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Quantum computers for high school: design of activities for an I SEE teaching module

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ABSTRACT

The thesis is situated within the I SEE project (Inclusive STEM Education to Enhance the capacity to aspire and imagine future careers), a triennial ERASMUS+ project involving six partners, started in 2016 and coordinated by the University's Department of Physics and Astronomy of the University of Bologna (https://iseeproject.eu/). The main goal of I SEE is the design of teaching approaches and modules on advanced interdisciplinary topics such as climate change, artificial intelligence and quantum computers for secondary school. The modules aim to: i) improve students' ability to imagine the future and to aspire to STEM careers; ii) develop transversal skills that allow students to play an active and conscious role in a global, fragile and constantly changing world.

The work of this thesis started from the analysis of an I SEE module on quantum computers realized by the Finnish partners of the project and consists in a revision and integration of their activities in order to solve some problems they encountered during the implementation. This revision aimed to build a better connection between quantum computers and future and to search for a global approach to lead students to understand the physics behind these new technologies without getting trapped in the technical details.

In particular, the work I have developed for the present thesis concerns the choice of teleportation as an emblematic case of the quantum protocol and sets as objectives: i) the comparison between the teleportation experiment and the circuit that realizes it, highlighting how the experiment can be reread in terms of logic gates and quantum circuits; ii) the educational transposition and the design of the teaching activity within the module.

Sommario

Il lavoro si inserisce all'interno di I SEE (Inclusive STEM Education to Enhance the capacity to aspire and imagine future careers), un progetto ERASMUS+ triennale che coinvolge sei partner, iniziato nel 2016 e coordinato dal Dipartimento di Fisica e Astronomia dell'Università di Bologna (https://iseeproject.eu/). Lo scopo principale di I SEE è la progettazione di approcci e moduli di insegnamento su temi interdisciplinari avanzati quali cambiamenti climatici, intelligenze artificiali e computer quantistici per la scuola secondaria di secondo grado. I moduli sono finalizzati a: i) migliorare la capacità degli studenti di immaginare il futuro e di aspirare a carriere in ambito STEM; ii) sviluppare competenze trasversali che permettano agli studenti di svolgere un ruolo attivo e consapevole in un mondo globale, fragile e in continuo mutamento.

Il lavoro di questa tesi si sviluppa a partire dall'analisi di un modulo I SEE sui computer quantistici realizzato dai partner finlandesi del progetto e consiste in una revisione e integrazione delle loro attività al fine di risolvere alcuni problemi da loro incontrati durante l'implementazione. Tale revisione era volta alla costruzione di una migliore connessione tra computer quantistici e futuro e alla ricerca di un approccio globale per portare gli studenti a comprendere la fisica alla base di queste nuove tecnologie senza rimanere intrappolati nei dettagli tecnici.

In particolare, il lavoro che ho sviluppato per la presente tesi riguarda la scelta del teletrasporto come caso emblematico del protocollo quantistico e si pone come obiettivi: i) il confronto tra l'esperimento del teletrasporto e circuito che lo realizza, mettendo in evidenza come l'esperimento possa essere riletto in termini di porte logiche e circuiti quantistici; ii) la trasposizione e la progettazione dell'attività didattica all'interno del modulo.

Introduction

We are currently experiencing a ‘second quantum revolution’ (Riedel, Max F. et al, 2017). We are, indeed, living in a world that is getting ready to be populated by quantum computers and quantum networks. The presence of these new technologies, based on the laws of quantum physics, is becoming increasingly important, representing a real resource, from the opportunity to solve new global problems faster and more effectively to the possibility of creating new jobs. But how can we understand the core and potential of these new technologies? How can young people realize the tide of opportunities that surround them?

Currently, at secondary school level, there are many projects to introduce quantum physics, but very few, if any, educational projects on the impact of quantum physics on the society. This is one of the main goals of the Erasmus+ I SEE project (Inclusive STEM Education to Enhance the capacity to aspire and imagine future careers) coordinated by the University of Bologna and started in 2016 (http://www.iseeproject.eu/)

The present thesis is situated within this project and aims to design an innovative approach and teaching module to foster students’ ability both to grasp the essence of quantum physics and quantum technologies, and to imagine their future implications.

The research work carried out within this thesis started from the analysis of an I SEE module developed by the Finnish partners of the project, and consists of a revision and integration of their activities, in order to solve some problems they encountered in their implementations. These problems concern: a) the construction of better connections between quantum technologies and future; b) the explicit search for a global approach that could lead the students to grasp what quantum logical gates, circuits, algorithms, simulators and computers are, without getting trapped in technical details.

In particular, the main focus of this thesis concerns the choice of teleportation as an emblematic case of quantum protocol and the issue of designing a teaching activity aimed to highlight how a quantum experiment can be re-read in terms of logical gates and quantum circuits.

The I SEE module we designed has been implemented within of the PLS (Piano Lauree Scientifiche) laboratory, organized by the department of Physics and Astronomy of University of Bologna in February-March 2019 (still ongoing). The module implementation will last about 20 hours (six weeks with 3-hours weekly sessions) and is involving 27 secondary students (16-17 years old) from different schools.

The thesis is articulated in three chapters and the conclusions.

The first chapter includes an analysis of the literature, so as to outline the state of art about the social relevance of quantum applications, the teaching of quantum physics at secondary school level, the available materials on quantum computers that can be used for their educational transposition. Then, the main concepts of quantum physics in quantum computation are described and finally we illustrate the approach we chose to revise the Finnish module and design the new activities.

The second chapter is specifically focused on the I SEE project. After a general description of its goals and structure, the Finnish module of quantum computers is described in detail. The last part of this chapter concerns the description of the Italian module and how the I SEE Italian group addressed the problems pointed out by the Finnish partners.

The third chapter represents the original core of this thesis and includes my main contribution to the research work. In particular the case of teleportation is described both for its physical contents and features, and for its epistemological and educational value. After a careful analysis of the teleportation protocol, we report how we realized its educational transposition and the teaching activities that we designed.

In the conclusion we discuss the main results and the main reactions that the students showed when exposed to the teleportation activities.

Chapter 1

The state of art on quantum computers in STEM education

In this chapter we argue why quantum computers are a crucial topic in STEM education. We then stress the educational problem we had to address in order to fill the gap that exists between the hyper-specialized treatment of this topic in university and research texts and the qualitative description that can be found in popular books. The main core of the chapter is, however, the presentation of the minimal concepts that are needed to understand quantum computation.

1.1 Quantum in STEM education

The term "STEM education" refers to teaching and learning in disciplines related to science, technology, engineering and mathematics (the acronym STEM refers to Science, Technology, Engineering and Mathematics). Over the years, several STEM cross-cutting courses and activities have been designed for students of all levels of education, from infancy to university, and in all educational environments (from formal to non-formal and open schooling contexts) (Gonzalez & Kuenzi, 2012).

STEM education was initially born under the pressure of economic and market needs. In fact, since 2000, in the United States, the pressure of innovation and development has been stressing the need to have a unified perspective on disciplines, inasmuch as the "new economy" requires more and more advanced information and telecommunication technologies.

The idea of STEM education progressively assumed the role of driver of change in basic scientific education so as to respond to the identified criticalities in current curricular and formal teaching: the results of the PISA (International Student Evaluation Program) and TIMSS (Trend in International Mathematics and Scientific Studies) tests, designed to monitor the level of preparation of students in science and mathematics, revealed important disciplinary gaps and the inability of a large number of students to obtain the expected results. In response to the so-called "PISA shock", a progressive orientation of educational policies and curricula towards STEM disciplines

has been boosted, in order to encourage and trigger substantial changes in the educational approach within the whole school system.
Currently STEM education is frequently mentioned as a fruitful perspective to fill the so-called "skill gap" between the concepts learned in formal education (schools and universities) and the skills required by the labour market and societal stakeholders. Indeed, the complexity of the current social, environmental, political and economical problems requires a multi-perspective and multidisciplinary approach.

Within this framework, the group of Bologna and its partners within the I SEE Erasmus+ project (section §2.1) has been developing an approach in science education aimed to value STEM education as a way to prepare the young people to deal with global unsustainability, uncertainty of the future, social liquidity. The I SEE project is built on the belief that STEM education can support young people in projecting themselves into the future as agents and active persons, citizens and professionals, and open their minds to future possibilities (Branchetti et al, 2018). In this direction, the project developed teaching modules on topics like climate change and artificial intelligence, that have been chosen because of their relevance for the development of both STEM and "future-scaffolding" skills.

The third module developed within the I SEE project concerns quantum computing. This is not only a perfect example of a STEM topic, where science, technology, engineering and mathematics find their place, but is also future-relevant in many different sectors, a global challenge in which also Europe is trying to play a role. In particular, on invitation of the Commissioner for Digital Economy and Society and the Minister of Economic Affairs in The Netherlands, a European team wrote a "Quantum Manifesto" to formulate a common strategy for Europe to stay at the front of the second Quantum Revolution (https://ec.europa.eu/futurium/en/content/quantum-manifesto-quantum-technologies) . The Manifesto has been officially released on 17-18 May 2016 at the Quantum Europe Conference in Amsterdam and Delft. On the basis of the Quantum Manifesto (de Touzalin, Marcus, Heijman, Cirac, Murray & Calarco, 2016), the European Commission launched a €1 billion Flagship-scale Initiative in Quantum Technology (European Cloud Initiative, 2016). As asserted in the "The European quantum technologies flagship programme" (Riedel, Max F., et al, 2017) the current quantum revolution follows the revolution that led the fundamental laws of the microscopic world to be discovered and the quantum theory formulated in the beginning of the XX Century. In the years following the first revolution, different technologies were designed (lasers and transistors), which can be understood and developed only with the help of quantum mechanics (for example the band structure of a semiconductor or the nature of a coherent state). However such technologies are based on mass effects, where many quantum degrees of freedom are manipulated at the same time. The second quantum revolution concerns instead technologies that can directly act on an individual quantum state and make use of quantum properties, such as superposition principle and entanglement. This revolution has been triggered by at least two different factors. This first one concerns the increasing number of start-ups that have been founded to offer quantum technologies to very specialized markets (for instance quantum cryptography devices and software are already sold to governments, banks and other customers with the highest security requirements). The second, and more important, factor concerns the large investment in quantum technologies of big global companies, including Google, IBM5, Intel, Microsoft and Toshiba. They are attracting "the best talents that only a couple of years ago had only the choice between the pursuit of an academic career and the abandonment of the field" (Riedel, Max F., et al, 2017). Governments are also taking a cue from the trend and launching large funding programs in the field (UK: http://uknqt.epsrc.ac.uk/; Netherlands: www.qutech.nl; Germany: www.qutega.de). In addition to quantum computing, quantum communication is particularly at the top of the agenda of many countries, especially in China, that is planning to invest heavily, on a larger scale than the European fleet, and has recently launched a satellite with quantum communication devices (Gibney, 2016).

The strong urgency for Europe to keep up with quantum technologies global developments is felt by many experts and decision makers. This urge was expressed also in the Quantum Manifesto, endorsed by over 3500 stakeholders from abroad community of industries, research institutes and scientists in Europe (de Touzalin, et al., 2016).

In light of these claims, quantum computing represents a real frontier topic, whose conceptual/technological breakthrough can guide students to explore their personal future(s) and future societies.

1.2 Quantum computer in literature

Quantum computers are slowly entering more and more into daily-lives and society is starting to feel that the change will be radical and will invest many fields, from politics to society, from the economics to scientific research.
"The invasion" of these new technologies represents a real possibility (new developing sectors, new careers), but to enable citizens to perceive these opportunities it is necessary to start to think about how science education can contribute to develop the skills needed to grasp the conceptual basis, the potential and/or the social implications of these new technologies.

Currently there are very few, if any, educational projects and materials that aim to introduce demanding quantum applications like quantum computers in secondary schools.

At the university level quantum computing is mainly addressed in master physics courses, where it is possible to use highly sophisticated and advanced mathematical and conceptual tools. In these courses, concepts/topics like qubits, quantum computation and simulation, algorithmic complexity, the Deustch - Josza algorithm, the entropy of Shannon and von Neumann etc, can be addressed formally and after a deep introduction of quantum physics.

The literature on quantum computing appears, then, very polarized: on one hand we have popular books where quantum computers are qualitatively described, on the other we have highly specialized texts where the discussion on quantum computers grounds on very advanced physics knowledge and formalisms. Moreover, almost all the published research papers are highly specialized and accessible only to experts, and it is difficult to find broad reviews that frame the specific studies within a global picture. The hyper-specificity sometimes affects even the communication between researchers of different areas of the same discipline; it is therefore a problem that is not only interdisciplinary, but also intra-disciplinary and, because of this hyper-specialization, the research papers are often very short and full of implicit concepts.

In front of this literature, the goal to design teaching materials on quantum computers or secondary school students implied us to address two types of barriers:

a) *structural barriers*, that concern the necessity to provide students with sophisticated conceptual tools based on quantum physics;
b) *contextual barriers*, that derive from the hyper-specificity of the available materials on quantum computers, which did not make easy to find a global view on the educational potential of this topic.

In light of this analysis, the two main research questions we had to address in order to design a module of quantum computer for secondary school students are:

a) what approach to quantum physics can we choose, by taking into account the fact that the target of the module are students attending the fourth year of secondary school (11^{th} grade, 16-17 years old) who had not previously studied quantum physics?

b) what global view can we point out in order to analyse the current materials on quantum computers and flesh out not only its conceptual essence but also its epistemological, educational and social value?

1.3 Quantum mechanics for Quantum computation

The intent of this section is to show the pivotal points on which we based our reconstruction of quantum physics and to highlight the perspective on which the teaching activities have been developed. The following sections present an overview of the main concepts of quantum physics, that we used to design the module: the qubit, the superposition principle, the measurement, the entanglement. The qubit is the simplest quantum system and implements all the principles and postulates of quantum mechanics. This seems something obvious, but at the same time highlights that, to understand the essence of qubits and quantum computers, it is necessary to know "all" quantum mechanics, or at least to have an advanced knowledge of it. In the next section we present qubit as a physical object, and then, in section 1.3.2, qubit is discussed as a mathematical object, together with the concepts of superposition principle and quantum measurement. Section 1.3.3 concerns the concept of entanglement and Bell states, of particular interest for the development of the activities object of this dissertation (see Chapter 3). In section 1.3.4 the issues of reversibility and computational complexity are addressed.

1.3.1 Qubit as the simplest quantum physical state

From a physical point of view, a qubit describes an arbitrary two-state physical variable, as for example the two different polarizations of a photon, the alignment of a nuclear spin in a uniform magnetic field, or two orbital degrees of an electron.

The choice we made is to present the qubit through the Stern and Gerlach experiment for the discovery of spin (Gerlach & Stern, 1922).

In the original Stern-Gerlach experiment the silver atoms are produced and expelled from an oven and pass through a magnetic field of appropriate shape, intensity and oriented transversely to the trajectory of the particle (figure 2.4). At the output of the magnet, the position of each atom is recorded. Classically we would have expected a continuous spatial distribution of atoms coming out from the Stern-Gerlach magnets. Instead, what emerges is that atoms arrive only in two separated spots, which proves that the magnetic dipole moment of the atoms is quantized, that is, it has only discrete values, multiples of a certain fundamental quantity.

In the following reasoning, we will follow the text of Nielsen & Chuang (2002) from which we took the structure of the argument and key sentences.

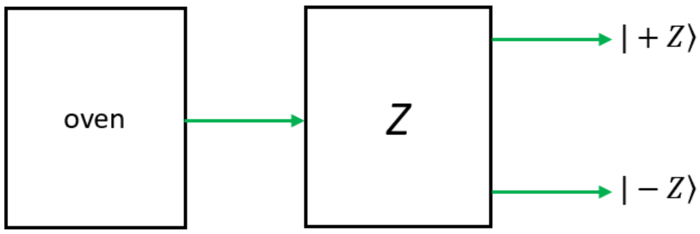

figure 1.1: Abstract schematic of the Stern–Gerlach experiment. Hot hydrogen atoms are beamed from an oven through a magnetic field, causing a deflection either up $|+Z\rangle$ or down $|-Z\rangle$.

Suppose now to connect two Stern-Gerlach devices in cascade, as shown in *figure 1.2*. Proceed with blocking exit $|-Z\rangle$ from the first Stern-Gerlach apparatus, while the exit

$|+Z\rangle$ is sent through a second device oriented along the $\widehat{+x}$ axis. A detector is positioned at the final output to measure the distribution of atoms along the $\widehat{+x}$ axis.

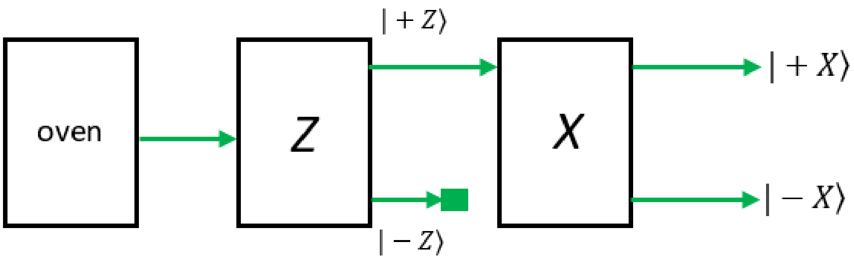

figure 1.2

What we observe experimentally is that there are two peaks of equal intensity. This result suggests that the atoms can have definite magnetic moments along each axis, independently and that each atom passing through the second apparatus can be described as being in a state we might write as $|+Z\rangle$ $|+X\rangle$ or $|+Z\rangle|-X\rangle$ to indicate the two values for spin that might be observed.

Another experiment, shown in *figure 1.3*, can test this hypothesis by sending one beam of the previous output through a second $\hat{z}$ oriented Stern–Gerlach apparatus. If the atoms had retained their $|+Z\rangle$ orientation, then the output would be expected to have only one peak, at the $|+Z\rangle$ output. However, again two beams are observed at the final output, of equal intensity. Thus, the conclusion would seem to be that, contrary to classical expectations, a $|+Z\rangle$ state consists of equal portions of $|+X\rangle$ and $|-X\rangle$ states, and a $|+X\rangle$ state consists of equal portions of $|+Z\rangle$ and $|-Z\rangle$ states. Similar conclusions can be reached if the Stern–Gerlach apparatus is aligned along some other axis, like the $\hat{y}$ axis.

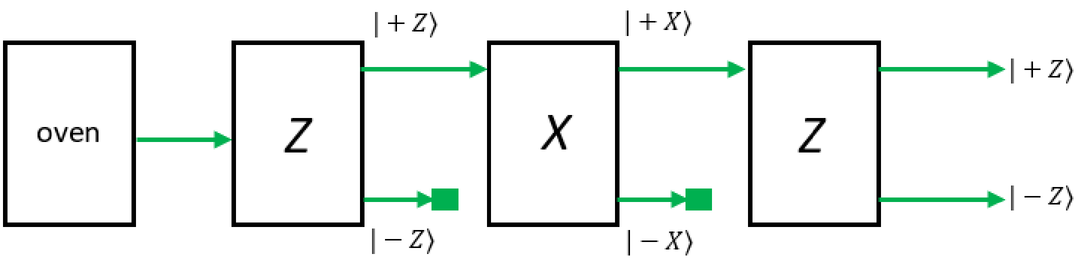

figure 1.3

The qubit model provides a simple explanation of this experimental behaviour. Taking a small step forward (section §1.1.2), let $|0\rangle$ and $|1\rangle$ be the states of a qubit, and make the assignments

$$|+Z\rangle \quad \leftarrow \quad |0\rangle$$

$$|-Z\rangle \quad \leftarrow \quad |1\rangle$$

$$|+X\rangle \quad \leftarrow \quad \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|-X\rangle \quad \leftarrow \quad \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Then the results of the Stern–Gerlach experiments can be explained by assuming that the z Stern–Gerlach apparatus measures the spin in the computational basis $|0\rangle$, $|1\rangle$, and the x Stern–Gerlach apparatus measures the spin in the computational basis $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$, $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ (Nielsen & Chuang, 2002).

1.3.2 Qubit as the simplest mathematical model of a quantum state

The qubit can be seen as a simplest mathematical object, characterized by certain specific properties. Treating qubits as abstract entities gives the freedom to construct a general theory of quantum computation and quantum information which does not depend upon a specific system for its realization. From an informational point of view, whilst classical bits can have only 0 or 1 state, a qubit can assume the states $|0\rangle, |1\rangle$ - represented in the usual braket notation - or a state represented by a linear combination of them:

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad (1.1)$$

where α and β are complex numbers and $|0\rangle$ and $|1\rangle$ are known as computational basis states, an orthonormal basis for the vector space. The possibility to build superposition states as $|\varphi\rangle$ in eq. 1.1 comes, mathematically, from the linearity of Hilbert spaces, where quantum states are defined.

It is possible to examine a bit to determine whether it is in the state 0 or 1. Rather remarkably, it is not possible to measure a qubit to determine its quantum state and, hence, the values of α and β. When we measure a qubit, we get either the result 0, with probability $|\alpha|^2$, or the result 1, with probability $|\beta|^2$. Naturally, $|\alpha|^2 + |\beta|^2 = 1$, since the probabilities must sum to one. Geometrically, we can interpret this as the condition on the qubit state to be normalized to 1. Thus, in general, a qubit state is a unitary vector in a two-dimensional complex vector space.

"The possibility of a qubit to be in a superposition state is of course counter-intuitive. A classical bit is like a coin: either heads or tails up. For imperfect coins, there may be intermediate states like having it balanced on an edge, but those can be disregarded in the ideal case. By contrast, a qubit can exist in a continuum of states between $|0\rangle$ and $|1\rangle$ – until it is observed" (Nielsen & Chuang, 2002).

A useful picture to think about qubits is the following geometric representation.

Because $|\alpha|^2 + |\beta|^2 = 1$, we may rewrite Equation (1.1) as:

$$|\psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right) \qquad (1.2)$$

where θ, φ and γ are real numbers. It is possible to ignore the factor of $e^{i\gamma}$, because it has no observable effects, and for that reason we can effectively write

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \qquad (1.3)$$

The numbers θ and φ define a point on the unit three-dimensional sphere, as shown in *figure 1.4*.

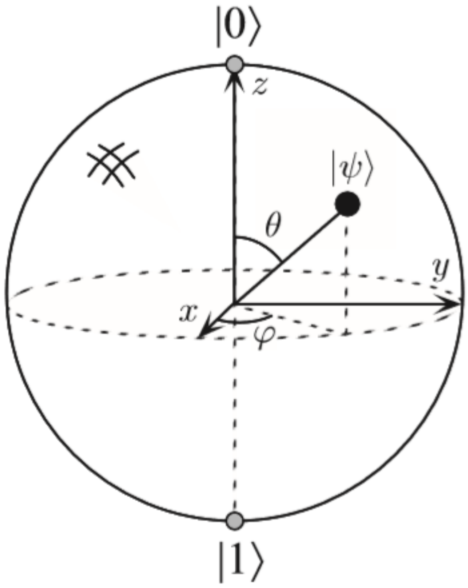

figure 1.4: Bloch sphere representation of a qubit (Nielsen & Chuang, 2002).

This sphere is often called the Bloch sphere; it provides a useful means to visualize the state of a single qubit. Many of the operations on single qubits are neatly described within the Bloch sphere picture. However, this representation is limited, since there is no simple generalization of the Bloch sphere known for multiple qubits.

Behind these introductory lines lies the first postulate of quantum mechanics:

> **Postulate 1**: Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the state space of the system. The system is completely described by its state vector, which is a unit vector in the system's state space. (Nielsen & Chuang, 2002)

It is possible to think of quantum mechanics in terms of computation: "similarly to the way a classical computer is built from an electrical circuit containing wires and logic gates, a quantum computer is built from a quantum circuit containing wires (even if in this case they do not necessarily represent physical cables for transmitting the information) and elementary quantum gates to manipulate the quantum information." (Nielsen & Chuang, 2002). In fact, the quantum logic gates take the state of a qubit and process into another state of the same Hilbert space. The principal single qubit logic gates are: X, Y, Z and Hadamard gates.

The matrix corresponding to the quantum NOT is called for historical reasons X and is defined by X Pauli matrix:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

In fact, it can be verified that the application of X to a qubit $\alpha|0\rangle + \beta|1\rangle$ (written in vector notation) is

$$X \left[\frac{\alpha}{\beta} \right] = \left[\frac{\beta}{\alpha} \right]$$

It corresponds to a rotation of the Bloch sphere around the $\hat{x}$ axis by π (*figure 1.5*). It maps $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$. Due to this nature, it is sometimes called bit-flip.

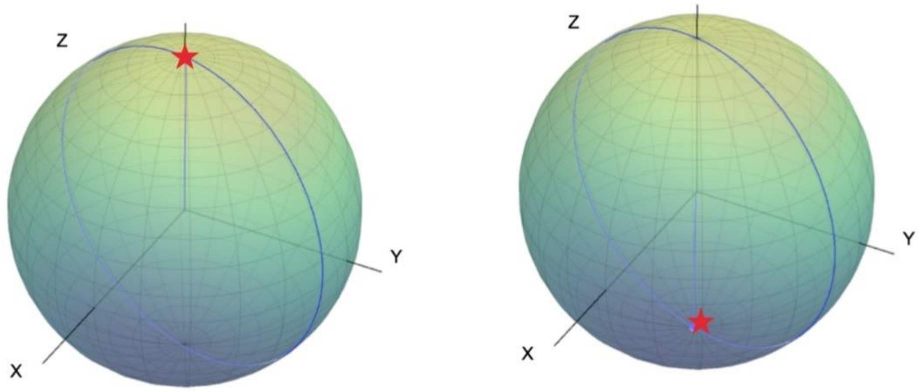

figure 1.5: Visualization of the X gate on the Bloch sphere[1]

The logic gate Z is described by the Z Pauli matrix

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

which acts only on the component $|1\rangle$ and exchanges its sign (*figure 1.6*).

