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Comparison between fault location methods in distribution grid using PMU measurements

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This thesis is for my FAMILY, who have always supported and believed in me

Contents

Abstract

The introduction of Phasor Measurement Unit (PMU) in distribution network has an important innovation that brings the possibility of having in real time the complete knowledge of the full network with higher accuracy.
In the following chapters is analized the possibility to derive the position of a fault along a line using measurements performed by a limited number of PMU connected to strategic nodes.
Aim of this thesis is to analize and compare different possible use of PMU for fault location and undestrand which are the problematics that have to be solved for real implementation. Firstly is provided an introduction on the main features of the instrument focusing on the definition of the synchrophasor and because it is important to improve the quality of the measurements providing some accuracy indexes used to define such parameter.
Then, different methods for fault location in distribution network are described concentrating the discussion on the algorithm provided for the calculation and finally, is introduced a method for stategic PMU positioning inside the network and then a comparison between the methods is performed emphasizing the advantages and disadvantages brings from each method, focusing on the limitations that can occur due to not easily accessible input data, flexibility of each method to different types of fault and extension of the theorical model to a real network with different components and higher vastity.

Chapter 1

PMU

A Phasor Measurement Unit (PMU) is an innovative device, used in measurement application, that allows to measure both voltage and current phasors amplitude and phase, frequency and ROCOF (Rate Of Change Of Frequency) in power networks, using a source of time synchronization to tag each measurements with the corresponding time instant. This last concept is the very important innovation introduced with this device because permit to make measurements from differents part of the network synchronized with reduced measurements errors.

1.1 History

The concept of phasor was introduced from Charles Proteus Steinmentz in 1893 [1] that introduced it as a simplified matematical description of the waveforms of alternating current electricity. This concept is then evolved in 1988 in to the real time phasor measurement introduced with the invention of the PMU from Dr. Arun G. Phadke and Dr. James S. Thorp at Virginia Tech. [2] The necessity of synchronized sampling appear firstly in the field of the protection systems in which data samples were used in different and distant substations. This necessity translates in the invention, at the Virginia tech, of the symmetrical component distance relay that introduced to the invention of the PMU. The number of manifacturers that decide to introduce the PMU concept in their devices increase rapidly in time reaching tens of producers now and modern electronic devices used in electric substation are packed with PMU functionalities [3]. In parallel to the technical advence, a standardization process for the usage of the PMU was needed and then introduced from IEEE. The first version of the PMU standard was published in 1995 and then further revisions led to the current version defined in 2014. These standards introduce over the time design solutions for the manifacturers, giving specifications under steady state and dynamic test conditions; they also specify the indexes for the errors calculation that can affect the results of a mesurement performed with the

PMU. The standard IEEE C37.118.1 [4] introduces two performance classes: a P-class, for applications requiring fast responce like protections and an M-class, for applications requiring higher measurement accuracy. The IEEE C37.242 is also an another important milestone for the standardization, it was realesed in 2013 and describe a guide for calibration, testing and installation.

1.2 PMU fundamentals

1.2.1 Phasor and Synchrophasor, frequency and ROCOF

Considering a generic AC signal $x(t)$, a cosinusoidal signal with constant frequency and magnitude [2]:

$$x(t) = X_m cos(\omega t + \varphi_0) \tag{1.1}$$

where X_m is the peak value, $\omega = 2\pi f$ is the system angular frequency and φ_0 is the initial phase of the signal, the corresponding phasor representation [5] of $x(t)$ is:

$$\overline{X} = \frac{X_m}{\sqrt{2}} e^{i\varphi_0} = \frac{X_m}{\sqrt{2}} (cos\varphi_0 + isin\varphi_0) = X_r + iX_i \tag{1.2}$$

with X_r and X_i real and imaginary part of the phasor representation respectively.
The phasor phase angle φ_0 is strictly related to initial time instant used as a reference, is so important to define the starting time with high precision and for this reason, the introduced concept of synchrophasor [6] play a very important role because its estimation is based on the same idea underlying the phasor but usign the Coordinated Universal Time (UTC) as a time reference for the calculation of the phase angle. In this way, two synchrophasors, calculated in two different points of the network, can be easily compared. The convention wants that φ_0 has to be zero when the maximum of $x(t)$ occurs at the UTC time reference T_r choosen for the measurement.
The original signal is the real part of the phasor rotated at the signal angular frequency:

$$x(t) = \Re[\sqrt{2}\overline{X} e^{i2\pi f t}] \tag{1.3}$$

The PMU has to be able to measure not only the phasor componenents but also the frequency f and the ROCOF defined as follows:

$$ROCOF = \frac{\partial f}{\partial t} \tag{1.4}$$

it must be referred to the UTC reporting time T_r.

1.2.2 Source of synchronization

The technique used to define the time reference in distribution systems are multiple. There is the necessity to introduce the definition of *clock*, that is a circuitry to measure, keep and indicate time; it is composed by two parts: an oscillating device for defining a reference time interval and a counter device that counts the intervals provides the time indication. There are different time scale used for different applications. For distributed measurements the most common are two:

- **Coordinated Universal Time (UTC)**, it is based on measurements performed with atomic clocks. It is the time standard all over the world and divides time into the current definition of days, hours, minutes and seconds. Thanks to the use of leap second all units below the second are constant and all above are of variable duration.
  It is calculated subtracting from the TAI (International Atomic Time)[1] the accumulated leap second with occasional discontinuities due to irregular UTC days lenght. The computation of the time interval elapsed between two UTC timestamps is impossible without knowing how many leap second occur (tabled) in between therefore apllications requiring high precision in years long time intervals often uses TAI instead. UTC does not change in relation to each jurisdiction. UTC±00:00, with Greenwich time, is the reference from which all other times zones times are calculated, spacing from UTC-12:00 (west) to UTC+14:00 (east);

- **Global Positioning System (GPS)**, it is based on atomic clocks installed on satellites that form the GPS constellation. This time scale is similar to the TAI one but here is adopted a compensantion for relativistic effects that permitt to have perfect synchronization with the atomic clock present in the devices on the ground with which the satellites communicate. In this method the leap seconds is not implemented. It is elaborated by the US DoD originally each of twentyfour satellites were equipped with four atomic clocks with a frequency of 10.23 MHz for the relativistic compensation [7]. A receiver is able to provide a clock offset of 19 ns [8] with reference clock allowing high accuracy, being ideal for long distance implementation.

1.2.3 Accuracy indexes

To describe the performances of the PMU are utilized some indexes that define the measurement accuracy and are easy to be calculated.
It's easy to understand that the accuracy in synchrophasor measurements can be quan-

[1]TAI, froom the French Temps Atomique International, is based on the mean time provided by 200 atomic clocks located all around the world in 70 international laboratories.

tified by means of the deviations of both the phasor amplitude and phase angle measurements. The following indexes are so defined:

- *RAE=Realtive Amplitude Error*$\stackrel{\Delta}{=} \frac{\tilde{a}-a}{a} = \frac{\Delta a}{a}$;
- *PhE=Phase angle Error*$\stackrel{\Delta}{=} (\tilde{\varphi} - \varphi) = \Delta\varphi$

where $\tilde{a}$ and $\tilde{\varphi}$ are respectively the amplitude and phase angle measured value, a and φ are the nominal values.
Such errors are function of time and their maximum absolute values are tipically used as indexes of PMU performance. Very important is the so called *Total Vector Error (TVE)* that summarize the deviations of both the phasor parmeters:

$$TVE \triangleq \frac{|\tilde{X} - \overline{X}|}{|\overline{X}|} = \sqrt{\frac{(\tilde{X}_r - X_r)^2 + (\tilde{X}_i - X_i)^2}{(X_r - X_i)^2}} \tag{1.5}$$

is also possibile to define it in term of amplitude and phase angle errors:

$$TVE = \frac{|\tilde{a}e^{i\tilde{\varphi}} - ae^{i\varphi}|}{a} \cong \sqrt{(\frac{\Delta a}{a})^2 + \Delta\varphi} \tag{1.6}$$

where the last approximation is valid only for small values of Δa and $\Delta\varphi$.
Following the IEEE standard C37.118.1 [9] for synchrophasor measurements, the max allowed value for the TVE is 1%.
PMU measure also frequency and ROCOF, is so necessary to consider the indexes related to these parameters through the following expressions provided by standard [4,10]:

- *FE=Absolute Frequency Error*$\stackrel{\Delta}{=} |\tilde{f} - f| = |\Delta f|$
- *RFE=Absolute ROCOF Error*$\stackrel{\Delta}{=} |RO\tilde{C}OF - ROCOF| = |\Delta ROCOF|$

where another time the tilde indicates measured values.
Each of these indexes is calculated starting from the knoledge of the nominal values used as reference ones, calculated by means of simulated signals when testing only the PMU algorithms and by testing signals from calibrators when the whole PMU accuracy has to be verified.

1.2.4 Data rates

An important parameter to check the Standard [11] compliance for PMU is the data output rate, or more in general the time needed to provide the outputs from the elaboration of the inputs.
A high reporting rate indeed is required to the PMU to achieve an efficient gathering and

transmission of measurements over the wide area network. Usually PMU can operate at different reporting rates, integer multiples of the nominal frequency.
Values of reporting rates allowed by Standards are reported in figure 1.1 expressed in [frame/s].

System Frequency	50 Hz			60 Hz					
Reporting rates [frames/second]	10	25	50	10	12	15	20	30	60

Figure 1.1: Reporting rates table [11]

Once the compliance for a certain value of reporting rate is verified, it holds for all slower rates values.
From standard [11] some PMU parameters that define time performances can be derived:

- **Response time**, elapsed time between two steady- state measuments with a step change in input;
- **Delay time**, is the time interval between the instant when a step change is applied to the input of a PMU and measurement time when the stepped parameter achieves a value that is halfway the initial and final steady-state values. The purpose of this evaluation is to verify that the time tagging of the synchrophasor measurement has been properly compensated for filtering system group delay, so that the delay will be near to zero;
- **Reporting latency**, is the time delay from when an event occurs on the power system to the time that is reported in data. This latency includes many factors, such as the window over which data is gathered to perform a measurement, the estimation method, the measurement filtering, the PMU processing time and so forth. PMU reporting latency is defined as the maximum time interval between the data report time and the time when data becomes available at the PMU output;
- **Reporting rate**, is an integer number representing the rate of data outputs of PMUs;
- **Reporting Times**, for a given reporting rate, with N frames per second, where N is a positive integer, the reporting time shall be evenly spaced through each second with frame member 0 coincident with the UTC second rollover. These reporting time tags are used to define the instantaneous values of synchrophasor.

