_B$ in the case where the seesaw mechanism is put in by hand so that there are no new degrees of freedom in the theory besides the RHNs, i.e. the standard case, and in the case of interest for this thesis, where the seesaw mechanism is generated dinamically and the presence of a high energy $U(1)_{B-L}$ implies the existence of new degrees of freedom ($\phi$ and $\theta$) coupled to the RHNs.
We consider the parameter space where $m_\phi > 2M_1$ so that the scalar $\phi$ can decay into $2N_1$ and the scenario where $\phi$ is in thermal equilibrium with the thermal bath, so that the new contribution to the Boltzmann equation is precisely the one computed in Sec. (4.8).
We work with the Casas-Ibarra parameterization for the type I seesaw. So, what we need to specify besides the parameters of the PMNS matrix and the lightest neutrino mass $m_{1/3}$ are the six degrees of freedom of the orthogonal matrix R defined in Eq. (2.25) and the RHNs masses, or in alternative the washout parameter $K_1$ and the RHNs masses. Moreover, the lightest RHN $N_1$ mass is choosen to be $M_1 = 10^{14}$ and we work in the approximation of hierarchical RHN masses, i.e $M_1 \ll M_2 \ll M_3$ (all the parameters used are shown in Append. (F.2)); this choice is justified by the fact that this parameter space for $M_i$'s is closely related to the GUT scale. We then analyze some cases in the scenario of zero $N_1$ initial abundance, i.e $Y_{N_1}(0) = 0$ and also some case of thermal initial abundance, i.e $Y_{N_1}(0) = Y_{N_1}^{eq}$.
What we aim to obtain is to open up the parameter space of standard LG so that thermal LG becomes more viable, thanks to the presence of these new decay channel $\phi \to N_1 N_1$. We are going to consider some cases in the strong washout scenario, where we fix $K_1 = 10$ and some cases in the weak and very weak scenario, respectively with $K_1 = 0.1$ and $K_1 = 0.01$, each for different values of the decay parameter $K_\phi$.

## 4.11 The Strong Washout Case, $K_1 = 10$

Here we consider the strong washout scenario with three different cases for the decay parameter $K_\phi$: $K_\phi \gg K_1$, $K_\phi \sim K_1$ and $K_\phi \ll K_1$.

### 4.11.1 The Case with $K_\phi \gg K_1$

Fig. (4.3) shows the $N_1$ number density (left panel) and the absolute value of the baryon asymmetry (right panel) as a function of $z = M_1/T$ and both in the standard case (green) and in the modified one (blue).
Here the decay parameter $K_\phi$ is bigger than the washout one, $K_1$, and, as a result, the main role of the scalar $\phi$ is to help $N_1$ to thermalize with the bath faster, as indicated

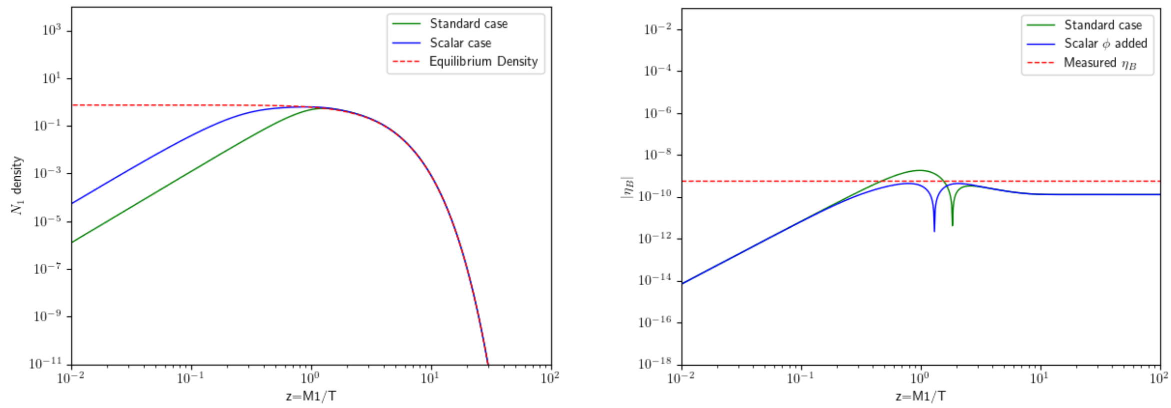



FIGURE 4.3: The left panel shows the lightest Right Handed Neutrino $N_1$ number density both in the standard case (green line) and in the case with the scalar $\phi$ decay added (blue line) and in the case of vanishing initial abundance for the RHN density. The right panel shows instead the absolute value of the BAU, $|\eta_B|$, in the standard case (green) and in the modified one (blue). Both are represented as a function of $z = M_1/T$. We considered $M_1 = 10^{14}$ GeV, $K_1 = 10$ (strong washout) and $K_\phi \simeq 150$. The dashed red lines represents the $N_1$ equilibrium number density (left) and the measured BAU today (right).

by the difference in the behaviour of the blue line with respect to the green one in the left panel, confirming what expected for such a large value of the decay parameter $K_\phi$. For what concern the BAU $\eta_B$, we see from the right panel of Fig. (4.3) that there are no significant differences between the standard case and the modified one, especially for the prediction of the baryon asymmetry $\eta_B$ today, which is given by the constant value of the asymmetry at late time, i.e for $z \gg 1$.
This is because in the strong washout regime any lepton asymmetry which is created before the out of equilibrium $N_1$ decay is washed out and since the differences in the $N_1$ densities in the two cases are relevant for $z < 1$, this results in a similar behaviour for $\eta_B$ in the two situations.
We note also the presence of the characteristic dip in the behaviour of the BAU $\eta_B$; what we can see here is that, in the modified case, the dip happens earlier. This is due to the fact that the $N_1$ number density reach faster its equilibrium distribution.

### 4.11.2 The Case with $K_\phi \sim K_1$

The obtained results for the case where the decay parameter $K_\phi \sim K_1$ are shown in the upper figures in Fig. (4.4).
What we see here is a situation similar to the one in Fig. (4.3): for what concern the $N_1$ number densities we see that, as in the previous case, the scalar is helping the $N_1$'s to reach their equilibrium distribution, but since in this case the decay parameter is smaller

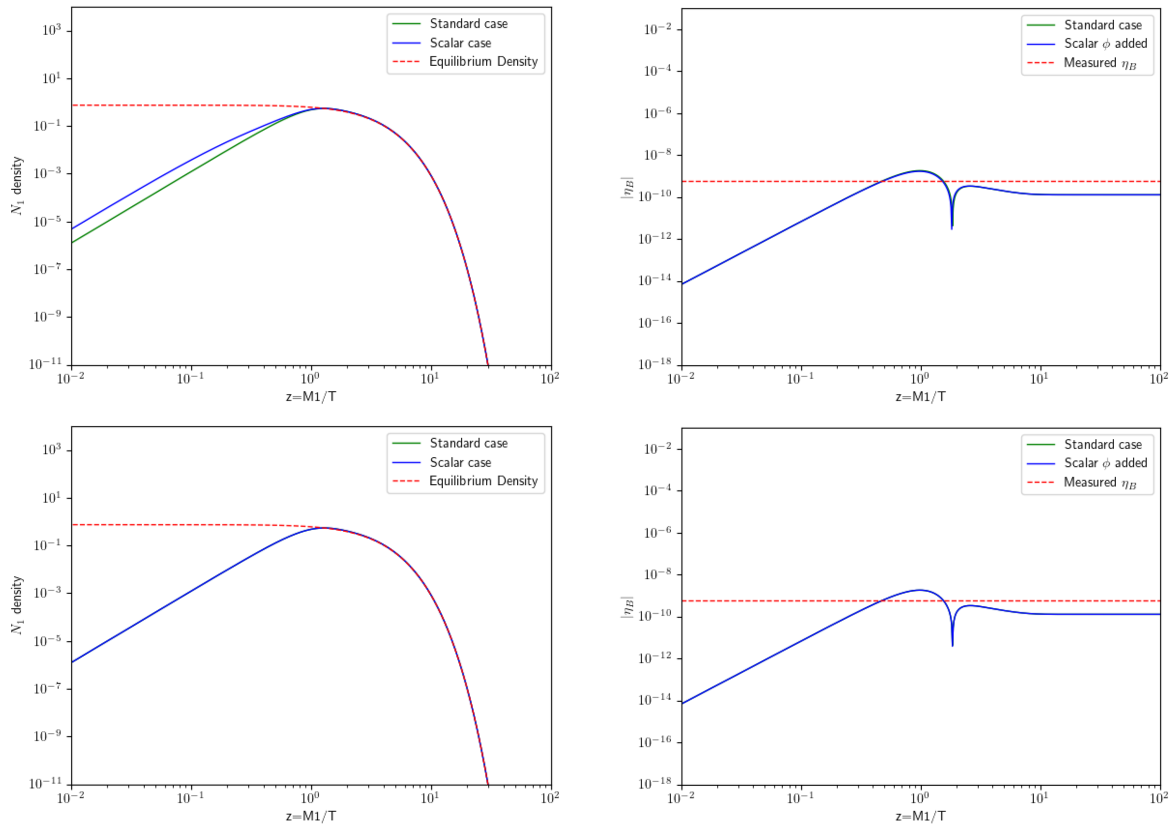



FIGURE 4.4: The figure represents what happens for $K_1 = 10$ and $M_1 = 10^{14}$ GeV in the case of vanishing initial abundance for the RHN density. For the upper $K_\phi \simeq 10$, for the lower ones $K_\phi \simeq 0.02$. The behaviour of the lightest RHN number density is shown both in the standard case (green) and in the modified one (blue) is showed in the left panels. The BAU is showed in the right panels both for the standard (green) and the modified (blue) cases. All the figures are represented as a function of $z = M_1/T$. The dashed red lines represents the $N_1$ equilibrium number density (left) and the BAU today (right).

of an order of magnitude with respect to the one considered in Subsec. (4.11.1), then also its effect on the $N_1$ density are smaller. Also the difference in the baryon asymmetry of the Universe become smaller between the two cases, as we can see from the figure on the right in Fig. (4.4).

### 4.11.3 The Case with $K_\phi \ll K_1$

The obtained results for the case where the decay parameter $K_\phi \ll K_1$ are shown in the lower figures of Fig. (4.4). In this case, the presence of the scalar is not even influencing the $N_1$ number density and the baryon asymmetry of the Universe $\eta_B$. This is due to having a small decay parameter $K_\phi$, given by a Yukawa coupling $Y_1^R \simeq 0.008$ for $m_\phi = 4M_1$. In this regime, the strength of the scalar decay is too small to give a measurable modification to the standard case situation.

For what concerns the strong washout scenario, we have seen that, for different values of the decay parameter there can be significant modifications to the $N_1$ number density. However, since also in the standard case the $N_1$'s are able to reach their equilibrium distribution before $z = 1$ and in the strong washout scenario every lepton asymmetry which is created before the $N_1$ decoupling from the thermal bath is erased, the difference in the prediction for the BAU today between the two models is small. It is also worth pointing out that with the parameters and the mass scales chosen in these example, the asymmetry $\eta_B$ predicted is slightly less than the measured one, represented by the dashed line in the figures on the left. Nevertheless, it should be possible to vary the other parameters, e.g. the mass $M_1$, and obtain then the present observed value for the BAU.

## 4.12 The Weak Washout Regime, $K_1 = 0.1$ and $K_1 = 0.01$

What we want to study now is what changes between the standard case and the one with the $\phi$ decay included when the washout is weak. We are going to look first at $K_1 = 0.1$ and then at $K_1 = 0.01$. In the weak washout regime what happens in the standard case is that the Yukawa interactions between the Higgs, $N_1$'s and the leptons are so weak that the $N_1$ number density does not reach its equilibrium number density during the evolution of the Universe; at the same time, and for the same reason, every lepton asymmetry which is created also before the out-of-equilibrium decay of $N_1$'s is not erased by the inverse decay, differently from what happens in the strong washout regime. In fact, what we observe in Fig. (4.5) is that the RHN interaction are not strong enough to take its density to the equilibrium one (dashed red line), a part from the case where $\phi$ is strongly coupled to $N_1$, i.e. $K_\phi \simeq 150$ (low panels in Fig. (4.5)). We notice also here the presence of the dip in the right panels, typical of the vanishing initial abundance situations.
Since we have seen in Sec. (4.11) that the presence of the scalar in our model is useful to increase the number density of $N_1$, what we expect in this case is to be able to find new regions of viable thermal Leptogenesis in the weak washout regime. As done in Sec. (4.11), we study what happens for different values of the decay parameter $K_\phi$.

### 4.12.1 The Case with $K_1 = 0.1$ and $K_\phi \simeq 20$

In this case, $K_\phi \gg K_1$; this value of the decay parameter has been obtained fixing $m_\phi = 10M_1$ and $\mathrm{Y}^1_\mathrm{R} \simeq 0.2$. The results are shown in the two upper figures of Fig. (4.5). As we can see comparing blue (modified case) and green (standard case) lines in the left

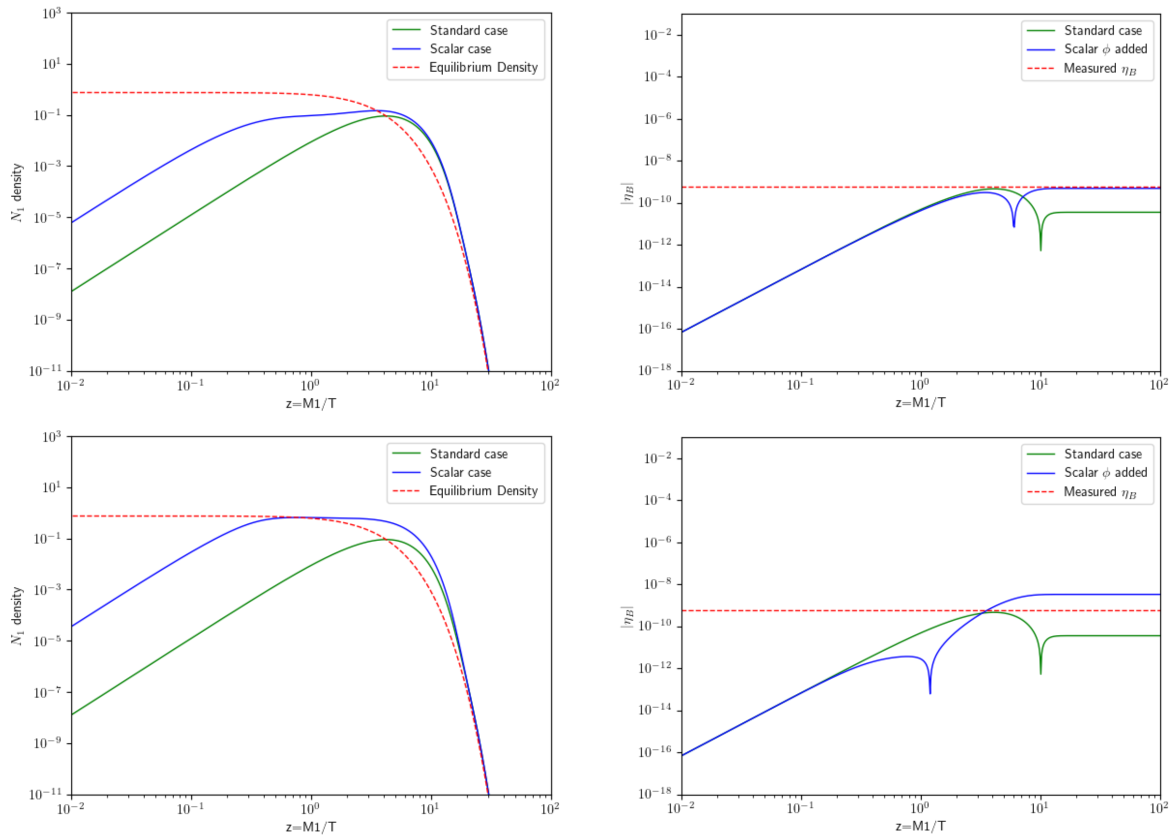



FIGURE 4.5: The four panels represent what happens for $K_1 = 0.1$ and $M_1 = 10^{14}$ GeV in the case of vanishing initial abundance for the RHN density. For the upper ones $K_\phi \simeq 20$, for the lower ones $K_\phi \simeq 150$. The behaviour of the lightest RHN number density is shown both in the standard case (green) and in the modified one (blue) is showed in the left panels. The BAU is showed in the right panels both for the standard (green) and the modified (blue) cases. All the figures are represented as a function of $z = M_1/T$. The dashed red lines represents the $N_1$ equilibrium number density (left) and the BAU today (right).

figure, thanks to the $\phi$ decay the $N_1$ number density is increased at smaller $z$ pointing towards the equilibrium distribution (red dashed line on the left). Notice that in both cases the RHN density is not able to reach the equilibrium distribution.

If we look to the figure on the right, the one which represents $\eta_B$, we can see that, differently from what happens in the strong washout regime, now the presence of the scalar field also modifies the prediction for the baryon asymmetry of the Universe. More precisely, we can see that, with this particular choice of the parameters, the prediction of our model is very close to the measured value of $\eta_B$ today Eq. (3.1), given by the dashed red line. A part from that, we have been able to increase the prediction for the baryon asymmetry of almost two order of magnitude. What happens if we further increase the decay strength?

### 4.12.2 The Case with $K_1 = 0.1$ and $K_\phi \simeq 150$

Now we look at what happens if we increase more the decay parameter $K_\phi$. We fix it to $K_\phi \simeq 150$ considering $m_\phi = 7M_1$ and $\mathrm{Y}_R^1 = 0.6$. The results are shown in the lower figures of Fig. (4.5).

What differs from the case studied in Subsec. (4.12.1) is that now the strength of the scalar decay is enough to take the $N_1$ right handed neutrinos at equilibrium with the thermal bath before $z = 1$. This leads to a strong increase in the $N_1$ number density with important consequences for the resulting baryon asymmetry of the Universe. In fact, looking to the picture on the right, one sees that now there is an overproduction of baryon asymmetry $\eta_B$. This means that the parameter space has been opened up: the right amount of baryon asymmetry can be reached lowering other parameters in the model responsible for CP violation, e.g lowering the scale of the right handed neutrino masses, or the CP violating phases.

When instead one looks at the opposite direction, trying to decrease the decay parameter $K_\phi$ considering it $\sim K_1$ or smaller, what happens is that we go back to the standard case in some sort of "continuous limit"; there is no significant change in the $N_1$ number density and in the prediction for the baryon asymmetry of the Universe $\eta_B$.

### 4.12.3 The Case with $K_1 = 0.01$ and $K_\phi \gg 1$

We want see here what happens when we lower the value of the washout parameter to $K_1 = 0.01$.

The results are shown in Fig. (4.6). What one sees is that the standard case prediction changes with respect to the case Subsec. (4.12.1). In particular, the one for the baryon asymmetry of the Universe is in this case lower of almost two orders of magnitude. Nevertheless, when considering the presence of the scalar $\phi$ and its decay, it turns out that we obtain a strong enhancement in the production of baryon asymmetry $\eta_B$, in particular reaching the measured amount of it for our specific choice of the decay parameter $K_\phi \simeq 20$ (upper figures in Fig. (4.6)). Increasing the value of the decay parameter to $K_\phi \simeq 200$ leads to a consistent thermalization of $N_1$'s in the Early Universe when $z < 1$ and to an overproduction of baryon asymmetry in the Universe, as shown in the lower two figures in Fig. (4.6). This again means that we are widening the space of the parameters where the type-I seesaw thermal LG is viable.

## 4.13 A Case with a Very Weak Washout

As last case for what concern the regime of vanishing initial abundance of the $N_1$ number density, we give also a look to a situation with a washout factor of $K_1 = 10^{-3}$, to see if

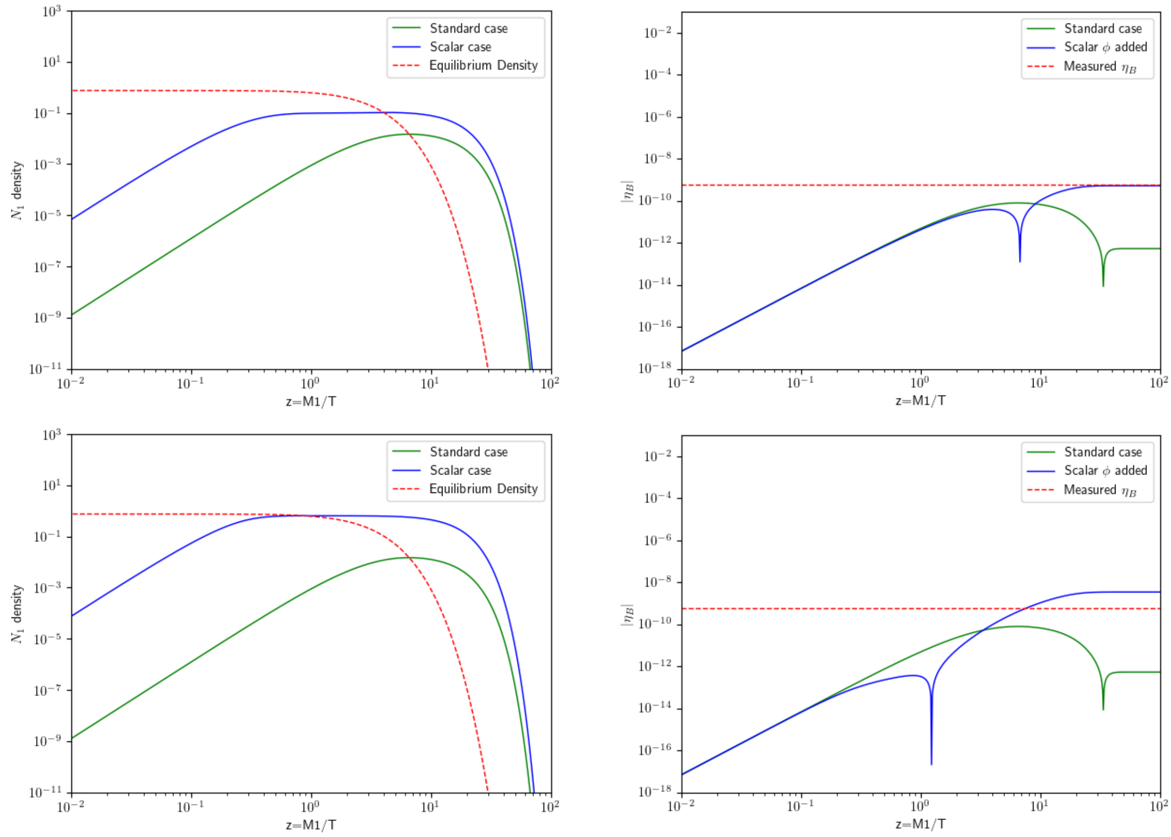



FIGURE 4.6: The four panels represent what happens for $K_1 = 0.01$ and $M_1 = 10^{14}$ GeV in the case of vanishing initial abundance for the RHN density. For the upper ones $K_\phi \simeq 20$, for the lower ones $K_\phi \simeq 200$. The behaviour of the lightest RHN number density is shown both in the standard case (green) and in the modified one (blue) is showed in the left panels. The BAU is showed in the right panels both for the standard (green) and the modified (blue) cases. All the figures are represented as a function of $z = M_1/T$. The dashed red lines represents the $N_1$ equilibrium number density (left) and the BAU today (right).