[1] The pictures of the operations on the Bloch sphere are taken from: https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/blochsphere/blochsphere.html

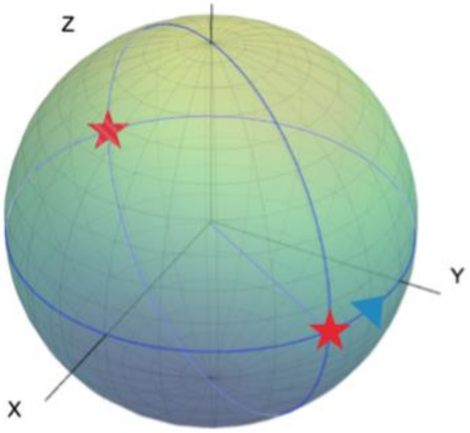

figure 1.6: Visualization of the Z gate on the Bloch sphere

The logic gate Y is described by the Y Pauli matrix

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

It corresponds to a rotation around the $\hat{y}$ axis of the Bloch sphere by π. It maps $|0\rangle$ to $i|1\rangle$ and $|1\rangle$ to $-i|0\rangle$ (*figure 1.7*).

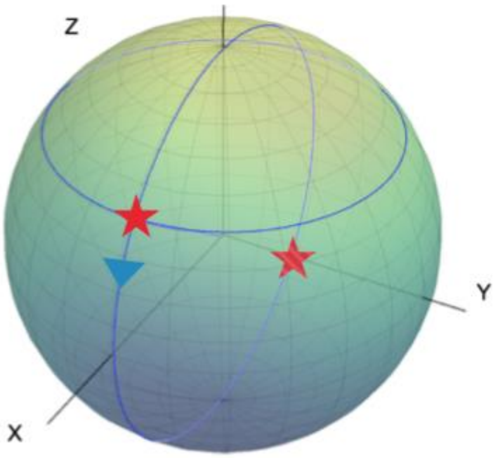

figure 1.7: Visualization of the Y gate on the Bloch sphere

The Hadamard gate acts on a single qubit. It maps the basis state $|0\rangle$ to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ which means that a measurement will have equal probabilities to become 1 or 0 (*figure 1.8*). It represents a rotation of π around the axis $\frac{\hat{x}+\hat{z}}{\sqrt{2}}$. Equivalently, it is the combination of two rotations, π about the $\hat{z}$ axis followed by $\frac{\pi}{2}$ about the $\hat{y}$ axis. It is represented by the Hadamard matrix:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

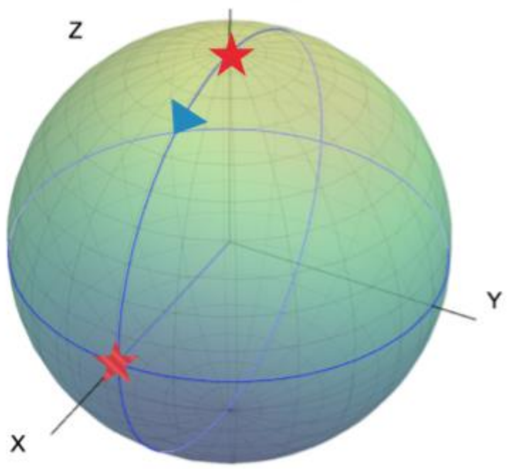

figure 1.8: Visualization of the Hadamard gate on the Bloch sphere

Next figure shows the circuit representation of X, Z and H gates.

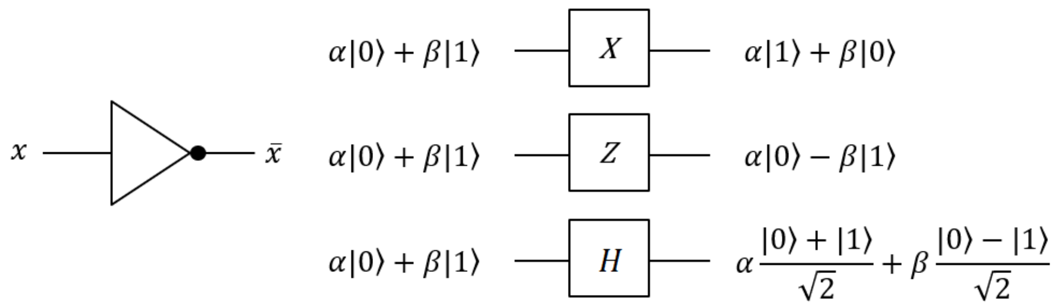

figure 1.9: Single bit (left) and qubit (right) logic gates.

Behind the concept of a logical gate lies the second postulate of quantum mechanics (Nielsen & Chuang, 2002):

> **Postulate 2**: The evolution of a closed quantum system is described by a unitary transformation. That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2,

$$|\psi'\rangle = U|\psi\rangle \quad (1.6)$$

Postulate 2 requires that the system being described it is not interacting in any way with other systems. Of course, all systems (except the Universe as a whole) interact with other systems, to a degree. Nevertheless, there are systems which can be described to a good approximation as closed system, and which are described by unitary evolution to some good approximation. Furthermore, at least in principle, every open system can be described as part of a larger closed system (the Universe) which is undergoing unitary evolution.

Postulate 2 describes how the quantum states of a closed quantum system at two different times are related. A more refined version of this postulate can be given to describe the evolution of a quantum system in continuous time (Nielsen & Chuang, 2002):

> **Postulate 2'**: The time evolution of the state of a closed quantum system is described by the Schrödinger equation,
>
> $$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle \quad (1.7)$$
>
> In this equation, $\hbar$ is a physical constant known as Planck's constant whose value must be experimentally determined. In practice, it is common to absorb the factor $\hbar$ into H, effectively setting $\hbar = 1$. H is a fixed Hermitian operator known as the Hamiltonian of the closed system.

The connection between the Hamiltonian picture of dynamics, Postulate 2', and the unitary operator picture, Postulate 2, is in the solution to Schrödinger's equation, which is easily verified to be:

$$|\psi(t_2)\rangle = e^{-\frac{iH(t_2-t_1)}{\hbar}}|\psi(t_1)\rangle = U(t_1,t_2)|\psi(t_1)\rangle \quad (1.8)$$

where

$$U(t_1,t_2) \equiv e^{-\frac{iH(t_2-t_1)}{\hbar}} \quad (1.9)$$

Any unitary operator U can be realized in the form $U = e^{iK}$ for some Hermitian operator K. There is therefore a one-to-one correspondence between the discrete-time description of dynamics using unitary operators, and the continuous time description using Hamiltonians.

In quantum computation and quantum information it is possible often to speak of applying a unitary operator to a particular quantum system. For example, in the context of quantum circuits we may speak of applying the unitary gate X to a single qubit.

"*Doesn't this contradict what we said earlier, about unitary operators describing the evolution of a closed quantum system? After all, if we are 'applying' a unitary operator, then that implies that there is an external 'we' who is interacting with the quantum system, and the system is not closed. Generally, for many systems like this it turns out to be possible to write down a time-varying Hamiltonian for a quantum system, in which the Hamiltonian for the system is not a constant, but varies according to some parameters which are under an experimentalist's control, and which may be changed during the course of an experiment. The system is not, therefore, closed, but it does evolve according to Schrödinger's equation with a time-varying Hamiltonian, to some good approximation*" (Nielsen & Chuang, 2002).

Let us now return to the logical gates. A circuit is a sequence of logic gates, that transform the quantum state. But one of the most important operations, different from the others, is the measurement one, whose circuit representation is

figure 1.10: quantum circuit symbol for measurement

As previously described, this operation converts a single qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ into a probabilistic classical bit M (distinguished from a qubit by drawing it as a double-line wire), which is 0 with probability $|\alpha|^2$, or 1 with probability $|\beta|^2$.

We saw that closed quantum systems evolve according to unitary evolution. The third Postulate provides a means for describing the effects of measurements on quantum systems.

Postulate 3 (Nielsen & Chuang, 2002): Quantum measurements are described by a collection $\{M_m\}$ of measurement operators. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then the probability that result m occurs is given by

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle \quad (1.10)$$

and the state of the system after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}} \quad (1.11)$$

The measurement operators satisfy the completeness equation,

$$\sum_m M_m^\dagger M_m = I \quad (1.12)$$

The completeness equation expresses the fact that probabilities sum to one:

$$1 = \sum_m p(m) = \sum_m \langle\psi|M_m^\dagger M_m|\psi\rangle \quad (1.13)$$

This equation being satisfied for all $|\psi\rangle$ is equivalent to the completeness equation. Consider the example of the measurement of a qubit in the computational basis. This is a measurement on a single qubit with two outcomes defined by the two measurement operators $M_0 = |0\rangle\langle 0|$, $M_1 = |1\rangle\langle 1|$. Each measurement operator is Hermitian, and $M_0^2 = M_0$, $M_1^2 = M_1$. Thus the completeness relation is obeyed, $I = M_0^\dagger M_0 + M_1^\dagger M_1 = M_0 + M_1$. Suppose the state being measured is $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Then the probability of obtaining measurement outcome 0 is

$$p(0) = \langle\psi|M_m^\dagger M_m|\psi\rangle = \langle\psi|M_0|\psi\rangle = |\alpha|^2 \quad (1.14)$$

Similarly, the probability of obtaining the measurement outcome 1 is $p(1) = |\beta|^2$. The state after measurement in the two cases is therefore

$$\frac{M_0|\psi\rangle}{|a|} = \frac{a}{|a|}|0\rangle \quad , \quad \frac{M_1|\psi\rangle}{|b|} = \frac{b}{|b|}|1\rangle \quad (1.15)$$

Measuring devices are quantum mechanical systems, so the quantum system being measured and the measuring device together are part of a larger, isolated, quantum mechanical system.

So far we considered only a single qubit system. Suppose now to have a system of two qubits. If these were two classical bits, then there would be four possible states, 00, 01, 10, and 11. Correspondingly, a two qubit system has four computational basis states denoted $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. A pair of qubits can exist in a superposition of these four states, so the quantum state of two qubits involves a complex coefficient – amplitude – with each computational basis state, so that the state vector describing the two qubits is

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \quad (1.4)$$

Similar to the case for a single qubit, the measurement result x (00, 01, 10 or 11) occurs with probability $|\alpha_x|^2$, with the state of the qubits after the measurement being $|x\rangle$. Again, the condition for the probabilities to sum to one is also expressed by the normalization condition $\sum_{x\in\{0,1\}^2} |\alpha_x|^2 = 1$, where the notation '$\{0,1\}^2$' means "the set of strings of length two with each letter being either zero or one" (Nielsen & Chuang, 2002). For a two qubit system, we can measure just a subset of the qubits: measuring the first qubit alone, for example, gives 0 with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$, leaving the post-measurement state

$$|\psi'\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}} \quad (1.5)$$

where the post-measurement state is re-normalized by the factor $|\alpha_{00}|^2 + |\alpha_{01}|^2$ so that it still satisfies the normalization condition.

A prototypical two-qubit quantum logic gate is the controlled-not gate or CNOT gate. This gate has two input qubits, known as the control qubit and the target qubit, respectively. The circuit representation for the is shown in *figure 1.11*; the top line represents the control qubit, while the bottom line represents the target qubit.

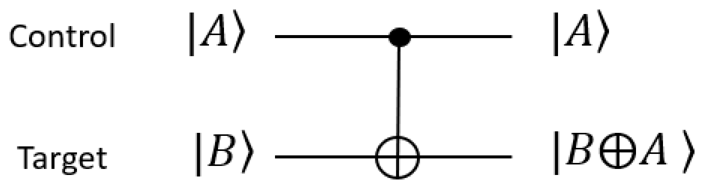

figure 1.11: CNOT gate

The action of the gate may be described as follows. If the control qubit is set to 0, then the target qubit is left alone. If the control qubit is set to 1, then the target qubit is flipped. In equations:

$$|00\rangle \rightarrow |00\rangle;\ |01\rangle \rightarrow |01\rangle;\ |10\rangle \rightarrow |11\rangle;\ |11\rangle \rightarrow |10\rangle$$

Yet another way of describing the action of the is to give a matrix representation U_{CN}:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

It can be easily verified that the first column of U_{CN} describes the transformation that occurs to $|00\rangle$, and similarly for the other computational basis states, $|01\rangle$, $|10\rangle$, and $|11\rangle$. As for the single qubit case, the requirement that probability be conserved is expressed in the fact that U_{CN} is a unitary matrix, that is, ${U_{CN}}^{\dagger}U_{CN} = I$.

We have therefore seen how the very concept of qubit and state transformation, which represent the simplest quantum system and its evolution, still require a profound knowledge about some quantum physics concepts, touching the theory at its very core.

1.3.3 Entanglement

The last part of this section in dedicated to an all quantum feature: the entanglement.
Einstein Podolsky and Rosen in 1935 published an article called "Can quantum-mechanical description of physical reality be considered complete?" in which they proposed a mental experiment to show that, in order to preserve the principle of locality, quantum mechanics should have been necessarily incomplete (Einstein,

Albert, Boris Podolsky, and Nathan Rosen, 1935). The essence of the argument is that if measurements on two widely separated particles cannot influence each other, then the quantum mechanics of an ingeniously prepared two particle system can lead to conclude that the physical properties of each particle are really there, they are elements of reality, in the authors' words.

We choose here to follow the treatment of the book "Quantum Mechanics: A Paradigms Approach" (David H. McIntyre & Corinne A. Manogue, Janet Tate) from which we took the structure of the argument, the picture and key sentences.

The experimental situation is depicted in *figure 1.12*

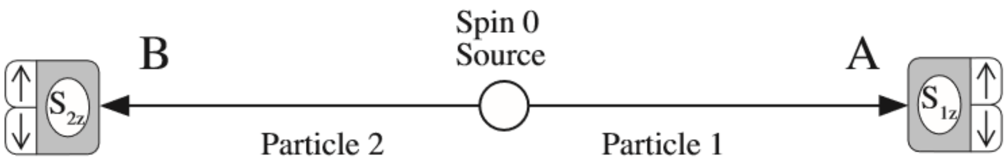

figure 1.12: Einstein-Podolsky-Rosen gedanken experiment, version of the EPR experiment is due to David Bohm and has been updated by N. David Mermin (David H. McIntyre & Corinne A. Manogue, Janet Tate)

An unstable particle with spin 0 decays into two spin $-1/2$ particles which, by conservation of angular momentum, must have opposite spin components and, by conservation of linear momentum, must travel in opposite directions. For example, a neutral pi meson decays into an electron and a positron: $\pi^0 \rightarrow e^- + e^+$. Observers A and B are on opposite sides of the decaying particle and each has a Stern-Gerlach apparatus to measure the spin component of the particle headed in its direction. Whenever one observer measures spin up along a given direction, then the other observer measures spin down along that same direction. The quantum state of this two-particle system is

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2) \quad (1.16)$$

An observer A measures the spin component of particle 1 and observer B measures the spin component of particle 2. The probability that observer A measures particle 1 to be spin up is 50% and the probability for spin down is 50%. The 50-50 split is the same for observer B. For a large ensemble of decays, each observer records a random sequence of spin up and spin down results, with a 50/50 ratio. However, because of the correlation between the spin components of the two particles, if observer A measures spin up (i.e., $S_{1z} = {}^{+\hbar}/_{2}$), then we can predict with 100% certainty that the result of observer B's measurement will be spin down ($S_{2z} = {}^{-\hbar}/_{2}$). The result is that even though each observer records a random sequence of ups and downs, the two sets of results are perfectly anti-correlated. The state $|\psi\rangle$ in equation (1.24), that produces this strange mixture of random and correlated measurement results, is known as an *entangled state*. The spins of the two particles are entangled with each other and produce this perfect correlation between the measurements of observer A and observer B.

Imagine that the two observers are separated by a large distance, with observer B slightly farther from the decay source than observer A. Once observer A has made the measurement $S_{1z} = {}^{+\hbar}/_{2}$, we know that the measurement by observer B in the next instant will be spin down ($S_{2z} = {}^{-\hbar}/_{2}$). It is possible to conclude that the state $|\psi\rangle$ in equation (1.24) instantaneously collapses onto the state $|+\rangle_1|-\rangle_2$, and the measurement by observer A has somehow produced the measurement by observer B. Einstein referred to this as "spooky action at a distance". The result that observer B records is still random, but its randomness is perfectly anti-correlated with the random result of A. So, there is no problem with faster communication of light here, because there is no information transmitted between the two observers. The EPR argument claims that because we can predict a measurement result with 100% certainty (e.g., $S_{2z} = {}^{-\hbar}/_{2}$), then that result must be a "real" property of the particle—it must be an element of reality. Since the particles are widely separated, this element of reality must be independent of what observer A does, and therefore must always exist. The independence of the elements of the reality of the two particles is called the Einstein locality principle and is a fundamental assumption of the EPR argument.

The correlation of spin measurements of the two observers is independent of the choice of direction of the measurement, assuming the same direction for both observers. That is, if observer A measures the x-component of spin and records $S_{1z} = {}^{+\hbar}/_{2}$, then we know with absolutely certainty that observer B will measure $S_{2z} = {-\hbar}/{2}$. However, quantum mechanics maintains that we can know only one spin component at a time for a single particle. EPR concludes that quantum mechanics is an incomplete description of physical reality because it does not describe all the elements of the particle reality (David H. McIntyre & Corinne A. Manogue, Janet Tate).

From a computational point of view two entangled qubits are described by the Bell states. The projection of two states in a Bell state is obtained by sequencing two logic gates: a Hadamard and a CNOT gates (*figure 1.13*).

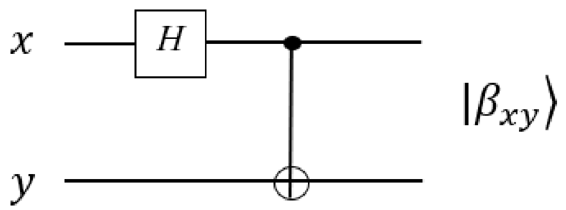

figure 1.13: Quantum circuit to create Bell states

This circuit works in this way: first, the Hadamard transform the top qubit in a superposition; this then acts as a control input to the CNOT, and the target gets inverted only when the control is 1. The output states are

$$\beta_{00} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$\beta_{01} = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
$$\beta_{10} = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$
$$\beta_{11} = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

known also as EPR states or EPR pairs.

These states are responsible for many surprises in quantum computation and quantum information. It is the key ingredient in quantum teleportation and super dense coding, and the prototype for many other interesting quantum states.

1.3.4 A little bit more: Reversibility and Complexity

The purpose of the last section was to highlight how the concepts of qubits and logical gates represent an implementation of all the main principles of quantum mechanics. However, quantum computing is not limited to qubits, operations on them and to the concept of entanglement. The quantum computer conceals a much wider world.
For example, let us consider classical and quantum logic gates and the concept of reversibility/irreversibility. A function is said to be reversible if, given its output, it is always possible to determine back its input, which is the case when there is a one-to-one relationship between input and output states. If the space of states is finite, such a function is a permutation. Logical reversibility implies conservation of information.
When several input states are mapped onto the same output state, then the function is irreversible, since it is impossible by only knowing the final state to find back the initial one. In Boolean algebra, NOT is reversible, while Boolean functions like AND, OR, XOR are irreversible, since they map 2 input states into 1 output state (*figure 1.14*).

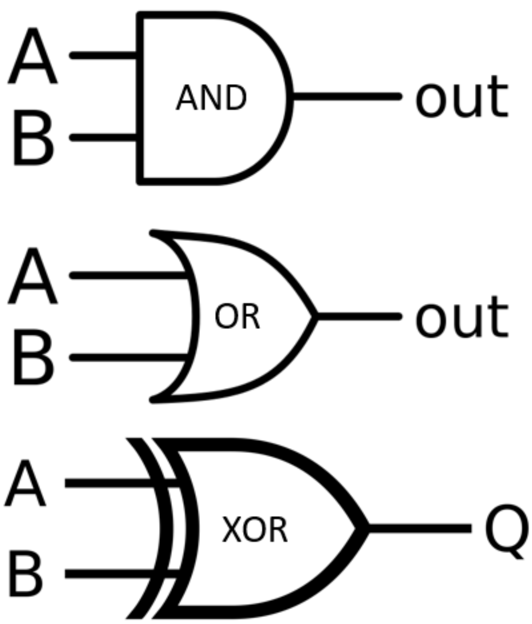

figure 1.14: classical logic gates

Quantum logic gates, instead, are in principle reversible, because in standard quantum mechanics closed systems evolve by unitary transformations, which are objective and invertible. So, if a logic gate is irreversible some of the input information is lost irretrievably when the gate operates; that is, some of the information has been erased by the gate. Conversely, in a reversible computation no information is ever erased, because the input can always be recovered from the output. Thus, saying that a computation is reversible is equivalent to say that no information is erased during the computation. A question then arises: if classical logic gates are irreversible and quantum gates are reversible, how can a classical quantum computer perform like a quantum computer?

Rolf Landauer, a German physicist who studied irreversibility and heat production in computing process (1961), noticed that any irreversible computation may be transformed into a reversible one by embedding it into a larger computation where no information is lost, e.g. by replicating every output in the input (’sources’) and every input in the output (’sinks’). The substantial idea is therefore to add ‘ancilla’ and ‘garbage’ bits prepared in states either 0 or 1, are not directly important to the computation.
So this little stratagem allows to make the computation irreversible (Nielsen & Chuang, 2002).

Another important question is: what kind of problems can a quantum computer solve compared to the classical one? Are there any limitations on which computational problems can be performed? The world of computational complexity opens up here. There are several classes of problem complexity of. A class of complexity can be thought of as a collection of computational problems, each of which shares some common characteristics with respect to the computational resources necessary to solve such problems. The four main classes are the following:

- P is the class of computational problems that can be solved quickly on computer computers.
- NP is the class of problems that have solutions that can be easily controlled on a classic computer.
- PSPACE consists of those problems that can be solved using resources of small spatial dimensions (i.e. the computer is "small"), but not necessarily in time (the

"long" calculations are satisfactory). It is thought to be strictly larger than P and NP, but this has never been demonstrated.

- BPP is the class of problems that can be solved using randomized algorithms in polynomial time, if a limited probability of error (for example 1/4) is allowed in the solution of the problem. It is believed that BPP is, even more than P, the class of problems that should be considered efficiently soluble on a classical computer.

The difference between P and NP classes is fundamental to structural diversity of a classical computer and a quantum one, in terms of their capacity to solve a problem. For example, this difference is behind the problem of integer factorization on which the RSA public-key cryptosystems is based. Indeed there is no quick way to solve this problem on a classic computer, which suggests that the problem is not in P. On the other hand, if someone says that a certain number p is a factor of n, then you can quickly check if it is correct by dividing p by n, so factorization is a problem in NP.

It is clear that P is a subset of NP, since the ability to solve a problem implies the ability to verify potential solutions. What is not clear is whether or not there are problems in NPs that are not in P. Determining whether these two classes are different is perhaps the most unresolved problem in theoretical computer science:

$$P \neq NP.$$

Many experts believe that NP contains problems that are not in P. In particular, there is an important subclass of NP problems, NP-complete problems, which are of particular importance for two reasons. First of all, there are thousands of problems, many very important, that are known to be NP-complete. Secondly, every given NP-complete problem is in a sense “at least as difficult as” all the other problems in NP. More precisely, an algorithm meant to solve a specific NP specific problem can be adapted to solve any other problem in NP, with a small overhead. In particular, if $P = NP$, it will follow that no NP-complete problem can be solved efficiently on a classical computer.

The resolution of NP problems on classical computer requires exponential times; the same problem, solved with an adequate algorithm on quantum computers requires polynomial times, that is it can be solved much more quickly (Nielsen and Chuang, 2002).

A typical example is the factoring problem, which requires to write a whole number N as a product of primes. This kind of problem is an example of NP problem, so it cannot be solved in reasonable time in a classical computer. Shor's algorithm, a quantum algorithm, solves this problem by reducing it to instances of the order-finding problem. This algorithm is considered efficient because it uses resources bounded by a polynomial in the number of digits of N. (Knill et al., 2002).

BQP is the class of all computational problems that can be solved efficiently on a quantum computer, where a limited probability of error is allowed. Where exactly BQP fits with respect to P, NP and PSPACE is still unknown. What we know is that quantum computers can solve all problems efficiently, but there are no problems outside of PSPACE that can solve efficiently. Therefore, BQP is between P and PSPACE. An important implication is that if it is shown that quantum computers are strictly more powerful than classical computers, then it will follow that P is not equal to PSPACE.

So, where are the BQP problems located? What is known is that quantum computers can solve all problems in P efficiently, but that there are no problems outside of PSPACE that can solve efficiently. Therefore, BQP is between P and PSPACE, as shown in *figure 1.15*.

