Alma Mater Studiorum - Università di Bologna

Dipartimento di Fisica e Astronomia
Corso di Laurea in Astronomia

ENERGY TRANSPORT MECHANISMS IN ASTROPHYSICS

Tesi di laurea

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Dedication my family

Abstract

To enable humans to observe, understand, and analyze celestial bodies, celestial bodies must emit energy from themselves and transmit them to the earth through free space, such as light, gravitational waves, or other things. Therefore, humans can analyze the content of celestial bodies based on the obtained information to understand the universe. This process can be called the energy transport process. He has exerted an important role in the communication between the earth and the universe, and we will discuss the main energy transport mechanisms in the stellar atmosphere in detail.

Contents

Chapter 1

Introduction

The energy transport mechanism and process of energy in astrophysics is not a single process. The universe has provided us with unlimited possibilities from the very beginning. The universe is a place full of miracles.

We can acquire a lot of information from the universe. We can capture gravitational waves, X-rays, etc. from the earth. In addition, we can also analyze the spectrum to get some elemental composition of outer space, and even some information. For example, the redshift of the estimated age of the universe is inseparable from information.

The universe needs to pass these messages to us and naturally needs a" channel", which is the process of transportation. The process of transporting energy may be extremely long, and perhaps when we capture this information, the star that sent the message has died.

The state of the universe does not remain unique. It is expanding or contracting or in a static state, and their energy transfer methods are naturally different. In the case of a white dwarf, electrons dominate, and conductive transport is the main energy transport method. However, in other, at that time, radiation transportation is much more important. For me today, I want to concentrate on these several methods of cosmic energy transportation.

1. Radiation transport
2. Conductive transportation
3. Convection transportation

They may not represent everything, but they are typical enough to give us a preliminary understanding of the universe.

Chapter 2

Radiative transport

The radiation propagating in the extended medium undergoes a continuous emission and absorption process, consequently changing its intensity and spectral distribution. This part of the radiation energy is undoubtedly provided by photons. In the current part, we focus on the radiation transport equation.

2.1 Radiation transport in stellar

When performing radiation transmission, the path of photons is not a regular path, but a random path called a random walk. The mean free path l of a photon is the typical distance traveled by a photon before interacting with a particle, and is provided by:

$$l = \frac{1}{\sigma \cdot n} \tag{2.1}$$

In the formulan is the density of particles in the unit of volume and σ represents the cross section related to the probability of the interaction (absorption /diffusion) between a photon and a particle

Referring to Fig. 2.1, it is possible to evaluate the overall distance d traveled by a photon:

$$\vec{d} = \overrightarrow{\ell_1} + \overrightarrow{\ell_2} + \overrightarrow{\ell_3} + \ldots + \overrightarrow{\ell_N} \tag{2.2}$$

Taking the dot product of d with itself we can obtain:

$$\begin{aligned}
\vec{d} \cdot \vec{d} &= \overrightarrow{\ell_1} \cdot \overrightarrow{\ell_1} + \overrightarrow{\ell_1} \cdot \overrightarrow{\ell_2} + \ldots + \overrightarrow{\ell_1} \cdot \overrightarrow{\ell_N} && (2.3)\\
&+ \overrightarrow{\ell_2} \cdot \overrightarrow{\ell_1} + \overrightarrow{\ell_2} \cdot \overrightarrow{\ell_2} + \ldots + \overrightarrow{\ell_2} \cdot \overrightarrow{\ell_N} && (2.4)\\
&+ \ldots + \overrightarrow{\ell_N} \cdot \overrightarrow{\ell_1} + \overrightarrow{\ell_N} \cdot \overrightarrow{\ell_2} + \ldots + \ell_N \cdot \overrightarrow{\ell_N} && (2.5)\\
&= \sum_{i=1}^{N} \overrightarrow{\ell_{j=1}^{N}} \overrightarrow{\ell_i} \cdot \overrightarrow{\ell_j} && (2.6)
\end{aligned}$$