Leptogenesis can be also viable here and under which conditions. The results are shown in Fig. (4.7).
As we can see, the presence of $\phi$ again increases the $N_1$'s number density in the Early Universe but not enough to take them to the equilibrium density. But the most important consequence comes from the prediction for the baryon asymmetry of the Universe: while in the standard case lowering the washout parameter leads to lower and lower values for the predicted $\eta_B$, when we add the contribution of the $\phi$-decay we are able to match the measurements. And again, there is some part of the parameter space, mostly obtained by increasing $K_\phi$, where we can get an overproduction of baryon asymmetry.

In the weak washout scenario, our model modified with the addition of a massive scalar $\phi$ coupled to the right handed neutrinos is able to open the parameter space

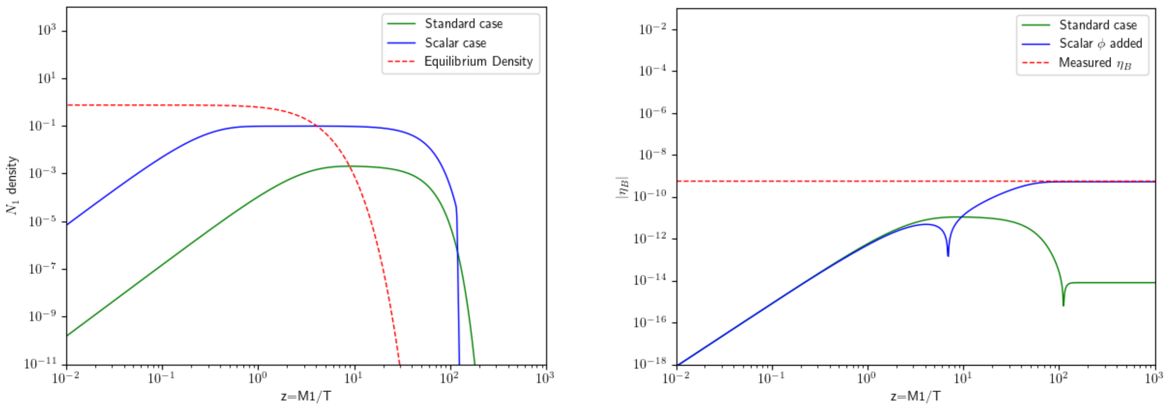



FIGURE 4.7: The figure shows the results for the case where $K_1 = 0.001$, $M_1 = 10^{14}$ and $K_\phi \simeq 20$ in the case of vanishing initial abundance for the RHN density. The left panel shows the lightest RHN number density both in the standard case (green) and in modified one (blue). The right panel shows the absolute value of the BAU, $|\eta_B|$, again in the two cases, the standard (green) and the modified one (blue). Both the quantities are represented as a function of $z = M_1/T$. The dashed red lines represent the $N_1$ equilibrium number density (left) and the measured BAU today (right).

where thermal LG is viable, and also it makes the process almost independent from the precise value of the washout parameter $K_1$ as long as it remains $< 1$.

## 4.14 The Case with Thermal Initial Abundance

So far we mainly focused on the case of thermal LG with zero initial abundance, i.e with $Y_{N_1}(0) = 0$. There is also an other situation which is usually considered in thermal LG, and it's the one where one uses as initial condition for the $N_1$ number density their equilibrium one, i.e $Y_{N_1}(z_i) = Y_{N_1}^{eq} = 3/4$.
We want to give a look to some cases also within this scenario to understand if something interesting and different from the standard case happens. Since it has turned out that differences from the standard case arises when the decay parameter $K_\phi \gg 1$, we focus here on that regime. The results are shown in Fig. (4.8), while other choice of the parameter space are presented in Append. (E.2). It turns out that in this situation, we are not even able to distinguish the standard case (green line) and the modified case (blue line) in the figures. This is because the main role of the scalar decay, i.e. bringing the $N_1$'s at their thermal equilibrium distribution, has become here useless since the right handed neutrino density is the equilibrium one from the beginning. As a consequence, nothing changes with respect to the standard picture also for the prediction of the baryon asymmetry of the Universe. For what concern the behaviour of the baryon asymmetry, in this case we do not see the dip any more; this time neutrinos are already at equilibrium

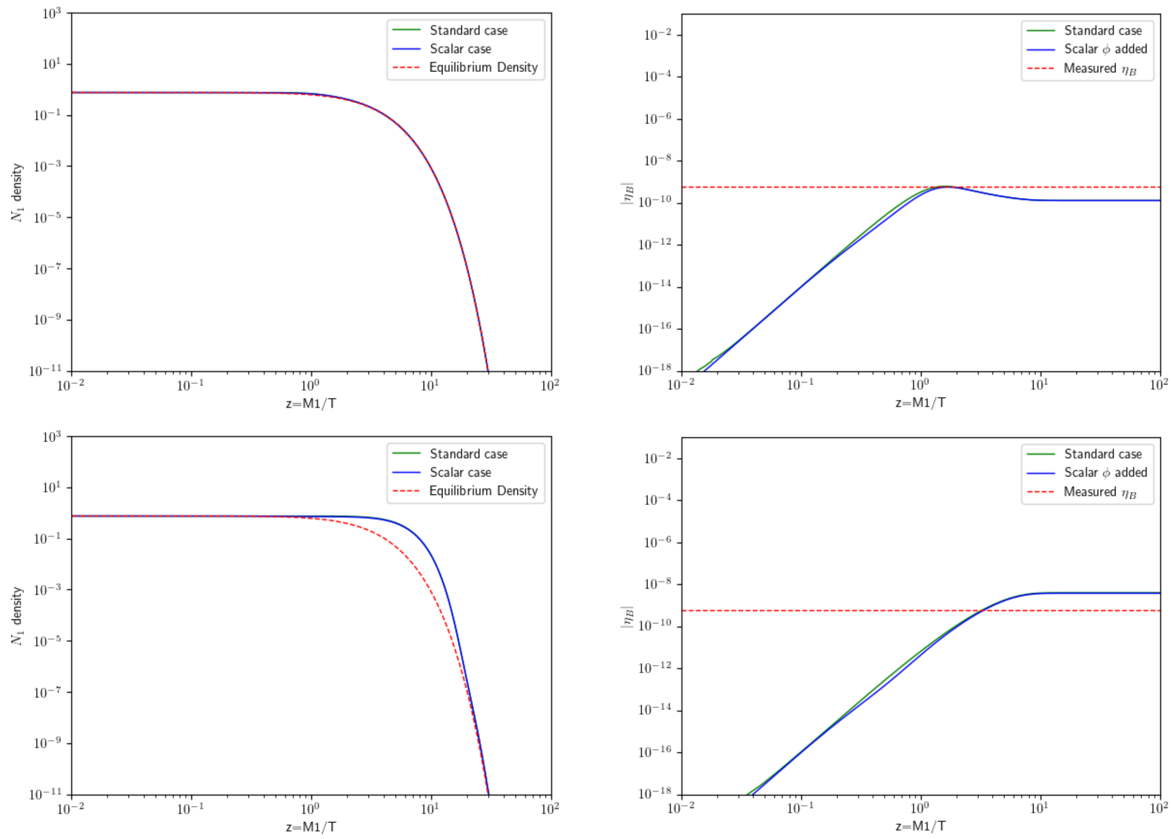



FIGURE 4.8: The four panels represent what happens for $M_1 = 10^{14}$ GeV in the case of thermal initial abundance for the RHN density. For the upper ones $K_1 = 10$ and $K_\phi \simeq 2 \cdot 10^2$, for the lower ones $K_1 = 0.1$ and again $K_\phi \simeq 2 \cdot 10^2$. The behaviour of the lightest RHN number density is shown both in the standard case (green) and in the modified one (blue) is showed in the left panels. The BAU is showed in the right panels both for the standard (green) and the modified (blue) cases. All the figures are represented as a function of $z = M_1/T$. The dashed red lines represents the $N_1$ equilibrium number density (left) and the BAU today (right).

and so change of sign happens between the period of their production in the thermal bath and the one of their out-of-equilibrium decays.
We also point out that, for the particular choice of the parameters in Fig. (4.8), the strong washout scenario leads to a baryon asymmetry which is smaller then the measured one, while the weak washout one leads to an overproduction of it. Nevertheless, also in this case it is possible to obtained the measured value for the BAU varying the parameters of the model.

## 4.15 Summary of the Results

We showed that the presence of the scalar decay $\phi \to N_1 N_1$ can have very interesting and important consequences on thermal LG and on the production of a baryon asymmetry in the Universe. In the strong washout scenario, due to the fact that every baryon asymmetry created for $z \ll 1$ is erased, the prediction of our model are not different from the standard scenario. Anyway, we showed that a modification in the $N_1$ number density occurs if the scalar is strongly coupled to it, i.e. when $K_\phi \gg 1$. Different is the case of the weak washout scenario. We showed that in situations where the standard picture is predicting a smaller value for the BAU, our addition of the scalar degrees of freedom is enough to solve the situation and obtain the right amount of asymmetry, or even an overproduction. More precisely if the standard picture predicts lower values of the asymmetry for lower values of the washout, then our modified model accounts for an enhancement in the BAU, depending on the value of $K_\phi$ and regardless of the value of the washout parameter $K_1$: as shown, with $K_\phi \simeq 20$, we obtain the right amount of BAU, $\eta_B$, while with $K_\phi \simeq 200$, we obtain an overproduction of it. The consequence of having an overproduction of BAU is that the right amount of it can be obtained by lowering other parameters in the theory, e.g. the RHNs mass scale. This happens because the scalar decay increases a lot the $N_1$ number density during the evolution of the Universe, this resulting in more out-of-equilibrium decays and, in a strong enhancement of the BAU, since, being in the weak washout regime, every asymmetry which is created is not erased by the inverse process. We showed also that, with the thermal initial abundance condition for $N_1$, almost nothing changes with respect to the standard picture, both in the strong and in the weak washout. This happens the scalar is not modifying at all the $N_1$ number density, which is the equilibrium one from the beginning.

# Chapter 5

# Conclusions and Outlooks

In this thesis we have studied the consequences on thermal LG of a model where, besides the SM particle content and symmetries, an arbitrary number of right handed neutrinos $\nu_R$'s, a complex scalar singlet S and a global $U(1)_{B-L}$ are added.
What we first wanted to do was to be able to generate right handed neutrinos masses dinamically thanks to the spontaneous breaking of the $U(1)_{B-L}$ symmetry. In this way one is able to account and predict the existence of active neutrino masses. The model has been mainly analyzed in Chapter 4 and what we found out is that, at energies above the EWSSB and below the $U(1)$ breaking, it predicts the existence of two new scalar degrees of freedom which interact with the right handed neutrinos $N_i$ and the SM Higgs. As expected, we have also been able to predict light neutrino masses through the standard seesaw relation, which now can be written as a function of the two different scales of symmetry breaking $v$ and $v_s$, and the Yukawa couplings $Y_D$ and $Y_R$, as shown in Eq. (4.31).

The new interactions have consequences on the LG process, which have been studied looking both at the CP asymmetry factor and at the Boltzmann equations. From the point of view of the CP asymmetry factor, even if there are new 1-loop diagrams which interfer with the tree level one giving new sources of CP violation, we showed that in our specific case the new contributions vanish, due to the lack of possibility of having mixing, mediated by the scalar, between RHNs, as shown in Sec. (4.7).
Something more interesting happens when we look at the consequences of the new interactions in the Boltzmann equations. We considered here the case where $m_\phi > 2M_i$, so that it is possible for the scalar to decay into a couple of the lightest right handed neutrinos during the evolution of the Universe. Also, we considered the scalar $\phi$ at equilibrium with the thermal bath in the early Universe. We looked then at some cases of the strong washout regime and of the weak one, mainly focusing on the scenario of vanishing initial abundance for the $N_1$'s. We found that no significant modifications to the BAU predictions are present in the strong washout regime, and this has to do with the main role of the scalar in the early Universe, which is to increase and/or thermalize the $N_1$

number density for $z \ll 1$; in the strong washout scenario anything that happens to the asymmetry before $z = 1$ is erased by the strong washout. Similar conclusions hold for the cases with thermal initial abundance assumed for the RHNs. Significant differences came out when looking at the weak washout regime when $K_\phi \gg 1$: in the standard case and for fixed $M_1$, the predicted value of BAU decreases with the washout factor, leading to obtain less asymmetry than observed. Instead, when we couple the scalar $\phi$, thanks to its decay, the population of $N_1$'s at early times increases and, even if it does not reach the equilibrium one, it gives origin to the right amount of baryon asymmetry, or even an overproduction of it. This happens beacause in the weak washout regime, also the asymmetry created for $z \ll 1$ gives its contribution to the relic abundance of $\eta_B$. Moreover, having been able to obtain an asymmetry overproduction means that we have been able to open up the parameter space: now one could obtain the right amount of baryon asymmetry changing other parameters in the theory and being less restrictive on CPV requirements.

Studying the parameter space for the Yukawa coupling between $N_1$ and $\phi$, $Y_R^1$, what one sees is that, to obtain the results described above, $Y_R^1 \geq 0.1$ is needed, considering a mass for the scalar of $4M_1 \leq m_\phi \leq 10M_1$.

So, we can conclude that the addition of a complex scalar particle S to the standard type I seesaw picture can, under the right condition, give us the correct amount of BAU or even an overproduction of it, opening the possibility of viable LG also by changing some of the other parameters in the theory.

At this point, it would be interesting to:

i. Try to include in the analysis of the Boltzmann Equations next to leading order contributions involving the scalars, for example $2 \longleftrightarrow 2$ scatterings both with the heavy scalar $\phi$ and with the Majoron $\theta$, similarly to what has been done in [18].

ii. See what happens in terms of Leptogenesis when the number of RHNs is fixed to $n = 3$ and the $B - L$ is considered local, i.e. a $U(1)_{B-L}$ gauge symmetry, since $U(1)_{B-L}$ is not an anomalous symmetry. Gauging $B - L$ means to predict a new massive gauge boson, for example, which can interact with the particles of the theory. Also, $U(1)_{B-L}$ SSB in the early Universe can be a first-order phase transition, with the consequent production of a stochastic gravitational wave radiation [101][102] which could be detected also by the next generation of interferometer, e.g. LISA [103]. And then one could also see under which conditions this model can be embedded into Unified models like Left-Right symmetric theories [46] [11] [12] or Grand Unified Theories.

iii. It could also be interesting to study what changes quantitatively in the CP asymmetry factor when one considers the presence of more than one scalar singlet in the theory, avoiding the possibility of diagonalizing at the same time the RHNs mass matrix and the Yukawa couplings between the scalars and the sterile neutrinos. This can be the case of Unified Theories, whose scalar sectors contains more than one scalar which is a singlet under the SM interactions [104].

iv. It can also be interesting to see what are the consequences of a model like this in the framework of LG via oscillations (ARS leptogenesis) [51], where the right handed neutrinos scale is much lower than the one considered here and the CP violation comes from a possible RHNs oscillating behaviour.

We leave this further improvements to future work.

# Appendix A

# Identities

## A.1 Robertson-Walker Metric and Christoffel Symbols

Christoffel symbols are quantities (not tensorial) which describes a metric connection. Their definition can be given in different ways, one way can be to define a coordinate system $x^\mu$ with correspondent tangent vectors $\frac{\partial}{\partial x^\mu}$ and a metric tensor $g_{\mu\nu} \equiv \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}$, then the Christoffel symbols can be written as

$$g_{\mu\sigma}\Gamma^\mu_{\nu\rho} \equiv \Gamma_{\sigma\nu\rho} = \frac{1}{2}(\partial_\rho g_{\sigma\nu} + \partial_\nu g_{\sigma\rho} - \partial_\sigma g_{\nu\rho}). \tag{A.1}$$

The Friedmann-Lemaitre-Robertson-Walker metric is instead defined as, in a space time with zero curvature,

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2) \tag{A.2}$$

where we considered a metric with a signature $(+, -, -, -)$, the speed of light $c = 1$ and $(t, x, y, z)$ are the time and spatial coordinates. $a(t)$ is instead the scale factor which describes the expansion of the UNiverse and which is related to the Hubble factor by $H \equiv \dot{a}/a$.

## A.2 Identities for the Majorana Fermions

$$N = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} a_p^s u_p^s e^{-ipx} + a_p^{s\dagger} v_p^s e^{ipx} \tag{A.3}$$

$$\bar{N} = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} a_p^{s\dagger} \bar{u}_p^s e^{ipx} + a_p^s \bar{v}_p^s e^{-ipx} \tag{A.4}$$

## A.3 $\gamma^\mu$'s traces identities

The following identities for the traces involving $\gamma^\mu$ matrices hold:

i. $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$

ii. $\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = -4i\epsilon^{\mu\nu\rho\sigma}$

iii. $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho] = \mathrm{Tr}[\gamma^\mu] = 0$

iv. $\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho] = \mathrm{Tr}[\gamma^5\gamma^\mu] = 0$

v. $\mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$

vi. $\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu] = 0$

# Appendix B

# Standard Leptogenesis Computations

## B.1 The Denominator of the CP Asymmetry Factor

The piece we need to compute is

$$2\sum_{\alpha}\int d\Pi_{l_\alpha\phi}(2\pi)^4\delta^4(P_i-P_f)|\mathcal{M}(i\to f)|^2 \tag{B.1}$$

Working in the CM frame, where $P_i=(E_i,\vec{p}_i)=(M_i,\vec{0})$ in the center of mass (CM) frame.

From (3.3.2), we can write,

$$\langle f|\,T\,|i\rangle=\langle L_\alpha H|-iy_D^{*\alpha i}\int d^4x\bar{L}_\alpha P_R\tilde{H}N_i\,|N_i\rangle\simeq -iy_D^{*\alpha i}\bar{u}_\alpha P_R u_i \tag{B.2}$$

where we gave as understood $(2\pi)^4\delta^4$ factors.
We can then identify $\mathcal{M}(i\to f)=-iy_D^{*\alpha i}\bar{u}_\alpha P_R u_i$ and $\mathcal{M}^\dagger(i\to f)=iy_D^{\alpha i}\bar{u}_i P_L u_\alpha$. Then,

$$\begin{aligned}\frac{1}{2}\sum_{spins}|\mathcal{M}(i\to f)|^2&=\frac{1}{2}|y_D^{\alpha i}|^2Tr\Big\{P_L\not{p}_\alpha P_R(\not{p}_i+M_i)\Big\}=\\&=\frac{1}{2}|y_D^{\alpha i}|^2\frac{1}{2}Tr\Big\{\not{p}_\alpha\not{p}_i\Big\}=\\&=|y_D^{\alpha i}|^2(p_\alpha\cdot p_i)=|y_D^{\alpha i}|^2\frac{M_i^2}{2},\end{aligned} \tag{B.3}$$

since from the kinematics we have that $p_\alpha^\mu=\left(\frac{M_i}{2},\frac{M_i}{2}\cos\alpha,\frac{M_i}{2}\sin\alpha\right)$.

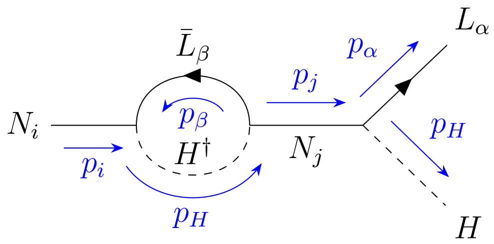
The integral then becomes

$$\begin{aligned}&\sum_{\alpha}|\mathcal{Y}_D^{\alpha i}|^2M_i^2\frac{1}{(2\pi)^2}\int\frac{d^3p_\alpha}{2E_\alpha}\frac{d^3p_H}{2E_H}\delta^3(\vec{p}_i-\vec{p}_\alpha-\vec{p}_H)\delta(M_i-E_\alpha-E_H)=\\&\sum_{\alpha}|\mathcal{Y}_D^{\alpha i}|^2M_i^2\frac{1}{(2\pi)^2}\int\frac{d^3p_\alpha}{4E_\alpha^2}\delta(M_i-2E_\alpha)=\frac{1}{8\pi}\sum_{\alpha}|\mathcal{Y}_D^{\alpha i}|^2M_i^2\end{aligned} \tag{B.4}$$

and its represent the value of the denominator.

### B.1.1 The Self-energy (or Wave Diagram) Contribution 1

We want to compute the contributions to the CP-asymmetry given by the interference between the tree level $N_i$ decay and the following one-loop diagram,




From (3.23) we need for the CP asymmetry the following amplitudes. $\langle k| T |f\rangle^*$ becomes $\langle H'\bar{L}_\beta| T |HL_\alpha\rangle^*$ and reads [cit. Peskin and Schroeder]

$$\langle H'\bar{L}_\beta| \int d^4x d^4y\, T\Big\{ \Big[-iy_{\beta j}\bar{N}_j P_L \tilde{H}^\dagger L_\beta\Big]_y \Big[-iy_{\alpha j}\bar{N}_j P_L \tilde{H}^\dagger L_\alpha\Big]_x \Big\} |HL_\alpha\rangle^* \tag{B.5}$$

The symbol $T$ into the integral represents the Time Ordering operator which, using Wick Theorem [20], can be transformed into a set of Normal Ordered products of operators plus their contractions; since Normal Ordered operator annihilates the vacuum, then only the contractions give a non-zero contribution to the amplitude of the process. Moreover, the contractions of the scalars $H$, $H'$ give only phase factors which enters in the conservation of 4-momentum after the spatial integrations. Noticing also that $\big[\bar{N}_j P_L L_\beta\big]_x = [\bar{N}_j P_L L_\beta]_x^T = L^T_{\beta,x} P_L^T \bar{N}^T_{j,x}$ and writing $\bar{N}_j^T = C^{-1}N_j$ where $C$ is the Charge conjugation operator and the relation holds for Majorana fermions, then we can see the propagator for the Right Handed Neutrinos appearing in its operatorial definition: $N_{j,x}\bar{N}_{j,y} \equiv S(q) = \frac{i}{\not{q}-M_j} = \frac{i(\not{q}+M_j)}{q^2-M_j^2}$.[20]

So, [1]

$$\begin{aligned}
\langle H'\bar{L}_\beta| T |HL_\alpha\rangle^* &\simeq -y^*_{\beta j}y^*_{\alpha j} \langle \bar{L}_\beta| L_x^T P_L^T C^{-1}\frac{i(\not{q}+M_j)}{q^2-M_j^2} P_L L_y |L_\alpha\rangle^* = \\
&= i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2-M_j^2} \langle \bar{L}_\beta| L_x^T P_L^T C^{-1}(\not{q}+M_j) P_L L_y |L_\alpha\rangle^* = \\
&= i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2-M_j^2} \big(v_\beta^T P_L^T C^{-1}(\not{q}+M_j) P_L u_\alpha\big)^* = \\
&= i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2-M_j^2} \big(\bar{u}_\beta C^T P_L^T C^{-1}(\not{q}+M_j) P_L u_\alpha\big)^* = \\
&= -i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2-M_j^2} \big(\bar{u}_\beta P_L(\not{q}+M_j) P_L u_\alpha\big)^*
\end{aligned} \tag{B.6}$$

[1] We are neglecting here the space integration, the integration over the q momenta in the definition of the propagator and in the result we give as understood $(2\pi)$ factors and $\delta(\sum p)$ reflecting the 4-momentum conservation in the process.