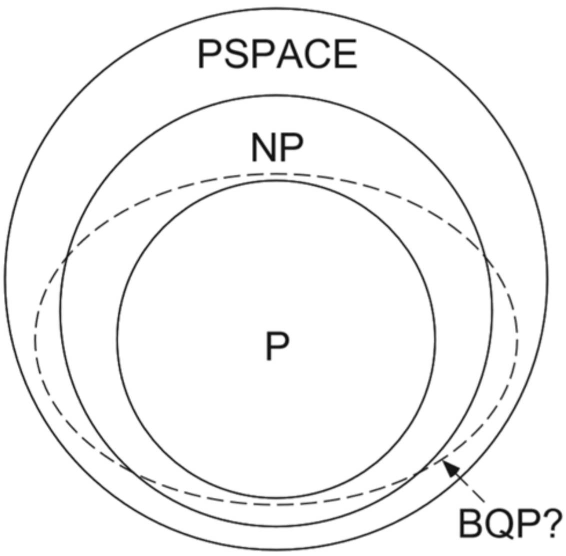

figure 1.15: The relationship between classical and quantum complexity classes. Quantum computers can quickly solve any problem in P, and it is known that they can't

solve problems outside of PSPACE quickly. Where quantum computers fit between P and PSPACE is not known, in part because we don't even know whether PSPACE is bigger than P (Nielsen & Chuang, 2002).

1.4 Approaches for teaching quantum physics at secondary level

As can be seen from the previous sections, the physics behind the "simple" concept of qubit and its transformations touches most of the postulates of quantum physics and quantum physics represents a challenge for high school students. The problem we had to address was to identify the key concepts for understanding the essence and the potential of quantum computers, taking into account our target: secondary school students to whom quantum mechanics has never been introduced.
The problem of how introduce quantum physics into secondary school is not a new problem in the physics education research literature. There have been several proposals, which raised as an alternative to the classical historical approach. These other designed paths have been based on a (i) *logical-philosophical* and/or a (ii) *phenomenological* approach.

The *logical-philosophical* approach (i) arises from the current structure of quantum theory, from its 'axiomatic' structure (Haber-Schaim, 1975; Lawrence, 1996). While mathematical formalism cannot be fully developed at a secondary school level, the main ideas can be understood by focusing on the concept of spin, which has no classical counterpart, and Pauli matrices, since it is "*impossible to understand quantum physics without mastering its mathematical structures*" (Pospiesch, 1999). By introducing the concept of spin from scratch, it is possible to move on to the superposition principle and to other elements of the axiomatic structure of quantum physics, without appealing to semi-classical representations. The application of the formalism to various experimental configurations (such as the Stern-Gerlach experiments) aims to support students to understand the connection between theoretical and experimental dimensions. This approach is effective for introducing the fundamental aspects of the quantum physics, as superposition principle, indeterminacy, complementarity, entanglement, etc. It also underlines the feasibility of teaching this type of formalism in secondary schools.

The *phenomenological* approach (ii), aims to build the theoretical framework on the logical base of what is observed from the experiments themselves, through an analysis of experimental results. One is based on Ghirardi's introduction to quantum physics (Ghirardi, 1997), whose argumentation is based on experiments about the polarization of light carried out with Polaroid filters and bi-refringent crystals. A second one was proposed by the research group in Physics Education at the University of Pavia (Malgieri, 2015). The teaching proposal is based on Feynman's paths method, and it benefits from the support of interactive simulations created with the open-source software GeoGebra.

In this context, Bologna's research group developed two proposals. The first one (Levrini & Fantini, 2013) was designed to create a rich and complex learning environment, where students can navigate between different personal trajectories; it was divided into two parts, each one characterized by a different approach (historical-philosophical in the first, phenomenological and formal in the second). The common thread was the concept of "object" from the "old quantum physics" to its systematization through the interpretation of the experiments of Stern-Gerlach with the notation of Dirac for the states and matrices of Pauli.
A second proposal was developed by a group of researchers from the Department of Physics and Astronomy, in collaboration with the CNR-IMM of Bologna, to be implemented in a laboratory for secondary volunteer students. The laboratory was part of the activities of the Plan of Scientific Degrees (PLS) and aimed to provide students with the opportunity to understand the essential elements of the quantum perspective, starting from "The most beautiful experiment of physics" (MBE), i.e. the experiment of the double slit with single electrons, initially made in Bologna in 1974 (Lulli, 2013), (Levrini, Lulli, Bertozzi, Ercolessi, Matteucci, Monzoni & Pecori, 2014, Stefanini, 2013; Lodovico, 2016; Ravaioli, 2016). The main feature of the path was its multidimensionality, being the epistemological, formal, logical, experimental and applicative aspects of quantum physics discussed and critically analyzed.

These approaches generally brought to a real and remarkable enhancement in students' comprehension of quantum foundations, in the sense that students appear to make strong progresses in solving problems and exercises concerning genuine and deep quantum concepts (Michelini et al., 2010).

For this teaching module on quantum computing, we chose to use a *spin-first* approach to the introduction of quantum physics (section §3.2), using the Stern-Gerlach experiments to derive a quantum description of the world and to introduce to the new logic.

1.5 The barrier of hyper specialization

As mentioned in section 1.2, we identified some *contextual problems* to the didactical transposition of quantum computing for high schools, that derive from the hyper-specificity of the topic. The published papers are highly specialized and they are often accessible only to experts.
The contemporary world is characterized by a wide, deep and serious gap between the personal knowledge and the problems that the world requires to face. The greatest challenge is therefore to understand how to deal with increasingly multidisciplinary, multidimensional, global problems, starting from the fragmented picture of knowledge that traditional disciplines carry along. This hyper-specialization prevents us from seeing the global picture as well as the essential elements. Nowadays, crucial issues, like quantum protocols, are posed and addressed in their specific context, but their social, educational and epistemological value appears as soon as they are posed into a global context. At the same time, the separation of disciplines makes it impossible to grasp the intrinsic fabric of "interweaving" issues, that is, etymologically, complex. The challenge of globality is therefore at the same time a challenge of complexity. In fact, there is complexity when the different components that make up a whole (such as the economic, the political, the sociological, the psychological, the emotional, the mythological) are inseparable and when there are non-linear interactions between the parts and the whole and between the whole and the parts.

The characteristic developments of our Century push us to face, more and more often and ineluctably, the challenges of complexity. “The disciplinary developments of the sciences have brought about the advantages of the division of labor, but at the same time have caused the disadvantages of super-specialization, compartmentalization and division of knowledge” (Morin E., 2000). Thus, the challenge we addressed in this research work concerns the identification, among the great amount of material, of a global approach that could help us to look and reconstruct the details without, at the same time, losing a general vision. The global approach we found, which then became one of the main threads of the developed module, is the comparison between experiment and algorithm (section §2.3). This approach was therefore an instrument of analysis that allowed us to find the key example, the teleportation (chapter 3). Through the global approach we have designed the activity and we focused both on the problem in depth, in its technicalities, both on its globality and its complexity.

Chapter 2

I SEE and the module on quantum computers

In this chapter, we introduce to the context that constitutes the framework of our research for this thesis: the I SEE European project and, in particular, a teaching-learning module on quantum computing. In the section 2.1, we present the core ideas of the project and the general structure of the I SEE module. Then, we focus on the module on quantum computing presenting that designed, within the project, at the University of Helsinki (section 2.2). In the third section, we present the Italian revision of the module. We present how these modules realize and implement the main design principle of the project: the futurization of science education basing on the disciplinary aspects. We dedicate the very last section of this chapter to the description our educational choice for the introduction of quantum physics: the spin-first approach.

2.1 The I SEE project

I SEE (Inclusive STEM Educating to Enhance the capacity to aspire and imagine future careers) is a triennial Erasmus + project, started in September 2016, coordinated by the Department of Physics and Astronomy of the University of Bologna. The strategic partnership is composed by institutions coming from four different countries: Italy, Finland, Iceland and the United Kingdom. In particular, the partners are: two universities (the University of Bologna and the University of Helsinki), three secondary schools (the "A. Einstein" Lyceum of Rimini, the Normal Lyceum of Helsinki and the Hamrahlid College of Reykjavik), an Icelandic environmental NGO, an association of English teachers (Association for Science Education) and a private foundation in Bologna (Golinelli Foundation).

The project aims to contribute to the complex and articulates debate on the integration of STEM disciplines in curricula, taking the original perspective of addressing, through the lens of science education, the issue of imagination of possible futures as a key to encourage the students to aspire to STEM careers. The STEM perspective arises in response to the need, stressed by the productive and entrepreneurial world, to fill the so-called "skill gap" between the concepts learned in schools and universities and the skills required by the labour market (BusinessEurope, 2012). To address this problem, the I SEE approach aims to favour not only the learning of a broader spectrum of disciplinary knowledge in different STEM areas, but also the development of interdisciplinary skills. Indeed, the complexity of current scientific and societal problems more and more requires professionals able to manage multi-perspective and multidisciplinary issues. Consequently, the teaching of S-T-E-M disciplines in school and out-of-school contexts should be revised in order to highlight the connections between them and foster abilities to recognise specificities, differences and integration areas.

By connecting the disciplines, intertwining and comparing their different epistemologies and practices, the I SEE project aims not only to improve the teaching and learning of scientific disciplines, but it also pursues the more general goal to create a texture that could enable the students to face an emerging social problem: to understand the role of science in this era dominated by social acceleration and uncertainty (Rosa, 2013). In fact, on one hand, the increasingly rapid evolution in science and technology contributes to the feeling of disorientation, uncertainty and lack of a future horizon; on the other, thanks to the types of modelling strategies and concepts that scientists developed and its epistemological structure, sciences, and physics in particular, can play the role of privileged mediators between past, present and future. The issue of time is intrinsic to physics that has been developed through history also to rationally manage the fear of the future and of the unknown. The first conception of time that students encounter when approaching physics is usually the Newtonian one that has at its core the determinism: the exact knowledge of the initial conditions and of the laws of evolution – mainly linear differential equations – determine the exact knowledge of the future. Even if the classical physics is the most studied in secondary school curricula, the Newtonian-deterministic paradigm is not the only one developed by physics: quantum physics and science of complex systems introduce new ways of conceiving time and future in terms of multiple possibilities, where uncertainty plays a crucial role.

The I SEE project takes up the challenge of *futurizing* STEM education and gathers it as an opportunity to transform the role of education into a lab to prepare the young generation to manage uncertainty.

As Branchetti and colleagues stress in the 2018 paper "The I SEE project: An approach to futurize STEM education", the current problems related to environmental and social sustainability are very demanding and have important implications for education. The role of the education is to prepare students for their future, but this "systemic global dysfunction" (Lotz-Sisitka, Wals, Kronlid, & McGarry, 2015) jeopardizes the grounds on which education is based inducing a strong feeling of an uncertain future. In order to reduce people's anxiety and fears, education over the years has proceeded to "defuturize" the future, that means depriving it of some of its main features like uncertainty, possibility and impossibility to determine what will happen, highlighting on the opposite the value of discipline to predict the unknown. It is precisely due to this background that the I SEE project states as main purpose to "futurize" the scientific education.

In light of these problems and of a society characterized by strong acceleration and a constant change, science education must play a critical role in making understandable the global crises and, at the same time, it has the task to overcome the barrier of students' lack of interest in and bias against STEM subject. In this direction, the I SEE project proposes to create an approach that addresses issues posed by global unsustainability, the uncertainty of the future and social liquidity and by the irrelevance of STEM education for young people and their future. The project goal is to design innovative approaches and teaching modules to encourage students' capacities to imagine the future and to foster students' identities as capable persons and citizens in a global, fragile and changing world. In particular, the project aims to outline a STEM education approach centred on the development of what Branchetti et al. call "*future-scaffolding skills*" i.e. skills that render science learning relevant from different points of view (personally, socially, professionally and scientifically) and enhance students' capacity to aspire to future careers in STEM and imagine themselves as active agents of change. Future-scaffolding skills can be developed within STEM education and support students to talk and to think about the future.

The I SEE teaching-learning modules

In order to develop future-scaffolding skills, the partnership have designed and implemented innovative teaching-learning modules on cross-cutting and contemporary topics: climate change, artificial intelligence and quantum computers. The issues have been selected by the partnership for their future relevance and because they all are controversial themes that present challenges for present and future societies, for the environment, and for working life. Coherently with the core ideas of the project, these issues are selected and addressed as future-oriented scientific issues (Levrini, Tasquier, Branchetti & Barelli, submitted). Despite the differences among the selected topics, all the modules share a common structure that highlights their specific future orientation. The *figure 2.1* shows the structure of an I SEE teaching-learning module. From left to right five blocks of activities are represented: i) activities of encountering with the focal topic and future thinking; ii) laboratory activities to link epistemological knowledge and practice, conceptual knowledge and inquiry practice; iii) "bridging" activities; iv) future-oriented activities; v) action competence activities.

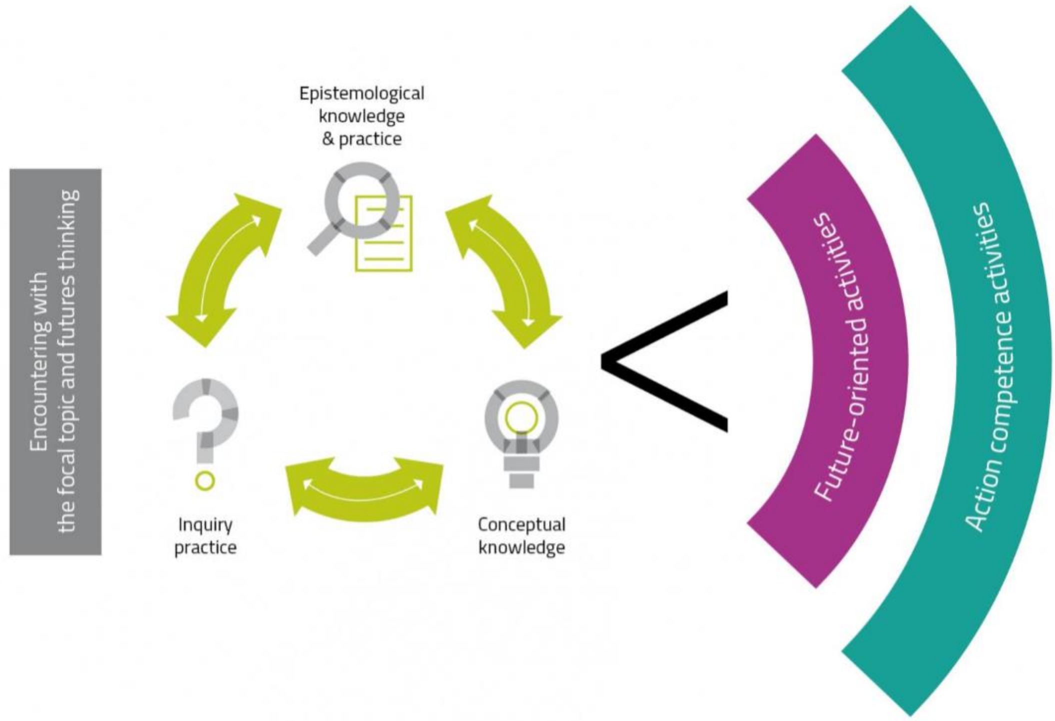

figure 2.1: Main structure of the I SEE teaching-learning modules.

Let us consider the structure in more details. The module begins with students encountering the scientific issue under exam and the basics of futures thinking. After having introduced the global disciplinary picture, the connections between it and the future are highlighted, in order to develop a level of awareness about the implications in many different dimensions and the impact on scientific research, politics, economy and society. In this phase the students are introduced also to the discipline of Futures Studies (FS), a branch of social sciences that has grown in the last ten years thanks to the intense participation of experts in many different areas: not only social scientists and philosophers but also academics in the STEM, economics and politics. The main concepts of the FS are introduced, e.g. the plurality of futures, the difference between probable, plausible, possible and desirable futures, the concept of scenario, the difference between forecast, foresight and anticipation of the futures. This phase of the module usually consists of plenary lectures but can foresee also group activity to encourage the exploration of the multiple dimensions of the future-oriented scientific issue.

After the first phase, teaching activities are carried out in order to explore the topic in more depth. In *figure 2.1* this phase is represented with a circle that mutually connects the three intertwined dimensions of science: i) conceptual knowledge; ii) epistemological knowledge and practice; iii) inquiry practice. The conceptual knowledge concerns the disciplinary content knowledge about the topic under exam. The dimension of epistemological knowledge and practice refers to epistemic practices such as modelling, arguing, and explaining: researches in the field of science education have shown that it fosters a deep and meaningful learning (Chinn, 2018; Tasquier, Levrini & Dillon, 2016). The dimension of inquiry practice relates to practices typical of experimental investigations such as posing questions, formulating hypotheses, designing inquiry, triggering peer-to-peer interaction, recognizing modelling as a process of isolating a particular phenomenon, and moving from models to experiments and vice versa. This phase of the module foresees laboratory activities and dialogic lectures in which the dynamical relationship among these three dimensions is implemented and highlighted.

On the right of *figure 2.1* are depicted the most specifically future-oriented parts of the module. These are developed in order to move from disciplinary knowledge and practices to the development of future-scaffolding skills and action competences. The activities that allow the transition from the most disciplinary to the most future-oriented parts of the module are the ones in the third section, represented with a “ < ” sign in *figure 2.1*. These activities, which consist of dialogic lectures, group works and discussions, have the role of re-reading the disciplinary concepts introduced in the previous parts of the module so as to highlight the future-related concepts intrinsic to the issue, with specific regard to the models of causal explanation. In this part of the module the panorama of forms of causality is enriched with the introduction of the perspective of the science of complex systems. Previous researches within the I SEE project have shown that the problematization of linear causality, determinism and reductionism, in favour of explanations that include circular causalities, emergent phenomena and deterministic chaos can foster students' attitudes to thinking and talking about the future (Barelli, 2017; Barelli, Branchetti, Tasquier, Albertazzi & Levrini, 2018; Levrini et al., under review; Tasquier, Branchetti & Levrini, under review).

The fourth part of the module aims to promote in students the development of skills that allow them to engage with the imagination of *probable, plausible, possible and desirable* scenarios. This distinction being already introduced in the first part of the module, the goal of these activities is to move from the idea that only one future, a predictable one, exists to the imagination of the existence of a plurality of futures and to the variety of ways to reach every depicted scenario. A special emphasis is given to the futures' cone, reported in *figure 2.2*, as an instrument to visually represent the opening-up of possibilities in which the desirable scenarios plays a crucial role: they encourage students to discuss and put into play their values and desires, their idiosyncratic preferences, their skills and their cultural points of view, for imagining their favourite future scenarios.

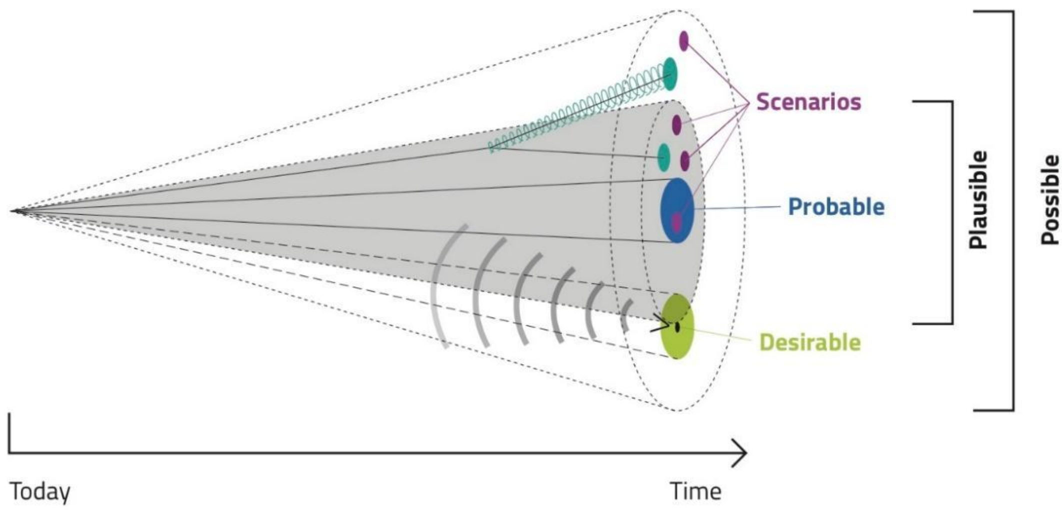

figure 2.2: The futures' cone

The spectrum of the activities in this part of the module is wide and various: activities enlarge students' imagination about possible future STEM careers; activities to select of a problem to be solved in a desirable future; activities to imagine feasible solutions to that problem.

The last part relates to action competence and aims to stimulate awareness of the plurality of perspectives in decision-making processes and support students in expanding their ethical consideration by making intentional decisions and taking deliberate actions. With the activities of this section, students are given the task of deciding collectively on a problem, determining how to investigate and address it, allowing them to participate differently and to bring into play different skills and interests, with respect to cultural diversities. These activities have the characteristic of activating a dynamic back and forth between the present and the future, which in *figure 2.2* is represented by the backward propulsion.

2.2 Finnish teaching module on Quantum Computing

The module on quantum computers, object of this thesis, was firstly designed and developed, within I SEE, at the University of Helsinki. It was structured to be implemented in two weekends. In order to promote the connection between the topic and the future, the Finnish researchers chose to dedicate part of each day to the disciplinary contents and part to the future. The table 2.1 shows the chronological structure of the Finnish module, divided in conceptual/epistemological disciplinary activities and future-oriented activities.

Table 2.1: Structure of the Finnish module on quantum computing

	Conceptual / epistemological	**Future-oriented**
1st weekend	**Saturday**	
	Electronic computer Information as bits - binary exercises	Future projects Basics of creative thinking
	Sunday	

	Components of a computer Operations of a computer Algorithms - electronics homework	“Back to the future” activity
2nd weekend	**Saturday**	
	Introduction to QM - quantum exercises	Mapping the problem Scenarios
	Sunday	
	Quantum computing - quantum homework	Backcasting activity

From a conceptual/epistemological point of view the teaching module was driven by some main choices:

1) to compare the classical and the quantum computers on a mathematical, logical, and technological perspective. The first weekend was entirely dedicated to classical computers. On Saturday, a brief history of electronic computers and their functioning was presented, focusing on mathematical aspects of computers and introducing information in terms of binary systems. The second part of the lesson was dedicated to information as bits, proposing exercises of conversion from the decimal numeric system to the binary numeric system and *vice versa*, introducing some arithmetic operations with bits. On Sunday, the lecture was focused on how a computer is made, explaining its various parts and how they work. In a second moment, the transistor logical gates and their logical operations were introduced, leading to the concept of algorithm as composed by universal classical logical gates. As examples of algorithms the tic tac toe and battleship were shown. All of these elements will be recalled in the second week-end while speaking about a quantum computer.

2) to introduce the new logic of quantum physics with a quasi- *spin-first* approach, so as to avoid any reference to the properties of classical objects. The logic presented, in fact, was exactly the same as the one built with sequential Stern-Gerlach experiments (or with a Mach-Zender interferometer), but the spin’s orientations (the photon polarization) were substituted with the use of shapes (square, triangles) and colors (red and blue). At the end of the presentation of the new the

superposition principle was introduced as the core of the quantum theoretical description of reality.

3) the use of the Deustch's algorithm as a simple example of quantum algorithms. On Sunday, similarly to what was done with classical computers, students were introduced to the new basic element of the system, the qubit. Then, after a brief description of multi-qubit systems, students are guided toward quantum computers in terms of the algorithm and circuits. Through the IBM's simulations, the teachers showed how an algorithm can be built by combining in sequence different quantum gates. In particular the Hadamard and CNOT gates, and measurement operator with its circuit symbol, were introduced. As an example, it was used the Deustch's algorithm, solving the problem posed both in classical and in quantum mechanical way. The teacher closed this part by stressing that so far with quantum simulators we can answer only to some kinds of questions and that the future is still to be written.

From the future point of view, the Finnish researchers proposed different activities, mainly aimed to reach two goals.

a. The first goal concerns he widening of imagination, fostering students to think out of boxes and to use creativity.
   The activities of the first week-end were "Basic of creative thinking" and "Back to the future". The first one consists of a presentation of the foundations of creativity: technical, theoretical, methodological knowledge; inner motivation; creative thinking skills which permit people to approach problems in a flexible and imaginative way. At the same time, they also showed how paradoxes are embedded in creativity, showing that, to solve a problem, i) the eye of a beginner can change the perspective of an expert, ii) to tackle with a discipline (usually characterized by some rigidity and strict rules) it is necessary to have some degrees of freedom, iii) to play a game professionalism is needed and iv) improvisation is necessary as much as planning, and *vice versa*. The second activity, "Back to the future", is a sort of challenge for students. Four movie clips from the homonym film (produced in 1980 and set in 2015) were shown. Students had to pay attention to the

similarities and differences between the setting of the film and the present and, at the same time, to individuate the correct predictions, so as to think about what it means to make predictions about the future based on the technologies of the present time.