1.2.5 Steady-state and dynamic condition in power systems

A power system that works under ideal conditions is in a sinusoidal steady-state characterized by a nominal frequency, usually 50 or 60 Hz. In reality this never appens, in fact voltage and current signals differ from these ideal conditions in both fundamental frequency and distorted waveform.
For what concern the frequency, the system works in a narrow band around the nominal frequency but in particular situation it is possible that the frequency assume a value very different from the nominal one. The PMU has to be able to follow such frequency variations and to measure accurately the synchrophasor [6].
Other problems like loads and generators output, key operating parameters can change continually causing different disturbances like harmonics, intherharmonics, transient components and power swings[1]. These disturbances can affect the voltage and current measurements performed by the PMU.
Figure 1.2 gives a classification of the not sinusoidal events in power systems:

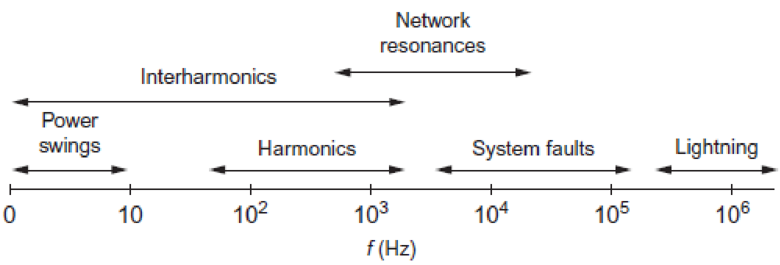

Figure 1.2: Non sinusoidal events classification [6]

All the high frequency not sinusoidal events, like ones that arise from lightning, are easily removable from input signal using filtering stage. The harmonics and interharmonics are disturbances that affect the input band of the PMU introducing distortions that can

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- *Harmonics and interharmonics*, generally produced by power electronic devices and not linear loads. Harmonics are at a frequency multiple of the foundamental, interharmonics are at all frequencies that are not integer of the fundamental [12];
- *System faults and travelling waves*, produce step changes in the voltage and current waveforms and generate very high frequency components in the signal;
- *Lightning and travelling waves*, cause very fast transients with frequencys that may be higher than 10^6 Hz;
- *Power swing*, generated by the superposition of different waveforms with different frequencies caused from an inequelibrium between generation and load.

further affect the synchrophasor estimation. Transient conditions like fault and switching operations, are particular situations that the PMU should evaluate in the shortest possible time. A PMU can not avoid to be subjected to a transitory, during these events the measurements are not usefull so PMU transient behaviour should be limited both in amplitude and in duration. Power swings imply a deviation from the sinusoidal conditions, they are low frequency variations and have to be followed accurately by the PMU. Considering an amplitude modulated sinusoidal signal, amplitude A_1 and frequency f, with the modulating signal as a different sinusoid, amplitude A_2 and frequency Δf their representation in the time and frequency domain are:

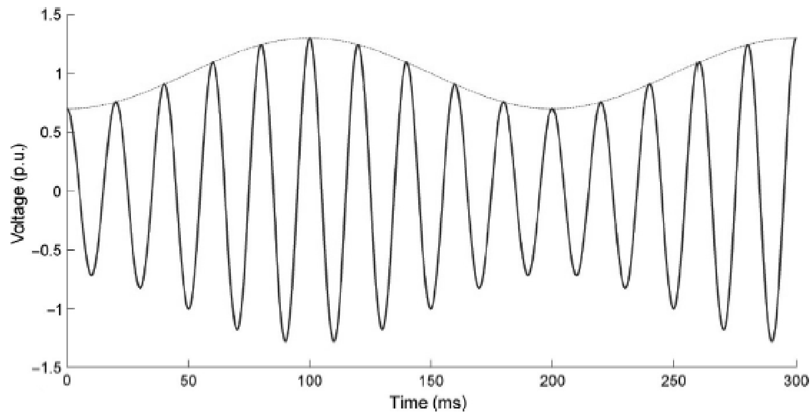

Figure 1.3: Amplitude modulated signal in the time domain [6]

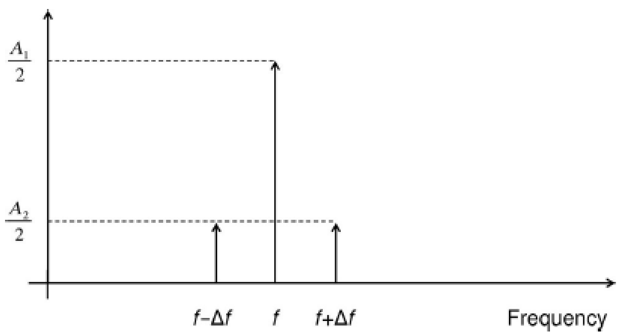

Figure 1.4: Amplitude modulated signal in the frequency domain

In effort with [13], to calculate the phasor amplitude, the signal considered shall be the modulating one that has a frequency in the range $[0.1 - 10]$ Hz but the observed signal range is $|f - \Delta f, f + \Delta f|$, in this range interharmonics close to the fundamental frequency of the signal could be erroneously confuse as part of the measured synchrophasor causing incorrect results. PMUs are fundamental for the tracing of the transient behaviours and to do that is introduced the concept of *dynamic synchrophasor*, referred to the UTC time reference frame [14]:

$$\overline{X}(t) = \frac{X_m g(t)}{\sqrt{2}} e^{i\varphi(t)} \tag{1.7}$$

where:

- $X_m g(t)$ is the modulated signal magnitude;
- $\varphi(t)$ is a real function that describe the phase dynamics.

This is a generalization of the basic concept of phasor usefull in case which both the amplitude and the phase angle are not constant. The dynamic phasor is so very important because permitt to folow the non sinusoidal conditions of interest highlighting phasor amplitude and phase time changing. Is so defined a passband around f_0 where all the frequency component outside it are considered disturbances. The UTC reference frame here substitute the reference measured instant for the concept of initial time.
The dynamic model gives a reference frame ideal to the implementation of accuracy indexes estimation algorithms allowing to perform synchrophasors evaluations taking in to account all possible dynamics that can affect the signal.

1.3 Architecture of a PMU

Starting from the initial apparatus the architecture of a PMU, differently from the one of the GPS, is not change a lot. An analog signal arrives as an input in to a signal

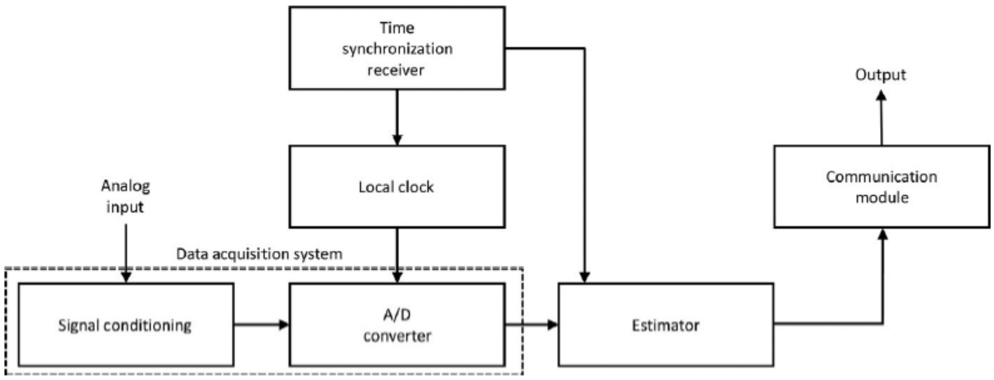

Figure 1.5: General architecture of a PMU [15]

conditioning circuit module that performs the adaptation of that signal to adequate for Analog to Digital converter (A/D converter). The conditioning circuit include also an anti-aliasing filter action performed by a low pass filter with adaptable cut-off frequency to decrease signal bandwidth before the analog to digital conversion. These two modules compose the so called Data acquisition system, is important that during the acquisition no relevant delays nor distortion are added to the signal to be acquired because delays can affect both measurement synchronization and repoprting latency; particular attention is also given to avoid significant modifications around the system fundamental frequency hence a flat response around 50 Hz (or 60 Hz) is necessary to reduce errors on estimated phasors. The Analog to Digital converter has a sampling frequency f_c [16] related to the filter frequency response too.
Particular attention must be payed on the conversion time of this converter that has to elaborate input signal heavily affecting the overall computation latency of the PMU. It is also critical to deal with synchronized acquisition; to find the best way to correlate acquired samples with UTC time reference, two different architectural approaches are possible:

- **Synchronized source clock acquisition**, the synchronization source provides the time synchronization with a good level of accuracy that today for the PMU are around hundreds or nanoseconds. The basic output of the synchronization source is a digital signal that provides the Pulse-Per-Second (PPL) that is the accurate information about the start of the UTC second.

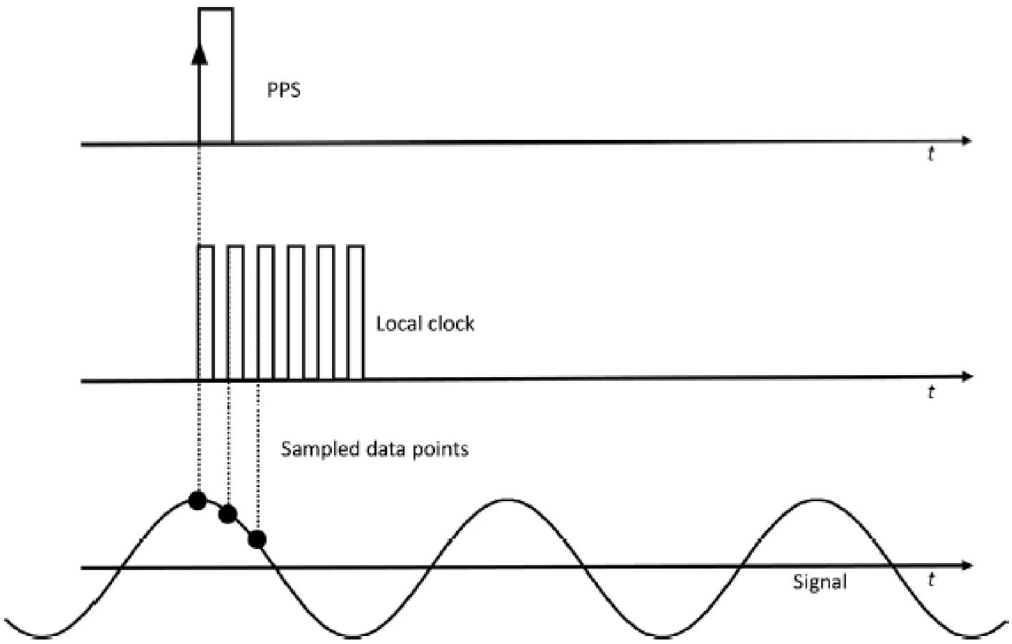

Figure 1.6: Synchronized source clock acquisition [15]

The local clock in the PMU is triggered by the PPS to start the measurement process at a well defined time instant. The term local clock refers to the time reference dissemination inside the PMU and, in particular, refers to the time source used for the scheduling of the acquisition process that can include a system of different clocks inside the boards. The local clock, which control the acquisition has to be locked to the PPS so that the alignment and the pace of the acquisition are linked to the UTC; due to the locking the acquired samples fall in known time instant and so a direct relationship with the reporting time has been created.

- **Free running acquisition**, is an approach less accurate respect to the previous described and presents few drawbacks. It can be important when the local clock of the acquisition system is not accessible or tunable from external synchronization sources, being the only solution available. The sampling clock is independent from the reference clock and the obtained samples are not located exactly on the desired predetermined instants [17]. The sampling frequency is not limited by the UTC and an offset clearly arise between the reference time of the measurement and the nearest sample.