where we used the fact that $v = C\bar{u}^T$ which means that $v^T = \bar{u}C^T$ and the relations between $C$ and $\gamma^5$: $C\gamma^5 C^{-1} = (\gamma^5)^T$, i.e $C^{-1}(\gamma^5)^T C = \gamma^5$, and $C^T = C^{-1} = -C$. Also, the $\not{q}$ piece do not contribute since $P_L \not{q} = \not{q} P_R$ and $P_R P_L = 0$, so

$$\begin{aligned}
\left\langle H'\bar{L}_\beta\right| T \left|HL_\alpha\right\rangle^* &\simeq -i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}(\bar{u}_\beta P_L M_j P_L u_\alpha)^* = \\
&= -i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}(\bar{u}_\beta M_j P_L u_\alpha)^\dagger = \\
&= -i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}u^\dagger_\alpha P^\dagger_L M_j (\gamma^0)^\dagger u_\beta = \\
&= -i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}u^\dagger_\alpha M_j P_L \gamma^0 u_\beta = \\
&= -i\frac{y^*_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\bar{u}_\alpha M_j P_R u_\beta
\end{aligned} \tag{B.7}$$

The other two amplitudes we need to compute are $T_{if} \equiv \langle i| T |f\rangle$ and $T_{ki} \equiv \langle k| T |i\rangle$:

$$\begin{aligned}
T_{ki} = \left\langle H'\bar{L}_\beta\right| T \left|N_i\right\rangle &= \left\langle H'\bar{L}_\beta\right| \int d^4x T\left\{\left[-i\mathcal{Y}_{\beta i}\bar{N}_i P_L H^\dagger L_\beta\right]_x\right\} \left|N_i\right\rangle \simeq \\
\simeq \left\langle \bar{L}_\beta\right| - iy_{\beta i}\bar{N}_i P_L L_\beta \left|N_i\right\rangle &= -iy_{\beta i}\left\langle \bar{L}_\beta\right| L^T_\beta P^T_L C^{-1} N_i \left|N_i\right\rangle = \\
= -iy_{\beta i}v^T_\beta P^T_L C^{-1}u_i &= -iy_{\beta i}\bar{u}_\beta C^T P^T_L C^{-1}u_i = iy_{\beta i}\bar{u}_\beta P_L u_i
\end{aligned} \tag{B.8}$$

and

$$\begin{aligned}
T_{if} = \langle N_i| T \left|L_\alpha H\right\rangle &= \langle N_i| \int d^4x T\left\{\left[-iy_{\alpha i}\bar{N}_i P_L H^\dagger L_\alpha\right]_x\right\} \left|L_\alpha H\right\rangle \simeq \\
&\simeq -iy_{\alpha i}\bar{u}_i P_L u_\alpha
\end{aligned} \tag{B.9}$$

Now we need to multiply the three different pieces and to consider the sum over the final spin states and the average over the initial spin states, that is

$$\begin{aligned}
&\frac{1}{2}\sum_{spins}(-iy_{\alpha i}\bar{u}_i P_L u_\alpha)\left(-iy^*_{\beta j}y^*_{\alpha j}\bar{u}_\alpha P_R \frac{M_j}{q^2 - M_j^2}u_\beta\right)(iy_{\beta i}\bar{u}_\beta P_L u_i) = \\
&= \frac{1}{2}\sum_{spins}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{q^2 - M_j^2}M_j(-i)(\bar{u}_i P_L u_\alpha \bar{u}_\alpha P_R u_\beta \bar{u}_\beta P_L u_i) = \\
&= \frac{1}{2}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{q^2 - M_j^2}M_j(-i)\,\mathrm{Tr}\left\{\bar{u}_i P_L \not{p}_\alpha P_R \not{p}_\beta P_L u_i\right\} = \\
&= \frac{1}{2}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{q^2 - M_j^2}M_j(-i)\,\mathrm{Tr}\left\{P_L \not{p}_\alpha \not{p}_\beta (\not{p}_i + M_i)\right\}
\end{aligned} \tag{B.10}$$

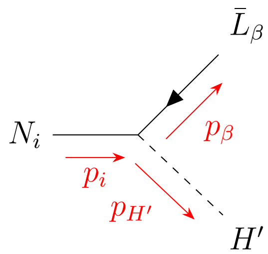
where we used that $\sum_{spins} u(p)\bar{u}(p) = \not{p} + m$ and the ciclicity of the trace operator. Now, writing explicitly $P_L = \frac{1}{2}(1 - \gamma^5)$ and considering trace identities involving $\gamma^\mu$ matrices, it all becomes

$$
\begin{aligned}
&\left(\frac{-i}{4}\right)\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{q^2-M_j^2}M_j\,\mathrm{Tr}\Big\{(1-\gamma^5)\not{p}_\alpha\not{p}_\beta(\not{p}_i+M_i)\Big\} = \\
&=\left(\frac{-i}{4}\right)\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{q^2-M_j^2}M_jM_i4(p_\alpha\cdot p_\beta) = \\
&=(-i)\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{q^2-M_j^2}M_jM_i(p_\alpha\cdot p_\beta),
\end{aligned}
\tag{B.11}
$$

since all the traces with odd $\gamma^\mu$ matrices, odd $\gamma^\mu$'s and a $\gamma^5$ and 2 $\gamma^\mu$'s and a $\gamma^5$ vanishes. Now we need to evaluate the results in (B.11) using kinematics.

### The Phase Space of the Process

We use 4-momentum conservation to find the relation for the 4-momentum of the particles in a specific reference frame, i.e the center of mass frame (CM frame), and then we use them to evaluate the Lorentz invariant quantity computed in (B.25). Since the loop particles must be on-shell, we can evaluate the kinematics for all the three different processes. We start from,




In the CM frame we have

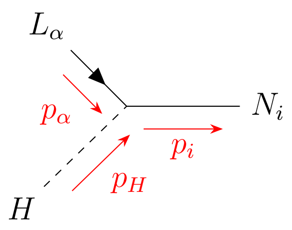
$$p_i^\mu = (M_i, \vec{0});$$

this means that $E_\beta + E_{H'} = E_i = M_i$ and $\vec{p}_\beta + \vec{p}_{H'} = \vec{p}_i = \vec{0}$. We can then use the relativistic energy-momentum relation, which holds for on-shell particles, and write $E_\beta = \sqrt{|\vec{p}_\beta|^2 + m_\beta^2} = |\vec{p}_\beta|$ and $E_{H'} = \sqrt{|\vec{p}_{H'}|^2 + m_{H'}^2} = |\vec{p}_{H'}| = |\vec{p}_\beta|$ since we are in the unbroken phase for the Higgs potential and both the masses of the lepton and the Higgs are zero. At this point we turn out with

$$p_\beta^\mu = \left(\frac{M_i}{2}, \vec{p}_\beta\right)$$

$$p_{H'}^\mu = \left(\frac{M_i}{2}, -\vec{p}_\beta\right)$$

The same holds for the diagram $L_\alpha H \to N_i$,

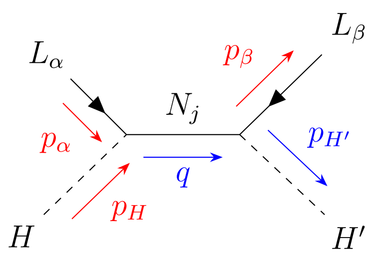



where $E_\alpha = |\vec{p}_\beta|$ and $E_H = |\vec{p}_H| = |\vec{p}_\alpha|$ so we can write

$$p^\mu_\alpha = \left(\frac{M_i}{2}, \vec{p}_\alpha\right)$$

$$p^\mu_H = \left(\frac{M_i}{2}, -\vec{p}_\alpha\right).$$

Now, the only thing that remain to explicit is the 4-momentum of the particle exchanged in the s-channel of the third diagram, i.e. $L_\alpha H \to \bar{L}_\beta H'$,




for which it holds that $q^\mu = p^\mu_\alpha + p^\mu_H = p^\mu_\beta + p^\mu_{H'} = \left(M_i, \vec{0}\right)$. Substituting these relations in (B.11) it becomes,

$$\begin{aligned} &(-i)\frac{y_{\alpha i} y^*_{\beta j} y^*_{\alpha j} y_{\beta i}}{M_i^2 - M_j^2} M_i M_j \left(\frac{M_i^2}{4} - \vec{p}_\alpha \cdot \vec{p}_\beta\right) = \\ &= (-i)\frac{y_{\alpha i} y^*_{\beta j} y^*_{\alpha j} y_{\beta i}}{M_i^2 - M_j^2} \frac{M_i^3 M_j}{4}(1 - \cos\theta_{\alpha\beta}), \end{aligned} \tag{B.12}$$

considering that $q^2 = M_i^2$ and $\vec{p}_\beta \cdot \vec{p}_\alpha = \frac{M_i^2}{4}\cos\theta_{\alpha\beta}$, where $\theta_{\alpha\beta}$ is the spatial angle [2] between $L_\alpha$ and $L_\beta$. What we are missing now to find our result is the integration over the phase space of the process.

### The Phase Space Integration

As we can see from (3.27), to obtain the value of the CP asymmetry factor we need to perform first the phase space integration over the intermediate states, i.e over $d\Pi_{L_\beta,H'}$,

[2] We have spherical symmetry and so the process can be described spatially only in terms of this angle.

and then over the one of the final states, i.e over $d\Pi_{L_\alpha,H}$. First, let's look at

$$\begin{aligned}
&\int d\Pi_{L_\beta,H'}(-i)\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{4}(1-\cos\theta_{\alpha\beta}) = \\
&= \int \frac{d^3p_\beta}{(2\pi)^3 2E_\beta}\frac{d^3p_{H'}}{(2\pi)^3 2E_{H'}}(2\pi)^4\delta^4(p_i^\mu - p_\beta^\mu - p_{H'}^\mu)\cdot \\
&\cdot (-i)\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{4}(1-\cos\theta_{\alpha\beta}) = \\
&= \int \frac{d^3p_\beta}{(2\pi)^3 2E_\beta}\frac{d^3p_{H'}}{(2\pi)^3 2E_{H'}}(2\pi)^4\delta^3(\vec{p}_\beta + \vec{p}_{H'})\delta(E_i - E_{H'} - E_\beta)\cdot \\
&\cdot (-i)\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{4}(1-\cos\theta_{\alpha\beta})
\end{aligned} \tag{B.13}$$

where we used the definition of $d\Pi_i$ and of the 4D $\delta^4$. We can first perform the integration of the $\delta^3(\vec{p}_\beta + \vec{p}_{H'})$ which fixes $\vec{p}_{H'} = -\vec{p}_\beta$ and as a consequence $E_{H'} = E_\beta$. We obtain,

$$\frac{-i}{(2\pi)^2}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{4}\int\frac{d^3p_\beta}{4E_\beta^2}\delta^0(E_i - 2E_\beta)(1-\cos\theta_{\alpha\beta}) \tag{B.14}$$

We can perform a rotation of the reference frame to have one of the axis aligned with the emission direction of the $L_\alpha$ state; in this way $\cos\theta_{\alpha\beta} \to \cos\theta_\beta$ and the integral contains variables related to $L_\beta$ only. We also pass to spherical coordinates $|p_\beta| \equiv p_\beta$, $\theta_\beta$ and $\phi_\beta$, or equivalently using the solid angle $\Omega_\beta$, with $d\Omega = d\phi_\beta d\cos\theta_\beta$

$$\begin{aligned}
&\frac{-i}{(2\pi)^2}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{4}\int\frac{d\Omega_\beta p_\beta^2 dp_\beta}{4E_\beta^2}\delta^0(E_i - 2E_\beta)(1-\cos\theta_\beta) \\
&\frac{-i}{(2\pi)^2}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{4}\int\frac{d\phi_\beta d\cos\theta_\beta p_\beta^2 dp_\beta}{4E_\beta^2}\delta^0(E_i - 2E_\beta)(1-\cos\theta_\beta)
\end{aligned} \tag{B.15}$$

With a change of variable in the integral, using the fact that $E_\beta = p_\beta$ and so $dE_\beta = dp_\beta$, we obtain

$$\begin{aligned}
&\frac{-i}{(2\pi)^2}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{4}(2\pi)\int_0^\infty\int_{-1}^{+1} d\cos\theta_\beta dE_\beta\frac{1}{4}\delta^0(E_i - 2E_\beta)(1-\cos\theta_\beta) = \\
&= \frac{-i}{2\pi}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}\frac{M_i^3M_j}{16}\left(\int_0^\infty\int_{-1}^{+1} d\cos\theta_\beta dE_\beta\delta^0(E_i - 2E_\beta) + \right. \\
&\left. - \int_0^\infty\int_{-1}^{+1} d\cos\theta_\beta dE_\beta\delta^0(E_i - 2E_\beta)\cos\theta_\beta\right) = \frac{-i}{32\pi}\frac{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}}{M_i^2-M_j^2}M_i^3M_j
\end{aligned} \tag{B.16}$$

since the second integral vanishes and the first gives a factor of 1/2.

Now what is missing is the integration over $d\Pi_{L_\alpha,H}$, which, after taking care of a minus sign and a factor of 2, becomes

$$\begin{aligned}\int d\Pi_{L_\alpha,H}\frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}\}}{M_i^2-M_j^2}M_i^3M_j = \int \frac{d^3p_\alpha}{(2\pi)^3 2E_\alpha}\frac{d^3p_H}{(2\pi)^3 2E_H}\cdot\\ \cdot(2\pi)^4\delta^4(p_i^\mu-p_\alpha^\mu-p_H^\mu)\frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}\}}{M_i^2-M_j^2}M_i^3M_j.\end{aligned} \tag{B.17}$$

Taking out from the integral everything that does not depend on $p_\alpha$ and $p_H$ and integrating the $\delta$'s, we obtain

$$\begin{aligned}\frac{1}{(2\pi)^2}\frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}\}}{M_i^2-M_j^2}M_i^3M_j\int\frac{d^3p_\alpha}{4E_\alpha^2}\delta^0(E_i-2E_\alpha) =\\ =\left(\frac{1}{8\pi}\right)\frac{1}{16\pi}\mathrm{Im}\{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}\}M_i^2\frac{M_j}{M_i}\frac{1}{1-M_j^2/M_i^2}\end{aligned} \tag{B.18}$$

where passing from the first line to the second one again we passed to spherical coordinates and performed a change of variables using $E_\alpha = |p_\alpha| \equiv p_\alpha$. Considering now the denominator from (B.4) and defining $z \equiv M_j^2/M_i^2$, we finally obtain

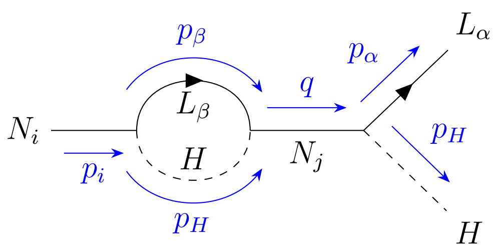
$$\epsilon_{i\alpha}^{wave-1} = \frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y^*_{\beta j}y^*_{\alpha j}y_{\beta i}\}}{\sum_\alpha |y_{\alpha i}|^2}\frac{\sqrt{z}}{1-z} \tag{B.19}$$

This result is in agreement with the one obtained in [88] if one takes into account the two different degrees of freedom of the lepton doublet, i.e if one multiplies the result for a factor of 2. one considers the factor of 2 and also the sum over the flavours $\beta$ for the lepton running in the loop and the one over the flavours $j$ for the right handed neutrino $N_j$ in the propagator, considering $j \neq i$, the result becomes

$$\epsilon_{i\alpha}^{wave-1} = \frac{1}{8\pi}\frac{1}{(\mathrm{Y}^\dagger\mathrm{Y})_{ii}}\sum_{j\neq i}\mathrm{Im}\Big\{\big(\mathrm{Y}^\dagger\mathrm{Y}\big)_{ji}\mathrm{Y}_{\alpha i}\mathrm{Y}^*_{\alpha j}\Big\}\frac{\sqrt{z}}{1-z} \tag{B.20}$$

### B.1.2 The Self-Energy (or Wave Diagram) Contribution 2

Now, the loop diagram we need to consider is



and we need to compute $T_{if}$, $T_{ki}$ and $T^*_{kf}$ again, which in this case are $\langle N_i| T |L_\alpha H\rangle$, $\langle L_\beta H'| T |N_i\rangle$ and $\langle L_\beta H'| T |L_\alpha H\rangle^*$. $\langle N_i| T |L_\alpha H\rangle$ has already been computed in (B.1.1), while the other two pieces need a more detailed analysis.

Let's start from computing $\langle L_\beta H'| T |N_i\rangle$:

$$\begin{aligned}\langle L_\beta H'| T |N_i\rangle &= \langle L_\beta H'| \int d^4x \mathrm{T}\Big\{[-iy^*_{\beta i}\bar{L}_\beta P_R \tilde{H} N_i]_x\Big\} |N_i\rangle \simeq \\ &\simeq \langle L_\beta| - iy^*_{\beta i}\bar{L}_\beta P_R N_i |N_i\rangle = -iy^*_{\beta i}\bar{u}_\beta P_R u_i\end{aligned} \tag{B.21}$$

and then $\langle L_\beta H'| T |L_\alpha H\rangle^*$:

$$\begin{aligned}\langle L_\beta H'| T |L_\alpha H\rangle^* = \langle L_\beta H'|& \\ &\int d^4x d^4y \mathrm{T}\Big\{[-iy^*_{\beta j}\bar{L}_\beta P_R\tilde{H}N_j]_y[-iy_{\alpha j}\bar{N}_j P_L \tilde{H} L_\alpha]_x\Big\} |L_\alpha H\rangle^* \simeq \\ &\langle L_\beta H'| [-iy^*_{\beta j}\bar{L}_\beta P_R\tilde{H}N_j]_y[-iy_{\alpha j}\bar{N}_j P_L \tilde{H} L_\alpha]_x |L_\alpha H\rangle^* = \\ &= \langle L_\beta| (-i)^2 y^*_{\beta j} y_{\alpha j}\bar{L}_\beta P_R N_{j,y}\bar{N}_{j,x} P_L L_\alpha |L_\alpha\rangle^* = \\ &= -y_{\beta j}y^*_{\alpha j}\langle L_\beta| \bar{L}_\beta P_R \frac{i(\not{q} + M_j)}{q^2 - M_j^2} P_L L_\alpha |L_\alpha\rangle^* = \\ &= -\frac{y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\langle L_\beta| \bar{L}_\beta P_R i(\not{q} + M_j) P_L L_\alpha |L_\alpha\rangle^*\end{aligned} \tag{B.22}$$

considering that $P_R\not{q} = \not{q}P_L$ and $P_R P_L = 0$,

$$\begin{aligned}\langle L_\beta H'| T |L_\alpha H\rangle^* &= \frac{(+i)y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\langle L_\beta| \bar{L}_\beta \not{q} P_L L_\alpha |L_\alpha\rangle^* = \\ &= \frac{(+i)y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\left(\bar{u}_\beta \not{q} P_L u_\alpha\right)^* = \frac{(i)y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\left(\bar{u}_\beta \not{q} P_L u_\alpha\right)^\dagger = \\ &= \frac{(i)y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\Big(u^\dagger_\alpha P^\dagger_L \not{q}^\dagger (\gamma^0)^\dagger u_\beta\Big) = \frac{(i)y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\left(u^\dagger_\alpha P_L \gamma^0 \not{q} u_\beta\right) = \\ &= \frac{(i)y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\bar{u}_\alpha P_R \not{q} u_\beta\end{aligned} \tag{B.23}$$

since $\not{q}^\dagger(\gamma^0)^\dagger = \not{q}^\dagger\gamma^0 = \gamma^0\not{q}$. Again, multiplying the three different pieces and considering the sum over the final spin states and the average over the initial spin states, that is

$$\begin{aligned}&\frac{1}{2}\sum_{spins}(-iy_{\alpha i}\bar{u}_i P_L u_\alpha)\left(\frac{(i)y_{\beta j}y^*_{\alpha j}}{q^2 - M_j^2}\bar{u}_\alpha P_R \not{q} u_\beta\right)\left(-iy^*_{\beta i}\bar{u}_\beta P_R u_i\right) = \\ &= \left(\frac{-i}{2}\right)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{q^2 - M_j^2}\,\mathrm{Tr}\Big\{\bar{u}_i P_L \not{p}_\alpha P_R \not{q} \not{p}_\beta P_R u_i\Big\} = \\ &= \left(\frac{-i}{2}\right)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{q^2 - M_j^2}\,\mathrm{Tr}\Big\{P_L \not{p}_\alpha \not{q} \not{p}_\beta (\not{p}_i + M_i)\Big\}\end{aligned} \tag{B.24}$$

where we used that $\sum_{spins} u(p)\bar{u}(p) = \not{p} + m$ and the ciclicity of the trace operator. Now, writing explicitly $P_L = \frac{1}{2}(1-\gamma^5)$ and considering trace identities involving $\gamma^\mu$ matrices, it all becomes

$$\begin{aligned}
&\left(\frac{-i}{2}\right)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{q^2-M_j^2}\frac{1}{2}\,\mathrm{Tr}\Big\{(1-\gamma^5)\not{p}_\alpha\not{q}\not{p}_\beta\not{p}_i + (1-\gamma^5)\not{p}_\alpha\not{q}\not{p}_\beta M_i\Big\} = \\
&= \left(\frac{-i}{4}\right)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{q^2-M_j^2} p_{\alpha,\mu}q_\nu p_{\beta,\rho}p_{i,\sigma}(4g^{\mu\nu}g^{\rho\sigma} + 4g^{\mu\sigma}g^{\rho\nu} - 4g^{\mu\rho}g^{\nu\sigma}) = \\
&= (-i)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{q^2-M_j^2}((p_\alpha\cdot q)(p_\beta\cdot p_i) + (p_\alpha\cdot p_i)(p_\beta\cdot q) - (p_\alpha\cdot p_\beta)(p_i\cdot q)),
\end{aligned} \tag{B.25}$$

since all the traces with an odd number of $\gamma$ matrices vanishes and so the traces with an odd number of $\gamma$'s plus a $\gamma^5$ matrix. The piece related to $\mathrm{Tr}\{\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\}$ gives a contribution $\propto \epsilon^{\mu\nu\rho\sigma}$[3] and it also vanishes for symmetry reasons. Now we need to evaluate the results in (B.25) using kinematics.

### The Kinematics of the Process

As before, we use 4-momentum conservation to find the relation for the 4-momentum of the particles in the (CM frame), and then we use them to evaluate the Lorentz invariant quantity computed in (B.25). The phase space analysis for this second wave diagram contribution to the CP asymmetry is analogous to the one carried out in (B.1.1), so we can skip it and go directly to the results.

It holds that,

$$\begin{aligned}
p_i^\mu &= (M_i, \vec{0}); \\
p_\beta^\mu &= \left(\frac{M_i}{2}, \vec{p}_\beta\right); \\
p_{H'}^\mu &= \left(\frac{M_i}{2}, -\vec{p}_\beta\right); \\
p_\alpha^\mu &= \left(\frac{M_i}{2}, \vec{p}_\alpha\right) \\
p_H^\mu &= \left(\frac{M_i}{2}, -\vec{p}_\alpha\right).
\end{aligned}$$

and again for the s-channel diagram $q^\mu = p_\alpha^\mu + p_H^\mu = p_\beta^\mu + p_{H'}^\mu = \left(M_i, \vec{0}\right)$ and so $q^2 = M_i^2$.

[3] $\epsilon^{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor.

Substituting these relations in (B.25) it becomes,

$$\begin{aligned}&(-i)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{M_i^2-M_j^2}\left(\frac{M_i^4}{4}+\frac{M_i^4}{4}-M_i^2\left(\frac{M_i^2}{4}-\vec{p_\beta}\cdot\vec{p_\alpha}\right)\right)=\\&=(-i)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{M_i^2-M_j^2}\frac{M_i^4}{4}(1+\cos\theta_{\alpha\beta}),\end{aligned}\tag{B.26}$$

where again $\vec{p_\beta}\cdot\vec{p_\alpha}=\frac{M_i^2}{4}\cos\theta_{\alpha\beta}$ and $\theta_{\alpha\beta}$ is the angle between the two leptons $L_\alpha$ and $L_\beta$. Now we need to perform the phase space integration as done in (B.1.1).

