Alma Mater Studiorum · University of Bologna

School of Science
Department of Physics and Astronomy
Master Degree in Physics

TOY MODEL OF QUANTUM MECHANICS MODIFIED BY GRAVITY

Supervisor:
Prof. Alexandr Kamenchtchik

Submitted by:
Sara Cesari

Academic Year 2021/2022

Abstract

This thesis contributes to the debate on the outstanding issue of quantum gravity, fitting into the context of the upcoming related experimental era. We part ways with the conventional view of effective field theory, where General Relativity adapts to quantum laws, in favour of a transitional approach from quantum to classical regime at low energies.
In the first part we challenge the implications of a framework imposing quantization on the non-linear specificity of gravity. A stream of research about gravitation preventing massive objects to fluctuate and violating quantum mechanics is examined afterwards.
In line with this argumentation, we introduce a recent alternative to standard quantum gravity, the Correlated WorldLine (CWL) theory by Stamp, with the new perspectives it brings to the table.
In the core part we perform numerical analysis on toy models inspired by Stamp's approach with simplified method. Our goal is to mimic the above models to evaluate their reliability by heuristically predicting what kind of effects might surface and at what mass scales.
The last part is dedicated to briefly summarize results and identify possible development of related research.

Contents

Chapter 1

Introduction

1.1 Origin of mainstream quantum gravity

Two major breakthroughs still echo from the first half of last century: Quantum Mechanics (QM) and General Relativity (GR). One pertaining the dynamics of the micro-world, the other regulating gravitation up to celestial bodies. Experiments conducted over the years have cemented both theories at low energies, to the point one cannot undertake significant steps in modern theoretical physics by disregarding entirely their contribution. Each one stands on its own as a pillar of its corresponding domain.
A question that naturally follows - and many already have been asking themselves - is: how could they possibly merge into a larger, unified theory? The answer is one of the greatest challenges for XXI century physicists, for it requires fulfilling two demanding tasks: internal consistency between distant concepts and empirical proof accessible to experiments. As Rovelli put it [1], Quantum Gravity is comparable to the quest for the Holy Grail; the term is mainly given to theories of gravitation with non-negligible quantum effects, but is also more generic, related to the open issue.
The thirties were the cradle of Quantum Gravity, with the first deliberate works by Rosenfeld [2] and Bronstein [3], in a very different context from our own [4]. The former author was fairly optimistic about quantizing the gravitational field in a way that parallels electrodynamics. However, five years later, the latter found in a detailed study that this is easily possible provided that one remains within the framework of special relativity. Extension to the domain of GR requires one to question ordinary notion of the classical world.
A couple decades after this evidence of an apparent conflict between QM and GR, another was highlighted by Feynman [5]: the superposition principle, underpinning mechanics of the microscopic scales, seems to fail when extrapolated to the macroscopic scales. What is commonly known as "The Schrödinger's cat paradox" [6] seems to clash with our own experience of massive bodies, as in they are perfectly localized and do not fluctuate.

Rosenfeld himself [7] came to hypothesize that the seemingly classical nature of gravity renders its quantization pointless, contradicting his previous hopes.
One thing must be underlined, though. The vast majority of attempts to elaborate a unified theory assume that the space-time metric needs to be quantized, notably in what Stamp [11] refers to as "Orthodox Quantum Gravity": assuming QM to be universally valid, some UV quantum theory, such as well-known String Theory or Loop Quantum Gravity, reduces to GR, or a perturbed GR, in the low-energy limit [8]. Quantum correction are too small to induce any experimental proof of conflict between QM and IR gravity at all. To the present day, that is the most common opinion, often along with the the idea only Planck energy scales ($\sim 10^{19}$ GeV) allow heavily quantum and gravitational phenomena to coexist. If so, any aspiring candidate would be de facto untestable.

1.2 Reasons for a different approach

As is often the case, another take on the matter evolved in parallel fashion. A plethora of authors identified a possible solution to the conflict in a breakdown of QM at low energies. Among these theories, some ascribe the violation of the superposition principle to some phenomenon of gravitational nature. We are particularly interested in this line of reasoning, which brought to the more recent efforts by Stamp and Wilson-Gerow [10-13]: linear behaviour of QM is disrupted by a mechanism called "path bunching", whereby gravitational correlations arise for some crossover mass and the system of interest rapidly switches to classical behaviour.
Before diving deeper into the topic that inspired our work, clarification of the choice is in order. Let us probe some arguments against the quantization of gravity:

(i) *Quantum superposition of space-times*: As a matter of fact, gravitation strongly differs from other fundamental forces. Deeply embedded in the very structure of reality, it is involved in interactions as well as sets the framework of said interactions. Penrose [14] suggested a thought-experiment to test the stability of a quantum superposition of two slightly different mass distributions of the same object, like a sort of virtual Schrödinger's cat.
According to pure QM, any linear combinations of those two states is still a stationary state, perfectly stable in time. When one tries to add the effects of gravitational fields generated by the distributions, theoretical dilemmas needs to be attended to with extreme caution.
First of all, one experiences a conflict of principles. Ordinary QM allows decomposition into each state involved in the superposition; each wave function corresponds to a different particle location. On the other hand, GR covariance forbids distinction of locations: space-time geometries arising from different set of coordinates would be all just one.

Also, there would be no way to tell apart different states, for each configuration would be identical. Stamp proposes this perplexity again [11], one finds that trying to map between varieties unrelated to each other is quite a hard job. The foundation of GR at low energies basically strips every meaning from quantum interference.
Moreover, Penrose highlights the precarious stability of the superposed state. Two distinct metrics mean two distinct geometries of the states, thus ill-defined time-translation operator and related energy. What kind of physics one can expect to do in such a predicament? (A possible interpretation will be the subject of chapter 2).

(ii) *Acausality*: After the first approaches in the 1930's, much effort has been put into quantization of Einstein gravity, starting from generating functional theories with $g^{\tilde{\mu\nu}}(x)$ behaving like a field. The add of quantum fluctuations, though, causes instabilities to the functional integral that undermine the causal structure of space-time [15-19] .

(iii) *Uncertain Feynman diagrams*: The alleged universality of quantum gravity perturbative models with an effective approach at low energies is threatened by the ambiguity in quantum corrections calculation [20]. One has to extend GR to some generalized theory to make things work. The propagator of the gravitational field has no unequivocal origin, it could be derived from a higher derivative, non Einsteinian term of the action, failing to ensure decoupling of unphysical ghost at low energies.

(iv) *Philosophical standpoint*: The problem of unification is far from trivial. All the technical difficulties encountered raise the question whether there is actual compatibility between QM and GR. Theoretically speaking, the risk of losing the global vision on the problem of quantum gravity increases by excluding a priori any hypothesis aside the conventional route, referred to as psi-incomplete quantum gravity models (PIQG) [21]. The latter appeals to a compelling need of considering the universe as a homogeneous entity - although not all theories following this approach aspire to such a thing -, in order to understand it better. That is something deeply entailed in human nature, unifying things with the naive illusion of having control.
There are still two other options left: either gravity and quanta are part of something bigger and not yet within our grasp, in which case only time has the answer; or the unification is to be understood as describing correlation and transition between different domains. This second case has equal ontological dignity than any other.
Furthermore, one has to distinguish between progress in theoretical vs empirical field. New kinds of mathematical structures of standard quantum gravity must prove physical significance only through experimentation. Same naturally goes for competing theories, but the scarcity of technical means of the past prevented to guide researches through the leading path. Unless one manages to establish that PIQG models all fail to describe reality, success of the main research programme will not suffice to argue in favour of itself.

1.3 Introducing our study

The debate on shortcomings of theories still thrives and the final say is something definitely not within the reach of this work. This stance might suggest that further discussion could be vain, sterile. However, we are at the dawn of a new type of experimental era. The time is right and research ripe for refining. It is of utmost importance, for the sake of intellectual honesty and pragmatism, to provide what could be an answer, if the popular idea proves ineffective. Despite the great expectations for experiments that could definitively confirm the quantum nature of gravity by forcing massive objects into superposition [22, 23], there is still room for manoeuvre in testing - both theoretically and empirically - how it holds at large mass scales. The absence of clear evidence of QM validity in this regime might suggest some destructive gravitational interference that permanently impairs quantum decoherence, and that is what we intend to investigate.
A realistic aim for our work is to validate this different current of thought and ensure it has some legitimate points via heuristic analisys. The thesis is built on accepting Stamp's argumentation, but approaches the matter with an easier and more intuitive perspective, for a family of basic non relativistic models - namely, the free particle and the harmonic oscillator - that feature a gravitational self-interacting term. Our work consists in estimating a crossover mass for this term to become leading in behaviour and what kind of effects might be entailed at such scales. There are different scenarios that can happen:

- the effects of gravity are so little one can consider the system to remain basically quantum in behaviour;
- the effects of gravity become dominant very soon, disrupting quantum laws almost immediately;
- one is able to discern an intermediate transitional phase in a particular range of masses.

The work is organized as follows. What has been made in the field of alternative quantum gravity, i.e. gravity breaking down QM, will be presented more in detail in chapter 2, accompanied by potential directions for future experiments; chapter 3 will outline the main features of CWL theory by Stamp and its predictions; chapter 4 will yield numerical evidence for the critical mass in our systems, the threshold when non-linear gravitational effects predicted by Stamp start to appear, followed by concluding remarks in chapter 4.2.3.

Chapter 2

Review of Gravity-based Models modifying Quantum Mechanics

As proof of the relevance of the theme of this thesis as of today, there have been recent interest over the past few years in reviewing models with this kind of alternative approach to challenging QM [11, 13, 24-28]. We intend to follow the same steps, particularly the analysis in ref. [28] to create order in the matter.
Aside from the possible entanglement with gravity, modifying QM seems a logical as well as an inherent conclusion to the problem of how to take measurements, as stated by the "father" of QM himself and others [6, 29-32]. However, to reiterate the concept, the lack of experimental data that testify the quantum nature of gravity at microscopic scales and the failure at macroscopic superposition - as of now - opens the way to reconsider the different role of gravity at larger scales. Einsteinian torsion-less GR has passed all of the tests [69-37] trying to draw discrepancies out of it. We therefore proceed to examine a category of models that features the following proprieties:

1. gravity is a classical object and is treated as such as far as possible;
2. non-relativistic;
3. GR and QM are fundamentally incompatible theories at all scales. M_P does not seal the limit under which perturbatively quantized general relativity is consistent (as stated by EFT), it is just a threshold when departures from QM are so large it fails entirely and becomes non-linear [36];
4. evolution is regulated by dynamics of states or density matrices defined on a Hilbert space.

A convenient way to illustrate this family of models is by pointing at what is the main concept behind them. Here follows a division in four groups:

2.1 Uncertainty principle theories

Some of the proposals take their cue from the notorious "Heisenberg's uncertainty principle" [38], which is one of the leading proprieties of QM. Basically, there is a limit to what one can measure in a quantum system; despite knowing the initial condition of pairs of conjugate physical quantities, there is no way to simultaneously predict the exact value of both observables. A random variable is then introduced that does not disrupt linearity in $|\psi\rangle$ evolution, but causes decoherence.