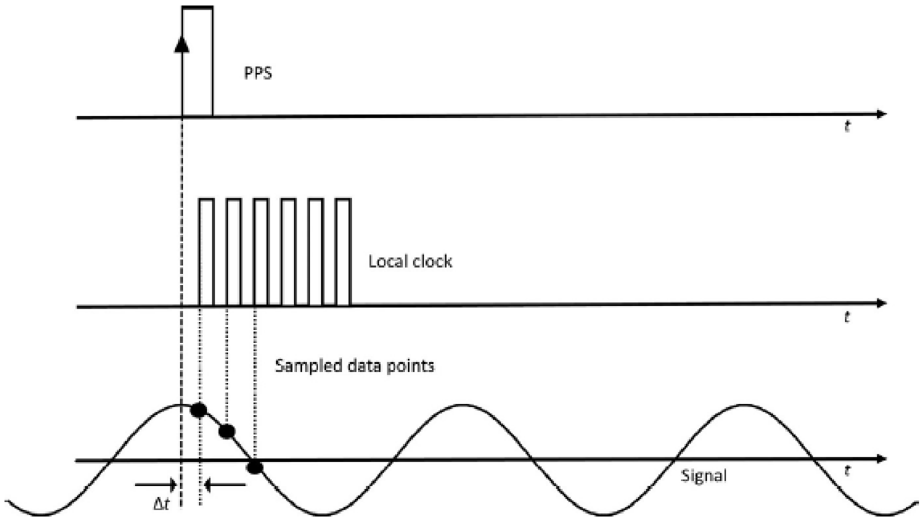

Figure 1.7: Free running acquisition [15]

The offset Δt is clearly limited by half the sampling period and thus the impact on the phase measurement due to the offset is lower for a higher sampling frequency. This offset has to be compensated to obtain an accurate synchrophasor evaluation. The phase compensation becomes:

$$\Delta\varphi = 2\pi\tilde{f}\tilde{\Delta}t \qquad (1.8)$$

where $\tilde{f}$ and $\tilde{\Delta}t$ are respectively the estimates of the frequency and of the time offset.

Thanks to the time synchronization this module respresents the main distinction from a tradional measurements device. It makes available a time source all over the system and controls the synchronization of all internal clocks with measurements.
The synchronization source can be both internal and external, relying on satellites directly or indirectly.

This module perform very important different tasks:

- *Gives common UTC reference*;
- *Gives measurement reporting instant*;
- *Trigger the acquisition*;
- *Gives time quality information.*

The clock circuit is composed by an oscillator that defines the time reference intervals and a counter counting them to provide the time indication. The most used time scales are, as sad in previous sections, the UTC and the GPS, the use of the latter for synchronization allows high accuracy for long distance applications but a synchronization backup is necessary due to possible deterioration, jamming and spoofing signals.
For PMU applications, IEEE C37.118 Standard [18] prescribes the unmodulated DCLS (Direct Current Level Shift) version use, becauase it provides the best time accuracy, as the leading edge of the first maker pulse is well defined and can be generated with an accuracy of few tens of nanoseconds, given a good source clock[15].
The estimator module represents the stage in which synchrophasor, frequency and ROCOF, coming from algorithms results, are implemented in the process unit. The PMU processing capability and its speed are fundamental because of the need of latency minimization and of the real time scheduling, with the PMU reporting rate as an important constraints for computation.
Being a smart device the PMU must be provided of a communication module with low latency but high reporting rate to PDCs, this is possible thanks to the enancements in the communication technologies and the introduction of fibre optic and wireless connections. The Standard IEEE C37.118.2 [19] gives a set of message types and PMU packet formats, but PMUs' communications protocols are continuosly updated due to security and interoperability issues of the overall network.

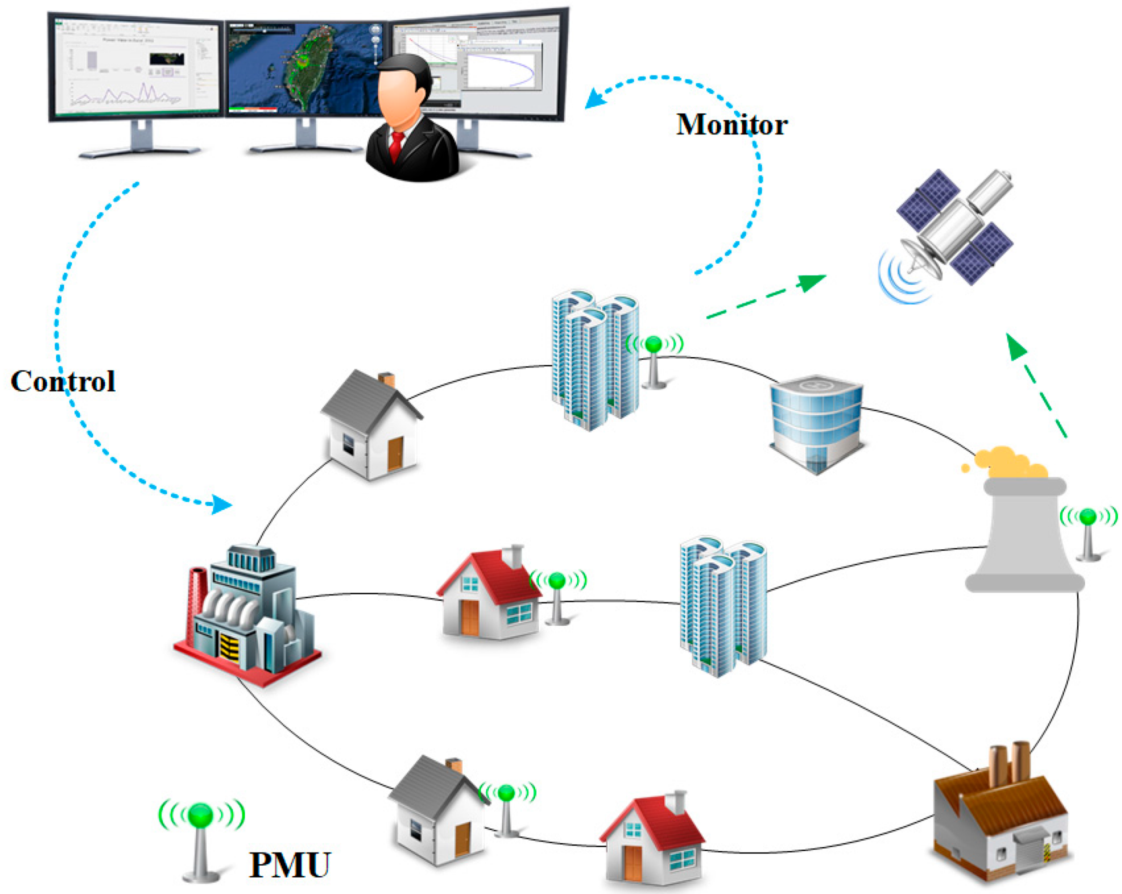

Figure 1.8: PMU in the network

1.4 PMU in the network

Figure 1.8 shows how PMU are integrated inside the power network. PMU are located in different key points of the power grid that permitt to have a complete coverage and knowledge of the all network parameters[1]. The measurements performed by the various PMU are transmitted following the Standard IEEE C37.118.2 [19] to a Phasor Data Concentrators (PDS) that collects them time ordered and send them to higher level PDCs [2].
Different utilities PDCs may be connected to a single central PDC, in order to provide an overview of the entire grid therefore a trustworthy comunication network is required to the correct functioning of a wide area PMUs network and can be a dedicated network

[1]The complete knowledge of the state of the power network is reached when the values of the magnitudes and of the phase angles of the voltages of the each node of the grid are known.

or part of the corporate data network of the power utility, that may carry other user traffic.
Other users can affect the behaviour of the wide area control system, is so important to understand the effects of the network traffic characteristic on the power system operation, that can cause both latency and packet-loss probability depending on network congestion. The bandwidth of the communication link define the max data rate and the latency of the switches and routers directly impact performances and reliability of the network.

1.4.1 PDC generalities

These are devices that act as a nodes in a communication networks collecting data from many PMUs to create sets of measurements of the wide area, correlated by time tags.
The aim of PDCs in a real time scenario is to align the data from PMUs using the timestamp included in the input streams, with high performances required in terms of latency and reliability.
From local PDCs, one or more streams of data reach higher level PDCs, depending on the wideness of the monitored area and on the network topology, with transmissions that may be performed at different rates with different targets. The Standard IEC 61850

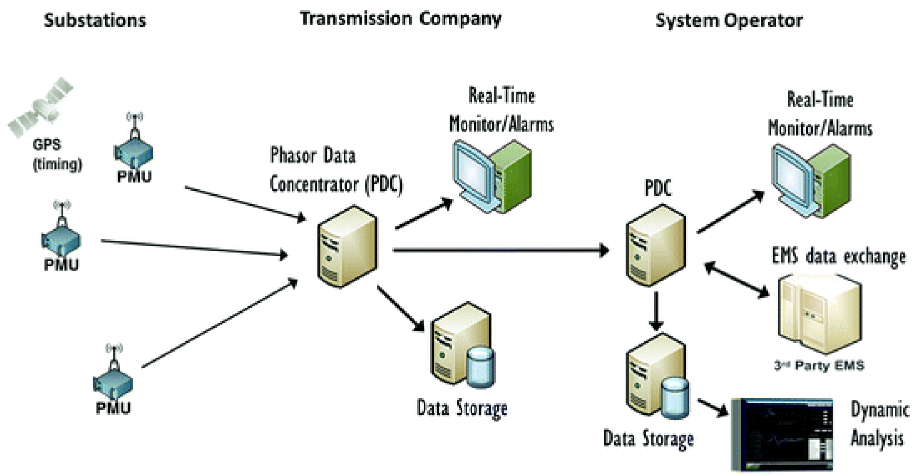

Figure 1.9: PDCs communication network

[20] defines two communication levels: Station and Process Busses, defining instead of a topology, a set of requirements that communication network must satisfy [21,22]. In a wide perspective, the Standard IEC 61850-90-5 [23] defines data models for synchrophasor real time transmission secure and scalable over a wide area network, between PMUs and PDCs, both local and central ones. PDCs are also used for quality check and overall monitoring for disturbances; at many levels they are also used for verifiction, validation and storage of specific data packets into database.

1.4.2 Communication generalities

The communication system among different substations has been standardized by IEC 61850 [20] ensuring interpolability by a serial protocol, standardizing both data objects and their accesses. Most common services are the GOOSE (Generic Object-Oriented System Event) and the SV (Sampled Value), with the first used for information transmissions with strict requirements in time terms, the second instead is used for rapid transmission of flows of current or voltage samples as in circuit breakers release.
The implementation of a cyclic data transmission with multiple sending through a continuous communication between IEDs is a security criterion adopted to ensure the correct reception of data.
The structure implemented for communication sees as highest level the station one, connected by the station bus to protection, control and monitoring IEDs bay units, transmitting control information.
The MMs protocol is used for transferring data between the station level and bay IEDs, while GOOSE is the service used for transferring data from bay to bay.
the process bus connects the bay units to operating devices in the field using services such as SV for transmitting measurements samples for protection purposes.
The process level, placed at the bottom of the structure, realizes IEDs of circuit breakers, disconnectors and their relative connections.

1.5 Simulink PMU block

Simulink is a program very usefull for the representation of a distribution line. Through this program is possible to simulate different situations that can occur during normal operating conditions, making possible the implementation of ideal and real conditions. A particular package of blocks called *simscape* is specific for electrotechinal applications; inside it is possible to find blocks that permitt to represent all components that can be found in a distribution network.