We continue to derive

$$
\begin{aligned}
d^2 &= N\ell^2 + \ell^2 [\cos\theta_{12} + \cos\theta_{13} + \ldots + \cos\theta_{1N} && (2.7)\\
&+ \cos\theta_{21} + \cos\theta_{23} + \ldots + \cos\theta_{2N} && (2.8)\\
&+ \ldots + \cos\theta_{N1} + \cos\theta_{N2} + \ldots + \cos\theta_{N(N-1)} && (2.9)\\
&= N\ell^2 + \ell^2 \textstyle\sum_{i=1}^{N} \sum_{\substack{j=1\\ j\neq i}}^{N} \cos\theta_{ij} && (2.10)
\end{aligned}
$$

For a large number of exchanges, N, the sum of the cosines tends to zero:

$$d^2 = Nl^2 \tag{2.11}$$

we got:

$$d = l\sqrt{N} \tag{2.12}$$

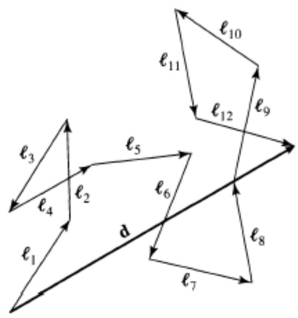

Figure 2.1: Random displacement motion diagram of photons

Based on this result, it is obvious that we can see that the efficiency of energy transmission through this kind of radiation is not high, or even very low. For example, if the photons produced by the core of the sun are sent to the surface, they travel at the speed of light under the premise that it also takes 10^6 years.This efficiency is extremely low

2.2 Equation of radiative transport

The defined F_{rad} is the energy flow, per unit of time and surface, the moment dp transported by the radiation to the volume element:

$$dp = \frac{dF_{\text{rad}}}{c} \tag{2.13}$$

We know from (2.1.2) that l is the mean free path of photons, where the pressure applied to dr by dp and radiation is exactly the opposite of d F_{rad}, Therefre, we can express the formula as:

$$dP_{rad} = -\frac{F_{rad}}{c}\frac{dr}{l} \tag{2.14}$$

We now introduce a new quantity, opacity. We will discuss this new variable in detail later. At this time, we can rewrite the formula:

$$\frac{dP_{rad}}{dr} = -\frac{\kappa\rho}{c}F_{rad} \tag{2.15}$$

because:

$$P_{rad} = \frac{1}{3}aT^4 \tag{2.16}$$

We can rewrite the formula as:

$$\frac{dP_{rad}}{dr} = \frac{4}{3}aT^3\frac{dT}{dr} \tag{2.17}$$

$$a = \frac{4\sigma}{c} \tag{2.18}$$

a is constant and σ is Boltzmann's constant, We connect the above equations and make them equal:

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}F_{rad} \tag{2.19}$$

Because we know

$$F_{rad} = \frac{L_r}{4\pi r^2} \tag{2.20}$$

Therefore, in the end we got the final formula, the thermal gradient of radiative transmission can be expressed as:

$$\left.\frac{dT}{dr}\right|_{\mathrm{rad}} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2} \tag{2.21}$$

This equation describes the basic concept of radiative transmission. To make the equation meaningful, the left side of the equation cannot be zero. In the second part, we introduce a special and important concept, namely opacity. In the third part, we will discuss opacity in detail.

Chapter 3

Opacity

The opacity of stellar matter refers to a measure of the resistance offered by matter to the passage of radiation. Opacity is caused by a multiplicity of atomic processes that schematically they can be of two types:

True absorption: The photon is absorbed and loses its identity (it no longer reap pears at the frequency it had when it was absorbed).

Diffusion: There is a decrease in flux in the observed direction because a deviation of the photon is produced.

These are processes that contribute to the decrease in the flux of radiation of a given frequency.

The processes that lead to the opacity of a ray of light and to a decrease in its intensity may come from an absorption of the photons that constitute it (photons in this case disappear and their energy is transformed into thermal energy of the gas) or in a scattering of the photon in another direction (so we see less of it). If these processes vary slowly with the wavelength, then the absorption coefficient kl slowly depends on the wavelength and the absorption is continuous in the spectrum. Otherwise, if it varies rapidly, we have the formation of lines. Essentially there are four physical mechanisms that lead to absorption or scattering of photons, respectively, Bound- Bound(BB), Bound-Free(BF), Free-Free(FF) and Scattering

3.1 Bound-Bound(BB)

Let an electron be given in an energy state E_1 which absorbs a photon and moves into an energy state E_2 .

$$hv_{BB} = E_2 - E_1 \tag{3.1}$$

Although this is an absorption process, it is selective, because the transition of an electron from a given energy level to a higher energy level always requires a photon of a given frequency. However, inside the stars, this is not important, because most of the atoms are ionized (so few electrons are in a bound state).