### The Phase Space Integration

Again, we need to perform the two phase space integration, the one over the phase space of the intermediate states $L_\beta$ and $H'$ and the one over the phase space of the final states $L_\alpha$ and $H$. The computation are very close to the one carried on in (B.1.1). The integration over the intermediate states gives us

$$\begin{aligned}&\int d\Pi_{L_\beta,H'}(-i)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{M_i^2-M_j^2}\frac{M_i^4}{4}(1+\cos\theta_{\alpha\beta})=\\&=\int\frac{d^3p_\beta}{(2\pi)^3 2E_\beta}\frac{d^3p_{H'}}{(2\pi)^3 2E_{H'}}(2\pi)^4\delta^4(p_i^\mu-p_\beta^\mu-p_{H'}^\mu)(-i)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{M_i^2-M_j^2}\frac{M_i^4}{4}(1+\cos\theta_{\alpha\beta})\end{aligned}\tag{B.27}$$

taking out from the integral everything that does not depend on the integration variables and since the $\cos\theta_{\alpha\beta}$ gives again zero contribution,

$$\begin{aligned}&\frac{1}{(2\pi)^2}(-i)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{M_i^2-M_j^2}\frac{M_i^4}{4}\int\frac{d^3p_\beta}{2E_\beta}\frac{d^3p_{H'}}{2E_{H'}}\delta^4(p_i^\mu-p_\beta^\mu-p_{H'}^\mu)(1+\cos\theta_{\alpha\beta})=\\&=\frac{1}{(2\pi)^2}(-i)\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{M_i^2-M_j^2}\frac{M_i^4}{4}\cdot\left(\frac{\pi}{2}\right)=\frac{-i}{32\pi}M_i^4\frac{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}}{M_i^2-M_j^2}\end{aligned}\tag{B.28}$$

And then the integration over the final states $L_\alpha$ and $H$:

$$\begin{aligned}&\int d\Pi_{L_\alpha,H}\frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}\}}{M_i^2-M_j^2}M_i^4=\frac{1}{(2\pi)^2}\frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}\}}{M_i^2-M_j^2}M_i^4\cdot\\&\cdot\int\frac{d^3p_\alpha}{2E_\alpha}\frac{d^3p_H}{2E_H}\delta^4(p_i^\mu-p_\alpha^\mu-p_H^\mu)=\frac{1}{(2\pi)^2}\frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}\}}{M_i^2-M_j^2}M_i^4\cdot\left(\frac{\pi}{2}\right)=\\&=\left(\frac{1}{8\pi}\right)\frac{1}{16\pi}\frac{\mathrm{Im}\{y_{\alpha i}y_{\beta j}y^*_{\alpha j}y^*_{\beta i}\}}{M_i^2-M_j^2}M_i^4\end{aligned}\tag{B.29}$$

Considering then the denominator from (B.4) we find the final result

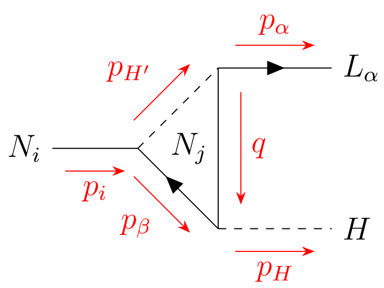
$$\epsilon_{i\alpha}^{wave-2} = \frac{1}{16\pi} \frac{\mathrm{Im}\{y_{\alpha i} y_{\beta j} y^*_{\alpha j} y^*_{\beta i}\}}{\sum_\alpha |y_{\alpha i}|^2} \frac{1}{1-z}, \tag{B.30}$$

where again $z \equiv M_j^2/M_i^2$. Again, if one considers the factor of 2 and also the sum over the flavours $\beta$ for the lepton running in the loop and the one over the flavours $j$ for the right handed neutrino $N_j$ in the propagator, considering $j \neq i$, the result becomes

$$\epsilon_{i\alpha}^{wave-2} = \frac{1}{8\pi} \frac{1}{(\mathrm{Y}^\dagger \mathrm{Y})_{ii}} \sum_{j \neq i} \mathrm{Im}\left\{ \left(\mathrm{Y}^\dagger \mathrm{Y}\right)_{ij} \mathrm{Y}_{\alpha i} \mathrm{Y}^*_{\alpha j} \right\} \frac{1}{1-z} \tag{B.31}$$

### B.1.3 The Vertex Diagram Contribution

The vertex contribution to the CP asymmetry is given by the interference between the tree-level $N_i$ decay to $L_\alpha$ and $H$ and the following triangular diagram.




The various amplitudes read:

$$\left\langle H' \bar{L}_\beta \right| T \left| H L_\alpha \right\rangle^* =$$

$$\left\langle H' \bar{L}_\beta \right| \int d^4x d^4y T \left\{ \left[ -i y_{\beta j} \bar{N} P_L H^\dagger L \right]_y \left[ -i y_{\alpha j} \bar{N} P_L H^\dagger L \right]_x \right\} \left| H L_\alpha \right\rangle^*,$$

using Wick Theorem we remain only with the Normal Ordered products of operators and their contractions. Since Normal Ordered operator annihilates the vacuum, then we can consider only the contractions. The contraction of the scalars $H$, $H'$ give only phase factors which enters in the conservation of 4-momentum after the performing the spatial integrations. Noticing that $\left[\bar{N}_j P_L L\right]_x = [\bar{N}_j P_L L]_x^T = L_x^T P_L^T \bar{N}_{j,x}^T$ and writing $\bar{N}_j^T = C^{-1} N_j$ where $C$ is the Charge conjugation operator, then we can see the propagator for the Right Handed Neutrinos appear as an operator $N_{j,x} \bar{N}_{j,y} \equiv S(q) = \frac{i}{\not{q} - M_j} = \frac{i(\not{q}+M_j)}{q^2 - M_j^2}$. So, [4]

[4] We are neglecting here the space integration, the integration over the q momenta in the definition of the propagator and in the result we give as understood $(2\pi)$ factors and $\delta(\sum p)$ reflecting the 4-momentum conservation in the process.

$$\langle H'\bar{L}_\beta | T | HL_\alpha\rangle^* \simeq$$

$$\simeq y^*_{\beta j} y^*_{\alpha j} \langle \bar{L}_\beta | L^T_x P^T_L \frac{i(\not{q} + M_j)}{q^2 - M^2_j} P_L L_y | L_\alpha\rangle^* =$$

$$= -i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j} \langle \bar{L}_\beta | L^T_x P^T_L C^{-1}(\not{q} + M_j) P_L L_y | L_\alpha\rangle^* =$$

$$= -i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j} \left(v^T_\beta P^T_L C^{-1}(\not{q} + M_j) P_L u_\alpha\right)^* =$$

$$= -i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j} \left(\bar{u}_\beta C^T P^T_L C^{-1}(\not{q} + M_j) P_L u_\alpha\right)^* =$$

$$= i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j} \left(\bar{u}_\beta P_L(\not{q} + M_j) P_L u_\alpha\right)^*,$$

where we used the fact that $v = C\bar{u}^T$ which means that $v^T = \bar{u}C^T$ and the relations between $C$ and $\gamma^5$: $C\gamma^5 C^{-1} = (\gamma^5)^T$, i.e $C^{-1}(\gamma^5)^T C = \gamma^5$, and $C^T = C^{-1} = -C$. Also, the $\not{q}$ piece do not contribute since $P_R P_L = 0$, so

$$\langle H'\bar{L}_\beta | T | HL_\alpha\rangle^* \simeq i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j}(\bar{u}_\beta P_L M_j P_L u_\alpha)^* =$$

$$= i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j}(\bar{u}_\beta P_L M_j u_\alpha)^\dagger = -i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j} u^\dagger_\alpha M_j P^\dagger_L (\gamma^0)^\dagger u_\beta =$$

$$= -i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j} u^\dagger_\alpha M_j P_L \gamma^0 u_\beta = -i\frac{y^*_{\beta j} y^*_{\alpha j}}{q^2 - M^2_j} \bar{u}_\alpha M_j P_R u_\beta$$

Then we need

$$\langle H'\bar{L}_\beta | T | N_i\rangle = \langle H'\bar{L}_\beta | \int d^4x T\left\{\left[-iy_{\beta i}\bar{N}_i P_L H^\dagger L_\beta\right]_x\right\} | N_i\rangle \simeq$$

$$\simeq \langle \bar{L}_\beta | -iy_{\beta i}\bar{N}_i P_L L_\beta | N_i\rangle = -iy_{\beta i}\langle \bar{L}_\beta | L^T_\beta P^T_L C^{-1} N_i | N_i\rangle = -iy_{\beta i} v^T_\beta P^T_L C^{-1} u_i =$$

$$= -iy_{\beta i}\bar{u}_\beta C^T P^T_L C^{-1} u_i = iy_{\beta i}\bar{u}_\beta P_L u_i$$

and with the same procedure we can write

$$\langle N_i | T | L_\alpha H\rangle = \langle N_i | \int d^4x T\left\{\left[-iy_{\alpha i}\bar{N}_i P_L H^\dagger L_\alpha\right]_x\right\} | L_\alpha H\rangle \simeq$$

$$\simeq -iy_{\alpha i}\bar{u}_i P_L u_\alpha$$

as obtained in (B.1.1). Summing and averaging over spin states,

$$\frac{1}{2}\sum_{spins}(-i)\frac{y^*_{\beta j}y^*_{\alpha j}y_{\beta i}y_{\alpha i}}{q^2-M_j^2}M_j\,\bar{u}_\alpha P_R u_\beta \bar{u}_\beta P_L u_i \bar{u}_i P_L u_\alpha =$$

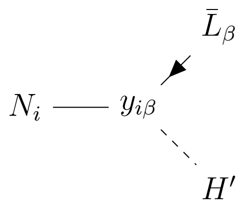
$$= \frac{-i}{2}\frac{y^*_{\beta j}y^*_{\alpha j}y_{\beta i}y_{\alpha i}}{q^2-M_j^2}M_j\,Tr\Big\{P_R \not{p}_\beta P_L(\not{p}_i + M_i)P_L \not{p}_\alpha\Big\} =$$

$$= \frac{-i}{2}\frac{y^*_{\beta j}y^*_{\alpha j}y_{\beta i}y_{\alpha i}}{q^2-M_j^2}M_j M_i\,Tr\Big\{\not{p}_\beta \not{p}_\alpha P_R\Big\} =$$

$$= (-i)\frac{y^*_{\beta j}y^*_{\alpha j}y_{\beta i}y_{\alpha i}}{q^2-M_j^2}M_j M_i(p_\beta \cdot p_\alpha)$$

Now we need to study the kinematics of the process and to perform the integration over the phase space.

**The Kinematics of the Process**



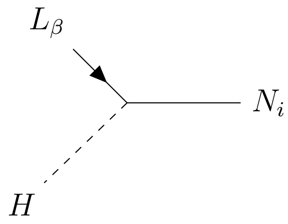

From 4-momentum conservation we have that $p_i^\mu = p_\beta^\mu + p_{H'}^\mu$. Working in the Centre of Mass Frame we can write $p_i^\mu = (M_i, \vec{0})$; this means that $E_\beta + E_{H'} = E_i = M_i$ and $\vec{p}_\beta + \vec{p}_{H'} = \vec{p}_i = \vec{0}$. So, since the particles in the loop must be produced on-shell to have a non-vanishing CP asymmetry, we can also use the relativistic energy momentum relation and write $E_\beta = \sqrt{|\vec{p}_\beta|^2 + m_\beta^2} = |\vec{p}_\beta|$ and $E_{H'} = \sqrt{|\vec{p}_{H'}|^2 + m_{H'}^2} = |\vec{p}_{H'}| = |\vec{p}_\beta|$ (we are in the unbroken phase for the Higgs potential and both the masses of the lepton and the Higgs are zero), in this way obtaining

$$p_\beta^\mu = \left(\frac{M_i}{2}, \vec{p}_\beta\right)$$

$$p_{H'}^\mu = \left(\frac{M_i}{2}, -\vec{p}_\beta\right)$$

.

The same holds for the diagram $L_\alpha H \to N_i$,

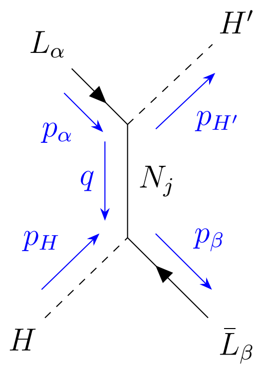



where we can write

$$p_\alpha^\mu = \left(\frac{M_i}{2}, \vec{p}_\alpha\right)$$

$$p_H^\mu = \left(\frac{M_i}{2}, -\vec{p}_\alpha\right)$$

since $E_\alpha = |\vec{p}_\beta|$ and $E_H = |\vec{p}_H| = |\vec{p}_\alpha|$.
Now, the only thing that remain to explicit is the 4-momentum of the particle exchanged in the t-channel of the third diagram, i.e. $L_\alpha H \to \bar{L}_\beta H'$,




for which it holds that $q^\mu = p_\alpha^\mu - p_{H'}^\mu = (0, p_\alpha^x - p_{H'}^x, p_\alpha^y - p_{H'}^y)$

### The Phase Space Integration

For computing the complete contribution we need to evaluate

$$\int \frac{d^3 p_\beta}{(2\pi)^3 2E_\beta} \frac{d^3 p_{H'}}{(2\pi)^3 2E_{H'}} (2\pi)^4 \delta^4(p_i - p_\beta - p_{H'})(-i) y_{\beta j}^* y_{\alpha j}^* y_{\beta i} y_{\alpha i} M_j M_i \frac{(p_\beta \cdot p_\alpha)}{q^2 - M_j^2}$$

where we can write

$$p_\beta \cdot p_\alpha = E_\beta E_\alpha - \vec{p}_\beta \cdot \vec{p}_\alpha = E_\beta E_\alpha - |\vec{p}_\beta||\vec{p}_\alpha| \cos\theta_\beta = \frac{M_i^2}{4}(1 - \cos\theta_\beta)$$

where $\beta$ is the angle at which the anti-lepton $\bar{L}_\beta$ is emitted in the $N_i$-decay since we have the freedom to perform a rotation of the reference frame and choose the reference frame where the lepton $L_\alpha$ is emitted in the $N_i$-decay at an angle $\alpha = 0$.

We also need to compute $q^2$, where

$$q_\mu q^\mu = -(p^x_\alpha - p^x_{H'})^2 - (p^y_\alpha - p^y_{H'})^2 = -\Bigg( (p^x_\alpha)^2 + (p^x_{H'})^2 - 2(p^x_\alpha)(p^x_{H'}) + (p^y_\alpha)^2 +$$

$$+(p^y_{H'})^2 - 2(p^y_\alpha)(p^y_{H'}) \Bigg) = -\left( \frac{M_i^2}{4} + \frac{M_i^2}{4} - 2\frac{M_i^2}{4}\cos(\pi + \theta_\beta) \right) = -\frac{M_i^2}{2}(1+\cos\theta_\beta)$$

In this way the amplitude becomes

$$(-i) y^*_{\beta j} y^*_{\alpha j} y_{\beta i} y_{\alpha i} M_j M_i \frac{\frac{M_i^2}{4}(1-\cos\theta_\beta)}{-\frac{M_i^2}{2}(1+\cos\theta_\beta) - M_j^2} =$$

$$= \frac{(+i)}{2} y^*_{\beta j} y^*_{\alpha j} y_{\beta i} y_{\alpha i} M_i M_j \frac{(1-\cos\theta_\beta)}{1+\cos\theta_\beta + \frac{2M_j^2}{M_i^2}}$$

And now we have to perform the integral considering the constraints given by the conservation laws.

$$\frac{(+i)}{2} y^*_{\beta j} y^*_{\alpha j} y_{\beta i} y_{\alpha i} M_i M_j \int \frac{d^3 p_\beta}{(2\pi)^3 2E_\beta} \frac{d^3 p_{H'}}{(2\pi)^3 2E_{H'}} (2\pi)^4 \delta^3(\vec{p}_i - \vec{p}_\beta - \vec{p}_{H'}) \times$$

$$\times \delta^0(M_i - E_\beta - E_{H'}) \frac{(1-\cos\theta_\beta)}{1+\cos\theta_\beta + \frac{2M_j^2}{M_i^2}} = \frac{(+i)}{2} \frac{y^*_{\beta j} y^*_{\alpha j} y_{\beta i} y_{\alpha i} M_j M_i}{(2\pi)^2} \times$$

$$\times \int \frac{d^3 p_\beta d^3 p_{H'}}{2E_\beta 2E_{H'}} \delta^3(\vec{p}_i - \vec{p}_\beta - \vec{p}_{H'}) \delta^0(M_i - E_\beta - E_{H'}) \frac{(1-\cos\theta_\beta)}{1+\cos\theta_\beta + \frac{2M_j^2}{M_i^2}}$$

Integrating out the $\delta^3(\vec{p}_i - \vec{p}_\beta - \vec{p}_{H'})$ which fixes $\vec{p}_\beta = -\vec{p}_{H'}$ and $E_\beta = E_{H'}$,

$$\frac{(+i)}{2} \frac{y^*_{\beta j} y^*_{\alpha j} y_{\beta i} y_{\alpha i} M_j M_i}{(2\pi)^2} \int \frac{d^3 p_\beta}{4E_\beta^2} \delta^0(M_i - 2E_\beta) \frac{(1-\cos\theta_\beta)}{1+\cos\theta_\beta + \frac{2M_j^2}{M_i^2}}$$

Passing to spherical coordinates, i.e. writing $d^3 p_\beta = d\Omega_\beta |\vec{p}_\beta|^2 d|\vec{p}_\beta| = d\phi_\beta d\cos\theta_\beta E_\beta^2 dE_\beta$, the integral becomes

$$\frac{(+i)}{8} \frac{y^*_{\beta j} y^*_{\alpha j} y_{\beta i} y_{\alpha i} M_j M_i}{(2\pi)^2} \int d\phi_\beta d\cos\theta_\beta dE_\beta \delta^0(M_i - 2E_\beta) \frac{(1-\cos\theta_\beta)}{1+\cos\theta_\beta + \frac{2M_j^2}{M_i^2}}$$

The integral over $d\phi_\beta$ gives a factor of $2\pi$ and the one over $dE_\beta$ gives a $\frac{1}{2}$ factor thanks to the $\delta^0(M_i - 2E_\beta)$,

$$\frac{(+i) y^*_{\beta j} y^*_{\alpha j} y_{\beta i} y_{\alpha i} M_j M_i}{32\pi} \int d\cos\theta_\beta \frac{(1-\cos\theta_\beta)}{1+\cos\theta_\beta + \frac{2M_j^2}{M_i^2}}$$

The last is an integral which can be computed analytically; defining $x \equiv \cos\theta_\beta$ and $z \equiv M_j^2/M_i^2$,

$$\int dx \frac{(1-x)}{1+x+2z} = -2-2(1+z)\ln z + 2(1+z)\ln(1+z) = -2\left[1+(1+z)\ln\left(\frac{z}{1+z}\right)\right] =$$

$$= -2\left[1-(1+z)\ln\left(\frac{1+z}{z}\right)\right] = -2\left[1-(1+z)\ln\left(1+\frac{1}{z}\right)\right]$$

The final result can be written as

$$(-i)\frac{y^*_{\alpha j}\left[Y^\dagger Y\right]_{ji} y_{\alpha i} M_j M_i}{16\pi}\left[1-(1+z)\ln\left(1+\frac{1}{z}\right)\right] \tag{B.32}$$

To obtain the CP asymmetry as defined above we need now to perform the integration over the final state phase space, i.e. $\frac{d^3p_\alpha}{2E_\alpha}\frac{d^3p_H}{2E_H}$, and to consider the denominator contribution. The phase space integral will be,

$$\mathrm{Im}\{y^{\alpha j\star}[Y^\dagger Y]_{ji} y^{\alpha i}\}\frac{M_i M_j}{8\pi}\left[1-(1+z)\ln\left(1+\frac{1}{z}\right)\right]\cdot$$
$$\cdot \int \frac{d^3p_\alpha}{(2\pi)^3 2E_\alpha}\frac{d^3p_H}{(2\pi)^3 2E_H}(2\pi)^4\delta^3(\vec{p_i}-\vec{p_\alpha}-\vec{p}_H)\delta^0(M_i-E_\alpha-E_H)$$

and since nothing in the amplitude depends on $\vec{p_\alpha}$ and $\vec{p}_H$ the integral will contribute up to a numerical factor.

$$\int \frac{d^3p_\alpha}{(2\pi)^3 2E_\alpha}\frac{d^3p_H}{(2\pi)^3 2E_H}(2\pi)^4\delta^3(\vec{p_i}-\vec{p_\alpha}-\vec{p}_H)\delta^0(M_i-E_\alpha-E_H) =$$
$$= \frac{1}{(2\pi)^2}\int \frac{d^3p_\alpha}{4E_\alpha^2}\delta^0(M_i-2E_\alpha) = \frac{1}{(2\pi)^2}\int \frac{d\Omega_\alpha|\vec{p_\alpha}|^2 d|\vec{p_\alpha}|}{4E_\alpha^2}\delta^0(M_i-2E_\alpha)$$

Now, since $|\vec{p_\alpha}| = E_\alpha$ and $\int d\Omega_\alpha = 4\pi$, the integral becomes

$$\frac{1}{(2\pi)^2}(4\pi)\int \frac{E_\alpha^2 dE_\alpha}{4E_\alpha^2}\delta^0(M_i-2E_\alpha) = \frac{1}{4\pi}\int dE_\alpha \delta^0(M_i-2E_\alpha) = \frac{1}{8\pi} \tag{B.33}$$

The final form of the CP asymmetry contribution given by B.1.3 can be written putting together and B.4:

$$\begin{aligned}\epsilon_{i\alpha}^{vertex} &= \frac{\mathrm{Im}\{y^{\alpha j\star}[Y^\dagger Y]_{ji} y^{\alpha i}\}\frac{M_i M_j}{16\pi}\left[1-(1+z)\ln\left(1+\frac{1}{z}\right)\right]\cdot\left(\frac{1}{8\pi}\right)}{\frac{1}{8\pi}\sum_\alpha |y^{\alpha i}|^2 M_i^2} \\ &= \frac{1}{8\pi}\frac{\mathrm{Im}\{y^{\alpha j\star}[Y^\dagger Y]_{ji} y^{\alpha i}\}}{\sum_\alpha |y^{\alpha i}|^2}\sqrt{z}\left[1-(1+z)\ln\left(1+\frac{1}{z}\right)\right]\end{aligned} \tag{B.34}$$

where $z$ has been defined as $z \equiv \frac{M_j^2}{M_i^2}$.