Presumably, the first to apply this kind of approach to QM and GR was Károlyházy [39-41]. The idea is to interpret equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \langle \psi | \hat{T}_{\mu\nu} \psi \rangle, \tag{2.1}$$

differently from conventional quantum gravity. In this context the lhs inherits the uncertainties of the rhs. In other words, the quantum fluctuations of the stress-energy tensor become fluctuations of the space-time metric and Riemann tensor, consequently getting larger with increasing mass. One has, in essence, a family of random configurations of the metric $g_{\mu\nu}$, each characterized by an associated length s with at least a minimum value of the estimating error of

$$\Delta s \sim \sqrt[3]{\ell_P^2 s}. \tag{2.2}$$

This uncertainties implies a decay time for a quantum state to experience decoherence in position, which Károlyházy characterizes for the case of a homogeneous sphere of variable radius.
It has been then noticed via a perturbative approach [42] that this uncertainty of the metric modifies the Schrödinger equation with a stochastic potential, which leads to a non-Markovian master equation. Unfortunately, Károlyházy himself was not sure about the soundness of the model for smaller values of the computed cut-off [39] and the aforementioned stochastic corrections have been proven senselessly broad first [43], then correlated to decay of high-energy particles in X-ray range [44-47].

Then, the first Diósi's model suggested that - similarly to Károlyházy's on the basis of eq. (2.1) - no quantum measure can be performed exactly on the gravitational field [48] because of an uncertainty - generated by the system being in different quantum states - estimated via

$$\delta \tilde{\boldsymbol{g}} \approx \frac{G\hbar}{VT}, \tag{2.3}$$

where the field is defined by

$$\boldsymbol{g}(\boldsymbol{r}) = -\nabla\phi(\boldsymbol{r}) \tag{2.4}$$

and VT is a space-time volume. Here also followed a perturbative study of the Schrödinger equation that provided a master equation [49].
After that came the idea from Penrose [14], which differs from Károlyházy's in that is not based on GR, but rather on the Newtonian limit of a superposition of masses and space-times. In this configuration, each point in a gravitational field is subjected to two different accelerations that generate the following uncertainty in evaluation of local energy

$$\Delta E = \frac{1}{G}\int d\boldsymbol{r}\,(\boldsymbol{g}_1(\boldsymbol{r})-\boldsymbol{g}_2(\boldsymbol{r}))^2 = 4\pi G\int d\boldsymbol{r}\int d\boldsymbol{r'}\,\frac{\rho_1(\boldsymbol{r'})-\rho_2(\boldsymbol{r'})(\rho_1(\boldsymbol{r})-\rho_2(\boldsymbol{r}))}{|\boldsymbol{r}-\boldsymbol{r'}|}, \tag{2.5}$$

where expression (2.4) features a potential satisfying the Poisson equation. Eq. (2.5) leads to ill-definition of the time-translation operator and is used to derive the duration of the superposed state, which is a great threat to the equivalence principle [50]. Despite the obvious issues of divergence and limitation in the choice of mass distribution of expression (2.5), there has been some proposal to test the accuracy of this eventual decay time [51, 52] and interestingly enough, it is theoretically very similar to what further studies from Diósi brought, which are the subject of next subsection.

2.2 Stochastic collapse theories

Penrose did not imbibe on details of the mechanism found, but was the first to try to describe general features of a bigger theory, suggesting that its nature could possibly be of intrinsic decoherence of any quantum object coupled to gravity. Afterwards, a new wave of models came, and with it what we have already seen as the retelling of Károlyházy's proposal. This approach involves an external source of stochastic noise violating QM and therefore leading to non-linear dynamics of collapse of the wave-function, which is a very specific phenomenon that could possibly explain why QM is affected by measuring limitation. Here the main building block is the density matrix.

The so called "Diósi-Penrose" model - which is not a joint work but just an expression that essentially gathers the two above proposals from the respective authors with a new perspective - is the first example of this approach. Perfecting earlier found master equation, Diósi added a stochastic term [53] that brought to the following collapse model

$$d|\psi(t)\rangle = \Big[-\frac{i}{\hbar}\hat{H}dt + \int d\boldsymbol{x}(\hat{\rho}(\boldsymbol{r}) - \langle\hat{\rho}(\boldsymbol{r})_t\rangle)dW_t(\boldsymbol{r}) - \\ -\frac{G}{2\hbar}\int d\boldsymbol{r}\int d\boldsymbol{r'}\frac{(\hat{\rho}(\boldsymbol{r}) - \langle\hat{\rho}(\boldsymbol{r})_t\rangle)(\hat{\rho}(\boldsymbol{r'}) - \langle\hat{\rho}(\boldsymbol{r'})_t\rangle)}{|\boldsymbol{r}-\boldsymbol{r'}|}\Big]|\psi(t)\rangle, \quad (2.6)$$

which de-coheres with the same definite time as Penrose's model save for a negligible factor. If one excludes the back-reaction on the quantum system all works, provided that one employs a finite radial mass distribution, but Ghirardi et al. [54, 55] highlighted that the kinetic energy values go out of control for concentrated masses. This restrictive feature renders the model difficult to test [56] and with a lower bound on the diffusion radius of the mass distribution that has been calculated for astronomical bodies [57-60]. It was suggested to add dissipation as a solution [61].

Another model built on an added noise with zero average is the one by Adler [62,63]. It is based upon the assumption that the space-time fluctuations must have a complex part to induce the collapse of the wave-function and evolves according to the weak-field approximation Hamiltonian

$$\hat{H} = \hat{H}_0 + \xi c^2\int d\boldsymbol{x}\hat{\rho}(\boldsymbol{x})w(\boldsymbol{x},t), \quad (2.7)$$

where the noise is given by w and its strength by ξ. This model required adding some non-linear terms to resolve the non Hermiticity [64], so a further study provided the relative master equation, which is the generalization of Károlyházy's and first Diósi's models. Its real part can be reduced to the master equation of CSL model in ref. [54].

The latest Diósi's work [65] aims to overcome the obstacles of his previous proposals, namely: (a) the absence of a mechanism that could eliminate the existence of states other than well-located wave-functions for the Schrödinger equation modified by a Newtonian potential [66] and (b) the annoying presence in the collapsed state of continuous extra energy injections which violate, in fact, the energy conservation principles [53], as we already said earlier.
The brand new model is then a stochastic Schrödinger-Newton equation

$$\frac{d|\Psi\rangle}{dt} = -\frac{i}{\hbar}(\hat{H}+\hat{V}_\Psi)|\Psi\rangle - \frac{G}{2\hbar}\int d\boldsymbol{r}\int d\boldsymbol{r'}\frac{(\hat{\rho}_c(\boldsymbol{r})\hat{\rho}_c(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|}|\Psi\rangle + \frac{e^{-i\pi/4}}{\hbar}\int d\boldsymbol{r}\hat{\rho}_c(\boldsymbol{r})\Phi(\boldsymbol{r})|\Psi\rangle, \quad (2.8)$$

where the exponential factor in the stochastic potential destroys the fluctuations in the momentum with the add of the self-attracting potential, a ploy Diósi had previously guessed [67]. The price to pay is a non-linear master equation, something that might invalidate the model [68], but the rapid decay of the superposed state could prevent from detecting faults [69]. A pro is this is the first proposal addressing at least partially the energy-momentum conservation, but how it could provide directions for actual tests is still unclear and needs further refinements.

2.3 Non-linear Schrödinger Equation theories

These semi-classical models feature a deterministic equation

$$i\hbar \frac{d}{dt}\psi_t(\boldsymbol{x},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 - Gm^2 \int d\boldsymbol{y} \frac{|\psi_t(\boldsymbol{y},t)|^2}{|\boldsymbol{x}-\boldsymbol{y}|}\right)\psi_t(\boldsymbol{x},t), \tag{2.9}$$

which is the one-body case - extendable to many-bodies, and are based on the assumption that eq. (2.1) is the classical background in which dynamics of quantum matter takes place. The Newtonian term controls self-attraction of various parts of the wave-function and pulls against the constant widening of the wave packet, bringing it to a halt at some point, based on the mass involved.

Refs. [66, 70, 30] are probably the first attempts at trying this route and shortly after faced sharp counterarguments [68, 71-74], such as instantaneous signals and energy conservation, mainly deriving from maintaining standard structures of QM (Hilbert space and relative operators, states and system of measuring).
Despite the evident issues and the weak effects predicted, there has been some proposals to set experiments in which detecting disturbance to the quantum states of the system may be possible [75, 76]. Kibble, though, had a prominent role in the analysis of this problematic models [77-79], proposing a one of a kind sort of solution. One should entirely abandon cumbersome addition of terms to eq. (2.9) in favour of a class of generalized QFT models adapting to the non-linear peculiarities of GR. Nevertheless, his efforts did not provide a complete perturbation theory but rather warned about the challenges of such a task.

2.4 Stochastic gravity theories

A more recent stream of research, it began as a solution to earlier semi-classical proposals issues. Einstein equation (2.1) is substituted by its generalization, which introduces stochastic noise that prevents super-luminal signals and features a dynamics of continuous measurements of the position and a consequent feedback Hamiltonian [80-82].

Authors of ref. [83] explained how to build such a model with the positions of the particles of the system recorded as the average plus random fluctuations that cause noise:

$$r_j(t) = \langle \hat{x}_j(t) \rangle + \frac{\hbar}{\sqrt{\gamma}_j} w_j(t). \tag{2.10}$$

This is a way to control the result of measurements and thus is called a feedback protocol, where the quantum interaction is replaced by a classic-quantum one. The mechanism inspired the work that brought to light the KTM model [84], which features a pair of particles in a harmonic oscillator with induced decoherence by the noise in the measurements process, reduced via gravitational interaction.
A Lindblad master equation was extrapolated from the model [85] and attempts at generalizing it to more pairs of particles proved challenging and not in line with experiments [85, 86].

A similar approach was put forward by Tilloy and Diósi [87]. The authors work around the limitation imposed by eq. (2.9) with the repeated measurement protocol that generates stochastic fluctuations everywhere in the space-time background, not just where the particles are located - differently from the KTM model, which is why it is not the reduction of TD model. Thus, the gravitational source is now

$$\langle \psi \hat{\rho}(\boldsymbol{r}) \psi \rangle + \hbar \int d\boldsymbol{s} \gamma^{-1}(\boldsymbol{r} - \boldsymbol{s}) \delta \rho_t(\boldsymbol{s}) \tag{2.11}$$

where the decoherence linked to γ can be minimized and obtain the master equation of DP model, with the perk of automatically recovering the Newtonian interaction and not having to add it by hand.