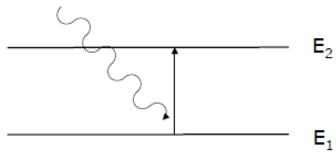

Figure 3.1: Bound-Bound

3.2 Bound-Free(BF)

An electron placed in a bound state of energy E1 absorbs a photon and can position itself on a free orbit (of energy E_3). This process is called “photoionization”

$$\mathrm{hv_{BF}} \quad = \quad \mathrm{E_3} - \mathrm{E_1} > \chi_{\mathrm{ion}} \tag{3.2}$$

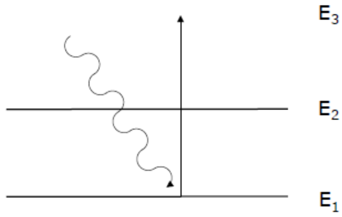

Figure 3.2: Bound-Free

Any energy above the Xion ionization potential is fine. Then, the electron (depending on the conditions of the environment in which it is found) may lose part of its kinetic energy due to collision with the surrounding particles and may be recaptured by another atom (recombination) by emitting a photon less energy. This process degrades energy.

3.3 Free-Free(FF)

A free electron with energy E_3 absorbs a photon of energy $h\nu_{FF}$ and moves to another free state of energy E_3 In this case, there are no restrictions on the energy of the photon

$$hv_{FF} \quad = \quad E_4 - E_3 \tag{3.3}$$

In this case, there are no restrictions on the energy of the photon. The only necessary condition for this phenomenon to occur is that the electron is in the electric field of an ion, the presence of which guarantees the conservation of the momentum.

3.4 Scattering

From the classical perspective, the process can be represented with a collision between two particles (photon-electron). If the energy of the photon of frequency v_s

is:

$$hv_s << mc^2 \tag{3.4}$$

where m is the mass of the particle with which the photon collides. Then, the particle practically does not move.

Subsequently, the particle is practically not moving. This is almost always true in stellar interiors, except in very high temperature areas.

The effect of this process is to delay the photon's exit from the star because its direction is constantly changed, decreasing the intensity of the radiation beam in the direction of propagation.

3.5 Conclusion

Therefore, opacity plays an important and decisive role in energy transport. As the opacity or the probability of productivity increases inside a star, the radiation temperature gradient increases, and radiation is no longer an effective way to transfer energy. At this time, the radiation balance is unstable. So at this time, the star produces convection, and the opacity seems to play a role in connecting, connecting the way of concentrated energy transportation.

Because we have already discussed the transport mechanism of radiation, we will discuss convection and other transport mechanisms next. In this regard, opacity still plays an important role.

Chapter 4

Conductive Transport

A white dwarf, also called a degenerate dwarf, is a stellar nuclear remnant mainly composed of electronically degenerate matter. It is a special existence in the universe. In the second chapter, we discussed t radiation transportation. Although electrons have more energy than photons, conductive transmission is not the main effective mechanism for energy transmission inside stars. In special circumstances, as we said, among white dwarfs, because white dwarfs are in a degraded environment, the average degree of freedom of electrons becomes larger. Consequently, conductive transmission becomes effective. The energy flow associated with conduction is:

$$F_{\text{cond}} = -\Lambda \frac{dT}{dr} \tag{4.1}$$

where Λ is a factor determined by the theory of degenerate gases.

$$\Lambda = \frac{4acT^3}{3\kappa_c \rho} \tag{4.2}$$

In the previous article, we obtained the expression of Frad (2.21) When conduction becomes important, an overall flow F is defined as:

$$F = F_{rad} + F_{\text{cond}} \tag{4.3}$$

In the end we can get the expression of F as:

$$F = -\frac{4ac}{3}\frac{\kappa}{\rho}T^3\frac{dT}{dr} \tag{4.4}$$

Chapter 5

Convective transport

In addition, convection is also an energy transmission mechanism, which can be triggered when the thermal gradient exceeds a certain critical value, causing to increase heat and mass flow of matter, leading to convection as an energy transmission mechanism. In order to describe this mechanism, we make an assumption. This assumption is adiabatic, which is an ideal state. We may also use some equations about ideal gases.