2. The second goal concerns the achievement of action competences so as to assume an active role in the present and orient the future. The activity about the "Futures projects" aimed precisely at this. Some societal open problems were initially presented, such as the waste emergency, the climate change, the request for a new type of security, and so on. The students were asked to choose a challenge with which they felt particularly involved, and then, they were divided in groups based on the chosen topic and invited to think about possible strategies to address it. In the activity students were fostered to analyse their topic trying to think and reflect about:
   - what kind of assumptions they found behind their strategies;
   - the eventual presence of rules, and if and how they can be broken;
   - the relationship between logic and intuition;
   - how they could take more ricks reducing the fear of failure.

   The second weekend was entirely dedicated to activities aimed to guide the students to imagine and build scenarios on the basis the analyses of the challenges they had previously chosen. In the activity "Mapping the problem", they worked in groups to reflect on a particular aspect of the challenge, investigating its possible impacts on society, policy, economy etc., and trying to find connections with the development of quantum computers. They were asked to argue and discuss their analysis with the classmates and defend it from attacks. The focus of this part is the identification of the possible consequences and leverage points of the challenges, estimating how much easy/hard it can be to affect different aspects of the whole system.

   The future activity of Sunday was dedicated to build hypothetical "Scenarios" in 2035 in which the problems chosen from the students are solved. The experts introduced the futures cones, pointing out three ways to think about the future (as shown in *figure 2.3*) and fostering students to grasp the differences between them.

Three Ways to Think about Future

Futures	Forces	Thinking	Techniques
Expected (baseline)	• Constants • Trends	• Definite • Scientific	• Historical analogy • Extrapolation
Plausible (alternative)	• Discontinuities • Surprises	• Speculative • Imaginative	• Scenarios • Simulation
Preferable (visionary)	• Choices • Images	• Aspirational • Empowered	• Visioning • Planning

figure 2.3: table shown ways of thinking about three possible futures

Starting from the description of the scenarios, the students were then guided in a back-casting action, that is a step by step reconstruction of the actions that back in time possibly led to the change, individuating their impact on the system, the hypothetical obstacles to their realization, and finding out ideas to overcome them. Some of the topics chosen by the students are colonisation of Mars, genetic engineering, ecological energy production, recycling, privacy in the era of Internet and pensions crisis. The aim of this activity is to foster students' action competences, leading them to play an active role in the present, looking towards a future in which they would like to live.

The Finnish partners analysed the teaching module from different perspectives, pointing out, among other results, that the course aimed: i) to sow seeds in order to start growing students' awareness and imagination; ii) to provide students with skills to think in a more precise and positive way to the future, expanding their horizons to new ideas; iii) to provide a vision of the future less random and full of possibilities.
Some of the students were interviewed at the end of the course, and their answers seem to show a positive feedback regarding the future-oriented purposes of the module. Some expressions were, for example:

"*I thought about the future or tried to predict it, it was like really haphazard it doesn't matter if what I predicted actually happens, but just the fact that I have thought about it, helps me prepare for… whatever comes. So, like, I feel like that way I learned a new way of thinking, like a new process*"

> “Well, *my future is in my hands, so I can influence it very much. Of course there are things, like if there are accidents or something, but .. well, they don’t really stop me, there’s always a way to do what you want to do*”.

Some students were very interested in the subject and the STEM disciplines in general, as expressed here:

> “*I realized that I could also be like a leader type of person who works with people, like even if I wanted to work with science, I don’t need to work alone, like doing some computer work. I’ve always wanted to work with people, but then I’ve thought I have to choose? But now I was like, maybe I can combine them*”.

> “There were so many people, like smart and nice people, so I thought that since they have all these good ideas and if they really put them into practice... then I think maybe the world could become a better place. So, it gave me .. like, a positive feeling”.

Expressions like “a new way of thinking”, “future is in my hands”, “positive thinking”, show that the module helped the students to diminish the sense of disorientation and to increase a proactive attitude towards the future. Some preliminary observations drawn from the Finnish research group about the impact of the teaching module are the followings:

- the students acquired a more positive and broader vision of the future;
- the students adopted new ways and techniques of thought: creative thinking, scenario thinking, systemic thinking;
- the students were confident in their ability to influence their future and, to some extent, their global future (mainly through career choice);
- the course has also expanded the point of view of some students on science careers (even imagining jobs that do not yet exist);

- the students were able to imagine how to solve evil problems with ICT.

Along these observations, they highlighted also two main difficulties emerging from students' responses, that need to be investigated and improved. The first one is about the concept of algorithm, and in particular the Deustch's algorithm. The reaction that the students showed is a sense of disorientation. Despite the Deustch's algorithm is one of the simplest, they found it very complicated. The teachers did not individuate particular reasons for this, but they hypothesized that it could regard the kind of problem that the algorithm is meant to solve, with the new logic that is required, or/and with the difficulties inherent the formalism. The research group of the University of Helsinki is studying the issue with the aim of finding other ways to explain the algorithm and re-propose the module.

A second important issue they pointed out regards the connection between the quantum computing conceptual/epistemological issues and the future-oriented part of the module, that seemed to them to be a little bit weak and not properly made explicit.

2.3 Italian teaching module on Quantum Computing

In order to develop the Italian module, a team with different competences has been established. The team was comprised of:

- a theoretical physicist, prof. Elisa Ercolessi, expert in quantum computing;
- a researcher in physics education, prof. Olivia Levrini;
- two post-doc students, one in mathematics education, dr. Laura Branchetti, one in physics education, dr. Giulia Tasquier;
- three PhD students, one in Physics (dr. Giovanni Ravaioli), one in Computer Science (dr. Michael Lodi) and one in Data science and Computation (dr. Eleonora Barelli)
- one bachelor student, Roberta Spada;
- one master student, Sara Satanassi
- a secondary school teacher with professional expertise in classical computing architectures and algorithms, prof. Paola Fantini;

- two teachers of “A. Einstein” Lyceum of Rimini, prof. Michela Clementi and prof. Fabio Filippi.

The team met regularly from September 2018 in order to design the activities to implement during the module. During these meetings, we analyzed the literature regarding quantum computers and the Finnish module, appropriating their choices in order to decide what to keep and points on which to reason and propose revisions. The work has been divided between the group members on the line of personal skills and research interests, and the overall process was supervised and coordinated by prof. Olivia Levrini.

Prof. Elisa Ercolessi built an introduction to the new logic of quantum physics at the base of quantum computers. Prof. Paola Fantini was responsible for building an overview of the history of classical computers so as to open the door to these new technologies. Roberta Spada and prof. Michela Clementi studied the social and scientific impact of quantum computers, using the Quantum Manifesto and other official documents. Dr. Laura Branchetti and dr. Eleonora Barelli studied the connection between the quantum computing and the future, respectively through the game theory and the science of complex systems.

I took part to all the meetings and to the whole process of the module design. I was responsible, together with dr. Giovanni Ravaioli, of the reconstruction of the comparison between the teleportation experiment and the teleportation protocol, and to the design of the corresponding activity described in chapter 3.

The entire teaching module was designed as a revision of the experience carried out in Finland. We maintained a similar backbone and some main conceptual choices. In particular, we chose on the line of the Finnish module to:

a) compare classical and quantum computing from a mathematical, logical and technological perspective;
b) use a *spin-first* approach to introduce the new logic of quantum physics;
c) present an example of a quantum algorithm;
d) bring out the conceptual and future-oriented activities in parallel.

However, the differences in the temporal structure of the module and the difficulties highlighted by the Finnish partners led us to make some different choices, also based

on the work shared within our research meetings. Two main choices have been made with respect to the Finnish module.
The first one regards the connection between quantum computers and future, that the partners found to be not as strong as it was meant to be in their module. This may concern the fact that even today it is not easy to see the real potential of quantum technologies. There are already several algorithms that can be implemented on a quantum computer, but there is not yet a quantum technology powerful enough to compete with the classic processors. Furthermore, most of the problems can be solved with quantum simulators, but they are usually very specific problems whose impact on society is not easy to predict and interpret. Thus, in order to strengthen the connection between the topic and future we chose to develop some specific future activities that explore the impact of quantum computer in the society, through the introduction of the Quantum Manifesto and a synthetic presentation of some developing applications of quantum computing (see further activity "Quantum Computing &…").
Our second choice concerns the quantum algorithm. The Finnish group proposed the Deustch's algorithm, but they found some complications. In fact, despite it is one of the simplest quantum computing algorithms, the students found it very complicated. This maybe can deal with the kind of problem that the algorithm is proposed to solve or/and with the difficulties inherent to the formalism. The Finnish group is now studying the issue with the aim of finding other ways to explain the algorithm and re-propose the module. In light of this problem we chose to propose another algorithm, the teleportation protocol, comparing one of its experimental implementations (Ursin et al., 2004) with the algorithmic representation of the teleportation protocol. This choice, together with the recovery of a paper called "an ancient rope-and-pulley computer is unearthed in the jungle of Apraphul" (Dewdney, Ak., 1988), allowed us to build a synthetic and organized approach to read, interpret, and reconstruct the conceptual breakthrough of quantum computing.

The context for the implementation of the teaching module was the PLS Project (Piano Lauree Scientifiche), a national plan in Italy that supports the students' enrolment in scientific degrees (physics, mathematics, biology, chemistry, geology). It has the dual purpose of encouraging the study of scientific disciplines, offering students the opportunity to get closer to advanced research topics, and to acquire basic skills on the "profession of the scientist". The Department of Physics and Astronomy at the University of Bologna has a tradition of support to this project, hosting more than a hundred of secondary school students every year in the laboratories that cover different subjects. Each laboratory lasts in general five or six weeks with three-hours weekly sessions; our module has been developed in six meetings of three hours each. Table 2.2 shows the chronological sequence of lectures and activities. 26 upper secondary school students, from different schools of Emilia-Romagna, enrolled to the quantum computing course and participated to the lectures that took place in February-March 2019.

Table 2.2: Structure of the Italian module on quantum computing

Day	Lectures	Future activities
1°	▪ History of Computers ▪ Physics of quantum computers	▪ Introduction to future's cone
2°	▪ Introduction to multi-qubit systems and entanglement ▪ Cryptography	▪ Future-oriented activity "quantum computing &…"
3°	▪ Quantum teleportation	▪ Delivery of students' outputs on "quantum computing &"
4°	▪ Classical and quantum problems ▪ Predict, simulate and build future scenarios ▪ Game theory: which interactions between agents?	▪ "Back to the future"
5°	▪ Futures and Action competence activity	
6°	▪ Delivery of students' outputs on futures and action competence activity	

The module has been designed according to the structure of the I SEE modules, articulated in the five aforementioned phases (section §2.1).

The encountering with the topic of quantum computing was realized with the lecture "History of computers" by prof. Paola Fantini and with the first part of the lecture "Physics of quantum computers" by prof. Elisa Ercolessi – the division of the lecture in two parts being clarified and motivated in the next paragraphs. The encountering with the issue of future was realized introducing the future' cone and carrying out the

future-oriented activity “Quantum computing &…”, led by Roberta Spada and prof. Michela Clementi.
The intertwining between conceptual knowledge, epistemological knowledge and practice, and inquiry practice was realized during four lectures: i) second part of “Physics of quantum computers”, held by prof. Elisa Ercolessi; ii) “Introduction to multi-qubit systems and entanglement”, held by prof. Elisa Ercolessi, iii) “Cryptography”, held by prof. Elisa Ercolessi, iv) “Quantum teleportation”, held by me; iv) “Classical and quantum problems”, held by dr. Laura Branchetti. In these lectures, students coped with specific conceptual aspects of the issue and were guided to recognize the different dimensions involved (e.g. experimental, logical, formal, applicative).

The bridging from the disciplinary aspects to the most explicitly future-oriented activities was realized during dialogic lectures (“Predict, simulate and build future scenarios” by dr. Eleonora Barelli and “Game theory: which interactions between agents?” by dr. Laura Branchetti): the students were divided in groups and had to discuss about a computational simulation of a complex system and about a problem of game theory.

The future-oriented and the action-competence activities are merged in this module: the activities were readapted from the Finnish module and encourage students to synthesize the disciplinary learned concepts to deal with a problem in the future. After having been grouped according to their preference toward a theme or another (e.g. health and wellbeing, war and conflict, work and unemployment), the students are asked to choose a problem, to imagine a desirable future scenario for 2040 in which the problem has been solved, and to retrace, through the 2040-2019 timeline, possible actions and choices that could lead to that scenario.

In the following section, we present the detailed design of the module according to two main threads, that guided the construction of its conceptual and epistemological structure:

a. the comparison between the experimental and the computational dimensions;
b. the connection between the topic and the future.

The pursuit of these strands allowed, in our opinion, to give a meaningful organization of the conceptual knowledge and to tie it to the imagination of the future.

2.3.1 Experiments and computation

The comparison between the experimental and the computational dimension has determined our global approach to the topic and has become a tool for an educational reconstruction. We identified three different levels on which to build the (deepened further in the section §3.2, in which we have applied them to the case of teleportation as an example): *narrative*, that allows to keep an overall view without getting lost in the details, *symbolic/logical,* that consists in the truth table and in symbolic form of representation, and of mechanism, that deals with how things work. These three dimensions allowed us to go through some technical and theoretical details without getting lost, building a synthetic image of the evolution of quantum computing and of its conceptual breakthrough. Let us see how this thread was developed in the module, with specific reference to the encountering lectures ("History of Computers" by prof. Paola Fantini and "Physics of quantum computers" by prof. Elisa Ercolessi) and to the first lecture of the conceptual-epistemological-inquiry part ("Introduction to multi-qubit systems and entanglement" by prof. Elisa Ercolessi).

The lecture "History of Computers" is an overview of the history of classical computers and computation, in which the lecturer showed a "correlation between the things we can compute and the physics". The focus of the lecture became soon the concept of information and of processing of information. The entire speech was built around the following flow:

$$input\ information\ \rightarrow processing\ \rightarrow output\ information \quad (2.1)$$

The speaker stressed the fact that communication consists of i) the encoding of information in terms of bits, ii) its elaboration, that consists in projecting the input, operating on it and returning it legibly, and iii) the final transmission. In order to explain how the elaboration of information is possible, prof. Fantini introduced the concept of algorithm and started to build the comparison between an algorithm and a physics experiment, mentioning a paper titled "An ancient rope-and-pulley computer is unearthed in the jungle of Apraphul" written by Dewdney (Dewdney, Ak., 1988). The paper narrates that, in the imagined island of Apraphul, some "archaeologists of informatics" discovered the ruins of an ingenious system of ropes and pulleys, and considered it the first computer in history. This ingenious system of ropes and pulleys can be interpreted as the mechanical counterpart of logic gates. Moving back and forth in history, the lecturer showed to students how early computers – so similar to the mythological system in Apraphul – evolved in modern laptops, going through a process of miniaturization of the main components.

The analysis of the lecture in terms of narrative, symbolic/logical and mechanism levels shows that these levels can be declined for both the experimental and the computational dimension. With the term experiment we mean in this case the set of pulleys and ropes on Apraphul island, while with algorithm the sequence of operations performed by logic gates. In table 2.3 is reported the synthesis of the analysis that we discuss in the followings.

Table 2.3: Level analysis of classical computation

Level	**Experiment**	**Algorithm**
Narrative	Apraphul Island	/
Symbolic/logical	Single tools (boxes) and how they process a bit	Logic gates and true table
Mechanism	Mechanical ropes and pulleys	/

As the table shows, the narrative level is held by the lecturer's storytelling about Apraphul. The island is populated of boxes, whose aim is to manipulate information (logic level) and inside them there are systems of ropes and pulleys that carry out the processing (level of mechanism). The experiment represents the state of the art of classical computing in its early days. The algorithm can be interpreted as the evolution of the experiment. As the table show, the narrative and the mechanical levels are not present. In fact, since the various components were becoming smaller and smaller, losing the sense of mechanism, hardware and software of the computers changed very fast with the technology progresses and the old architecture is replaced by the most modern computers, ever more powerful and light. Only the logic level remains, in term of logical gates and the operations that they perform. The actual state of art of quantum computers is analogue to the island of Apraphul for the classical computers. Many algorithms based on a new logic have already been developed. However there are not sufficiently powerful quantum computers to take advantage of quantum physics in the description of certain systems. During this overview students are introduced to logic gates as signal manipulators. In particular, the three universal logic gates (NOT, AND, OR) have been introduced with their respective circuit symbols and truth tables.

The following lecture "Physics of quantum computers", held by prof. Ercolessi, aims to introduce the basics of quantum physics to make students grasp how quantum computers work and their potentialities. The speaker started by reconsidering the flow 2.1, introduce in the previous lecture, and re-read it in a quantum mechanical way:

$$preparation\ of\ a\ state\ \rightarrow trasformation\ \rightarrow measurement \quad (2.2)$$

The preparation of a quantum state consists in three steps: i) the encoding of the information in a string of qubit ($input\ information$), ii) the transformation to the processing of information through operations on the qubit string following an algorithm ($processing$), iii) the measurement to the reading of the processed bit string ($output\ information$).

After having introduced the main characteristics of the quantum world adopting a spin-first approach and referring to the Stern-Gerlach apparatus (see section 2.4 for details), prof. Ercolessi introduced the qubit model as a binary system (spin up and spin down) and the superposition principle through the use of the mathematical representation of Block's sphere (*figure 2.4*). Here the encountering phase of the module ends, and the students are introduced the most specific conceptual part of the module.

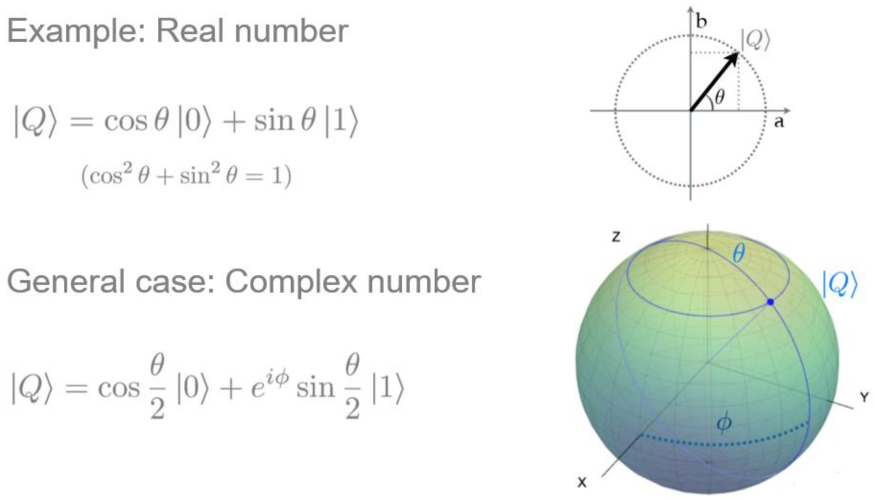

figure 2.4: Block's representation

Analogously to the *processing* occurring with classical computers, for quantum computers the *trasformation* is possible through logical gates. The logic gates are introduced, as well as in classical case, for their role of transforming a state into an another. After having presented the representation of qubits on the Block's sphere and after the introduction of measure and of collapse, it was possible to give another interpretation of logic gate as unitary transformation. In these terms, a transformation can be seen as a rotation of a vector in the abstract space of Block's sphere. In particular the logical gates introduced are X, Y, Z and Hadamard gates, with their truth table (*figure 2.5*).

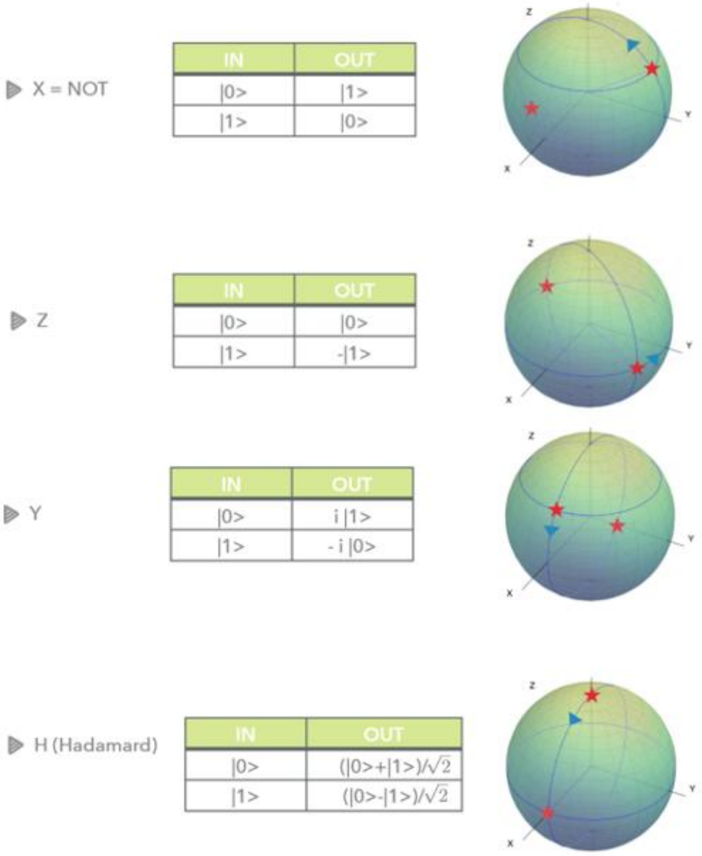

figure 2.5: logical gates represented as rotations in Block's sphere.

The third lecture ("Introduction to multi-qubit systems and entanglement" by prof. Ercolessi) was designed to analyse in more depth the novelty of the concept of measurement in quantum physics, introducing two-qubit systems and entanglement. These concepts served as a base for the following activities of the module in which quantum cryptography and teleportation were addressed and presented as applications of the previously introduced concepts of two-qubit system and entanglement. These activities will be discussed in detail in sections 3.2 and 3.3.

The analysis of the "Physics of quantum computers" and "Introduction to multi-qubit systems and entanglement" lectures according to the narrative, symbolic/logical and mechanism levels is synthetized in table 2.4. It allowed an interpretation of the role of quantum experiments in terms of logic gates, re-attaching vice versa to quantum simulators and computers their structural nature of experiments.

table 2.4: Level analysis of quantum computation

Level	**Experiment**	**Algorithm**
Narrative	Alice and Bob	Alice and Bob
Symbolic/logic	Experimental setup as states processor	Logic gates and truth tables
Of mechanism	Single tool and how it can manipulate states	/

The narrative level is represented by the narration of Alice and Bob, that usually share a pair of entangled photons. Both in cryptography and in teleportation lectures, this stratagem has been recalled to contextualize the problems to be solved. As will be shown in section 3.1, the symbolic/logical level consists for the algorithm in logic gates and true tables, while for the experiment in the set of tools that modify a state. The level of mechanism is identified in how a single tool can manipulates information.

2.3.2 Quantum computing and future

In order to establish the connection between quantum computers and future, we developed through the module both disciplinary contents and future-oriented activities. As in the Finnish module, the students were encouraged to think about a world linked with quantum networks and populated by quantum computers able to manage more and more data in less time, and to describe systems that follow the laws of quantum mechanics.

With the goal of making students understand the real potential of the new technologies, we decided to distance a little from the Finnish module designing the activity “Quantum computers &…”. This activity, positioned in the encountering phase of the module, aims to suggest the feeling that quantum computers represent a real possibility both for their innumerable applications and potentials, and for the new jobs that they could create. In this regard, the Quantum Manifesto (de Touzalin et al., 2016) presented to the students. On invitation of Mr. Günther Oettinger (Commissioner for Digital Economy and Society) and Mr. Henk Kamp (Minister of Economic Affairs in The

Netherlands), a European team has been working on a "Quantum Manifesto" to formulate a common strategy for Europe to stay at the front of the second Quantum Revolution. The Manifesto will be officially released on 17-18 May 2016 at the Quantum Europe Conference that The Netherlands is organizing in Amsterdam in cooperation with the European Commission and the QuTech centre in Delft. The Quantum Manifesto calls upon Member States and the European Commission to launch a €1 billion Flagship-scale Initiative in Quantum Technology, preparing for a start in 2018 within the European H2020 research and innovation framework programme (https://ec.europa.eu/futurium/en/content/quantum-manifesto-quantum-technologies). The manifesto marks a line time that foresees the development of applications in four sectors (communication, simulations, sensors and quantum computers) up to 2035. After a brief presentation we provided the students with information sheets about the impact on four fields: society, politics, economics and research; each sheet was equipped with many additional online resources as links. In *figure 2.6* and *figure 2.7* we report two examples of sheets we prepared for students, one about the implications on scientific-technological research and the other about the implications on society.

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QC e Ricerca Scientifico-Tecnologica

Alcune definizioni:

- ***Simulatore quantistico***: sistema quantistico controllabile (ovvero per cui è possibile variare alcuni parametri) usato per simulare altri sistemi quantistici.
- ***DQS (simulatori quantistici digitali)***: sistema in grado di riprodurre un algoritmo quantistico per simulare un altro sistema. Ne sono un esempio i computer quantistici, che sono simulatori universali in quanto *potenzialmente* in grado di riprodurre qualsiasi sistema.
- ***AQS (sistemi quantistici analogici)***: sistemi quantistici concreti che ne replicano altri. Possono simulare una classe ristretta di sistemi o fenomeni di interesse. Trattandosi di sistemi fisici concreti, la misura delle proprietà del sistema può essere effettuata direttamente su questo, mentre nel caso dei DQS lo stato viene manipolato attraverso algoritmi per poterne trarre informazioni utili. Negli ultimi anni sono stati proposti diversi modelli di AQS, la maggior parte dei quali è basata sulla fisica della materia condensata (atomi neutri in reticoli ottici, ioni raffreddati e intrappolati, circuiti superconduttori, etc.).
- ***Sensori quantistici***: dispositivi che sfruttano il principio di sovrapposizione e/o l'entanglement per raggiungere una maggiore sensibilità e risoluzione nella misura di campi gravitazionali, campi magnetici, intervalli di tempo o costanti fondamentali della fisica.