We are not blind to faults of these models. Whether it is challenging to promote them to a fully relativistic theory, or the meaning of constraints to resolve divergences is unclear, or the noise is not introduced providing how it could interact with other forces involved, it seems there is still some work to do. However, many years of these calculations have been lacking a solid experimental base to guide them. In a sense, they humbly set their sights high, hoping for the new generations of physicists to have access to new technology. We can consider all the efforts as a stepping stone to what could be a promising way.

Chapter 3

Correlated WordLine Theory

We have seen how all models with the aim of modifying QM preserving the structure of GR encountered serious hurdles we can summarize as follows [11]:

too conservative: those which remain anchored to restricting language of QM bring unnecessary intricacy (that is also the case of standard quantum gravity, holding onto the idea QM is unalterable at any scale);

too drastic: those which bypass any contemporary verified theory, such as QFT.

The individual success of QM and GR demands they are not treated as modular theories [27]. Somehow, they need to be intertwined in their entirety via some definite mechanism, which can be observed through experiments, but also has to be predicted from the underlying new theory, in line with physical principals. That is precisely what CWL theory seeks.
Let us see, then, how it is different from the proposals seen so far. The main proprieties are:

1. gravity is still a classical entity and rules the macroscopic domain, with a lower bound corresponding to a crossover mass m_c, after which invalidation of QM is expected;

2. fully relativistic. One can obtain the non-relativistic limit, but the structure is entirely based on QFT (basically realizing the intuition by Kibble et al. [77-79]);

3. the equivalence principle from GR provides the key point for incorporating gravity in the model;

4. all the GR apparatus is in a context of wordline formalism;

5. evolution is regulated by standard Feynman path integrals.

In order to convey the uniqueness of this model, we provide an overview of the underlying reasoning.
The starting point for Stamp is a series of considerations about the phenomenon of decoherence [10]. The most common models under the scope of environmental decoherence are Spin-boson and Central Spin. With the term "environmental" it is assumed that a quantum system of interest is entangled to and interacts with an external system, which eventually destroys quantum interference. Unfortunately, these models show intrinsic issues linked to the purely quantum nature of the process involved and may not be exhaustive in describing all types of decoherence. The reason of this scepticism is experiments on coherence in large systems as in ref. [89] may be considered a test of QM validity, but do not add anything new at the fundamental level. Is the system large enough to be considered macroscopic? Is it possible to devise a situation in which QM actually fails? Stamps remarks that the possible solutions is intrinsic decoherence of the system, a possibility not to be theoretically excluded, also due to the issues about macroscopic "Cat states" [90] and already mentioned inconsistencies of a union between GR and QM [14, 72]. If one is able to pinpoint what forces are involved in the disruption of a superposed state of an isolated system, they must automatically bring to a spontaneous collapse of QM laws.
Therefore, one's mission becomes to search for some new kind of mechanism and to distinguish its signs from already known traces of environmental sources of decoherence during experiments. The inherent difficulty is similar to that of models seen in last chapter, because they add ad hoc terms that predict departures from quantum behaviour other than environmental decoherence. The key difference now is there is no need to summon interactions outside of the system, but a gravitational contribute is to be explained nonetheless.

So, what is the foundation of this new proposal? An outline is given in [10, 13, 88]. Firstly, there is a definition of adequate path integrals, which have two benefits with respects to the wave-functions:

- they emphasize the equations of motions and coherence because of the quantum phase factors featuring the action;
- the paths not followed by the system are automatically included and weighted accordingly in the calculation of the total amplitude, giving a more general formulation.

Standard QM path integral for a non relativistic particle is

$$K(x_2, t_2; x_1, t_1) = \int_{x_1}^{x_2} \mathcal{D}x e^{\frac{i}{\hbar} S[x]}, \tag{3.1}$$

while the same case for 2-paths particle in CWL theory, which can be visualized as a couple of rings, is

$$K(x_2,t_2;x_1,t_1) = \int_{x_1}^{x_2} \mathcal{D}x_1 \int_{x_1}^{x_2} \mathcal{D}x_2 e^{\frac{i}{\hbar}(S[x_1]+S[x_2]+S_{corr}[x_1,x_2])}. \tag{3.2}$$

The CWL propagator that takes into account also the interaction between the two paths. How so? What actually fixes the theory is the indistinguishability of the paths as stated in the generalized equivalence principle [11]. It can be interpreted here as the fact if one cannot tell identical particles apart, the concept must extend to paths of the same particle. Not only to a pair of paths, but to all the possible pairs of paths, which leads to the generalization for a scalar field

$$\mathcal{Z}[J] = \prod_{n=1}^{\infty} \int \mathcal{D}g^{(n)} e^{-nS_G[g^{(n)}]} \prod_{k=1}^{n} \int \mathcal{D}\phi_k^{(n)} e^{-S[\phi_k^{(n)}|g^{(n)}]} \tag{3.3}$$

where

$$S_G[g] = \frac{1}{16\pi G} \int d^4x \sqrt{g}R \tag{3.4}$$

is the Einstein-Hilbert action. One obtains the standard QFT when gravity is irrelevant and the classical Einstein equation. This new correlation is a peculiar feature, which means now the paths are not anymore independent, but correlated and thus, cannot be independently summed, that is the reason why the product form is correct and guarantees consistency with GR in the classical limit.

The complete form of the generating functional of a relativistic scalar field is written as

$$\mathcal{Q}[J] = \sum_{n=1}^{\infty} \prod_{k=1}^{n} \frac{1}{n!} \int \mathcal{D}\tilde{g}^{\mu\nu} \int \mathcal{D}\phi_k e^{iS_G/\hbar \Delta[\tilde{g}^{\mu\nu}]} e^{\frac{i}{n\hbar}\sum_k (S[g,\phi_k]+\int d^4x\, J(x)\phi_k(x))} \tag{3.5}$$

from which one can derive the n-point correlation functions, which exist at any point of any path. The gravitational correlator, then, is what allows quantum communication between paths via the metric density, for it defines the relative phase of the superposed state. This phenomenon takes the name of "path-bunching", in that the superposition is broken because all the paths tend to bunch together on the classical trajectories. In other words, a particle along a path is subjected not only to its own fields, but also to that of the others.

How to construct all the necessary tools for a complete CWL theory is extensively discussed in [11, 12, 13], so we limit ourselves to refer the main findings and limits:

- the theory may be renormalizable;
- in conventional QM probability is associated to the square module of the wave-function, but here, because of the new terms correlating parts of the density matrix how to physically interpret measurement is quite unclear still;

- in a perturbative study, the first order approximation in the gravitational coupling seems a no-loop theory. Further investigation on higher orders correction to n-point functions with large n is required in order to see path-bunching;
- when it comes to basic models, path-bunching is not experienced by quantum oscillators and particles with a single classical path, but there is a shift from quantum behaviour for two-paths systems;
- given the primary role of mass and the absence of entanglement with a binary system of particles, a study on a microscopic composite solid (silica) is undertaken and results in two characteristic frequencies and time scales for detecting path-bunching [13, 91] near the range of LISA.

Chapter 4

Systems modelling and Data Analysis

We believe that trying to illustrate the seriousness of the line of arguments seen in the highly complex model by Stamp can be a constructive contribution to the bigger picture, even if just limiting ourselves to the heuristic method.
We intend to employ all the language regarding standard quantum mechanics, with the square module of wave-functions interpreted as a probability, as usual. After setting the quantum model, we proceed to couple a semi-classical gravitational term to the model and study how quantum decoherence is afflicted as a consequence.
So, how do we build the simplest possible physical model? The answer lies at the beginning of everyone's journey through physics, that is, classical mechanics. A body free to move in space is the first kind of kinematic system one can think of. Such basic example can help us if we imagine a quantum particle as said body. More precisely, an extended non-relativistic particle, with a spherical mass distribution in space. We then add the possibility different parts of this distribution interact with each other via gravitation.
Along the way, we also re-frame the theoretical groundwork, while keeping the mathematical structure fairly concise and initial condition unvaried. We employ the harmonic oscillator model and again we modify the quantum base with a self-interacting term, this time in particular distinguishing between a global and a local kind of interaction.
Framework developing, numerical testing and analysis of results for both systems are discussed in this chapter.

4.1 Free non-relativistic quantum particle model

4.1.1 The propagator in one dimension

First of all, let us find the propagator for a particle of mass m in one dimension (on $\mathbb{R}$):

$$\langle x|e^{-\frac{i\hat{H}t}{\hbar}}|x'\rangle, \tag{4.1}$$

where the Hamiltonian is

$$\hat{H} = \frac{\hat{p}^2}{2m}. \tag{4.2}$$

We shall use the following formula:

$$\hat{x}|x_0\rangle = x_0|x_0\rangle, \tag{4.3}$$

where $|x_0\rangle$ is an eigenstate of the operator of the coordinate $\hat{x}$ with the eigenvalue x_0. The eigenstates $\hat{x}$ are normalized as

$$\langle x_1|x_2\rangle = \delta(x_1 - x_2). \tag{4.4}$$

If $|\psi\rangle$ is a quantum state (a vector in the corresponding Hilbert space), then

$$\langle x|\psi\rangle \tag{4.5}$$

is this state in the coordinate representation. Respectively,

$$\hat{p}|p_0\rangle = p_0|p_0\rangle, \tag{4.6}$$
$$\langle p_1|p_2\rangle = \delta(p_1 - p_2), \tag{4.7}$$

where $\hat{p}$ is a momentum operator. $\langle x|p\rangle$ is a eigenvector of the momentum operator in the coordinate representation:

$$\langle x|p\rangle = \frac{e^{\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}. \tag{4.8}$$

Then

$$\langle p|x\rangle = \frac{e^{-\frac{ipx}{\hbar}}}{\sqrt{2\pi\hbar}}. \tag{4.9}$$

The identity operator can be represented as

$$\hat{I} = \int_{-\infty}^{\infty} dx|x\rangle\langle x| = \int_{-\infty}^{\infty} dp|p\rangle\langle p|. \tag{4.10}$$

Then

$$\langle x|e^{-\frac{i\hat{H}t}{\hbar}}|x'\rangle = \int_{-\infty}^{\infty} dp\langle x|e^{-\frac{i\hat{H}t}{\hbar}}|p\rangle\langle p|x'\rangle \tag{4.11}$$
$$= \int_{-\infty}^{\infty} dp\langle x|e^{-\frac{i\hat{p}^2 t}{2m\hbar}}|p\rangle\langle p|x'\rangle. \tag{4.12}$$

However,

$$e^{-\frac{i\hat{p}^2 t}{2m\hbar}}|p\rangle = e^{-\frac{ip^2 t}{2m\hbar}}|p\rangle. \tag{4.13}$$

Hence,

$$\langle x|e^{-\frac{i\hat{H}t}{\hbar}}|x'\rangle = \int_{-\infty}^{\infty} dp e^{-\frac{ip^2 t}{2m\hbar}}\langle x|p\rangle\langle p|x'\rangle. \tag{4.14}$$

Now, using the formulae (4.8) and (4.9), we obtain

$$\langle x|e^{-\frac{i\hat{H}t}{\hbar}}|x'\rangle = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dp e^{-\frac{ip^2 t}{2m\hbar}} e^{\frac{ip(x-x')}{\hbar}}. \tag{4.15}$$