Summing over the flavours $\beta$ and $j$ with $j \neq i$, we obtain

$$\epsilon_{i\alpha}^{vertex} = \frac{1}{8\pi} \frac{1}{\left(\mathrm{Y}^{\dagger}\mathrm{Y}\right)_{ii}} \mathrm{Im}\left\{ \left(Y^{\dagger}Y\right)_{ji} \mathrm{Y}_{\alpha j}^{\star} \mathrm{Y}_{\alpha i} \right\} \sqrt{z} \left[ 1 - (1+z) \ln\left(1 + \frac{1}{z}\right) \right] \tag{B.35}$$

# Appendix C

# Spontaneous Symmetry Breaking in The Model

## C.1 The $U(1)_{B-L}$ Symmetry Breaking

We want to find the minima of the potential

$$\begin{aligned} V_{scalars}(H,\mathrm{S}) &= m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \Lambda(H^\dagger H)(\mathrm{S}^\dagger \mathrm{S}) - \mu^2(S^\dagger S) + \lambda_s(\mathrm{S}^\dagger \mathrm{S})^2 \\ &= m^2|H|^2 + \lambda|H|^4 + \Lambda|H|^2|\mathrm{S}|^2 - \mu^2|\mathrm{S}|^2 + \lambda_s|\mathrm{S}|^4. \end{aligned} \tag{C.1}$$

Taking first derivatives in the fields we obtain,

$$\frac{\partial V}{\partial |H|} = 2m^2|H| + 4\lambda|H|^3 + 2\Lambda|H||\mathrm{S}|^2 = 0 \tag{C.2}$$

$$\frac{\partial V}{\partial |\mathrm{S}|} = -2\mu^2|\mathrm{S}| + 4\lambda_s|\mathrm{S}|^3 + 2\Lambda|H|^2|\mathrm{S}| = 0 \tag{C.3}$$

There is only a minus sign of difference which makes a great difference in the outcome. The condition for the stationary points now read

$$|H| = 0 \qquad\qquad |H|^2 = -\frac{m^2 + \Lambda|\mathrm{S}|^2}{2\lambda} \tag{C.4}$$

$$|\mathrm{S}| = 0 \qquad\qquad |\mathrm{S}|^2 = \frac{\mu^2 - \Lambda|\mathrm{H}|^2}{2\lambda_s} \tag{C.5}$$

and since for $H$ the only viable solution is $|H| = 0$ we obtain now two stationary points in

$$|H| = 0 \qquad\qquad |\mathrm{S}| = 0 \tag{C.6}$$

$$|H| = 0 \qquad\qquad |\mathrm{S}|^2 = \frac{\mu^2}{2\lambda_s} \equiv \frac{v_s^2}{2} \tag{C.7}$$

To find if these two stationary points represents maxima or minima of the potential we can look at second derivatives,

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 + 12\lambda |H|^2 + 2\Lambda |\mathrm{S}|^2 \tag{C.8}$$

$$\frac{\partial^2 V}{\partial |\mathrm{S}|^2} = -2\mu^2 + 12\lambda_s |\mathrm{S}|^2 + 2\Lambda |H|^2 \tag{C.9}$$

$$\frac{\partial^2 V}{\partial |\mathrm{S}| \partial |H|} = \frac{\partial^2 V}{\partial |H| \partial |\mathrm{S}|} = 4\Lambda |H||\mathrm{S}| \tag{C.10}$$

When we evaluate them for the point $|H| = 0$, $|\mathrm{S}| = 0$ we obtain

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 \qquad\qquad \frac{\partial^2 V}{\partial |\mathrm{S}|^2} = -2\mu^2 \tag{C.11}$$

$$\frac{\partial^2 V}{\partial |\mathrm{S}| \partial |H|} = \frac{\partial^2 V}{\partial |H| \partial |\mathrm{S}|} = 0 \tag{C.12}$$

The Hessian matrix is indefinite for $|H| = 0$ and $|\mathrm{S}| = 0$, i.e. it has one positive and one negative value; this means that this above configuration is no more a minimum, as it was before SSB, now it's a saddle point and it can not represent the vacuum of our system. Anyway, for the stationary point $|H| = 0$, $|\mathrm{S}|^2 = \frac{\mu^2}{2\lambda_s} \equiv \frac{v_s^2}{2}$ we have

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 \qquad\qquad \frac{\partial^2 V}{\partial |\mathrm{S}|^2} = 4\mu^2 = 4\lambda_s v_s^2 \tag{C.13}$$

$$\frac{\partial^2 V}{\partial |\mathrm{S}| \partial |H|} = \frac{\partial^2 V}{\partial |H| \partial |\mathrm{S}|} = 0 \tag{C.14}$$

We can see that the Hessian matrix in this point is positive definite, identifying it as a minimum for the potential and so also as a good configuration for the vacuum of our system.

We want now to minimize the potential written in terms of the $\phi$ and $\theta$ fields, where $\mathrm{S} = \frac{\phi + i\theta}{\sqrt{2}}$, and

$$\begin{aligned} V_{scalars}(H, \phi, \theta) = m^2 |H|^2 + \lambda |H|^4 + \frac{\Lambda}{2} |H|^2 (\phi^2 + \theta^2) \\ -\frac{\mu^2}{2}(\phi^2 + \theta^2) + \frac{\lambda_s}{4}(\phi^2 + \theta^2)^2. \end{aligned} \tag{C.15}$$

The stationary conditions here read

$$\frac{\partial V}{\partial |H|} = 2|H| \left( m^2 + 2\lambda |H|^2 + \frac{\Lambda}{2}(\phi^2 + \theta^2) \right) = 0 \tag{C.16}$$

$$\frac{\partial V}{\partial \phi} = \phi\big(-\mu^2 + \lambda_s(\phi^2+\theta^2) + \Lambda|H|^2\big) = 0 \tag{C.17}$$

$$\frac{\partial V}{\partial \theta} = \theta\big(-\mu^2 + \lambda_s(\phi^2+\theta^2) + \Lambda|H|^2\big) = 0 \tag{C.18}$$

Electroweak symmetry is not broken at this stage so we have $|H| = 0$ at the minimum. Putting these together with the stationary conditions for $\phi(x)$ and $\theta(x)$, we obtain

$$\phi = 0 \qquad\qquad \phi^2+\theta^2 = \frac{\mu^2}{2\lambda_s} \equiv v_s^2 \tag{C.19}$$

$$\theta = 0 \qquad\qquad \phi^2+\theta^2 = \frac{\mu^2}{2\lambda_s} \equiv v_s^2 \tag{C.20}$$

which reflects the ones obtained for |S| in (4.2.2).

To understand the nature of this stationary points we need to evaluate the Hessian matrix for the various cases,

$$\{|H| = 0;\ \theta = 0;\ \phi = 0\} \tag{C.21}$$

$$\big\{|H| = 0;\ \theta = 0;\ \phi^2 = v_s^2\big\} \tag{C.22}$$

$$\big\{|H| = 0;\ \theta^2 = v_s^2;\ \phi = 0\big\} \tag{C.23}$$

$$\big\{|H| = 0;\ \phi,\theta\ s.t.\ \phi^2+\theta^2 = 0\big\} \tag{C.24}$$

For the second derivatives we have

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 + 12\lambda|H|^2 + \Lambda(\phi^2+\theta^2) \tag{C.25}$$

$$\frac{\partial^2 V}{\partial \phi^2} = -\mu^2 + 3\lambda_s\phi^2 + \lambda_s\theta^2 + \Lambda|H|^2 \tag{C.26}$$

$$\frac{\partial^2 V}{\partial \theta^2} = -\mu^2 + \lambda_s\phi^2 + 3\lambda_s\theta^2 + \Lambda|H|^2 \tag{C.27}$$

$$\frac{\partial^2 V}{\partial |H|\partial\theta} = \frac{\partial^2 V}{\partial\theta\partial|H|} = 2\Lambda|H|\theta \tag{C.28}$$

$$\frac{\partial^2 V}{\partial |H|\partial\phi} = \frac{\partial^2 V}{\partial\phi\partial|H|} = 2\Lambda|H|\phi \tag{C.29}$$

$$\frac{\partial^2 V}{\partial\phi\partial\theta} = \frac{\partial^2 V}{\partial\theta\partial\phi} = 2\lambda_s\phi\theta \tag{C.30}$$

From the previous analysis we expect that (C.21) does not represent the minimum for the system; in fact evaluating the second derivatives for $\{|H| = 0;\, \theta = 0;\, \phi = 0\}$ we obtain

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 \qquad \frac{\partial^2 V}{\partial \phi^2} = -\mu^2 \qquad \frac{\partial^2 V}{\partial \theta^2} = -\mu^2 \tag{C.31}$$

$$\frac{\partial^2 V}{\partial |H| \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial |H|} = 0 \qquad \frac{\partial^2 V}{\partial |H| \partial \phi} = \frac{\partial^2 V}{\partial \phi \partial |H|} = 0 \qquad \frac{\partial^2 V}{\partial \phi \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial \phi} = 0 \tag{C.32}$$

This represent a saddle point as expected. Now, let's see what happens to the other three conditions which should corresponds to the minimum of the system.
For $\{|H| = 0;\, \theta = 0;\, \phi^2 = v_s^2\}$ we have,

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 + \Lambda v_s^2 \qquad \frac{\partial^2 V}{\partial \phi^2} = -\mu^2 + 3\lambda_s v_s^2 = 2\mu^2 \tag{C.33}$$

$$\frac{\partial^2 V}{\partial \theta^2} = -\mu^2 + \lambda_s v_s^2 = 0 \qquad \frac{\partial^2 V}{\partial |H| \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial |H|} = 0 \tag{C.34}$$

$$\frac{\partial^2 V}{\partial |H| \partial \phi} = \frac{\partial^2 V}{\partial \phi \partial |H|} = 0 \qquad \frac{\partial^2 V}{\partial \phi \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial \phi} = 0 \tag{C.35}$$

We obtain an Hessian semi-definite positive which corresponds to a minimum of the potential with a massless mode represented by the oscillations of the $\theta$ field. We have found the Goldstone mode.
For $\{|H| = 0;\, \theta^2 = v_s^2;\, \phi = 0\}$ we have,

$$\frac{\partial^2 V}{\partial |H|^2} = 2m^2 + \Lambda v_s^2 \qquad \frac{\partial^2 V}{\partial \phi^2} = 0 \qquad \frac{\partial^2 V}{\partial \theta^2} = 2\mu^2 \tag{C.36}$$

$$\frac{\partial^2 V}{\partial |H| \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial |H|} = 0 \qquad \frac{\partial^2 V}{\partial |H| \partial \phi} = \frac{\partial^2 V}{\partial \phi \partial |H|} = 0 \qquad \frac{\partial^2 V}{\partial \phi \partial \theta} = \frac{\partial^2 V}{\partial \theta \partial \phi} = 0 \tag{C.37}$$

We obtain an Hessian semi-definite positive which corresponds to a minimum of the potential with a massless mode which now is represented by the oscillations of the $\phi$ field. We always have a Goldstone boson.

The third case, i.e. the one with $\{|H| = 0;\, \phi, \theta\, s.t.\, \phi^2 + \theta^2 = v_s^2\}$, corresponds to consider the minimum of the potential in a configuration where both to $\phi$ and $\theta$ are $\neq 0$, i.e to give a vacuum expectation value both to $\phi$ and $\theta$. We will have

$$\phi = v_s \cos\alpha \qquad \theta = v_s \sin\alpha \tag{C.38}$$

and this reduces to a mixing between the two scalars degrees of freedom which can be eliminated through a rotation of the frame in the $\phi$,$\theta$ space. This case is studied in detail in the next section.

## C.2 Diagonalizing the Potential

To be as general as possible we are going to show also what happens if we choose the vacuum in such a way that both $\theta$ and $\phi$ have a different from zero vacuum expectation value. In this case we can write

$$\begin{aligned} \mathrm{S} &= \frac{1}{\sqrt{2}}(v_s e^{i\alpha_s} + \phi + i\theta) = \frac{1}{\sqrt{2}}(v_s \cos\alpha_s + i v_s \sin\alpha_s + \phi + i\theta) = \\ &= \frac{1}{\sqrt{2}}((v_s \cos\alpha_s + \phi) + i(v_s \sin\alpha_s + \theta)) \end{aligned} \tag{C.39}$$

which corresponds to (C.38). Inserting it in the potential we obtain

$$\begin{gathered} V_{scalars}(H,\phi,\theta) = m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \\ +\frac{\Lambda}{2}(H^\dagger H)(v_s^2 + \phi^2 + \theta^2 + v_s e^{-i\alpha_s}\phi + i v_s e^{-i\alpha_s}\theta + v_s e^{i\alpha_s}\phi - i v_s e^{i\alpha_s}\theta) + \\ +\lambda_s(\frac{1}{2}(v_s^2 + \phi^2 + \theta^2 + v_s e^{-i\alpha_s}\phi + i v_s e^{-i\alpha_s}\theta + v_s e^{i\alpha_s}\phi - i v_s e^{i\alpha_s}\theta) - \frac{v_s^2}{2})^2 \end{gathered}$$

$$\begin{gathered} V_{scalars}(H,\phi,\theta) = m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \\ +\frac{\Lambda}{2}(H^\dagger H)(v_s^2 + \phi^2 + \theta^2 + v_s(e^{i\alpha_s} + e^{-i\alpha_s})\phi - i v_s(e^{i\alpha_s} - e^{-i\alpha_s})\theta) + \\ +\frac{\lambda_s}{4}(\phi^2 + \theta^2 + v_s(v_s e^{i\alpha_s} + e^{-i\alpha_s})\phi - i v_s(e^{i\alpha_s} - e^{-i\alpha_s})\theta)^2 \end{gathered}$$

$$\begin{aligned} V_{scalars}(H,\mathrm{S}) =& m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \frac{\Lambda}{2}(H^\dagger H)v_s^2 + \frac{\Lambda}{2}(H^\dagger H)\phi^2 + \frac{\Lambda}{2}(H^\dagger H)\theta^2 \\ &+\Lambda v_s \cos\alpha_s (H^\dagger H)\phi + \Lambda v_s \sin\alpha_s (H^\dagger H)\theta + \frac{\lambda_s}{4}\phi^4 + \lambda_s v_s^2 \cos^2\alpha_s \phi^2 + \\ &+\frac{\lambda_s}{4}\theta^4 + \lambda_s v_s^2 \sin^2\alpha_s \theta^2 + \lambda_s v_s \cos\alpha_s \phi^3 + \frac{\lambda_s}{2}\phi^2\theta^2 + \lambda_s v_s \cos\alpha_s \phi\theta^2 + \\ &+ \lambda_s v_s \sin\alpha_s \theta^3 + \lambda_s v_s \sin\alpha_s \theta\phi^2 + 2\lambda_s v_s^2 \sin\alpha_s \cos\alpha_s \phi\theta \end{aligned} \tag{C.40}$$

In this potential we can recognize the following "quadratic" part in the real and complex degrees of freedom of S:

$$V(H,\phi,\theta) \supset \lambda_s v_s^2 \cos^2\alpha_s \phi^2 + \lambda_s v_s^2 \sin^2\alpha_s \theta^2 + 2\lambda_s v_s^2 \sin\alpha_s \cos\alpha_s \phi\theta$$

which we have to diagonalize to find the mass eigenstates. To do so we rewrite this part of the potential through a mass matrix $\mathrm{M}_S$,

$$V(H,\mathrm{S}) \supset \frac{1}{2}[\phi,\theta]\mathrm{M}_S \begin{bmatrix} \phi \\ \theta \end{bmatrix} \tag{C.41}$$

where

$$\mathrm{M}_S = \begin{bmatrix} 2\lambda_s v_s^2 \cos^2\alpha_s & 2\lambda_s v_s^2 \sin\alpha_s \cos\alpha_s \\ 2\lambda_s v_s^2 \sin\alpha_s \cos\alpha_s & 2\lambda_s v_s^2 \sin^2\alpha_s \end{bmatrix} \tag{C.42}$$

Being $\mathrm{M}_S$ symmetric, we can diagonalize it through an orthogonal matrix O such that $\mathrm{O}\mathrm{D}_S\mathrm{O}^T = \mathrm{M}_S$. Solving the equation $\det(\mathrm{M}_S - \lambda 1_{2\times 2})$, we find the eigenvalues of our mass matrix $\mathrm{M}_S$ to be $\lambda_1 = 2\lambda_s v_s^2$ and $\lambda_2 = 0$, corresponding to the physical massive scalar and to the Nambu-Goldstone boson generated by the $U(1)_{B-L}$ breaking. Now, since we want to find the mass eigenstates, we need to consider the explicit form of the diagonalizing matrix which is,

$$\mathrm{O} = \begin{bmatrix} \cos\alpha_s & -\sin\alpha_s \\ \sin\alpha_s & \cos\alpha_s \end{bmatrix} \tag{C.43}$$

We can use the matrix O to write,

$$V(H,\phi,\theta) \supset \frac{1}{2}[\phi,\theta]\mathrm{O}\mathrm{D}_S\mathrm{O}^T \begin{bmatrix} \phi \\ \theta \end{bmatrix} \tag{C.44}$$

where $\mathrm{D}_S$ is explicitly given by,

$$\mathrm{D}_S = \begin{bmatrix} 2\lambda_s v_s^2 & 0 \\ 0 & 0 \end{bmatrix} \tag{C.45}$$

and the mass eigenstates can be defined as

$$\begin{bmatrix} \Phi \\ \Theta \end{bmatrix} = \mathrm{O}^T \begin{bmatrix} \phi \\ \theta \end{bmatrix} = \begin{bmatrix} \cos\alpha_s & \sin\alpha_s \\ -\sin\alpha_s & \cos\alpha_s \end{bmatrix} \begin{bmatrix} \phi \\ \theta \end{bmatrix} = \begin{bmatrix} \cos\alpha_s\phi + \sin\alpha_s\theta \\ -\sin\alpha_s\phi + \cos\alpha_s\theta \end{bmatrix} \tag{C.46}$$

The inverse relations are given by

$$\begin{bmatrix} \phi \\ \theta \end{bmatrix} = \begin{bmatrix} \cos\alpha_s\Phi - \sin\alpha_s\Theta \\ \sin\alpha_s\Phi + \cos\alpha_s\Theta \end{bmatrix} \tag{C.47}$$

and we can use this last ones to rewrite the potential in C.40 in terms of the mass eigenstates.

$$\begin{aligned} V(H,\Phi,\Theta) = \left(m^2 + \frac{\Lambda}{2}v_s^2\right)(H^\dagger H) + \lambda(H^\dagger H)^2 + \frac{\Lambda}{2}(H^\dagger H)\Phi^2 + \\ + \frac{\Lambda}{2}(H^\dagger H)\Theta^2 + \Lambda v_s(H^\dagger H)\Phi + \lambda_s v_s^2\Phi^2 + \frac{\lambda_s}{2}\Phi^2\Theta^2 + \lambda_s v_s\Phi^3 + \\ + \lambda_s v_s\Phi\Theta^3 + \frac{\lambda_s}{4}\Phi^4 + \frac{\lambda_s}{4}\Theta^4 \end{aligned} \tag{C.48}$$

As it should be due to the $U(1)_{B-L}$ symmetry of our system, the potential above, written in terms of $\Phi$ and $\Theta$ is completely equivalent to the one in (4.3). So, in the following treatment we will assume that only $\phi$ has a $\neq 0$ vacuum expectation value, i.e. $\phi_0 = v_s$ and $\theta_0 = 0$, since this is the most general case.

## C.3 Electroweak Symmetry Breaking in the U(1) Model

It turns out that an easier way to study the minimum configuration of the potential after EWSB is to start directly from (4.18) and to consider that for having EWSB there must be also $m^2 < 0$. Then we send $m^2 \to -m^2$ and, considering the parameterization for the Higgs field in (4.34), we rewrite the potential as,

$$V(h,\phi,\theta) = -\frac{m^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{\Lambda}{4}h^2(\phi^2+\theta^2) - \frac{\mu^2}{2}(\phi^2+\theta^2) + \frac{\lambda_s}{4}(\phi^2+\theta^2)^2. \tag{C.49}$$

and looking for the minima starting from here. The system of first derivatives is,

$$\partial_h V = -m^2 h + \lambda h^3 + \frac{\Lambda}{2}h(\phi^2+\theta^2) = h\left(-m^2 + \lambda h^2 + \frac{\Lambda}{2}(\phi^2+\theta^2)\right) = 0, \tag{C.50}$$

$$\partial_\phi V = -\mu^2\phi + \lambda_s\phi(\phi^2+\theta^2) + \frac{\Lambda}{2}h^2\phi = \phi\left(-\mu^2 + \lambda_s(\phi^2+\theta^2) + \frac{\Lambda}{2}h^2\right) = 0, \tag{C.51}$$

$$\partial_\theta V = -\mu^2\theta + \lambda_s\theta(\phi^2+\theta^2) + \frac{\Lambda}{2}h^2\theta = \theta\left(-\mu^2 + \lambda_s(\phi^2+\theta^2) + \frac{\Lambda}{2}h^2\right) = 0, \tag{C.52}$$

We have that: (C.50) is satisfied for $h_0 = 0$ and

$$h_0^2 = \frac{m^2 - \frac{\Lambda}{2}(\phi^2+\theta^2)}{\lambda}, \tag{C.53}$$

(C.51) is satisfied for $\phi_0 = 0$ and

$$\phi_0^2 + \theta_0^2 = \frac{\mu^2 - \frac{\Lambda}{2}h^2}{\lambda_s} \tag{C.54}$$

and (C.52) is satisfied for $\theta_0 = 0$ and

$$\phi_0^2 + \theta_0^2 = \frac{\mu^2 - \frac{\Lambda}{2}h^2}{\lambda_s}. \tag{C.55}$$

Again (C.54) and (C.55) reflects the $U(1)$ invariance of the theory.

To understand if these points represents (local) minima, (local) maxima o saddle points, we need to perform the second derivatives and study the form of the Hessian.

The system of second derivatives is,

$$\partial^2_{hh}V = -m^2 + 3\lambda h^2 + \frac{\Lambda}{2}(\phi^2 + \theta^2), \tag{C.56}$$

$$\partial^2_{\phi\phi}V = -\mu^2 + 3\lambda_s\phi^2 + \lambda_s\theta^2 + \frac{\Lambda}{2}h^2, \tag{C.57}$$

$$\partial^2_{\theta\theta}V = -\mu^2 + \lambda_s\phi^2 + 3\lambda_s\theta^2 + \frac{\Lambda}{2}h^2, \tag{C.58}$$

$$\partial^2_{h\phi}V = \partial^2_{\phi h}V = \Lambda h\phi, \tag{C.59}$$

$$\partial^2_{h\theta}V = \partial^2_{\theta h}V = \Lambda h\theta, \tag{C.60}$$

$$\partial^2_{\phi\theta}V = \partial^2_{\theta\phi}V = 2\lambda_s\phi\theta. \tag{C.61}$$

At this point we need to substitute the values of the stationary points found above and to see if the Hessian matrix in those points is positive (semi-)definite, negative (semi-)definite or indefinite. We expect the minimum to be in the configuration where (C.54) (or (C.55)) and (C.53). At this point we obtain, after having chosen the condition of $\theta_0 = 0$,

$$H(h_0, \phi_0, \theta_0) = \begin{bmatrix} \frac{2\lambda\lambda_s m^2 - \Lambda\lambda\mu^2}{\lambda\lambda_s - \frac{\Lambda^2}{4}} & \frac{\Lambda\sqrt{\lambda\lambda_s m^2\mu^2 - \frac{\Lambda}{2}(\lambda_s m^4 + \lambda\mu^4) + \frac{\Lambda^2}{4}m^2\mu^2}}{\lambda\lambda_s - \frac{\Lambda^2}{4}} & 0 \\ \frac{\Lambda\sqrt{\lambda\lambda_s m^2\mu^2 - \frac{\Lambda}{2}(\lambda_s m^4 + \lambda\mu^4) + \frac{\Lambda^2}{4}m^2\mu^2}}{\lambda\lambda_s - \frac{\Lambda^2}{4}} & \frac{2\lambda\lambda_s\mu^2 - \Lambda\lambda_s m^2}{\lambda\lambda_s - \frac{\Lambda^2}{4}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{C.62}$$

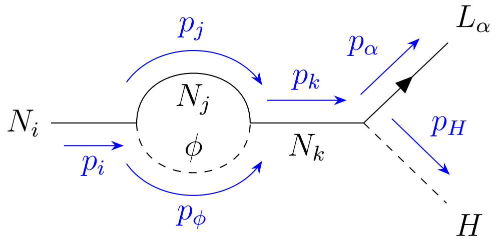
This is the result we were looking for: there is a region in the parameter space for which this Hessian matrix is semipositive definite and represents a minimum for the system. We can say more, there is a 0 eigenvalue corresponding to $\partial_{\theta\theta}$ and this is related to the existence of the Goldstone boson $\theta$, i.e the massless mode, which remain so even after Elwctroweak symmetry breaking. And we also see that there is a mixing between the other two scalars degrees of freedom $h$ and $\phi$, i.e the two massive scalars mixes. The usual SM Higgs is no more a mass eigenstate of our model, we should diagonalize the Hessian matrix to find the two mass eigenstates; however, the mixing is small if $m_\phi >> m_h$ and can be neglected.