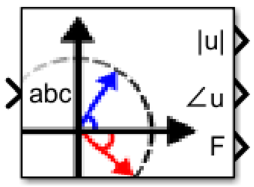

Figure 1.10: PMU block [24]

Simulink provides a dedicated block for the PMU that can be used to perform all type of operation for what it is concerned. This block receive as input a voltage or current

three-phase signal that is composed by the combination of each phase signal waveforms in to a unique one.
The output of this block are the canonical parameters for what a PMU is projected:

- *Phasor magnitude*, indicates with the symbol $|u|$, of the positive-sequence component of the three-phase input signal at the fundamental frequency in the same units as the input signal.
- *Phasor phase*, indicate with the symbol $\angle u$, in degree, relative to the PLL phase of the positive-sequence component of the three-phase input signal at the fundamental frequency.
- *Frequency*, in Hz, relative to the PLL phase of the positive-sequence component of the three-phase input signal at the fundamental frequency.

To initialize the simulation, this block requires to be defined the value of the *nominal frequency*, the *sampling rate* in point/cycle and the *reporting rate factor* that is a parameter used to multiply to sample time and calculate the reporting rate.

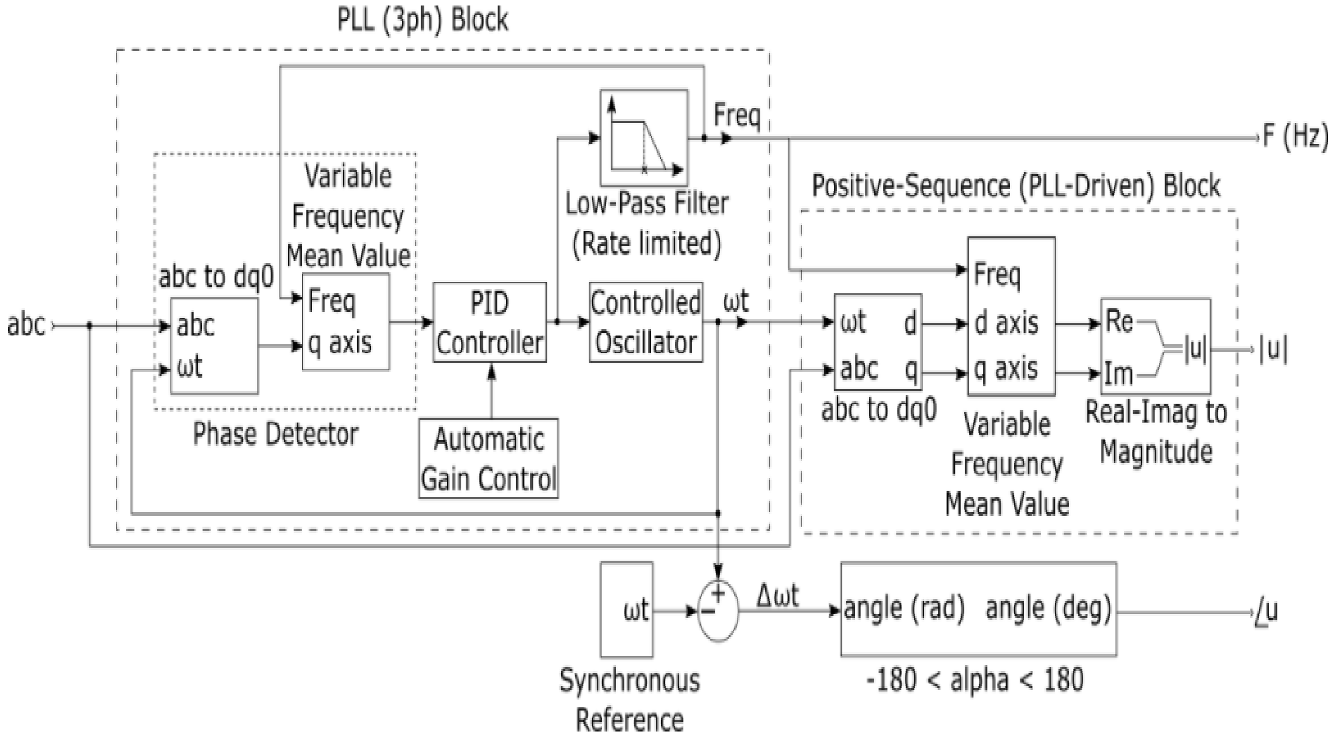

Figure 1.11: PMU block zoom [24]

Figure 1.11 describe the block circuit used inside the block PMU of Simulink tocalculate the output parameters. The algorithm used for the measurements of phasor is the

Phase-Locked Loop (PLL) which compute the positive sequence component of the three-phase input signal over a period of one cycle of fundamental frequency given by the input signal.

1.5.1 PLL

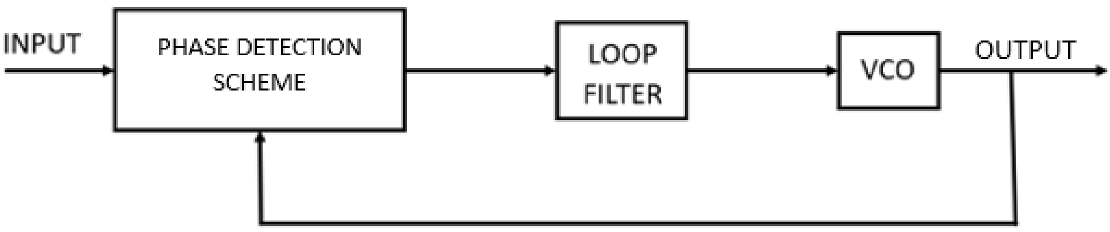

Figure 1.12: Single phase PLL scheme [25]

Figure 1.12 describe the scheme of a single phase PLL. It is composed by three parts:

- **VCO (Voltage Controller Oscillator)**, that generates an oscillation with a frequency proportional to the one of the input signal;
- **Phase Detection Scheme**, is a multiplicator;
- **Loop Filter**, is a low pass filter.

The filter has to reduce as much as possible the double frequency ripple produced by the generic input signal in the loop that unfortunately can not be complitely eliminated; this disturb make the solution not useful for power system application. To avoid this

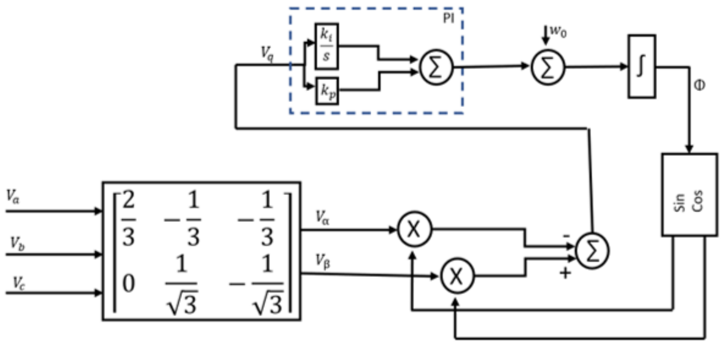

Figure 1.13: Three-phase PLL scheme [25]

problem is introduced the Three-phase scheme of the PLL shown in figure 1.13.
The three-phase algorithm is able to provide the phase estimation under both phase and magnitude unbalances, including also offset and harmonic distortions.
A reference called *$\alpha\beta$-reference frame* is defined, in which the PLL reach the synchronization after a translation of the natural *abc-reference frame*. The phase angle $\theta_{\alpha\beta,PLL}$ is estimated using trygonometric equations [26].

$$\theta_{error} \approx [sin(\theta_{grid}) * cos(\theta_{\alpha\beta,PLL}) - sin(\theta_{\alpha\beta,PLL}) * cos(\theta_{grid})] \tag{1.9}$$

where θ_{grid} is the actual grid voltage phase angle; this equation is valid only if θ_{error} is small.
The extraction of the phase value is achieved monitoring θ_{error} to track the zero reference with a PI controller.
If the grid is operating under balanced conditions this algorithm is able to estimate accurately the phase angle but under unbalanced conditions it lost accuracy; this loss can be compensated reducing the bandwidth, but it results in a slower dynamic response [27-28]. To improve the performances of the three-phase PLL scheme under unbalanced conditions solution called *Enhanced PLL* is implemented.
The most famous mathematical tool for estimation purposes is the so called *Discrete Fourier Transform* (DFT), whose algorithm is the following:

$$X_s(f) = \sum_{n=0}^{N-1} x[n]e^{\frac{-j2\pi nk}{N}} \tag{1.10}$$

where N is the total number of samples and $X_s(f)$ is the signal of the samples in the frequency domain. The DFT is a windowed-based method that requires high attention in window-centering to avoid errors, while PLL is not window-based thus it does not present corrections or windowing issues.
When harmonics or noises are present in the system causing high distorsions, the usage of multi units of PLL is recommended to avoid errors in the phasors. This procedure not heavely affects the circuit complexity thanks to the high scalability of each unit.

Chapter 2

Proposed fault location methods

The advent of the PMU in the world of electrical networks has arise the interest of different companies for the possibility of identify the location of a generic fault that occurs in the system with a very short working time and with high accuracy, thanks to the introduction of the synchrophasor evaluated in real time by the PMU.
The main challenge for this application is find an algorithm that can be able to operate in a distribution network ; reach the objective is not easy due to the complexity of such part of the electric system (high number of sparse load connected, bidirectional power flow,...).
Here theoretical proposal to be possible solution are analyzed.

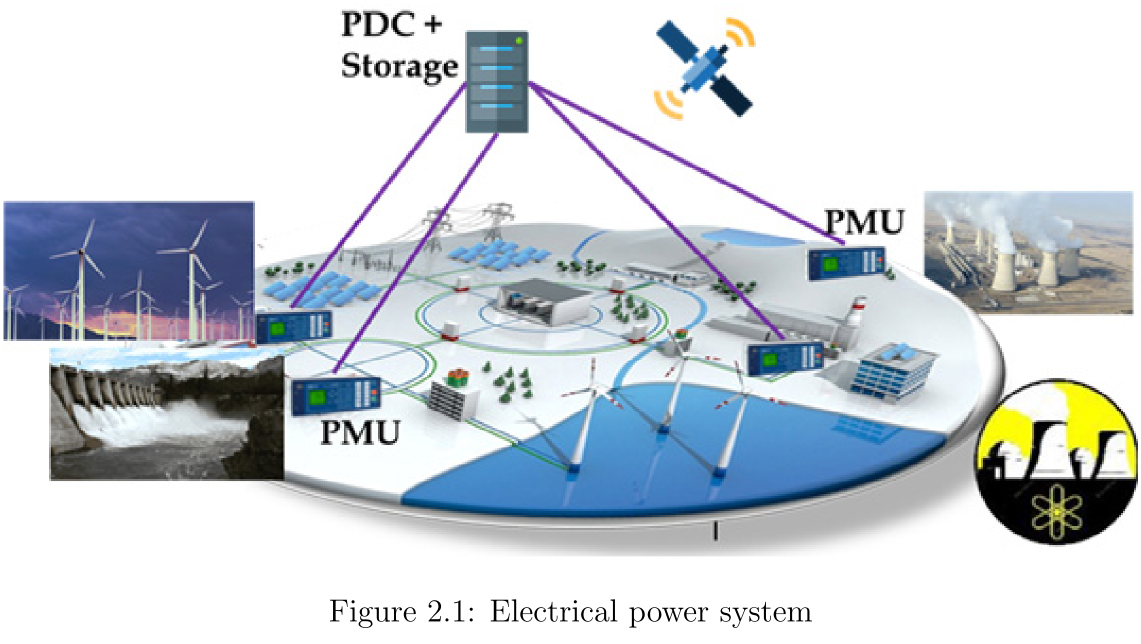

Figure 2.1: Electrical power system

2.1 Method I

The approach used in this method is based on the assumption that different parameters of the line are considered known. For the identification of the fault location are used the values of the voltage and current in pre-fault and during-fault conditions measured from different PMUs positioned in various nodes of the network [29].