We need to introduce an important equation, the ideal gas equation, to facilitate a series of calculations.

$$P = \frac{k_B \rho T}{\mu H} \tag{5.1}$$

μ is the average molecular weight of the considered group and H is the core of the core hydrogen (mass of the proton).

5.1 Adiabatic gradient and Schwarzschild criterion

As shown in Figure (5.1), we assume that a substance moves radially outward from the inside of the star, and its displacement distance is dr. The substance expands adiabatically until its internal pressure and the pressure of the surrounding substance reach equilibrium.

We will now discuss whether the substance moves down to its original position or will continue to move upward. If it moves downward, the radiation balance is stable at this time. If it continues to move upward, convection occurs.

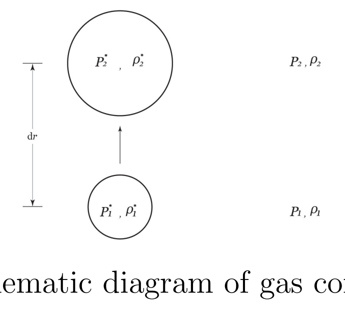

Figure 5.1: Schematic diagram of gas convection stability

Because material transportation is adiabatic transportation, so:

$$\rho_1* \quad = \quad \rho_1 \tag{5.2}$$

$$P_1* \quad = \quad P_1 \tag{5.3}$$

$$T_1* \quad = \quad T_1 \tag{5.4}$$

After being disturbed, the pressure is in equilibrium, but the density in the material body and the surrounding density are not used, and the density inside the material is determined by adiabatic expansion:

$$\rho_2^* \quad = \quad \left(\frac{P_2^*}{P_1^*}\right)^{\frac{1}{\gamma}} \rho_1^* \tag{5.5}$$

Equation of state in the case of adiabatic transformation:

$$P \propto \rho^\gamma \quad \text{so} \quad \gamma \quad = \quad \frac{c_P}{c_V} \tag{5.6}$$

γ is the adiabatic index and for highly ionized gases γ=5/3

If the density inside the substance is greater than the density around it, the gravity on the substance will increase, and the substance will receive a downward force, so it gradually moves to its original position.

$$\rho_2^* > \rho_2 \tag{5.7}$$

Any disturbance will be prevented, so it is a stable state at this time. And because we said before that matter is transported in adiabatic state, we can derive the formula:

$$\rho_2^* \quad = \quad \rho_1^* \left(\frac{P_2^*}{P_1^*}\right)^{\frac{1}{\gamma}} > \rho_2 \tag{5.8}$$

We deduced that in steady state, ρ_2 is less than ρ_1

$$\rho_1 \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} > \rho_2 \tag{5.9}$$

$$\rho_1 \left(\frac{P_1 + \frac{dP}{dr}dr}{P_1}\right)^{\frac{1}{\gamma}} > \rho_1 + \frac{d\rho}{dr}dr \tag{5.10}$$

$$\rho_1 \left(1 + \frac{1}{P_1}\left(\frac{dP}{dr}dr\right)\right)^{\frac{1}{\gamma}} > \rho_1 + \frac{d\rho}{dr}dr \tag{5.11}$$

We can continue to rewrite this complex relationship as:

$$\frac{1}{\gamma}\frac{\rho_1}{P_1}\frac{dP}{dr}dr > \frac{d\rho}{dr}dr \tag{5.12}$$

If we use the ideal gas state equation (5.1) and replace the density gradient with temperature gradient and pressure gradient, we can find the equation:

$$\left(1 - \frac{1}{\gamma}\right)\frac{1}{P}\left|\frac{dP}{dr}\right| > \frac{1}{T}\left|\frac{dT}{dr}\right| \tag{5.13}$$

We thus get the expression of the adiabatic gradient:

$$\left.\frac{dT}{dr}\right|_{ad} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} \tag{5.14}$$