# Appendix D

# CP Asymmetry Contribution from the $U(1)$ Scalars

### D.0.1 Wave Function Corrections by $\phi$

We want to compute the injection in CP asymmetry given by the interference between the tree-level $N_i$ decay and the following 1-loop diagram,




It's important to notice that since we need to produce on-shell loop particles to have a non-zero CP asymmetry injection, the condition $M_i > M_j + m_\phi$ must hold. For example, we won't have CP injection in the decay of the lightest RHN, since it's not able to produce a more massive $N_j$.

#### The Phase Space of the Process

We need to find a relation for the 4-momentum of all the different particles involved into the process, i.e. $N_i$, $N_j$, $N_k$, $\phi$, $H$, $L_\alpha$. We can go through it considering the fact that the 4-momentum is conserved in each vertex and also considering that we need the particles running in the loop to be on-shell in order to obtain a non-vanishing CP asymmetry, as in the case of the standard contributions computed in (3.3.3). So, the following relations hold,

$$p_i^\mu = p_j^\mu + p_\phi^\mu \tag{D.1}$$

$$p_j^\mu + p_\phi^\mu = p_k^\mu \tag{D.2}$$

$$p_k^\mu = p_\alpha^\mu + p_H^\mu \tag{D.3}$$

Working in the center of mass frame of the decaying particle $N_i$, we can write

$$p_i^\mu = (M_i, \vec{0})$$

and then, considering D.1 we can write $M_i = E_j + E_\phi$ and $\vec{p}_j + \vec{p}_\phi = \vec{0}$ where also $E_j = \sqrt{|\vec{p}_j|^2 + M_j^2}$ and $E_\phi = \sqrt{|\vec{p}_\phi|^2 + m_\phi^2} = \sqrt{|\vec{p}_j|^2 + m_\phi^2}$. From D.1 and D.2 we have that $p_i^\mu = p_k^\mu$ and putting together D.1, D.2 and D.3 we find the conditions $M_i = E_\alpha + E_H$ and $\vec{p}_\alpha + \vec{p}_H = \vec{0}$. Since Electroweak Symmetry Breaking has not yet happened, both $m_\alpha = 0$ and $m_H = 0$ and so the energy-momentum relations stand for $E_\alpha = |\vec{p}_\alpha|$ and $E_H = |\vec{p}_H|$ implying $E_\alpha = E_H = \frac{M_i}{2}$.

So,

$$p_j^\mu = (E_j, \vec{p}_j) \tag{D.4}$$

$$p_\phi^\mu = (E_\phi, -\vec{p}_j) \tag{D.5}$$

$$p_k^\mu = (M_i, \vec{0}) \tag{D.6}$$

$$p_\alpha^\mu = \left(\frac{M_i}{2}, \frac{M_i}{2}\hat{p}_\alpha\right) \tag{D.7}$$

$$p_H^\mu = \left(\frac{M_i}{2}, -\frac{M_i}{2}\hat{p}_\alpha\right) \tag{D.8}$$

We can use the energy-momentum relation to obtain an explicit form for $E_j$, $E_\phi$ and $|\vec{p}_j|$. From

$$M_i = E_j + E_\phi = \sqrt{|\vec{p}_j|^2 + M_j^2} + \sqrt{|\vec{p}_j|^2 + m_\phi^2}$$

we can obtain

$$|\vec{p}_j| = \sqrt{\frac{(M_i^2 + M_j^2 - m_\phi^2)^2}{4M_i^2} - M_j^2} = \sqrt{\frac{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2M_j^2 - 2M_i^2m_\phi^2 - 2M_j^2m_\phi^2}{4M_i^2}} \tag{D.9}$$

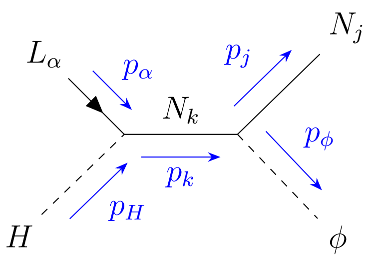
And then,

$$E_j = \frac{M_i^2 + M_j^2 - m_\phi^2}{2M_i} \tag{D.10}$$

$$\begin{aligned} E_\phi &= \sqrt{\frac{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2M_j^2 - 2M_i^2m_\phi^2 - 2M_j^2m_\phi^2}{4M_i^2} + m_\phi^2} = \\ &= \sqrt{\frac{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2M_j^2 + 2M_i^2m_\phi^2 - 2M_j^2m_\phi^2}{4M_i^2}} = \frac{M_i^2 - M_j^2 + m_\phi^2}{2M_i}. \end{aligned} \tag{D.11}$$

Now that we have the kinematics relations we have to compute the different amplitudes needed for computing the CP asymmetry factor $\epsilon_{i\alpha}$.

We start from computing $T_{kf}^*$, which in this case is given by $\langle N_j\phi|\, T\, |L_\alpha H\rangle^\star$. The related Feynmann Diagram is,



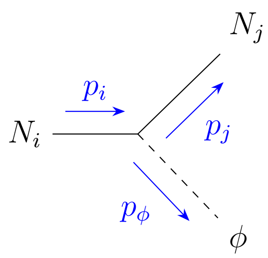

We need to evaluate, from the Lagrangian (4.33),

$$\begin{aligned}
&\langle N_j\phi|\int d^4x d^4y\Big[-\frac{i}{2\sqrt{2}}Y_R^{jk}\bar{N}_j N_k\phi\Big]_y\left[-iY_D^{k\alpha}\bar{N}_k P_L H^\dagger L_\alpha\right]_x|L_\alpha H\rangle \simeq \\
&\simeq -\frac{Y_R^{jk}Y_D^{k\alpha}}{2\sqrt{2}}\int d^4xd^4y\,\langle N_j|\,\bar{N}_{j,y}N_{k,y}\bar{N}_{k,x}P_L L_{\alpha,x}\,|L_\alpha\rangle \simeq \\
&\simeq -\frac{Y_R^{jk}Y_D^{k\alpha}}{2\sqrt{2}}\frac{i}{p_k^2-M_k^2}(\bar{u}_j(\not{p}_k+M_k)P_L u_\alpha)
\end{aligned} \tag{D.12}$$

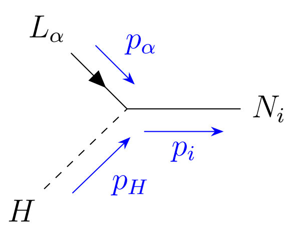
where we neglected the $(2\pi)$ factors and the $\delta^4(P_f-P_k)$ coming from the $\int d^4xd^4y$ integration reflecting the 4-momentum conservation. Since we need $\langle N_j\phi|\,T\,|L_\alpha H\rangle^\star$, we have to take the complex conjugate of (D.12) which is

$$\begin{aligned}
\langle N_j\phi|\,T\,|L_\alpha H\rangle^\star \simeq \frac{i}{2\sqrt{2}}\frac{Y_R^{jk\star}Y_D^{k\alpha\star}}{p_k^2-M_k^2}(\bar{u}_j(\not{p}_k+M_k)P_L u_\alpha)^\star = \\
= \frac{i}{2\sqrt{2}}\frac{Y_R^{jk\star}Y_D^{k\alpha\star}}{p_k^2-M_k^2}(\bar{u}_j(\not{p}_k+M_k)P_L u_\alpha)^\dagger = \frac{i}{2\sqrt{2}}\frac{Y_R^{jk\star}Y_D^{k\alpha\star}}{p_k^2-M_k^2}\bar{u}_\alpha P_R(\not{p}_k+M_K)u_j.
\end{aligned} \tag{D.13}$$

And now it's the turn of $T_{ki}=\langle N_j\phi|\,T\,|N_i\rangle$ and $T_{if}=\langle N_i|\,T\,|L_\alpha H\rangle$. The related Feynmann diagrams are respectively,




and



Again, starting from 4.33,

$$\begin{aligned} \langle N_j\phi| \, T \, |N_i\rangle &= \langle N_j\phi| \int d^4x \left[-\frac{i}{2\sqrt{2}} Y_R^{ji} \bar{N}_j N_i \phi\right]_x |N_i\rangle \simeq \\ &\simeq -\frac{i}{2\sqrt{2}} Y_R^{ji} \int d^4x \, \langle N_j| \, \bar{N}_{j,x} N_{i,x} \, |N_i\rangle \simeq \\ &\simeq -\frac{i}{2\sqrt{2}} Y_R^{ji} \bar{u}_j u_i, \end{aligned} \tag{D.14}$$

and we can then recall from (B.9) that

$$\langle N_i| \, T \, |L_\alpha H\rangle \simeq -i Y_D^{\alpha i} \bar{u}_i P_L u_\alpha. \tag{D.15}$$

The contribution to the CP asymmetry is given by 3.27 and so we need to multiply together D.13, D.14, D.15, consider the sum/average over the internal degrees of freedom and to integrate over the loop particles $N_j$, $\phi$ phase spaces. So, since the initial state is a spin-$\frac{1}{2}$ particle, we have,

$$\begin{aligned} &\frac{1}{2} \sum_{spins} \frac{(-i)}{8} \frac{Y_R^{jk\star} Y_D^{k\alpha\star} Y_R^{ji} y_D^{i\alpha}}{p_k^2 - M_k^2} \bar{u}_\alpha P_R(\not{p}_k + M_k) u_j \bar{u}_j u_i \bar{u}_i P_L u_\alpha = \\ = \frac{(-i)}{16} \frac{Y_R^{jk\star} Y_D^{k\alpha\star} Y_R^{ji} Y_D^{i\alpha}}{p_k^2 - M_k^2} \, &\mathrm{Tr}\Big\{ \bar{u}_\alpha P_R(\not{p}_k + M_k) u_j \bar{u}_j u_i \bar{u}_i P_L u_\alpha \Big\} = \frac{(-i)}{16} \frac{Y_R^{jk\star} Y_D^{k\alpha\star} Y_R^{ji} Y_D^{i\alpha}}{p_k^2 - M_k^2} \cdot \\ &\cdot \mathrm{Tr}\Big\{ P_R(\not{p}_k + M_K)(\not{p}_j + M_j)(\not{p}_i + M_i) P_L \not{p}_\alpha \Big\} \end{aligned} \tag{D.16}$$

Let's focus on the trace part:

$$\begin{aligned} &\mathrm{Tr}\Big\{ P_R(\not{p}_k + M_K)(\not{p}_j + M_j)(\not{p}_i + M_i)\not{p}_\alpha \Big\} = \\ &= \frac{1}{2} \mathrm{Tr}\Big[ (1+\gamma^5)(\not{p}_k \not{p}_j + \not{p}_k M_j + M_k \not{p}_j + M_k M_j)(\not{p}_i \not{p}_\alpha + M_i \not{p}_\alpha) \Big] = \\ &= \frac{1}{2} \mathrm{Tr}\Big[ (1+\gamma^5)(\not{p}_k \not{p}_j \not{p}_i \not{p}_\alpha + M_i \not{p}_k \not{p}_j \not{p}_\alpha + M_j \not{p}_k \not{p}_i \not{p}_\alpha + M_j M_i \not{p}_k \not{p}_\alpha \\ &\quad + M_k \not{p}_j \not{p}_i \not{p}_\alpha + M_k M_i \not{p}_j \not{p}_\alpha + M_k M_j \not{p}_i \not{p}_\alpha + M_k M_j M_i \not{p}_\alpha) \Big] \end{aligned}$$

We recognize the presence of some different types of traces involving the $\gamma^\mu$ matrices and the $\gamma^5$ one and we can use the identities from (A.3) which holds in a 4-dimensional spacetime. Some pieces vanish and others give scalar products,

$$\begin{aligned}&\frac{1}{2}\Big(\mathrm{Tr}\Big[\not{p}_k\not{p}_j\not{p}_i\not{p}_\alpha\Big]+\mathrm{Tr}\Big[\gamma^5\not{p}_k\not{p}_j\not{p}_i\not{p}_\alpha\Big]+M_iM_j\,\mathrm{Tr}\Big[\not{p}_k\not{p}_\alpha\Big]+\\&+M_kM_i\,\mathrm{Tr}\Big[\not{p}_j\not{p}_\alpha\Big]+M_kM_j\,\mathrm{Tr}\Big[\not{p}_i\not{p}_\alpha\Big]\Big)=\\&=2[(p_k\cdot p_j)(p_i\cdot p_\alpha)+(p_k\cdot p_\alpha)(p_i\cdot p_j)-(p_k\cdot p_i)(p_j\cdot p_\alpha)+\\&+M_iM_j(p_k\cdot p_\alpha)+M_iM_k(p_j\cdot p_\alpha)+M_kM_j(p_i\cdot p_\alpha)]\end{aligned}\tag{D.17}$$

The $\mathrm{Tr}\left[\gamma^5\not{p}_k\not{p}_j\not{p}_i\not{p}_\alpha\right]$ vanishes because it's proportional to $\epsilon^{kji\alpha}p_kp_jp_ip_\alpha$ and goes to zero for symmetry reasons.

Using the results found in D.0.1 we can write

$$(p_k\cdot p_j)=M_iE_j\qquad(p_i\cdot p_\alpha)=\frac{M_i^2}{2}\qquad(p_k\cdot p_\alpha)=\frac{M_i^2}{2}\tag{D.18}$$

$$(p_i\cdot p_j)=M_iE_j\qquad(p_k\cdot p_i)=M_i^2\qquad(p_j\cdot p_\alpha)=\frac{E_jM_i}{2}-\vec{p}_j\vec{p}_\alpha\tag{D.19}$$

and then D.17 becomes

$$2\left(\frac{M_i^3E_j}{2}+\frac{M_i^3E_j}{2}-\left(M_i^2-M_iM_k\right)\left(\frac{M_iE_j}{2}-\vec{p}_j\vec{p}_\alpha\right)+\frac{M_i^3M_j}{2}+\frac{M_i^2M_jM_k}{2}\right)\tag{D.20}$$

Considering D.10, D.7 and D.9 we have,

$$\begin{aligned}\vec{p}_j\vec{p}_\alpha&=\frac{M_i}{2}\sqrt{\frac{M_i^4+M_j^4+M_k^4-2M_i^2M_j^2-2M_i^2m_\phi^2-2M_j^2m_\phi^2}{4M_i^2}}\cos\gamma=\\&=\frac{1}{4}\sqrt{M_i^4+M_j^4+M_k^4-2M_i^2M_j^2-2M_i^2m_\phi^2-2M_j^2m_\phi^2}\cos\gamma\end{aligned}\tag{D.21}$$

where $\gamma$ is the angle between $N_j$ and $L_\alpha$. Then we can rearrange D.20,

$$\begin{aligned}&M_i^3\left(M_j+\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)+M_i^2M_k\left(M_j-\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)+\\&+\frac{1}{2}M_i(M_i+M_k)\sqrt{M_i^4+M_j^4+M_k^4-2M_i^2M_j^2-2M_i^2m_\phi^2-2M_j^2m_\phi^2}\cos\gamma\end{aligned}\tag{D.22}$$

The CP asymmetry will be then proportional to

$$\begin{aligned}&\left(\frac{1}{8}\right)\mathrm{Im}\left\{\mathrm{Y}_R^{jk\star}\mathrm{Y}_D^{k\alpha\star}\mathrm{Y}_R^{ji}\mathrm{Y}_D^{i\alpha}\right\}\frac{1}{M_i^2-M_k^2}\left[M_i^3\left(M_j+\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)+\right.\\&+M_i^2M_k\left(M_j-\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)+\\&\left.+\frac{1}{2}M_i(M_i+M_k)\sqrt{M_i^4+M_j^4+M_k^4-2M_i^2M_j^2-2M_i^2m_\phi^2-2M_j^2m_\phi^2}\cos\gamma\right]\end{aligned}\tag{D.23}$$

Now, we need to perform the integral first in the phase space of the intermediate states $N_j$,$\phi$ and then on that of the final ones $H$,$L_\alpha$.

$$
\begin{aligned}
&\left(\frac{1}{8}\right)\mathrm{Im}\left\{\mathrm{Y}_R^{jk\star}\mathrm{Y}_D^{k\alpha\star}\mathrm{Y}_R^{ji}\mathrm{Y}_D^{i\alpha}\right\}\frac{1}{M_i^2-M_k^2}\cdot\\
&\cdot\left[\left(M_i^3\left(M_j+\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)+M_i^2M_k\left(M_j-\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)\right)\cdot\right.\\
&\cdot\int\frac{d^3p_jd^3p_\phi}{2E_j(2\pi)^3 2E_\phi(2\pi)^3}(2\pi)^4\delta^0(M_i-E_j-E_\phi)\delta^3(\vec{p}_i-\vec{p}_j-\vec{p}_\phi)+\\
&+\frac{1}{2}M_i(M_i+M_k)\sqrt{M_i^4+M_j^4+m_\phi^4-2M_i^2M_j^2-2M_i^2m_\phi^2-2M_j^2m_\phi^2}\cdot\\
&\left.\cdot\int\frac{d^3p_jd^3p_\phi}{2E_j(2\pi)^3 2E_\phi(2\pi)^3}(2\pi)^4\delta^0(M_i-E_j-E_\phi)\delta^3(\vec{p}_i-\vec{p}_j-\vec{p}_\phi)\cos\gamma_j\right]
\end{aligned}
\tag{D.24}
$$

where $\gamma_j$ is the angle at which the Right Handed Neutrino $N_j$ comes out. Thanks to the symmetries of the problem, we are free to perform a rotation of the reference frame and select the frame where the exit angle of $L_\alpha$ is equal to 0, i.e. $\gamma \to \gamma_j$

**The Phase Space Integral**

This time the phase space integral is more difficult than the one we faced in B.1.1 because of the presence of non-zero mass particles. Again, we need to perform two different integration, one without and the other with the angular dependance on $\cos\gamma_j$. The first step is to integrate out the $\delta^3(\vec{p}_i-\vec{p}_j-\vec{p}_\phi)$, which sets $\vec{p}_j=\vec{p}_\phi$ but not $E_j=E_\phi$ because they have different masses;

$$
\begin{aligned}
&\int\frac{d^3p_jd^3p_\phi}{2E_j2E_\phi}\frac{1}{(2\pi)^2}\delta^0(M_i-E_j-E_\phi)\delta^3(\vec{p}_i-\vec{p}_j-\vec{p}_\phi)=\\
&=\int\frac{d^3p_j}{4E_jE_\phi}\frac{1}{(2\pi)^2}\delta^0(M_i-E_j-E_\phi)
\end{aligned}
\tag{D.25}
$$

which, after passing to spherical coordinates and redefining $|\vec{p}_j|\equiv x$,

$$
\begin{aligned}
&\int \frac{d\Omega_j x^2 dx}{4E_j(x)E_\phi(x)}\frac{1}{(2\pi)^2}\delta^0(M_i - E_j(x) - E_\phi(x)) = \\
&= \frac{1}{4\pi}\int_0^\infty \frac{x^2 dx}{E_j(x)E_\phi(x)}\delta^0(M_i - E_j(x) - E_\phi(x)) = \\
&= \frac{1}{8\pi}\left(\int_0^\infty \frac{x^2 dx}{E_j(x)E_\phi(x)}\delta^0(M_i - E_j(x) - E_\phi(x)) + \right. \\
&\left. + \int_0^\infty \frac{x^2 dz}{E_j(x)E_\phi(x)}\delta^0(M_i - E_j(x) - E_\phi(x))\right) = \\
&= \frac{1}{8\pi}\int_{-\infty}^\infty \frac{x^2 dx}{E_j(x)E_\phi(x)}\delta^0(M_i - E_j(x) - E_\phi(x)) = \\
&= \frac{1}{8\pi}\int_{-\infty}^\infty \frac{x^2 dx}{\sqrt{x^2+M_j^2}\sqrt{x^2+m_\phi^2}}\delta^0(M_i - \sqrt{x^2+M_j^2} - \sqrt{x^2+m_\phi^2})
\end{aligned}
\tag{D.26}
$$

And then if we define

$$
g(x) = M_i - \sqrt{x^2+M_j^2} - \sqrt{x^2+m_\phi^2}
\tag{D.27}
$$

with

$$
|g'(x)| = x\left(\frac{\sqrt{x^2+M_j^2}+\sqrt{x^2+m_\phi^2}}{\sqrt{(x^2+M_j^2)(x^2+m_\phi^2)}}\right)
\tag{D.28}
$$

Multiplying and dividing D.28 into the integral we obtain

$$
\begin{aligned}
&\frac{1}{8\pi}\int_{-\infty}^\infty \frac{x dx}{\sqrt{x^2+M_j^2}+\sqrt{x^2+m_\phi^2}}\cdot \\
&\cdot\delta^0(M_i - \sqrt{x^2+M_j^2} - \sqrt{x^2+m_\phi^2})\left(x\frac{\sqrt{x^2+M_j^2}+\sqrt{x^2+m_\phi^2}}{\sqrt{(x^2+M_j^2)(x^2+m_\phi^2)}}\right) = \\
&= \frac{1}{8\pi}\int_{-\infty}^\infty \frac{x dx}{\sqrt{x^2+M_j^2}+\sqrt{x^2+m_\phi^2}}\delta^0(g(x))|g'(x)|
\end{aligned}
\tag{D.29}
$$

To solve this integral we can now use the following relation for the Dirac $\delta$ function

$$
\int_{\mathrm{R}} \delta(g(x)) f(g(x))|g'(x)|dx = \int_{g(\mathrm{R})} \delta(u) f(u) du = f(0)
\tag{D.30}
$$

and D.29 becomes

$$
\frac{1}{8\pi}\frac{x}{\sqrt{x^2+M_j^2}+\sqrt{x^2+m_\phi^2}}\bigg|_{g(x)=0}
\tag{D.31}
$$

The condition $g(x) = 0$ comes directly from the kinematic of the process, and it fixes the variable $x$, which has been defined to be $x \equiv |\vec{p}_j|$. So, from (D.9), $x$ must be

$$x = |\vec{p}_j| = \sqrt{\frac{(M_i^2 + M_j^2 - m_\phi^2)^2}{4M_i^2} - M_j^2} \tag{D.32}$$

and D.31 becomes

$$\frac{1}{8\pi} \frac{\sqrt{\frac{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2 M_j^2 - 2M_i^2 m_\phi^2 - 2M_j^2 m_{\phi 2}}{4M_i^2}}}{\sqrt{m_\phi^2 + \frac{(M_i^2 + M_j^2 - m_\phi^2)^2}{4M_i^2} - M_j^2} + \sqrt{m_j^2 + \frac{(M_i^2 + M_j^2 - m_\phi^2)^2}{4M_i^2} - M_j^2}} \tag{D.33}$$