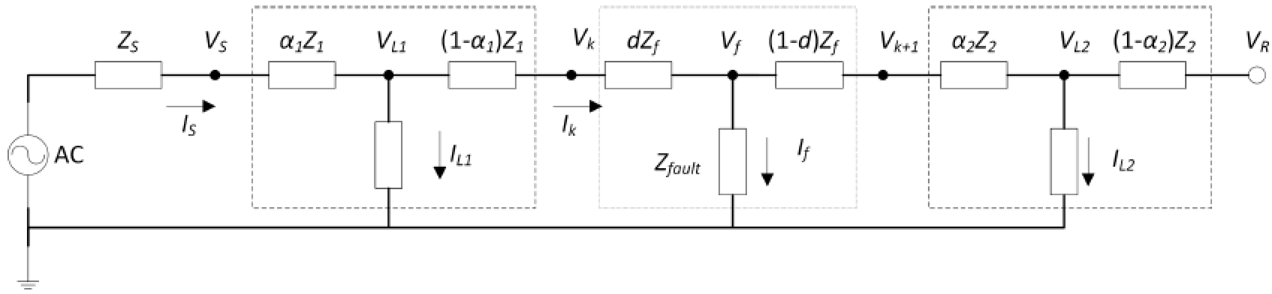

Figure 2.2: Circuital scheme [29]

Referring to figure 2.2, the fault occurs on the line between the node *k* and *k+1*.
Two PMUs are positioned at nodes S, where there is the substation and node R, the end of the feeder.
The loads that are linked between nodes *S - k* and *k+1 - R*, are modeled as a single unknown load in such parts of the line.
The voltages and currents at nodes k and k+1 can be calculated before the fault only if the location of the equivalents loads are known. In the line *S - k* the position of the load is expressed through the coefficient α_1 that is a percentage of the total lenght of the considered branch; for the loads linked to the line *k+1 - R* is used the same logic but the coefficient is α_2.
The impedances of the two lines are considered known and indicated with Z_1 and Z_2 for the line *S - k* and *k+1 - R* respectively. Z_f is the impedance of the assumed faulted line. This method considers that loads are not directly measured or known and when they varies, their spatial distribution remains roughly constant. The nodes L_1 and L_2 indicate the point in wich such loads are linked.
Using the Ohm's Law and the Kirchhoff Current Law (KLC) the voltages at the nodes where there are not provided PMUs are calculated:

$$V_{L_1} = V_S - \alpha_1 Z_1 I_S \tag{2.1}$$

$$V_{L_2} = V_R \tag{2.2}$$

$$V_k = V_{L_1} - (1-\alpha_1) Z_1 I_k \tag{2.3}$$

$$V_{k+1} = V_R + \alpha_2 Z_2 I_{L_2} \tag{2.4}$$

$$I_k = I_S - I_{L_1} \tag{2.5}$$

In pre-fault conditions there is no current dispersion in the line bbetween nodes k and $k+1$ so is possible to define other two equations:

$$I_k = I_{L_2} \tag{2.6}$$

$$V_k - V_{k+1} = Z_f I_{L_2} \tag{2.7}$$

This set of equations is used to solve a linear system in 7 variables that are: V_{L_1}, V_{L_2}, V_k, V_{k+1}, I_k, I_{L_1}, I_{L_2}.
Considering now the during-fault conditions, the equations from (2.1) to (2.5) continue to maintain their validity assuming, as already mentioned above, that the coefficients α_i do not change.
Now there is the necessity to introduce a model that allows to estimate the currents of the loads in during-fault conditions, such model used is the ZIP one [30] where the parameters $Z_\% + I_\% + P_\% = 1$ describe the portion of the load that is constant impedance, current and power, respectively.
Then the following equations are introduced:

$$|I_{L_{i,during}}|' = |I_{L_{i,pre}}|(|\frac{V_{L_{i,during}}}{V_{L_{i,pre}}}|Z_\% + I_\% + |\frac{V_{L_{i,pre}}}{V_{L_{i,during}}}|P_\%) \tag{2.8}$$

$$\angle I'_{L_{i,during}} = \angle V_{L_{i,during}} - (\angle V_{L_{i,pre}} - \angle I_{L_{i,pre}}) \tag{2.9}$$

where i indicates the number of the loads considered (in the circuit it can be 1 or 2), the apostrophe indicates estimate quantities.
All values used in these new equations are known and calculted from the previous set in both pre and during fault conditions.
After the fault the equations 2.6 and 2.7 no longer hold, they are so substituited with the following new ones:

$$V_f = V_k - dZ_f I_k \tag{2.10}$$

$$V_{k+1} = V_f - (1-d)Z_f(I_S - I_{L_1} - I_f) \tag{2.11}$$

$$I_k = I_f + I_{L_2} \tag{2.12}$$

The estimates are then included:

$$I_{L_1} = I^{'}_{L_1} \tag{2.13}$$

$$I_{L_2} = I^{'}_{L_2} \tag{2.14}$$

These equations form a system that can be solved and where the variables are: V_{L_1}, V_{L_2}, V_k, V_f, V_{k+1}, I_{L_1}, I_k, I_f, I_{L_2}, d. The index d can then be used to calculte the distance of the fault point.

2.2 Method II

There is a theorem that allows to simplify the analysis of an unbalanced three-phase power systems under both normal and abnormal conditions.
The Fortescues theorem describes this possibility that is based on the idea that an asymmetrical set of N phasors can be expressed as a linear combination of N symmetrical sets of phasors by means of a complex linear transformation. This theorem is based on superposition principle, so it is applicable to linear power systems only, or to linear approximations of non-linear power systems.
In the most common application of this method to a three-phase system, the resulting symmetrical components are referred to as direct (or positive), inverse (or negative) and zero (or homopolar). The analysis of power system is much simpler in the domain of symmetrical components, because the resulting equations are mutually linearly independent if the circuit itself is balanced.
The mathematical representation of this concept, applied to a three-phase system is the following:

$$\begin{bmatrix} F_a \\ F_b \\ F_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_0 \end{bmatrix} \tag{2.15}$$

where the first member is a vector that contain the three-phase phasors triplet, the 3×3 matrix is called *transformation matrix* and the last vector contain the triplet of phasors of the positive (1), negative (2) and homopolar (0) sequences.
The two coefficients a and a^2 are:

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

In method II is considered that the positive sequence network is always present for every type of fault that can occur, is so analysed only the positive sequence component network at the moment of fault [31].

2.2.1 Faulty feeder identification

The first challenge is to identify which is the faulty feeder. Considering a network composed by different feeders connected to a common point, called branch point, a PMU is installed at the end of each feeder and making the voltages and currents at these points known. Taking into account figure 2.3, the PMU is positioned at the end (point 2).

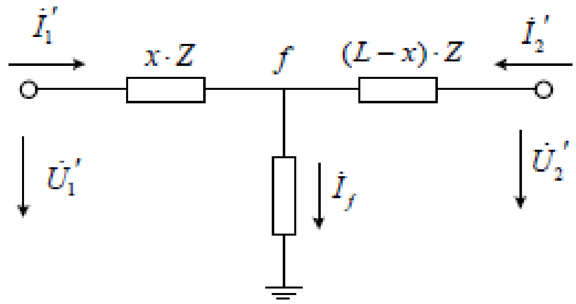

Figure 2.3: Simplified circuital scheme [31]

is so possible to calculate the voltage at the first end of the feeder :

$$\dot{U}_1 = \dot{U}_2 + LZz \cdot \dot{I}_2 \tag{2.16}$$

where L and Z are respectively the total lenght of the feeder and the impedance per unit lenght of such feeder. When the fault occurs at the point f, distant x from the feeder head (point 1), the voltage of such point becomes:

$$\dot{U}_1' = \dot{U}_2' + (L-x)Z \cdot \dot{I}_2' + xZ \cdot \dot{I}_f \tag{2.17}$$

where I_f is the fault current.
The fault feeder is unknown and so is necessary to calculate the headend voltage at the same branch point, that can be estimated through the equation 2.16:

$$\dot{U}_{1,f}' = \dot{U}_2' + LZ \cdot \dot{I}_2' \tag{2.18}$$

The faulty branch that contains faulty feeder is not taken in to account, there is so an error between the calculated and the true value of the head end voltage:

$$\Delta\dot{U}' = \dot{U}_1' - \dot{U}_{1,f}' = xZ \cdot \dot{I}_f \tag{2.19}$$

This error is calculated for each feeder.
Because the calculated value on faulty feeder has an error from the true one, the faulty feeder is then localized.

2.2.2 Accurate fault location

Starting from the first part just described is possible to determine an accurate position of the fault in the faulty feeder. The equations of a line can be described from the following reletionship:

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} cosh(\gamma L) & Z_c sinh(\gamma L) \\ \frac{sinh(\gamma L)}{Z_c} & cos(\gamma L) \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ \dot{I}_2 \end{bmatrix} \tag{2.20}$$

where:

- $Z_c = \sqrt{\frac{Z}{Y}}$, is the positive sequence impedance of the line;
- $\gamma = \sqrt{ZY}$, is the propagation coefficient of the line;
- Z, Y are respectively the line impedance per unit lenght and the ground admittance per unit lenght;

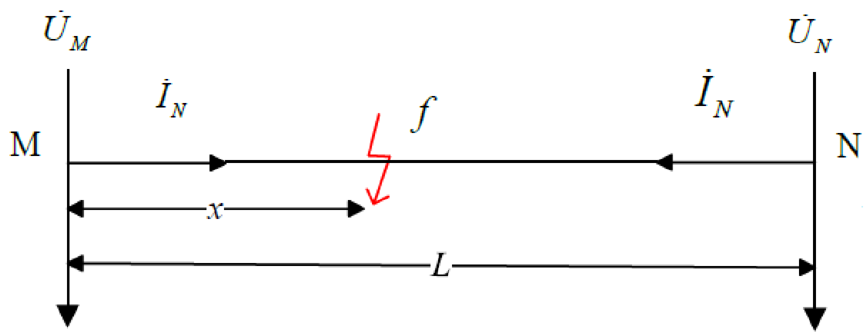

Figure 2.4: Faulty feeder representation [31]

The head and end point of the considered feeder are now indicated with M and N respectively. Thanks to equation 2.20 is possible to express the voltage phasor of the fault point in therm of $\dot{U_M}$ and $\dot{I_M}$ or, equivalently, using $\dot{U_N}$ and $\dot{I_N}$:

$$\dot{U}_{M,f}(x) = \dot{U}_M cosh(\gamma x) - \dot{I}_M Z_c sinh(\gamma x) \tag{2.21}$$

$$\dot{U}_{N,f}(x) = \dot{U}_N cosh[\gamma(L-x)] - \dot{I}_N Z_c sinh[\gamma(L-x)] \tag{2.22}$$

These two voltages are referred to the same point so they must be equal $\dot{U}_{M,f}(x) =$

$\dot{U}_{N,f}(x)$; using the exponential expression of the hyperbolic functions:

$$cosh(x) = \frac{1}{2}(e^{\gamma x} + e^{-\gamma x}) \tag{2.23}$$

$$sinh(x) = \frac{1}{2}(e^{\gamma x} - e^{-\gamma x}) \tag{2.24}$$

is then possible to extrapolate the variables dependent from the position x:

$$e^{2\gamma x} = \frac{(\dot{U}_N - \dot{I}_N Z_c)e^{\gamma L} - (\dot{U}_M + \dot{I}_M Z_c)}{-(\dot{U}_N + \dot{I}_N Z_c)e^{-\gamma L} + (\dot{U}_M - \dot{I}_M Z_c)} = A + jB \tag{2.25}$$

where A and B are the real and imaginary part of the complex number.
The propagation constant can be expressed as a complex number, $\gamma = \alpha + j\beta$, so equalizing the real and imaginary parts on both sides of the equation:

$$2\beta x = arctan(\frac{B}{a}) \tag{2.26}$$

$$x = \frac{1}{2\beta} arctan(\frac{B}{A}) \tag{2.27}$$

the fault distance is finally calculated.