Finally, calculating the Gaussian integral, we arrive at

$$\langle x|e^{-\frac{i\hat{H}t}{\hbar}}|x'\rangle = \sqrt{\frac{m}{2\pi i\hbar t}}\exp\left(\frac{im(x-x')^2}{2\hbar t}\right). \tag{4.16}$$

4.1.2 1D dynamics

Let us choose the initial state (at $t = 0$) for the quantum particle as follows:

$$\langle x|\psi\rangle = \psi(x,t)|_{t=0} = \frac{1}{\pi^{1/4}\sqrt{\sigma}}\exp\left(-\frac{x^2}{2\sigma^2}\right). \tag{4.17}$$

Then, using the contraction of this initial state (4.17) with the propagator (4.16), we obtain the expression for the state of the particle at an arbitrary moment of time t:

$$\begin{aligned}\psi(x,t) &= \int dx' \langle x|e^{-\frac{i\hat{H}t}{\hbar}}|x'\rangle\langle x'|\psi\rangle \\ &= \sqrt{\frac{m}{2\pi i\hbar t}}\frac{1}{\pi^{1/4}}\int dx' \exp\left(\frac{im(x-x')^2}{2\hbar t} - \frac{x'^2}{\sigma^2}\right) \\ &= \frac{1}{\pi^{1/4}}\sqrt{\frac{m\sigma}{m\sigma^2 + i\hbar t}}\exp\left(-\frac{m}{2(m\sigma^2+i\hbar t)}x^2\right).\end{aligned} \tag{4.18}$$

The probability density is equal now to

$$\rho(x,t) = \bar{\psi}(x,t)\psi(x,t) = \frac{1}{\sqrt{\pi}}\sqrt{\frac{m^2\sigma^2}{m^2\sigma^4+\hbar^2 t^2}}\exp\left(-\frac{m^2\sigma^2}{m^2\sigma^4+\hbar^2 t^2}x^2\right). \tag{4.19}$$

One can see that the dispersion of the corresponding Gaussian packet is growing with time. Namely, from the form of the expression (4.19) we see that

$$\sigma(t) = \sigma\sqrt{1+\frac{\hbar^2 t^2}{m^2\sigma^4}}. \tag{4.20}$$

Let us note also that we can consider a backward evolution when t is decreasing. There is the invariance with respect to the time inversion $t \to -t$ and it is the moment $t = 0$ when the dispersion is minimal.

4.1.3 3D dynamics

For the free particle the evolution in three space direction is independent, being that the hamiltonian

$$\hat{H} = \frac{\hat{p_x}^2}{2m} + \frac{\hat{p_y}^2}{2m} + \frac{\hat{p_z}^2}{2m} \tag{4.21}$$

is separable. Hence, if we choose as the initial state the isotropic state

$$\langle x, y, z|\psi\rangle = \psi(x, y, z, t)|_{t=0} = \frac{1}{\pi^{3/4}\sqrt{\sigma^3}} \exp\left(-\frac{x^2 + y^2 + z^2}{2\sigma^2}\right), \tag{4.22}$$

then its time evolution will be given by

$$\psi(x, y, z, t) = \frac{1}{\pi^{3/4}} \left(\frac{m\sigma}{m\sigma^2 + i\hbar t}\right)^{3/2} \exp\left(-\frac{m}{2(m\sigma^2 + i\hbar t)}(x^2 + y^2 + z^2)\right). \tag{4.23}$$

Respectively, the probability density is equal to

$$\rho(x, y, z, t) = \frac{1}{\pi^{3/2}} \left(\frac{m^2\sigma^2}{m^2\sigma^4 + \hbar^2 t^2}\right)^{3/2} \exp\left(-\frac{m^2\sigma^2}{m^2\sigma^4 + \hbar^2 t^2}(x^2 + y^2 + z^2)\right). \tag{4.24}$$

4.1.4 Including the effect of the gravitation

Considering the gravitational interaction as a very weak one, we can try to calculate the self-interacting gravitational energy of the particle at the moment t using the expression (4.24). Then

$$E_{grav}(t) = -\frac{Gm^2}{2} \int dx_1 dy_1 dz_1 dx_2 dy_2 dz_2 \frac{\rho(x_1, y_1, z_1, t)\rho(x_2, y_2, z_2, t)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}, \tag{4.25}$$

where G is the Newton gravitational constant.
What does this expression mean? Looking closely at its form, we are surely reminded of the Newtonian potential. From a geometrical point of view, one has to abandon the idea of the particle as a point-like object well placed in space and imagine a sphere of infinite volume. If we consider the distribution of the mass (4.24), all we have at our disposal is the probability to find mass in every point of the sphere, higher going to the centre, where the particle should be, and lower further away from it. Here we need to quantify how each point of the sphere interacts via gravitation with another one, treating the particle as an indefinitely extended object.
We cannot calculate this function analytically, but we can calculate it numerically at different values of the parameter t and then we can try to find an analytic function,

which fits well enough the numerically found function (4.25). Then we can substitute this function of time into the Schrödinger equation, adding it to the Hamiltonian and see how the solution of the Schrödinger equations will be influenced by this correction.
The Hamiltonian of the three particle contains only the kinetic term

$$\hat{H} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right). \tag{4.26}$$

We can calculate the energy of the particle

$$E_{kin} = \langle\psi|\hat{H}|\psi\rangle = \frac{3\hbar^2}{4m\sigma^2}. \tag{4.27}$$

One can confront the expression (4.27) with the energy of the gravitational self-interaction, obtained numerically, and see on what mass and initial dispersion scale the peak of gravity becomes comparable to the energy of the particle, producing some effects on the model. We establish a reasonable threshold for this crossover mass m_c to lie at the point when the gravitational energy reaches around an order of magnitude lower than the kinetic energy.

Let us start by choosing the cgs as system of units, so all the constants are set accordingly:

$$G = 6.67 \cdot 10^{-8}\, cm^3 g^{-1} s^{-2}, \tag{4.28}$$

$$\hbar = 1.05 \cdot 10^{-27}\, erg\, s. \tag{4.29}$$

The composite rectangular integration method is straightforward and we decide to utilize it for our purpose. The normalization condition

$$\iiint \rho(x, y, z, t) dx dy dz = 1 \tag{4.30}$$

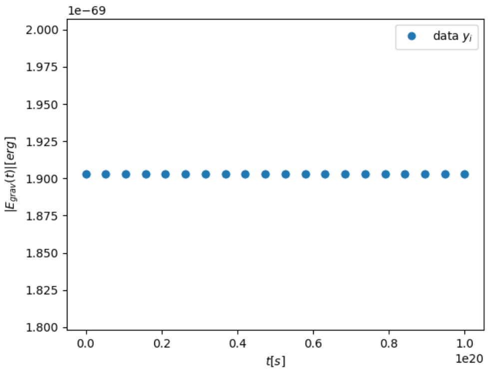
serves as a testing ground. For an integral in three variables such as in eq. (4.30) we are free to adjust the precision by changing the number of intervals N and thus iteration points $N + 1$ over which the algorithm calculates the values of the function. It is fairly easy to obtain the exact value for the total probability taking negligible time.
However, when we try to implement the same algorithm with the same precision to solve the integral in (4.25), problems arise that need to be addressed.
Calculation time grows excessively because of the intrinsic complexity of the problem. The number of operations with t fixed and $N = 1000$ increases from 10^9 to 10^{18}. We are forced to reduce N to 50 as a compromise, having (4.30) underestimated at 0.94.
Other issues ensue when one approximates the infinite limits for integration to some

large value. This leads to an intricate work of selecting orders of magnitude of the dispersion bigger than the accuracy factor, otherwise there is no contribution to the integral. Something analogous happens from iterating the process for growing values of t, due to which the increasingly flatter Gaussian from a certain point onwards simply vanishes. The strategy to work around this inefficient method is exploiting the fact 99% of the area subtended from a Gaussian is within a region of six times the dispersion (4.20).
With the above precondition in mind, we approach the calculation of self-energy (4.25) to see what happens as soon as the wave packet starts to spread. The set up is that of a test free particle in space, each time varying the initial uncertainty of localization. Fig. 4.1 demonstrates that a particle maintains its quantum nature more strongly the narrower we choose its initial mass distribution to be, as evidenced by an increasingly faster decay in time - even though the peak value of this contribution is inversely proportional to the dispersion. That is a very interesting observation in that one might be led to think that a gravitational mechanism based on an extended mass distribution would always feed itself. That is actually the case when the dispersion is at large scales as in (a), the self-interaction diffuse all over the space and remains unchanged with time passing by.
This, though, is a necessary but not sufficient condition, as reported in tab. 4.1, because it all depends also on how massive the distribution is. In essence, self-gravity does not imply actual modifications on the model unless the particle fluctuates in a region wider than some crossover scale of standard deviation of the starting Gaussian packet, referred to as σ_c, has sufficiently large mass, referred to as m_c, and its self-gravitating waterfall effect passes the test of time.

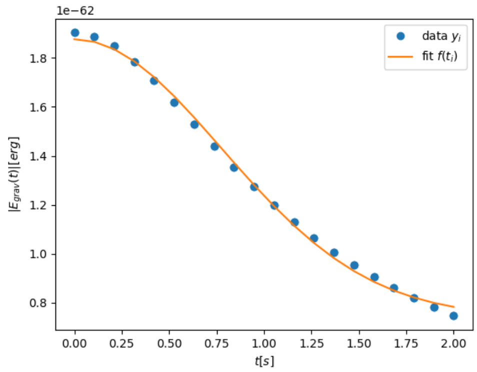
σ_c (cm)	m_c (g)	$\lvert E_{grav}(t=0)\rvert$ (erg)
$10^{34}\,(10^6 \cdot R_H)$	$9.11 \cdot 10^{-28}\,(m_e)$	$1.90 \cdot 10^{-96}$
10^{15} (Solar System $\varnothing$)	$3 \cdot 10^{-21}$ (3 Fullerene-C_{60})	$2.11 \cdot 10^{-64}$
1 (Human finger)	$3 \cdot 10^{-16}$ (Human adenovirus)	$2.11 \cdot 10^{-39}$
10^{-3} (Cloud water droplet)	$3 \cdot 10^{-15}$ (E. Coli genome)	$2.11 \cdot 10^{-34}$
10^{-6} (Extreme UV λ)	$3 \cdot 10^{-14}$ (Large virus)	$2.11 \cdot 10^{-29}$
$10^{-12}\,(10 \cdot r_c)$	$3 \cdot 10^{-11}$ (Yeast cell)	$2.11 \cdot 10^{-19}$
10^{-20} (σ 1 MeV ν)	10^{-9} (Birch pollen)	$2.35 \cdot 10^{-6}$
$2.21 \cdot 10^{-32}\,(\ell_P)$	$10^{-5}\,(M_P)$	$1.06 \cdot 10^{14}$
10^{-40}	$5 \cdot 10^{-3}$ (Mosquito)	$5.87 \cdot 10^{27}$

Table 4.1: Rough estimates of the scales when the gravitational term value is around one order of magnitude lower than the kinetic energy for a self-interacting particle.