La simulazione quantistica trova applicazione in svariati ambiti scientifico-tecnologici, dalla fisica atomica e dello stato solido, fino alla fisica delle alte energie e alla cosmologia, ma non solo.
Infatti, poichè consentirà di analizzare le proprietà di sistemi specifici, individuare sistemi con desiderate proprietà o realizzare nuovi composti con desiderate proprietà, essa permetterà di progettare nuovi farmaci, come pure nuovi materiali

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con importanti proprietà (superconduttori ad alta temperatura) che potranno essere utilizzati in molteplici settori, tra cui quello energetico o dei trasporti.
In campo ambientale, uno tra i cosiddetti *world's critical problems* indirizzati dal QC è la progettazione di un sistema efficiente di cattura del carbonio dall'atmosfera.
Un'ampia accessibilità all'utilizzo dei computer quantistici da un lato e alla conoscenza scientifica su di essi dall'altro è dunque necessaria per evitare la creazione di monopoli tra le compagnie farmaceutiche, come tra le grandi aziende o gli enti governativi. È per affrontare queste sfide che occorre formare nuove generazioni di tecnici e scienziati.

Link	Descrizione
https://www.youtube.com/watch?v=qarc7AA4-wM	Breve video introduttivo alle sfide della simulazione quantistica
https://www.microsoft.com/en-us/research/blog/problems-will-solve-quantum-computer/	Articolo che mostra che un QC può essere utilizzato per svelare le dinamiche di reazione in sistemi chimici complessi. Per esempio, l'analisi del complesso enzimatico nitrogenasi (al di fuori delle possibilità del più potente fra i supercomputer) avrà importanti ricadute nella produzione di fertilizzanti.
https://qt.eu/discover/applications-of-qt/fertilizer-and-other-quantum-computer-chemistry/	Sito che illustra gli obiettivi della chimica computazionale nello sviluppo di modelli matematici che simulino comportamento e proprietà di sistemi chimici complessi.
https://home.cern/news/news/computing/exploring-quantum-computing-high-energy-physics	QC per le sfide della fisica delle alte energie.
https://www.cnr.it/it/news/8292/sensori-quantistici-superconduttivi-per-comprendere-la-progressione-della-sla	Utilizzo dei sensori quantistici superconduttivi nello studio funzionale del cervello.
https://qt.eu/app/uploads/2018/04/93056_Quantum-Manifesto_WEB.pdf	Quantum Manifesto

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figure 2.6: Sheet about Quantum computing and scientific-technological research

I-SEE

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QC e Società

Alcune definizioni:

- ***Ottimizzazione quantistica:*** branca delle tecnologie quantistiche che si occupa di migliorare gli algoritmi già esistenti di *machine learning* per ottenere nuove soluzioni, non solo grazie alla maggior velocità di calcolo del QC, ma anche grazie all'approccio dato dai qubit, differente da quello dato dai bit classici. Le tecniche per ottimizzare gli algoritmi di *machine learning* sono dette di *quantum annealing*.

Il principio di ottimizzazione ha molteplici applicazioni in vari campi, non solamente in quello scientifici o ingegneristico. Si prospetta che l'uso massiccio di tali algoritmi, della robotica e del *machine learning* modificherà radicalmente gli attuali concetti di professione e di industria.
Le professioni meccaniche, manuali e di calcolo potrebbero essere affidate a macchine, lasciando all'uomo mestieri creativi e di realizzazione e coordinamento delle macchine. È la cosiddetta "industria 4.0": il computer quantistico potrebbe velocizzarne l'arrivo?

Alcuni esempi:

- **Ottimizzazione del traffico:** La Volkswagen, in collaborazione con la D-Wave, un'azienda che sviluppa tecnologie quantistiche, sta studiando un metodo per predire i punti di una città in cui potrebbero crearsi ingorghi, per poter indirizzare le auto verso altre strade in cui è meno probabile che l'ingorgo si formi. Questo richiede l'analisi di una mole di dati che nessun supercomputer attualmente esistente sarebbe in grado di gestire.
- **Ottimizzazione per la medicina:** una clinica statunitense ha collaborato con D-Wave per sviluppare algoritmi volti ad ottimizzare le tecniche di determinazione del dosaggio di radiazione ottimale per il paziente, a seconda del tipo di tumore, dello stadio e delle caratteristiche del paziente stesso.

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- **Ottimizzazione degli annunci online:** la Recruit Communications, un'azienda di risorse umane, ha sviluppato un algoritmo di ottimizzazione che abbina consumatore e annuncio e permette alle aziende che utilizzano annunci online di aumentare il proprio CTR (*Click-through-rate*), ovvero il rapporto tra il numero di volte che un utente preme sull'annuncio e il numero in cui esso compare. Questo è uno dei tassi che più influisce su quanto viene pagato il gestore del sito web che pubblica l'annuncio per conto dell'azienda.

Link	Descrizione
https://ai.google/research/teams/applied-science/quantum-ai/	Pagina della Google AI con aree di ricerca nella Quantum AI e applicazioni a breve termine
https://qz.com/1323987/quantum-computing-could-put-a-stop-to-traffic-jams/	Articolo che spiega il principio alla base dell'ottimizzazione del traffico
https://media.vw.com/en-us/releases/1098	Comunicato stampa della Volkswagen
https://www.dwavesys.com/sites/default/files/VW.pdf	Slide che spiegano l'algoritmo di ottimizzazione del traffico nel dettaglio
https://www.dwavesys.com/sites/default/files/Radiotherapy-Optimization-Roswell-Park_0.pdf	Slide sull'ottimizzazione nel campo della radioterapia
https://www.dwavesys.com/press-releases/recruit-communications-and-d-wave-collaborate-apply-quantum-computing-marketing	Comunicato stampa della D-Wave sul lavoro della Recruit Communications per l'ottimizzazione degli annunci online
https://www.dwavesys.com/sites/default/files/RCO_0_0.pdf	Slide sull'algoritmo di ottimizzazione degli annunci
https://qt.eu/app/uploads/2018/04/93056_Quantum-Manifesto_WEB.pdf	Quantum Manifesto

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figure 2.7: Sheet about Quantum computing and society

We asked the students, divided in little groups, to choose a field, explore the links and the information contained, and try to identify the connections among other fields and possible areas. On this basis they were asked to build a map connecting different domains and problems (*figure 2.3*). The template of the map was developed on the basis of the Quantum Manifesto and other papers (Preskill, John, 2000; Möller & Vuik, 2017).

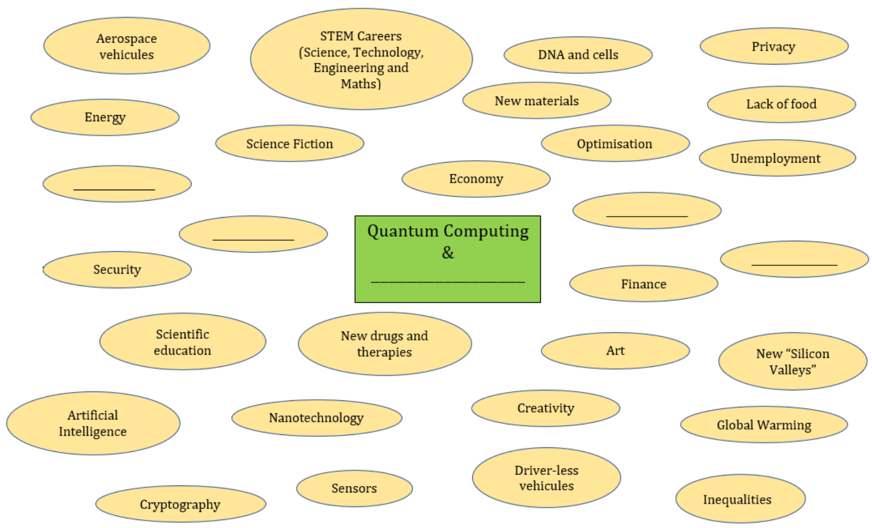


figure 2.8: Maps of connection of different domain and problems

At the beginning of the third day, the students delivered the map they had produced and presented their work to the class, explaining the connections and the aspects they considered particularly interesting.

2.4 The spin-first approach

As proposed by Finnish partners, the introduction to quantum physics has been done in our module through the quantum concept of spin and state, in order to immediately break with the classical properties and avoid dangerous analogies: we refer to this educational choice as the spin-first approach. Through the set-up of Stern-Gerlach experiment, the students were what it actually means to "prepare" a state and are introduced to the new logic of quantum physics. A schematic representation of the apparatus is reported in *figure 2.9*:

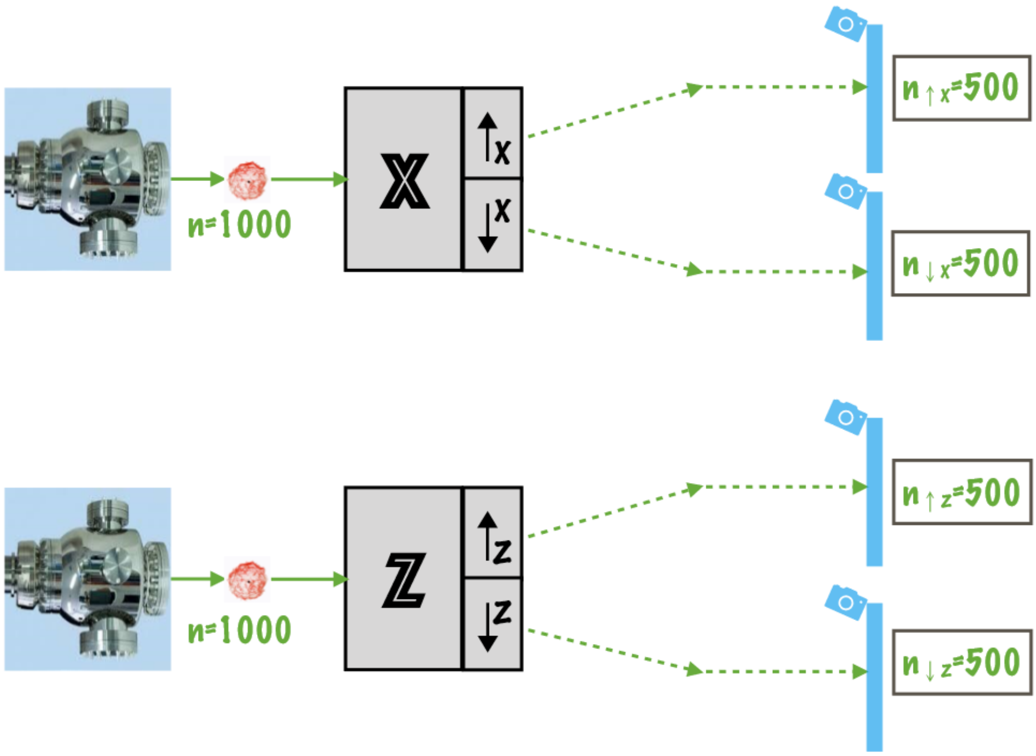


figure 2.9: Schema of the Stern–Gerlach apparatus.

The Stern-Gerlach apparatus was the base for a series of exercises, during the lecture "Physics of quantum computers", with which the lecturer, prof. Ercolessi, challenged the students. Supposed to have 1000 particles passing through Stern-Gerlach devices, the speaker invited the students to think about the number of particles in different cases. The first exercise is the one in *figure 2.4*. At the output of the apparatus one can expect 500 particles occupying the "top" position ($|\uparrow\rangle$) and 500 at the "bottom" position $|\downarrow\rangle$. In the second exercise, we have supposed to connect two Stern-Gerlach devices in cascade, as shown in *figure 2.10*. The $|\downarrow\rangle$ exit from the first Stern-Gerlach apparatus is blocked, while the $|\uparrow\rangle$ one is sent through a second device oriented along on the same axis. A detector positioned at the final output measures the distribution of atoms.

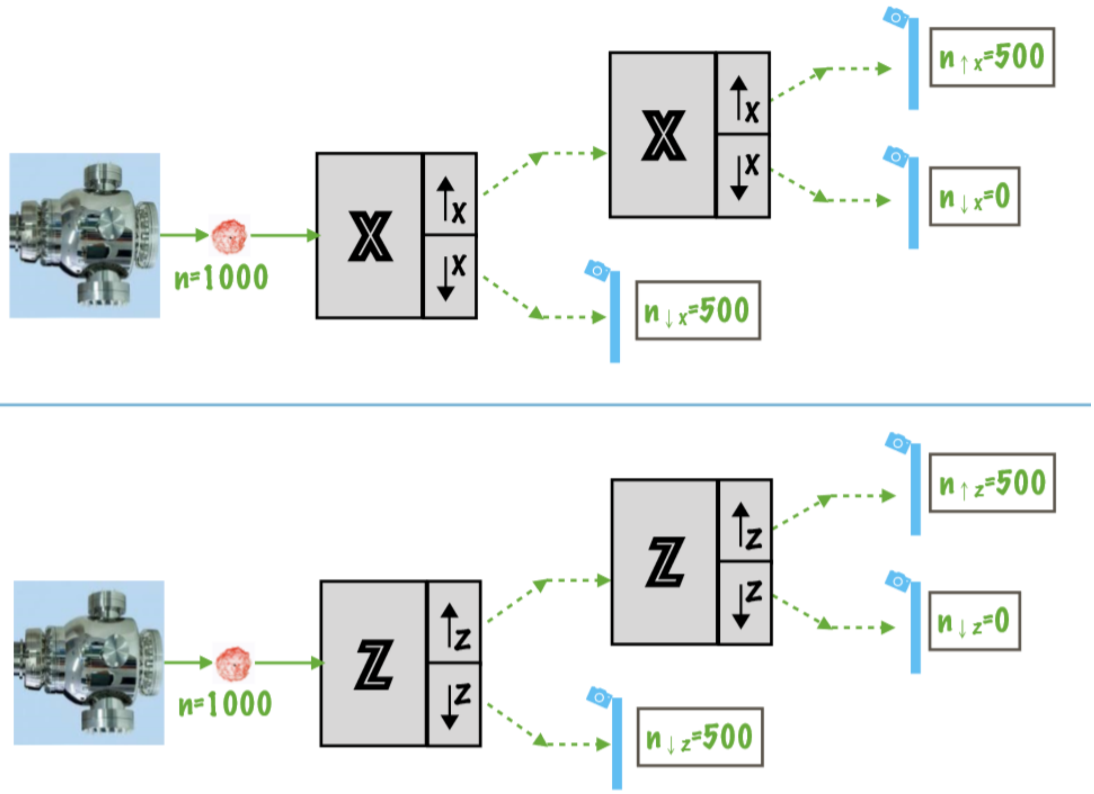


figure 2.10: N.2 exercise.

The final output is not represented by two equal distributions, as would be expected in the classical case. All the particles end in the upper position, as if having prepared the system in one of the two possible states ($|\uparrow\rangle$) affects the output of the second apparatus. This means that we will find 500 particles in a position and 0 in the other.

The third case is represented in *figure 2.11*. Unlike the previous case the final experimental result consists in two peaks of equal intensity, which means 250 particles in one position and 250 in the other. Since the two apparatuses are oriented in different direction, the preparation of the first one in a certain state does not influence the second one.

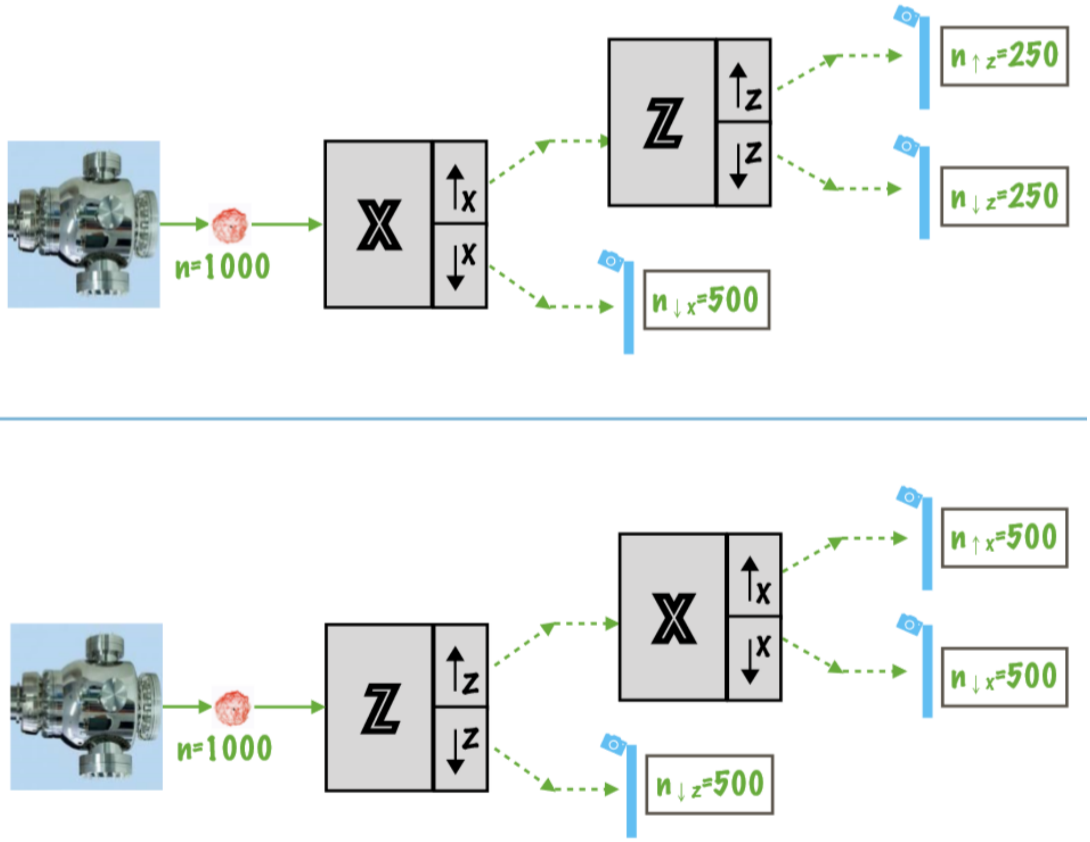


figure 2.11: N.3 exercise.

The fourth case is represented in *figure 2.12*. If the atoms had retained their $|+Z\rangle$ orientation, then the output would be expected to have only one peak, at the $|+Z\rangle$ output. However, again two beams are observed at the final output, of equal intensity (125 particles each). Thus, the conclusion would seem to disagree with the classical expectations: the presence of the X apparatus seems to make the first device "forget" that he already prepared the state along the $\hat{z}$ axis in a certain way.

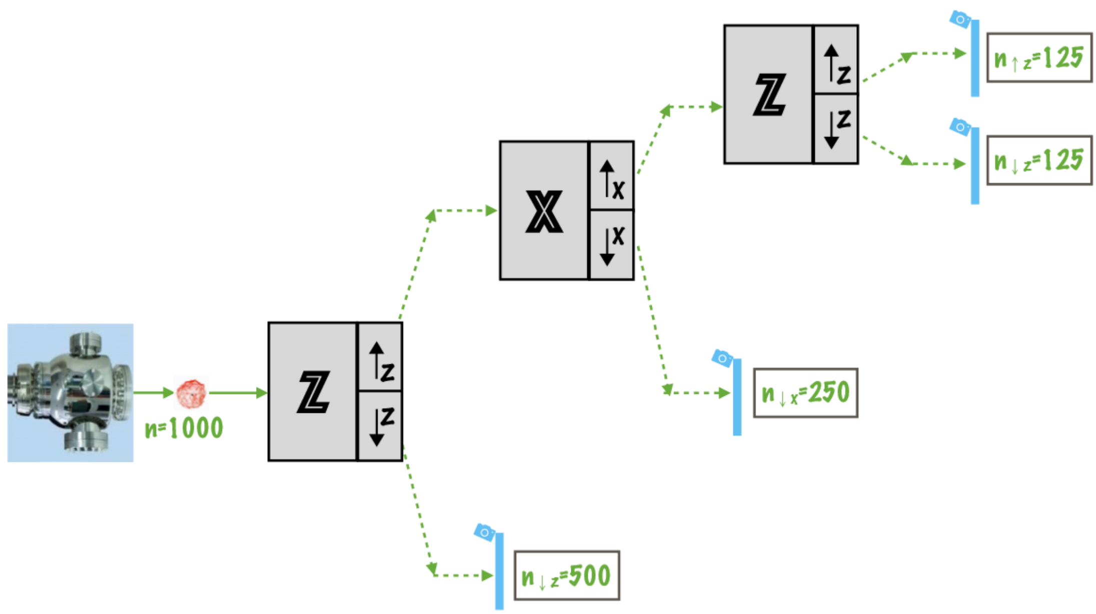


figure 2.12: N.4 exercise

The last case (*figure 2.13*) is similar to the precedent with the difference that we do not block the exit of the apparatus that acts on the $\hat{x}$ axis, i.e. all the particles enter in the final device. This means that, at the end, two different distributions are detected, i.e. 500 particles in one position and 0 in another: it is as if the central apparatus was not even there.

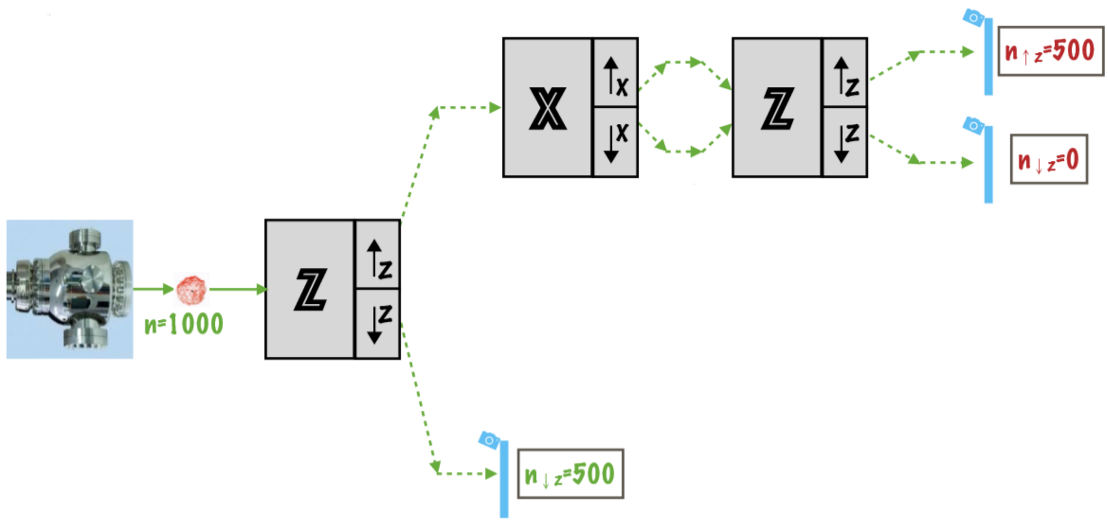


Figure 2.13: N.5 exercise

The Stern-Gerlach experiment enabled students to recognize there is something missing that could help to describe this apparently "strange" behaviour of particles: the superposition principle. Translated in terms of our discussion, the state can be described as a linear combination of spin up and down, so $|\varphi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$. It turns out to be in $|\uparrow\rangle$ or in $|\downarrow\rangle$ only when it passes through the last device Z and we actually measure it.

Chapter 3

The emblematic case of teleportation

In this chapter, we focus on the specificities of our original work of educational reconstruction of quantum teleportation. The goal of the study we carried out was to design a lecture on this topic as part of the teaching-learning I SEE module on quantum computing introduced in chapter 2. This activity of design required many steps that are organized in this chapter in three sections. In the first one, we describe the experiment on teleportation and its reconceptualization as circuit; then, we make the designing criteria and methods for a didactical transposition explicit; finally, we describe and analyse the lecture we designed.

3.1 The experiment on teleportation and its reconceptualization

This section is dedicated to one of the main threads that characterize the entire module, the connection between experiment and circuit. In particular, we have analysed the teleportation protocol both from an experimental and a logical/circuital perspective, so as to establish a comparison between them and to highlight how an experiment can be re-read in terms of logic gates.

We selected one of the first experiments on teleportation, developed by the group of Zeilinger in 2004 (Ursin et al., 2004). In this experiment, the state of a photon (in term of its polarization) was teleported from one shore to the other of the Danube.

We chose to include teleportation in the module because it represents an important demonstration of what Einstein-Podolsky-Rosen highlighted in their famous article of 1935 ("Can quantum-mechanical description of physical reality be considered complete?") and that *Schrödinger, in* "Die gegenwärtige Situation in der Quantenmechanik" (1935), *called entanglement.* The demonstration of the effectiveness of teleportation is not only an evidence of this "all quantum feature" and of the principle of non-locality, but also a step toward a different application of quantum computation and quantum information: the quantum internet.

The first challenge we had to face concerns the physical content. The teleportation algorithm is not particularly complicated from a mathematical point of view but, in order to establish the comparison between the experiment and the circuit, we needed to make them *comparable*. By *comparable* we mean that each step of the algorithm had to be interpreted in physical terms and, *vice versa*, that each passage of the experiment had to be interpreted in logical terms.