2.3 Method III

This method is based on the coupling parameters identification using two terminal data. Due to the difficulties to have a priori knowledge of the line parameters, they are calculated by characteristic loop voltage equation based on pre-fault PMU recording data [32].

2.3.1 Single-phase scenario

A first approach to this method is given considering the simpliest case as the single phase one.

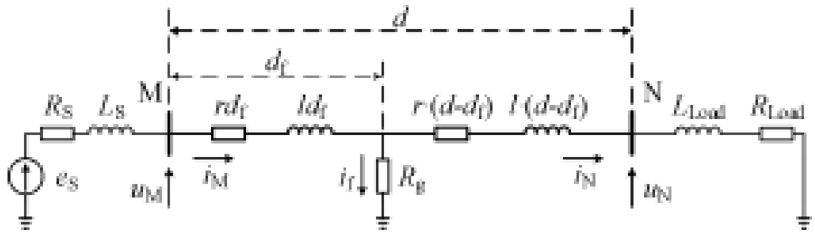

Figure 2.5: Circuital scheme [32]

Considering figure 2.4 the fault occurs at point f, d is the distance between these two points and d_f is the distance of the point f from M. The electrical parameters are :

- u_M, i_M, istantaneous voltage and current at the point M after fault occurs;
- u_N, i_N, istantaneous voltage and current at the point N after fault occurs;
- e_s is the istantaneous values of the power potential when fault occurs;
- r, l, are the resistance and inductance per unit lenght;
- R_S, L_S, are the equivalent resistance and inductance of the system;
- R_Load, L_Load, are the equivalent resistance and inductance of the load;
- R_g, is the grounding resistance.

From the Kirchhoff Voltage Law (KVL):

$$u_M = rd_f i_M + ld_f \cdot (\frac{di_M}{dt}) + r \cdot (d - d_f) \cdot i_N + l(d - d_f) \cdot (\frac{di_N}{dt}) + u_N \tag{2.28}$$

$$u_M - u_N = rd_f \cdot (i_M - i_N) + ld_f \cdot (\frac{di_M}{dt} - \frac{di_N}{dt}) + rdi_N + ld \cdot (\frac{di_N}{dt}) \tag{2.29}$$

$$d_f = \frac{u_M - u_N - rdi_N - ld \cdot (\frac{di_N}{dt})}{r(i_M - i_N) + l \cdot (\frac{di_M}{dt} - \frac{di_N}{dt})} \tag{2.30}$$

The parameters u_M, u_N, i_M, i_N are measured and the parameters *r, l, d* are usually known so fault location d_f can be easely measured.

2.3.2 Three-phase scenario

The fault location identification is performed without the introduction of the phase-mode transformation.

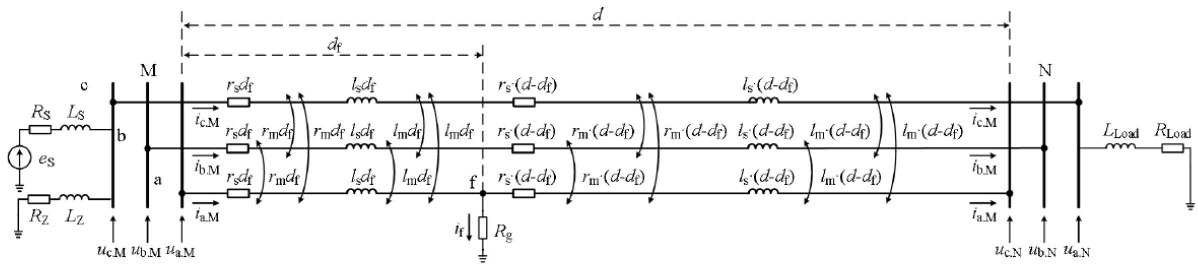

Figure 2.6: Circuital scheme [32]

where the parameters are:

- $u_{i,M}$, are the istantaneous voltages of the M bus, where $i = a, b, c$ indicate the three phases;
- $i_{i,M}$, are the istantaneous currents of the M bus, where $i = a, b, c$ indicate the three phases;
- $u_{i,N}$, are the istantaneous voltages of the N bus, where $i = a, b, c$ indicate the three phases;
- $i_{i,N}$, are the istantaneous currents of the N bus, where $i = a, b, c$ indicate the three phases;
- r_s, r_m, l_s, l_m, are respectively the self and mutual resistances and the self and mutual inductances per unit lenght;

- R_Z, L_Z, are the equivalent resistance and inductance of the grounding transformer and grounding device;
- d is the distance between point M and N.

Using the KVL in the time domain is possible to express the voltage equation fro each phase; considering only *phase a* for sake of semplicity:

$$u_{a,M}-u_{a,N}=r_s d_f p_{1,a}+r_m d_f p_{2,a}+l_s d_f p_{3,a}+l_m d_f p_{4,a}+r_s d p_{5,a}+r_m d p_{6,a}+l_s d p_{7,a}+l_m d p_{8,a} \tag{2.31}$$

where:

- $p_{1,a}=i_{a,M}-i_{a,N}$;
- $p_{2,a}=i_{b,M}+i_{c,M}-i_{b,N}-i_{c,N}$;
- $p_{3,a}=\frac{di_{a,M}}{dt}-\frac{di_{a,N}}{dt}$;
- $p_{4,a}=\frac{di_{b,M}}{dt}+\frac{di_{c,M}}{dt}-\frac{di_{b,N}}{dt}-\frac{di_{c,N}}{dt}$;
- $p_{5,a}=i_{a,N}$;
- $p_{6,a}=i_{b,N}+i_{c,N}$;
- $p_{7,a}=\frac{di_{a,N}}{dt}$;
- $p_{8,a}=\frac{di_{b,N}}{dt}+\frac{di_{c,N}}{dt}$;

according to figure 2.6, is assumed that a fault occurs at point f distant d_f from point M.
Introducing now the full lenght parameters of the line equation 2.31 can be rewritten as:

$$y_a(n)=R_s q_{1,a}(n)+R_m q_{2,a}(n)+L_s q_{3,a}(n)++L_m q_{4,a}(n) \tag{2.32}$$

where:

- $y_a(n)=u_{a,M}(n)-u_{a,N}(n)$;
- $q_{1,a}(n)=i_{a,M}$;
- $q_{2,a}(n)=i_{b,N}(n)+i_{c,M}$;
- $q_{3,a}(n)=\frac{di_{a,M}(n)}{dt}=\frac{i_{a,M}(n+1)-i_{a,M}(n-1)}{T_s}$;
- $q_{4,a}(n)=\frac{di_{b,M}(n)}{dt}+\frac{di_{c,M}(n)}{dt}=\frac{[i_{b,M}(n+1)-i_{,M}(n-1)]+[i_{c,M}(n+1)-i_{c,M}(n-1)]}{T_s}$.

Parameter n indicates the number of iteration considered, this because the calculation of $y_a(n)$ is repeated different times to recave the values of the line parameters without has a priory knowledge of their values that can be a problem in real application. The last two parameters are then expressed using midpoint difference instead of differential, T_s is the sampling period considered. There are 4 variables $[R_s, R_m, L_s, L_m]$, and are necesary four recording time instant to define a set of four equations; the start time of the PMU is taken as the fault time and the fault recording data of the previous power frequency cycle, of about 20 ms, before the fault time is substituited.
The number of sampling power is larger then four and so a certain redundancy in the parameters identification can occurs.
To increase the accuracy of the parameters estimation a set of constraints on their value can be introduced:

- $k_{R_s,min} k_d r_s \leqslant R_s \leqslant k_{R_s,max} k_d r_s$;
- $k_{R_m,min} k_d r_m \leqslant R_m \leqslant k_{R_m,max} k_d r_m$;
- $k_{L_s,min} k_d l_s \leqslant L_s \leqslant k_{L_s,max} k_d l_s$;
- $k_{L_m,min} k_d l_m \leqslant L_m \leqslant k_{L_m,max} k_d l_m$;

The various *k-parameters* are the constraints coefficients, the values of the per unit lenght parametrs pf the line can be obtained through other line ledgers similar to the one just described.
Coming back to the equation 2.31 the only unknown parameter is d_f, the fault distance, that can be know evaluate.

2.4 Method IV

This method aim to define an algorithm that can be used in new active distribution network[1] [33].

2.4.1 Before the failure

The current phase can assume very different values in pre and during failure conditions. Considering to have different PMUs installed in a line, is possible to measure the phasors of the current at both ends of the line and evaluate their difference.

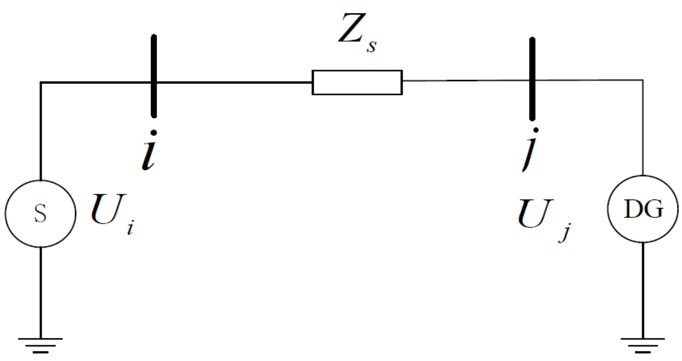

Figure 2.7: Pre-fault circuit [33]

Figure 2.7 shows a schematic situation of a line with the head connected to the power grid and the end to a DG (distributed generation). The parameter utilized in this representation are:

- $\dot{U}_i = |U_i|\angle\theta_i$, that is the voltage phasor of the power grid;
- $\dot{U}_j = |U_j|\angle\theta_i$, that is the voltage phasor of the DG;
- $Z_s = R_s + jX_s$, represents the impedance of the line.

The currents that flow in the line are:

- $\dot{I}_{i,pre}$, that flows from node i to node j;
- $\dot{I}_{j,pre}$, that flows from node j to node i.