Trying to simplify this expression we obtain

$$\begin{aligned}
&\left(\frac{1}{8\pi}\right) \frac{\sqrt{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2 M_j^2 - 2M_i^2 m_\phi^2 - 2M_j^2 m_\phi^2}}{\sqrt{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2 M_j^2 + 2M_i^2 m_\phi^2 - 2M_j^2 m_\phi^2} + M_i^2 + M_j^2 - m_\phi^2} = \\
&= \left(\frac{1}{8\pi}\right) \frac{\sqrt{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2 M_j^2 - 2M_i^2 m_\phi^2 - 2M_j^2 m_\phi^2}}{M_i^2 - M_j^2 + m_\phi^2 + M_i^2 + M_j^2 - m_\phi^2} = \\
&= \left(\frac{1}{8\pi}\right) \frac{\sqrt{M_i^4 + M_j^4 + m_\phi^4 - 2M_i^2 M_j^2 - 2M_i^2 m_\phi^2 - 2M_j^2 m_\phi^2}}{2M_i^2}
\end{aligned}$$

The other integration from D.24 is

$$\begin{aligned}
&\int \frac{d^3 p_j d^3 p_\phi}{2E_j 2E_\phi} \frac{1}{(2\pi)^2} \delta^0(M_i - E_j - E_\phi) \delta^3(\vec{p}_i - \vec{p}_j - \vec{p}_\phi) \cos\gamma_j = \\
&= \int \frac{d^3 p_j}{4E_j E_\phi} \frac{1}{(2\pi)^2} \delta^0(M_i - E_j - E_\phi) \cos\gamma_j = \\
&= \int \frac{d\Omega_{\gamma_j} x^2 dx}{4E_j(x) E_\phi(x)} \frac{1}{(2\pi)^2} \delta^0(g(x)) \cos\gamma_j
\end{aligned} \tag{D.34}$$

where $x$ and $g(x)$ are defined as in (D.26) and (D.27). We can identify two terms, one analogous to the one in (D.25) multiplied by the integration over the solid angle

$$\int d\phi_j d\cos\gamma_j \cos\gamma_j = (2\pi) \int d\cos\gamma_j \cos\gamma_j = 0. \tag{D.35}$$

So this second piece vanishes and (D.24) can be rewritten as

$$\begin{aligned}
&\left(\frac{1}{8}\right)\mathrm{Im}\Big\{Y_R^{jk\star}Y_D^{k\alpha\star}Y_R^{ji}Y_D^{i\alpha}\Big\}\frac{1}{M_i^2-M_k^2}\cdot\\
&\cdot\left[M_i^3\left(M_j+\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)+M_i^2M_k\left(M_j-\frac{M_i^2+M_j^2-m_\phi^2}{2M_i}\right)\right]\cdot\\
&\cdot\left(\frac{1}{8\pi}\right)\frac{\sqrt{M_i^4+M_j^4+m_\phi^4-2M_i^2M_j^2-2M_i^2m_\phi^2-2M_j^2m_\phi^2}}{2M_i^2}=\\
&=\left(\frac{1}{64\pi}\right)\mathrm{Im}\Big\{Y_R^{jk\star}Y_D^{k\alpha\star}Y_R^{ji}Y_D^{i\alpha}\Big\}\frac{1}{M_i^2-M_k^2}\cdot\\
&\cdot\left[M_i^2(M_iM_j+M_kM_j)+\frac{\left(M_i^2+M_j^2-m_\phi^2\right)(M_i^2-M_iM_k)}{2}\right]\cdot\\
&\cdot\frac{\sqrt{M_i^4+M_j^4+m_\phi^4-2M_i^2M_j^2-2M_i^2m_\phi^2-2M_j^2m_\phi^2}}{2M_i^2}=\\
&=\left(\frac{1}{64\pi}\right)\mathrm{Im}\Big\{Y_R^{jk\star}Y_D^{k\alpha\star}Y_R^{ji}Y_D^{i\alpha}\Big\}\frac{1}{M_i^2\left(1-\frac{M_k^2}{M_i^2}\right)}\cdot\\
&\cdot M_i^2\frac{\sqrt{1+\frac{M_j^4}{M_i^4}+\frac{m_\phi^4}{M_i^4}-\frac{2M_i^2M_j^2}{M_i^4}-\frac{2M_i^2m_\phi^2}{M_i^4}-\frac{2M_j^2m_\phi^2}{M_i^4}}}{2M_i^2}\cdot\\
&\cdot M_i^4\left[\left(\frac{M_j}{M_i}+\frac{M_kM_j}{M_i^2}\right)+\frac{1}{2}\left(1+\frac{M_j^2}{M_i^2}-\frac{m_\phi^2}{M_i^2}\right)\left(1-\frac{M_k}{M_i}\right)\right],
\end{aligned}\tag{D.36}$$

Some $M_i$ factors simplify and defining

$$\sigma_i=\frac{m_\phi^2}{M_i^2},\qquad\qquad r_{ji}=\frac{M_j^2}{M_i^2},\tag{D.37}$$

$$\rho_{ij}=(1-r_{ji}-\sigma_i)^2-4r_{ji}\sigma_i.\tag{D.38}$$

we can rewrite the amplitude (D.36) in terms of this new functions,

$$\begin{aligned}
&\left(\frac{1}{128\pi}\right)\mathrm{Im}\Big\{Y_R^{jk\star}Y_D^{k\alpha\star}Y_R^{ji}Y_D^{i\alpha}\Big\}\frac{M_i^2}{1-r_{ki}}\bigg[\sqrt{r_{ji}}(1+\sqrt{r_{ki}})+\\
&+\frac{1}{2}(1+r_{ji}-\sigma_i)(1-\sqrt{r_{ki}})\bigg]\sqrt{1+r_{ji}^2+\sigma_i^2-2r_{ji}-2\sigma_i-2\sigma_ir_{ji}}=\\
&=\left(\frac{1}{128\pi}\right)\mathrm{Im}\Big\{Y_R^{jk\star}Y_D^{k\alpha\star}Y_R^{ji}Y_D^{i\alpha}\Big\}\frac{M_i^2}{1-r_{ki}}\bigg[\sqrt{r_{ji}}(1+\sqrt{r_{ki}})+\\
&+\frac{1}{2}(1+r_{ji}-\sigma_i)(1-\sqrt{r_{ki}})\bigg]\sqrt{\rho_{ij}}=-2Im\Big\{\left[T^\dagger T\right]_{fi}T_{if}\Big\}.
\end{aligned}\tag{D.39}$$

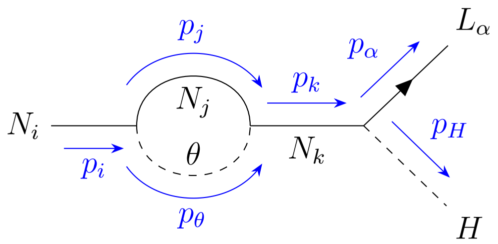
Now what is missing is the integration over $d\Pi_{L_\alpha,H}$; since nothing in (D.39) depends on

$\vec{p}_\alpha$ or $\vec{p}_H$, then it will all result in a multiplicative factor of $\left(\frac{1}{8\pi}\right)$ as showed in (B.33). When we also take into account the contribution from the denominator (B.4) and the sum over the RHN flavours in the loop and in the propagator, we obtain

$$\begin{aligned}\epsilon_{i\alpha}^{wave,\phi} = \left(\frac{1}{128\pi}\right)\frac{1}{\sum_\alpha |Y_D^{\alpha i}|^2}\sum_{j,k\neq i}\mathrm{Im}\Big\{Y_R^{jk\star}Y_D^{k\alpha\star}Y_R^{ji}Y_D^{i\alpha}\Big\}\frac{1}{1-r_{ki}}\cdot \\ \cdot\left[\sqrt{r_{ji}}(1+\sqrt{r_{ki}})+\frac{1}{2}(1+r_{ji}-\sigma_i)(1-\sqrt{r_{ki}})\right]\sqrt{\rho_{ij}}.\end{aligned} \tag{D.40}$$

### D.0.2 Wave Function Corrections by $\theta$

Let's see if something different happens when we consider the CP-odd degrees of freedom of the $U(1)_{B-L}$ scalar, i.e the Majoron $\theta$. This particle is massless and so in principle the kinematical condition for the on-shell loop is satisfied. In principle we don't expect a $\epsilon_{i\alpha}^{wave,\phi} \neq 0$ because the lack of mixing in the RHNs-$\theta$ interaction leads to a combination of Yukawa couplings which is real. The 1-loop diagram we are considering in this case is,




The kinematical condition in this case becomes $M_i > M_j + m_\theta$, but with $m_\theta = 0$. Again, the contribution to $\epsilon_{i\alpha}^{wave,\theta}$ will be related to $T^*_{kf} = \langle N_j\theta|\,T\,|L_\alpha H\rangle^*$, $T_{if} = \langle N_i|\,T\,|L_\alpha H\rangle$, already computed in (B.9) and $T_{ki} = \langle N_j\theta|\,T\,|N_i\rangle$. Reading out the terms from the Lagrangian in (4.33) we have that[1]

$$\begin{aligned}T_{ki} = \langle k|\,T\,|i\rangle = \langle N_j\theta|\int d^4x\frac{Y_R^{ij}}{2\sqrt{2}}\bar{N}\gamma^5 N\theta\,|N_i\rangle \simeq \\ \simeq \frac{Y_R^{ij}}{2\sqrt{2}}\langle N_j|\,\bar{N}\gamma^5 N\,|N_i\rangle = \frac{Y_R^{ij}}{2\sqrt{2}}\bar{u}_j\gamma^5 u_i,\end{aligned} \tag{D.41}$$

[1]The $i$ factor in the amplitude disappear when multipied with the $(-i)$ present in the Lagrangian.

and

$$\begin{aligned}
T^*_{kf} &= \langle k|\, T\, |f\rangle^* = \\
&= \langle N_j\theta| \int d^4x d^4y \left[\frac{\Upsilon^{kj}_R}{2\sqrt{2}}\bar{N}\gamma^5 N\theta\right]_y \left[-i\bar{N}\Upsilon^{k\alpha}_D P_L \tilde{H}^\dagger L\right]_x |L_\alpha H\rangle^* \simeq \\
&\simeq i\frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \langle N_j|\, \bar{N}\gamma^5 N_y \bar{N}_x P_L L\, |L_\alpha\rangle^* = \\
&= i\frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \langle N_j|\, \bar{N}\gamma^5 \frac{i}{\not{p}_k - M_k} P_L L\, |L_\alpha\rangle^* = \\
&= \frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \frac{1}{p_k^2 - M_k^2} \langle N_j|\, \bar{N}\gamma^5(\not{p}_k + M_k) P_L L\, |L_\alpha\rangle^* = \\
&= \frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \frac{1}{p_k^2 - M_k^2} \Big(\bar{u}_j \gamma^5 (\not{p}_k + M_k) P_L u_\alpha\Big)^* = \\
&= \frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \frac{1}{p_k^2 - M_k^2} \Big(\bar{u}_j \gamma^5 (\not{p}_k + M_k) P_L u_\alpha\Big)^\dagger = \\
&= \frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \frac{1}{p_k^2 - M_k^2} u^\dagger_\alpha P_L \big(p_{k,\mu}(\gamma^\mu)^\dagger + M_k\big)\gamma^5\gamma^0 u_j,
\end{aligned} \tag{D.42}$$

Considering the commutation relation between $\gamma^\mu$ and $\gamma^5$ and the definition of $(\gamma^\mu)^\dagger$, we can write all as

$$\begin{aligned}
&- \frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \frac{1}{p_k^2 - M_k^2} u^\dagger_\alpha P_L \big(p_{k,\mu}(\gamma^\mu)^\dagger + M_k\big)\gamma^0\gamma^5 u_j \\
&= -\frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \frac{1}{p_k^2 - M_k^2} \bar{u}_\alpha P_R \Big(-\not{p}_k + M_k\Big) u_j
\end{aligned} \tag{D.43}$$

And now, multiplying the three pieces together and considering the sum/average over the internal degrees of freedom, we obtain

$$\begin{aligned}
&\frac{1}{2}\sum_{spins} \left(-i\Upsilon^{\alpha i}_D \bar{u}_i P_L u_\alpha\right) \left(-\frac{\Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D}{2\sqrt{2}} \frac{1}{p_k^2 - M_k^2} \bar{u}_\alpha P_R \Big(-\not{p}_k + M_k\Big) u_j\right) \left(\frac{\Upsilon^{ij}_R}{2\sqrt{2}} \bar{u}_j \gamma^5 u_i\right) = \\
&= \frac{-i}{16} \frac{\Upsilon^{\alpha i}_D \Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D \Upsilon^{ij}_R}{p_k^2 - M_k^2} \mathrm{Tr}\Big\{\bar{u}_i P_L u_\alpha \bar{u}_\alpha P_R \Big(\not{p}_k - M_k\Big) u_j \bar{u}_j \gamma^5 u_i\Big\} = \\
&= \frac{-i}{16} \frac{\Upsilon^{\alpha i}_D \Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D \Upsilon^{ij}_R}{p_k^2 - M_k^2} \mathrm{Tr}\Big\{P_L \not{p}_\alpha \Big(\not{p}_k - M_k\Big)\Big(\not{p}_j + M_j\Big)\gamma^5\Big(\not{p}_i + M_i\Big)\Big\} = \\
&= \frac{-i}{16} \frac{\Upsilon^{\alpha i}_D \Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D \Upsilon^{ij}_R}{p_k^2 - M_k^2} \mathrm{Tr}\Big\{\gamma^5 P_L \not{p}_\alpha \Big(\not{p}_k - M_k\Big)\Big(\not{p}_j + M_j\Big)\Big(-\not{p}_i + M_i\Big)\Big\} = \\
&= \frac{-i}{16} \frac{\Upsilon^{\alpha i}_D \Upsilon^{kj,*}_R \Upsilon^{k\alpha,*}_D \Upsilon^{ij}_R}{p_k^2 - M_k^2} \mathrm{Tr}\Big\{-P_L \not{p}_\alpha \Big(\not{p}_k - M_k\Big)\Big(\not{p}_j + M_j\Big)\Big(-\not{p}_i + M_i\Big)\Big\}
\end{aligned} \tag{D.44}$$

For the complete contribution we should study the kinematics of the process and to compute the phase space integrals, a very similar procedure to the one carried on in (D.0.1). Anyway, we can already see what happens when we consider diagonal RHNs Yukawa couplings, i.e if $\mathrm{Y}_R^{ij} = \delta^i_j \mathrm{Y}_R^{ij} \equiv \mathrm{Y}_R^i$; when we start with an $|N_i\rangle$ initial state, then (D.44) takes the form

$$\begin{aligned}
&\frac{-i}{16}\frac{\left(\mathrm{Y}_D^{i\alpha,*}\mathrm{Y}_D^{i\alpha}\right)\left(\mathrm{Y}_R^{i,*}\mathrm{Y}_R^{i}\right)}{p_i^2 - M_i^2}\,\mathrm{Tr}\Big\{-P_L \not{p}_\alpha \Big(\not{p}_i - M_i\Big)\Big(\not{p}_i + M_i\Big)\Big(-\not{p}_i + M_i\Big)\Big\} = \\
&= \frac{-i}{16}\frac{|\mathrm{Y}_D^{i\alpha}|^2|\mathrm{Y}_R^{i}|^2}{p_i^2 - M_i^2}\,\mathrm{Tr}\Big\{-P_L \not{p}_\alpha \Big(\not{p}_i - M_i\Big)\Big(\not{p}_i + M_i\Big)\Big(-\not{p}_i + M_i\Big)\Big\} = \\
&= \frac{-i}{16}\frac{|\mathrm{Y}_D^{i\alpha}|^2|\mathrm{Y}_R^{i}|^2}{p_i^2 - M_i^2}\,\mathrm{Tr}\Big\{-P_L \not{p}_\alpha (p_i^2 - M_i^2)\Big(-\not{p}_i + M_i\Big)\Big\} = \\
&= \frac{-i}{16}|\mathrm{Y}_D^{i\alpha}|^2|\mathrm{Y}_R^{i}|^2\,\mathrm{Tr}\Big\{-P_L \not{p}_\alpha \Big(-\not{p}_i + M_i\Big)\Big\}.
\end{aligned} \tag{D.45}$$

We can see that (D.45) is identically 0 because $\mathrm{Im}\Big\{|\mathrm{Y}_D^{i\alpha}|^2|\mathrm{Y}_R^{i}|^2\Big\}$ is and so also this contribution vanishes.

# Appendix E

# Boltzmann Equations: $\phi \to 2N_1$

Here we find the expression for the Boltzmann Equation (**??**). The thermal cross section $\gamma^{ab...}_{ij...}$ is defined by:

$$\begin{aligned} \gamma^{ab...}_{ij...} &= \gamma(a+b+... \to i+j+...) \equiv \\ &\equiv \int d\Pi_a f_a^{eq} d\Pi_b f_b^{eq} ... |\mathcal{M}(a+b+... \to i+j+...)|^2 (2\pi)^4 \delta^4 d\Pi_i d\Pi_j ... \end{aligned} \tag{E.1}$$

The equation we need to solve is, from (**??**)

$$\left(\dot{Y}_{N_1}\right)_{\phi-decay} = [\phi \longleftrightarrow N_1 N_1] \simeq (1 - y^2_{N_1})\gamma_\phi^{N_1 N_1} \tag{E.2}$$

So, we need to compute the thermal cross section $\gamma_\phi^{N_1 N_1}$ and for that we need first to compute the amplitude squared $|\mathcal{M}(\phi \to N_1 N_1)|^2$. The relevant piece in the Lagrangian (4.33) is

$$\mathcal{L}_{int} = -\frac{\mathrm{Y_R}}{2\sqrt{2}} \bar{N} N \phi \tag{E.3}$$

and so

$$\mathcal{M}(\phi \to N_1 N_1) = \langle N_1 N_1 | \left[ -\frac{\mathrm{Y_R}}{2\sqrt{2}} \bar{N} N \phi \right] |\phi\rangle = -\frac{Y_R^1}{2\sqrt{2}} \bar{u}_1 v_1 \tag{E.4}$$

and

$$\mathcal{M}^\dagger(\phi \to N_1 N_1) = -\frac{Y_R^1}{2\sqrt{2}} \bar{v}_1 u_1. \tag{E.5}$$

Summing and averaging over internal states we obtain

$$\begin{aligned} \sum_{spins} \mathcal{M}^\dagger \mathcal{M} &= \frac{(Y_R^1)^2}{8} \mathrm{Tr}\{\bar{v}_1 u_1 \bar{u}_1 v_1\} = \frac{(Y_R^1)^2}{8} \mathrm{Tr}\Big\{(\not{p}_1 + M_1)(\not{p}_2 - M_1)\Big\} = \\ &= \frac{(Y_R^1)^2}{8} \mathrm{Tr}\Big\{\not{p}_1 \not{p}_2 + M_1 \not{p}_2 + \not{p}_1 M_2 - M_1^2\Big\} = \frac{(Y_R^1)^2}{2} (p_1 \cdot p_2 - M_1^2) \end{aligned} \tag{E.6}$$

after having used the ciclicity of the trace and the spinor identities. Working in the CM frame and considering 4-momentum conservation, we can write the relations for the kinematical quantities of the process, i.e

$$p^\mu_\phi = \left(m_\phi, \vec{0}\right)$$

$$p^\mu_1 = (E_1, \vec{p}_1)$$

$$p^\mu_2 = (E_2, -\vec{p}_1)$$

with $E_1 = \sqrt{|\vec{p}_1|^2 + M_1^2}$ and $E_2 = \sqrt{|\vec{p}_2|^2 + M_1^2} = \sqrt{|\vec{p}_1|^2 + M_1^2} = E_1$, and since $E_1 + E_2 = 2E_1 = m_\phi$ we can write

$$E_1 = E_2 = \frac{m_\phi}{2} \tag{E.7}$$

and

$$|\vec{p}_1|^2 = |\vec{p}_2|^2 \equiv |\vec{p}|^2 = \frac{m_\phi^2}{4} - M_1^2. \tag{E.8}$$

Considering that

$$p_1 \cdot p_2 = E_1 E_2 + |\vec{p}|^2 = E_1^2 + |\vec{p}|^2 = \frac{m_\phi^2}{2} - M_1^2, \tag{E.9}$$

the amplitude can be written as

$$\sum_{spins} \mathcal{M}^\dagger \mathcal{M} = \frac{(Y_R^1)^2}{2}\left(\frac{m_\phi^2}{2} - 2M_1^2\right) = \frac{(Y_R^1)^2}{4} m_\phi^2 \left(1 - 4\frac{M_1^2}{m_\phi^2}\right). \tag{E.10}$$

We want now to compute the decay width of the scalar $\phi$, because we are going to use it to define an adimensional parameter $K_S \equiv \frac{\Gamma_D^\phi}{H_1 = H(T=M_1)}$ which represents a measurement of the strength of the sclar decay relative to the Hubble parameter at the time when the temperature of the thermal bath is $T = M_1$. The decay width is defined by

$$\begin{aligned}\Gamma_D^\phi \equiv \int d\Pi_{NN} \frac{1}{2E_p} \sum_{spins} \mathcal{M}^\dagger \mathcal{M} = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{1}{2m_\phi}(2\pi)^4 \cdot \\ \cdot \, \delta^4(p^\mu_\phi - p^\mu_1 - p^\mu_2) \frac{(Y_R^1)^2}{4} m_\phi^2 \left(1 - 4\frac{M_1^2}{m_\phi^2}\right)\end{aligned} \tag{E.11}$$

where $E_p$ is the energy of the process. After the integration of the $\delta^3(\vec{p}_\phi - \vec{p}_1 - \vec{p}_2)$ we remain with

$$\frac{1}{(2\pi)^2} \frac{(Y_R^1)^2}{32} m_\phi \left(1 - 4\frac{M_1^2}{m_\phi^2}\right) \int \frac{d^3p_1}{E_1^2} \delta^0(m_\phi - 2E_1). \tag{E.12}$$

Using spherical coordinates and defining $|\vec{p}_1| = p$ and $E \equiv E_1$ we obtain

$$\frac{1}{(2\pi)^2} \frac{(Y_R^1)^2}{32} m_\phi \left(1 - 4\frac{M_1^2}{m_\phi^2}\right) \int \frac{d\Omega p^2 dp}{E^2} \delta^0(m_\phi - 2E). \tag{E.13}$$

One can then use the relation $p^2 = E^2 - M_1^2$ to perform a change of variables in the integral: $EdE = pdp$ and

$$dp = \frac{EdE}{p} = \frac{EdE}{\sqrt{E^2 - M_1^2}} \tag{E.14}$$

so that the integral becomes

$$\begin{aligned}
&\frac{1}{(2\pi)^2}\frac{(Y_R^1)^2}{32} m_\phi \left(1 - \frac{4M_1^2}{m_\phi^2}\right)(4\pi)\int \frac{E^2 - M_1^2}{E^2}\frac{EdE}{\sqrt{E^2 - M_1^2}}\delta^0(m_\phi - 2E) = \\
&= \frac{(Y_R^1)^2}{32\pi} m_\phi \left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\sqrt{E^2 - M_1^2}}{E}\Bigg|_{E=\frac{m_\phi}{2}} = \frac{(Y_R^1)^2}{32\pi} m_\phi \left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\sqrt{\frac{m_\phi^2}{4} - M_1^2}}{\frac{m_\phi}{2}} = \\
&= \frac{(Y_R^1)^2}{32\pi} m_\phi \left(1 - \frac{4M_1^2}{m_\phi^2}\right)^{\frac{3}{2}} = \Gamma_D^\phi
\end{aligned} \tag{E.15}$$