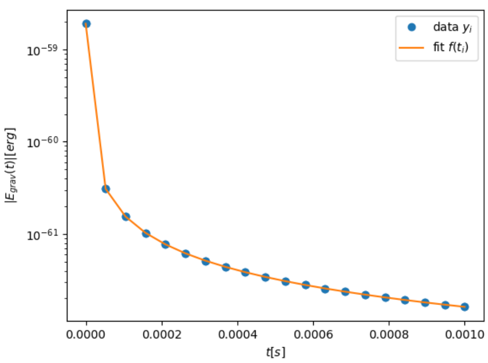
Tab. 4.1 collects data relative to fig. 4.2. The horizontal line symbolizes the imaginary



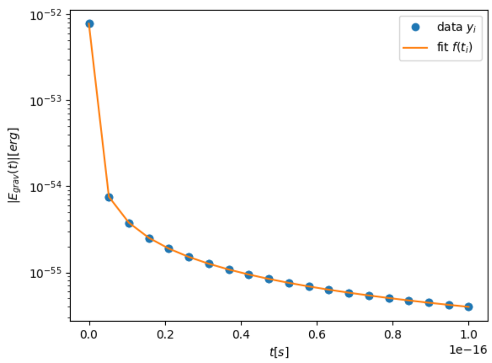
(a) $\sigma = 10^7 \ cm$



(b) $\sigma = 1 \ cm, \ f(t_i) = A + B \cdot \exp\left(-\frac{t^2}{2C^2}\right)$



(c) $\sigma = 10^{-3}\, cm, \ f(t_i) = A - B \cdot \arctan(Ct)$



(d) $\sigma = 10^{-10}\, cm, \ f(t_i) = A - B \cdot \arctan(Ct)$

Figure 4.1: Time evolution of the gravitational energy of a self-interacting particle at different values of the initial dispersion σ of its wave packet.

threshold we chose between the quantum regime (lower half of the graph) and classical regime (upper half of the graph), which is the point where the maximum self-gravitational interaction energy - when time decay did not happened yet - is 10% of kinetic energy. Three features really stand out:

- with increasing mass, the system has higher probability to experience transition to classical behaviour, while small values of mass and dispersion reinforce its quantum nature. Macroscopic masses in general are primary of classical behaviour, so this conclusion seems reasonable;
- in order to abandon the quantum regime, the distribution of the particle of microscopic mass requires the wave packet to be wide spread. Vice-versa, a well located distribution must contain a macroscopic object;
- the model with $\sigma = \lambda_C = 2\pi\hbar/mc$ behaves slightly differently because of the direct dependance on the mass of the dispersion. Gravitational effects, in this case, emerge at Planck's scale.

Finally, the data collected show that the particle system may not be the most functional with the purpose of experimentation. The time decay may prevent detecting departures from QM, thus we need a more stable system.

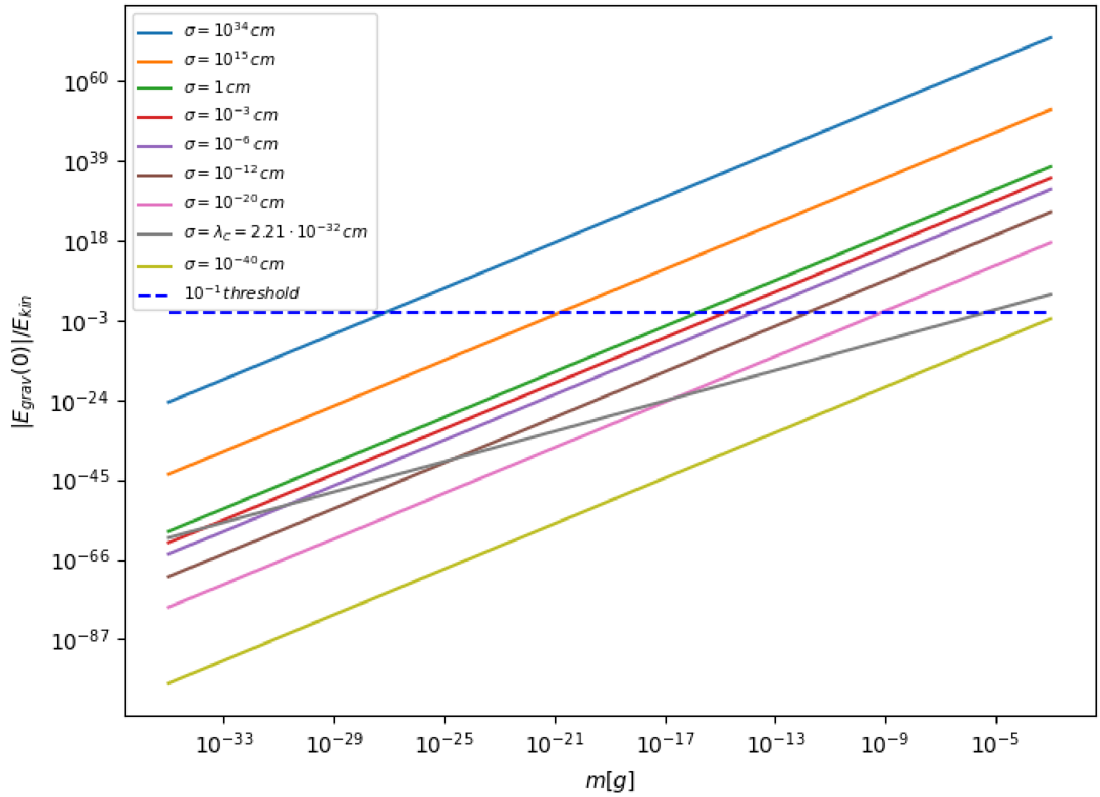


Figure 4.2: Susceptibility to classical self-gravity of the quantum particle model for variable mass scales. On the Y axis the ratio between the maximum self-interacting energy term and the free quantum kinetic energy.

4.2 Non-relativistic quantum harmonic oscillator model

4.2.1 1D case

Let us proceed with the analysis of another simple system, following the procedure seen for the free particle.
The Hamiltonian of the harmonic oscillator is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2}. \tag{4.31}$$

The initial state (at $t = 0$) for a quantum system is the ground state

$$\psi_0(x,t)|_{t=0} = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right), \tag{4.32}$$

with square module

$$\rho_0(x,t)|_{t=0} = \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \exp\left(-\frac{m\omega x^2}{\hbar}\right). \tag{4.33}$$

Expression (4.33) has the shape of a normalized Gaussian distribution, given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{4.34}$$

where the dispersion is

$$\sigma = \sqrt{\frac{\hbar}{2m\omega}}. \tag{4.35}$$

This is a very interesting expression when compared to the maximum amplitude of a classic harmonic oscillator with the same energy as the quantum one. To see that, we imagine the classic system as if it was a pulsating sphere. At a turning point, the total energy - i.e. the potential energy - equals the vacuum energy of the quantum oscillator:

$$\frac{\hbar\omega}{2} = \frac{m\omega^2 x_0^2}{2}. \tag{4.36}$$

The radius expands at a maximum of

$$x_0 = \sqrt{\frac{\hbar}{m\omega}}, \tag{4.37}$$

which represents the amplitude of the oscillator, obtained reversing eq. (4.36). We notice there is just a $1/\sqrt{2}$ factor between (4.37) and (4.35).
The evolution of the state (4.32) at an arbitrary time t is:

$$\psi_0(x,t) = e^{-iE_0 t/\hbar}\psi_0(x,0) = e^{-i\omega t/2}\psi_0(x,0), \tag{4.38}$$

whereas there is no variation in the associated probability density, due to the property of stationary states, depending on time exclusively in the phase factor:

$$\begin{aligned}\rho_0(x,t) &= |\psi_0(x,t)|^2 \\ &= \bar{\psi}_0(x,t)\psi_0(x,t) \\ &= e^{i\omega t/2}e^{-i\omega t/2}\bar{\psi}_0(x,0)\psi_0(x,0) \\ &= |\psi_0(x,0)|^2 \\ &= \rho_0(x,t)|_{t=0} \\ &= P.\end{aligned} \tag{4.39}$$

4.2.2 3D case

Extending the study to the three-dimensional case, the Hamiltonian of the system can be written as the sum of three unidimensional commuting harmonic oscillators:

$$\hat{H} = \sum_{i=1}^{3} \frac{1}{2m}\left(\hat{p_i}^2 + m^2\omega^2\hat{x_i}^2\right). \tag{4.40}$$

The wave function of the ground state becomes

$$\psi_0(x,y,z,t)|_{t=0} = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} \exp\left(-\frac{m\omega}{2\hbar}(x^2+y^2+z^2)\right) \tag{4.41}$$

and the relative probability density simply follows:

$$\rho_0(x,y,z,t)|_{t=0} = \left(\frac{m\omega}{\pi\hbar}\right)^{3/2} \exp\left(-\frac{m\omega}{\hbar}(x^2+y^2+z^2)\right). \tag{4.42}$$

The Gaussian form

$$\rho_0(x,y,z,0) = \frac{1}{(2\pi)^{3/2}\sigma^3} \exp\left(-\frac{1}{2\sigma^2}(x^2+y^2+z^2)\right) \tag{4.43}$$

is, again, evident in (4.42), with dispersion still defined by (4.35).
The vacuum state at an arbitrary time t is produced by

$$\psi_0(x,y,z,t) = e^{-i\omega t/2}\psi_0(x,y,z,0), \tag{4.44}$$

and its square module is derived directly to obtain the probability density:

$$\rho_0(x,y,z) = |\psi_0(x,y,z,t)|^2. \tag{4.45}$$

As is the case with the one-dimensional oscillator, distribution (4.45) equals (4.43) over any time, owing to the annihilation between phases of each state and its conjugate. That is the main feature, beneficial for the stability of the self-gravity term, which is the next step.