Let us now consider the physical experiment, whose representation, shown in *figure 3.1*, is borrowed from Ursin and colleagues (Ursin et al, 2004)

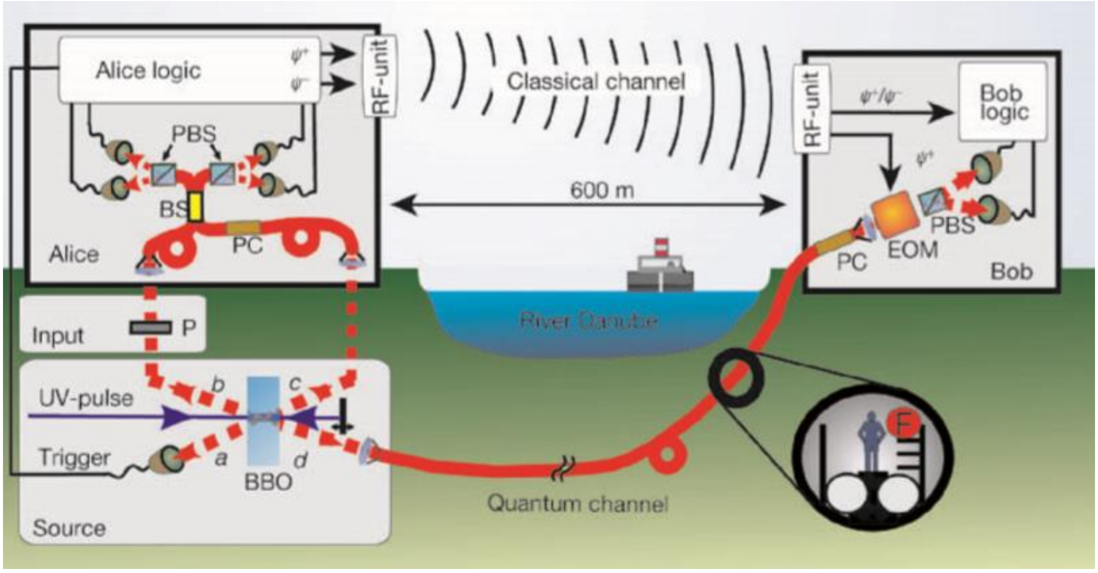


figure 3.1: set up of teleportation experiment (Ursin et al., 2004)

A pulsed laser (wavelength 394 nm; rate 76 MHz) is used to pump a β-barium borate (BBO) non-linear crystal and, hence, to generate the first entangled photon pair c and d by parametric conversion. C is the photon that goes to Alice and d the photon that goes to Bob. For reflection of the pulsed light on a mirror, another pair of entangled photons, a and b, are produced: a serves as a trigger and b, passing through a polarizer, comes to be in the superposition state $|\psi\rangle_b = (\alpha|0\rangle + \beta|1\rangle)_b$ that Alice wants to teleport to Bob. Therefore, the initial state of the system is:

$$|\psi\rangle = |\psi\rangle_b|\beta_{11}\rangle_{cd} = (\alpha|0\rangle + \beta|1\rangle)_b\left(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right)_{cd} =$$

$$= \alpha|0\rangle_b\frac{|0\rangle_c|1\rangle_d - |1\rangle_c|0\rangle_d}{\sqrt{2}} + \beta|1\rangle_b\frac{|0\rangle_c|1\rangle_d - |1\rangle_c|0\rangle_d}{\sqrt{2}}$$

Coupling b and c photons, we have:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\alpha|00\rangle_{bc}|1\rangle_d - \alpha|01\rangle_{bc}|0\rangle_d + \beta|10\rangle_{bc}|1\rangle_d - \beta|11\rangle_{bc}|0\rangle_d) \qquad (3.1)$$

Photons b and c are guided into a single-mode optical-fibre beam splitter (BS). This is connected to polarizing beam splitters (PBS) in order to allow Bell-state measurement. The four Bell states are:

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

With simple calculations, it turns out that:

$$|00\rangle = \frac{|\Phi^+\rangle + |\Phi^-\rangle}{\sqrt{2}}$$

$$|11\rangle = \frac{|\Phi^+\rangle - |\Phi^-\rangle}{\sqrt{2}}$$

$$|01\rangle = \frac{|\Psi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}$$

$$|10\rangle = \frac{|\Psi^+\rangle - |\Psi^-\rangle}{\sqrt{2}}$$

Replacing these states in (3.1), we obtain:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\alpha\left(\frac{|\Phi^+\rangle + |\Phi^-\rangle}{\sqrt{2}}\right)_{bc}|1\rangle_d - \alpha\left(\frac{|\Psi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}\right)_{bc}|0\rangle_d + \beta\left(\frac{|\Psi^+\rangle - |\Psi^-\rangle}{\sqrt{2}}\right)_{bc}|1\rangle_d - \beta\left(\frac{|\Phi^+\rangle - |\Phi^-\rangle}{\sqrt{2}}\right)_{bc}|0\rangle_d\right) =$$

$$= \frac{1}{2}[|\Phi^+\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d) + |\Phi^-\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d) - |\Psi^+\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) - |\Psi^-\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d)] \qquad (3.2)$$

This is an important step, since it shows that teleportation can occur if and only if it is possible to make a Bell-state measurement, that means to measure a coincidence of photons in Alice's position.

Making a Bell measurement on two states means to project them on one of the Bell states. Theorically, the probability to find each state is:

$$P(|\Phi^+\rangle_{bc}) = P(|\Phi^-\rangle_{bc}) = P(|\Psi^+\rangle_{bc}) = P(|\Psi^-\rangle_{bc}) = 25\%$$

Nevertheless, by construction, for this specific experimental set-up, the only two possible Bell states are either $|\Psi^-\rangle_{bc}$ or $|\Psi^+\rangle_{bc}$, which can be distinguished one from the other by Alice's logical electronics (Bell state measurement). Alice's result is then transmitted through a classical microwave channel (RF unit); table 3.1 shows the two possible results of the Bell measurement that Alice, with the same probability, can obtain and the corresponding state of Bob's photon.

table 3.1: Alice's state and corresponding Bob's state

Cases	Alice	Bob
1	$\|\Psi^-\rangle_{bc}$	$(\alpha\|0\rangle_d + \beta\|1\rangle_d)$
2	$\|\Psi^+\rangle_{bc}$	$(\alpha\|0\rangle_d - \beta\|1\rangle_d)$

Knowing the state of Bob's photon, a transformation can be operated with the electro-optic modulator (EOM) to transform the state of photon d into the desired Alice's input state of photon b, so that the teleportation is complete. The latter are unitary transformations that, in the case of photons, correspond to rotation of polarization or phase displacements, obtained by applying a voltage pulse on the EOM.

As Bennett and colleagues stated in their 1993 paper, "*the spin-exchange method of sending full information to Bob still lumps classical and nonclassical information together in a single transmission*" (Bennett et al., 1993), as *figure 3.1* shows. Indeed, as they demonstrated, the full information of Alice encoded in her state is composed by two parts, "*one purely classical and the other purely nonclassical*", and it is sent to Bob through two different channels. This observation, combined with the fact that the state of Alice is destroyed during process, ensures that information does not travel to higher speeds than speed of light. Thus the second principle of relativity is not violated, and it ensures that the state is not cloned, as the no-cloning theorem requires.

Let us consider now the circuit not only as an abstract representation of the experiment, but as a special re-reading of it in terms of logic gates. In *figure 3.2*, the circuit of quantum teleportation is reported.

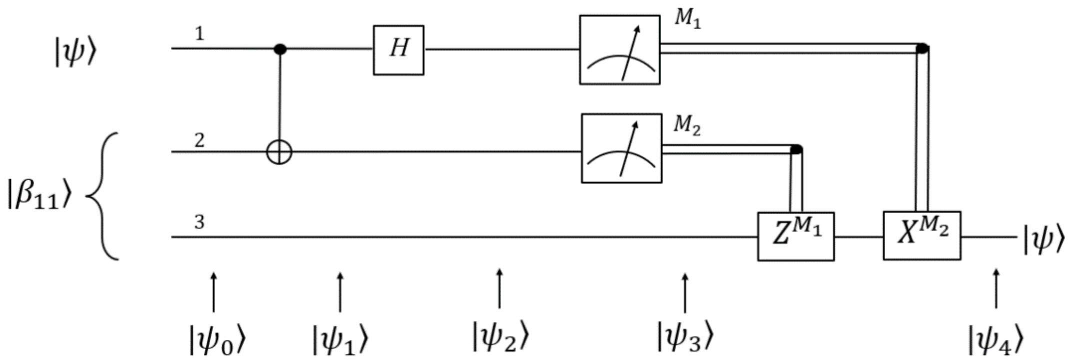


figure 3.2: teleportation circuit

In this representation it is possible to identify five different moments given by the states $|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$ and $|\psi_4\rangle$.

The state $|\psi_0\rangle$ describes the initial state of the system and it is the product of $|\psi\rangle$ and $|\beta_{11}\rangle$, where the first is the state that has to be teleported ($|\psi\rangle_1 = (\alpha|0\rangle_1 + \beta|1\rangle_1)$ and the latter is one of the four Bell states:

$$|\psi_0\rangle = |\psi\rangle_1|\beta_{11}\rangle_{23} = (\alpha|0\rangle_1 + \beta|1\rangle_1)\left(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right)_{23}$$

$$= \frac{1}{\sqrt{2}}[\alpha|0\rangle_1(|01\rangle - |10\rangle)_{23} + \beta|1\rangle_1(|01\rangle - |10\rangle)_{23}] \quad (3.3)$$

As well as in the experiment, where it is necessary to make a Bell measurement on the photons b and c in order to have the teleportation, also in the algorithm it is necessary to project the photons 1 and 2 in a Bell state. This is possible through the use of two logic gates in sequence, a CNOT, having as input photons 1 and 2, and a Hadamard gate on photon 1.

The CNOT gate has two input qubits, known as the *control* qubit and the *target* qubit, respectively. The circuit representation for the CNOT is shown in *figure 3*; the top line represents the *control* qubit, while the bottom line represents the *target*.

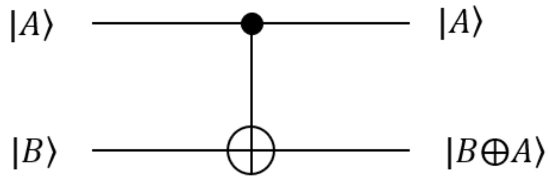


figure 3.3: CNOT gate

The action performed by the logical gate is the following: if the control qubit is set on 0, then the *target* qubit is left as it is; if the *control* qubit is set on 1, then the *target* qubit is flipped. Formally, this means:

$$|00\rangle \rightarrow |00\rangle\,, |01\rangle \rightarrow |01\rangle\,, |10\rangle \rightarrow |11\rangle\,, |11\rangle \rightarrow |10\rangle.$$

Therefore, if CNOT gate is applied on photons 1 and 2, (3.3) becomes:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle_1(|01\rangle - |10\rangle)_{23} + \beta|1\rangle_1(|11\rangle - |00\rangle)_{23}] \quad (3.4)$$

In order to complete the projection on a Bell state, a Hadamard gate is applied to photon 1. This gate is about a single qubit gate and transforms the state in the following way:

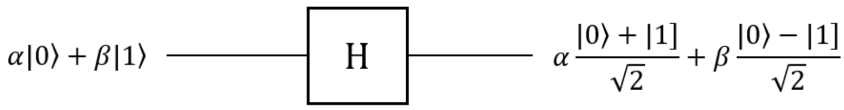


figure 3.4: Hadamard Gate

Therefore, (3.4) becomes:

$$|\psi_2\rangle = \frac{1}{2}[\alpha(|0\rangle_1 + |1\rangle_1)(|01\rangle - |10\rangle)_{23} + \beta(|0\rangle_1 - |1\rangle_1)(|11\rangle - |00\rangle)_{23}] \quad (3.5)$$

Reorganizing the terms of (3.5), we obtain:

$$|\psi_2\rangle = \frac{1}{2}[|00\rangle_{12}(\alpha|1\rangle_3 - \beta|0\rangle_3) - |01\rangle_{12}(\alpha|0\rangle_3 - \beta|1\rangle_3)$$
$$+ |10\rangle_{12}(\alpha|1\rangle_3 + \beta|0\rangle_3) - |11\rangle_{12}(\alpha|0\rangle_3 + \beta|1\rangle_3)] \quad (3.6)$$

In (3.6) the first term represents Alice's qubit ($|00\rangle_{12}, \ldots, |11\rangle_{12}$) and the second Bob's qubit.

Depending on Alice's Measurement, Bob's qubit will be in one of four possible states:

$$|00\rangle_{12} \rightarrow |\psi_3(00)\rangle \equiv [\alpha|1\rangle_3 - \beta|0\rangle_3]$$
$$|01\rangle_{12} \rightarrow |\psi_3(01)\rangle \equiv [\alpha|0\rangle_3 - \beta|1\rangle_3]$$
$$|10\rangle_{12} \rightarrow |\psi_3(10)\rangle \equiv [\alpha|1\rangle_3 + \beta|0\rangle_3]$$
$$|11\rangle_{12} \rightarrow |\psi_3(11)\rangle \equiv [\alpha|0\rangle_3 + \beta|1\rangle_3]$$

As in the physics experiment, also here Bob needs to know the result of Alice's measurement to complete teleportation.

If Alice makes the measure and gets $|11\rangle$, Bob will not have to do anything, because his qubit is already in the right state. If, instead, Alice gets $|10\rangle$, Bob will have to apply the X gate. If Alice gets $|01\rangle$, Bob will apply the Z gate. Finally if Alice's result is $|00\rangle$, Bob will apply both X and Z. X and Z are two single-qubit gates that work respectively as depicted in *figures 3.5* and *3.6*.

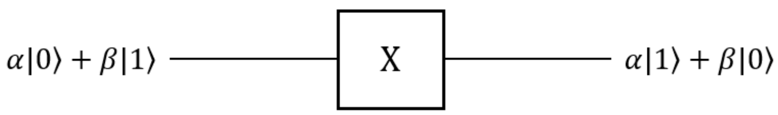


figure 3.5: X gate

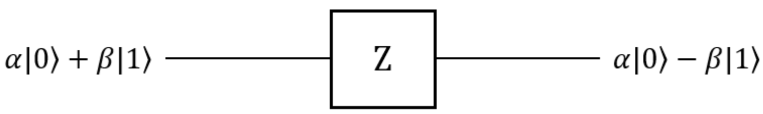


figure 3.6: Z gate

In summary, in order recover the state $|\psi_4\rangle = \alpha|0\rangle + \beta|1\rangle$ successfully, Bob will have to apply the unitary transformation $Z^{M_2}X^{M_1}$ to his qubit.

3.2 Criteria and methods for a didactical transposition

In the previous section, we described the analogy between the logical structure of a physics experiment and its representation in terms of logic gates, highlighting how the quantum states can be manipulated both from an experimental and a computational point of view. Here, we want to highlight the epistemological value of this comparison and, hence, to present the design approach we chose to exploit such value from an educational point of view.

The first epistemological element of interest is the possibility to highlight differences and connections between the conceptual tools and argumentation schemes embedded, on one hand, in the experimental apparatuses and, on the other, in the circuit, allowing one to become a lens for the other, and *vice versa*.

In order to value this epistemological aspect, we identified three crucial steps of the teleportation protocol that, despite being equivalent in terms of physical results both in the experimental set up and in the circuital realization, are expressed with a different formalism, symbolic form, and trigger different epistemological approaches.

The first important moment is the projection of the two photons (b and c) in a Bell state. From the experimental point of view, states are modified by a particular setup (BSA) composed by a polarizer controller, a single-mode optical-fibre beam splitter (BS) connected to four polarizing beam splitters (PBS, for Bell-state measurement). From a mathematical point of view, the states of the photons are manipulated as shown in the equations 3.1 and 3.2. The analogous in the circuit is realized by sequencing two logic gates, the CNOT and the Hadamard gate, whose logic is showed in the equations 3.4 and 3.5.

The second crucial moment regards Alice's measurement. This step in the circuit is represented simply by the quantum symbol for measurement reported in *figure 3.6*.

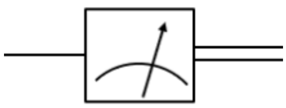


figure 3.7: symbol for measurement in the quantum circuit

In the experiment, the measurement is carried out through the combination of PBS and the four detectors. In the circuit, there are not particular conditions or constraints for which teleportation takes place, while in the experiment it occurs only if the four detectors measure a coincidence, so only if all four work.

The third moment consists in the unitary transformation that Bob has to perform in order to recover the state, after that Alice has communicated the results of her measurement via classical channel. In the experiment, the state is modified by applying a voltage pulse through the EOM (electro-optic modulator), whilst in the circuit the signal has to be passed through X and Z gates that correspond to rotations of the state in the Block's sphere.

The focus on these moments and their comparison stresses the logical interpretation of the experimental apparatuses as ways to act, transform and interpret physical signals: this is the real essence of a quantum simulation, and this is why in the research community it has taken the role of an implicit epistemological tool that blends scientific vocabularies and guides the scientific investigation. Furthermore, this can provide the students both with a synthetic picture of the quantum model of the phenomenon, and with a grounded sense of the experimental mechanism.

The second epistemological and educational element of interest concerns the two different narrative schemes. Indeed, the two representations are structurally focused on different aspects of the quantum model of the phenomenon, and stimulate the formation of different kinds of imagery and explanations.

The experimental approach suggests to follow the events and the photons in a *space-time order*, that of course allows to grasp the counter-intuitive essence of entanglement as a "spooky action at a distance". The circuital approach, instead, suggests a *holistic/systemic view* of the phenomenon, allowing to have a global picture of the entire system. From an educational perspective this can have a positive impact; in fact, as Mannila & Koponen (2001) showed, "students are used to direct their attention to properties of entities (particle, bodies, etc.), create images and draw pictures, where illustrations concentrate on the behaviour of entities. A similar approach is very difficult in quantum physics where the properties of basic entities are difficult to approach, and one should really concentrate on properties of phenomena" and foster a proper "conceptual shift to form a new ontology".

In order to value the two epistemological aspects, we decided to design the teaching activity in three parts: i) the first part aims to show the teleportation experiment by Ursin and colleagues in 2004 and to present how state's teleportation takes place, physically speaking; ii) the second part aims to present the circuit that carries out the teleportation protocol and to stress its correspondence with the experiment; iii) the third aims to discuss teleportation applications and future activities.

Each part foresees the argumentation to be developed along a three-levels structure, that is along the following three levels:

a) the narrative;
b) the logical;
c) the technical / mechanical.

In the experiment, (first part) the narrative level consists of building a story: "Alice and Bob, before leaving, exchange a pair of entangled photons, after a few years Alice, who has obtained a second photon, decides to send to Bob the status of her new photon, how can she do?". This level is important in order to create a scenario and to contextualize the problem to be solved. Via classical channel it is impossible that Alice manages to send its state because the qubit contains an infinite number of classical information (its state varies in a continuous space), so she would take infinite time to communicate it to Bob: Alice needs quantum teleportation to solve this task. From the point of view of contents, this level fosters the understanding of the difference between classical and quantum information and how the introduction of a new logic to solve a concrete problem becomes fundamental. The logical level refers both to the logic of the experiment and the logic of the circuit. Let us consider the first part of the activity in which students meet the physical apparatus (*figure 3.1*) and how Alice's task can be solved from a physical point of view. In order to show the students the logic of the experiment, we stressed four crucial moments of the logic of the experiment:

1. the production of two pairs of entangled photons;
2. the projection of two photons, initially not entangled, in a Bell state;

3. Alice’s measurement and the communication through classical channel;
4. Bob’s operation to recover the initial state of Alice, after knowing her results.

In this part, the students are guided to recognise how entanglement acquires meaning in the specific case of teleportation, introducing the idea of remote action, and they start to concretize its potential toward its application in quantum communication. These key four steps of teleportation are kept together by the narrative level that helps building a big picture of the situation. Within this picture, the technical level is switched on in order to point out the experimental tools needed to transform Alice’s state and teleport it to Bob. Since the original experimental setup is very complicated, we had to address the problem of how we could simplify it, without losing essential elements. The result is the setup made by five blocks, that are described in details in the next section and that concern: i) the production of two pairs of entangled photons through pulsed laser, non-linear crystal and a mirror; ii) the experimental set up composed by polarizer and beam splitter needed to create an entanglement relation between two initial non entangled photons; iii) measurement of Alice’s state through polarized beam splitter (PBS) and detectors; iv) communication via classical channel with microwave channel; v) Bob’s application of a voltage pulse to the EOM.

In the second part of the activity, dedicated to the teleportation circuit (*figure 3.2*), the narrative level is still present and the circuit is stressed to represent a way to transform the experiment into a quantum simulator. Here the logical level refers to the logic of the circuit and special attention is paid to present the circuit as a way to flesh out the logical structure that stays behind the experiment. The circuit is then turned into the playground to get acquainted with new logic, by coping with the concepts seen in the first two days. For these purposes, the representation of the circuit is shown and step by step, together with the students, the mathematical passages are reconstructed, demonstrating that Alice's status has actually been teleported to Bob. As we will see in the next section, the formalism is simpler than the one shown in the previous paragraph. The entangled photons are chosen in the Bell state $\beta_{00} = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$, and not in β_{11}, in order to find, by developing the calculation, the initial state $\alpha|0\rangle + \beta|1\rangle$ corresponding to the first Alice’s measure. We have finally decided to present the mathematical steps both to demonstrate formally that the teleportation takes place and to show that manipulating information formally corresponds to manipulating the states in an equation.

Summing up, the interaction between the narrative, logical and technical/mechanical levels has a special potential to stress, from an epistemological point of view, the meaning of teleportation. Indeed, this multi-layered structure provides, at the same time, imaginative, technical and logical tools to grasp the sense of quantum teleportation as state transformation. In particular, the narrative level contextualizes the problem and highlights the requirement of a new physics and a new logic, the logical and the technical/mechanical levels provide the necessary conceptual and formal tools to follow the process of state transformation. Moreover from an epistemological point of view the "experimental method" and the "computational method" can be stressed as different ways to solve the same task: even though they are equivalent if compared from the results, they represent two different ways to looking at a task, each of them is characterized by its own language, symbolic forms of representation and formalism. Their comparison points out the double nature, physical and computational, of quantum states: these two methods are apparently different, but they give sense to information, in the sense that they attach it both physical meaning (information as photons' polarization) and mathematical meaning (information as a qubit).

At the same time, in line with one of the goals of I SEE project to develop "STEM competences", the comparison between experiment and circuit allowed us to highlight the interdisciplinarity of the topic. The interaction of the three levels build the scaffold on which disciplines raise, each one with its own specificity, making the teleportation a real STEM topic: from information and its processing that is interpreted both physically and mathematically, to the engineering aspects of the experimental set up, to the technological aspects of the possible applications.

3.3 Analysis of the lecture on quantum teleportation

In this section we analyse in more details the lecture I presented to the students during the third day of the course on quantum computing with the lenses introduced in the previous section.

From the beginning we had highlighted the two main purposes of the activity:

- To show teleportation experiment as a concrete application of the entanglement;
- To re-read the experiment in terms of a circuit.

Immediately, we resumed the story introduced during the lecture "History of computers" by prof. Fantini and the three levels of its articulation: the narrative level represented by the story of Apraphul island, the symbolic/logical level represented by the logical gates and the level of mechanism represented by the system of pulleys and ropes. To facilitate the comparison with the quantum teleportation story, we marked the three levels with a color code as it is shown in *figure 3.8*.

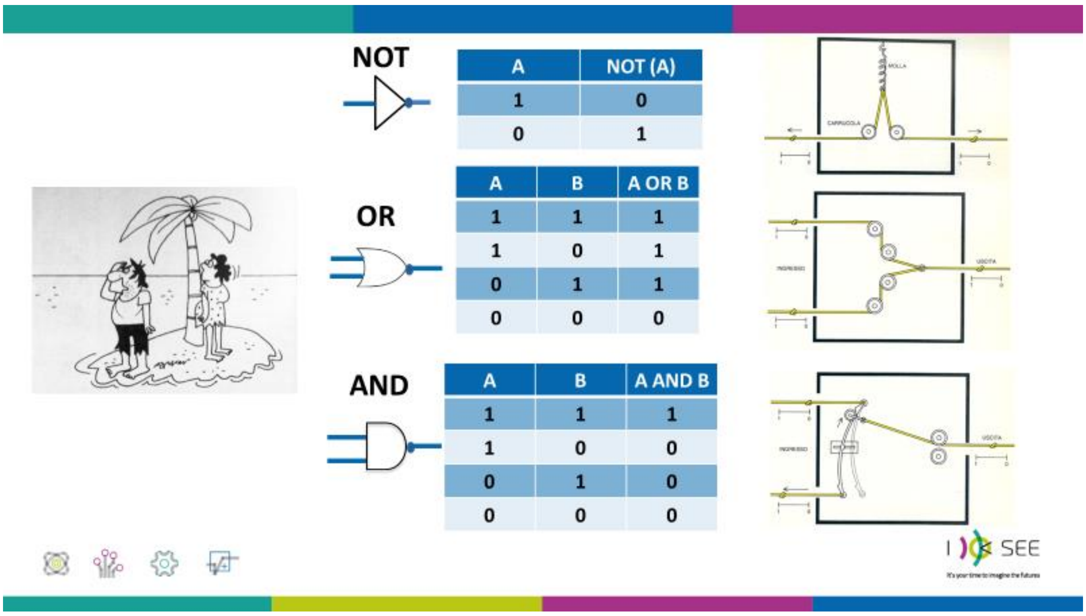


figure 3.8: three levels in the Story of computer lecture: in green the narrative level; in blue the symbolic/logical and in pink the mechanism one.