[1]The distribution network is called active due to the introduction of the *distributed generation* (DG) in the low-voltage side of the network. The therm DG indicates the introduction of small generators (for example photovoltaic panels plant) in the network with the aim to guarantee the electric production.

Thanks to the simple analysis of the circuit in figure 2.7 they can be expressed in therm of the voltage phasors, considering that the reference direction is from node i to node j:

$$I_{i,pre} = \frac{\dot{U}_i - \dot{U}_j}{Z_s} = \sqrt{\frac{U_i^2 + U_j^2 - 2U_iU_jcos(\theta_i - \theta_j)}{R_s^2 + X_s^2}} \angle(\theta_{\Delta U} - \theta_s); \tag{2.33}$$

where $\theta_{\Delta U}$ is an angle obtained from the difference of the phases of the voltage phasors and θ_s is the phase of the phasor of the line impedance:

$$\theta_{\Delta U} = arctan(\frac{U_isin\theta_i - U_jsin\theta_j}{U_icos\theta_i - U_jcos\theta_j});$$

$$\theta_s = arctan(\frac{X_s}{R_s});$$

2.4.2 After the failure

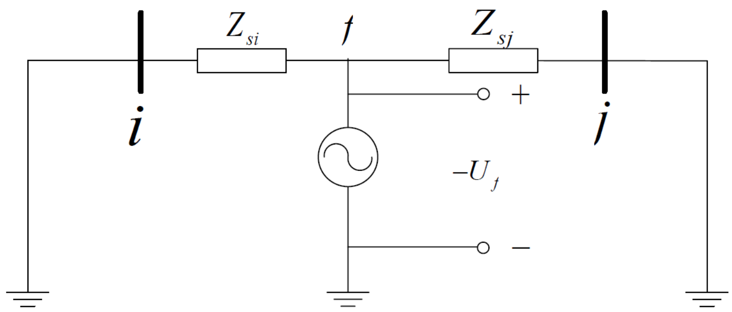

Figure 2.8: After-fault circuit [33]

Figure 2.8 shows the equivalent circuit of the line, considered in the previous paragraph, for after fault conditions. The fault point is named f and so the line is divided in two parts. The phasor $\dot{U}_f = |U_f|\angle\theta_f$ reprents the voltage at the point f before the failure. The line impedance is then defined for each of the two part of the line: $Z_{s,i}$ and $Z_{s,j}$ are respectively related to part *i-f* and *f-j*; they can be expressed in therm of the total line impedance and of a coefficient α that is a percentage of the total lenght of the line: $\dot{Z}_{s,i} = \alpha\dot{Z}_s$ and $\dot{Z}_{s,j} = (1-\alpha)\dot{Z}_s$. The currents that flow from i to f and from j to f *during* the failure are respectively $\dot{I}_{i-f}$ and $\dot{I}_{j-f}$:

$$\dot{I}_{i-f} = \frac{\dot{U}_f}{\dot{Z}_{s,i}} = \frac{\dot{U}_f}{\sqrt{R_{s,i}^2 + X_{s,i}^2}} \angle(\theta_f - \theta_{s,i}) \tag{2.34}$$

where:

$$\theta_{s,i} = arctg(\frac{X_{s,i}}{R_{s,i}})$$

$$\theta_{i-f} = \theta_{\Delta U} - \theta_{s,i}$$

and:

$$\dot{I}_{j-f} = \frac{\dot{U}_f}{\dot{Z}_{s,j}} = \frac{\dot{U}_f}{\sqrt{R_{s,j}^2 + X_{s,j}^2}} \angle(\theta_f - \theta_{s,j}) \tag{2.35}$$

where

$$\theta_{s,j} = arctg(\frac{X_{s,j}}{R_{s,j}})$$

$$\theta_{j-f} = \theta_f - \theta_{s,j}$$

Now the currents that flow through the two buses *after* the failure are:

$$\dot{I}_i = \dot{I}_{i,pre} + \dot{I}_{i-f} = \frac{\dot{U}_i - \dot{U}_j}{Z_s} + \frac{\dot{U}_f}{Z_{s,i}} \tag{2.36}$$

$$\dot{I}_j = \dot{I}_{j,pre} + \dot{I}_{j-f} = \frac{\dot{U}_i - \dot{U}_j}{Z_s} + \frac{-\dot{U}_f}{Z_{s,i}} \tag{2.37}$$

where:

$$\theta_{I_i} = arctg(\frac{U_i sin\theta_i - U_j sin\theta_j + \alpha U_f sin\theta_f}{U_i cos\theta_i cos\theta_i - U_j cos\theta_j + \alpha U_f cos\theta_f}) - arctg(\frac{R_s}{X_s}) \tag{2.38}$$

$$\theta_{I_j} = arctg(\frac{U_i sin\theta_i - U_j sin\theta_j - (1-\alpha) U_f sin\theta_f}{U_i cos\theta_i cos\theta_i - U_j cos\theta_j - (1-\alpha) U_f cos\theta_f}) - arctg(\frac{R_s}{X_s}) \tag{2.39}$$

taking the difference between these two angles is obtained:

$$\theta_{If} = arctg(\frac{U_i sin\theta_i - U_j sin\theta_j - (1-\alpha) U_f sin\theta_f}{U_i cos\theta_i cos\theta_i - U_j cos\theta_j - (1-\alpha) U_f cos\theta_f}) - arctg(\frac{U_i sin\theta_i - U_j sin\theta_j + \alpha U_f sin\theta_f}{U_i cos\theta_i cos\theta_i - U_j cos\theta_j + \alpha U_f cos\theta_f}) \tag{2.40}$$

this angle assume a value that is pratically equal to π. Apllying this procedure to different section of the line, if the value of the last angle falls in the interval [0;π] means that the considered section is subjected to a fault. The value of θ_{If} can be calculated through a PMU and so is possible to recave the corresponding value of α to locate the fault.

2.5 Method V

Like in method IV, here is considered the case in which are present distributed generators.

2.5.1 Possible failure points

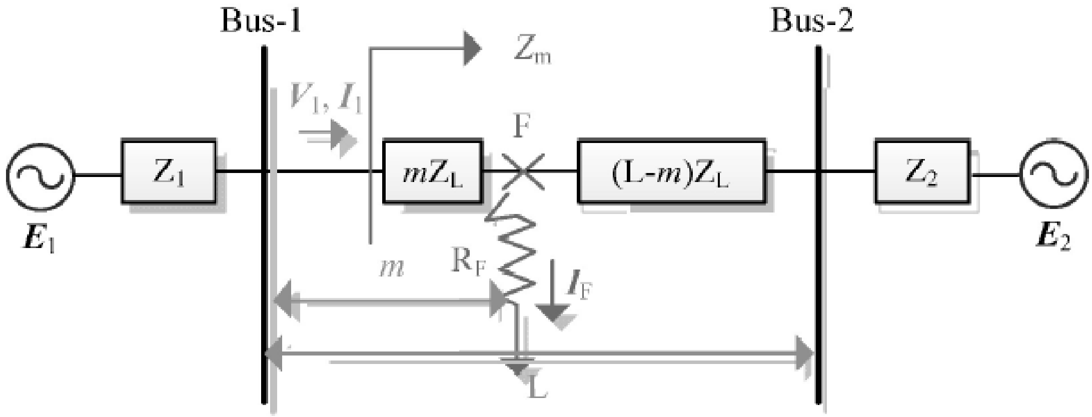

Figure 2.9: Simplified circuit [34]

Figure 2.9 describes a simple circuit that represents a portion of a line between two buses. A PMU is installed in bus 1 so the value of V_1 and I_1 are known, L is the total lenght of the section considered. Z_m is the measurement impedance [35] and its value can be calcultaed as follows:

$$Z_m = \frac{\dot{V}_1}{\dot{I}_1} = mZ_L + R_F(\frac{\dot{I}_F}{\dot{I}_1}) \tag{2.41}$$

R_F is the transition resistance. If we consider that the fault current $\dot{I}_F$ and the current $\dot{I}_1$ are in phase the transition resistance is a real number, the value of the fault distance m can be calculated through the following expression:

$$m = \frac{Im\frac{\dot{V}_1}{\dot{I}_1}}{I_m Z_L} \tag{2.42}$$

In real conditions these two currents are not in phase with each other and so equation 2.42 leads to a result that presents a large measurement error. Is so defined a new equation to solve the fault distance:

$$m^2 - k_1 m + k_2 - k_3 R_F = 0 \tag{2.43}$$

where the K_i coefficients are:

- $k_1 = 1 + \frac{Z_2}{Z_L} + (\frac{\dot{V}_1}{Z_L \times \dot{I}_1}) = a + ib$
- $k_2 = \frac{\dot{V}_1}{Z_L \times \dot{I}_1}(1 + \frac{Z_2}{Z_L}) = c + id$
- $k_3 = \frac{\Delta \dot{I}_1}{Z_L \times \dot{I}_1}(1 + \frac{Z_2 + Z_1}{Z_L}) = e + if$

is so possible to obtain the fault distance solving the second degree equation:

$$m = \frac{(a - \frac{eb}{f}) \pm \sqrt{(a - \frac{eb}{f})^2 - 4(c - \frac{ed}{f})}}{2} \tag{2.44}$$

If the value of m is lower then the value of L of the considered section a possible fault point is identified. This process is repeated for every branche of the line where the voltage bus in which there isn't a PMU is calculted with the following expression, here applied for the case of bus 2 in figure 2.9:

$$\dot{V}_2 = \dot{V}_1 - LZ_L\dot{I}_1 \tag{2.45}$$

2.5.2 True failure point identification

The procedure just described leads to the identification of different possible failure points along the line; there is the necessity of discard false points.
Firstly is take in to account the phase angle of the voltage phasor measured from the PMU at bus 1, it is then expressed as a sum of two measured components:

$$\theta_1 = arg(\frac{\dot{V}_1}{\dot{V}_F}) = arg(\frac{\dot{V}_1}{\dot{I}_1}) + arg(\frac{\dot{I}_1}{\dot{V}_F}) = arg(\frac{\dot{V}_1}{\dot{I}_1}) + arg(\frac{\dot{I}_1}{R_F\dot{I}_F}) = arg(\frac{\dot{V}_1}{\dot{I}_1}) + arg(\frac{\dot{I}_1}{\dot{I}_F}) \tag{2.46}$$

The last equality of equation 2.46 introduce the phase angle between the measured and fault currents, this angle can not be directly calculated, is so necesssary to introduce the passage to the sequence components to describe the line parameters.
In this new scenario the fault sequence current is in-phase with the current of the protections. This means that sequence current measured at bus 1 and fault sequence current are in-phase. Is then provided the following expression:

$$arg(\frac{\dot{I}_1}{\dot{V}_F}) = arg(\frac{\dot{I}_1}{C\dot{I}_1^2}) \approx= arg(\frac{\dot{I}_1}{\dot{I}_1^2}) \tag{2.47}$$

where C is a coefficient of the negative sequence current distribution. Using equation 2.47 is so possible to calculate the vale of θ_1.
Considering now a network with different branches, in which are present different nodes and in which are identified different possible fault points, two PMUs are installed at the two ends of the network; is so possible to provide the value of the θ_i of the two nodes in which is provided a PMU,for bus 1:

$$\theta_1 = arg(\frac{\dot{V_1}}{\dot{I_1}}) + arg(\frac{\dot{I_1}}{\dot{I_1^2}}) = arg(\frac{\dot{V_1}}{\dot{I_1^2}}) \tag{2.48}$$

once obtained the value of θ_i for the two ends are then calculated the fault distances from the two buses S_i that are used to discard false fault points:

$$S_i = \frac{\theta_i}{2\pi f} \tag{2.49}$$

where i indicates the end buses and f is the power frequency.