4.2.3 Including the effect of gravitation

By the same approach at the start of section 4.1.4, we modify the model with the contribution of a gravitational self-interaction calculated in this way:

$$E_{glob} = -\frac{Gm^2}{2} \int dx_1 dy_1 dz_1 dx_2 dy_2 dz_2 \frac{\rho_0(x_1, y_1, z_1)\rho_0(x_2, y_2, z_2)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}. \quad (4.46)$$

We are dealing with an interaction on global scale, since (4.46) features integration over whole space for each coordinate. The Hamiltonian (4.40) is now complicated by the contribution of energy (4.46). The corresponding solution of time-dependent Schrödinger equation is now

$$\psi_{glob}(x, y, z, t) = e^{-iE_{glob}t}\psi_0(x, y, z, t). \quad (4.47)$$

Using the reduced Planck's constant defined in (4.29) and the angular frequency

$$\omega = 2\pi f, \quad (4.48)$$

one can find eigenvalues of the separable Hamiltonian (4.40):

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) = \hbar\omega\left(n_x + n_y + n_z + \frac{1}{2}\right). \quad (4.49)$$

The ground state of the harmonic oscillator subsequently is

$$E_0 = \frac{\hbar\omega}{2}. \quad (4.50)$$

We must not neglect the normalization condition

$$\iiint \rho_0(x, y, z)dxdydz = 1. \quad (4.51)$$

To get a picture as complete as possible, we change perspective on the nature of self-gravity. This time let us assume the interaction occurs at local level. One needs to calculate the radial energy density

$$E_{loc}(r) = -\frac{Gm^2}{2} \int dr' \frac{\rho_0(r)\rho_0(r')}{\sqrt{(r - r')^2}}, \quad (4.52)$$

keeping r fixed and leveraging the spherical symmetry, where the radius is defined by

$$r \equiv |r|,$$
$$|r| = \sqrt{x^2 + y^2 + z^2}, \quad (4.53)$$

and the probability density is always (4.43) with the coordinate change (4.53). Analogous to the previous sections, the gravitational Hamiltonian adds to the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = (H_{osc} + H_{loc})\, \psi, \tag{4.54}$$

with solution

$$\psi_{loc}(r,t) = e^{-iE_{loc}(r)t}\psi_0(r,t). \tag{4.55}$$

In order to obtain the gravitational energy, we need to integrate the radial distribution over the whole space,

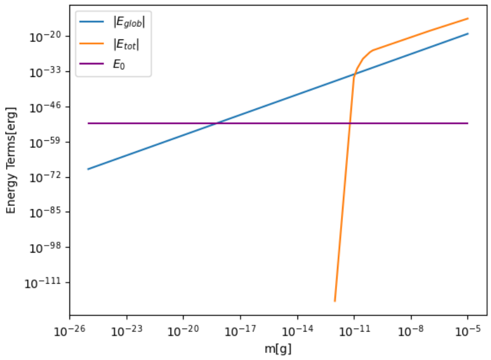
$$E_{tot} = \int_0^\infty 4\pi r^2 \rho_0(r) E_{loc}(r) dr. \tag{4.56}$$

This way, there is everything one needs to study the transition.

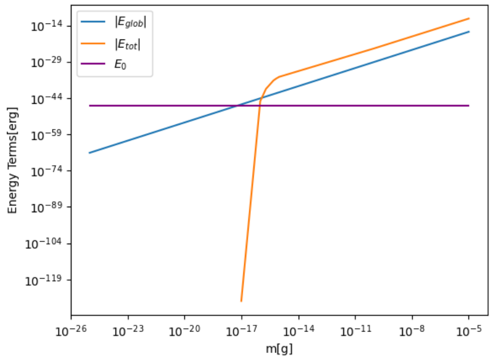
At this point, one can compare how these two gravitational terms eventually affect the quantum dynamics of the system at hand. The approach involves fixing the value of the parameter ω, which also fixes the vacuum energy of the oscillator (see eq. (4.50)), and search the crossover mass m_c and, via eq. (4.35), related crossover dispersion σ_c, established when gravity contribution reaches approximately a tenth of the value of the quantum energy, just like before.
Data at different parametrizations are showed in fig. 4.3. They all represent the trend of both self-gravity components with respect to a fixed quantum energy (horizontal line), each at different energy scales. One can consider the part above the quantum energy as the classical regime, and the lower part as the quantum regime. Let us sum up what is noticeable:

- higher gravitational corrections are found for the largest masses;
- the global self-interaction evolves with respect to the crescent mass in a logarithmic fashion more straightforward than the local one, which features a cut-off and thus appears more rapidly;
- the global method modifies the model at lower masses than the local method in cases (a), (b) and (e), while the local method reaches first the classical regime in cases (c) and (d).

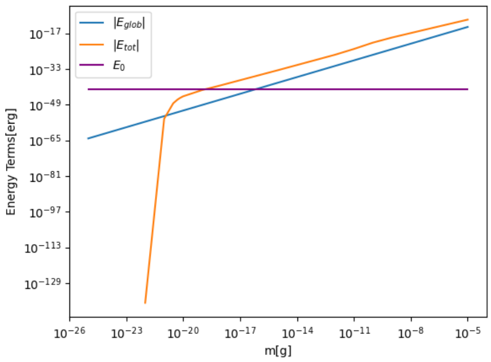
Some examples of crossover data are listed in tab. 4.2 and tab. 4.3. We immediately observe that:



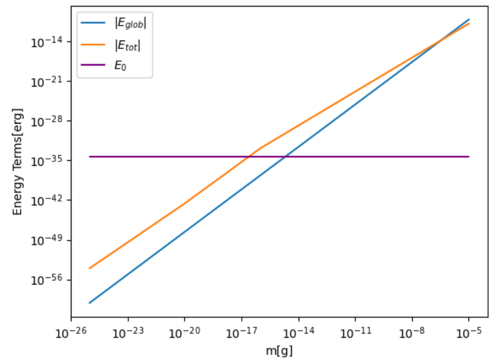
(a) $\omega = 10^{-25}\, rad \cdot s^{-1}$ *and* $E_0 = 5.27 \cdot 10^{-53}$ *erg*



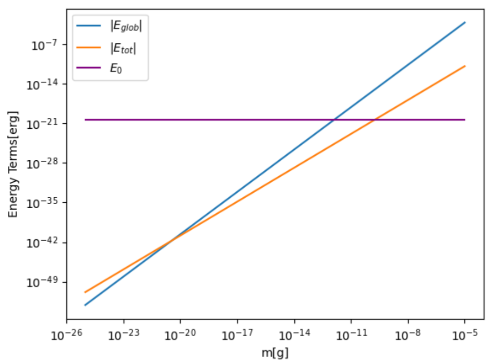
(b) $\omega = 10^{-20}\, rad \cdot s^{-1}$ *and* $E_0 = 5.27 \cdot 10^{-48}$ *erg*



(c) $\omega = 10^{-15}\, rad \cdot s^{-1}$ *and* $E_0 = 5.27 \cdot 10^{-43}$ *erg*



(d) $\omega = 10^{-7}\, rad \cdot s^{-1}$ *and* $E_0 = 5.27 \cdot 10^{-35}$ *erg*



(e) $\omega = 10^{7}\, rad \cdot s^{-1}$ *and* $E_0 = 5.27 \cdot 10^{-21}$ *erg*

Figure 4.3: Behaviour of energy components at fixed frequencies and varying masses for the harmonic oscillator.

- in accordance with the inverse proportionality dictated by eq. (4.35), the gravitational corrections relative to largest masses for the global method are felt by the most compact wave packets, as it was for the particle model;
- we are able to determine the range in which the local method starts featuring gravitational corrections at lower masses than the global - namely the range $\sim 10^{-18} - 10^{-13}$ g, between $\omega \sim 10^{-18} - 1\, rad \cdot s^{-1}$, which means approximately a band between 10^{-1} aHz and 10^{-1} Hz, provided the oscillator is confined in a region between $\approx 10^{-7} - 10^{4}$ cm.
- in the second table there is a sort of lower bound on the mass around 10^{-19} g and an upper bound on the spreading of the packet around 10^{5} cm.

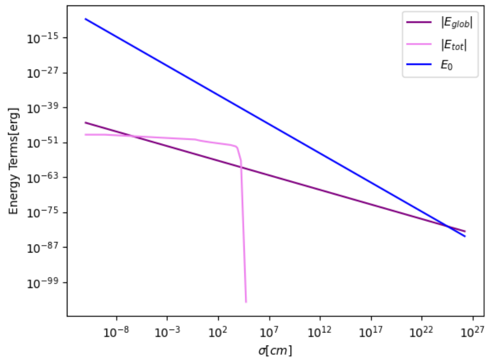
This latest fact arouses our curiosity and we further advance in the study, this time performing the same analysis except in the starting point, which is fixed mass and varying ω.

ω (rad s^{-1})	σ_c (cm)	m_c (g)	$\lvert E_{glob}\rvert$ (erg)
10^{-52}	$1.78 \cdot 10^{24}$ (1/2 Mpc)	$1.67 \cdot 10^{-24}$ (proton)	$2.60 \cdot 10^{-80}$
10^{-33}	$7.26 \cdot 10^{12}$ (Mercury aphelion)	10^{-20} (protein)	$2.29 \cdot 10^{-61}$
10^{-19}	$3.25 \cdot 10^{4}$ (Eiffel Tower)	$5 \cdot 10^{-18}$ (ribosome)	$1.28 \cdot 10^{-47}$
10^{-18}	$7.26 \cdot 10^{3}$ (football field)	10^{-17} (TMV)	$2.29 \cdot 10^{-46}$
10^{-3} (LISA)	$7.26 \cdot 10^{-6}$ (antibody)	10^{-14} (large virus)	$2.29 \cdot 10^{-31}$
1 (LISA)	$1.03 \cdot 10^{-7}$ (DNA)	$5 \cdot 10^{-14}$	$4.04 \cdot 10^{-28}$
10^{7}	$7.26 \cdot 10^{-12}$ (atomic nucleus)	10^{-12} (E. Coli)	$2.29 \cdot 10^{-21}$
$5.58 \cdot 10^{14}$(ΔE_1 HCl)	$1.78 \cdot 10^{-16}$ (LIGO)	$3 \cdot 10^{-11}$ (cell)	$8.41 \cdot 10^{-14}$
10^{42}	$7.26 \cdot 10^{-33}$ (ℓ_P)	10^{-5} (M_P)	$2.29 \cdot 10^{14}$

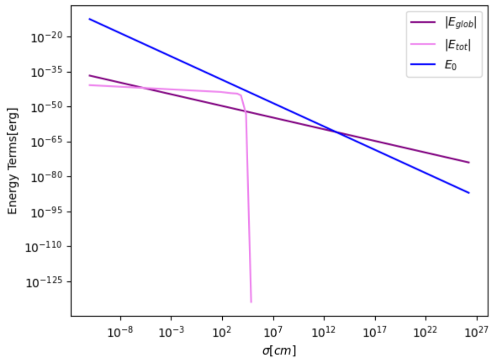
Table 4.2: Rough estimates of the scales when the gravitational term value is around one order of magnitude lower than the vacuum energy for a global self-interacting harmonic oscillator

This leads to fig. 4.4. The quantum regime pertains the lower left corner and it is divided by the ground state energy (blue line) from the classical regime, that pertains the diametrically opposed corner. The graphs re-affirm what has already been observed, there is a clear cut-off for how spread can the wave-packet be to feel modification via self-gravity. In terms of dispersion, the model can be considered more heavily quantum the more they are small. These results for the harmonic oscillator, too, have enough sense, considering what we know about standard QM.