I then introduced the students to teleportation through the story of Alice and Bob, and we described them the problem to be solved: Alice, after having exchange a pair of photons entangled with Bob and after having obtained another photon, wants to transfer the state of the latter to Bob. How can she do that? Students were then invited to pay attention to the fact that this problem needs quantum teleportation to be solved.

At this point, we introduced the experiment using the setup in *figure 3.1*. It was explained to them that it was realized by Zeilinger and colleagues in 2004, who have demonstrated that they can teleport a quantum state, in this case the polarization of a photon, from one side of the Danube to the other, through the use of optical fibers.

To facilitate the students maintaining a global vision on the topic, without getting lost in the technicalities, we decided to maintain the narrative level (see *figure 3.9*) and use it to stress that the subject of the teleportation is the *state*: Alice and Bob, before dividing, exchanged a pair of entangled photons; Alice wanted to teleport the state of a further photon that she has been procured.

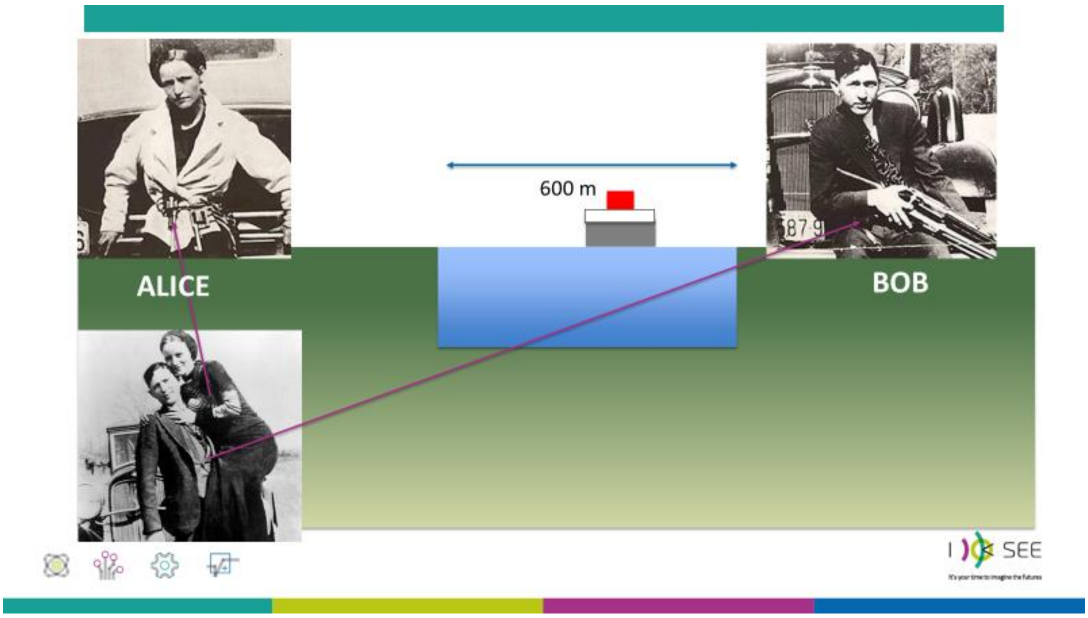


figure 3.9: narrative level in teleportation

We now led the students into the logic of the experiment (*figure 3.10*) and showed, in the picture, what represented experimentally the production of two pairs of entangled photons: c and d, a and b. However, of these four photons we considered only b, c and d, where c and d are the pair of entangled photons that Alice and Bob exchanged previously, and b is the photon whose state is going to be teleported, instead a act as a trigger, communicating to Alice that the two pair of entangled photons are correctly produced. Photon a provides only the information that the two photon pairs have been correctly produced.

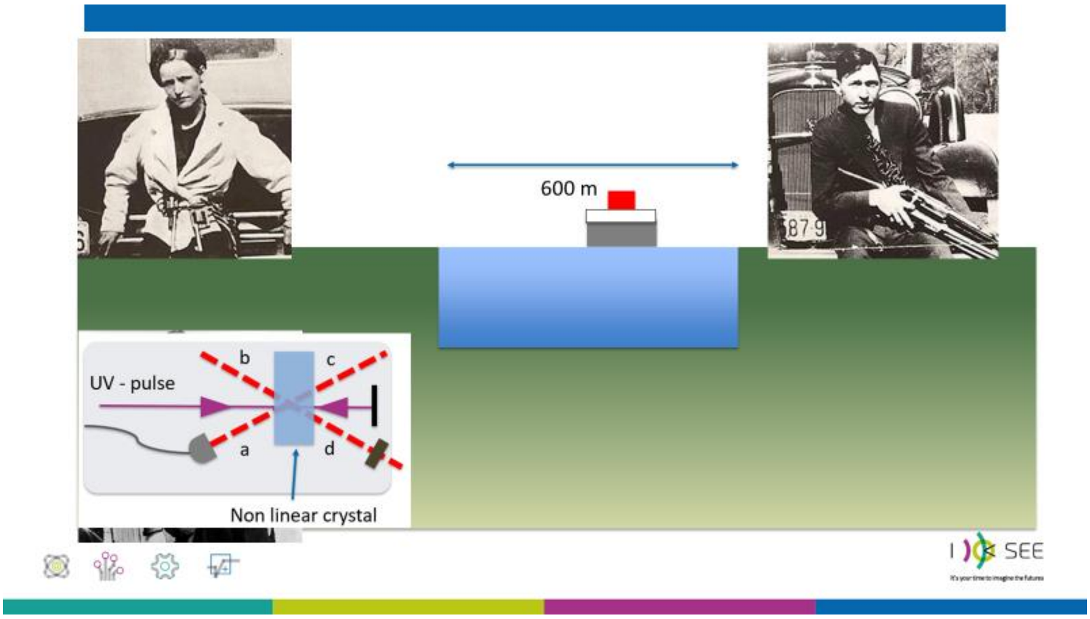


figure 3.10: logical level of experiment, production of pair of entangled photons

Then, we moved on to the level of mechanism to show them what represents, in the picture, the production of two pairs of entangled photons (*figure 3.11*). The parametric conversion is really complicated so we have highlighted only the essential elements (a pulsed light beam, a non-linear crystal and a mirror) and that the production of entangled photons derives from a double interaction with non-linear crystal, first c and d , then a and b.

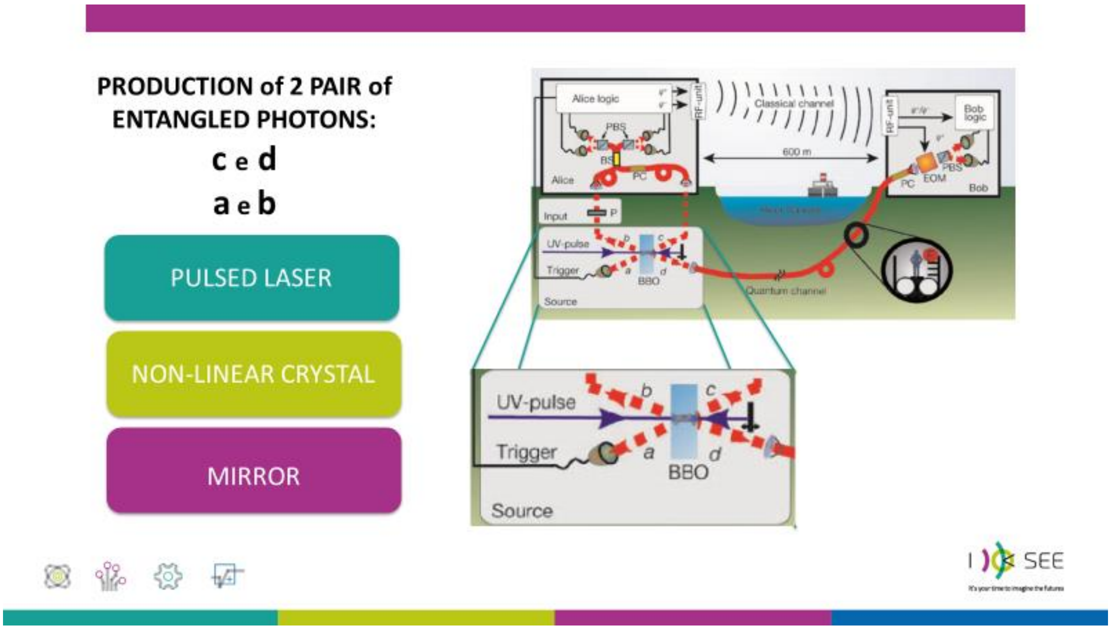


figure 3.11: level of mechanism in teleportation, production of two pair of entangled photons

We then came back to the logical level of the experiment by following the photons *b* and *c* which, through optical fibers, are transported to the Alice station where, in order for teleportation to occur, they must be made entangled i.e. projected into a Bell state (*figure 3.12*).

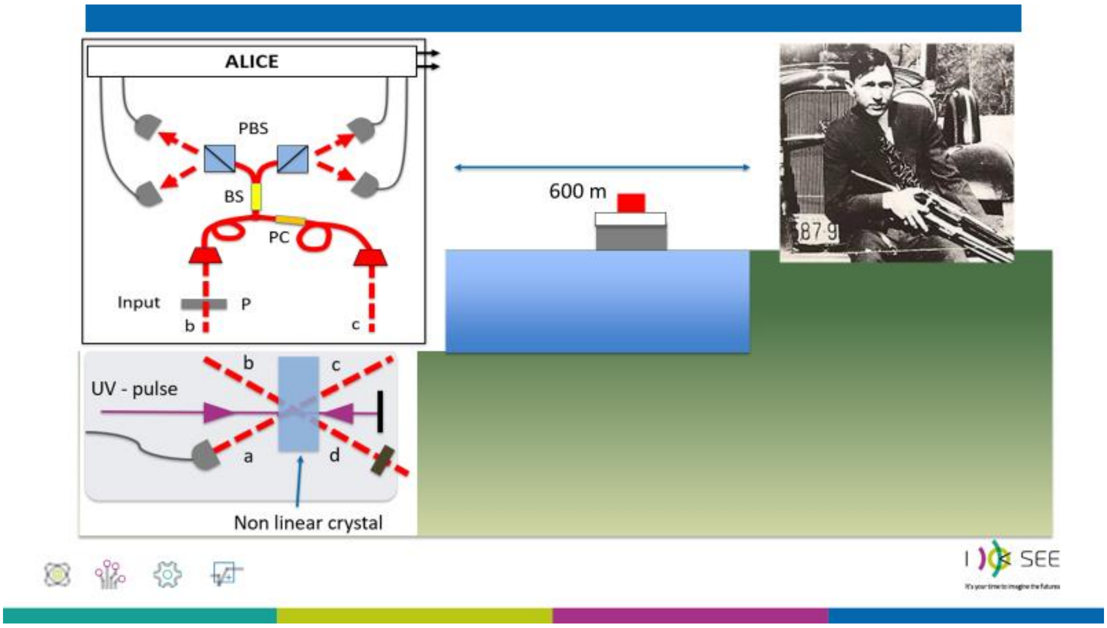


figure 3.12: logical level of experiment, projection in *b* and *c* in a Bell state

Coming back to the level of mechanism, we explained to the students that the photon *b* initially passes through a polarizer, which prepares it in the state to be teleported. A series of tools (including a polarization controller and a beam splitter) manipulate states so that photons *b* and *c* become entangled. It is possible to know that they have been made entangled if and only if the four detectors detect photons simultaneously (*figure 3.13*).

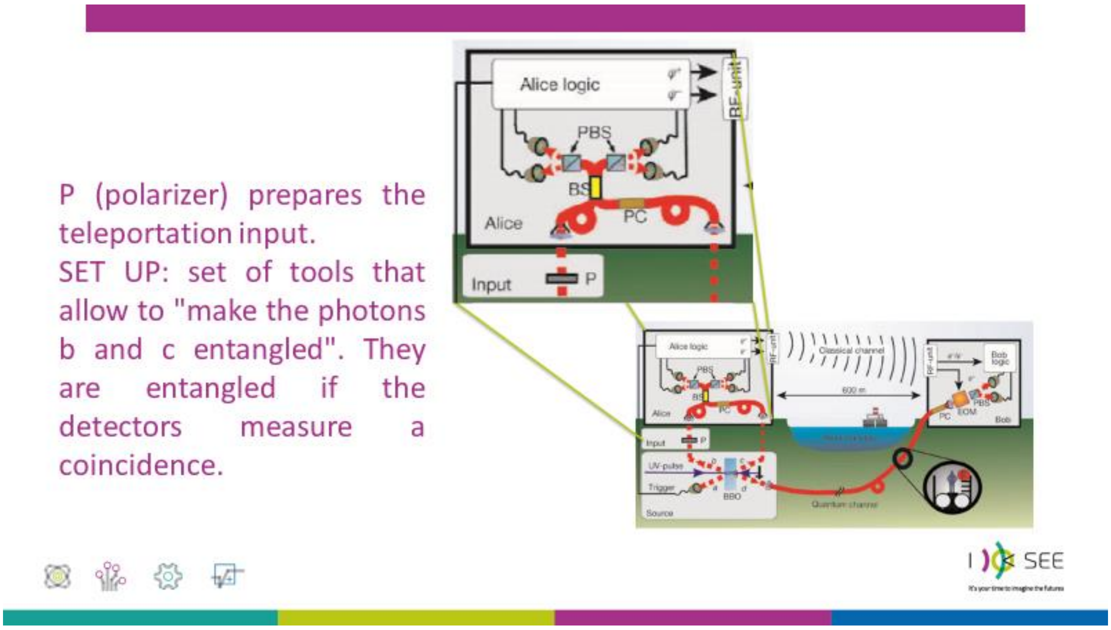


figure 3.13: level of mechanism, projection in a Bell state

We have shown that, returning to the logical level, the photon d, in the meantime, is transported to the Bob station through optical fibers. Alice, through PBS and detectors, measures the state of her two photons and communicates the result of measurement to Bob, by a classical channel, so that Bob can recover the initial state on the basis on the outcome of her measure (*figure 3.14*).

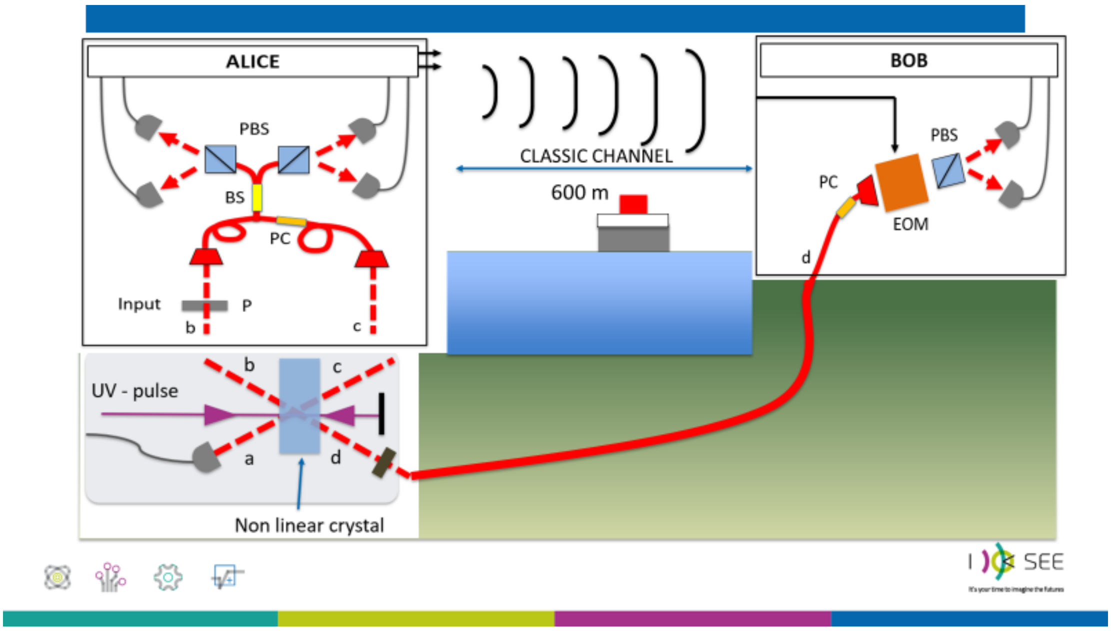


figure 3.14: logical level, communication of outputs and recovery of initial state

Through the level of mechanism, we mentioned that the classical channel is represented by microwave and that, for recovering the initial state, Bob has to apply a voltage to EOM. Because of the reduced speed of light in the optical fibre channel (two thirds of the speed of light in the air and through the air), the classic signal reaches the other laboratory 1,5 μs before the arrival of the photon d (*figure 3.15*).

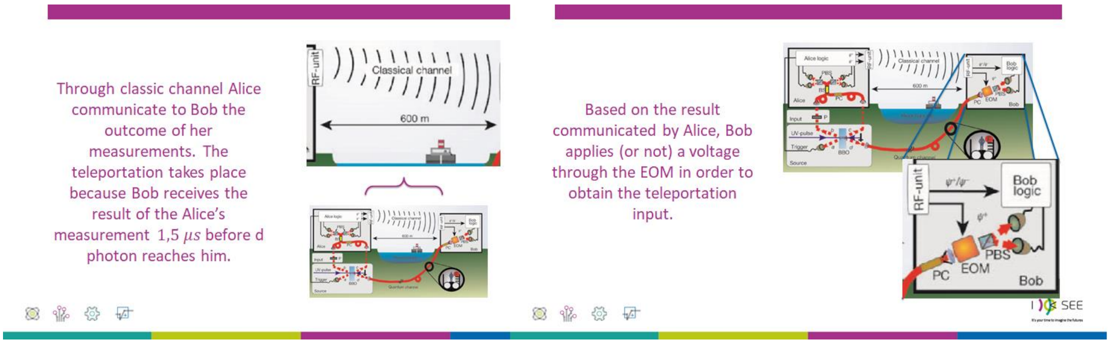


figure 3.15: level of mechanism, communication of outputs and recovery of initial state

Here we finished the first part of the lesson and introduced the circuit in the form shown in *figure 3.2*. Trying to make a connection with what we had just seen, we explained that $|\psi\rangle = \alpha|0\rangle_b + \beta|1\rangle_b$ is the state to be teleported corresponding to the photon b, $|\beta_{00}\rangle$ is the state of Bell that describes the relationship of entanglement between b and c (projection in a Bell state).

The logical level in this part predominates over the others and we used that to show students, step by step, how the various parts of the experimental set-up can be translated into logical gates.

We started to follow the circuit and see with students how the logical gates that appear in the circuit modify the state. Initially we explained that the initial state of the total system, $|\psi_0\rangle$ is the product between $|\psi\rangle$ and $|\beta_{00}\rangle$.

As *figure 3.16* shows, we immediately reconnected this state to the experiment: the first thing that happened was the creation of an entangled relationship between the photons b and c and that, from a circuital point of view, it is possible to reproduce that by putting in sequence a CNOT and a H gates.

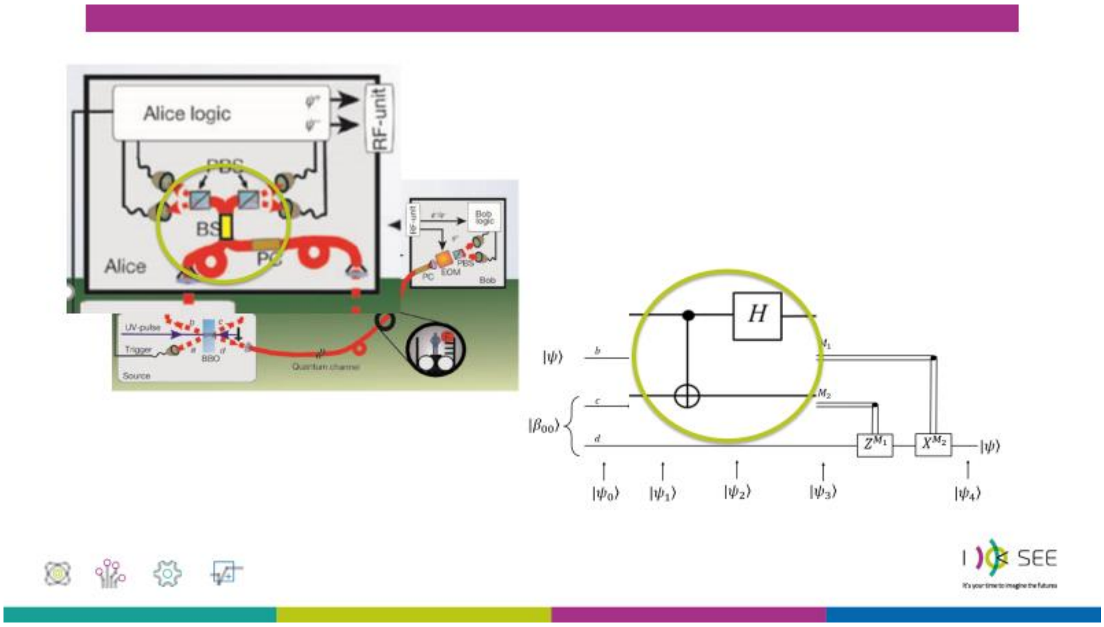


figure 3.16: comparison projection in Bell state in algorithm and circuit

Step by step and in a dialogic way, the whole class was involved in the calculus of the evolution of the overall state, passing through a CNOT and then to H gates (*figure 3.17*).

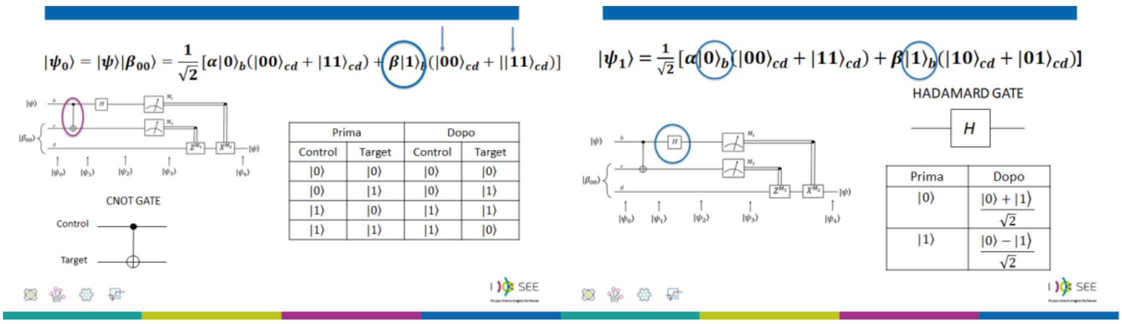


figure 3.17: logical level of circuit

After doing the calculations and obtaining the following state for the system

$$|\psi_2\rangle = \frac{1}{2}[|00\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d) + |01\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d) +$$
$$+|10\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) + |11\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d)]$$

we came back to the parallelism and showed the students what, in the experiment, corresponds to the symbol of quantum logical gate for measurement (*figure 3.18*).

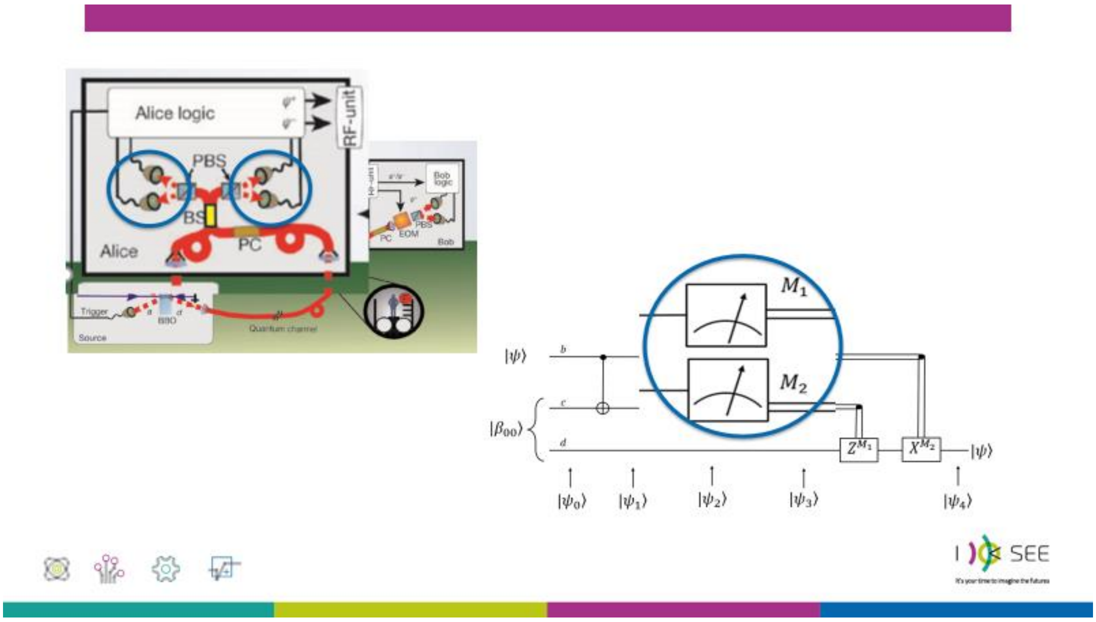


figure 3.18: comparison of measurement in the experiment and in the circuit

Always following the logic of the experiment we focused students' attention on the fact that, once the measurement is complete, Alice must communicate her outcome to Bob, who, as in the experimental case applied a voltage to the EOM, in case of the circuit applies the X and/or Z gates (*figure 3.19*).