We continue to write this equation:

$$\frac{dT}{dr}\frac{dr}{dP}\frac{P}{T} = \left(1 - \frac{1}{\gamma}\right) \tag{5.15}$$

$$\frac{d\log T}{d\log P} \equiv \nabla \tag{5.16}$$

then:

$$\nabla_{ad} = \left(1 - \frac{1}{\gamma}\right) \tag{5.17}$$

So the criterion becomes:

$$\nabla_{ad} > \nabla_{rad} \tag{5.18}$$

This relationship is called the Schwarzschild criterion, which represents the conditions under which the gradient occurs without convection. Therefore, there are two situations:

$$1.\ \nabla_{ad} < \nabla_{\mathrm{rad}} \text{ Convection occurs} \tag{5.19}$$

$$2.\ \nabla_{\mathrm{ad}} > \nabla_{\mathrm{rad}} \text{ No convection} \tag{5.20}$$

5.2 Convective flow equation

In the universe, we cannot accurately measure precise values, so we need to use MLT (Mixing Length Theory), used for the parameterization of the mean free path of convection cells:

$$l = \alpha H_p \tag{5.21}$$

α is the selected parameter, H_p is the distance corresponding to which the pressure varies by a factor of 1 / e.

Then:

$$\frac{1}{H_P} = -\frac{1}{P}\frac{dP}{dr} \tag{5.22}$$

$$\frac{dT}{dr} - \left(\frac{dT}{dr}\right)_{ad} \equiv \frac{d\Delta T}{dr} \tag{5.23}$$

The flux, which increases with the radiative temperature gradient and is dominant in the most internal areas where the rate of nuclear energy production strongly depends on the temperature. So the flow energy transportation equation we want to get is:

$$F_{conv} = c_P \Delta T \frac{1}{2}\rho v \tag{5.24}$$

ΔT is the temperature difference between the substance and the environmentc_pΔT is the thermal energy per gram of matter transported by the convective cell moving at constant pressure,ρv is mass flow.

We can get some formulas about ΔT:

$$\Delta T = \frac{d\Delta T}{dr} \cdot \ell \tag{5.25}$$

Next we can start to derive the equationWe can express the speed as:

$$v = \sqrt{2gl\frac{\Delta T}{T}} \tag{5.26}$$

We continue:

$$F_{conv} = \frac{1}{2}\rho v c_P \left(\left.\frac{dT}{dr}\right|_{ad} - \frac{dT}{dr} \right) \cdot \ell \tag{5.27}$$

$$F_{conv} = \frac{1}{2}\rho v c_P \ell \left(\left.\frac{dT}{dP}\right|_{ad} - \frac{dT}{dP} \right) \frac{dP}{dr}\frac{P}{T}\frac{T}{P} \tag{5.28}$$

Let's try to organize the formula:

$$F_{conv} = \frac{1}{2}\rho v c_P \frac{\ell}{H_P}\left(\nabla - \nabla_{ad}\right) T \tag{5.29}$$

In the end we got what we wanted:

$$F_{conv} = \frac{1}{2}\rho v c_P \alpha \left(\nabla - \nabla_{ad}\right) T \tag{5.30}$$

If we want to get the General equation of flow equation, we need to continue to derive,We need to express the radiant flux as a temperature gradient function:

$$F_{\mathrm{rad}} = -\frac{4ac}{3\kappa\rho} T^3 \frac{dT}{dr} \tag{5.31}$$

$$F_{rad} = -\frac{4ac}{3\kappa\rho} T^3 \frac{dT}{dP}\frac{dP}{dr}\frac{T}{P}\frac{P}{T} \tag{5.32}$$

$$F_{rad} = \frac{4ac}{3\kappa\rho}\frac{T^4}{H_p}\nabla \tag{5.33}$$

Because we know:

$$F = F_{rad} + F_{conv} \tag{5.34}$$

so:

$$F = \tfrac{4ac}{3\kappa\rho}\tfrac{T^4}{H_P}\nabla + \tfrac{1}{2}\rho v c_P \alpha \left(\nabla - \nabla_{ad}\right) T \tag{5.35}$$

$$F = \frac{L_r}{4\pi r^2} \tag{5.36}$$

We get the general equation.

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