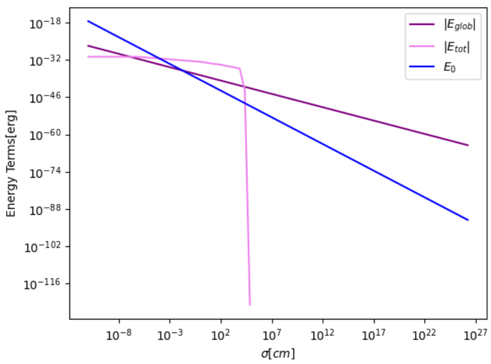
Differences in accuracy of algorithms should not be overlooked. Expression (4.56) requires less calculation time compared to (4.46), thus allowing much more accurate



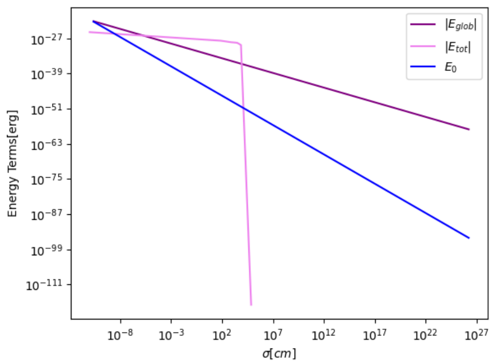
(a) $m = 10^{-24}\, g$



(b) $m = 10^{-20}\, g$



(c) $m = 10^{-15} \quad g$



(d) $m = 10^{-12} \quad g$

Figure 4.4: Behaviour of energy components at fixed masses and varying dispersions for the harmonic oscillator.

ω (rad s^{-1})	σ_c (cm)	m_c (g)	$\lvert E_{tot} \rvert$ (erg)
10^{-40}	$6.42 \cdot 10^4$	$1.28 \cdot 10^3$ (Cat)	$7.99 \cdot 10^{-69}$
10^{-25}	$3.38 \cdot 10^4$	$3.50 \cdot 10^{-12}$ (E. Coli)	$1.39 \cdot 10^{-53}$
10^{-19}	$2.24 \cdot 10^4$	$1.05 \cdot 10^{-17}$	$1.42 \cdot 10^{-47}$
10^{-18}	$1.87 \cdot 10^4$ (Giza pyramid)	$1.50 \cdot 10^{-18}$ (BMV)	$1.05 \cdot 10^{-46}$
10^{-15}	$2.57 \cdot 10^3$ (Whale)	$8 \cdot 10^{-20}$ (Protein)	$2.15 \cdot 10^{-43}$
10^{-7}	$2.29 \cdot 10^{-2}$ (Paramecium)	10^{-17}	$1.26 \cdot 10^{-35}$
1	$1.03 \cdot 10^{-7}$ (DNA)	$5 \cdot 10^{-14}$	$3.42 \cdot 10^{-28}$
10^3	$7.26 \cdot 10^{-10}$ (X-ray λ)	10^{-12}	$1.37 \cdot 10^{-25}$
10^9	$2.29 \cdot 10^{-14}$ (Proton radius)	10^{-9} (Birch pollen)	$1.37 \cdot 10^{-19}$

Table 4.3: Rough estimates of the mass scales when the gravitational term value is around one order of magnitude lower than the vacuum energy for a local self-interacting harmonic oscillator.

performance than the other routes discussed (we are able to feature $N = 300$ in the algorithm). That means the local model finds values with less underestimation.

Conclusions

This work aimed to contribute, at least partially, to the discussion on quantum gravity, from a different angle with respect to the traditional approach.
On one side, QM and GR are allegedly considered to be compatible with each other, provided one remains in a low-energy regime, where renormalization issues do not arise. That is the stand of EFT (effective field theories), a reduction of some other UV untestable theory. On the other, the conflict between QM and GR is taken as unsolvable because of the inconsistencies regarding superposition of large masses and GR behaviour at short distances. The only way to deal with this second option is to build a model predicting drastic changes for QM at large mass scales in order not to clash with GR.
We covered many years of attempts at building semi-classical and stochastic collapse models explaining why and how a quantum system would lose coherence via a gravitational mechanism and we could not help but bring to light entailed fallacies. The only proposal based on relativistic QFT and an intrinsic process compatible with GR principles that goes beyond QM is CWL theory, which predicts gravitational correlations between paths and makes them bunch together along the classical trajectories for large masses.
This idea directed our endeavours and, in order to validate the whole school of thought, we proposed an intuitive study based on a family of quantum toy models simulating what happens in CWL theory, in search for changes to their dynamics at macroscopic masses. We analysed whether is it possible to break quantum behaviour in the case of (i) a single particle and (ii) a harmonic oscillator, by introducing in the respective Hamiltonians a gravitational self-interacting term between two spherical Gaussian mass distributions. The oscillator model features both a global and a local self-interaction.
The main insight is effects on the systems depend largely on the size and mass of the body at hand. The analysis predicts an interference of the coherence of the wave packet for the largest well-localized masses, which tend to favour the transition to classical dynamics, while the smallest favour the quantum nature, unless they are dislocated. Concerning the particle, the feasibility of this model is subjected also to the parameter of time, which renders it the most difficult to test. As for the oscillator, each version show a slightly different behaviour. They produce gravitational induced decoherence more rapidly with the mass in different ranges of other parameters, and the local one presents a cut-off which restricts the model to a minimum mass required and a maximum width.

There are three ways in which our models can be improved and tested: (i) consider more than two mass distributions in the gravitational self-interacting term, mimicking the idea of replicas furthered in the formulation CWL theory, (ii) set the starting wave-packet with a distribution other than Gaussian and (iii) abandon the spherical symmetry. The importance of exploring alternative models at this particular time resides in different facts: (i) to compete with conventional quantum gravity, in case experimental tests falsified it, we need a self-consistent alternative, (ii) distinguish among them and what eventually different outcomes they predict and (iii) on the horizon of future sophisticated experiments, even if proved to be the wrong approach at quantum gravity, they may provide interesting challenges for furthering capabilities of available technology.

Bibliography

[1] C. Rovelli, "Quantum gravity," (Scholarpedia, 3(5):7117 2008)

[2] L. Rosenfeld, "Über die Gravitationswirkungen des Lichtes," *Z. Physik* **65**, 589–599 (1930); also L. Rosenfeld, "Zur Quantelung der Wellenfelder", *Ann. Physik* **5**, 113-152 (1930)

[3] M. P. Bronstein, "Quantentheorie Schwacher Gravitationsfelder," *Phys. Zh. Sowjet.*, **9**: 140–157 (1936)

[4] G.E. Gorelik, "Matvei Bronstein and quantum gravity: 70th anniversary of the unsolved problem," *Phys. Usp.* **48** 039-1053 (2005)

[5] R.P. Feynman and others, in *"Report from Chapel Hill conference"* ed. C.M. DeWitt, D. Rickles (1957), particularly session VIII, sections 22, 23; see also R.P. Feynman, *"Feynman Lectures on Gravitation"*, particularly Ch. 1 (Addison-Wesley, 1995)

[6] E. Schrödinger, "Die gegenwärtige Situation in der Quantenmechanik," Naturwissenschaften **23**, 807 (1935)

[7] L. Rosenfeld, "On quantization of fields", *Nucl. Phys.* **40** 353-356 (1963)

[8] J. F. Donoghue, "General relativity as an effective field theory: The leading quantum corrections," *Phys. Rev.* D**50**, 3874 (1994), and "The effective field theory treatment of quantum gravity," *AIP Conference Proceedings* **1483**, 73 (2012); C.P. Burgess, "Quantum Gravity in Everyday Life: General Relativity as an Effective Field Theory," *Living Rev. Relativity* **7**, 5 (2004)

[9] A. O. Barvinsky, G.A. Vilkovisky, "The Generalized Schwinger-Dewitt Technique in Gauge Theories and Quantum Gravity," *Phys. Rep.* **119**, 1-74 (1985)

[10] P.C.E. Stamp, "Environmental Decoherence versus Intrinsic Decoherence," *Phil. Trans. Roy. Soc. A* **370**, 4429-4453 (2012)

[11] P.C.E. Stamp, "Rationale for a Correlated Worldline Theory of Quantum Gravity," *New J. Phys.* **17** 065017 (2015)

[12] Jordan Wilson-Gerow and P. C. E. Stamp, "Paths and States in the Correlated Worldline Theory of Quantum Gravity," (2020)

[13] J. Wilson-Gerow, "A study of the quantum-to-classical transition in gravity, and a study of the consequences of constraints in gauge theory path-integrals," *University of British Columbia* (2021)

[14] R. Penrose, "On Gravity's role in Quantum State Reduction," *Gen. Rel. Grav.* **28**, 581–600 (1996)

[15] G.'t Hooft, M. Veltman, "One loop divergencies in the theory of gravitation," *Ann. Poincare Phys. Theor. A* **20**, 69 (1974)

[16] M.H. Goroff, A. Sagnotti, "Quantum Gravity at two loops," *Phys. Lett. B* **60** 81 (1985); M.H. Goroff, A. Sagnotti, "The Ultraviolet Behavior of Einstein Gravity," *Nucl. Phys. B* **266**, 709 (1986); A.E.M. van de Ven, "Two loop quantum gravity," *Nucl. Phys. B* **378**, 309 (1992)

[17] S. Carlip, "Quantum gravity: A Progress report," *Rep. Prog. Phys.* **64**, 885 (2001)

[18] R.P. Woodard, "How Far Are We from the Quantum Theory of Gravity?," *Rep. Prog. Phys.* 72, 126002 (2009)

[19] R.M.Wald, "General Relativity", sec. 14 *Univ of Chicago Press*, (1984)

[20] I.L. Shapiro, "Polemic Notes on IR Perturbative Quantum Gravity," *Int. J. Mod. Phys.* bf A24, 557 (2009)

[21] E. Adlam, "Tabletop Experiments for Quantum Gravity Are Also Tests of the Interpretation of Quantum Mechanics." *Found Phys* **52**, 115 (2022)

[22] S. Bose, A. Mazumdar, G. Morley, H. Ulbricht, M. Toros, M. Pasternostro, A. Geraci, P. Barker, M.S. Kim, and G. Milburn, "Spin entanglement witness for quantum gravity," *Phys. Rev. Let.*, **119** (24), (2017)

[23] C. Marletto and V. Vedral, "Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity," *Phys. Rev. Let.*, **119** (24), (2017)

[24] S. Carlip, "Is quantum gravity necessary?," *Classical and Quantum Gravity* **25**, 154010, (2008)

[25] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, "Models of wave-function collapse, underlying theories, and experimental tests," *Rev. Mod. Phys.* **85**, pp. 471-527, (2013)

[26] A. Bassi, A. Großardt, and H. Ulbricht, "Gravitational decoherence," *Classical and Quantum Gravity*, vol. 34, no. 19, p. 193002, (2017)

[27] D. Carney, P. C. E. Stamp, and J. M. Taylor, "Tabletop experiments for quantum gravity: a user's manual," *Classical and Quantum Gravity*, **36**, no. 3, p. 034001, (2019)

[28] S. Donadi, A. Bassi, "Seven nonstandard models coupling quantum matter and gravity," *AVS Quantum Sci.* **4** 2 (2022)

[29] A. J. Leggett, "Macroscopic Quantum Systems and the Quantum Theory of Measurement," *Progress of Theoretical Physics Supplement* **69**, 80-100 (1980)

[30] S. Weinberg, "Precision Tests of Quantum Mechanics," *Phys. Rev. Lett.* **62**, 485 (1989)

[31] J. S. Bell, "Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy" (Cambridge university press, 2004).