At this point we can define the scalar decay parameter $K_\phi$ as:

$$K_\phi = \frac{\Gamma_D^\phi}{H_1} = \left(\frac{1}{32\pi}\right)(Y_R^1)^2 \frac{m_\phi M_{PL}}{1.66\sqrt{g_\star}M_1^2}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)^{\frac{3}{2}}, \tag{E.16}$$

What we need to do now is to compute the thermal cross section $\gamma_\phi^{N_1N_1}$ to conclude our analysis on the Boltzmann Equation. In this specific case $\gamma$ is given by,

$$\begin{aligned}
\gamma_\phi^{N_1N_1} &= \int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_\phi}{(2\pi)^3 2E_\phi}(2\pi)^4\delta^3(\vec{p}_\phi - \vec{p}_1 - \vec{p}_2)\cdot \\
&\cdot \delta^0(E_\phi - E_1 - E_2)e^{-\frac{E_\phi}{T}}|\mathcal{M}(\phi \to N_1N_1)|^2 = \\
&= \int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_\phi}{(2\pi)^3 2E_\phi}(2\pi)^4\delta^3(\vec{p}_1 + \vec{p}_2)\cdot \\
&\cdot \delta^0(E_\phi - E_1 - E_2)e^{-\frac{E_\phi}{T}}\frac{(Y_R^1)^2}{4}m_\phi^2\left(1 - 4\frac{M_1^2}{m_\phi^2}\right),
\end{aligned} \tag{E.17}$$

the $d^3p_2$ integration fixes $\vec{p}_2 = -\vec{p}_1$ and $E_1 = E_2$,

$$\begin{aligned}
&\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_\phi}{(2\pi)^3 2E_\phi}(2\pi)^4 \cdot \\
&\cdot \delta^3(\vec{p}_1+\vec{p}_2)\delta^0(E_\phi - E_1 - E_2)e^{-\frac{E_\phi}{T}}\frac{(Y_R^1)^2}{4}m_\phi^2\left(1-4\frac{M_1^2}{m_\phi^2}\right) = \\
&= \frac{(Y_R^1)^2}{4}m_\phi^2\left(1-4\frac{M_1^2}{m_\phi^2}\right)\int \frac{d^3p_1}{(2\pi)^3 4E_1^2}\frac{d^3p_\phi}{(2\pi)^3 2E_\phi}e^{-\frac{E_\phi}{T}}\delta^0(E_\phi - 2E_1) = \\
&= \frac{(Y_R^1)^2}{4}m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\int \frac{d\Omega|\vec{p}_1|^2 d|\vec{p}_1|}{(2\pi)^3 4E_1^2}\frac{d^3p_\phi}{(2\pi)^3 2E_\phi}e^{-\frac{E_\phi}{T}}\delta^0(E_\phi - 2E_1)
\end{aligned} \tag{E.18}$$

and again, considering the change of variable (E.14) and that $\int d\Omega = 4\pi$ we obtain

$$\left(\frac{1}{32\pi}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\int \frac{\sqrt{E_1^2-M_1^2}dE_1}{E_1}\frac{d^3p_\phi}{E_\phi}e^{-\frac{E_\phi}{T}}\delta^0(E_\phi - 2E_1). \tag{E.19}$$

We can now integrate over $E_1$ eliminating the last $\delta$, to get

$$\begin{aligned}
&\left(\frac{1}{32\pi}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\left(\frac{1}{2}\right)\int \frac{d^3p_\phi}{E_\phi}e^{-\frac{E_\phi}{T}}\frac{\sqrt{\frac{E_\phi^2}{4}-M_1^2}}{\frac{E_\phi}{2}} = \\
&= \left(\frac{1}{32\pi}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\left(\frac{1}{2}\right)\int d^3p_\phi e^{-\frac{E_\phi}{T}}\frac{\sqrt{E_\phi^2-4M_1^2}}{E_\phi^2}
\end{aligned} \tag{E.20}$$

and since $p_\phi^2 = E_\phi^2 - m_\phi^2$ and as in (E.14) $dp_\phi = \frac{E_\phi dE_\phi}{\sqrt{E_\phi^2-m_\phi^2}}$[1] and $d^3p_\phi = d\Omega p_\phi^2 dp_\phi$,

$$\begin{aligned}
&\left(\frac{1}{32\pi}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\left(\frac{1}{2}\right)\cdot \\
&\cdot \int d\Omega \int_{m_\phi}^{\infty}\frac{\left(E_\phi^2-m_\phi^2\right)E_\phi dE_\phi}{\sqrt{E_\phi^2-m_\phi^2}}e^{-\frac{E_\phi}{T}}\frac{\sqrt{E_\phi^2-4M_1^2}}{E_\phi^2} = \\
&= \left(\frac{1}{16}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\cdot \\
&\cdot \int_{m_\phi}^{\infty} e^{-\frac{E_\phi}{T}}E_\phi\sqrt{1-\frac{m_\phi^2}{E_\phi^2}}\sqrt{1-\frac{4M_1^2}{E_\phi^2}}dE_\phi.
\end{aligned} \tag{E.21}$$

At this point it's worth noticing that the variable $E_\phi$ runs from $m_\phi$ to $+\infty$, and we need also to satisfy the condition $m_\phi \geq 2M_1$ in order to have the right kinematics for the scalar

[1] Again we can define $|\vec{p}_\phi| = p_\phi$

decay we are studying. So, if we work in the regime where $m_\phi >> M_1$, we can work with the approximation $\sqrt{1-\frac{4M_1^2}{E_\phi^2}} \simeq 1$ valid for all the values of the $E_\phi$ range in the integral. If we want to have a numerical estimation, already considering $m_\phi = 3M_1$, we obtain that $0.75 \leq \sqrt{1-\frac{4M_1^2}{E_\phi^2}} \leq 1$ and for $m_\phi = 4M_1$, we obtain that $0.86 \leq \sqrt{1-\frac{4M_1^2}{E_\phi^2}} \leq 1$.

In this limit we can write the integral in (E.21) as

$$\simeq \left(\frac{1}{16}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\int_{m_\phi}^{\infty} e^{-\frac{E_\phi}{T}}E_\phi\sqrt{1-\frac{m_\phi^2}{E_\phi^2}}dE_\phi \tag{E.22}$$

Now we define the variable $x \equiv E_\phi/T$, so that $E_\phi = Tx$ and $dE_\phi = Tdx$ and we rewrite (E.22) as

$$\begin{aligned}&\left(\frac{1}{16}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\int_{m_\phi/T}^{\infty} e^{-x}(xT)\sqrt{1-\frac{m_\phi^2}{T^2x^2}}Tdx = \\ &= \left(\frac{1}{16}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)T^2\int_{m_\phi/T}^{\infty} e^{-x}\sqrt{x^2-\frac{m_\phi^2}{T^2}}dx\end{aligned} \tag{E.23}$$

To solve the integral we then write $\frac{m_\phi}{T} \equiv \alpha$ so that

$$\int_{m_\phi/T}^{\infty} e^{-x}\sqrt{x^2-\frac{m_\phi^2}{T^2}}dx = \int_{\alpha}^{\infty} e^{-x}\sqrt{x^2-\alpha^2}dx \tag{E.24}$$

which solution can be written in terms of the modified Bessel functions of the second kind $\mathcal{K}_1$, i.e

$$\int_{\alpha}^{\infty} e^{-x}\sqrt{x^2-\alpha^2}dx = \alpha\mathcal{K}_1(\alpha). \tag{E.25}$$

If we recall the definition of the variable $z \equiv \frac{M_1}{T}$ given in (3.3.6), then $\alpha = \frac{m_\phi}{M_1}z$ and $\gamma_\phi^{N_1N_1}$ can be finally written as

$$\begin{aligned}&\left(\frac{1}{16}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^2\left(1-\frac{4M_1^2}{m_\phi^2}\right)\frac{M_1^2}{z^2}\frac{m_\phi}{M_1}z\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right) = \\ &= \left(\frac{1}{16}\right)\frac{1}{(2\pi)^3}(Y_R^1)^2 m_\phi^3 M_1\left(1-\frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{z} = \gamma_\phi^{N_1N_1}\end{aligned} \tag{E.26}$$

The time variation of the lightest RHN yield $Y_1$ due to the $\phi$-decay will be given by (4.45),

$$\left(\dot{Y}_{N_1}\right)_{\phi-decay} = \frac{sH_1}{z}\left(\frac{dY_{N_1}}{dz}\right)_{\phi-decay} =$$
$$= \left(1 - \frac{Y_{N_1}^2}{Y_{eq,N_1}^2}\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{z} = \tag{E.27}$$
$$= \frac{1}{Y_{eq,N_1}^2}\left(Y_{eq,N_1}^2 - Y_{N_1}^2\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{z},$$

while the variation in terms of $z$ turns out to be

$$\left(\frac{dY_{N_1}}{dz}\right)_{\phi-decay} = \frac{1}{Y_{eq,N_1}^2}\left(Y_{eq,N_1}^2 - Y_{N_1}^2\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{sH_1} \tag{E.28}$$

What we want to do now is first to rewrite this result in terms of the number of $N_1$ particles, which is the product of the $N_1$ density and the comoving volume $a^3$ and which we are going to call for simplicity $N_1 \equiv n_1 a^3$. The entropy density and the comoving volume are related by

$$s = \frac{2\pi^2}{45}\frac{1}{a^3}, \tag{E.29}$$

and so

$$Y_{N_1} \equiv \frac{n_1}{s} = \frac{45}{2\pi^2}n_1 a^3 = \frac{45}{2\pi^2}N_1. \tag{E.30}$$

In this way (E.28) can be rewritten as

$$\left(\frac{45}{2\pi^2}\right)\left(\frac{dN_1}{dz}\right)_{\phi-decay} = \frac{s^2}{n_{eq,1}^2}\left(\frac{n_{eq,1}^2}{s^2} - \frac{n_1^2}{s^2}\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{sH_1} =$$
$$= \frac{s}{n_{eq,1}^2}\left(\left(\frac{45}{2\pi^2}\right)^2(N_{eq,1}^2 - N_1^2)\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{H_1} =$$
$$= \left(\frac{2\pi^2}{45}\right)\frac{1}{N_{eq,1}n_{eq,1}}\left(\left(\frac{45}{2\pi^2}\right)^2(N_{eq,1}^2 - N_1^2)\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{H_1} \tag{E.31}$$

and we can see that the $2\pi^2/45$ factors go away. The final result reads

$$\left(\frac{dN_1}{dz}\right)_{\phi-decay} = \frac{1}{N_{eq,1}n_{eq,1}}\left(N_{eq,1}^2 - N_1^2\right)\frac{(Y_R^1)^2 m_\phi^3 M_1}{16(2\pi)^3}\left(1 - \frac{4M_1^2}{m_\phi^2}\right)\frac{\mathcal{K}_1\left(\frac{m_\phi}{M_1}z\right)}{H_1} \tag{E.32}$$

where $n_{eq,1}$ is computed using the Maxwell-Boltzmann statistics, i.e $f_{eq}^{MB}(p) = e^{-E/T}$, since it makes a small difference with respect to using the more complicated Fermi statistics [2], and it's defined by [22]

$$n_{eq,1} \equiv \frac{g(=2)}{(2\pi)^3} \int d^3p f_{eq}^{MB}(p) = \frac{T^3}{\pi^2} z^2 \mathcal{K}_2(z) = \frac{M_1^3}{\pi^2} \frac{\mathcal{K}_2(z)}{z}. \tag{E.33}$$

What we need to do now is to rewrite this result in terms of the $K_\phi$ parameter defined in (E.16). Substituting for $n_{eq,1}$ we obtain

$$\begin{aligned} \left(\frac{dN_1}{dz}\right)_{\phi-decay} &= \frac{(Y_R^1)^2 m_\phi^3}{(128\pi) M_1^2 H_1} \left(1 - \frac{4M_1^2}{m_\phi^2}\right) z \frac{\mathcal{K}_1(z\frac{m_\phi}{M_1})}{\mathcal{K}_2(z)} \frac{\left(N_{eq,1}^2 - N_1^2\right)}{N_{eq,1}} = \\ &= \frac{1}{\sqrt{1 - \frac{4M_1^2}{m_\phi^2}}} \left(\frac{m_\phi}{2M_1}\right)^2 K_S z \frac{\mathcal{K}_1(z\frac{m_\phi}{M_1})}{\mathcal{K}_2(z)} \frac{\left(N_{eq,1}^2 - N_1^2\right)}{N_{eq,1}}. \end{aligned} \tag{E.34}$$

## E.1 The Case with $K_1 = 0.1$ and $K_\phi \simeq K_1$

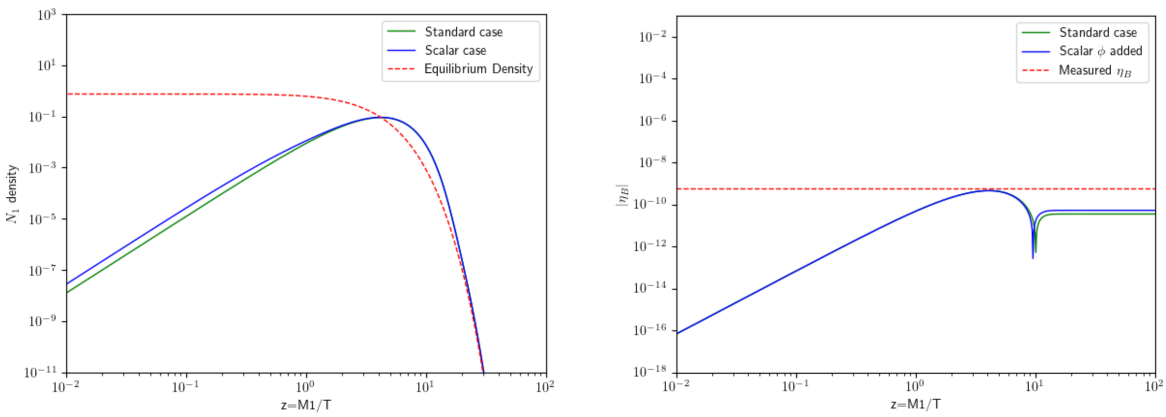
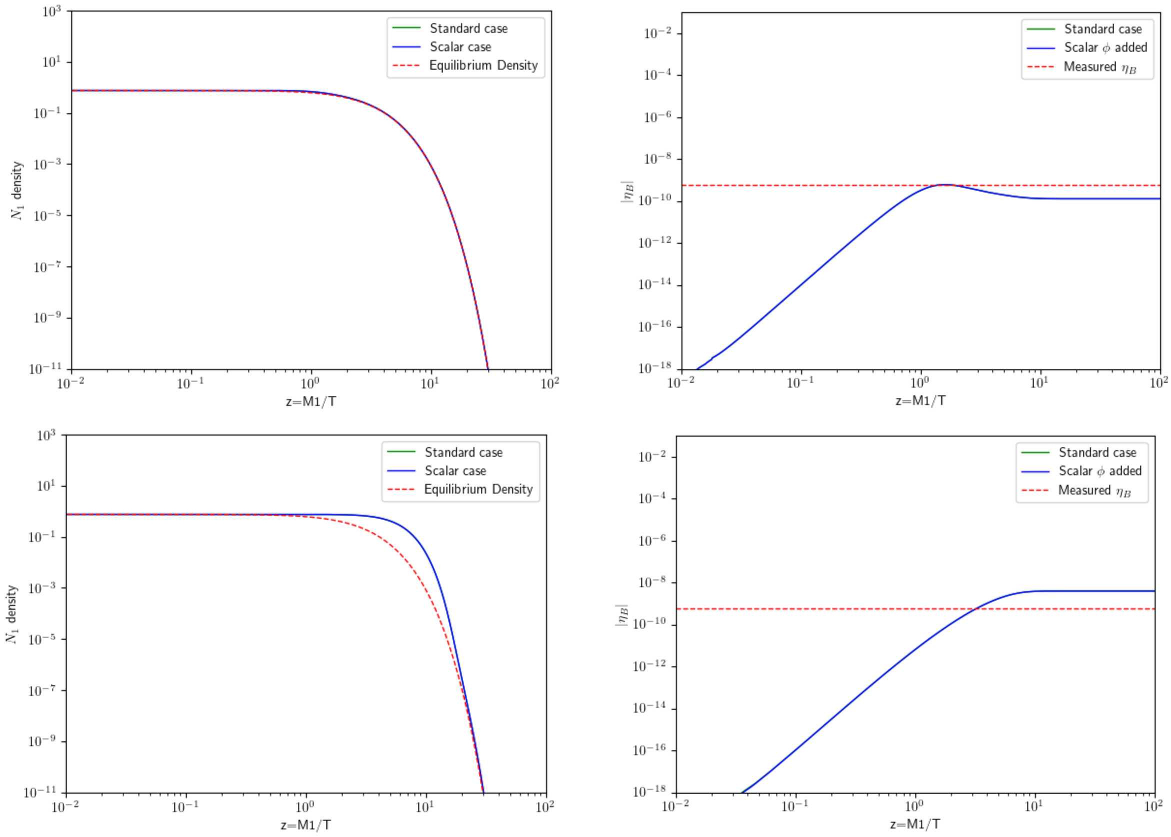



FIGURE E.1: The figure on the left shows the behaviour of the lightest Right Handed Neutrino $N_1$ number density (on the left) both in the standard case (green line) and in the case with the scalar $\phi$ decay added (blue line). The figure on the right shows instead the absolute value of the baryon asymmetry of the Universe $|\eta_B|$ (on the right), where again the two cases, the standard one (green) and the modified one (blue) are plotted. Both the $N_1$ number density and the $|\eta_B|$ are represented as a function of $z = M_1/T$. We considered $M_1 = 10^{14}$, the washout factor $K_1 = 0.1$ (weak washout regime) and the decay parameter $K_\phi \simeq 0.1$. The dashed red lines represents the $N_1$ equilibrium number density (on the left) and the measured baryon asymmetry of the Universe today (on the right).

[2] The difference in $n_{eq}$ is of order of 10% at $T = m$

## E.2 The Thermal Initial Abundance Case

We show here the results for the case with thermal initial abundance for the $N_1$ number density in the case where the scalar decay parameter is $K_\phi << 1$, in particular in the upper figures we have $K_1 = 10$ and $K_\phi \simeq 0.1$ while in the lower ones $K_1 = 0.1$ and $K_\phi \simeq 0.01$.




We can notice that when the decay parameter is $K_\phi << 1$, the presence of the scalar does not affect the standard picture as expected, in fact the green and blue lines cannot even be distinguished.

# Appendix F

# ULYSSES

## F.1 ULYSSES: Universal LeptogeneSiS Equation Solver

ULYSSES [105] is a python package that calculates the baryon asymmetry from leptogenesis in the context of a type-I seesaw mechanism. The code solves semi-classical Boltzmann equations for points in the parameter space chosen by the user. The ULYSSES code solves Boltzmann equations in terms of number density of particles, using the conversion from the $B-L$ number density to the baryon-to-photon ratio:

$$\eta_B \equiv \frac{N_B}{N_\gamma^{\mathrm{rec}}} = a_{\mathrm{sph}} \frac{N_{B-L}}{N_\gamma^{\mathrm{rec}}} = \frac{28}{79}\frac{1}{27} N_{B-L} = 0.013 N_{B-L}, \tag{F.1}$$

where $N_{B-L}$ is the final B-L asymmetry, $a_{\mathrm{sph}} = 28/79$ is the SM sphaleron factor and the $1/27$ factor derives from diluition in our normalization conventions. New physics can change the sphaleron factor, for instance in the MSSM we have $a_{\mathrm{sph}} = 8/23$. ULYSSES can solve LG equations in different context and scenarios: it can consider density matrix equations with 1,2 or 3 RHNs, taking into account transitions between different flavours and it can work with different combinations of flavours and RHNs considered. In this thesis project we considered the so-called $1BE1F$ model, the one which works the one-flavour regime and consider the decay of only 1, i.e. the lightest, RHN.

The built in code has been modified to account for the contributions to the RHN number density due to the scalar decay.

## F.2 Tables Of the Parameters Used for the Numerical Analysis

We put here the parameters used for our numerical simulations. What changes between one case and the other ones is the value of the washout factor $K_1$ and of the $\phi$ mas, $m_\phi$ and coupling $Y_R^1$. We work with $M_1 = 10^14$ GeV.

| Parameter | Unit | Code input | Value |
|---|---|---|---|
| $\delta$ | [°] | delta | 217 |
| $\alpha_{21}$ | [°] | a21 | 0 |
| $\alpha_{31}$ | [°] | a31 | 0 |
| $\theta_{23}$ | [°] | t23 | 49.2 |
| $\theta_{12}$ | [°] | t12 | 33.44 |
| $\theta_{13}$ | [°] | t13 | 8.57 |
| $x_1$ | [°] | x1 | 180 |
| $y_1$ | [°] | y1 | 1.4 |
| $x_2$ | [°] | x2 | 180 |
| $y_2$ | [°] | y2 | 11.2 |
| $x_3$ | [°] | x3 | 180 |
| $y_3$ | [°] | y3 | 11 |
| $\log_{10}(m_{1/3})$ | [eV] | m | -100 |
| $\log_{10}(M_1)$ | [GeV] | M1 | 14 |
| $\log_{10}(M_2)$ | [GeV] | M2 | 15 |
| $\log_{10}(M_3)$ | [GeV] | M3 | 16 |

**The Case with $K_1 = 10$, $K_\phi \simeq 150$**

In this case, $K_1 = 10$, $m_\phi = 10^{15}$ GeV and $Y_R^1 = 0.5$.

**The Case with $K_1 = 10$, $K_\phi \simeq 10$**

In this case, $K_1 = 10$, $m_\phi = 10^{15}$ GeV and $Y_R^1 = 0.13$.

**The Case with $K_1 = 10$, $K_\phi \simeq 0.02$**

In this case, $K_1 = 10$, $m_\phi = 4 \cdot 10^{14}$ GeV and $Y_R^1 = 0.008$.

**The Case with $K_1 = 0.1$, $K_\phi \simeq 20$**

In this case, $K_1 = 0.1$, $m_\phi = 10^{15}$ GeV and $Y_R^1 = 0.17$.

**The Case with $K_1 = 0.1$, $K_\phi \simeq 150$**

In this case, $K_1 = 0.1$, $m_\phi = 7 \cdot 10^{14}$ GeV and $Y_R^1 = 0.6$.

**The Case with $K_1 = 0.01$, $K_\phi \simeq 20$**

In this case, $K_1 = 0.01$, $m_\phi = 10^{15}$ GeV and $Y_R^1 = 0.18$.

**The Case with $K_1 = 0.01$, $K_\phi \simeq 200$**

In this case, $K_1 = 0.01$, $m_\phi = 10^{15}$ GeV and $Y_R^1 = 0.6$.

**The Case with $K_1 = 0.001$, $K_\phi \simeq 20$**

In this case, $K_1 = 0.001$, $m_\phi = 10^{15}$ GeV and $Y_R^1 = 0.18$.

**The Case with Thermal Initial Ab.**

In these two cases we considered $m_\phi = 10^{15}$ GeV and $Y_R^1 = 0.6$, and then, respectively, $K_1 = 10$ and $K_1 = 0.1$.

Notice that the parameters in the table are the same for the strong washout and the weak one cases; this is because fixing by hand the washout parameter lead to a modification in the six unknown parameters of the orthogonal matrix $R$ of the Casas-Ibarra parameterization.

# Acknowledgements

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