Chapter 3

Real implementation considerations and problematics

As already mentioned in previous chapters, the distribution network topology is changed along the years and its complexity is increased. The advent of PMU plays and important role in the possibility of have a complete knowledge of the network in real time.
As just said the complete knowledge of a network is reached when the voltage amplitude ang phase of each bus of the network are known. The simpliest way to solve this problem is to locate a PMU in every bus, but this could be complicated due to the high vastity of the distribution network and also for economic reasons.

3.1 PMU positioning

Is possible to resume buses configurations that can be found inside the distribution network and define for them an efficient positioning methodology for the PMU.

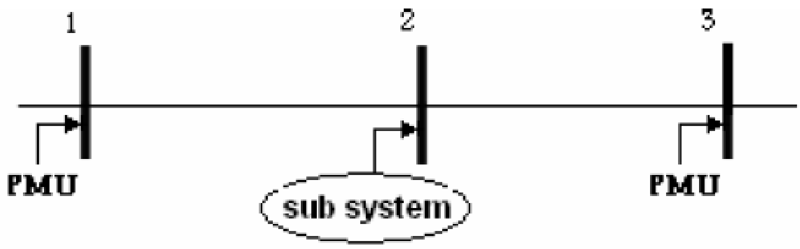

Figure 3.1: Configuration I [36]

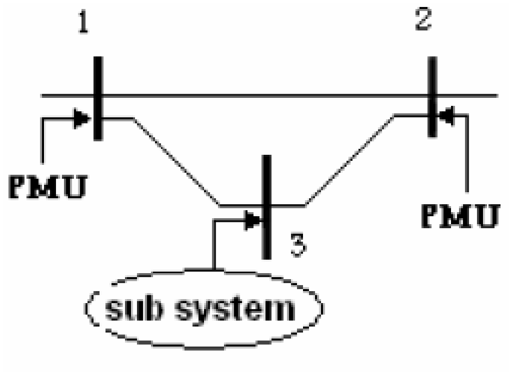

Figure 3.2: Configuration II [36]

Figures 3.1, 3.2 describe the simpliest buses configurations, they are composed by three nodes and the PMUs installed are two in both cases. The knowledge of the parameters at the bus in which is not provided a PMU can be calculated through the KVL knowing the line parameters (impedance, admittance,...).
There are other three possible configurations that can be found inside a distribution network.

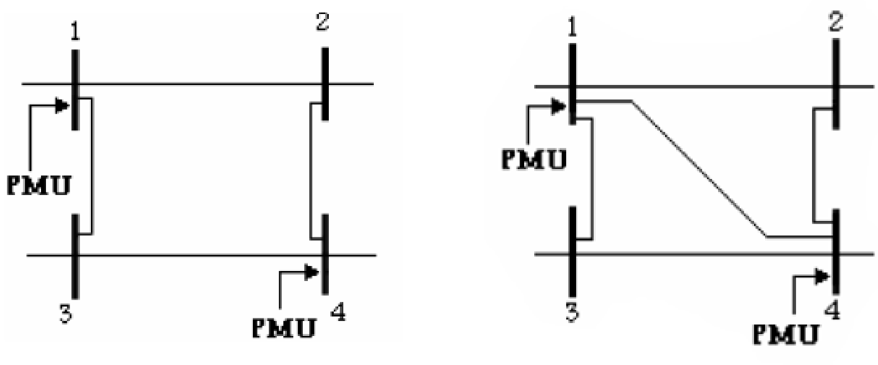

Figure 3.3: Configuration III and IV [36]

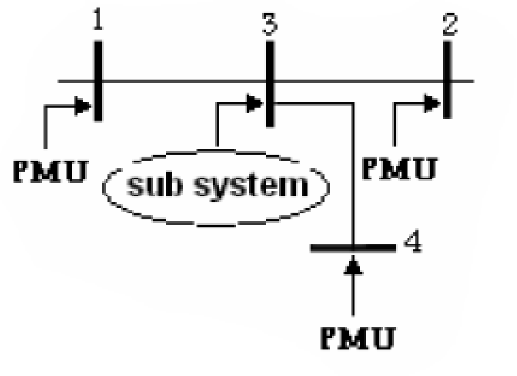

Figure 3.4: Configuration V [36]

3.2 Input data

Chapter 2 describes different methods for fault identification in distribution network using PMU, the input data that each method necessity are:

- **Method I**:
  - Network topology
  - Nodes connected to a PMU
  - Line lenght
  - Pre-fault phasors of the phase voltages and currents measured from PMUs (V_S, I_S, V_R)
  - During-fault phasors of the voltages and currents from PMU measurements
  - Lateral-load position along a branch of the line between two nodes (coefficients α_i)
  - ZIP model of each lateral load (coefficients $Z_{\%}$, $I_{\%}$, $P_{\%}$)
  - Line parameters
- **Method II**:
  - Network topology
  - Nodes connected to a PMU
  - Line lenght
  - **Faulty feeder identification**:
    * Pre and during fault phasors of the positive sequence components of voltages and currents of the end node measured from PMU

    - Fault current
    - Line impedances
  - **Accurate fault location**:
    - During-fault phasors of the positive sequence componentsof voltages and currents of both two ends of the line measured from PMU
    - Line impedances and admittances

- **Method III**:
  - Network topology
  - Nodes connected to a PMU
  - Line lenght
  - Pre and during fault phasors of the phase voltages and currents measured from PMUs
  - Value of the k_i coefficients used to define the constraints for the calculation of the line parameters
  - Sampling period
  - Number of iteration

- **Method IV**:
  - Network topology
  - Nodes connected to a PMU
  - Line lenght
  - Pre and after fault phasors of the phase voltages and curretns measured from PMUs
  - Line parameters
  - Location of distributed generators (DG)

- **Method V**:
  - Network topology
  - Nodes connected to a PMU
  - Line lenght
  - Pre and during fault phasors of the phase voltages and currents measured from PMUs
  - Line parameters
  - Value of the fault transition resistance (R_F)

3.3 Implementation analysis

The introduction of PMU in distribution network results more difficult compared to the case of transmission network. In the latter one the line can be schematized as a three-phase line between two nodes where are positioned the PMU; there are different methodologies that define, for transmission application, an algorithm for the fault location using PMU, these are all very similar to each other and much simplier to be understood since the circuital scheme that is used as the model in the theorical explanation of the method is very similar to the real one.
In distribution application this concept doesn't holds since, as already mentioned, this part of the network is composed by a large variety of components and the number of nodes is higher, increasing the number of parameters that must be known to have the complete knowledge of the network.
The first objective to be reach from fault location methods for distribution grid is the possibility of obtaining all the information, positioning a limited number of PMU in strategic nodes of the network.
The proposed methods use circuital law, like Ohm's one and KVL, to determine the values of the unknown parameters not directly measured from a PMU.
Method I is the only one described, that provides a circuit model able to better represents a possible part of the distribution network, with the hypothesis of grouping all loads between two nodes in a unique one (called *lateral load*); of these ones only the knowledge of the ZIP model parameters is required.
To solve the problem of network representation a compromise was introduced linked to the loads model because these parameters could not be easily accessible to the operator.
For what concern the other methods, they represent the network, in the theorical part, in a simpliest way (much similar to the case of transmission network) using only a model with two nodes where a PMU is provided for both of them. Problematic could be introduced when these algorithm will be applied to a complex network due to presence of lateral lines and loads or DG.
The presence of distributed generators is introduced in the analysis of *Methods IV and V* but these consider only lines included between two nodes where are linked the supply generator and the distributed generators; the need may arise to adapt these algorithms to the lines that do not present DG in all ending nodes.
From the data analysis of each method is possible to understand that some parameters are necessary for all of them. Input data could be an another limitation of algorithms implementation due to the difficulty to have a priori knowledge of some of them. This is the case of the ZIP parameters, as already mentioned for *Method I*, or the impedance, admittance values of the different line brances. Is too difficult to reach an accurate knowledge of the values of such parameters, during normal operating conditions and particularly when faults occur, because different influence quantities change them from the scheduled values. Temperature, aging, electromegnetic field and weathering cause

undesired modifications of the line apparatus and so of their parameters.
This means that the necessity to know such data to use these methods could be a limitations for the implementation and could give rise to problems related to the measurements accuracy.
Method III introduce a possible solution for this problematic providing in its algorithm a part dedicated to the estimation of the line impedances. Also for this case a compromise was introduced because the estimation procedure increase the computational load and to reduce it is introduced the knowledge necessity of some coefficients linked to boundary constraints of the apparathus manifacturing that in some cases may be difficult to know a priori.
A further limitation, related to the concept of input parameters, is linked to the necessity of *Method V* to know the value of the transition resistance of the fault that may be very difficult to reach in most cases.
Lines can be affected by different types of fault like:

- Three-phase to ground or not;
- Single-phase to ground or not;
- Double phase to ground or not;
- Temporary disconnection of a load that can cause unbalances;
- Frequency fluctuations
- Lightnings overvoltages;
- Supply interruption;
- Others...

A fault localization algorithm could be able to work under all of these situations and in same cases has to be able to recognize which is the problem type.
The methods of chapter 2 are not all described for every type of fault, in some cases the description is given only for a single scenario.
Method I is based on the assumption that the line is affected by a three-phase to ground fault, is also indicated that in the case of a double or single line to ground fault there is the necessity to introduce one or two additional equations to the algorithm respectively, that have to set to zero the fault current in the healthy phases. *Method III* analizes the single phase to ground fault but is able to operate also for the double and three phase one. For what concern other applications, different from the canonical types of fault, informations and specifications are not provided but thanks to the different parameters measurable with the PMU they could be implemented starting from these realizations.

From this analysis is possible to understand that:

- To avoid the need for a PMU in each node of the network is necessary a priori knowledge of a large amount of parameters, this could be the first limitation to the applicability of the algorithms due to the not ideality of the real case for what concern the accuracy of the requested input data;

- To increase the flexibility of the methods to be able to operate under different conditions and recognize different types of fault, algorithms must be developed to be adapted to the different situations that can occur in distribution network;

- *Method I and III* are the most advanced and flexible and are the only ones that provide solutions for implementation problems. *Method IV and V* take in to account an important part of the distribution network, the distributed generators, that are increasing in number and dimensions introducing problematics that a state estimation evaluation through PMU could be able to reduce increasing the accuracy in measurements thanks to the synchronization of the data from different points along the line.

Conclusions

The purpouse of this elaboarte is the introduction and description of different methodologies for fault location in distribution grid. The five proposed methods cover different possible cases of study that can be found in a real network scenario.
Each method present limitations for the application in real conditions due to the necessity of a priori knowledge of different parameters necessary to execute the algorithm or due to the limitated flexibility for the application to different fault conditions.
Starting from these methods will be possible to define new metodologies for fault location using PMU able to overcome and solve problems described above. The introduction of the PMU in distribution grid will made possible to reach a real-time knowledge of the network state during different conditions, increase the measurements accuracy and prevent possible unfavorable situations that could will cause huge damages to the infrastructures.

List of Figures

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