[32] G. C. Ghirardi, A. Rimini, and T. Weber, "Unified dynamics for microscopic and macroscopic systems," *Phys. Rev. D* **34**, 470 (1986)

[33] T.P. Sotiriou, V. Faraoni, "Theories Of Gravity," *Rev. Mod. Phys.* **82**, 451 (2010)

[34] S. Capozziello, M. de Laurentis, "Extended Theories of Gravity," *Phys. Rep.* **509**, 167 (2011)

[35] S. Nojiri, S.D. Odintsov, "Unified cosmic history in modified gravity: From F(R) theory to Lorentz non-invariant models," *Phys. Rep.* **505**, 59 (2011)

[36] N. Rosen, "The relation between classical and quantum mechanics," *American Journal of Physics*, **32**, 8, 597-600, (1964)

[37] T. Clifton et al., "Modified gravity and cosmology," *Phys. Rep.* **513**, 1 (2012)

[38] W. Heisenberg, "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", *Zeitschrift für Physik*, **43** (3–4), 172–198, (1927)

[39] F. Károlyházy, "Gravitation and quantum mechanics of macroscopic objects," *Il Nuovo Cimento A (1965-1970)*, **42**, 2, 390-402, (1966)

[40] F. Károlyházy, A. Frenkel, and B. Lukács, "On the Possibility of Observing the Eventual Breakdown of the Superposition Principle" (in Physics as Natural Philosophy, 204, edited by A. Simony and H. Feschbach, MIT Press, Cambridge,MA, 1982)

[41] F. Károlyházy, A. Frenkel, and B. Lukács, "On the possible role of gravity in the reduction of the wave function," (in *Quantum Concepts in Space and Time* (R. Penrose and C. J. Isham, eds.), 1-109, Oxford University Press, 1986)

[42] S. Bera, S. Donadi, K. Lochan, and T. P. Singh, "A comparison between models of gravity induced decoherence," *Foundations of Physics* **45**, 1537 (2015)

[43] L. Diósi and B. Lukács, "Calculation of X-ray signals from Károlyházy hazy space-time," *Phys. Lett. A* **181**, 366-368 (1993)

[44] Q. Fu, "Spontaneous radiation of free electrons in a nonrelativistic collapse model," *Phys. Rev. A* **56**, 1806 (1997)

[45] S. L. Adler and F. M. Ramazanoglu, "Photon-emission rate from atomic systems in the CSL model," *Journal of Physics* A **42**, 109801 (2009)

[46] S. L. Adler, A. Bassi, and S. Donadi, "On spontaneous photon emission in collapse models," *J. Phys. A: Math. Theor.* **46**, 245304 (2013)

[47] S. Donadi and A. Bassi, "The emission of electromagnetic radiation from a quantum system interacting with an external noise: a general result," *J. Phys. A: Math. Theor.* **48**, 035305 (2014)

[48] L. Diósi and B. Lukács, "In Favor of a Newtonian Quantum Gravity," *Annalen der Physik* **499**, 488 (1987)

[49] L. Diósi, "A universal master equation for the gravitational violation of quantum mechanics," *Phys. Lett.* A **120**, 377 (1987)

[50] R. Penrose, "On the Gravitization of Quantum Mechanics 1: Quantum State Reduction," *Foundations of Physics* **44**, 557-575 (2014)

[51] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, "Towards Quantum Superpositions of a Mirror," *Phys. Rev. Lett.* **91**, 130401 (2003)

[52] R. Howl, R. Penrose, and I. Fuentes, "Exploring the unification of quantum theory and general relativity with a Bose–Einstein condensate" *New J. Phys.* **21**, 043047 (2019)

[53] L. Diósi, "Models for universal reduction of macroscopic quantum fluctuations," *Phys. Rev. A* **40**, 1165 (1989)

[54] G. Ghirardi, R. Grassi, and A. Rimini, "Continuous-spontaneous-reduction model involving gravity," *Phys. Rev. A* **42**, 1057 (1990)

[55] P. Pearle, "Combining stochastic dynamical state-vector reduction with spontaneous localization," *Phys. Rev. A* **39**, 2277 (1989)

[56] S. Donadi, D.-A. Deckert, and A. Bassi, "On the spontaneous emission of electromagnetic radiation in the csl model," *Annals of Physics* **340**, 1, 70-86, (2014)

[57] B. Helou, B. Slagmolen, D. E. McClelland, and Y. Chen, "LISA pathfinder appreciably constrains collapse models," *Phys. Rev. D* **95**, 084054 (2017)

[58] A. Tilloy and T. M. Stace, "Neutron Star Heating Constraints on Wave-Function Collapse Models," *Phys. Rev. Lett.* **123**, 080402 (2019)

[59] S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein, and A. Bassi, "Underground test of gravity-related wave function collapse," *Nature Physics* **17**, 74 (2021)

[60] A. Vinante and H. Ulbricht, "Gravity-related collapse of the wave function and spontaneous heating: Revisiting the experimental bounds," *AVS Quantum Science* **3**, 045602 (2021)

[61] M. Bahrami, A. Smirne, and A. Bassi, "Role of gravity in the collapse of a wave function: A probe into the Diósi-Penrose model," *Phys. Rev. A* **90**, 062105 (2014)

[62] S. L. Adler, "Quantum theory as an emergent phenomenon: The statistical mechanics of matrix models as the precursor of quantum field theory" (Cambridge University Press, 2004).

[63] S. Adler, "Gravitation and the noise needed in objective reduction models" (in Quantum Nonlocality and Reality: 50 Years of Bell's Theorem, ed Mary Bell and Shan Gao, Cambridge University Press, 2016)

[64] G. Gasbarri, M. Toros, S. Donadi, and A. Bassi, "Gravity induced wave function collapse," *Phys. Rev. D* **96**, 104013 (2017)

[65] L. Diósi, "Schrödinger–Newton equation with spontaneous wave function collapse," *Quantum Reports* **4** 566-573 (2022)

[66] L. Diósi, "Gravitation and quantum-mechanical localization of macro-objects," *Phys. Let. A* **105**, 4-5, 199-202, (1984)

[67] L. Diósi, "Spontaneous Wave Function Collapse with Frame Dragging and Induced Gravity," *Quantum Reports* **1**, 277 (2019)

[68] N. Gisin, "Stochastic quantum dynamics and relativity," *Helv. Phys. Acta* **62**, 363 (1989)

[69] A. Großardt, "Three little paradoxes: making sense of semiclassical gravity," *AVS Quantum Science* **4**, 010502 (2022)

[70] I. Bialnicki-Birula, J Mycielski, "Nonlinear wave mechanics," *Ann. Phys.* **100**, 62 (1976)

[71] K. Eppley and E. Hannah, "The necessity of quantizing the gravitational fi eld," *Foundations of Physics* **7**, 51-68, (1977)

[72] W. G. Unruh, "Quantum Theory of Gravity: Essays in Honor of the 60th Birthday of Bryce S DeWitt" (S. Christensen, ed., Bristol: Hilger, 1983)

[73] J. Polchinski, "Weinberg's nonlinear quantum mechanics and the Einstein-Podolsky-Rosen paradox," *Phys. Rev. Lett.* **66**, 397 (1991)

[74] M. Bahrami, A. Großardt, S. Donadi, and A. Bassi, "The Schrödinger-Newton equation and its foundations," *New Jou. Phys.* **16**, 115007 (2014)

[75] H. Yang, H. Miao, D.-S. Lee, B. Helou, and Y. Chen, "Macroscopic Quantum Mechanics in a Classical Spacetime," *Phys. Rev. Lett.* **110**, 170401 (2013)

[76] A. Großardt, J. Bateman, H. Ulbricht, and A. Bassi, "Optomechanical test of the Schrödinger-Newton equation," *Phys. Rev. D* **93**, 096003 (2016)

[77] T.W.B. Kibble, "Relativistic models of nonlinear quantum mechanics," *Comm. Math. Phys.* **64**, 73 (1978)

[78] T.W.B. Kibble, S. Randjbar-Daemi, "Nonlinear Coupling of Quantum Theory and Classical Gravity," *J. Phys A* **13**, 141 (1980)

[79] T.W.B. Kibble, "Is a Semiclassical Theory of Gravity Viable?," in *Quantum Gravity 2: A Second Oxford Symposium*, C.J. Isham, R. Penrose, D.W. Sciama eds., Oxford Univeristy Press, (1981)

[80] B. L. Hu, "Stochastic gravity," *Int. J. Theoretical Physics* **38**, 2987 (1999)

[81] B. L. Hu and E. Verdaguer, "Stochastic gravity: Theory and applications," *Living Reviews in Relativity* **11**, 3 (2008)

[82] B. L. Hu and E. Verdaguer, "Semiclassical and Stochastic Gravity: Quantum Field Effects on Curved Spacetime," (in Cambridge Monographs on Mathematical Physics series, Cambridge University Press, 2020)

[83] K. Jacobs and D. A. Steck, "A Straightforward Introduction to Continuous Quantum Measurement," *Contemporary Physics* **47**, 279 (2006)

[84] D. Kafri, J. Taylor, and G. Milburn, "A classical channel model for gravitational decoherence," *New J. Phys.* **16**, 065020 (2014)

[85] J. L. Gaona-Reyes, M. Carlesso, and A. Bassi, "Gravitational interaction through a feedback mechanism," *Phys. Rev. D* **103**, 056011 (2021)

[86] N. Altamirano, P. Corona-Ugalde, R. B.Mann, andM. Zych, "Gravity is not a pairwise local classical channel," *Classical and Quantum Gravity* **35**, 145005 (2018)

[87] A. Tilloy and L. Diósi, "Sourcing semiclassical gravity from spontaneously localized quantum matter," *Phys. Rev. D* **93**, 024026 (2016)

[88] A. O. Barvinsky, D. Carney, and P. C. E. Stamp, "Structure of correlatedworldline theories of quantum gravity," *Phys. Rev. D* **98**, 084052, (2018)

[89] Chiorescu, I., Nakamura, Y., Harmans, C.J.P.M., Mooij, J.E., "Coherent quantum dynamics of a superconducting flux qubit," *Science* **299**, 1869-1871 (2003)

[90] Leggett, A.J., "Testing the limits of quantum mechanics: motivation, state of play, prospects", *J. Phys. Cond. Mat.* **14**, R415 (2002)

[91] P. Amaro-Seoane et al., "Low-frequency gravitational-wave science with eLISA/NGO," *Class. Quantum Grav.* **29**, 124016, (2012)