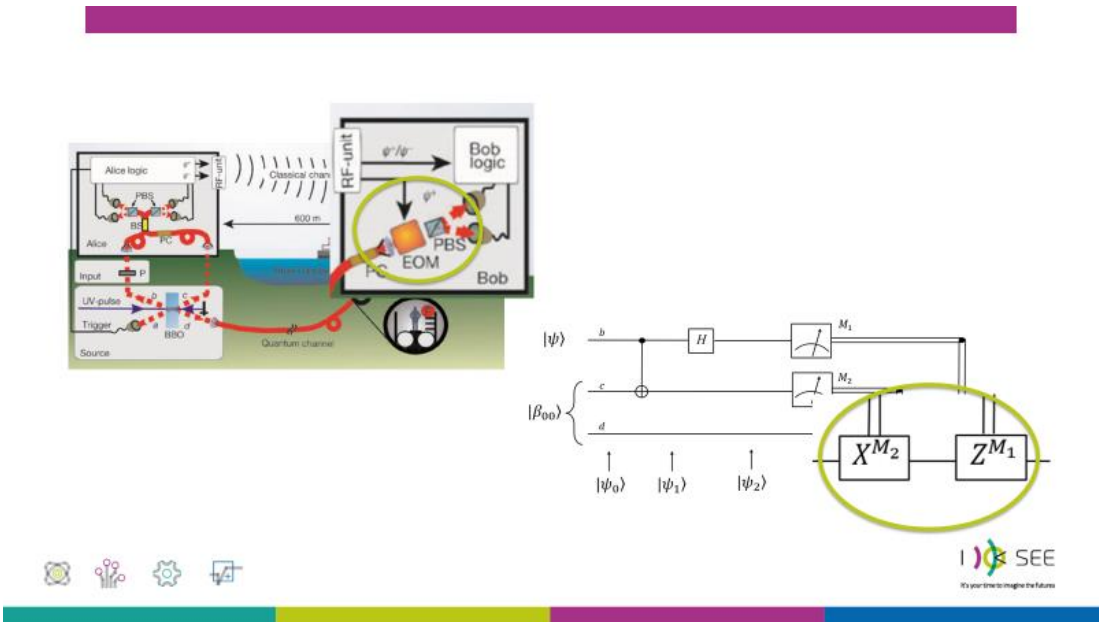


figure 3.19: recover of teleportation input in the experiment and in the circuit

This part of the reasoning was particularly challenging for the students, since they were asked to apply the learned concept of measurement and state collapse to understand what Bob would have obtained if Alice had measured $|00\rangle, |01\rangle,\ |10\rangle$ and $|11\rangle$. We asked them to recognize which gate had to be applied (X or Z) to complete the teleportation (*figure 3.20*).

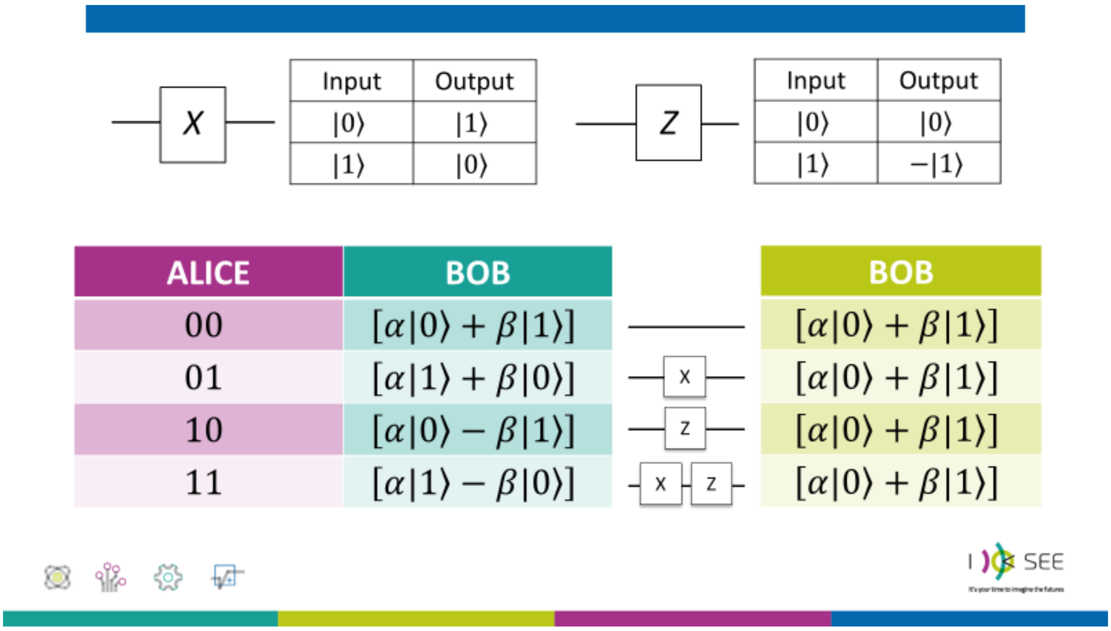


figure 3.20: application of logic gates to recover the teleportation input

The last part of the activity was dedicated to the development of reflections about the implications of teleportation to quantum internet and its potentialities. In order to understand how a quantum network can be created, we introduced the concepts of

i. maximally entangled states;

ii. quantum repeater.

We explained them that the first concept is important because the entanglement is fragile, since the decoherence due to the interaction of the quantum system with the environment, quantum noise and absorption, dispersion and non-linearity phenomena within the fiber could destroy this quantum bond. It was therefore presented the students a fairly simple video showing distillation as a way to make two states maximally entangled and how diamonds, or rather the spins of his carbon atoms, could be used to store information.

We have introduced quantum repeater as something that is able to extend the quantum communication interval between sender and receiver. It was then shown that, if you want to transmit information between two network nodes distant 200 km (too far for direct transmission), it is necessary to:

- create two entangled qubits between the first endpoint and the repeater (100 km away) and
- create two further entangled qubits between the repeater and the second endpoint (100 km away).

By teleportation, the quantum repeater transfers the qubit that is entangled with the first endpoint to the second endpoint, forming an entangled link.

We showed that the development of a quantum internet is important not only to have a secure network, but also because, having quantum computers large dimensions and requiring temperatures close to 0 K, it gives the possibility of a remote access to a quantum computer by cloud computing.

We concluded the activity showing to students that we are not so far from the realization of quantum internet. Indeed, the research group of Qutech at the University of Delft is expected to realize, by 2020, the first quantum internet that will connect four Dutch cities.

Conclusions

The thesis is situated within the I SEE project, with the final purpose of designing activities concerning quantum computers. These activities aimed to develop basic quantum concepts needed to grasp the essence of these new technologies and to promote their connection with the future.

The first problem that we had to address was to identify the key concepts of quantum physics needed to understand the new logic on which these new technologies are based. Through the analysis of the literature, previous research works carried out on teaching/learning quantum physics in Bologna (Lodovico, 2016, Ravaioli, 2016; Levrini & Fantini, 2013), and of the Finnish module, we identified four focal points - the qubit, the superposition principle, the measure, the entanglement. Then, we decided to choose a simplified *spin-first* approach, designed together with prof. Elisa Ercolessi, to introduce them. The simplifications had to take into account that the module was targeted to students attending the fourth year of secondary school (11th grade, 16-17 years old) who had not already studied quantum physics.

The second problem we had to address was to find a global view to analyse the current materials on quantum computers, that appear very fragmented and hyper-specialized, and flesh out not only its conceptual essence but also its epistemological and educational value. In the words of Edgar Morin, we had to face directly the paradox of one contemporary challenge that consists of solving increasing multidisciplinary, transversal and multidimensional problems starting from a fragmented knowledge (Morin E., 2000).

The global view we identified can be briefly described by the motto "re-reading a quantum experiment as a quantum circuit". This view informed the overall design of the module, since the first lesson focused on the history of classical computers, and it found its crucial phase when we led the students to compare the experiment on teleportation and its circuit. The educational reconstruction of this experiment and the design of a teaching activity on it were the core of this thesis.

From the interaction between three levels (narrative, logical and of mechanism), we found that the comparison between experiment and circuit to could be a powerful epistemological tool from two different perspectives:

- the *first* concerns the possibility to highlight differences and connections between the conceptual tools and argumentation schemes embedded, on one hand, in the experimental apparatuses and, on the other, in the circuit, allowing one to become a lens for the other, and *vice versa*.

- the *second* concerns the two different narrative schemes. Indeed, the two representations are structurally focused on different aspects of the quantum model of the phenomenon, and stimulate the formation of different kinds of imagery and explanations.

The activity has been realized on February 19^{th} and a week after we asked the students to answer some questions about the contents and the approach to teleportation they encountered.

In spite of the intrinsic difficulties of the subject, they students found the comparison very helpful to capture what we mean today with quantum logical gates, algorithm, simulator and computer and they found very engaging the exercise of calculating the state's transformations through the logical gates. We also had the impression that the relation with the quantum internet helped to strengthen the connections between quantum computers and future, since it widened the span of socially relevant implications.

While we are finishing the writing of this thesis, the implementation is still ongoing, so we will be able only in some weeks to really check the impact of the module on students' imagination. In any case, the design of the activity and its test in class were a strongly stimulating experience and we do believe that it can provide a significant contribution to the development of the educational materials aimed to prepare the young generation for the second quantum revolution.

Annex A:

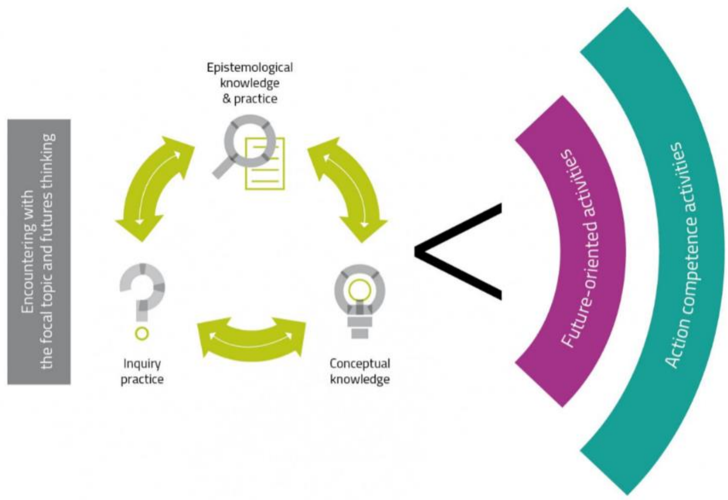
At the end of the teleportation activity I realized a sheet with the I SEE format that presents: i) the conceptual, epistemological and social/emotional goals, ii) a detailed description of the dialogic lesson and the iii) teaching method that characterized the activity.

Quantum Computers

ACTIVITY 1

Teleportation as a comparison between experiment and circuit

<table>
<tr><td>Position in the module

Encountering with the focal issue</td><td>

The activity aims to grasp a how an experiment can be re-read in terms of circuit, by using a comparison between the experiment, which realize the teleportation, and the standard teleportation protocol, making a step forward the idea of simulation. It reinforces the concept of entanglement and stress on the presence of a “new” logic and on the notion of information encoded in the qubit, which represent a real resource.</td></tr>
</table>

<table>
<tr><td></td><td>A particular focus is on:

experimental tools as manipulator of information
simulation of an experiment
logic gates as manipulator of signal and information
potentialities of teleportation
</td></tr>
<tr><td>Goals</td><td>conceptual

to understand that a state is described by the superposition of states ($|H\rangle$ and $|V\rangle$, $|\uparrow\rangle$ and $|\downarrow\rangle$, $|0\rangle$ and $|1\rangle$), which represent information
to reinforce the concept of entanglement and “spooky action at distance”
to understand the logic of experiment

creation of two pair of entangled photons (c, d and a, b)
projection of photons b and c in a Bell state
measure and communication, via classical channel, of outcomes
operation to recover the initial state

to understand that experimental tools manipulate the overall state of system so that teleportation occur
to get confidence with circuits representation
to understand that nowadays a new logic, the quantum mechanical logic, is needed to solve kinds of problems
to get confidence with the new logic and a new formalism
to get confidence with new type of logic gates

CNOT
Hadamard
X
Z

to understand the effect of a measure in quantum mechanics
to start to understand how an experiment can be interpret and re-read with logic gates
to start to understand the importance of simulation looking at a concrete example
to understand that manipulate a state correspond to manipulate an information
to understand that teleportation opens new opportunities whose impact span different
</td></tr>
</table>

	dimensions (political, social, economic, ethical, environmental, professional…) • to understand that the entanglement and its “spoky action at distance” represent a real resource for many application • to understand how entanglement can represent a turning point for the development of quantum internet ***epistemological*** • to recognize there are some problems that could be solved only with quantum physics • to begin to recognize how it is possible reinterpret an experiment in terms of logic gates • to recognize that experiment and circuit are two ways to solve the same task • the role of simulation • to begin to recognize the impact and the scope of application based on quantum mechanics ***social/emotional*** to begin to reflect on the potentialities and risks of quantum computers and quantum internet according to their own world view and values to enlarge imagination about possible future STEM careers to get personally involved in class discussion according to their ideas sharing their points of view
Time required	One hour

<table>
<tr>
<td>Materials</td>
<td>
Slides for the dialogic lesson

recovery of levels of analysis used in the first lesson

narrative
logic
of mechanism

Presentation of Alice and Bob narration to contextualize the task and
Focus on the fact that the task is resolvable only with a quantum teleportation
Presentation of the physical set up stressing on the logic of the experiment:

creation of two pairs of entangled photons
projection of two photons, initially non entangled, in a Bell state
Alice's measurement and the communications through classical channel
Bob's operation to recover the initial Alice's state, after knowing Alice's results

Presentation of the mechanism and of the tools that permit teleportation in the chosen experiment:

non-linear crystal and pulsed laser to produce two pair of entangle state
Bell state analysis through the use of beam splitter (BS)
Measure of Alice's state through polarized beam splitter and detectors
Communication of Alice's state via classical channel
Application of e tension to the EOM based on Alice's measure in order to recover the initial state

Presentation of the scheme of teleportation protocol
Focus on calculation following what happens to the state passing through logic gates and comparison step by step between experiment and circuit

consecution of CNOT and Hadamard gate in order to project two non-entangled state in a Bell state and focus on the part of the experiment corresponding to
measure operator in the circuit and the corresponding Alice's action
Communication of measure in the circuit (00, 01, 10, 11) and in the experiment
Application of X and Z gates and corresponding application of a tension to EOM in order to recover the initial teleportation state

</td>
</tr>
</table>

<table>
<tr><td></td><td>Presentation of quantum networks as the bases of
cloud computing
quantum internet
Presentation of the main ingredients for quantum internet:
presentation of entanglement as a fragile link depending on different environmental conditions (as thermal noise)
concept of maximally entangled state through a video showing how obtained
the use of quantum repeater in order to extend the range of quantum communication between sender and receiver</td></tr>
<tr><td>Teaching methods</td><td>A dialogic lesson

The teacher fosters each student to take active part in the dialogic lesson, get involved especially when the mathematical passages are presented and take care that all the class is engaged in the collective activity.
Three different level (narrative, logical and technical) are presented</td></tr>
<tr><td>Tips for teachers from previous classroom experiences</td><td>Students seemed very interested in the subject.
The part of the experiment, both the logical level and the mechanism level, was not immediately easy to follow, but at the end of the discussion the students seem to be convinced. During the second part, that of the circuit, the students seemed very engaged, they got involved especially with the logic part.

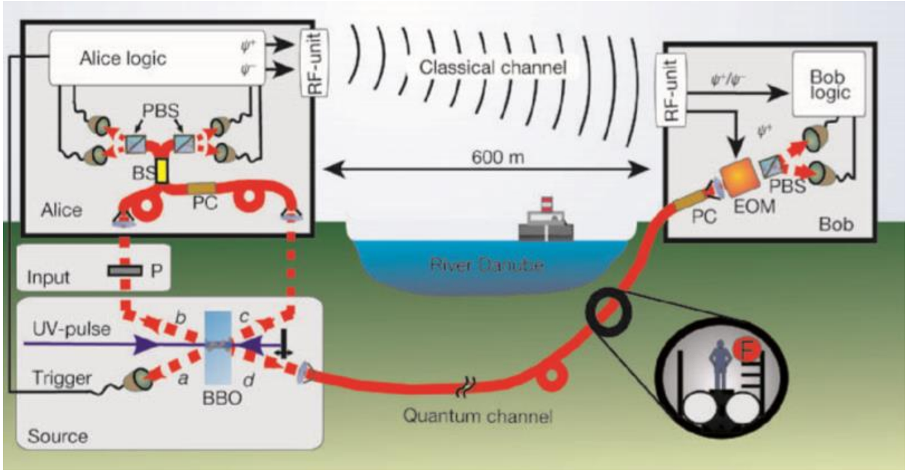
The final part has helped the students to understand and realize the potential of teleportation in terms of internet quantum.</td></tr>
<tr><td>Additional resources</td><td>Sites of university of Delft: https://qutech.nl/</td></tr>
</table>

Annex B

In annex the exercise that we proposed to the students on the fourth day.

Nome e Cognome:

Esercizio 1: descrivi quello che ti ricordi dell'esperimento del teletrasporto



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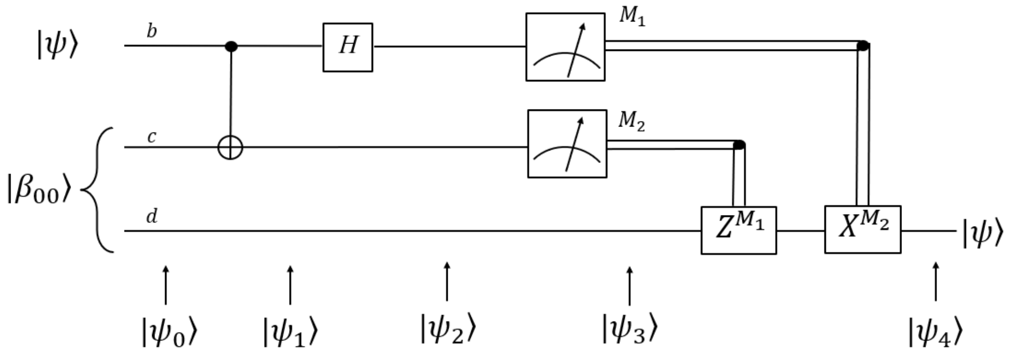
Esercizio 2: descrivi quello che ti ricordi del circuito che ralizza il teletrasporto del teletrasporto



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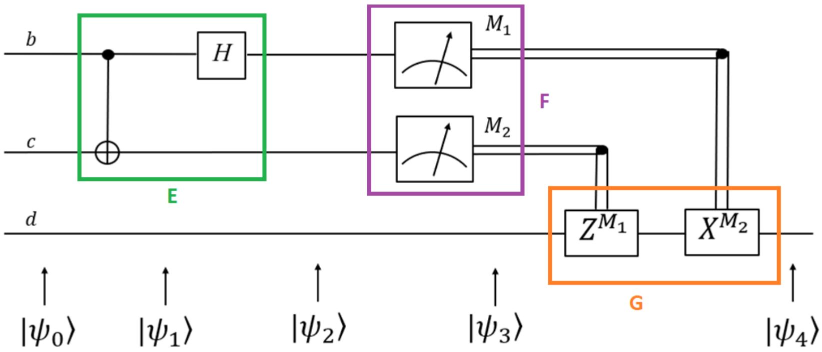
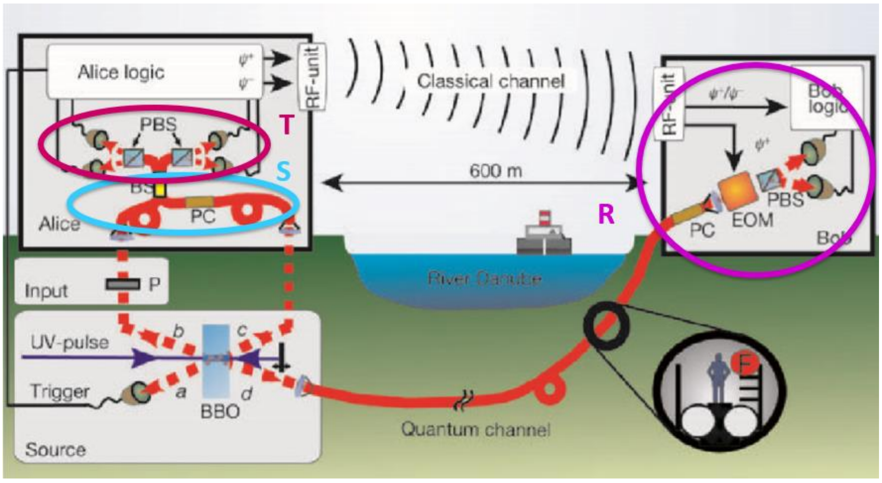
Esercizio 2: Consideriamo ora assieme il circuito e l'esperimento:





Associa le parti del circuito all'esperimento?

$$E \;\rightarrow$$
$$F \;\rightarrow$$
$$G \;\rightarrow$$

Il circuito e l'esperimento sono analoghi? Che differenze vedi tra le due rappresentazioni?

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Come viene processata l'informazione nell'esperimento? E nel circuito?

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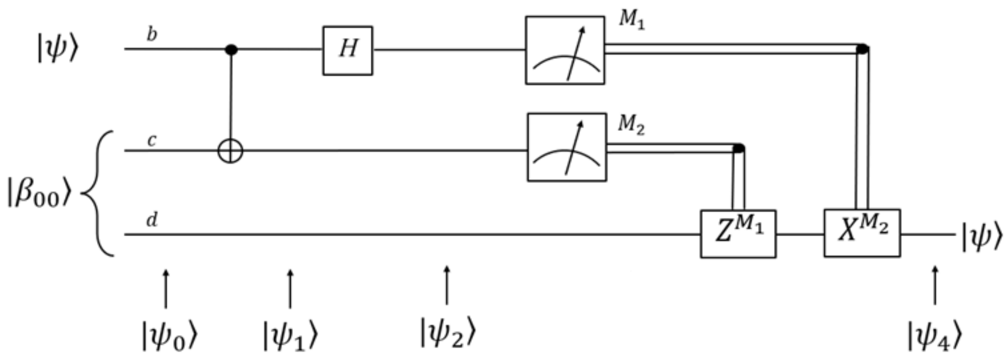
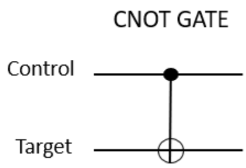
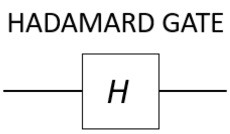
(FACOLTATIVO) Esercizio 1: Consideriamo il circuito del teletrasporto



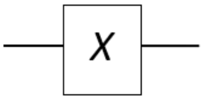
Ricordando che:



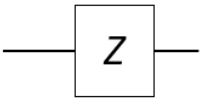
Prima		Dopo	
Control	**Target**	**Control**	**Target**
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$



Prima	Dopo
$\|0\rangle$	$\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$
$\|1\rangle$	$\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$



Prima	Dopo
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$



Prima	Dopo
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$-\|1\rangle$

(i) $\frac{1}{2}[|00\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d) + |01\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d) + |10\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) + |11\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d)]$

(l) $|01\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d)$

(m) $\frac{1}{\sqrt{2}}[\alpha|0\rangle_b(|00\rangle_{cd} + |11\rangle_{cd}) + \beta|1\rangle_b(|10\rangle_{cd} + |01\rangle_{cd})]$

(n) $(\alpha|0\rangle_b + \beta|1\rangle_b)\ \frac{|00\rangle_{cd}+|11\rangle_{cd}}{\sqrt{2}}$

Associa le equazioni ai vari momenti del circuito.

$|\psi_0\rangle =$

$|\psi_1\rangle =$

$|\psi_2\rangle =$

$|\psi_4\rangle =$

Qual è l'informazione da teletrasportare? Cosa vuole dire processare l'informazione nel circuito?

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Ringraziamenti

“Conta solo il cammino, perché solo lui è duraturo e non lo scopo, che risulta essere soltanto l’illusione del viaggio”.

Antoine de Saint-Exupery

Anche se la fine di questo percorso è sicuramente un traguardo molto importante, il viaggio che mi ha condotto qui è stato di gran lunga più interessante. È giunto il momento di ringraziare le persone che ho conosciuto e che, lungo la strada, sono state compagne, amiche, insegnati e sostenitrici. Non sono mai stata molto brava con queste cose, per cui lo farò a modo mio.

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Sitography

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[13] http://uknqt.epsrc.ac.uk/

[14]https://www.businesseurope.eu/sites/buseur/files/media/imported/2011-00855-E.pdf

[14]https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/blochsphere/blochsphere.html